



Intel® Math Kernel Library

Developer Reference

Revision: 011

MKL 2017

Legal Information

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Introducing the Intel® Math Kernel Library

The Intel® Math Kernel Library (Intel® MKL) improves performance with math routines for software applications that solve large computational problems. Intel MKL provides BLAS and LAPACK linear algebra routines, fast Fourier transforms, vectorized math functions, random number generation functions, and other functionality.

NOTE

It is your responsibility when using Intel MKL to ensure that input data has the required format and does not contain invalid characters. These can cause unexpected behavior of the library.

The library requires subroutine and function parameters to be valid before being passed. While some Intel MKL routines do limited checking of parameter errors, your application should check for NULL pointers, for example.

Intel MKL is optimized for the latest Intel processors, including processors with multiple cores (see the *Intel MKL Release Notes* for the full list of supported processors). Intel MKL also performs well on non-Intel processors.

For more details about functionality provided by Intel MKL, see the [Function Domains](#) section.

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Getting Help and Support

Intel provides a support web site that contains a rich repository of self help information, including getting started tips, known product issues, product errata, license information, user forums, and more. Visit the Intel MKL support website at <http://www.intel.com/software/products/support/>.

What's New

This Developer Reference documents Intel® Math Kernel Library (Intel® MKL) 2017 Update 2 release for the C interface.

NOTE

This publication, the *Intel Math Kernel Library Developer Reference*, was previously known as the *Intel Math Kernel Library Reference Manual*.

The following function domains were updated with new functions, enhancements to the existing functionality, or improvements to the existing documentation:

- Deep Neural Network Functions have been updated to support:
 - Asymmetric border types. For more details, see [dnnConvolutionCreate](#) and [dnnPoolingCreate](#).
 - Selection of the computation method to perform batch normalization. For more details, see the description of the new function [dnnBatchNormalizationCreate_v2](#).
- Support functions have been added to specify and return the number of partitions along the leading dimension of the output matrix for parallel `?gemm` functions. For more details, see [mkl_set_num_stripes](#) and [mkl_get_num_stripes](#).
- Support of ScaLAPACK by the progress routine has been implemented. For more details, see [mkl_progress](#) and [mkl_set_progress](#).

The manual has also been updated to reflect other enhancements to the product, and minor improvements and error corrections have been made.

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Notational Conventions

This manual uses the following terms to refer to operating systems:

| | |
|-------------|--|
| Windows* OS | This term refers to information that is valid on all supported Windows* operating systems. |
| Linux* OS | This term refers to information that is valid on all supported Linux* operating systems. |
| macOS* | This term refers to information that is valid on Intel®-based systems running the macOS* operating system. |

This manual uses the following notational conventions:

- Routine name shorthand (for example, `?ungqr` instead of `cungqr/zungqr`).
- Font conventions used for distinction between the text and the code.

Routine Name Shorthand

For shorthand, names that contain a question mark "?" represent groups of routines with similar functionality. Each group typically consists of routines used with four basic data types: single-precision real, double-precision real, single-precision complex, and double-precision complex. The question mark is used to indicate any or all possible varieties of a function; for example:

| | |
|--------------------|--|
| <code>?swap</code> | Refers to all four data types of the vector-vector <code>?swap</code> routine: <code>sswap</code> , <code>dswap</code> , <code>cswap</code> , and <code>zswap</code> . |
|--------------------|--|

Font Conventions

The following font conventions are used:

| | |
|---|---|
| <code>lowercase courier</code> | Code examples: <code>a[k+i][j] = matrix[i][j];</code> data types; for example, <code>const float*</code> |
| <code>lowercase courier mixed with UpperCase courier</code> | Function names; for example, <code>vmlSetMode</code> |
| <code>lowercase courier italic</code> | Variables in arguments and parameters description. For example, <code>incx</code> . |
| <code>*</code> | Used as a multiplication symbol in code examples and equations and where required by the programming language syntax. |

Function Domains

NOTE

It is your responsibility when using Intel MKL to ensure that input data has the required format and does not contain invalid characters. These can cause unexpected behavior of the library.

The library requires subroutine and function parameters to be valid before being passed. While some Intel MKL routines do limited checking of parameter errors, your application should check for NULL pointers, for example.

The Intel® Math Kernel Library includes Fortran routines and functions optimized for Intel® processor-based computers running operating systems that support multiprocessing. In addition to the Fortran interface, Intel MKL includes a C-language interface for the Discrete Fourier transform functions, as well as for the Vector Mathematics and Vector Statistics functions. For hardware and software requirements to use Intel MKL, see *Intel® MKL Release Notes*.

BLAS Routines

The BLAS routines and functions are divided into the following groups according to the operations they perform:

- **BLAS Level 1 Routines** perform operations of both addition and reduction on vectors of data. Typical operations include scaling and dot products.
- **BLAS Level 2 Routines** perform matrix-vector operations, such as matrix-vector multiplication, rank-1 and rank-2 matrix updates, and solution of triangular systems.
- **BLAS Level 3 Routines** perform matrix-matrix operations, such as matrix-matrix multiplication, rank-k update, and solution of triangular systems.

Starting from release 8.0, Intel® MKL also supports the Fortran 95 interface to the BLAS routines.

Starting from release 10.1, a number of **BLAS-like Extensions** are added to enable the user to perform certain data manipulation, including matrix in-place and out-of-place transposition operations combined with simple matrix arithmetic operations.

Sparse BLAS Routines

The **Sparse BLAS Level 1 Routines and Functions** and **Sparse BLAS Level 2 and Level 3 Routines** routines and functions operate on sparse vectors and matrices. These routines perform vector operations similar to the BLAS Level 1, 2, and 3 routines. The Sparse BLAS routines take advantage of vector and matrix sparsity: they allow you to store only non-zero elements of vectors and matrices. Intel MKL also supports Fortran 95 interface to Sparse BLAS routines.

LAPACK Routines

The Intel® Math Kernel Library fully supports the LAPACK 3.6 set of computational, driver, auxiliary and utility routines.

The original versions of LAPACK from which that part of Intel MKL was derived can be obtained from <http://www.netlib.org/lapack/index.html>. The authors of LAPACK are E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen.

The **LAPACK routines** can be divided into the following groups according to the operations they perform:

- Routines for solving systems of linear equations, factoring and inverting matrices, and estimating condition numbers (see **LAPACK Routines: Linear Equations**).

- Routines for solving least squares problems, eigenvalue and singular value problems, and Sylvester's equations (see [LAPACK Routines: Least Squares and Eigenvalue Problems](#)).

Starting from release 8.0, Intel MKL also supports the Fortran 95 interface to LAPACK computational and driver routines. This interface provides an opportunity for simplified calls of LAPACK routines with fewer required arguments.

Sparse Solver Routines

Direct sparse solver routines in Intel MKL (see [Chapter 8](#)) solve symmetric and symmetrically-structured sparse matrices with real or complex coefficients. For symmetric matrices, these Intel MKL subroutines can solve both positive-definite and indefinite systems. Intel MKL includes a solver based on the PARDISO* sparse solver, referred to as Intel MKL PARDISO, as well as an alternative set of user callable direct sparse solver routines.

If you use the Intel MKL PARDISO sparse solver, please cite:

O.Schenk and K.Gartner. Solving unsymmetric sparse systems of linear equations with PARDISO. *J. of Future Generation Computer Systems*, 20(3):475-487, 2004.

Intel MKL provides also an iterative sparse solver (see [Chapter 8](#)) that uses Sparse BLAS level 2 and 3 routines and works with different sparse data formats.

Extended Eigensolver Routines

The [Extended Eigensolver RCI Routines](#) is a set of high-performance numerical routines for solving standard ($Ax = \lambda x$) and generalized ($Ax = \lambda Bx$) eigenvalue problems, where A and B are symmetric or Hermitian. It yields all the eigenvalues and eigenvectors within a given search interval. It is based on the Feast algorithm, an innovative fast and stable numerical algorithm presented in [\[Polizzi09\]](#), which deviates fundamentally from the traditional Krylov subspace iteration based techniques (Arnoldi and Lanczos algorithms [\[Bai00\]](#)) or other Davidson-Jacobi techniques [\[Sleijpen96\]](#). The Feast algorithm is inspired by the density-matrix representation and contour integration technique in quantum mechanics.

It is free from orthogonalization procedures. Its main computational tasks consist of solving very few inner independent linear systems with multiple right-hand sides and one reduced eigenvalue problem orders of magnitude smaller than the original one. The Feast algorithm combines simplicity and efficiency and offers many important capabilities for achieving high performance, robustness, accuracy, and scalability on parallel architectures. This algorithm is expected to significantly augment numerical performance in large-scale modern applications.

Some of the characteristics of the Feast algorithm [\[Polizzi09\]](#) are:

- Converges quickly in 2-3 iterations with very high accuracy
- Naturally captures all eigenvalue multiplicities
- No explicit orthogonalization procedure
- Can reuse the basis of pre-computed subspace as suitable initial guess for performing outer-refinement iterations

This capability can also be used for solving a series of eigenvalue problems that are close one another.

- The number of internal iterations is independent of the size of the system and the number of eigenpairs in the search interval
- The inner linear systems can be solved either iteratively (even with modest relative residual error) or directly

VM Functions

The Vector Mathematics functions (see [Chapter 9](#)) include a set of highly optimized implementations of certain computationally expensive core mathematical functions (power, trigonometric, exponential, hyperbolic, etc.) that operate on vectors of real and complex numbers.

Application programs that might significantly improve performance with VM include nonlinear programming software, integrals computation, and many others. VM provides interfaces both for Fortran and C languages.

Statistical Functions

Vector Statistics (VS) contains three sets of functions (see [Chapter 10](#)) providing:

- Pseudorandom, quasi-random, and non-deterministic random number generator subroutines implementing basic continuous and discrete distributions. To provide best performance, the VS subroutines use calls to highly optimized Basic Random Number Generators (BRNGs) and a set of vector mathematical functions.
- A wide variety of convolution and correlation operations.
- Initial statistical analysis of raw single and double precision multi-dimensional datasets.

Fourier Transform Functions

The Intel® MKL multidimensional Fast Fourier Transform (FFT) functions with mixed radix support (see [Chapter 11](#)) provide uniformity of discrete Fourier transform computation and combine functionality with ease of use. Both Fortran and C interface specification are given. There is also a cluster version of FFT functions, which runs on distributed-memory architectures and is provided only for Intel® 64 and Intel® Many Integrated Core architectures.

The FFT functions provide fast computation via the FFT algorithms for arbitrary lengths. See *the Intel® MKL Developer Guide* for the specific radices supported.

Partial Differential Equations Support

Intel® MKL provides tools for solving Partial Differential Equations (PDE) (see [Chapter 13](#)). These tools are Trigonometric Transform interface routines and Poisson Solver.

The Trigonometric Transform routines may be helpful to users who implement their own solvers similar to the Intel MKL Poisson Solver. The users can improve performance of their solvers by using fast sine, cosine, and staggered cosine transforms implemented in the Trigonometric Transform interface.

The Poisson Solver is designed for fast solving of simple Helmholtz, Poisson, and Laplace problems. The Trigonometric Transform interface, which underlies the solver, is based on the Intel MKL FFT interface (refer to [Chapter 11](#)), optimized for Intel® processors.

Support Functions

The Intel® MKL support functions (see [Chapter 15](#)) are used to support the operation of the Intel MKL software and provide basic information on the library and library operation, such as the current library version, timing, setting and measuring of CPU frequency, error handling, and memory allocation.

Starting from release 10.0, the Intel MKL support functions provide additional threading control.

Starting from release 10.1, Intel MKL selectively supports a *Progress Routine* feature to track progress of a lengthy computation and/or interrupt the computation using a callback function mechanism. The user application can define a function called `mkl_progress` that is regularly called from the Intel MKL routine supporting the progress routine feature. See [the Progress Routines](#) section in [Chapter 15](#) for reference. Refer to a specific LAPACK or DSS/PARDISO function description to see whether the function supports this feature or not.

Intel® Math Kernel Library for Deep Neural Networks (Intel® MKL-DNN)

Intel® Math Kernel Library (Intel® MKL) functions for Deep Neural Networks (DNN functions) is a collection of performance primitives for Deep Neural Networks (DNN) applications optimized for Intel® architecture. The implementation of DNN functions includes a limited set of primitives used in the AlexNet topology.

The primitives implement forward and backward passes for several convolution, pooling, normalization, activation, and multi-dimensional transposition operations.

Intel MKL DNN primitives implement a plain C application programming interface (API) that can be used in the existing C/C++ DNN frameworks, as well as in custom DNN applications.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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Performance Enhancements

The Intel® Math Kernel Library has been optimized by exploiting both processor and system features and capabilities. Special care has been given to those routines that most profit from cache-management techniques. These especially include matrix-matrix operation routines such as `dgemm()`.

In addition, code optimization techniques have been applied to minimize dependencies of scheduling integer and floating-point units on the results within the processor.

The major optimization techniques used throughout the library include:

- Loop unrolling to minimize loop management costs
- Blocking of data to improve data reuse opportunities
- Copying to reduce chances of data eviction from cache
- Data prefetching to help hide memory latency
- Multiple simultaneous operations (for example, dot products in `dgemm`) to eliminate stalls due to arithmetic unit pipelines
- Use of hardware features such as the SIMD arithmetic units, where appropriate

These are techniques from which the arithmetic code benefits the most.

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Parallelism

In addition to the performance enhancements discussed above, Intel® MKL offers performance gains through parallelism provided by the symmetric multiprocessing performance (SMP) feature. You can obtain improvements from SMP in the following ways:

- One way is based on user-managed threads in the program and further distribution of the operations over the threads based on data decomposition, domain decomposition, control decomposition, or some other parallelizing technique. Each thread can use any of the Intel MKL functions (except for the deprecated `?lacon` LAPACK routine) because the library has been designed to be thread-safe.
- Another method is to use the FFT and BLAS level 3 routines. They have been parallelized and require no alterations of your application to gain the performance enhancements of multiprocessing. Performance using multiple processors on the level 3 BLAS shows excellent scaling. Since the threads are called and managed within the library, the application does not need to be recompiled thread-safe.

- Yet another method is to use *tuned LAPACK routines*. Currently these include the single- and double precision flavors of routines for *QR* factorization of general matrices, triangular factorization of general and symmetric positive-definite matrices, solving systems of equations with such matrices, as well as solving symmetric eigenvalue problems.

For instructions on setting the number of available processors for the BLAS level 3 and LAPACK routines, see *Intel® MKL Developer Guide*.

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C Datatypes Specific to Intel MKL

The `mk1_types.h` file defines datatypes specific to Intel MKL.

| C/C++ Type | Fortran Type | LP32 Equivalent (Size in Bytes) | LP64 Equivalent (Size in Bytes) | ILP64 Equivalent (Size in Bytes) |
|---|---------------------------------|---|---|---|
| MKL_INT (MKL integer) | INTEGER (default INTEGER) | C/C++: int Fortran: INTEGER*4 (4 bytes) | C/C++: int Fortran: INTEGER*4 (4 bytes) | C/C++: long long (or define MKL_ILP64 macros Fortran: INTEGER*8 (8 bytes) |
| MKL_UINT (MKL unsigned integer) | N/A | C/C++: unsigned int (4 bytes) | C/C++: unsigned int (4 bytes) | C/C++: unsigned long long (8 bytes) |
| MKL_LONG (MKL long integer) | N/A | C/C++: long (4 bytes) | C/C++: long (Windows: 4 bytes) (Linux, Mac: 8 bytes) | C/C++: long (8 bytes) |
| MKL_Complex8 (Like C99 complex float) | COMPLEX*8 | (8 bytes) | (8 bytes) | (8 bytes) |
| MKL_Complex16 (Like C99 complex double) | COMPLEX*16 | (16 bytes) | (16 bytes) | (16 bytes) |

You can redefine datatypes specific to Intel MKL. One reason to do this is if you have your own types which are binary-compatible with Intel MKL datatypes, with the same representation or memory layout. To redefine a datatype, use one of these methods:

- Insert the `#define` statement redefining the datatype before the `mkl.h` header file `#include` statement. For example,

```
#define MKL_INT size_t
#include "mkl.h"
```

- Use the compiler `-D` option to redefine the datatype. For example,

```
...-DMKL_INT=size_t...
```

NOTE

As the user, if you redefine Intel MKL datatypes you are responsible for making sure that your definition is compatible with that of Intel MKL. If not, it might cause unpredictable results or crash the application.

BLAS and Sparse BLAS Routines

This chapter describes the Intel® Math Kernel Library implementation of the BLAS and Sparse BLAS routines, and BLAS-like extensions. The routine descriptions are arranged in several sections:

- [BLAS Level 1 Routines](#) (vector-vector operations)
- [BLAS Level 2 Routines](#) (matrix-vector operations)
- [BLAS Level 3 Routines](#) (matrix-matrix operations)
- [Sparse BLAS Level 1 Routines](#) (vector-vector operations).
- [Sparse BLAS Level 2 and Level 3 Routines](#) (matrix-vector and matrix-matrix operations)
- [BLAS-like Extensions](#)

Each section presents the routine and function group descriptions in alphabetical order by routine or function group name; for example, the `?asum` group, the `?axpy` group. The question mark in the group name corresponds to different character codes indicating the data type (`s`, `d`, `c`, and `z` or their combination); see [Routine Naming Conventions](#).

When BLAS or Sparse BLAS routines encounter an error, they call the error reporting routine `xerbla`.

In BLAS Level 1 groups `i?amax` and `i?amin`, an "i" is placed before the data-type indicator and corresponds to the index of an element in the vector. These groups are placed in the end of the BLAS Level 1 section.

BLAS Routines

Naming Conventions for BLAS Routines

BLAS routine names have the following structure:

`<character> <name> <mod> ()`

The `<character>` field indicates the data type:

| | |
|----------------|---------------------------|
| <code>s</code> | real, single precision |
| <code>c</code> | complex, single precision |
| <code>d</code> | real, double precision |
| <code>z</code> | complex, double precision |

Some routines and functions can have combined character codes, such as `sc` or `dz`.

For example, the function `scasum` uses a complex input array and returns a real value.

The `<name>` field, in BLAS level 1, indicates the operation type. For example, the BLAS level 1 routines `?dot`, `?rot`, `?swap` compute a vector dot product, vector rotation, and vector swap, respectively.

In BLAS level 2 and 3, `<name>` reflects the matrix argument type:

| | |
|-----------------|---------------------|
| <code>ge</code> | general matrix |
| <code>gb</code> | general band matrix |
| <code>sy</code> | symmetric matrix |

| | |
|----|------------------------------------|
| sp | symmetric matrix (packed storage) |
| sb | symmetric band matrix |
| he | Hermitian matrix |
| hp | Hermitian matrix (packed storage) |
| hb | Hermitian band matrix |
| tr | triangular matrix |
| tp | triangular matrix (packed storage) |
| tb | triangular band matrix. |

The *<mod>* field, if present, provides additional details of the operation. BLAS level 1 names can have the following characters in the *<mod>* field:

| | |
|----|---------------------------------------|
| c | conjugated vector |
| u | unconjugated vector |
| g | Givens rotation construction |
| m | modified Givens rotation |
| mg | modified Givens rotation construction |

BLAS level 2 names can have the following characters in the *<mod>* field:

| | |
|----|---|
| mv | matrix-vector product |
| sv | solving a system of linear equations with a single unknown vector |
| r | rank-1 update of a matrix |
| r2 | rank-2 update of a matrix. |

BLAS level 3 names can have the following characters in the *<mod>* field:

| | |
|-----|--|
| mm | matrix-matrix product |
| sm | solving a system of linear equations with multiple unknown vectors |
| rk | rank- <i>k</i> update of a matrix |
| r2k | rank-2 <i>k</i> update of a matrix. |

The examples below illustrate how to interpret BLAS routine names:

| | |
|--------|---|
| ddot | <i><d></i> <i><dot></i> : double-precision real vector-vector dot product |
| cdotc | <i><c></i> <i><dot></i> <i><c></i> : complex vector-vector dot product, conjugated |
| scasum | <i><sc></i> <i><asum></i> : sum of magnitudes of vector elements, single precision real output and single precision complex input |
| cdotu | <i><c></i> <i><dot></i> <i><u></i> : vector-vector dot product, unconjugated, complex |
| sgemv | <i><s></i> <i><ge></i> <i><mv></i> : matrix-vector product, general matrix, single precision |

`ztrmm` `<z> <tr> <mm>`: matrix-matrix product, triangular matrix, double-precision complex.

Sparse BLAS level 1 naming conventions are similar to those of BLAS level 1. For more information, see [Naming Conventions](#).

C Interface Conventions for BLAS Routines

CBLAS, the C interface to the Basic Linear Algebra Subprograms (BLAS), provides a C language interface to BLAS routines for Intel MKL. While you can call the Fortran implementation of BLAS, for coding in C the CBLAS interface has some advantages such as allowing you to specify column-major or row-major ordering with the `layout` parameter.

For more information about calling Fortran routines from C in general, and specifically about calling BLAS and CBLAS routines, see "Mixed-language Programming with the Intel Math Kernel Library" in the *Intel Math Kernel Library Developer Guide*.

NOTE

This reference contains syntax in C for both the CBLAS interface and the Fortran BLAS routines.

In CBLAS, the Fortran routine names are prefixed with `cblas_` (for example, `dasum` becomes `cblas_dasum`). Names of all CBLAS functions are in lowercase letters.

Complex functions `?dotc` and `?dotu` become CBLAS subroutines (void functions); they return the complex result via a void pointer, added as the last parameter. CBLAS names of these functions are suffixed with `_sub`. For example, the BLAS function `cdotc` corresponds to `cblas_cdotc_sub`.

WARNING

Users of the CBLAS interface should be aware that the CBLAS are just a C interface to the BLAS, which is based on the FORTRAN standard and subject to the FORTRAN standard restrictions. In particular, the output parameters should not be referenced through more than one argument.

NOTE

This interface is not implemented in the Sparse BLAS Level 2 and Level 3 routines.

The arguments of CBLAS functions comply with the following rules:

- Input arguments are declared with the `const` modifier.
- Non-complex scalar input arguments are passed by value.
- Complex scalar input arguments are passed as void pointers.
- Array arguments are passed by address.
- BLAS character arguments are replaced by the appropriate enumerated type.
- Level 2 and Level 3 routines acquire an additional parameter of type `CBLAS_LAYOUT` as their first argument. This parameter specifies whether two-dimensional arrays are row-major (`CblasRowMajor`) or column-major (`CblasColMajor`).

Enumerated Types

The CBLAS interface uses the following enumerated types:

```
enum CBLAS_LAYOUT {
    CblasRowMajor=101, /* row-major arrays */
    CblasColMajor=102}; /* column-major arrays */
enum CBLAS_TRANSPOSE {
    CblasNoTrans=111, /* trans='N' */
    CblasTrans=112, /* trans='T' */
```

```

    CblasConjTrans=113}; /* trans='C' */
enum CBLAS_UPLO {
    CblasUpper=121,      /* uplo ='U' */
    CblasLower=122};   /* uplo ='L' */
enum CBLAS_DIAG {
    CblasNonUnit=131,   /* diag ='N' */
    CblasUnit=132};    /* diag ='U' */
enum CBLAS_SIDE {
    CblasLeft=141,     /* side ='L' */
    CblasRight=142};  /* side ='R' */

```

Matrix Storage Schemes for BLAS Routines

Matrix arguments of BLAS and CBLAS routines can use the following storage schemes:

- *Full storage*: a matrix A is stored in a two-dimensional array a , with the matrix element A_{ij} stored in the array element $a[i + j*lda]$ for column-major layout and $a[j + i*lda]$ for row-major layout, where lda is the leading dimension for the array.
- *Packed storage* scheme allows you to store symmetric, Hermitian, or triangular matrices more compactly. For column-major layout, the upper or lower triangle of the matrix is packed by columns in a one dimensional array. For row-major layout, the upper or lower triangle of the matrix is packed by rows in a one dimensional array.
- *Band storage*: a band matrix is stored compactly in a two-dimensional array. For column-major layout, columns of the matrix are stored in the corresponding columns of the array, and diagonals of the matrix are stored in a specific row of the array. For row-major layout, rows of the matrix are stored in the corresponding rows of the array, and diagonals of the matrix are stored in a specific column of the array.

For more information on matrix storage schemes, see [Matrix Arguments](#) in Appendix B.

Row-Major and Column-Major Layout

The BLAS routines follow the Fortran convention of storing two-dimensional arrays using column-major layout. When calling BLAS routines from C, remember that they require arrays to be in column-major format, not the row-major format that is the convention for C. Unless otherwise specified, the psuedo-code examples for the BLAS routines illustrate matrices stored using column-major layout.

The CBLAS interface allows you to specify either column-major or row-major layout for BLAS Level 2 and Level 3 routines, by setting the `layout` parameter to `CblasColMajor` or `CblasRowMajor`.

BLAS Level 1 Routines and Functions

BLAS Level 1 includes routines and functions, which perform vector-vector operations. [Table “BLAS Level 1 Routine Groups and Their Data Types”](#) lists the BLAS Level 1 routine and function groups and the data types associated with them.

BLAS Level 1 Routine and Function Groups and Their Data Types

| Routine or Function Group | Data Types | Description |
|---------------------------|--------------|---|
| <code>cblas_?asum</code> | s, d, sc, dz | Sum of vector magnitudes (functions) |
| <code>cblas_?axpy</code> | s, d, c, z | Scalar-vector product (routines) |
| <code>cblas_?copy</code> | s, d, c, z | Copy vector (routines) |
| <code>cblas_?dot</code> | s, d | Dot product (functions) |
| <code>cblas_?sdot</code> | sd, d | Dot product with double precision (functions) |

| Routine or Function Group | Data Types | Description |
|---------------------------|--------------------|---|
| <code>cblas_?dotc</code> | c, z | Dot product conjugated (functions) |
| <code>cblas_?dotu</code> | c, z | Dot product unconjugated (functions) |
| <code>cblas_?nrm2</code> | s, d, sc, dz | Vector 2-norm (Euclidean norm) (functions) |
| <code>cblas_?rot</code> | s, d, cs, zd | Plane rotation of points (routines) |
| <code>cblas_?rotg</code> | s, d, c, z | Generate Givens rotation of points (routines) |
| <code>cblas_?rotm</code> | s, d | Modified Givens plane rotation of points (routines) |
| <code>cblas_?rotmg</code> | s, d | Generate modified Givens plane rotation of points (routines) |
| <code>cblas_?scal</code> | s, d, c, z, cs, zd | Vector-scalar product (routines) |
| <code>cblas_?swap</code> | s, d, c, z | Vector-vector swap (routines) |
| <code>cblas_i?amax</code> | s, d, c, z | Index of the maximum absolute value element of a vector (functions) |
| <code>cblas_i?amin</code> | s, d, c, z | Index of the minimum absolute value element of a vector (functions) |
| <code>cblas_?cabs1</code> | s, d | Auxiliary functions, compute the absolute value of a complex number of single or double precision |

cblas_?asum

Computes the sum of magnitudes of the vector elements.

Syntax

```
float cblas_sasum (const MKL_INT n, const float *x, const MKL_INT incx);
float cblas_scasum (const MKL_INT n, const void *x, const MKL_INT incx);
double cblas_dasum (const MKL_INT n, const double *x, const MKL_INT incx);
double cblas_dzasum (const MKL_INT n, const void *x, const MKL_INT incx);
```

Include Files

- `mk1.h`

Description

The `?asum` routine computes the sum of the magnitudes of elements of a real vector, or the sum of magnitudes of the real and imaginary parts of elements of a complex vector:

$$\text{res} = |\text{Re}x_1| + |\text{Im}x_1| + |\text{Re}x_2| + |\text{Im}x_2| + \dots + |\text{Re}x_n| + |\text{Im}x_n|,$$

where x is a vector with n elements.

Input Parameters

n Specifies the number of elements in vector x .

x

Array, size at least $(1 + (n-1) * \text{abs}(incx))$.

incx Specifies the increment for indexing vector *x*.

Return Values

Contains the sum of magnitudes of real and imaginary parts of all elements of the vector.

cblas_?axpy

Computes a vector-scalar product and adds the result to a vector.

Syntax

```
void cblas_saxpy (const MKL_INT n, const float a, const float *x, const MKL_INT incx, float *y, const MKL_INT incy);
```

```
void cblas_daxpy (const MKL_INT n, const double a, const double *x, const MKL_INT incx, double *y, const MKL_INT incy);
```

```
void cblas_caxpy (const MKL_INT n, const void *a, const void *x, const MKL_INT incx, void *y, const MKL_INT incy);
```

```
void cblas_zaxpy (const MKL_INT n, const void *a, const void *x, const MKL_INT incx, void *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

The ?axpy routines perform a vector-vector operation defined as

```
y := a*x + y
```

where:

a is a scalar

x and *y* are vectors each with a number of elements that equals *n*.

Input Parameters

n Specifies the number of elements in vectors *x* and *y*.

a Specifies the scalar *a*.

x Array, size at least $(1 + (n-1) * \text{abs}(incx))$.

incx Specifies the increment for the elements of *x*.

y Array, size at least $(1 + (n-1) * \text{abs}(incy))$.

incy Specifies the increment for the elements of *y*.

Output Parameters

y Contains the updated vector *y*.

cblas_?copy

Copies vector to another vector.

Syntax

```
void cblas_scopy (const MKL_INT n, const float *x, const MKL_INT incx, float *y, const
MKL_INT incy);
```

```
void cblas_dcopy (const MKL_INT n, const double *x, const MKL_INT incx, double *y,
const MKL_INT incy);
```

```
void cblas_ccopy (const MKL_INT n, const void *x, const MKL_INT incx, void *y, const
MKL_INT incy);
```

```
void cblas_zcopy (const MKL_INT n, const void *x, const MKL_INT incx, void *y, const
MKL_INT incy);
```

Include Files

- mkl.h

Description

The ?copy routines perform a vector-vector operation defined as

```
y = x,
```

where x and y are vectors.

Input Parameters

| | |
|--------|---|
| n | Specifies the number of elements in vectors x and y . |
| x | Array, size at least $(1 + (n-1)*abs(incx))$. |
| $incx$ | Specifies the increment for the elements of x . |
| y | Array, size at least $(1 + (n-1)*abs(incy))$. |
| $incy$ | Specifies the increment for the elements of y . |

Output Parameters

| | |
|-----|--|
| y | Contains a copy of the vector x if n is positive. Otherwise, parameters are unaltered. |
|-----|--|

cblas_?dot

Computes a vector-vector dot product.

Syntax

```
float cblas_sdot (const MKL_INT n, const float *x, const MKL_INT incx, const float *y,
const MKL_INT incy);
```

```
double cblas_ddot (const MKL_INT n, const double *x, const MKL_INT incx, const double
*y, const MKL_INT incy);
```

Include Files

- `mkl.h`

Description

The `?dot` routines perform a vector-vector reduction operation defined as

$$res = \sum_{i=1}^n x_i * y_i,$$

where x_i and y_i are elements of vectors x and y .

Input Parameters

| | |
|-------------------|---|
| <code>n</code> | Specifies the number of elements in vectors x and y . |
| <code>x</code> | Array, size at least $(1+(n-1)*abs(incx))$. |
| <code>incx</code> | Specifies the increment for the elements of x . |
| <code>y</code> | Array, size at least $(1+(n-1)*abs(incy))$. |
| <code>incy</code> | Specifies the increment for the elements of y . |

Return Values

The result of the dot product of x and y , if n is positive. Otherwise, returns 0.

`cblas_?sdot`

Computes a vector-vector dot product with double precision.

Syntax

```
float cblas_sdsdot (const MKL_INT n, const float sb, const float *sx, const MKL_INT incx, const float *sy, const MKL_INT incy);
```

```
double cblas_dsdot (const MKL_INT n, const float *sx, const MKL_INT incx, const float *sy, const MKL_INT incy);
```

Include Files

- `mkl.h`

Description

The `?sdot` routines compute the inner product of two vectors with double precision. Both routines use double precision accumulation of the intermediate results, but the `sdsdot` routine outputs the final result in single precision, whereas the `dsdot` routine outputs the double precision result. The function `sdsdot` also adds scalar value sb to the inner product.

Input Parameters

| | |
|----------------|---|
| <code>n</code> | Specifies the number of elements in the input vectors sx and sy . |
|----------------|---|

| | |
|---------------|---|
| <i>sb</i> | Single precision scalar to be added to inner product (for the function <code>sdsdot</code> only). |
| <i>sx, sy</i> | Arrays, size at least $(1+(n-1)*abs(incx))$ and $(1+(n-1)*abs(incy))$, respectively. Contain the input single precision vectors. |
| <i>incx</i> | Specifies the increment for the elements of <i>sx</i> . |
| <i>incy</i> | Specifies the increment for the elements of <i>sy</i> . |

Return Values

The result of the dot product of *sx* and *sy* (with *sb* added for `sdsdot`), if *n* is positive. Otherwise, returns *sb* for `sdsdot` and 0 for `dsdot`.

cblas_?dotc

Computes a dot product of a conjugated vector with another vector.

Syntax

```
void cblas_cdotc_sub (const MKL_INT n, const void *x, const MKL_INT incx, const void *y, const MKL_INT incy, void *dotc);
```

```
void cblas_zdotc_sub (const MKL_INT n, const void *x, const MKL_INT incx, const void *y, const MKL_INT incy, void *dotc);
```

Include Files

- `mkl.h`

Description

The `?dotc` routines perform a vector-vector operation defined as:

$$res = \sum_{i=1}^n conjg(x_i) * y_i,$$

where x_i and y_i are elements of vectors *x* and *y*.

Input Parameters

| | |
|-------------|---|
| <i>n</i> | Specifies the number of elements in vectors <i>x</i> and <i>y</i> . |
| <i>x</i> | Array, size at least $(1 + (n-1)*abs(incx))$. |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . |
| <i>y</i> | Array, size at least $(1 + (n-1)*abs(incy))$. |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . |

Output Parameters

| | |
|-------------|--|
| <i>dotc</i> | Contains the result of the dot product of the conjugated <i>x</i> and unconjugated <i>y</i> , if <i>n</i> is positive. Otherwise, it contains 0. |
|-------------|--|

cblas_?dotu

Computes a vector-vector dot product.

Syntax

```
void cblas_cdotu_sub (const MKL_INT n, const void *x, const MKL_INT incx, const void *y, const MKL_INT incy, void *dotu);
```

```
void cblas_zdotu_sub (const MKL_INT n, const void *x, const MKL_INT incx, const void *y, const MKL_INT incy, void *dotu);
```

Include Files

- mkl.h

Description

The ?dotu routines perform a vector-vector reduction operation defined as

$$res = \sum_{i=1}^n x_i * y_i$$

where x_i and y_i are elements of complex vectors x and y .

Input Parameters

| | |
|--------|---|
| n | Specifies the number of elements in vectors x and y . |
| x | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. |
| $incx$ | Specifies the increment for the elements of x . |
| y | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. |
| $incy$ | Specifies the increment for the elements of y . |

Output Parameters

| | |
|--------|---|
| $dotu$ | Contains the result of the dot product of x and y , if n is positive. Otherwise, it contains 0. |
|--------|---|

cblas_?nrm2

Computes the Euclidean norm of a vector.

Syntax

```
float cblas_snorm2 (const MKL_INT n, const float *x, const MKL_INT incx);
```

```
double cblas_dnorm2 (const MKL_INT n, const double *x, const MKL_INT incx);
```

```
float cblas_scnrm2 (const MKL_INT n, const void *x, const MKL_INT incx);
```

```
double cblas_dznrm2 (const MKL_INT n, const void *x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

The `?nrm2` routines perform a vector reduction operation defined as

```
res = ||x||,
```

where:

x is a vector,

res is a value containing the Euclidean norm of the elements of x .

Input Parameters

| | |
|--------|---|
| n | Specifies the number of elements in vector x . |
| x | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. |
| $incx$ | Specifies the increment for the elements of x . |

Return Values

The Euclidean norm of the vector x .

`cblas_?rot`

Performs rotation of points in the plane.

Syntax

```
void cblas_srot (const MKL_INT n, float *x, const MKL_INT incx, float *y, const MKL_INT
incy, const float c, const float s);
```

```
void cblas_drot (const MKL_INT n, double *x, const MKL_INT incx, double *y, const
MKL_INT incy, const double c, const double s);
```

```
void cblas_csrot (const MKL_INT n, void *x, const MKL_INT incx, void *y, const MKL_INT
incy, const float c, const float s);
```

```
void cblas_zdrot (const MKL_INT n, void *x, const MKL_INT incx, void *y, const MKL_INT
incy, const double c, const double s);
```

Include Files

- `mkl.h`

Description

Given two complex vectors x and y , each vector element of these vectors is replaced as follows:

$$x_i = c * x_i + s * y_i$$

$$y_i = c * y_i - s * x_i$$

Input Parameters

| | |
|--------|---|
| n | Specifies the number of elements in vectors x and y . |
| x | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. |
| $incx$ | Specifies the increment for the elements of x . |

| | |
|-------------|---|
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . |
| <i>c</i> | A scalar. |
| <i>s</i> | A scalar. |

Output Parameters

| | |
|----------|---|
| <i>x</i> | Each element is replaced by $c*x + s*y$. |
| <i>y</i> | Each element is replaced by $c*y - s*x$. |

cblas_?rotg

Computes the parameters for a Givens rotation.

Syntax

```
void cblas_srotg (float *a, float *b, float *c, float *s);  
void cblas_drotg (double *a, double *b, double *c, double *s);  
void cblas_crotg (void *a, const void *b, float *c, void *s);  
void cblas_zrotg (void *a, const void *b, double *c, void *s);
```

Include Files

- mkl.h

Description

Given the Cartesian coordinates (a, b) of a point, these routines return the parameters c , s , r , and z associated with the Givens rotation. The parameters c and s define a unitary matrix such that:

$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

The parameter z is defined such that if $|a| > |b|$, z is s ; otherwise if c is not 0 z is $1/c$; otherwise z is 1.

Input Parameters

| | |
|----------|---|
| <i>a</i> | Provides the x-coordinate of the point <i>p</i> . |
| <i>b</i> | Provides the y-coordinate of the point <i>p</i> . |

Output Parameters

| | |
|----------|---|
| <i>a</i> | Contains the parameter r associated with the Givens rotation. |
| <i>b</i> | Contains the parameter z associated with the Givens rotation. |
| <i>c</i> | Contains the parameter c associated with the Givens rotation. |
| <i>s</i> | Contains the parameter s associated with the Givens rotation. |

cblas_?rotm

Performs modified Givens rotation of points in the plane.

Syntax

```
void cblas_srotm (const MKL_INT n, float *x, const MKL_INT incx, float *y, const
MKL_INT incy, const float *param);
```

```
void cblas_drotm (const MKL_INT n, double *x, const MKL_INT incx, double *y, const
MKL_INT incy, const double *param);
```

Include Files

- mkl.h

Description

Given two vectors x and y , each vector element of these vectors is replaced as follows:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = H \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

for $i=1$ to n , where H is a modified Givens transformation matrix whose values are stored in the `param[1]` through `param[4]` array. See discussion on the `param` argument.

Input Parameters

`n` Specifies the number of elements in vectors x and y .

`x` Array, size at least $(1 + (n - 1) * \text{abs}(incx))$.

`incx` Specifies the increment for the elements of x .

`y` Array, size at least $(1 + (n - 1) * \text{abs}(incy))$.

`incy` Specifies the increment for the elements of y .

`param` Array, size 5.

The elements of the `param` array are:

`param[0]` contains a switch, `flag`. `param[1-4]` contain h_{11} , h_{21} , h_{12} , and h_{22} , respectively, the components of the array H .

Depending on the values of `flag`, the components of H are set as follows:

$$flag = -1.0: H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$flag = 0.0: H = \begin{bmatrix} 1.0 & h_{12} \\ h_{21} & 1.0 \end{bmatrix}$$

$$flag = 1.0: H = \begin{bmatrix} h_{11} & 1.0 \\ -1.0 & h_{22} \end{bmatrix}$$

$$flag = -2.0: H = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

In the last three cases, the matrix entries of 1.0, -1.0, and 0.0 are assumed based on the value of *flag* and are not required to be set in the *param* vector.

Output Parameters

x Each element $x[i]$ is replaced by $h_{11} * x[i] + h_{12} * y[i]$.

y Each element $y[i]$ is replaced by $h_{21} * x[i] + h_{22} * y[i]$.

cblas_?rotmg

Computes the parameters for a modified Givens rotation.

Syntax

```
void cblas_srotmg (float *d1, float *d2, float *x1, const float y1, float *param);
void cblas_drotmg (double *d1, double *d2, double *x1, const double y1, double *param);
```

Include Files

- mkl.h

Description

Given Cartesian coordinates (*x1*, *y1*) of an input vector, these routines compute the components of a modified Givens transformation matrix *H* that zeros the *y*-component of the resulting vector:

$$\begin{bmatrix} x1 \\ 0 \end{bmatrix} = H \begin{bmatrix} x1\sqrt{d1} \\ y1\sqrt{d2} \end{bmatrix}$$

Input Parameters

d1 Provides the scaling factor for the *x*-coordinate of the input vector.

d2 Provides the scaling factor for the *y*-coordinate of the input vector.

x1 Provides the *x*-coordinate of the input vector.

y1 Provides the *y*-coordinate of the input vector.

Output Parameters

d1 Provides the first diagonal element of the updated matrix.

d2 Provides the second diagonal element of the updated matrix.

x1 Provides the *x*-coordinate of the rotated vector before scaling.

param Array, size 5.
The elements of the *param* array are:
param[0] contains a switch, *flag*. the other array elements *param*[1-4] contain the components of the array *H*: h_{11} , h_{21} , h_{12} , and h_{22} , respectively.
Depending on the values of *flag*, the components of *H* are set as follows:

$$flag = -1.0: H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

$$flag = 0.0: H = \begin{bmatrix} 1.0 & h_{12} \\ h_{21} & 1.0 \end{bmatrix}$$

$$flag = 1.0: H = \begin{bmatrix} h_{11} & 1.0 \\ -1.0 & h_{22} \end{bmatrix}$$

$$flag = -2.0: H = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$

In the last three cases, the matrix entries of 1.0, -1.0, and 0.0 are assumed based on the value of *flag* and are not required to be set in the *param* vector.

cblas_?scal

Computes the product of a vector by a scalar.

Syntax

```
void cblas_sscal (const MKL_INT n, const float a, float *x, const MKL_INT incx);
void cblas_dscal (const MKL_INT n, const double a, double *x, const MKL_INT incx);
void cblas_cscal (const MKL_INT n, const void *a, void *x, const MKL_INT incx);
void cblas_zscal (const MKL_INT n, const void *a, void *x, const MKL_INT incx);
void cblas_csscal (const MKL_INT n, const float a, void *x, const MKL_INT incx);
void cblas_zdscal (const MKL_INT n, const double a, void *x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

The ?scal routines perform a vector operation defined as

$$x = a*x$$

where:

a is a scalar, *x* is an *n*-element vector.

Input Parameters

| | |
|-------------|---|
| <i>n</i> | Specifies the number of elements in vector <i>x</i> . |
| <i>a</i> | Specifies the scalar <i>a</i> . |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . |

Output Parameters

x Updated vector *x*.

cblas_?swap

Swaps a vector with another vector.

Syntax

```
void cblas_sswap (const MKL_INT n, float *x, const MKL_INT incx, float *y, const MKL_INT incy);
```

```
void cblas_dswap (const MKL_INT n, double *x, const MKL_INT incx, double *y, const MKL_INT incy);
```

```
void cblas_cswap (const MKL_INT n, void *x, const MKL_INT incx, void *y, const MKL_INT incy);
```

```
void cblas_zswap (const MKL_INT n, void *x, const MKL_INT incx, void *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

Given two vectors *x* and *y*, the *?swap* routines return vectors *y* and *x* swapped, each replacing the other.

Input Parameters

n Specifies the number of elements in vectors *x* and *y*.

x Array, size at least $(1 + (n-1)*abs(incx))$.

incx Specifies the increment for the elements of *x*.

y Array, size at least $(1 + (n-1)*abs(incy))$.

incy Specifies the increment for the elements of *y*.

Output Parameters

x Contains the resultant vector *x*, that is, the input vector *y*.

y Contains the resultant vector *y*, that is, the input vector *x*.

cblas_i?amax

Finds the index of the element with maximum absolute value.

Syntax

```
CBLAS_INDEX cblas_isamax (const MKL_INT n, const float *x, const MKL_INT incx);
```

```
CBLAS_INDEX cblas_idamax (const MKL_INT n, const double *x, const MKL_INT incx);
```

```
CBLAS_INDEX cblas_icamax (const MKL_INT n, const void *x, const MKL_INT incx);
```

```
CBLAS_INDEX cblas_izamax (const MKL_INT n, const void *x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

Given a vector x , the $i?amax$ functions return the position of the vector element $x[i]$ that has the largest absolute value for real flavors, or the largest sum $|\operatorname{Re}(x[i])| + |\operatorname{Im}(x[i])|$ for complex flavors.

If n is not positive, 0 is returned.

If more than one vector element is found with the same largest absolute value, the index of the first one encountered is returned.

Input Parameters

| | |
|--------|---|
| n | Specifies the number of elements in vector x . |
| x | Array, size at least $(1+(n-1)*\operatorname{abs}(incx))$. |
| $incx$ | Specifies the increment for the elements of x . |

Return Values

Returns the position of vector element that has the largest absolute value such that $x[index-1]$ has the largest absolute value.

cblas_i?amin

Finds the index of the element with the smallest absolute value.

Syntax

```
CBLAS_INDEX cblas_isamin (const MKL_INT n, const float *x, const MKL_INT incx);
CBLAS_INDEX cblas_idamin (const MKL_INT n, const double *x, const MKL_INT incx);
CBLAS_INDEX cblas_icamin (const MKL_INT n, const void *x, const MKL_INT incx);
CBLAS_INDEX cblas_izamin (const MKL_INT n, const void *x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

Given a vector x , the $i?amin$ functions return the position of the vector element $x[i]$ that has the smallest absolute value for real flavors, or the smallest sum $|\operatorname{Re}(x[i])| + |\operatorname{Im}(x[i])|$ for complex flavors.

If n is not positive, 0 is returned.

If more than one vector element is found with the same smallest absolute value, the index of the first one encountered is returned.

Input Parameters

| | |
|-----|--|
| n | On entry, n specifies the number of elements in vector x . |
| x | Array, size at least $(1+(n-1)*\operatorname{abs}(incx))$. |

incx Specifies the increment for the elements of *x*.

Return Values

Indicates the position of vector element with the smallest absolute value such that $x[index-1]$ has the smallest absolute value.

cblas_?cabs1

Computes absolute value of complex number.

Syntax

```
float cblas_scabs1 (const void *z);
double cblas_dcabs1 (const void *z);
```

Include Files

- mkl.h

Description

The `?cabs1` is an auxiliary routine for a few BLAS Level 1 routines. This routine performs an operation defined as

```
res = |Re(z)| + |Im(z)|,
```

where *z* is a scalar, and *res* is a value containing the absolute value of a complex number *z*.

Input Parameters

z Scalar.

Return Values

The absolute value of a complex number *z*.

BLAS Level 2 Routines

This section describes BLAS Level 2 routines, which perform matrix-vector operations. [Table "BLAS Level 2 Routine Groups and Their Data Types"](#) lists the BLAS Level 2 routine groups and the data types associated with them.

BLAS Level 2 Routine Groups and Their Data Types

| Routine Groups | Data Types | Description |
|--------------------------|------------|---|
| <code>cblas_?gbmv</code> | s, d, c, z | Matrix-vector product using a general band matrix |
| <code>cblas?_gemv</code> | s, d, c, z | Matrix-vector product using a general matrix |
| <code>cblas_?ger</code> | s, d | Rank-1 update of a general matrix |
| <code>cblas_?gerc</code> | c, z | Rank-1 update of a conjugated general matrix |
| <code>cblas_?geru</code> | c, z | Rank-1 update of a general matrix, unconjugated |
| <code>cblas_?hbmV</code> | c, z | Matrix-vector product using a Hermitian band matrix |
| <code>cblas_?hemv</code> | c, z | Matrix-vector product using a Hermitian matrix |

| Routine Groups | Data Types | Description |
|--------------------------|------------|--|
| <code>cblas_?her</code> | c, z | Rank-1 update of a Hermitian matrix |
| <code>cblas_?her2</code> | c, z | Rank-2 update of a Hermitian matrix |
| <code>cblas_?hpmv</code> | c, z | Matrix-vector product using a Hermitian packed matrix |
| <code>cblas_?hpr</code> | c, z | Rank-1 update of a Hermitian packed matrix |
| <code>cblas_?hpr2</code> | c, z | Rank-2 update of a Hermitian packed matrix |
| <code>cblas_?sbmv</code> | s, d | Matrix-vector product using symmetric band matrix |
| <code>cblas_?spmv</code> | s, d | Matrix-vector product using a symmetric packed matrix |
| <code>cblas_?spr</code> | s, d | Rank-1 update of a symmetric packed matrix |
| <code>cblas_?spr2</code> | s, d | Rank-2 update of a symmetric packed matrix |
| <code>cblas_?symv</code> | s, d | Matrix-vector product using a symmetric matrix |
| <code>cblas_?syr</code> | s, d | Rank-1 update of a symmetric matrix |
| <code>cblas_?syr2</code> | s, d | Rank-2 update of a symmetric matrix |
| <code>cblas_?tbmv</code> | s, d, c, z | Matrix-vector product using a triangular band matrix |
| <code>cblas_?tbsv</code> | s, d, c, z | Solution of a linear system of equations with a triangular band matrix |
| <code>cblas_?tpmv</code> | s, d, c, z | Matrix-vector product using a triangular packed matrix |
| <code>cblas_?tpsv</code> | s, d, c, z | Solution of a linear system of equations with a triangular packed matrix |
| <code>cblas_?trmv</code> | s, d, c, z | Matrix-vector product using a triangular matrix |
| <code>cblas_?trsv</code> | s, d, c, z | Solution of a linear system of equations with a triangular matrix |

`cblas_?gbmv`

Computes a matrix-vector product using a general band matrix

Syntax

```
void cblas_sgbmv (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const MKL_INT kl, const MKL_INT ku, const float alpha, const float *a, const MKL_INT lda, const float *x, const MKL_INT incx, const float beta, float *y, const MKL_INT incy);
```

```
void cblas_dgbmv (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const MKL_INT kl, const MKL_INT ku, const double alpha, const double *a, const MKL_INT lda, const double *x, const MKL_INT incx, const double beta, double *y, const MKL_INT incy);
```

```
void cblas_cgbmv (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const MKL_INT kl, const MKL_INT ku, const void *alpha, const void *a, const MKL_INT lda, const void *x, const MKL_INT incx, const void *beta, void *y, const MKL_INT incy);
```

```
void cblas_zgbmv (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE trans, const MKL_INT
m, const MKL_INT n, const MKL_INT kl, const MKL_INT ku, const void *alpha, const void
*a, const MKL_INT lda, const void *x, const MKL_INT incx, const void *beta, void *y,
const MKL_INT incy);
```

Include Files

- mkl.h

Description

The `?gbmv` routines perform a matrix-vector operation defined as

$$y := \alpha * A * x + \beta * y,$$

or

$$y := \alpha * A' * x + \beta * y,$$

or

$$y := \alpha * \text{conjg}(A') * x + \beta * y,$$

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*n* band matrix, with *kl* sub-diagonals and *ku* super-diagonals.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>trans</i> | Specifies the operation: If <i>trans</i> =CblasNoTrans, then $y := \alpha * A * x + \beta * y$ If <i>trans</i> =CblasTrans, then $y := \alpha * A' * x + \beta * y$ If <i>trans</i> =CblasConjTrans, then $y := \alpha * \text{conjg}(A') * x + \beta * y$ |
| <i>m</i> | Specifies the number of rows of the matrix <i>A</i> . The value of <i>m</i> must be at least zero. |
| <i>n</i> | Specifies the number of columns of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>kl</i> | Specifies the number of sub-diagonals of the matrix <i>A</i> . The value of <i>kl</i> must satisfy $0 \leq kl$. |
| <i>ku</i> | Specifies the number of super-diagonals of the matrix <i>A</i> . The value of <i>ku</i> must satisfy $0 \leq ku$. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>a</i> | Array, size $lda * n$. |

Layout = CblasColMajor: Before entry, the leading $(kl + ku + 1)$ by n part of the array a must contain the matrix of coefficients. This matrix must be supplied column-by-column, with the leading diagonal of the matrix in row (ku) of the array, the first super-diagonal starting at position 1 in row $(ku - 1)$, the first sub-diagonal starting at position 0 in row $(ku + 1)$, and so on. Elements in the array a that do not correspond to elements in the band matrix (such as the top left ku by ku triangle) are not referenced.

The following program segment transfers a band matrix from conventional full matrix storage ($matrix$, with leading dimension ldm) to band storage (a , with leading dimension lda):

```
for (j = 0; j < n; j++) {
    k = ku - j;
    for (i = max(0, j-ku); i < min(m, j+kl+1); i++) {
        a[(k+i) + j*lda] = matrix[i + j*ldm];
    }
}
```

Layout = CblasRowMajor: Before entry, the leading $(kl + ku + 1)$ by m part of the array a must contain the matrix of coefficients. This matrix must be supplied row-by-row, with the leading diagonal of the matrix in column (kl) of the array, the first super-diagonal starting at position 0 in column $(kl + 1)$, the first sub-diagonal starting at position 1 in row $(kl - 1)$, and so on. Elements in the array a that do not correspond to elements in the band matrix (such as the top left kl by kl triangle) are not referenced.

The following program segment transfers a band matrix from row-major full matrix storage ($matrix$, with leading dimension ldm) to band storage (a , with leading dimension lda):

```
for (i = 0; i < m; i++) {
    k = kl - i;
    for (j = max(0, i-kl); j < min(n, i+ku+1); j++) {
        a[(k+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

| | |
|-------------|--|
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. The value of lda must be at least $(kl + ku + 1)$. |
| x | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$ when $trans=CblasNoTrans$, and at least $(1 + (m - 1) * \text{abs}(incx))$ otherwise. Before entry, the array x must contain the vector x . |
| <i>incx</i> | Specifies the increment for the elements of x . $incx$ must not be zero. |
| <i>beta</i> | Specifies the scalar beta. When $beta$ is equal to zero, then y need not be set on input. |
| y | Array, size at least $(1 + (m - 1) * \text{abs}(incy))$ when $trans=CblasNoTrans$ and at least $(1 + (n - 1) * \text{abs}(incy))$ otherwise. Before entry, the incremented array y must contain the vector y . |
| <i>incy</i> | Specifies the increment for the elements of y . The value of $incy$ must not be zero. |

Output Parameters

y Updated vector *y*.

cblas_?gemv

Computes a matrix-vector product using a general matrix

Syntax

```
void cblas_sgemv (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const float alpha, const float *a, const MKL_INT lda, const float *x, const MKL_INT incx, const float beta, float *y, const MKL_INT incy);
```

```
void cblas_dgemv (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const double alpha, const double *a, const MKL_INT lda, const double *x, const MKL_INT incx, const double beta, double *y, const MKL_INT incy);
```

```
void cblas_cgemv (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const void *alpha, const void *a, const MKL_INT lda, const void *x, const MKL_INT incx, const void *beta, void *y, const MKL_INT incy);
```

```
void cblas_zgemv (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const void *alpha, const void *a, const MKL_INT lda, const void *x, const MKL_INT incx, const void *beta, void *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

The ?gemv routines perform a matrix-vector operation defined as

$$y := \alpha A x + \beta y,$$

or

$$y := \alpha A' x + \beta y,$$

or

$$y := \alpha \text{conjg}(A') x + \beta y,$$

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*n* matrix.

Input Parameters

Layout Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor).

trans Specifies the operation:
 if *trans*=CblasNoTrans, then $y := \alpha A x + \beta y$;
 if *trans*=CblasTrans, then $y := \alpha A' x + \beta y$;

| | |
|----------|---|
| | if $trans=CblasConjTrans$, then $y := \alpha *conjg(A')*x + \beta*y$. |
| m | Specifies the number of rows of the matrix A . The value of m must be at least zero. |
| n | Specifies the number of columns of the matrix A . The value of n must be at least zero. |
| α | Specifies the scalar α . |
| a | Array, size $lda*k$. For $Layout = CblasColMajor$, k is n . Before entry, the leading m -by- n part of the array a must contain the matrix A . For $Layout = CblasRowMajor$, k is m . Before entry, the leading n -by- m part of the array a must contain the matrix A . |
| lda | Specifies the leading dimension of a as declared in the calling (sub)program. For $Layout = CblasColMajor$, the value of lda must be at least $\max(1, m)$. For $Layout = CblasRowMajor$, the value of lda must be at least $\max(1, n)$. |
| x | Array, size at least $(1+(n-1)*abs(incx))$ when $trans=CblasNoTrans$ and at least $(1+(m-1)*abs(incx))$ otherwise. Before entry, the incremented array x must contain the vector x . |
| $incx$ | Specifies the increment for the elements of x . The value of $incx$ must not be zero. |
| β | Specifies the scalar β . When β is set to zero, then y need not be set on input. |
| y | Array, size at least $(1+(m-1)*abs(incy))$ when $trans=CblasNoTrans$ and at least $(1+(n-1)*abs(incy))$ otherwise. Before entry with non-zero β , the incremented array y must contain the vector y . |
| $incy$ | Specifies the increment for the elements of y . The value of $incy$ must not be zero. |

Output Parameters

| | |
|-----|----------------------|
| y | Updated vector y . |
|-----|----------------------|

cblas_?ger

Performs a rank-1 update of a general matrix.

Syntax

```
void cblas_sger (const CBLAS_LAYOUT Layout, const MKL_INT m, const MKL_INT n, const float alpha, const float *x, const MKL_INT incx, const float *y, const MKL_INT incy, float *a, const MKL_INT lda);
```

```
void cblas_dger (const CBLAS_LAYOUT Layout, const MKL_INT m, const MKL_INT n, const
double alpha, const double *x, const MKL_INT incx, const double *y, const MKL_INT
incy, double *a, const MKL_INT lda);
```

Include Files

- mkl.h

Description

The `?ger` routines perform a matrix-vector operation defined as

```
 $A := \alpha x y^T + A,$ 
```

where:

α is a scalar,

x is an m -element vector,

y is an n -element vector,

A is an m -by- n general matrix.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>m</i> | Specifies the number of rows of the matrix A . The value of m must be at least zero. |
| <i>n</i> | Specifies the number of columns of the matrix A . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>x</i> | Array, size at least $(1 + (m - 1) * \text{abs}(incx))$. Before entry, the incremented array x must contain the m -element vector x . |
| <i>incx</i> | Specifies the increment for the elements of x . The value of $incx$ must not be zero. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array y must contain the n -element vector y . |
| <i>incy</i> | Specifies the increment for the elements of y . The value of $incy$ must not be zero. |
| <i>a</i> | Array, size $lda * k$. For $Layout = \text{CblasColMajor}$, k is n . Before entry, the leading m -by- n part of the array a must contain the matrix A . For $Layout = \text{CblasRowMajor}$, k is m . Before entry, the leading n -by- m part of the array a must contain the matrix A . |
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. |

For $Layout = CblasColMajor$, the value of lda must be at least $\max(1, m)$.

For $Layout = CblasRowMajor$, the value of lda must be at least $\max(1, n)$.

Output Parameters

a Overwritten by the updated matrix.

`cblas_?gerc`

Performs a rank-1 update (conjugated) of a general matrix.

Syntax

```
void cblas_cgerc (const CBLAS_LAYOUT Layout, const MKL_INT m, const MKL_INT n, const
void *alpha, const void *x, const MKL_INT incx, const void *y, const MKL_INT incy,
void *a, const MKL_INT lda);
```

```
void cblas_zgerc (const CBLAS_LAYOUT Layout, const MKL_INT m, const MKL_INT n, const
void *alpha, const void *x, const MKL_INT incx, const void *y, const MKL_INT incy,
void *a, const MKL_INT lda);
```

Include Files

- `mkl.h`

Description

The `?gerc` routines perform a matrix-vector operation defined as

$$A := \alpha * x * \text{conj}(y') + A,$$

where:

α is a scalar,

x is an m -element vector,

y is an n -element vector,

A is an m -by- n matrix.

Input Parameters

| | |
|----------|---|
| $Layout$ | Specifies whether two-dimensional array storage is row-major (<code>CblasRowMajor</code>) or column-major (<code>CblasColMajor</code>). |
| m | Specifies the number of rows of the matrix A . The value of m must be at least zero. |
| n | Specifies the number of columns of the matrix A . The value of n must be at least zero. |
| α | Specifies the scalar α . |

| | |
|-------------|---|
| <i>x</i> | Array, size at least $(1 + (m - 1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>m</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array <i>y</i> must contain the <i>n</i> -element vector <i>y</i> . |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . The value of <i>incy</i> must not be zero. |
| <i>a</i> | Array, size <i>lda</i> * <i>k</i> . For <i>Layout</i> = CblasColMajor, <i>k</i> is <i>n</i> . Before entry, the leading <i>m</i> -by- <i>n</i> part of the array <i>a</i> must contain the matrix <i>A</i> . For <i>Layout</i> = CblasRowMajor, <i>k</i> is <i>m</i> . Before entry, the leading <i>n</i> -by- <i>m</i> part of the array <i>a</i> must contain the matrix <i>A</i> . |
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. For <i>Layout</i> = CblasColMajor, the value of <i>lda</i> must be at least $\max(1, m)$. For <i>Layout</i> = CblasRowMajor, the value of <i>lda</i> must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------|------------------------------------|
| <i>a</i> | Overwritten by the updated matrix. |
|----------|------------------------------------|

cblas_?geru

Performs a rank-1 update (unconjugated) of a general matrix.

Syntax

```
void cblas_cgeru (const CBLAS_LAYOUT Layout, const MKL_INT m, const MKL_INT n, const void *alpha, const void *x, const MKL_INT incx, const void *y, const MKL_INT incy, void *a, const MKL_INT lda);
```

```
void cblas_zgeru (const CBLAS_LAYOUT Layout, const MKL_INT m, const MKL_INT n, const void *alpha, const void *x, const MKL_INT incx, const void *y, const MKL_INT incy, void *a, const MKL_INT lda);
```

Include Files

- mkl.h

Description

The ?geru routines perform a matrix-vector operation defined as

$$A := \alpha * x * y^T + A,$$

where:

α is a scalar,
 x is an m -element vector,
 y is an n -element vector,
 A is an m -by- n matrix.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>m</i> | Specifies the number of rows of the matrix A . The value of m must be at least zero. |
| <i>n</i> | Specifies the number of columns of the matrix A . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>x</i> | Array, size at least $(1 + (m - 1) * \text{abs}(incx))$. Before entry, the incremented array x must contain the m -element vector x . |
| <i>incx</i> | Specifies the increment for the elements of x . The value of $incx$ must not be zero. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array y must contain the n -element vector y . |
| <i>incy</i> | Specifies the increment for the elements of y . The value of $incy$ must not be zero. |
| <i>a</i> | Array, size $lda * k$. For <i>Layout</i> = CblasColMajor, k is n . Before entry, the leading m -by- n part of the array a must contain the matrix A . For <i>Layout</i> = CblasRowMajor, k is m . Before entry, the leading n -by- m part of the array a must contain the matrix A . |
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. For <i>Layout</i> = CblasColMajor, the value of lda must be at least $\max(1, m)$. For <i>Layout</i> = CblasRowMajor, the value of lda must be at least $\max(1, n)$. |

Output Parameters

a Overwritten by the updated matrix.

cblas_?hbmV

Computes a matrix-vector product using a Hermitian band matrix.

Syntax

```
void cblas_chbmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const MKL_INT k, const void *alpha, const void *a, const MKL_INT lda, const void *x,
const MKL_INT incx, const void *beta, void *y, const MKL_INT incy);
```

```
void cblas_zhbmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const MKL_INT k, const void *alpha, const void *a, const MKL_INT lda, const void *x,
const MKL_INT incx, const void *beta, void *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

The ?hbmv routines perform a matrix-vector operation defined as $y := \alpha A x + \beta y$,

where:

α and β are scalars,

x and y are n -element vectors,

A is an n -by- n Hermitian band matrix, with k super-diagonals.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the Hermitian band matrix A is used: If <i>uplo</i> = CblasUpper, then the upper triangular part of the matrix A is used. If <i>uplo</i> = CblasLower, then the low triangular part of the matrix A is used. |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>k</i> | For <i>uplo</i> = CblasUpper: Specifies the number of super-diagonals of the matrix A . For <i>uplo</i> = CblasLower: Specifies the number of sub-diagonals of the matrix A . The value of k must satisfy $0 \leq k$. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>a</i> | Array, size $lda * n$. <i>Layout</i> = CblasColMajor: Before entry with <i>uplo</i> = CblasUpper, the leading $(k + 1)$ by n part of the array a must contain the upper triangular band part of the Hermitian matrix. The matrix must be supplied column-by-column, with the leading diagonal of the matrix in row k of the array, the first super-diagonal starting at position 1 in row $(k - 1)$, and so on. The top left k by k triangle of the array a is not referenced. |

The following program segment transfers the upper triangular part of a Hermitian band matrix from conventional full matrix storage (*matrix*, with leading dimension *ldm*) to band storage (*a*, with leading dimension *lda*):

```
for (j = 0; j < n; j++) {
    m = k - j;
    for (i = max(0, j - k); i <= j; i++) {
        a[(m+i) + j*lda] = matrix[i + j*ldm];
    }
}
```

Before entry with *uplo* = *CblasLower*, the leading $(k + 1)$ by n part of the array *a* must contain the lower triangular band part of the Hermitian matrix, supplied column-by-column, with the leading diagonal of the matrix in row 0 of the array, the first sub-diagonal starting at position 0 in row 1, and so on. The bottom right k by k triangle of the array *a* is not referenced.

The following program segment transfers the lower triangular part of a Hermitian band matrix from conventional full matrix storage (*matrix*, with leading dimension *ldm*) to band storage (*a*, with leading dimension *lda*):

```
for (j = 0; j < n; j++) {
    m = -j;
    for (i = j; i < min(n, j + k + 1); i++) {
        a[(m+i) + j*lda] = matrix[i + j*ldm];
    }
}
```

Layout = *CblasRowMajor*:

Before entry with *uplo* = *CblasUpper*, the leading $(k + 1)$ -by- n part of array *a* must contain the upper triangular band part of the Hermitian matrix. The matrix must be supplied row-by-row, with the leading diagonal of the matrix in column 0 of the array, the first super-diagonal starting at position 0 in column 1, and so on. The bottom right k -by- k triangle of array *a* is not referenced.

The following program segment transfers the upper triangular part of a Hermitian band matrix from row-major full matrix storage (*matrix* with leading dimension *ldm*) to row-major band storage (*a*, with leading dimension *lda*):

```
for (i = 0; i < n; i++) {
    m = -i;
    for (j = i; j < MIN(n, i+k+1); j++) {
        a[(m+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

Before entry with *uplo* = *CblasLower*, the leading $(k + 1)$ -by- n part of array *a* must contain the lower triangular band part of the Hermitian matrix, supplied row-by-row, with the leading diagonal of the matrix in column k of the array, the first sub-diagonal starting at position 1 in column $k-1$, and so on. The top left k -by- k triangle of array *a* is not referenced.

The following program segment transfers the lower triangular part of a Hermitian row-major band matrix from row-major full matrix storage (*matrix*, with leading dimension *ldm*) to row-major band storage (*a*, with leading dimension *lda*):

```
for (i = 0; i < n; i++) {
    m = k - i;
    for (j = max(0, i-k); j <= i; j++) {
        a[(m+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

The imaginary parts of the diagonal elements need not be set and are assumed to be zero.

| | |
|-------------|--|
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. The value of <i>lda</i> must be at least $(k + 1)$. |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incx}))$. Before entry, the incremented array <i>x</i> must contain the vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incy}))$. Before entry, the incremented array <i>y</i> must contain the vector <i>y</i> . |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . The value of <i>incy</i> must not be zero. |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Overwritten by the updated vector <i>y</i> . |
|----------|--|

cblas_?hemv

Computes a matrix-vector product using a Hermitian matrix.

Syntax

```
void cblas_chemv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const void *alpha, const void *a, const MKL_INT lda, const void *x, const MKL_INT
incx, const void *beta, void *y, const MKL_INT incy);
```

```
void cblas_zhemv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const void *alpha, const void *a, const MKL_INT lda, const void *x, const MKL_INT
incx, const void *beta, void *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

The `?hemv` routines perform a matrix-vector operation defined as

$$y := \alpha * A * x + \beta * y,$$

where:

α and β are scalars,

x and y are n -element vectors,

A is an n -by- n Hermitian matrix.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (<code>CblasRowMajor</code>) or column-major (<code>CblasColMajor</code>). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array a is used. If $uplo = CblasUpper$, then the upper triangular of the array a is used. If $uplo = CblasLower$, then the low triangular of the array a is used. |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>a</i> | Array, size $lda * n$. Before entry with $uplo = CblasUpper$, the leading n -by- n upper triangular part of the array a must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of a is not referenced. Before entry with $uplo = CblasLower$, the leading n -by- n lower triangular part of the array a must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of a is not referenced. The imaginary parts of the diagonal elements need not be set and are assumed to be zero. |
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. The value of lda must be at least $\max(1, n)$. |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array x must contain the n -element vector x . |
| <i>incx</i> | Specifies the increment for the elements of x . The value of $incx$ must not be zero. |
| <i>beta</i> | Specifies the scalar β . When β is supplied as zero then y need not be set on input. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array y must contain the n -element vector y . |
| <i>incy</i> | Specifies the increment for the elements of y . The value of $incy$ must not be zero. |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Overwritten by the updated vector y . |
|----------|---|

cblas_?her

Performs a rank-1 update of a Hermitian matrix.

Syntax

```
void cblas_cher (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const float alpha, const void *x, const MKL_INT incx, void *a, const MKL_INT lda);
```

```
void cblas_zher (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const double alpha, const void *x, const MKL_INT incx, void *a, const MKL_INT lda);
```

Include Files

- mkl.h

Description

The ?her routines perform a matrix-vector operation defined as

$$A := \alpha x \text{conjg}(x') + A,$$

where:

alpha is a real scalar,

x is an *n*-element vector,

A is an *n*-by-*n* Hermitian matrix.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array <i>a</i> is used. If <i>uplo</i> = CblasUpper, then the upper triangular of the array <i>a</i> is used. If <i>uplo</i> = CblasLower, then the low triangular of the array <i>a</i> is used. |
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>x</i> | Array, dimension at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>a</i> | Array, size <i>lda</i> * <i>n</i> . Before entry with <i>uplo</i> = CblasUpper, the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of <i>a</i> is not referenced. Before entry with <i>uplo</i> = CblasLower, the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of <i>a</i> is not referenced. |

The imaginary parts of the diagonal elements need not be set and are assumed to be zero.

lda

Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least $\max(1, n)$.

Output Parameters

a

With *uplo* = CblasUpper, the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated matrix.

With *uplo* = CblasLower, the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated matrix.

The imaginary parts of the diagonal elements are set to zero.

cblas_?her2

Performs a rank-2 update of a Hermitian matrix.

Syntax

```
void cblas_cher2 (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const void *alpha, const void *x, const MKL_INT incx, const void *y, const MKL_INT
incy, void *a, const MKL_INT lda);
```

```
void cblas_zher2 (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const void *alpha, const void *x, const MKL_INT incx, const void *y, const MKL_INT
incy, void *a, const MKL_INT lda);
```

Include Files

- mkl.h

Description

The ?her2 routines perform a matrix-vector operation defined as

$$A := \alpha * x * \text{conjg}(y') + \text{conjg}(\alpha) * y * \text{conjg}(x') + A,$$

where:

alpha is a scalar,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* Hermitian matrix.

Input Parameters

Layout

Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor).

uplo

Specifies whether the upper or lower triangular part of the array *a* is used.

If *uplo* = CblasUpper, then the upper triangular of the array *a* is used.

If *uplo* = CblasLower, then the low triangular of the array *a* is used.

n

Specifies the order of the matrix *A*. The value of *n* must be at least zero.

| | |
|--------------|---|
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array <i>y</i> must contain the <i>n</i> -element vector <i>y</i> . |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . The value of <i>incy</i> must not be zero. |
| <i>a</i> | Array, size <i>lda</i> * <i>n</i> . Before entry with <i>uplo</i> = CblasUpper, the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of <i>a</i> is not referenced. Before entry with <i>uplo</i> = CblasLower, the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of <i>a</i> is not referenced. The imaginary parts of the diagonal elements need not be set and are assumed to be zero. |
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. The value of <i>lda</i> must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | With <i>uplo</i> = CblasUpper, the upper triangular part of the array <i>a</i> is overwritten by the upper triangular part of the updated matrix. With <i>uplo</i> = CblasLower, the lower triangular part of the array <i>a</i> is overwritten by the lower triangular part of the updated matrix. The imaginary parts of the diagonal elements are set to zero. |
|----------|---|

cblas_?hpmv

Computes a matrix-vector product using a Hermitian packed matrix.

Syntax

```
void cblas_chpmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const void *alpha, const void *ap, const void *x, const MKL_INT incx, const void
*beta, void *y, const MKL_INT incy);
```

```
void cblas_zhpmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const void *alpha, const void *ap, const void *x, const MKL_INT incx, const void
*beta, void *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

The ?hpmv routines perform a matrix-vector operation defined as

$$y := \alpha * A * x + \beta * y,$$

where:

α and β are scalars,

x and y are n -element vectors,

A is an n -by- n Hermitian matrix, supplied in packed form.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array ap . If $uplo = CblasUpper$, then the upper triangular part of the matrix A is supplied in the packed array ap . If $uplo = CblasLower$, then the low triangular part of the matrix A is supplied in the packed array ap . |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>ap</i> | Array, size at least $((n * (n + 1)) / 2)$. For $Layout = CblasColMajor$: Before entry with $uplo = CblasUpper$, the array ap must contain the upper triangular part of the Hermitian matrix packed sequentially, column-by-column, so that $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{1,2}$ and $A_{2,2}$ respectively, and so on. Before entry with $uplo = CblasLower$, the array ap must contain the lower triangular part of the Hermitian matrix packed sequentially, column-by-column, so that $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{2,1}$ and $A_{3,1}$ respectively, and so on. For $Layout = CblasRowMajor$: Before entry with $uplo = CblasUpper$, the array ap must contain the upper triangular part of the Hermitian matrix packed sequentially, row-by-row, $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{1,2}$ and $A_{1,3}$ respectively, and so on. Before entry with $uplo = CblasLower$, the array ap must contain the lower triangular part of the Hermitian matrix packed sequentially, row-by-row, so that $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{2,1}$ and $A_{2,2}$ respectively, and so on. The imaginary parts of the diagonal elements need not be set and are assumed to be zero. |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incx}))$. Before entry, the incremented array x must contain the n -element vector x . |
| <i>incx</i> | Specifies the increment for the elements of x . |

The value of *incx* must not be zero.

beta

Specifies the scalar *beta*.

When *beta* is equal to zero then *y* need not be set on input.

y

Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array *y* must contain the *n*-element vector *y*.

incy

Specifies the increment for the elements of *y*.

The value of *incy* must not be zero.

Output Parameters

y

Overwritten by the updated vector *y*.

cblas_?hpr

Performs a rank-1 update of a Hermitian packed matrix.

Syntax

```
void cblas_chpr (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const float alpha, const void *x, const MKL_INT incx, void *ap);
```

```
void cblas_zhpr (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const double alpha, const void *x, const MKL_INT incx, void *ap);
```

Include Files

- mkl.h

Description

The ?hpr routines perform a matrix-vector operation defined as

$$A := \alpha * x * \text{conjg}(x') + A,$$

where:

alpha is a real scalar,

x is an *n*-element vector,

A is an *n*-by-*n* Hermitian matrix, supplied in packed form.

Input Parameters

Layout

Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor).

uplo

Specifies whether the upper or lower triangular part of the matrix *A* is supplied in the packed array *ap*.

If *uplo* = CblasUpper, the upper triangular part of the matrix *A* is supplied in the packed array *ap*.

If *uplo* = CblasLower, the low triangular part of the matrix *A* is supplied in the packed array *ap*.

| | |
|--------------|---|
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . <i>incx</i> must not be zero. |
| <i>ap</i> | <p>Array, size at least $((n * (n + 1)) / 2)$.</p> <p>For <i>Layout</i> = CblasColMajor:</p> <p>Before entry with <i>uplo</i> = CblasUpper, the array <i>ap</i> must contain the upper triangular part of the Hermitian matrix packed sequentially, column-by-column, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{1,2}$ and $A_{2,2}$ respectively, and so on.</p> <p>Before entry with <i>uplo</i> = CblasLower, the array <i>ap</i> must contain the lower triangular part of the Hermitian matrix packed sequentially, column-by-column, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{2,1}$ and $A_{3,1}$ respectively, and so on.</p> <p>For <i>Layout</i> = CblasRowMajor:</p> <p>Before entry with <i>uplo</i> = CblasUpper, the array <i>ap</i> must contain the upper triangular part of the Hermitian matrix packed sequentially, row-by-row, <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{1,2}$ and $A_{1,3}$ respectively, and so on.</p> <p>Before entry with <i>uplo</i> = CblasLower, the array <i>ap</i> must contain the lower triangular part of the Hermitian matrix packed sequentially, row-by-row, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{2,1}$ and $A_{2,2}$ respectively, and so on.</p> <p>The imaginary parts of the diagonal elements need not be set and are assumed to be zero.</p> |

Output Parameters

| | |
|-----------|--|
| <i>ap</i> | <p>With <i>uplo</i> = CblasUpper, overwritten by the upper triangular part of the updated matrix.</p> <p>With <i>uplo</i> = CblasLower, overwritten by the lower triangular part of the updated matrix.</p> <p>The imaginary parts of the diagonal elements are set to zero.</p> |
|-----------|--|

cblas_?hpr2

Performs a rank-2 update of a Hermitian packed matrix.

Syntax

```
void cblas_chpr2 (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const void *alpha, const void *x, const MKL_INT incx, const void *y, const MKL_INT
incy, void *ap);
```

```
void cblas_zhpr2 (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const void *alpha, const void *x, const MKL_INT incx, const void *y, const MKL_INT
incy, void *ap);
```

Include Files

- mkl.h

Description

The ?hpr2 routines perform a matrix-vector operation defined as

$$A := \alpha * x * \text{conjg}(y') + \text{conjg}(\alpha) * y * \text{conjg}(x') + A,$$

where:

alpha is a scalar,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* Hermitian matrix, supplied in packed form.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the matrix <i>A</i> is supplied in the packed array <i>ap</i> . If <i>uplo</i> = CblasUpper, then the upper triangular part of the matrix <i>A</i> is supplied in the packed array <i>ap</i> . If <i>uplo</i> = CblasLower, then the low triangular part of the matrix <i>A</i> is supplied in the packed array <i>ap</i> . |
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>x</i> | Array, dimension at least $(1 + (n - 1) * \text{abs}(\text{incx}))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incy}))$. Before entry, the incremented array <i>y</i> must contain the <i>n</i> -element vector <i>y</i> . |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . The value of <i>incy</i> must not be zero. |
| <i>ap</i> | Array, size at least $((n * (n + 1)) / 2)$. For <i>Layout</i> = CblasColMajor: Before entry with <i>uplo</i> = CblasUpper, the array <i>ap</i> must contain the upper triangular part of the Hermitian matrix packed sequentially, column-by-column, so that <i>ap</i> [0] contains $A_{1,1}$, <i>ap</i> [1] and <i>ap</i> [2] contain $A_{1,2}$ and $A_{2,2}$ respectively, and so on. |

Before entry with `uplo = CblasLower`, the array `ap` must contain the lower triangular part of the Hermitian matrix packed sequentially, column-by-column, so that `ap[0]` contains $A_{1,1}$, `ap[1]` and `ap[2]` contain $A_{2,1}$ and $A_{3,1}$ respectively, and so on.

For `Layout = CblasRowMajor`:

Before entry with `uplo = CblasUpper`, the array `ap` must contain the upper triangular part of the Hermitian matrix packed sequentially, row-by-row, `ap[0]` contains $A_{1,1}$, `ap[1]` and `ap[2]` contain $A_{1,2}$ and $A_{1,3}$ respectively, and so on.

Before entry with `uplo = CblasLower`, the array `ap` must contain the lower triangular part of the Hermitian matrix packed sequentially, row-by-row, so that `ap[0]` contains $A_{1,1}$, `ap[1]` and `ap[2]` contain $A_{2,1}$ and $A_{2,2}$ respectively, and so on.

The imaginary parts of the diagonal elements need not be set and are assumed to be zero.

Output Parameters

`ap`

With `uplo = CblasUpper`, overwritten by the upper triangular part of the updated matrix.

With `uplo = CblasLower`, overwritten by the lower triangular part of the updated matrix.

The imaginary parts of the diagonal elements need are set to zero.

`cblas_?sbmv`

Computes a matrix-vector product using a symmetric band matrix.

Syntax

```
void cblas_ssbmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const MKL_INT k, const float alpha, const float *a, const MKL_INT lda, const float *x,
const MKL_INT incx, const float beta, float *y, const MKL_INT incy);
```

```
void cblas_dsbmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const MKL_INT k, const double alpha, const double *a, const MKL_INT lda, const double
*x, const MKL_INT incx, const double beta, double *y, const MKL_INT incy);
```

Include Files

- `mk1.h`

Description

The `?sbmv` routines perform a matrix-vector operation defined as

$$y := \alpha * A * x + \beta * y,$$

where:

α and β are scalars,

x and y are n -element vectors,

A is an n -by- n symmetric band matrix, with k super-diagonals.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the band matrix <i>A</i> is used: if <i>uplo</i> = CblasUpper - upper triangular part; if <i>uplo</i> = CblasLower - low triangular part. |
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>k</i> | Specifies the number of super-diagonals of the matrix <i>A</i> . The value of <i>k</i> must satisfy $0 \leq k$. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>a</i> | Array, size <i>lda</i> * <i>n</i> . Before entry with <i>uplo</i> = CblasUpper, the leading (<i>k</i> + 1) by <i>n</i> part of the array <i>a</i> must contain the upper triangular band part of the symmetric matrix, supplied column-by-column, with the leading diagonal of the matrix in row <i>k</i> of the array, the first super-diagonal starting at position 1 in row (<i>k</i> - 1), and so on. The top left <i>k</i> by <i>k</i> triangle of the array <i>a</i> is not referenced. |

The following program segment transfers the upper triangular part of a symmetric band matrix from conventional full matrix storage (*matrix*, with leading dimension *ldm*) to band storage (*a*, with leading dimension *lda*):

```
for (j = 0; j < n; j++) {
    m = k - j;
    for (i = max( 0, j - k); i <= j; i++) {
        a[(m+i) + j*lda] = matrix[i + j*ldm];
    }
}
```

Before entry with *uplo* = CblasLower, the leading (*k* + 1) by *n* part of the array *a* must contain the lower triangular band part of the symmetric matrix, supplied column-by-column, with the leading diagonal of the matrix in row 0 of the array, the first sub-diagonal starting at position 0 in row 1, and so on. The bottom right *k* by *k* triangle of the array *a* is not referenced.

The following program segment transfers the lower triangular part of a symmetric band matrix from conventional full matrix storage (*matrix*, with leading dimension *ldm*) to band storage (*a*, with leading dimension *lda*):

```
for (j = 0; j < n; j++) {
    m = -j;
    for (i = j; i < min(n, j + k + 1); i++) {
        a[(m+i) + j*lda] = matrix[i + j*ldm];
    }
}
```

Layout = CblasRowMajor:

Before entry with *uplo* = CblasUpper, the leading (*k* + 1)-by-*n* part of array *a* must contain the upper triangular band part of the symmetric matrix. The matrix must be supplied row-by-row, with the leading diagonal

of the matrix in column 0 of the array, the first super-diagonal starting at position 0 in column 1, and so on. The bottom right k -by- k triangle of array a is not referenced.

The following program segment transfers the upper triangular part of a symmetric band matrix from row-major full matrix storage ($matrix$ with leading dimension ldm) to row-major band storage (a , with leading dimension lda):

```
for (i = 0; i < n; i++) {
    m = -i;
    for (j = i; j < MIN(n, i+k+1); j++) {
        a[(m+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

Before entry with $uplo = CblasLower$, the leading $(k + 1)$ -by- n part of array a must contain the lower triangular band part of the symmetric matrix, supplied row-by-row, with the leading diagonal of the matrix in column k of the array, the first sub-diagonal starting at position 1 in column $k-1$, and so on. The top left k -by- k triangle of array a is not referenced.

The following program segment transfers the lower triangular part of a symmetric row-major band matrix from row-major full matrix storage ($matrix$, with leading dimension ldm) to row-major band storage (a , with leading dimension lda):

```
for (i = 0; i < n; i++) {
    m = k - i;
    for (j = max(0, i-k); j <= i; j++) {
        a[(m+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

| | |
|-------------|--|
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. The value of lda must be at least $(k + 1)$. |
| x | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incx}))$. Before entry, the incremented array x must contain the vector x . |
| <i>incx</i> | Specifies the increment for the elements of x . The value of <i>incx</i> must not be zero. |
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| y | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incy}))$. Before entry, the incremented array y must contain the vector y . |
| <i>incy</i> | Specifies the increment for the elements of y . The value of <i>incy</i> must not be zero. |

Output Parameters

| | |
|-----|---|
| y | Overwritten by the updated vector y . |
|-----|---|

cblas_?spmv

Computes a matrix-vector product using a symmetric packed matrix.

Syntax

```
void cblas_sspmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const float alpha, const float *ap, const float *x, const MKL_INT incx, const float
beta, float *y, const MKL_INT incy);
```

```
void cblas_dspmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const double alpha, const double *ap, const double *x, const MKL_INT incx, const double
beta, double *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

The ?spmv routines perform a matrix-vector operation defined as

$$y := \alpha * A * x + \beta * y,$$

where:

α and β are scalars,

x and y are n -element vectors,

A is an n -by- n symmetric matrix, supplied in packed form.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array ap . If $uplo = CblasUpper$, then the upper triangular part of the matrix A is supplied in the packed array ap . If $uplo = CblasLower$, then the low triangular part of the matrix A is supplied in the packed array ap . |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>ap</i> | Array, size at least $((n * (n + 1)) / 2)$. For $Layout = CblasColMajor$: Before entry with $uplo = CblasUpper$, the array ap must contain the upper triangular part of the symmetric matrix packed sequentially, column-by-column, so that $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{1,2}$ and $A_{2,2}$ respectively, and so on. Before entry with $uplo = CblasLower$, the array ap must contain the lower triangular part of the symmetric matrix packed sequentially, column-by-column, so that $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{2,1}$ and $A_{3,1}$ respectively, and so on. |

For *Layout* = CblasRowMajor:

Before entry with *uplo* = CblasUpper, the array *ap* must contain the upper triangular part of the symmetric matrix packed sequentially, row-by-row, *ap*[0] contains $A_{1,1}$, *ap*[1] and *ap*[2] contain $A_{1,2}$ and $A_{1,3}$ respectively, and so on. Before entry with *uplo* = CblasLower, the array *ap* must contain the lower triangular part of the symmetric matrix packed sequentially, row-by-row, so that *ap*[0] contains $A_{1,1}$, *ap*[1] and *ap*[2] contain $A_{2,1}$ and $A_{2,2}$ respectively, and so on.

| | |
|-------------|---|
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>beta</i> | Specifies the scalar <i>beta</i> . When <i>beta</i> is supplied as zero, then <i>y</i> need not be set on input. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array <i>y</i> must contain the <i>n</i> -element vector <i>y</i> . |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . The value of <i>incy</i> must not be zero. |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Overwritten by the updated vector <i>y</i> . |
|----------|--|

cblas_?spr

Performs a rank-1 update of a symmetric packed matrix.

Syntax

```
void cblas_sspr (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const float alpha, const float *x, const MKL_INT incx, float *ap);
```

```
void cblas_dspr (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const double alpha, const double *x, const MKL_INT incx, double *ap);
```

Include Files

- mkl.h

Description

The ?spr routines perform a matrix-vector operation defined as

$$a := \alpha * x * x' + A,$$

where:

alpha is a real scalar,

x is an *n*-element vector,

A is an *n*-by-*n* symmetric matrix, supplied in packed form.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | <p>Specifies whether the upper or lower triangular part of the matrix <i>A</i> is supplied in the packed array <i>ap</i>.</p> <p>If <i>uplo</i> = CblasUpper, then the upper triangular part of the matrix <i>A</i> is supplied in the packed array <i>ap</i>.</p> <p>If <i>uplo</i> = CblasLower, then the low triangular part of the matrix <i>A</i> is supplied in the packed array <i>ap</i>.</p> |
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incx}))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | <p>Specifies the increment for the elements of <i>x</i>.</p> <p>The value of <i>incx</i> must not be zero.</p> |
| <i>ap</i> | <p>For <i>Layout</i> = CblasColMajor:</p> <p>Before entry with <i>uplo</i> = CblasUpper, the array <i>ap</i> must contain the upper triangular part of the symmetric matrix packed sequentially, column-by-column, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{1,2}$ and $A_{2,2}$ respectively, and so on.</p> <p>Before entry with <i>uplo</i> = CblasLower, the array <i>ap</i> must contain the lower triangular part of the symmetric matrix packed sequentially, column-by-column, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{2,1}$ and $A_{3,1}$ respectively, and so on.</p> <p>For <i>Layout</i> = CblasRowMajor:</p> <p>Before entry with <i>uplo</i> = CblasUpper, the array <i>ap</i> must contain the upper triangular part of the symmetric matrix packed sequentially, row-by-row, <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{1,2}$ and $A_{1,3}$ respectively, and so on.</p> <p>Before entry with <i>uplo</i> = CblasLower, the array <i>ap</i> must contain the lower triangular part of the symmetric matrix packed sequentially, row-by-row, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{2,1}$ and $A_{2,2}$ respectively, and so on.</p> |

Output Parameters

| | |
|-----------|---|
| <i>ap</i> | <p>With <i>uplo</i> = CblasUpper, overwritten by the upper triangular part of the updated matrix.</p> <p>With <i>uplo</i> = CblasLower, overwritten by the lower triangular part of the updated matrix.</p> |
|-----------|---|

cblas_?spr2

Performs a rank-2 update of a symmetric packed matrix.

Syntax

```
void cblas_sspr2 (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const float alpha, const float *x, const MKL_INT incx, const float *y, const MKL_INT
incy, float *ap);
```

```
void cblas_dspr2 (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const double alpha, const double *x, const MKL_INT incx, const double *y, const MKL_INT
incy, double *ap);
```

Include Files

- mkl.h

Description

The ?spr2 routines perform a matrix-vector operation defined as

$$A := \alpha x x^T + \alpha y y^T + A,$$

where:

α is a scalar,

x and y are n -element vectors,

A is an n -by- n symmetric matrix, supplied in packed form.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the matrix A is supplied in the packed array ap . If $uplo = CblasUpper$, then the upper triangular part of the matrix A is supplied in the packed array ap . If $uplo = CblasLower$, then the low triangular part of the matrix A is supplied in the packed array ap . |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array x must contain the n -element vector x . |
| <i>incx</i> | Specifies the increment for the elements of x . The value of $incx$ must not be zero. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array y must contain the n -element vector y . |

| | |
|-------------|---|
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . The value of <i>incy</i> must not be zero. |
| <i>ap</i> | <p>For <i>Layout</i> = CblasColMajor:</p> <p>Before entry with <i>uplo</i> = CblasUpper, the array <i>ap</i> must contain the upper triangular part of the symmetric matrix packed sequentially, column-by-column, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{1,2}$ and $A_{2,2}$ respectively, and so on.</p> <p>Before entry with <i>uplo</i> = CblasLower, the array <i>ap</i> must contain the lower triangular part of the symmetric matrix packed sequentially, column-by-column, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{2,1}$ and $A_{3,1}$ respectively, and so on.</p> <p>For <i>Layout</i> = CblasRowMajor:</p> <p>Before entry with <i>uplo</i> = CblasUpper, the array <i>ap</i> must contain the upper triangular part of the symmetric matrix packed sequentially, row-by-row, <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{1,2}$ and $A_{1,3}$ respectively, and so on.</p> <p>Before entry with <i>uplo</i> = CblasLower, the array <i>ap</i> must contain the lower triangular part of the symmetric matrix packed sequentially, row-by-row, so that <i>ap</i>[0] contains $A_{1,1}$, <i>ap</i>[1] and <i>ap</i>[2] contain $A_{2,1}$ and $A_{2,2}$ respectively, and so on.</p> |

Output Parameters

| | |
|-----------|---|
| <i>ap</i> | <p>With <i>uplo</i> = CblasUpper, overwritten by the upper triangular part of the updated matrix.</p> <p>With <i>uplo</i> = CblasLower, overwritten by the lower triangular part of the updated matrix.</p> |
|-----------|---|

cblas_?symv

Computes a matrix-vector product for a symmetric matrix.

Syntax

```
void cblas_ssymv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const float alpha, const float *a, const MKL_INT lda, const float *x, const MKL_INT incx,
const float beta, float *y, const MKL_INT incy);
```

```
void cblas_dsymv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const double alpha, const double *a, const MKL_INT lda, const double *x, const MKL_INT incx,
const double beta, double *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

The ?symv routines perform a matrix-vector operation defined as

$$y := \alpha A x + \beta y,$$

where:

alpha and *beta* are scalars,
x and *y* are *n*-element vectors,
A is an *n*-by-*n* symmetric matrix.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array <i>a</i> is used. If <i>uplo</i> = CblasUpper, then the upper triangular part of the array <i>a</i> is used. If <i>uplo</i> = CblasLower, then the low triangular part of the array <i>a</i> is used. |
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>a</i> | Array, size <i>lda</i> * <i>n</i> . Before entry with <i>uplo</i> = CblasUpper, the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> must contain the upper triangular part of the symmetric matrix <i>A</i> and the strictly lower triangular part of <i>a</i> is not referenced. Before entry with <i>uplo</i> = CblasLower, the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> must contain the lower triangular part of the symmetric matrix <i>A</i> and the strictly upper triangular part of <i>a</i> is not referenced. |
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. The value of <i>lda</i> must be at least $\max(1, n)$. |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incx}))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>beta</i> | Specifies the scalar <i>beta</i> . When <i>beta</i> is supplied as zero, then <i>y</i> need not be set on input. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(\text{incy}))$. Before entry, the incremented array <i>y</i> must contain the <i>n</i> -element vector <i>y</i> . |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . The value of <i>incy</i> must not be zero. |

Output Parameters

y Overwritten by the updated vector *y*.

cblas_?syr

Performs a rank-1 update of a symmetric matrix.

Syntax

```
void cblas_ssyrr (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const float alpha, const float *x, const MKL_INT incx, float *a, const MKL_INT lda);
```

```
void cblas_dsyrr (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const double alpha, const double *x, const MKL_INT incx, double *a, const MKL_INT lda);
```

Include Files

- mkl.h

Description

The ?syrr routines perform a matrix-vector operation defined as

$$A := \alpha x x^T + A,$$

where:

alpha is a real scalar,

x is an *n*-element vector,

A is an *n*-by-*n* symmetric matrix.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array <i>a</i> is used. If <i>uplo</i> = CblasUpper, then the upper triangular part of the array <i>a</i> is used. If <i>uplo</i> = CblasLower, then the low triangular part of the array <i>a</i> is used. |
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>x</i> | Array, size at least $(1 + (n-1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>a</i> | Array, size <i>lda</i> * <i>n</i> . Before entry with <i>uplo</i> = CblasUpper, the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> must contain the upper triangular part of the symmetric matrix <i>A</i> and the strictly lower triangular part of <i>a</i> is not referenced. Before entry with <i>uplo</i> = CblasLower, the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> must contain the lower triangular part of the symmetric matrix <i>A</i> and the strictly upper triangular part of <i>a</i> is not referenced. |
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. The value of <i>lda</i> must be at least $\max(1, n)$. |

Output Parameters

a With *uplo* = CblasUpper, the upper triangular part of the array *a* is overwritten by the upper triangular part of the updated matrix.

With *uplo* = CblasLower, the lower triangular part of the array *a* is overwritten by the lower triangular part of the updated matrix.

cblas_?syr2

Performs a rank-2 update of symmetric matrix.

Syntax

```
void cblas_ssy2 (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const float alpha, const float *x, const MKL_INT incx, const float *y, const MKL_INT
incy, float *a, const MKL_INT lda);
```

```
void cblas_dsy2 (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const MKL_INT n,
const double alpha, const double *x, const MKL_INT incx, const double *y, const MKL_INT
incy, double *a, const MKL_INT lda);
```

Include Files

- mkl.h

Description

The ?syr2 routines perform a matrix-vector operation defined as

$$A := \alpha x x^T + \alpha y y^T + A,$$

where:

alpha is a scalar,

x and *y* are *n*-element vectors,

A is an *n*-by-*n* symmetric matrix.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array <i>a</i> is used. If <i>uplo</i> = CblasUpper, then the upper triangular part of the array <i>a</i> is used. If <i>uplo</i> = CblasLower, then the low triangular part of the array <i>a</i> is used. |
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . |

| | |
|-------------|---|
| | The value of <i>incx</i> must not be zero. |
| <i>y</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incy))$. Before entry, the incremented array <i>y</i> must contain the <i>n</i> -element vector <i>y</i> . |
| <i>incy</i> | Specifies the increment for the elements of <i>y</i> . The value of <i>incy</i> must not be zero. |
| <i>a</i> | Array, size <i>lda</i> * <i>n</i> . Before entry with <i>uplo</i> = CblasUpper, the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of <i>a</i> is not referenced. Before entry with <i>uplo</i> = CblasLower, the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of <i>a</i> is not referenced. |
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. The value of <i>lda</i> must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | With <i>uplo</i> = CblasUpper, the upper triangular part of the array <i>a</i> is overwritten by the upper triangular part of the updated matrix. With <i>uplo</i> = CblasLower, the lower triangular part of the array <i>a</i> is overwritten by the lower triangular part of the updated matrix. |
|----------|--|

cblas_?tbmv

Computes a matrix-vector product using a triangular band matrix.

Syntax

```
void cblas_stbmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const MKL_INT k, const
float *a, const MKL_INT lda, float *x, const MKL_INT incx);
```

```
void cblas_dtbmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const MKL_INT k, const
double *a, const MKL_INT lda, double *x, const MKL_INT incx);
```

```
void cblas_ctbmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const MKL_INT k, const
void *a, const MKL_INT lda, void *x, const MKL_INT incx);
```

```
void cblas_ztbmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const MKL_INT k, const
void *a, const MKL_INT lda, void *x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

The ?tbmv routines perform one of the matrix-vector operations defined as

$x := A*x$, or $x := A'*x$, or $x := \text{conjg}(A')*x$,

where:

x is an n -element vector,

A is an n -by- n unit, or non-unit, upper or lower triangular band matrix, with $(k + 1)$ diagonals.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the matrix A is an upper or lower triangular matrix: <i>uplo</i> = CblasUpper if <i>uplo</i> = CblasLower, then the matrix is low triangular. |
| <i>trans</i> | Specifies the operation: if <i>trans</i> =CblasNoTrans, then $x := A*x$; if <i>trans</i> =CblasTrans, then $x := A'*x$; if <i>trans</i> =CblasConjTrans, then $x := \text{conjg}(A')*x$. |
| <i>diag</i> | Specifies whether the matrix A is unit triangular: if <i>diag</i> = CblasUnit then the matrix is unit triangular; if <i>diag</i> = CblasNonUnit, then the matrix is not unit triangular. |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>k</i> | On entry with <i>uplo</i> = CblasUpper specifies the number of super-diagonals of the matrix A . On entry with <i>uplo</i> = CblasLower, k specifies the number of sub-diagonals of the matrix a . The value of k must satisfy $0 \leq k$. |
| <i>a</i> | Array, size $lda*n$. <i>Layout</i> = CblasColMajor: Before entry with <i>uplo</i> = CblasUpper, the leading $(k + 1)$ by n part of the array a must contain the upper triangular band part of the matrix of coefficients, supplied column-by-column, with the leading diagonal of the matrix in row k of the array, the first super-diagonal starting at position 1 in row $(k - 1)$, and so on. The top left k by k triangle of the array a is not referenced. The following program segment transfers an upper triangular band matrix from conventional full matrix storage (<i>matrix</i> , with leading dimension <i>ldm</i>) to band storage (a , with leading dimension <i>lda</i>): |

```

for (j = 0; j < n; j++) {
    m = k - j;
    for (i = max(0, j - k); i <= j; i++) {
        a[(m+i) + j*lda] = matrix[i + j*ldm];
    }
}

```

Before entry with `uplo = CblasLower`, the leading $(k + 1)$ by n part of the array `a` must contain the lower triangular band part of the matrix of coefficients, supplied column-by-column, with the leading diagonal of the matrix in row 0 of the array, the first sub-diagonal starting at position 0 in row 1, and so on. The bottom right k by k triangle of the array `a` is not referenced. The following program segment transfers a lower triangular band matrix from conventional full matrix storage (`matrix`, with leading dimension `ldm`) to band storage (`a`, with leading dimension `lda`):

```
for (j = 0; j < n; j++) {
    m = -j;
    for (i = j; i < min(n, j + k + 1); i++) {
        a[(m+i) + j*lda] = matrix[i + j*ldm];
    }
}
```

Note that when `diag = CblasUnit`, the elements of the array `a` corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity.

Layout = `CblasRowMajor`:

Before entry with `uplo = CblasUpper`, the leading $(k + 1)$ -by- n part of array `a` must contain the upper triangular band part of the matrix of coefficients. The matrix must be supplied row-by-row, with the leading diagonal of the matrix in column 0 of the array, the first super-diagonal starting at position 0 in column 1, and so on. The bottom right k -by- k triangle of array `a` is not referenced.

The following program segment transfers the upper triangular part of a Hermitian band matrix from row-major full matrix storage (`matrix` with leading dimension `ldm`) to row-major band storage (`a`, with leading dimension `lda`):

```
for (i = 0; i < n; i++) {
    m = -i;
    for (j = i; j < MIN(n, i+k+1); j++) {
        a[(m+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

Before entry with `uplo = CblasLower`, the leading $(k + 1)$ -by- n part of array `a` must contain the lower triangular band part of the matrix of coefficients, supplied row-by-row, with the leading diagonal of the matrix in column k of the array, the first sub-diagonal starting at position 1 in column $k-1$, and so on. The top left k -by- k triangle of array `a` is not referenced.

The following program segment transfers the lower triangular part of a Hermitian row-major band matrix from row-major full matrix storage (`matrix`, with leading dimension `ldm`) to row-major band storage (`a`, with leading dimension `lda`):

```
for (i = 0; i < n; i++) {
    m = k - i;
    for (j = max(0, i-k); j <= i; j++) {
        a[(m+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

| | |
|-------------|---|
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. The value of <i>lda</i> must be at least $(k + 1)$. |
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |

Output Parameters

| | |
|----------|--|
| <i>x</i> | Overwritten with the transformed vector <i>x</i> . |
|----------|--|

cblas_?tbsv

Solves a system of linear equations whose coefficients are in a triangular band matrix.

Syntax

```
void cblas_stbsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const MKL_INT k, const
float *a, const MKL_INT lda, float *x, const MKL_INT incx);
```

```
void cblas_dtbsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const MKL_INT k, const
double *a, const MKL_INT lda, double *x, const MKL_INT incx);
```

```
void cblas_ctbsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const MKL_INT k, const
void *a, const MKL_INT lda, void *x, const MKL_INT incx);
```

```
void cblas_ztbsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const MKL_INT k, const
void *a, const MKL_INT lda, void *x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

The ?tbsv routines solve one of the following systems of equations:

$A*x = b$, or $A'*x = b$, or $\text{conjg}(A')*x = b$,

where:

b and *x* are *n*-element vectors,

A is an *n*-by-*n* unit, or non-unit, upper or lower triangular band matrix, with $(k + 1)$ diagonals.

The routine does not test for singularity or near-singularity.

Such tests must be performed before calling this routine.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
|---------------|---|

| | |
|--------------|---|
| <i>uplo</i> | <p>Specifies whether the matrix <i>A</i> is an upper or lower triangular matrix:</p> <p>if <i>uplo</i> = <code>CblasUpper</code> the matrix is upper triangular;</p> <p>if <i>uplo</i> = <code>CblasLower</code>, the matrix is low triangular.</p> |
| <i>trans</i> | <p>Specifies the system of equations:</p> <p>if <i>trans</i>=<code>CblasNoTrans</code>, then $A*x = b$;</p> <p>if <i>trans</i>=<code>CblasTrans</code>, then $A'*x = b$;</p> <p>if <i>trans</i>=<code>CblasConjTrans</code>, then $\text{conjg}(A')*x = b$.</p> |
| <i>diag</i> | <p>Specifies whether the matrix <i>A</i> is unit triangular:</p> <p>if <i>diag</i> = <code>CblasUnit</code> then the matrix is unit triangular;</p> <p>if <i>diag</i> = <code>CblasNonUnit</code>, then the matrix is not unit triangular.</p> |
| <i>n</i> | Specifies the order of the matrix <i>A</i> . The value of <i>n</i> must be at least zero. |
| <i>k</i> | <p>On entry with <i>uplo</i> = <code>CblasUpper</code>, <i>k</i> specifies the number of super-diagonals of the matrix <i>A</i>. On entry with <i>uplo</i> = <code>CblasLower</code>, <i>k</i> specifies the number of sub-diagonals of the matrix <i>A</i>.</p> <p>The value of <i>k</i> must satisfy $0 \leq k$.</p> |
| <i>a</i> | <p>Array, size <i>lda</i>*<i>n</i>.</p> <p><i>Layout</i> = <code>CblasColMajor</code>:</p> <p>Before entry with <i>uplo</i> = <code>CblasUpper</code>, the leading (<i>k</i> + 1) by <i>n</i> part of the array <i>a</i> must contain the upper triangular band part of the matrix of coefficients, supplied column-by-column, with the leading diagonal of the matrix in row <i>k</i> of the array, the first super-diagonal starting at position 1 in row (<i>k</i> - 1), and so on. The top left <i>k</i> by <i>k</i> triangle of the array <i>a</i> is not referenced.</p> <p>The following program segment transfers an upper triangular band matrix from conventional full matrix storage (<i>matrix</i>, with leading dimension <i>ldm</i>) to band storage (<i>a</i>, with leading dimension <i>lda</i>):</p> <pre>for (j = 0; j < n; j++) { m = k - j; for (i = max(0, j - k); i <= j; i++) { a[(m+i) + j*lda] = matrix[i + j*ldm]; } }</pre> <p>Before entry with <i>uplo</i> = <code>CblasLower</code>, the leading (<i>k</i> + 1) by <i>n</i> part of the array <i>a</i> must contain the lower triangular band part of the matrix of coefficients, supplied column-by-column, with the leading diagonal of the matrix in row 0 of the array, the first sub-diagonal starting at position 0 in row 1, and so on. The bottom right <i>k</i> by <i>k</i> triangle of the array <i>a</i> is not referenced.</p> |

The following program segment transfers a lower triangular band matrix from conventional full matrix storage (*matrix*, with leading dimension *ldm*) to band storage (*a*, with leading dimension *lda*):

```
for (j = 0; j < n; j++) {
    m = -j;
    for (i = j; i < min(n, j + k + 1); i++) {
        a[(m+i) + j*lda] = matrix[i + j*ldm];
    }
}
```

When *diag* = *CblasUnit*, the elements of the array *a* corresponding to the diagonal elements of the matrix are not referenced, but are assumed to be unity.

Layout = *CblasRowMajor*:

Before entry with *uplo* = *CblasUpper*, the leading (*k* + 1)-by-*n* part of array *a* must contain the upper triangular band part of the matrix of coefficients. The matrix must be supplied row-by-row, with the leading diagonal of the matrix in column 0 of the array, the first super-diagonal starting at position 0 in column 1, and so on. The bottom right *k*-by-*k* triangle of array *a* is not referenced.

The following program segment transfers the upper triangular part of a Hermitian band matrix from row-major full matrix storage (*matrix* with leading dimension *ldm*) to row-major band storage (*a*, with leading dimension *lda*):

```
for (i = 0; i < n; i++) {
    m = -i;
    for (j = i; j < MIN(n, i+k+1); j++) {
        a[(m+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

Before entry with *uplo* = *CblasLower*, the leading (*k* + 1)-by-*n* part of array *a* must contain the lower triangular band part of the matrix of coefficients, supplied row-by-row, with the leading diagonal of the matrix in column *k* of the array, the first sub-diagonal starting at position 1 in column *k*-1, and so on. The top left *k*-by-*k* triangle of array *a* is not referenced.

The following program segment transfers the lower triangular part of a Hermitian row-major band matrix from row-major full matrix storage (*matrix*, with leading dimension *ldm*) to row-major band storage (*a*, with leading dimension *lda*):

```
for (i = 0; i < n; i++) {
    m = k - i;
    for (j = max(0, i-k); j <= i; j++) {
        a[(m+j) + i*lda] = matrix[j + i*ldm];
    }
}
```

lda

Specifies the leading dimension of *a* as declared in the calling (sub)program. The value of *lda* must be at least (*k* + 1).

x

Array, size at least $(1 + (n - 1) * \text{abs}(\text{incx}))$. Before entry, the incremented array *x* must contain the *n*-element right-hand side vector *b*.

incx Specifies the increment for the elements of *x*.
The value of *incx* must not be zero.

Output Parameters

x Overwritten with the solution vector *x*.

cblas_?tpmv

Computes a matrix-vector product using a triangular packed matrix.

Syntax

```
void cblas_stpmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const float *ap, float
*x, const MKL_INT incx);
```

```
void cblas_dtpmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const double *ap, double
*x, const MKL_INT incx);
```

```
void cblas_ctpmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const void *ap, void
*x, const MKL_INT incx);
```

```
void cblas_ztpmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const void *ap, void
*x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

The ?tpmv routines perform one of the matrix-vector operations defined as

$x := A*x$, or $x := A'*x$, or $x := \text{conjg}(A')*x$,

where:

x is an *n*-element vector,

A is an *n*-by-*n* unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

Input Parameters

Layout Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor).

uplo Specifies whether the matrix *A* is upper or lower triangular:
uplo = CblasUpper
if *uplo* = CblasLower, then the matrix is low triangular.

trans Specifies the operation:
if *trans*=CblasNoTrans, then $x := A*x$;

| | |
|-------------|---|
| | <p>if $trans=CblasTrans$, then $x := A'*x$;</p> <p>if $trans=CblasConjTrans$, then $x := conjg(A')*x$.</p> |
| <i>diag</i> | <p>Specifies whether the matrix A is unit triangular:</p> <p>if $diag = CblasUnit$ then the matrix is unit triangular;</p> <p>if $diag = CblasNonUnit$, then the matrix is not unit triangular.</p> |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>ap</i> | <p>Array, size at least $((n*(n + 1))/2)$.</p> <p>For $Layout = CblasColMajor$:</p> <p>Before entry with $uplo = CblasUpper$, the array ap must contain the upper triangular matrix packed sequentially, column-by-column, so that respectively, and so on. Before entry with $uplo = CblasLower$ $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{1,2}$ and $A_{2,2}$, the array ap must contain the lower triangular matrix packed sequentially, column-by-column, so that $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{2,1}$ and $A_{3,1}$ respectively, and so on. When $diag = CblasUnit$, the diagonal elements of a are not referenced, but are assumed to be unity.</p> <p>For $Layout = CblasRowMajor$:</p> <p>Before entry with $uplo = CblasUpper$, the array ap must contain the upper triangular matrix packed sequentially, row-by-row, $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{1,2}$ and $A_{1,3}$ respectively, and so on.</p> <p>Before entry with $uplo = CblasLower$, the array ap must contain the lower triangular matrix packed sequentially, row-by-row, so that $ap[0]$ contains $A_{1,1}$, $ap[1]$ and $ap[2]$ contain $A_{2,1}$ and $A_{2,2}$ respectively, and so on.</p> |
| <i>x</i> | Array, size at least $(1 + (n - 1)*abs(incx))$. Before entry, the incremented array x must contain the n -element vector x . |
| <i>incx</i> | <p>Specifies the increment for the elements of x.</p> <p>The value of $incx$ must not be zero.</p> |

Output Parameters

| | |
|----------|---|
| <i>x</i> | Overwritten with the transformed vector x . |
|----------|---|

cblas_?tpsv

Solves a system of linear equations whose coefficients are in a triangular packed matrix.

Syntax

```
void cblas_stpsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const float *ap, float
*x, const MKL_INT incx);
```

```
void cblas_dtpsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const double *ap, double
*x, const MKL_INT incx);
```

```
void cblas_ctpsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const void *ap, void
*x, const MKL_INT incx);
```

```
void cblas_ztpsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const void *ap, void
*x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

The ?tpsv routines solve one of the following systems of equations

$A*x = b$, or $A'*x = b$, or $\text{conjg}(A')*x = b$,

where:

b and x are n -element vectors,

A is an n -by- n unit, or non-unit, upper or lower triangular matrix, supplied in packed form.

This routine does not test for singularity or near-singularity.

Such tests must be performed before calling this routine.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the matrix A is upper or lower triangular: <i>uplo</i> = CblasUpper if <i>uplo</i> = CblasLower, then the matrix is low triangular. |
| <i>trans</i> | Specifies the system of equations: if <i>trans</i> =CblasNoTrans, then $A*x = b$; if <i>trans</i> =CblasTrans, then $A'*x = b$; if <i>trans</i> =CblasConjTrans, then $\text{conjg}(A')*x = b$. |
| <i>diag</i> | Specifies whether the matrix A is unit triangular: if <i>diag</i> = CblasUnit then the matrix is unit triangular; if <i>diag</i> = CblasNonUnit , then the matrix is not unit triangular. |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>ap</i> | Array, size at least $((n*(n + 1))/2)$. For <i>Layout</i> = CblasColMajor: Before entry with <i>uplo</i> = CblasUpper, the array <i>ap</i> must contain the upper triangular part of the triangular matrix packed sequentially, column-by-column, so that <i>ap</i> [0] contains $A_{1,1}$, <i>ap</i> [1] and <i>ap</i> [2] contain $A_{1,2}$ and $A_{2,2}$ respectively, and so on. |

Before entry with `uplo = CblasLower`, the array `ap` must contain the lower triangular part of the triangular matrix packed sequentially, column-by-column, so that `ap[0]` contains $A_{1,1}$, `ap[1]` and `ap[2]` contain $A_{2,1}$ and $A_{3,1}$ respectively, and so on.

For `Layout = CblasRowMajor`:

Before entry with `uplo = CblasUpper`, the array `ap` must contain the upper triangular part of the triangular matrix packed sequentially, row-by-row, `ap[0]` contains $A_{1,1}$, `ap[1]` and `ap[2]` contain $A_{1,2}$ and $A_{1,3}$ respectively, and so on. Before entry with `uplo = CblasLower`, the array `ap` must contain the lower triangular part of the triangular matrix packed sequentially, row-by-row, so that `ap[0]` contains $A_{1,1}$, `ap[1]` and `ap[2]` contain $A_{2,1}$ and $A_{2,2}$ respectively, and so on.

When `diag = CblasUnit`, the diagonal elements of `a` are not referenced, but are assumed to be unity.

`x` Array, size at least $(1 + (n - 1) * \text{abs}(\text{incx}))$. Before entry, the incremented array `x` must contain the n -element right-hand side vector `b`.

`incx` Specifies the increment for the elements of `x`.
The value of `incx` must not be zero.

Output Parameters

`x` Overwritten with the solution vector `x`.

`cblas_?trmv`

Computes a matrix-vector product using a triangular matrix.

Syntax

```
void cblas_strmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const float *a, const
MKL_INT lda, float *x, const MKL_INT incx);
```

```
void cblas_dtrmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const double *a, const
MKL_INT lda, double *x, const MKL_INT incx);
```

```
void cblas_ctrmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const void *a, const
MKL_INT lda, void *x, const MKL_INT incx);
```

```
void cblas_ztrmv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const void *a, const
MKL_INT lda, void *x, const MKL_INT incx);
```

Include Files

- `mkl.h`

Description

The `?trmv` routines perform one of the following matrix-vector operations defined as

$x := A^*x$, or $x := A'x$, or $x := \text{conjg}(A')x$,

where:

x is an n -element vector,

A is an n -by- n unit, or non-unit, upper or lower triangular matrix.

Input Parameters

| | |
|---------------|---|
| <i>layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the matrix A is upper or lower triangular: <i>uplo</i> = CblasUpper if <i>uplo</i> = CblasLower, then the matrix is low triangular. |
| <i>trans</i> | Specifies the operation: if <i>trans</i> =CblasNoTrans, then $x := A*x$; if <i>trans</i> =CblasTrans, then $x := A'*x$; if <i>trans</i> =CblasConjTrans, then $x := \text{conjg}(A')*x$. |
| <i>diag</i> | Specifies whether the matrix A is unit triangular: if <i>diag</i> = CblasUnit then the matrix is unit triangular; if <i>diag</i> = CblasNonUnit, then the matrix is not unit triangular. |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |
| <i>a</i> | Array, size $lda*n$. Before entry with <i>uplo</i> = CblasUpper, the leading n -by- n upper triangular part of the array a must contain the upper triangular matrix and the strictly lower triangular part of a is not referenced. Before entry with <i>uplo</i> = CblasLower, the leading n -by- n lower triangular part of the array a must contain the lower triangular matrix and the strictly upper triangular part of a is not referenced. When <i>diag</i> = CblasUnit, the diagonal elements of a are not referenced either, but are assumed to be unity. |
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. The value of <i>lda</i> must be at least $\max(1, n)$. |
| <i>x</i> | Array, size at least $(1 + (n - 1)*\text{abs}(\text{incx}))$. Before entry, the incremented array x must contain the n -element vector x . |
| <i>incx</i> | Specifies the increment for the elements of x . The value of <i>incx</i> must not be zero. |

Output Parameters

| | |
|----------|---|
| <i>x</i> | Overwritten with the transformed vector x . |
|----------|---|

cblas_?trsv

Solves a system of linear equations whose coefficients are in a triangular matrix.

Syntax

```
void cblas_strsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const float *a, const
MKL_INT lda, float *x, const MKL_INT incx);
```

```
void cblas_dtrsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const double *a, const
MKL_INT lda, double *x, const MKL_INT incx);
```

```
void cblas_ctrsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const void *a, const
MKL_INT lda, void *x, const MKL_INT incx);
```

```
void cblas_ztrsv (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const CBLAS_DIAG diag, const MKL_INT n, const void *a, const
MKL_INT lda, void *x, const MKL_INT incx);
```

Include Files

- mkl.h

Description

The `?trsv` routines solve one of the systems of equations:

$$A*x = b, \text{ or } A'*x = b, \text{ or } \text{conjg}(A')*x = b,$$

where:

b and x are n -element vectors,

A is an n -by- n unit, or non-unit, upper or lower triangular matrix.

The routine does not test for singularity or near-singularity.

Such tests must be performed before calling this routine.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the matrix A is upper or lower triangular: <i>uplo</i> = CblasUpper if <i>uplo</i> = CblasLower, then the matrix is low triangular. |
| <i>trans</i> | Specifies the systems of equations: if <i>trans</i> =CblasNoTrans, then $A*x = b$; if <i>trans</i> =CblasTrans, then $A'*x = b$; if <i>trans</i> =CblasConjTrans, then $\text{conjg}(A')*x = b$. |
| <i>diag</i> | Specifies whether the matrix A is unit triangular: if <i>diag</i> = CblasUnit then the matrix is unit triangular; if <i>diag</i> = CblasNonUnit, then the matrix is not unit triangular. |
| <i>n</i> | Specifies the order of the matrix A . The value of n must be at least zero. |

| | |
|-------------|---|
| <i>a</i> | Array, size $lda*n$. Before entry with $uplo = CblasUpper$, the leading n -by- n upper triangular part of the array a must contain the upper triangular matrix and the strictly lower triangular part of a is not referenced. Before entry with $uplo = CblasLower$, the leading n -by- n lower triangular part of the array a must contain the lower triangular matrix and the strictly upper triangular part of a is not referenced. When $diag = CblasUnit$, the diagonal elements of a are not referenced either, but are assumed to be unity. |
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. The value of lda must be at least $\max(1, n)$. |
| <i>x</i> | Array, size at least $(1 + (n - 1)*abs(incx))$. Before entry, the incremented array x must contain the n -element right-hand side vector b . |
| <i>incx</i> | Specifies the increment for the elements of x . The value of $incx$ must not be zero. |

Output Parameters

| | |
|----------|--|
| <i>x</i> | Overwritten with the solution vector x . |
|----------|--|

BLAS Level 3 Routines

BLAS Level 3 routines perform matrix-matrix operations. [Table "BLAS Level 3 Routine Groups and Their Data Types"](#) lists the BLAS Level 3 routine groups and the data types associated with them.

BLAS Level 3 Routine Groups and Their Data Types

| Routine Group | Data Types | Description |
|---------------------------|------------|--|
| <code>cblas_?gemm</code> | s, d, c, z | Computes a matrix-matrix product with general matrices. |
| <code>cblas_?hemm</code> | c, z | Computes a matrix-matrix product where one input matrix is Hermitian. |
| <code>cblas_?herk</code> | c, z | Performs a Hermitian rank-k update. |
| <code>cblas_?her2k</code> | c, z | Performs a Hermitian rank-2k update. |
| <code>cblas_?symm</code> | s, d, c, z | Computes a matrix-matrix product where one input matrix is symmetric. |
| <code>cblas_?syrk</code> | s, d, c, z | Performs a symmetric rank-k update. |
| <code>cblas_?syr2k</code> | s, d, c, z | Performs a symmetric rank-2k update. |
| <code>cblas_?trmm</code> | s, d, c, z | Computes a matrix-matrix product where one input matrix is triangular. |
| <code>cblas_?trsm</code> | s, d, c, z | Solves a triangular matrix equation. |

Symmetric Multiprocessing Version of Intel® MKL

Many applications spend considerable time executing BLAS routines. This time can be scaled by the number of processors available on the system through using the symmetric multiprocessing (SMP) feature built into the Intel MKL Library. The performance enhancements based on the parallel use of the processors are available without any programming effort on your part.

To enhance performance, the library uses the following methods:

- The BLAS functions are blocked where possible to restructure the code in a way that increases the localization of data reference, enhances cache memory use, and reduces the dependency on the memory bus.
- The code is distributed across the processors to maximize parallelism.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

cblas_?gemm

Computes a matrix-matrix product with general matrices.

Syntax

```
void cblas_sgemm (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE transa, const
CBLAS_TRANSPOSE transb, const MKL_INT m, const MKL_INT n, const MKL_INT k, const float
alpha, const float *a, const MKL_INT lda, const float *b, const MKL_INT ldb, const
float beta, float *c, const MKL_INT ldc);

void cblas_dgemm (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE transa, const
CBLAS_TRANSPOSE transb, const MKL_INT m, const MKL_INT n, const MKL_INT k, const double
alpha, const double *a, const MKL_INT lda, const double *b, const MKL_INT ldb, const
double beta, double *c, const MKL_INT ldc);

void cblas_cgemm (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE transa, const
CBLAS_TRANSPOSE transb, const MKL_INT m, const MKL_INT n, const MKL_INT k, const void
*alpha, const void *a, const MKL_INT lda, const void *b, const MKL_INT ldb, const void
*beta, void *c, const MKL_INT ldc);

void cblas_zgemm (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE transa, const
CBLAS_TRANSPOSE transb, const MKL_INT m, const MKL_INT n, const MKL_INT k, const void
*alpha, const void *a, const MKL_INT lda, const void *b, const MKL_INT ldb, const void
*beta, void *c, const MKL_INT ldc);
```

Include Files

- mkl.h

Description

The ?gemm routines compute a scalar-matrix-matrix product and add the result to a scalar-matrix product, with general matrices. The operation is defined as

$$C := \alpha * \text{op}(A) * \text{op}(B) + \beta * C,$$

where:

op(X) is one of op(X) = X, or op(X) = X^T, or op(X) = X^H,

alpha and beta are scalars,

A, B and C are matrices:

$\text{op}(A)$ is an m -by- k matrix,

$\text{op}(B)$ is a k -by- n matrix,

C is an m -by- n matrix.

See also

- `?gemm` for the Fortran language interface to this routine
- `?gemm3m`, BLAS-like extension routines, that use matrix multiplication for similar matrix-matrix operations

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>transa</i> | Specifies the form of $\text{op}(A)$ used in the matrix multiplication: if <i>transa</i> =CblasNoTrans, then $\text{op}(A) = A$; if <i>transa</i> =CblasTrans, then $\text{op}(A) = A^T$; if <i>transa</i> =CblasConjTrans, then $\text{op}(A) = A^H$. |
| <i>transb</i> | Specifies the form of $\text{op}(B)$ used in the matrix multiplication: if <i>transb</i> =CblasNoTrans, then $\text{op}(B) = B$; if <i>transb</i> =CblasTrans, then $\text{op}(B) = B^T$; if <i>transb</i> =CblasConjTrans, then $\text{op}(B) = B^H$. |
| <i>m</i> | Specifies the number of rows of the matrix $\text{op}(A)$ and of the matrix C . The value of <i>m</i> must be at least zero. |
| <i>n</i> | Specifies the number of columns of the matrix $\text{op}(B)$ and the number of columns of the matrix C . The value of <i>n</i> must be at least zero. |
| <i>k</i> | Specifies the number of columns of the matrix $\text{op}(A)$ and the number of rows of the matrix $\text{op}(B)$. The value of <i>k</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |

| | | |
|----------------------------------|---|---|
| <i>a</i> | <i>transa</i> =CblasNoTrans | <i>transa</i> =CblasTrans or <i>transa</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | Array, size $lda*k$. Before entry, the leading m -by- k part of the array <i>a</i> must contain the matrix <i>A</i> . | Array, size $lda*m$. Before entry, the leading k -by- m part of the array <i>a</i> must contain the matrix <i>A</i> . |
| <i>Layout</i> = CblasRowMajor | Array, size $lda*m$. | Array, size $lda*k$. Before entry, the leading m -by- k part of the array <i>a</i> must contain the matrix <i>A</i> . |

Before entry, the leading k -by- m part of the array a must contain the matrix A .

lda

Specifies the leading dimension of a as declared in the calling (sub)program.

| | | |
|----------------------------------|---|---|
| | <i>transa</i> =CblasNoTrans | <i>transa</i> =CblasTrans or <i>transa</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | <i>lda</i> must be at least $\max(1, m)$. | <i>lda</i> must be at least $\max(1, k)$ |
| <i>Layout</i> = CblasRowMajor | <i>lda</i> must be at least $\max(1, k)$ | <i>lda</i> must be at least $\max(1, m)$. |

b

| | | |
|----------------------------------|---|--|
| | <i>transb</i> =CblasNoTrans | <i>transb</i> =CblasTrans or <i>transb</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | Array, size <i>ldb</i> by n . Before entry, the leading k -by- n part of the array b must contain the matrix B . | Array, size <i>ldb</i> by k . Before entry the leading n -by- k part of the array b must contain the matrix B . |
| <i>Layout</i> = CblasRowMajor | Array, size <i>ldb</i> by k . Before entry the leading n -by- k part of the array b must contain the matrix B . | Array, size <i>ldb</i> by n . Before entry, the leading k -by- n part of the array b must contain the matrix B . |

ldb

Specifies the leading dimension of b as declared in the calling (sub)program.

| | | |
|----------------------------------|---|---|
| | <i>transb</i> =CblasNoTrans | <i>transb</i> =CblasTrans or <i>transb</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | <i>ldb</i> must be at least $\max(1, k)$. | <i>ldb</i> must be at least $\max(1, n)$. |
| <i>Layout</i> = CblasRowMajor | <i>ldb</i> must be at least $\max(1, n)$. | <i>ldb</i> must be at least $\max(1, k)$. |

beta

Specifies the scalar *beta*.

When *beta* is equal to zero, then c need not be set on input.

c

| | |
|--|---|
| <p><i>Layout</i> = CblasColMajor</p> | <p>Array, size <i>ldc</i> by <i>n</i>. Before entry, the leading <i>m</i>-by-<i>n</i> part of the array <i>c</i> must contain the matrix <i>C</i>, except when <i>beta</i> is equal to zero, in which case <i>c</i> need not be set on entry.</p> |
| <p><i>Layout</i> = CblasRowMajor</p> | <p>Array, size <i>ldc</i> by <i>m</i>. Before entry, the leading <i>n</i>-by-<i>m</i> part of the array <i>c</i> must contain the matrix <i>C</i>, except when <i>beta</i> is equal to zero, in which case <i>c</i> need not be set on entry.</p> |

ldc

Specifies the leading dimension of *c* as declared in the calling (sub)program.

| | |
|-------------------------------|--|
| <i>Layout</i> = CblasColMajor | <i>ldc</i> must be at least $\max(1, m)$. |
| <i>Layout</i> = CblasRowMajor | <i>ldc</i> must be at least $\max(1, n)$. |

Output Parameters

c

Overwritten by the *m*-by-*n* matrix $(\alpha * \text{op}(A) * \text{op}(B) + \beta * C)$.

Example

For examples of routine usage, see the code in the Intel MKL installation directory:

- cblas_sgemm: examples\cblas\source\cblas_sgemmx.c
- cblas_dgemm: examples\cblas\source\cblas_dgemmx.c
- cblas_cgemm: examples\cblas\source\cblas_cgemmx.c
- cblas_zgemm: examples\cblas\source\cblas_zgemmx.c

cblas_?hemm

Computes a matrix-matrix product where one input matrix is Hermitian.

Syntax

```
void cblas_chemm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const MKL_INT m, const MKL_INT n, const void *alpha, const void *a, const
MKL_INT lda, const void *b, const MKL_INT ldb, const void *beta, void *c, const
MKL_INT ldc);
```

```
void cblas_zhemm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const MKL_INT m, const MKL_INT n, const void *alpha, const void *a, const
MKL_INT lda, const void *b, const MKL_INT ldb, const void *beta, void *c, const
MKL_INT ldc);
```

Include Files

- mkl.h

Description

The `?hemm` routines compute a scalar-matrix-matrix product using a Hermitian matrix A and a general matrix B and add the result to a scalar-matrix product using a general matrix C . The operation is defined as

```
 $C := \alpha * A * B + \beta * C$ 
```

or

```
 $C := \alpha * B * A + \beta * C,$ 
```

where:

α and β are scalars,

A is a Hermitian matrix,

B and C are m -by- n matrices.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>side</i> | Specifies whether the Hermitian matrix A appears on the left or right in the operation as follows: if $side = \text{CblasLeft}$, then $C := \alpha * A * B + \beta * C$; if $side = \text{CblasRight}$, then $C := \alpha * B * A + \beta * C$. |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the Hermitian matrix A is used: If $uplo = \text{CblasUpper}$, then the upper triangular part of the Hermitian matrix A is used. If $uplo = \text{CblasLower}$, then the low triangular part of the Hermitian matrix A is used. |
| <i>m</i> | Specifies the number of rows of the matrix C . The value of m must be at least zero. |
| <i>n</i> | Specifies the number of columns of the matrix C . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>a</i> | Array, size $lda * ka$, where ka is m when $side = \text{CblasLeft}$ and is n otherwise. Before entry with $side = \text{CblasLeft}$, the m -by- m part of the array a must contain the Hermitian matrix, such that when $uplo = \text{CblasUpper}$, the leading m -by- m upper triangular part of the array a must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of a is not referenced, and when $uplo = \text{CblasLower}$, the leading m -by- m lower triangular part of the array a must contain the lower triangular part of the Hermitian matrix, and the strictly upper triangular part of a is not referenced. Before entry with $side = \text{CblasRight}$, the n -by- n part of the array a must contain the Hermitian matrix, such that when $uplo = \text{CblasUpper}$, the leading n -by- n upper triangular part of the array a must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part |

of a is not referenced, and when $uplo = CblasLower$, the leading n -by- n lower triangular part of the array a must contain the lower triangular part of the Hermitian matrix, and the strictly upper triangular part of a is not referenced. The imaginary parts of the diagonal elements need not be set, they are assumed to be zero.

| | |
|-------------|--|
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub) program. When $side = CblasLeft$ then lda must be at least $\max(1, m)$, otherwise lda must be at least $\max(1, n)$. |
| <i>b</i> | For $Layout = CblasColMajor$: array, size $ldb*n$. The leading m -by- n part of the array b must contain the matrix B . For $Layout = CblasRowMajor$: array, size $ldb*m$. The leading n -by- m part of the array b must contain the matrix B . |
| <i>ldb</i> | Specifies the leading dimension of b as declared in the calling (sub)program. When $Layout = CblasColMajor$, ldb must be at least $\max(1, m)$; otherwise, ldb must be at least $\max(1, n)$. |
| <i>beta</i> | Specifies the scalar $beta$. When $beta$ is supplied as zero, then c need not be set on input. |
| <i>c</i> | For $Layout = CblasColMajor$: array, size $ldc*n$. Before entry, the leading m -by- n part of the array c must contain the matrix C , except when $beta$ is zero, in which case c need not be set on entry. For $Layout = CblasRowMajor$: array, size $ldc*m$. Before entry, the leading n -by- m part of the array c must contain the matrix C , except when $beta$ is zero, in which case c need not be set on entry. |
| <i>ldc</i> | Specifies the leading dimension of c as declared in the calling (sub)program. When $Layout = CblasColMajor$, ldc must be at least $\max(1, m)$; otherwise, ldc must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | Overwritten by the m -by- n updated matrix. |
|----------|---|

cblas_?herk

Performs a Hermitian rank- k update.

Syntax

```
void cblas_cherk (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const float alpha, const void
*a, const MKL_INT lda, const float beta, void *c, const MKL_INT ldc);

void cblas_zherk (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const double alpha, const void
*a, const MKL_INT lda, const double beta, void *c, const MKL_INT ldc);
```

Include Files

- mkl.h

Description

The `?herk` routines perform a rank- k matrix-matrix operation using a general matrix A and a Hermitian matrix C . The operation is defined as

$$C := \alpha * A * A^H + \beta * C,$$

or

$$C := \alpha * A^H * A + \beta * C,$$

where:

α and β are real scalars,

C is an n -by- n Hermitian matrix,

A is an n -by- k matrix in the first case and a k -by- n matrix in the second case.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array c is used. If $uplo = CblasUpper$, then the upper triangular part of the array c is used. If $uplo = CblasLower$, then the low triangular part of the array c is used. |
| <i>trans</i> | Specifies the operation: if $trans=CblasNoTrans$, then $C := \alpha * A * A^H + \beta * C$; if $trans=CblasConjTrans$, then $C := \alpha * A^H * A + \beta * C$. |
| <i>n</i> | Specifies the order of the matrix C . The value of n must be at least zero. |
| <i>k</i> | With $trans=CblasNoTrans$, k specifies the number of columns of the matrix A , and with $trans=CblasConjTrans$, k specifies the number of rows of the matrix A . The value of k must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |

a

| | $trans=CblasNoTrans$ | $trans=CblasConjTrans$ |
|-------------------------------|---|---|
| <i>Layout = CblasColMajor</i> | Array, size $lda * k$. Before entry, the leading n -by- k part of the array a must contain the matrix A . | Array, size $lda * n$. Before entry, the leading k -by- n part of the array a must contain the matrix A . |
| <i>Layout = CblasRowMajor</i> | Array, size $lda * n$. Before entry, the leading k -by- n part of the array a must contain the matrix A . | Array, size $lda * k$. Before entry, the leading n -by- k part of the array a must contain the matrix A . |

lda

| | <i>trans</i> =CblasNoTrans | <i>trans</i> =CblasConjTrans |
|-------------------------------|--|--|
| <i>Layout</i> = CblasColMajor | <i>lda</i> must be at least $\max(1, n)$. | <i>lda</i> must be at least $\max(1, k)$ |
| <i>Layout</i> = CblasRowMajor | <i>lda</i> must be at least $\max(1, k)$ | <i>lda</i> must be at least $\max(1, n)$. |

beta

Specifies the scalar *beta*.

c

Array, size *ldc* by *n*.

Before entry with *uplo* = CblasUpper, the leading *n*-by-*n* upper triangular part of the array *c* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *c* is not referenced.

Before entry with *uplo* = CblasLower, the leading *n*-by-*n* lower triangular part of the array *c* must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of *c* is not referenced.

The imaginary parts of the diagonal elements need not be set, they are assumed to be zero.

ldc

Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least $\max(1, n)$.

Output Parameters

c

With *uplo* = CblasUpper, the upper triangular part of the array *c* is overwritten by the upper triangular part of the updated matrix.

With *uplo* = CblasLower, the lower triangular part of the array *c* is overwritten by the lower triangular part of the updated matrix.

The imaginary parts of the diagonal elements are set to zero.

cblas_?her2k

Performs a Hermitian rank-2k update.

Syntax

```
void cblas_cher2k (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const void *alpha, const void
*a, const MKL_INT lda, const void *b, const MKL_INT ldb, const float beta, void *c,
const MKL_INT ldc);
```

```
void cblas_zher2k (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const void *alpha, const void
*a, const MKL_INT lda, const void *b, const MKL_INT ldb, const double beta, void *c,
const MKL_INT ldc);
```

Include Files

- mkl.h

Description

The `zher2k` routines perform a rank-2k matrix-matrix operation using general matrices A and B and a Hermitian matrix C . The operation is defined as

$$C := \alpha * A * B^H + \text{conjg}(\alpha) * B * A^H + \beta * C,$$

or

$$C := \alpha * A^H * B + \text{conjg}(\alpha) * B^H * A + \beta * C,$$

where:

α is a scalar and β is a real scalar,

C is an n -by- n Hermitian matrix,

A and B are n -by- k matrices in the first case and k -by- n matrices in the second case.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array c is used. If $uplo = \text{CblasUpper}$, then the upper triangular of the array c is used. If $uplo = \text{CblasLower}$, then the low triangular of the array c is used. |
| <i>trans</i> | Specifies the operation: if $trans = \text{CblasNoTrans}$, then $C := \alpha * A * B^H + \alpha * B * A^H + \beta * C$; if $trans = \text{CblasConjTrans}$, then $C := \alpha * A^H * B + \alpha * B^H * A + \beta * C$. |
| <i>n</i> | Specifies the order of the matrix C . The value of n must be at least zero. |
| <i>k</i> | With $trans = \text{CblasNoTrans}$ specifies the number of columns of the matrix A , and with $trans = \text{CblasConjTrans}$, k specifies the number of rows of the matrix A . The value of k must be at least equal to zero. |
| <i>alpha</i> | Specifies the scalar α . |

a

| | $trans = \text{CblasNoTrans}$ | $trans = \text{CblasConjTrans}$ |
|---------------------------------|---|---|
| $Layout = \text{CblasColMajor}$ | Array, size $lda * k$. Before entry, the leading n -by- k part of the array a must contain the matrix A . | Array, size $lda * n$. Before entry, the leading k -by- n part of the array a must contain the matrix A . |
| $Layout = \text{CblasRowMajor}$ | Array, size $lda * n$. Before entry, the leading k -by- n part of the array a must contain the matrix A . | Array, size $lda * k$. Before entry, the leading n -by- k part of the array a must contain the matrix A . |

lda

Specifies the leading dimension of *a* as declared in the calling (sub)program.

| | <i>trans</i> =CblasNoTrans | <i>trans</i> =CblasConjTrans |
|-------------------------------|--|--|
| <i>Layout</i> = CblasColMajor | <i>lda</i> must be at least $\max(1, n)$. | <i>lda</i> must be at least $\max(1, k)$ |
| <i>Layout</i> = CblasRowMajor | <i>lda</i> must be at least $\max(1, k)$ | <i>lda</i> must be at least $\max(1, n)$. |

beta

Specifies the scalar *beta*.

b

| | <i>trans</i> =CblasNoTrans | <i>trans</i> =CblasConjTrans |
|-------------------------------|---|---|
| <i>Layout</i> = CblasColMajor | Array, size <i>ldb</i> * <i>k</i> . Before entry, the leading <i>n</i> -by- <i>k</i> part of the array <i>b</i> must contain the matrix <i>B</i> . | Array, size <i>ldb</i> * <i>n</i> . Before entry, the leading <i>k</i> -by- <i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i> . |
| <i>Layout</i> = CblasRowMajor | Array, size <i>lda</i> * <i>n</i> . Before entry, the leading <i>k</i> -by- <i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i> . | Array, size <i>lda</i> * <i>k</i> . Before entry, the leading <i>n</i> -by- <i>k</i> part of the array <i>b</i> must contain the matrix <i>B</i> . |

ldb

Specifies the leading dimension of *a* as declared in the calling (sub)program.

| | <i>trans</i> =CblasNoTrans | <i>trans</i> =CblasConjTrans |
|-------------------------------|--|--|
| <i>Layout</i> = CblasColMajor | <i>ldb</i> must be at least $\max(1, n)$. | <i>ldb</i> must be at least $\max(1, k)$ |
| <i>Layout</i> = CblasRowMajor | <i>ldb</i> must be at least $\max(1, k)$ | <i>ldb</i> must be at least $\max(1, n)$. |

c

Array, size *ldc* by *n*.

Before entry with *uplo* = CblasUpper, the leading *n*-by-*n* upper triangular part of the array *c* must contain the upper triangular part of the Hermitian matrix and the strictly lower triangular part of *c* is not referenced.

Before entry with *uplo* = CblasLower, the leading *n*-by-*n* lower triangular part of the array *c* must contain the lower triangular part of the Hermitian matrix and the strictly upper triangular part of *c* is not referenced.

The imaginary parts of the diagonal elements need not be set, they are assumed to be zero.

ldc

Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least $\max(1, n)$.

Output Parameters

c With *uplo* = CblasUpper, the upper triangular part of the array *c* is overwritten by the upper triangular part of the updated matrix.

With *uplo* = CblasLower, the lower triangular part of the array *c* is overwritten by the lower triangular part of the updated matrix.

The imaginary parts of the diagonal elements are set to zero.

cblas_?symm

Computes a matrix-matrix product where one input matrix is symmetric.

Syntax

```
void cblas_ssymm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO uplo, const MKL_INT m, const MKL_INT n, const float alpha, const float *a, const MKL_INT lda, const float *b, const MKL_INT ldb, const float beta, float *c, const MKL_INT ldc);
```

```
void cblas_dsymm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO uplo, const MKL_INT m, const MKL_INT n, const double alpha, const double *a, const MKL_INT lda, const double *b, const MKL_INT ldb, const double beta, double *c, const MKL_INT ldc);
```

```
void cblas_csymm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO uplo, const MKL_INT m, const MKL_INT n, const void *alpha, const void *a, const MKL_INT lda, const void *b, const MKL_INT ldb, const void *beta, void *c, const MKL_INT ldc);
```

```
void cblas_zsymm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO uplo, const MKL_INT m, const MKL_INT n, const void *alpha, const void *a, const MKL_INT lda, const void *b, const MKL_INT ldb, const void *beta, void *c, const MKL_INT ldc);
```

Include Files

- mkl.h

Description

The ?symm routines compute a scalar-matrix-matrix product with one symmetric matrix and add the result to a scalar-matrix product. The operation is defined as

$$C := \alpha A * B + \beta A * C,$$

or

$$C := \alpha B * A + \beta B * C,$$

where:

alpha and *beta* are scalars,

A is a symmetric matrix,

B and *C* are *m*-by-*n* matrices.

Input Parameters

| | |
|---------------|--|
| <i>layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>side</i> | Specifies whether the symmetric matrix <i>A</i> appears on the left or right in the operation: if <i>side</i> = CblasLeft, then $C := \alpha * A * B + \beta * C$; if <i>side</i> = CblasRight, then $C := \alpha * B * A + \beta * C$. |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the symmetric matrix <i>A</i> is used: if <i>uplo</i> = CblasUpper, then the upper triangular part is used; if <i>uplo</i> = CblasLower, then the lower triangular part is used. |
| <i>m</i> | Specifies the number of rows of the matrix <i>C</i> . The value of <i>m</i> must be at least zero. |
| <i>n</i> | Specifies the number of columns of the matrix <i>C</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>a</i> | Array, size <i>lda</i> * <i>ka</i> , where <i>ka</i> is <i>m</i> when <i>side</i> = CblasLeft and is <i>n</i> otherwise. Before entry with <i>side</i> = CblasLeft, the <i>m</i> -by- <i>m</i> part of the array <i>a</i> must contain the symmetric matrix, such that when <i>uplo</i> = CblasUpper, the leading <i>m</i> -by- <i>m</i> upper triangular part of the array <i>a</i> must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of <i>a</i> is not referenced, and when <i>side</i> = CblasLeft, the leading <i>m</i> -by- <i>m</i> lower triangular part of the array <i>a</i> must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of <i>a</i> is not referenced. Before entry with <i>side</i> = CblasRight, the <i>n</i> -by- <i>n</i> part of the array <i>a</i> must contain the symmetric matrix, such that when <i>uplo</i> = CblasUpper array <i>a</i> must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of <i>a</i> is not referenced, and when <i>side</i> = CblasLeft, the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of <i>a</i> is not referenced. |
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. When <i>side</i> = CblasLeft then <i>lda</i> must be at least $\max(1, m)$, otherwise <i>lda</i> must be at least $\max(1, n)$. |
| <i>b</i> | For <i>Layout</i> = CblasColMajor: array, size <i>ldb</i> * <i>n</i> . The leading <i>m</i> -by- <i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i> . For <i>Layout</i> = CblasRowMajor: array, size <i>ldb</i> * <i>m</i> . The leading <i>n</i> -by- <i>m</i> part of the array <i>b</i> must contain the matrix <i>B</i> |

| | |
|-------------|--|
| <i>ldb</i> | Specifies the leading dimension of <i>b</i> as declared in the calling (sub)program. When <i>Layout</i> = CblasColMajor, <i>ldb</i> must be at least $\max(1, m)$; otherwise, <i>ldb</i> must be at least $\max(1, n)$. |
| <i>beta</i> | Specifies the scalar <i>beta</i> . When <i>beta</i> is set to zero, then <i>c</i> need not be set on input. |
| <i>c</i> | For <i>Layout</i> = CblasColMajor: array, size <i>ldc</i> * <i>n</i> . Before entry, the leading <i>m</i> -by- <i>n</i> part of the array <i>c</i> must contain the matrix <i>C</i> , except when <i>beta</i> is zero, in which case <i>c</i> need not be set on entry. For <i>Layout</i> = CblasRowMajor: array, size <i>ldc</i> * <i>m</i> . Before entry, the leading <i>n</i> -by- <i>m</i> part of the array <i>c</i> must contain the matrix <i>C</i> , except when <i>beta</i> is zero, in which case <i>c</i> need not be set on entry. |
| <i>ldc</i> | Specifies the leading dimension of <i>c</i> as declared in the calling (sub)program. When <i>Layout</i> = CblasColMajor, <i>ldc</i> must be at least $\max(1, m)$; otherwise, <i>ldc</i> must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | Overwritten by the <i>m</i> -by- <i>n</i> updated matrix. |
|----------|---|

cblas_?syrk

Performs a symmetric rank-k update.

Syntax

```
void cblas_ssyrc (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const float alpha, const float
*a, const MKL_INT lda, const float beta, float *c, const MKL_INT ldc);

void cblas_dsyrk (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const double alpha, const
double *a, const MKL_INT lda, const double beta, double *c, const MKL_INT ldc);

void cblas_csyrk (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const void *alpha, const void
*a, const MKL_INT lda, const void *beta, void *c, const MKL_INT ldc);

void cblas_zsyrk (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const void *alpha, const void
*a, const MKL_INT lda, const void *beta, void *c, const MKL_INT ldc);
```

Include Files

- mkl.h

Description

The ?syrk routines perform a rank-k matrix-matrix operation for a symmetric matrix *C* using a general matrix *A*. The operation is defined as

$$C := \alpha * A * A' + \beta * C,$$

or

$$C := \alpha * A' * A + \beta * C,$$

where:

α and β are scalars,

C is an n -by- n symmetric matrix,

A is an n -by- k matrix in the first case and a k -by- n matrix in the second case.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array c is used. If $uplo = CblasUpper$, then the upper triangular part of the array c is used. If $uplo = CblasLower$, then the low triangular part of the array c is used. |
| <i>trans</i> | Specifies the operation: if $trans=CblasNoTrans$, then $C := \alpha * A * A' + \beta * C$; if $trans=CblasTrans$, then $C := \alpha * A' * A + \beta * C$; if $trans=CblasConjTrans$, then $C := \alpha * A' * A + \beta * C$. |
| <i>n</i> | Specifies the order of the matrix C . The value of n must be at least zero. |
| <i>k</i> | On entry with $trans=CblasNoTrans$, k specifies the number of columns of the matrix a , and on entry with $trans=CblasTrans$ or $trans=CblasConjTrans$, k specifies the number of rows of the matrix a . The value of k must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |
| <i>a</i> | Array, size $lda * ka$, where ka is k when $trans=CblasNoTrans$, and is n otherwise. Before entry with $trans=CblasNoTrans$, the leading n -by- k part of the array a must contain the matrix A , otherwise the leading k -by- n part of the array a must contain the matrix A . |

| | | |
|-------------------------------|---|---|
| | $trans=CblasNoTrans$ | $trans=CblasConjTrans$ |
| <i>Layout = CblasColMajor</i> | Array, size $lda * k$. Before entry, the leading n -by- k part of the array a must contain the matrix A . | Array, size $lda * n$. Before entry, the leading k -by- n part of the array a must contain the matrix A . |
| <i>Layout = CblasRowMajor</i> | Array, size $lda * n$. Before entry, the leading k -by- n part of the array a must contain the matrix A . | Array, size $lda * k$. Before entry, the leading n -by- k part of the array a must contain the matrix A . |

| | | |
|------------|----------------------|------------------------|
| <i>lda</i> | $trans=CblasNoTrans$ | $trans=CblasConjTrans$ |
|------------|----------------------|------------------------|

| | | |
|-----------------|-----------------------------|-----------------------------|
| <i>Layout</i> = | <i>lda</i> must be at least | <i>lda</i> must be at least |
| CblasColMajor | max(1, <i>n</i>). | max(1, <i>k</i>) |
| <i>Layout</i> = | <i>lda</i> must be at least | <i>lda</i> must be at least |
| CblasRowMajor | max(1, <i>k</i>) | max(1, <i>n</i>). |

| | |
|-------------|--|
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>c</i> | Array, size <i>ldc</i> * <i>n</i> . Before entry with <i>uplo</i> = CblasUpper, the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>c</i> must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of <i>c</i> is not referenced. Before entry with <i>uplo</i> = CblasLower, the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>c</i> must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of <i>c</i> is not referenced. |
| <i>ldc</i> | Specifies the leading dimension of <i>c</i> as declared in the calling (sub)program. The value of <i>ldc</i> must be at least max(1, <i>n</i>). |

Output Parameters

| | |
|----------|--|
| <i>c</i> | With <i>uplo</i> = CblasUpper, the upper triangular part of the array <i>c</i> is overwritten by the upper triangular part of the updated matrix. With <i>uplo</i> = CblasLower, the lower triangular part of the array <i>c</i> is overwritten by the lower triangular part of the updated matrix. |
|----------|--|

cblas_?syr2k

Performs a symmetric rank-2k update.

Syntax

```
void cblas_ssy2k (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const float alpha, const float
*a, const MKL_INT lda, const float *b, const MKL_INT ldb, const float beta, float *c,
const MKL_INT ldc);
```

```
void cblas_dsyr2k (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const double alpha, const
double *a, const MKL_INT lda, const double *b, const MKL_INT ldb, const double beta,
double *c, const MKL_INT ldc);
```

```
void cblas_csyr2k (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const void *alpha, const void
*a, const MKL_INT lda, const void *b, const MKL_INT ldb, const void *beta, void *c,
const MKL_INT ldc);
```

```
void cblas_zsyr2k (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE trans, const MKL_INT n, const MKL_INT k, const void *alpha, const void
*a, const MKL_INT lda, const void *b, const MKL_INT ldb, const void *beta, void *c,
const MKL_INT ldc);
```

Include Files

- mkl.h

Description

The `?syr2k` routines perform a rank-2k matrix-matrix operation for a symmetric matrix C using general matrices A and B . The operation is defined as

$$C := \alpha * A * B' + \alpha * B * A' + \beta * C,$$

or

$$C := \alpha * A' * B + \alpha * B' * A + \beta * C,$$

where:

α and β are scalars,

C is an n -by- n symmetric matrix,

A and B are n -by- k matrices in the first case, and k -by- n matrices in the second case.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array c is used. If $uplo = CblasUpper$, then the upper triangular part of the array c is used. If $uplo = CblasLower$, then the low triangular part of the array c is used. |
| <i>trans</i> | Specifies the operation: if $trans=CblasNoTrans$, then $C := \alpha * A * B' + \alpha * B * A' + \beta * C$; if $trans=CblasTrans$, then $C := \alpha * A' * B + \alpha * B' * A + \beta * C$; if $trans=CblasConjTrans$, then $C := \alpha * A' * B + \alpha * B' * A + \beta * C$. |
| <i>n</i> | Specifies the order of the matrix C . The value of n must be at least zero. |
| <i>k</i> | On entry with $trans=CblasNoTrans$, k specifies the number of columns of the matrices A and B , and on entry with $trans=CblasTrans$ or $trans=CblasConjTrans$, k specifies the number of rows of the matrices A and B . The value of k must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . |

a

| | <i>trans=CblasNoTrans</i> | <i>trans=CblasConjTrans</i> |
|-------------------------------|---|---|
| <i>Layout = CblasColMajor</i> | Array, size $lda * k$. Before entry, the leading n -by- k part of the array a must contain the matrix A . | Array, size $lda * n$. Before entry, the leading k -by- n part of the array a must contain the matrix A . |
| <i>Layout = CblasRowMajor</i> | Array, size $lda * n$. | Array, size $lda * k$. |

| | |
|--|--|
| Before entry, the leading k -by- n part of the array a must contain the matrix A . | Before entry, the leading n -by- k part of the array a must contain the matrix A . |
|--|--|

lda

Specifies the leading dimension of a as declared in the calling (sub)program.

| | | |
|----------------------------------|--|--|
| | <i>trans</i> =CblasNoTrans | <i>trans</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | <i>lda</i> must be at least $\max(1, n)$. | <i>lda</i> must be at least $\max(1, k)$ |
| <i>Layout</i> = CblasRowMajor | <i>lda</i> must be at least $\max(1, k)$ | <i>lda</i> must be at least $\max(1, n)$. |

b

| | | |
|----------------------------------|---|---|
| | <i>trans</i> =CblasNoTrans | <i>trans</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | Array, size $ldb*k$. Before entry, the leading n -by- k part of the array b must contain the matrix B . | Array, size $ldb*n$. Before entry, the leading k -by- n part of the array b must contain the matrix B . |
| <i>Layout</i> = CblasRowMajor | Array, size $lda*n$. Before entry, the leading k -by- n part of the array b must contain the matrix B . | Array, size $lda*k$. Before entry, the leading n -by- k part of the array b must contain the matrix B . |

ldb

Specifies the leading dimension of a as declared in the calling (sub)program.

| | | |
|----------------------------------|--|--|
| | <i>trans</i> =CblasNoTrans | <i>trans</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | <i>ldb</i> must be at least $\max(1, n)$. | <i>ldb</i> must be at least $\max(1, k)$ |
| <i>Layout</i> = CblasRowMajor | <i>ldb</i> must be at least $\max(1, k)$ | <i>ldb</i> must be at least $\max(1, n)$. |

beta

Specifies the scalar *beta*.

c

Array, size $ldc* n$. Before entry with *uplo* = CblasUpper, the leading n -by- n upper triangular part of the array c must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of c is not referenced.

Before entry with *uplo* = CblasLower, the leading n -by- n lower triangular part of the array c must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of c is not referenced.

ldc Specifies the leading dimension of *c* as declared in the calling (sub)program. The value of *ldc* must be at least $\max(1, n)$.

Output Parameters

c With *uplo* = CblasUpper, the upper triangular part of the array *c* is overwritten by the upper triangular part of the updated matrix.
 With *uplo* = CblasLower, the lower triangular part of the array *c* is overwritten by the lower triangular part of the updated matrix.

cblas_?trmm

Computes a matrix-matrix product where one input matrix is triangular.

Syntax

```
void cblas_strmm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const CBLAS_TRANSPOSE transa, const CBLAS_DIAG diag, const MKL_INT m, const
MKL_INT n, const float alpha, const float *a, const MKL_INT lda, float *b, const
MKL_INT ldb);
```

```
void cblas_dtrmm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const CBLAS_TRANSPOSE transa, const CBLAS_DIAG diag, const MKL_INT m, const
MKL_INT n, const double alpha, const double *a, const MKL_INT lda, double *b, const
MKL_INT ldb);
```

```
void cblas_ctrmm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const CBLAS_TRANSPOSE transa, const CBLAS_DIAG diag, const MKL_INT m, const
MKL_INT n, const void *alpha, const void *a, const MKL_INT lda, void *b, const MKL_INT
ldb);
```

```
void cblas_ztrmm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const CBLAS_TRANSPOSE transa, const CBLAS_DIAG diag, const MKL_INT m, const
MKL_INT n, const void *alpha, const void *a, const MKL_INT lda, void *b, const MKL_INT
ldb);
```

Include Files

- mkl.h

Description

The ?trmm routines compute a scalar-matrix-matrix product with one triangular matrix. The operation is defined as

$$B := \alpha * \text{op}(A) * B$$

or

$$B := \alpha * B * \text{op}(A)$$

where:

alpha is a scalar,

B is an *m*-by-*n* matrix,

A is a unit, or non-unit, upper or lower triangular matrix

op(*A*) is one of op(*A*) = *A*, or op(*A*) = *A'*, or op(*A*) = conjg(*A'*).

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>side</i> | Specifies whether $\text{op}(A)$ appears on the left or right of B in the operation: if $\text{side} = \text{CblasLeft}$, then $B := \alpha * \text{op}(A) * B$; if $\text{side} = \text{CblasRight}$, then $B := \alpha * B * \text{op}(A)$. |
| <i>uplo</i> | Specifies whether the matrix A is upper or lower triangular: $\text{uplo} = \text{CblasUpper}$ if $\text{uplo} = \text{CblasLower}$, then the matrix is low triangular. |
| <i>transa</i> | Specifies the form of $\text{op}(A)$ used in the matrix multiplication: if $\text{transa} = \text{CblasNoTrans}$, then $\text{op}(A) = A$; if $\text{transa} = \text{CblasTrans}$, then $\text{op}(A) = A'$; if $\text{transa} = \text{CblasConjTrans}$, then $\text{op}(A) = \text{conjg}(A')$. |
| <i>diag</i> | Specifies whether the matrix A is unit triangular: if $\text{diag} = \text{CblasUnit}$ then the matrix is unit triangular; if $\text{diag} = \text{CblasNonUnit}$, then the matrix is not unit triangular. |
| <i>m</i> | Specifies the number of rows of B . The value of m must be at least zero. |
| <i>n</i> | Specifies the number of columns of B . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar α . When α is zero, then a is not referenced and b need not be set before entry. |
| <i>a</i> | Array, size lda by k , where k is m when $\text{side} = \text{CblasLeft}$ and is n when $\text{side} = \text{CblasRight}$. Before entry with $\text{uplo} = \text{CblasUpper}$, the leading k by k upper triangular part of the array a must contain the upper triangular matrix and the strictly lower triangular part of a is not referenced. Before entry with $\text{uplo} = \text{CblasLower}$, the leading k by k lower triangular part of the array a must contain the lower triangular matrix and the strictly upper triangular part of a is not referenced. When $\text{diag} = \text{CblasUnit}$, the diagonal elements of a are not referenced either, but are assumed to be unity. |
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. When $\text{side} = \text{CblasLeft}$, then lda must be at least $\max(1, m)$, when $\text{side} = \text{CblasRight}$, then lda must be at least $\max(1, n)$. |
| <i>b</i> | For $\text{Layout} = \text{CblasColMajor}$: array, size $ldb * n$. Before entry, the leading m -by- n part of the array b must contain the matrix B . For $\text{Layout} = \text{CblasRowMajor}$: array, size $ldb * m$. Before entry, the leading n -by- m part of the array b must contain the matrix B . |

ldb Specifies the leading dimension of *b* as declared in the calling (sub)program. When *Layout* = CblasColMajor, *ldb* must be at least $\max(1, m)$; otherwise, *ldb* must be at least $\max(1, n)$.

Output Parameters

b Overwritten by the transformed matrix.

cblas_?trsm

Solves a triangular matrix equation.

Syntax

```
void cblas_strsm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const CBLAS_TRANSPOSE transa, const CBLAS_DIAG diag, const MKL_INT m, const
MKL_INT n, const float alpha, const float *a, const MKL_INT lda, float *b, const
MKL_INT ldb);
```

```
void cblas_dtrsm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const CBLAS_TRANSPOSE transa, const CBLAS_DIAG diag, const MKL_INT m, const
MKL_INT n, const double alpha, const double *a, const MKL_INT lda, double *b, const
MKL_INT ldb);
```

```
void cblas_ctrsm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const CBLAS_TRANSPOSE transa, const CBLAS_DIAG diag, const MKL_INT m, const
MKL_INT n, const void *alpha, const void *a, const MKL_INT lda, void *b, const MKL_INT
ldb);
```

```
void cblas_ztrsm (const CBLAS_LAYOUT Layout, const CBLAS_SIDE side, const CBLAS_UPLO
uplo, const CBLAS_TRANSPOSE transa, const CBLAS_DIAG diag, const MKL_INT m, const
MKL_INT n, const void *alpha, const void *a, const MKL_INT lda, void *b, const MKL_INT
ldb);
```

Include Files

- mkl.h

Description

The ?trsm routines solve one of the following matrix equations:

$$\text{op}(A) * X = \alpha * B,$$

or

$$X * \text{op}(A) = \alpha * B,$$

where:

alpha is a scalar,

X and *B* are *m*-by-*n* matrices,

A is a unit, or non-unit, upper or lower triangular matrix

op(*A*) is one of op(*A*) = *A*, or op(*A*) = *A'*, or op(*A*) = conjg(*A'*).

The matrix *B* is overwritten by the solution matrix *X*.

Input Parameters

| | |
|---------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>side</i> | Specifies whether $\text{op}(A)$ appears on the left or right of X in the equation: if <i>side</i> = CblasLeft, then $\text{op}(A) * X = \alpha * B$; if <i>side</i> = CblasRight, then $X * \text{op}(A) = \alpha * B$. |
| <i>uplo</i> | Specifies whether the matrix A is upper or lower triangular: <i>uplo</i> = CblasUpper if <i>uplo</i> = CblasLower, then the matrix is low triangular. |
| <i>transa</i> | Specifies the form of $\text{op}(A)$ used in the matrix multiplication: if <i>transa</i> =CblasNoTrans, then $\text{op}(A) = A$; if <i>transa</i> =CblasTrans; if <i>transa</i> =CblasConjTrans, then $\text{op}(A) = \text{conjg}(A')$. |
| <i>diag</i> | Specifies whether the matrix A is unit triangular: if <i>diag</i> = CblasUnit then the matrix is unit triangular; if <i>diag</i> = CblasNonUnit , then the matrix is not unit triangular. |
| <i>m</i> | Specifies the number of rows of B . The value of m must be at least zero. |
| <i>n</i> | Specifies the number of columns of B . The value of n must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . When <i>alpha</i> is zero, then a is not referenced and b need not be set before entry. |
| <i>a</i> | Array, size $lda * k$, where k is m when <i>side</i> = CblasLeft and is n when <i>side</i> = CblasRight. Before entry with <i>uplo</i> = CblasUpper, the leading k by k upper triangular part of the array a must contain the upper triangular matrix and the strictly lower triangular part of a is not referenced. Before entry with <i>uplo</i> = CblasLower lower triangular part of the array a must contain the lower triangular matrix and the strictly upper triangular part of a is not referenced. When <i>diag</i> = CblasUnit, the diagonal elements of a are not referenced either, but are assumed to be unity. |
| <i>lda</i> | Specifies the leading dimension of a as declared in the calling (sub)program. When <i>side</i> = CblasLeft, then <i>lda</i> must be at least $\max(1, m)$, when <i>side</i> = CblasRight, then <i>lda</i> must be at least $\max(1, n)$. |
| <i>b</i> | For <i>Layout</i> = CblasColMajor: array, size $ldb * n$. Before entry, the leading m -by- n part of the array b must contain the matrix B . For <i>Layout</i> = CblasRowMajor: array, size $ldb * m$. Before entry, the leading n -by- m part of the array b must contain the matrix B . |

ldb Specifies the leading dimension of *b* as declared in the calling (sub)program. When *Layout* = `CblasColMajor`, *ldb* must be at least $\max(1, m)$; otherwise, *ldb* must be at least $\max(1, n)$.

Output Parameters

b Overwritten by the solution matrix *X*.

Sparse BLAS Level 1 Routines

This section describes Sparse BLAS Level 1, an extension of BLAS Level 1 included in the Intel® Math Kernel Library beginning with the Intel MKL release 2.1. Sparse BLAS Level 1 is a group of routines and functions that perform a number of common vector operations on sparse vectors stored in compressed form.

Sparse vectors are those in which the majority of elements are zeros. Sparse BLAS routines and functions are specially implemented to take advantage of vector sparsity. This allows you to achieve large savings in computer time and memory. If *nz* is the number of non-zero vector elements, the computer time taken by Sparse BLAS operations will be $O(nz)$.

Vector Arguments

Compressed sparse vectors. Let *a* be a vector stored in an array, and assume that the only non-zero elements of *a* are the following:

```
a[k1], a[k2], a[k3] . . . a[knz],
```

where *nz* is the total number of non-zero elements in *a*.

In Sparse BLAS, this vector can be represented in compressed form by two arrays, *x* (values) and *indx* (indices). Each array has *nz* elements:

```
x[0]=a[k1], x[1]=a[k2], . . . x[nz-1]= a[knz],
```

```
indx[0]=k1, indx[1]=k2, . . . indx[nz-1]= knz.
```

Thus, a sparse vector is fully determined by the triple (*nz*, *x*, *indx*). If you pass a negative or zero value of *nz* to Sparse BLAS, the subroutines do not modify any arrays or variables.

Full-storage vectors. Sparse BLAS routines can also use a vector argument fully stored in a single array (a full-storage vector). If *y* is a full-storage vector, its elements must be stored contiguously: the first element in *y*[0], the second in *y*[1], and so on. This corresponds to an increment *incy* = 1 in BLAS Level 1. No increment value for full-storage vectors is passed as an argument to Sparse BLAS routines or functions.

Naming Conventions for Sparse BLAS Routines

Similar to BLAS, the names of Sparse BLAS subprograms have prefixes that determine the data type involved: *s* and *d* for single- and double-precision real; *c* and *z* for single- and double-precision complex respectively.

If a Sparse BLAS routine is an extension of a "dense" one, the subprogram name is formed by appending the suffix *i* (standing for *indexed*) to the name of the corresponding "dense" subprogram. For example, the Sparse BLAS routine `saxpyi` corresponds to the BLAS routine `saxpy`, and the Sparse BLAS function `cdotci` corresponds to the BLAS function `cdotc`.

Routines and Data Types

Routines and data types supported in the Intel MKL implementation of Sparse BLAS are listed in [Table "Sparse BLAS Routines and Their Data Types"](#).

Sparse BLAS Routines and Their Data Types

| Routine/ Function | Data Types | Description |
|---------------------------|------------|--|
| <code>cblas_?axpyi</code> | s, d, c, z | Scalar-vector product plus vector (routines) |
| <code>cblas_?doti</code> | s, d | Dot product (functions) |
| <code>cblas_?dotci</code> | c, z | Complex dot product conjugated (functions) |
| <code>cblas_?dotui</code> | c, z | Complex dot product unconjugated (functions) |
| <code>cblas_?gthr</code> | s, d, c, z | Gathering a full-storage sparse vector into compressed form $nz, x, indx$ (routines) |
| <code>cblas_?gthrz</code> | s, d, c, z | Gathering a full-storage sparse vector into compressed form and assigning zeros to gathered elements in the full-storage vector (routines) |
| <code>cblas_?roti</code> | s, d | Givens rotation (routines) |
| <code>cblas_?sctr</code> | s, d, c, z | Scattering a vector from compressed form to full-storage form (routines) |

BLAS Level 1 Routines That Can Work With Sparse Vectors

The following BLAS Level 1 routines will give correct results when you pass to them a compressed-form array x (with the increment $incx=1$):

| | |
|---------------------------|--|
| <code>cblas_?asum</code> | sum of absolute values of vector elements |
| <code>cblas_?copy</code> | copying a vector |
| <code>cblas_?nrm2</code> | Euclidean norm of a vector |
| <code>cblas_?scal</code> | scaling a vector |
| <code>cblas_i?amax</code> | index of the element with the largest absolute value for real flavors, or the largest sum $ \operatorname{Re}(x[i]) + \operatorname{Im}(x[i]) $ for complex flavors. |
| <code>cblas_i?amin</code> | index of the element with the smallest absolute value for real flavors, or the smallest sum $ \operatorname{Re}(x[i]) + \operatorname{Im}(x[i]) $ for complex flavors. |

The result i returned by `i?amax` and `i?amin` should be interpreted as index in the compressed-form array, so that the largest (smallest) value is $x[i-1]$; the corresponding index in full-storage array is $indx[i-1]$.

You can also call `cblas_?rotg` to compute the parameters of Givens rotation and then pass these parameters to the Sparse BLAS routines `cblas_?roti`.

`cblas_?axpyi`

Adds a scalar multiple of compressed sparse vector to a full-storage vector.

Syntax

```
void cblas_saxpyi (const MKL_INT nz, const float a, const float *x, const MKL_INT *indx, float *y);
```

```
void cblas_daxpyi (const MKL_INT nz, const double a, const double *x, const MKL_INT *indx, double *y);
```

```
void cblas_caxpyi (const MKL_INT nz, const void *a, const void *x, const MKL_INT *indx, void *y);
```

```
void cblas_zaxpyi (const MKL_INT nz, const void *a, const void *x, const MKL_INT *indx, void *y);
```

Include Files

- mkl.h

Description

The `?axpyi` routines perform a vector-vector operation defined as

```
 $y := a*x + y$ 
```

where:

a is a scalar,

x is a sparse vector stored in compressed form,

y is a vector in full storage form.

The `?axpyi` routines reference or modify only the elements of y whose indices are listed in the array $indx$.

The values in $indx$ must be distinct.

Input Parameters

| | |
|--------|--|
| nz | The number of elements in x and $indx$. |
| a | Specifies the scalar a . |
| x | Array, size at least nz . |
| $indx$ | Specifies the indices for the elements of x . Array, size at least nz . |
| y | Array, size at least $\max(indx[i])$. |

Output Parameters

| | |
|-----|-----------------------------------|
| y | Contains the updated vector y . |
|-----|-----------------------------------|

cblas_?doti

Computes the dot product of a compressed sparse real vector by a full-storage real vector.

Syntax

```
float cblas_sdoti (const MKL_INT nz, const float *x, const MKL_INT *indx, const float *y);
```

```
double cblas_ddoti (const MKL_INT nz, const double *x, const MKL_INT *indx, const double *y);
```

Include Files

- mkl.h

Description

The `?doti` routines return the dot product of x and y defined as

```
res = x[0]*y[indx[0]] + x[1]*y[indx[1]] +...+ x[nz-1]*y[indx[nz-1]]
```

where the triple $(nz, x, indx)$ defines a sparse real vector stored in compressed form, and y is a real vector in full storage form. The functions reference only the elements of y whose indices are listed in the array $indx$. The values in $indx$ must be distinct.

Input Parameters

| | |
|--------|--|
| nz | The number of elements in x and $indx$. |
| x | Array, size at least nz . |
| $indx$ | Specifies the indices for the elements of x . Array, size at least nz . |
| y | Array, size at least $\max(indx[i])$. |

Output Parameters

| | |
|-------|---|
| res | Contains the dot product of x and y , if nz is positive. Otherwise, res contains 0. |
|-------|---|

`cblas_?dotci`

Computes the conjugated dot product of a compressed sparse complex vector with a full-storage complex vector.

Syntax

```
void cblas_cdotci_sub (const MKL_INT nz, const void *x, const MKL_INT *indx, const void *y, void *dotui);
```

```
void cblas_zdotci_sub (const MKL_INT nz, const void *x, const MKL_INT *indx, const void *y, void *dotui);
```

Include Files

- `mkl.h`

Description

The `?dotci` routines return the dot product of x and y defined as

```
conjg(x[0])*y[indx[0]] + ... + conjg(x[nz-1])*y[indx[nz-1]]
```

where the triple $(nz, x, indx)$ defines a sparse complex vector stored in compressed form, and y is a real vector in full storage form. The functions reference only the elements of y whose indices are listed in the array $indx$. The values in $indx$ must be distinct.

Input Parameters

| | |
|------|--|
| nz | The number of elements in x and $indx$. |
|------|--|

| | |
|-------------|--|
| <i>x</i> | Array, size at least <i>nz</i> . |
| <i>indx</i> | Specifies the indices for the elements of <i>x</i> . Array, size at least <i>nz</i> . |
| <i>y</i> | Array, size at least $\max(\text{indx}[i])$. |

Output Parameters

| | |
|--------------|--|
| <i>dotui</i> | Contains the conjugated dot product of <i>x</i> and <i>y</i> , if <i>nz</i> is positive. Otherwise, it contains 0. |
|--------------|--|

cblas_?dotui

Computes the dot product of a compressed sparse complex vector by a full-storage complex vector.

Syntax

```
void cblas_cdotui_sub (const MKL_INT nz, const void *x, const MKL_INT *indx, const void *y, void *dotui);
```

```
void cblas_zdotui_sub (const MKL_INT nz, const void *x, const MKL_INT *indx, const void *y, void *dotui);
```

Include Files

- `mkl.h`

Description

The `?dotui` routines return the dot product of *x* and *y* defined as

```
res = x[0]*y[indx[0]] + x[1]*y[indx[1]] + ... + x[nz - 1]*y[indx[nz - 1]]
```

where the triple (*nz*, *x*, *indx*) defines a sparse complex vector stored in compressed form, and *y* is a real vector in full storage form. The functions reference only the elements of *y* whose indices are listed in the array *indx*. The values in *indx* must be distinct.

Input Parameters

| | |
|-------------|--|
| <i>nz</i> | The number of elements in <i>x</i> and <i>indx</i> . |
| <i>x</i> | Array, size at least <i>nz</i> . |
| <i>indx</i> | Specifies the indices for the elements of <i>x</i> . Array, size at least <i>nz</i> . |
| <i>y</i> | Array, size at least $\max(\text{indx}[i])$. |

Output Parameters

| | |
|--------------|---|
| <i>dotui</i> | Contains the dot product of <i>x</i> and <i>y</i> , if <i>nz</i> is positive. Otherwise, <i>res</i> contains 0. |
|--------------|---|

cblas_?gthr

Gathers a full-storage sparse vector's elements into compressed form.

Syntax

```
void cblas_sgthr (const MKL_INT nz, const float *y, float *x, const MKL_INT *indx);
void cblas_dgthr (const MKL_INT nz, const double *y, double *x, const MKL_INT *indx);
void cblas_cgthr (const MKL_INT nz, const void *y, void *x, const MKL_INT *indx);
void cblas_zgthr (const MKL_INT nz, const void *y, void *x, const MKL_INT *indx);
```

Include Files

- mkl.h

Description

The `?gthr` routines gather the specified elements of a full-storage sparse vector y into compressed form(nz , x , $indx$). The routines reference only the elements of y whose indices are listed in the array $indx$:

$x[i] = y[indx[i]]$, for $i=0,1,\dots,nz-1$.

Input Parameters

| | |
|--------|--|
| nz | The number of elements of y to be gathered. |
| $indx$ | Specifies indices of elements to be gathered. Array, size at least nz . |
| y | Array, size at least $\max(indx[i])$. |

Output Parameters

| | |
|-----|--|
| x | Array, size at least nz . Contains the vector converted to the compressed form. |
|-----|--|

cblas_?gthrz

Gathers a sparse vector's elements into compressed form, replacing them by zeros.

Syntax

```
void cblas_sgthrz (const MKL_INT nz, float *y, float *x, const MKL_INT *indx);
void cblas_dgthrz (const MKL_INT nz, double *y, double *x, const MKL_INT *indx);
void cblas_cgthrz (const MKL_INT nz, void *y, void *x, const MKL_INT *indx);
void cblas_zgthrz (const MKL_INT nz, void *y, void *x, const MKL_INT *indx);
```

Include Files

- mkl.h

Description

The `?gthrz` routines gather the elements with indices specified by the array `indx` from a full-storage vector `y` into compressed form (`nz`, `x`, `indx`) and overwrite the gathered elements of `y` by zeros. Other elements of `y` are not referenced or modified (see also `?gthr`).

Input Parameters

| | |
|-------------------|---|
| <code>nz</code> | The number of elements of <code>y</code> to be gathered. |
| <code>indx</code> | Specifies indices of elements to be gathered. Array, size at least <code>nz</code> . |
| <code>y</code> | Array, size at least $\max(\text{indx}[i])$. |

Output Parameters

| | |
|----------------|---|
| <code>x</code> | Array, size at least <code>nz</code> . Contains the vector converted to the compressed form. |
| <code>y</code> | The updated vector <code>y</code> . |

`cblas_?roti`

Applies Givens rotation to sparse vectors one of which is in compressed form.

Syntax

```
void cblas_sroti (const MKL_INT nz, float *x, const MKL_INT *indx, float *y, const float c, const float s);
```

```
void cblas_droti (const MKL_INT nz, double *x, const MKL_INT *indx, double *y, const double c, const double s);
```

Include Files

- `mkl.h`

Description

The `?roti` routines apply the Givens rotation to elements of two real vectors, `x` (in compressed form `nz`, `x`, `indx`) and `y` (in full storage form):

```
x[i] = c*x[i] + s*y[indx[i]]
```

```
y[indx[i]] = c*y[indx[i]] - s*x[i]
```

The routines reference only the elements of `y` whose indices are listed in the array `indx`. The values in `indx` must be distinct.

Input Parameters

| | |
|-----------------|--|
| <code>nz</code> | The number of elements in <code>x</code> and <code>indx</code> . |
| <code>x</code> | Array, size at least <code>nz</code> . |

| | |
|-------------|--|
| <i>indx</i> | Specifies the indices for the elements of <i>x</i> . Array, size at least <i>nz</i> . |
| <i>y</i> | Array, size at least $\max(\text{indx}[i])$. |
| <i>c</i> | A scalar. |
| <i>s</i> | A scalar. |

Output Parameters

| | |
|-----------------------|---------------------|
| <i>x</i> and <i>y</i> | The updated arrays. |
|-----------------------|---------------------|

cblas_?sctr

Converts compressed sparse vectors into full storage form.

Syntax

```
void cblas_ssctr (const MKL_INT nz, const float *x, const MKL_INT *indx, float *y);
void cblas_dsctr (const MKL_INT nz, const double *x, const MKL_INT *indx, double *y);
void cblas_csctr (const MKL_INT nz, const void *x, const MKL_INT *indx, void *y);
void cblas_zsctr (const MKL_INT nz, const void *x, const MKL_INT *indx, void *y);
```

Include Files

- mkl.h

Description

The `?sctr` routines scatter the elements of the compressed sparse vector (*nz*, *x*, *indx*) to a full-storage vector *y*. The routines modify only the elements of *y* whose indices are listed in the array *indx*:

$y[\text{indx}[i]] = x[i]$, for $i=0,1,\dots,nz-1$.

Input Parameters

| | |
|-------------|---|
| <i>nz</i> | The number of elements of <i>x</i> to be scattered. |
| <i>indx</i> | Specifies indices of elements to be scattered. Array, size at least <i>nz</i> . |
| <i>x</i> | Array, size at least <i>nz</i> . Contains the vector to be converted to full-storage form. |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Array, size at least $\max(\text{indx}[i])$. Contains the vector <i>y</i> with updated elements. |
|----------|--|

Sparse BLAS Level 2 and Level 3 Routines

This section describes Sparse BLAS Level 2 and Level 3 routines included in the Intel® Math Kernel Library (Intel® MKL). Sparse BLAS Level 2 is a group of routines and functions that perform operations between a sparse matrix and dense vectors. Sparse BLAS Level 3 is a group of routines and functions that perform operations between a sparse matrix and dense matrices.

The terms and concepts required to understand the use of the Intel MKL Sparse BLAS Level 2 and Level 3 routines are discussed in the [Linear Solvers Basics](#) appendix.

The Sparse BLAS routines can be useful to implement iterative methods for solving large sparse systems of equations or eigenvalue problems. For example, these routines can be considered as building blocks for [Iterative Sparse Solvers based on Reverse Communication Interface \(RCI ISS\)](#) described in the Chapter 8 of the manual.

Intel MKL provides Sparse BLAS Level 2 and Level 3 routines with typical (or conventional) interface similar to the interface used in the NIST* Sparse BLAS library [[Rem05](#)].

Some software packages and libraries (the [PARDISO* Solver](#) used in Intel MKL, *Sparskit 2* [[Saad94](#)], the Compaq* Extended Math Library (CXML)[[CXML01](#)]) use different (early) variation of the compressed sparse row (CSR) format and support only Level 2 operations with simplified interfaces. Intel MKL provides an additional set of Sparse BLAS Level 2 routines with similar simplified interfaces. Each of these routines operates only on a matrix of the fixed type.

The routines described in this section support both one-based indexing and zero-based indexing of the input data (see details in the section [One-based and Zero-based Indexing](#)).

Naming Conventions in Sparse BLAS Level 2 and Level 3

Each Sparse BLAS Level 2 and Level 3 routine has a six- or eight-character base name preceded by the prefix `mkl_` or `mkl_cspblas_`.

The routines with typical (conventional) interface have six-character base names in accordance with the template:

```
mkl_<character> <data> <operation>( )
```

The routines with simplified interfaces have eight-character base names in accordance with the templates:

```
mkl_<character> <data> <mtype> <operation>( )
```

for routines with one-based indexing; and

```
mkl_cspblas_<character> <data><mtype><operation>( )
```

for routines with zero-based indexing.

The `<character>` field indicates the data type:

| | |
|---|---------------------------|
| s | real, single precision |
| c | complex, single precision |
| d | real, double precision |
| z | complex, double precision |

The `<data>` field indicates the sparse matrix storage format (see section [Sparse Matrix Storage Formats](#)):

| | |
|-----|--|
| coo | coordinate format |
| csr | compressed sparse row format and its variations |
| csc | compressed sparse column format and its variations |

| | |
|------------------|--|
| <code>dia</code> | diagonal format |
| <code>sky</code> | skyline storage format |
| <code>bsr</code> | block sparse row format and its variations |

The `<operation>` field indicates the type of operation:

| | |
|-----------------|---|
| <code>mv</code> | matrix-vector product (Level 2) |
| <code>mm</code> | matrix-matrix product (Level 3) |
| <code>sv</code> | solving a single triangular system (Level 2) |
| <code>sm</code> | solving triangular systems with multiple right-hand sides (Level 3) |

The field `<mtype>` indicates the matrix type:

| | |
|-----------------|--|
| <code>ge</code> | sparse representation of a general matrix |
| <code>sy</code> | sparse representation of the upper or lower triangle of a symmetric matrix |
| <code>tr</code> | sparse representation of a triangular matrix |

Sparse Matrix Storage Formats for Sparse BLAS Routines

The current version of Intel MKL Sparse BLAS Level 2 and Level 3 routines support the following point entry [Duff86] storage formats for sparse matrices:

- *compressed sparse row* format (CSR) and its variations;
- *compressed sparse column* format (CSC);
- *coordinate* format;
- *diagonal* format;
- *skyline* storage format;

and one block entry storage format:

- *block sparse row* format (BSR) and its variations.

For more information see "Sparse Matrix Storage Formats" in Appendix A.

Intel MKL provides auxiliary routines - [matrix converters](#) - that convert sparse matrix from one storage format to another.

Routines and Supported Operations

This section describes operations supported by the Intel MKL Sparse BLAS Level 2 and Level 3 routines. The following notations are used here:

A is a sparse matrix;
 B and C are dense matrices;
 D is a diagonal scaling matrix;
 x and y are dense vectors;
 α and β are scalars;

$\text{op}(A)$ is one of the possible operations:

$\text{op}(A) = A;$

$\text{op}(A) = A^T$ - transpose of A ;
 $\text{op}(A) = A^H$ - conjugated transpose of A .

$\text{inv}(\text{op}(A))$ denotes the inverse of $\text{op}(A)$.

The Intel MKL Sparse BLAS Level 2 and Level 3 routines support the following operations:

- computing the vector product between a sparse matrix and a dense vector:

```
y := alpha*op(A)*x + beta*y
```

- solving a single triangular system:

```
y := alpha*inv(op(A))*x
```

- computing a product between sparse matrix and dense matrix:

```
C := alpha*op(A)*B + beta*C
```

- solving a sparse triangular system with multiple right-hand sides:

```
C := alpha*inv(op(A))*B
```

Intel MKL provides an additional set of the Sparse BLAS Level 2 routines with *simplified interfaces*. Each of these routines operates on a matrix of the fixed type. The following operations are supported:

- computing the vector product between a sparse matrix and a dense vector (for general and symmetric matrices):

```
y := op(A)*x
```

- solving a single triangular system (for triangular matrices):

```
y := inv(op(A))*x
```

Matrix type is indicated by the field `<mtype>` in the routine name (see section [Naming Conventions in Sparse BLAS Level 2 and Level 3](#)).

NOTE

The routines with simplified interfaces support only four sparse matrix storage formats, specifically:

- CSR format in the 3-array variation accepted in the direct sparse solvers and in the CXML;
- diagonal format accepted in the CXML;
- coordinate format;
- BSR format in the 3-array variation.

Note that routines with both typical (conventional) and simplified interfaces use the same computational kernels that work with certain internal data structures.

The Intel MKL Sparse BLAS Level 2 and Level 3 routines do not support in-place operations.

Complete list of all routines is given in the [“Sparse BLAS Level 2 and Level 3 Routines”](#).

Interface Consideration

One-Based and Zero-Based Indexing

The Intel MKL Sparse BLAS Level 2 and Level 3 routines support one-based and zero-based indexing of data arrays.

Routines with typical interfaces support zero-based indexing for the following sparse data storage formats: CSR, CSC, BSR, and COO. Routines with simplified interfaces support zero based indexing for the following sparse data storage formats: CSR, BSR, and COO. See the complete list of [Sparse BLAS Level 2 and Level 3 Routines](#).

The one-based indexing uses the convention of starting array indices at 1. The zero-based indexing uses the convention of starting array indices at 0. For example, indices of the 5-element array x can be presented in case of one-based indexing as follows:

Element index: 1 2 3 4 5

Element value: 1.0 5.0 7.0 8.0 9.0

and in case of zero-based indexing as follows:

Element index: 0 1 2 3 4

Element value: 1.0 5.0 7.0 8.0 9.0

The detailed descriptions of the one-based and zero-based variants of the sparse data storage formats are given in the "[Sparse Matrix Storage Formats](#)" in Appendix A.

Most parameters of the routines are identical for both one-based and zero-based indexing, but some of them have certain differences. The following table lists all these differences.

| Parameter | One-based Indexing | Zero-based Indexing |
|--------------|--|---|
| <i>val</i> | Array containing non-zero elements of the matrix A , its length is $.pntrb[m] - pntrb[1]$ | Array containing non-zero elements of the matrix A , its length is $.pntrb[m-1] - pntrb[0]$ |
| <i>pntrb</i> | Array of length m . This array contains row indices, such that $pntrb[i] - pntrb[1]+1$ is the first index of row i in the arrays <i>val</i> and <i>indx</i> | Array of length m . This array contains row indices, such that $pntrb[i] - pntrb[0]$ is the first index of row i in the arrays <i>val</i> and <i>indx</i> . |
| <i>pntrb</i> | Array of length m . This array contains row indices, such that $pntrb[i] - pntrb[1]$ is the last index of row i in the arrays <i>val</i> and <i>indx</i> . | Array of length m . This array contains row indices, such that $pntrb[i] - pntrb[0]-1$ is the last index of row i in the arrays <i>val</i> and <i>indx</i> . |
| <i>ia</i> | Array of length $m + 1$, containing indices of elements in the array a , such that $ia[i]$ is the index in the array a of the first non-zero element from the row i . The value of the last element $ia[m + 1]$ is equal to the number of non-zeros plus one. | Array of length $m+1$, containing indices of elements in the array a , such that $ia[i]$ is the index in the array a of the first non-zero element from the row i . The value of the last element $ia[m]$ is equal to the number of non-zeros. |
| <i>ldb</i> | Specifies the leading dimension of b as declared in the calling (sub)program. | Specifies the second dimension of b as declared in the calling (sub)program. |
| <i>ldc</i> | Specifies the leading dimension of c as declared in the calling (sub)program. | Specifies the second dimension of c as declared in the calling (sub)program. |

Differences Between Intel MKL and NIST* Interfaces

The Intel MKL Sparse BLAS Level 3 routines have the following conventional interfaces:

`mkl_xyyyymm(transa, m, n, k, alpha, matdescra, arg(A), b, ldb, beta, c, ldc)`, for matrix-matrix product;

`mkl_xyyyysm(transa, m, n, alpha, matdescra, arg(A), b, ldb, c, ldc)`, for triangular solvers with multiple right-hand sides.

Here x denotes data type, and yyy - sparse matrix data structure (storage format).

The analogous NIST* Sparse BLAS (NSB) library routines have the following interfaces:

`xyyyymm(transa, m, n, k, alpha, descra, arg(A), b, ldb, beta, c, ldc, work, lwork)`, for matrix-matrix product;

`xyyyism(transa, m, n, unitd, dv, alpha, descra, arg(A), b, ldb, beta, c, ldc, work, lwork)`, for triangular solvers with multiple right-hand sides.

Some similar arguments are used in both libraries. The argument *transa* indicates what operation is performed and is slightly different in the NSB library (see Table “Parameter *transa*”). The arguments *m* and *k* are the number of rows and column in the matrix *A*, respectively, *n* is the number of columns in the matrix *C*. The arguments *alpha* and *beta* are scalar *alpha* and *beta* respectively (*beta* is not used in the Intel MKL triangular solvers.) The arguments *b* and *c* are rectangular arrays with the leading dimension *ldb* and *ldc*, respectively. `arg(A)` denotes the list of arguments that describe the sparse representation of *A*.

Parameter *transa*

| | MKL interface | NSB interface | Operation |
|-----------|---------------|---------------|--------------------------------|
| data type | char * | INTEGER | |
| value | N or n | 0 | $op(A) = A$ |
| | T or t | 1 | $op(A) = A^T$ |
| | C or c | 2 | $op(A) = A^T$ or $op(A) = A^H$ |

Parameter *matdescra*

The parameter *matdescra* describes the relevant characteristic of the matrix *A*. This manual describes *matdescra* as an array of six elements in line with the NIST* implementation. However, only the first four elements of the array are used in the current versions of the Intel MKL Sparse BLAS routines. Elements *matdescra*[4] and *matdescra*[5] are reserved for future use. Note that whether *matdescra* is described in your application as an array of length 6 or 4 is of no importance because the array is declared as a pointer in the Intel MKL routines. To learn more about declaration of the *matdescra* array, see the Sparse BLAS examples located in the Intel MKL installation directory: `examples/spblas/` for C. The table below lists elements of the parameter *matdescra*, their Fortran values, and their meanings. The parameter *matdescra* corresponds to the argument *descra* from NSB library.

Possible Values of the Parameter *matdescra* [*descra* - 1]

| | MKL interface | | NSB interface | Matrix characteristics |
|-------------|----------------------|----------------------|-------------------|-------------------------------------|
| | one-based indexing | zero-based indexing | | |
| data type | char * | char * | int * | |
| 1st element | <i>matdescra</i> [1] | <i>matdescra</i> [0] | <i>descra</i> [0] | matrix structure |
| value | G | G | 0 | general |
| | S | S | 1 | symmetric ($A = A^T$) |
| | H | H | 2 | Hermitian ($A = (A^H)$) |
| | T | T | 3 | triangular |
| | A | A | 4 | skew(anti)-symmetric ($A = -A^T$) |
| | D | D | 5 | diagonal |

| | MKL interface | | NSB interface | Matrix characteristics |
|-------------------|----------------------|----------------------|-------------------|----------------------------------|
| 2nd element value | <i>matdescra</i> [2] | <i>matdescra</i> [1] | <i>descra</i> [1] | upper/lower triangular indicator |
| | L | L | 1 | lower |
| | U | U | 2 | upper |
| 3rd element value | <i>matdescra</i> [3] | <i>matdescra</i> [2] | <i>descra</i> [2] | main diagonal type |
| | N | N | 0 | non-unit |
| | U | U | 1 | unit |
| 4th element value | <i>matdescra</i> [4] | <i>matdescra</i> [3] | <i>descra</i> [3] | type of indexing |
| | F | | 1 | one-based indexing |
| | | C | 0 | zero-based indexing |

In some cases possible element values of the parameter *matdescra* depend on the values of other elements. The [Table "Possible Combinations of Element Values of the Parameter *matdescra*"](#) lists all possible combinations of element values for both multiplication routines and triangular solvers.

Possible Combinations of Element Values of the Parameter *matdescra*

| Routines | <i>matdescra</i> [0] | <i>matdescra</i> [1] | <i>matdescra</i> [2] | <i>matdescra</i> [3] |
|--|----------------------|----------------------|----------------------|-------------------------|
| Multiplication Routines | G | ignored | ignored | F (default) or C |
| | S or H | L (default) | N (default) | F (default) or C |
| | S or H | L (default) | U | F (default) or C |
| | S or H | U | N (default) | F (default) or C |
| | S or H | U | U | F (default) or C |
| | A | L (default) | ignored | F (default) or C |
| | A | U | ignored | F (default) or C |
| Multiplication Routines and Triangular Solvers | T | L | U | F (default) or C |
| | T | L | N | F (default) or C |
| | T | U | U | F (default) or C |
| | T | U | N | F (default) or C |
| | D | ignored | N (default) | F (default) or C |
| | D | ignored | U | F (default) or C |

For a matrix in the skyline format with the main diagonal declared to be a unit, diagonal elements must be stored in the sparse representation even if they are zero. In all other formats, diagonal elements can be stored (if needed) in the sparse representation if they are not zero.

Operations with Partial Matrices

One of the distinctive features of the Intel MKL Sparse BLAS routines is a possibility to perform operations only on partial matrices composed of certain parts (triangles and the main diagonal) of the input sparse matrix. It can be done by setting properly first three elements of the parameter *matdescra*.

An arbitrary sparse matrix *A* can be decomposed as

$$A = L + D + U$$

where L is the strict lower triangle of A , U is the strict upper triangle of A , D is the main diagonal.

Table "Output Matrices for Multiplication Routines" shows correspondence between the output matrices and values of the parameter `matdescra` for the sparse matrix A for multiplication routines.

Output Matrices for Multiplication Routines

| <code>matdescra[0]</code> | <code>matdescra[1]</code> | <code>matdescra[2]</code> | Output Matrix |
|---------------------------|---------------------------|---------------------------|--|
| G | ignored | ignored | $\alpha * \text{op}(A) * x + \beta * y$ $\alpha * \text{op}(A) * B + \beta * C$ |
| S or H | L | N | $\alpha * \text{op}(L+D+L') * x + \beta * y$ $\alpha * \text{op}(L+D+L') * B + \beta * C$ |
| S or H | L | U | $\alpha * \text{op}(L+I+L') * x + \beta * y$ $\alpha * \text{op}(L+I+L') * B + \beta * C$ |
| S or H | U | N | $\alpha * \text{op}(U'+D+U) * x + \beta * y$ $\alpha * \text{op}(U'+D+U) * B + \beta * C$ |
| S or H | U | U | $\alpha * \text{op}(U'+I+U) * x + \beta * y$ $\alpha * \text{op}(U'+I+U) * B + \beta * C$ |
| T | L | U | $\alpha * \text{op}(L+I) * x + \beta * y$ $\alpha * \text{op}(L+I) * B + \beta * C$ |
| T | L | N | $\alpha * \text{op}(L+D) * x + \beta * y$ $\alpha * \text{op}(L+D) * B + \beta * C$ |
| T | U | U | $\alpha * \text{op}(U+I) * x + \beta * y$ $\alpha * \text{op}(U+I) * B + \beta * C$ |
| T | U | N | $\alpha * \text{op}(U+D) * x + \beta * y$ $\alpha * \text{op}(U+D) * B + \beta * C$ |
| A | L | ignored | $\alpha * \text{op}(L-L') * x + \beta * y$ $\alpha * \text{op}(L-L') * B + \beta * C$ |
| A | U | ignored | $\alpha * \text{op}(U-U') * x + \beta * y$ $\alpha * \text{op}(U-U') * B + \beta * C$ |
| D | ignored | N | $\alpha * D * x + \beta * y$ $\alpha * D * B + \beta * C$ |
| D | ignored | U | $\alpha * x + \beta * y$ $\alpha * B + \beta * C$ |

Table "Output Matrices for Triangular Solvers" shows correspondence between the output matrices and values of the parameter `matdescra` for the sparse matrix A for triangular solvers.

Output Matrices for Triangular Solvers

| <code>matdescra[0]</code> | <code>matdescra[1]</code> | <code>matdescra[2]</code> | Output Matrix |
|---------------------------|---------------------------|---------------------------|--|
| T | L | N | $\alpha * \text{inv}(\text{op}(L)) * x$ $\alpha * \text{inv}(\text{op}(L)) * B$ |
| T | L | U | $\alpha * \text{inv}(\text{op}(L)) * x$ |

| matdescra[0] | matdescra[1] | matdescra[2] | Output Matrix |
|---------------------|---------------------|---------------------|--|
| | | | $alpha*inv(op(L))*B$ |
| T | U | N | $alpha*inv(op(U))*x$ $alpha*inv(op(U))*B$ |
| T | U | U | $alpha*inv(op(U))*x$ $alpha*inv(op(U))*B$ |
| D | ignored | N | $alpha*inv(D)*x$ $alpha*inv(D)*B$ |
| D | ignored | U | $alpha*x$ $alpha*B$ |

Sparse BLAS Level 2 and Level 3 Routines.

Table "Sparse BLAS Level 2 and Level 3 Routines" lists the sparse BLAS Level 2 and Level 3 routines described in more detail later in this section.

Sparse BLAS Level 2 and Level 3 Routines

| Routine/Function | Description |
|---|---|
| Simplified interface, one-based indexing | |
| <code>mkl_?csrgev</code> | Computes matrix - vector product of a sparse general matrix in the CSR format (3-array variation) |
| <code>mkl_?bsrgev</code> | Computes matrix - vector product of a sparse general matrix in the BSR format (3-array variation). |
| <code>mkl_?coogev</code> | Computes matrix - vector product of a sparse general matrix in the coordinate format. |
| <code>mkl_?diagev</code> | Computes matrix - vector product of a sparse general matrix in the diagonal format. |
| <code>mkl_?csrsv</code> | Computes matrix - vector product of a sparse symmetrical matrix in the CSR format (3-array variation) |
| <code>mkl_?bsrsv</code> | Computes matrix - vector product of a sparse symmetrical matrix in the BSR format (3-array variation). |
| <code>mkl_?coosv</code> | Computes matrix - vector product of a sparse symmetrical matrix in the coordinate format. |
| <code>mkl_?diasv</code> | Computes matrix - vector product of a sparse symmetrical matrix in the diagonal format. |
| <code>mkl_?csrtrsv</code> | Triangular solvers with simplified interface for a sparse matrix in the CSR format (3-array variation). |
| <code>mkl_?bsrtrsv</code> | Triangular solver with simplified interface for a sparse matrix in the BSR format (3-array variation). |
| <code>mkl_?cootrsv</code> | Triangular solvers with simplified interface for a sparse matrix in the coordinate format. |

| Routine/Function | Description |
|--|--|
| <code>mkl_?diatrsv</code> | Triangular solvers with simplified interface for a sparse matrix in the diagonal format. |
| Simplified interface, zero-based indexing | |
| <code>mkl_cspblas_?csrgemv</code> | Computes matrix - vector product of a sparse general matrix in the CSR format (3-array variation) with zero-based indexing. |
| <code>mkl_cspblas_?bsrgemv</code> | Computes matrix - vector product of a sparse general matrix in the BSR format (3-array variation) with zero-based indexing. |
| <code>mkl_cspblas_?coogemv</code> | Computes matrix - vector product of a sparse general matrix in the coordinate format with zero-based indexing. |
| <code>mkl_cspblas_?csrsymv</code> | Computes matrix - vector product of a sparse symmetrical matrix in the CSR format (3-array variation) with zero-based indexing. |
| <code>mkl_cspblas_?bsrsymv</code> | Computes matrix - vector product of a sparse symmetrical matrix in the BSR format (3-array variation) with zero-based indexing. |
| <code>mkl_cspblas_?coosymv</code> | Computes matrix - vector product of a sparse symmetrical matrix in the coordinate format with zero-based indexing. |
| <code>mkl_cspblas_?csrtrsv</code> | Triangular solvers with simplified interface for a sparse matrix in the CSR format (3-array variation) with zero-based indexing. |
| <code>mkl_cspblas_?bsrtrsv</code> | Triangular solver with simplified interface for a sparse matrix in the BSR format (3-array variation) with zero-based indexing. |
| <code>mkl_cspblas_?cootrsv</code> | Triangular solver with simplified interface for a sparse matrix in the coordinate format with zero-based indexing. |

Typical (conventional) interface, one-based and zero-based indexing

| | |
|-------------------------|--|
| <code>mkl_?csrsv</code> | Computes matrix - vector product of a sparse matrix in the CSR format. |
| <code>mkl_?bsrsv</code> | Computes matrix - vector product of a sparse matrix in the BSR format. |
| <code>mkl_?cscmv</code> | Computes matrix - vector product for a sparse matrix in the CSC format. |
| <code>mkl_?coomv</code> | Computes matrix - vector product for a sparse matrix in the coordinate format. |
| <code>mkl_?csrsv</code> | Solves a system of linear equations for a sparse matrix in the CSR format. |
| <code>mkl_?bsrsv</code> | Solves a system of linear equations for a sparse matrix in the BSR format. |
| <code>mkl_?cscsv</code> | Solves a system of linear equations for a sparse matrix in the CSC format. |

| Routine/Function | Description |
|-------------------------|--|
| <code>mkl_?coosv</code> | Solves a system of linear equations for a sparse matrix in the coordinate format. |
| <code>mkl_?csrmm</code> | Computes matrix - matrix product of a sparse matrix in the CSR format |
| <code>mkl_?bsrmm</code> | Computes matrix - matrix product of a sparse matrix in the BSR format. |
| <code>mkl_?cscmm</code> | Computes matrix - matrix product of a sparse matrix in the CSC format |
| <code>mkl_?coomm</code> | Computes matrix - matrix product of a sparse matrix in the coordinate format. |
| <code>mkl_?csrsm</code> | Solves a system of linear matrix equations for a sparse matrix in the CSR format. |
| <code>mkl_?bsrsm</code> | Solves a system of linear matrix equations for a sparse matrix in the BSR format. |
| <code>mkl_?cscsm</code> | Solves a system of linear matrix equations for a sparse matrix in the CSC format. |
| <code>mkl_?coosm</code> | Solves a system of linear matrix equations for a sparse matrix in the coordinate format. |

Typical (conventional) interface, one-based indexing

| | |
|-------------------------|---|
| <code>mkl_?diamv</code> | Computes matrix - vector product of a sparse matrix in the diagonal format. |
| <code>mkl_?skymv</code> | Computes matrix - vector product for a sparse matrix in the skyline storage format. |
| <code>mkl_?diasv</code> | Solves a system of linear equations for a sparse matrix in the diagonal format. |
| <code>mkl_?skysv</code> | Solves a system of linear equations for a sparse matrix in the skyline format. |
| <code>mkl_?diamm</code> | Computes matrix - matrix product of a sparse matrix in the diagonal format. |
| <code>mkl_?skymm</code> | Computes matrix - matrix product of a sparse matrix in the skyline storage format. |
| <code>mkl_?diasm</code> | Solves a system of linear matrix equations for a sparse matrix in the diagonal format. |
| <code>mkl_?skysm</code> | Solves a system of linear matrix equations for a sparse matrix in the skyline storage format. |

Auxiliary routines

Matrix converters

| | |
|--------------------------|---|
| <code>mkl_?dnscsr</code> | Converts a sparse matrix in uncompressed representation to CSR format (3-array variation) and vice versa. |
| <code>mkl_?csrcoo</code> | Converts a sparse matrix in CSR format (3-array variation) to coordinate format and vice versa. |

| Routine/Function | Description |
|-------------------------------|---|
| <code>mkl_?csrbsr</code> | Converts a sparse matrix in CSR format to BSR format (3-array variations) and vice versa. |
| <code>mkl_?csrcsc</code> | Converts a sparse matrix in CSR format to CSC format and vice versa (3-array variations). |
| <code>mkl_?csrdia</code> | Converts a sparse matrix in CSR format (3-array variation) to diagonal format and vice versa. |
| <code>mkl_?csrsky</code> | Converts a sparse matrix in CSR format (3-array variation) to sky line format and vice versa. |
| Operations on sparse matrices | |
| <code>mkl_?csradd</code> | Computes the sum of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing. |
| <code>mkl_?csrmultcsr</code> | Computes the product of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing. |
| <code>mkl_?csrmultd</code> | Computes product of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing. The result is stored in the dense matrix. |

`mkl_?csrgemv`

Computes matrix - vector product of a sparse general matrix stored in the CSR format (3-array variation) with one-based indexing.

Syntax

```
void mkl_scsrgemv (const char *transa , const MKL_INT *m , const float *a , const MKL_INT *ia , const MKL_INT *ja , const float *x , float *y );
```

```
void mkl_dcsrgemv (const char *transa , const MKL_INT *m , const double *a , const MKL_INT *ia , const MKL_INT *ja , const double *x , double *y );
```

```
void mkl_ccsrgemv (const char *transa , const MKL_INT *m , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zcsrgemv (const char *transa , const MKL_INT *m , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- `mkl.h`

Description

The `mkl_?csrgemv` routine performs a matrix-vector operation defined as

$$y := A * x$$

or

$$y := A^T * x,$$

where:

x and y are vectors,

A is an m -by- m sparse square matrix in the CSR format (3-array variation), A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|---------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then as $y := A*x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := A^T*x$, |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>a</i> | Array containing non-zero elements of the matrix <i>A</i> . Its length is equal to the number of non-zero elements in the matrix <i>A</i> . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ia</i> | Array of length $m + 1$, containing indices of elements in the array <i>a</i> , such that $ia[i] - ia[0]$ is the index in the array <i>a</i> of the first non-zero element from the row <i>i</i> . The value of the last element $ia[m] - ia[0]$ is equal to the number of non-zeros. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ja</i> | Array containing the column indices plus one for each non-zero element of the matrix <i>A</i> . Its length is equal to the length of the array <i>a</i> . Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>x</i> | Array, size is <i>m</i> . On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least <i>m</i> . On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_?bsrgemv

Computes matrix - vector product of a sparse general matrix stored in the BSR format (3-array variation) with one-based indexing.

Syntax

```
void mkl_sbsrgemv (const char *transa , const MKL_INT *m , const MKL_INT *lb , const
float *a , const MKL_INT *ia , const MKL_INT *ja , const float *x , float *y );
void mkl_dbsrgemv (const char *transa , const MKL_INT *m , const MKL_INT *lb , const
double *a , const MKL_INT *ia , const MKL_INT *ja , const double *x , double *y );
void mkl_cbsrgemv (const char *transa , const MKL_INT *m , const MKL_INT *lb , const
MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex8 *x ,
MKL_Complex8 *y );
```

```
void mkl_zbsrgemv (const char *transa , const MKL_INT *m , const MKL_INT *lb , const
MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex16 *x ,
MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?bsrgemv` routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := AT*x,
```

where:

x and y are vectors,

A is an m -by- m block sparse square matrix in the BSR format (3-array variation), A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|---------------|---|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then the matrix-vector product is computed as $y := A*x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as $y := A^T*x$, |
| <i>m</i> | Number of block rows of the matrix A . |
| <i>lb</i> | Size of the block in the matrix A . |
| <i>a</i> | Array containing elements of non-zero blocks of the matrix A . Its length is equal to the number of non-zero blocks in the matrix A multiplied by $lb*lb$. Refer to <i>values</i> array description in BSR Format for more details. |
| <i>ia</i> | Array of length $(m + 1)$, containing indices of block in the array a , such that $ia[i] - ia[0]$ is the index in the array a of the first non-zero element from the row i . The value of the last element $ia[m] - ia[0]$ is equal to the number of non-zero blocks. Refer to <i>rowIndex</i> array description in BSR Format for more details. |
| <i>ja</i> | Array containing the column indices plus one for each non-zero block in the matrix A . Its length is equal to the number of non-zero blocks of the matrix A . Refer to <i>columns</i> array description in BSR Format for more details. |
| <i>x</i> | Array, size $(m*lb)$. |

On entry, the array x must contain the vector x .

Output Parameters

y Array, size at least $(m \cdot lb)$.
On exit, the array y must contain the vector y .

mkl_?coogemv

Computes matrix-vector product of a sparse general matrix stored in the coordinate format with one-based indexing.

Syntax

```
void mkl_scoogemv (const char *transa , const MKL_INT *m , const float *val , const
MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const float *x , float
*y );
```

```
void mkl_dcoogemv (const char *transa , const MKL_INT *m , const double *val , const
MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const double *x , double
*y );
```

```
void mkl_ccoogemv (const char *transa , const MKL_INT *m , const MKL_Complex8 *val ,
const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const MKL_Complex8
*x , MKL_Complex8 *y );
```

```
void mkl_zcoogemv (const char *transa , const MKL_INT *m , const MKL_Complex16 *val ,
const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const
MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?coogemv` routine performs a matrix-vector operation defined as

```
 $y := A * x$ 
```

or

```
 $y := A^T * x,$ 
```

where:

x and y are vectors,

A is an m -by- m sparse square matrix in the coordinate format, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

$transa$ Specifies the operation.

If *transa* = 'N' or 'n', then the matrix-vector product is computed as $y := A*x$

If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as $y := A^T*x$,

| | |
|---------------|---|
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>val</i> | Array of length <i>nnz</i> , contains non-zero elements of the matrix <i>A</i> in the arbitrary order. Refer to <i>values</i> array description in Coordinate Format for more details. |
| <i>rowind</i> | Array of length <i>nnz</i> , contains the row indices plus one for each non-zero element of the matrix <i>A</i> . Refer to <i>rows</i> array description in Coordinate Format for more details. |
| <i>colind</i> | Array of length <i>nnz</i> , contains the column indices plus one for each non-zero element of the matrix <i>A</i> . Refer to <i>columns</i> array description in Coordinate Format for more details. |
| <i>nnz</i> | Specifies the number of non-zero element of the matrix <i>A</i> . Refer to <i>nnz</i> description in Coordinate Format for more details. |
| <i>x</i> | Array, size is <i>m</i> . One entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least <i>m</i> . On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_?diagemv

Computes matrix - vector product of a sparse general matrix stored in the diagonal format with one-based indexing.

Syntax

```
void mkl_sdiagemv (const char *transa , const MKL_INT *m , const float *val , const
MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const float *x , float
*y );

void mkl_ddiagemv (const char *transa , const MKL_INT *m , const double *val , const
MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const double *x , double
*y );

void mkl_cdiagemv (const char *transa , const MKL_INT *m , const MKL_Complex8 *val ,
const MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const MKL_Complex8
*x , MKL_Complex8 *y );

void mkl_zdiagemv (const char *transa , const MKL_INT *m , const MKL_Complex16 *val ,
const MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const MKL_Complex16
*x , MKL_Complex16 *y );
```

Include Files

- `mkl.h`

Description

The `mkl_?diagemv` routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := AT*x,
```

where:

x and y are vectors,

A is an m -by- m sparse square matrix in the diagonal storage format, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|---------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $y := A*x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := A^T*x$, |
| <i>m</i> | Number of rows of the matrix A . |
| <i>val</i> | Two-dimensional array of size $lval*ndiag$, contains non-zero diagonals of the matrix A . Refer to <i>values</i> array description in Diagonal Storage Scheme for more details. |
| <i>lval</i> | Leading dimension of val , $lval \geq m$. Refer to <i>lval</i> description in Diagonal Storage Scheme for more details. |
| <i>idiag</i> | Array of length <i>ndiag</i> , contains the distances between main diagonal and each non-zero diagonals in the matrix A . Refer to <i>distance</i> array description in Diagonal Storage Scheme for more details. |
| <i>ndiag</i> | Specifies the number of non-zero diagonals of the matrix A . |
| <i>x</i> | Array, size is m . On entry, the array x must contain the vector x . |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Array, size at least m . On exit, the array y must contain the vector y . |
|----------|--|

mkl_?csrsvmv

Computes matrix - vector product of a sparse symmetrical matrix stored in the CSR format (3-array variation) with one-based indexing.

Syntax

```
void mkl_scsrsvmv (const char *uplo , const MKL_INT *m , const float *a , const MKL_INT *ia , const MKL_INT *ja , const float *x , float *y );
```

```
void mkl_dcsrsvmv (const char *uplo , const MKL_INT *m , const double *a , const MKL_INT *ia , const MKL_INT *ja , const double *x , double *y );
```

```
void mkl_ccsrsvmv (const char *uplo , const MKL_INT *m , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zcsrsvmv (const char *uplo , const MKL_INT *m , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?csrsvmv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and y are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the CSR format (3-array variation).

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|-------------|---|
| <i>uplo</i> | Specifies whether the upper or low triangle of the matrix A is used. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix A is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix A is used. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>a</i> | Array containing non-zero elements of the matrix A . Its length is equal to the number of non-zero elements in the matrix A . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ia</i> | Array of length $m + 1$, containing indices of elements in the array a , such that $ia[i] - ia[0]$ is the index in the array a of the first non-zero element from the row i . The value of the last element $ia[m] - ia[0]$ is equal to the number of non-zeros. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details. |

| | |
|-----------|---|
| <i>ja</i> | Array containing the column indices plus one for each non-zero element of the matrix <i>A</i> . Its length is equal to the length of the array <i>a</i> . Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>x</i> | Array, size is <i>m</i> . On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least <i>m</i> . On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_?bsrsymv

Computes matrix-vector product of a sparse symmetrical matrix stored in the BSR format (3-array variation) with one-based indexing.

Syntax

```
void mkl_sbsrsymv (const char *uplo , const MKL_INT *m , const MKL_INT *lb , const
float *a , const MKL_INT *ia , const MKL_INT *ja , const float *x , float *y );
void mkl_dbsrsymv (const char *uplo , const MKL_INT *m , const MKL_INT *lb , const
double *a , const MKL_INT *ia , const MKL_INT *ja , const double *x , double *y );
void mkl_cbsrsymv (const char *uplo , const MKL_INT *m , const MKL_INT *lb , const
MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex8 *x ,
MKL_Complex8 *y );
void mkl_zbsrsymv (const char *uplo , const MKL_INT *m , const MKL_INT *lb , const
MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex16 *x ,
MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?bsrsymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and *y* are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the BSR format (3-array variation).

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|-------------|--|
| <i>uplo</i> | Specifies whether the upper or low triangle of the matrix <i>A</i> is considered. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix <i>A</i> is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix <i>A</i> is used. |
| <i>m</i> | Number of block rows of the matrix <i>A</i> . |
| <i>lb</i> | Size of the block in the matrix <i>A</i> . |
| <i>a</i> | Array containing elements of non-zero blocks of the matrix <i>A</i> . Its length is equal to the number of non-zero blocks in the matrix <i>A</i> multiplied by <i>lb*lb</i> . Refer to <i>values</i> array description in BSR Format for more details. |
| <i>ia</i> | Array of length $(m + 1)$, containing indices of block in the array <i>a</i> , such that $ia[i] - ia[0]$ is the index in the array <i>a</i> of the first non-zero element from the row <i>i</i> . The value of the last element $ia[m] - ia[0]$ is equal to the number of non-zero blocks. Refer to <i>rowIndex</i> array description in BSR Format for more details. |
| <i>ja</i> | Array containing the column indices plus one for each non-zero block in the matrix <i>A</i> . Its length is equal to the number of non-zero blocks of the matrix <i>A</i> . Refer to <i>columns</i> array description in BSR Format for more details. |
| <i>x</i> | Array, size $(m*lb)$. On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least $(m*lb)$. On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_?coosymv

Computes matrix - vector product of a sparse symmetrical matrix stored in the coordinate format with one-based indexing.

Syntax

```
void mkl_scoosymv (const char *uplo , const MKL_INT *m , const float *val , const
MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const float *x , float
*y );
```

```
void mkl_dcoosymv (const char *uplo , const MKL_INT *m , const double *val , const
MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const double *x , double
*y );
```

```
void mkl_ccoosymv (const char *uplo , const MKL_INT *m , const MKL_Complex8 *val ,
const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const MKL_Complex8
*x , MKL_Complex8 *y );
```

```
void mkl_zcoosymv (const char *uplo , const MKL_INT *m , const MKL_Complex16 *val ,
const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const
MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- `mkl.h`

Description

The `mkl_?coosymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and y are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the coordinate format.

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|---------------------|--|
| <code>uplo</code> | Specifies whether the upper or low triangle of the matrix A is used. If <code>uplo = 'U' or 'u'</code> , then the upper triangle of the matrix A is used. If <code>uplo = 'L' or 'l'</code> , then the low triangle of the matrix A is used. |
| <code>m</code> | Number of rows of the matrix A . |
| <code>val</code> | Array of length <code>nnz</code> , contains non-zero elements of the matrix A in the arbitrary order. Refer to <code>values</code> array description in Coordinate Format for more details. |
| <code>rowind</code> | Array of length <code>nnz</code> , contains the row indices plus one for each non-zero element of the matrix A . Refer to <code>rows</code> array description in Coordinate Format for more details. |
| <code>colind</code> | Array of length <code>nnz</code> , contains the column indices plus one for each non-zero element of the matrix A . Refer to <code>columns</code> array description in Coordinate Format for more details. |
| <code>nnz</code> | Specifies the number of non-zero element of the matrix A . Refer to <code>nnz</code> description in Coordinate Format for more details. |
| <code>x</code> | Array, size is m . On entry, the array <code>x</code> must contain the vector x . |

Output Parameters

| | |
|----------------|---|
| <code>y</code> | Array, size at least m . On exit, the array <code>y</code> must contain the vector y . |
|----------------|---|

mkl_?diasymv

Computes matrix - vector product of a sparse symmetrical matrix stored in the diagonal format with one-based indexing.

Syntax

```
void mkl_sdiasymv (const char *uplo , const MKL_INT *m , const float *val , const
MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const float *x , float
*y );
```

```
void mkl_ddiasymv (const char *uplo , const MKL_INT *m , const double *val , const
MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const double *x , double
*y );
```

```
void mkl_cdiasymv (const char *uplo , const MKL_INT *m , const MKL_Complex8 *val ,
const MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const MKL_Complex8
*x , MKL_Complex8 *y );
```

```
void mkl_zdiasymv (const char *uplo , const MKL_INT *m , const MKL_Complex16 *val ,
const MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const MKL_Complex16
*x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?diasymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and y are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix.

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|-------------|--|
| <i>uplo</i> | Specifies whether the upper or low triangle of the matrix A is used. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix A is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix A is used. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>val</i> | Two-dimensional array of size <i>lval</i> by <i>ndiag</i> , contains non-zero diagonals of the matrix A . Refer to <i>values</i> array description in Diagonal Storage Scheme for more details. |
| <i>lval</i> | Leading dimension of <i>val</i> , $lval \geq m$. Refer to <i>lval</i> description in Diagonal Storage Scheme for more details. |

| | |
|--------------|---|
| <i>idiag</i> | Array of length <i>ndiag</i> , contains the distances between main diagonal and each non-zero diagonals in the matrix <i>A</i> . Refer to <i>distance</i> array description in Diagonal Storage Scheme for more details. |
| <i>ndiag</i> | Specifies the number of non-zero diagonals of the matrix <i>A</i> . |
| <i>x</i> | Array, size is <i>m</i> . On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least <i>m</i> . On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_?csrtrsv

Triangular solvers with simplified interface for a sparse matrix in the CSR format (3-array variation) with one-based indexing.

Syntax

```
void mkl_scsrtrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const float *a , const MKL_INT *ia , const MKL_INT *ja , const float *x ,
float *y );
```

```
void mkl_dcsrtrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const double *a , const MKL_INT *ia , const MKL_INT *ja , const double
*x , double *y );
```

```
void mkl_ccsrtrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const
MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zcsrtrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const
MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?csrtrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the CSR format (3 array variation):

$$\bar{A}^*y = x$$

or

$$\bar{A}^T y = x,$$

where:

x and *y* are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of *A*.

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

uplo Specifies whether the upper or low triangle of the matrix A is used.
 If *uplo* = 'U' or 'u', then the upper triangle of the matrix A is used.
 If *uplo* = 'L' or 'l', then the low triangle of the matrix A is used.

transa Specifies the system of linear equations.
 If *transa* = 'N' or 'n', then $A*y = x$
 If *transa* = 'T' or 't' or 'C' or 'c', then $A^T*y = x$,

diag Specifies whether A is unit triangular.
 If *diag* = 'U' or 'u', then A is a unit triangular.
 If *diag* = 'N' or 'n', then A is not unit triangular.

m Number of rows of the matrix A .

a Array containing non-zero elements of the matrix A . Its length is equal to the number of non-zero elements in the matrix A . Refer to *values* array description in [Sparse Matrix Storage Formats](#) for more details.

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

ia Array of length $m + 1$, containing indices of elements in the array *a*, such that $ia[i] - ia[0]$ is the index in the array *a* of the first non-zero element from the row i . The value of the last element $ia[m] - ia[0]$ is equal to the number of non-zeros. Refer to *rowIndex* array description in [Sparse Matrix Storage Formats](#) for more details.

ja Array containing the column indices plus one for each non-zero element of the matrix A .

Its length is equal to the length of the array *a*. Refer to *columns* array description in [Sparse Matrix Storage Formats](#) for more details.

NOTE

Column indices must be sorted in increasing order for each row.

x Array, size is m .
 On entry, the array *x* must contain the vector x .

Output Parameters

y Array, size at least m .
Contains the vector y .

`mkl_?bsrtrsv`

Triangular solver with simplified interface for a sparse matrix stored in the BSR format (3-array variation) with one-based indexing.

Syntax

```
void mkl_sbsrtrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_INT *lb , const float *a , const MKL_INT *ia , const MKL_INT
*ja , const float *x , float *y );
```

```
void mkl_dbsrtrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_INT *lb , const double *a , const MKL_INT *ia , const MKL_INT
*ja , const double *x , double *y );
```

```
void mkl_cbsrtrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_INT *lb , const MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zbsrtrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_INT *lb , const MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- `mkl.h`

Description

The `mkl_?bsrtrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the BSR format (3-array variation) :

```
 $y := A * x$ 
```

or

```
 $y := A^T * x,$ 
```

where:

x and y are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

$uplo$ Specifies the upper or low triangle of the matrix A is used.
If $uplo = 'U'$ or $'u'$, then the upper triangle of the matrix A is used.

If $uplo = 'L'$ or $'l'$, then the low triangle of the matrix A is used.

$transa$

Specifies the operation.

If $transa = 'N'$ or $'n'$, then the matrix-vector product is computed as $y := A*x$

If $transa = 'T'$ or $'t'$ or $'C'$ or $'c'$, then the matrix-vector product is computed as $y := A^T*x$.

$diag$

Specifies whether A is a unit triangular matrix.

If $diag = 'U'$ or $'u'$, then A is a unit triangular.

If $diag = 'N'$ or $'n'$, then A is not a unit triangular.

m

Number of block rows of the matrix A .

lb

Size of the block in the matrix A .

a

Array containing elements of non-zero blocks of the matrix A . Its length is equal to the number of non-zero blocks in the matrix A multiplied by $lb*lb$. Refer to *values* array description in [BSR Format](#) for more details.

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

ia

Array of length $(m + 1)$, containing indices of block in the array a , such that $ia[I] - ia[0]$ is the index in the array a of the first non-zero element from the row I . The value of the last element $ia[m] - ia[0]$ is equal to the number of non-zero blocks. Refer to *rowIndex* array description in [BSR Format](#) for more details.

ja

Array containing the column indices plus one for each non-zero block in the matrix A .

Its length is equal to the number of non-zero blocks of the matrix A . Refer to *columns* array description in [BSR Format](#) for more details.

x

Array, size $(m*lb)$.

On entry, the array x must contain the vector x .

Output Parameters

y

Array, size at least $(m*lb)$.

On exit, the array y must contain the vector y .

mkl_?cootrsv

Triangular solvers with simplified interface for a sparse matrix in the coordinate format with one-based indexing.

Syntax

```
void mkl_scootrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const float *val , const MKL_INT *rowind , const MKL_INT *colind , const
MKL_INT *nnz , const float *x , float *y );
```

```
void mkl_dcootrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const double *val , const MKL_INT *rowind , const MKL_INT *colind , const
MKL_INT *nnz , const double *x , double *y );
```

```
void mkl_ccootrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_Complex8 *val , const MKL_INT *rowind , const MKL_INT *colind ,
const MKL_INT *nnz , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zcootrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_Complex16 *val , const MKL_INT *rowind , const MKL_INT *colind ,
const MKL_INT *nnz , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?cootrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the coordinate format:

$$A*y = x$$

or

$$A^T*y = x,$$

where:

x and y are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|---------------|--|
| <i>uplo</i> | Specifies whether the upper or low triangle of the matrix A is considered. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix A is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix A is used. |
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $A*y = x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $A^T*y = x$, |

| | |
|---------------|---|
| <i>diag</i> | Specifies whether <i>A</i> is unit triangular. If <i>diag</i> = 'U' or 'u', then <i>A</i> is unit triangular. If <i>diag</i> = 'N' or 'n', then <i>A</i> is not unit triangular. |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>val</i> | Array of length <i>nnz</i> , contains non-zero elements of the matrix <i>A</i> in the arbitrary order. Refer to <i>values</i> array description in Coordinate Format for more details. |
| <i>rowind</i> | Array of length <i>nnz</i> , contains the row indices plus one for each non-zero element of the matrix <i>A</i> . Refer to <i>rows</i> array description in Coordinate Format for more details. |
| <i>colind</i> | Array of length <i>nnz</i> , contains the column indices plus one for each non-zero element of the matrix <i>A</i> . Refer to <i>columns</i> array description in Coordinate Format for more details. |
| <i>nnz</i> | Specifies the number of non-zero element of the matrix <i>A</i> . Refer to <i>nnz</i> description in Coordinate Format for more details. |
| <i>x</i> | Array, size is <i>m</i> . On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least <i>m</i> . Contains the vector <i>y</i> . |
|----------|---|

mkl_?diatrsv

Triangular solvers with simplified interface for a sparse matrix in the diagonal format with one-based indexing.

Syntax

```
void mkl_sdiatrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const float *val , const MKL_INT *lval , const MKL_INT *idiag , const
MKL_INT *ndiag , const float *x , float *y );
```

```
void mkl_ddiatrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const double *val , const MKL_INT *lval , const MKL_INT *idiag , const
MKL_INT *ndiag , const double *x , double *y );
```

```
void mkl_cdiatrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_Complex8 *val , const MKL_INT *lval , const MKL_INT *idiag ,
const MKL_INT *ndiag , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zdiatrsv (const char *uplo , const char *transa , const char *diag , const
MKL_INT *m , const MKL_Complex16 *val , const MKL_INT *lval , const MKL_INT *idiag ,
const MKL_INT *ndiag , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- `mkl.h`

Description

The `mkl_?diatrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the diagonal format:

$$A^*y = x$$

or

$$A^T*y = x,$$

where:

x and y are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|---------------|--|
| <i>uplo</i> | Specifies whether the upper or low triangle of the matrix A is used. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix A is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix A is used. |
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $A^*y = x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $A^T*y = x$, |
| <i>diag</i> | Specifies whether A is unit triangular. If <i>diag</i> = 'U' or 'u', then A is unit triangular. If <i>diag</i> = 'N' or 'n', then A is not unit triangular. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>val</i> | Two-dimensional array of size <i>lval</i> by <i>ndiag</i> , contains non-zero diagonals of the matrix A . Refer to <i>values</i> array description in Diagonal Storage Scheme for more details. |
| <i>lval</i> | Leading dimension of <i>val</i> , $lval \geq m$. Refer to <i>lval</i> description in Diagonal Storage Scheme for more details. |
| <i>idiag</i> | Array of length <i>ndiag</i> , contains the distances between main diagonal and each non-zero diagonals in the matrix A . |

NOTE

All elements of this array must be sorted in increasing order.

Refer to *distance* array description in [Diagonal Storage Scheme](#) for more details.

ndiag Specifies the number of non-zero diagonals of the matrix *A*.

x Array, size is *m*.
On entry, the array *x* must contain the vector *x*.

Output Parameters

y Array, size at least *m*.
Contains the vector *y*.

mkl_cspblas_?csrgermv

Computes matrix - vector product of a sparse general matrix stored in the CSR format (3-array variation) with zero-based indexing.

Syntax

```
void mkl_cspblas_scsrgermv (const char *transa , const MKL_INT *m , const float *a ,
const MKL_INT *ia , const MKL_INT *ja , const float *x , float *y );
void mkl_cspblas_dcsrgermv (const char *transa , const MKL_INT *m , const double *a ,
const MKL_INT *ia , const MKL_INT *ja , const double *x , double *y );
void mkl_cspblas_ccsrgermv (const char *transa , const MKL_INT *m , const MKL_Complex8
*a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex8 *x , MKL_Complex8 *y );
void mkl_cspblas_zcsrgermv (const char *transa , const MKL_INT *m , const MKL_Complex16
*a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex16 *x , MKL_Complex16
*y );
```

Include Files

- mkl.h

Description

The `mkl_cspblas_?csrgermv` routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := AT*x,
```

where:

x and *y* are vectors,

A is an *m*-by-*m* sparse square matrix in the CSR format (3-array variation) with zero-based indexing, *A*^T is the transpose of *A*.

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

| | |
|---------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then the matrix-vector product is computed as $y := A*x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as $y := A^T*x$, |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>a</i> | Array containing non-zero elements of the matrix <i>A</i> . Its length is equal to the number of non-zero elements in the matrix <i>A</i> . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ia</i> | Array of length $m + 1$, containing indices of elements in the array <i>a</i> , such that <i>ia</i> [<i>I</i>] is the index in the array <i>a</i> of the first non-zero element from the row <i>I</i> . The value of the last element <i>ia</i> [<i>m</i>] is equal to the number of non-zeros. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ja</i> | Array containing the column indices for each non-zero element of the matrix <i>A</i> . Its length is equal to the length of the array <i>a</i> . Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>x</i> | Array, size is <i>m</i> . One entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least <i>m</i> . On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_cspblas_?bsrgemv

Computes matrix - vector product of a sparse general matrix stored in the BSR format (3-array variation) with zero-based indexing.

Syntax

```
void mkl_cspblas_sbsrgemv (const char *transa , const MKL_INT *m , const MKL_INT *lb ,
const float *a , const MKL_INT *ia , const MKL_INT *ja , const float *x , float *y );
void mkl_cspblas_dbsrgemv (const char *transa , const MKL_INT *m , const MKL_INT *lb ,
const double *a , const MKL_INT *ia , const MKL_INT *ja , const double *x , double
*y );
void mkl_cspblas_cbsrgemv (const char *transa , const MKL_INT *m , const MKL_INT *lb ,
const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex8 *x ,
MKL_Complex8 *y );
void mkl_cspblas_zbsrgemv (const char *transa , const MKL_INT *m , const MKL_INT *lb ,
const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex16
*x , MKL_Complex16 *y );
```

Include Files

- `mkl.h`

Description

The `mkl_cspblas_?bsrgemv` routine performs a matrix-vector operation defined as

$$y := A*x$$

or

$$y := A^T*x,$$

where:

x and y are vectors,

A is an m -by- m block sparse square matrix in the BSR format (3-array variation) with zero-based indexing, A^T is the transpose of A .

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

| | |
|---------------|---|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then the matrix-vector product is computed as $y := A*x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as $y := A^T*x$, |
| <i>m</i> | Number of block rows of the matrix A . |
| <i>lb</i> | Size of the block in the matrix A . |
| <i>a</i> | Array containing elements of non-zero blocks of the matrix A . Its length is equal to the number of non-zero blocks in the matrix A multiplied by $lb*lb$. Refer to <i>values</i> array description in BSR Format for more details. |
| <i>ia</i> | Array of length $(m + 1)$, containing indices of block in the array a , such that $ia[i]$ is the index in the array a of the first non-zero element from the row i . The value of the last element $ia[m]$ is equal to the number of non-zero blocks. Refer to <i>rowIndex</i> array description in BSR Format for more details. |
| <i>ja</i> | Array containing the column indices for each non-zero block in the matrix A . Its length is equal to the number of non-zero blocks of the matrix A . Refer to <i>columns</i> array description in BSR Format for more details. |
| <i>x</i> | Array, size $(m*lb)$. On entry, the array x must contain the vector x . |

Output Parameters

y Array, size at least $(m \cdot lb)$.
On exit, the array *y* must contain the vector *y*.

mkl_cspblas_?coogemv

Computes matrix - vector product of a sparse general matrix stored in the coordinate format with zero-based indexing.

Syntax

```
void mkl_cspblas_scoogemv (const char *transa , const MKL_INT *m , const float *val ,
const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const float *x ,
float *y );
```

```
void mkl_cspblas_dcoogemv (const char *transa , const MKL_INT *m , const double *val ,
const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const double *x ,
double *y );
```

```
void mkl_cspblas_ccoogemv (const char *transa , const MKL_INT *m , const MKL_Complex8
*val , const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const
MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_cspblas_zcoogemv (const char *transa , const MKL_INT *m , const MKL_Complex16
*val , const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const
MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_cspblas_dcoogemv` routine performs a matrix-vector operation defined as

```
y := A*x
```

or

```
y := AT*x,
```

where:

x and *y* are vectors,

A is an *m*-by-*m* sparse square matrix in the coordinate format with zero-based indexing, *A*^T is the transpose of *A*.

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

transa Specifies the operation.
If *transa* = 'N' or 'n', then the matrix-vector product is computed as
y := *A***x*

If *transa* = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as $y := A^T * x$.

| | |
|---------------|--|
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>val</i> | Array of length <i>nnz</i> , contains non-zero elements of the matrix <i>A</i> in the arbitrary order. Refer to <i>values</i> array description in Coordinate Format for more details. |
| <i>rowind</i> | Array of length <i>nnz</i> , contains the row indices for each non-zero element of the matrix <i>A</i> . Refer to <i>rows</i> array description in Coordinate Format for more details. |
| <i>colind</i> | Array of length <i>nnz</i> , contains the column indices for each non-zero element of the matrix <i>A</i> . Refer to <i>columns</i> array description in Coordinate Format for more details. |
| <i>nnz</i> | Specifies the number of non-zero element of the matrix <i>A</i> . Refer to <i>nnz</i> description in Coordinate Format for more details. |
| <i>x</i> | Array, size is <i>m</i> . On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least <i>m</i> . On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_cspblas_?csrsmv

Computes matrix-vector product of a sparse symmetrical matrix stored in the CSR format (3-array variation) with zero-based indexing.

Syntax

```
void mkl_cspblas_scsrsmv (const char *uplo , const MKL_INT *m , const float *a , const
MKL_INT *ia , const MKL_INT *ja , const float *x , float *y );

void mkl_cspblas_dcsrsmv (const char *uplo , const MKL_INT *m , const double *a ,
const MKL_INT *ia , const MKL_INT *ja , const double *x , double *y );

void mkl_cspblas_ccsrsmv (const char *uplo , const MKL_INT *m , const MKL_Complex8
*a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex8 *x , MKL_Complex8 *y );

void mkl_cspblas_zcsrsmv (const char *uplo , const MKL_INT *m , const MKL_Complex16
*a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex16 *x , MKL_Complex16
*y );
```

Include Files

- mkl.h

Description

The `mkl_cspblas_?csrsvmv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and y are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the CSR format (3-array variation) with zero-based indexing.

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

| | |
|-------------------|---|
| <code>uplo</code> | Specifies whether the upper or low triangle of the matrix A is used. If <code>uplo = 'U' or 'u'</code> , then the upper triangle of the matrix A is used. If <code>uplo = 'L' or 'l'</code> , then the low triangle of the matrix A is used. |
| <code>m</code> | Number of rows of the matrix A . |
| <code>a</code> | Array containing non-zero elements of the matrix A . Its length is equal to the number of non-zero elements in the matrix A . Refer to <code>values</code> array description in Sparse Matrix Storage Formats for more details. |
| <code>ia</code> | Array of length $m + 1$, containing indices of elements in the array <code>a</code> , such that <code>ia[i]</code> is the index in the array <code>a</code> of the first non-zero element from the row i . The value of the last element <code>ia[m]</code> is equal to the number of non-zeros. Refer to <code>rowIndex</code> array description in Sparse Matrix Storage Formats for more details. |
| <code>ja</code> | Array containing the column indices for each non-zero element of the matrix A . Its length is equal to the length of the array <code>a</code> . Refer to <code>columns</code> array description in Sparse Matrix Storage Formats for more details. |
| <code>x</code> | Array, size is m . On entry, the array <code>x</code> must contain the vector x . |

Output Parameters

| | |
|----------------|---|
| <code>y</code> | Array, size at least m . On exit, the array <code>y</code> must contain the vector y . |
|----------------|---|

`mkl_cspblas_?bsrsvmv`

Computes matrix-vector product of a sparse symmetrical matrix stored in the BSR format (3-arrays variation) with zero-based indexing.

Syntax

```
void mkl_cspblas_sbsrsymv (const char *uplo , const MKL_INT *m , const MKL_INT *lb ,
const float *a , const MKL_INT *ia , const MKL_INT *ja , const float *x , float *y );
void mkl_cspblas_dbsrsymv (const char *uplo , const MKL_INT *m , const MKL_INT *lb ,
const double *a , const MKL_INT *ia , const MKL_INT *ja , const double *x , double
*y );
void mkl_cspblas_cbsrsymv (const char *uplo , const MKL_INT *m , const MKL_INT *lb ,
const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex8 *x ,
MKL_Complex8 *y );
void mkl_cspblas_zbsrsymv (const char *uplo , const MKL_INT *m , const MKL_INT *lb ,
const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_Complex16
*x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_cspblas_?bsrsymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and y are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the BSR format (3-array variation) with zero-based indexing.

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

| | |
|-------------------|--|
| <code>uplo</code> | Specifies whether the upper or low triangle of the matrix A is used. If <code>uplo = 'U' or 'u'</code> , then the upper triangle of the matrix A is used. If <code>uplo = 'L' or 'l'</code> , then the low triangle of the matrix A is used. |
| <code>m</code> | Number of block rows of the matrix A . |
| <code>lb</code> | Size of the block in the matrix A . |
| <code>a</code> | Array containing elements of non-zero blocks of the matrix A . Its length is equal to the number of non-zero blocks in the matrix A multiplied by $lb*lb$. Refer to <code>values</code> array description in BSR Format for more details. |

| | |
|-----------|--|
| <i>ia</i> | Array of length $(m + 1)$, containing indices of block in the array <i>a</i> , such that <i>ia</i> [<i>i</i>] is the index in the array <i>a</i> of the first non-zero element from the row <i>i</i> . The value of the last element <i>ia</i> [<i>m</i>] is equal to the number of non-zero blocks. Refer to <i>rowIndex</i> array description in BSR Format for more details. |
| <i>ja</i> | Array containing the column indices for each non-zero block in the matrix <i>A</i> . Its length is equal to the number of non-zero blocks of the matrix <i>A</i> . Refer to <i>columns</i> array description in BSR Format for more details. |
| <i>x</i> | Array, size $(m \cdot lb)$. On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least $(m \cdot lb)$. On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_cspblas_?coosymv

Computes matrix - vector product of a sparse symmetrical matrix stored in the coordinate format with zero-based indexing .

Syntax

```
void mkl_cspblas_scoosymv (const char *uplo , const MKL_INT *m , const float *val ,
const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const float *x ,
float *y );
```

```
void mkl_cspblas_dcoosymv (const char *uplo , const MKL_INT *m , const double *val ,
const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const double *x ,
double *y );
```

```
void mkl_cspblas_ccoosymv (const char *uplo , const MKL_INT *m , const MKL_Complex8
*val , const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const
MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_cspblas_zcoosymv (const char *uplo , const MKL_INT *m , const MKL_Complex16
*val , const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const
MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_cspblas_?coosymv` routine performs a matrix-vector operation defined as

```
y := A*x
```

where:

x and *y* are vectors,

A is an upper or lower triangle of the symmetrical sparse matrix in the coordinate format with zero-based indexing.

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

| | |
|---------------|---|
| <i>uplo</i> | Specifies whether the upper or low triangle of the matrix <i>A</i> is used. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix <i>A</i> is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix <i>A</i> is used. |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>val</i> | Array of length <i>nnz</i> , contains non-zero elements of the matrix <i>A</i> in the arbitrary order. Refer to <i>values</i> array description in Coordinate Format for more details. |
| <i>rowind</i> | Array of length <i>nnz</i> , contains the row indices for each non-zero element of the matrix <i>A</i> . Refer to <i>rows</i> array description in Coordinate Format for more details. |
| <i>colind</i> | Array of length <i>nnz</i> , contains the column indices for each non-zero element of the matrix <i>A</i> . Refer to <i>columns</i> array description in Coordinate Format for more details. |
| <i>nnz</i> | Specifies the number of non-zero element of the matrix <i>A</i> . Refer to <i>nnz</i> description in Coordinate Format for more details. |
| <i>x</i> | Array, size is <i>m</i> . On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least <i>m</i> . On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

mkl_cspblas_?csrtrsv

Triangular solvers with simplified interface for a sparse matrix in the CSR format (3-array variation) with zero-based indexing.

Syntax

```
void mkl_cspblas_scsrtrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const float *a , const MKL_INT *ia , const MKL_INT *ja , const float
*x , float *y );

void mkl_cspblas_dcsrtrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const double *a , const MKL_INT *ia , const MKL_INT *ja , const
double *x , double *y );

void mkl_cspblas_ccsrtrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_cspblas_zcsrtrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_cspblas_?csrtrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the CSR format (3-array variation) with zero-based indexing:

$$A*y = x$$

or

$$A^T*y = x,$$

where:

x and y are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

| | |
|---------------|---|
| <i>uplo</i> | Specifies whether the upper or low triangle of the matrix A is used. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix A is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix A is used. |
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $A*y = x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $A^T*y = x$, |
| <i>diag</i> | Specifies whether matrix A is unit triangular. If <i>diag</i> = 'U' or 'u', then A is unit triangular. If <i>diag</i> = 'N' or 'n', then A is not unit triangular. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>a</i> | Array containing non-zero elements of the matrix A . Its length is equal to the number of non-zero elements in the matrix A . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

ia Array of length $m+1$, containing indices of elements in the array *a*, such that *ia*[*i*] is the index in the array *a* of the first non-zero element from the row *i*. The value of the last element *ia*[*m*] is equal to the number of non-zeros. Refer to *rowIndex* array description in [Sparse Matrix Storage Formats](#) for more details.

ja Array containing the column indices for each non-zero element of the matrix *A*.
Its length is equal to the length of the array *a*. Refer to *columns* array description in [Sparse Matrix Storage Formats](#) for more details.

NOTE

Column indices must be sorted in increasing order for each row.

x Array, size is *m*.
On entry, the array *x* must contain the vector *x*.

Output Parameters

y Array, size at least *m*.
Contains the vector *y*.

mkl_cspblas_?bsrtrsv

Triangular solver with simplified interface for a sparse matrix stored in the BSR format (3-array variation) with zero-based indexing.

Syntax

```
void mkl_cspblas_sbsrtrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const MKL_INT *lb , const float *a , const MKL_INT *ia , const
MKL_INT *ja , const float *x , float *y );
```

```
void mkl_cspblas_dbsrtrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const MKL_INT *lb , const double *a , const MKL_INT *ia , const
MKL_INT *ja , const double *x , double *y );
```

```
void mkl_cspblas_cbsrtrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const MKL_INT *lb , const MKL_Complex8 *a , const MKL_INT *ia ,
const MKL_INT *ja , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_cspblas_zbsrtrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const MKL_INT *lb , const MKL_Complex16 *a , const MKL_INT *ia ,
const MKL_INT *ja , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mk1_cspblas_?bsrtrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the BSR format (3-array variation) with zero-based indexing:

```
y := A*x
```

or

```
y := AT*x,
```

where:

x and y are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

| | |
|---------------|--|
| <i>uplo</i> | Specifies the upper or low triangle of the matrix A is used. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix A is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix A is used. |
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then the matrix-vector product is computed as $y := A*x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as $y := A^T*x$. |
| <i>diag</i> | Specifies whether matrix A is unit triangular or not. If <i>diag</i> = 'U' or 'u', A is unit triangular. If <i>diag</i> = 'N' or 'n', A is not unit triangular. |
| <i>m</i> | Number of block rows of the matrix A . |
| <i>lb</i> | Size of the block in the matrix A . |
| <i>a</i> | Array containing elements of non-zero blocks of the matrix A . Its length is equal to the number of non-zero blocks in the matrix A multiplied by $lb*lb$. Refer to <i>values</i> array description in BSR Format for more details. |

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

| | |
|-----------|--|
| <i>ia</i> | Array of length $(m + 1)$, containing indices of block in the array <i>a</i> , such that <i>ia</i> [<i>I</i>] is the index in the array <i>a</i> of the first non-zero element from the row <i>I</i> . The value of the last element <i>ia</i> [<i>m</i>] is equal to the number of non-zero blocks. Refer to <i>rowIndex</i> array description in BSR Format for more details. |
| <i>ja</i> | Array containing the column indices for each non-zero block in the matrix <i>A</i> . Its length is equal to the number of non-zero blocks of the matrix <i>A</i> . Refer to <i>columns</i> array description in BSR Format for more details. |
| <i>x</i> | Array, size $(m \cdot lb)$. On entry, the array <i>x</i> must contain the vector <i>x</i> . |

Output Parameters

| | |
|----------|---|
| <i>y</i> | Array, size at least $(m \cdot lb)$. On exit, the array <i>y</i> must contain the vector <i>y</i> . |
|----------|---|

[mkl_cspblas_?cootrsv](#)

Triangular solvers with simplified interface for a sparse matrix in the coordinate format with zero-based indexing .

Syntax

```
void mkl_cspblas_scootrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const float *val , const MKL_INT *rowind , const MKL_INT *colind ,
const MKL_INT *nnz , const float *x , float *y );

void mkl_cspblas_dcootrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const double *val , const MKL_INT *rowind , const MKL_INT *colind ,
const MKL_INT *nnz , const double *x , double *y );

void mkl_cspblas_ccootrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const MKL_Complex8 *val , const MKL_INT *rowind , const MKL_INT
*colind , const MKL_INT *nnz , const MKL_Complex8 *x , MKL_Complex8 *y );

void mkl_cspblas_zcootrsv (const char *uplo , const char *transa , const char *diag ,
const MKL_INT *m , const MKL_Complex16 *val , const MKL_INT *rowind , const MKL_INT
*colind , const MKL_INT *nnz , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_cspblas_?cootrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the coordinate format with zero-based indexing:

$$A \cdot y = x$$

or

$$A^T \cdot y = x,$$

where:

x and y are vectors,

A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only zero-based indexing of the input arrays.

Input Parameters

| | |
|---------------|--|
| <i>uplo</i> | Specifies whether the upper or low triangle of the matrix A is considered. If <i>uplo</i> = 'U' or 'u', then the upper triangle of the matrix A is used. If <i>uplo</i> = 'L' or 'l', then the low triangle of the matrix A is used. |
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $A*y = x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $A^T*y = x$, |
| <i>diag</i> | Specifies whether A is unit triangular. If <i>diag</i> = 'U' or 'u', then A is unit triangular. If <i>diag</i> = 'N' or 'n', then A is not unit triangular. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>val</i> | Array of length <i>nnz</i> , contains non-zero elements of the matrix A in the arbitrary order. Refer to <i>values</i> array description in Coordinate Format for more details. |
| <i>rowind</i> | Array of length <i>nnz</i> , contains the row indices for each non-zero element of the matrix A . Refer to <i>rows</i> array description in Coordinate Format for more details. |
| <i>colind</i> | Array of length <i>nnz</i> , contains the column indices for each non-zero element of the matrix A . Refer to <i>columns</i> array description in Coordinate Format for more details. |
| <i>nnz</i> | Specifies the number of non-zero element of the matrix A . Refer to <i>nnz</i> description in Coordinate Format for more details. |
| <i>x</i> | Array, size is <i>m</i> . On entry, the array <i>x</i> must contain the vector x . |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Array, size at least <i>m</i> . Contains the vector y . |
|----------|--|

mkl_?csrvm

Computes matrix - vector product of a sparse matrix stored in the CSR format.

Syntax

```
void mkl_scsrvm (const char *transa , const MKL_INT *m , const MKL_INT *k , const float
*alpha , const char *matdescra , const float *val , const MKL_INT *indx , const MKL_INT
*pntrb , const MKL_INT *pntre , const float *x , const float *beta , float *y );
```

```
void mkl_dcsrvm (const char *transa , const MKL_INT *m , const MKL_INT *k , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *indx , const
MKL_INT *pntrb , const MKL_INT *pntre , const double *x , const double *beta , double
*y );
```

```
void mkl_ccsrvm (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex8 *x , const
MKL_Complex8 *beta , MKL_Complex8 *y );
```

```
void mkl_zcsrvm (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex16 *x , const
MKL_Complex16 *beta , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?csrvm` routine performs a matrix-vector operation defined as

```
y := alpha*A*x + beta*y
```

or

```
y := alpha*AT*x + beta*y,
```

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix in the CSR format, *A^T* is the transpose of *A*.

NOTE

This routine supports a CSR format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|---------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $y := \alpha A x + \beta y$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha A^T x + \beta y$, |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |

| | |
|------------------|--|
| <i>k</i> | Number of columns of the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing non-zero elements of the matrix <i>A</i> . Its length is $pntrb[m-1] - pntrb[0]$. Refer to <i>values</i> array description in CSR Format for more details. |
| <i>indx</i> | For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix <i>A</i> . For zero-based indexing, array containing the column indices for each non-zero element of the matrix <i>A</i> . Its length is equal to length of the <i>val</i> array. Refer to <i>columns</i> array description in CSR Format for more details. |
| <i>pntrb</i> | Array of length <i>m</i> . This array contains row indices, such that $pntrb[i] - pntrb[0]$ is the first index of row <i>i</i> in the arrays <i>val</i> and <i>indx</i> . Refer to <i>pointerb</i> array description in CSR Format for more details. |
| <i>pntrr</i> | Array of length <i>m</i> . This array contains row indices, such that $pntrr[i] - pntrb[0]-1$ is the last index of row <i>i</i> in the arrays <i>val</i> and <i>indx</i> . Refer to <i>pointerE</i> array description in CSR Format for more details. |
| <i>x</i> | Array, size at least <i>k</i> if <i>transa</i> = 'N' or 'n' and at least <i>m</i> otherwise. On entry, the array <i>x</i> must contain the vector <i>x</i> . |
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>y</i> | Array, size at least <i>m</i> if <i>transa</i> = 'N' or 'n' and at least <i>k</i> otherwise. On entry, the array <i>y</i> must contain the vector <i>y</i> . |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Overwritten by the updated vector <i>y</i> . |
|----------|--|

mkl_?bsrmv

Computes matrix - vector product of a sparse matrix stored in the BSR format.

Syntax

```
void mkl_sbsrmv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_INT *lb , const float *alpha , const char *matdescra , const float *val , const
MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntrc , const float *x , const
float *beta , float *y );
```

```
void mkl_dbsrmv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_INT *lb , const double *alpha , const char *matdescra , const double *val , const
MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntrc , const double *x , const
double *beta , double *y );
```

```
void mkl_cbsrmv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_INT *lb , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8
*val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntrc , const
MKL_Complex8 *x , const MKL_Complex8 *beta , MKL_Complex8 *y );
```

```
void mkl_zbsrmv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_INT *lb , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16
*val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntrc , const
MKL_Complex16 *x , const MKL_Complex16 *beta , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?bsrmv` routine performs a matrix-vector operation defined as

$$y := \alpha A^*x + \beta y$$

or

$$y := \alpha A^T x + \beta y,$$

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* block sparse matrix in the BSR format, A^T is the transpose of *A*.

NOTE

This routine supports a BSR format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|---------------|---|
| <i>transa</i> | <p>Specifies the operation.</p> <p>If <i>transa</i> = 'N' or 'n', then the matrix-vector product is computed as</p> $y := \alpha A^*x + \beta y$ <p>If <i>transa</i> = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as</p> $y := \alpha A^T x + \beta y,$ |
| <i>m</i> | Number of block rows of the matrix <i>A</i> . |

| | |
|------------------|--|
| <i>k</i> | Number of block columns of the matrix <i>A</i> . |
| <i>lb</i> | Size of the block in the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing elements of non-zero blocks of the matrix <i>A</i> . Its length is equal to the number of non-zero blocks in the matrix <i>A</i> multiplied by $lb*lb$. Refer to <i>values</i> array description in BSR Format for more details. |
| <i>indx</i> | For one-based indexing, array containing the column indices plus one for each non-zero block of the matrix <i>A</i> . For zero-based indexing, array containing the column indices for each non-zero block of the matrix <i>A</i> . Its length is equal to the number of non-zero blocks in the matrix <i>A</i> . Refer to <i>columns</i> array description in BSR Format for more details. |
| <i>pntrb</i> | Array of length <i>m</i> . This array contains row indices, such that $pntrb[i] - pntrb[0]$ is the first index of block row <i>i</i> in the array <i>indx</i> . Refer to <i>pointerB</i> array description in BSR Format for more details. |
| <i>pntrE</i> | Array of length <i>m</i> . For zero-based indexing this array contains row indices, such that $pntrE[i] - pntrb[0] - 1$ is the last index of block row <i>i</i> in the array <i>indx</i> . Refer to <i>pointerE</i> array description in BSR Format for more details. |
| <i>x</i> | Array, size at least $(k*lb)$ if <i>transa</i> = 'N' or 'n', and at least $(m*lb)$ otherwise. On entry, the array <i>x</i> must contain the vector <i>x</i> . |
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>y</i> | Array, size at least $(m*lb)$ if <i>transa</i> = 'N' or 'n', and at least $(k*lb)$ otherwise. On entry, the array <i>y</i> must contain the vector <i>y</i> . |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Overwritten by the updated vector <i>y</i> . |
|----------|--|

mkl_?cscmv

Computes matrix-vector product for a sparse matrix in the CSC format.

Syntax

```
void mkl_scscmv (const char *transa , const MKL_INT *m , const MKL_INT *k , const float
*alpha , const char *matdescra , const float *val , const MKL_INT *indx , const MKL_INT
*pntrb , const MKL_INT *pntre , const float *x , const float *beta , float *y );
```

```
void mkl_dcscmv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *indx , const
MKL_INT *pntrb , const MKL_INT *pntre , const double *x , const double *beta , double
*y );
```

```
void mkl_ccscmv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex8 *x , const
MKL_Complex8 *beta , MKL_Complex8 *y );
```

```
void mkl_zcscmv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex16 *x , const
MKL_Complex16 *beta , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?cscmv` routine performs a matrix-vector operation defined as

$$y := \alpha A^*x + \beta y$$

or

$$y := \alpha A^T x + \beta y,$$

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix in compressed sparse column (CSC) format, A^T is the transpose of *A*.

NOTE

This routine supports CSC format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|---------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $y := \alpha A^*x + \beta y$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha A^T x + \beta y,$ |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>k</i> | Number of columns of the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |

| | |
|------------------|---|
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing non-zero elements of the matrix <i>A</i> . Its length is $pntrb[k-1] - pntrb[0]$. Refer to <i>values</i> array description in CSC Format for more details. |
| <i>indx</i> | For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix <i>A</i> . For zero-based indexing, array containing the column indices for each non-zero element of the matrix <i>A</i> . Its length is equal to length of the <i>val</i> array. Refer to <i>rows</i> array description in CSC Format for more details. |
| <i>pntrb</i> | Array of length <i>k</i> . This array contains column indices, such that $pntrb[i] - pntrb[0] + 1$ is the first index of column <i>i</i> in the arrays <i>val</i> and <i>indx</i> . Refer to <i>pointerb</i> array description in CSC Format for more details. |
| <i>pntrb</i> | Array of length <i>k</i> . For one-based indexing this array contains column indices, such that $pntrb[i] - pntrb[1]$ is the last index of column <i>i</i> in the arrays <i>val</i> and <i>indx</i> . For zero-based indexing this array contains column indices, such that $pntrb[i] - pntrb[1] - 1$ is the last index of column <i>i</i> in the arrays <i>val</i> and <i>indx</i> . Refer to <i>pointerE</i> array description in CSC Format for more details. |
| <i>x</i> | Array, size at least <i>k</i> if <i>transa</i> = 'N' or 'n' and at least <i>m</i> otherwise. On entry, the array <i>x</i> must contain the vector <i>x</i> . |
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>y</i> | Array, size at least <i>m</i> if <i>transa</i> = 'N' or 'n' and at least <i>k</i> otherwise. On entry, the array <i>y</i> must contain the vector <i>y</i> . |

Output Parameters

y Overwritten by the updated vector *y*.

mkl_?coomv

Computes matrix - vector product for a sparse matrix in the coordinate format.

Syntax

```
void mkl_scoomv (const char *transa , const MKL_INT *m , const MKL_INT *k , const float *alpha , const char *matdescra , const float *val , const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const float *x , const float *beta , float *y );
```

```
void mkl_dcoomv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *rowind ,
const MKL_INT *colind , const MKL_INT *nnz , const double *x , const double *beta ,
double *y );
```

```
void mkl_ccoomv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*rowind , const MKL_INT *colind , const MKL_INT *nnz , const MKL_Complex8 *x , const
MKL_Complex8 *beta , MKL_Complex8 *y );
```

```
void mkl_zcoomv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*rowind , const MKL_INT *colind , const MKL_INT *nnz , const MKL_Complex16 *x , const
MKL_Complex16 *beta , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?coomv` routine performs a matrix-vector operation defined as

```
 $y := \alpha A x + \beta y$ 
```

or

```
 $y := \alpha A^T x + \beta y,$ 
```

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix in compressed coordinate format, A^T is the transpose of *A*.

NOTE

This routine supports a coordinate format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $y := \alpha A x + \beta y$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha A^T x + \beta y,$ |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>k</i> | Number of columns of the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |

| | |
|---------------|--|
| <i>val</i> | <p>Array of length <i>nnz</i>, contains non-zero elements of the matrix <i>A</i> in the arbitrary order.</p> <p>Refer to <i>values</i> array description in Coordinate Format for more details.</p> |
| <i>rowind</i> | <p>Array of length <i>nnz</i>.</p> <p>For one-based indexing, contains the row indices plus one for each non-zero element of the matrix <i>A</i>.</p> <p>For zero-based indexing, contains the row indices for each non-zero element of the matrix <i>A</i>.</p> <p>Refer to <i>rows</i> array description in Coordinate Format for more details.</p> |
| <i>colind</i> | <p>Array of length <i>nnz</i>.</p> <p>For one-based indexing, contains the column indices plus one for each non-zero element of the matrix <i>A</i>.</p> <p>For zero-based indexing, contains the column indices for each non-zero element of the matrix <i>A</i>.</p> <p>Refer to <i>columns</i> array description in Coordinate Format for more details.</p> |
| <i>nnz</i> | <p>Specifies the number of non-zero element of the matrix <i>A</i>.</p> <p>Refer to <i>nnz</i> description in Coordinate Format for more details.</p> |
| <i>x</i> | <p>Array, size at least <i>k</i> if <i>transa</i> = 'N' or 'n' and at least <i>m</i> otherwise. On entry, the array <i>x</i> must contain the vector <i>x</i>.</p> |
| <i>beta</i> | <p>Specifies the scalar <i>beta</i>.</p> |
| <i>y</i> | <p>Array, size at least <i>m</i> if <i>transa</i> = 'N' or 'n' and at least <i>k</i> otherwise. On entry, the array <i>y</i> must contain the vector <i>y</i>.</p> |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Overwritten by the updated vector <i>y</i> . |
|----------|--|

mkl_?csrsv

Solves a system of linear equations for a sparse matrix in the CSR format.

Syntax

```
void mkl_scsrsv (const char *transa , const MKL_INT *m , const float *alpha , const
char *matdescra , const float *val , const MKL_INT *indx , const MKL_INT *pntrb , const
MKL_INT *pntre , const float *x , float *y );
```

```
void mkl_dcsrsv (const char *transa , const MKL_INT *m , const double *alpha , const
char *matdescra , const double *val , const MKL_INT *indx , const MKL_INT *pntrb ,
const MKL_INT *pntre , const double *x , double *y );
```

```
void mkl_ccsrsv (const char *transa , const MKL_INT *m , const MKL_Complex8 *alpha ,
const char *matdescra , const MKL_Complex8 *val , const MKL_INT *indx , const MKL_INT
*pntrb , const MKL_INT *pntre , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zcsrsv (const char *transa , const MKL_INT *m , const MKL_Complex16 *alpha ,
const char *matdescra , const MKL_Complex16 *val , const MKL_INT *indx , const MKL_INT
*pntrb , const MKL_INT *pntrc , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?csrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the CSR format:

```
y := alpha*inv(A)*x
```

or

```
y := alpha*inv(AT)*x,
```

where:

alpha is scalar, *x* and *y* are vectors, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A^T* is the transpose of *A*.

NOTE

This routine supports a CSR format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $y := \alpha \cdot \text{inv}(A) \cdot x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha \cdot \text{inv}(A^T) \cdot x$, |
| <i>m</i> | Number of columns of the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing non-zero elements of the matrix <i>A</i> . Its length is <code>pntrc[m - 1] - pntrb[0]</code> . Refer to <i>values</i> array description in CSR Format for more details. |

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

indx For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix *A*. For zero-based indexing, array containing the column indices for each non-zero element of the matrix *A*. Its length is equal to length of the *val* array.

Refer to *columns* array description in [CSR Format](#) for more details.

NOTE

Column indices must be sorted in increasing order for each row.

pntrb Array of length *m*.
This array contains row indices, such that *pntrb*[*i*] - *pntrb*[0] is the first index of row *i* in the arrays *val* and *indx*.

Refer to *pointerb* array description in [CSR Format](#) for more details.

pntrE Array of length *m*.
This array contains row indices, such that *pntrE*[*i*] - *pntrb*[0] - 1 is the last index of row *i* in the arrays *val* and *indx*.

Refer to *pointerE* array description in [CSR Format](#) for more details.

x Array, size at least *m*.
On entry, the array *x* must contain the vector *x*. The elements are accessed with unit increment.

y Array, size at least *m*.
On entry, the array *y* must contain the vector *y*. The elements are accessed with unit increment.

Output Parameters

y Contains solution vector *x*.

mkl_?bsrsv

Solves a system of linear equations for a sparse matrix in the BSR format.

Syntax

```
void mkl_sbsrsv (const char *transa , const MKL_INT *m , const MKL_INT *lb , const
float *alpha , const char *matdescra , const float *val , const MKL_INT *indx , const
MKL_INT *pntrb , const MKL_INT *pntrE , const float *x , float *y );
```

```
void mkl_dbsrsv (const char *transa , const MKL_INT *m , const MKL_INT *lb , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *indx , const
MKL_INT *pntrb , const MKL_INT *pntrE , const double *x , double *y );
```

```
void mkl_cbsrsv (const char *transa , const MKL_INT *m , const MKL_INT *lb , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntrE , const MKL_Complex8 *x ,
MKL_Complex8 *y );
```

```
void mkl_zbsrsv (const char *transa , const MKL_INT *m , const MKL_INT *lb , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex16 *x ,
MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?bsrsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the BSR format:

```
y := alpha*inv(A)*x
```

or

```
y := alpha*inv(AT)* x,
```

where:

alpha is scalar, *x* and *y* are vectors, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A*^T is the transpose of *A*.

NOTE

This routine supports a BSR format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $y := \alpha \cdot \text{inv}(A) \cdot x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha \cdot \text{inv}(A^T) \cdot x$, |
| <i>m</i> | Number of block columns of the matrix <i>A</i> . |
| <i>lb</i> | Size of the block in the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing elements of non-zero blocks of the matrix <i>A</i> . Its length is equal to the number of non-zero blocks in the matrix <i>A</i> multiplied by $lb \cdot lb$. Refer to the <i>values</i> array description in BSR Format for more details. |

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

| | |
|--------------|---|
| <i>indx</i> | For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix <i>A</i> . For zero-based indexing, array containing the column indices for each non-zero element of the matrix <i>A</i> . Its length is equal to the number of non-zero blocks in the matrix <i>A</i> . Refer to the <i>columns</i> array description in BSR Format for more details. |
| <i>pntrb</i> | Array of length <i>m</i> . This array contains row indices, such that $pntrb[i] - pntrb[0]$ is the first index of block row <i>i</i> in the array <i>indx</i> . Refer to <i>pointerB</i> array description in BSR Format for more details. |
| <i>pntrc</i> | Array of length <i>m</i> . For one-based indexing this array contains row indices, such that $pntrc[i] - pntrb[1]$ is the last index of block row <i>i</i> in the array <i>indx</i> . For zero-based indexing this array contains row indices, such that $pntrc[i] - pntrb[0] - 1$ is the last index of block row <i>i</i> in the array <i>indx</i> . Refer to <i>pointerE</i> array description in BSR Format for more details. |
| <i>x</i> | Array, size at least $(m \cdot lb)$. On entry, the array <i>x</i> must contain the vector <i>x</i> . The elements are accessed with unit increment. |
| <i>y</i> | Array, size at least $(m \cdot lb)$. On entry, the array <i>y</i> must contain the vector <i>y</i> . The elements are accessed with unit increment. |

Output Parameters

| | |
|----------|-------------------------------------|
| <i>y</i> | Contains solution vector <i>x</i> . |
|----------|-------------------------------------|

mkl_?cscsv

Solves a system of linear equations for a sparse matrix in the CSC format.

Syntax

```
void mkl_scscsv (const char *transa , const MKL_INT *m , const float *alpha , const char *matdescra , const float *val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntrc , const float *x , float *y );
```

```
void mkl_dcscsv (const char *transa , const MKL_INT *m , const double *alpha , const char *matdescra , const double *val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntrc , const double *x , double *y );
```

```
void mkl_ccscsv (const char *transa , const MKL_INT *m , const MKL_Complex8 *alpha ,
const char *matdescra , const MKL_Complex8 *val , const MKL_INT *indx , const MKL_INT
*pntrb , const MKL_INT *pntre , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zcscsv (const char *transa , const MKL_INT *m , const MKL_Complex16 *alpha ,
const char *matdescra , const MKL_Complex16 *val , const MKL_INT *indx , const MKL_INT
*pntrb , const MKL_INT *pntre , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?cscsv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the CSC format:

```
 $y := \alpha \cdot \text{inv}(A) \cdot x$ 
```

or

```
 $y := \alpha \cdot \text{inv}(A^T) \cdot x,$ 
```

where:

α is scalar, x and y are vectors, A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports a CSC format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $y := \alpha \cdot \text{inv}(A) \cdot x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha \cdot \text{inv}(A^T) \cdot x,$ |
| <i>m</i> | Number of columns of the matrix A . |
| <i>alpha</i> | Specifies the scalar α . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing non-zero elements of the matrix A . Its length is $\text{pntre}[m-1] - \text{pntrb}[0]$. Refer to <i>values</i> array description in CSC Format for more details. |

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

indx

For one-based indexing, array containing the row indices plus one for each non-zero element of the matrix *A*.

For zero-based indexing, array containing the row indices for each non-zero element of the matrix *A*.

Its length is equal to length of the *val* array.

Refer to *columns* array description in [CSC Format](#) for more details.

NOTE

Row indices must be sorted in increasing order for each column.

pntrb

Array of length *m*.

This array contains column indices, such that $pntrb[i] - pntrb[0]$ is the first index of column *i* in the arrays *val* and *indx*.

Refer to *pointerb* array description in [CSC Format](#) for more details.

pntrE

Array of length *m*.

This array contains column indices, such that $pntrE[i] - pntrb[0] - 1$ is the last index of column *i* in the arrays *val* and *indx*.

Refer to *pointerE* array description in [CSC Format](#) for more details.

x

Array, size at least *m*.

On entry, the array *x* must contain the vector *x*. The elements are accessed with unit increment.

y

Array, size at least *m*.

On entry, the array *y* must contain the vector *y*. The elements are accessed with unit increment.

Output Parameters*y*

Contains the solution vector *x*.

mkl_?coosv

Solves a system of linear equations for a sparse matrix in the coordinate format.

Syntax

```
void mkl_scoosv (const char *transa , const MKL_INT *m , const float *alpha , const
char *matdescra , const float *val , const MKL_INT *rowind , const MKL_INT *colind ,
const MKL_INT *nnz , const float *x , float *y );
```

```
void mkl_dcoosv (const char *transa , const MKL_INT *m , const double *alpha , const
char *matdescra , const double *val , const MKL_INT *rowind , const MKL_INT *colind ,
const MKL_INT *nnz , const double *x , double *y );
```

```
void mkl_ccoosv (const char *transa , const MKL_INT *m , const MKL_Complex8 *alpha ,
const char *matdescra , const MKL_Complex8 *val , const MKL_INT *rowind , const MKL_INT
*colind , const MKL_INT *nnz , const MKL_Complex8 *x , MKL_Complex8 *y );

void mkl_zcoosv (const char *transa , const MKL_INT *m , const MKL_Complex16 *alpha ,
const char *matdescra , const MKL_Complex16 *val , const MKL_INT *rowind , const
MKL_INT *colind , const MKL_INT *nnz , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?coosv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the coordinate format:

```
 $y := \alpha \cdot \text{inv}(A) \cdot x$ 
```

or

```
 $y := \alpha \cdot \text{inv}(A^T) \cdot x,$ 
```

where:

α is scalar, x and y are vectors, A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports a coordinate format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $y := \alpha \cdot \text{inv}(A) \cdot x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha \cdot \text{inv}(A^T) \cdot x,$ |
| <i>m</i> | Number of rows of the matrix A . |
| <i>alpha</i> | Specifies the scalar α . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array of length <i>nnz</i> , contains non-zero elements of the matrix A in the arbitrary order. Refer to <i>values</i> array description in Coordinate Format for more details. |
| <i>rowind</i> | Array of length <i>nnz</i> . For one-based indexing, contains the row indices plus one for each non-zero element of the matrix A . |

For zero-based indexing, contains the row indices for each non-zero element of the matrix A .

Refer to `rows` array description in [Coordinate Format](#) for more details.

`colind`

Array of length `nnz`.

For one-based indexing, contains the column indices plus one for each non-zero element of the matrix A .

For zero-based indexing, contains the column indices for each non-zero element of the matrix A .

Refer to `columns` array description in [Coordinate Format](#) for more details.

`nnz`

Specifies the number of non-zero element of the matrix A .

Refer to `nnz` description in [Coordinate Format](#) for more details.

`x`

Array, size at least m .

On entry, the array `x` must contain the vector x . The elements are accessed with unit increment.

`y`

Array, size at least m .

On entry, the array `y` must contain the vector y . The elements are accessed with unit increment.

Output Parameters

`y`

Contains solution vector x .

`mkl_?csrmm`

Computes matrix - matrix product of a sparse matrix stored in the CSR format.

Syntax

```
void mkl_scsrmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const float *alpha , const char *matdescra , const float *val , const
MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const float *b , const
MKL_INT *ldb , const float *beta , float *c , const MKL_INT *ldc );
```

```
void mkl_dcsrmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const double *alpha , const char *matdescra , const double *val , const
MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const double *b , const
MKL_INT *ldb , const double *beta , double *c , const MKL_INT *ldc );
```

```
void mkl_ccsrmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8
*val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const
MKL_Complex8 *b , const MKL_INT *ldb , const MKL_Complex8 *beta , MKL_Complex8 *c ,
const MKL_INT *ldc );
```

```
void mkl_zcsrmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16
*val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const
MKL_Complex16 *b , const MKL_INT *ldb , const MKL_Complex16 *beta , MKL_Complex16 *c ,
const MKL_INT *ldc );
```

Include Files

- `mkl.h`

Description

The `mkl_?csrmm` routine performs a matrix-matrix operation defined as

$$C := \alpha * A * B + \beta * C$$

or

$$C := \alpha * A^T * B + \beta * C$$

or

$$C := \alpha * A^H * B + \beta * C,$$

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in compressed sparse row (CSR) format, A^T is the transpose of *A*, and A^H is the conjugate transpose of *A*.

NOTE

This routine supports a CSR format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $C := \alpha * A * B + \beta * C$, If <i>transa</i> = 'T' or 't', then $C := \alpha * A^T * B + \beta * C$, If <i>transa</i> = 'C' or 'c', then $C := \alpha * A^H * B + \beta * C$. |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>n</i> | Number of columns of the matrix <i>C</i> . |
| <i>k</i> | Number of columns of the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing non-zero elements of the matrix <i>A</i> . For zero-based indexing its length is $pntrc[m-1] - pntrb[0]$. Refer to <i>values</i> array description in CSR Format for more details. |
| <i>indx</i> | For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix <i>A</i> . |

For zero-based indexing, array containing the column indices for each non-zero element of the matrix A .

Its length is equal to length of the `val` array.

Refer to `columns` array description in [CSR Format](#) for more details.

`pntrb`

Array of length m .

This array contains row indices, such that `pntrb[I] - pntrb[0]` is the first index of row I in the arrays `val` and `indx`.

Refer to `pointerb` array description in [CSR Format](#) for more details.

`pntrE`

Array of length m .

This array contains row indices, such that `pntrE[I] - pntrb[0] - 1` is the last index of row I in the arrays `val` and `indx`.

Refer to `pointerE` array description in [CSR Format](#) for more details.

`b`

Array, size `ldb` by at least n for non-transposed matrix A and at least m for transposed for one-based indexing, and (at least k for non-transposed matrix A and at least m for transposed, `ldb`) for zero-based indexing.

On entry with `transa='N'` or `'n'`, the leading k -by- n part of the array `b` must contain the matrix B , otherwise the leading m -by- n part of the array `b` must contain the matrix B .

`ldb`

Specifies the leading dimension of `b` for one-based indexing, and the second dimension of `b` for zero-based indexing, as declared in the calling (sub)program.

`beta`

Specifies the scalar `beta`.

`c`

Array, size `ldc` by n for one-based indexing, and (m , `ldc`) for zero-based indexing.

On entry, the leading m -by- n part of the array `c` must contain the matrix C , otherwise the leading k -by- n part of the array `c` must contain the matrix C .

`ldc`

Specifies the leading dimension of `c` for one-based indexing, and the second dimension of `c` for zero-based indexing, as declared in the calling (sub)program.

Output Parameters

`c`

Overwritten by the matrix $(\alpha * A * B + \beta * C)$, $(\alpha * A^T * B + \beta * C)$, or $(\alpha * A^H * B + \beta * C)$.

mkl_?bsrmm

Computes matrix - matrix product of a sparse matrix stored in the BSR format.

Syntax

```
void mkl_sbsrmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_INT *lb , const float *alpha , const char *matdescra , const
float *val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const
float *b , const MKL_INT *ldb , const float *beta , float *c , const MKL_INT *ldc );

void mkl_dbsrmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_INT *lb , const double *alpha , const char *matdescra , const
double *val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const
double *b , const MKL_INT *ldb , const double *beta , double *c , const MKL_INT *ldc );

void mkl_cbsrmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_INT *lb , const MKL_Complex8 *alpha , const char *matdescra ,
const MKL_Complex8 *val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT
*pntre , const MKL_Complex8 *b , const MKL_INT *ldb , const MKL_Complex8 *beta ,
MKL_Complex8 *c , const MKL_INT *ldc );

void mkl_zbsrmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_INT *lb , const MKL_Complex16 *alpha , const char *matdescra ,
const MKL_Complex16 *val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT
*pntre , const MKL_Complex16 *b , const MKL_INT *ldb , const MKL_Complex16 *beta ,
MKL_Complex16 *c , const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?bsrmm` routine performs a matrix-matrix operation defined as

```
 $C := \alpha * A * B + \beta * C$ 
```

or

```
 $C := \alpha * A^T * B + \beta * C$ 
```

or

```
 $C := \alpha * A^H * B + \beta * C,$ 
```

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in block sparse row (BSR) format, A^T is the transpose of *A*, and A^H is the conjugate transpose of *A*.

NOTE

This routine supports a BSR format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|---------------|---|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then the matrix-matrix product is computed as $C := \alpha * A * B + \beta * C$ |
|---------------|---|

If $transa = 'T'$ or $'t'$, then the matrix-vector product is computed as
 $C := \alpha * A^T * B + \beta * C$

If $transa = 'C'$ or $'c'$, then the matrix-vector product is computed as
 $C := \alpha * A^H * B + \beta * C,$

| | |
|-------------|--|
| m | Number of block rows of the matrix A . |
| n | Number of columns of the matrix C . |
| k | Number of block columns of the matrix A . |
| lb | Size of the block in the matrix A . |
| α | Specifies the scalar α . |
| $matdescra$ | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter $matdescra$ ($descra$)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter $matdescra$" . |
| val | Array containing elements of non-zero blocks of the matrix A . Its length is equal to the number of non-zero blocks in the matrix A multiplied by $lb * lb$. Refer to the $values$ array description in BSR Format for more details. |
| $indx$ | For one-based indexing, array containing the column indices plus one for each non-zero block in the matrix A . For zero-based indexing, array containing the column indices for each non-zero block in the matrix A . Its length is equal to the number of non-zero blocks in the matrix A . Refer to the $columns$ array description in BSR Format for more details. |
| $pntrb$ | Array of length m . This array contains row indices, such that $pntrb[I] - pntrb[0]$ is the first index of block row I in the array $indx$. Refer to $pointerB$ array description in BSR Format for more details. |
| $pntrb$ | Array of length m . This array contains row indices, such that $pntrb[I] - pntrb[0] - 1$ is the last index of block row I in the array $indx$. Refer to $pointerE$ array description in BSR Format for more details. |
| b | Array, size ldb by at least n for non-transposed matrix A and at least m for transposed for one-based indexing, and (at least k for non-transposed matrix A and at least m for transposed, ldb) for zero-based indexing. On entry with $transa='N'$ or $'n'$, the leading n -by- k block part of the array b must contain the matrix B , otherwise the leading m -by- n block part of the array b must contain the matrix B . |
| ldb | Specifies the leading dimension (in blocks) of b as declared in the calling (sub)program. |

| | |
|-------------|--|
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>c</i> | Array, size $ldc * n$ for one-based indexing, size $k * ldc$ for zero-based indexing. On entry, the leading <i>m</i> -by- <i>n</i> block part of the array <i>c</i> must contain the matrix <i>C</i> , otherwise the leading <i>n</i> -by- <i>k</i> block part of the array <i>c</i> must contain the matrix <i>C</i> . |
| <i>ldc</i> | Specifies the leading dimension (in blocks) of <i>c</i> as declared in the calling (sub)program. |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Overwritten by the matrix $(\alpha * A * B + \beta * C)$ or $(\alpha * A^T * B + \beta * C)$ or $(\alpha * A^H * B + \beta * C)$. |
|----------|--|

mkl_?cscmm

Computes matrix-matrix product of a sparse matrix stored in the CSC format.

Syntax

```
void mkl_scscmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const float *alpha , const char *matdescra , const float *val , const
MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const float *b , const
MKL_INT *ldb , const float *beta , float *c , const MKL_INT *ldc );
```

```
void mkl_dcscmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const double *alpha , const char *matdescra , const double *val , const
MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const double *b , const
MKL_INT *ldb , const double *beta , double *c , const MKL_INT *ldc );
```

```
void mkl_ccscmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8
*val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const
MKL_Complex8 *b , const MKL_INT *ldb , const MKL_Complex8 *beta , MKL_Complex8 *c ,
const MKL_INT *ldc );
```

```
void mkl_zcscmm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16
*val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const
MKL_Complex16 *b , const MKL_INT *ldb , const MKL_Complex16 *beta , MKL_Complex16 *c ,
const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?cscmm` routine performs a matrix-matrix operation defined as

```
C :=  $\alpha * A * B + \beta * C$ 
```

or

```
C :=  $\alpha * A^T * B + \beta * C$ ,
```

or

$$C := \alpha * A^H * B + \beta * C,$$

where:

α and β are scalars,

B and C are dense matrices, A is an m -by- k sparse matrix in compressed sparse column (CSC) format, A^T is the transpose of A , and A^H is the conjugate transpose of A .

NOTE

This routine supports CSC format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $C := \alpha * A * B + \beta * C$ If <i>transa</i> = 'T' or 't', then $C := \alpha * A^T * B + \beta * C$, If <i>transa</i> = 'C' or 'c', then $C := \alpha * A^H * B + \beta * C$ |
| <i>m</i> | Number of rows of the matrix A . |
| <i>n</i> | Number of columns of the matrix C . |
| <i>k</i> | Number of columns of the matrix A . |
| <i>alpha</i> | Specifies the scalar α . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing non-zero elements of the matrix A . Its length is $pntrb[k-1] - pntrb[0]$. Refer to <i>values</i> array description in CSC Format for more details. |
| <i>indx</i> | For one-based indexing, array containing the row indices plus one for each non-zero element of the matrix A . For zero-based indexing, array containing the column indices for each non-zero element of the matrix A . Its length is equal to length of the <i>val</i> array. Refer to <i>rows</i> array description in CSC Format for more details. |
| <i>pntrb</i> | Array of length k . This array contains column indices, such that $pntrb[i] - pntrb[0]$ is the first index of column i in the arrays <i>val</i> and <i>indx</i> . Refer to <i>pointerb</i> array description in CSC Format for more details. |
| <i>pntrc</i> | Array of length k . |

This array contains column indices, such that $pntre[i] - pntxb[0] - 1$ is the last index of column i in the arrays val and $indx$.

Refer to *pointerE* array description in [CSC Format](#) for more details.

| | |
|-------------|---|
| <i>b</i> | <p>Array, size ldb by at least n for non-transposed matrix A and at least m for transposed for one-based indexing, and (at least k for non-transposed matrix A and at least m for transposed, ldb) for zero-based indexing.</p> <p>On entry with $transa = 'N'$ or $'n'$, the leading k-by-n part of the array b must contain the matrix B, otherwise the leading m-by-n part of the array b must contain the matrix B.</p> |
| <i>ldb</i> | <p>Specifies the leading dimension of b for one-based indexing, and the second dimension of b for zero-based indexing, as declared in the calling (sub)program.</p> |
| <i>beta</i> | <p>Specifies the scalar $beta$.</p> |
| <i>c</i> | <p>Array, size ldc by n for one-based indexing, and (m, ldc) for zero-based indexing.</p> <p>On entry, the leading m-by-n part of the array c must contain the matrix C, otherwise the leading k-by-n part of the array c must contain the matrix C.</p> |
| <i>ldc</i> | <p>Specifies the leading dimension of c for one-based indexing, and the second dimension of c for zero-based indexing, as declared in the calling (sub)program.</p> |

Output Parameters

| | |
|----------|---|
| <i>c</i> | <p>Overwritten by the matrix $(alpha * A * B + beta * C)$ or $(alpha * A^T * B + beta * C)$ or $(alpha * A^H * B + beta * C)$.</p> |
|----------|---|

mkl_?coomm

Computes matrix-matrix product of a sparse matrix stored in the coordinate format.

Syntax

```
void mkl_scoomm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const float *alpha , const char *matdescra , const float *val , const
MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const float *b , const
MKL_INT *ldb , const float *beta , float *c , const MKL_INT *ldc );

void mkl_dcoomm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const double *alpha , const char *matdescra , const double *val , const
MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const double *b , const
MKL_INT *ldb , const double *beta , double *c , const MKL_INT *ldc );

void mkl_ccoomm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8
*val , const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const
MKL_Complex8 *b , const MKL_INT *ldb , const MKL_Complex8 *beta , MKL_Complex8 *c ,
const MKL_INT *ldc );
```

```
void mkl_zcoomm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16
*val , const MKL_INT *rowind , const MKL_INT *colind , const MKL_INT *nnz , const
MKL_Complex16 *b , const MKL_INT *ldb , const MKL_Complex16 *beta , MKL_Complex16 *c ,
const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?coomm` routine performs a matrix-matrix operation defined as

$$C := \alpha * A * B + \beta * C$$

or

$$C := \alpha * A^T * B + \beta * C,$$

or

$$C := \alpha * A^H * B + \beta * C,$$

where:

alpha and *beta* are scalars,

B and *C* are dense matrices, *A* is an *m*-by-*k* sparse matrix in the coordinate format, A^T is the transpose of *A*, and A^H is the conjugate transpose of *A*.

NOTE

This routine supports a coordinate format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $C := \alpha * A * B + \beta * C$ If <i>transa</i> = 'T' or 't', then $C := \alpha * A^T * B + \beta * C,$ If <i>transa</i> = 'C' or 'c', then $C := \alpha * A^H * B + \beta * C.$ |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>n</i> | Number of columns of the matrix <i>C</i> . |
| <i>k</i> | Number of columns of the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array of length <i>nnz</i> , contains non-zero elements of the matrix <i>A</i> in the arbitrary order. |

| | |
|---------------|--|
| | Refer to <i>values</i> array description in Coordinate Format for more details. |
| <i>rowind</i> | <p>Array of length <i>nnz</i>.</p> <p>For one-based indexing, contains the row indices plus one for each non-zero element of the matrix <i>A</i>.</p> <p>For zero-based indexing, contains the row indices for each non-zero element of the matrix <i>A</i>.</p> <p>Refer to <i>rows</i> array description in Coordinate Format for more details.</p> |
| <i>colind</i> | <p>Array of length <i>nnz</i>.</p> <p>For one-based indexing, contains the column indices plus one for each non-zero element of the matrix <i>A</i>.</p> <p>For zero-based indexing, contains the column indices for each non-zero element of the matrix <i>A</i>.</p> <p>Refer to <i>columns</i> array description in Coordinate Format for more details.</p> |
| <i>nnz</i> | <p>Specifies the number of non-zero element of the matrix <i>A</i>.</p> <p>Refer to <i>nnz</i> description in Coordinate Format for more details.</p> |
| <i>b</i> | <p>Array, size <i>ldb</i> by at least <i>n</i> for non-transposed matrix <i>A</i> and at least <i>m</i> for transposed for one-based indexing, and (at least <i>k</i> for non-transposed matrix <i>A</i> and at least <i>m</i> for transposed, <i>ldb</i>) for zero-based indexing.</p> <p>On entry with <i>transa</i> = 'N' or 'n', the leading <i>k</i>-by-<i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i>, otherwise the leading <i>m</i>-by-<i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i>.</p> |
| <i>ldb</i> | Specifies the leading dimension of <i>b</i> for one-based indexing, and the second dimension of <i>b</i> for zero-based indexing, as declared in the calling (sub)program. |
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>c</i> | <p>Array, size <i>ldc</i> by <i>n</i> for one-based indexing, and (<i>m</i>, <i>ldc</i>) for zero-based indexing.</p> <p>On entry, the leading <i>m</i>-by-<i>n</i> part of the array <i>c</i> must contain the matrix <i>C</i>, otherwise the leading <i>k</i>-by-<i>n</i> part of the array <i>c</i> must contain the matrix <i>C</i>.</p> |
| <i>ldc</i> | Specifies the leading dimension of <i>c</i> for one-based indexing, and the second dimension of <i>c</i> for zero-based indexing, as declared in the calling (sub)program. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | Overwritten by the matrix $(\alpha * A * B + \beta * C)$, $(\alpha * A^T * B + \beta * C)$, or $(\alpha * A^H * B + \beta * C)$. |
|----------|---|

mkl_?csrsm

Solves a system of linear matrix equations for a sparse matrix in the CSR format.

Syntax

```
void mkl_scsrsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const float
*alpha , const char *matdescra , const float *val , const MKL_INT *indx , const MKL_INT
*pntrb , const MKL_INT *pntre , const float *b , const MKL_INT *ldb , float *c , const
MKL_INT *ldc );
```

```
void mkl_dcsrsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *indx , const
MKL_INT *pntrb , const MKL_INT *pntre , const double *b , const MKL_INT *ldb , double
*c , const MKL_INT *ldc );
```

```
void mkl_ccsrsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex8 *b , const
MKL_INT *ldb , MKL_Complex8 *c , const MKL_INT *ldc );
```

```
void mkl_zcsrsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex16 *b , const
MKL_INT *ldb , MKL_Complex16 *c , const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?csrsm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the CSR format:

$$C := \alpha \operatorname{inv}(A) * B$$

or

$$C := \alpha \operatorname{inv}(A^T) * B,$$

where:

α is scalar, B and C are dense matrices, A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports a CSR format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|---------------|--|
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $C := \alpha \operatorname{inv}(A) * B$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $C := \alpha \operatorname{inv}(A^T) * B,$ |
| <i>m</i> | Number of columns of the matrix A . |
| <i>n</i> | Number of columns of the matrix C . |
| <i>alpha</i> | Specifies the scalar α . |

| | |
|------------------|---|
| <i>matdescra</i> | <p>Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>".</p> |
| <i>val</i> | <p>Array containing non-zero elements of the matrix <i>A</i>. For zero-based indexing its length is $pntrb[m-1] - pntrb[0]$. Refer to <i>values</i> array description in CSR Format for more details.</p> <hr/> <p>NOTE The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right). No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.</p> <hr/> |
| <i>indx</i> | <p>For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix <i>A</i>. For zero-based indexing, array containing the column indices for each non-zero element of the matrix <i>A</i>. Its length is equal to length of the <i>val</i> array. Refer to <i>columns</i> array description in CSR Format for more details.</p> <hr/> <p>NOTE Column indices must be sorted in increasing order for each row.</p> <hr/> |
| <i>pntrb</i> | <p>Array of length <i>m</i>. This array contains row indices, such that $pntrb[i] - pntrb[0]$ is the first index of row <i>i</i> in the arrays <i>val</i> and <i>indx</i>. Refer to <i>pointerb</i> array description in CSR Format for more details.</p> |
| <i>pntrE</i> | <p>Array of length <i>m</i>. For zero-based indexing this array contains row indices, such that $pntrE[i] - pntrb[0] - 1$ is the last index of row <i>i</i> in the arrays <i>val</i> and <i>indx</i>. Refer to <i>pointerE</i> array description in CSR Format for more details.</p> |
| <i>b</i> | <p>Array, size $ldb * n$ for one-based indexing, and (m, ldb) for zero-based indexing. On entry the leading <i>m</i>-by-<i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i>.</p> |
| <i>ldb</i> | <p>Specifies the leading dimension of <i>b</i> for one-based indexing, and the second dimension of <i>b</i> for zero-based indexing, as declared in the calling (sub)program.</p> |

ldc Specifies the leading dimension of *c* for one-based indexing, and the second dimension of *c* for zero-based indexing, as declared in the calling (sub)program.

Output Parameters

c Array, size *ldc* by *n* for one-based indexing, and (*m*, *ldc*) for zero-based indexing.
The leading *m*-by-*n* part of the array *c* contains the output matrix *C*.

mkl_?cscsm

Solves a system of linear matrix equations for a sparse matrix in the CSC format.

Syntax

```
void mkl_scscsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const float
*alpha , const char *matdescra , const float *val , const MKL_INT *indx , const MKL_INT
*pntrb , const MKL_INT *pntre , const float *b , const MKL_INT *ldb , float *c , const
MKL_INT *ldc );
```

```
void mkl_dcscsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *indx , const
MKL_INT *pntrb , const MKL_INT *pntre , const double *b , const MKL_INT *ldb , double
*c , const MKL_INT *ldc );
```

```
void mkl_ccscsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex8 *b , const
MKL_INT *ldb , MKL_Complex8 *c , const MKL_INT *ldc );
```

```
void mkl_zcscsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*indx , const MKL_INT *pntrb , const MKL_INT *pntre , const MKL_Complex16 *b , const
MKL_INT *ldb , MKL_Complex16 *c , const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?cscsm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the CSC format:

```
 $C := \alpha \text{inv}(A) * B$ 
```

or

```
 $C := \alpha \text{inv}(A^T) * B,$ 
```

where:

alpha is scalar, *B* and *C* are dense matrices, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of *A*.

NOTE

This routine supports a CSC format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the system of equations. If <i>transa</i> = 'N' or 'n', then $C := \alpha * \text{inv}(A) * B$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $C := \alpha * \text{inv}(A^T) * B$, |
| <i>m</i> | Number of columns of the matrix A. |
| <i>n</i> | Number of columns of the matrix C. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing non-zero elements of the matrix A. For zero-based indexing its length is $\text{pntrb}[m] - \text{pntrb}[0]$. Refer to <i>values</i> array description in CSC Format for more details. |

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

| | |
|-------------|---|
| <i>indx</i> | For one-based indexing, array containing the row indices plus one for each non-zero element of the matrix A. For zero-based indexing, array containing the row indices for each non-zero element of the matrix A. Refer to <i>rows</i> array description in CSC Format for more details. |
|-------------|---|

NOTE

Row indices must be sorted in increasing order for each column.

| | |
|--------------|--|
| <i>pntrb</i> | Array of length <i>m</i> . This array contains column indices, such that $\text{pntrb}[I] - \text{pntrb}[0]$ is the first index of column <i>I</i> in the arrays <i>val</i> and <i>indx</i> . Refer to <i>pointerb</i> array description in CSC Format for more details. |
| <i>pntrc</i> | Array of length <i>m</i> . This array contains column indices, such that $\text{pntrc}[I] - \text{pntrb}[1] - 1$ is the last index of column <i>I</i> in the arrays <i>val</i> and <i>indx</i> . |

Refer to *pointerE* array description in [CSC Format](#) for more details.

| | |
|------------|--|
| <i>b</i> | Array, size <i>ldb</i> by <i>n</i> for one-based indexing, and (<i>m</i> , <i>ldb</i>) for zero-based indexing. On entry the leading <i>m</i> -by- <i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i> . |
| <i>ldb</i> | Specifies the leading dimension of <i>b</i> for one-based indexing, and the second dimension of <i>b</i> for zero-based indexing, as declared in the calling (sub)program. |
| <i>ldc</i> | Specifies the leading dimension of <i>c</i> for one-based indexing, and the second dimension of <i>c</i> for zero-based indexing, as declared in the calling (sub)program. |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Array, size <i>ldc</i> by <i>n</i> for one-based indexing, and (<i>m</i> , <i>ldc</i>) for zero-based indexing. The leading <i>m</i> -by- <i>n</i> part of the array <i>c</i> contains the output matrix <i>C</i> . |
|----------|--|

mkl_?coosm

Solves a system of linear matrix equations for a sparse matrix in the coordinate format.

Syntax

```
void mkl_scoosm (const char *transa , const MKL_INT *m , const MKL_INT *n , const float
*alpha , const char *matdescra , const float *val , const MKL_INT *rowind , const
MKL_INT *colind , const MKL_INT *nnz , const float *b , const MKL_INT *ldb , float
*c , const MKL_INT *ldc );
```

```
void mkl_dcoosm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *rowind ,
const MKL_INT *colind , const MKL_INT *nnz , const double *b , const MKL_INT *ldb ,
double *c , const MKL_INT *ldc );
```

```
void mkl_ccoosm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*rowind , const MKL_INT *colind , const MKL_INT *nnz , const MKL_Complex8 *b , const
MKL_INT *ldb , MKL_Complex8 *c , const MKL_INT *ldc );
```

```
void mkl_zcoosm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*rowind , const MKL_INT *colind , const MKL_INT *nnz , const MKL_Complex16 *b , const
MKL_INT *ldb , MKL_Complex16 *c , const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?coosm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the coordinate format:

```
C := alpha*inv(A)*B
```

or

```
C := alpha*inv(AT)*B,
```

where:

alpha is scalar, *B* and *C* are dense matrices, *A* is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, *A*^T is the transpose of *A*.

NOTE

This routine supports a coordinate format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|---|
| <i>transa</i> | <p>Specifies the system of linear equations.</p> <p>If <i>transa</i> = 'N' or 'n', then the matrix-matrix product is computed as $C := \text{alpha} * \text{inv}(A) * B$</p> <p>If <i>transa</i> = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as $C := \text{alpha} * \text{inv}(A^T) * B$,</p> |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>n</i> | Number of columns of the matrix <i>C</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | <p>Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)". Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>".</p> |
| <i>val</i> | <p>Array of length <i>nnz</i>, contains non-zero elements of the matrix <i>A</i> in the arbitrary order.</p> <p>Refer to <i>values</i> array description in Coordinate Format for more details.</p> |
| <i>rowind</i> | <p>Array of length <i>nnz</i>.</p> <p>For one-based indexing, contains the row indices plus one for each non-zero element of the matrix <i>A</i>.</p> <p>For zero-based indexing, contains the row indices for each non-zero element of the matrix <i>A</i>.</p> <p>Refer to <i>rows</i> array description in Coordinate Format for more details.</p> |
| <i>colind</i> | <p>Array of length <i>nnz</i>.</p> <p>For one-based indexing, contains the column indices plus one for each non-zero element of the matrix <i>A</i></p> <p>For zero-based indexing, contains the row indices for each non-zero element of the matrix <i>A</i></p> <p>Refer to <i>columns</i> array description in Coordinate Format for more details.</p> |
| <i>nnz</i> | Specifies the number of non-zero element of the matrix <i>A</i> . |

Refer to *nnz* description in [Coordinate Format](#) for more details.

| | |
|------------|---|
| <i>b</i> | <p>Array, size <i>ldb</i> by <i>n</i> for one-based indexing, and (<i>m</i>, <i>ldb</i>) for zero-based indexing.</p> <p>Before entry the leading <i>m</i>-by-<i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i>.</p> |
| <i>ldb</i> | <p>Specifies the leading dimension of <i>b</i> for one-based indexing, and the second dimension of <i>b</i> for zero-based indexing, as declared in the calling (sub)program.</p> |
| <i>ldc</i> | <p>Specifies the leading dimension of <i>c</i> for one-based indexing, and the second dimension of <i>c</i> for zero-based indexing, as declared in the calling (sub)program.</p> |

Output Parameters

| | |
|----------|---|
| <i>c</i> | <p>Array, size <i>ldc</i> by <i>n</i> for one-based indexing, and (<i>m</i>, <i>ldc</i>) for zero-based indexing.</p> <p>The leading <i>m</i>-by-<i>n</i> part of the array <i>c</i> contains the output matrix <i>C</i>.</p> |
|----------|---|

mkl_?bsrsm

Solves a system of linear matrix equations for a sparse matrix in the BSR format.

Syntax

```
void mkl_sbsrsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *lb , const float *alpha , const char *matdescra , const float *val , const
MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const float *b , const
MKL_INT *ldb , float *c , const MKL_INT *ldc );
```

```
void mkl_dbsrsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *lb , const double *alpha , const char *matdescra , const double *val , const
MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const double *b , const
MKL_INT *ldb , double *c , const MKL_INT *ldc );
```

```
void mkl_cbsrsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *lb , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8
*val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const
MKL_Complex8 *b , const MKL_INT *ldb , MKL_Complex8 *c , const MKL_INT *ldc );
```

```
void mkl_zbsrsm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *lb , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16
*val , const MKL_INT *indx , const MKL_INT *pntrb , const MKL_INT *pntre , const
MKL_Complex16 *b , const MKL_INT *ldb , MKL_Complex16 *c , const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?bsrsm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the BSR format:

$$C := \alpha * \text{inv}(A) * B$$

or

$$C := \alpha * \text{inv}(A^T) * B,$$

where:

α is scalar, B and C are dense matrices, A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports a BSR format both with one-based indexing and zero-based indexing.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then the matrix-matrix product is computed as $C := \alpha * \text{inv}(A) * B$. If <i>transa</i> = 'T' or 't' or 'C' or 'c', then the matrix-vector product is computed as $C := \alpha * \text{inv}(A^T) * B$. |
| <i>m</i> | Number of block columns of the matrix A . |
| <i>n</i> | Number of columns of the matrix C . |
| <i>lb</i> | Size of the block in the matrix A . |
| <i>alpha</i> | Specifies the scalar α . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Array containing elements of non-zero blocks of the matrix A . Its length is equal to the number of non-zero blocks in the matrix A multiplied by $lb * lb$. Refer to the <i>values</i> array description in BSR Format for more details. |

NOTE

The non-zero elements of the given row of the matrix must be stored in the same order as they appear in the row (from left to right).

No diagonal element can be omitted from a sparse storage if the solver is called with the non-unit indicator.

| | |
|-------------|---|
| <i>indx</i> | For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix A . For zero-based indexing, array containing the column indices for each non-zero element of the matrix A . Its length is equal to the number of non-zero blocks in the matrix A . |
|-------------|---|

| | |
|--------------|---|
| | Refer to the <i>columns</i> array description in BSR Format for more details. |
| <i>pntrb</i> | <p>Array of length <i>m</i>.</p> <p>This array contains row indices, such that <i>pntrb[i] - pntrb[0]</i> is the first index of block row <i>i</i> in the array <i>indx</i>.</p> <p>Refer to <i>pointerB</i> array description in BSR Format for more details.</p> |
| <i>pntrE</i> | <p>Array of length <i>m</i>.</p> <p>This array contains row indices, such that <i>pntrE[i] - pntrb[0] - 1</i> is the last index of block row <i>i</i> in the arrays <i>val</i> and <i>indx</i>.</p> <p>Refer to <i>pointerE</i> array description in BSR Format for more details.</p> |
| <i>b</i> | <p>Array, size <i>ldb* n</i> for one-based indexing, size <i>m* ldb</i> for zero-based indexing.</p> <p>On entry the leading <i>m-by-n</i> part of the array <i>b</i> must contain the matrix <i>B</i>.</p> |
| <i>ldb</i> | Specifies the leading dimension (in blocks) of <i>b</i> as declared in the calling (sub)program. |
| <i>ldc</i> | Specifies the leading dimension (in blocks) of <i>c</i> as declared in the calling (sub)program. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | <p>Array, size <i>ldc* n</i> for one-based indexing, size <i>m* ldc</i> for zero-based indexing.</p> <p>The leading <i>m-by-n</i> part of the array <i>c</i> contains the output matrix <i>C</i>.</p> |
|----------|---|

mkl_?diamv

Computes matrix - vector product for a sparse matrix in the diagonal format with one-based indexing.

Syntax

```
void mkl_sdiamv (const char *transa , const MKL_INT *m , const MKL_INT *k , const float
*alpha , const char *matdescra , const float *val , const MKL_INT *lval , const MKL_INT
*idiag , const MKL_INT *ndiag , const float *x , const float *beta , float *y );

void mkl_ddiamv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *lval , const
MKL_INT *idiag , const MKL_INT *ndiag , const double *x , const double *beta , double
*y );

void mkl_cdiamv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*lval , const MKL_INT *idiag , const MKL_INT *ndiag , const MKL_Complex8 *x , const
MKL_Complex8 *beta , MKL_Complex8 *y );

void mkl_zdiamv (const char *transa , const MKL_INT *m , const MKL_INT *k , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*lval , const MKL_INT *idiag , const MKL_INT *ndiag , const MKL_Complex16 *x , const
MKL_Complex16 *beta , MKL_Complex16 *y );
```

Include Files

- `mkl.h`

Description

The `mkl_?diamv` routine performs a matrix-vector operation defined as

$$y := \alpha * A * x + \beta * y$$

or

$$y := \alpha * A^T * x + \beta * y,$$

where:

alpha and *beta* are scalars,

x and *y* are vectors,

A is an *m*-by-*k* sparse matrix stored in the diagonal format, A^T is the transpose of *A*.

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $y := \alpha * A * x + \beta * y$, If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha * A^T * x + \beta * y$. |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>k</i> | Number of columns of the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Two-dimensional array of size <i>lval</i> by <i>ndiag</i> , contains non-zero diagonals of the matrix <i>A</i> . Refer to <i>values</i> array description in Diagonal Storage Scheme for more details. |
| <i>lval</i> | Leading dimension of <i>val</i> , $lval \geq m$. Refer to <i>lval</i> description in Diagonal Storage Scheme for more details. |
| <i>idiag</i> | Array of length <i>ndiag</i> , contains the distances between main diagonal and each non-zero diagonals in the matrix <i>A</i> . Refer to <i>distance</i> array description in Diagonal Storage Scheme for more details. |
| <i>ndiag</i> | Specifies the number of non-zero diagonals of the matrix <i>A</i> . |

| | |
|--------|---|
| x | Array, size at least k if $transa = 'N'$ or $'n'$, and at least m otherwise. On entry, the array x must contain the vector x . |
| $beta$ | Specifies the scalar $beta$. |
| y | Array, size at least m if $transa = 'N'$ or $'n'$, and at least k otherwise. On entry, the array y must contain the vector y . |

Output Parameters

| | |
|-----|---|
| y | Overwritten by the updated vector y . |
|-----|---|

mkl_?skymv

Computes matrix - vector product for a sparse matrix in the skyline storage format with one-based indexing.

Syntax

```
void mkl_sskymv (const char *transa , const MKL_INT *m , const MKL_INT *k , const float *alpha , const char *matdescra , const float *val , const MKL_INT *pntr , const float *x , const float *beta , float *y );
```

```
void mkl_dskymv (const char *transa , const MKL_INT *m , const MKL_INT *k , const double *alpha , const char *matdescra , const double *val , const MKL_INT *pntr , const double *x , const double *beta , double *y );
```

```
void mkl_cskymv (const char *transa , const MKL_INT *m , const MKL_INT *k , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT *pntr , const MKL_Complex8 *x , const MKL_Complex8 *beta , MKL_Complex8 *y );
```

```
void mkl_zskymv (const char *transa , const MKL_INT *m , const MKL_INT *k , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT *pntr , const MKL_Complex16 *x , const MKL_Complex16 *beta , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?skymv` routine performs a matrix-vector operation defined as

```
 $y := alpha * A * x + beta * y$ 
```

or

```
 $y := alpha * A^T * x + beta * y,$ 
```

where:

$alpha$ and $beta$ are scalars,

x and y are vectors,

A is an m -by- k sparse matrix stored using the skyline storage scheme, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $y := \alpha * A * x + \beta * y$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha * A^T * x + \beta * y$, |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>k</i> | Number of columns of the matrix <i>A</i> . |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |

NOTE

General matrices (*matdescra*[0]='G') is not supported.

| | |
|-------------|--|
| <i>val</i> | Array containing the set of elements of the matrix <i>A</i> in the skyline profile form. If <i>matdescrsa</i> [1]= 'L', then <i>val</i> contains elements from the low triangle of the matrix <i>A</i> . If <i>matdescrsa</i> [1]= 'U', then <i>val</i> contains elements from the upper triangle of the matrix <i>A</i> . Refer to <i>values</i> array description in Skyline Storage Scheme for more details. |
| <i>pntr</i> | Array of length (<i>m</i> + 1) for lower triangle, and (<i>k</i> + 1) for upper triangle. It contains the indices specifying in the <i>val</i> the positions of the first element in each row (column) of the matrix <i>A</i> . Refer to <i>pointers</i> array description in Skyline Storage Scheme for more details. |
| <i>x</i> | Array, size at least <i>k</i> if <i>transa</i> = 'N' or 'n' and at least <i>m</i> otherwise. On entry, the array <i>x</i> must contain the vector <i>x</i> . |
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>y</i> | Array, size at least <i>m</i> if <i>transa</i> = 'N' or 'n' and at least <i>k</i> otherwise. On entry, the array <i>y</i> must contain the vector <i>y</i> . |

Output Parameters

| | |
|----------|--|
| <i>y</i> | Overwritten by the updated vector <i>y</i> . |
|----------|--|

mkl_?diasv

Solves a system of linear equations for a sparse matrix in the diagonal format with one-based indexing.

Syntax

```
void mkl_sdiasv (const char *transa , const MKL_INT *m , const float *alpha , const
char *matdescra , const float *val , const MKL_INT *lval , const MKL_INT *idiag , const
MKL_INT *ndiag , const float *x , float *y );
```

```
void mkl_ddiasv (const char *transa , const MKL_INT *m , const double *alpha , const
char *matdescra , const double *val , const MKL_INT *lval , const MKL_INT *idiag ,
const MKL_INT *ndiag , const double *x , double *y );
```

```
void mkl_cdiasv (const char *transa , const MKL_INT *m , const MKL_Complex8 *alpha ,
const char *matdescra , const MKL_Complex8 *val , const MKL_INT *lval , const MKL_INT
*idiag , const MKL_INT *ndiag , const MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zdiasv (const char *transa , const MKL_INT *m , const MKL_Complex16 *alpha ,
const char *matdescra , const MKL_Complex16 *val , const MKL_INT *lval , const MKL_INT
*idiag , const MKL_INT *ndiag , const MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?diasv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix stored in the diagonal format:

```
 $y := \alpha \text{inv}(A) * x$ 
```

or

```
 $y := \alpha \text{inv}(A^T) * x,$ 
```

where:

α is scalar, x and y are vectors, A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|---------------|--|
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $y := \alpha \text{inv}(A) * x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha \text{inv}(A^T) * x,$ |
| <i>m</i> | Number of rows of the matrix A . |
| <i>alpha</i> | Specifies the scalar α . |

| | |
|------------------|--|
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table “Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)” . Possible combinations of element values of this parameter are given in Table “Possible Combinations of Element Values of the Parameter <i>matdescra</i>” . |
| <i>val</i> | Two-dimensional array of size <i>lval</i> by <i>ndiag</i> , contains non-zero diagonals of the matrix <i>A</i> . Refer to <i>values</i> array description in Diagonal Storage Scheme for more details. |
| <i>lval</i> | Leading dimension of <i>val</i> , $lval \geq m$. Refer to <i>lval</i> description in Diagonal Storage Scheme for more details. |
| <i>idiag</i> | Array of length <i>ndiag</i> , contains the distances between main diagonal and each non-zero diagonals in the matrix <i>A</i> . |

NOTE

All elements of this array must be sorted in increasing order.

Refer to *distance* array description in [Diagonal Storage Scheme](#) for more details.

| | |
|--------------|---|
| <i>ndiag</i> | Specifies the number of non-zero diagonals of the matrix <i>A</i> . |
| <i>x</i> | Array, size at least <i>m</i> . On entry, the array <i>x</i> must contain the vector <i>x</i> . The elements are accessed with unit increment. |
| <i>y</i> | Array, size at least <i>m</i> . On entry, the array <i>y</i> must contain the vector <i>y</i> . The elements are accessed with unit increment. |

Output Parameters

| | |
|----------|-------------------------------------|
| <i>y</i> | Contains solution vector <i>x</i> . |
|----------|-------------------------------------|

mkl_?skysv

Solves a system of linear equations for a sparse matrix in the skyline format with one-based indexing.

Syntax

```
void mkl_sskysv (const char *transa , const MKL_INT *m , const float *alpha , const
char *matdescra , const float *val , const MKL_INT *pntr , const float *x , float *y );
void mkl_dskysv (const char *transa , const MKL_INT *m , const double *alpha , const
char *matdescra , const double *val , const MKL_INT *pntr , const double *x , double
*y );
void mkl_cskysv (const char *transa , const MKL_INT *m , const MKL_Complex8 *alpha ,
const char *matdescra , const MKL_Complex8 *val , const MKL_INT *pntr , const
MKL_Complex8 *x , MKL_Complex8 *y );
```

```
void mkl_zskysv (const char *transa , const MKL_INT *m , const MKL_Complex16 *alpha ,
const char *matdescra , const MKL_Complex16 *val , const MKL_INT *pntr , const
MKL_Complex16 *x , MKL_Complex16 *y );
```

Include Files

- mkl.h

Description

The `mkl_?skysv` routine solves a system of linear equations with matrix-vector operations for a sparse matrix in the skyline storage format:

```
 $y := \alpha \text{inv}(A) * x$ 
```

or

```
 $y := \alpha \text{inv}(A^T) * x,$ 
```

where:

α is scalar, x and y are vectors, A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $y := \alpha \text{inv}(A) * x$ If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $y := \alpha \text{inv}(A^T) * x,$ |
| <i>m</i> | Number of rows of the matrix A . |
| <i>alpha</i> | Specifies the scalar α . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |

NOTE

General matrices (*matdescra*[0]='G') is not supported.

| | |
|------------|---|
| <i>val</i> | Array containing the set of elements of the matrix A in the skyline profile form. If <i>matdescra</i> [2]= 'L', then <i>val</i> contains elements from the low triangle of the matrix A . If <i>matdescsa</i> [2]= 'U', then <i>val</i> contains elements from the upper triangle of the matrix A . |
|------------|---|

Refer to *values* array description in [Skyline Storage Scheme](#) for more details.

| | |
|-------------|--|
| <i>pntr</i> | <p>Array of length $(m + 1)$ for lower triangle, and $(k + 1)$ for upper triangle.</p> <p>It contains the indices specifying in the <i>val</i> the positions of the first element in each row (column) of the matrix A. Refer to <i>pointers</i> array description in Skyline Storage Scheme for more details.</p> |
| <i>x</i> | <p>Array, size at least <i>m</i>.</p> <p>On entry, the array <i>x</i> must contain the vector <i>x</i>. The elements are accessed with unit increment.</p> |
| <i>y</i> | <p>Array, size at least <i>m</i>.</p> <p>On entry, the array <i>y</i> must contain the vector <i>y</i>. The elements are accessed with unit increment.</p> |

Output Parameters

| | |
|----------|-------------------------------------|
| <i>y</i> | Contains solution vector <i>x</i> . |
|----------|-------------------------------------|

mkl_?diamm

Computes matrix-matrix product of a sparse matrix stored in the diagonal format with one-based indexing.

Syntax

```
void mkl_sdiamm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const float *alpha , const char *matdescra , const float *val , const
MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const float *b , const
MKL_INT *ldb , const float *beta , float *c , const MKL_INT *ldc );
```

```
void mkl_ddiamm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const double *alpha , const char *matdescra , const double *val , const
MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const double *b , const
MKL_INT *ldb , const double *beta , double *c , const MKL_INT *ldc );
```

```
void mkl_cdiamm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8
*val , const MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const
MKL_Complex8 *b , const MKL_INT *ldb , const MKL_Complex8 *beta , MKL_Complex8 *c ,
const MKL_INT *ldc );
```

```
void mkl_zdiamm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16
*val , const MKL_INT *lval , const MKL_INT *idiag , const MKL_INT *ndiag , const
MKL_Complex16 *b , const MKL_INT *ldb , const MKL_Complex16 *beta , MKL_Complex16 *c ,
const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?diamm` routine performs a matrix-matrix operation defined as

$$C := \alpha * A * B + \beta * C$$

or

$$C := \alpha * A^T * B + \beta * C,$$

or

$$C := \alpha * A^H * B + \beta * C,$$

where:

α and β are scalars,

B and C are dense matrices, A is an m -by- k sparse matrix in the diagonal format, A^T is the transpose of A , and A^H is the conjugate transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $C := \alpha * A * B + \beta * C$, If <i>transa</i> = 'T' or 't', then $C := \alpha * A^T * B + \beta * C$, If <i>transa</i> = 'C' or 'c', then $C := \alpha * A^H * B + \beta * C$. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>n</i> | Number of columns of the matrix C . |
| <i>k</i> | Number of columns of the matrix A . |
| <i>alpha</i> | Specifies the scalar α . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Two-dimensional array of size <i>lval</i> by <i>ndiag</i> , contains non-zero diagonals of the matrix A . Refer to <i>values</i> array description in Diagonal Storage Scheme for more details. |
| <i>lval</i> | Leading dimension of <i>val</i> , $lval \geq m$. Refer to <i>lval</i> description in Diagonal Storage Scheme for more details. |
| <i>idiag</i> | Array of length <i>ndiag</i> , contains the distances between main diagonal and each non-zero diagonals in the matrix A . Refer to <i>distance</i> array description in Diagonal Storage Scheme for more details. |
| <i>ndiag</i> | Specifies the number of non-zero diagonals of the matrix A . |

| | |
|-------------|--|
| <i>b</i> | Array, size $ldb * n$. On entry with <i>transa</i> = 'N' or 'n', the leading k -by- n part of the array <i>b</i> must contain the matrix <i>B</i> , otherwise the leading m -by- n part of the array <i>b</i> must contain the matrix <i>B</i> . |
| <i>ldb</i> | Specifies the leading dimension of <i>b</i> as declared in the calling (sub)program. |
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>c</i> | Array, size ldc by n . On entry, the leading m -by- n part of the array <i>c</i> must contain the matrix <i>C</i> , otherwise the leading k -by- n part of the array <i>c</i> must contain the matrix <i>C</i> . |
| <i>ldc</i> | Specifies the leading dimension of <i>c</i> as declared in the calling (sub)program. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | Overwritten by the matrix $(\alpha * A * B + \beta * C)$, $(\alpha * A^T * B + \beta * C)$, or $(\alpha * A^H * B + \beta * C)$. |
|----------|---|

mkl_?skymm

Computes matrix-matrix product of a sparse matrix stored using the skyline storage scheme with one-based indexing.

Syntax

```
void mkl_sskymm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const float *alpha , const char *matdescra , const float *val , const
MKL_INT *pntr , const float *b , const MKL_INT *ldb , const float *beta , float *c ,
const MKL_INT *ldc );
```

```
void mkl_dskymm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const double *alpha , const char *matdescra , const double *val , const
MKL_INT *pntr , const double *b , const MKL_INT *ldb , const double *beta , double
*c , const MKL_INT *ldc );
```

```
void mkl_cskymm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8
*val , const MKL_INT *pntr , const MKL_Complex8 *b , const MKL_INT *ldb , const
MKL_Complex8 *beta , MKL_Complex8 *c , const MKL_INT *ldc );
```

```
void mkl_zskymm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16
*val , const MKL_INT *pntr , const MKL_Complex16 *b , const MKL_INT *ldb , const
MKL_Complex16 *beta , MKL_Complex16 *c , const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?skymm` routine performs a matrix-matrix operation defined as

$$C := \alpha * A * B + \beta * C$$

or

$$C := \alpha * A^T * B + \beta * C,$$

or

$$C := \alpha * A^H * B + \beta * C,$$

where:

α and β are scalars,

B and C are dense matrices, A is an m -by- k sparse matrix in the skyline storage format, A^T is the transpose of A , and A^H is the conjugate transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the operation. If <i>transa</i> = 'N' or 'n', then $C := \alpha * A * B + \beta * C$, If <i>transa</i> = 'T' or 't', then $C := \alpha * A^T * B + \beta * C$, If <i>transa</i> = 'C' or 'c', then $C := \alpha * A^H * B + \beta * C$. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>n</i> | Number of columns of the matrix C . |
| <i>k</i> | Number of columns of the matrix A . |
| <i>alpha</i> | Specifies the scalar α . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |

NOTE

General matrices (*matdescra* [0]='G') is not supported.

| | |
|------------|---|
| <i>val</i> | Array containing the set of elements of the matrix A in the skyline profile form. If <i>matdescrsa</i> [2]= 'L', then <i>val</i> contains elements from the low triangle of the matrix A . If <i>matdescrsa</i> [2]= 'U', then <i>val</i> contains elements from the upper triangle of the matrix A . |
|------------|---|

Refer to *values* array description in [Skyline Storage Scheme](#) for more details.

pntr Array of length $(m + 1)$ for lower triangle, and $(k + 1)$ for upper triangle. It contains the indices specifying the positions of the first element of the matrix *A* in each row (for the lower triangle) or column (for upper triangle) in the *val* array such that $val[pntr[i] - 1]$ is the first element in row or column $i + 1$. Refer to *pointers* array description in [Skyline Storage Scheme](#) for more details.

b Array, size $ldb * n$.
On entry with *transa* = 'N' or 'n', the leading k -by- n part of the array *b* must contain the matrix *B*, otherwise the leading m -by- n part of the array *b* must contain the matrix *B*.

ldb Specifies the leading dimension of *b* as declared in the calling (sub)program.

beta Specifies the scalar *beta*.

c Array, size ldc by n .
On entry, the leading m -by- n part of the array *c* must contain the matrix *C*, otherwise the leading k -by- n part of the array *c* must contain the matrix *C*.

ldc Specifies the leading dimension of *c* as declared in the calling (sub)program.

Output Parameters

c Overwritten by the matrix $(alpha * A * B + beta * C)$, $(alpha * A^T * B + beta * C)$, or $(alpha * A^H * B + beta * C)$.

mkl_?diasm

Solves a system of linear matrix equations for a sparse matrix in the diagonal format with one-based indexing.

Syntax

```
void mkl_sdiasm (const char *transa , const MKL_INT *m , const MKL_INT *n , const float
*alpha , const char *matdescra , const float *val , const MKL_INT *lval , const MKL_INT
*idiag , const MKL_INT *ndiag , const float *b , const MKL_INT *ldb , float *c , const
MKL_INT *ldc );
```

```
void mkl_ddiasm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
double *alpha , const char *matdescra , const double *val , const MKL_INT *lval , const
MKL_INT *idiag , const MKL_INT *ndiag , const double *b , const MKL_INT *ldb , double
*c , const MKL_INT *ldc );
```

```
void mkl_cdiasm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT
*lval , const MKL_INT *idiag , const MKL_INT *ndiag , const MKL_Complex8 *b , const
MKL_INT *ldb , MKL_Complex8 *c , const MKL_INT *ldc );
```

```
void mkl_zdiasm (const char *transa , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT
*lval , const MKL_INT *idiag , const MKL_INT *ndiag , const MKL_Complex16 *b , const
MKL_INT *ldb , MKL_Complex16 *c , const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?diasm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the diagonal format:

$$C := \alpha * \text{inv}(A) * B$$

or

$$C := \alpha * \text{inv}(A^T) * B,$$

where:

α is scalar, B and C are dense matrices, A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|------------------|--|
| <i>transa</i> | Specifies the system of linear equations. If <i>transa</i> = 'N' or 'n', then $C := \alpha * \text{inv}(A) * B$, If <i>transa</i> = 'T' or 't' or 'C' or 'c', then $C := \alpha * \text{inv}(A^T) * B$. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>n</i> | Number of columns of the matrix C . |
| <i>alpha</i> | Specifies the scalar α . |
| <i>matdescra</i> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <i>matdescra</i> (<i>descra</i>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <i>matdescra</i>" . |
| <i>val</i> | Two-dimensional array of size <i>lval</i> by <i>ndiag</i> , contains non-zero diagonals of the matrix A . Refer to <i>values</i> array description in Diagonal Storage Scheme for more details. |
| <i>lval</i> | Leading dimension of <i>val</i> , $lval \geq m$. Refer to <i>lval</i> description in Diagonal Storage Scheme for more details. |
| <i>idiag</i> | Array of length <i>ndiag</i> , contains the distances between main diagonal and each non-zero diagonals in the matrix A . |

NOTE

All elements of this array must be sorted in increasing order.

Refer to *distance* array description in [Diagonal Storage Scheme](#) for more details.

| | |
|--------------|--|
| <i>ndiag</i> | Specifies the number of non-zero diagonals of the matrix <i>A</i> . |
| <i>b</i> | Array, size <i>ldb</i> * <i>n</i> . On entry the leading <i>m</i> -by- <i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i> . |
| <i>ldb</i> | Specifies the leading dimension of <i>b</i> as declared in the calling (sub)program. |
| <i>ldc</i> | Specifies the leading dimension of <i>c</i> as declared in the calling (sub)program. |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Array, size <i>ldc</i> by <i>n</i> . The leading <i>m</i> -by- <i>n</i> part of the array <i>c</i> contains the matrix <i>C</i> . |
|----------|--|

mkl_?skysm

Solves a system of linear matrix equations for a sparse matrix stored using the skyline storage scheme with one-based indexing.

Syntax

```
void mkl_sskysm (const char *transa , const MKL_INT *m , const MKL_INT *n , const float *alpha , const char *matdescra , const float *val , const MKL_INT *pntr , const float *b , const MKL_INT *ldb , float *c , const MKL_INT *ldc );
```

```
void mkl_dskysm (const char *transa , const MKL_INT *m , const MKL_INT *n , const double *alpha , const char *matdescra , const double *val , const MKL_INT *pntr , const double *b , const MKL_INT *ldb , double *c , const MKL_INT *ldc );
```

```
void mkl_cskysm (const char *transa , const MKL_INT *m , const MKL_INT *n , const MKL_Complex8 *alpha , const char *matdescra , const MKL_Complex8 *val , const MKL_INT *pntr , const MKL_Complex8 *b , const MKL_INT *ldb , MKL_Complex8 *c , const MKL_INT *ldc );
```

```
void mkl_zskysm (const char *transa , const MKL_INT *m , const MKL_INT *n , const MKL_Complex16 *alpha , const char *matdescra , const MKL_Complex16 *val , const MKL_INT *pntr , const MKL_Complex16 *b , const MKL_INT *ldb , MKL_Complex16 *c , const MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?skysm` routine solves a system of linear equations with matrix-matrix operations for a sparse matrix in the skyline storage format:

$$C := \alpha * \text{inv}(A) * B$$

or

$$C := \alpha * \text{inv}(A^T) * B,$$

where:

α is scalar, B and C are dense matrices, A is a sparse upper or lower triangular matrix with unit or non-unit main diagonal, A^T is the transpose of A .

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|------------------------|--|
| <code>transa</code> | Specifies the system of linear equations. If <code>transa = 'N' or 'n'</code> , then $C := \alpha * \text{inv}(A) * B$, If <code>transa = 'T' or 't' or 'C' or 'c'</code> , then $C := \alpha * \text{inv}(A^T) * B$, |
| <code>m</code> | Number of rows of the matrix A . |
| <code>n</code> | Number of columns of the matrix C . |
| <code>alpha</code> | Specifies the scalar α . |
| <code>matdescra</code> | Array of six elements, specifies properties of the matrix used for operation. Only first four array elements are used, their possible values are given in Table "Possible Values of the Parameter <code>matdescra</code> (<code>descra</code>)" . Possible combinations of element values of this parameter are given in Table "Possible Combinations of Element Values of the Parameter <code>matdescra</code>" . |

NOTE

General matrices (`matdescra[0]='G'`) is not supported.

| | |
|-------------------|---|
| <code>val</code> | Array containing the set of elements of the matrix A in the skyline profile form. If <code>matdescrsa[2]= 'L'</code> , then <code>val</code> contains elements from the low triangle of the matrix A . If <code>matdescrsa[2]= 'U'</code> , then <code>val</code> contains elements from the upper triangle of the matrix A . Refer to <code>values</code> array description in Skyline Storage Scheme for more details. |
| <code>pntr</code> | Array of length $(m + 1)$ for lower triangle, and $(n + 1)$ for upper triangle. |

It contains the indices specifying the positions of the first element of the matrix A in each row (for the lower triangle) or column (for upper triangle) in the `val` array such that `val[ptr[i] - 1]` is the first element in row or column $i + 1$. Refer to `pointers` array description in [Skyline Storage Scheme](#) for more details.

`b` Array, size `ldb * n`.
On entry the leading m -by- n part of the array `b` must contain the matrix B .

`ldb` Specifies the leading dimension of `b` as declared in the calling (sub)program.

`ldc` Specifies the leading dimension of `c` as declared in the calling (sub)program.

Output Parameters

`c` Array, size `ldc` by `n`.
The leading m -by- n part of the array `c` contains the matrix C .

mkl_?dnscsr

Convert a sparse matrix in uncompressed representation to the CSR format and vice versa.

Syntax

```
void mkl_ddnscsr (const MKL_INT *job , const MKL_INT *m , const MKL_INT *n , double
*adns , const MKL_INT *lda , double *acsr , MKL_INT *ja , MKL_INT *ia , MKL_INT
*info );

void mkl_sdnscsr (const MKL_INT *job , const MKL_INT *m , const MKL_INT *n , float
*adns , const MKL_INT *lda , float *acsr , MKL_INT *ja , MKL_INT *ia , MKL_INT *info );

void mkl_cdnscsr (const MKL_INT *job , const MKL_INT *m , const MKL_INT *n ,
MKL_Complex8 *adns , const MKL_INT *lda , MKL_Complex8 *acsr , MKL_INT *ja , MKL_INT
*ia , MKL_INT *info );

void mkl_zdnscsr (const MKL_INT *job , const MKL_INT *m , const MKL_INT *n ,
MKL_Complex16 *adns , const MKL_INT *lda , MKL_Complex16 *acsr , MKL_INT *ja , MKL_INT
*ia , MKL_INT *info );
```

Include Files

- `mkl.h`

Description

This routine converts a sparse matrix A between formats: stored as a rectangular array (dense representation) and stored using compressed sparse row (CSR) format (3-array variation).

Input Parameters

`job` Array, contains the following conversion parameters:

- `job[0]`: Conversion type.

- If $job[0]=0$, the rectangular matrix A is converted to the CSR format;
- if $job[0]=1$, the rectangular matrix A is restored from the CSR format.
- $job[1]$: index base for the rectangular matrix A .
 - If $job[1]=0$, zero-based indexing for the rectangular matrix A is used;
 - if $job[1]=1$, one-based indexing for the rectangular matrix A is used.
- $job[2]$: Index base for the matrix in CSR format.
 - If $job[2]=0$, zero-based indexing for the matrix in CSR format is used;
 - if $job[2]=1$, one-based indexing for the matrix in CSR format is used.
- $job[3]$: Portion of matrix.
 - If $job[3]=0$, $adns$ is a lower triangular part of matrix A ;
 - If $job[3]=1$, $adns$ is an upper triangular part of matrix A ;
 - If $job[3]=2$, $adns$ is a whole matrix A .
- $job[4]=nzmax$: maximum number of the non-zero elements allowed if $job[0]=0$.
- $job[5]$: job indicator for conversion to CSR format.
 - If $job[5]=0$, only array ia is generated for the output storage.
 - If $job[5]>0$, arrays $acsr$, ia , ja are generated for the output storage.

m Number of rows of the matrix A .

n Number of columns of the matrix A .

$adns$ (input/output)

If the conversion type is from uncompressed to CSR, on input $adns$ contains an uncompressed (dense) representation of matrix A .

lda Specifies the leading dimension of $adns$ as declared in the calling (sub)program.

For zero-based indexing of A , lda must be at least $\max(1, n)$.

For one-based indexing of A , lda must be at least $\max(1, m)$.

$acsr$ (input/output)

If conversion type is from CSR to uncompressed, on input $acsr$ contains the non-zero elements of the matrix A . Its length is equal to the number of non-zero elements in the matrix A . Refer to [values](#) array description in [Sparse Matrix Storage Formats](#) for more details.

ja (input/output). If conversion type is from CSR to uncompressed, on input for zero-based indexing of A ja contains the column indices plus one for each non-zero element of the matrix A . For one-based indexing of A ja contains the column indices for each non-zero element of the matrix A .

Its length is equal to the length of the array *acsr*. Refer to *columns* array description in [Sparse Matrix Storage Formats](#) for more details.

ia

(input/output). Array of length $m + 1$.

If conversion type is from CSR to uncompressed, on input for zero-based indexing of *A* *ia* contains indices of elements in the array *acsr*, such that $ia[i] - 1$ is the index in the array *acsr* of the first non-zero element from the row *i*. For one-based indexing of *A* *ia* contains indices of elements in the array *acsr*, such that $ia[i]$ is the index in the array *acsr* of the first non-zero element from the row *i*.

The value of $ia[m] - ia[0]$ is equal to the number of non-zeros. Refer to *rowIndex* array description in [Sparse Matrix Storage Formats](#) for more details.

Output Parameters

adns

If conversion type is from CSR to uncompressed, on output *adns* contains the uncompressed (dense) representation of matrix *A*.

acsr, ja, ia

If conversion type is from uncompressed to CSR, on output *acsr*, *ja*, and *ia* contain the compressed sparse row (CSR) format (3-array variation) of matrix *A* (see [Sparse Matrix Storage Formats](#) for a description of the storage format).

info

Integer info indicator only for restoring the matrix *A* from the CSR format.

If $info=0$, the execution is successful.

If $info=i$, the routine is interrupted processing the *i*-th row because there is no space in the arrays *acsr* and *ja* according to the value *nzmax*.

mkl_?csrcoo

Converts a sparse matrix in the CSR format to the coordinate format and vice versa.

Syntax

```
void mkl_scsrcoo (const MKL_INT *job , const MKL_INT *n , float *acsr , MKL_INT *ja ,
MKL_INT *ia , MKL_INT *nnz , float *acoo , MKL_INT *rowind , MKL_INT *colind , MKL_INT
*info );
```

```
void mkl_dcsrcoo (const MKL_INT *job , const MKL_INT *n , double *acsr , MKL_INT *ja ,
MKL_INT *ia , MKL_INT *nnz , double *acoo , MKL_INT *rowind , MKL_INT *colind ,
MKL_INT *info );
```

```
void mkl_ccsrcoo (const MKL_INT *job , const MKL_INT *n , MKL_Complex8 *acsr , MKL_INT
*ja , MKL_INT *ia , MKL_INT *nnz , MKL_Complex8 *acoo , MKL_INT *rowind , MKL_INT
*colind , MKL_INT *info );
```

```
void mkl_zcsrcoo (const MKL_INT *job , const MKL_INT *n , MKL_Complex16 *acsr , MKL_INT
*ja , MKL_INT *ia , MKL_INT *nnz , MKL_Complex16 *acoo , MKL_INT *rowind , MKL_INT
*colind , MKL_INT *info );
```

Include Files

- mkl.h

Description

This routine converts a sparse matrix A stored in the compressed sparse row (CSR) format (3-array variation) to coordinate format and vice versa.

Input Parameters

| | |
|------------|---|
| <i>job</i> | <p>Array, contains the following conversion parameters:</p> <p><i>job</i>[0]</p> <p>If <i>job</i>[0]=0, the matrix in the CSR format is converted to the coordinate format;</p> <p>if <i>job</i>[0]=1, the matrix in the coordinate format is converted to the CSR format.</p> <p>if <i>job</i>[0]=2, the matrix in the coordinate format is converted to the CSR format, and the column indices in CSR representation are sorted in the increasing order within each row.</p> <p><i>job</i>[1]</p> <p>If <i>job</i>[1]=0, zero-based indexing for the matrix in CSR format is used;</p> <p>if <i>job</i>[1]=1, one-based indexing for the matrix in CSR format is used.</p> <p><i>job</i>[2]</p> <p>If <i>job</i>[2]=0, zero-based indexing for the matrix in coordinate format is used;</p> <p>if <i>job</i>[2]=1, one-based indexing for the matrix in coordinate format is used.</p> <p><i>job</i>[4]</p> <p><i>job</i>[4]=<i>nzmax</i> - maximum number of the non-zero elements allowed if <i>job</i>[0]=0.</p> <p><i>job</i>[5] - job indicator.</p> <p>For conversion to the coordinate format:</p> <p>If <i>job</i>[5]=1, only array <i>rowind</i> is filled in for the output storage.</p> <p>If <i>job</i>[5]=2, arrays <i>rowind</i>, <i>colind</i> are filled in for the output storage.</p> <p>If <i>job</i>[5]=3, all arrays <i>rowind</i>, <i>colind</i>, <i>acoo</i> are filled in for the output storage.</p> <p>For conversion to the CSR format:</p> <p>If <i>job</i>[5]=0, all arrays <i>acsr</i>, <i>ja</i>, <i>ia</i> are filled in for the output storage.</p> <p>If <i>job</i>[5]=1, only array <i>ia</i> is filled in for the output storage.</p> <p>If <i>job</i>[5]=2, then it is assumed that the routine already has been called with the <i>job</i>[5]=1, and the user allocated the required space for storing the output arrays <i>acsr</i> and <i>ja</i>.</p> |
| <i>n</i> | Dimension of the matrix A . |
| <i>nnz</i> | Specifies the number of non-zero elements of the matrix A for <i>job</i> [0]≠0. Refer to <i>nnz</i> description in Coordinate Format for more details. |

| | |
|---------------------|---|
| <code>acsr</code> | (input/output) Array containing non-zero elements of the matrix <i>A</i> . Its length is equal to the number of non-zero elements in the matrix <i>A</i> . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <code>ja</code> | (input/output). For <code>job[1] = 1</code> (one-based indexing for the matrix in CSR format), array containing the column indices plus one for each non-zero element of the matrix <i>A</i> . For <code>job[1] = 0</code> (zero-based indexing for the matrix in CSR format), array containing the column indices for each non-zero element of the matrix <i>A</i> . Its length is equal to the length of the array <code>acsr</code> . Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details. |
| <code>ia</code> | (input/output). Array of length $n + 1$, containing indices of elements in the array <code>acsr</code> , such that <code>ia[i] - ia[0]</code> is the index in the array <code>acsr</code> of the first non-zero element from the row <i>i</i> . The value of the last element <code>ia[n] - ia[0]</code> is equal to the number of non-zeros plus one. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details. |
| <code>acoo</code> | (input/output) Array containing non-zero elements of the matrix <i>A</i> . Its length is equal to the number of non-zero elements in the matrix <i>A</i> . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <code>rowind</code> | (input/output). Array of length <code>nnz</code> , contains the row indices for each non-zero element of the matrix <i>A</i> . Refer to <i>rows</i> array description in Coordinate Format for more details. |
| <code>colind</code> | (input/output). Array of length <code>nnz</code> , contains the column indices for each non-zero element of the matrix <i>A</i> . Refer to <i>columns</i> array description in Coordinate Format for more details. |

Output Parameters

| | |
|-------------------|---|
| <code>nnz</code> | Returns the number of converted elements of the matrix <i>A</i> for <code>job[0]=0</code> . |
| <code>info</code> | Integer info indicator only for converting the matrix <i>A</i> from the CSR format. If <code>info=0</code> , the execution is successful. If <code>info=1</code> , the routine is interrupted because there is no space in the arrays <code>acoo</code> , <code>rowind</code> , <code>colind</code> according to the value <code>nzmax</code> . |

mkl_?csrbsr

Converts a square sparse matrix in the CSR format to the BSR format and vice versa.

Syntax

```
void mkl_scsrbsr (const MKL_INT *job , const MKL_INT *m , const MKL_INT *mblk , const
MKL_INT *ldabsr , float *acsr , MKL_INT *ja , MKL_INT *ia , float *absr , MKL_INT
*jab , MKL_INT *iab , MKL_INT *info );
```

```

void mkl_dcsrbsr (const MKL_INT *job , const MKL_INT *m , const MKL_INT *mblk , const
MKL_INT *ldabsr , double *acsr , MKL_INT *ja , MKL_INT *ia , double *absr , MKL_INT
*jab , MKL_INT *iab , MKL_INT *info );

void mkl_ccsrbsr (const MKL_INT *job , const MKL_INT *m , const MKL_INT *mblk , const
MKL_INT *ldabsr , MKL_Complex8 *acsr , MKL_INT *ja , MKL_INT *ia , MKL_Complex8
*absr , MKL_INT *jab , MKL_INT *iab , MKL_INT *info );

void mkl_zcsrbsr (const MKL_INT *job , const MKL_INT *m , const MKL_INT *mblk , const
MKL_INT *ldabsr , MKL_Complex16 *acsr , MKL_INT *ja , MKL_INT *ia , MKL_Complex16
*absr , MKL_INT *jab , MKL_INT *iab , MKL_INT *info );

```

Include Files

- mkl.h

Description

This routine converts a square sparse matrix A stored in the compressed sparse row (CSR) format (3-array variation) to the block sparse row (BSR) format and vice versa.

Input Parameters

job

Array, contains the following conversion parameters:

job[0]

If *job*[0]=0, the matrix in the CSR format is converted to the BSR format;
if *job*[0]=1, the matrix in the BSR format is converted to the CSR format.

job[1]

If *job*[1]=0, zero-based indexing for the matrix in CSR format is used;
if *job*[1]=1, one-based indexing for the matrix in CSR format is used.

job[2]

If *job*[2]=0, zero-based indexing for the matrix in the BSR format is used;
if *job*[2]=1, one-based indexing for the matrix in the BSR format is used.

job[3] is only used for conversion to CSR format. By default, the converter saves the blocks without checking whether an element is zero or not. If *job*[3]=1, then the converter only saves non-zero elements in blocks.

job[5] - job indicator.

For conversion to the BSR format:

If *job*[5]=0, only arrays *jab*, *iab* are generated for the output storage.

If *job*[5]>0, all output arrays *absr*, *jab*, and *iab* are filled in for the output storage.

If *job*[5]=-1, *iab*[*m*] - *iab*[0] returns the number of non-zero blocks.

For conversion to the CSR format:

If *job*[5]=0, only arrays *ja*, *ia* are generated for the output storage.

m

Actual row dimension of the matrix A for convert to the BSR format; block row dimension of the matrix A for convert to the CSR format.

| | |
|---------------|---|
| <i>mblk</i> | Size of the block in the matrix <i>A</i> . |
| <i>ldabsr</i> | Leading dimension of the array <i>absr</i> as declared in the calling program. <i>ldabsr</i> must be greater than or equal to <i>mblk*mblk</i> . |
| <i>acsr</i> | (input/output) Array containing non-zero elements of the matrix <i>A</i> . Its length is equal to the number of non-zero elements in the matrix <i>A</i> . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ja</i> | (input/output). Array containing the column indices for each non-zero element of the matrix <i>A</i> . Its length is equal to the length of the array <i>acsr</i> . Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ia</i> | (input/output). Array of length $m + 1$, containing indices of elements in the array <i>acsr</i> , such that $ia[I] - iab[0]$ is the index in the array <i>acsr</i> of the first non-zero element from the row <i>I</i> . The value of $ia[m] - iab[0]$ is equal to the number of non-zeros. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>absr</i> | (input/output) Array containing elements of non-zero blocks of the matrix <i>A</i> . Its length is equal to the number of non-zero blocks in the matrix <i>A</i> multiplied by <i>mblk*mblk</i> . Refer to <i>values</i> array description in BSR Format for more details. |
| <i>jab</i> | (input/output). Array containing the column indices for each non-zero block of the matrix <i>A</i> . Its length is equal to the number of non-zero blocks of the matrix <i>A</i> . Refer to <i>columns</i> array description in BSR Format for more details. |
| <i>iab</i> | (input/output). Array of length $(m + 1)$, containing indices of blocks in the array <i>absr</i> , such that $iab[i] - iab[0]$ is the index in the array <i>absr</i> of the first non-zero element from the <i>i</i> -th row. The value of $iab[m]$ is equal to the number of non-zero blocks. Refer to <i>rowIndex</i> array description in BSR Format for more details. |

Output Parameters

| | |
|-------------|---|
| <i>info</i> | Integer info indicator only for converting the matrix <i>A</i> from the CSR format. If <i>info</i> =0, the execution is successful. If <i>info</i> =1, it means that <i>mblk</i> is equal to 0. If <i>info</i> =2, it means that <i>ldabsr</i> is less than <i>mblk*mblk</i> and there is no space for all blocks. |
|-------------|---|

mkl_?csrsc

Converts a square sparse matrix in the CSR format to the CSC format and vice versa.

Syntax

```
void mkl_dcsrcsc (const MKL_INT *job , const MKL_INT *n , double *acsr , MKL_INT *ja ,
MKL_INT *ia , double *acsc , MKL_INT *jal , MKL_INT *ial , MKL_INT *info );

void mkl_scsrcsc (const MKL_INT *job , const MKL_INT *n , float *acsr , MKL_INT *ja ,
MKL_INT *ia , float *acsc , MKL_INT *jal , MKL_INT *ial , MKL_INT *info );

void mkl_ccsrcsc (const MKL_INT *job , const MKL_INT *n , MKL_Complex8 *acsr , MKL_INT
*ja , MKL_INT *ia , MKL_Complex8 *acsc , MKL_INT *jal , MKL_INT *ial , MKL_INT *info );

void mkl_zcsrcsc (const MKL_INT *job , const MKL_INT *n , MKL_Complex16 *acsr , MKL_INT
*ja , MKL_INT *ia , MKL_Complex16 *acsc , MKL_INT *jal , MKL_INT *ial , MKL_INT
*info );
```

Include Files

- mkl.h

Description

This routine converts a square sparse matrix A stored in the compressed sparse row (CSR) format (3-array variation) to the compressed sparse column (CSC) format and vice versa.

Input Parameters

job

Array, contains the following conversion parameters:

job[0]

If *job*[0]=0, the matrix in the CSR format is converted to the CSC format;
if *job*[0]=1, the matrix in the CSC format is converted to the CSR format.

job[1]

If *job*[1]=0, zero-based indexing for the matrix in CSR format is used;
if *job*[1]=1, one-based indexing for the matrix in CSR format is used.

job[2]

If *job*[2]=0, zero-based indexing for the matrix in the CSC format is used;
if *job*[2]=1, one-based indexing for the matrix in the CSC format is used.

job[5] - job indicator.

For conversion to the CSC format:

If *job*[5]=0, only arrays *jal*, *ial* are filled in for the output storage.

If *job*[5]≠0, all output arrays *acsc*, *jal*, and *ial* are filled in for the output storage.

For conversion to the CSR format:

If *job*[5]=0, only arrays *ja*, *ia* are filled in for the output storage.

If *job*[5]≠0, all output arrays *acsr*, *ja*, and *ia* are filled in for the output storage.

m

Dimension of the square matrix A .

acsr

(input/output)

Array containing non-zero elements of the square matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in [Sparse Matrix Storage Formats](#) for more details.

ja (input/output). Array containing the column indices for each non-zero element of the matrix *A*.

Its length is equal to the length of the array *acsr*. Refer to *columns* array description in [Sparse Matrix Storage Formats](#) for more details.

ia (input/output). Array of length $m + 1$, containing indices of elements in the array *acsr*, such that $ia[i] - ia[0]$ is the index in the array *acsr* of the first non-zero element from the row *i*. The value of $ia[m] - ia[0]$ is equal to the number of non-zeros. Refer to *rowIndex* array description in [Sparse Matrix Storage Formats](#) for more details.

acsc (input/output)

Array containing non-zero elements of the square matrix *A*. Its length is equal to the number of non-zero elements in the matrix *A*. Refer to *values* array description in [Sparse Matrix Storage Formats](#) for more details.

jal (input/output). Array containing the row indices for each non-zero element of the matrix *A*.

Its length is equal to the length of the array *acsc*. Refer to *columns* array description in [Sparse Matrix Storage Formats](#) for more details.

ial (input/output). Array of length $m + 1$, containing indices of elements in the array *acsc*, such that $ial[i] - ial[0]$ is the index in the array *acsc* of the first non-zero element from the column *i*. The value of $ial[m] - ial[0]$ is equal to the number of non-zeros. Refer to *rowIndex* array description in [Sparse Matrix Storage Formats](#) for more details.

Output Parameters

info This parameter is not used now.

mkl_?csrdia

Converts a sparse matrix in the CSR format to the diagonal format and vice versa.

Syntax

```
void mkl_dcsrdia (const MKL_INT *job , const MKL_INT *n , double *acsr , MKL_INT *ja ,
MKL_INT *ia , double *adia , const MKL_INT *ndiag , MKL_INT *distance , MKL_INT
*idiag , double *acsr_rem , MKL_INT *ja_rem , MKL_INT *ia_rem , MKL_INT *info );
```

```
void mkl_scsrdia (const MKL_INT *job , const MKL_INT *n , float *acsr , MKL_INT *ja ,
MKL_INT *ia , float *adia , const MKL_INT *ndiag , MKL_INT *distance , MKL_INT
*idiag , float *acsr_rem , MKL_INT *ja_rem , MKL_INT *ia_rem , MKL_INT *info );
```

```
void mkl_ccsrdia (const MKL_INT *job , const MKL_INT *n , MKL_Complex8 *acsr , MKL_INT
*ja , MKL_INT *ia , MKL_Complex8 *adia , const MKL_INT *ndiag , MKL_INT *distance ,
MKL_INT *idiag , MKL_Complex8 *acsr_rem , MKL_INT *ja_rem , MKL_INT *ia_rem , MKL_INT
*info );
```

```
void mkl_zcsrda (const MKL_INT *job , const MKL_INT *n , MKL_Complex16 *acsr , MKL_INT
*ja , MKL_INT *ia , MKL_Complex16 *adia , const MKL_INT *ndiag , MKL_INT *distance ,
MKL_INT *idiag , MKL_Complex16 *acsr_rem , MKL_INT *ja_rem , MKL_INT *ia_rem , MKL_INT
*info );
```

Include Files

- mkl.h

Description

This routine converts a sparse matrix A stored in the compressed sparse row (CSR) format (3-array variation) to the diagonal format and vice versa.

Input Parameters

job

Array, contains the following conversion parameters:

job[0]

If *job*[0]=0, the matrix in the CSR format is converted to the diagonal format;

if *job*[0]=1, the matrix in the diagonal format is converted to the CSR format.

job[1]

If *job*[1]=0, zero-based indexing for the matrix in CSR format is used;

if *job*[1]=1, one-based indexing for the matrix in CSR format is used.

job[2]

If *job*[2]=0, zero-based indexing for the matrix in the diagonal format is used;

if *job*[2]=1, one-based indexing for the matrix in the diagonal format is used.

job[5] - job indicator.

For conversion to the diagonal format:

If *job*[5]=0, diagonals are not selected internally, and *acsr_rem*, *ja_rem*, *ia_rem* are not filled in for the output storage.

If *job*[5]=1, diagonals are not selected internally, and *acsr_rem*, *ja_rem*, *ia_rem* are filled in for the output storage.

If *job*[5]=10, diagonals are selected internally, and *acsr_rem*, *ja_rem*, *ia_rem* are not filled in for the output storage.

If *job*[5]=11, diagonals are selected internally, and *csr_rem*, *ja_rem*, *ia_rem* are filled in for the output storage.

For conversion to the CSR format:

If *job*[5]=0, each entry in the array *adia* is checked whether it is zero. Zero entries are not included in the array *acsr*.

If *job*[5]≠0, each entry in the array *adia* is not checked whether it is zero.

m

Dimension of the matrix A .

| | |
|---------------------------------|--|
| <i>acsr</i> | (input/output) Array containing non-zero elements of the matrix A. Its length is equal to the number of non-zero elements in the matrix A. Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ja</i> | (input/output). Array containing the column indices for each non-zero element of the matrix A. Its length is equal to the length of the array <i>acsr</i> . Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ia</i> | (input/output). Array of length $m + 1$, containing indices of elements in the array <i>acsr</i> , such that $ia[i] - ia[0]$ is the index in the array <i>acsr</i> of the first non-zero element from the row i . The value of $ia[m] - ia[0]$ is equal to the number of non-zeros. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>adia</i> | (input/output) Array of size $(ndiag * idiag)$ containing diagonals of the matrix A. The key point of the storage is that each element in the array <i>adia</i> retains the row number of the original matrix. To achieve this diagonals in the lower triangular part of the matrix are padded from the top, and those in the upper triangular part are padded from the bottom. |
| <i>ndiag</i> | Specifies the leading dimension of the array <i>adia</i> as declared in the calling (sub)program, must be at least $\max(1, m)$. |
| <i>distance</i> | Array of length <i>idiag</i> , containing the distances between the main diagonal and each non-zero diagonal to be extracted. The distance is positive if the diagonal is above the main diagonal, and negative if the diagonal is below the main diagonal. The main diagonal has a distance equal to zero. |
| <i>idiag</i> | Number of diagonals to be extracted. For conversion to diagonal format on return this parameter may be modified. |
| <i>acsr_rem, ja_rem, ia_rem</i> | Remainder of the matrix in the CSR format if it is needed for conversion to the diagonal format. |

Output Parameters

info This parameter is not used now.

mkl_?csrsky

Converts a sparse matrix in CSR format to the skyline format and vice versa.

Syntax

```
void mkl_dcsrsky (const MKL_INT *job , const MKL_INT *m , double *acsr , MKL_INT *ja ,
MKL_INT *ia , double *asky , MKL_INT *pointers , MKL_INT *info );
```

```
void mkl_scsrsky (const MKL_INT *job , const MKL_INT *m , float *acsr , MKL_INT *ja ,
MKL_INT *ia , float *asky , MKL_INT *pointers , MKL_INT *info );
```

```
void mkl_ccsrsky (const MKL_INT *job , const MKL_INT *m , MKL_Complex8 *acsr , MKL_INT
*ja , MKL_INT *ia , MKL_Complex8 *asky , MKL_INT *pointers , MKL_INT *info );
void mkl_zcsrsky (const MKL_INT *job , const MKL_INT *m , MKL_Complex16 *acsr , MKL_INT
*ja , MKL_INT *ia , MKL_Complex16 *asky , MKL_INT *pointers , MKL_INT *info );
```

Include Files

- mkl.h

Description

This routine converts a sparse matrix A stored in the compressed sparse row (CSR) format (3-array variation) to the skyline format and vice versa.

Input Parameters

job

Array, contains the following conversion parameters:

job[0]

If *job*[0]=0, the matrix in the CSR format is converted to the skyline format;

if *job*[0]=1, the matrix in the skyline format is converted to the CSR format.

job[1]

If *job*[1]=0, zero-based indexing for the matrix in CSR format is used;

if *job*[1]=1, one-based indexing for the matrix in CSR format is used.

job[2]

If *job*[2]=0, zero-based indexing for the matrix in the skyline format is used;

if *job*[2]=1, one-based indexing for the matrix in the skyline format is used.

job[3]

For conversion to the skyline format:

If *job*[3]=0, the upper part of the matrix A in the CSR format is converted.

If *job*[3]=1, the lower part of the matrix A in the CSR format is converted.

For conversion to the CSR format:

If *job*[3]=0, the matrix is converted to the upper part of the matrix A in the CSR format.

If *job*[3]=1, the matrix is converted to the lower part of the matrix A in the CSR format.

job[4]

job[4]=*nzmax* - maximum number of the non-zero elements of the matrix A if *job*[0]=0.

job[5] - job indicator.

Only for conversion to the skyline format:

If *job*[5]=0, only arrays *pointers* is filled in for the output storage.

If `job[5]=1`, all output arrays `asky` and `pointers` are filled in for the output storage.

| | |
|-----------------------|---|
| <code>m</code> | Dimension of the matrix <i>A</i> . |
| <code>acsr</code> | (input/output) Array containing non-zero elements of the matrix <i>A</i> . Its length is equal to the number of non-zero elements in the matrix <i>A</i> . Refer to <code>values</code> array description in Sparse Matrix Storage Formats for more details. |
| <code>ja</code> | (input/output). Array containing the column indices for each non-zero element of the matrix <i>A</i> . Its length is equal to the length of the array <code>acsr</code> . Refer to <code>columns</code> array description in Sparse Matrix Storage Formats for more details. |
| <code>ia</code> | (input/output). Array of length $m + 1$, containing indices of elements in the array <code>acsr</code> , such that $ia[i] - ia[0]$ is the index in the array <code>acsr</code> of the first non-zero element from the row <i>i</i> . The value of $ia[m] - ia[0]$ is equal to the number of non-zeros. Refer to <code>rowIndex</code> array description in Sparse Matrix Storage Formats for more details. |
| <code>asky</code> | (input/output) Array, for a lower triangular part of <i>A</i> it contains the set of elements from each row starting from the first non-zero element to and including the diagonal element. For an upper triangular matrix it contains the set of elements from each column of the matrix starting with the first non-zero element down to and including the diagonal element. Encountered zero elements are included in the sets. Refer to <code>values</code> array description in Skyline Storage Format for more details. |
| <code>pointers</code> | (input/output). Array with dimension $(m+1)$, where <i>m</i> is number of rows for lower triangle (columns for upper triangle), $pointers[i-1] - pointers[0]$ gives the index of element in the array <code>asky</code> that is first non-zero element in row (column) <i>i</i> . The value of <code>pointers[m]</code> is set to $nnz + pointers[0]$, where <i>nnz</i> is the number of elements in the array <code>asky</code> . Refer to <code>pointers</code> array description in Skyline Storage Format for more details |

Output Parameters

| | |
|-------------------|--|
| <code>info</code> | Integer info indicator only for converting the matrix <i>A</i> from the CSR format. If <code>info=0</code> , the execution is successful. If <code>info=1</code> , the routine is interrupted because there is no space in the array <code>asky</code> according to the value <code>nzmax</code> . |
|-------------------|--|

mkl_?csradd

Computes the sum of two matrices stored in the CSR format (3-array variation) with one-based indexing.

Syntax

```
void mkl_dcsradd (const char *trans , const MKL_INT *request , const MKL_INT *sort ,
const MKL_INT *m , const MKL_INT *n , double *a , MKL_INT *ja , MKL_INT *ia , const
double *beta , double *b , MKL_INT *jb , MKL_INT *ib , double *c , MKL_INT *jc ,
MKL_INT *ic , const MKL_INT *nzmax , MKL_INT *info );
```

```
void mkl_scsradd (const char *trans , const MKL_INT *request , const MKL_INT *sort ,
const MKL_INT *m , const MKL_INT *n , float *a , MKL_INT *ja , MKL_INT *ia , const
float *beta , float *b , MKL_INT *jb , MKL_INT *ib , float *c , MKL_INT *jc , MKL_INT
*ic , const MKL_INT *nzmax , MKL_INT *info );
```

```
void mkl_ccsradd (const char *trans , const MKL_INT *request , const MKL_INT *sort ,
const MKL_INT *m , const MKL_INT *n , MKL_Complex8 *a , MKL_INT *ja , MKL_INT *ia ,
const MKL_Complex8 *beta , MKL_Complex8 *b , MKL_INT *jb , MKL_INT *ib , MKL_Complex8
*c , MKL_INT *jc , MKL_INT *ic , const MKL_INT *nzmax , MKL_INT *info );
```

```
void mkl_zcsradd (const char *trans , const MKL_INT *request , const MKL_INT *sort ,
const MKL_INT *m , const MKL_INT *n , MKL_Complex16 *a , MKL_INT *ja , MKL_INT *ia ,
const MKL_Complex16 *beta , MKL_Complex16 *b , MKL_INT *jb , MKL_INT *ib ,
MKL_Complex16 *c , MKL_INT *jc , MKL_INT *ic , const MKL_INT *nzmax , MKL_INT *info );
```

Include Files

- mkl.h

Description

The `mkl_?csradd` routine performs a matrix-matrix operation defined as

$$C := A + \text{beta} * \text{op}(B)$$

where:

A , B , C are the sparse matrices in the CSR format (3-array variation).

$\text{op}(B)$ is one of $\text{op}(B) = B$, or $\text{op}(B) = B^T$, or $\text{op}(B) = B^H$

beta is a scalar.

The routine works correctly if and only if the column indices in sparse matrix representations of matrices A and B are arranged in the increasing order for each row. If not, use the parameter `sort` (see below) to reorder column indices and the corresponding elements of the input matrices.

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|----------------|---|
| <i>trans</i> | Specifies the operation. If <i>trans</i> = 'N' or 'n', then $C := A + \text{beta} * B$ If <i>trans</i> = 'T' or 't', then $C := A + \text{beta} * B^T$ If <i>trans</i> = 'C' or 'c', then $C := A + \text{beta} * B^H$. |
| <i>request</i> | If <i>request</i> =0, the routine performs addition. The memory for the output arrays <i>ic</i> , <i>jc</i> , <i>c</i> must be allocated beforehand. |

If $request=1$, the routine only computes the values of the array ic of length $m + 1$. The memory for the ic array must be allocated beforehand. On exit the value $ic[m] - 1$ is the actual number of the elements in the arrays c and jc .

If $request=2$, after the routine is called previously with the parameter $request=1$ and after the output arrays jc and c are allocated in the calling program with length at least $ic[m] - 1$, the routine performs addition.

$sort$

Specifies the type of reordering. If this parameter is not set (default), the routine does not perform reordering.

If $sort=1$, the routine arranges the column indices ja for each row in the increasing order and reorders the corresponding values of the matrix A in the array a .

If $sort=2$, the routine arranges the column indices jb for each row in the increasing order and reorders the corresponding values of the matrix B in the array b .

If $sort=3$, the routine performs reordering for both input matrices A and B .

m

Number of rows of the matrix A .

n

Number of columns of the matrix A .

a

Array containing non-zero elements of the matrix A . Its length is equal to the number of non-zero elements in the matrix A . Refer to *values* array description in [Sparse Matrix Storage Formats](#) for more details.

ja

Array containing the column indices plus one for each non-zero element of the matrix A . For each row the column indices must be arranged in the increasing order.

The length of this array is equal to the length of the array a . Refer to *columns* array description in [Sparse Matrix Storage Formats](#) for more details.

ia

Array of length $m + 1$, containing indices of elements in the array a , such that $ia[i] - ia[0]$ is the index in the array a of the first non-zero element from the row i . The value of the last element $ia[m]$ is equal to the number of non-zero elements of the matrix A plus one. Refer to *rowIndex* array description in [Sparse Matrix Storage Formats](#) for more details.

$beta$

Specifies the scalar $beta$.

b

Array containing non-zero elements of the matrix B . Its length is equal to the number of non-zero elements in the matrix B . Refer to *values* array description in [Sparse Matrix Storage Formats](#) for more details.

jb

Array containing the column indices plus one for each non-zero element of the matrix B . For each row the column indices must be arranged in the increasing order.

The length of this array is equal to the length of the array b . Refer to *columns* array description in [Sparse Matrix Storage Formats](#) for more details.

| | |
|--------------|---|
| <i>ib</i> | <p>Array of length $m + 1$ when <i>trans</i> = 'N' or 'n', or $n + 1$ otherwise.</p> <p>This array contains indices of elements in the array <i>b</i>, such that $ib[i] - ib[0]$ is the index in the array <i>b</i> of the first non-zero element from the row <i>i</i>. The value of the last element $ib[m]$ or $ib[n]$ is equal to the number of non-zero elements of the matrix <i>B</i> plus one. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details.</p> |
| <i>nzmax</i> | <p>The length of the arrays <i>c</i> and <i>jc</i>.</p> <p>This parameter is used only if <i>request</i>=0. The routine stops calculation if the number of elements in the result matrix <i>C</i> exceeds the specified value of <i>nzmax</i>.</p> |

Output Parameters

| | |
|-------------|---|
| <i>c</i> | <p>Array containing non-zero elements of the result matrix <i>C</i>. Its length is equal to the number of non-zero elements in the matrix <i>C</i>. Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details.</p> |
| <i>jc</i> | <p>Array containing the column indices plus one for each non-zero element of the matrix <i>C</i>.</p> <p>The length of this array is equal to the length of the array <i>c</i>. Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details.</p> |
| <i>ic</i> | <p>Array of length $m + 1$, containing indices of elements in the array <i>c</i>, such that $ic[i] - ic[0]$ is the index in the array <i>c</i> of the first non-zero element from the row <i>i</i>. The value of the last element $ic[m]$ is equal to the number of non-zero elements of the matrix <i>C</i> plus one. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details.</p> |
| <i>info</i> | <p>If <i>info</i>=0, the execution is successful.</p> <p>If <i>info</i>=<i>I</i>>0, the routine stops calculation in the <i>I</i>-th row of the matrix <i>C</i> because number of elements in <i>C</i> exceeds <i>nzmax</i>.</p> <p>If <i>info</i>=-1, the routine calculates only the size of the arrays <i>c</i> and <i>jc</i> and returns this value plus 1 as the last element of the array <i>ic</i>.</p> |

mkl_?csrmultcsr

Computes product of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing.

Syntax

```
void mkl_dcsrcmultcsr (const char *trans , const MKL_INT *request , const MKL_INT
*sort , const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , double *a , MKL_INT
*ja , MKL_INT *ia , double *b , MKL_INT *jb , MKL_INT *ib , double *c , MKL_INT *jc ,
MKL_INT *ic , const MKL_INT *nzmax , MKL_INT *info );

void mkl_scsrcmultcsr (const char *trans , const MKL_INT *request , const MKL_INT
*sort , const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , float *a , MKL_INT
*ja , MKL_INT *ia , float *b , MKL_INT *jb , MKL_INT *ib , float *c , MKL_INT *jc ,
MKL_INT *ic , const MKL_INT *nzmax , MKL_INT *info );
```

```
void mkl_ccsrmultcsr (const char *trans , const MKL_INT *request , const MKL_INT
*sort , const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , MKL_Complex8 *a ,
MKL_INT *ja , MKL_INT *ia , MKL_Complex8 *b , MKL_INT *jb , MKL_INT *ib , MKL_Complex8
*c , MKL_INT *jc , MKL_INT *ic , const MKL_INT *nzmax , MKL_INT *info );
```

```
void mkl_zcsrmultcsr (const char *trans , const MKL_INT *request , const MKL_INT
*sort , const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , MKL_Complex16 *a ,
MKL_INT *ja , MKL_INT *ia , MKL_Complex16 *b , MKL_INT *jb , MKL_INT *ib ,
MKL_Complex16 *c , MKL_INT *jc , MKL_INT *ic , const MKL_INT *nzmax , MKL_INT *info );
```

Include Files

- mkl.h

Description

The `mkl_?csrmultcsr` routine performs a matrix-matrix operation defined as

$$C := \text{op}(A) * B$$

where:

A, B, C are the sparse matrices in the CSR format (3-array variation);

$\text{op}(A)$ is one of $\text{op}(A) = A$, or $\text{op}(A) = A^T$, or $\text{op}(A) = A^H$.

You can use the parameter `sort` to perform or not perform reordering of non-zero entries in input and output sparse matrices. The purpose of reordering is to rearrange non-zero entries in compressed sparse row matrix so that column indices in compressed sparse representation are sorted in the increasing order for each row.

The following table shows correspondence between the value of the parameter `sort` and the type of reordering performed by this routine for each sparse matrix involved:

| Value of the parameter <i>sort</i> | Reordering of A (arrays <i>a, ja, ia</i>) | Reordering of B (arrays <i>b, jb, ib</i>) | Reordering of C (arrays <i>c, jc, ic</i>) |
|--|---|---|---|
| 1 | yes | no | yes |
| 2 | no | yes | yes |
| 3 | yes | yes | yes |
| 4 | yes | no | no |
| 5 | no | yes | no |
| 6 | yes | yes | no |
| 7 | no | no | no |
| arbitrary value not equal to 1, 2, ..., 7 | no | no | yes |

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

trans

Specifies the operation.

If *trans* = 'N' or 'n', then $C := A * B$

If *trans* = 'T' or 't' or 'C' or 'c', then $C := A^T * B$.

| | |
|----------------|--|
| <i>request</i> | <p>If <i>request</i>=0, the routine performs multiplication, the memory for the output arrays <i>ic</i>, <i>jc</i>, <i>c</i> must be allocated beforehand.</p> <p>If <i>request</i>=1, the routine computes only values of the array <i>ic</i> of length $m + 1$, the memory for this array must be allocated beforehand. On exit the value $ic[m] - 1$ is the actual number of the elements in the arrays <i>c</i> and <i>jc</i>.</p> <p>If <i>request</i>=2, the routine has been called previously with the parameter <i>request</i>=1, the output arrays <i>jc</i> and <i>c</i> are allocated in the calling program and they are of the length $ic[m] - 1$ at least.</p> |
| <i>sort</i> | Specifies whether the routine performs reordering of non-zeros entries in input and/or output sparse matrices (see table above). |
| <i>m</i> | Number of rows of the matrix <i>A</i> . |
| <i>n</i> | Number of columns of the matrix <i>A</i> . |
| <i>k</i> | Number of columns of the matrix <i>B</i> . |
| <i>a</i> | Array containing non-zero elements of the matrix <i>A</i> . Its length is equal to the number of non-zero elements in the matrix <i>A</i> . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ja</i> | <p>Array containing the column indices plus one for each non-zero element of the matrix <i>A</i>. For each row the column indices must be arranged in the increasing order.</p> <p>The length of this array is equal to the length of the array <i>a</i>. Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details.</p> |
| <i>ia</i> | <p>Array of length $m + 1$.</p> <p>This array contains indices of elements in the array <i>a</i>, such that $ia[i] - ia[0]$ is the index in the array <i>a</i> of the first non-zero element from the row <i>i</i>. The value of the last element $ia[m]$ is equal to the number of non-zero elements of the matrix <i>A</i> plus one. Refer to <i>rowIndex</i> array description in Sparse Matrix Storage Formats for more details.</p> |
| <i>b</i> | Array containing non-zero elements of the matrix <i>B</i> . Its length is equal to the number of non-zero elements in the matrix <i>B</i> . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>jb</i> | <p>Array containing the column indices plus one for each non-zero element of the matrix <i>B</i>. For each row the column indices must be arranged in the increasing order.</p> <p>The length of this array is equal to the length of the array <i>b</i>. Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details.</p> |
| <i>ib</i> | Array of length $n + 1$ when <i>trans</i> = 'N' or 'n', or $m + 1$ otherwise. |

This array contains indices of elements in the array b , such that $ib[i] - ib[0]$ is the index in the array b of the first non-zero element from the row i . The value of the last element $ib[n]$ or $ib[m]$ is equal to the number of non-zero elements of the matrix B plus one. Refer to `rowIndex` array description in [Sparse Matrix Storage Formats](#) for more details.

`nzmax`

The length of the arrays c and jc .

This parameter is used only if `request=0`. The routine stops calculation if the number of elements in the result matrix C exceeds the specified value of `nzmax`.

Output Parameters

`c`

Array containing non-zero elements of the result matrix C . Its length is equal to the number of non-zero elements in the matrix C . Refer to `values` array description in [Sparse Matrix Storage Formats](#) for more details.

`jc`

Array containing the column indices plus one for each non-zero element of the matrix C .

The length of this array is equal to the length of the array c . Refer to `columns` array description in [Sparse Matrix Storage Formats](#) for more details.

`ic`

Array of length $m + 1$ when `trans = 'N'` or `'n'`, or $n + 1$ otherwise.

This array contains indices of elements in the array c , such that $ic[i] - ic[0]$ is the index in the array c of the first non-zero element from the row i . The value of the last element $ic[m]$ or $ic[n]$ is equal to the number of non-zero elements of the matrix C plus one. Refer to `rowIndex` array description in [Sparse Matrix Storage Formats](#) for more details.

`info`

If `info=0`, the execution is successful.

If `info=I>0`, the routine stops calculation in the I -th row of the matrix C because number of elements in C exceeds `nzmax`.

If `info=-1`, the routine calculates only the size of the arrays c and jc and returns this value plus 1 as the last element of the array ic .

mkl_?csrmultd

Computes product of two sparse matrices stored in the CSR format (3-array variation) with one-based indexing. The result is stored in the dense matrix.

Syntax

```
void mkl_dcsmultd (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , double *a , MKL_INT *ja , MKL_INT *ia , double *b , MKL_INT *jb , MKL_INT
*ib , double *c , MKL_INT *ldc );
```

```
void mkl_scsmultd (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , float *a , MKL_INT *ja , MKL_INT *ia , float *b , MKL_INT *jb , MKL_INT
*ib , float *c , MKL_INT *ldc );
```

```
void mkl_ccsrmultd (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , MKL_Complex8 *a , MKL_INT *ja , MKL_INT *ia , MKL_Complex8 *b , MKL_INT
*jb , MKL_INT *ib , MKL_Complex8 *c , MKL_INT *ldc );
```

```
void mkl_zcsrmultd (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_INT *k , MKL_Complex16 *a , MKL_INT *ja , MKL_INT *ia , MKL_Complex16 *b , MKL_INT
*jb , MKL_INT *ib , MKL_Complex16 *c , MKL_INT *ldc );
```

Include Files

- mkl.h

Description

The `mkl_?csrmultd` routine performs a matrix-matrix operation defined as

$$C := \text{op}(A) * B$$

where:

A , B are the sparse matrices in the CSR format (3-array variation), C is dense matrix;

$\text{op}(A)$ is one of $\text{op}(A) = A$, or $\text{op}(A) = A^T$, or $\text{op}(A) = A^H$.

The routine works correctly if and only if the column indices in sparse matrix representations of matrices A and B are arranged in the increasing order for each row.

NOTE

This routine supports only one-based indexing of the input arrays.

Input Parameters

| | |
|--------------|--|
| <i>trans</i> | Specifies the operation. If <i>trans</i> = 'N' or 'n', then $C := A * B$ If <i>trans</i> = 'T' or 't' or 'C' or 'c', then $C := A^T * B$. |
| <i>m</i> | Number of rows of the matrix A . |
| <i>n</i> | Number of columns of the matrix A . |
| <i>k</i> | Number of columns of the matrix B . |
| <i>a</i> | Array containing non-zero elements of the matrix A . Its length is equal to the number of non-zero elements in the matrix A . Refer to <i>values</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ja</i> | Array containing the column indices plus one for each non-zero element of the matrix A . For each row the column indices must be arranged in the increasing order. The length of this array is equal to the length of the array <i>a</i> . Refer to <i>columns</i> array description in Sparse Matrix Storage Formats for more details. |
| <i>ia</i> | Array of length $m + 1$ when <i>trans</i> = 'N' or 'n', or $n + 1$ otherwise. |

This array contains indices of elements in the array a , such that $ia[i] - ia[0]$ is the index in the array a of the first non-zero element from the row i . The value of the last element $ia[m]$ or $ia[n]$ is equal to the number of non-zero elements of the matrix A plus one. Refer to `rowIndex` array description in [Sparse Matrix Storage Formats](#) for more details.

b Array containing non-zero elements of the matrix B . Its length is equal to the number of non-zero elements in the matrix B . Refer to `values` array description in [Sparse Matrix Storage Formats](#) for more details.

jb Array containing the column indices plus one for each non-zero element of the matrix B . For each row the column indices must be arranged in the increasing order.

The length of this array is equal to the length of the array b . Refer to `columns` array description in [Sparse Matrix Storage Formats](#) for more details.

ib Array of length $m + 1$.

This array contains indices of elements in the array b , such that $ib[i] - ib[0]$ is the index in the array b of the first non-zero element from the row i . The value of the last element $ib[m]$ is equal to the number of non-zero elements of the matrix B plus one. Refer to `rowIndex` array description in [Sparse Matrix Storage Formats](#) for more details.

Output Parameters

c Array containing non-zero elements of the result matrix C .

ldc Specifies the leading dimension of the dense matrix C as declared in the calling (sub)program. Must be at least $\max(m, 1)$ when `trans = 'N'` or `'n'`, or $\max(1, n)$ otherwise.

Inspector-executor Sparse BLAS Routines

The inspector-executor API for Sparse BLAS divides operations into two stages: analysis and execution. During the initial analysis stage, the API inspects the matrix sparsity pattern and applies matrix structure changes. In the execution stage, subsequent routine calls reuse this information in order to improve performance.

The inspector-executor API supports key Sparse BLAS operations for iterative sparse solvers:

- Sparse matrix-vector multiplication
- Sparse matrix-matrix multiplication with a sparse or dense result
- Solution of triangular systems
- Sparse matrix addition

Naming conventions in Inspector-executor Sparse BLAS Routines

The Inspector-executor Sparse BLAS API routine names use the following convention:

```
mkl_sparse_[<character>]<operation>[_<format>]
```

The `<character>` field indicates the data type:

| | |
|---|---------------------------|
| s | real, single precision |
| c | complex, single precision |
| d | real, double precision |
| z | complex, double precision |

The data type is included in the name only if the function accepts dense matrix or scalar floating point parameters.

The *<operation>* field indicates the type of operation:

| | |
|------------------------------|--|
| create | create matrix handle |
| copy | create a copy of matrix handle |
| convert | convert matrix between sparse formats |
| export | export matrix from internal representation to CSR or BSR format |
| destroy | frees memory allocated for matrix handle |
| set_ <i><op></i> _hint | provide information about number of upcoming compute operations and operation type for optimization purposes, where <i><op></i> is mv, sv, mm, sm, or memory |
| optimize | analyze the matrix using hints and store optimization information in matrix handle |
| mv | compute sparse matrix-vector product |
| mm | compute sparse matrix by dense matrix product (batch mv) |
| sppmm/sppmmd | compute sparse matrix by sparse matrix product and store the result as a sparse/dense matrix |
| trsv | solve a triangular system |
| trsm | solve a triangular system with multiple right-hand sides |
| add | compute sum of two sparse matrices |

The *<format>* field indicates the sparse matrix storage format:

| | |
|-----|---|
| coo | coordinate format |
| csr | compressed sparse row format plus variations |
| csc | compressed sparse column format plus variations |
| bsr | block sparse row format plus variations |

The format is included in the function name only if the function parameters include an explicit sparse matrix in one of the conventional sparse matrix formats.

Sparse Matrix Storage Formats for Inspector-executor Sparse BLAS Routines

Inspector-executor Sparse BLAS routines support four conventional sparse matrix storage formats:

- compressed sparse row format (CSR) plus variations

- compressed sparse column format (CSC) plus variations
- coordinate format (COO)
- block sparse row format (BSR) plus variations

Computational routines operate on a matrix handle that stores a matrix in CSR or BSR formats. Other formats should be converted to CSR or BSR format before calling any computational routines. For more information see [Sparse Matrix Storage Formats](#).

Supported Inspector-executor Sparse BLAS Operations

The Inspector-executor Sparse BLAS API can perform several operations involving sparse matrices. These notations are used in the description of the operations:

- A , G , V are sparse matrices
- B and C are dense matrices
- x and y are dense vectors
- $alpha$ and $beta$ are scalars

$op(A)$ represents a possible transposition of matrix A

$$\begin{aligned} op(A) &= A \\ op(A) &= A^T - \text{transpose of } A \\ op(A) &= A^H - \text{conjugate transpose of } A \end{aligned}$$

$op(A)^{-1}$ denotes the inverse of $op(A)$.

The Inspector-executor Sparse BLAS routines support the following operations:

- computing the vector product between a sparse matrix and a dense vector:

$$y := alpha * op(A) * x + beta * y$$

- solving a single triangular system:

$$y := alpha * inv(op(A)) * x$$

- computing a product between a sparse matrix and a dense matrix:

$$C := alpha * op(A) * B + beta * C$$

- computing a product between sparse matrices with a sparse result:

$$V := alpha * op(A) * G$$

- computing a product between sparse matrices with a dense result:

$$C := alpha * op(A) * G$$

- computing a sum of sparse matrices with a sparse result:

$$V := alpha * op(A) + G$$

- solving a sparse triangular system with multiple right-hand sides:

$$C := alpha * inv(op(A)) * B$$

Matrix manipulation routines

The [Matrix Manipulation routines](#) table lists the Matrix Manipulation routines and the data types associated with them.

Matrix Manipulation Routines and Their Data Types

| Routine or Function Group | Data Types | Description |
|--------------------------------------|------------|--|
| <code>mkl_sparse_?_create_csr</code> | s, d, c, z | Creates a handle for a CSR format matrix. |
| <code>mkl_sparse_?_create_csc</code> | s, d, c, z | Creates a handle for a CSC format matrix. |
| <code>mkl_sparse_?_create_coo</code> | s, d, c, z | Creates a handle for a matrix in COO format. |
| <code>mkl_sparse_?_create_bsr</code> | s, d, c, z | Creates a handle for a matrix in BSR format. |
| <code>mkl_sparse_copy</code> | NA | Creates a copy of a matrix handle. |
| <code>mkl_sparse_destroy</code> | NA | Frees memory allocated for matrix handle. |
| <code>mkl_sparse_convert_csr</code> | NA | Converts internal matrix representation to CSR format. |
| <code>mkl_sparse_convert_bsr</code> | NA | Converts internal matrix representation to BSR format or changes BSR block size. |
| <code>mkl_sparse_?_export_csr</code> | s, d, c, z | Exports CSR matrix from internal representation. |
| <code>mkl_sparse_?_export_bsr</code> | s, d, c, z | Exports BSR matrix from internal representation. |
| <code>mkl_sparse_?_set_value</code> | s, d, c, z | Changes a single value of matrix in internal representation. |

`mkl_sparse_?_create_csr`

Creates a handle for a CSR format matrix.

Syntax

```
sparse_status_t mkl_sparse_s_create_csr (sparse_matrix_t *A, sparse_index_base_t indexing, MKL_INT rows, MKL_INT cols, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT *col_indx, float *values);
```

```
sparse_status_t mkl_sparse_d_create_csr (sparse_matrix_t *A, sparse_index_base_t indexing, MKL_INT rows, MKL_INT cols, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT *col_indx, double *values);
```

```
sparse_status_t mkl_sparse_c_create_csr (sparse_matrix_t *A, sparse_index_base_t indexing, MKL_INT rows, MKL_INT cols, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT *col_indx, MKL_Complex8 *values);
```

```
sparse_status_t mkl_sparse_z_create_csr (sparse_matrix_t *A, sparse_index_base_t indexing, MKL_INT rows, MKL_INT cols, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT *col_indx, MKL_Complex16 *values);
```

Include Files

- `mkl_spblas.h`

Description

The `mkl_sparse_?_create_csr` routine creates a handle for an m -by- k matrix A in CSR format.

Input Parameters

| | |
|-------------------|--|
| <i>indexing</i> | Indicates how input arrays are indexed. |
| | <p>SPARSE_INDEX_BASE_Z Zero-based (C-style) indexing: indices start at 0.</p> <p>SPARSE_INDEX_BASE_O One-based (Fortran-style) indexing: indices start at 1.</p> |
| <i>rows</i> | Number of rows of matrix <i>A</i> . |
| <i>cols</i> | Number of columns of matrix <i>A</i> . |
| <i>rows_start</i> | <p>Array of length at least <i>m</i>. This array contains row indices, such that $rows_start[i] - rows_start[0]$ is the first index of row <i>i</i> in the arrays <i>values</i> and <i>col_indx</i>.</p> <p>Refer to <i>pointerb</i> array description in CSR Format for more details.</p> |
| <i>rows_end</i> | <p>Array of at least length <i>m</i>. This array contains row indices, such that $rows_end[i] - rows_start[0] - 1$ is the last index of row <i>i</i> in the arrays <i>values</i> and <i>col_indx</i>.</p> <p>Refer to <i>pointerE</i> array description in CSR Format for more details.</p> |
| <i>col_indx</i> | <p>For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix <i>A</i>. For zero-based indexing, array containing the column indices for each non-zero element of the matrix <i>A</i>. Its length is at least $rows_end[rows - 1] - rows_start[0]$.</p> |
| <i>values</i> | <p>Array containing non-zero elements of the matrix <i>A</i>. Its length is equal to length of the <i>col_indx</i> array.</p> <p>Refer to <i>values</i> array description in CSR Format for more details.</p> |

Output Parameters

| | |
|----------|---|
| <i>A</i> | Handle containing internal data for subsequent Inspector-executor Sparse BLAS operations. |
|----------|---|

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|------------------------------|--|
| SPARSE_STATUS_SUCCESS | The operation was successful. |
| SPARSE_STATUS_NOT_INITIALIZE | The routine encountered an empty handle or matrix array. |
| D | |
| SPARSE_STATUS_ALLOC_FAILED | Internal memory allocation failed. |
| SPARSE_STATUS_INVALID_VALUE | The input parameters contain an invalid value. |
| SPARSE_STATUS_EXECUTION_FAIL | Execution failed. |
| ED | |
| SPARSE_STATUS_INTERNAL_ERROR | An error in algorithm implementation occurred. |

SPARSE_STATUS_NOT_SUPPORTED The requested operation is not supported.

mkl_sparse_?_create_csc

Creates a handle for a CSC format matrix.

Syntax

```
sparse_status_t mkl_sparse_s_create_csc (sparse_matrix_t *A, sparse_index_base_t
indexing, MKL_INT rows, MKL_INT cols, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT
*col_indx, float *values);
```

```
sparse_status_t mkl_sparse_d_create_csc (sparse_matrix_t *A, sparse_index_base_t
indexing, MKL_INT rows, MKL_INT cols, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT
*col_indx, double *values);
```

```
sparse_status_t mkl_sparse_c_create_csc (sparse_matrix_t *A, sparse_index_base_t
indexing, MKL_INT rows, MKL_INT cols, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT
*col_indx, MKL_Complex8 *values);
```

```
sparse_status_t mkl_sparse_z_create_csc (sparse_matrix_t *A, sparse_index_base_t
indexing, MKL_INT rows, MKL_INT cols, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT
*col_indx, MKL_Complex16 *values);
```

Include Files

- mkl_spblas.h

Description

The `mkl_sparse_?_create_csc` routine creates a handle for an m -by- k matrix A in CSC format.

Input Parameters

| | |
|-------------------|---|
| <i>indexing</i> | Indicates how input arrays are indexed. SPARSE_INDEX_BASE_Z Zero-based (C-style) indexing: indices start at ERO 0. SPARSE_INDEX_BASE_O One-based (Fortran-style) indexing: indices NE start at 1. |
| <i>rows</i> | Number of rows of the matrix A . |
| <i>cols</i> | Number of columns of the matrix A . |
| <i>rows_start</i> | Array of length at least m . This array contains row indices, such that $rows_start[i] - rows_start[0]$ is the first index of row i in the arrays $values$ and col_indx . Refer to <i>pointerb</i> array description in CSC Format for more details. |
| <i>rows_end</i> | Array of at least length m . This array contains row indices, such that $rows_end[i] - rows_start[0] - 1$ is the last index of row i in the arrays $values$ and col_indx . Refer to <i>pointerE</i> array description in CSC Format for more details. |

| | |
|-----------------------|--|
| <code>col_indx</code> | For one-based indexing, array containing the column indices plus one for each non-zero element of the matrix <i>A</i> . For zero-based indexing, array containing the column indices for each non-zero element of the matrix <i>A</i> . Its length is at least <code>rows_end[rows - 1] - rows_start[0]</code> . |
| <code>values</code> | Array containing non-zero elements of the matrix <i>A</i> . Its length is equal to length of the <code>col_indx</code> array. Refer to <code>values</code> array description in CSC Format for more details. |

Output Parameters

| | |
|----------------|----------------------------------|
| <code>A</code> | Handle containing internal data. |
|----------------|----------------------------------|

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|---|--|
| <code>SPARSE_STATUS_SUCCESS</code> | The operation was successful. |
| <code>SPARSE_STATUS_NOT_INITIALIZE</code> | The routine encountered an empty handle or matrix array. |
| <code>D</code> | |
| <code>SPARSE_STATUS_ALLOC_FAILED</code> | Internal memory allocation failed. |
| <code>SPARSE_STATUS_INVALID_VALUE</code> | The input parameters contain an invalid value. |
| <code>SPARSE_STATUS_EXECUTION_FAIL</code> | Execution failed. |
| <code>ED</code> | |
| <code>SPARSE_STATUS_INTERNAL_ERROR</code> | An error in algorithm implementation occurred. |
| <code>SPARSE_STATUS_NOT_SUPPORTED</code> | The requested operation is not supported. |

`mkl_sparse_?_create_coo`

Creates a handle for a matrix in COO format.

Syntax

```
sparse_status_t mkl_sparse_s_create_coo (sparse_matrix_t *A, sparse_index_base_t indexing, MKL_INT rows, MKL_INT cols, MKL_INT nnz, MKL_INT *row_indx, MKL_INT *col_indx, float *values);
```

```
sparse_status_t mkl_sparse_d_create_coo (sparse_matrix_t *A, sparse_index_base_t indexing, MKL_INT rows, MKL_INT cols, MKL_INT nnz, MKL_INT *row_indx, MKL_INT *col_indx, double *values);
```

```
sparse_status_t mkl_sparse_c_create_coo (sparse_matrix_t *A, sparse_index_base_t indexing, MKL_INT rows, MKL_INT cols, MKL_INT nnz, MKL_INT *row_indx, MKL_INT *col_indx, MKL_Complex8 *values);
```

```
sparse_status_t mkl_sparse_z_create_coo (sparse_matrix_t *A, sparse_index_base_t indexing, MKL_INT rows, MKL_INT cols, MKL_INT nnz, MKL_INT *row_indx, MKL_INT *col_indx, MKL_Complex16 *values);
```

Include Files

- `mkl_splblas.h`

Description

The `mkl_sparse_?_create_coo` routine creates a handle for an m -by- k matrix A in COO format.

Input Parameters

| | |
|-----------------|--|
| <i>indexing</i> | Indicates how input arrays are indexed. |
| | <p><code>SPARSE_INDEX_BASE_Z</code> Zero-based (C-style) indexing: indices start at 0.</p> <p><code>SPARSE_INDEX_BASE_ONE</code> One-based (Fortran-style) indexing: indices start at 1.</p> |
| <i>rows</i> | Number of rows of matrix A . |
| <i>cols</i> | Number of columns of matrix A . |
| <i>nnz</i> | Specifies the number of non-zero elements of the matrix A . Refer to <i>nnz</i> description in Coordinate Format for more details. |
| <i>row_indx</i> | Array of length <i>nnz</i> , containing the row indices for each non-zero element of matrix A . Refer to <i>rows</i> array description in Coordinate Format for more details. |
| <i>col_indx</i> | Array of length <i>nnz</i> , containing the column indices for each non-zero element of matrix A . Refer to <i>columns</i> array description in Coordinate Format for more details. |
| <i>values</i> | Array of length <i>nnz</i> , containing the non-zero elements of matrix A in arbitrary order. Refer to <i>values</i> array description in Coordinate Format for more details. |

Output Parameters

| | |
|----------|----------------------------------|
| <i>A</i> | Handle containing internal data. |
|----------|----------------------------------|

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|---|--|
| <code>SPARSE_STATUS_SUCCESS</code> | The operation was successful. |
| <code>SPARSE_STATUS_NOT_INITIALIZE</code> | The routine encountered an empty handle or matrix array. |
| <code>D</code> | |
| <code>SPARSE_STATUS_ALLOC_FAILED</code> | Internal memory allocation failed. |
| <code>SPARSE_STATUS_INVALID_VALUE</code> | The input parameters contain an invalid value. |
| <code>SPARSE_STATUS_EXECUTION_FAIL</code> | Execution failed. |
| <code>ED</code> | |
| <code>SPARSE_STATUS_INTERNAL_ERROR</code> | An error in algorithm implementation occurred. |
| <code>SPARSE_STATUS_NOT_SUPPORTED</code> | The requested operation is not supported. |

mkl_sparse_?_create_bsr

Creates a handle for a matrix in BSR format.

Syntax

```

sparse_status_t mkl_sparse_s_create_bsr (sparse_matrix_t *A, sparse_index_base_t
indexing, sparse_layout_t block_layout, MKL_INT rows, MKL_INT cols, MKL_INT
block_size, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT *col_indx, float *values);

sparse_status_t mkl_sparse_d_create_bsr (sparse_matrix_t *A, sparse_index_base_t
indexing, sparse_layout_t block_layout, MKL_INT rows, MKL_INT cols, MKL_INT
block_size, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT *col_indx, double *values);

sparse_status_t mkl_sparse_c_create_bsr (sparse_matrix_t *A, sparse_index_base_t
indexing, sparse_layout_t block_layout, MKL_INT rows, MKL_INT cols, MKL_INT
block_size, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT *col_indx, MKL_Complex8
*values);

sparse_status_t mkl_sparse_z_create_bsr (sparse_matrix_t *A, sparse_index_base_t
indexing, sparse_layout_t block_layout, MKL_INT rows, MKL_INT cols, MKL_INT
block_size, MKL_INT *rows_start, MKL_INT *rows_end, MKL_INT *col_indx, MKL_Complex16
*values);

```

Include Files

- mkl_spblas.h

Description

The `mkl_sparse_?_create_bsr` routine creates a handle for an m -by- k matrix A in BSR format.

Input Parameters

| | |
|-------------------------------|---|
| <i>indexing</i> | Indicates how input arrays are indexed. |
| SPARSE_INDEX_BASE_Z ERO | Zero-based (C-style) indexing: indices start at 0. |
| SPARSE_INDEX_BASE_O NE | One-based (Fortran-style) indexing: indices start at 1. |
| <i>block_layout</i> | Specifies layout of blocks: |
| SPARSE_LAYOUT_ROW_M AJOR | Storage of elements of blocks uses row major layout. |
| SPARSE_LAYOUT_COLUMN MAJOR | Storage of elements of blocks uses column major layout. |

NOTE

If you specify `SPARSE_INDEX_BASE_ZERO` for *indexing*, you must use `SPARSE_LAYOUT_ROW_MAJOR` for *block_layout*. Similarly, if you specify `SPARSE_INDEX_BASE_ONE` for *indexing*, you must use `SPARSE_LAYOUT_COLUMN_MAJOR` for *block_layout*. Otherwise `mkl_sparse?_create_bsr` returns `SPARSE_STATUS_NOT_SUPPORTED`.

| | |
|-------------------------|---|
| <code>rows</code> | Number of block rows of matrix <i>A</i> . |
| <code>cols</code> | Number of block columns of matrix <i>A</i> . |
| <code>block_size</code> | Size of blocks in matrix <i>A</i> . |
| <code>rows_start</code> | <p>Array of length <i>m</i>. This array contains row indices, such that <code>rows_start[i] - rows_start[0]</code> is the first index of block row <i>i</i> in the arrays <code>values</code> and <code>col_indx</code>.</p> <p>Refer to <code>pointerb</code> array description in CSR Format for more details.</p> |
| <code>rows_end</code> | <p>Array of length <i>m</i>. This array contains row indices, such that <code>rows_end[i] - rows_start[0] - 1</code> is the last index of block row <i>i</i> in the arrays <code>values</code> and <code>col_indx</code>.</p> <p>Refer to <code>pointerE</code> array description in CSR Format for more details.</p> |
| <code>col_indx</code> | <p>For one-based indexing, array containing the column indices plus one for each non-zero block of the matrix <i>A</i>. For zero-based indexing, array containing the column indices for each non-zero block of the matrix <i>A</i>. Its length is <code>rows_end[rows - 1] - rows_start[0]</code>.</p> |
| <code>values</code> | <p>Array containing non-zero elements of the matrix <i>A</i>. Its length is equal to length of the <code>col_indx</code> array multiplied by <code>block_size*block_size</code>.</p> <p>Refer to the <code>values</code> array description in BSR Format for more details.</p> |

Output Parameters

`A` Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|---|--|
| <code>SPARSE_STATUS_SUCCESS</code> | The operation was successful. |
| <code>SPARSE_STATUS_NOT_INITIALIZE</code> | The routine encountered an empty handle or matrix array. |
| <code>D</code> | |
| <code>SPARSE_STATUS_ALLOC_FAILED</code> | Internal memory allocation failed. |
| <code>SPARSE_STATUS_INVALID_VALUE</code> | The input parameters contain an invalid value. |
| <code>SPARSE_STATUS_EXECUTION_FAIL</code> | Execution failed. |
| <code>ED</code> | |

SPARSE_STATUS_INTERNAL_ERROR An error in algorithm implementation occurred.

SPARSE_STATUS_NOT_SUPPORTED The requested operation is not supported.

mkl_sparse_copy

Creates a copy of a matrix handle.

Syntax

```
sparse_status_t mkl_sparse_copy (const sparse_matrix_t source, struct matrix_descr
descr, sparse_matrix_t *dest);
```

Include Files

- mkl_spblas.h

Description

The `mkl_sparse_copy` routine creates a copy of a matrix handle.

Input Parameters

| | |
|-------------------------------------|--|
| <i>source</i> | Specifies handle containing internal data. |
| <i>descr</i> | Structure specifying sparse matrix properties. |
| | <code>sparse_matrix_type_t type</code> - Specifies the type of a sparse matrix: |
| SPARSE_MATRIX_TYPE_GENERAL | The matrix is processed as is. |
| SPARSE_MATRIX_TYPE_SYMMETRIC | The matrix is symmetric (only the requested triangle is processed). |
| SPARSE_MATRIX_TYPE_HERMITIAN | The matrix is Hermitian (only the requested triangle is processed). |
| SPARSE_MATRIX_TYPE_TRIANGULAR | The matrix is triangular (only the requested triangle is processed). |
| SPARSE_MATRIX_TYPE_DIAGONAL | The matrix is diagonal (only diagonal elements are processed). |
| SPARSE_MATRIX_TYPE_BLOCK_TRIANGULAR | The matrix is block-triangular (only requested triangle is processed). (Applies to BSR format only.) |
| SPARSE_MATRIX_TYPE_BLOCK_DIAGONAL | The matrix is block-diagonal (only diagonal blocks are processed). (Applies to BSR format only.) |
| | <code>sparse_fill_mode_t mode</code> - Specifies the triangular matrix part for symmetric, Hermitian, triangular, and block-triangular matrices: |
| SPARSE_FILL_MODE_LOWER | The lower triangular matrix part is processed. |

SPARSE_FILL_MODE_UP PER The upper triangular matrix part is processed.

sparse_diag_type_t *diag* - Specifies diagonal type for non-general matrices:

SPARSE_DIAG_NON_UNIT T Diagonal elements might not be equal to one.

SPARSE_DIAG_UNIT Diagonal elements are equal to one.

Output Parameters

dest Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

SPARSE_STATUS_SUCCESS The operation was successful.

SPARSE_STATUS_NOT_INITIALIZE D The routine encountered an empty handle or matrix array.

SPARSE_STATUS_ALLOC_FAILED Internal memory allocation failed.

SPARSE_STATUS_INVALID_VALUE The input parameters contain an invalid value.

SPARSE_STATUS_EXECUTION_FAILED ED Execution failed.

SPARSE_STATUS_INTERNAL_ERROR An error in algorithm implementation occurred.

SPARSE_STATUS_NOT_SUPPORTED The requested operation is not supported.

mkl_sparse_destroy

Frees memory allocated for matrix handle.

Syntax

```
sparse_status_t mkl_sparse_destroy (sparse_matrix_t A);
```

Include Files

- `mkl_spblas.h`

Description

The `mkl_sparse_destroy` routine frees memory allocated for matrix handle.

NOTE

You must free memory allocated for matrices after completing use of them. The `mkl_sparse_destroy` provides a utility to do so.

Input Parameters

A Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

`SPARSE_STATUS_SUCCESS` The operation was successful.

`SPARSE_STATUS_NOT_INITIALIZE` The routine encountered an empty handle or matrix array.

`D`

`SPARSE_STATUS_ALLOC_FAILED` Internal memory allocation failed.

`SPARSE_STATUS_INVALID_VALUE` The input parameters contain an invalid value.

`SPARSE_STATUS_EXECUTION_FAIL` Execution failed.

`ED`

`SPARSE_STATUS_INTERNAL_ERROR` An error in algorithm implementation occurred.

`SPARSE_STATUS_NOT_SUPPORTED` The requested operation is not supported.

`mkl_sparse_convert_csr`

Converts internal matrix representation to CSR format.

Syntax

```
sparse_status_t mkl_sparse_convert_csr (const sparse_matrix_t source,
sparse_operation_t operation, sparse_matrix_t *dest);
```

Include Files

- `mkl_spblas.h`

Description

The `mkl_sparse_convert_csr` routine converts internal matrix representation to CSR format.

Input Parameters

source Handle containing internal data.

operation Specifies operation `op()` on input matrix.

`SPARSE_OPERATION_NO` Non-transpose, $op(A) = A$.
`N_TRANSPOSE`

`SPARSE_OPERATION_TR` Transpose, $op(A) = A^T$.
`ANSPOSE`

`SPARSE_OPERATION_CO` Conjugate transpose, $op(A) = A^H$.
`NJUGATE_TRANSPOSE`

Output Parameters

dest Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

`SPARSE_STATUS_SUCCESS` The operation was successful.

`SPARSE_STATUS_NOT_INITIALIZE` The routine encountered an empty handle or matrix array.
D

`SPARSE_STATUS_ALLOC_FAILED` Internal memory allocation failed.

`SPARSE_STATUS_INVALID_VALUE` The input parameters contain an invalid value.

`SPARSE_STATUS_EXECUTION_FAIL` Execution failed.
ED

`SPARSE_STATUS_INTERNAL_ERROR` An error in algorithm implementation occurred.

`SPARSE_STATUS_NOT_SUPPORTED` The requested operation is not supported.

`mkl_sparse_convert_bsr`

Converts internal matrix representation to BSR format or changes BSR block size.

Syntax

```
sparse_status_t mkl_sparse_convert_bsr (const sparse_matrix_t source, MKL_INT
block_size, sparse_layout_t block_layout, sparse_operation_t operation, sparse_matrix_t
*dest);
```

Include Files

- `mkl_spblas.h`

Description

The `mkl_sparse_convert_bsr` routine converts internal matrix representation to BSR format or changes BSR block size.

Input Parameters

source Handle containing internal data.

block_size Size of the block in the output structure.

block_layout Specifies layout of blocks:

`SPARSE_LAYOUT_ROW_MAJOR` Storage of elements of blocks uses row major layout.

`SPARSE_LAYOUT_COLUMN_MAJOR` Storage of elements of blocks uses column major layout.

operation Specifies operation `op()` on input matrix.

SPARSE_OPERATION_NO **Non-transpose**, $op(A) = A$.
N_TRANSPOSE

SPARSE_OPERATION_TR **Transpose**, $op(A) = A^T$.
ANSPOSE

SPARSE_OPERATION_CO **Conjugate transpose**, $op(A) = A^H$.
NJUGATE_TRANSPOSE

Output Parameters

dest Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

SPARSE_STATUS_SUCCESS The operation was successful.

SPARSE_STATUS_NOT_INITIALIZE The routine encountered an empty handle or matrix array.
D

SPARSE_STATUS_ALLOC_FAILED Internal memory allocation failed.

SPARSE_STATUS_INVALID_VALUE The input parameters contain an invalid value.

SPARSE_STATUS_EXECUTION_FAIL Execution failed.
ED

SPARSE_STATUS_INTERNAL_ERROR An error in algorithm implementation occurred.

SPARSE_STATUS_NOT_SUPPORTED The requested operation is not supported.

mkl_sparse_?_export_csr

Exports CSR matrix from internal representation.

Syntax

```
sparse_status_t mkl_sparse_s_export_csr (const sparse_matrix_t *source,
sparse_index_base_t *indexing, MKL_INT *rows, MKL_INT *cols, MKL_INT **rows_start,
MKL_INT **rows_end, MKL_INT **col_indx, float **values);
```

```
sparse_status_t mkl_sparse_d_export_csr (const sparse_matrix_t *source,
sparse_index_base_t *indexing, MKL_INT *rows, MKL_INT *cols, MKL_INT **rows_start,
MKL_INT **rows_end, MKL_INT **col_indx, double **values);
```

```
sparse_status_t mkl_sparse_c_export_csr (const sparse_matrix_t *source,
sparse_index_base_t *indexing, MKL_INT *rows, MKL_INT *cols, MKL_INT **rows_start,
MKL_INT **rows_end, MKL_INT **col_indx, MKL_Complex8 **values);
```

```
sparse_status_t mkl_sparse_z_export_csr (const sparse_matrix_t *source,
sparse_index_base_t *indexing, MKL_INT *rows, MKL_INT *cols, MKL_INT **rows_start,
MKL_INT **rows_end, MKL_INT **col_indx, MKL_Complex16 **values);
```

Include Files

- mkl_splblas.h

Description

If the matrix specified by the *source* handle is in CSR format, the `mkl_sparse_?_export_csr` routine exports an m -by- k matrix A in CSR format matrix from the internal representation. The routine returns pointers to the internal representation and does not allocate additional memory.

NOTE

Since the exported data is a copy of an internally stored structure, any changes made to it have no effect on subsequent Inspector-executor Sparse BLAS operations.

If the matrix is not already in CSR format, the routine returns `SPARSE_STATUS_INVALID_VALUE`.

Input Parameters

source Handle containing internal data.

Output Parameters

indexing Indicates how input arrays are indexed.

`SPARSE_INDEX_BASE_Z` Zero-based (C-style) indexing: indices start at 0.

`SPARSE_INDEX_BASE_ONE` One-based (Fortran-style) indexing: indices start at 1.

rows Number of rows of the matrix *source*.

cols Number of columns of the matrix *source*.

rows_start Pointer to array of length m . This array contains row indices, such that $rows_start[i] - rows_start[0]$ is the first index of row i in the arrays *values* and *col_indx*.

Refer to *pointerb* array description in [CSR Format](#) for more details.

rows_end Pointer to array of length m . This array contains row indices, such that $rows_end[i] - rows_start[0] - 1$ is the last index of row i in the arrays *values* and *col_indx*.

Refer to *pointerE* array description in [CSR Format](#) for more details.

col_indx For one-based indexing, pointer to array containing the column indices plus one for each non-zero element of the matrix *source*. For zero-based indexing, pointer to array containing the column indices for each non-zero element of the matrix *source*. Its length is $rows_end[rows - 1] - rows_start[0]$.

values Pointer to array containing non-zero elements of the matrix A . Its length is equal to length of the *col_indx* array.

Refer to *values* array description in [CSR Format](#) for more details.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|------------------------------------|--|
| SPARSE_STATUS_SUCCESS | The operation was successful. |
| SPARSE_STATUS_NOT_INITIALIZE D | The routine encountered an empty handle or matrix array. |
| SPARSE_STATUS_ALLOC_FAILED | Internal memory allocation failed. |
| SPARSE_STATUS_INVALID_VALUE | The input parameters contain an invalid value. |
| SPARSE_STATUS_EXECUTION_FAIL ED | Execution failed. |
| SPARSE_STATUS_INTERNAL_ERROR | An error in algorithm implementation occurred. |
| SPARSE_STATUS_NOT_SUPPORTED | The requested operation is not supported. |

mkl_sparse_?_export_bsr

Exports BSR matrix from internal representation.

Syntax

```
sparse_status_t mkl_sparse_s_export_bsr (const sparse_matrix_t source,  
sparse_index_base_t *indexing, sparse_layout_t *block_layout, MKL_INT *rows, MKL_INT  
*cols, MKL_INT *block_size, MKL_INT **rows_start, MKL_INT **rows_end, MKL_INT  
**col_indx, float **values);
```

```
sparse_status_t mkl_sparse_d_export_bsr (const sparse_matrix_t source,  
sparse_index_base_t *indexing, sparse_layout_t *block_layout, MKL_INT *rows, MKL_INT  
*cols, MKL_INT *block_size, MKL_INT **rows_start, MKL_INT **rows_end, MKL_INT  
**col_indx, double **values);
```

```
sparse_status_t mkl_sparse_c_export_bsr (const sparse_matrix_t source,  
sparse_index_base_t *indexing, sparse_layout_t *block_layout, MKL_INT *rows, MKL_INT  
*cols, MKL_INT *block_size, MKL_INT **rows_start, MKL_INT **rows_end, MKL_INT  
**col_indx, MKL_Complex8 **values);
```

```
sparse_status_t mkl_sparse_z_export_bsr (const sparse_matrix_t source,  
sparse_index_base_t *indexing, sparse_layout_t *block_layout, MKL_INT *rows, MKL_INT  
*cols, MKL_INT *block_size, MKL_INT **rows_start, MKL_INT **rows_end, MKL_INT  
**col_indx, MKL_Complex16 **values);
```

Include Files

- mkl_spblas.h

Description

If the matrix specified by the *source* handle is in BSR format, the `mkl_sparse_?_export_bsr` routine exports an m -by- k matrix A in BSR format from the internal representation. The routine returns pointers to the internal representation and does not allocate additional memory.

NOTE

Since the exported data is a copy of an internally stored structure, any changes made to it have no effect on subsequent Inspector-executor Sparse BLAS operations.

If the matrix is not already in BSR format, the routine returns `SPARSE_STATUS_INVALID_VALUE`.

Input Parameters

source Handle containing internal data.

Output Parameters

indexing Indicates how input arrays are indexed.

SPARSE_INDEX_BASE_Z
ERO Zero-based (C-style) indexing: indices start at 0.

SPARSE_INDEX_BASE_O
NE One-based (Fortran-style) indexing: indices start at 1.

block_layout Specifies layout of blocks:

SPARSE_LAYOUT_ROW_M
AJOR Storage of elements of blocks uses row major layout.

SPARSE_LAYOUT_COLUMN_MAJOR
N_MAJOR Storage of elements of blocks uses column major layout.

rows Number of block rows of the matrix *source*.

cols Number of columns of the matrix *source*. Number of block columns of matrix *source*.

block_size Size of the block in matrix *source*.

rows_start Pointer to array of length *m*. This array contains row indices, such that $rows_start[i] - rows_start[0]$ is the first index of block row *i* in the arrays *values* and *col_indx*.

Refer to *pointerb* array description in [CSR Format](#) for more details.

rows_end Pointer to array of length *m*. This array contains row indices, such that $rows_end[i] - rows_start[0] - 1$ is the last index of block row *i* in the arrays *values* and *col_indx*.

Refer to *pointerE* array description in [CSR Format](#) for more details.

col_indx For one-based indexing, pointer to array containing the column indices plus one for each non-zero blocks of the matrix *source*. For zero-based indexing, pointer to array containing the column indices for each non-zero blocks of the matrix *source*. Its length is $rows_end[m - 1] - rows_start[0]$.

values Pointer to array containing non-zero elements of matrix *source*. Its length is equal to length of the *col_indx* array multiplied by $block_size * block_size$.

Refer to the *values* array description in [BSR Format](#) for more details.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

SPARSE_STATUS_SUCCESS The operation was successful.

`SPARSE_STATUS_NOT_INITIALIZE` The routine encountered an empty handle or matrix array.
D

`SPARSE_STATUS_ALLOC_FAILED` Internal memory allocation failed.

`SPARSE_STATUS_INVALID_VALUE` The input parameters contain an invalid value.

`SPARSE_STATUS_EXECUTION_FAIL` Execution failed.
ED

`SPARSE_STATUS_INTERNAL_ERROR` An error in algorithm implementation occurred.

`SPARSE_STATUS_NOT_SUPPORTED` The requested operation is not supported.

mkl_sparse_?_set_value

Changes a single value of matrix in internal representation.

Syntax

```
sparse_status_t mkl_sparse_s_set_value (sparse_matrix_t A, MKL_INT row, MKL_INT col, float value);  
  
sparse_status_t mkl_sparse_d_set_value (sparse_matrix_t A, MKL_INT row, MKL_INT col, double value);  
  
sparse_status_t mkl_sparse_c_set_value (sparse_matrix_t A, MKL_INT row, MKL_INT col, MKL_Complex8 value);  
  
sparse_status_t mkl_sparse_z_set_value (sparse_matrix_t A, MKL_INT row, MKL_INT col, MKL_Complex16 value);
```

Include Files

- `mkl_spblas.h`

Description

Use the `mkl_sparse_?_set_value` routine to change a single value of a matrix in internal Inspector-executor Sparse BLAS format.

Input Parameters

| | |
|--------------------|---|
| <code>A</code> | Specifies handle containing internal data. |
| <code>row</code> | Indicates row of matrix in which to set value. |
| <code>col</code> | Indicates column of matrix in which to set value. |
| <code>value</code> | Indicates value |

Output Parameters

| | |
|----------------|---|
| <code>A</code> | Handle containing modified internal data. |
|----------------|---|

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|-----------------------------------|--|
| SPARSE_STATUS_SUCCESS | The operation was successful. |
| SPARSE_STATUS_NOT_INITIALIZE D | The routine encountered an empty handle or matrix array. |
| SPARSE_STATUS_INVALID_VALUE | The input parameters contain an invalid value. |
| SPARSE_STATUS_INTERNAL_ERROR | An error in algorithm implementation occurred. |

Inspector-executor Sparse BLAS Analysis Routines

Analysis Routines and Their Data Types

| Routine or Function Group | Description |
|--|---|
| mkl_sparse_set_mv_hint | Provides estimate of number and type of upcoming matrix-vector operations. |
| mkl_sparse_set_sv_hint | Provides estimate of number and type of upcoming triangular system solver operations. |
| mkl_sparse_set_mm_hint | Provides estimate of number and type of upcoming matrix-matrix multiplication operations. |
| mkl_sparse_set_sm_hint | Provides estimate of number and type of upcoming triangular matrix solve with multiple right hand sides operations. |
| mkl_sparse_set_memory_hint | Provides memory requirements for performance optimization purposes. |
| mkl_sparse_optimize | Analyzes matrix structure and performs optimizations using the hints provided in the handle. |

Optimization Notice

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[mkl_sparse_set_mv_hint](#)

Provides estimate of number and type of upcoming matrix-vector operations.

Syntax

```
sparse_status_t mkl_sparse_set_mv_hint (sparse_matrix_t A, sparse_operation_t
operation, struct matrix_descr descr, MKL_INT expected_calls);
```

Include Files

- `mkl_spblas.h`

Description

Use the `mkl_sparse_set_mv_hint` routine to provide the Inspector-executor Sparse BLAS API an estimate of the number of upcoming matrix-vector multiplication operations for performance optimization, and specify whether or not to perform an operation on the matrix.

Optimization Notice

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Input Parameters

operation

Specifies operation `op()` on input matrix.

`SPARSE_OPERATION_NO` Non-transpose, $op(A) = A$.
`N_TRANSPOSE`

`SPARSE_OPERATION_TR` Transpose, $op(A) = A^T$.
`ANSPOSE`

`SPARSE_OPERATION_CO` Conjugate transpose, $op(A) = A^H$.
`NJUGATE_TRANSPOSE`

descr

Structure specifying sparse matrix properties.

`sparse_matrix_type_t` *type* - Specifies the type of a sparse matrix:

`SPARSE_MATRIX_TYPE_` The matrix is processed as is.
`GENERAL`

`SPARSE_MATRIX_TYPE_` The matrix is symmetric (only the requested
`SYMMETRIC` triangle is processed).

`SPARSE_MATRIX_TYPE_` The matrix is Hermitian (only the requested
`HERMITIAN` triangle is processed).

`SPARSE_MATRIX_TYPE_` The matrix is triangular (only the requested
`TRIANGULAR` triangle is processed).

`SPARSE_MATRIX_TYPE_` The matrix is diagonal (only diagonal elements
`DIAGONAL` are processed).

`SPARSE_MATRIX_TYPE_` The matrix is block-triangular (only requested
`BLOCK_TRIANGULAR` triangle is processed). (Applies to BSR format only.)

`SPARSE_MATRIX_TYPE_` The matrix is block-diagonal (only diagonal
`BLOCK_DIAGONAL` blocks are processed). (Applies to BSR format only.)

`sparse_fill_mode_t mode` - Specifies the triangular matrix part for symmetric, Hermitian, triangular, and block-triangular matrices:

`SPARSE_FILL_MODE_LO` The lower triangular matrix part is processed.
WER

`SPARSE_FILL_MODE_UP` The upper triangular matrix part is processed.
PER

`sparse_diag_type_t diag` - Specifies diagonal type for non-general matrices:

`SPARSE_DIAG_NON_UNIT` Diagonal elements might not be equal to one.
T

`SPARSE_DIAG_UNIT` Diagonal elements are equal to one.

`expected_calls`

Number of expected calls to execution routine.

Output Parameters

`A` Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

`SPARSE_STATUS_SUCCESS` The operation was successful.

`SPARSE_STATUS_NOT_INITIALIZE` The routine encountered an empty handle or matrix array.
D

`SPARSE_STATUS_ALLOC_FAILED` Internal memory allocation failed.

`SPARSE_STATUS_INVALID_VALUE` The input parameters contain an invalid value.

`SPARSE_STATUS_EXECUTION_FAIL` Execution failed.
ED

`SPARSE_STATUS_INTERNAL_ERROR` An error in algorithm implementation occurred.

`SPARSE_STATUS_NOT_SUPPORTED` The requested operation is not supported.

`mkl_sparse_set_sv_hint`

Provides estimate of number and type of upcoming triangular system solver operations.

Syntax

```
sparse_status_t mkl_sparse_set_sv_hint (sparse_matrix_t A, sparse_operation_t
operation, struct matrix_descr descr, MKL_INT expected_calls);
```

Include Files

- `mkl_splblas.h`

Description

The `mkl_sparse_sv_hint` routine provides an estimate of the number of upcoming triangular system solver operations and type of these operations for performance optimization.

Optimization Notice

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Input Parameters

| | |
|------------------|--|
| <i>operation</i> | <p>Specifies operation <code>op()</code> on input matrix.</p> <p><code>SPARSE_OPERATION_NO</code> Non-transpose, $op(A) = A$. <code>N_TRANSPOSE</code></p> <p><code>SPARSE_OPERATION_TR</code> Transpose, $op(A) = A^T$. <code>ANSPOSE</code></p> <p><code>SPARSE_OPERATION_CO</code> Conjugate transpose, $op(A) = A^H$. <code>NJUGATE_TRANSPOSE</code></p> |
| <i>descr</i> | <p>Structure specifying sparse matrix properties.</p> <p><code>sparse_matrix_type_t</code> <i>type</i> - Specifies the type of a sparse matrix:</p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is processed as is. <code>GENERAL</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is symmetric (only the requested triangle is processed). <code>SYMMETRIC</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is Hermitian (only the requested triangle is processed). <code>HERMITIAN</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is triangular (only the requested triangle is processed). <code>TRIANGULAR</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is diagonal (only diagonal elements are processed). <code>DIAGONAL</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is block-triangular (only requested triangle is processed). (Applies to BSR format only.) <code>BLOCK_TRIANGULAR</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is block-diagonal (only diagonal blocks are processed). (Applies to BSR format only.) <code>BLOCK_DIAGONAL</code></p> |

`sparse_fill_mode_t mode` - Specifies the triangular matrix part for symmetric, Hermitian, triangular, and block-triangular matrices:

`SPARSE_FILL_MODE_LO` The lower triangular matrix part is processed.
WER

`SPARSE_FILL_MODE_UP` The upper triangular matrix part is processed.
PER

`sparse_diag_type_t diag` - Specifies diagonal type for non-general matrices:

`SPARSE_DIAG_NON_UNIT` Diagonal elements might not be equal to one.
T

`SPARSE_DIAG_UNIT` Diagonal elements are equal to one.

`expected_calls` Number of expected calls to execution routine.

Output Parameters

`A` Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

`SPARSE_STATUS_SUCCESS` The operation was successful.

`SPARSE_STATUS_NOT_INITIALIZE` The routine encountered an empty handle or matrix array.
D

`SPARSE_STATUS_ALLOC_FAILED` Internal memory allocation failed.

`SPARSE_STATUS_INVALID_VALUE` The input parameters contain an invalid value.

`SPARSE_STATUS_EXECUTION_FAIL` Execution failed.
ED

`SPARSE_STATUS_INTERNAL_ERROR` An error in algorithm implementation occurred.

`SPARSE_STATUS_NOT_SUPPORTED` The requested operation is not supported.

`mkl_sparse_set_mm_hint`

Provides estimate of number and type of upcoming matrix-matrix multiplication operations.

Syntax

```
sparse_status_t mkl_sparse_set_mm_hint (sparse_matrix_t A, sparse_operation_t
operation, struct matrix_descr descr, sparse_layout_t layout, MKL_INT
dense_matrix_size, MKL_INT expected_calls);
```

Include Files

- `mkl_spblas.h`

Description

The `mkl_sparse_set_mm_hint` routine provides an estimate of the number of upcoming matrix-matrix multiplication operations and type of these operations for performance optimization purposes.

Optimization Notice

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Notice revision #20110804

Input Parameters

operation

Specifies operation `op()` on input matrix.

`SPARSE_OPERATION_NO` Non-transpose, $op(A) = A$.
`N_TRANSPOSE`

`SPARSE_OPERATION_TR` Transpose, $op(A) = A^T$.
`ANSPOSE`

`SPARSE_OPERATION_CO` Conjugate transpose, $op(A) = A^H$.
`NJUGATE_TRANSPOSE`

descr

Structure specifying sparse matrix properties.

`sparse_matrix_type_t` *type* - Specifies the type of a sparse matrix:

`SPARSE_MATRIX_TYPE_` The matrix is processed as is.
`GENERAL`

`SPARSE_MATRIX_TYPE_` The matrix is symmetric (only the requested
`SYMMETRIC` triangle is processed).

`SPARSE_MATRIX_TYPE_` The matrix is Hermitian (only the requested
`HERMITIAN` triangle is processed).

`SPARSE_MATRIX_TYPE_` The matrix is triangular (only the requested
`TRIANGULAR` triangle is processed).

`SPARSE_MATRIX_TYPE_` The matrix is diagonal (only diagonal elements
`DIAGONAL` are processed).

`SPARSE_MATRIX_TYPE_` The matrix is block-triangular (only requested
`BLOCK_TRIANGULAR` triangle is processed). (Applies to BSR format only.)

`SPARSE_MATRIX_TYPE_` The matrix is block-diagonal (only diagonal
`BLOCK_DIAGONAL` blocks are processed). (Applies to BSR format only.)

`sparse_fill_mode_t mode` - Specifies the triangular matrix part for symmetric, Hermitian, triangular, and block-triangular matrices:

`SPARSE_FILL_MODE_LO` The lower triangular matrix part is processed.
WER

`SPARSE_FILL_MODE_UP` The upper triangular matrix part is processed.
PER

`sparse_diag_type_t diag` - Specifies diagonal type for non-general matrices:

`SPARSE_DIAG_NON_UNI` Diagonal elements might not be equal to one.
T

`SPARSE_DIAG_UNIT` Diagonal elements are equal to one.

`layout`

Specifies layout of elements:

`SPARSE_LAYOUT_COLUMN` Storage of elements uses column major layout.
N_MAJOR

`SPARSE_LAYOUT_ROW_M` Storage of elements uses row major layout.
A_JOR

`dense_matrix_size`

Number of columns in dense matrix.

`expected_calls`

Number of expected calls to execution routine.

Output Parameters

A

Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

`SPARSE_STATUS_SUCCESS` The operation was successful.

`SPARSE_STATUS_NOT_INITIALIZE` The routine encountered an empty handle or matrix array.
D

`SPARSE_STATUS_ALLOC_FAILED` Internal memory allocation failed.

`SPARSE_STATUS_INVALID_VALUE` The input parameters contain an invalid value.

`SPARSE_STATUS_EXECUTION_FAIL` Execution failed.
ED

`SPARSE_STATUS_INTERNAL_ERROR` An error in algorithm implementation occurred.

`SPARSE_STATUS_NOT_SUPPORTED` The requested operation is not supported.

`mkl_sparse_set_sm_hint`

Provides estimate of number and type of upcoming triangular matrix solve with multiple right hand sides operations.

Syntax

```
sparse_status_t mkl_sparse_set_sm_hint (sparse_matrix_t A, sparse_operation_t
operation, struct matrix_descr descr, sparse_layout_t layout, MKL_INT
dense_matrix_size, MKL_INT expected_calls);
```

Include Files

- mkl_splblas.h

Description

The `mkl_sparse_set_sm_hint` routine provides an estimate of the number of upcoming triangular matrix solve with multiple right hand sides operations and type of these operations for performance optimization purposes.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

operation

Specifies operation `op()` on input matrix.

`SPARSE_OPERATION_NO` Non-transpose, $op(A) = A$.
`N_TRANSPOSE`

`SPARSE_OPERATION_TR` Transpose, $op(A) = A^T$.
`ANSPOSE`

`SPARSE_OPERATION_CO` Conjugate transpose, $op(A) = A^H$.
`NJUGATE_TRANSPOSE`

descr

Structure specifying sparse matrix properties.

`sparse_matrix_type_t type` - Specifies the type of a sparse matrix:

`SPARSE_MATRIX_TYPE_` The matrix is processed as is.
`GENERAL`

`SPARSE_MATRIX_TYPE_` The matrix is symmetric (only the requested
`SYMMETRIC` triangle is processed).

`SPARSE_MATRIX_TYPE_` The matrix is Hermitian (only the requested
`HERMITIAN` triangle is processed).

`SPARSE_MATRIX_TYPE_` The matrix is triangular (only the requested
`TRIANGULAR` triangle is processed).

SPARSE_MATRIX_TYPE_ DIAGONAL The matrix is diagonal (only diagonal elements are processed).

SPARSE_MATRIX_TYPE_ BLOCK_TRIANGULAR The matrix is block-triangular (only requested triangle is processed). (Applies to BSR format only.)

SPARSE_MATRIX_TYPE_ BLOCK_DIAGONAL The matrix is block-diagonal (only diagonal blocks are processed). (Applies to BSR format only.)

`sparse_fill_mode_t mode` - Specifies the triangular matrix part for symmetric, Hermitian, triangular, and block-triangular matrices:

SPARSE_FILL_MODE_LO WER The lower triangular matrix part is processed.

SPARSE_FILL_MODE_UP PER The upper triangular matrix part is processed.

`sparse_diag_type_t diag` - Specifies diagonal type for non-general matrices:

SPARSE_DIAG_NON_UNIT T Diagonal elements might not be equal to one.

SPARSE_DIAG_UNIT Diagonal elements are equal to one.

`layout`

Specifies layout of elements:

SPARSE_LAYOUT_COLUMN_MAJOR N_MAJOR Storage of elements uses column major layout.

SPARSE_LAYOUT_ROW_MAJOR AJOR Storage of elements uses row major layout.

`dense_matrix_size`

Number of right-hand-side.

`expected_calls`

Number of expected calls to execution routine.

Output Parameters

A

Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

SPARSE_STATUS_SUCCESS The operation was successful.

SPARSE_STATUS_NOT_INITIALIZE D The routine encountered an empty handle or matrix array.

SPARSE_STATUS_ALLOC_FAILED Internal memory allocation failed.

SPARSE_STATUS_INVALID_VALUE The input parameters contain an invalid value.

`SPARSE_STATUS_EXECUTION_FAIL` Execution failed.
`ED`

`SPARSE_STATUS_INTERNAL_ERROR` An error in algorithm implementation occurred.

`SPARSE_STATUS_NOT_SUPPORTED` The requested operation is not supported.

mkl_sparse_set_memory_hint

Provides memory requirements for performance optimization purposes.

Syntax

```
sparse_status_t mkl_sparse_set_memory_hint (sparse_matrix_t A, sparse_memory_usage_t policy);
```

Include Files

- `mkl_spblas.h`

Description

The `mkl_sparse_set_memory_hint` routine allocates additional memory for further performance optimization purposes.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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Input Parameters

| | |
|---------------------------------------|--|
| <code>policy</code> | Specify memory utilization policy for optimization routine using these types: |
| <code>SPARSE_MEMORY_NONE</code> | Routine can allocate memory only for auxiliary structures (such as for workload balancing); the amount of memory is proportional to vector size. |
| <code>SPARSE_MEMORY_AGGRESSIVE</code> | Default. |
| | Routine can allocate memory up to the size of matrix <i>A</i> for converting into the appropriate sparse format. |

Output Parameters

`A` Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|---|--|
| <code>SPARSE_STATUS_SUCCESS</code> | The operation was successful. |
| <code>SPARSE_STATUS_NOT_INITIALIZE</code> D | The routine encountered an empty handle or matrix array. |
| <code>SPARSE_STATUS_ALLOC_FAILED</code> | Internal memory allocation failed. |
| <code>SPARSE_STATUS_INVALID_VALUE</code> | The input parameters contain an invalid value. |
| <code>SPARSE_STATUS_EXECUTION_FAIL</code> ED | Execution failed. |
| <code>SPARSE_STATUS_INTERNAL_ERROR</code> | An error in algorithm implementation occurred. |
| <code>SPARSE_STATUS_NOT_SUPPORTED</code> | The requested operation is not supported. |

`mkl_sparse_optimize`

Analyzes matrix structure and performs optimizations using the hints provided in the handle.

Syntax

```
sparse_status_t mkl_sparse_optimize (sparse_matrix_t A);
```

Include Files

- `mkl_splblas.h`

Description

The `mkl_sparse_optimize` routine analyzes matrix structure and performs optimizations using the hints provided in the handle. Generally, specifying a higher number of expected operations allows for more aggressive and time consuming optimizations.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

`A` Handle containing internal data.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|------------------------------------|--|
| SPARSE_STATUS_SUCCESS | The operation was successful. |
| SPARSE_STATUS_NOT_INITIALIZE D | The routine encountered an empty handle or matrix array. |
| SPARSE_STATUS_ALLOC_FAILED | Internal memory allocation failed. |
| SPARSE_STATUS_INVALID_VALUE | The input parameters contain an invalid value. |
| SPARSE_STATUS_EXECUTION_FAIL ED | Execution failed. |
| SPARSE_STATUS_INTERNAL_ERROR | An error in algorithm implementation occurred. |
| SPARSE_STATUS_NOT_SUPPORTED | The requested operation is not supported. |

Inspector-executor Sparse BLAS Execution Routines

Execution Routines and Their Data Types

| Routine or Function Group | Data Types | Description |
|-----------------------------------|------------|--|
| mkl_sparse_?_mv | s, d, c, z | Computes a sparse matrix-vector product. |
| mkl_sparse_?_trsv | s, d, c, z | Solves a system of linear equations for a square sparse matrix. |
| mkl_sparse_?_mm | s, d, c, z | Computes the product of a sparse matrix and a dense matrix. |
| mkl_sparse_?_trsm | s, d, c, z | Solves a system of linear equations with multiple right hand sides for a square sparse matrix. |
| mkl_sparse_?_add | s, d, c, z | Computes sum of two sparse matrices. |
| mkl_sparse_spmv | s, d, c, z | Computes the product of two sparse matrices and stores the result as a sparse matrix. |
| mkl_sparse_?_spmv | s, d, c, z | Computes the product of two sparse matrices and stores the result as a dense matrix. |

[mkl_sparse_?_mv](#)

Computes a sparse matrix-vector product.

Syntax

```
sparse_status_t mkl_sparse_s_mv (sparse_operation_t operation, float alpha, const
sparse_matrix_t A, struct matrix_descr descr, const float *x, float beta, float *y);
sparse_status_t mkl_sparse_d_mv (sparse_operation_t operation, double alpha, const
sparse_matrix_t A, struct matrix_descr descr, const double *x, double beta, double *y);
sparse_status_t mkl_sparse_c_mv (sparse_operation_t operation, MKL_Complex8 alpha,
const sparse_matrix_t A, struct matrix_descr descr, const MKL_Complex8 *x, MKL_Complex8
beta, MKL_Complex8 *y);
sparse_status_t mkl_sparse_z_mv (sparse_operation_t operation, MKL_Complex16 alpha,
const sparse_matrix_t A, struct matrix_descr descr, const MKL_Complex16 *x,
MKL_Complex16 beta, MKL_Complex16 *y);
```

Include Files

- `mkl_splblas.h`

Description

The `mkl_sparse_?_mv` routine computes a sparse matrix-vector product defined as

$$y := \alpha * \text{op}(A) * x + \beta * y$$

where:

alpha and *beta* are scalars, *x* and *y* are vectors, and *A* is a matrix handle of a matrix with *m* rows and *k* columns.

Input Parameters

| | |
|------------------|---|
| <i>operation</i> | Specifies operation <code>op()</code> on input matrix. |
| | <p><code>SPARSE_OPERATION_NO</code> Non-transpose, $\text{op}(A) = A$.</p> <p><code>N_TRANSPOSE</code></p> <p><code>SPARSE_OPERATION_TR</code> Transpose, $\text{op}(A) = A^T$.</p> <p><code>ANSPOSE</code></p> <p><code>SPARSE_OPERATION_CO</code> Conjugate transpose, $\text{op}(A) = A^H$.</p> <p><code>NJUGATE_TRANSPOSE</code></p> |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>A</i> | Handle containing sparse matrix in internal data structure. |
| <i>descr</i> | Structure specifying sparse matrix properties. |
| | <p><code>sparse_matrix_type_t</code> <i>type</i> - Specifies the type of a sparse matrix:</p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is processed as is.</p> <p><code>GENERAL</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is symmetric (only the requested triangle is processed).</p> <p><code>SYMMETRIC</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is Hermitian (only the requested triangle is processed).</p> <p><code>HERMITIAN</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is triangular (only the requested triangle is processed).</p> <p><code>TRIANGULAR</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is diagonal (only diagonal elements are processed).</p> <p><code>DIAGONAL</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is block-triangular (only requested triangle is processed). (Applies to BSR format only.)</p> <p><code>BLOCK_TRIANGULAR</code></p> <p><code>SPARSE_MATRIX_TYPE_</code> The matrix is block-diagonal (only diagonal blocks are processed). (Applies to BSR format only.)</p> <p><code>BLOCK_DIAGONAL</code></p> <p><code>sparse_fill_mode_t</code> <i>mode</i> - Specifies the triangular matrix part for symmetric, Hermitian, triangular, and block-triangular matrices:</p> |

SPARSE_FILL_MODE_LO The lower triangular matrix part is processed.
WER

SPARSE_FILL_MODE_UP The upper triangular matrix part is processed.
PER

sparse_diag_type_t *diag* - Specifies diagonal type for non-general matrices:

SPARSE_DIAG_NON_UNIT Diagonal elements might not be equal to one.
T

SPARSE_DIAG_UNIT Diagonal elements are equal to one.

x Array with size equal to the number of columns of *A* if *operation* = SPARSE_OPERATION_NON_TRANSPOSE and at least rows of *A* otherwise. On entry, the array *x* must contain the vector *x*.

beta Specifies the scalar *beta*.

y Array with size at least *m* if *operation*=SPARSE_OPERATION_NON_TRANSPOSE and at least *k* otherwise. On entry, the array *y* must contain the vector *y*.

Output Parameters

y Overwritten by the updated vector *y*.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

SPARSE_STATUS_SUCCESS The operation was successful.

SPARSE_STATUS_NOT_INITIALIZE The routine encountered an empty handle or matrix array.
D

SPARSE_STATUS_ALLOC_FAILED Internal memory allocation failed.

SPARSE_STATUS_INVALID_VALUE The input parameters contain an invalid value.

SPARSE_STATUS_EXECUTION_FAILED Execution failed.
ED

SPARSE_STATUS_INTERNAL_ERROR An error in algorithm implementation occurred.

SPARSE_STATUS_NOT_SUPPORTED The requested operation is not supported.

mkl_sparse_?_trsv

Solves a system of linear equations for a triangular sparse matrix.

Syntax

```
sparse_status_t mkl_sparse_s_trsv (sparse_operation_t operation, float alpha, const
sparse_matrix_t A, struct matrix_descr descr, const float *x, float *y);
```

```

sparse_status_t mkl_sparse_d_trsv (sparse_operation_t operation, double alpha, const
sparse_matrix_t A, struct matrix_descr descr, const double *x, double *y);

sparse_status_t mkl_sparse_c_trsv (sparse_operation_t operation, MKL_Complex8 alpha,
const sparse_matrix_t A, struct matrix_descr descr, const MKL_Complex8 *x, MKL_Complex8
*y);

sparse_status_t mkl_sparse_z_trsv (sparse_operation_t operation, MKL_Complex16 alpha,
const sparse_matrix_t A, struct matrix_descr descr, const MKL_Complex16 *x,
MKL_Complex16 *y);

```

Include Files

- mkl_spblas.h

Description

The `mkl_sparse_?_trsv` routine solves a system of linear equations for a matrix:

$$\text{op}(A)*y = \text{alpha} * x$$

where A is a triangular sparse matrix, alpha is a scalar, and x and y are vectors.

Input Parameters

| | |
|------------------|--|
| <i>operation</i> | Specifies operation <code>op()</code> on input matrix. |
| | <p>SPARSE_OPERATION_NO Non-transpose, $\text{op}(A) = A$. N_TRANSPOSE</p> <p>SPARSE_OPERATION_TR Transpose, $\text{op}(A) = A^T$. ANSPOSE</p> <p>SPARSE_OPERATION_CO Conjugate transpose, $\text{op}(A) = A^H$. NJUGATE_TRANSPOSE</p> |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>A</i> | Handle containing sparse matrix in internal data structure. |
| <i>descr</i> | Structure specifying sparse matrix properties. |
| | <p><code>sparse_matrix_type_t type</code> - Specifies the type of a sparse matrix:</p> <p>SPARSE_MATRIX_TYPE_ The matrix is processed as is. GENERAL</p> <p>SPARSE_MATRIX_TYPE_ The matrix is symmetric (only the requested SYMMETRIC triangle is processed).</p> <p>SPARSE_MATRIX_TYPE_ The matrix is Hermitian (only the requested HERMITIAN triangle is processed).</p> <p>SPARSE_MATRIX_TYPE_ The matrix is triangular (only the requested TRIANGULAR triangle is processed).</p> <p>SPARSE_MATRIX_TYPE_ The matrix is diagonal (only diagonal elements DIAGONAL are processed).</p> |

SPARSE_MATRIX_TYPE_BLOCK_TRIANGULAR The matrix is block-triangular (only requested triangle is processed). (Applies to BSR format only.)

SPARSE_MATRIX_TYPE_BLOCK_DIAGONAL The matrix is block-diagonal (only diagonal blocks are processed). (Applies to BSR format only.)

`sparse_fill_mode_t mode` - Specifies the triangular matrix part for symmetric, Hermitian, triangular, and block-triangular matrices:

SPARSE_FILL_MODE_LOWER The lower triangular matrix part is processed.

SPARSE_FILL_MODE_UPPER The upper triangular matrix part is processed.

`sparse_diag_type_t diag` - Specifies diagonal type for non-general matrices:

SPARSE_DIAG_NON_UNIT Diagonal elements might not be equal to one.

SPARSE_DIAG_UNIT Diagonal elements are equal to one.

`x` Array of size at least m , where m is the number of rows of matrix A . On entry, the array `x` must contain the vector x .

Output Parameters

`y` Array of size at least m containing the solution to the system of linear equations.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

SPARSE_STATUS_SUCCESS The operation was successful.

SPARSE_STATUS_NOT_INITIALIZED The routine encountered an empty handle or matrix array.

SPARSE_STATUS_ALLOC_FAILED Internal memory allocation failed.

SPARSE_STATUS_INVALID_VALUE The input parameters contain an invalid value.

SPARSE_STATUS_EXECUTION_FAILED Execution failed.

SPARSE_STATUS_INTERNAL_ERROR An error in algorithm implementation occurred.

SPARSE_STATUS_NOT_SUPPORTED The requested operation is not supported.

`mkl_sparse_?_mm`

Computes the product of a sparse matrix and a dense matrix.

Syntax

```
sparse_status_t mkl_sparse_s_mm (sparse_operation_t operation, float alpha, const
sparse_matrix_t A, struct matrix_descr descr, sparse_layout_t layout, const float *x,
MKL_INT columns, MKL_INT ldx, float beta, float *y, MKL_INT ldy);
```

```
sparse_status_t mkl_sparse_d_mm (sparse_operation_t operation, double alpha, const
sparse_matrix_t A, struct matrix_descr descr, sparse_layout_t layout, const double *x,
MKL_INT columns, MKL_INT ldx, double beta, double *y, MKL_INT ldy);
```

```
sparse_status_t mkl_sparse_c_mm (sparse_operation_t operation, MKL_Complex8 alpha,
const sparse_matrix_t A, struct matrix_descr descr, sparse_layout_t layout, const
MKL_Complex8 *x, MKL_INT columns, MKL_INT ldx, MKL_Complex8 beta, MKL_Complex8 *y,
MKL_INT ldy);
```

```
sparse_status_t mkl_sparse_z_mm (sparse_operation_t operation, MKL_Complex16 alpha,
const sparse_matrix_t A, struct matrix_descr descr, sparse_layout_t layout, const
MKL_Complex16 *x, MKL_INT columns, MKL_INT ldx, MKL_Complex16 beta, MKL_Complex16 *y,
MKL_INT ldy);
```

Include Files

- mkl_spblas.h

Description

The `mkl_sparse_?_mm` routine performs a matrix-matrix operation:

```
 $y := \alpha * \text{op}(A) * x + \beta * y$ 
```

where α and β are scalars, A is a sparse matrix, and x and y are dense matrices.

The `mkl_sparse_?_mm` and `mkl_sparse_?_trsm` routines support these configurations:

| | | |
|--|--|--|
| | Column-major dense matrix: $layout =$ SPARSE_LAYOUT_COLUMN_MAJ OR | Row-major dense matrix: $layout =$ SPARSE_LAYOUT_ROW_MAJOR |
| 0-based sparse matrix: SPARSE_INDEX_BASE_ZERO | CSR BSR: general non-transposed matrix multiplication only | All formats |
| 1-based sparse matrix: SPARSE_INDEX_BASE_ONE | All formats | CSR BSR: general non-transposed matrix multiplication only |

Input Parameters

operation

Specifies operation `op()` on input matrix.

SPARSE_OPERATION_NO Non-transpose, $\text{op}(A) = A$.
N_TRANSPOSE

SPARSE_OPERATION_TR Transpose, $\text{op}(A) = A^T$.
ANSPOSE

SPARSE_OPERATION_CO Conjugate transpose, $\text{op}(A) = A^H$.
NJUGATE_TRANSPOSE

| | |
|---------------|--|
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |
| <i>A</i> | Handle containing sparse matrix in internal data structure. |
| <i>descr</i> | <p>Structure specifying sparse matrix properties.</p> <p><code>sparse_matrix_type_t</code> <i>type</i> - Specifies the type of a sparse matrix:</p> <p><code>SPARSE_MATRIX_TYPE_GENERAL</code> The matrix is processed as is.</p> <p><code>SPARSE_MATRIX_TYPE_SYMMETRIC</code> The matrix is symmetric (only the requested triangle is processed).</p> <p><code>SPARSE_MATRIX_TYPE_HERMITIAN</code> The matrix is Hermitian (only the requested triangle is processed).</p> <p><code>SPARSE_MATRIX_TYPE_TRIANGULAR</code> The matrix is triangular (only the requested triangle is processed).</p> <p><code>SPARSE_MATRIX_TYPE_DIAGONAL</code> The matrix is diagonal (only diagonal elements are processed).</p> <p><code>SPARSE_MATRIX_TYPE_BLOCK_TRIANGULAR</code> The matrix is block-triangular (only requested triangle is processed). (Applies to BSR format only.)</p> <p><code>SPARSE_MATRIX_TYPE_BLOCK_DIAGONAL</code> The matrix is block-diagonal (only diagonal blocks are processed). (Applies to BSR format only.)</p> <p><code>sparse_fill_mode_t</code> <i>mode</i> - Specifies the triangular matrix part for symmetric, Hermitian, triangular, and block-triangular matrices:</p> <p><code>SPARSE_FILL_MODE_LOWER</code> The lower triangular matrix part is processed.</p> <p><code>SPARSE_FILL_MODE_UPPER</code> The upper triangular matrix part is processed.</p> <p><code>sparse_diag_type_t</code> <i>diag</i> - Specifies diagonal type for non-general matrices:</p> <p><code>SPARSE_DIAG_NON_UNIT</code> Diagonal elements might not be equal to one.</p> <p><code>SPARSE_DIAG_UNIT</code> Diagonal elements are equal to one.</p> |
| <i>layout</i> | <p>Describes the storage scheme for the dense matrix:</p> <p><code>SPARSE_LAYOUT_COLUMN_MAJOR</code> Storage of elements uses column major layout.</p> <p><code>SPARSE_LAYOUT_ROW_MAJOR</code> Storage of elements uses row major layout.</p> |
| <i>x</i> | Array of size at least <i>rows*cols</i> . |

| | | |
|--|---|---|
| | <i>layout</i> = SPARSE_LAYOUT_COLUMN_MAJOR | <i>layout</i> = SPARSE_LAYOUT_ROW_MAJOR |
| <i>rows</i> (number of rows in <i>x</i>) | <i>ldx</i> | If $op(A) = A$, number of columns in A If $op(A) = A^T$, number of rows in A |
| <i>cols</i> (number of columns in <i>x</i>) | <i>columns</i> | <i>ldx</i> |

columns Number of columns of matrix *y*.

ldx Specifies the leading dimension of matrix *x*.

beta Specifies the scalar *beta*

y Array of size at least *rows***cols*, where

| | | |
|--|---|---|
| | <i>layout</i> = SPARSE_LAYOUT_COLUMN_MAJOR | <i>layout</i> = SPARSE_LAYOUT_ROW_MAJOR |
| <i>rows</i> (number of rows in <i>y</i>) | <i>ldy</i> | If $op(A) = A$, number of columns in A If $op(A) = A^T$, number of rows in A |
| <i>cols</i> (number of columns in <i>y</i>) | <i>columns</i> | <i>ldy</i> |

Output Parameters

y Overwritten by the updated matrix *y*.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

- SPARSE_STATUS_SUCCESS The operation was successful.
- SPARSE_STATUS_NOT_INITIALIZE The routine encountered an empty handle or matrix array.
- D
- SPARSE_STATUS_ALLOC_FAILED Internal memory allocation failed.
- SPARSE_STATUS_INVALID_VALUE The input parameters contain an invalid value.
- SPARSE_STATUS_EXECUTION_FAIL Execution failed.
- ED
- SPARSE_STATUS_INTERNAL_ERROR An error in algorithm implementation occurred.
- SPARSE_STATUS_NOT_SUPPORTED The requested operation is not supported.

mkl_sparse_?_trsm

Solves a system of linear equations with multiple right hand sides for a triangular sparse matrix.

Syntax

```
sparse_status_t mkl_sparse_s_trsm (sparse_operation_t operation, float alpha, const
sparse_matrix_t A, struct matrix_descr descr, sparse_layout_t layout, const float *x,
MKL_INT columns, MKL_INT ldx, float *y, MKL_INT ldy);
```

```
sparse_status_t mkl_sparse_d_trsm (sparse_operation_t operation, double alpha, const
sparse_matrix_t A, struct matrix_descr descr, sparse_layout_t layout, const double *x,
MKL_INT columns, MKL_INT ldx, double *y, MKL_INT ldy);
```

```
sparse_status_t mkl_sparse_c_trsm (sparse_operation_t operation, MKL_Complex8 alpha,
const sparse_matrix_t A, struct matrix_descr descr, sparse_layout_t layout, const
MKL_Complex8 *x, MKL_INT columns, MKL_INT ldx, MKL_Complex8 *y, MKL_INT ldy);
```

```
sparse_status_t mkl_sparse_z_trsm (sparse_operation_t operation, MKL_Complex16 alpha,
const sparse_matrix_t A, struct matrix_descr descr, sparse_layout_t layout, const
MKL_Complex16 *x, MKL_INT columns, MKL_INT ldx, MKL_Complex16 *y, MKL_INT ldy);
```

Include Files

- mkl_spblas.h

Description

The `mkl_sparse_?_trsm` routine solves a system of linear equations with multiple right hand sides for a triangular sparse matrix:

$$y := \alpha * \text{inv}(\text{op}(A)) * x$$

where:

alpha is a scalar, *x* and *y* are dense matrices, and *A* is a sparse matrix.

The `mkl_sparse_?_mm` and `mkl_sparse_?_trsm` routines support these configurations:

| | | |
|--|---|--|
| | Column-major dense matrix: <i>layout</i> = SPARSE_LAYOUT_COLUMN_MAJ OR | Row-major dense matrix: <i>layout</i> = SPARSE_LAYOUT_ROW_MAJOR |
| 0-based sparse matrix: SPARSE_INDEX_BASE_ZERO | CSR BSR: general non-transposed matrix multiplication only | All formats |
| 1-based sparse matrix: SPARSE_INDEX_BASE_ONE | All formats | CSR BSR: general non-transposed matrix multiplication only |

Input Parameters

operation Specifies operation `op()` on input matrix.

SPARSE_OPERATION_NO Non-transpose, `op(A) = A`.
N_TRANSPOSE

| | | |
|---------------|-----------------------------------|--|
| | SPARSE_OPERATION_TR | Transpose, $\text{op}(A) = A^T$. |
| | ANSPOSE | |
| | SPARSE_OPERATION_CO | Conjugate transpose, $\text{op}(A) = A^H$. |
| | NJUGATE_TRANSPOSE | |
| <i>alpha</i> | | Specifies the scalar <i>alpha</i> . |
| <i>A</i> | | Handle containing sparse matrix in internal data structure. |
| <i>descr</i> | | Structure specifying sparse matrix properties. |
| | <code>sparse_matrix_type_t</code> | <i>type</i> - Specifies the type of a sparse matrix: |
| | SPARSE_MATRIX_TYPE_ | The matrix is processed as is. |
| | GENERAL | |
| | SPARSE_MATRIX_TYPE_ | The matrix is symmetric (only the requested |
| | SYMMETRIC | triangle is processed). |
| | SPARSE_MATRIX_TYPE_ | The matrix is Hermitian (only the requested |
| | HERMITIAN | triangle is processed). |
| | SPARSE_MATRIX_TYPE_ | The matrix is triangular (only the requested |
| | TRIANGULAR | triangle is processed). |
| | SPARSE_MATRIX_TYPE_ | The matrix is diagonal (only diagonal elements |
| | DIAGONAL | are processed). |
| | SPARSE_MATRIX_TYPE_ | The matrix is block-triangular (only requested |
| | BLOCK_TRIANGULAR | triangle is processed). (Applies to BSR format |
| | | only.) |
| | SPARSE_MATRIX_TYPE_ | The matrix is block-diagonal (only diagonal |
| | BLOCK_DIAGONAL | blocks are processed. (Applies to BSR format |
| | | only.) |
| | <code>sparse_fill_mode_t</code> | <i>mode</i> - Specifies the triangular matrix part for |
| | | symmetric, Hermitian, triangular, and block-triangular matrices: |
| | SPARSE_FILL_MODE_LO | The lower triangular matrix part is processed. |
| | WER | |
| | SPARSE_FILL_MODE_UP | The upper triangular matrix part is processed. |
| | PER | |
| | <code>sparse_diag_type_t</code> | <i>diag</i> - Specifies diagonal type for non-general |
| | | matrices: |
| | SPARSE_DIAG_NON_UNI | Diagonal elements might not be equal to one. |
| | T | |
| | SPARSE_DIAG_UNIT | Diagonal elements are equal to one. |
| <i>layout</i> | | Describes the storage scheme for the dense matrix: |
| | SPARSE_LAYOUT_COLUM | Storage of elements uses column major layout. |
| | N_MAJOR | |

SPARSE_LAYOUT_ROW_M Storage of elements uses row major layout.
AJOR

x Array of size at least *rows*cols*.

| | | |
|--|---|--|
| | <i>layout =</i> SPARSE_LAYOUT_COLUMN_MAJOR | <i>layout =</i> SPARSE_LAYOUT_ROW_MAJOR |
| <i>rows</i> (number of rows in <i>x</i>) | <i>ldx</i> | number of rows in A |
| <i>cols</i> (number of columns in <i>x</i>) | <i>columns</i> | <i>ldx</i> |

On entry, the array *x* must contain the matrix *x*.

columns Number of columns in matrix *y*.

ldx Specifies the leading dimension of matrix *x*.

y Array of size at least *rows*cols*, where

| | | |
|--|---|--|
| | <i>layout =</i> SPARSE_LAYOUT_COLUMN_MAJOR | <i>layout =</i> SPARSE_LAYOUT_ROW_MAJOR |
| <i>rows</i> (number of rows in <i>y</i>) | <i>ldy</i> | number of rows in A |
| <i>cols</i> (number of columns in <i>y</i>) | <i>columns</i> | <i>ldy</i> |

Output Parameters

y Overwritten by the updated matrix *y*.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|------------------------------|--|
| SPARSE_STATUS_SUCCESS | The operation was successful. |
| SPARSE_STATUS_NOT_INITIALIZE | The routine encountered an empty handle or matrix array. |
| D | |
| SPARSE_STATUS_ALLOC_FAILED | Internal memory allocation failed. |
| SPARSE_STATUS_INVALID_VALUE | The input parameters contain an invalid value. |
| SPARSE_STATUS_EXECUTION_FAIL | Execution failed. |
| ED | |
| SPARSE_STATUS_INTERNAL_ERROR | An error in algorithm implementation occurred. |
| SPARSE_STATUS_NOT_SUPPORTED | The requested operation is not supported. |

mkl_sparse_?_add

Computes sum of two sparse matrices.

Syntax

```
sparse_status_t mkl_sparse_s_add (sparse_operation_t operation, const sparse_matrix_t
A, float alpha, const sparse_matrix_t B, sparse_matrix_t *C);
```

```
sparse_status_t mkl_sparse_d_add (sparse_operation_t operation, const sparse_matrix_t
A, double alpha, const sparse_matrix_t B, sparse_matrix_t *C);
```

```
sparse_status_t mkl_sparse_c_add (sparse_operation_t operation, const sparse_matrix_t
A, MKL_Complex8 alpha, const sparse_matrix_t B, sparse_matrix_t *C);
```

```
sparse_status_t mkl_sparse_z_add (sparse_operation_t operation, const sparse_matrix_t
A, MKL_Complex16 alpha, const sparse_matrix_t B, sparse_matrix_t *C);
```

Include Files

- mkl_spblas.h

Description

The `mkl_sparse_?_add` routine performs a matrix-matrix operation:

```
 $C := \alpha * \text{op}(A) + B$ 
```

where *alpha* is a scalar and *A*, *B*, and *C* are sparse matrices.

Input Parameters

A Handle containing a sparse matrix in internal data structure.

alpha Specifies the scalar *alpha*.

operation Specifies operation `op()` on input matrix.

SPARSE_OPERATION_NO Non-transpose, $\text{op}(A) = A$.
N_TRANSPOSE

SPARSE_OPERATION_TR Transpose, $\text{op}(A) = A^T$.
ANSPOSE

SPARSE_OPERATION_CO Conjugate transpose, $\text{op}(A) = A^H$.
NJUGATE_TRANSPOSE

B Handle containing a sparse matrix in internal data structure.

Output Parameters

C Handle containing the resulting sparse matrix in internal data structure.

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

SPARSE_STATUS_SUCCESS The operation was successful.

SPARSE_STATUS_NOT_INITIALIZE The routine encountered an empty handle or matrix array.
D

| | |
|--|--|
| <code>SPARSE_STATUS_ALLOC_FAILED</code> | Internal memory allocation failed. |
| <code>SPARSE_STATUS_INVALID_VALUE</code> | The input parameters contain an invalid value. |
| <code>SPARSE_STATUS_EXECUTION_FAIL</code> <code>ED</code> | Execution failed. |
| <code>SPARSE_STATUS_INTERNAL_ERROR</code> | An error in algorithm implementation occurred. |
| <code>SPARSE_STATUS_NOT_SUPPORTED</code> | The requested operation is not supported. |

mkl_sparse_spm

Computes the product of two sparse matrices and stores the result as a sparse matrix.

Syntax

```
sparse_status_t mkl_sparse_spm (sparse_operation_t operation, const sparse_matrix_t A,  
const sparse_matrix_t B, sparse_matrix_t *C);
```

Include Files

- `mkl_splblas.h`

Description

The `mkl_sparse_spm` routine performs a matrix-matrix operation:

```
C := op(A) *B
```

where *A*, *B*, and *C* are sparse matrices.

Input Parameters

| | |
|------------------|---|
| <i>operation</i> | Specifies operation <code>op()</code> on input matrix. <code>SPARSE_OPERATION_NO</code> Non-transpose, $op(A) = A$. <code>N_TRANSPOSE</code> <code>SPARSE_OPERATION_TR</code> Transpose, $op(A) = A^T$. <code>ANSPOSE</code> <code>SPARSE_OPERATION_CO</code> Conjugate transpose, $op(A) = A^H$. <code>NJUGATE_TRANSPOSE</code> |
| <i>A</i> | Handle containing a sparse matrix in internal data structure. |
| <i>B</i> | Handle containing a sparse matrix in internal data structure. |

Output Parameters

| | |
|----------|---|
| <i>C</i> | Handle containing the resulting sparse matrix in internal data structure. |
|----------|---|

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|------------------------------------|-------------------------------|
| <code>SPARSE_STATUS_SUCCESS</code> | The operation was successful. |
|------------------------------------|-------------------------------|

SPARSE_STATUS_NOT_INITIALIZE The routine encountered an empty handle or matrix array.
D

SPARSE_STATUS_ALLOC_FAILED Internal memory allocation failed.

SPARSE_STATUS_INVALID_VALUE The input parameters contain an invalid value.

SPARSE_STATUS_EXECUTION_FAIL Execution failed.
ED

SPARSE_STATUS_INTERNAL_ERROR An error in algorithm implementation occurred.

SPARSE_STATUS_NOT_SUPPORTED The requested operation is not supported.

mkl_sparse_?_spmmd

Computes the product of two sparse matrices and stores the result as a dense matrix.

Syntax

```
sparse_status_t mkl_sparse_s_spmmd (sparse_operation_t operation, const sparse_matrix_t
A, const sparse_matrix_t B, sparse_layout_t layout, float *C, MKL_INT ldc);

sparse_status_t mkl_sparse_d_spmmd (sparse_operation_t operation, const sparse_matrix_t
A, const sparse_matrix_t B, sparse_layout_t layout, double *C, MKL_INT ldc);

sparse_status_t mkl_sparse_c_spmmd (sparse_operation_t operation, const sparse_matrix_t
A, const sparse_matrix_t B, sparse_layout_t layout, MKL_Complex8 *C, MKL_INT ldc);

sparse_status_t mkl_sparse_z_spmmd (sparse_operation_t operation, const sparse_matrix_t
A, const sparse_matrix_t B, sparse_layout_t layout, MKL_Complex16 *C, MKL_INT ldc);
```

Include Files

- mkl_spblas.h

Description

The `mkl_sparse_?_spmmd` routine performs a matrix-matrix operation:

```
C := op(A)*B
```

where A and B are sparse matrices and C is a dense matrix.

Input Parameters

| | |
|------------------|--|
| <i>operation</i> | Specifies operation <code>op()</code> on input matrix. SPARSE_OPERATION_NO Non-transpose, $\text{op}(A) = A$. N_TRANSPOSE SPARSE_OPERATION_TR Transpose, $\text{op}(A) = A^T$. ANSPOSE SPARSE_OPERATION_CO Conjugate transpose, $\text{op}(A) = A^H$. NJUGATE_TRANSPOSE |
| <i>A</i> | Handle containing a sparse matrix in internal data structure. |
| <i>B</i> | Handle containing a sparse matrix in internal data structure. |

| | |
|---------------|--|
| <i>layout</i> | Describes the storage scheme for the dense matrix: |
| | SPARSE_LAYOUT_COLUMN_MAJOR Storage of elements uses column major layout. |
| | SPARSE_LAYOUT_ROW_MAJOR Storage of elements uses row major layout. |
| <i>ldC</i> | Leading dimension of matrix C. |

Output Parameters

| | |
|----------|---|
| <i>C</i> | Resulting dense matrix stored in internal data structure. |
|----------|---|

Return Values

The function returns a value indicating whether the operation was successful or not, and why.

| | |
|--------------------------------|--|
| SPARSE_STATUS_SUCCESS | The operation was successful. |
| SPARSE_STATUS_NOT_INITIALIZE | The routine encountered an empty handle or matrix array. |
| D | |
| SPARSE_STATUS_ALLOC_FAILED | Internal memory allocation failed. |
| SPARSE_STATUS_INVALID_VALUE | The input parameters contain an invalid value. |
| SPARSE_STATUS_EXECUTION_FAILED | Execution failed. |
| ED | |
| SPARSE_STATUS_INTERNAL_ERROR | An error in algorithm implementation occurred. |
| SPARSE_STATUS_NOT_SUPPORTED | The requested operation is not supported. |

BLAS-like Extensions

Intel MKL provides C and Fortran routines to extend the functionality of the BLAS routines. These include routines to compute vector products, matrix-vector products, and matrix-matrix products.

Intel MKL also provides routines to perform certain data manipulation, including matrix in-place and out-of-place transposition operations combined with simple matrix arithmetic operations. Transposition operations are Copy As Is, Conjugate transpose, Transpose, and Conjugate. Each routine adds the possibility of scaling during the transposition operation by giving some *alpha* and/or *beta* parameters. Each routine supports both row-major orderings and column-major orderings.

Table “BLAS-like Extensions” lists these routines.

The <?> symbol in the routine short names is a precision prefix that indicates the data type:

| | |
|----------|---------------|
| <i>s</i> | float |
| <i>d</i> | double |
| <i>c</i> | MKL_Complex8 |
| <i>z</i> | MKL_Complex16 |

BLAS-like Extensions

| Routine | Data Types | Description |
|---------------------------------------|-------------------|---|
| <code>cblas_?axpby</code> | s, d, c, z | Scales two vectors, adds them to one another and stores result in the vector (routines). |
| <code>cblas_?gemmt</code> | s, d, c, z | Computes a matrix-matrix product with general matrices but updates only the upper or lower triangular part of the result matrix. |
| <code>cblas_?gemm3m</code> | c, z | Computes a scalar-matrix-matrix product using matrix multiplications and adds the result to a scalar-matrix product. |
| <code>cblas_?gemm_batch</code> | s, d, c, z | Computes scalar-matrix-matrix products and adds the results to scalar matrix products for groups of general matrices. |
| <code>cblas_? gemm3m_batch</code> | c, z | Computes a scalar-matrix-matrix product using matrix multiplications and adds the result to a scalar-matrix product. |
| <code>mkl_?imatcopy</code> | s, d, c, z | Performs scaling and in-place transposition/copying of matrices. |
| <code>mkl_?omatcopy</code> | s, d, c, z | Performs scaling and out-of-place transposition/copying of matrices. |
| <code>mkl_?omatcopy2</code> | s, d, c, z | Performs two-strided scaling and out-of-place transposition/copying of matrices. |
| <code>mkl_?omatadd</code> | s, d, c, z | Performs scaling and sum of two matrices including their out-of-place transposition/copying. |
| <code>cblas_?gemm_alloc</code> | s, d | Allocates storage for a packed matrix. |
| <code>?cblas_gemm_pack</code> | s, d | Performs scaling and packing of the matrix into the previously allocated buffer. |
| <code>cblas_? gemm_compute</code> | s, d | Computes a matrix-matrix product with general matrices where one or both input matrices are stored in a packed data structure and adds the result to a scalar-matrix product. |
| <code>cblas_?gemm_free</code> | s, d | Frees the storage previously allocated for the packed matrix. |

`cblas_?axpby`

Scales two vectors, adds them to one another and stores result in the vector.

Syntax

```
void cblas_saxpby (const MKL_INT n, const float a, const float *x, const MKL_INT incx,
const float b, float *y, const MKL_INT incy);
```

```
void cblas_daxpby (const MKL_INT n, const double a, const double *x, const MKL_INT
incx, const double b, double *y, const MKL_INT incy);
```

```
void cblas_caxpby (const MKL_INT n, const void *a, const void *x, const MKL_INT incx,
const void *b, void *y, const MKL_INT incy);
```

```
void cblas_zaxpby (const MKL_INT n, const void *a, const void *x, const MKL_INT incx,  
const void *b, void *y, const MKL_INT incy);
```

Include Files

- mkl.h

Description

The `?axpby` routines perform a vector-vector operation defined as

```
 $y := a*x + b*y$ 
```

where:

a and b are scalars

x and y are vectors each with n elements.

Input Parameters

| | |
|--------|---|
| n | Specifies the number of elements in vectors x and y . |
| a | Specifies the scalar a . |
| x | Array, size at least $(1 + (n-1)*abs(incx))$. |
| $incx$ | Specifies the increment for the elements of x . |
| b | Specifies the scalar b . |
| y | Array, size at least $(1 + (n-1)*abs(incy))$. |
| $incy$ | Specifies the increment for the elements of y . |

Output Parameters

| | |
|-----|-----------------------------------|
| y | Contains the updated vector y . |
|-----|-----------------------------------|

Example

For examples of routine usage, see the code in the Intel MKL installation directory:

- `cblas_saxpby`: `examples\cblas\source\cblas_saxpbyx.c`
- `cblas_daxpby`: `examples\cblas\source\cblas_daxpbyx.c`
- `cblas_caxpby`: `examples\cblas\source\cblas_caxpbyx.c`
- `cblas_zaxpby`: `examples\cblas\source\cblas_zaxpbyx.c`

`cblas_?gemmt`

Computes a matrix-matrix product with general matrices but updates only the upper or lower triangular part of the result matrix.

Syntax

```
void cblas_sgemmt (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const  
CBLAS_TRANSPOSE transa, const CBLAS_TRANSPOSE transb, const MKL_INT n, const MKL_INT k,  
const float alpha, const float *a, const MKL_INT lda, const float *b, const MKL_INT  
ldb, const float beta, float *c, const MKL_INT ldc);
```

```

void cblas_dgemmt (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE transa, const CBLAS_TRANSPOSE transb, const MKL_INT n, const MKL_INT k,
const double alpha, const double *a, const MKL_INT lda, const double *b, const MKL_INT
ldb, const double beta, double *c, const MKL_INT ldc);

void cblas_cgemmt (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE transa, const CBLAS_TRANSPOSE transb, const MKL_INT n, const MKL_INT k,
const void *alpha, const void *a, const MKL_INT lda, const void *b, const MKL_INT ldb,
const void *beta, void *c, const MKL_INT ldc);

void cblas_zgemmt (const CBLAS_LAYOUT Layout, const CBLAS_UPLO uplo, const
CBLAS_TRANSPOSE transa, const CBLAS_TRANSPOSE transb, const MKL_INT n, const MKL_INT k,
const void *alpha, const void *a, const MKL_INT lda, const void *b, const MKL_INT ldb,
const void *beta, void *c, const MKL_INT ldc);

```

Include Files

- mkl.h

Description

The `?gemmt` routines compute a scalar-matrix-matrix product with general matrices and add the result to the upper or lower part of a scalar-matrix product. These routines are similar to the `?gemm` routines, but they only access and update a triangular part of the square result matrix (see Application Notes below).

The operation is defined as

$$C := \alpha * \text{op}(A) * \text{op}(B) + \beta * C,$$

where:

$\text{op}(X)$ is one of $\text{op}(X) = X$, or $\text{op}(X) = X^T$, or $\text{op}(X) = X^H$,

α and β are scalars,

A , B and C are matrices:

$\text{op}(A)$ is an n -by- k matrix,

$\text{op}(B)$ is a k -by- n matrix,

C is an n -by- n upper or lower triangular matrix.

Input Parameters

| | |
|---------------|---|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array c is used. If $uplo = 'U'$ or $'u'$, then the upper triangular part of the array c is used. If $uplo = 'L'$ or $'l'$, then the lower triangular part of the array c is used. |
| <i>transa</i> | Specifies the form of $\text{op}(A)$ used in the matrix multiplication: if $transa = 'N'$ or $'n'$, then $\text{op}(A) = A$; if $transa = 'T'$ or $'t'$, then $\text{op}(A) = A^T$; if $transa = 'C'$ or $'c'$, then $\text{op}(A) = A^H$. |
| <i>transb</i> | Specifies the form of $\text{op}(B)$ used in the matrix multiplication: if $transb = 'N'$ or $'n'$, then $\text{op}(B) = B$; |

if $transb = 'T'$ or $'t'$, then $op(B) = B^T$;

if $transb = 'C'$ or $'c'$, then $op(B) = B^H$.

| | |
|---------|--|
| n | Specifies the order of the matrix C . The value of n must be at least zero. |
| k | Specifies the number of columns of the matrix $op(A)$ and the number of rows of the matrix $op(B)$. The value of k must be at least zero. |
| $alpha$ | Specifies the scalar $alpha$. |
| a | Array, size lda by ka , where ka is k when $transa = 'N'$ or $'n'$, and is n otherwise. Before entry with $transa = 'N'$ or $'n'$, the leading n -by- k part of the array a must contain the matrix A , otherwise the leading k -by- n part of the array a must contain the matrix A . |
| lda | Specifies the leading dimension of a as declared in the calling (sub)program. When $transa = 'N'$ or $'n'$, then lda must be at least $\max(1, n)$, otherwise lda must be at least $\max(1, k)$. |
| b | Array, size ldb by kb , where kb is n when $transb = 'N'$ or $'n'$, and is k otherwise. Before entry with $transb = 'N'$ or $'n'$, the leading k -by- n part of the array b must contain the matrix B , otherwise the leading n -by- k part of the array b must contain the matrix B . |
| ldb | Specifies the leading dimension of b as declared in the calling (sub)program. When $transb = 'N'$ or $'n'$, then ldb must be at least $\max(1, k)$, otherwise ldb must be at least $\max(1, n)$. |
| $beta$ | Specifies the scalar $beta$. When $beta$ is equal to zero, then c need not be set on input. |
| c | Array, size ldc by n . Before entry with $uplo = 'U'$ or $'u'$, the leading n -by- n upper triangular part of the array c must contain the upper triangular part of the matrix C and the strictly lower triangular part of c is not referenced. Before entry with $uplo = 'L'$ or $'l'$, the leading n -by- n lower triangular part of the array c must contain the lower triangular part of the matrix C and the strictly upper triangular part of c is not referenced. When $beta$ is equal to zero, then c need not be set on input. |
| ldc | Specifies the leading dimension of c as declared in the calling (sub)program. The value of ldc must be at least $\max(1, n)$. |

Output Parameters

| | |
|-----|--|
| c | When $uplo = 'U'$ or $'u'$, the upper triangular part of the array c is overwritten by the upper triangular part of the updated matrix. When $uplo = 'L'$ or $'l'$, the lower triangular part of the array c is overwritten by the lower triangular part of the updated matrix. |
|-----|--|

Application Notes

These routines only access and update the upper or lower triangular part of the result matrix. This can be useful when the result is known to be symmetric; for example, when computing a product of the form $C := \alpha * B * S * B^T + \beta * C$, where S and C are symmetric matrices and B is a general matrix. In this case, first compute $A := B * S$ (which can be done using the corresponding `?symm` routine), then compute $C := \alpha * A * B^T + \beta * C$ using the `?gemmt` routine.

cblas_?gemm3m

Computes a scalar-matrix-matrix product using matrix multiplications and adds the result to a scalar-matrix product.

Syntax

```
void cblas_cgemm3m (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE transa, const
CBLAS_TRANSPOSE transb, const MKL_INT m, const MKL_INT n, const MKL_INT k, const void
*alpha, const void *a, const MKL_INT lda, const void *b, const MKL_INT ldb, const void
*beta, void *c, const MKL_INT ldc);
```

```
void cblas_zgemm3m (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE transa, const
CBLAS_TRANSPOSE transb, const MKL_INT m, const MKL_INT n, const MKL_INT k, const void
*alpha, const void *a, const MKL_INT lda, const void *b, const MKL_INT ldb, const void
*beta, void *c, const MKL_INT ldc);
```

Include Files

- mkl.h

Description

The `?gemm3m` routines perform a matrix-matrix operation with general complex matrices. These routines are similar to the `?gemm` routines, but they use fewer matrix multiplication operations (see *Application Notes* below).

The operation is defined as

$$C := \alpha * \text{op}(A) * \text{op}(B) + \beta * C,$$

where:

$\text{op}(x)$ is one of $\text{op}(x) = x$, or $\text{op}(x) = x'$, or $\text{op}(x) = \text{conjg}(x')$,

α and β are scalars,

A , B and C are matrices:

$\text{op}(A)$ is an m -by- k matrix,

$\text{op}(B)$ is a k -by- n matrix,

C is an m -by- n matrix.

Input Parameters

Layout Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor).

transa Specifies the form of $\text{op}(A)$ used in the matrix multiplication:
if $\text{transa} = \text{CblasNoTrans}$, then $\text{op}(A) = A$;

if *transa*=CblasTrans, then $\text{op}(A) = A'$;
 if *transa*=CblasConjTrans, then $\text{op}(A) = \text{conjg}(A')$.

transb

Specifies the form of $\text{op}(B)$ used in the matrix multiplication:

if *transb*=CblasNoTrans, then $\text{op}(B) = B$;
 if *transb*=CblasTrans, then $\text{op}(B) = B'$;
 if *transb*=CblasConjTrans, then $\text{op}(B) = \text{conjg}(B')$.

m

Specifies the number of rows of the matrix $\text{op}(A)$ and of the matrix *C*. The value of *m* must be at least zero.

n

Specifies the number of columns of the matrix $\text{op}(B)$ and the number of columns of the matrix *C*.

The value of *n* must be at least zero.

k

Specifies the number of columns of the matrix $\text{op}(A)$ and the number of rows of the matrix $\text{op}(B)$.

The value of *k* must be at least zero.

alpha

Specifies the scalar *alpha*.

a

| | <i>transa</i> =CblasNoTrans | <i>transa</i> =CblasTrans or <i>transa</i> =CblasConjTrans |
|----------------------------------|---|---|
| <i>Layout</i> = CblasColMajor | Array, size <i>lda</i> * <i>k</i> . Before entry, the leading <i>m</i> -by- <i>k</i> part of the array <i>a</i> must contain the matrix <i>A</i> . | Array, size <i>lda</i> * <i>m</i> . Before entry, the leading <i>k</i> -by- <i>m</i> part of the array <i>a</i> must contain the matrix <i>A</i> . |
| <i>Layout</i> = CblasRowMajor | Array, size <i>lda</i> * <i>m</i> . Before entry, the leading <i>k</i> -by- <i>m</i> part of the array <i>a</i> must contain the matrix <i>A</i> . | Array, size <i>lda</i> * <i>k</i> . Before entry, the leading <i>m</i> -by- <i>k</i> part of the array <i>a</i> must contain the matrix <i>A</i> . |

lda

Specifies the leading dimension of *a* as declared in the calling (sub)program.

| | <i>transa</i> =CblasNoTrans | <i>transa</i> =CblasTrans or <i>transa</i> =CblasConjTrans |
|----------------------------------|--|---|
| <i>Layout</i> = CblasColMajor | <i>lda</i> must be at least $\max(1, m)$. | <i>lda</i> must be at least $\max(1, k)$ |
| <i>Layout</i> = CblasRowMajor | <i>lda</i> must be at least $\max(1, k)$ | <i>lda</i> must be at least $\max(1, m)$. |

b

| | | |
|----------------------------------|---|--|
| | <i>transb</i> =CblasNoTrans | <i>transb</i> =CblasTrans or <i>transb</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | Array, size <i>ldb</i> by <i>n</i> . Before entry, the leading <i>k</i> -by- <i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i> . | Array, size <i>ldb</i> by <i>k</i> . Before entry the leading <i>n</i> -by- <i>k</i> part of the array <i>b</i> must contain the matrix <i>B</i> . |
| <i>Layout</i> = CblasRowMajor | Array, size <i>ldb</i> by <i>k</i> . Before entry the leading <i>n</i> -by- <i>k</i> part of the array <i>b</i> must contain the matrix <i>B</i> . | Array, size <i>ldb</i> by <i>n</i> . Before entry, the leading <i>k</i> -by- <i>n</i> part of the array <i>b</i> must contain the matrix <i>B</i> . |

ldb

Specifies the leading dimension of *b* as declared in the calling (sub)program.

| | | |
|----------------------------------|---|---|
| | <i>transb</i> =CblasNoTrans | <i>transb</i> =CblasTrans or <i>transb</i> =CblasConjTrans |
| <i>Layout</i> = CblasColMajor | <i>ldb</i> must be at least $\max(1, k)$. | <i>ldb</i> must be at least $\max(1, n)$. |
| <i>Layout</i> = CblasRowMajor | <i>ldb</i> must be at least $\max(1, n)$. | <i>ldb</i> must be at least $\max(1, k)$. |

beta

Specifies the scalar *beta*.

When *beta* is equal to zero, then *c* need not be set on input.

c

| | |
|----------------------------------|--|
| <i>Layout</i> = CblasColMajor | Array, size <i>ldc</i> by <i>n</i> . Before entry, the leading <i>m</i> -by- <i>n</i> part of the array <i>c</i> must contain the matrix <i>C</i> , except when <i>beta</i> is equal to zero, in which case <i>c</i> need not be set on entry. |
| <i>Layout</i> = CblasRowMajor | Array, size <i>ldc</i> by <i>m</i> . Before entry, the leading <i>n</i> -by- <i>m</i> part of the array <i>c</i> must contain the matrix <i>C</i> , except when <i>beta</i> is equal to zero, in which case <i>c</i> need not be set on entry. |

ldc

Specifies the leading dimension of *c* as declared in the calling (sub)program.

| | |
|-------------------------------|--|
| <i>Layout</i> = CblasColMajor | <i>ldc</i> must be at least $\max(1, m)$. |
| <i>Layout</i> = CblasRowMajor | <i>ldc</i> must be at least $\max(1, n)$. |

Output Parameters

c

Overwritten by the *m*-by-*n* matrix $(\alpha * \text{op}(A) * \text{op}(B) + \beta * C)$.

Application Notes

These routines perform a complex matrix multiplication by forming the real and imaginary parts of the input matrices. This uses three real matrix multiplications and five real matrix additions instead of the conventional four real matrix multiplications and two real matrix additions. The use of three real matrix multiplications reduces the time spent in matrix operations by 25%, resulting in significant savings in compute time for large matrices.

If the errors in the floating point calculations satisfy the following conditions:

$$fl(x \text{ op } y) = (x \text{ op } y) (1 + \delta), |\delta| \leq u, \text{ op} = *, /, fl(x \pm y) = x(1 + \alpha) \pm y(1 + \beta), |\alpha|, |\beta| \leq u$$

then for an n -by- n matrix $\hat{C} = fl(C_1 + iC_2) = fl((A_1 + iA_2)(B_1 + iB_2)) = \hat{C}_1 + i\hat{C}_2$, the following bounds are satisfied:

$$\|\hat{C}_1 - C_1\| \leq 2(n+1)u \|A\|_\infty \|B\|_\infty + O(u^2),$$

$$\|\hat{C}_2 - C_2\| \leq 4(n+4)u \|A\|_\infty \|B\|_\infty + O(u^2),$$

where $\|A\|_\infty = \max(\|A_1\|_\infty, \|A_2\|_\infty)$, and $\|B\|_\infty = \max(\|B_1\|_\infty, \|B_2\|_\infty)$.

Thus the corresponding matrix multiplications are stable.

cblas_?gemm_batch

Computes scalar-matrix-matrix products and adds the results to scalar matrix products for groups of general matrices.

Syntax

```
void cblas_sgemm_batch (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE* transa_array,
const CBLAS_TRANSPOSE* transb_array, const MKL_INT* m_array, const MKL_INT* n_array,
const MKL_INT* k_array, const float* alpha_array, const float **a_array, const MKL_INT*
lda_array, const float **b_array, const MKL_INT* ldb_array, const float* beta_array,
float **c_array, const MKL_INT* ldc_array, const MKL_INT group_count, const MKL_INT*
group_size);
```

```
void cblas_dgemm_batch (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE* transa_array,
const CBLAS_TRANSPOSE* transb_array, const MKL_INT* m_array, const MKL_INT* n_array,
const MKL_INT* k_array, const double* alpha_array, const double **a_array, const
MKL_INT* lda_array, const double **b_array, const MKL_INT* ldb_array, const double*
beta_array, double **c_array, const MKL_INT* ldc_array, const MKL_INT group_count,
const MKL_INT* group_size);
```

```
void cblas_cgemm_batch (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE* transa_array,
const CBLAS_TRANSPOSE* transb_array, const MKL_INT* m_array, const MKL_INT* n_array,
const MKL_INT* k_array, const void *alpha_array, const void **a_array, const MKL_INT*
lda_array, const void **b_array, const MKL_INT* ldb_array, const void *beta_array, void
**c_array, const MKL_INT* ldc_array, const MKL_INT group_count, const MKL_INT*
group_size);
```

```
void cblas_zgemm_batch (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE* transa_array,
const CBLAS_TRANSPOSE* transb_array, const MKL_INT* m_array, const MKL_INT* n_array,
const MKL_INT* k_array, const void *alpha_array, const void **a_array, const MKL_INT*
lda_array, const void **b_array, const MKL_INT* ldb_array, const void *beta_array, void
**c_array, const MKL_INT* ldc_array, const MKL_INT group_count, const MKL_INT*
group_size);
```

Include Files

- `mkl.h`

Description

The `?gemm_batch` routines perform a series of matrix-matrix operations with general matrices. They are similar to the `?gemm` routine counterparts, but the `?gemm_batch` routines perform matrix-matrix operations with groups of matrices, processing a number of groups at once. The groups contain matrices with the same parameters.

The operation is defined as

```
idx = 0
for i = 0..group_count - 1
  alpha and beta in alpha_array[i] and beta_array[i]
  for j = 0..group_size[i] - 1
    A, B, and C matrix in a_array[idx], b_array[idx], and c_array[idx]
    C := alpha*op(A)*op(B) + beta*C,
    idx = idx + 1
  end for
end for
```

where:

$\text{op}(X)$ is one of $\text{op}(X) = X$, or $\text{op}(X) = X^T$, or $\text{op}(X) = X^H$,

alpha and *beta* are scalar elements of *alpha_array* and *beta_array*,

A, *B* and *C* are matrices such that for *m*, *n*, and *k* which are elements of *m_array*, *n_array*, and *k_array*:

$\text{op}(A)$ is an *m*-by-*k* matrix,

$\text{op}(B)$ is a *k*-by-*n* matrix,

C is an *m*-by-*n* matrix.

A, *B*, and *C* represent matrices stored at addresses pointed to by *a_array*, *b_array*, and *c_array*, respectively. The number of entries in *a_array*, *b_array*, and *c_array* is *total_batch_count* = the sum of all of the *group_size* entries.

See also [gemm](#) for a detailed description of multiplication for general matrices and [?gemm3m_batch](#), BLAS-like extension routines for similar matrix-matrix operations.

NOTE

Error checking is not performed for Intel MKL Windows* single dynamic libraries for the `?gemm_batch` routines.

Input Parameters

| | |
|---------------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>transa_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $\text{transa}_i = \text{transa_array}[i]$ specifies the form of $\text{op}(A)$ used in the matrix multiplication:</p> <p>if $\text{transa}_i = \text{CblasNoTrans}$, then $\text{op}(A) = A$;</p> <p>if $\text{transa}_i = \text{CblasTrans}$, then $\text{op}(A) = A^T$;</p> <p>if $\text{transa}_i = \text{CblasConjTrans}$, then $\text{op}(A) = A^H$.</p> |

| | |
|---------------------|--|
| <i>transb_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $transb_i = transb_array[i]$ specifies the form of $op(B_i)$ used in the matrix multiplication:</p> <p>if $transb_i = CblasNoTrans$, then $op(B) = B$;</p> <p>if $transb_i = CblasTrans$, then $op(B) = B^T$;</p> <p>if $transb_i = CblasConjTrans$, then $op(B) = B^H$.</p> |
| <i>m_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $m_i = m_array[i]$ specifies the number of rows of the matrix $op(A)$ and of the matrix <i>C</i>.</p> <p>The value of each element of <i>m_array</i> must be at least zero.</p> |
| <i>n_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $n_i = n_array[i]$ specifies the number of columns of the matrix $op(B)$ and the number of columns of the matrix <i>C</i>.</p> <p>The value of each element of <i>n_array</i> must be at least zero.</p> |
| <i>k_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $k_i = k_array[i]$ specifies the number of columns of the matrix $op(A)$ and the number of rows of the matrix $op(B)$.</p> <p>The value of each element of <i>k_array</i> must be at least zero.</p> |
| <i>alpha_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $alpha_array[i]$ specifies the scalar $alpha_i$.</p> |
| <i>a_array</i> | <p>Array, size <i>total_batch_count</i>, of pointers to arrays used to store <i>A</i> matrices.</p> |
| <i>lda_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $lda_i = lda_array[i]$ specifies the leading dimension of the array storing matrix <i>A</i> as declared in the calling (sub)program.</p> |

| | | |
|-----------------|---|---|
| | <i>transa_i</i> =CblasNoTrans | <i>transa_i</i> =CblasTrans or <i>transa_i</i> =CblasConjTrans |
| <i>Layout</i> = | <i>lda_i</i> must be at least CblasColMajor $\max(1, m_i)$. | <i>lda_i</i> must be at least $\max(1, k_i)$ |
| <i>Layout</i> = | <i>lda_i</i> must be at least CblasRowMajor $\max(1, k_i)$ | <i>lda_i</i> must be at least $\max(1, m_i)$. |

| | |
|------------------|---|
| <i>b_array</i> | <p>Array, size <i>total_batch_count</i>, of pointers to arrays used to store <i>B</i> matrices.</p> |
| <i>ldb_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $ldb_i = ldb_array[i]$ specifies the leading dimension of the array storing matrix <i>B</i> as declared in the calling (sub)program.</p> |

| | | |
|-----------------|---|---|
| | <i>transb_i</i> =CblasNoTrans | <i>transb_i</i> =CblasTrans or <i>transb_i</i> =CblasConjTrans |
| <i>Layout</i> = | <i>ldb_i</i> must be at least CblasColMajor $\max(1, k_i)$. | <i>ldb_i</i> must be at least $\max(1, n_i)$. |

| | | |
|-----------------|---|---|
| <i>Layout</i> = | <i>ldb_i</i> must be at least | <i>ldb_i</i> must be at least |
| CblasRowMajor | $\max(1, n_i)$. | $\max(1, k_i)$. |

| | |
|--------------------|---|
| <i>beta_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, <i>beta_array</i>[<i>i</i>] specifies the scalar <i>beta_i</i>.</p> <p>When <i>beta_i</i> is equal to zero, then <i>C</i> matrices in group <i>i</i> need not be set on input.</p> |
| <i>c_array</i> | <p>Array, size <i>total_batch_count</i>, of pointers to arrays used to store <i>C</i> matrices.</p> |
| <i>ldc_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, <i>ldc_i</i> = <i>ldc_array</i>[<i>i</i>] specifies the leading dimension of all arrays storing matrix <i>C</i> in group <i>i</i> as declared in the calling (sub)program.</p> <p>When <i>Layout</i> = CblasColMajor/<i>ldc_i</i> must be at least $\max(1, m_i)$.</p> <p>When <i>Layout</i> = CblasRowMajor/<i>ldc_i</i> must be at least $\max(1, n_i)$.</p> |
| <i>group_count</i> | <p>Specifies the number of groups. Must be at least 0.</p> |
| <i>group_size</i> | <p>Array of size <i>group_count</i>. The element <i>group_size</i>[<i>i</i>] specifies the number of matrices in group <i>i</i>. Each element in <i>group_size</i> must be at least 0.</p> |

Output Parameters

| | |
|----------------|---|
| <i>c_array</i> | <p>Overwritten by the <i>m_i</i>-by-<i>n_i</i> matrix ($\alpha_i \text{op}(A) \text{op}(B) + \beta_i C$) for group <i>i</i>.</p> |
|----------------|---|

cblas_?gemm3m_batch

Computes scalar-matrix-matrix products and adds the results to scalar matrix products for groups of general matrices.

Syntax

```
void cblas_cgemm3m_batch (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE*
transa_array, const CBLAS_TRANSPOSE* transb_array, const MKL_INT* m_array, const
MKL_INT* n_array, const MKL_INT* k_array, const void *alpha_array, const void
**a_array, const MKL_INT* lda_array, const void **b_array, const MKL_INT* ldb_array,
const void *beta_array, void **c_array, const MKL_INT* ldc_array, const MKL_INT
group_count, const MKL_INT* group_size);
```

```
void cblas_zgemm3m_batch (const CBLAS_LAYOUT Layout, const CBLAS_TRANSPOSE*
transa_array, const CBLAS_TRANSPOSE* transb_array, const MKL_INT* m_array, const
MKL_INT* n_array, const MKL_INT* k_array, const void *alpha_array, const void
**a_array, const MKL_INT* lda_array, const void **b_array, const MKL_INT* ldb_array,
const void *beta_array, void **c_array, const MKL_INT* ldc_array, const MKL_INT
group_count, const MKL_INT* group_size);
```

Include Files

- mkl.h

Description

The `?gemm3m_batch` routines perform a series of matrix-matrix operations with general matrices. They are similar to the `?gemm3m` routine counterparts, but the `?gemm3m_batch` routines perform matrix-matrix operations with groups of matrices, processing a number of groups at once. The groups contain matrices with the same parameters. The `?gemm3m_batch` routines use fewer matrix multiplications than the `?gemm_batch` routines, as described in the *Application Notes*.

The operation is defined as

```

idx = 0
for i = 0..group_count - 1
  alpha and beta in alpha_array[i] and beta_array[i]
  for j = 0..group_size[i] - 1
    A, B, and C matrix in a_array[idx], b_array[idx], and c_array[idx]
    C := alpha*op(A)*op(B) + beta*C,
    idx = idx + 1
  end for
end for
end for

```

where:

$\text{op}(X)$ is one of $\text{op}(X) = X$, or $\text{op}(X) = X^T$, or $\text{op}(X) = X^H$,

alpha and *beta* are scalar elements of *alpha_array* and *beta_array*,

A, *B* and *C* are matrices such that for *m*, *n*, and *k* which are elements of *m_array*, *n_array*, and *k_array*:

$\text{op}(A)$ is an *m*-by-*k* matrix,

$\text{op}(B)$ is a *k*-by-*n* matrix,

C is an *m*-by-*n* matrix.

A, *B*, and *C* represent matrices stored at addresses pointed to by *a_array*, *b_array*, and *c_array*, respectively. The number of entries in *a_array*, *b_array*, and *c_array* is *total_batch_count* = the sum of all the *group_size* entries.

See also [gemm](#) for a detailed description of multiplication for general matrices and [gemm_batch](#), BLAS-like extension routines for similar matrix-matrix operations.

NOTE

Error checking is not performed for Intel MKL Windows* single dynamic libraries for the `?gemm3m_batch` routines.

Input Parameters

| | |
|---------------------|--|
| <i>Layout</i> | Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor). |
| <i>transa_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $\text{transa}_i = \text{transa_array}[i]$ specifies the form of $\text{op}(A)$ used in the matrix multiplication:</p> <p>if $\text{transa}_i = \text{CblasNoTrans}$, then $\text{op}(A) = A$;</p> <p>if $\text{transa}_i = \text{CblasTrans}$, then $\text{op}(A) = A^T$;</p> <p>if $\text{transa}_i = \text{CblasConjTrans}$, then $\text{op}(A) = A^H$.</p> |
| <i>transb_array</i> | <p>Array of size <i>group_count</i>. For the group <i>i</i>, $\text{transb}_i = \text{transb_array}[i]$ specifies the form of $\text{op}(B_i)$ used in the matrix multiplication:</p> |

if $transb_i = \text{CblasNoTrans}$, then $op(B) = B$;
 if $transb_i = \text{CblasTrans}$, then $op(B) = B^T$;
 if $transb_i = \text{CblasConjTrans}$, then $op(B) = B^H$.

m_array Array of size *group_count*. For the group *i*, $m_i = m_array[i]$ specifies the number of rows of the matrix $op(A)$ and of the matrix *C*.

The value of each element of *m_array* must be at least zero.

n_array Array of size *group_count*. For the group *i*, $n_i = n_array[i]$ specifies the number of columns of the matrix $op(B)$ and the number of columns of the matrix *C*.

The value of each element of *n_array* must be at least zero.

k_array Array of size *group_count*. For the group *i*, $k_i = k_array[i]$ specifies the number of columns of the matrix $op(A)$ and the number of rows of the matrix $op(B)$.

The value of each element of *k_array* must be at least zero.

alpha_array Array of size *group_count*. For the group *i*, *alpha_array*[*i*] specifies the scalar α_i .

a_array Array, size *total_batch_count*, of pointers to arrays used to store *A* matrices.

lda_array Array of size *group_count*. For the group *i*, $lda_i = lda_array[i]$ specifies the leading dimension of the array storing matrix *A* as declared in the calling (sub)program.

| | | |
|-----------------|---|---|
| | $transa_i = \text{CblasNoTrans}$ | $transa_i = \text{CblasTrans}$ or $transa_i = \text{CblasConjTrans}$ |
| <i>Layout</i> = | <i>lda_i</i> must be at least $\max(1, m_i)$. | <i>lda_i</i> must be at least $\max(1, k_i)$ |
| <i>Layout</i> = | <i>lda_i</i> must be at least $\max(1, k_i)$ | <i>lda_i</i> must be at least $\max(1, m_i)$ |

b_array Array, size *total_batch_count*, of pointers to arrays used to store *B* matrices.

ldb_array Array of size *group_count*. For the group *i*, $ldb_i = ldb_array[i]$ specifies the leading dimension of the array storing matrix *B* as declared in the calling (sub)program.

| | | |
|-----------------|---|---|
| | $transb_i = \text{CblasNoTrans}$ | $transb_i = \text{CblasTrans}$ or $transb_i = \text{CblasConjTrans}$ |
| <i>Layout</i> = | <i>ldb_i</i> must be at least $\max(1, k_i)$. | <i>ldb_i</i> must be at least $\max(1, n_i)$. |

| | | |
|-----------------|---|---|
| <i>Layout</i> = | <i>ldb_i</i> must be at least | <i>ldb_i</i> must be at least |
| CblasRowMajor | $\max(1, n_i)$. | $\max(1, k_i)$. |

beta_array For the group *i*, *beta_array*[*i*] specifies the scalar *beta_i*.
When *beta_i* is equal to zero, then *C* matrices in group *i* need not be set on input.

c_array Array, size *total_batch_count*, of pointers to arrays used to store *C* matrices.

ldc_array Array of size *group_count*. For the group *i*, *ldc_i* = *ldc_array*[*i*] specifies the leading dimension of all arrays storing matrix *C* in group *i* as declared in the calling (sub)program.
When *Layout* = CblasColMajor *ldc_i* must be at least $\max(1, m_i)$.
When *Layout* = CblasRowMajor *ldc_i* must be at least $\max(1, n_i)$.

group_count Specifies the number of groups. Must be at least 0.

group_size Array of size *group_count*. The element *group_size*[*i*] specifies the number of matrices in group *i*. Each element in *group_size* must be at least 0.

Output Parameters

c_array Overwritten by the *m_i*-by-*n_i* matrix (*alpha_i**_{op}(*A*)*_{op}(*B*) + *beta_i***C*) for group *i*.

Application Notes

These routines perform a complex matrix multiplication by forming the real and imaginary parts of the input matrices. This uses three real matrix multiplications and five real matrix additions instead of the conventional four real matrix multiplications and two real matrix additions. The use of three real matrix multiplications reduces the time spent in matrix operations by 25%, resulting in significant savings in compute time for large matrices.

If the errors in the floating point calculations satisfy the following conditions:

$$fl(x \text{ op } y) = (x \text{ op } y) (1 + \delta), |\delta| \leq u, \text{ op} = *, /, fl(x \pm y) = x(1 + \alpha) \pm y(1 + \beta), |\alpha|, |\beta| \leq u$$

then for an *n*-by-*n* matrix $\hat{C} = fl(C_1 + iC_2) = fl((A_1 + iA_2)(B_1 + iB_2)) = \hat{C}_1 + i\hat{C}_2$, the following bounds are satisfied:

$$\|\hat{C}_1 - C_1\| \leq 2(n+1)u \|A\|_\infty \|B\|_\infty + O(u^2),$$

$$\|\hat{C}_2 - C_2\| \leq 4(n+4)u \|A\|_\infty \|B\|_\infty + O(u^2),$$

where $\|A\|_\infty = \max(\|A_1\|_\infty, \|A_2\|_\infty)$, and $\|B\|_\infty = \max(\|B_1\|_\infty, \|B_2\|_\infty)$.

Thus the corresponding matrix multiplications are stable.

mkl_?imatcopy

Performs scaling and in-place transposition/copying of matrices.

Syntax

```
void mkl_simatcopy (const char ordering, const char trans, size_t rows, size_t cols,
const float alpha, float * AB, size_t lda, size_t ldb);
```

```
void mkl_dimatcopy (const char ordering, const char trans, size_t rows, size_t cols,
const double alpha, double * AB, size_t lda, size_t ldb);
```

```
void mkl_cimatcopy (const char ordering, const char trans, size_t rows, size_t cols,
const MKL_Complex8 alpha, MKL_Complex8 * AB, size_t lda, size_t ldb);
```

```
void mkl_zimatcopy (const char ordering, const char trans, size_t rows, size_t cols,
const MKL_Complex16 alpha, MKL_Complex16 * AB, size_t lda, size_t ldb);
```

Include Files

- mkl.h

Description

The `mkl_?imatcopy` routine performs scaling and in-place transposition/copying of matrices. A transposition operation can be a normal matrix copy, a transposition, a conjugate transposition, or just a conjugation. The operation is defined as follows:

$$AB := \alpha * \text{op}(AB).$$

NOTE

Different arrays must not overlap.

Input Parameters

| | |
|-----------------|--|
| <i>ordering</i> | Ordering of the matrix storage. If <i>ordering</i> = 'R' or 'r', the ordering is row-major. If <i>ordering</i> = 'C' or 'c', the ordering is column-major. |
| <i>trans</i> | Parameter that specifies the operation type. If <i>trans</i> = 'N' or 'n', $\text{op}(AB) = AB$ and the matrix <i>AB</i> is assumed unchanged on input. If <i>trans</i> = 'T' or 't', it is assumed that <i>AB</i> should be transposed. If <i>trans</i> = 'C' or 'c', it is assumed that <i>AB</i> should be conjugate transposed. If <i>trans</i> = 'R' or 'r', it is assumed that <i>AB</i> should be only conjugated. If the data is real, then <i>trans</i> = 'R' is the same as <i>trans</i> = 'N', and <i>trans</i> = 'C' is the same as <i>trans</i> = 'T'. |
| <i>rows</i> | The number of rows in matrix <i>AB</i> before the transpose operation. |
| <i>cols</i> | The number of columns in matrix <i>AB</i> before the transpose operation. |
| <i>ab</i> | Array. |
| <i>alpha</i> | This parameter scales the input matrix by <i>alpha</i> . |

| | |
|------------|---|
| <i>lda</i> | <p>Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the source matrix; measured in the number of elements.</p> <p>This parameter must be at least <i>rows</i> if <i>ordering</i> = 'C' or 'c', and $\max(1, cols)$ otherwise.</p> |
| <i>ldb</i> | <p>Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the destination matrix; measured in the number of elements.</p> <p>To determine the minimum value of <i>ldb</i> on output, consider the following guideline:</p> <p>If <i>ordering</i> = 'C' or 'c', then</p> <ul style="list-style-type: none">• If <i>trans</i> = 'T' or 't' or 'C' or 'c', this parameter must be at least $\max(1, cols)$• If <i>trans</i> = 'N' or 'n' or 'R' or 'r', this parameter must be at least $\max(1, rows)$ <p>If <i>ordering</i> = 'R' or 'r', then</p> <ul style="list-style-type: none">• If <i>trans</i> = 'T' or 't' or 'C' or 'c', this parameter must be at least $\max(1, rows)$• If <i>trans</i> = 'N' or 'n' or 'R' or 'r', this parameter must be at least $\max(1, cols)$ |

Output Parameters

| | |
|-----------|---|
| <i>ab</i> | <p>Array.</p> <p>Contains the matrix <i>AB</i>.</p> |
|-----------|---|

Interfaces

mkl_?omatcopy

Performs scaling and out-place transposition/copying of matrices.

Syntax

```
void mkl_somatcopy (char ordering, char trans, size_t rows, size_t cols, const float alpha, const float * A, size_t lda, float * B, size_t ldb);  
void mkl_domatcopy (char ordering, char trans, size_t rows, size_t cols, const double alpha, const double * A, size_t lda, double * B, size_t ldb);  
void mkl_comatcopy (char ordering, char trans, size_t rows, size_t cols, const MKL_Complex8 alpha, const MKL_Complex8 * A, size_t lda, MKL_Complex8 * B, size_t ldb);  
void mkl_zomatcopy (char ordering, char trans, size_t rows, size_t cols, const MKL_Complex16 alpha, const MKL_Complex16 * A, size_t lda, MKL_Complex16 * B, size_t ldb);
```

Include Files

- mkl.h

Description

The `mkl_omatcopy` routine performs scaling and out-of-place transposition/copying of matrices. A transposition operation can be a normal matrix copy, a transposition, a conjugate transposition, or just a conjugation. The operation is defined as follows:

$$B := \alpha * \text{op}(A)$$

NOTE

Different arrays must not overlap.

Input Parameters

| | |
|-----------------|--|
| <i>ordering</i> | Ordering of the matrix storage. If <i>ordering</i> = 'R' or 'r', the ordering is row-major. If <i>ordering</i> = 'C' or 'c', the ordering is column-major. |
| <i>trans</i> | Parameter that specifies the operation type. If <i>trans</i> = 'N' or 'n', $\text{op}(A) = A$ and the matrix <i>A</i> is assumed unchanged on input. If <i>trans</i> = 'T' or 't', it is assumed that <i>A</i> should be transposed. If <i>trans</i> = 'C' or 'c', it is assumed that <i>A</i> should be conjugate transposed. If <i>trans</i> = 'R' or 'r', it is assumed that <i>A</i> should be only conjugated. If the data is real, then <i>trans</i> = 'R' is the same as <i>trans</i> = 'N', and <i>trans</i> = 'C' is the same as <i>trans</i> = 'T'. |
| <i>rows</i> | The number of rows in matrix <i>B</i> (the destination matrix). |
| <i>cols</i> | The number of columns in matrix <i>B</i> (the destination matrix). |
| <i>alpha</i> | This parameter scales the input matrix by <i>alpha</i> . |
| <i>a</i> | Array. |
| <i>lda</i> | If <i>ordering</i> = 'R' or 'r', <i>lda</i> represents the number of elements in array <i>a</i> between adjacent rows of matrix <i>A</i> ; <i>lda</i> must be at least equal to the number of columns of matrix <i>A</i> . If <i>ordering</i> = 'C' or 'c', <i>lda</i> represents the number of elements in array <i>a</i> between adjacent columns of matrix <i>A</i> ; <i>lda</i> must be at least equal to the number of row in matrix <i>A</i> . |
| <i>b</i> | Array. |
| <i>ldb</i> | If <i>ordering</i> = 'R' or 'r', <i>ldb</i> represents the number of elements in array <i>b</i> between adjacent rows of matrix <i>B</i> . <ul style="list-style-type: none"> If <i>trans</i> = 'T' or 't' or 'C' or 'c', <i>ldb</i> must be at least equal to <i>rows</i>. |

- If *trans* = 'N' or 'n' or 'R' or 'r', *ldb* must be at least equal to *cols*.

If *ordering* = 'C' or 'c', *ldb* represents the number of elements in array *b* between adjacent columns of matrix *B*.

- If *trans* = 'T' or 't' or 'C' or 'c', *ldb* must be at least equal to *cols*.
- If *trans* = 'N' or 'n' or 'R' or 'r', *ldb* must be at least equal to *rows*.

Output Parameters

b Array, size at least *m*.
Contains the destination matrix.

Interfaces

mkl_?omatcopy2

Performs two-strided scaling and out-of-place transposition/copying of matrices.

Syntax

```
void mkl_somatcopy2 (char ordering, char trans, size_t rows, size_t cols, const float alpha, const float * A, size_t lda, size_t stridea, float * B, size_t ldb, size_t strideb);
```

```
void mkl_domatcopy2 (char ordering, char trans, size_t rows, size_t cols, const double alpha, const double * A, size_t lda, size_t stridea, double * B, size_t ldb, size_t strideb);
```

```
void mkl_comatcopy2 (char ordering, char trans, size_t rows, size_t cols, const MKL_Complex8 alpha, const MKL_Complex8 * A, size_t lda, size_t stridea, MKL_Complex8 * B, size_t ldb, size_t strideb);
```

```
void mkl_zomatcopy2 (char ordering, char trans, size_t rows, size_t cols, const MKL_Complex16 alpha, const MKL_Complex16 * A, size_t lda, size_t stridea, MKL_Complex16 * B, size_t ldb, size_t strideb);
```

Include Files

- mkl.h

Description

The `mkl_?omatcopy2` routine performs two-strided scaling and out-of-place transposition/copying of matrices. A transposition operation can be a normal matrix copy, a transposition, a conjugate transposition, or just a conjugation. The operation is defined as follows:

$$B := \alpha *_{op}(A)$$

Normally, matrices in the BLAS or LAPACK are specified by a single stride index. For instance, in the column-major order, $A(2,1)$ is stored in memory one element away from $A(1,1)$, but $A(1,2)$ is a leading dimension away. The leading dimension in this case is at least the number of rows of the source matrix. If a matrix has two strides, then both $A(2,1)$ and $A(1,2)$ may be an arbitrary distance from $A(1,1)$.

NOTE

Different arrays must not overlap.

Input Parameters

| | |
|-----------------|---|
| <i>ordering</i> | Ordering of the matrix storage. If <i>ordering</i> = 'R' or 'r', the ordering is row-major. If <i>ordering</i> = 'C' or 'c', the ordering is column-major. |
| <i>trans</i> | Parameter that specifies the operation type. If <i>trans</i> = 'N' or 'n', $op(A) = A$ and the matrix <i>A</i> is assumed unchanged on input. If <i>trans</i> = 'T' or 't', it is assumed that <i>A</i> should be transposed. If <i>trans</i> = 'C' or 'c', it is assumed that <i>A</i> should be conjugate transposed. If <i>trans</i> = 'R' or 'r', it is assumed that <i>A</i> should be only conjugated. If the data is real, then <i>trans</i> = 'R' is the same as <i>trans</i> = 'N', and <i>trans</i> = 'C' is the same as <i>trans</i> = 'T'. |
| <i>rows</i> | The number of rows in matrix <i>B</i> (the destination matrix). |
| <i>cols</i> | The number of columns in matrix <i>B</i> (the destination matrix). |
| <i>alpha</i> | This parameter scales the input matrix by <i>alpha</i> . |
| <i>a</i> | Array. |
| <i>lda</i> | If <i>ordering</i> = 'R' or 'r', <i>lda</i> represents the number of elements in array <i>a</i> between adjacent rows of matrix <i>A</i> ; <i>lda</i> must be at least equal to the number of columns of matrix <i>A</i> . If <i>ordering</i> = 'C' or 'c', <i>lda</i> represents the number of elements in array <i>a</i> between adjacent columns of matrix <i>A</i> ; <i>lda</i> must be at least 1 and not more than the number of columns in matrix <i>A</i> . |
| <i>stridea</i> | If <i>ordering</i> = 'R' or 'r', <i>stridea</i> represents the number of elements in array <i>a</i> between adjacent columns of matrix <i>A</i> . <i>stridea</i> must be at least 1 and not more than the number of columns in matrix <i>A</i> . If <i>ordering</i> = 'C' or 'c', <i>stridea</i> represents the number of elements in array <i>a</i> between adjacent rows of matrix <i>A</i> . <i>stridea</i> must be at least equal to the number of columns in matrix <i>A</i> . |
| <i>b</i> | Array. |
| <i>ldb</i> | If <i>ordering</i> = 'R' or 'r', <i>ldb</i> represents the number of elements in array <i>b</i> between adjacent rows of matrix <i>B</i> . <ul style="list-style-type: none"> If <i>trans</i> = 'T' or 't' or 'C' or 'c', <i>ldb</i> must be at least equal to $rows/strideb$. |

- If *trans* = 'N' or 'n' or 'R' or 'r', *ldb* must be at least equal to *cols/strideb*.

If *ordering* = 'C' or 'c', *ldb* represents the number of elements in array *b* between adjacent columns of matrix *B*.

- If *trans* = 'T' or 't' or 'C' or 'c', *ldb* must be at least 1 and not more than *rows/strideb*.
- If *trans* = 'N' or 'n' or 'R' or 'r', *ldb* must be at least 1 and not more than *cols/strideb*.

strideb

If *ordering* = 'R' or 'r', *strideb* represents the number of elements in array *b* between adjacent columns of matrix *B*.

- If *trans* = 'T' or 't' or 'C' or 'c', *strideb* must be at least 1 and not more than *rows* (the number of rows in matrix *B*).
- If *trans* = 'N' or 'n' or 'R' or 'r', *strideb* must be at least 1 and not more than *cols* (the number of columns in matrix *B*).

If *ordering* = 'C' or 'c', *strideb* represents the number of elements in array *b* between adjacent rows of matrix *B*.

- If *trans* = 'T' or 't' or 'C' or 'c', *strideb* must be at least equal to *rows* (the number of rows in matrix *B*).
- If *trans* = 'N' or 'n' or 'R' or 'r', *strideb* must be at least equal to *cols* (the number of columns in matrix *B*).

Output Parameters

b Array, size at least *m*.
Contains the destination matrix.

Interfaces

mkl_?omatadd

Scales and sums two matrices including in addition to performing out-of-place transposition operations.

Syntax

```
void mkl_somatadd (char ordering, char transa, char transb, size_t m, size_t n, const float alpha, const float * A, size_t lda, const float beta, const float * B, size_t ldb, float * C, size_t ldc);
```

```
void mkl_domatadd (char ordering, char transa, char transb, size_t m, size_t n, const double alpha, const double * A, size_t lda, const double beta, const double * B, size_t ldb, double * C, size_t ldc);
```

```
void mkl_comatadd (char ordering, char transa, char transb, size_t m, size_t n, const MKL_Complex8 alpha, const MKL_Complex8 * A, size_t lda, const MKL_Complex8 beta, const MKL_Complex8 * B, size_t ldb, MKL_Complex8 * C, size_t ldc);
```

```
void mkl_zomatadd (char ordering, char transa, char transb, size_t m, size_t n, const
MKL_Complex16 alpha, const MKL_Complex16 * A, size_t lda, const MKL_Complex16 beta,
const MKL_Complex16 * B, size_t ldb, MKL_Complex16 * C, size_t ldc);
```

Include Files

- mkl.h

Description

The `mkl_?omatadd` routine scales and adds two matrices, as well as performing out-of-place transposition operations. A transposition operation can be no operation, a transposition, a conjugate transposition, or a conjugation (without transposition). The following out-of-place memory movement is done:

$$C := \alpha * \text{op}(A) + \beta * \text{op}(B)$$

where the `op(A)` and `op(B)` operations are transpose, conjugate-transpose, conjugate (no transpose), or no transpose, depending on the values of `transa` and `transb`. If no transposition of the source matrices is required, m is the number of rows and n is the number of columns in the source matrices A and B . In this case, the output matrix C is m -by- n .

NOTE

Note that different arrays must not overlap.

Input Parameters

| | |
|-----------------|---|
| <i>ordering</i> | Ordering of the matrix storage. If <i>ordering</i> = 'R' or 'r', the ordering is row-major. If <i>ordering</i> = 'C' or 'c', the ordering is column-major. |
| <i>transa</i> | Parameter that specifies the operation type on matrix A . If <i>transa</i> = 'N' or 'n', $\text{op}(A)=A$ and the matrix A is assumed unchanged on input. If <i>transa</i> = 'T' or 't', it is assumed that A should be transposed. If <i>transa</i> = 'C' or 'c', it is assumed that A should be conjugate transposed. If <i>transa</i> = 'R' or 'r', it is assumed that A should be conjugated (and not transposed). If the data is real, then <i>transa</i> = 'R' is the same as <i>transa</i> = 'N', and <i>transa</i> = 'C' is the same as <i>transa</i> = 'T'. |
| <i>transb</i> | Parameter that specifies the operation type on matrix B . If <i>transb</i> = 'N' or 'n', $\text{op}(B)=B$ and the matrix B is assumed unchanged on input. If <i>transb</i> = 'T' or 't', it is assumed that B should be transposed. If <i>transb</i> = 'C' or 'c', it is assumed that B should be conjugate transposed. If <i>transb</i> = 'R' or 'r', it is assumed that B should be conjugated (and not transposed). |

If the data is real, then `transb = 'R'` is the same as `transb = 'N'`, and `transb = 'C'` is the same as `transb = 'T'`.

| | |
|--------------------|--|
| <code>m</code> | The number of matrix rows in <code>op(A)</code> , <code>op(B)</code> , and <code>C</code> . |
| <code>n</code> | The number of matrix columns in <code>op(A)</code> , <code>op(B)</code> , and <code>C</code> . |
| <code>alpha</code> | This parameter scales the input matrix by <code>alpha</code> . |
| <code>a</code> | Array. |
| <code>lda</code> | Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the source matrix <code>A</code> ; measured in the number of elements. For <code>ordering = 'C'</code> or <code>'c'</code> : when <code>transa = 'N', 'n', 'R', or 'r'</code> , <code>lda</code> must be at least <code>max(1, m)</code> ; otherwise <code>lda</code> must be <code>max(1, n)</code> . For <code>ordering = 'R'</code> or <code>'r'</code> : when <code>transa = 'N', 'n', 'R', or 'r'</code> , <code>lda</code> must be at least <code>max(1, n)</code> ; otherwise <code>lda</code> must be <code>max(1, m)</code> . |
| <code>beta</code> | This parameter scales the input matrix by <code>beta</code> . |
| <code>b</code> | Array. |
| <code>ldb</code> | Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the source matrix <code>B</code> ; measured in the number of elements. For <code>ordering = 'C'</code> or <code>'c'</code> : when <code>transa = 'N', 'n', 'R', or 'r'</code> , <code>ldb</code> must be at least <code>max(1, m)</code> ; otherwise <code>ldb</code> must be <code>max(1, n)</code> . For <code>ordering = 'R'</code> or <code>'r'</code> : when <code>transa = 'N', 'n', 'R', or 'r'</code> , <code>ldb</code> must be at least <code>max(1, n)</code> ; otherwise <code>ldb</code> must be <code>max(1, m)</code> . |
| <code>ldc</code> | Distance between the first elements in adjacent columns (in the case of the column-major order) or rows (in the case of the row-major order) in the destination matrix <code>C</code> ; measured in the number of elements. If <code>ordering = 'C'</code> or <code>'c'</code> , then <code>ldc</code> must be at least <code>max(1, m)</code> , otherwise <code>ldc</code> must be at least <code>max(1, n)</code> . |

Output Parameters

| | |
|----------------|--------|
| <code>c</code> | Array. |
|----------------|--------|

Interfaces

`cblas_?gemm_alloc`

Allocates storage for a packed matrix.

Syntax

```
float* cblas_sgemm_alloc (const CBLAS_IDENTIFIER identifier, const MKL_INT m, const MKL_INT n, const MKL_INT k);
```

```
double* cblas_dgemm_alloc (const CBLAS_IDENTIFIER identifier, const MKL_INT m, const MKL_INT n, const MKL_INT k);
```

Include Files

- `mkl.h`

Description

The `cblas_?gemm_alloc` routine is one of a set of related routines that enable use of an internal packed storage. Call the `cblas_?gemm_alloc` routine first to allocate storage for a packed matrix structure to be used in subsequent calls, ultimately to compute

$$C := \alpha * \text{op}(A) * \text{op}(B) + \beta * C,$$

where:

$\text{op}(X)$ is one of the operations $\text{op}(X) = X$, $\text{op}(X) = X^T$, or $\text{op}(X) = X^H$,

α and β are scalars,

A , B , and C are matrices:

$\text{op}(A)$ is an m -by- k matrix,

$\text{op}(B)$ is a k -by- n matrix,

C is an m -by- n matrix.

The `cblas_?gemm_alloc` routine is not supported on Windows* OS for the IA-32 architecture with single dynamic library linking.

Input Parameters

| | |
|-------------------|---|
| <i>identifier</i> | Specifies which matrix is to be packed: If <i>identifier</i> = <code>CblasAMatrix</code> , the routine allocates storage to pack matrix A . If <i>identifier</i> = <code>CblasBMatrix</code> , the routine allocates storage to pack matrix B . |
| <i>m</i> | Specifies the number of rows of matrix $\text{op}(A)$ and of the matrix C . The value of m must be at least zero. |
| <i>n</i> | Specifies the number of columns of matrix $\text{op}(B)$ and the number of columns of matrix C . The value of n must be at least zero. |
| <i>k</i> | Specifies the number of columns of matrix $\text{op}(A)$ and the number of rows of matrix $\text{op}(B)$. The value of k must be at least zero. |

Return Values

The function returns the allocated storage.

See Also

[cblas_?gemm_pack](#) Performs scaling and packing of the matrix into the previously allocated buffer.

[cblas_?gemm_compute](#) Computes a matrix-matrix product with general matrices where one or both input matrices are stored in a packed data structure and adds the result to a scalar-matrix product.

[cblas_?gemm_free](#) Frees the storage previously allocated for the packed matrix.

[cblas_?gemm](#) for a detailed description of general matrix multiplication.

cblas_?gemm_pack

Performs scaling and packing of the matrix into the previously allocated buffer.

Syntax

```
void cblas_sgemm_pack (const CBLAS_LAYOUT Layout, const CBLAS_IDENTIFIER identifier,
const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const MKL_INT k, const
float alpha, const float *src, const MKL_INT ld, float *dest);
```

```
void cblas_dgemm_pack (const CBLAS_LAYOUT Layout, const CBLAS_IDENTIFIER identifier,
const CBLAS_TRANSPOSE trans, const MKL_INT m, const MKL_INT n, const MKL_INT k, const
double alpha, const double *src, const MKL_INT ld, double *dest);
```

Include Files

- mkl.h

Description

The `cblas_?gemm_pack` routine is one of a set of related routines that enable use of an internal packed storage. Call `cblas_?gemm_pack` after successfully calling `cblas_?gemm_alloc`. The `cblas_?gemm_pack` scales the identified matrix by α and packs it into the buffer allocated previously with `cblas_?gemm_alloc`.

The `cblas_?gemm_pack` routine performs this operation:

$$dest := \alpha * op(src) \text{ as part of the computation } C := \alpha * op(A) * op(B) + \beta * C$$

where:

$op(X)$ is one of the operations $op(X) = X$, $op(X) = X^T$, or $op(X) = X^H$,

α and β are scalars,

src is a matrix,

A , B , and C are matrices

$op(src)$ is an m -by- k matrix if $identifier = CblasAMatrix$,

$op(src)$ is a k -by- n matrix if $identifier = CblasBMatrix$,

$dest$ is an internal packed storage buffer.

NOTE

You must use the same value of the `Layout` parameter for the entire sequence of related `cblas_?gemm_pack` and `cblas_?gemm_compute` calls.

For best performance, use the same number of threads for packing and for computing.

If packing for both A and B matrices, you must use the same number of threads for packing A as for packing B .

Input Parameters

`Layout` Specifies whether two-dimensional array storage is row-major (CblasRowMajor) or column-major (CblasColMajor).

`identifier` Specifies which matrix is to be packed:
If `identifier = CblasAMatrix`, the routine allocates storage to pack matrix A .

If *identifier* = `CblasBMatrix`, the routine allocates storage to pack matrix *B*.

trans

Specifies the form of `op(src)` used in the packing:

If *trans* = `CblasNoTrans` `op(src)` = *src*.

If *trans* = `CblasTrans` `op(src)` = *src*^T.

If *trans* = `CblasConjTrans` `op(src)` = *src*^H.

m

Specifies the number of rows of the matrix `op(A)` and of the matrix *C*. The value of *m* must be at least zero.

n

Specifies the number of columns of the matrix `op(B)` and the number of columns of the matrix *C*. The value of *n* must be at least zero.

k

Specifies the number of columns of the matrix `op(A)` and the number of rows of the matrix `op(B)`. The value of *k* must be at least zero.

alpha

Specifies the scalar *alpha*.

src

Array:

| | <i>identifier</i> = <code>CblasAMatrix</code> | | <i>identifier</i> = <code>CblasBMatrix</code> | |
|--|---|---|---|---|
| | <i>trans</i> = <code>CblasNoTrans</code> | <i>trans</i> = <code>CblasTrans</code> OR <i>trans</i> = <code>CblasConjTrans</code> | <i>trans</i> = <code>CblasNoTrans</code> | <i>trans</i> = <code>CblasTrans</code> OR <i>trans</i> = <code>CblasConjTrans</code> |
| <i>Layout</i> = <code>CblasColMajor</code> | Size <i>ld</i> * <i>k</i> . Before entry, the leading <i>m</i> -by- <i>k</i> part of the array <i>src</i> must contain the matrix <i>A</i> . | Size <i>ld</i> * <i>m</i> . Before entry, the leading <i>k</i> -by- <i>m</i> part of the array <i>src</i> must contain the matrix <i>A</i> . | Size <i>ld</i> * <i>n</i> . Before entry, the leading <i>k</i> -by- <i>n</i> part of the array <i>src</i> must contain the matrix <i>B</i> . | Size <i>ld</i> * <i>k</i> . Before entry, the leading <i>n</i> -by- <i>k</i> part of the array <i>src</i> must contain the matrix <i>B</i> . |
| <i>Layout</i> = <code>CblasRowMajor</code> | Size <i>ld</i> * <i>m</i> . Before entry, the leading <i>k</i> -by- <i>m</i> part of the array <i>src</i> | Size <i>ld</i> * <i>k</i> . Before entry, the leading <i>m</i> -by- <i>k</i> part of the array <i>src</i> | Size <i>ld</i> * <i>k</i> . Before entry, the leading <i>n</i> -by- <i>k</i> part of the array <i>src</i> must contain the matrix <i>B</i> . | Size <i>ld</i> * <i>n</i> . Before entry, the leading <i>k</i> -by- <i>n</i> part of the array <i>src</i> must contain the matrix <i>B</i> . |

| | | | | |
|--|---|---|---|---|
| | <i>identifier</i> = CblasAMatrix | | <i>identifier</i> = CblasBMatrix | |
| | <i>trans</i> = CblasNoTrans | <i>trans</i> = CblasTrans OR <i>trans</i> = CblasConjTrans | <i>trans</i> = CblasNoTrans | <i>trans</i> = CblasTrans OR <i>trans</i> = CblasConjTrans |
| | must contain the matrix <i>A</i> . | must contain the matrix <i>A</i> . | | |

ld

Specifies the leading dimension of *src* as declared in the calling (sub)program.

| | | | | |
|--------------------------------------|---|---|---|---|
| | <i>identifier</i> = CblasAMatrix | | <i>identifier</i> = CblasBMatrix | |
| | <i>trans</i> = CblasNoTrans | <i>trans</i> = CblasTrans OR <i>trans</i> = CblasConjTrans | <i>trans</i> = CblasNoTrans | <i>trans</i> = CblasTrans OR <i>trans</i> = CblasConjTrans |
| <i>Layout</i> = CblasColMajor | <i>ld</i> must be at least $\max(1, m)$. | <i>ld</i> must be at least $\max(1, k)$. | <i>ld</i> must be at least $\max(1, k)$. | <i>ld</i> must be at least $\max(1, n)$. |
| <i>Layout</i> = CblasRowMajor | <i>ld</i> must be at least $\max(1, k)$. | <i>ld</i> must be at least $\max(1, m)$. | <i>ld</i> must be at least $\max(1, n)$. | <i>ld</i> must be at least $\max(1, k)$. |

dest

Scaled and packed internal storage buffer.

Output Parameters

dest

Overwritten by the matrix $\alpha * op(src)$.

See Also

[cblas_?gemm_alloc](#) Allocates storage for a packed matrix.

[cblas_?gemm_compute](#) Computes a matrix-matrix product with general matrices where one or both input matrices are stored in a packed data structure and adds the result to a scalar-matrix product.

[cblas_?gemm_free](#) Frees the storage previously allocated for the packed matrix.

[cblas_?gemm](#) for a detailed description of general matrix multiplication.

cblas_?gemm_compute

Computes a matrix-matrix product with general matrices where one or both input matrices are stored in a packed data structure and adds the result to a scalar-matrix product.

Syntax

```
void cblas_sgemm_compute (const CBLAS_LAYOUT Layout, const MKL_INT transa, const
MKL_INT transb, const MKL_INT m, const MKL_INT n, const MKL_INT k, const float *a,
const MKL_INT lda, const float *b, const MKL_INT ldb, const float beta, float *c,
const MKL_INT ldc);
```

```
void cblas_dgemm_compute (const CBLAS_LAYOUT Layout, const MKL_INT transa, const
MKL_INT transb, const MKL_INT m, const MKL_INT n, const MKL_INT k, const double *a,
const MKL_INT lda, const double *b, const MKL_INT ldb, const double beta, double *c,
const MKL_INT ldc);
```

Include Files

- mkl.h

Description

The `cblas_?gemm_compute` routine is one of a set of related routines that enable use of an internal packed storage. After calling `cblas_?gemm_pack` call `cblas_?gemm_compute` to compute

$$C := \text{op}(A) * \text{op}(B) + \text{beta} * C,$$

where:

$\text{op}(X)$ is one of the operations $\text{op}(X) = X$, $\text{op}(X) = X^T$, or $\text{op}(X) = X^H$,

beta is a scalar,

A , B , and C are matrices:

$\text{op}(A)$ is an m -by- k matrix,

$\text{op}(B)$ is a k -by- n matrix,

C is an m -by- n matrix.

NOTE

You must use the same value of the `Layout` parameter for the entire sequence of related `cblas_?gemm_pack` and `cblas_?gemm_compute` calls.

For best performance, use the same number of threads for packing and for computing.

If packing for both A and B matrices, you must use the same number of threads for packing A as for packing B .

Input Parameters

| | |
|---------------------|---|
| <code>Layout</code> | Specifies whether two-dimensional array storage is row-major (<code>CblasRowMajor</code>) or column-major (<code>CblasColMajor</code>). |
| <code>transa</code> | Specifies the form of $\text{op}(A)$ used in the matrix multiplication, one of the <code>CBLAS_TRANSPOSE</code> or <code>CBLAS_STORAGE</code> enumerated types: If <code>transa = CblasNoTrans</code> $\text{op}(A) = A$. |

If $transa = \text{CblasTrans}$ $\text{op}(A) = A^T$.

If $transa = \text{CblasConjTrans}$ $\text{op}(A) = A^H$.

If $transa = \text{CblasPacked}$ the matrix in array a is packed and lda is ignored.

$transb$

Specifies the form of $\text{op}(B)$ used in the matrix multiplication, one of the `CBLAS_TRANSPOSE` or `CBLAS_STORAGE` enumerated types:

If $transb = \text{CblasNoTrans}$ $\text{op}(B) = B$.

If $transb = \text{CblasTrans}$ $\text{op}(B) = B^T$.

If $transb = \text{CblasConjTrans}$ $\text{op}(B) = B^H$.

If $transb = \text{CblasPacked}$ the matrix in array b is packed and ldb is ignored.

m

Specifies the number of rows of the matrix $\text{op}(A)$ and of the matrix C . The value of m must be at least zero.

n

Specifies the number of columns of the matrix $\text{op}(B)$ and the number of columns of the matrix C . The value of n must be at least zero.

k

Specifies the number of columns of the matrix $\text{op}(A)$ and the number of rows of the matrix $\text{op}(B)$. The value of k must be at least zero.

a

Array:

| | | | |
|--|--|--|--|
| | $transa =$ <code>CblasNoTrans</code> | $transa =$ <code>CblasTrans</code> or $transa =$ <code>CblasConjTrans</code> | $transa =$ <code>CblasPacked</code> |
| $Layout =$ <code>CblasColMajor</code> | Size $lda * k$. Before entry, the leading m -by- k part of the array a must contain the matrix A . | Size $lda * m$. Before entry, the leading k -by- m part of the array a must contain the matrix A . | Stored in internal packed format. |
| $Layout =$ <code>CblasRowMajor</code> | Size $lda * m$. Before entry, the leading k -by- m part of the array a must contain the matrix A . | Size $lda * k$. Before entry, the leading m -by- k part of the array a must contain the matrix A . | Stored in internal packed format. |

lda

Specifies the leading dimension of a as declared in the calling (sub)program.

| | | | |
|--|---|---|--|
| | $transa =$ <code>CblasNoTrans</code> | $transa =$ <code>CblasTrans</code> or $transa =$ <code>CblasConjTrans</code> | $transa =$ <code>CblasPacked</code> |
|--|---|---|--|

| | | | |
|----------------------------------|--|--|------------------------|
| <i>Layout</i> = CblasColMajor | <i>lda</i> must be at least $\max(1, m)$. | <i>lda</i> must be at least $\max(1, k)$. | <i>lda</i> is ignored. |
| <i>Layout</i> = CblasRowMajor | <i>lda</i> must be at least $\max(1, k)$. | <i>lda</i> must be at least $\max(1, m)$. | <i>lda</i> is ignored. |

b

Array:

| | | | |
|----------------------------------|--|--|-----------------------------------|
| | <i>transb</i> = CblasNoTrans | <i>transb</i> = CblasTrans or <i>transb</i> = CblasConjTrans | <i>transb</i> = CblasPacked |
| <i>Layout</i> = CblasColMajor | Size $ldb \cdot n$. Before entry, the leading k -by- n part of the array <i>b</i> must contain the matrix <i>B</i> . | Size $ldb \cdot k$. Before entry, the leading n -by- k part of the array <i>b</i> must contain the matrix <i>B</i> . | Stored in internal packed format. |
| <i>Layout</i> = CblasRowMajor | Size $ldb \cdot k$. Before entry, the leading n -by- k part of the array <i>b</i> must contain the matrix <i>B</i> . | Size $ldb \cdot n$. Before entry, the leading k -by- n part of the array <i>b</i> must contain the matrix <i>B</i> . | Stored in internal packed format. |

ldb

Specifies the leading dimension of *b* as declared in the calling (sub)program.

| | | | |
|----------------------------------|--|---|--------------------------------|
| | <i>transb</i> = CblasNoTrans | <i>transb</i> = CblasTrans or <i>transb</i> = CblasConjTrans | <i>transb</i> = CblasPacked |
| <i>Layout</i> = CblasColMajor | <i>ldb</i> must be at least $\max(1, k)$. | <i>ldb</i> must be at least $\max(1, n)$. | <i>ldb</i> is ignored. |
| <i>Layout</i> = CblasRowMajor | <i>ldb</i> must be at least $\max(1, n)$. | <i>ldb</i> must be at least $\max(1, k)$. | <i>ldb</i> is ignored. |

beta

Specifies the scalar *beta*. When *beta* is equal to zero, then *c* need not be set on input.

c

Array:

| | |
|----------------------------------|---|
| <i>Layout</i> = CblasColMajor | Size $ldc*n$. Before entry, the leading m -by- n part of the array c must contain the matrix C , except when $beta$ is equal to zero, in which case c need not be set on entry. |
| <i>Layout</i> = CblasRowMajor | Size $ldc*m$. Before entry, the leading n -by- m part of the array c must contain the matrix C , except when $beta$ is equal to zero, in which case c need not be set on entry. |

ldc

Specifies the leading dimension of c as declared in the calling (sub)program.

| | |
|-------------------------------|--|
| <i>Layout</i> = CblasColMajor | <i>ldc</i> must be at least $\max(1, m)$. |
| <i>Layout</i> = CblasRowMajor | <i>ldc</i> must be at least $\max(1, n)$. |

Output Parameters

c

Overwritten by the m -by- n matrix $op(A) * op(B) + beta * C$.

See Also

[cblas_?gemm_alloc](#) Allocates storage for a packed matrix.

[cblas_?gemm_pack](#) Performs scaling and packing of the matrix into the previously allocated buffer.

[cblas_?gemm_free](#) Frees the storage previously allocated for the packed matrix.

[cblas_?gemm](#) for a detailed description of general matrix multiplication.

cblas_?gemm_free

Frees the storage previously allocated for the packed matrix.

Syntax

```
void cblas_sgemm_free (float *dest);
```

```
void cblas_dgemm_free (double *dest);
```

Include Files

- mkl.h

Description

The `cblas_?gemm_free` routine is one of a set of related routines that enable use of an internal packed storage. Call the `cblas_?gemm_free` routine last to release storage for the packed matrix structure allocated with `cblas_?gemm_alloc`.

Input Parameters

dest

Previously allocated storage.

LAPACK Routines

This chapter describes the Intel® Math Kernel Library implementation of routines from the LAPACK package that are used for solving systems of linear equations, linear least squares problems, eigenvalue and singular value problems, and performing a number of related computational tasks. The library includes LAPACK routines for both real and complex data. Routines are supported for systems of equations with the following types of matrices:

- general
- banded
- symmetric or Hermitian positive-definite (full, packed, and rectangular full packed (RFP) storage)
- symmetric or Hermitian positive-definite banded
- symmetric or Hermitian indefinite (both full and packed storage)
- symmetric or Hermitian indefinite banded
- triangular (full, packed, and RFP storage)
- triangular banded
- tridiagonal
- diagonally dominant tridiagonal.

NOTE

Different arrays used as parameters to Intel MKL LAPACK routines must not overlap.

WARNING

LAPACK routines assume that input matrices do not contain IEEE 754 special values such as `INF` or `NaN` values. Using these special values may cause LAPACK to return unexpected results or become unstable.

C Interface Conventions for LAPACK Routines

The C interfaces are implemented for most of the Intel MKL LAPACK driver and computational routines.

Function Prototypes

Intel MKL supports four distinct floating-point precisions. Each corresponding prototype looks similar, usually differing only in the data type. C interface LAPACK function names follow the form `<?><name>`, where `<?>` is:

- `LAPACKE_s` for float
- `LAPACKE_d` for double
- `LAPACKE_c` for `lapack_complex_float`
- `LAPACKE_z` for `lapack_complex_double`

A specific example follows. To solve a system of linear equations with a packed Cholesky-factored Hermitian positive-definite matrix with complex precision, use the following:

```
lapack_int LAPACKE_c pptrs(int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float * ap, lapack_complex_float * b, lapack_int ldb);
```

Workspace Arrays

In contrast to the Fortran interface, the LAPACK C interface omits workspace parameters because workspace is allocated during runtime and released upon completion of the function operation.

If you prefer to allocate workspace arrays yourself, the LAPACK C interface provides alternate interfaces with *work* parameters. The name of the alternate interface is the same as the LAPACK C interface with *_work* appended. For example, the syntax for the singular value decomposition of a real bidiagonal matrix is:

```
Fortran:          call sbdsdc ( uplo, compq, n, d, e, u, ldu, vt, ldvt, q, iq,
                    work , iwork, info )

C LAPACK interface:  lapack_int LAPACKE_sbdsdc ( int matrix_layout, char uplo, char
                    compq, lapack_int n, float* d, float* e, float* u, lapack_int
                    ldu, float* vt, lapack_int ldvt, float* q, lapack_int* iq );

Alternate C LAPACK  lapack_int LAPACKE_sbdsdc_work( int matrix_layout, char uplo,
interface with work char compq, lapack_int n, float* d, float* e, float* u,
parameters:         lapack_int ldu, float* vt, lapack_int ldvt, float* q, lapack_int*
                    iq, float* work , lapack_int* iwork );
```

See the *install_dir/include/mkl_lapacke.h* file for the full list of alternative C LAPACK interfaces.

The Intel MKL Fortran-specific documentation contains details about workspace arrays.

Mapping Fortran Data Types against C Data Types

Fortran Data Types vs. C Data Types

| FORTRAN | C |
|---------------------------|-----------------------|
| INTEGER | lapack_int |
| LOGICAL | lapack_logical |
| REAL | float |
| DOUBLE PRECISION | double |
| COMPLEX | lapack_complex_float |
| COMPLEX*16/DOUBLE COMPLEX | lapack_complex_double |
| CHARACTER | char |

C Type Definitions

You can find type definitions specific to Intel MKL such as `MKL_INT`, `MKL_Complex8`, and `MKL_Complex16` in *install_dir/mkl_types.h*.

```
C types
#ifdef lapack_int
#define lapack_int MKL_INT
#endif

#ifdef lapack_logical
#define lapack_logical lapack_int
#endif
```

Complex Type Definitions Complex type for single precision:

```
#ifdef lapack_complex_float
#define lapack_complex_float MKL_Complex8
#endif
```

Complex type for double precision:

```
#ifndef lapack_complex_double
#define lapack_complex_double    MKL_Complex16
#endif
```

Matrix Layout Definitions

```
#define LAPACK_ROW_MAJOR    101
#define LAPACK_COL_MAJOR    102
```

See [Matrix Layout for LAPACK Routines](#) for an explanation of row-major order and column-major order storage.

Error Code Definitions

```
#define LAPACK_WORK_MEMORY_ERROR    -1010    /* Failed to allocate
memory
                                           for a working array */
#define LAPACK_TRANSPOSE_MEMORY_ERROR    -1011    /* Failed to allocate
memory
                                           for transposed matrix */
```

Matrix Layout for LAPACK Routines

There are two general methods of storing a two dimensional matrix in linear (one dimensional) memory: column-wise (column major order) or row-wise (row major order). Consider an M -by- N matrix A :

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,j_0+1} & a_{1,j_0+2} & \dots & a_{1,j_0+L-1} & a_{1,j_0+L} & \dots & a_{1,N} \\ a_{2,1} & a_{2,2} & \dots & a_{2,j_0+1} & a_{2,j_0+2} & \dots & a_{2,j_0+L-1} & a_{2,j_0+L} & \dots & a_{2,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{i_0+1,1} & a_{i_0+1,2} & \dots & a_{i_0+1,j_0+1} & a_{i_0+1,j_0+2} & \dots & a_{i_0+1,j_0+L-1} & a_{i_0+1,j_0+L} & \dots & a_{i_0+1,N} \\ a_{i_0+2,1} & a_{i_0+2,2} & \dots & a_{i_0+2,j_0+1} & a_{i_0+2,j_0+2} & \dots & a_{i_0+2,j_0+L-1} & a_{i_0+2,j_0+L} & \dots & a_{i_0+2,N} \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ a_{i_0+K-1,1} & a_{i_0+K-1,2} & \dots & a_{i_0+K-1,j_0+1} & a_{i_0+K-1,j_0+2} & \dots & a_{i_0+K-1,j_0+L-1} & a_{i_0+K-1,j_0+L} & \dots & a_{i_0+K-1,N} \\ a_{i_0+K,1} & a_{i_0+K,2} & \dots & a_{i_0+K,j_0+1} & a_{i_0+K,j_0+2} & \dots & a_{i_0+K,j_0+L-1} & a_{i_0+K,j_0+L} & \dots & a_{i_0+K,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{M,1} & a_{M,2} & \dots & a_{M,j_0+1} & a_{M,j_0+2} & \dots & a_{M,j_0+L-1} & a_{M,j_0+L} & \dots & a_{M,N} \end{bmatrix}$$

Column Major Layout

In column major layout the first index, i , of matrix elements $a_{i,j}$ changes faster than the second index when accessing sequential memory locations. In other words, for $1 \leq i < M$, if the element $a_{i,j}$ is stored in a specific location in memory, the element $a_{i+1,j}$ is stored in the next location, and, for $1 \leq j < N$, the element $a_{M,j}$ is stored in the location previous to element $a_{1,j+1}$. So the matrix elements are located in memory according to this sequence:

$$\{a_{1,1}a_{2,1} \dots a_{M,1}a_{1,2}a_{2,2} \dots a_{M,2} \dots \dots a_{1,N}a_{2,N} \dots a_{M,N}\}$$

Row Major Layout

In row major layout the second index, j , of matrix elements $a_{i,j}$ changes faster than the first index when accessing sequential memory locations. In other words, for $1 \leq j < N$, if the element $a_{i,j}$ is stored in a specific location in memory, the element $a_{i,j+1}$ is stored in the next location, and, for $1 \leq i < M$, the element $a_{i,N}$ is stored in the location previous to element $a_{i+1,1}$. So the matrix elements are located in memory according to this sequence:

$$\{a_{1,1}a_{1,2} \dots a_{1,N}a_{2,1}a_{2,2} \dots a_{2,N} \dots \dots a_{N,1}a_{N,2} \dots a_{M,N}\}$$

Leading Dimension Parameter

A leading dimension parameter allows use of LAPACK routines on a submatrix of a larger matrix. For example, the submatrix B can be extracted from the original matrix A defined previously:

$$B = \begin{bmatrix} a_{i_0+1,j_0+1} & a_{i_0+1,j_0+2} & \dots & a_{i_0+1,j_0+L-1} & a_{i_0+1,j_0+L} \\ a_{i_0+2,j_0+1} & a_{i_0+2,j_0+2} & \dots & a_{i_0+2,j_0+L-1} & a_{i_0+2,j_0+L} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i_0+K-1,j_0+1} & a_{i_0+K-1,j_0+2} & \dots & a_{i_0+K-1,j_0+L-1} & a_{i_0+K-1,j_0+L} \\ a_{i_0+K,j_0+1} & a_{i_0+K,j_0+2} & \dots & a_{i_0+K,j_0+L-1} & a_{i_0+K,j_0+L} \end{bmatrix}$$

B is formed from rows with indices $i_0 + 1$ to $i_0 + K$ and columns $j_0 + 1$ to $j_0 + L$ of matrix A . To specify matrix B , LAPACK routines require four parameters:

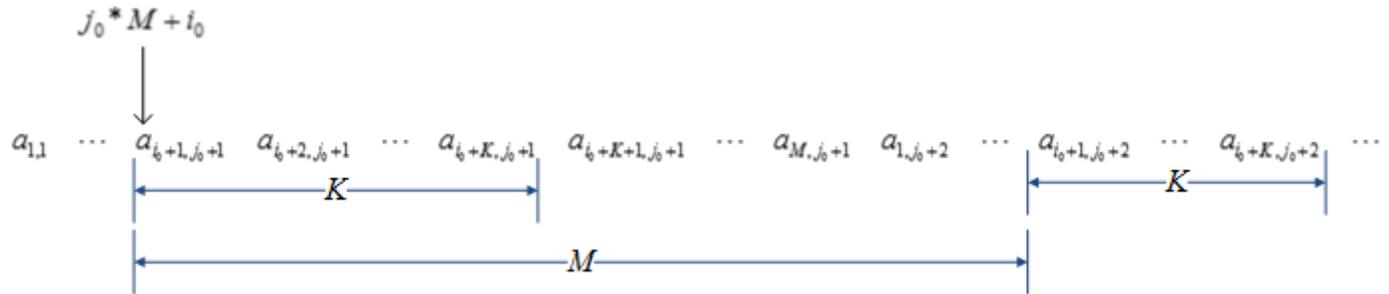
- the number of rows K ;
- the number of columns L ;
- a pointer to the start of the array containing elements of B ;
- the leading dimension of the array containing elements of B .

The leading dimension depends on the layout of the matrix:

- Column major layout

Leading dimension $ldb=M$, the number of rows of matrix A .

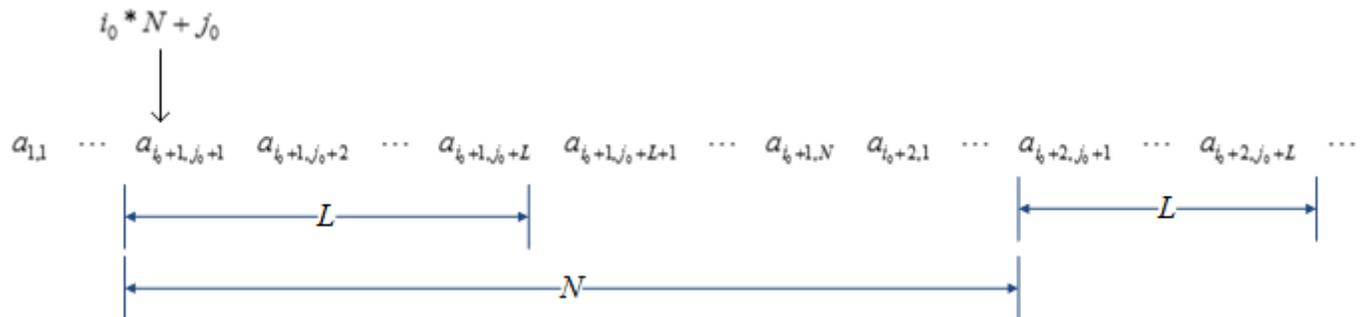
Starting address: offset by $i_0 + j_0 * ldb$ from $a_{1,1}$.



- Row major layout

Leading dimension $ldb=N$, the number of columns of matrix A .

Starting address: offset by $i_0 * ldb + j_0$ from $a_{1,1}$.



Matrix Storage Schemes for LAPACK Routines

LAPACK routines use the following matrix storage schemes:

- Full Storage
- Packed Storage
- Band Storage
- Rectangular Full Packed (RFP) Storage

Full Storage

Consider an m -by- n matrix A :

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,n} \\ a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{3,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & a_{m,3} & \cdots & a_{m,n} \end{pmatrix}$$

It is stored in a one-dimensional array a of length at least $lda*n$ for column major layout or $m*lda$ for row major layout. Element $a_{i,j}$ is stored as array element $a[k]$ where the mapping of $k(i, j)$ is defined as

- column major layout: $k(i, j) = i - 1 + (j - 1)*lda$
- row major layout: $k(i, j) = (i - 1)*lda + j - 1$

NOTE

Although LAPACK accepts parameter values of zero for matrix size, in general the size of the array used to store an m -by- n matrix A with leading dimension lda should be greater than or equal to $\max(1, n*lda)$ for column major layout and $\max(1, m*lda)$ for row major layout.

NOTE

Even though the array used to store a matrix is one-dimensional, for simplicity the documentation sometimes refers parts of the array such as rows, columns, upper and lower triangular part, and diagonals. These refer to the parts of the matrix stored within the array. For example, the lower triangle of array a is defined as the subset of elements $a[k(i, j)]$ with $i \geq j$.

Packed Storage

The packed storage format compactly stores matrix elements when only one part of the matrix, the upper or lower triangle, is necessary to determine all of the elements of the matrix. This is the case when the matrix is upper triangular, lower triangular, symmetric, or Hermitian. For an n -by- n matrix of one of these types, a linear array ap of length $n*(n + 1)/2$ is adequate. Two parameters define the storage scheme:

$matrix_layout$, which specifies column major (with the value `LAPACK_COL_MAJOR`) or row major (with the value `LAPACK_ROW_MAJOR`) matrix layout, and $uplo$, which specifies that the upper triangle (with the value 'U') or the lower triangle (with the value 'L') is stored.

Element $a_{i,j}$ is stored as array element $a[k]$ where the mapping of $k(i, j)$ is defined as

| | |
|---|--|
| $matrix_layout = \text{LAPACK_COL_MAJOR}$ | $matrix_layout = \text{LAPACK_ROW_MAJOR}$ |
| $uplo = 'U' \quad 1 \leq i \leq j \leq n \quad k(i, j) = i - 1 + j*(j - 1)/2$ | $k(i, j) = j - 1 + (i - 1)*(2*n - i)/2$ |

$$AB = \begin{pmatrix} & & & & a_{1,1} & a_{1,2} & \cdots & a_{1,k_u+1} \\ & & & & & a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,k_u+2} \\ & & & & & & a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & \cdots & a_{3,k_u+3} \\ & & & & & & & \ddots & \vdots & \vdots & \vdots & \vdots \\ a_{k_l+1,1} & a_{k_l+1,2} & \cdots & \cdots & & & & & & a_{k_l+1,k_l+1} & a_{k_l+1,k_l+2} & \cdots & a_{k_l+1,k_u+k_l+1} \\ a_{k_l+2,2} & a_{k_l+2,3} & \cdots & \cdots & & & & & & a_{k_l+2,k_l+2} & a_{k_l+2,k_l+3} & \cdots & a_{k_l+2,k_u+k_l+2} \\ a_{k_l+3,3} & a_{k_l+3,4} & \cdots & \cdots & & & & & & a_{k_l+3,k_l+3} & a_{k_l+3,k_l+4} & \cdots & a_{k_l+3,k_u+k_l+3} \\ \vdots & \vdots \end{pmatrix}$$

The number of columns of AB is $kl + ku + 1$, and the number of rows of AB is m .

NOTE

For both column major and row major layout, elements of the upper left triangle of AB are not used. Depending on the relationship of the dimensions m , n , kl , and ku , the lower right triangle might not be used.

- Converting the Packed Matrix to a One-Dimensional Array: The packed matrix AB is stored in a linear array ab as described in [Full Storage](#). The size of ab should be greater than or equal to the total number of elements of matrix AB : $ldab*n$ for column major layout or $ldab*m$ for row major layout. The leading dimension of ab , $ldab$, must be greater than or equal to $kl + ku + 1$ (and some routines require it to be even larger).

Element a_{ij} is stored as array element $a[k(i, j)]$ where the mapping of $k(i, j)$ is defined as

- column major layout: $k(i, j) = i + ku - j + (j - 1)*ldab$; $1 \leq j \leq n$, $\max(1, j - ku) \leq j \leq \min(m, j + kl)$
- row major layout: $k(i, j) = (i - j)*ldab + kl + j - 1$; $1 \leq i \leq m$, $\max(1, i - kl) \leq j \leq \min(n, i + ku)$

NOTE

Although LAPACK accepts parameter values of zero for matrix size, in general the size of the array should be greater than or equal to $\max(1, n*ldab)$ for column major layout and $\max(1, m*ldab)$ for row major layout.

Rectangular Full Packed Storage

A combination of full and packed storage, rectangular full packed storage can be used to store the upper or lower triangle of a matrix which is upper triangular, lower triangular, symmetric, or Hermitian. It offers the storage savings of packed storage plus the efficiency of using full storage Level 3 BLAS and LAPACK routines. Three parameters define the storage scheme: *matrix_layout*, which specifies column major (with the value `LAPACK_COL_MAJOR`) or row major (with the value `LAPACK_ROW_MAJOR`) matrix layout; *uplo*, which specifies that the upper triangle (with the value 'U') or the lower triangle (with the value 'L') is stored; and *transr*, which specifies normal (with the value 'N'), transpose (with the value 'T'), or conjugate transpose (with the value 'C') operation on the matrix.

Consider an N -by- N matrix A :

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & a_{1,2} & \cdots & a_{1,N-1} \\ a_{2,0} & a_{2,1} & a_{2,2} & \cdots & a_{2,N-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{N-1,0} & a_{N-1,1} & a_{N-1,2} & \cdots & a_{N-1,N-1} \end{pmatrix}$$

The upper or lower triangle of A can be stored in the array ap of length $N*(N + 1)/2$.

Additionally, define k as the integer part of $N/2$, such that $N=2*k$ if N is even, and $N=2*k + 1$ if N is odd.

Storing the matrix involves packing the matrix into a rectangular matrix, and then storing the matrix in a one-dimensional array. The size of rectangular matrix AP required for the N -by- N matrix A is $N + 1$ by $N/2$ for even N , and N by $(N + 1)/2$ for odd N .

These examples illustrate the rectangular full packed storage method.

- Upper triangular - $uplo = 'U'$

Consider a matrix A with $N = 6$:

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} & a_{0,4} & a_{0,5} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} \\ a_{4,0} & a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} \\ a_{5,0} & a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & a_{5,5} \end{pmatrix}$$

- Not transposed - $transr = 'N'$

The elements of the upper triangle of A can be packed in a matrix with the dimensions $(N + 1)$ -by- $(N/2) = 7$ by 3 :

$$AP = \begin{pmatrix} a_{0,3} & a_{0,4} & a_{0,5} \\ a_{1,3} & a_{1,4} & a_{1,5} \\ a_{2,3} & a_{2,4} & a_{2,5} \\ a_{3,3} & a_{3,4} & a_{3,5} \\ a_{0,0} & a_{4,4} & a_{4,5} \\ a_{0,1} & a_{1,1} & a_{5,5} \\ a_{0,2} & a_{1,2} & a_{2,2} \end{pmatrix}$$

- Transposed or conjugate transposed - $transr = 'T'$ or $transr = 'C'$

The elements of the upper triangle of A can be packed in a matrix with the dimensions $(N/2)$ by $(N + 1) = 3$ by 7 :

$$AP = \begin{pmatrix} a_{0,3} & a_{1,3} & a_{2,3} & a_{3,3} & a_{0,0} & a_{0,1} & a_{0,2} \\ a_{0,4} & a_{1,4} & a_{2,4} & a_{3,4} & a_{4,4} & a_{1,1} & a_{1,2} \\ a_{0,5} & a_{1,5} & a_{2,5} & a_{3,5} & a_{4,5} & a_{5,5} & a_{2,2} \end{pmatrix}$$

Consider a matrix A with $N = 5$:

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} & a_{0,4} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,0} & a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

- Not transposed - $transr = 'N'$

The elements of the upper triangle of A can be packed in a matrix with the dimensions (N) -by- $((N + 1)/2) = 5$ by 3 :

$$AP = \begin{pmatrix} a_{0,2} & a_{0,3} & a_{0,4} \\ a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,2} & a_{2,3} & a_{2,4} \\ a_{0,0} & a_{3,3} & a_{3,4} \\ a_{0,1} & a_{1,1} & a_{4,4} \end{pmatrix}$$

- Transposed or conjugate transposed - $transr = 'T'$ or $transr = 'C'$

The elements of the upper triangle of A can be packed in a matrix with the dimensions $((N+1)/2)$ by $(N) = 5$ by 3:

$$AP = \begin{pmatrix} a_{0,2} & a_{1,2} & a_{2,3} & a_{0,0} & a_{0,1} \\ a_{0,3} & a_{1,3} & a_{2,3} & a_{3,3} & a_{1,1} \\ a_{0,4} & a_{1,4} & a_{2,4} & a_{3,4} & a_{4,4} \end{pmatrix}$$

- Lower triangular - $uplo = 'L'$

Consider a matrix A with $N = 6$:

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} & a_{0,4} & a_{0,5} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} & a_{1,5} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} & a_{2,5} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & a_{3,5} \\ a_{4,0} & a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} \\ a_{5,0} & a_{5,1} & a_{5,2} & a_{5,3} & a_{5,4} & a_{5,5} \end{pmatrix}$$

- Not transposed - $transr = 'N'$

The elements of the lower triangle of A can be packed in a matrix with the dimensions $(N + 1)$ -by- $(N/2) = 7$ by 3:

$$AP = \begin{pmatrix} a_{3,3} & a_{4,3} & a_{5,3} \\ a_{0,0} & a_{4,4} & a_{5,4} \\ a_{1,0} & a_{1,1} & a_{5,5} \\ a_{2,0} & a_{2,1} & a_{2,2} \\ a_{3,0} & a_{3,1} & a_{3,2} \\ a_{3,0} & a_{4,1} & a_{4,2} \\ a_{5,0} & a_{5,1} & a_{5,2} \end{pmatrix}$$

- Transposed or conjugate transposed - $transr = 'T'$ or $transr = 'C'$

The elements of the lower triangle of A can be packed in a matrix with the dimensions $(N/2)$ by $(N + 1) = 3$ by 7:

$$AP = \begin{pmatrix} a_{3,3} & a_{0,0} & a_{1,0} & a_{2,0} & a_{3,0} & a_{4,0} & a_{5,0} \\ a_{4,3} & a_{4,4} & a_{1,1} & a_{2,1} & a_{3,1} & a_{4,1} & a_{5,1} \\ a_{5,3} & a_{5,4} & a_{5,5} & a_{2,2} & a_{3,2} & a_{4,2} & a_{5,2} \end{pmatrix}$$

Consider a matrix A with $N = 5$:

$$A = \begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} & a_{0,4} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,0} & a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,0} & a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

- Not transposed - *transr* = 'N'

The elements of the lower triangle of *A* can be packed in a matrix with the dimensions (N) -by- $((N+1)/2) = 5$ by 3:

$$AP = \begin{pmatrix} a_{0,0} & a_{3,3} & a_{4,3} \\ a_{1,0} & a_{1,1} & a_{4,4} \\ a_{2,0} & a_{2,1} & a_{2,2} \\ a_{3,0} & a_{3,1} & a_{3,2} \\ a_{4,0} & a_{4,1} & a_{4,2} \end{pmatrix}$$

- Transposed or conjugate transposed - *transr* = 'T' or *transr* = 'C'

The elements of the lower triangle of *A* can be packed in a matrix with the dimensions $((N+1)/2)$ by $(N) = 5$ by 3:

$$AP = \begin{pmatrix} a_{0,0} & a_{1,0} & a_{2,0} & a_{3,0} & a_{4,0} \\ a_{3,3} & a_{1,1} & a_{2,1} & a_{3,1} & a_{4,1} \\ a_{4,3} & a_{4,4} & a_{2,2} & a_{3,2} & a_{4,2} \end{pmatrix}$$

The packed matrix *AP* can be stored using column major layout or row major layout.

NOTE

The *matrix_layout* and *transr* parameters can specify the same storage scheme: for example, the storage scheme for *matrix_layout* = LAPACK_COL_MAJOR and *transr* = 'N' is the same as that for *matrix_layout* = LAPACK_ROW_MAJOR and *transr* = 'T'.

Element $a_{i,j}$ is stored as array element $ap[l]$ where the mapping of $l(i, j)$ is defined in the following tables.

- Column major layout: *matrix_layout* = LAPACK_COL_MAJOR a

| <i>transr</i> | <i>uplo</i> | N | l(i, j) = | i | j |
|---------------|-------------|-----------------------|---------------------------|----------------------------|-------------------------|
| 'N' | 'U' | 2*k | $(j - k) * (N + 1) + i$ | $0 \leq i < N$ | $\max(i, k) \leq j < N$ |
| | | | $i * (N + 1) + j + k + 1$ | $0 \leq i < k$ | $i \leq j < k$ |
| | 2*k + 1 | $(j - k) * N + i$ | $0 \leq i < N$ | $\max(i, k) \leq j < N$ | |
| | | $i * N + j + k + 1$ | $0 \leq i < k$ | $i \leq j < k$ | |
| 'L' | 2*k | $j * (N + 1) + i + 1$ | $0 \leq i < N$ | $0 \leq j \leq \min(i, k)$ | |

| <i>trans</i> <i>r</i> | <i>uplo</i> | N | l(i, j) = | i | j |
|--------------------------|-------------|-----------|---------------------------------|--------------------|----------------------------|
| | | | $(i - k) * (N + 1) + j - k$ | $k \leq i < N$ | $k \leq j \leq i$ |
| | | $2*k + 1$ | $j * N + i$ | $0 \leq i < N$ | $0 \leq j \leq \min(i, k)$ |
| | | | $(i - k) * N + j - k - 1$ | $k + 1 \leq i < N$ | $k + 1 \leq j \leq i$ |
| 'T' or 'C' | 'U' | $2*k$ | $i * k + j - k$ | $0 \leq i < N$ | $\max(i, k) \leq j < N$ |
| | | | $(j + k + 1) * k + i$ | $0 \leq i < k$ | $i \leq j < k$ |
| | | $2*k + 1$ | $i * (k + 1) + j - k$ | $0 \leq i < N$ | $\max(i, k) \leq j < N$ |
| | | | $(j + k + 1) * (k + 1) + i$ | $0 \leq i < k$ | $i \leq j < k$ |
| | 'L' | $2*k$ | $(i + 1) * k + j$ | $0 \leq i < N$ | $0 \leq j \leq \min(i, k)$ |
| | | | $(j - k) * k + i - k$ | $k \leq i < N$ | $k \leq j \leq i$ |
| | | $2*k + 1$ | $i * (k + 1) + j$ | $0 \leq i < N$ | $0 \leq j \leq \min(i, k)$ |
| | | | $(j - k - 1) * (k + 1) + i - k$ | $k + 1 \leq i < N$ | $k + 1 \leq j \leq i$ |

- Row major layout: *matrix_layout* = LAPACK_ROW_MAJOR

| <i>trans</i> <i>r</i> | <i>uplo</i> | N | l(i, j) = | i | j |
|--------------------------|-------------|-----------|---------------------------------|--------------------|----------------------------|
| 'N' | 'U' | $2*k$ | $i * k + j - k$ | $0 \leq i < N$ | $\max(i, k) \leq j < N$ |
| | | | $(k + j + 1) * k + i$ | $0 \leq i < k$ | $i \leq j < k$ |
| | | $2*k + 1$ | $i * (k + 1) + j - k$ | $0 \leq i < N$ | $\max(i, k) \leq j < N$ |
| | | | $(k + j + 1) * (k + 1) + i$ | $0 \leq i < k$ | $i \leq j < k$ |
| | 'L' | $2*k$ | $(i + 1) * k + j$ | $0 \leq i < N$ | $0 \leq j \leq \min(i, k)$ |
| | | | $(j - k) * k + i - k$ | $k \leq i < N$ | $k \leq j \leq i$ |
| | | $2*k + 1$ | $i * (k + 1) + j$ | $0 \leq i < N$ | $0 \leq j \leq \min(i, k)$ |
| | | | $(j - k - 1) * (k + 1) + i - k$ | $k + 1 \leq i < N$ | $k + 1 \leq j \leq i$ |
| 'T' or 'C' | 'U' | $2*k$ | $(j - k) * (N + 1) + i$ | $0 \leq i < N$ | $\max(i, k) \leq j < N$ |

| <i>trans</i> <i>r</i> | <i>uplo</i> | N | l(i, j) = | i | j |
|--------------------------|-------------|-----------|---------------------------|---------------------|----------------------------|
| | | | $i*(N + 1) + k + j + 1$ | $0 \leq i < k$ | $i \leq j < k$ |
| | | $2*k + 1$ | $(j - k)*N + i$ | $0 \leq i < N$ | $\max(i, k) \leq j < N$ |
| | | | $i*N + k + j + 1$ | $0 \leq i < k$ | $i \leq j < k$ |
| | | 'L' | $2*k$ | $j*(N + 1) + i + 1$ | $0 \leq i < N$ |
| | | | $(i - k)*(N + 1) + j - k$ | $k \leq i < N$ | $k \leq j \leq i$ |
| | $2*k + 1$ | | $j*N + i$ | $0 \leq i < N$ | $0 \leq j \leq \min(i, k)$ |
| | | | $(i - k)*N + j - k - 1$ | $k + 1 \leq i < N$ | $k + 1 \leq j \leq i$ |

NOTE

Although LAPACK accepts parameter values of zero for matrix size, in general the size of the array should be greater than or equal to $\max(1, N*(N + 1)/2)$.

Mathematical Notation for LAPACK Routines

Descriptions of LAPACK routines use the following notation:

A^H For an M -by- N matrix A , denotes the conjugate transposed N -by- M matrix with elements:

$$a_{i,j}^H = \overline{a_{j,i}}$$

For a real-valued matrix, $A^H = A^T$.

$x \cdot y$ The *dot product* of two vectors, defined as:

$$x \cdot y = \sum_i x_i \overline{y_i}$$

$Ax = b$ A system of linear equations with an n -by- n matrix $A = \{a_{ij}\}$, a right-hand side vector $b = \{b_i\}$, and an unknown vector $x = \{x_i\}$.

$AX = B$ A set of systems with a common matrix A and multiple right-hand sides. The columns of B are individual right-hand sides, and the columns of X are the corresponding solutions.

$|x|$ the vector with elements $|x_i|$ (absolute values of x_i).

$|A|$ the matrix with elements $|a_{ij}|$ (absolute values of a_{ij}).

$\|x\|_\infty = \max_i |x_i|$ The *infinity-norm* of the vector x .

$\|A\|_\infty = \max_i \sum_j |a_{ij}|$ The *infinity-norm* of the matrix A .

| | |
|------------------------------------|--|
| $\ A\ _1 = \max_j \sum_i a_{ij} $ | The <i>one-norm</i> of the matrix A . $\ A\ _1 = \ A^T\ _\infty = \ A^H\ _\infty$ |
| $\ x\ _2$ | The <i>2-norm</i> of the vector x : $\ x\ _2 = (\sum_i x_i ^2)^{1/2} = \ x\ _E$ (see the definition for <i>Euclidean norm</i> in this section). |
| $\ A\ _2$ | The <i>2-norm</i> (or <i>spectral norm</i>) of the matrix A . $\ A\ _2 = \max_i \sigma_i, \ A\ _2^2 = \max_{\ x\ _2=1} (Ax \cdot Ax)$ |
| $\ A\ _E$ | The <i>Euclidean norm</i> of the matrix A : $\ A\ _E^2 = \sum_i \sum_j a_{ij} ^2$. |
| $\kappa(A) = \ A\ \ A^{-1}\ $ | The <i>condition number</i> of the matrix A . |
| λ_j | <i>Eigenvalues</i> of the matrix A (for the definition of eigenvalues, see Eigenvalue Problems). |
| σ_j | <i>Singular values</i> of the matrix A . They are equal to square roots of the eigenvalues of $A^H A$. (For more information, see Singular Value Decomposition). |

Error Analysis

In practice, most computations are performed with rounding errors. Besides, you often need to solve a system $Ax = b$, where the data (the elements of A and b) are not known exactly. Therefore, it is important to understand how the data errors and rounding errors can affect the solution x .

Data perturbations. If x is the exact solution of $Ax = b$, and $x + \delta x$ is the exact solution of a perturbed problem $(A + \delta A)(x + \delta x) = (b + \delta b)$, then this estimate, given up to linear terms of perturbations, holds:

$$\frac{\|\delta x\|}{\|x\|} \leq \kappa(A) \left(\frac{\|\delta A\|}{\|A\|} + \frac{\|\delta b\|}{\|b\|} \right)$$

where $A + \delta A$ is nonsingular and

$$\kappa(A) = \|A\| \|A^{-1}\|.$$

In other words, relative errors in A or b may be amplified in the solution vector x by a factor $\kappa(A) = \|A\| \|A^{-1}\|$ called the *condition number* of A .

Rounding errors have the same effect as relative perturbations $c(n)\varepsilon$ in the original data. Here ε is the *machine precision*, defined as the smallest positive number x such that $1 + x > 1$; and $c(n)$ is a modest function of the matrix order n . The corresponding solution error is

$\|\delta x\| / \|x\| \leq c(n) \kappa(A) \varepsilon$. (The value of $c(n)$ is seldom greater than $10n$.)

NOTE

Machine precision depends on the data type used. For example, it is usually defined in the `float.h` file as `FLT_EPSILON` the `float` datatype and `DBL_EPSILON` for the `double` datatype.

Thus, if your matrix A is *ill-conditioned* (that is, its condition number $\kappa(A)$ is very large), then the error in the solution x can also be large; you might even encounter a complete loss of precision. LAPACK provides routines that allow you to estimate $\kappa(A)$ (see [Routines for Estimating the Condition Number](#)) and also give you a more precise estimate for the actual solution error (see [Refining the Solution and Estimating Its Error](#)).

LAPACK Linear Equation Routines

This section describes routines for performing the following computations:

- factoring the matrix (except for triangular matrices)
- equilibrating the matrix (except for RFP matrices)
- solving a system of linear equations
- estimating the condition number of a matrix (except for RFP matrices)
- refining the solution of linear equations and computing its error bounds (except for RFP matrices)
- inverting the matrix.

To solve a particular problem, you can call two or more [computational routines](#) or call a corresponding [driver routine](#) that combines several tasks in one call. For example, to solve a system of linear equations with a general matrix, call `?getrf` (*LU* factorization) and then `?getrs` (computing the solution). Then, call `?gerfs` to refine the solution and get the error bounds. Alternatively, use the driver routine `?gesvx` that performs all these tasks in one call.

LAPACK Linear Equation Computational Routines

Table "Computational Routines for Systems of Equations with Real Matrices" lists the LAPACK computational routines for factorizing, equilibrating, and inverting *real* matrices, estimating their condition numbers, solving systems of equations with real matrices, refining the solution, and estimating its error. Table "Computational Routines for Systems of Equations with Complex Matrices" lists similar routines for *complex* matrices.

Computational Routines for Systems of Equations with Real Matrices

| Matrix type, storage scheme | Factorize matrix | Equilibrate matrix | Solve system | Condition number | Estimate error | Invert matrix |
|---|----------------------|--|---------------------|---------------------|--|---------------------|
| general | <code>?getrf</code> | <code>?geequ,</code> <code>?geequb</code> | <code>?getrs</code> | <code>?gecon</code> | <code>?gerfs,</code> <code>?gerfsx</code> | <code>?getri</code> |
| general band | <code>?gbtrf</code> | <code>?gbequ,</code> <code>?gbequb</code> | <code>?gbtrs</code> | <code>?gbcon</code> | <code>?gbrfs,</code> <code>?gbrfsx</code> | |
| general tridiagonal | <code>?gttrf</code> | | <code>?gttrs</code> | <code>?gtcon</code> | <code>?gtrfs</code> | |
| diagonally dominant tridiagonal | <code>?dttrfb</code> | | <code>?dttrs</code> | | | |
| symmetric positive-definite | <code>?potrf</code> | <code>?poequ,</code> <code>?poequb</code> | <code>?potrs</code> | <code>?pocon</code> | <code>?porfs,</code> <code>?porfsx</code> | <code>?potri</code> |
| symmetric positive-definite, packed storage | <code>?pptrf</code> | <code>?ppequ</code> | <code>?pptrs</code> | <code>?ppcon</code> | <code>?pprfs</code> | <code>?pptri</code> |
| symmetric positive-definite, RFP storage | <code>?pftrf</code> | | <code>?pftrs</code> | | | <code>?pftri</code> |

| Matrix type, storage scheme | Factorize matrix | Equilibrate matrix | Solve system | Condition number | Estimate error | Invert matrix |
|--|---|-----------------------|-------------------|---------------------|--------------------|-------------------------------|
| symmetric positive-definite, band | ?pbtrf | ?pbequ | ?pbtrs | ?pbcon | ?pbrfs | |
| symmetric positive-definite, tridiagonal | ?pttrf | | ?pttrs | ?ptcon | ?ptrfs | |
| symmetric indefinite | ?sytrf | ?syequb | ?sytrs ?sytrs2 | ?sycon | ?syrfs, ?syrfsx | ?sytri ?sytri2 ?sytri2x |
| symmetric indefinite, packed storage | ?sptrf mkl_? spffrt2, mkl_? spffrtx | | ?sptrs | ?spcon | ?sprfs | ?sptri |
| triangular | | | ?trtrs | ?trcon | ?trrfs | ?trtri |
| triangular, packed storage | | | ?tptrs | ?tpcon | ?tprfs | ?tptri |
| triangular, RFP storage | | | | | | ?tftri |
| triangular band | | | ?tbtrs | ?tbcon | ?tbrfs | |

Computational Routines for Systems of Equations with Complex Matrices

| Matrix type, storage scheme | Factorize matrix | Equilibrate matrix | Solve system | Condition number | Estimate error | Invert matrix |
|---|---------------------|-----------------------|-----------------|---------------------|--------------------|---------------|
| general | ?getrf | ?geequ, ?geequb | ?getrs | ?gecon | ?gerfs, ?gerfsx | ?getri |
| general band | ?gbtrf | ?gbequ, ?gbequb | ?gbtrs | ?gbcon | ?gbrfs, ?gbrfsx | |
| general tridiagonal | ?gttrf | | ?gttrs | ?gtcon | ?gtrfs | |
| Hermitian positive-definite | ?potrf | ?poequ, ?poequb | ?potrs | ?pocon | ?porfs, ?porfsx | ?potri |
| Hermitian positive-definite, packed storage | ?pptrf | ?ppequ | ?pptrs | ?ppcon | ?pprfs | ?pptri |
| Hermitian positive-definite, RFP storage | ?pftrf | | ?pftrs | | | ?pftri |
| Hermitian positive-definite, band | ?pbtrf | ?pbequ | ?pbtrs | ?pbcon | ?pbrfs | |

| Matrix type, storage scheme | Factorize matrix | Equilibrate matrix | Solve system | Condition number | Estimate error | Invert matrix |
|--|--|-----------------------|---|---------------------|--|--|
| Hermitian positive-definite, tridiagonal | <code>?pttrf</code> | | <code>?pttrs</code> | <code>?ptcon</code> | <code>?ptrfs</code> | |
| Hermitian indefinite | <code>?hetrf</code> | <code>?heequb</code> | <code>?hetrs</code> <code>?hetrs2</code> | <code>?hecon</code> | <code>?herfs,</code> <code>?herfsx</code> | <code>?hetri</code> <code>?hetri2</code> <code>?hetri2x</code> |
| symmetric indefinite | <code>?sytrf</code> | <code>?syequb</code> | <code>?sytrs</code> <code>?sytrs2</code> | <code>?sycon</code> | <code>?syrfs,</code> <code>?syrfsx</code> | <code>?sytri</code> <code>?sytri2</code> <code>?sytri2x</code> |
| Hermitian indefinite, packed storage | <code>?hptrf</code> | | <code>?hptrs</code> | <code>?hpcon</code> | <code>?hprfs</code> | <code>?hptri</code> |
| symmetric indefinite, packed storage | <code>?sptrf</code> <code>mkl_?</code> <code>spffrt2,</code> <code>mkl_?</code> <code>spffrtx</code> | | <code>?sptrs</code> | <code>?spcon</code> | <code>?sprfs</code> | <code>?sptri</code> |
| triangular | | | <code>?trtrs</code> | <code>?trcon</code> | <code>?trrfs</code> | <code>?trtri</code> |
| triangular, packed storage | | | <code>?tptrs</code> | <code>?tpcon</code> | <code>?tprfs</code> | <code>?tptri</code> |
| triangular, RFP storage | | | | | | <code>?tftri</code> |
| triangular band | | | <code>?tbtrs</code> | <code>?tbcon</code> | <code>?tbrfs</code> | |

Matrix Factorization: LAPACK Computational Routines

This section describes the LAPACK routines for matrix factorization. The following factorizations are supported:

- *LU* factorization
- Cholesky factorization of real symmetric positive-definite matrices
- Cholesky factorization of real symmetric positive-definite matrices with pivoting
- Cholesky factorization of Hermitian positive-definite matrices
- Cholesky factorization of Hermitian positive-definite matrices with pivoting
- Bunch-Kaufman factorization of real and complex symmetric matrices
- Bunch-Kaufman factorization of Hermitian matrices.

You can compute:

- the *LU* factorization using full and band storage of matrices
- the Cholesky factorization using full, packed, RFP, and band storage
- the Bunch-Kaufman factorization using full and packed storage.

`?getrf`

Computes the *LU* factorization of a general *m*-by-*n* matrix.

Syntax

```
lapack_int LAPACKE_sgetrf (int matrix_layout , lapack_int m , lapack_int n , float *  
a , lapack_int lda , lapack_int * ipiv );
```

```
lapack_int LAPACKE_dgetrf (int matrix_layout , lapack_int m , lapack_int n , double *  
a , lapack_int lda , lapack_int * ipiv );
```

```
lapack_int LAPACKE_cgetrf (int matrix_layout , lapack_int m , lapack_int n ,  
lapack_complex_float * a , lapack_int lda , lapack_int * ipiv );
```

```
lapack_int LAPACKE_zgetrf (int matrix_layout , lapack_int m , lapack_int n ,  
lapack_complex_double * a , lapack_int lda , lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the LU factorization of a general m -by- n matrix A as

$$A = P*LU,$$

where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). The routine uses partial pivoting, with row interchanges.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows in the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns in A ; $n \geq 0$. |
| <i>a</i> | Array, size at least $\max(1, lda*n)$ for column-major layout or $\max(1, lda*m)$ for row-major layout. Contains the matrix A . |
| <i>lda</i> | The leading dimension of array a , which must be at least $\max(1, m)$ for column-major layout or $\max(1, n)$ for row-major layout. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | Overwritten by L and U . The unit diagonal elements of L are not stored. |
| <i>ipiv</i> | Array, size at least $\max(1, \min(m, n))$. The pivot indices; for $1 \leq i \leq \min(m, n)$, row i was interchanged with row $ipiv(i)$. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, u_{ii} is 0. The factorization has been completed, but U is exactly singular. Division by 0 will occur if you use the factor U for solving a system of linear equations.

Application Notes

The computed L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(\min(m, n)) \varepsilon P |L| |U|$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

The approximate number of floating-point operations for real flavors is

$$(2/3)n^3 \quad \text{If } m = n,$$

$$(1/3)n^2(3m-n) \quad \text{If } m > n,$$

$$(1/3)m^2(3n-m) \quad \text{If } m < n.$$

The number of operations for complex flavors is four times greater.

After calling this routine with $m = n$, you can call the following:

`?getrs` to solve $A^*X = B$ or $A^T X = B$ or $A^H X = B$

`?gecon` to estimate the condition number of A

`?getri` to compute the inverse of A .

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

mkl_?getrfnpi

Performs LU factorization (complete or incomplete) of a general matrix without pivoting.

Syntax

```
lapack_int LAPACKE_mkl_sgetrfnpi (int matrix_layout, lapack_int m, lapack_int n,
lapack_int nfact, float* a, lapack_int lda);
```

```
lapack_int LAPACKE_mkl_dgetrfnpi (int matrix_layout, lapack_int m, lapack_int n,
lapack_int nfact, double* a, lapack_int lda);
```

```
lapack_int LAPACKE_mkl_cgetrfnpi (int matrix_layout, lapack_int m, lapack_int n,
lapack_int nfact, lapack_complex_float* a, lapack_int lda);
```

```
lapack_int LAPACKE_mkl_zgetrfnpi (int matrix_layout, lapack_int m, lapack_int n,
lapack_int nfact, lapack_complex_double* a, lapack_int lda);
```

Include Files

- `mkl.h`

Description

The routine computes the LU factorization of a general m -by- n matrix A without using pivoting. It supports incomplete factorization. The factorization has the form:

$$A = L^*U,$$

where L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$).

Incomplete factorization has the form:

$$A = L * U + \tilde{A}$$

where L is lower trapezoidal with unit diagonal elements, U is upper trapezoidal, and \tilde{A} is the unfactored part of matrix A . See the application notes section for further details.

NOTE

Use `?getrf` if it is possible that the matrix is not diagonal dominant.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>m</code> | The number of rows in matrix A ; $m \geq 0$. |
| <code>n</code> | The number of columns in matrix A ; $n \geq 0$. |
| <code>nfact</code> | The number of rows and columns to factor; $0 \leq \text{nfact} \leq \min(m, n)$. Note that if $\text{nfact} < \min(m, n)$, incomplete factorization is performed. |
| <code>a</code> | Array of size at least $\text{lda} * n$ for column major layout and at least $\text{lda} * m$ for row major layout. Contains the matrix A . |
| <code>lda</code> | The leading dimension of array a . $\text{lda} \geq \max(1, m)$ for column major layout and $\text{lda} \geq \max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------------|--|
| <code>a</code> | Overwritten by L and U . The unit diagonal elements of L are not stored. When incomplete factorization is specified by setting $\text{nfact} < \min(m, n)$, a also contains the unfactored submatrix \tilde{A}_{22} . See the application notes section for further details. |
|----------------|--|

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

If `info = i`, u_{ij} is 0. The requested factorization has been completed, but U is exactly singular. Division by 0 will occur if factorization is completed and factor U is used for solving a system of linear equations.

Application Notes

The computed L and U are the exact factors of a perturbed matrix $A + E$, with

$$|E| \leq c(\min(m, n)) \epsilon |L| |U|$$

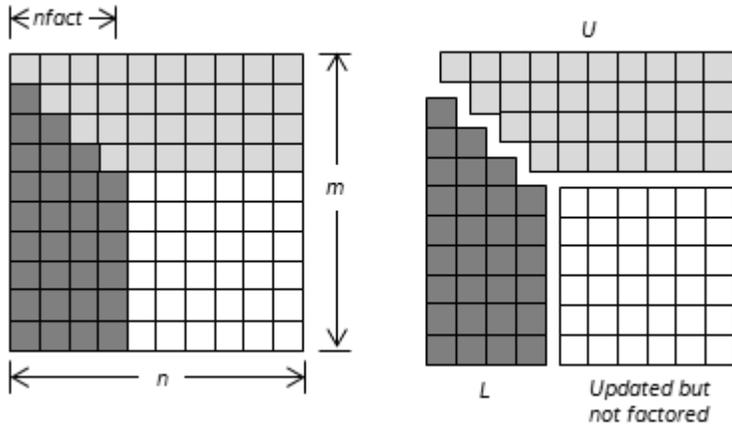
where $c(n)$ is a modest linear function of n , and ε is the machine precision.

The approximate number of floating-point operations for real flavors is

| | |
|---|--------------------------------|
| $(2/3)n^3$ | If $m = n = nfact$ |
| $(1/3)n^2(3m-n)$ | If $m > n = nfact$ |
| $(1/3)m^2(3n-m)$ | If $m = nfact < n$ |
| $(2/3)n^3 - (n-nfact)^3$ | If $m = n, nfact < \min(m, n)$ |
| $(1/3)(n^2(3m-n) - (n-nfact)^2(3m - 2nfact - n))$ | If $m > n > nfact$ |
| $(1/3)(m^2(3n-m) - (m-nfact)^2(3n - 2nfact - m))$ | If $nfact < m < n$ |

The number of operations for complex flavors is four times greater.

When incomplete factorization is specified, the first $nfact$ rows and columns are factored, with the update of the remaining rows and columns of A as follows:



If matrix A is represented as a block 2-by-2 matrix:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

where

- A_{11} is a square matrix of order $nfact$,
- A_{21} is an $(m - nfact)$ -by- $nfact$ matrix,
- A_{12} is an $nfact$ -by- $(n - nfact)$ matrix, and
- A_{22} is an $(m - nfact)$ -by- $(n - nfact)$ matrix.

The result is

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \cdot [U_1 \ U_2] + \begin{bmatrix} 0 & 0 \\ 0 & \tilde{A}_{22} \end{bmatrix}$$

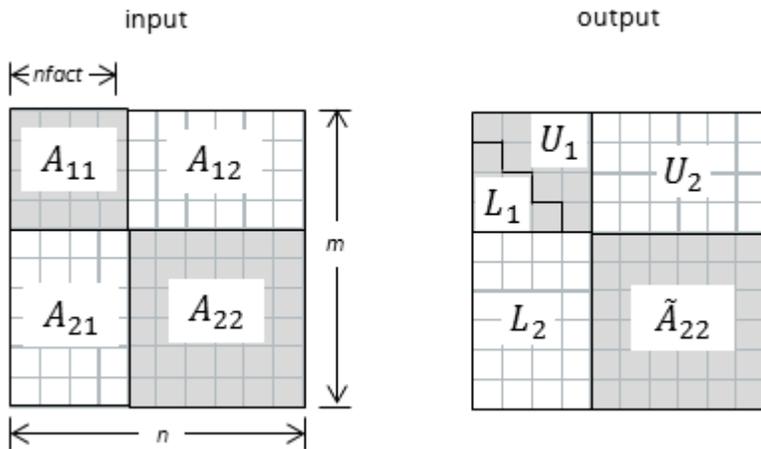
L_1 is a lower triangular square matrix of order $nfact$ with unit diagonal and U_1 is an upper triangular square matrix of order $nfact$. L_1 and U_1 result from LU factorization of matrix A_{11} : $A_{11} = L_1 U_1$.

L_2 is an $(m - nfact)$ -by- $nfact$ matrix and $L_2 = A_{21}U_1^{-1}$. U_2 is an $nfact$ -by- $(n - nfact)$ matrix and $U_2 = L_1^{-1}A_{12}$.

\tilde{A}_{22} is an $(m - nfact)$ -by- $(n - nfact)$ matrix and $\tilde{A}_{22} = A_{22} - L_2U_2$.

On exit, elements of the upper triangle U_1 are stored in place of the upper triangle of block A_{11} in array a ; elements of the lower triangle L_1 are stored in the lower triangle of block A_{11} in array a (unit diagonal

elements are not stored). Elements of L_2 replace elements of A_{21} ; U_2 replaces elements of A_{12} and \tilde{A}_{22} replaces elements of A_{22} .



?getrf2

Computes LU factorization using partial pivoting with row interchanges.

Syntax

```
lapack_int LAPACKE_sgetrf2 (int matrix_layout, lapack_int m, lapack_int n, float * a,
lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_dgetrf2 (int matrix_layout, lapack_int m, lapack_int n, double * a,
lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_cgetrf2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float * a, lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_zgetrf2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double * a, lapack_int lda, lapack_int * ipiv);
```

Include Files

- mkl.h

Description

?getrf2 computes an LU factorization of a general m -by- n matrix A using partial pivoting with row interchanges.

The factorization has the form

$$A = P * L * U$$

where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$), and U is upper triangular (upper trapezoidal if $m < n$).

This is the recursive version of the algorithm. It divides the matrix into four submatrices:

$$A = \begin{pmatrix} A11 & A12 \\ A21 & A22 \end{pmatrix}$$

where $A11$ is $n1$ by $n1$ and $A22$ is $n2$ by $n2$ with $n1 = \min(m, n)$, and $n2 = n - n1$.

The subroutine calls itself to factor $\begin{pmatrix} A11 \\ A12 \end{pmatrix}$,

do the swaps on $\begin{pmatrix} A12 \\ A22 \end{pmatrix}$, solve $A12$, update $A22$, then it calls itself to factor $A22$ and do the swaps on $A21$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix A. $m \geq 0$. |
| <i>n</i> | The number of columns of the matrix A. $n \geq 0$. |
| <i>a</i> | Array, size $lda*n$. On entry, the m -by- n matrix to be factored. |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1, m)$. |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | On exit, the factors L and U from the factorization $A = P * L * U$; the unit diagonal elements of L are not stored. |
| <i>ipiv</i> | Array, size $\min(m, n)$. The pivot indices; for $1 \leq i \leq \min(m, n)$, row i of the matrix was interchanged with row $ipiv[i - 1]$. |

Return Values

This function returns a value *info*.

= 0: successful exit

< 0: if $info = -i$, the i -th argument had an illegal value.

> 0: if $info = i$, $U_{i, i}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

?gbtrf

Computes the LU factorization of a general m -by- n band matrix.

Syntax

```
lapack_int LAPACKE_sgbtrf (int matrix_layout , lapack_int m , lapack_int n , lapack_int
kl , lapack_int ku , float * ab , lapack_int ldab , lapack_int * ipiv );
```

```
lapack_int LAPACKE_dgbtrf (int matrix_layout , lapack_int m , lapack_int n , lapack_int
kl , lapack_int ku , double * ab , lapack_int ldab , lapack_int * ipiv );
```

```
lapack_int LAPACKE_cgbtrf (int matrix_layout , lapack_int m , lapack_int n , lapack_int
kl , lapack_int ku , lapack_complex_float * ab , lapack_int ldab , lapack_int * ipiv );
lapack_int LAPACKE_zgbtrf (int matrix_layout , lapack_int m , lapack_int n , lapack_int
kl , lapack_int ku , lapack_complex_double * ab , lapack_int ldab , lapack_int *
ipiv );
```

Include Files

- mkl.h

Description

The routine forms the LU factorization of a general m -by- n band matrix A with kl non-zero subdiagonals and ku non-zero superdiagonals, that is,

$$A = P * L * U,$$

where P is a permutation matrix; L is lower triangular with unit diagonal elements and at most kl non-zero elements in each column; U is an upper triangular band matrix with $kl + ku$ superdiagonals. The routine uses partial pivoting, with row interchanges (which creates the additional kl superdiagonals in U).

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows in matrix A ; $m \geq 0$. |
| <i>n</i> | The number of columns in matrix A ; $n \geq 0$. |
| <i>kl</i> | The number of subdiagonals within the band of A ; $kl \geq 0$. |
| <i>ku</i> | The number of superdiagonals within the band of A ; $ku \geq 0$. |
| <i>ab</i> | Array, size at least $\max(1, ldab * n)$ for column-major layout or $\max(1, ldab * m)$ for row-major layout. The array <i>ab</i> contains the matrix A in band storage as described in Band Storage . |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> . ($ldab \geq 2 * kl + ku + 1$) |

Output Parameters

| | |
|-----------|---|
| <i>ab</i> | Overwritten with elements of L and U . U is stored as an upper triangular band matrix with $kl + ku$ superdiagonals, and L is stored as a lower triangular band matrix with kl subdiagonals (diagonal unit values are not stored). Since the output array has more nonzero elements than the initial matrix A , there are limitations on the value of <i>ldab</i> and the placement of elements of A in array <i>ab</i> . |
|-----------|---|

See Application Notes below for further details.

ipiv Array, size at least $\max(1, \min(m, n))$. The pivot indices; for $1 \leq i \leq \min(m, n)$, row i was interchanged with row $ipiv(i)$.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, u_{ii} is 0. The factorization has been completed, but U is exactly singular. Division by 0 will occur if you use the factor U for solving a system of linear equations.

Application Notes

The computed L and U are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(kl+ku+1) \varepsilon P |L| |U|$$

$c(k)$ is a modest linear function of k , and ε is the machine precision.

The total number of floating-point operations for real flavors varies between approximately $2n(ku+1)kl$ and $2n(kl+ku+1)kl$. The number of operations for complex flavors is four times greater. All these estimates assume that kl and ku are much less than $\min(m, n)$.

As described in [Band Storage](#), storage of a band matrix can be considered in two steps: packing band matrix elements into a matrix AB , then storing the elements in a linear array *ab* using a full storage scheme. The effect of the `?gbtrf` routine on matrix AB is illustrated by this example, for $m = n = 6$, $kl = 2$, $ku = 1$.

- `matrix_layout = LAPACK_COL_MAJOR`

| | |
|--|---|
| <p>On entry:</p> $AB = \begin{bmatrix} * & * & * & + & + & + \\ * & * & + & + & + & + \\ * & a_{1,2} & a_{2,3} & a_{3,4} & a_{4,5} & a_{5,6} \\ a_{1,1} & a_{2,2} & a_{3,3} & a_{4,4} & a_{5,5} & a_{6,6} \\ a_{2,1} & a_{3,2} & a_{4,3} & a_{5,4} & a_{6,5} & * \\ a_{3,1} & a_{4,2} & a_{5,3} & a_{6,4} & * & * \end{bmatrix}$ | <p>On exit:</p> $AB = \begin{bmatrix} * & * & * & u_{1,4} & u_{2,5} & u_{3,6} \\ * & * & u_{1,3} & u_{2,4} & u_{3,5} & u_{4,6} \\ * & u_{1,2} & u_{2,3} & u_{3,4} & u_{4,5} & u_{5,6} \\ u_{1,1} & u_{2,2} & u_{3,3} & u_{4,4} & u_{5,5} & u_{6,6} \\ l_{2,1} & l_{3,2} & l_{4,3} & l_{5,4} & l_{6,5} & * \\ l_{3,1} & l_{4,2} & l_{5,3} & l_{6,4} & * & * \end{bmatrix}$ |
|--|---|

- `matrix_layout = LAPACK_ROW_MAJOR`

| | |
|--|---|
| <p>On entry:</p> $AB = \begin{bmatrix} * & * & a_{1,1} & a_{1,2} & + & + \\ * & a_{2,1} & a_{2,2} & a_{2,3} & + & + \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} & + & + \\ a_{4,2} & a_{4,3} & a_{4,4} & a_{4,5} & + & * \\ a_{5,3} & a_{5,4} & a_{5,5} & a_{5,6} & * & * \\ a_{6,4} & a_{6,5} & a_{6,6} & * & * & * \end{bmatrix}$ | <p>On exit:</p> $AB = \begin{bmatrix} * & * & u_{1,1} & u_{1,2} & u_{1,3} & u_{1,4} \\ * & l_{2,1} & u_{2,2} & u_{2,3} & u_{2,4} & u_{2,5} \\ l_{3,1} & l_{3,2} & u_{3,3} & u_{3,4} & u_{3,5} & u_{3,6} \\ l_{4,2} & l_{4,3} & u_{4,4} & u_{4,5} & u_{4,6} & * \\ l_{5,3} & l_{5,4} & u_{5,5} & u_{5,6} & * & * \\ l_{6,4} & l_{6,5} & u_{6,6} & * & * & * \end{bmatrix}$ |
|--|---|

Elements marked * are not used; elements marked + need not be set on entry, but are required by the routine to store elements of U because of fill-in resulting from the row interchanges.

After calling this routine with $m = n$, you can call the following routines:

`gbtrs` to solve $A^*X = B$ or $A^T * X = B$ or $A^H * X = B$

`gbcon` to estimate the condition number of A .

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?gttrf

Computes the LU factorization of a tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_sgttrf (lapack_int n , float * dl , float * d , float * du , float *
du2 , lapack_int * ipiv );
```

```
lapack_int LAPACKE_dgttrf (lapack_int n , double * dl , double * d , double * du ,
double * du2 , lapack_int * ipiv );
```

```
lapack_int LAPACKE_cgttrf (lapack_int n , lapack_complex_float * dl ,
lapack_complex_float * d , lapack_complex_float * du , lapack_complex_float * du2 ,
lapack_int * ipiv );
```

```
lapack_int LAPACKE_zgttrf (lapack_int n , lapack_complex_double * dl ,
lapack_complex_double * d , lapack_complex_double * du , lapack_complex_double * du2 ,
lapack_int * ipiv );
```

Include Files

- `mkl.h`

Description

The routine computes the LU factorization of a real or complex tridiagonal matrix A using elimination with partial pivoting and row interchanges.

The factorization has the form

$$A = L^*U,$$

where L is a product of permutation and unit lower bidiagonal matrices and U is upper triangular with nonzeros in only the main diagonal and first two superdiagonals.

Input Parameters

n The order of the matrix A ; $n \geq 0$.

dl, d, du Arrays containing elements of A .

The array dl of dimension $(n - 1)$ contains the subdiagonal elements of A .

The array d of dimension n contains the diagonal elements of A .

The array du of dimension $(n - 1)$ contains the superdiagonal elements of A .

Output Parameters

| | |
|-------------|--|
| <i>dl</i> | Overwritten by the $(n-1)$ multipliers that define the matrix L from the LU factorization of A . |
| <i>d</i> | Overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A . |
| <i>du</i> | Overwritten by the $(n-1)$ elements of the first superdiagonal of U . |
| <i>du2</i> | Array, dimension $(n-2)$. On exit, <i>du2</i> contains $(n-2)$ elements of the second superdiagonal of U . |
| <i>ipiv</i> | Array, dimension (n) . The pivot indices: for $1 \leq i \leq n$, row i was interchanged with row $ipiv[i-1]$. $ipiv[i-1]$ is always i or $i+1$; $ipiv[i-1] = i$ indicates a row interchange was not required. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, u_{ii} is 0. The factorization has been completed, but U is exactly singular. Division by zero will occur if you use the factor U for solving a system of linear equations.

Application Notes

| | |
|--------|--|
| ?gbtrs | to solve $A * X = B$ or $A^T * X = B$ or $A^H * X = B$ |
| ?gbcon | to estimate the condition number of A . |

?dttfbb

Computes the factorization of a diagonally dominant tridiagonal matrix.

Syntax

```
void sdttrfb (const MKL_INT * n , float * dl , float * d , const float * du , MKL_INT * info );
void ddttrfb (const MKL_INT * n , double * dl , double * d , const double * du , MKL_INT * info );
void cdttrfb (const MKL_INT * n , MKL_Complex8 * dl , MKL_Complex8 * d , const MKL_Complex8 * du , MKL_INT * info );
void zdttrfb_ (const MKL_INT * n , MKL_Complex16 * dl , MKL_Complex16 * d , const MKL_Complex16 * du , MKL_INT * info );
```

Include Files

- mkl.h

Description

The `?dtttrfb` routine computes the factorization of a real or complex tridiagonal matrix A with the BABE (Burning At Both Ends) algorithm without pivoting. The factorization has the form

$$A = L_1 * U * L_2$$

where

- L_1 and L_2 are unit lower bidiagonal with k and $n - k - 1$ subdiagonal elements, respectively, where $k = n/2$, and
- U is an upper bidiagonal matrix with nonzeros in only the main diagonal and first superdiagonal.

Input Parameters

| | |
|-------------|---|
| n | The order of the matrix A ; $n \geq 0$. |
| dl, d, du | Arrays containing elements of A . The array dl of dimension $(n - 1)$ contains the subdiagonal elements of A . The array d of dimension n contains the diagonal elements of A . The array du of dimension $(n - 1)$ contains the superdiagonal elements of A . |

Output Parameters

| | |
|--------|---|
| dl | Overwritten by the $(n - 1)$ multipliers that define the matrix L from the LU factorization of A . |
| d | Overwritten by the n diagonal element reciprocals of the upper triangular matrix U from the factorization of A . |
| du | Overwritten by the $(n-1)$ elements of the superdiagonal of U . |
| $info$ | If $info = 0$, the execution is successful. If $info = -i$, the i -th parameter had an illegal value. If $info = i$, u_{ii} is 0. The factorization has been completed, but U is exactly singular. Division by zero will occur if you use the factor U for solving a system of linear equations. |

Application Notes

A diagonally dominant tridiagonal system is defined such that $|d_i| > |dl_{i-1}| + |du_i|$ for any i :

$$1 < i < n, \text{ and } |d_1| > |du_1|, |d_n| > |dl_{n-1}|$$

The underlying BABE algorithm is designed for diagonally dominant systems. Such systems are free from the numerical stability issue unlike the canonical systems that use elimination with partial pivoting (see `?gttrf`). The diagonally dominant systems are much faster than the canonical systems.

NOTE

- The current implementation of BABE has a potential accuracy issue on very small or large data close to the underflow or overflow threshold respectively. Scale the matrix before applying the solver in the case of such input data.
 - Applying the `?dtttrfb` factorization to non-diagonally dominant systems may lead to an accuracy loss, or false singularity detected due to no pivoting.
-

?potrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite matrix.

Syntax

```
lapack_int LAPACKE_spotrf (int matrix_layout , char uplo , lapack_int n , float * a ,
lapack_int lda );

lapack_int LAPACKE_dpotrf (int matrix_layout , char uplo , lapack_int n , double * a ,
lapack_int lda );

lapack_int LAPACKE_cpotrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda );

lapack_int LAPACKE_zpotrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda );
```

Include Files

- mkl.h

Description

The routine forms the Cholesky factorization of a symmetric positive-definite or, for complex data, Hermitian positive-definite matrix A :

$$\begin{array}{ll}
 A = U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} & \text{if } uplo = 'U' \\
 A = L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} & \text{if } uplo = 'L'
 \end{array}$$

where L is a lower triangular matrix and U is upper triangular.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored and how A is factored: If <i>uplo</i> = 'U', the array <i>a</i> stores the upper triangular part of the matrix A , and the strictly lower triangular part of the matrix is not referenced. If <i>uplo</i> = 'L', the array <i>a</i> stores the lower triangular part of the matrix A , and the strictly upper triangular part of the matrix is not referenced. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>a</i> | Array, size $\max(1, lda * n)$. The array <i>a</i> contains either the upper or the lower triangular part of the matrix A (see <i>uplo</i>). |

lda The leading dimension of *a*. Must be at least $\max(1, n)$.

Output Parameters

a The upper or lower triangular part of *a* is overwritten by the Cholesky factor *U* or *L*, as specified by *uplo*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = *-i*, parameter *i* had an illegal value.

If *info* = *i*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, and the factorization could not be completed. This may indicate an error in forming the matrix *A*.

Application Notes

If *uplo* = 'U', the computed factor *U* is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\varepsilon |U^H| |U|, |e_{ij}| \leq c(n)\varepsilon \sqrt{a_{ii}a_{jj}}$$

$c(n)$ is a modest linear function of *n*, and ε is the machine precision.

A similar estimate holds for *uplo* = 'L'.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors or $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

| | |
|---------------------|--|
| <code>?potrs</code> | to solve $A * X = B$ |
| <code>?pocon</code> | to estimate the condition number of <i>A</i> |
| <code>?potri</code> | to compute the inverse of <i>A</i> . |

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?potrf2

Computes Cholesky factorization using a recursive algorithm.

Syntax

```
lapack_int LAPACKE_spotrf2 (int matrix_layout, char uplo, lapack_int n, float * a,
lapack_int lda);
```

```
lapack_int LAPACKE_dpotrf2 (int matrix_layout, char uplo, lapack_int n, double * a,
lapack_int lda);
```

```
lapack_int LAPACKE_cpotrf2 (int matrix_layout, char uplo, lapack_int n,
lapack_complex_float * a, lapack_int lda);
```

```
lapack_int LAPACKE_zpotrf2 (int matrix_layout, char uplo, lapack_int n,
lapack_complex_double * a, lapack_int lda);
```

Include Files

- mkl.h

Description

`zpotrf2` computes the Cholesky factorization of a real or complex symmetric positive definite matrix A using the recursive algorithm.

The factorization has the form

for real flavors:

$$A = U^T * U, \text{ if } uplo = 'U', \text{ or}$$

$$A = L * L^T, \text{ if } uplo = 'L',$$

for complex flavors:

$$A = U^H * U, \text{ if } uplo = 'U',$$

$$\text{or } A = L * L^H, \text{ if } uplo = 'L',$$

where U is an upper triangular matrix and L is lower triangular.

This is the recursive version of the algorithm. It divides the matrix into four submatrices:

$$A = \begin{pmatrix} A11 & A12 \\ A21 & A22 \end{pmatrix}$$

where $A11$ is $n1$ by $n1$ and $A22$ is $n2$ by $n2$, with $n1 = n/2$ and $n2 = n-n1$.

The subroutine calls itself to factor $A11$. Update and scale $A21$ or $A12$, update $A22$ then call itself to factor $A22$.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | = 'U': Upper triangle of A is stored; = 'L': Lower triangle of A is stored. |
| <code>n</code> | The order of the matrix A . $n \geq 0$. |
| <code>a</code> | Array, size ($lda*n$). On entry, the symmetric matrix A . If <code>uplo = 'U'</code> , the leading n -by- n upper triangular part of a contains the upper triangular part of the matrix A , and the strictly lower triangular part of a is not referenced. If <code>uplo = 'L'</code> , the leading n -by- n lower triangular part of a contains the lower triangular part of the matrix A , and the strictly upper triangular part of a is not referenced. |
| <code>lda</code> | The leading dimension of the array a . $lda \geq \max(1, n)$. |

Output Parameters

a On exit, if *info* = 0, the factor *U* or *L* from the Cholesky factorization.
 For real flavors:
 $A = U^T * U$ or $A = L * L^T$;
 For complex flavors:
 $A = U^H * U$ or $A = L * L^H$.

Return Values

This function returns a value *info*.

= 0: successful exit

< 0: if *info* = -*i*, the *i*-th argument had an illegal value

> 0: if *info* = *i*, the leading minor of order *i* is not positive definite, and the factorization could not be completed.

?pstrf

Computes the Cholesky factorization with complete pivoting of a real symmetric (complex Hermitian) positive semidefinite matrix.

Syntax

```
lapack_int LAPACKE_spstrf( int matrix_layout, char uplo, lapack_int n, float* a,
lapack_int lda, lapack_int* piv, lapack_int* rank, float tol );

lapack_int LAPACKE_dpstrf( int matrix_layout, char uplo, lapack_int n, double* a,
lapack_int lda, lapack_int* piv, lapack_int* rank, double tol );

lapack_int LAPACKE_cpstrf( int matrix_layout, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_int* piv, lapack_int* rank, float
tol );

lapack_int LAPACKE_zpstrf( int matrix_layout, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_int* piv, lapack_int* rank, double
tol );
```

Include Files

- mkl.h

Description

The routine computes the Cholesky factorization with complete pivoting of a real symmetric (complex Hermitian) positive semidefinite matrix. The form of the factorization is:

$$\begin{aligned}
 P^T * A * P &= U^T * U, \text{ if } uplo = 'U' \text{ for real flavors,} \\
 P^T * A * P &= U^H * U, \text{ if } uplo = 'U' \text{ for complex flavors,} \\
 P^T * A * P &= L * L^T, \text{ if } uplo = 'L' \text{ for real flavors,} \\
 P^T * A * P &= L * L^H, \text{ if } uplo = 'L' \text{ for complex flavors,}
 \end{aligned}$$

where *P* is a permutation matrix stored as vector *piv*, and *U* and *L* are upper and lower triangular matrices, respectively.

This algorithm does not attempt to check that *A* is positive semidefinite. This version of the algorithm calls level 3 BLAS.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <code>uplo</code> = 'U', the array a stores the upper triangular part of the matrix A , and the strictly lower triangular part of the matrix is not referenced. If <code>uplo</code> = 'L', the array a stores the lower triangular part of the matrix A , and the strictly upper triangular part of the matrix is not referenced. |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>a</code> | Array a , size $\max(1, lda * n)$. The array a contains either the upper or the lower triangular part of the matrix A (see <code>uplo</code>). |
| <code>tol</code> | User defined tolerance. If <code>tol</code> < 0, then $n * \epsilon * \max(A_{k,k})$, where ϵ is the machine precision, will be used (see Error Analysis for the definition of machine precision). The algorithm terminates at the $(k-1)$ -st step, if the pivot $\leq tol$. |
| <code>lda</code> | The leading dimension of a ; at least $\max(1, n)$. |

Output Parameters

| | |
|-------------------|--|
| <code>a</code> | If <code>info</code> = 0, the factor U or L from the Cholesky factorization is as described in <i>Description</i> . |
| <code>piv</code> | Array, size at least $\max(1, n)$. The array <code>piv</code> is such that the nonzero entries are $P_{piv[k-1], k}$ ($1 \leq k \leq n$). |
| <code>rank</code> | The rank of a given by the number of steps the algorithm completed. |

Return Values

This function returns a value `info`.

If `info` = 0, the execution is successful.

If `info` = $-k$, the k -th argument had an illegal value.

If `info` > 0, the matrix A is either rank deficient with a computed rank as returned in `rank`, or is not positive semidefinite.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pstrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite matrix using the Rectangular Full Packed (RFP) format .

Syntax

```
lapack_int LAPACKE_spftrf (int matrix_layout , char transr , char uplo , lapack_int n ,
float * a );
```

```
lapack_int LAPACKE_dpftrf (int matrix_layout , char transr , char uplo , lapack_int n ,
double * a );
```

```
lapack_int LAPACKE_cpftrf (int matrix_layout , char transr , char uplo , lapack_int n ,
lapack_complex_float * a );
```

```
lapack_int LAPACKE_zpftrf (int matrix_layout , char transr , char uplo , lapack_int n ,
lapack_complex_double * a );
```

Include Files

- mkl.h

Description

The routine forms the Cholesky factorization of a symmetric positive-definite or, for complex data, a Hermitian positive-definite matrix A :

$$\begin{array}{ll}
 A = U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} & \text{if } uplo = 'U' \\
 A = L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} & \text{if } uplo = 'L'
 \end{array}$$

where L is a lower triangular matrix and U is upper triangular.

The matrix A is in the Rectangular Full Packed (RFP) format. For the description of the RFP format, see [Matrix Storage Schemes](#).

This is the block version of the algorithm, calling Level 3 BLAS.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>transr</i> | Must be 'N', 'T' (for real data) or 'C' (for complex data). If <i>transr</i> = 'N', the Normal <i>transr</i> of RFP A is stored. If <i>transr</i> = 'T', the Transpose <i>transr</i> of RFP A is stored. If <i>transr</i> = 'C', the Conjugate-Transpose <i>transr</i> of RFP A is stored. |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <i>uplo</i> = 'U', the array a stores the upper triangular part of the matrix A . If <i>uplo</i> = 'L', the array a stores the lower triangular part of the matrix A . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array, size $(n * (n + 1) / 2)$. The array a contains the matrix A in the RFP format. |

Output Parameters

a *a* is overwritten by the Cholesky factor *U* or *L*, as specified by *uplo* and *trans*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, and the factorization could not be completed. This may indicate an error in forming the matrix *A*.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pptrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite matrix using packed storage.

Syntax

```
lapack_int LAPACKE_spptrf (int matrix_layout , char uplo , lapack_int n , float * ap );
lapack_int LAPACKE_dpptrf (int matrix_layout , char uplo , lapack_int n , double *
ap );
lapack_int LAPACKE_cpptf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * ap );
lapack_int LAPACKE_zpptrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * ap );
```

Include Files

- mkl.h

Description

The routine forms the Cholesky factorization of a symmetric positive-definite or, for complex data, Hermitian positive-definite packed matrix *A*:

$$\begin{array}{ll}
 A = U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} & \text{if } uplo = 'U' \\
 A = L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} & \text{if } uplo = 'L'
 \end{array}$$

where *L* is a lower triangular matrix and *U* is upper triangular.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

| | |
|-------------|---|
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is packed in the array ap , and how A is factored: If $uplo = 'U'$, the array ap stores the upper triangular part of the matrix A , and A is factored as U^H*U . If $uplo = 'L'$, the array ap stores the lower triangular part of the matrix A ; A is factored as $L*L^H$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>ap</i> | Array, size at least $\max(1, n(n+1)/2)$. The array ap contains either the upper or the lower triangular part of the matrix A (as specified by $uplo$) in packed storage (see Matrix Storage Schemes). |

Output Parameters

| | |
|-----------|--|
| <i>ap</i> | Overwritten by the Cholesky factor U or L , as specified by $uplo$. |
|-----------|--|

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, the leading minor of order i (and therefore the matrix A itself) is not positive-definite, and the factorization could not be completed. This may indicate an error in forming the matrix A .

Application Notes

If $uplo = 'U'$, the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\varepsilon |U^H| |U|, |e_{ij}| \leq c(n)\varepsilon \sqrt{a_{ii}a_{jj}}$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

A similar estimate holds for $uplo = 'L'$.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors and $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

| | |
|------------------------|---|
| ?pptrs | to solve $A*X = B$ |
| ?ppcon | to estimate the condition number of A |
| ?pptri | to compute the inverse of A . |

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?pbtrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite band matrix.

Syntax

```
lapack_int LAPACKE_spbtrf (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , float * ab , lapack_int ldab );
```

```
lapack_int LAPACKE_dpbtrf (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , double * ab , lapack_int ldab );
```

```
lapack_int LAPACKE_cpbtrf (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_complex_float * ab , lapack_int ldab );
```

```
lapack_int LAPACKE_zpbtrf (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_complex_double * ab , lapack_int ldab );
```

Include Files

- mkl.h

Description

The routine forms the Cholesky factorization of a symmetric positive-definite or, for complex data, Hermitian positive-definite band matrix A :

$$\begin{aligned}
 A &= U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} && \text{if } uplo = 'U' \\
 A &= L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} && \text{if } uplo = 'L'
 \end{aligned}$$

where L is a lower triangular matrix and U is upper triangular.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored in the array ab , and how A is factored: If $uplo = 'U'$, the upper triangle of A is stored. If $uplo = 'L'$, the lower triangle of A is stored. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>kd</i> | The number of superdiagonals or subdiagonals in the matrix A ; $kd \geq 0$. |
| <i>ab</i> | Array, size $\max(1, ldab * n)$. The array ab contains either the upper or the lower triangular part of the matrix A (as specified by $uplo$) in band storage (see Matrix Storage Schemes). |
| <i>ldab</i> | The leading dimension of the array ab . ($ldab \geq kd + 1$) |

Output Parameters

ab The upper or lower triangular part of A (in band storage) is overwritten by the Cholesky factor U or L , as specified by *uplo*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, parameter i had an illegal value.

If *info* = i , the leading minor of order i (and therefore the matrix A itself) is not positive-definite, and the factorization could not be completed. This may indicate an error in forming the matrix A .

Application Notes

If *uplo* = 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(kd + 1)\varepsilon |U^H| |U|, |e_{ij}| \leq c(kd + 1)\varepsilon \sqrt{a_{ii}a_{jj}}$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

A similar estimate holds for *uplo* = 'L'.

The total number of floating-point operations for real flavors is approximately $n(kd+1)^2$. The number of operations for complex flavors is 4 times greater. All these estimates assume that kd is much less than n .

After calling this routine, you can call the following routines:

[?pbtrs](#) to solve $A*X = B$

[?pbcon](#) to estimate the condition number of A .

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?pttrf

Computes the factorization of a symmetric (Hermitian) positive-definite tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_spttrf( lapack_int n, float* d, float* e );
lapack_int LAPACKE_dpttrf( lapack_int n, double* d, double* e );
lapack_int LAPACKE_cpttrf( lapack_int n, float* d, lapack_complex_float* e );
lapack_int LAPACKE_zpttrf( lapack_int n, double* d, lapack_complex_double* e );
```

Include Files

- `mkl.h`

Description

The routine forms the factorization of a symmetric positive-definite or, for complex data, Hermitian positive-definite tridiagonal matrix A :

$A = L * D * L^T$ for real flavors, or

$A = L * D * L^H$ for complex flavors,

where D is diagonal and L is unit lower bidiagonal. The factorization may also be regarded as having the form $A = U^T * D * U$ for real flavors, or $A = U^H * D * U$ for complex flavors, where U is unit upper bidiagonal.

Input Parameters

| | |
|-----|--|
| n | The order of the matrix A ; $n \geq 0$. |
| d | Array, dimension (n). Contains the diagonal elements of A . |
| e | Array, dimension ($n - 1$). Contains the subdiagonal elements of A . |

Output Parameters

| | |
|-----|---|
| d | Overwritten by the n diagonal elements of the diagonal matrix D from the $L * D * L^T$ (for real flavors) or $L * D * L^H$ (for complex flavors) factorization of A . |
| e | Overwritten by the ($n - 1$) sub-diagonal elements of the unit bidiagonal factor L or U from the factorization of A . |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = - i , parameter i had an illegal value.

If *info* = i , the leading minor of order i (and therefore the matrix A itself) is not positive-definite; if $i < n$, the factorization could not be completed, while if $i = n$, the factorization was completed, but $d[n - 1] \leq 0$.

?sytrf

Computes the Bunch-Kaufman factorization of a symmetric matrix.

Syntax

```
lapack_int LAPACKE_ssytrf (int matrix_layout , char uplo , lapack_int n , float * a ,
lapack_int lda , lapack_int * ipiv );
```

```
lapack_int LAPACKE_dsytrf (int matrix_layout , char uplo , lapack_int n , double * a ,
lapack_int lda , lapack_int * ipiv );
```

```
lapack_int LAPACKE_csytrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda , lapack_int * ipiv );
```

```
lapack_int LAPACKE_zsytrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda , lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the factorization of a real/complex symmetric matrix A using the Bunch-Kaufman diagonal pivoting method. The form of the factorization is:

if $uplo = 'U'$, $A = U * D * U^T$
 if $uplo = 'L'$, $A = L * D * L^T$,

where A is the input matrix, U and L are products of permutation and triangular matrices with unit diagonal (upper triangular for U and lower triangular for L), and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D .

NOTE

This routine supports the Progress Routine feature. See [Progress Routines](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored and how A is factored: If $uplo = 'U'$, the array a stores the upper triangular part of the matrix A , and A is factored as $U * D * U^T$. If $uplo = 'L'$, the array a stores the lower triangular part of the matrix A , and A is factored as $L * D * L^T$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>a</i> | Array, size $\max(1, lda * n)$. The array a contains either the upper or the lower triangular part of the matrix A (see <i>uplo</i>). |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, n)$. |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | The upper or lower triangular part of a is overwritten by details of the block-diagonal matrix D and the multipliers used to obtain the factor U (or L). |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D . If $ipiv[i-1] = k > 0$, then d_{ii} is a 1-by-1 block, and the i -th row and column of A was interchanged with the k -th row and column. If $uplo = 'U'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and i -th row and column of A was interchanged with the m -th row and column. If $uplo = 'L'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and $(i+1)$ -th row and column of A was interchanged with the m -th row and column. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, D_{ii} is 0. The factorization has been completed, but *D* is exactly singular. Division by 0 will occur if you use *D* for solving a system of linear equations.

Application Notes

The 2-by-2 unit diagonal blocks and the unit diagonal elements of *U* and *L* are not stored. The remaining elements of *U* and *L* are stored in the corresponding columns of the array *a*, but additional row interchanges are required to recover *U* or *L* explicitly (which is seldom necessary).

If *ipiv*[*i*-1] = *i* for all $i = 1 \dots n$, then all off-diagonal elements of *U* (*L*) are stored explicitly in the corresponding elements of the array *a*.

If *uplo* = 'U', the computed factors *U* and *D* are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon P|U||D||U^T|P^T$$

$c(n)$ is a modest linear function of *n*, and ϵ is the machine precision. A similar estimate holds for the computed *L* and *D* when *uplo* = 'L'.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors or $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

| | |
|---------------------|--|
| <code>?sytrs</code> | to solve $A * X = B$ |
| <code>?sycon</code> | to estimate the condition number of <i>A</i> |
| <code>?sytri</code> | to compute the inverse of <i>A</i> . |

If *uplo* = 'U', then $A = U * D * U'$, where

$$U = P(n) * U(n) * \dots * P(k) * U(k) * \dots,$$

that is, *U* is a product of terms $P(k) * U(k)$, where

- *k* decreases from *n* to 1 in steps of 1 and 2.
- *D* is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks $D(k)$.
- $P(k)$ is a permutation matrix as defined by *ipiv*[*k*-1].
- *U*(*k*) is a unit upper triangular matrix, such that if the diagonal block $D(k)$ is of order *s* ($s = 1$ or 2), then

$$U(k) = \begin{pmatrix} I & v & 0 & \\ 0 & I & 0 & \\ 0 & 0 & I & \\ & & & \end{pmatrix} \begin{matrix} k - s \\ s \\ n - k \\ k - s \quad s \quad n - k \end{matrix}$$

If $s = 1$, $D(k)$ overwrites $A(k,k)$, and *v* overwrites $A(1:k-1,k)$.

If $s = 2$, the upper triangle of $D(k)$ overwrites $A(k-1,k-1)$, $A(k-1,k)$ and $A(k,k)$, and v overwrites $A(1:k-2,k-1:k)$.

If $uplo = 'L'$, then $A = L * D * L'$, where

```
L = P(1)*L(1)* ... *P(k)*L(k)*...
```

that is, L is a product of terms $P(k)*L(k)$, where

- k increases from 1 to n in steps of 1 and 2.
- D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks $D(k)$.
- $P(k)$ is a permutation matrix as defined by $ipiv(k)$.
- $L(k)$ is a unit lower triangular matrix, such that if the diagonal block $D(k)$ is of order s ($s = 1$ or 2), then

$$L(k) = \begin{pmatrix} I & 0 & 0 & & \\ 0 & I & 0 & & \\ 0 & v & I & & \\ & & & \ddots & \\ & & & & I \end{pmatrix} \begin{matrix} k-1 \\ s \\ n-k-s+1 \\ \\ \end{matrix}$$

If $s = 1$, $D(k)$ overwrites $A(k,k)$, and v overwrites $A(k+1:n,k)$.

If $s = 2$, the lower triangle of $D(k)$ overwrites $A(k,k)$, $A(k+1,k)$, and $A(k+1,k+1)$, and v overwrites $A(k+2:n,k:k+1)$.

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?sytrf_rook

Computes the bounded Bunch-Kaufman factorization of a symmetric matrix.

Syntax

```
lapack_int LAPACKE_ssytrf_rook (int matrix_layout, char uplo, lapack_int n, float * a,
lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_dsytrf_rook (int matrix_layout, char uplo, lapack_int n, double * a,
lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_csytrf_rook (int matrix_layout, char uplo, lapack_int n,
lapack_complex_float * a, lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_zsytrf_rook (int matrix_layout, char uplo, lapack_int n,
lapack_complex_double * a, lapack_int lda, lapack_int * ipiv);
```

Include Files

- `mkl.h`

Description

The routine computes the factorization of a real/complex symmetric matrix A using the bounded Bunch-Kaufman ("rook") diagonal pivoting method. The form of the factorization is:

if $uplo = 'U'$, $A = U * D * U^T$
 if $uplo = 'L'$, $A = L * D * L^T$,

where A is the input matrix, U and L are products of permutation and triangular matrices with unit diagonal (upper triangular for U and lower triangular for L), and D is a symmetric block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D .

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout for array <i>b</i> is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored and how A is factored: If $uplo = 'U'$, the array <i>a</i> stores the upper triangular part of the matrix A , and A is factored as $U * D * U^T$. If $uplo = 'L'$, the array <i>a</i> stores the lower triangular part of the matrix A , and A is factored as $L * D * L^T$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>a</i> | Array, size $lda * n$. The array <i>a</i> contains either the upper or the lower triangular part of the matrix A (see <i>uplo</i>). |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | The upper or lower triangular part of <i>a</i> is overwritten by details of the block-diagonal matrix D and the multipliers used to obtain the factor U (or L). |
| <i>ipiv</i> | If $ipiv(k) > 0$, then rows and columns k and $ipiv(k)$ were interchanged and $D_{k,k}$ is a 1-by-1 diagonal block. If $uplo = 'U'$ and $ipiv(k) < 0$ and $ipiv(k - 1) < 0$, then rows and columns k and $-ipiv(k)$ were interchanged, rows and columns $k - 1$ and $-ipiv(k - 1)$ were interchanged, and $D_{k-1:k, k-1:k}$ is a 2-by-2 diagonal block. If $uplo = 'L'$ and $ipiv(k) < 0$ and $ipiv(k + 1) < 0$, then rows and columns k and $-ipiv(k)$ were interchanged, rows and columns $k + 1$ and $-ipiv(k + 1)$ were interchanged, and $D_{k:k+1, k:k+1}$ is a 2-by-2 diagonal block. |

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, the *i*-th parameter had an illegal value.

If $info = i$, D_{ii} is 0. The factorization has been completed, but D is exactly singular. Division by 0 will occur if you use D for solving a system of linear equations.

Application Notes

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors or $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

| | |
|---|---|
| <code>?sytrs_rook</code> | to solve $A*X = B$ |
| <code>?sycon_rook</code> (Fortran only) | to estimate the condition number of A |
| <code>?sytri_rook</code> (Fortran only) | to compute the inverse of A . |

If `uplo = 'U'`, then $A = U*D*U'$, where

$$U = P(n)*U(n)* \dots *P(k)*U(k)*\dots,$$

that is, U is a product of terms $P(k)*U(k)$, where

- k decreases from n to 1 in steps of 1 and 2.
- D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks $D(k)$.
- $P(k)$ is a permutation matrix as defined by `ipiv[k-1]`.
- $U(k)$ is a unit upper triangular matrix, such that if the diagonal block $D(k)$ is of order s ($s = 1$ or 2), then

$$U(k) = \begin{pmatrix} I & v & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} \begin{matrix} k-s \\ s \\ n-k \end{matrix}$$

$k-s \quad s \quad n-k$

If $s = 1$, $D(k)$ overwrites $A(k,k)$, and v overwrites $A(1:k-1,k)$.

If $s = 2$, the upper triangle of $D(k)$ overwrites $A(k-1,k-1)$, $A(k-1,k)$ and $A(k,k)$, and v overwrites $A(1:k-2,k-1:k)$.

If `uplo = 'L'`, then $A = L*D*L'$, where

$$L = P(1)*L(1)* \dots *P(k)*L(k)*\dots,$$

that is, L is a product of terms $P(k)*L(k)$, where

- k increases from 1 to n in steps of 1 and 2.
- D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks $D(k)$.
- $P(k)$ is a permutation matrix as defined by `ipiv(k)`.
- $L(k)$ is a unit lower triangular matrix, such that if the diagonal block $D(k)$ is of order s ($s = 1$ or 2), then

$$L(k) = \begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & v & I \end{pmatrix} \begin{matrix} k-1 \\ s \\ n-k-s+1 \end{matrix}$$

$k-1 \quad s \quad n-k-s+1$

If $s = 1$, $D(k)$ overwrites $A(k,k)$, and v overwrites $A(k+1:n,k)$.

If $s = 2$, the lower triangle of $D(k)$ overwrites $A(k,k)$, $A(k+1,k)$, and $A(k+1,k+1)$, and v overwrites $A(k+2:n,k:k+1)$.

See Also

Matrix Storage Schemes for LAPACK Routines

?hetrf

Computes the Bunch-Kaufman factorization of a complex Hermitian matrix.

Syntax

```
lapack_int LAPACKE_chetrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda , lapack_int * ipiv );
```

```
lapack_int LAPACKE_zhetrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda , lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the factorization of a complex Hermitian matrix A using the Bunch-Kaufman diagonal pivoting method:

if $uplo='U'$, $A = U^*D*U^H$
if $uplo='L'$, $A = L^*D*L^H$,

where A is the input matrix, U and L are products of permutation and triangular matrices with unit diagonal (upper triangular for U and lower triangular for L), and D is a Hermitian block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D .

NOTE

This routine supports the Progress Routine feature. See [Progress Routines](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored and how A is factored: If $uplo = 'U'$, the array a stores the upper triangular part of the matrix A , and A is factored as U^*D*U^H . If $uplo = 'L'$, the array a stores the lower triangular part of the matrix A , and A is factored as L^*D*L^H . |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |

| | |
|------------|---|
| <i>a</i> | Array, size $\max(1, lda * n)$. The array <i>a</i> contains the upper or the lower triangular part of the matrix <i>A</i> (see <i>uplo</i>). |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | The upper or lower triangular part of <i>a</i> is overwritten by details of the block-diagonal matrix <i>D</i> and the multipliers used to obtain the factor <i>U</i> (or <i>L</i>). |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of <i>D</i> . If $ipiv[i-1] = k > 0$, then d_{ii} is a 1-by-1 block, and the <i>i</i> -th row and column of <i>A</i> was interchanged with the <i>k</i> -th row and column. If $uplo = 'U'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then <i>D</i> has a 2-by-2 block in rows/columns <i>i</i> and <i>i+1</i> , and <i>i</i> -th row and column of <i>A</i> was interchanged with the <i>m</i> -th row and column. If $uplo = 'L'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then <i>D</i> has a 2-by-2 block in rows/columns <i>i</i> and <i>i+1</i> , and (<i>i+1</i>)-th row and column of <i>A</i> was interchanged with the <i>m</i> -th row and column. |

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, parameter *i* had an illegal value.

If $info = i$, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular. Division by 0 will occur if you use *D* for solving a system of linear equations.

Application Notes

This routine is suitable for Hermitian matrices that are not known to be positive-definite. If *A* is in fact positive-definite, the routine does not perform interchanges, and no 2-by-2 diagonal blocks occur in *D*.

The 2-by-2 unit diagonal blocks and the unit diagonal elements of *U* and *L* are not stored. The remaining elements of *U* and *L* are stored in the corresponding columns of the array *a*, but additional row interchanges are required to recover *U* or *L* explicitly (which is seldom necessary).

If $ipiv[i-1] = i$ for all $i = 1 \dots n$, then all off-diagonal elements of *U* (*L*) are stored explicitly in the corresponding elements of the array *a*.

If $uplo = 'U'$, the computed factors *U* and *D* are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n) \varepsilon P |U| |D| |U^T| P^T$$

$c(n)$ is a modest linear function of *n*, and ε is the machine precision.

A similar estimate holds for the computed *L* and *D* when $uplo = 'L'$.

The total number of floating-point operations is approximately $(4/3)n^3$.

After calling this routine, you can call the following routines:

[?hetrs](#) to solve $A * X = B$

[?hecon](#) to estimate the condition number of A

[?hetri](#) to compute the inverse of A .

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

[?hetrf_rook](#)

Computes the bounded Bunch-Kaufman factorization of a complex Hermitian matrix.

Syntax

```
lapack_int LAPACKE_chetrf_rook (int matrix_layout, char uplo, lapack_int n,
lapack_complex_float * a, lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_zhetrf_rook (int matrix_layout, char uplo, lapack_int n,
lapack_complex_double * a, lapack_int lda, lapack_int * ipiv);
```

Include Files

- `mkl.h`

Description

The routine computes the factorization of a complex Hermitian matrix A using the bounded Bunch-Kaufman diagonal pivoting method:

if $uplo='U'$, $A = U^*D*U^H$
 if $uplo='L'$, $A = L^*D*L^H$,

where A is the input matrix, U (or L) is a product of permutation and unit upper (or lower) triangular matrices, and D is a Hermitian block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks.

This is the blocked version of the algorithm, calling Level 3 BLAS.

Input Parameters

matrix_layout Specifies whether matrix storage layout for array b is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

uplo Must be 'U' or 'L'.
 Indicates whether the upper or lower triangular part of A is stored:
 If $uplo = 'U'$, the array a stores the upper triangular part of the matrix A .
 If $uplo = 'L'$, the array a stores the lower triangular part of the matrix A .

n The order of matrix A ; $n \geq 0$.

a Array a , size $(lda*n)$
 The array a contains the upper or the lower triangular part of the matrix A (see *uplo*).

If $uplo = 'U'$, the leading n -by- n upper triangular part of a contains the upper triangular part of the matrix A , and the strictly lower triangular part of a is not referenced. If $uplo = 'L'$, the leading n -by- n lower triangular part of a contains the lower triangular part of the matrix A , and the strictly upper triangular part of a is not referenced.

lda

The leading dimension of a ; at least $\max(1, n)$.

Output Parameters

a

The block diagonal matrix D and the multipliers used to obtain the factor U or L (see Application Notes for further details).

$ipiv$

- If $uplo = 'U'$:

If $ipiv(k) > 0$, then rows and columns k and $ipiv(k)$ were interchanged and $D_{k,k}$ is a 1-by-1 diagonal block.

If $ipiv(k) < 0$ and $ipiv(k - 1) < 0$, then rows and columns k and $-ipiv(k)$ were interchanged and rows and columns $k - 1$ and $-ipiv(k - 1)$ were interchanged, $D_{k-1:k,k-1:k}$ is a 2-by-2 diagonal block.

- If $uplo = 'L'$:

If $ipiv(k) > 0$, then rows and columns k and $ipiv(k)$ were interchanged and $D_{k,k}$ is a 1-by-1 diagonal block.

If $ipiv(k) < 0$ and $ipiv(k + 1) < 0$, then rows and columns k and $-ipiv(k)$ were interchanged and rows and columns $k + 1$ and $-ipiv(k + 1)$ were interchanged, $D_{k:k+1,k:k+1}$ is a 2-by-2 diagonal block.

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = i$, D_{ii} is exactly 0. The factorization has been completed, but the block diagonal matrix D is exactly singular, and division by 0 will occur if you use D for solving a system of linear equations.

Application Notes

If $uplo = 'U'$, then $A = U * D * U^H$, where

$$U = P(n) * U(n) * \dots * P(k) * U(k) * \dots,$$

i.e., U is a product of terms $P(k) * U(k)$, where k decreases from n to 1 in steps of 1 or 2, and D is a block diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks $D(k)$. $P(k)$ is a permutation matrix as defined by $ipiv(k)$, and $U(k)$ is a unit upper triangular matrix, such that if the diagonal block $D(k)$ is of order s ($s = 1$ or 2), then

$$U(k) = \begin{matrix} & k-s & s & n-k \\ k-s & \begin{pmatrix} I & v & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix} \\ s & \\ n-k & \end{matrix}$$

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is packed in the array ap and how A is factored: If $uplo = 'U'$, the array ap stores the upper triangular part of the matrix A , and A is factored as $U^*D^*U^T$. If $uplo = 'L'$, the array ap stores the lower triangular part of the matrix A , and A is factored as $L^*D^*L^T$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>ap</i> | Array, size at least $\max(1, n(n+1)/2)$. The array ap contains the upper or the lower triangular part of the matrix A (as specified by $uplo$) in <i>packed storage</i> (see Matrix Storage Schemes). |

Output Parameters

| | |
|-------------|---|
| <i>ap</i> | The upper or lower triangle of A (as specified by $uplo$) is overwritten by details of the block-diagonal matrix D and the multipliers used to obtain the factor U (or L). |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D . If $ipiv[i-1] = k > 0$, then d_{ii} is a 1-by-1 block, and the i -th row and column of A was interchanged with the k -th row and column. If $uplo = 'U'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and i -th row and column of A was interchanged with the m -th row and column. If $uplo = 'L'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and $(i+1)$ -th row and column of A was interchanged with the m -th row and column. |

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, d_{ii} is 0. The factorization has been completed, but D is exactly singular. Division by 0 will occur if you use D for solving a system of linear equations.

Application Notes

The 2-by-2 unit diagonal blocks and the unit diagonal elements of U and L are not stored. The remaining elements of U and L overwrite elements of the corresponding columns of the array ap , but additional row interchanges are required to recover U or L explicitly (which is seldom necessary).

If $ipiv(i) = i$ for all $i = 1 \dots n$, then all off-diagonal elements of U (L) are stored explicitly in packed form.

If $uplo = 'U'$, the computed factors U and D are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\varepsilon P |U| |D| |U^T| P^T$$

$c(n)$ is a modest linear function of n , and ε is the machine precision. A similar estimate holds for the computed L and D when $uplo = 'L'$.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors or $(4/3)n^3$ for complex flavors.

After calling this routine, you can call the following routines:

| | |
|---------------------|---|
| <code>?sptrs</code> | to solve $A * X = B$ |
| <code>?spcon</code> | to estimate the condition number of A |
| <code>?sptri</code> | to compute the inverse of A . |

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?hptrf

Computes the Bunch-Kaufman factorization of a complex Hermitian matrix using packed storage.

Syntax

```
lapack_int LAPACKE_chptrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * ap , lapack_int * ipiv );

lapack_int LAPACKE_zhptrf (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * ap , lapack_int * ipiv );
```

Include Files

- `mkl.h`

Description

The routine computes the factorization of a complex Hermitian packed matrix A using the Bunch-Kaufman diagonal pivoting method:

```
if uplo='U', A = U * D * UH
if uplo='L', A = L * D * LH,
```

where A is the input matrix, U and L are products of permutation and triangular matrices with unit diagonal (upper triangular for U and lower triangular for L), and D is a Hermitian block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. U and L have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of D .

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is packed and how A is factored: If <i>uplo</i> = 'U', the array <i>ap</i> stores the upper triangular part of the matrix A , and A is factored as $U^*D^*U^H$. If <i>uplo</i> = 'L', the array <i>ap</i> stores the lower triangular part of the matrix A , and A is factored as $L^*D^*L^H$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>ap</i> | Array, size at least $\max(1, n(n+1)/2)$. The array <i>ap</i> contains the upper or the lower triangular part of the matrix A (as specified by <i>uplo</i>) in <i>packed storage</i> (see Matrix Storage Schemes). |

Output Parameters

| | |
|-------------|---|
| <i>ap</i> | The upper or lower triangle of A (as specified by <i>uplo</i>) is overwritten by details of the block-diagonal matrix D and the multipliers used to obtain the factor U (or L). |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D . If <i>ipiv</i> [$i-1$] = $k > 0$, then d_{ii} is a 1-by-1 block, and the i -th row and column of A was interchanged with the k -th row and column. If <i>uplo</i> = 'U' and <i>ipiv</i> [i] = <i>ipiv</i> [$i-1$] = $-m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and i -th row and column of A was interchanged with the m -th row and column. If <i>uplo</i> = 'L' and <i>ipiv</i> [i] = <i>ipiv</i> [$i-1$] = $-m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and $(i+1)$ -th row and column of A was interchanged with the m -th row and column. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = $-i$, parameter i had an illegal value.

If *info* = i , d_{ii} is 0. The factorization has been completed, but D is exactly singular. Division by 0 will occur if you use D for solving a system of linear equations.

Application Notes

The 2-by-2 unit diagonal blocks and the unit diagonal elements of U and L are not stored. The remaining elements of U and L are stored in the array ap , but additional row interchanges are required to recover U or L explicitly (which is seldom necessary).

If $ipiv[i-1] = i$ for all $i = 1 \dots n$, then all off-diagonal elements of U (L) are stored explicitly in the corresponding elements of the array a .

If $uplo = 'U'$, the computed factors U and D are the exact factors of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\varepsilon P|U||D||U^T|P^T$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

A similar estimate holds for the computed L and D when $uplo = 'L'$.

The total number of floating-point operations is approximately $(4/3)n^3$.

After calling this routine, you can call the following routines:

| | |
|---------------------|---|
| <code>?hptrs</code> | to solve $A * X = B$ |
| <code>?hpcon</code> | to estimate the condition number of A |
| <code>?hptri</code> | to compute the inverse of A . |

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

mkl_?spffrt2, mkl_?spffrtx

Computes the partial LDL^T factorization of a symmetric matrix using packed storage.

Syntax

```
void mkl_sspffrt2 (float *ap , const MKL_INT *n , const MKL_INT *ncolm , float *work ,
float *work2 );

void mkl_dspffrt2 (double *ap , const MKL_INT *n , const MKL_INT *ncolm , double
*work , double *work2 );

void mkl_cspffrt2 (MKL_Complex8 *ap , const MKL_INT *n , const MKL_INT *ncolm ,
MKL_Complex8 *work , MKL_Complex8 *work2 );

void mkl_zspffrt2 (MKL_Complex16 *ap , const MKL_INT *n , const MKL_INT *ncolm ,
MKL_Complex16 *work , MKL_Complex16 *work2 );

void mkl_sspffrtx (float *ap , const MKL_INT *n , const MKL_INT *ncolm , float *work ,
float *work2 );

void mkl_dspffrtx (double *ap , const MKL_INT *n , const MKL_INT *ncolm , double
*work , double *work2 );

void mkl_cspffrtx (MKL_Complex8 *ap , const MKL_INT *n , const MKL_INT *ncolm ,
MKL_Complex8 *work , MKL_Complex8 *work2 );

void mkl_zspffrtx (MKL_Complex16 *ap , const MKL_INT *n , const MKL_INT *ncolm ,
MKL_Complex16 *work , MKL_Complex16 *work2 );
```

Include Files

- `mk1.h`

Description

The routine computes the partial factorization $A = LDL^T$, where L is a lower triangular matrix and D is a diagonal matrix.

CAUTION

The routine assumes that the matrix A is factorizable. The routine does not perform pivoting and does not handle diagonal elements which are zero, which cause the routine to produce incorrect results without any indication.

Consider the matrix $A = \begin{pmatrix} a & b^T \\ b & C \end{pmatrix}$, where a is the element in the first row and first column of A , b is a column vector of size $n - 1$ containing the elements from the second through n -th column of A , C is the lower-right square submatrix of A , and I is the identity matrix.

The `mk1_?spffrt2` routine performs $ncolm$ successive factorizations of the form

$$A = \begin{pmatrix} a & b^T \\ b & C \end{pmatrix} = \begin{pmatrix} a & 0 \\ b & I \end{pmatrix} \begin{pmatrix} a^{-1} & 0 \\ 0 & C - ba^{-1}b^T \end{pmatrix} \begin{pmatrix} a & b^T \\ 0 & I \end{pmatrix}.$$

The `mk1_?spffrtx` routine performs $ncolm$ successive factorizations of the form

$$A = \begin{pmatrix} a & b^T \\ b & C \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ ba^{-1} & I \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & C - ba^{-1}b^T \end{pmatrix} \begin{pmatrix} 1 & (ba^{-1})^T \\ 0 & I \end{pmatrix}.$$

The approximate number of floating point operations performed by real flavors of these routines is $(1/6)*ncolm*(2*ncolm^2 - 6*ncolm*n + 3*ncolm + 6*n^2 - 6*n + 7)$.

The approximate number of floating point operations performed by complex flavors of these routines is $(1/3)*ncolm*(4*ncolm^2 - 12*ncolm*n + 9*ncolm + 12*n^2 - 18*n + 8)$.

Input Parameters

| | |
|--------------------|--|
| <i>ap</i> | Array, size at least $\max(1, n(n+1)/2)$. The array <i>ap</i> contains the lower triangular part of the matrix A in packed storage (see Matrix Storage Schemes for <code>uplo = 'L'</code>). |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>ncolm</i> | The number of columns to factor, $ncolm \leq n$. |
| <i>work, work2</i> | Workspace arrays, size of each at least n . |

Output Parameters

| | |
|-----------|--|
| <i>ap</i> | Overwritten by the factor L . The first $ncolm$ diagonal elements of the input matrix A are replaced with the diagonal elements of D . The subdiagonal elements of the first $ncolm$ columns are replaced with the corresponding elements of L . The rest of the input array is updated as indicated in the Description section. |
|-----------|--|

NOTE

Specifying $n_{colm} = n$ results in complete factorization $A = LDL^T$.

See Also[mkl_progress](#)[Matrix Storage Schemes for LAPACK Routines](#)**Solving Systems of Linear Equations: LAPACK Computational Routines**

This section describes the LAPACK routines for solving systems of linear equations. Before calling most of these routines, you need to factorize the matrix of your system of equations (see [Routines for Matrix Factorization](#) in this chapter). However, the factorization is not necessary if your system of equations has a triangular matrix.

?getrs

Solves a system of linear equations with an LU-factored square coefficient matrix, with multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_sgetrs (int matrix_layout , char trans , lapack_int n , lapack_int
nrhs , const float * a , lapack_int lda , const lapack_int * ipiv , float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_dgetrs (int matrix_layout , char trans , lapack_int n , lapack_int
nrhs , const double * a , lapack_int lda , const lapack_int * ipiv , double * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_cgetrs (int matrix_layout , char trans , lapack_int n , lapack_int
nrhs , const lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zgetrs (int matrix_layout , char trans , lapack_int n , lapack_int
nrhs , const lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mkl.h`

Description

The routine solves for X the following systems of linear equations:

$$A * X = B \quad \text{if } trans = 'N',$$

$$A^T * X = B \quad \text{if } trans = 'T',$$

$$A^H * X = B \quad \text{if } trans = 'C' \text{ (for complex matrices only).}$$

Before calling this routine, you must call [?getrf](#) to compute the LU factorization of A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', then $A^*X = B$ is solved for X . If <i>trans</i> = 'T', then $A^T * X = B$ is solved for X . If <i>trans</i> = 'C', then $A^H * X = B$ is solved for X . |
| <i>n</i> | The order of A ; the number of rows in B ($n \geq 0$). |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>a</i> | Array of size $\max(1, lda * n)$. The array <i>a</i> contains LU factorization of matrix A resulting from the call of <code>?getrf</code> . |
| <i>b</i> | Array of size $\max(1, ldb * nrhs)$ for column major layout, and $\max(1, ldb * n)$ for row major layout. The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by <code>?getrf</code> . |

Output Parameters

| | |
|----------|--|
| <i>b</i> | Overwritten by the solution matrix X . |
|----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

For each right-hand side b , the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n) \varepsilon P |L| |U|$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is $2n^2$ for real flavors and $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call `?gecon`.

To refine the solution and estimate the error, call `?gerfs`.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gbtrs

Solves a system of linear equations with an LU-factored band coefficient matrix, with multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_sgbtrs (int matrix_layout , char trans , lapack_int n , lapack_int
kl , lapack_int ku , lapack_int nrhs , const float * ab , lapack_int ldab , const
lapack_int * ipiv , float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dgbtrs (int matrix_layout , char trans , lapack_int n , lapack_int
kl , lapack_int ku , lapack_int nrhs , const double * ab , lapack_int ldab , const
lapack_int * ipiv , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_cgbtrs (int matrix_layout , char trans , lapack_int n , lapack_int
kl , lapack_int ku , lapack_int nrhs , const lapack_complex_float * ab , lapack_int
ldab , const lapack_int * ipiv , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zgbtrs (int matrix_layout , char trans , lapack_int n , lapack_int
kl , lapack_int ku , lapack_int nrhs , const lapack_complex_double * ab , lapack_int
ldab , const lapack_int * ipiv , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the following systems of linear equations:

$A * X = B$ if $trans = 'N'$,

$A^T * X = B$ if $trans = 'T'$,

$A^H * X = B$ if $trans = 'C'$ (for complex matrices only).

Here A is an LU -factored general band matrix of order n with kl non-zero subdiagonals and ku nonzero superdiagonals. Before calling this routine, call `?gbtrf` to compute the LU factorization of A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. |
| <i>n</i> | The order of <i>A</i> ; the number of rows in <i>B</i> ; $n \geq 0$. |
| <i>kl</i> | The number of subdiagonals within the band of <i>A</i> ; $kl \geq 0$. |
| <i>ku</i> | The number of superdiagonals within the band of <i>A</i> ; $ku \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ab</i> | Array <i>ab</i> size $\max(1, ldab*n)$ The array <i>ab</i> contains elements of the LU factors of the matrix <i>A</i> as returned by gbtrf . |
| <i>b</i> | Array <i>b</i> size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations. |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; $ldab \geq 2*kl + ku + 1$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?gbtrf . |

Output Parameters

b Overwritten by the solution matrix *X*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(kl + ku + 1)\epsilon P|L||U|$$

$c(k)$ is a modest linear function of *k*, and ϵ is the machine precision.

If x_0 is the true solution, the computed solution *x* satisfies this error bound:

$$\frac{\|x - x_0\|_{\infty}}{\|x\|_{\infty}} \leq c(kl + ku + 1) \text{cond}(A, x)\varepsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_{\infty} / \|x\|_{\infty} \leq \|A^{-1}\|_{\infty} \|A\|_{\infty} = \kappa_{\infty}(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_{\infty}(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_{\infty}(A)$.

The approximate number of floating-point operations for one right-hand side vector is $2n(ku + 2kl)$ for real flavors. The number of operations for complex flavors is 4 times greater. All these estimates assume that kl and ku are much less than $\min(m, n)$.

To estimate the condition number $\kappa_{\infty}(A)$, call `?gbcon`.

To refine the solution and estimate the error, call `?gbrfs`.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gttrs

Solves a system of linear equations with a tridiagonal coefficient matrix using the LU factorization computed by ?gttrf.

Syntax

```
lapack_int LAPACKE_sgttrs (int matrix_layout , char trans , lapack_int n , lapack_int
nrhs , const float * dl , const float * d , const float * du , const float * du2 ,
const lapack_int * ipiv , float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dgttrs (int matrix_layout , char trans , lapack_int n , lapack_int
nrhs , const double * dl , const double * d , const double * du , const double * du2 ,
const lapack_int * ipiv , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_cgttrs (int matrix_layout , char trans , lapack_int n , lapack_int
nrhs , const lapack_complex_float * dl , const lapack_complex_float * d , const
lapack_complex_float * du , const lapack_complex_float * du2 , const lapack_int *
ipiv , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zgttrs (int matrix_layout , char trans , lapack_int n , lapack_int
nrhs , const lapack_complex_double * dl , const lapack_complex_double * d , const
lapack_complex_double * du , const lapack_complex_double * du2 , const lapack_int *
ipiv , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the following systems of linear equations with multiple right hand sides:

$A * X = B$ if *trans* = 'N',

$A^T * X = B$ if *trans* = 'T',

$A^H * X = B$ if *trans* = 'C' (for complex matrices only).

Before calling this routine, you must call `?gttrf` to compute the LU factorization of A .

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout for array b is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>trans</code> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If $trans = 'N'$, then $A*X = B$ is solved for X . If $trans = 'T'$, then $A^T*X = B$ is solved for X . If $trans = 'C'$, then $A^H*X = B$ is solved for X . |
| <code>n</code> | The order of A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides, that is, the number of columns in B ; $nrhs \geq 0$. |
| <code>dl,d,du,du2</code> | Arrays: $dl(n-1)$, $d(n)$, $du(n-1)$, $du2(n-2)$. The array dl contains the $(n-1)$ multipliers that define the matrix L from the LU factorization of A . The array d contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A . The array du contains the $(n-1)$ elements of the first superdiagonal of U . The array $du2$ contains the $(n-2)$ elements of the second superdiagonal of U . |
| <code>b</code> | Array of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, n*ldb)$ for row major layout. Contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <code>ldb</code> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <code>ipiv</code> | Array, size (n) . The $ipiv$ array, as returned by <code>?gttrf</code> . |

Output Parameters

| | |
|----------------|--|
| <code>b</code> | Overwritten by the solution matrix X . |
|----------------|--|

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, parameter i had an illegal value.

Application Notes

For each right-hand side b , the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon P |L| |U|$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(kl + ku + 1) \text{cond}(A, x)\varepsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is $7n$ (including n divisions) for real flavors and $34n$ (including $2n$ divisions) for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call [?gtcon](#).

To refine the solution and estimate the error, call [?gtrfs](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?dtrs](#)

Solves a system of linear equations with a diagonally dominant tridiagonal coefficient matrix using the LU factorization computed by [?dttrfb](#).

Syntax

```
void sdttrs (const char * trans, const MKL_INT * n, const MKL_INT * nrhs, const float * dl, const float * d, const float * du, float * b, const MKL_INT * ldb, MKL_INT * info );
```

```
void ddttrs (const char * trans, const MKL_INT * n, const MKL_INT * nrhs, const double * dl, const double * d, const double * du, double * b, const MKL_INT * ldb, MKL_INT * info );
```

```
void cdttrs (const char * trans, const MKL_INT * n, const MKL_INT * nrhs, const MKL_Complex8 * dl, const MKL_Complex8 * d, const MKL_Complex8 * du, MKL_Complex8 * b, const MKL_INT * ldb, MKL_INT * info );
```

```
void zdttrs (const char * trans, const MKL_INT * n, const MKL_INT * nrhs, const MKL_Complex16 * dl, const MKL_Complex16 * d, const MKL_Complex16 * du, MKL_Complex16 * b, const MKL_INT * ldb, MKL_INT * info );
```

Include Files

- `mkl.h`

Description

The [?dtrs](#) routine solves the following systems of linear equations with multiple right hand sides for X :

$A^*X = B$ if *trans* = 'N',
 $A^T * X = B$ if *trans* = 'T',
 $A^H * X = B$ if *trans* = 'C' (for complex matrices only).

Before calling this routine, call `?dtrfb` to compute the factorization of *A*.

Input Parameters

trans Must be 'N' or 'T' or 'C'.
 Indicates the form of the equations solved for *X*:
 If *trans* = 'N', then $A^*X = B$.
 If *trans* = 'T', then $A^T * X = B$.
 If *trans* = 'C', then $A^H * X = B$.

n The order of *A*; $n \geq 0$.

nrhs The number of right-hand sides, that is, the number of columns in *B*; $nrhs \geq 0$.

dl, d, du Arrays: $dl(n-1)$, $d(n)$, $du(n-1)$.
 The array *dl* contains the $(n-1)$ multipliers that define the matrices L_1, L_2 from the factorization of *A*.
 The array *d* contains the n diagonal elements of the upper triangular matrix *U* from the factorization of *A*.
 The array *du* contains the $(n-1)$ elements of the superdiagonal of *U*.

b Array of size $\max(1, ldb * nrhs)$. Contains the matrix *B* whose columns are the right-hand sides for the systems of equations.

ldb The leading dimension of *b*; $ldb \geq \max(1, n)$.

Output Parameters

b Overwritten by the solution matrix *X*.

info If *info* = 0, the execution is successful.
 If *info* = -*i*, the *i*-th parameter had an illegal value.

?spotrs

Solves a system of linear equations with a Cholesky-factored symmetric (Hermitian) positive-definite coefficient matrix.

Syntax

```

lapack_int LAPACKE_spotrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const float * a , lapack_int lda , float * b , lapack_int ldb );

lapack_int LAPACKE_dpotsr (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const double * a , lapack_int lda , double * b , lapack_int ldb );
  
```

```
lapack_int LAPACKE_cpotrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_float * a , lapack_int lda , lapack_complex_float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_zpotrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_double * a , lapack_int lda , lapack_complex_double * b ,
lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the system of linear equations $A * X = B$ with a symmetric positive-definite or, for complex data, Hermitian positive-definite matrix A , given the Cholesky factorization of A :

$$A = U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} \quad \text{if } uplo = 'U'$$

$$A = L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} \quad \text{if } uplo = 'L'$$

where L is a lower triangular matrix and U is upper triangular. The system is solved with multiple right-hand sides stored in the columns of the matrix B .

Before calling this routine, you must call [potrf](#) to compute the Cholesky factorization of A .

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <i>uplo</i> = 'U', U is stored, where $A = U^T * U$ for real data, $A = U^H * U$ for complex data. If <i>uplo</i> = 'L', L is stored, where $A = L * L^T$ for real data, $A = L * L^H$ for complex data |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides ($nrhs \geq 0$). |
| <i>a</i> | Array A of size at least $\max(1, lda * n)$ The array a contains the factor U or L (see <i>uplo</i>) as returned by potrf . |
| <i>b</i> | The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. The size of b must be at least $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

b Overwritten by the solution matrix *X*.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

If *uplo* = 'U', the computed solution for each right-hand side *b* is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon |U^H| |U|$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

A similar estimate holds for *uplo* = 'L'. If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\varepsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$. The approximate number of floating-point operations for one right-hand side vector *b* is $2n^2$ for real flavors and $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call `?pocon`.

To refine the solution and estimate the error, call `?porfs`.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pftrs

Solves a system of linear equations with a Cholesky-factored symmetric (Hermitian) positive-definite coefficient matrix using the Rectangular Full Packed (RFP) format.

Syntax

```
lapack_int LAPACKE_spftrs (int matrix_layout , char transr , char uplo , lapack_int n ,
lapack_int nrhs , const float * a , float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dpftrs (int matrix_layout , char transr , char uplo , lapack_int n ,
lapack_int nrhs , const double * a , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_cpftrs (int matrix_layout , char transr , char uplo , lapack_int n ,
lapack_int nrhs , const lapack_complex_float * a , lapack_complex_float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_zpftrs (int matrix_layout , char transr , char uplo , lapack_int n ,
lapack_int nrhs , const lapack_complex_double * a , lapack_complex_double * b ,
lapack_int ldb );
```

Include Files

- `mkl.h`

Description

The routine solves a system of linear equations $A^*X = B$ with a symmetric positive-definite or, for complex data, Hermitian positive-definite matrix A using the Cholesky factorization of A :

$$\begin{aligned}
 A &= U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} && \text{if } uplo = 'U' \\
 A &= L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} && \text{if } uplo = 'L'
 \end{aligned}$$

Before calling `?pfttrs`, you must call `?pftrf` to compute the Cholesky factorization of A . L stands for a lower triangular matrix and U for an upper triangular matrix.

The matrix A is in the Rectangular Full Packed (RFP) format. For the description of the RFP format, see [Matrix Storage Schemes](#).

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>transr</code> | Must be 'N', 'T' (for real data) or 'C' (for complex data). If <code>transr = 'N'</code> , the untransposed factor of A is stored in RFP format. If <code>transr = 'T'</code> , the transposed factor of A is stored in RFP format. If <code>transr = 'C'</code> , the conjugate-transposed factor of A is stored in RFP format. |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo = 'U'</code> , U is stored, where $A = U^T * U$ for real data, $A = U^H * U$ for complex data. If <code>uplo = 'L'</code> , L is stored, where $A = L * L^T$ for real data, $A = L * L^H$ for complex data |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides, that is, the number of columns of the matrix B ; $nrhs \geq 0$. |
| <code>a</code> | Array <code>a</code> of size $\max(1, n * (n + 1) / 2)$. The array <code>a</code> contains, in the RFP format, the factor U or L obtained by factorization of matrix A . |
| <code>b</code> | The array <code>b</code> of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <code>ldb</code> | The leading dimension of <code>b</code> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

b The solution matrix X .

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, parameter *i* had an illegal value.

See Also

Matrix Storage Schemes for LAPACK Routines

?pptrs

Solves a system of linear equations with a packed Cholesky-factored symmetric (Hermitian) positive-definite coefficient matrix.

Syntax

```
lapack_int LAPACKE_spptrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const float * ap , float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dppts (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const double * ap , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_cppts (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_float * ap , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zppts (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_double * ap , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the system of linear equations $A * X = B$ with a packed symmetric positive-definite or, for complex data, Hermitian positive-definite matrix A , given the Cholesky factorization of A :

$$\begin{aligned} A &= U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} && \text{if } uplo = 'U' \\ A &= L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} && \text{if } uplo = 'L' \end{aligned}$$

where L is a lower triangular matrix and U is upper triangular. The system is solved with multiple right-hand sides stored in the columns of the matrix B .

Before calling this routine, you must call `?pptrf` to compute the Cholesky factorization of A .

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

uplo Must be 'U' or 'L'.

Indicates how the input matrix A has been factored:

If *uplo* = 'U', U is stored, where $A = U^T * U$ for real data, $A = U^H * U$ for complex data.

| | |
|---------|---|
| | If $uplo = 'L'$, L is stored, where $A = L*L^T$ for real data, $A = L*L^H$ for complex data |
| n | The order of matrix A ; $n \geq 0$. |
| $nrhs$ | The number of right-hand sides ($nrhs \geq 0$). |
| ap, b | The size of ap must be at least $\max(1, n(n+1)/2)$. The array ap contains the factor U or L , as specified by $uplo$, in <i>packed storage</i> (see Matrix Storage Schemes). |
| b | The array b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| ldb | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

b Overwritten by the solution matrix X .

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

Application Notes

If $uplo = 'U'$, the computed solution for each right-hand side b is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon |U^H| |U|$$

$c(n)$ is a modest linear function of n , and ϵ is the machine precision.

A similar estimate holds for $uplo = 'L'$.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is $2n^2$ for real flavors and $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call [?ppcon](#).

To refine the solution and estimate the error, call [?pprfs](#).

See Also

Matrix Storage Schemes for LAPACK Routines

?pbtrs

Solves a system of linear equations with a Cholesky-factored symmetric (Hermitian) positive-definite band coefficient matrix.

Syntax

```
lapack_int LAPACKE_spbtrs (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_int nrhs , const float * ab , lapack_int ldab , float * b , lapack_int
ldb );
```

```
lapack_int LAPACKE_dpbtrs (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_int nrhs , const double * ab , lapack_int ldab , double * b , lapack_int
ldb );
```

```
lapack_int LAPACKE_cpbtrs (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_int nrhs , const lapack_complex_float * ab , lapack_int ldab ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zpbtrs (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_int nrhs , const lapack_complex_double * ab , lapack_int ldab ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for real data a system of linear equations $A^*X = B$ with a symmetric positive-definite or, for complex data, Hermitian positive-definite *band* matrix A , given the Cholesky factorization of A :

$$\begin{array}{ll}
 A = U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} & \text{if } uplo = 'U' \\
 A = L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} & \text{if } uplo = 'L'
 \end{array}$$

where L is a lower triangular matrix and U is upper triangular. The system is solved with multiple right-hand sides stored in the columns of the matrix B .

Before calling this routine, you must call [?pbtrf](#) to compute the Cholesky factorization of A in the band storage form.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, U is stored in ab , where $A = U^T * U$ for real matrices and $A = U^H * U$ for complex matrices. If $uplo = 'L'$, L is stored in ab , where $A = L * L^T$ for real matrices and $A = L * L^H$ for complex matrices. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |

| | |
|-------------|--|
| <i>kd</i> | The number of superdiagonals or subdiagonals in the matrix <i>A</i> ; $kd \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ab</i> | Array <i>ab</i> is of size $\max(1, ldab*n)$. The array <i>ab</i> contains the Cholesky factor, as returned by the factorization routine, in <i>band storage</i> form. The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations. |
| <i>b</i> | The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations. The size of <i>b</i> is at least $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; $ldab \geq kd + 1$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

b Overwritten by the solution matrix *X*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(kd + 1)\epsilon P |U^H| |U| \text{ or } |E| \leq c(kd + 1)\epsilon P |L^H| |L|$$

$c(k)$ is a modest linear function of *k*, and ϵ is the machine precision.

If x_0 is the true solution, the computed solution *x* satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(kd + 1) \text{cond}(A, x) \epsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector is $4n*kd$ for real flavors and $16n*kd$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call [?pbcon](#).

To refine the solution and estimate the error, call [?pbrfs](#).

See Also

Matrix Storage Schemes for LAPACK Routines

?pttrs

Solves a system of linear equations with a symmetric (Hermitian) positive-definite tridiagonal coefficient matrix using the factorization computed by [?pttrf](#).

Syntax

```
lapack_int LAPACKE_spttrs( int matrix_layout, lapack_int n, lapack_int nrhs, const float* d, const float* e, float* b, lapack_int ldb );
```

```
lapack_int LAPACKE_dpttrs( int matrix_layout, lapack_int n, lapack_int nrhs, const double* d, const double* e, double* b, lapack_int ldb );
```

```
lapack_int LAPACKE_cpttrs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs, const float* d, const lapack_complex_float* e, lapack_complex_float* b, lapack_int ldb );
```

```
lapack_int LAPACKE_zpttrs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs, const double* d, const lapack_complex_double* e, lapack_complex_double* b, lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X a system of linear equations $A^*X = B$ with a symmetric (Hermitian) positive-definite tridiagonal matrix A . Before calling this routine, call [?pttrf](#) to compute the $L^*D^*L^T$ or $U^T^*D^*U$ for real data and the $L^*D^*L^H$ or $U^H^*D^*U$ factorization of A for complex data.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Used for <i>cpttrs</i> / <i>zpttrs</i> only. Must be 'U' or 'L'. Specifies whether the superdiagonal or the subdiagonal of the tridiagonal matrix A is stored and how A is factored: If <i>uplo</i> = 'U', the array <i>e</i> stores the conjugated values of the superdiagonal of U , and A is factored as $U^H^*D^*U$. If <i>uplo</i> = 'L', the array <i>e</i> stores the subdiagonal of L , and A is factored as $L^*D^*L^H$. |
| <i>n</i> | The order of A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, that is, the number of columns of the matrix B ; $nrhs \geq 0$. |
| <i>d</i> | Array, dimension (n). Contains the diagonal elements of the diagonal matrix D from the factorization computed by ?pttrf . |
| <i>e</i> | Array <i>e</i> is of size ($n - 1$). |

The array e contains the $(n - 1)$ sub-diagonal elements of the unit bidiagonal factor L or the conjugated values of the superdiagonal of U from the factorization computed by `?pttrf` (see `uplo`).

e, b

The array b contains the matrix B whose columns are the right-hand sides for the systems of equations.

The size of b is at least $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout.

ldb

The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

Output Parameters

b

Overwritten by the solution matrix X .

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

`?sytrs`

Solves a system of linear equations with a UDU^T - or LDL^T -factored symmetric coefficient matrix.

Syntax

```
lapack_int LAPACKE_ssytrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const float * a , lapack_int lda , const lapack_int * ipiv , float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_dsytrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const double * a , lapack_int lda , const lapack_int * ipiv , double * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_csytrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zsytrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mk1.h`

Description

The routine solves for X the system of linear equations $A * X = B$ with a symmetric matrix A , given the Bunch-Kaufman factorization of A :

if `uplo='U'`, $A = U * D * U^T$

if `uplo='L'`, $A = L * D * L^T$,

where U and L are upper and lower triangular matrices with unit diagonal and D is a symmetric block-diagonal matrix. The system is solved with multiple right-hand sides stored in the columns of the matrix B . You must supply to this routine the factor U (or L) and the array `ipiv` returned by the factorization routine [?sytrf](#).

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo = 'U'</code> , the array <code>a</code> stores the upper triangular factor U of the factorization $A = U * D * U^T$. If <code>uplo = 'L'</code> , the array <code>a</code> stores the lower triangular factor L of the factorization $A = L * D * L^T$. |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; $nrhs \geq 0$. |
| <code>ipiv</code> | Array, size at least $\max(1, n)$. The <code>ipiv</code> array, as returned by ?sytrf . |
| <code>a</code> | The array <code>a</code> of size $\max(1, lda * n)$ contains the factor U or L (see <code>uplo</code>). |
| <code>b</code> | The array <code>b</code> contains the matrix B whose columns are the right-hand sides for the system of equations. The size of <code>b</code> is at least $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout. |
| <code>lda</code> | The leading dimension of <code>a</code> ; $lda \geq \max(1, n)$. |
| <code>ldb</code> | The leading dimension of <code>b</code> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

`b` Overwritten by the solution matrix X .

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, parameter `i` had an illegal value.

Application Notes

For each right-hand side b , the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon P|U||D||U^T|P^T \text{ or } |E| \leq c(n)\varepsilon P|L||D||U^T|P^T$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\varepsilon$$

where $\text{cond}(A, x) = \frac{\|A^{-1}\|_\infty \|A\|_\infty \|x\|_\infty}{\|x\|_\infty} \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$.

The total number of floating-point operations for one right-hand side vector is approximately $2n^2$ for real flavors or $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call [?sycon](#).

To refine the solution and estimate the error, call [?sytrfs](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?sytrs_rook](#)

Solves a system of linear equations with a UDU- or LDL-factored symmetric coefficient matrix.

Syntax

```
lapack_int LAPACK_essytrs_rook (int matrix_layout, char uplo, lapack_int n, lapack_int
nrhs, const float * a, lapack_int lda, const lapack_int * ipiv, float * b, lapack_int
ldb);
```

```
lapack_int LAPACK_dsytrs_rook (int matrix_layout, char uplo, lapack_int n, lapack_int
nrhs, const double * a, lapack_int lda, const lapack_int * ipiv, double * b,
lapack_int ldb);
```

```
lapack_int LAPACK_csytrs_rook (int matrix_layout, char uplo, lapack_int n, lapack_int
nrhs, const lapack_complex_float * a, lapack_int lda, const lapack_int * ipiv,
lapack_complex_float * b, lapack_int ldb);
```

```
lapack_int LAPACK_zsytrs_rook (int matrix_layout, char uplo, lapack_int n, lapack_int
nrhs, const lapack_complex_double * a, lapack_int lda, const lapack_int * ipiv,
lapack_complex_double * b, lapack_int ldb);
```

Include Files

- mkl.h

Description

The routine solves a system of linear equations $A * X = B$ with a symmetric matrix A , using the factorization $A = U * D * U^T$ or $A = L * D * L^T$ computed by [?sytrf_rook](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout for array <i>b</i> is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix <i>A</i> has been factored: If <i>uplo</i> = 'U', the factorization is of the form $A = U^*D^*U^T$. If <i>uplo</i> = 'L', the factorization is of the form $A = L^*D^*L^T$. |
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?sytrf_rook . |
| <i>a, b</i> | Arrays: <i>a</i> , size (<i>lda</i> * <i>n</i>), <i>b</i> size (<i>ldb</i> * <i>nrhs</i>). The array <i>a</i> contains the block diagonal matrix <i>D</i> and the multipliers used to obtain <i>U</i> or <i>L</i> as computed by ?sytrf_rook (see <i>uplo</i>). The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the system of equations. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>b</i> | Overwritten by the solution matrix <i>X</i> . |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The total number of floating-point operations for one right-hand side vector is approximately $2n^2$ for real flavors or $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?hetrs](#)

Solves a system of linear equations with a UDU^T - or LDL^T -factored Hermitian coefficient matrix.

Syntax

```
lapack_int LAPACKE_chetrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zhetrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the system of linear equations $A \cdot X = B$ with a Hermitian matrix A , given the Bunch-Kaufman factorization of A :

if $uplo='U'$, $A = U \cdot D \cdot U^H$

if $uplo='L'$, $A = L \cdot D \cdot L^H$,

where U and L are upper and lower triangular matrices with unit diagonal and D is a symmetric block-diagonal matrix. The system is solved with multiple right-hand sides stored in the columns of the matrix B . You must supply to this routine the factor U (or L) and the array $ipiv$ returned by the factorization routine [?hetrf](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, the array a stores the upper triangular factor U of the factorization $A = U \cdot D \cdot U^H$. If $uplo = 'L'$, the array a stores the lower triangular factor L of the factorization $A = L \cdot D \cdot L^H$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The $ipiv$ array, as returned by ?hetrf . |
| <i>a</i> | The array a of size $\max(1, lda \cdot n)$ contains the factor U or L (see <i>uplo</i>). |
| <i>b</i> | The array b contains the matrix B whose columns are the right-hand sides for the system of equations. The size of b is at least $\max(1, ldb \cdot nrhs)$ for column major layout and $\max(1, ldb \cdot n)$ for row major layout. |

| | |
|------------|---|
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>b</i> | Overwritten by the solution matrix <i>X</i> . |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon P|U||D||U^H|P^T \text{ or } |E| \leq c(n)\varepsilon P|L||D||L^H|P^T$$

$c(n)$ is a modest linear function of *n*, and ε is the machine precision.

If x_0 is the true solution, the computed solution *x* satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\varepsilon$$

where $\text{cond}(A, x) = \frac{\|A^{-1}\| \|A\| \|x\|_\infty}{\|x\|_\infty} \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$.

The total number of floating-point operations for one right-hand side vector is approximately $8n^2$.

To estimate the condition number $\kappa_\infty(A)$, call [?hecon](#).

To refine the solution and estimate the error, call [?herfs](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?hetrs_rook](#)

Solves a system of linear equations with a UDU- or LDL-factored Hermitian coefficient matrix.

Syntax

```
lapack_int LAPACKE_chetrs_rook (int matrix_layout, char uplo, lapack_int n, lapack_int
nrhs, const lapack_complex_float * a, lapack_int lda, const lapack_int * ipiv,
lapack_complex_float * b, lapack_int ldb);
```

```
lapack_int LAPACKE_zhetrs_rook (int matrix_layout, char uplo, lapack_int n, lapack_int
nrhs, const lapack_complex_double * a, lapack_int lda, const lapack_int * ipiv,
lapack_complex_double * b, lapack_int ldb);
```

Include Files

- `mk1.h`

Description

The routine solves for a system of linear equations $A^*X = B$ with a complex Hermitian matrix A using the factorization $A = U^*D^*U^H$ or $A = L^*D^*L^H$ computed by [?hetrf_rook](#).

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout for array b is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo = 'U'</code> , the factorization is of the form $A = U^*D^*U^H$. If <code>uplo = 'L'</code> , the factorization is of the form $A = L^*D^*L^H$. |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; $nrhs \geq 0$. |
| <code>ipiv</code> | Array, size at least $\max(1, n)$. The <code>ipiv</code> array, as returned by ?hetrf_rook . |
| <code>a, b</code> | Arrays: a ($lda*n$), b ($ldb*nrhs$). The array a contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by ?hetrf_rook (see <code>uplo</code>). The array b contains the matrix B whose columns are the right-hand sides for the system of equations. |
| <code>lda</code> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <code>ldb</code> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

`b` Overwritten by the solution matrix X .

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

[?sytrs2](#)

Solves a system of linear equations with a UDU- or LDL-factored symmetric coefficient matrix.

Syntax

```
lapack_int LAPACKE_ssytrs2 (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const float * a , lapack_int lda , const lapack_int * ipiv , float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_dsytrs2 (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const double * a , lapack_int lda , const lapack_int * ipiv , double * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_csytrs2 (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zsytrs2 (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves a system of linear equations $A \cdot X = B$ with a symmetric matrix A using the factorization of A :

if $uplo='U'$, $A = U \cdot D \cdot U^T$

if $uplo='L'$, $A = L \cdot D \cdot L^T$

where

- U and L are upper and lower triangular matrices with unit diagonal
- D is a symmetric block-diagonal matrix.

The factorization is computed by `?sytrf`.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, the array a stores the upper triangular factor U of the factorization $A = U \cdot D \cdot U^T$. If $uplo = 'L'$, the array a stores the lower triangular factor L of the factorization $A = L \cdot D \cdot L^T$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>a</i> | The array a of size $\max(1, lda \cdot n)$ contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by <code>?sytrf</code> . |

| | |
|-------------|--|
| <i>b</i> | The array <i>b</i> contains the right-hand side matrix <i>B</i> . The size of <i>b</i> is at least $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ipiv</i> | Array of size <i>n</i> . The <i>ipiv</i> array contains details of the interchanges and the block structure of <i>D</i> as determined by <code>?sytrf</code> . |

Output Parameters

| | |
|----------|---|
| <i>b</i> | Overwritten by the solution matrix <i>X</i> . |
|----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

See Also

[?sytrf](#)

[Matrix Storage Schemes for LAPACK Routines](#)

[?hetrs2](#)

Solves a system of linear equations with a UDU- or LDL-factored Hermitian coefficient matrix.

Syntax

```
lapack_int LAPACKE_chetrs2 (int matrix_layout , char uplo , lapack_int n , lapack_int nrhs , const lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zhetrs2 (int matrix_layout , char uplo , lapack_int n , lapack_int nrhs , const lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves a system of linear equations $A*X = B$ with a complex Hermitian matrix *A* using the factorization of *A*:

if *uplo*='U', $A = U*D*U^H$

if *uplo*='L', $A = L*D*L^H$

where

- *U* and *L* are upper and lower triangular matrices with unit diagonal

- D is a Hermitian block-diagonal matrix.

The factorization is computed by `?hetrf`.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo = 'U'</code> , the array a stores the upper triangular factor U of the factorization $A = U^*D^*U^H$. If <code>uplo = 'L'</code> , the array a stores the lower triangular factor L of the factorization $A = L^*D^*L^H$. |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; $nrhs \geq 0$. |
| <code>a</code> | The array a of size $\max(1, lda*n)$ contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by <code>?hetrf</code> . |
| <code>b</code> | The array b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the right-hand side matrix B . |
| <code>lda</code> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <code>ldb</code> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <code>ipiv</code> | Array of size n . The <code>ipiv</code> array contains details of the interchanges and the block structure of D as determined by <code>?hetrf</code> . |

Output Parameters

| | |
|----------------|--|
| <code>b</code> | Overwritten by the solution matrix X . |
|----------------|--|

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful.

If `info = -i`, parameter i had an illegal value.

See Also

[?hetrf](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?sptrs

Solves a system of linear equations with a UDU- or LDL-factored symmetric coefficient matrix using packed storage.

Syntax

```
lapack_int LAPACKE_ssptrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const float * ap , const lapack_int * ipiv , float * b , lapack_int ldb );

lapack_int LAPACKE_dsptrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const double * ap , const lapack_int * ipiv , double * b , lapack_int ldb );

lapack_int LAPACKE_csptrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_float * ap , const lapack_int * ipiv , lapack_complex_float
* b , lapack_int ldb );

lapack_int LAPACKE_zsptrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_double * ap , const lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the system of linear equations $A * X = B$ with a symmetric matrix A , given the Bunch-Kaufman factorization of A :

if $uplo='U'$, $A = U * D * U^T$

if $uplo='L'$, $A = L * D * L^T$,

where U and L are upper and lower *packed* triangular matrices with unit diagonal and D is a symmetric block-diagonal matrix. The system is solved with multiple right-hand sides stored in the columns of the matrix B . You must supply the factor U (or L) and the array $ipiv$ returned by the factorization routine [?sptrf](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, the array ap stores the packed factor U of the factorization $A = U * D * U^T$. If $uplo = 'L'$, the array ap stores the packed factor L of the factorization $A = L * D * L^T$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The $ipiv$ array, as returned by ?sptrf . |
| <i>ap</i> | The dimension of array ap must be at least $\max(1, n(n+1)/2)$. The array ap contains the factor U or L , as specified by $uplo$, in <i>packed storage</i> (see Matrix Storage Schemes). |
| <i>b</i> | The array b contains the matrix B whose columns are the right-hand sides for the system of equations. The size of b is $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout. |

ldb The leading dimension of *b*; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

Output Parameters

b Overwritten by the solution matrix *X*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon P|U||D||U^T|P^T \text{ or } |E| \leq c(n)\varepsilon P|L||D||L^T|P^T$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\varepsilon$$

where $\text{cond}(A, x) = \| |A^{-1}||A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$.

The total number of floating-point operations for one right-hand side vector is approximately $2n^2$ for real flavors or $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call [?spcon](#).

To refine the solution and estimate the error, call [?sprfs](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?hptrs](#)

Solves a system of linear equations with a UDU- or LDL-factored Hermitian coefficient matrix using packed storage.

Syntax

```
lapack_int LAPACKE_chptrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_float * ap , const lapack_int * ipiv , lapack_complex_float
* b , lapack_int ldb );
```

```
lapack_int LAPACKE_zhptrs (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , const lapack_complex_double * ap , const lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mkl.h`

Description

The routine solves for X the system of linear equations $A * X = B$ with a Hermitian matrix A , given the Bunch-Kaufman factorization of A :

if `uplo='U'`, $A = U * D * U^H$

if `uplo='L'`, $A = L * D * L^H$,

where U and L are upper and lower *packed* triangular matrices with unit diagonal and D is a symmetric block-diagonal matrix. The system is solved with multiple right-hand sides stored in the columns of the matrix B .

You must supply to this routine the arrays `ap` (containing U or L) and `ipiv` in the form returned by the factorization routine `?hptrf`.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo = 'U'</code> , the array <code>ap</code> stores the packed factor U of the factorization $A = U * D * U^H$. If <code>uplo = 'L'</code> , the array <code>ap</code> stores the packed factor L of the factorization $A = L * D * L^H$. |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; $nrhs \geq 0$. |
| <code>ipiv</code> | Array, size at least $\max(1, n)$. The <code>ipiv</code> array, as returned by <code>?hptrf</code> . |
| <code>ap</code> | The dimension of array <code>ap</code> must be at least $\max(1, n(n+1)/2)$. The array <code>ap</code> contains the factor U or L , as specified by <code>uplo</code> , in <i>packed storage</i> (see Matrix Storage Schemes). |
| <code>b</code> | The array <code>b</code> contains the matrix B whose columns are the right-hand sides for the system of equations. The size of <code>b</code> is $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout. |
| <code>ldb</code> | The leading dimension of <code>b</code> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

`b` Overwritten by the solution matrix X .

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful.

If $info = -i$, parameter i had an illegal value.

Application Notes

For each right-hand side b , the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n) \varepsilon P |U| |D| |U^H| P^T \text{ or } |E| \leq c(n) \varepsilon P |L| |D| |L^H| P^T$$

$c(n)$ is a modest linear function of n , and ε is the machine precision.

If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x) \varepsilon$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$.

The total number of floating-point operations for one right-hand side vector is approximately $8n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call [?hpcon](#).

To refine the solution and estimate the error, call [?hprfs](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?trtrs](#)

Solves a system of linear equations with a triangular coefficient matrix, with multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_strtrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int nrhs , const float * a , lapack_int lda , float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_dtrtrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int nrhs , const double * a , lapack_int lda , double * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_ctrtrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int nrhs , const lapack_complex_float * a , lapack_int lda ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_ztrtrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int nrhs , const lapack_complex_double * a , lapack_int lda ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mkl.h`

Description

The routine solves for X the following systems of linear equations with a triangular matrix A , with multiple right-hand sides stored in B :

$A * X = B$ if *trans* = 'N',
 $A^T * X = B$ if *trans* = 'T',
 $A^H * X = B$ if *trans* = 'C' (for complex matrices only).

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

uplo Must be 'U' or 'L'.
 Indicates whether *A* is upper or lower triangular:
 If *uplo* = 'U', then *A* is upper triangular.
 If *uplo* = 'L', then *A* is lower triangular.

trans Must be 'N' or 'T' or 'C'.
 If *trans* = 'N', then $A * X = B$ is solved for *X*.
 If *trans* = 'T', then $A^T * X = B$ is solved for *X*.
 If *trans* = 'C', then $A^H * X = B$ is solved for *X*.

diag Must be 'N' or 'U'.
 If *diag* = 'N', then *A* is not a unit triangular matrix.
 If *diag* = 'U', then *A* is unit triangular: diagonal elements of *A* are assumed to be 1 and not referenced in the array *a*.

n The order of *A*; the number of rows in *B*; $n \geq 0$.

nrhs The number of right-hand sides; $nrhs \geq 0$.

a The array *a* contains the matrix *A*.
 The size of *a* is $\max(1, lda * n)$.

b The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.
 The size of *b* is $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout.

lda The leading dimension of *a*; $lda \geq \max(1, n)$.

ldb The leading dimension of *b*; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

Output Parameters

b Overwritten by the solution matrix *X*.

Return Values

This function returns a value *info*.

If `info=0`, the execution is successful.

If `info = -i`, parameter `i` had an illegal value.

Application Notes

For each right-hand side b , the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon |A|$$

$c(n)$ is a modest linear function of n , and ε is the machine precision. If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\varepsilon \text{ provided } c(n) \text{cond}(A, x)\varepsilon < 1$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector b is n^2 for real flavors and $4n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call `?trcon`.

To estimate the error in the solution, call `?trrfs`.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?tptrs

Solves a system of linear equations with a packed triangular coefficient matrix, with multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_stptrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int nrhs , const float * ap , float * b , lapack_int ldb );
lapack_int LAPACKE_dtptrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int nrhs , const double * ap , double * b , lapack_int ldb );
lapack_int LAPACKE_ctptrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int nrhs , const lapack_complex_float * ap , lapack_complex_float
* b , lapack_int ldb );
lapack_int LAPACKE_ztptrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int nrhs , const lapack_complex_double * ap ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mk1.h`

Description

The routine solves for X the following systems of linear equations with a packed triangular matrix A , with multiple right-hand sides stored in B :

$$A * X = B \quad \text{if } trans = 'N',$$

$$A^T * X = B \quad \text{if } trans = 'T',$$

$$A^H * X = B \quad \text{if } trans = 'C' \text{ (for complex matrices only).}$$

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If <i>uplo</i> = 'U', then A is upper triangular. If <i>uplo</i> = 'L', then A is lower triangular. |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. If <i>trans</i> = 'N', then $A * X = B$ is solved for X . If <i>trans</i> = 'T', then $A^T * X = B$ is solved for X . If <i>trans</i> = 'C', then $A^H * X = B$ is solved for X . |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', then A is not a unit triangular matrix. If <i>diag</i> = 'U', then A is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array <i>ap</i> . |
| <i>n</i> | The order of A ; the number of rows in B ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ap</i> | The dimension of array <i>ap</i> must be at least $\max(1, n(n+1)/2)$. The array <i>ap</i> contains the matrix A in <i>packed storage</i> (see Matrix Storage Schemes). |
| <i>b</i> | The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the system of equations. The size of <i>b</i> is $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>b</i> | Overwritten by the solution matrix X . |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon |A|$$

c(*n*) is a modest linear function of *n*, and ε is the machine precision.

If x_0 is the true solution, the computed solution *x* satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\varepsilon \text{ provided } c(n) \text{cond}(A, x)\varepsilon < 1$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector *b* is n^2 for real flavors and $4n^2$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call [?tpcon](#).

To estimate the error in the solution, call [?tprfs](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?tbtrs](#)

Solves a system of linear equations with a band triangular coefficient matrix, with multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_stbtrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int kd , lapack_int nrhs , const float * ab , lapack_int ldab ,
float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dtbtrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int kd , lapack_int nrhs , const double * ab , lapack_int ldab ,
double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_ctbtrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int kd , lapack_int nrhs , const lapack_complex_float * ab ,
lapack_int ldab , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_ztbtrs (int matrix_layout , char uplo , char trans , char diag ,
lapack_int n , lapack_int kd , lapack_int nrhs , const lapack_complex_double * ab ,
lapack_int ldab , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mkl.h`

Description

The routine solves for X the following systems of linear equations with a band triangular matrix A , with multiple right-hand sides stored in B :

$$\begin{aligned}
 A * X &= B && \text{if } trans = 'N', \\
 A^T * X &= B && \text{if } trans = 'T', \\
 A^H * X &= B && \text{if } trans = 'C' \text{ (for complex matrices only).}
 \end{aligned}$$

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If <i>uplo</i> = 'U', then A is upper triangular. If <i>uplo</i> = 'L', then A is lower triangular. |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. If <i>trans</i> = 'N', then $A * X = B$ is solved for X . If <i>trans</i> = 'T', then $A^T * X = B$ is solved for X . If <i>trans</i> = 'C', then $A^H * X = B$ is solved for X . |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', then A is not a unit triangular matrix. If <i>diag</i> = 'U', then A is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array <i>ab</i> . |
| <i>n</i> | The order of A ; the number of rows in B ; $n \geq 0$. |
| <i>kd</i> | The number of superdiagonals or subdiagonals in the matrix A ; $kd \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ab</i> | The array <i>ab</i> contains the matrix A in <i>band storage form</i> . The size of <i>ab</i> must be $\max(1, ldab * n)$ |
| <i>b</i> | The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations. The size of <i>b</i> is $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout. |
| <i>ldab</i> | The leading dimension of <i>ab</i> ; $ldab \geq kd + 1$. |

ldb The leading dimension of *b*; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

Output Parameters

b Overwritten by the solution matrix *X*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

For each right-hand side *b*, the computed solution is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\varepsilon|A|$$

$c(n)$ is a modest linear function of n , and ε is the machine precision. If x_0 is the true solution, the computed solution x satisfies this error bound:

$$\frac{\|x - x_0\|_\infty}{\|x\|_\infty} \leq c(n) \text{cond}(A, x)\varepsilon \text{ provided } c(n) \text{cond}(A, x)\varepsilon < 1$$

where $\text{cond}(A, x) = \| |A^{-1}| |A| |x| \|_\infty / \|x\|_\infty \leq \|A^{-1}\|_\infty \|A\|_\infty = \kappa_\infty(A)$.

Note that $\text{cond}(A, x)$ can be much smaller than $\kappa_\infty(A)$; the condition number of A^T and A^H might or might not be equal to $\kappa_\infty(A)$.

The approximate number of floating-point operations for one right-hand side vector *b* is $2n \cdot kd$ for real flavors and $8n \cdot kd$ for complex flavors.

To estimate the condition number $\kappa_\infty(A)$, call [?tbcon](#).

To estimate the error in the solution, call [?tbrfs](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

Estimating the Condition Number: LAPACK Computational Routines

This section describes the LAPACK routines for estimating the *condition number* of a matrix. The condition number is used for analyzing the errors in the solution of a system of linear equations (see [Error Analysis](#)). Since the condition number may be arbitrarily large when the matrix is nearly singular, the routines actually compute the *reciprocal* condition number.

[?gecon](#)

Estimates the reciprocal of the condition number of a general matrix in the 1-norm or the infinity-norm.

Syntax

```
lapack_int LAPACKE_sgecon( int matrix_layout, char norm, lapack_int n, const float* a,
lapack_int lda, float anorm, float* rcond );
```

```
lapack_int LAPACKE_dgecon( int matrix_layout, char norm, lapack_int n, const double* a,
lapack_int lda, double anorm, double* rcond );
```

```
lapack_int LAPACKE_cgecon( int matrix_layout, char norm, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float anorm, float* rcond );
```

```
lapack_int LAPACKE_zgecon( int matrix_layout, char norm, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a general matrix A in the 1-norm or infinity-norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$$

$$\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty = \kappa_1(A^T) = \kappa_1(A^H).$$

An estimate is obtained for $\|A^{-1}\|$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\| \|A^{-1}\|)$.

Before calling this routine:

- compute $anorm$ (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call [?getrf](#) to compute the LU factorization of A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>norm</i> | Must be '1' or 'O' or 'I'. If <i>norm</i> = '1' or 'O', then the routine estimates the condition number of matrix A in 1-norm. If <i>norm</i> = 'I', then the routine estimates the condition number of matrix A in infinity-norm. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | The array a contains the LU -factored matrix A , as returned by ?getrf . |
| <i>anorm</i> | The norm of the <i>original</i> matrix A (see Description). |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets $rcond = 0$ if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime $rcond$ is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A*x = b$ or $A^H*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2*n^2$ floating-point operations for real flavors and $8*n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gbcon

Estimates the reciprocal of the condition number of a band matrix in the 1-norm or the infinity-norm.

Syntax

```
lapack_int LAPACKE_sgbcon( int matrix_layout, char norm, lapack_int n, lapack_int kl,
lapack_int ku, const float* ab, lapack_int ldab, const lapack_int* ipiv, float anorm,
float* rcond );
```

```
lapack_int LAPACKE_dgbcon( int matrix_layout, char norm, lapack_int n, lapack_int kl,
lapack_int ku, const double* ab, lapack_int ldab, const lapack_int* ipiv, double
anorm, double* rcond );
```

```
lapack_int LAPACKE_cgbcon( int matrix_layout, char norm, lapack_int n, lapack_int kl,
lapack_int ku, const lapack_complex_float* ab, lapack_int ldab, const lapack_int* ipiv,
float anorm, float* rcond );
```

```
lapack_int LAPACKE_zgbcon( int matrix_layout, char norm, lapack_int n, lapack_int kl,
lapack_int ku, const lapack_complex_double* ab, lapack_int ldab, const lapack_int*
ipiv, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a general band matrix *A* in the 1-norm or infinity-norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$$

$$\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty = \kappa_1(A^T) = \kappa_1(A^H).$$

An estimate is obtained for $\|A^{-1}\|$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\| \|A^{-1}\|)$.

Before calling this routine:

- compute *anorm* (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call [?gbtrf](#) to compute the *LU* factorization of *A*.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>norm</i> | Must be '1' or 'O' or 'I'. If <i>norm</i> = '1' or 'O', then the routine estimates the condition number of matrix <i>A</i> in 1-norm. If <i>norm</i> = 'I', then the routine estimates the condition number of matrix <i>A</i> in infinity-norm. |
| <i>n</i> | The order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>kl</i> | The number of subdiagonals within the band of <i>A</i> ; $kl \geq 0$. |
| <i>ku</i> | The number of superdiagonals within the band of <i>A</i> ; $ku \geq 0$. |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> . ($ldab \geq 2*kl + ku + 1$). |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?gbtrf . |
| <i>ab</i> | The array <i>ab</i> of size $\max(1, ldab*n)$ contains the factored band matrix <i>A</i> , as returned by ?gbtrf . |
| <i>anorm</i> | The norm of the <i>original</i> matrix <i>A</i> (see Description). |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets <i>rcond</i> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <i>rcond</i> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than $10r$. A call to this routine involves solving a number of systems of linear equations $A*x = b$ or $A^H*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n(ku + 2kl)$ floating-point operations for real flavors and $8n(ku + 2kl)$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gtcon

Estimates the reciprocal of the condition number of a tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_sgtcon( char norm, lapack_int n, const float* dl, const float* d,
const float* du, const float* du2, const lapack_int* ipiv, float anorm, float* rcond );
```

```
lapack_int LAPACKE_dgtcon( char norm, lapack_int n, const double* dl, const double* d,
const double* du, const double* du2, const lapack_int* ipiv, double anorm, double*
rcond );
```

```
lapack_int LAPACKE_cgtcon( char norm, lapack_int n, const lapack_complex_float* dl,
const lapack_complex_float* d, const lapack_complex_float* du, const
lapack_complex_float* du2, const lapack_int* ipiv, float anorm, float* rcond );
```

```
lapack_int LAPACKE_zgtcon( char norm, lapack_int n, const lapack_complex_double* dl,
const lapack_complex_double* d, const lapack_complex_double* du, const
lapack_complex_double* du2, const lapack_int* ipiv, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a real or complex tridiagonal matrix A in the 1-norm or infinity-norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1$$

$$\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$$

An estimate is obtained for $\|A^{-1}\|$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\| \|A^{-1}\|)$.

Before calling this routine:

- compute $anorm$ (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call `?gttrf` to compute the LU factorization of A .

Input Parameters

| | |
|--------------------|---|
| <i>norm</i> | Must be '1' or 'O' or 'I'. If <i>norm</i> = '1' or 'O', then the routine estimates the condition number of matrix A in 1-norm. If <i>norm</i> = 'I', then the routine estimates the condition number of matrix A in infinity-norm. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>dl,d,du,du2</i> | Arrays: $dl(n-1)$, $d(n)$, $du(n-1)$, $du2(n-2)$. The array <i>dl</i> contains the $(n-1)$ multipliers that define the matrix L from the LU factorization of A as computed by <code>?gttrf</code> . The array <i>d</i> contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A . The array <i>du</i> contains the $(n-1)$ elements of the first superdiagonal of U . The array <i>du2</i> contains the $(n-2)$ elements of the second superdiagonal of U . |

ipiv Array, size (*n*). The array of pivot indices, as returned by `?gttrf`.

anorm The norm of the *original* matrix *A*(see *Description*).

Output Parameters

rcond An estimate of the reciprocal of the condition number. The routine sets *rcond*=0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

?pocon

Estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite matrix.

Syntax

```
lapack_int LAPACKE_spocon( int matrix_layout, char uplo, lapack_int n, const float* a,
lapack_int lda, float anorm, float* rcond );

lapack_int LAPACKE_dpocon( int matrix_layout, char uplo, lapack_int n, const double* a,
lapack_int lda, double anorm, double* rcond );

lapack_int LAPACKE_cpocon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float anorm, float* rcond );

lapack_int LAPACKE_zpocon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite matrix *A*:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \text{ (since } A \text{ is symmetric or Hermitian, } \kappa_\infty(A) = \kappa_1(A)\text{)}.$$

An estimate is obtained for $\|A^{-1}\|_1$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\|_1 \|A^{-1}\|_1)$.

Before calling this routine:

- compute $anorm$ (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call `?potrf` to compute the Cholesky factorization of A .

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo</code> = 'U', A is factored as $A = U^T * U$ for real flavors or $A = U^H * U$ for complex flavors, and U is stored. If <code>uplo</code> = 'L', A is factored as $A = L * L^T$ for real flavors or $A = L * L^H$ for complex flavors, and L is stored. |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>a</code> | The array a of size $\max(1, lda * n)$ contains the factored matrix A , as returned by <code>?potrf</code> . |
| <code>lda</code> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <code>anorm</code> | The norm of the <i>original</i> matrix A (see <i>Description</i>). |

Output Parameters

| | |
|--------------------|--|
| <code>rcond</code> | An estimate of the reciprocal of the condition number. The routine sets <code>rcond</code> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <code>rcond</code> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------------|--|

Return Values

This function returns a value `info`.

If `info` = 0, the execution is successful.

If `info` = $-i$, parameter i had an illegal value.

Application Notes

The computed `rcond` is never less than r (the reciprocal of the true condition number) and in practice is nearly always less than $10r$. A call to this routine involves solving a number of systems of linear equations $A * x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

`?ppcon`

Estimates the reciprocal of the condition number of a packed symmetric (Hermitian) positive-definite matrix.

Syntax

```
lapack_int LAPACKE_sppcon( int matrix_layout, char uplo, lapack_int n, const float* ap,
float anorm, float* rcond );
```

```
lapack_int LAPACKE_dppcon( int matrix_layout, char uplo, lapack_int n, const double*
ap, double anorm, double* rcond );
```

```
lapack_int LAPACKE_cppcon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* ap, float anorm, float* rcond );
```

```
lapack_int LAPACKE_zppcon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* ap, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a packed symmetric (Hermitian) positive-definite matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \text{ (since } A \text{ is symmetric or Hermitian, } \kappa_\infty(A) = \kappa_1(A)\text{)}.$$

An estimate is obtained for $\|A^{-1}\|_1$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\|_1 \|A^{-1}\|_1)$.

Before calling this routine:

- compute $anorm$ (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call `?pptrf` to compute the Cholesky factorization of A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, A is factored as $A = U^T * U$ for real flavors or $A = U^H * U$ for complex flavors, and U is stored. If $uplo = 'L'$, A is factored as $A = L * L^T$ for real flavors or $A = L * L^H$ for complex flavors, and L is stored. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>ap</i> | The array ap contains the packed factored matrix A , as returned by <code>?pptrf</code> . The dimension of ap must be at least $\max(1, n(n+1)/2)$. |
| <i>anorm</i> | The norm of the <i>original</i> matrix A (see <i>Description</i>). |

Output Parameters

rcond An estimate of the reciprocal of the condition number. The routine sets *rcond* = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pbcon

Estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite band matrix.

Syntax

```
lapack_int LAPACKE_spbcon( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
const float* ab, lapack_int ldab, float anorm, float* rcond );
```

```
lapack_int LAPACKE_dpbcon( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
const double* ab, lapack_int ldab, double anorm, double* rcond );
```

```
lapack_int LAPACKE_cpbcon( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
const lapack_complex_float* ab, lapack_int ldab, float anorm, float* rcond );
```

```
lapack_int LAPACKE_zpbcon( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
const lapack_complex_double* ab, lapack_int ldab, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite band matrix *A*:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \text{ (since } A \text{ is symmetric or Hermitian, } \kappa_\infty(A) = \kappa_1(A)\text{)}.$$

An estimate is obtained for $\|A^{-1}\|_1$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\|_1 \|A^{-1}\|_1)$.

Before calling this routine:

- compute *anorm* (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call [?pbtrf](#) to compute the Cholesky factorization of *A*.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo = 'U'</code> , A is factored as $A = U^T * U$ for real flavors or $A = U^H * U$ for complex flavors, and U is stored. If <code>uplo = 'L'</code> , A is factored as $A = L * L^T$ for real flavors or $A = L * L^H$ for complex flavors, and L is stored. |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>kd</code> | The number of superdiagonals or subdiagonals in the matrix A ; $kd \geq 0$. |
| <code>ldab</code> | The leading dimension of the array <code>ab</code> . ($ldab \geq kd + 1$). |
| <code>ab</code> | The array <code>ab</code> of size $\max(1, ldab * n)$ contains the factored matrix A in band form, as returned by <code>?pbtrf</code> . |
| <code>anorm</code> | The norm of the <i>original</i> matrix A (see <i>Description</i>). |

Output Parameters

| | |
|--------------------|--|
| <code>rcond</code> | An estimate of the reciprocal of the condition number. The routine sets <code>rcond = 0</code> if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <code>rcond</code> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------------|--|

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, parameter i had an illegal value.

Application Notes

The computed `rcond` is never less than r (the reciprocal of the true condition number) and in practice is nearly always less than $10r$. A call to this routine involves solving a number of systems of linear equations $A * x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $4 * n * (kd + 1)$ floating-point operations for real flavors and $16 * n * (kd + 1)$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?ptcon

Estimates the reciprocal of the condition number of a symmetric (Hermitian) positive-definite tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_sptcon( lapack_int n, const float* d, const float* e, float anorm, float* rcond );
```

```
lapack_int LAPACKE_dptcon( lapack_int n, const double* d, const double* e, double
anorm, double* rcond );
```

```
lapack_int LAPACKE_cptcon( lapack_int n, const float* d, const lapack_complex_float* e,
float anorm, float* rcond );
```

```
lapack_int LAPACKE_zptcon( lapack_int n, const double* d, const lapack_complex_double*
e, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine computes the reciprocal of the condition number (in the 1-norm) of a real symmetric or complex Hermitian positive-definite tridiagonal matrix using the factorization $A = L^*D^*L^T$ for real flavors and $A = L^*D^*L^H$ for complex flavors or $A = U^T*D^*U$ for real flavors and $A = U^H*D^*U$ for complex flavors computed by [?pttrf](#) :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \text{ (since } A \text{ is symmetric or Hermitian, } \kappa_\infty(A) = \kappa_1(A) \text{)}.$$

The norm $\|A^{-1}\|_1$ is computed by a direct method, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\|_1 \|A^{-1}\|_1)$.

Before calling this routine:

- compute *anorm* as $\|A\|_1 = \max_j \sum_i |a_{ij}|$
- call [?pttrf](#) to compute the factorization of *A*.

Input Parameters

| | |
|--------------|--|
| <i>n</i> | The order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>d</i> | Arrays, dimension (<i>n</i>). The array <i>d</i> contains the <i>n</i> diagonal elements of the diagonal matrix <i>D</i> from the factorization of <i>A</i> , as computed by ?pttrf ; |
| <i>e</i> | Array, size (<i>n</i> - 1). Contains off-diagonal elements of the unit bidiagonal factor <i>U</i> or <i>L</i> from the factorization computed by ?pttrf . |
| <i>anorm</i> | The 1- norm of the <i>original</i> matrix <i>A</i> (see <i>Description</i>). |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets <i>rcond</i> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <i>rcond</i> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed $rcond$ is never less than r (the reciprocal of the true condition number) and in practice is nearly always less than $10r$. A call to this routine involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $4*n*(kd + 1)$ floating-point operations for real flavors and $16*n*(kd + 1)$ for complex flavors.

?ssycon

Estimates the reciprocal of the condition number of a symmetric matrix.

Syntax

```
lapack_int LAPACKE_ssycon( int matrix_layout, char uplo, lapack_int n, const float* a,
lapack_int lda, const lapack_int* ipiv, float anorm, float* rcond );

lapack_int LAPACKE_dsycon( int matrix_layout, char uplo, lapack_int n, const double* a,
lapack_int lda, const lapack_int* ipiv, double anorm, double* rcond );

lapack_int LAPACKE_csycon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, const lapack_int* ipiv, float anorm, float*
rcond );

lapack_int LAPACKE_zsycon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, const lapack_int* ipiv, double anorm, double*
rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a symmetric matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \text{ (since } A \text{ is symmetric, } \kappa_\infty(A) = \kappa_1(A)\text{)}.$$

An estimate is obtained for $\|A^{-1}\|_1$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\|_1 \|A^{-1}\|_1)$.

Before calling this routine:

- compute $anorm$ (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call [?sytrf](#) to compute the factorization of A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, the array a stores the upper triangular factor U of the factorization $A = U*D*U^T$. If $uplo = 'L'$, the array a stores the lower triangular factor L of the factorization $A = L*D*L^T$. |

| | |
|--------------|--|
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>a</i> | The array <i>a</i> of size $\max(1, lda * n)$ contains the factored matrix <i>A</i> , as returned by <code>?sytrf</code> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The array <i>ipiv</i> , as returned by <code>?sytrf</code> . |
| <i>anorm</i> | The norm of the <i>original</i> matrix <i>A</i> (see <i>Description</i>). |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets <i>rcond</i> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <i>rcond</i> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A * x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

`?hecon`

Estimates the reciprocal of the condition number of a Hermitian matrix.

Syntax

```
lapack_int LAPACKE_checon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, const lapack_int* ipiv, float anorm, float*
rcond );
```

```
lapack_int LAPACKE_zhecon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, const lapack_int* ipiv, double anorm, double*
rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a Hermitian matrix *A*:

$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1$ (since A is Hermitian, $\kappa_\infty(A) = \kappa_1(A)$).

Before calling this routine:

- compute *anorm* (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call `?hetrf` to compute the factorization of A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <i>uplo</i> = 'U', the array a stores the upper triangular factor U of the factorization $A = U^*D^*U^H$. If <i>uplo</i> = 'L', the array a stores the lower triangular factor L of the factorization $A = L^*D^*L^H$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>a</i> | The array a of size $\max(1, lda*n)$ contains the factored matrix A , as returned by <code>?hetrf</code> . |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The array <i>ipiv</i> , as returned by <code>?hetrf</code> . |
| <i>anorm</i> | The norm of the <i>original</i> matrix A (see <i>Description</i>). |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets <i>rcond</i> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <i>rcond</i> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than r (the reciprocal of the true condition number) and in practice is nearly always less than $10r$. A call to this routine involves solving a number of systems of linear equations $A^*x = b$; the number is usually 5 and never more than 11. Each solution requires approximately $8n^2$ floating-point operations.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?spcon

Estimates the reciprocal of the condition number of a packed symmetric matrix.

Syntax

```
lapack_int LAPACKE_sspcon( int matrix_layout, char uplo, lapack_int n, const float* ap,
const lapack_int* ipiv, float anorm, float* rcond );
```

```
lapack_int LAPACKE_dspcon( int matrix_layout, char uplo, lapack_int n, const double*
ap, const lapack_int* ipiv, double anorm, double* rcond );
```

```
lapack_int LAPACKE_cspcon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* ap, const lapack_int* ipiv, float anorm, float* rcond );
```

```
lapack_int LAPACKE_zspcon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* ap, const lapack_int* ipiv, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a packed symmetric matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \text{ (since } A \text{ is symmetric, } \kappa_\infty(A) = \kappa_1(A)\text{)}.$$

An estimate is obtained for $\|A^{-1}\|_1$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\|_1 \|A^{-1}\|_1)$.

Before calling this routine:

- compute $anorm$ (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call [?sptrf](#) to compute the factorization of A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <i>uplo</i> = 'U', the array <i>ap</i> stores the packed upper triangular factor U of the factorization $A = U^*D^*U^T$. If <i>uplo</i> = 'L', the array <i>ap</i> stores the packed lower triangular factor L of the factorization $A = L^*D^*L^T$. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>ap</i> | The array <i>ap</i> contains the packed factored matrix A , as returned by ?sptrf . The dimension of <i>ap</i> must be at least $\max(1, n(n+1)/2)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The array <i>ipiv</i> , as returned by ?sptrf . |
| <i>anorm</i> | The norm of the <i>original</i> matrix A (see <i>Description</i>). |

Output Parameters

rcond An estimate of the reciprocal of the condition number. The routine sets *rcond* = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime *rcond* is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors and $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?hpcon

Estimates the reciprocal of the condition number of a packed Hermitian matrix.

Syntax

```
lapack_int LAPACKE_chpcon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* ap, const lapack_int* ipiv, float anorm, float* rcond );
```

```
lapack_int LAPACKE_zhpcon( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* ap, const lapack_int* ipiv, double anorm, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a Hermitian matrix *A*:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \text{ (since } A \text{ is Hermitian, } \kappa_\infty(A) = \kappa_1(A)\text{)}.$$

An estimate is obtained for $\|A^{-1}\|_1$, and the reciprocal of the condition number is computed as $rcond = 1 / (\|A\|_1 \|A^{-1}\|_1)$.

Before calling this routine:

- compute *anorm* (either $\|A\|_1 = \max_j \sum_i |a_{ij}|$ or $\|A\|_\infty = \max_i \sum_j |a_{ij}|$)
- call [?hptrf](#) to compute the factorization of *A*.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

| | |
|--------------|---|
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix <i>A</i> has been factored: If <i>uplo</i> = 'U', the array <i>ap</i> stores the packed upper triangular factor <i>U</i> of the factorization $A = U^*D^*U^T$. If <i>uplo</i> = 'L', the array <i>ap</i> stores the packed lower triangular factor <i>L</i> of the factorization $A = L^*D^*L^T$. |
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>ap</i> | The array <i>ap</i> contains the packed factored matrix <i>A</i> , as returned by ?hptrf . The dimension of <i>ap</i> must be at least $\max(1, n(n+1)/2)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The array <i>ipiv</i> , as returned by ?hptrf . |
| <i>anorm</i> | The norm of the <i>original</i> matrix <i>A</i> (see <i>Description</i>). |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets <i>rcond</i> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <i>rcond</i> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A^*x = b$; the number is usually 5 and never more than 11. Each solution requires approximately $8n^2$ floating-point operations.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?trcon](#)

Estimates the reciprocal of the condition number of a triangular matrix.

Syntax

```
lapack_int LAPACKE_strcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, const float* a, lapack_int lda, float* rcond );
```

```
lapack_int LAPACKE_dtrcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, const double* a, lapack_int lda, double* rcond );
```

```
lapack_int LAPACKE_ctrcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, const lapack_complex_float* a, lapack_int lda, float* rcond );
```

```
lapack_int LAPACKE_ztrcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, const lapack_complex_double* a, lapack_int lda, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a triangular matrix A in either the 1-norm or infinity-norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$$

$$\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty = \kappa_1(A^T) = \kappa_1(A^H).$$

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>norm</i> | Must be '1' or 'O' or 'I'. If <i>norm</i> = '1' or 'O', then the routine estimates the condition number of matrix A in 1-norm. If <i>norm</i> = 'I', then the routine estimates the condition number of matrix A in infinity-norm. |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If <i>uplo</i> = 'U', the array a stores the upper triangle of A , other array elements are not referenced. If <i>uplo</i> = 'L', the array a stores the lower triangle of A , other array elements are not referenced. |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', then A is not a unit triangular matrix. If <i>diag</i> = 'U', then A is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array a . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | The array a of size $\max(1, lda * n)$ contains the matrix A . |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets <i>rcond</i> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <i>rcond</i> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately n^2 floating-point operations for real flavors and $4n^2$ operations for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?tpcon

Estimates the reciprocal of the condition number of a packed triangular matrix.

Syntax

```
lapack_int LAPACKE_stpcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, const float* ap, float* rcond );
```

```
lapack_int LAPACKE_dtpcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, const double* ap, double* rcond );
```

```
lapack_int LAPACKE_ctpcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, const lapack_complex_float* ap, float* rcond );
```

```
lapack_int LAPACKE_ztpcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, const lapack_complex_double* ap, double* rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a packed triangular matrix *A* in either the 1-norm or infinity-norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$$

$$\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty = \kappa_1(A^T) = \kappa_1(A^H).$$

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

norm Must be '1' or 'O' or 'I'.

If *norm* = '1' or 'O', then the routine estimates the condition number of matrix *A* in 1-norm.

If *norm* = 'I', then the routine estimates the condition number of matrix *A* in infinity-norm.

| | |
|-------------|--|
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether <i>A</i> is upper or lower triangular: If <i>uplo</i> = 'U', the array <i>ap</i> stores the upper triangle of <i>A</i> in packed form. If <i>uplo</i> = 'L', the array <i>ap</i> stores the lower triangle of <i>A</i> in packed form. |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', then <i>A</i> is not a unit triangular matrix. If <i>diag</i> = 'U', then <i>A</i> is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array <i>ap</i> . |
| <i>n</i> | The order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>ap</i> | The array <i>ap</i> contains the packed matrix <i>A</i> . The dimension of <i>ap</i> must be at least $\max(1, n(n+1)/2)$. |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets <i>rcond</i> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <i>rcond</i> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A \cdot x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately n^2 floating-point operations for real flavors and $4n^2$ operations for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?tbcon

Estimates the reciprocal of the condition number of a triangular band matrix.

Syntax

```
lapack_int LAPACKE_stbcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, lapack_int kd, const float* ab, lapack_int ldab, float* rcond );
lapack_int LAPACKE_dtbcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, lapack_int kd, const double* ab, lapack_int ldab, double* rcond );
lapack_int LAPACKE_ctbcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, lapack_int kd, const lapack_complex_float* ab, lapack_int ldab, float*
rcond );
```

```
lapack_int LAPACKE_ztbcon( int matrix_layout, char norm, char uplo, char diag,
lapack_int n, lapack_int kd, const lapack_complex_double* ab, lapack_int ldab, double*
rcond );
```

Include Files

- mkl.h

Description

The routine estimates the reciprocal of the condition number of a triangular band matrix A in either the 1-norm or infinity-norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 = \kappa_\infty(A^T) = \kappa_\infty(A^H)$$

$$\kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty = \kappa_1(A^T) = \kappa_1(A^H).$$

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>norm</i> | Must be '1' or 'O' or 'I'. If <i>norm</i> = '1' or 'O', then the routine estimates the condition number of matrix A in 1-norm. If <i>norm</i> = 'I', then the routine estimates the condition number of matrix A in infinity-norm. |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If <i>uplo</i> = 'U', the array <i>ap</i> stores the upper triangle of A in packed form. If <i>uplo</i> = 'L', the array <i>ap</i> stores the lower triangle of A in packed form. |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', then A is not a unit triangular matrix. If <i>diag</i> = 'U', then A is unit triangular: diagonal elements are assumed to be 1 and not referenced in the array <i>ab</i> . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>kd</i> | The number of superdiagonals or subdiagonals in the matrix A ; $kd \geq 0$. |
| <i>ab</i> | The array <i>ab</i> of size $\max(1, ldab * n)$ contains the band matrix A . |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> . ($ldab \geq kd + 1$). |

Output Parameters

| | |
|--------------|--|
| <i>rcond</i> | An estimate of the reciprocal of the condition number. The routine sets <i>rcond</i> = 0 if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime <i>rcond</i> is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular. |
|--------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The computed *rcond* is never less than *r* (the reciprocal of the true condition number) and in practice is nearly always less than 10*r*. A call to this routine involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2*n(kd + 1)$ floating-point operations for real flavors and $8*n(kd + 1)$ operations for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

Refining the Solution and Estimating Its Error: LAPACK Computational Routines

This section describes the LAPACK routines for refining the computed solution of a system of linear equations and estimating the solution error. You can call these routines after factorizing the matrix of the system of equations and computing the solution (see [Routines for Matrix Factorization](#) and [Routines for Solving Systems of Linear Equations](#)).

?gerfs

Refines the solution of a system of linear equations with a general coefficient matrix and estimates its error.

Syntax

```
lapack_int LAPACKE_sgerfs( int matrix_layout, char trans, lapack_int n, lapack_int
nrhs, const float* a, lapack_int lda, const float* af, lapack_int ldaf, const
lapack_int* ipiv, const float* b, lapack_int ldb, float* x, lapack_int ldx, float*
ferr, float* berr );
```

```
lapack_int LAPACKE_dgerfs( int matrix_layout, char trans, lapack_int n, lapack_int
nrhs, const double* a, lapack_int lda, const double* af, lapack_int ldaf, const
lapack_int* ipiv, const double* b, lapack_int ldb, double* x, lapack_int ldx, double*
ferr, double* berr );
```

```
lapack_int LAPACKE_cgerfs( int matrix_layout, char trans, lapack_int n, lapack_int
nrhs, const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zgerfs( int matrix_layout, char trans, lapack_int n, lapack_int
nrhs, const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_double* b, lapack_int
ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A^*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a general matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \quad \text{such that} \quad (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?getrf](#)
- call the solver routine [?getrs](#).

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>trans</code> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <code>trans</code> = 'N', the system has the form $A^*X = B$. If <code>trans</code> = 'T', the system has the form $A^T*X = B$. If <code>trans</code> = 'C', the system has the form $A^H*X = B$. |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; $nrhs \geq 0$. |
| <code>a,af,b,x</code> | Arrays: <code>a</code> (size $\max(1, lda*n)$) contains the original matrix A , as supplied to ?getrf . <code>af</code> (size $\max(1, ldaf*n)$) contains the factored matrix A , as returned by ?getrf . <code>b</code> of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the right-hand side matrix B . <code>x</code> of size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout contains the solution matrix X . |
| <code>lda</code> | The leading dimension of <code>a</code> ; $lda \geq \max(1, n)$. |
| <code>ldaf</code> | The leading dimension of <code>af</code> ; $ldaf \geq \max(1, n)$. |
| <code>ldb</code> | The leading dimension of <code>b</code> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <code>ldx</code> | The leading dimension of <code>x</code> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <code>ipiv</code> | Array, size at least $\max(1, n)$. The <code>ipiv</code> array, as returned by ?getrf . |

Output Parameters

| | |
|--------------|---|
| x | The refined solution matrix X . |
| $ferr, berr$ | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5. Estimating the forward error involves solving a number of systems of linear equations $A*x = b$ with the same coefficient matrix A and different right hand sides b ; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?gerfsx](#)

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a general coefficient matrix A and provides error bounds and backward error estimates.

Syntax

```
lapack_int LAPACKE_sgerfsx( int matrix_layout, char trans, char equed, lapack_int n,
lapack_int nrhs, const float* a, lapack_int lda, const float* af, lapack_int ldaf,
const lapack_int* ipiv, const float* r, const float* c, const float* b, lapack_int
ldb, float* x, lapack_int ldx, float* rcond, float* berr, lapack_int n_err_bnds,
float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams, float* params );
```

```
lapack_int LAPACKE_dgerfsx( int matrix_layout, char trans, char equed, lapack_int n,
lapack_int nrhs, const double* a, lapack_int lda, const double* af, lapack_int ldaf,
const lapack_int* ipiv, const double* r, const double* c, const double* b, lapack_int
ldb, double* x, lapack_int ldx, double* rcond, double* berr, lapack_int n_err_bnds,
double* err_bnds_norm, double* err_bnds_comp, lapack_int nparams, double* params );
```

```
lapack_int LAPACKE_cgerfsx( int matrix_layout, char trans, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* af, lapack_int ldaf, const lapack_int* ipiv, const float* r,
const float* c, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* berr, lapack_int n_err_bnds, float*
err_bnds_norm, float* err_bnds_comp, lapack_int nparams, float* params );
```

```
lapack_int LAPACKE_zgerfsx( int matrix_layout, char trans, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* af, lapack_int ldaf, const lapack_int* ipiv, const double* r,
```

```
const double* c, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double*
x, lapack_int ldx, double* rcond, double* berr, lapack_int n_err_bnds, double*
err_bnds_norm, double* err_bnds_comp, lapack_int nparams, double* params );
```

Include Files

- mkl.h

Description

The routine improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed*, *r*, and *c* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

| | |
|---------------------------------|---|
| <i>matrix_layout</i> | Specifies whether two-dimensional array storage is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>trans</i> | Must be 'N', 'T', or 'C'. Specifies the form of the system of equations: If <i>trans</i> = 'N', the system has the form $A^*X = B$ (No transpose). If <i>trans</i> = 'T', the system has the form $A^T X = B$ (Transpose). If <i>trans</i> = 'C', the system has the form $A^H X = B$ (Conjugate transpose for complex flavors, Transpose for real flavors). |
| <i>equed</i> | Must be 'N', 'R', 'C', or 'B'. Specifies the form of equilibration that was done to <i>A</i> before calling this routine. If <i>equed</i> = 'N', no equilibration was done. If <i>equed</i> = 'R', row equilibration was done, that is, <i>A</i> has been premultiplied by <i>diag(r)</i> . If <i>equed</i> = 'C', column equilibration was done, that is, <i>A</i> has been postmultiplied by <i>diag(c)</i> . If <i>equed</i> = 'B', both row and column equilibration was done, that is, <i>A</i> has been replaced by $diag(r) * A * diag(c)$. The right-hand side <i>B</i> has been changed accordingly. |
| <i>n</i> | The number of linear equations; the order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns of the matrices <i>B</i> and <i>X</i> ; $nrhs \geq 0$. |
| <i>a</i> , <i>af</i> , <i>b</i> | Arrays: <i>a</i> (size $\max(1, lda * n)$), <i>af</i> (size $\max(1, ldaf * n)$), <i>b</i> (size $\max(1, ldb * nrhs)$) for column major layout and $\max(1, ldb * n)$ for row major layout). The array <i>a</i> contains the original <i>n</i> -by- <i>n</i> matrix <i>A</i> . |

The array *af* contains the factored form of the matrix *A*, that is, the factors *L* and *U* from the factorization $A = P * L * U$ as computed by `?getrf`.

The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.

| | |
|-------------------|--|
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Contains the pivot indices as computed by <code>?getrf</code> ; for row $1 \leq i \leq n$, row <i>i</i> of the matrix was interchanged with row <i>ipiv</i> (<i>i</i>). |
| <i>r, c</i> | <p>Arrays: <i>r</i> (size <i>n</i>), <i>c</i> (size <i>n</i>). The array <i>r</i> contains the row scale factors for <i>A</i>, and the array <i>c</i> contains the column scale factors for <i>A</i>.</p> <p><i>equed</i> = 'R' or 'B', <i>A</i> is multiplied on the left by <i>diag</i>(<i>r</i>); if <i>equed</i> = 'N' or 'C', <i>r</i> is not accessed.</p> <p>If <i>equed</i> = 'R' or 'B', each element of <i>r</i> must be positive.</p> <p>If <i>equed</i> = 'C' or 'B', <i>A</i> is multiplied on the right by <i>diag</i>(<i>c</i>); if <i>equed</i> = 'N' or 'R', <i>c</i> is not accessed.</p> <p>If <i>equed</i> = 'C' or 'B', each element of <i>c</i> must be positive.</p> <p>Each element of <i>r</i> or <i>c</i> should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.</p> |
| <i>ldb</i> | The leading dimension of the array <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>x</i> | <p>Array, of size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout.</p> <p>The solution matrix <i>X</i> as computed by <code>?getrs</code></p> |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <i>n_err_bnds</i> | Number of error bounds to return for each right hand side and each type (normwise or componentwise). See <i>err_bnds_norm</i> and <i>err_bnds_comp</i> descriptions in <i>Output Arguments</i> section below. |
| <i>nparams</i> | Specifies the number of parameters set in <i>params</i> . If ≤ 0 , the <i>params</i> array is never referenced and default values are used. |
| <i>params</i> | <p>Array, size <i>nparams</i>. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to <i>nparams</i> are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass <i>nparams</i> = 0, which prevents the source code from accessing the <i>params</i> argument.</p> <p><i>params</i>[0] : Whether to perform iterative refinement or not. Default: 1.0</p> |

| | |
|------|--|
| =0.0 | No refinement is performed and no error bounds are computed. |
| =1.0 | Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support double precision. |

(Other values are reserved for future use.)

params[1] : Maximum number of residual computations allowed for refinement.

Default 10.0

Aggressive Set to 100.0 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

Output Parameters

| | |
|----------------------|--|
| <i>x</i> | The improved solution matrix <i>X</i> . |
| <i>rcond</i> | Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix <i>A</i> after equilibration (if done). If <i>rcond</i> is less than the machine precision, in particular, if <i>rcond</i> = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned. |
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of <i>A</i> or <i>B</i> that makes x_j an exact solution. |
| <i>err_bnds_norm</i> | Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows: |

Normwise relative error in the *i*-th solution vector

$$\frac{\max_j |X_{true_{ji}} - X_{ji}|}{\max_j |X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.

- `err=1` "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors.
- `err=2` "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.
- `err=3` Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z=s*a$, where s scales each row by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error `err` is stored in:

- Column major layout: `err_bnds_norm[(err - 1)*nrhs + i - 1]`.
- Row major layout: `err_bnds_norm[err - 1 + (i - 1)*n_err_bnds]`

`err_bnds_comp`

Array of size `nrhs*n_err_bnds`. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the i -th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested (`params[2] = 0.0`), then `err_bnds_comp` is not accessed. If `n_err_bnds < 3`, then at most the first `n_err_bnds` columns of the `err_bnds_comp` array are returned.

| | |
|--------------|--|
| <i>err=1</i> | "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. |
| <i>err=2</i> | "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true. |
| <i>err=3</i> | Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are: |

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq \text{nrhs}$, and type of error *err* is stored in:

- Column major layout: $\text{err_bnds_comp}[(\text{err} - 1) * \text{nrhs} + i - 1]$.
- Row major layout: $\text{err_bnds_comp}[\text{err} - 1 + (i - 1) * n_err_bnds]$

params

Output parameter only if the input contains erroneous values, namely, in $\text{params}[0]$, $\text{params}[1]$, $\text{params}[2]$. In such a case, the corresponding elements of *params* are filled with default values on output.

Return Values

This function returns a value *info*.

If $\text{info} = 0$, the execution is successful. The solution to every right-hand side is guaranteed.

If $\text{info} = -i$, parameter i had an illegal value.

If $0 < \text{info} \leq n$: $U_{\text{info}, \text{info}}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; $\text{rcond} = 0$ is returned.

If $\text{info} = n+j$: The solution corresponding to the j -th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides k with $k > j$ may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested $\text{params}[2] = 0.0$, then the j -th

right-hand side is the first with a normwise error bound that is not guaranteed (the smallest j such that for column major layout $err_bnds_norm[j - 1] = 0.0$ or $err_bnds_comp[j - 1] = 0.0$; or for row major layout $err_bnds_norm[(j - 1)*n_err_bnds] = 0.0$ or $err_bnds_comp[(j - 1)*n_err_bnds] = 0.0$). See the definition of err_bnds_norm and err_bnds_comp for $err = 1$. To get information about all of the right-hand sides, check err_bnds_norm or err_bnds_comp .

See Also

Matrix Storage Schemes for LAPACK Routines

?gbrfs

Refines the solution of a system of linear equations with a general band coefficient matrix and estimates its error.

Syntax

```
lapack_int LAPACKE_sgbrfs( int matrix_layout, char trans, lapack_int n, lapack_int kl,
lapack_int ku, lapack_int nrhs, const float* ab, lapack_int ldab, const float* afb,
lapack_int ldafb, const lapack_int* ipiv, const float* b, lapack_int ldb, float* x,
lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dgbrfs( int matrix_layout, char trans, lapack_int n, lapack_int kl,
lapack_int ku, lapack_int nrhs, const double* ab, lapack_int ldab, const double* afb,
lapack_int ldafb, const lapack_int* ipiv, const double* b, lapack_int ldb, double* x,
lapack_int ldx, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cgbrfs( int matrix_layout, char trans, lapack_int n, lapack_int kl,
lapack_int ku, lapack_int nrhs, const lapack_complex_float* ab, lapack_int ldab, const
lapack_complex_float* afb, lapack_int ldafb, const lapack_int* ipiv, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* ferr, float* berr );
```

```
lapack_int LAPACKE_zgbrfs( int matrix_layout, char trans, lapack_int n, lapack_int kl,
lapack_int ku, lapack_int nrhs, const lapack_complex_double* ab, lapack_int ldab, const
lapack_complex_double* afb, lapack_int ldafb, const lapack_int* ipiv, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a band matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \quad \text{such that} \quad (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?gbtrf](#)
- call the solver routine [?gbtrs](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', the system has the form $A * X = B$. If <i>trans</i> = 'T', the system has the form $A^T * X = B$. If <i>trans</i> = 'C', the system has the form $A^H * X = B$. |
| <i>n</i> | The order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>kl</i> | The number of sub-diagonals within the band of <i>A</i> ; $kl \geq 0$. |
| <i>ku</i> | The number of super-diagonals within the band of <i>A</i> ; $ku \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ab,afb,b,x</i> | Arrays: <i>ab</i> (size $\max(1, ldab * n)$) contains the original band matrix <i>A</i> , as supplied to <code>?gbtrf</code> , but stored in rows from 1 to $kl + ku + 1$ for column major layout, and columns from 1 to $kl + ku + 1$ for row major layout. <i>afb</i> (size $\max(1, ldafb * n)$) contains the factored band matrix <i>A</i> , as returned by <code>?gbtrf</code> . <i>b</i> of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the right-hand side matrix <i>B</i> . <i>x</i> of size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout contains the solution matrix <i>X</i> . |
| <i>ldab</i> | The leading dimension of <i>ab</i> , $ldab \geq kl + ku + 1$. |
| <i>ldaafb</i> | The leading dimension of <i>afb</i> , $ldaafb \geq 2 * kl + ku + 1$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of <i>x</i> ; $ldx \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by <code>?gbtrf</code> . |

Output Parameters

| | |
|-------------------|---|
| <i>x</i> | The refined solution matrix <i>X</i> . |
| <i>ferr, berr</i> | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n(kl + ku)$ floating-point operations (for real flavors) or $16n(kl + ku)$ operations (for complex flavors). In addition, each step of iterative refinement involves $2n(4kl + 3ku)$ operations (for real flavors) or $8n(4kl + 3ku)$ operations (for complex flavors); the number of iterations may range from 1 to 5. Estimating the forward error involves solving a number of systems of linear equations $A^*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gbrfsx

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a banded coefficient matrix A and provides error bounds and backward error estimates.

Syntax

```
lapack_int LAPACKE_sgbrfsx( int matrix_layout, char trans, char equed, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, const float* ab, lapack_int ldab, const
float* afb, lapack_int ldafb, const lapack_int* ipiv, const float* r, const float* c,
const float* b, lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* berr,
lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams,
float* params );
```

```
lapack_int LAPACKE_dgbrfsx( int matrix_layout, char trans, char equed, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, const double* ab, lapack_int ldab,
const double* afb, lapack_int ldafb, const lapack_int* ipiv, const double* r, const
double* c, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* rcond,
double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double* err_bnds_comp,
lapack_int nparams, double* params );
```

```
lapack_int LAPACKE_cgbrfsx( int matrix_layout, char trans, char equed, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, const lapack_complex_float* ab,
lapack_int ldab, const lapack_complex_float* afb, lapack_int ldafb, const lapack_int*
ipiv, const float* r, const float* c, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* berr, lapack_int
n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams, float*
params );
```

```
lapack_int LAPACKE_zgbrfsx( int matrix_layout, char trans, char equed, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, const lapack_complex_double* ab,
lapack_int ldab, const lapack_complex_double* afb, lapack_int ldafb, const lapack_int*
ipiv, const double* r, const double* c, const lapack_complex_double* b, lapack_int
ldb, lapack_complex_double* x, lapack_int ldx, double* rcond, double* berr, lapack_int
n_err_bnds, double* err_bnds_norm, double* err_bnds_comp, lapack_int nparams, double*
params );
```

Include Files

- `mkl.h`

Description

The routine improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for `err_bnds_norm` and `err_bnds_comp` for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters `equed`, `r`, and `c` below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether two-dimensional array storage is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>trans</code> | Must be 'N', 'T', or 'C'. Specifies the form of the system of equations: If <code>trans</code> = 'N', the system has the form $A^*X = B$ (No transpose). If <code>trans</code> = 'T', the system has the form $A^T * X = B$ (Transpose). If <code>trans</code> = 'C', the system has the form $A^H * X = B$ (Conjugate transpose for complex flavors, Transpose for real flavors). |
| <code>equed</code> | Must be 'N', 'R', 'C', or 'B'. Specifies the form of equilibration that was done to A before calling this routine. If <code>equed</code> = 'N', no equilibration was done. If <code>equed</code> = 'R', row equilibration was done, that is, A has been premultiplied by $diag(r)$. If <code>equed</code> = 'C', column equilibration was done, that is, A has been postmultiplied by $diag(c)$. If <code>equed</code> = 'B', both row and column equilibration was done, that is, A has been replaced by $diag(r) * A * diag(c)$. The right-hand side B has been changed accordingly. |
| <code>n</code> | The number of linear equations; the order of the matrix A ; $n \geq 0$. |
| <code>kl</code> | The number of subdiagonals within the band of A ; $kl \geq 0$. |
| <code>ku</code> | The number of superdiagonals within the band of A ; $ku \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; the number of columns of the matrices B and X ; $nrhs \geq 0$. |
| <code>ab,afb,b</code> | The array <code>ab</code> of size $\max(1, ldab * n)$ contains the original matrix A in band storage, in rows from 1 to $kl + ku + 1$ for column major layout, and in columns from 1 to $kl + ku + 1$ for row major layout. |

| | |
|-------------------|---|
| | <p>The array <i>afb</i> of size $\max(1, l_{dafb} * n)$ contains details of the LU factorization of the banded matrix <i>A</i> as computed by <code>?gbtrf</code>.</p> <p>The array <i>b</i> of size $\max(1, l_{db} * nrhs)$ for column major layout and $\max(1, l_{db} * n)$ for row major layout contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations.</p> |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; $ldab \geq kl + ku + 1$. |
| <i>ldafb</i> | The leading dimension of the array <i>afb</i> ; $ldafb \geq 2 * kl + ku + 1$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Contains the pivot indices as computed by <code>?gbtrf</code> ; for row $1 \leq i \leq n$, row <i>i</i> of the matrix was interchanged with row <i>ipiv</i> [<i>i</i> -1]. |
| <i>r, c</i> | <p>Arrays: <i>r</i>(<i>n</i>), <i>c</i>(<i>n</i>). The array <i>r</i> contains the row scale factors for <i>A</i>, and the array <i>c</i> contains the column scale factors for <i>A</i>.</p> <p>If <i>equed</i> = 'R' or 'B', <i>A</i> is multiplied on the left by <i>diag</i>(<i>r</i>); if <i>equed</i> = 'N' or 'C', <i>r</i> is not accessed.</p> <p>If <i>equed</i> = 'R' or 'B', each element of <i>r</i> must be positive.</p> <p>If <i>equed</i> = 'C' or 'B', <i>A</i> is multiplied on the right by <i>diag</i>(<i>c</i>); if <i>equed</i> = 'N' or 'R', <i>c</i> is not accessed.</p> <p>If <i>equed</i> = 'C' or 'B', each element of <i>c</i> must be positive.</p> <p>Each element of <i>r</i> or <i>c</i> should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.</p> |
| <i>ldb</i> | The leading dimension of the array <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>x</i> | <p>Array, size $\max(1, l_{dx} * nrhs)$ for column major layout and $\max(1, l_{dx} * n)$ for row major layout.</p> <p>The solution matrix <i>X</i> as computed by <code>sgbtrs/dgbtrs</code> for real flavors or <code>cgbtrs/zgbtrs</code> for complex flavors.</p> |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <i>n_err_bnds</i> | Number of error bounds to return for each right-hand side and each type (normwise or componentwise). See <i>err_bnds_norm</i> and <i>err_bnds_comp</i> descriptions in <i>Output Arguments</i> section below. |
| <i>nparams</i> | Specifies the number of parameters set in <i>params</i> . If ≤ 0 , the <i>params</i> array is never referenced and default values are used. |
| <i>params</i> | Array, size <i>nparams</i> . Specifies algorithm parameters. If an entry is less than 0.0, that entry will be filled with the default value used for that parameter. Only positions up to <i>nparams</i> are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass <i>nparams</i> = 0, which prevents the source code from accessing the <i>params</i> argument. |

params[0] : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0 No refinement is performed and no error bounds are computed.

=1.0 Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support double precision.

(Other values are reserved for future use.)

params[1] : Maximum number of residual computations allowed for refinement.

Default 10.0

Aggressive Set to 100.0 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

Output Parameters

x The improved solution matrix *X*.

rcond Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

berr Array, size at least $\max(1, nrhs)$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of *A* or *B* that makes x_j an exact solution.

err_bnds_norm Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:

Normwise relative error in the *i*-th solution vector

$$\frac{\max_j |X_{true_{ji}} - X_{ji}|}{\max_j |X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix *Z* are

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * a$, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

The information for right-hand side *i*, where $1 \leq i \leq nrhs$, and type of error *err* is stored in:

- Column major layout: *err_bnds_norm*[(*err* - 1) * *nrhs* + *i* - 1].
- Row major layout: *err_bnds_norm*[*err* - 1 + (*i* - 1) * *n_err_bnds*]

err_bnds_comp

Array of size *nrhs* * *n_err_bnds*. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested (*params*[2] = 0.0), then *err_bnds_comp* is not accessed.

| | |
|--------------|---|
| <i>err=1</i> | "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. |
| <i>err=2</i> | "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true. |
| <i>err=3</i> | Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are |

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq \text{nrhs}$, and type of error *err* is stored in:

- Column major layout: $\text{err_bnds_comp}[(\text{err} - 1) * \text{nrhs} + i - 1]$.
- Row major layout: $\text{err_bnds_comp}[\text{err} - 1 + (i - 1) * n_err_bnds]$

params

Output parameter only if the input contains erroneous values, namely, in $\text{params}[0]$, $\text{params}[1]$, and $\text{params}[2]$. In such a case, the corresponding elements of *params* are filled with default values on output.

Return Values

This function returns a value *info*.

If $\text{info} = 0$, the execution is successful. The solution to every right-hand side is guaranteed.

If $\text{info} = -i$, parameter i had an illegal value.

If $0 < \text{info} \leq n$: $U_{\text{info}, \text{info}}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; $\text{rcond} = 0$ is returned.

If $\text{info} = n+j$: The solution corresponding to the j -th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides k with $k > j$ may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested $\text{params}[2] = 0.0$, then the j -th

right-hand side is the first with a normwise error bound that is not guaranteed (the smallest j such that for column major layout $err_bnds_norm[j - 1] = 0.0$ or $err_bnds_comp[j - 1] = 0.0$; or for row major layout $err_bnds_norm[(j - 1)*n_err_bnds] = 0.0$ or $err_bnds_comp[(j - 1)*n_err_bnds] = 0.0$). See the definition of err_bnds_norm and err_bnds_comp for $err = 1$. To get information about all of the right-hand sides, check err_bnds_norm or err_bnds_comp .

See Also

Matrix Storage Schemes for LAPACK Routines

?gtrfs

Refines the solution of a system of linear equations with a tridiagonal coefficient matrix and estimates its error.

Syntax

```
lapack_int LAPACKE_sgtrfs( int matrix_layout, char trans, lapack_int n, lapack_int
nrhs, const float* dl, const float* d, const float* du, const float* dlf, const float*
df, const float* duf, const float* du2, const lapack_int* ipiv, const float* b,
lapack_int ldb, float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dgtrfs( int matrix_layout, char trans, lapack_int n, lapack_int
nrhs, const double* dl, const double* d, const double* du, const double* dlf, const
double* df, const double* duf, const double* du2, const lapack_int* ipiv, const double*
b, lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cgtrfs( int matrix_layout, char trans, lapack_int n, lapack_int
nrhs, const lapack_complex_float* dl, const lapack_complex_float* d, const
lapack_complex_float* du, const lapack_complex_float* dlf, const lapack_complex_float*
df, const lapack_complex_float* duf, const lapack_complex_float* du2, const lapack_int*
ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zgtrfs( int matrix_layout, char trans, lapack_int n, lapack_int
nrhs, const lapack_complex_double* dl, const lapack_complex_double* d, const
lapack_complex_double* du, const lapack_complex_double* dlf, const
lapack_complex_double* df, const lapack_complex_double* duf, const
lapack_complex_double* du2, const lapack_int* ipiv, const lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a tridiagonal matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}|/|a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i|/|b_i| \leq \beta |b_i| \quad \text{such that } (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?gttrf](#)

- call the solver routine `?gttrs`.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', the system has the form $A * X = B$. If <i>trans</i> = 'T', the system has the form $A^T * X = B$. If <i>trans</i> = 'C', the system has the form $A^H * X = B$. |
| <i>n</i> | The order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, that is, the number of columns of the matrix <i>B</i> ; $nrhs \geq 0$. |
| <i>dl</i> | Array <i>dl</i> of size $n - 1$ contains the subdiagonal elements of <i>A</i> . |
| <i>d</i> | Array <i>d</i> of size n contains the diagonal elements of <i>A</i> . |
| <i>du</i> | Array <i>du</i> of size $n - 1$ contains the superdiagonal elements of <i>A</i> . |
| <i>dlf</i> | Array <i>dlf</i> of size $n - 1$ contains the $(n - 1)$ multipliers that define the matrix <i>L</i> from the <i>LU</i> factorization of <i>A</i> as computed by <code>?gttrf</code> . |
| <i>df</i> | Array <i>df</i> of size n contains the n diagonal elements of the upper triangular matrix <i>U</i> from the <i>LU</i> factorization of <i>A</i> . |
| <i>duf</i> | Array <i>duf</i> of size $n - 1$ contains the $(n - 1)$ elements of the first superdiagonal of <i>U</i> . |
| <i>du2</i> | Array <i>du2</i> of size $n - 2$ contains the $(n - 2)$ elements of the second superdiagonal of <i>U</i> . |
| <i>b</i> | Array <i>b</i> (size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout) contains the right-hand side matrix <i>B</i> . |
| <i>x</i> | Array <i>x</i> (size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$) contains the solution matrix <i>X</i> , as computed by <code>?gttrs</code> . |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by <code>?gttrf</code> . |

Output Parameters

| | |
|----------|--|
| <i>x</i> | The refined solution matrix <i>X</i> . |
|----------|--|

ferr, berr Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?porfs

Refines the solution of a system of linear equations with a symmetric (Hermitian) positive-definite coefficient matrix and estimates its error.

Syntax

```
lapack_int LAPACKE_sporfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const float* a, lapack_int lda, const float* af, lapack_int ldaf, const float* b,
lapack_int ldb, float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dporfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const double* a, lapack_int lda, const double* af, lapack_int ldaf, const double* b,
lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cporfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* af,
lapack_int ldaf, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float*
x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zporfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* af,
lapack_int ldaf, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double*
x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A \cdot X = B$ with a symmetric (Hermitian) positive definite matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A) x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?potrf](#)
- call the solver routine [?potrs](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', the upper triangle of <i>A</i> is stored. If <i>uplo</i> = 'L', the lower triangle of <i>A</i> is stored. |
| <i>n</i> | The order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>a</i> | Array <i>a</i> (size $\max(1, lda * n)$) contains the original matrix <i>A</i> , as supplied to <code>?potrf</code> . |
| <i>af</i> | Array <i>af</i> (size $\max(1, ldaf * n)$) contains the factored matrix <i>A</i> , as returned by <code>?potrf</code> . |
| <i>b</i> | Array <i>b</i> of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the right-hand side matrix <i>B</i> . The second dimension of <i>b</i> must be at least $\max(1, nrhs)$. |
| <i>x</i> | Array <i>x</i> of size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout contains the solution matrix <i>X</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-------------------|---|
| <i>x</i> | The refined solution matrix <i>X</i> . |
| <i>ferr, berr</i> | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5. Estimating the forward error involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?porfsx

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a symmetric/Hermitian positive-definite coefficient matrix A and provides error bounds and backward error estimates.

Syntax

```
lapack_int LAPACKE_sporfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const float* a, lapack_int lda, const float* af, lapack_int ldaf,
const float* s, const float* b, lapack_int ldb, float* x, lapack_int ldx, float*
rcond, float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp,
lapack_int nparams, float* params );
```

```
lapack_int LAPACKE_dporfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const double* a, lapack_int lda, const double* af, lapack_int ldaf,
const double* s, const double* b, lapack_int ldb, double* x, lapack_int ldx, double*
rcond, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, double* params );
```

```
lapack_int LAPACKE_cporfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* af, lapack_int ldaf, const float* s, const lapack_complex_float*
b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* rcond, float* berr,
lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams,
float* params );
```

```
lapack_int LAPACKE_zporfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* af, lapack_int ldaf, const double* s, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, double* params );
```

Include Files

- mkl.h

Description

The routine improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed* and *s* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether two-dimensional array storage is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>equed</i> | Must be 'N' or 'Y'. Specifies the form of equilibration that was done to A before calling this routine. If <i>equed</i> = 'N', no equilibration was done. If <i>equed</i> = 'Y', both row and column equilibration was done, that is, A has been replaced by $diag(s) * A * diag(s)$. The right-hand side B has been changed accordingly. |
| <i>n</i> | The number of linear equations; the order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns of the matrices B and X ; $nrhs \geq 0$. |
| <i>a</i> | The array <i>a</i> (size $\max(1, lda * n)$) contains the symmetric/Hermitian matrix A as specified by <i>uplo</i> . If <i>uplo</i> = 'U', the leading n -by- n upper triangular part of <i>a</i> contains the upper triangular part of the matrix A and the strictly lower triangular part of <i>a</i> is not referenced. If <i>uplo</i> = 'L', the leading n -by- n lower triangular part of <i>a</i> contains the lower triangular part of the matrix A and the strictly upper triangular part of <i>a</i> is not referenced. |
| <i>af</i> | The array <i>af</i> (size $\max(1, ldaf * n)$) contains the triangular factor L or U from the Cholesky factorization $A = U^T * U$ or $A = L * L^T$ as computed by spotrf for real flavors or dspotrf for complex flavors. |
| <i>b</i> | The array <i>b</i> (size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout) contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>s</i> | Array of size n . The array <i>s</i> contains the scale factors for A . If <i>equed</i> = 'N', <i>s</i> is not accessed. If <i>equed</i> = 'Y', each element of <i>s</i> must be positive. Each element of <i>s</i> should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable. |

| | | | | | | | | | |
|-------------------|---|------|--|------|--|---------|------|------------|---|
| <i>ldb</i> | The leading dimension of the array <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. | | | | | | | | |
| <i>x</i> | <p>Array, size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout.</p> <p>The solution matrix <i>X</i> as computed by <code>?potrs</code></p> | | | | | | | | |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. | | | | | | | | |
| <i>n_err_bnds</i> | Number of error bounds to return for each right hand side and each type (normwise or componentwise). See <i>err_bnds_norm</i> and <i>err_bnds_comp</i> descriptions in <i>Output Arguments</i> section below. | | | | | | | | |
| <i>nparams</i> | Specifies the number of parameters set in <i>params</i> . If ≤ 0 , the <i>params</i> array is never referenced and default values are used. | | | | | | | | |
| <i>params</i> | <p>Array, size <i>nparams</i>. Specifies algorithm parameters. If an entry is less than 0.0, that entry will be filled with the default value used for that parameter. Only positions up to <i>nparams</i> are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass <i>nparams</i> = 0, which prevents the source code from accessing the <i>params</i> argument.</p> <p><i>params</i>[0] : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).</p> <table border="0" style="margin-left: 20px;"> <tr> <td style="padding-right: 20px;">=0.0</td> <td>No refinement is performed and no error bounds are computed.</td> </tr> <tr> <td style="padding-right: 20px;">=1.0</td> <td>Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support double precision.</td> </tr> </table> <p>(Other values are reserved for future use.)</p> <p><i>params</i>[1] : Maximum number of residual computations allowed for refinement.</p> <table border="0" style="margin-left: 20px;"> <tr> <td style="padding-right: 20px;">Default</td> <td>10.0</td> </tr> <tr> <td style="padding-right: 20px;">Aggressive</td> <td>Set to 100.0 to permit convergence using approximate factorizations or factorizations other than <i>LU</i>. If the factorization uses a technique other than Gaussian elimination, the guarantees in <i>err_bnds_norm</i> and <i>err_bnds_comp</i> may no longer be trustworthy.</td> </tr> </table> <p><i>params</i>[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).</p> | =0.0 | No refinement is performed and no error bounds are computed. | =1.0 | Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support double precision. | Default | 10.0 | Aggressive | Set to 100.0 to permit convergence using approximate factorizations or factorizations other than <i>LU</i> . If the factorization uses a technique other than Gaussian elimination, the guarantees in <i>err_bnds_norm</i> and <i>err_bnds_comp</i> may no longer be trustworthy. |
| =0.0 | No refinement is performed and no error bounds are computed. | | | | | | | | |
| =1.0 | Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support double precision. | | | | | | | | |
| Default | 10.0 | | | | | | | | |
| Aggressive | Set to 100.0 to permit convergence using approximate factorizations or factorizations other than <i>LU</i> . If the factorization uses a technique other than Gaussian elimination, the guarantees in <i>err_bnds_norm</i> and <i>err_bnds_comp</i> may no longer be trustworthy. | | | | | | | | |

Output Parameters

| | |
|----------------------|--|
| <i>x</i> | The improved solution matrix X . |
| <i>rcond</i> | Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix A after equilibration (if done). If <i>rcond</i> is less than the machine precision, in particular, if <i>rcond</i> = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned. |
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution. |
| <i>err_bnds_norm</i> | <p>Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:</p> <p>Normwise relative error in the i-th solution vector</p> $\frac{\max_j X_{true_{ji}} - X_{ji} }{\max_j X_{ji} }$ <p>The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.</p> |
| <i>err=1</i> | "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. |
| <i>err=2</i> | "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true. |
| <i>err=3</i> | Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are |

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * a$, where s scales each row by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error err is stored in:

- Column major layout: $err_bnds_norm[(err - 1) * nrhs + i - 1]$.
- Row major layout: $err_bnds_norm[err - 1 + (i - 1) * n_err_bnds]$

`err_bnds_comp`

Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the i -th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested ($params[2] = 0.0$), then `err_bnds_comp` is not accessed.

- `err=1` "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors.
- `err=2` "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.
- `err=3` Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error err is stored in:

- Column major layout: `err_bnds_comp[(err - 1)*nrhs + i - 1]`.
- Row major layout: `err_bnds_comp[err - 1 + (i - 1)*n_err_bnds]`

`params`

Output parameter only if the input contains erroneous values, namely in `params[0]`, `params[1]`, or `params[2]`. In such a case, the corresponding elements of `params` are filled with default values on output.

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful. The solution to every right-hand side is guaranteed.

If `info = -i`, parameter i had an illegal value.

If $0 < info \leq n$: $U_{info,info}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; `rcond = 0` is returned.

If `info = n+j`: The solution corresponding to the j -th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides k with $k > j$ may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested `params[2] = 0.0`, then the j -th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest j such that for column major layout `err_bnds_norm[j - 1] = 0.0` or `err_bnds_comp[j - 1] = 0.0`; or for row major layout `err_bnds_norm[(j - 1)*n_err_bnds] = 0.0` or `err_bnds_comp[(j - 1)*n_err_bnds] = 0.0`). See the definition of `err_bnds_norm` and `err_bnds_comp` for `err = 1`. To get information about all of the right-hand sides, check `err_bnds_norm` or `err_bnds_comp`.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pprfs

Refines the solution of a system of linear equations with a symmetric (Hermitian) positive-definite coefficient matrix stored in a packed format and estimates its error.

Syntax

```
lapack_int LAPACKE_spprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const float* ap, const float* afp, const float* b, lapack_int ldb, float* x,
lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dpprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const double* ap, const double* afp, const double* b, lapack_int ldb, double* x,
lapack_int ldx, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cpprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* ap, const lapack_complex_float* afp, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* ferr, float* berr );
```

```
lapack_int LAPACKE_zpprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* ap, const lapack_complex_double* afp, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A \cdot X = B$ with a symmetric (Hermitian) positive definite matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution

$$\|x - x_e\|_{\infty} / \|x\|_{\infty}$$

where x_e is the exact solution.

Before calling this routine:

- call the factorization routine [?pptrf](#)
- call the solver routine [?pptrs](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ap</i> | <i>ap</i> contains the original matrix A in a packed format, as supplied to ?pptrf . The dimension of <i>ap</i> must be at least $\max(1, n(n+1)/2)$. |
| <i>afp</i> | <i>afp</i> contains the factored matrix A in a packed format, as returned by ?pptrf . The dimension of <i>afp</i> must be at least $\max(1, n(n+1)/2)$. |
| <i>b</i> | Array <i>b</i> of size $\max(1, ldb \cdot nrhs)$ for column major layout and $\max(1, ldb \cdot n)$ for row major layout contains the right-hand side matrix B . |
| <i>x</i> | Array <i>x</i> of size $\max(1, ldx \cdot nrhs)$ for column major layout and $\max(1, ldx \cdot n)$ for row major layout contains the solution matrix X . |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

ldx The leading dimension of *x*; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

x The refined solution matrix *X*.

ferr, berr Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations $A^*x = b$; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pbrfs

Refines the solution of a system of linear equations with a band symmetric (Hermitian) positive-definite coefficient matrix and estimates its error.

Syntax

```
lapack_int LAPACKE_spbtrfs( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const float* ab, lapack_int ldab, const float* afb, lapack_int ldafb,
const float* b, lapack_int ldb, float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_dpbrfs( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const double* ab, lapack_int ldab, const double* afb, lapack_int
ldafb, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* ferr,
double* berr );

lapack_int LAPACKE_cpbrfs( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const lapack_complex_float* ab, lapack_int ldab, const
lapack_complex_float* afb, lapack_int ldafb, const lapack_complex_float* b, lapack_int
ldb, lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_zpbrfs( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
lapack_int nrhs, const lapack_complex_double* ab, lapack_int ldab, const
lapack_complex_double* afb, lapack_int ldafb, const lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- `mkl.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A * X = B$ with a symmetric (Hermitian) positive definite band matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\| |x - x_e| |_{\infty} / \|x| |_{\infty}$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?pbtrf](#)
- call the solver routine [?pbtrs](#).

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo = 'U'</code> , the upper triangle of A is stored. If <code>uplo = 'L'</code> , the lower triangle of A is stored. |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>kd</code> | The number of superdiagonals or subdiagonals in the matrix A ; $kd \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; $nrhs \geq 0$. |
| <code>ab</code> | Array <code>ab</code> (size $\max(ldab * n)$) contains the original band matrix A , as supplied to ?pbtrf . |
| <code>afb</code> | Array <code>afb</code> (size $\max(ldafb * n)$) contains the factored band matrix A , as returned by ?pbtrf . |
| <code>b</code> | Array <code>b</code> of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the right-hand side matrix B . |
| <code>x</code> | Array <code>x</code> of size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout contains the solution matrix X . |
| <code>ldab</code> | The leading dimension of <code>ab</code> ; $ldab \geq kd + 1$. |
| <code>ldafb</code> | The leading dimension of <code>afb</code> ; $ldafb \geq kd + 1$. |
| <code>ldb</code> | The leading dimension of <code>b</code> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

ldx The leading dimension of *x*; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

x The refined solution matrix *X*.

ferr, berr Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $8n*kd$ floating-point operations (for real flavors) or $32n*kd$ operations (for complex flavors). In addition, each step of iterative refinement involves $12n*kd$ operations (for real flavors) or $48n*kd$ operations (for complex flavors); the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $4n*kd$ floating-point operations for real flavors or $16n*kd$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?ptrfs

Refines the solution of a system of linear equations with a symmetric (Hermitian) positive-definite tridiagonal coefficient matrix and estimates its error.

Syntax

```
lapack_int LAPACKE_sptrfs( int matrix_layout, lapack_int n, lapack_int nrhs, const
float* d, const float* e, const float* df, const float* ef, const float* b, lapack_int
ldb, float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dptrfs( int matrix_layout, lapack_int n, lapack_int nrhs, const
double* d, const double* e, const double* df, const double* ef, const double* b,
lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cptrfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const float* d, const lapack_complex_float* e, const float* df, const
lapack_complex_float* ef, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zptrfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const double* d, const lapack_complex_double* e, const double* df, const
lapack_complex_double* ef, const lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- `mkl.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A^*X = B$ with a symmetric (Hermitian) positive definite tridiagonal matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine `?pttrf`
- call the solver routine `?pttrs`.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Used for complex flavors only. Must be 'U' or 'L'. Specifies whether the superdiagonal or the subdiagonal of the tridiagonal matrix A is stored and how A is factored: If <code>uplo = 'U'</code> , the array <code>e</code> stores the superdiagonal of A , and A is factored as $U^H * D * U$. If <code>uplo = 'L'</code> , the array <code>e</code> stores the subdiagonal of A , and A is factored as $L * D * L^H$. |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; $nrhs \geq 0$. |
| <code>d</code> | The array <code>d</code> (size n) contains the n diagonal elements of the tridiagonal matrix A . |
| <code>df</code> | The array <code>df</code> (size n) contains the n diagonal elements of the diagonal matrix D from the factorization of A as computed by <code>?pttrf</code> . |
| <code>e, ef, b, x</code> | The array <code>e</code> (size $n - 1$) contains the $(n - 1)$ off-diagonal elements of the tridiagonal matrix A (see <code>uplo</code>). The array <code>ef</code> (size $n - 1$) contains the $(n - 1)$ off-diagonal elements of the unit bidiagonal factor U or L from the factorization computed by <code>?pttrf</code> (see <code>uplo</code>). The array <code>b</code> of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the matrix B whose columns are the right-hand sides for the systems of equations. |

The array x of size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout contains the solution matrix X as computed by `?pttrs`.

ldb The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

ldx The leading dimension of x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

x The refined solution matrix X .

ferr, berr Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

`?syrf`s

Refines the solution of a system of linear equations with a symmetric coefficient matrix and estimates its error.

Syntax

```
lapack_int LAPACKE_ssyrf( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const float* a, lapack_int lda, const float* af, lapack_int ldaf, const lapack_int*
ipiv, const float* b, lapack_int ldb, float* x, lapack_int ldx, float* ferr, float*
berr );
```

```
lapack_int LAPACKE_dsyrf( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const double* a, lapack_int lda, const double* af, lapack_int ldaf, const lapack_int*
ipiv, const double* b, lapack_int ldb, double* x, lapack_int ldx, double* ferr,
double* berr );
```

```
lapack_int LAPACKE_csyrf( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zsyrf( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_double* b, lapack_int
ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- `mkl.h`

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A \cdot X = B$ with a symmetric full-storage matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?sytrf](#)
- call the solver routine [?sytrs](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>a</i> | Array <i>a</i> (size $\max(1, lda * n)$) contains the original matrix A , as supplied to ?sytrf . |
| <i>af</i> | Array <i>af</i> (size $\max(1, ldaf * n)$) contains the factored matrix A , as returned by ?sytrf . |
| <i>b</i> | Array <i>b</i> of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the right-hand side matrix B . |
| <i>x</i> | Array <i>x</i> of size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout contains the solution matrix X . |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?sytrf . |

Output Parameters

| | |
|--------------|---|
| x | The refined solution matrix X . |
| $ferr, berr$ | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5. Estimating the forward error involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?syrfsx

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a symmetric indefinite coefficient matrix A and provides error bounds and backward error estimates.

Syntax

```
lapack_int LAPACKE_ssyrfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const float* a, lapack_int lda, const float* af, lapack_int ldaf,
const lapack_int* ipiv, const float* s, const float* b, lapack_int ldb, float* x,
lapack_int ldx, float* rcond, float* berr, lapack_int n_err_bnds, float*
err_bnds_norm, float* err_bnds_comp, lapack_int nparams, float* params );
```

```
lapack_int LAPACKE_dsyrfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const double* a, lapack_int lda, const double* af, lapack_int ldaf,
const lapack_int* ipiv, const double* s, const double* b, lapack_int ldb, double* x,
lapack_int ldx, double* rcond, double* berr, lapack_int n_err_bnds, double*
err_bnds_norm, double* err_bnds_comp, lapack_int nparams, double* params );
```

```
lapack_int LAPACKE_csyrfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* af, lapack_int ldaf, const lapack_int* ipiv, const float* s,
const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float*
err_bnds_comp, lapack_int nparams, float* params );
```

```
lapack_int LAPACKE_zsyrfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* af, lapack_int ldaf, const lapack_int* ipiv, const double* s,
```

```
const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int
ldx, double* rcond, double* berr, lapack_int n_err_bnds, double* err_bnds_norm,
double* err_bnds_comp, lapack_int nparams, double* params );
```

Include Files

- mkl.h

Description

The routine improves the computed solution to a system of linear equations when the coefficient matrix is symmetric indefinite, and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed* and *s* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether two-dimensional array storage is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of <i>A</i> is stored: If <i>uplo</i> = 'U', the upper triangle of <i>A</i> is stored. If <i>uplo</i> = 'L', the lower triangle of <i>A</i> is stored. |
| <i>equed</i> | Must be 'N' or 'Y'. Specifies the form of equilibration that was done to <i>A</i> before calling this routine. If <i>equed</i> = 'N', no equilibration was done. If <i>equed</i> = 'Y', both row and column equilibration was done, that is, <i>A</i> has been replaced by $diag(s) * A * diag(s)$. The right-hand side <i>B</i> has been changed accordingly. |
| <i>n</i> | The number of linear equations; the order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns of the matrices <i>B</i> and <i>X</i> ; $nrhs \geq 0$. |
| <i>a, af, b</i> | The array <i>a</i> (size $\max(1, lda * n)$) contains the symmetric/Hermitian matrix <i>A</i> as specified by <i>uplo</i> . If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of <i>a</i> contains the upper triangular part of the matrix <i>A</i> and the strictly lower triangular part of <i>a</i> is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i> -by- <i>n</i> lower triangular part of <i>a</i> contains the lower triangular part of the matrix <i>A</i> and the strictly upper triangular part of <i>a</i> is not referenced. The array <i>af</i> (size $\max(1, lda * n)$) contains the triangular factor <i>L</i> or <i>U</i> from the Cholesky factorization $A = U^T * U$ or $A = L * L^T$ as computed by ssytrf for real flavors or dsytrf for complex flavors. |

| | |
|----------------|---|
| | The array b (size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout) contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| lda | The leading dimension of a ; $lda \geq \max(1, n)$. |
| $ldaf$ | The leading dimension of af ; $ldaf \geq \max(1, n)$. |
| $ipiv$ | Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D as determined by <code>ssytrf</code> for real flavors or <code>dsytrf</code> for complex flavors. |
| s | Array, size (n) . The array s contains the scale factors for A . If $equed = 'N'$, s is not accessed. If $equed = 'Y'$, each element of s must be positive. Each element of s should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable. |
| ldb | The leading dimension of the array b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| x | Array, of size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout. The solution matrix X as computed by <code>?sytrs</code> |
| ldx | The leading dimension of the output array x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| n_err_bnds | Number of error bounds to return for each right hand side and each type (normwise or componentwise). See <code>err_bnds_norm</code> and <code>err_bnds_comp</code> descriptions in <i>Output Arguments</i> section below. |
| $nparams$ | Specifies the number of parameters set in <code>params</code> . If ≤ 0 , the <code>params</code> array is never referenced and default values are used. |
| $params$ | Array, size $nparams$. Specifies algorithm parameters. If an entry is less than 0.0, that entry will be filled with the default value used for that parameter. Only positions up to $nparams$ are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass $nparams = 0$, which prevents the source code from accessing the <code>params</code> argument. <code>params[0]</code> : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors). =0.0 No refinement is performed and no error bounds are computed. |

=1.0 Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support double precision.

(Other values are reserved for future use.)

params[1] : Maximum number of residual computations allowed for refinement.

Default 10.0

Aggressive Set to 100.0 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

Output Parameters

x The improved solution matrix *X*.

rcond Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

berr Array, size at least $\max(1, nrhs)$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of *A* or *B* that makes x_j an exact solution.

err_bnds_norm Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:

Normwise relative error in the *i*-th solution vector

$$\frac{\max_j |X_{true_{ji}} - X_{ji}|}{\max_j |X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix *Z* are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

The information for right-hand side *i*, where $1 \leq i \leq \text{nrhs}$, and type of error *err* is stored in:

- Column major layout: *err_bnds_norm*[(*err* - 1)**nrhs* + *i* - 1].
- Row major layout: *err_bnds_norm*[*err* - 1 + (*i* - 1)**n_err_bnds*]

err_bnds_comp

Array of size *nrhs***n_err_bnds*. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{|X_{\text{true}_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested (*params*[2] = 0.0), then *err_bnds_comp* is not accessed.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and

$\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3

Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error *err* is stored in:

- Column major layout: $err_bnds_comp[(err - 1) * nrhs + i - 1]$.
- Row major layout: $err_bnds_comp[err - 1 + (i - 1) * n_err_bnds]$

params

Output parameter only if the input contains erroneous values, namely, in $params[0]$, $params[1]$, $params[2]$. In such a case, the corresponding elements of *params* are filled with default values on output.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.

If *info* = -*i*, parameter *i* had an illegal value.

If $0 < info \leq n$: $U_{info,info}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; $rcond = 0$ is returned.

If *info* = $n + j$: The solution corresponding to the j -th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides k with $k > j$ may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested $params[2] = 0.0$, then the j -th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest j such that for column major layout $err_bnds_norm[j - 1] = 0.0$ or $err_bnds_comp[j - 1] = 0.0$; or for row major layout $err_bnds_norm[(j - 1) * n_err_bnds] = 0.0$ or $err_bnds_comp[(j - 1) * n_err_bnds] = 0.0$). See the definition of *err_bnds_norm* and *err_bnds_comp* for *err* = 1. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?herfs

Refines the solution of a system of linear equations with a complex Hermitian coefficient matrix and estimates its error.

Syntax

```
lapack_int LAPACKE_cherfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* a, lapack_int lda, const lapack_complex_float* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zherfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* a, lapack_int lda, const lapack_complex_double* af,
lapack_int ldaf, const lapack_int* ipiv, const lapack_complex_double* b, lapack_int
ldb, lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A \cdot X = B$ with a complex Hermitian full-storage matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \quad \text{such that } (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?hetrf](#)
- call the solver routine [?hetrs](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>a, af, b, x</i> | Arrays: <i>a</i> (size max(1, $lda \cdot n$)) contains the original matrix A , as supplied to ?hetrf . <i>af</i> (size max(1, $ldaf \cdot n$)) contains the factored matrix A , as returned by ?hetrf . |

b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the right-hand side matrix *B*.
x of size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout contains the solution matrix *X*.

| | |
|-------------|---|
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?hetrf . |

Output Parameters

| | |
|-------------------|---|
| <i>x</i> | The refined solution matrix <i>X</i> . |
| <i>ferr, berr</i> | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $16n^2$ operations. In addition, each step of iterative refinement involves $24n^2$ operations; the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations $A*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $8n^2$ floating-point operations.

The real counterpart of this routine is [?ssyrfs/?dsyrfs](#)

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?herfsx](#)

Uses extra precise iterative refinement to improve the solution to the system of linear equations with a symmetric indefinite coefficient matrix A and provides error bounds and backward error estimates.

Syntax

```
lapack_int LAPACKE_cherfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* af, lapack_int ldaf, const lapack_int* ipiv, const float* s,
```

```
const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float*
err_bnds_comp, lapack_int nparams, float* params );
```

```
lapack_int LAPACKE_zherfsx( int matrix_layout, char uplo, char equed, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* af, lapack_int ldaf, const lapack_int* ipiv, const double* s,
const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int
ldx, double* rcond, double* berr, lapack_int n_err_bnds, double* err_bnds_norm,
double* err_bnds_comp, lapack_int nparams, double* params );
```

Include Files

- mkl.h

Description

The routine improves the computed solution to a system of linear equations when the coefficient matrix is Hermitian indefinite, and provides error bounds and backward error estimates for the solution. In addition to a normwise error bound, the code provides a maximum componentwise error bound, if possible. See comments for *err_bnds_norm* and *err_bnds_comp* for details of the error bounds.

The original system of linear equations may have been equilibrated before calling this routine, as described by the parameters *equed* and *s* below. In this case, the solution and error bounds returned are for the original unequilibrated system.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether two-dimensional array storage is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of <i>A</i> is stored: If <i>uplo</i> = 'U', the upper triangle of <i>A</i> is stored. If <i>uplo</i> = 'L', the lower triangle of <i>A</i> is stored. |
| <i>equed</i> | Must be 'N' or 'Y'. Specifies the form of equilibration that was done to <i>A</i> before calling this routine. If <i>equed</i> = 'N', no equilibration was done. If <i>equed</i> = 'Y', both row and column equilibration was done, that is, <i>A</i> has been replaced by $diag(s) * A * diag(s)$. The right-hand side <i>B</i> has been changed accordingly. |
| <i>n</i> | The number of linear equations; the order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns of the matrices <i>B</i> and <i>X</i> ; $nrhs \geq 0$. |
| <i>a, af, b</i> | The array <i>a</i> of size $\max(1, lda * n)$ contains the Hermitian matrix <i>A</i> as specified by <i>uplo</i> . If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of <i>a</i> contains the upper triangular part of the matrix <i>A</i> and the strictly lower |

triangular part of a is not referenced. If $uplo = 'L'$, the leading n -by- n lower triangular part of a contains the lower triangular part of the matrix A and the strictly upper triangular part of a is not referenced.

The array af of size $\max(1, ldaf*n)$ contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization $A = U*D*U^T$ or $A = L*D*L^T$ as computed by `ssytrf` for `cherfsx` or `dsytrf` for `zherfsx`.

The array b of size $\max(1, ldb*nrhs)$ for row major layout and $\max(1, ldb*n)$ for column major layout contains the matrix B whose columns are the right-hand sides for the systems of equations.

| | |
|-------------------------|--|
| <code>lda</code> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <code>ldaf</code> | The leading dimension of af ; $ldaf \geq \max(1, n)$. |
| <code>ipiv</code> | Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D as determined by <code>ssytrf</code> for real flavors or <code>dsytrf</code> for complex flavors. |
| <code>s</code> | <p>Array, size (n). The array s contains the scale factors for A.</p> <p>If $equed = 'N'$, s is not accessed.</p> <p>If $equed = 'Y'$, each element of s must be positive.</p> <p>Each element of s should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.</p> |
| <code>ldb</code> | The leading dimension of the array b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <code>x</code> | <p>Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.</p> <p>The solution matrix X as computed by <code>?hetrs</code></p> |
| <code>ldx</code> | The leading dimension of the output array x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <code>n_err_bnds</code> | Number of error bounds to return for each right hand side and each type (normwise or componentwise). See <code>err_bnds_norm</code> and <code>err_bnds_comp</code> descriptions in <i>Output Arguments</i> section below. |
| <code>nparams</code> | Specifies the number of parameters set in <code>params</code> . If ≤ 0 , the <code>params</code> array is never referenced and default values are used. |
| <code>params</code> | Array, size <code>nparams</code> . Specifies algorithm parameters. If an entry is less than 0.0, that entry will be filled with the default value used for that parameter. Only positions up to <code>nparams</code> are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass <code>nparams = 0</code> , which prevents the source code from accessing the <code>params</code> argument. |

params[0] : Whether to perform iterative refinement or not. Default: 1.0 (for *cherfsx*), 1.0D+0 (for *zherfsx*).

| | |
|------|--|
| =0.0 | No refinement is performed and no error bounds are computed. |
| =1.0 | Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support double precision. |

(Other values are reserved for future use.)

params[1] : Maximum number of residual computations allowed for refinement.

| | |
|------------|---|
| Default | 10 |
| Aggressive | Set to 100 to permit convergence using approximate factorizations or factorizations other than <i>LU</i> . If the factorization uses a technique other than Gaussian elimination, the guarantees in <i>err_bnds_norm</i> and <i>err_bnds_comp</i> may no longer be trustworthy. |

params[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

Output Parameters

| | |
|----------------------|--|
| <i>x</i> | The improved solution matrix <i>X</i> . |
| <i>rcond</i> | Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix <i>A</i> after equilibration (if done). If <i>rcond</i> is less than the machine precision, in particular, if <i>rcond</i> = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned. |
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of <i>A</i> or <i>B</i> that makes x_j an exact solution. |
| <i>err_bnds_norm</i> | <p>Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:</p> <p>Normwise relative error in the <i>i</i>-th solution vector</p> $\frac{\max_j X_{true_{ji}} - X_{ji} }{\max_j X_{ji} }$ <p>The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.</p> |

- `err=1` "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for `cherfsx` and $\sqrt{n} * \text{dlamch}(\epsilon)$ for `zherfsx`.
- `err=2` "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for `cherfsx` and $\sqrt{n} * \text{dlamch}(\epsilon)$ for `zherfsx`. This error bound should only be trusted if the previous boolean is true.
- `err=3` Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z=s*a$, where s scales each row by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error `err` is stored in:

- Column major layout: `err_bnds_norm[(err - 1)*nrhs + i - 1]`.
- Row major layout: `err_bnds_norm[err - 1 + (i - 1)*n_err_bnds]`

`err_bnds_comp`

Array of size `nrhs*n_err_bnds`. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the i -th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested (`params[2] = 0.0`), then `err_bnds_comp` is not accessed.

- `err=1` "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for `cherfsx` and $\sqrt{n} * \text{dlamch}(\epsilon)$ for `zherfsx`.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for *cherfsx* and $\sqrt{n} * \text{dlamch}(\epsilon)$ for *zherfsx*. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix *Z* are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where *x* is the solution for the current right-hand side and *s* scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of *z* are approximately 1.

The information for right-hand side *i*, where $1 \leq i \leq \text{nrhs}$, and type of error *err* is stored in:

- Column major layout: *err_bnds_comp*[(*err* - 1) * *nrhs* + *i* - 1].
- Row major layout: *err_bnds_comp*[*err* - 1 + (*i* - 1) * *n_err_bnds*]

params

Output parameter only if the input contains erroneous values, namely, in *params*[0], *params*[1], *params*[2]. In such a case, the corresponding elements of *params* are filled with default values on output.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.

If *info* = -*i*, parameter *i* had an illegal value.

If $0 < \text{info} \leq n$: $U_{\text{info}, \text{info}}$ is exactly zero. The factorization has been completed, but the factor *D* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.

If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params*[2] = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that for column major layout *err_bnds_norm*[*j* - 1] = 0.0 or *err_bnds_comp*[*j* - 1] = 0.0; or for row major layout *err_bnds_norm*[(*j* - 1) * *n_err_bnds*] = 0.0 or *err_bnds_comp*[(*j* - 1) * *n_err_bnds*] = 0.0). See the definition of *err_bnds_norm* and *err_bnds_comp* for *err* = 1. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?sprfs

Refines the solution of a system of linear equations with a packed symmetric coefficient matrix and estimates the solution error.

Syntax

```
lapack_int LAPACKE_ssprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const float* ap, const float* afp, const lapack_int* ipiv, const float* b, lapack_int
ldb, float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dsprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const double* ap, const double* afp, const lapack_int* ipiv, const double* b,
lapack_int ldb, double* x, lapack_int ldx, double* ferr, double* berr );
```

```
lapack_int LAPACKE_csprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* ap, const lapack_complex_float* afp, const lapack_int*
ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zsprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* ap, const lapack_complex_double* afp, const lapack_int*
ipiv, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x,
lapack_int ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A \cdot X = B$ with a packed symmetric matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A) x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?sprtrf](#)
- call the solver routine [?sprtrs](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |

| | |
|-----------------|--|
| ap, afp, b, x | <p>Arrays:</p> <p>ap of size $\max(1, n(n+1)/2)$ contains the original packed matrix A, as supplied to <code>?sptrf</code>.</p> <p>afp of size $\max(1, n(n+1)/2)$ contains the factored packed matrix A, as returned by <code>?sptrf</code>.</p> <p>b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the right-hand side matrix B.</p> <p>x of size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout contains the solution matrix X.</p> |
| ldb | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| ldx | The leading dimension of x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq \max(1, nrhs)$ for row major layout. |
| $ipiv$ | Array, size at least $\max(1, n)$. The $ipiv$ array, as returned by <code>?sptrf</code> . |

Output Parameters

| | |
|--------------|---|
| x | The refined solution matrix X . |
| $ferr, berr$ | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector. |

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

Application Notes

The bounds returned in $ferr$ are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $4n^2$ floating-point operations (for real flavors) or $16n^2$ operations (for complex flavors). In addition, each step of iterative refinement involves $6n^2$ operations (for real flavors) or $24n^2$ operations (for complex flavors); the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations $A*x = b$; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately $2n^2$ floating-point operations for real flavors or $8n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?hprfs

Refines the solution of a system of linear equations with a packed complex Hermitian coefficient matrix and estimates the solution error.

Syntax

```
lapack_int LAPACKE_chprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_float* ap, const lapack_complex_float* afp, const lapack_int*
ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zhprfs( int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
const lapack_complex_double* ap, const lapack_complex_double* afp, const lapack_int*
ipiv, const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x,
lapack_int ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine performs an iterative refinement of the solution to a system of linear equations $A \cdot X = B$ with a packed complex Hermitian matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \quad \text{such that } (A + \delta A)x = (b + \delta b).$$

Finally, the routine estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine:

- call the factorization routine [?hptrf](#)
- call the solver routine [?hptrs](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ap, afp, b, x</i> | Arrays: <i>ap</i> of size $\max(1, n(n+1)/2)$ contains the original packed matrix A , as supplied to ?hptrf . <i>afp</i> of size $\max(1, n(n+1)/2)$ contains the factored packed matrix A , as returned by ?hptrf . <i>b</i> of size $\max(1, ldb \cdot nrhs)$ for column major layout and $\max(1, ldb \cdot n)$ for row major layout contains the right-hand side matrix B . <i>x</i> of size $\max(1, ldx \cdot nrhs)$ for column major layout and $\max(1, ldx \cdot n)$ for row major layout contains the solution matrix X . |

| | |
|-------------|---|
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?hptrf . |

Output Parameters

| | |
|-------------------|---|
| <i>x</i> | The refined solution matrix <i>X</i> . |
| <i>ferr, berr</i> | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

For each right-hand side, computation of the backward error involves a minimum of $16n^2$ operations. In addition, each step of iterative refinement involves $24n^2$ operations; the number of iterations may range from 1 to 5.

Estimating the forward error involves solving a number of systems of linear equations $A^*x = b$; the number is usually 4 or 5 and never more than 11. Each solution requires approximately $8n^2$ floating-point operations.

The real counterpart of this routine is [?ssprfs](#)/[?dsprfs](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?trrfs](#)

Estimates the error in the solution of a system of linear equations with a triangular coefficient matrix.

Syntax

```
lapack_int LAPACKE_strrfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const float* a, lapack_int lda, const float* b,
lapack_int ldb, const float* x, lapack_int ldx, float* ferr, float* berr );

lapack_int LAPACKE_dtrrfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const double* a, lapack_int lda, const double* b,
lapack_int ldb, const double* x, lapack_int ldx, double* ferr, double* berr );

lapack_int LAPACKE_ctrfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* b, lapack_int ldb, const lapack_complex_float* x, lapack_int ldx,
float* ferr, float* berr );
```

```
lapack_int LAPACKE_ztrrfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* b, lapack_int ldb, const lapack_complex_double* x, lapack_int
ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine estimates the errors in the solution to a system of linear equations $A^*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a triangular matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A)x = (b + \delta b).$$

The routine also estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine, call the solver routine `?trtrs`.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If <i>uplo</i> = 'U', then A is upper triangular. If <i>uplo</i> = 'L', then A is lower triangular. |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', the system has the form $A^*X = B$. If <i>trans</i> = 'T', the system has the form $A^T*X = B$. If <i>trans</i> = 'C', the system has the form $A^H*X = B$. |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', then A is not a unit triangular matrix. If <i>diag</i> = 'U', then A is unit triangular: diagonal elements of A are assumed to be 1 and not referenced in the array a . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>a, b, x</i> | Arrays: a (size $\max(1, lda * n)$) contains the upper or lower triangular matrix A , as specified by <i>uplo</i> . |

b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the right-hand side matrix *B*.

x of size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout contains the solution matrix *X*.

| | |
|------------|---|
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

ferr, berr Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

A call to this routine involves, for each right-hand side, solving a number of systems of linear equations $A*x = b$; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately n^2 floating-point operations for real flavors or $4n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?tprfs

Estimates the error in the solution of a system of linear equations with a packed triangular coefficient matrix.

Syntax

```
lapack_int LAPACKE_stprfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const float* ap, const float* b, lapack_int ldb, const
float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dtpdfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const double* ap, const double* b, lapack_int ldb,
const double* x, lapack_int ldx, double* ferr, double* berr );
```

```
lapack_int LAPACKE_ctprfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const lapack_complex_float* ap, const
lapack_complex_float* b, lapack_int ldb, const lapack_complex_float* x, lapack_int ldx,
float* ferr, float* berr );
```

```
lapack_int LAPACKE_ztprfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int nrhs, const lapack_complex_double* ap, const
lapack_complex_double* b, lapack_int ldb, const lapack_complex_double* x, lapack_int
ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine estimates the errors in the solution to a system of linear equations $A^*X = B$ or $A^T X = B$ or $A^H X = B$ with a packed triangular matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A)x = (b + \delta b).$$

The routine also estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine, call the solver routine [?tprfs](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If <i>uplo</i> = 'U', then A is upper triangular. If <i>uplo</i> = 'L', then A is lower triangular. |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', the system has the form $A^*X = B$. If <i>trans</i> = 'T', the system has the form $A^T X = B$. If <i>trans</i> = 'C', the system has the form $A^H X = B$. |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', A is not a unit triangular matrix. If <i>diag</i> = 'U', A is unit triangular: diagonal elements of A are assumed to be 1 and not referenced in the array <i>ap</i> . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ap, b, x</i> | Arrays: <i>ap</i> $\max(1, n(n + 1)/2)$ contains the upper or lower triangular matrix A , as specified by <i>uplo</i> . |

b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the right-hand side matrix *B*.

x of size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout contains the solution matrix *X*.

ldb The leading dimension of *b*; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

ldx The leading dimension of *x*; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

ferr, berr Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

Application Notes

The bounds returned in *ferr* are not rigorous, but in practice they almost always overestimate the actual error.

A call to this routine involves, for each right-hand side, solving a number of systems of linear equations $A*x = b$; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately n^2 floating-point operations for real flavors or $4n^2$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?tbrfs

Estimates the error in the solution of a system of linear equations with a triangular band coefficient matrix.

Syntax

```
lapack_int LAPACKE_stbrfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const float* ab, lapack_int ldab, const
float* b, lapack_int ldb, const float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dtbrfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const double* ab, lapack_int ldab, const
double* b, lapack_int ldb, const double* x, lapack_int ldx, double* ferr, double*
berr );
```

```
lapack_int LAPACKE_ctbrfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const lapack_complex_float* ab,
lapack_int ldab, const lapack_complex_float* b, lapack_int ldb, const
lapack_complex_float* x, lapack_int ldx, float* ferr, float* berr );
```

```
lapack_int LAPACKE_ztbrfs( int matrix_layout, char uplo, char trans, char diag,
lapack_int n, lapack_int kd, lapack_int nrhs, const lapack_complex_double* ab,
lapack_int ldab, const lapack_complex_double* b, lapack_int ldb, const
lapack_complex_double* x, lapack_int ldx, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine estimates the errors in the solution to a system of linear equations $A^*X = B$ or $A^T*X = B$ or $A^H*X = B$ with a triangular band matrix A , with multiple right-hand sides. For each computed solution vector x , the routine computes the *component-wise backward error* β . This error is the smallest relative perturbation in elements of A and b such that x is the exact solution of the perturbed system:

$$|\delta a_{ij}| \leq \beta |a_{ij}|, \quad |\delta b_i| \leq \beta |b_i| \text{ such that } (A + \delta A)x = (b + \delta b).$$

The routine also estimates the *component-wise forward error* in the computed solution $\|x - x_e\|_\infty / \|x\|_\infty$ (here x_e is the exact solution).

Before calling this routine, call the solver routine `?tbrs`.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If <i>uplo</i> = 'U', then A is upper triangular. If <i>uplo</i> = 'L', then A is lower triangular. |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', the system has the form $A^*X = B$. If <i>trans</i> = 'T', the system has the form $A^T*X = B$. If <i>trans</i> = 'C', the system has the form $A^H*X = B$. |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', A is not a unit triangular matrix. If <i>diag</i> = 'U', A is unit triangular: diagonal elements of A are assumed to be 1 and not referenced in the array <i>ab</i> . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>kd</i> | The number of super-diagonals or sub-diagonals in the matrix A ; $kd \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>ab, b, x</i> | Arrays: |

ab (size $\max(1, ldab*n)$) contains the upper or lower triangular matrix A , as specified by $uplo$, in band storage format.

b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the right-hand side matrix B .

x of size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout contains the solution matrix X .

| | |
|--------|--|
| $ldab$ | The leading dimension of the array ab ; $ldab \geq kd + 1$. |
| ldb | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| ldx | The leading dimension of x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|--------------|---|
| $ferr, berr$ | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and backward errors, respectively, for each solution vector. |
|--------------|---|

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

Application Notes

The bounds returned in $ferr$ are not rigorous, but in practice they almost always overestimate the actual error.

A call to this routine involves, for each right-hand side, solving a number of systems of linear equations $A*x = b$; the number of systems is usually 4 or 5 and never more than 11. Each solution requires approximately $2n*kd$ floating-point operations for real flavors or $8n*kd$ operations for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

Matrix Inversion: LAPACK Computational Routines

It is seldom necessary to compute an explicit inverse of a matrix. In particular, do not attempt to solve a system of equations $Ax = b$ by first computing A^{-1} and then forming the matrix-vector product $x = A^{-1}b$. Call a solver routine instead (see [Routines for Solving Systems of Linear Equations](#)); this is more efficient and more accurate.

However, matrix inversion routines are provided for the rare occasions when an explicit inverse matrix is needed.

?getri

Computes the inverse of an LU-factored general matrix.

Syntax

```
lapack_int LAPACKE_sgetri (int matrix_layout , lapack_int n , float * a , lapack_int lda , const lapack_int * ipiv );
```

```
lapack_int LAPACKE_dgetri (int matrix_layout , lapack_int n , double * a , lapack_int lda , const lapack_int * ipiv );
```

```
lapack_int LAPACKE_cgetri (int matrix_layout , lapack_int n , lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv );
```

```
lapack_int LAPACKE_zgetri (int matrix_layout , lapack_int n , lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a general matrix A . Before calling this routine, call [?getrf](#) to factorize A .

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array a (size $\max(1, lda * n)$) contains the factorization of the matrix A , as returned by ?getrf : $A = P * L * U$. The second dimension of a must be at least $\max(1, n)$. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?getrf . |

Output Parameters

| | |
|----------|--|
| <i>a</i> | Overwritten by the n -by- n matrix $\text{inv}(A)$. |
|----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the *i*-th diagonal element of the factor U is zero, U is singular, and the inversion could not be completed.

Application Notes

The computed inverse X satisfies the following error bound:

$$\|XA - I\| \leq c(n) \varepsilon \|X\| \|P\| \|L\| \|U\|,$$

where $c(n)$ is a modest linear function of n ; ε is the machine precision; I denotes the identity matrix; P , L , and U are the factors of the matrix factorization $A = P * L * U$.

The total number of floating-point operations is approximately $(4/3)n^3$ for real flavors and $(16/3)n^3$ for complex flavors.

See Also

Matrix Storage Schemes for LAPACK Routines

?potri

Computes the inverse of a symmetric (Hermitian) positive-definite matrix using the Cholesky factorization.

Syntax

```
lapack_int LAPACKE_spotri (int matrix_layout , char uplo , lapack_int n , float * a ,
lapack_int lda );
```

```
lapack_int LAPACKE_dpotri (int matrix_layout , char uplo , lapack_int n , double * a ,
lapack_int lda );
```

```
lapack_int LAPACKE_cpotri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda );
```

```
lapack_int LAPACKE_zpotri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a symmetric positive definite or, for complex flavors, Hermitian positive-definite matrix A . Before calling this routine, call [?potrf](#) to factorize A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array a (size $\max(1, lda * n)$). Contains the factorization of the matrix A , as returned by ?potrf . |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | Overwritten by the upper or lower triangle of the inverse of A . |
|----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = $-i$, parameter i had an illegal value.

If $info = i$, the i -th diagonal element of the Cholesky factor (and therefore the factor itself) is zero, and the inversion could not be completed.

Application Notes

The computed inverse X satisfies the following error bounds:

$$\|XA - I\|_2 \leq c(n) \varepsilon \kappa_2(A), \quad \|AX - I\|_2 \leq c(n) \varepsilon \kappa_2(A),$$

where $c(n)$ is a modest linear function of n , and ε is the machine precision; I denotes the identity matrix.

The 2-norm $\|A\|_2$ of a matrix A is defined by $\|A\|_2 = \max_{x \cdot x=1} (Ax \cdot Ax)^{1/2}$, and the condition number $\kappa_2(A)$ is defined by $\kappa_2(A) = \|A\|_2 \|A^{-1}\|_2$.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

See Also

Matrix Storage Schemes for LAPACK Routines

?pftri

Computes the inverse of a symmetric (Hermitian) positive-definite matrix in RFP format using the Cholesky factorization.

Syntax

```
lapack_int LAPACKE_spftri (int matrix_layout , char transr , char uplo , lapack_int n ,
float * a );
```

```
lapack_int LAPACKE_dpfttri (int matrix_layout , char transr , char uplo , lapack_int n ,
double * a );
```

```
lapack_int LAPACKE_cpfttri (int matrix_layout , char transr , char uplo , lapack_int n ,
lapack_complex_float * a );
```

```
lapack_int LAPACKE_zpfttri (int matrix_layout , char transr , char uplo , lapack_int n ,
lapack_complex_double * a );
```

Include Files

- mkl.h

Description

The routine computes the inverse $inv(A)$ of a symmetric positive definite or, for complex data, Hermitian positive-definite matrix A using the Cholesky factorization:

$$\begin{aligned} A &= U^T * U \text{ for real data, } A = U^H * U \text{ for complex data} && \text{if } uplo='U' \\ A &= L * L^T \text{ for real data, } A = L * L^H \text{ for complex data} && \text{if } uplo='L' \end{aligned}$$

Before calling this routine, call [?pfttrf](#) to factorize A .

The matrix A is in the Rectangular Full Packed (RFP) format. For the description of the RFP format, see [Matrix Storage Schemes](#).

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

transr Must be 'N', 'T' (for real data) or 'C' (for complex data).

If $transr = 'N'$, the Normal $transr$ of RFP U (if $uplo = 'U'$) or L (if $uplo = 'L'$) is stored.

If $transr = 'T'$, the Transpose $transr$ of RFP U (if $uplo = 'U'$) or L (if $uplo = 'L'$) is stored.

If $transr = 'C'$, the Conjugate-Transpose $transr$ of RFP U (if $uplo = 'U'$) or L (if $uplo = 'L'$) is stored.

$uplo$

Must be 'U' or 'L'.

Indicates how the input matrix A has been factored:

If $uplo = 'U'$, $A = U^T * U$ for real data or $A = U^H * U$ for complex data, and U is stored.

If $uplo = 'L'$, $A = L * L^T$ for real data or $A = L * L^H$ for complex data, and L is stored.

n

The order of the matrix A ; $n \geq 0$.

a

Array, size $(n * (n + 1) / 2)$. The array a contains the factor U or L matrix A in the RFP format.

Output Parameters

a

The symmetric/Hermitian inverse of the original matrix in the same storage format.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, the (i, i) element of the factor U or L is zero, and the inverse could not be computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pptri

Computes the inverse of a packed symmetric (Hermitian) positive-definite matrix using Cholesky factorization.

Syntax

```
lapack_int LAPACKE_spptri (int matrix_layout , char uplo , lapack_int n , float * ap );
```

```
lapack_int LAPACKE_dpptri (int matrix_layout , char uplo , lapack_int n , double * ap );
```

```
lapack_int LAPACKE_cpptri (int matrix_layout , char uplo , lapack_int n , lapack_complex_float * ap );
```

```
lapack_int LAPACKE_zpptri (int matrix_layout , char uplo , lapack_int n , lapack_complex_double * ap );
```

Include Files

- `mkl.h`

Description

The routine computes the inverse $\text{inv}(A)$ of a symmetric positive definite or, for complex flavors, Hermitian positive-definite matrix A in *packed* form. Before calling this routine, call `?pptrf` to factorize A .

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular factor is stored in <code>ap</code> : If <code>uplo = 'U'</code> , then the upper triangular factor is stored. If <code>uplo = 'L'</code> , then the lower triangular factor is stored. |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>ap</code> | Array, size at least $\max(1, n(n+1)/2)$. Contains the factorization of the packed matrix A , as returned by <code>?pptrf</code> . The dimension <code>ap</code> must be at least $\max(1, n(n+1)/2)$. |

Output Parameters

| | |
|-----------------|---|
| <code>ap</code> | Overwritten by the packed n -by- n matrix $\text{inv}(A)$. |
|-----------------|---|

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful.

If `info = -i`, parameter i had an illegal value.

If `info = i`, the i -th diagonal element of the Cholesky factor (and therefore the factor itself) is zero, and the inversion could not be completed.

Application Notes

The computed inverse X satisfies the following error bounds:

$$\|XA - I\|_2 \leq c(n) \varepsilon \kappa_2(A), \quad \|AX - I\|_2 \leq c(n) \varepsilon \kappa_2(A),$$

where $c(n)$ is a modest linear function of n , and ε is the machine precision; I denotes the identity matrix.

The 2-norm $\|A\|_2$ of a matrix A is defined by $\|A\|_2 = \max_{x \cdot x = 1} (Ax \cdot Ax)^{1/2}$, and the condition number $\kappa_2(A)$ is defined by $\kappa_2(A) = \|A\|_2 \|A^{-1}\|_2$.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?sytri

Computes the inverse of a symmetric matrix using $U^*D^*U^T$ or $L^*D^*L^T$ Bunch-Kaufman factorization.

Syntax

```
lapack_int LAPACKE_ssytri (int matrix_layout , char uplo , lapack_int n , float * a ,
lapack_int lda , const lapack_int * ipiv );

lapack_int LAPACKE_dsytri (int matrix_layout , char uplo , lapack_int n , double * a ,
lapack_int lda , const lapack_int * ipiv );

lapack_int LAPACKE_csytri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv );

lapack_int LAPACKE_zsytri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a symmetric matrix A . Before calling this routine, call [?sytrf](#) to factorize A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <i>uplo</i> = 'U', the array a stores the Bunch-Kaufman factorization $A = U^*D^*U^T$. If <i>uplo</i> = 'L', the array a stores the Bunch-Kaufman factorization $A = L^*D^*L^T$. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | $a(\text{size } \max(1, \text{lda} * n))$ contains the factorization of the matrix A , as returned by ?sytrf . |
| <i>lda</i> | The leading dimension of a ; $\text{lda} \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?sytrf . |

Output Parameters

| | |
|----------|--|
| <i>a</i> | Overwritten by the n -by- n matrix $\text{inv}(A)$. |
|----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, the i -th diagonal element of D is zero, D is singular, and the inversion could not be completed.

Application Notes

The computed inverse X satisfies the following error bounds:

$$|D^*U^T*P^T*X*P*U - I| \leq c(n)\epsilon(|D||U^T|P^T|X|P|U| + |D||D^{-1}|)$$

for $uplo = 'U'$, and

$$|D^*L^T*P^T*X*P*L - I| \leq c(n)\epsilon(|D||L^T|P^T|X|P|L| + |D||D^{-1}|)$$

for $uplo = 'L'$. Here $c(n)$ is a modest linear function of n , and ϵ is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?hetri

*Computes the inverse of a complex Hermitian matrix using $U^*D^*U^H$ or $L^*D^*L^H$ Bunch-Kaufman factorization.*

Syntax

```
lapack_int LAPACKE_chetri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv );

lapack_int LAPACKE_zhetri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the inverse $inv(A)$ of a complex Hermitian matrix A . Before calling this routine, call [?hetrf](#) to factorize A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, the array a stores the Bunch-Kaufman factorization $A = U^*D^*U^H$. If $uplo = 'L'$, the array a stores the Bunch-Kaufman factorization $A = L^*D^*L^H$. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |

| | |
|-------------|--|
| <i>a</i> , | Array <i>a</i> (size $\max(1, lda * n)$) contains the factorization of the matrix <i>A</i> , as returned by ?hetrf . The second dimension of <i>a</i> must be at least $\max(1, n)$. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?hetrf . |

Output Parameters

| | |
|----------|--|
| <i>a</i> | Overwritten by the <i>n</i> -by- <i>n</i> matrix $\text{inv}(A)$. |
|----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the *i*-th diagonal element of *D* is zero, *D* is singular, and the inversion could not be completed.

Application Notes

The computed inverse *X* satisfies the following error bounds:

$$|D * U^H * P^T * X * P * U - I| \leq c(n) \varepsilon (|D| |U^H| |P^T| |X| |P| |U| + |D| |D^{-1}|)$$

for *uplo* = 'U', and

$$|D * L^H * P^T * X * P * L - I| \leq c(n) \varepsilon (|D| |L^H| |P^T| |X| |P| |L| + |D| |D^{-1}|)$$

for *uplo* = 'L'. Here $c(n)$ is a modest linear function of *n*, and ε is the machine precision; *I* denotes the identity matrix.

The total number of floating-point operations is approximately $(8/3)n^3$ for complex flavors.

The real counterpart of this routine is [?sytri](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

[?sytri2](#)

Computes the inverse of a symmetric indefinite matrix through allocating memory and calling [?sytri2x](#).

Syntax

```
lapack_int LAPACKE_ssytri2 (int matrix_layout , char uplo , lapack_int n , float * a ,
lapack_int lda , const lapack_int * ipiv );

lapack_int LAPACKE_dsytri2 (int matrix_layout , char uplo , lapack_int n , double * a ,
lapack_int lda , const lapack_int * ipiv );

lapack_int LAPACKE_csytri2 (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv );

lapack_int LAPACKE_zsytri2 (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv );
```

Include Files

- `mk1.h`

Description

The routine computes the inverse $\text{inv}(A)$ of a symmetric indefinite matrix A using the factorization $A = U^*D^*U^T$ or $A = L^*D^*L^T$ computed by `?sytrf`.

The `?sytri2` routine allocates a temporary buffer before calling `?sytri2x` that actually computes the inverse.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <code>uplo = 'U'</code> , the array a stores the factorization $A = U^*D^*U^T$. If <code>uplo = 'L'</code> , the array a stores the factorization $A = L^*D^*L^T$. |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>a</code> | Array a (size $\max(1, lda * n)$) contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as returned by <code>?sytrf</code> . |
| <code>lda</code> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <code>ipiv</code> | Array, size at least $\max(1, n)$. Details of the interchanges and the block structure of D as returned by <code>?sytrf</code> . |

Output Parameters

| | |
|----------------|--|
| <code>a</code> | If <code>info = 0</code> , the symmetric inverse of the original matrix. If <code>uplo = 'U'</code> , the upper triangular part of the inverse is formed and the part of A below the diagonal is not referenced. If <code>uplo = 'L'</code> , the lower triangular part of the inverse is formed and the part of A above the diagonal is not referenced. |
|----------------|--|

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

If `info = i`, $D(i,i) = 0$; D is singular and its inversion could not be computed.

See Also

[?sytrf](#)

[?sytri2x](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?hetri2

Computes the inverse of a Hermitian indefinite matrix through allocating memory and calling ?hetri2x.

Syntax

```
lapack_int LAPACKE_chetri2 (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv );

lapack_int LAPACKE_zhetri2 (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a Hermitian indefinite matrix A using the factorization $A = U^*D^*U^H$ or $A = L^*D^*L^H$ computed by ?hetrf.

The ?hetri2 routine allocates a temporary buffer before calling ?hetri2x that actually computes the inverse.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <i>uplo</i> = 'U', the array a stores the factorization $A = U^*D^*U^H$. If <i>uplo</i> = 'L', the array a stores the factorization $A = L^*D^*L^H$. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array a (size $\max(1, lda*n)$) contains the block diagonal matrix D and the multipliers used to obtain the factor U or L as returned by ?sytrf. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Details of the interchanges and the block structure of D as returned by ?hetrf. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | If <i>info</i> = 0, the inverse of the original matrix. If <i>uplo</i> = 'U', the upper triangular part of the inverse is formed and the part of A below the diagonal is not referenced. If <i>uplo</i> = 'L', the lower triangular part of the inverse is formed and the part of A above the diagonal is not referenced. |
|----------|---|

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, $D(i,i) = 0$; D is singular and its inversion could not be computed.

See Also

[?hetrf](#)

[?hetri2x](#)

[Matrix Storage Schemes for LAPACK Routines](#)

[?sytri2x](#)

Computes the inverse of a symmetric indefinite matrix after [?sytri2](#) allocates memory.

Syntax

```
lapack_int LAPACKE_ssytri2x (int matrix_layout , char uplo , lapack_int n , float * a ,
lapack_int lda , const lapack_int * ipiv , lapack_int nb );
```

```
lapack_int LAPACKE_dsytri2x (int matrix_layout , char uplo , lapack_int n , double *
a , lapack_int lda , const lapack_int * ipiv , lapack_int nb );
```

```
lapack_int LAPACKE_csytri2x (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv , lapack_int nb );
```

```
lapack_int LAPACKE_zsytri2x (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv , lapack_int nb );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a symmetric indefinite matrix A using the factorization $A = U * D * U^T$ or $A = L * D * L^T$ computed by [?sytrf](#).

The [?sytri2x](#) actually computes the inverse after the [?sytri2](#) routine allocates memory before calling [?sytri2x](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, the array a stores the factorization $A = U * D * U^T$. If $uplo = 'L'$, the array a stores the factorization $A = L * D * L^T$. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array a (size $\max(1, lda * n)$) contains the nb (block size) diagonal matrix D and the multipliers used to obtain the factor U or L as returned by ?sytrf . The second dimension of a must be at least $\max(1, n)$. |

| | |
|-------------|---|
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Details of the interchanges and the <i>nb</i> structure of <i>D</i> as returned by <code>?sytrf</code> . |
| <i>nb</i> | Block size. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | If <i>info</i> = 0, the symmetric inverse of the original matrix. If <i>info</i> = 'U', the upper triangular part of the inverse is formed and the part of <i>A</i> below the diagonal is not referenced. If <i>info</i> = 'L', the lower triangular part of the inverse is formed and the part of <i>A</i> above the diagonal is not referenced. |
|----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, $D_{ii} = 0$; *D* is singular and its inversion could not be computed.

See Also

[?sytrf](#)

[?sytri2](#)

[Matrix Storage Schemes for LAPACK Routines](#)

[?hetri2x](#)

Computes the inverse of a Hermitian indefinite matrix after ?hetri2 allocates memory.

Syntax

```
lapack_int LAPACKE_chetri2x (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda , const lapack_int * ipiv , lapack_int nb );
lapack_int LAPACKE_zhetri2x (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda , const lapack_int * ipiv , lapack_int nb );
```

Include Files

- `mkl.h`

Description

The routine computes the inverse $\text{inv}(A)$ of a Hermitian indefinite matrix *A* using the factorization $A = U * D * U^H$ or $A = L * D * L^H$ computed by `?hetrf`.

The `?hetri2x` actually computes the inverse after the `?hetri2` routine allocates memory before calling `?hetri2x`.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix <i>A</i> has been factored: If <i>uplo</i> = 'U', the array <i>a</i> stores the factorization $A = U * D * U^H$. If <i>uplo</i> = 'L', the array <i>a</i> stores the factorization $A = L * D * L^H$. |
| <i>n</i> | The order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>a</i> | Arrays <i>a</i> (size $\max(1, lda * n)$) contains the <i>nb</i> (block size) diagonal matrix <i>D</i> and the multipliers used to obtain the factor U or L as returned by ?hetrf. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Details of the interchanges and the <i>nb</i> structure of <i>D</i> as returned by ?hetrf. |
| <i>nb</i> | Block size. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | If <i>info</i> = 0, the symmetric inverse of the original matrix. If <i>info</i> = 'U', the upper triangular part of the inverse is formed and the part of <i>A</i> below the diagonal is not referenced. If <i>info</i> = 'L', the lower triangular part of the inverse is formed and the part of <i>A</i> above the diagonal is not referenced. |
|----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, $D_{ii} = 0$; *D* is singular and its inversion could not be computed.

See Also

[?hetrf](#)

[?hetri2](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?sptri

Computes the inverse of a symmetric matrix using $U * D * U^T$ or $L * D * L^T$ Bunch-Kaufman factorization of matrix in packed storage.

Syntax

```
lapack_int LAPACKE_ssptri (int matrix_layout , char uplo , lapack_int n , float * ap ,
const lapack_int * ipiv );

lapack_int LAPACKE_dsptri (int matrix_layout , char uplo , lapack_int n , double * ap ,
const lapack_int * ipiv );

lapack_int LAPACKE_csptri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * ap , const lapack_int * ipiv );

lapack_int LAPACKE_zsptri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * ap , const lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a packed symmetric matrix A . Before calling this routine, call [?sptrf](#) to factorize A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If <i>uplo</i> = 'U', the array <i>ap</i> stores the Bunch-Kaufman factorization $A = U^T D U$. If <i>uplo</i> = 'L', the array <i>ap</i> stores the Bunch-Kaufman factorization $A = L D L^T$. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>ap</i> | Arrays <i>ap</i> (size $\max(1, n(n+1)/2)$) contains the factorization of the matrix A , as returned by ?sptrf . |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The <i>ipiv</i> array, as returned by ?sptrf . |

Output Parameters

| | |
|-----------|---|
| <i>ap</i> | Overwritten by the matrix $\text{inv}(A)$ in packed form. |
|-----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the *i*-th diagonal element of D is zero, D is singular, and the inversion could not be completed.

Application Notes

The computed inverse X satisfies the following error bounds:

$$|D*U^T*P^T*X*P*U - I| \leq c(n)\varepsilon(|D||U^T|P^T|X|P|U| + |D||D^{-1}|)$$

for $uplo = 'U'$, and

$$|D*L^T*P^T*X*P*L - I| \leq c(n)\varepsilon(|D||L^T|P^T|X|P|L| + |D||D^{-1}|)$$

for $uplo = 'L'$. Here $c(n)$ is a modest linear function of n , and ε is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(2/3)n^3$ for real flavors and $(8/3)n^3$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?hptri

*Computes the inverse of a complex Hermitian matrix using $U*D*U^H$ or $L*D*L^H$ Bunch-Kaufman factorization of matrix in packed storage.*

Syntax

```
lapack_int LAPACKE_chptri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * ap , const lapack_int * ipiv );
```

```
lapack_int LAPACKE_zhptri (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * ap , const lapack_int * ipiv );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a complex Hermitian matrix A using packed storage. Before calling this routine, call [?hptrf](#) to factorize A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix A has been factored: If $uplo = 'U'$, the array ap stores the packed Bunch-Kaufman factorization $A = U*D*U^H$. If $uplo = 'L'$, the array ap stores the packed Bunch-Kaufman factorization $A = L*D*L^H$. |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>ap</i> | Array ap (size $\max(1, n(n+1)/2)$) contains the factorization of the matrix A , as returned by ?hptrf . |

ipiv Array, size at least $\max(1, n)$.
The *ipiv* array, as returned by [?hptrf](#).

Output Parameters

ap Overwritten by the matrix $\text{inv}(A)$.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the *i*-th diagonal element of *D* is zero, *D* is singular, and the inversion could not be completed.

Application Notes

The computed inverse *X* satisfies the following error bounds:

$$|D^*U^H*P^T*X*P*U - I| \leq c(n)\varepsilon(|D||U^H|P^T|X|P|U| + |D||D^{-1}|)$$

for *uplo* = 'U', and

$$|D*L^H*P^T*X*P*L - I| \leq c(n)\varepsilon(|D||L^H|P^T|X|P*L| + |D||D^{-1}|)$$

for *uplo* = 'L'. Here $c(n)$ is a modest linear function of *n*, and ε is the machine precision; *I* denotes the identity matrix.

The total number of floating-point operations is approximately $(8/3)n^3$.

The real counterpart of this routine is [?sptri](#).

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?trtri

Computes the inverse of a triangular matrix.

Syntax

```
lapack_int LAPACKE_strtri (int matrix_layout , char uplo , char diag , lapack_int n ,
float * a , lapack_int lda );
```

```
lapack_int LAPACKE_dtrtri (int matrix_layout , char uplo , char diag , lapack_int n ,
double * a , lapack_int lda );
```

```
lapack_int LAPACKE_ctrtri (int matrix_layout , char uplo , char diag , lapack_int n ,
lapack_complex_float * a , lapack_int lda );
```

```
lapack_int LAPACKE_ztrtri (int matrix_layout , char uplo , char diag , lapack_int n ,
lapack_complex_double * a , lapack_int lda );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a triangular matrix *A*.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates whether A is upper or lower triangular: If <code>uplo</code> = 'U', then A is upper triangular. If <code>uplo</code> = 'L', then A is lower triangular. |
| <code>diag</code> | Must be 'N' or 'U'. If <code>diag</code> = 'N', then A is not a unit triangular matrix. If <code>diag</code> = 'U', A is unit triangular: diagonal elements of A are assumed to be 1 and not referenced in the array a . |
| <code>n</code> | The order of the matrix A ; $n \geq 0$. |
| <code>a</code> | Array: . Contains the matrix A . |
| <code>lda</code> | The first dimension of a ; $lda \geq \max(1, n)$. |

Output Parameters

| | |
|----------------|---|
| <code>a</code> | Overwritten by the matrix <code>inv(A)</code> . |
|----------------|---|

Return Values

This function returns a value `info`.

If `info` = 0, the execution is successful.

If `info` = $-i$, parameter i had an illegal value.

If `info` = i , the i -th diagonal element of A is zero, A is singular, and the inversion could not be completed.

Application Notes

The computed inverse X satisfies the following error bounds:

$$\|XA - I\| \leq c(n)\epsilon \|X\| \|A\|$$

$$\|XA - I\| \leq c(n)\epsilon \|A^{-1}\| \|A\| \|X\|,$$

where $c(n)$ is a modest linear function of n ; ϵ is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors and $(4/3)n^3$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?tftri

Computes the inverse of a triangular matrix stored in the Rectangular Full Packed (RFP) format.

Syntax

```
lapack_int LAPACKE_stftri (int matrix_layout , char transr , char uplo , char diag ,
lapack_int n , float * a );
```

```

lapack_int LAPACKE_dtftri (int matrix_layout , char transr , char uplo , char diag ,
lapack_int n , double * a );

lapack_int LAPACKE_ctftri (int matrix_layout , char transr , char uplo , char diag ,
lapack_int n , lapack_complex_float * a );

lapack_int LAPACKE_ztftri (int matrix_layout , char transr , char uplo , char diag ,
lapack_int n , lapack_complex_double * a );

```

Include Files

- mkl.h

Description

Computes the inverse of a triangular matrix A stored in the Rectangular Full Packed (RFP) format. For the description of the RFP format, see [Matrix Storage Schemes](#).

This is the block version of the algorithm, calling Level 3 BLAS.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>transr</i> | Must be 'N', 'T' (for real data) or 'C' (for complex data). If <i>transr</i> = 'N', the Normal <i>transr</i> of RFP A is stored. If <i>transr</i> = 'T', the Transpose <i>transr</i> of RFP A is stored. If <i>transr</i> = 'C', the Conjugate-Transpose <i>transr</i> of RFP A is stored. |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of RFP A is stored: If <i>uplo</i> = 'U', the array <i>a</i> stores the upper triangular part of the matrix A . If <i>uplo</i> = 'L', the array <i>a</i> stores the lower triangular part of the matrix A . |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', then A is not a unit triangular matrix. If <i>diag</i> = 'U', A is unit triangular: diagonal elements of A are assumed to be 1 and not referenced in the array <i>a</i> . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array, size $\max(1, n*(n+1)/2)$. The array <i>a</i> contains the matrix A in the RFP format. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | The (triangular) inverse of the original matrix in the same storage format. |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, $A_{i,i}$ is exactly zero. The triangular matrix is singular and its inverse cannot be computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?tptri

Computes the inverse of a triangular matrix using packed storage.

Syntax

```
lapack_int LAPACKE_stptri (int matrix_layout , char uplo , char diag , lapack_int n ,
float * ap );
```

```
lapack_int LAPACKE_dtptri (int matrix_layout , char uplo , char diag , lapack_int n ,
double * ap );
```

```
lapack_int LAPACKE_ctptri (int matrix_layout , char uplo , char diag , lapack_int n ,
lapack_complex_float * ap );
```

```
lapack_int LAPACKE_ztptri (int matrix_layout , char uplo , char diag , lapack_int n ,
lapack_complex_double * ap );
```

Include Files

- mkl.h

Description

The routine computes the inverse $\text{inv}(A)$ of a packed triangular matrix *A*.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether <i>A</i> is upper or lower triangular: If <i>uplo</i> = 'U', then <i>A</i> is upper triangular. If <i>uplo</i> = 'L', then <i>A</i> is lower triangular. |
| <i>diag</i> | Must be 'N' or 'U'. If <i>diag</i> = 'N', then <i>A</i> is not a unit triangular matrix. If <i>diag</i> = 'U', <i>A</i> is unit triangular: diagonal elements of <i>A</i> are assumed to be 1 and not referenced in the array <i>ap</i> . |
| <i>n</i> | The order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>ap</i> | Array, size at least $\max(1, n(n+1)/2)$. |

Contains the packed triangular matrix A .

Output Parameters

ap Overwritten by the packed n -by- n matrix $\text{inv}(A)$.

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, the i -th diagonal element of A is zero, A is singular, and the inversion could not be completed.

Application Notes

The computed inverse X satisfies the following error bounds:

$$|XA - I| \leq c(n)\epsilon |X| |A|$$

$$|X - A^{-1}| \leq c(n)\epsilon |A^{-1}| |A| |X|,$$

where $c(n)$ is a modest linear function of n ; ϵ is the machine precision; I denotes the identity matrix.

The total number of floating-point operations is approximately $(1/3)n^3$ for real flavors and $(4/3)n^3$ for complex flavors.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

Matrix Equilibration: LAPACK Computational Routines

Routines described in this section are used to compute scaling factors needed to equilibrate a matrix. Note that these routines do not actually scale the matrices.

?gseequ

Computes row and column scaling factors intended to equilibrate a general matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_sgeequ( int matrix_layout, lapack_int m, lapack_int n, const float*
a, lapack_int lda, float* r, float* c, float* rowcnd, float* colcnd, float* amax );
```

```
lapack_int LAPACKE_dgeequ( int matrix_layout, lapack_int m, lapack_int n, const double*
a, lapack_int lda, double* r, double* c, double* rowcnd, double* colcnd, double*
amax );
```

```
lapack_int LAPACKE_cgeequ( int matrix_layout, lapack_int m, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float* r, float* c, float* rowcnd, float*
colcnd, float* amax );
```

```
lapack_int LAPACKE_zgeequ( int matrix_layout, lapack_int m, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double* r, double* c, double* rowcnd,
double* colcnd, double* amax );
```

Include Files

- `mkl.h`

Description

The routine computes row and column scalings intended to equilibrate an m -by- n matrix A and reduce its condition number. The output array r returns the row scale factors and the array c the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix B with elements $b_{ij}=r[i-1]*a_{ij}*c[j-1]$ have absolute value 1.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>m</code> | The number of rows of the matrix A ; $m \geq 0$. |
| <code>n</code> | The number of columns of the matrix A ; $n \geq 0$. |
| <code>a</code> | Array: size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout. Contains the m -by- n matrix A whose equilibration factors are to be computed. |
| <code>lda</code> | The leading dimension of a ; $lda \geq \max(1, m)$. |

Output Parameters

| | |
|---------------------|--|
| <code>r, c</code> | Arrays: r (size m), c (size n). If $info = 0$, or $info > m$, the array r contains the row scale factors of the matrix A . If $info = 0$, the array c contains the column scale factors of the matrix A . |
| <code>rowcnd</code> | If $info = 0$ or $info > m$, <code>rowcnd</code> contains the ratio of the smallest $r[i]$ to the largest $r[i]$. |
| <code>colcnd</code> | If $info = 0$, <code>colcnd</code> contains the ratio of the smallest $c[i]$ to the largest $c[i]$. |
| <code>amax</code> | Absolute value of the largest element of the matrix A . |

Return Values

This function returns a value `info`.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, $i > 0$, and

$i \leq m$, the i -th row of A is exactly zero;

$i > m$, the $(i-m)$ th column of A is exactly zero.

Application Notes

All the components of r and c are restricted to be between `SMLNUM` = smallest safe number and `BIGNUM`= largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of A but works well in practice.

`SMLNUM` and `BIGNUM` are parameters representing machine precision. You can use the `?lamch` routines to compute them. For example, compute single precision values of `SMLNUM` and `BIGNUM` as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

If `rowcnd` \geq 0.1 and `amax` is neither too large nor too small, it is not worth scaling by r .

If `colcnd` \geq 0.1, it is not worth scaling by c .

If `amax` is very close to `SMLNUM` or very close to `BIGNUM`, the matrix A should be scaled.

See Also

Error Analysis

Matrix Storage Schemes for LAPACK Routines

?geequb

Computes row and column scaling factors restricted to a power of radix to equilibrate a general matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_sgeequb( int matrix_layout, lapack_int m, lapack_int n, const float*
a, lapack_int lda, float* r, float* c, float* rowcnd, float* colcnd, float* amax );
```

```
lapack_int LAPACKE_dgeequb( int matrix_layout, lapack_int m, lapack_int n, const
double* a, lapack_int lda, double* r, double* c, double* rowcnd, double* colcnd,
double* amax );
```

```
lapack_int LAPACKE_cgeequb( int matrix_layout, lapack_int m, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float* r, float* c, float* rowcnd, float*
colcnd, float* amax );
```

```
lapack_int LAPACKE_zgeequb( int matrix_layout, lapack_int m, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double* r, double* c, double* rowcnd,
double* colcnd, double* amax );
```

Include Files

- mkl.h

Description

The routine computes row and column scalings intended to equilibrate an m -by- n general matrix A and reduce its condition number. The output array r returns the row scale factors and the array c - the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix B with elements $b_{i,j} = r[i-1]*a_{i,j}*c[j-1]$ have an absolute value of at most the radix.

$r[i-1]$ and $c[j-1]$ are restricted to be a power of the radix between `SMLNUM` = smallest safe number and `BIGNUM` = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of a but works well in practice.

SMLNUM and *BIGNUM* are parameters representing machine precision. You can use the `?lamch` routines to compute them. For example, compute single precision values of *SMLNUM* and *BIGNUM* as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

This routine differs from `?geequ` by restricting the scaling factors to a power of the radix. Except for over- and underflow, scaling by these factors introduces no additional rounding errors. However, the scaled entries' magnitudes are no longer equal to approximately 1 but lie between $\sqrt{\text{radix}}$ and $1/\sqrt{\text{radix}}$.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix <i>A</i> ; $m \geq 0$. |
| <i>n</i> | The number of columns of the matrix <i>A</i> ; $n \geq 0$. |
| <i>a</i> | Array: size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout. Contains the <i>m</i> -by- <i>n</i> matrix <i>A</i> whose equilibration factors are to be computed. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, m)$. |

Output Parameters

| | |
|---------------|---|
| <i>r, c</i> | Arrays: $r(m), c(n)$. If <i>info</i> = 0, or <i>info</i> > <i>m</i> , the array <i>r</i> contains the row scale factors for the matrix <i>A</i> . If <i>info</i> = 0, the array <i>c</i> contains the column scale factors for the matrix <i>A</i> . |
| <i>rowcnd</i> | If <i>info</i> = 0 or <i>info</i> > <i>m</i> , <i>rowcnd</i> contains the ratio of the smallest $r[i]$ to the largest $r[i]$. If <i>rowcnd</i> ≥ 0.1 , and <i>amax</i> is neither too large nor too small, it is not worth scaling by <i>r</i> . |
| <i>colcnd</i> | If <i>info</i> = 0, <i>colcnd</i> contains the ratio of the smallest $c[i]$ to the largest $c[i]$. If <i>colcnd</i> ≥ 0.1 , it is not worth scaling by <i>c</i> . |
| <i>amax</i> | Absolute value of the largest element of the matrix <i>A</i> . If <i>amax</i> is very close to <i>SMLNUM</i> or very close to <i>BIGNUM</i> , the matrix should be scaled. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, $i > 0$, and

$i \leq m$, the *i*-th row of *A* is exactly zero;

$i > m$, the (*i*-*m*)-th column of *A* is exactly zero.

See Also

Error Analysis

Matrix Storage Schemes for LAPACK Routines

?gbequ

Computes row and column scaling factors intended to equilibrate a banded matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_sgbequ( int matrix_layout, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, const float* ab, lapack_int ldab, float* r, float* c, float*
rowcnd, float* colcnd, float* amax );
```

```
lapack_int LAPACKE_dgbequ( int matrix_layout, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, const double* ab, lapack_int ldab, double* r, double* c, double*
rowcnd, double* colcnd, double* amax );
```

```
lapack_int LAPACKE_cgbequ( int matrix_layout, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, const lapack_complex_float* ab, lapack_int ldab, float* r, float*
c, float* rowcnd, float* colcnd, float* amax );
```

```
lapack_int LAPACKE_zgbequ( int matrix_layout, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, const lapack_complex_double* ab, lapack_int ldab, double* r,
double* c, double* rowcnd, double* colcnd, double* amax );
```

Include Files

- mkl.h

Description

The routine computes row and column scalings intended to equilibrate an m -by- n band matrix A and reduce its condition number. The output array r returns the row scale factors and the array c the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix B with elements $b_{ij}=r[i - 1]*a_{ij}*c[j - 1]$ have absolute value 1.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix A ; $m \geq 0$. |
| <i>n</i> | The number of columns of the matrix A ; $n \geq 0$. |
| <i>kl</i> | The number of subdiagonals within the band of A ; $kl \geq 0$. |
| <i>ku</i> | The number of superdiagonals within the band of A ; $ku \geq 0$. |
| <i>ab</i> | Array, size $\max(1, ldab*n)$ for column major layout and $\max(1, ldab*m)$ for row major layout. Contains the original band matrix A . |
| <i>ldab</i> | The leading dimension of ab ; $ldab \geq kl+ku+1$. |

Output Parameters

| | |
|---------------------|--|
| <i>r</i> , <i>c</i> | Arrays: <i>r</i> (size <i>m</i>), <i>c</i> (size <i>n</i>). If <i>info</i> = 0, or <i>info</i> > <i>m</i> , the array <i>r</i> contains the row scale factors of the matrix <i>A</i> . If <i>info</i> = 0, the array <i>c</i> contains the column scale factors of the matrix <i>A</i> . |
| <i>rowcnd</i> | If <i>info</i> = 0 or <i>info</i> > <i>m</i> , <i>rowcnd</i> contains the ratio of the smallest <i>r</i> [<i>i</i>] to the largest <i>r</i> [<i>i</i>]. |
| <i>colcnd</i> | If <i>info</i> = 0, <i>colcnd</i> contains the ratio of the smallest <i>c</i> [<i>i</i>] to the largest <i>c</i> [<i>i</i>]. |
| <i>amax</i> | Absolute value of the largest element of the matrix <i>A</i> . |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i* and

i ≤ *m*, the *i*-th row of *A* is exactly zero;

i > *m*, the (*i*-*m*)th column of *A* is exactly zero.

Application Notes

All the components of *r* and *c* are restricted to be between *SMLNUM* = smallest safe number and *BIGNUM* = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of *A* but works well in practice.

SMLNUM and *BIGNUM* are parameters representing machine precision. You can use the `?lamch` routines to compute them. For example, compute single precision values of *SMLNUM* and *BIGNUM* as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

If *rowcnd* ≥ 0.1 and *amax* is neither too large nor too small, it is not worth scaling by *r*.

If *colcnd* ≥ 0.1, it is not worth scaling by *c*.

If *amax* is very close to *SMLNUM* or very close to *BIGNUM*, the matrix *A* should be scaled.

See Also

[Error Analysis](#)

[Matrix Storage Schemes for LAPACK Routines](#)

[?gbequb](#)

Computes row and column scaling factors restricted to a power of radix to equilibrate a banded matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_sgbequb( int matrix_layout, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, const float* ab, lapack_int ldab, float* r, float* c, float*
rowcnd, float* colcnd, float* amax );
```

```
lapack_int LAPACKE_dgbequb( int matrix_layout, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, const double* ab, lapack_int ldab, double* r, double* c, double*
rowcnd, double* colcnd, double* amax );
```

```
lapack_int LAPACKE_cgbequb( int matrix_layout, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, const lapack_complex_float* ab, lapack_int ldab, float* r, float*
c, float* rowcnd, float* colcnd, float* amax );
```

```
lapack_int LAPACKE_zgbequb( int matrix_layout, lapack_int m, lapack_int n, lapack_int
kl, lapack_int ku, const lapack_complex_double* ab, lapack_int ldab, double* r,
double* c, double* rowcnd, double* colcnd, double* amax );
```

Include Files

- mkl.h

Description

The routine computes row and column scalings intended to equilibrate an m -by- n banded matrix A and reduce its condition number. The output array r returns the row scale factors and the array c - the column scale factors. These factors are chosen to try to make the largest element in each row and column of the matrix B with elements $b_{i,j} = r[i-1] * a_{i,j} * c[j-1]$ have an absolute value of at most the radix.

$r[i]$ and $c[j]$ are restricted to be a power of the radix between $SMLNUM$ = smallest safe number and $BIGNUM$ = largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of a but works well in practice.

$SMLNUM$ and $BIGNUM$ are parameters representing machine precision. You can use the `?slamch` routines to compute them. For example, compute single precision values of $SMLNUM$ and $BIGNUM$ as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

This routine differs from `?gbequ` by restricting the scaling factors to a power of the radix. Except for over- and underflow, scaling by these factors introduces no additional rounding errors. However, the scaled entries' magnitudes are no longer equal to approximately 1 but lie between $\sqrt{\text{radix}}$ and $1/\sqrt{\text{radix}}$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix A ; $m \geq 0$. |
| <i>n</i> | The number of columns of the matrix A ; $n \geq 0$. |
| <i>kl</i> | The number of subdiagonals within the band of A ; $kl \geq 0$. |
| <i>ku</i> | The number of superdiagonals within the band of A ; $ku \geq 0$. |
| <i>ab</i> | Array: size $\max(1, ldab * n)$ for column major layout and $\max(1, ldab * m)$ for row major layout |
| <i>ldab</i> | The leading dimension of a ; $ldab \geq \max(1, m)$. |

Output Parameters

r, c Arrays: r (size m), c (size n).

If $info = 0$, or $info > m$, the array r contains the row scale factors for the matrix A .

If $info = 0$, the array c contains the column scale factors for the matrix A .

$rowcnd$

If $info = 0$ or $info > m$, $rowcnd$ contains the ratio of the smallest $r(i)$ to the largest $r(i)$. If $rowcnd \geq 0.1$, and $amax$ is neither too large nor too small, it is not worth scaling by r .

$colcnd$

If $info = 0$, $colcnd$ contains the ratio of the smallest $c[i]$ to the largest $c[i]$. If $colcnd \geq 0.1$, it is not worth scaling by c .

$amax$

Absolute value of the largest element of the matrix A . If $amax$ is very close to `SMLNUM` or `BIGNUM`, the matrix should be scaled.

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, the i -th diagonal element of A is nonpositive.

$i \leq m$, the i -th row of A is exactly zero;

$i > m$, the $(i-m)$ -th column of A is exactly zero.

See Also

[Error Analysis](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?poequ

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive definite matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_spoequ( int matrix_layout, lapack_int n, const float* a, lapack_int lda, float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_dpoequ( int matrix_layout, lapack_int n, const double* a, lapack_int lda, double* s, double* scond, double* amax );
```

```
lapack_int LAPACKE_cpoequ( int matrix_layout, lapack_int n, const lapack_complex_float* a, lapack_int lda, float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_zpoequ( int matrix_layout, lapack_int n, const lapack_complex_double* a, lapack_int lda, double* s, double* scond, double* amax );
```

Include Files

- `mkl.h`

Description

The routine computes row and column scalings intended to equilibrate a symmetric (Hermitian) positive-definite matrix A and reduce its condition number (with respect to the two-norm). The output array s returns scale factors such that contains

$$\frac{1}{\sqrt{a_{i,i}}}$$

These factors are chosen so that the scaled matrix B with elements $B_{i,j}=s[i-1]*A_{i,j}*s[j-1]$ has diagonal elements equal to 1.

This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array: size $\max(1, lda*n)$. Contains the n -by- n symmetric or Hermitian positive definite matrix A whose scaling factors are to be computed. Only the diagonal elements of A are referenced. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |

Output Parameters

| | |
|--------------|--|
| <i>s</i> | Array, size n . If <i>info</i> = 0, the array s contains the scale factors for A . |
| <i>scond</i> | If <i>info</i> = 0, <i>scond</i> contains the ratio of the smallest $s[i]$ to the largest $s[i]$. |
| <i>amax</i> | Absolute value of the largest element of the matrix A . |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = $-i$, parameter i had an illegal value.

If *info* = i , the i -th diagonal element of A is nonpositive.

Application Notes

If $scond \geq 0.1$ and *amax* is neither too large nor too small, it is not worth scaling by s .

If *amax* is very close to SMLNUM or very close to BIGNUM, the matrix A should be scaled.

See Also

[Error Analysis](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?poequb

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive definite matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_spoequb( int matrix_layout, lapack_int n, const float* a, lapack_int lda, float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_dpoequb( int matrix_layout, lapack_int n, const double* a, lapack_int lda, double* s, double* scond, double* amax );
```

```
lapack_int LAPACKE_cpoequb( int matrix_layout, lapack_int n, const lapack_complex_float* a, lapack_int lda, float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_zpoequb( int matrix_layout, lapack_int n, const lapack_complex_double* a, lapack_int lda, double* s, double* scond, double* amax );
```

Include Files

- mkl.h

Description

The routine computes row and column scalings intended to equilibrate a symmetric (Hermitian) positive-definite matrix A and reduce its condition number (with respect to the two-norm).

These factors are chosen so that the scaled matrix B with elements $B_{i,j}=s[i-1]*A_{i,j}*s[j-1]$ has diagonal elements equal to 1. $s[i - 1]$ is a power of two nearest to, but not exceeding $1/\sqrt{A_{i,i}}$.

This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array: size $\max(1, lda*n)$. Contains the n -by- n symmetric or Hermitian positive definite matrix A whose scaling factors are to be computed. Only the diagonal elements of A are referenced. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, m)$. |

Output Parameters

| | |
|--------------|---|
| <i>s</i> | Array, size (n). If <i>info</i> = 0, the array s contains the scale factors for A . |
| <i>scond</i> | If <i>info</i> = 0, <i>scond</i> contains the ratio of the smallest $s[i]$ to the largest $s[i]$. If $scond \geq 0.1$, and <i>amax</i> is neither too large nor too small, it is not worth scaling by s . |

amax Absolute value of the largest element of the matrix *A*. If *amax* is very close to `SMLNUM` or `BIGNUM`, the matrix should be scaled.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

See Also

[Error Analysis](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?ppequ

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive definite matrix in packed storage and reduce its condition number.

Syntax

```
lapack_int LAPACKE_sppequ( int matrix_layout, char uplo, lapack_int n, const float* ap,
float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_dppequ( int matrix_layout, char uplo, lapack_int n, const double*
ap, double* s, double* scond, double* amax );
```

```
lapack_int LAPACKE_cppequ( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* ap, float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_zppequ( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* ap, double* s, double* scond, double* amax );
```

Include Files

- `mkl.h`

Description

The routine computes row and column scalings intended to equilibrate a symmetric (Hermitian) positive definite matrix *A* in packed storage and reduce its condition number (with respect to the two-norm). The output array *s* returns scale factors such that contains

$$s(i) = \frac{1}{\sqrt{a_{i,i}}}$$

These factors are chosen so that the scaled matrix *B* with elements $b_{ij}=s[i-1]*a_{ij}*s[j-1]$ has diagonal elements equal to 1.

This choice of *s* puts the condition number of *B* within a factor *n* of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of <i>A</i> is packed in the array <i>ap</i> : If <i>uplo</i> = 'U', the array <i>ap</i> stores the upper triangular part of the matrix <i>A</i> . If <i>uplo</i> = 'L', the array <i>ap</i> stores the lower triangular part of the matrix <i>A</i> . |
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>ap</i> | Array, size at least $\max(1, n(n+1)/2)$. The array <i>ap</i> contains the upper or the lower triangular part of the matrix <i>A</i> (as specified by <i>uplo</i>) in <i>packed storage</i> (see Matrix Storage Schemes). |

Output Parameters

| | |
|--------------|---|
| <i>s</i> | Array, size (<i>n</i>). If <i>info</i> = 0, the array <i>s</i> contains the scale factors for <i>A</i> . |
| <i>scond</i> | If <i>info</i> = 0, <i>scond</i> contains the ratio of the smallest <i>s</i> [<i>i</i>] to the largest <i>s</i> [<i>i</i>]. |
| <i>amax</i> | Absolute value of the largest element of the matrix <i>A</i> . |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

Application Notes

If $scond \geq 0.1$ and *amax* is neither too large nor too small, it is not worth scaling by *s*.

If *amax* is very close to SMLNUM or very close to BIGNUM, the matrix *A* should be scaled.

See Also

[Error Analysis](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?pbequ

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive-definite band matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_spbequ( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
const float* ab, lapack_int ldab, float* s, float* acond, float* amax );
```

```

lapack_int LAPACKE_dpbequ( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
const double* ab, lapack_int ldab, double* s, double* scond, double* amax );

lapack_int LAPACKE_cpbequ( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
const lapack_complex_float* ab, lapack_int ldab, float* s, float* scond, float* amax );

lapack_int LAPACKE_zpbequ( int matrix_layout, char uplo, lapack_int n, lapack_int kd,
const lapack_complex_double* ab, lapack_int ldab, double* s, double* scond, double*
amax );

```

Include Files

- mkl.h

Description

The routine computes row and column scalings intended to equilibrate a symmetric (Hermitian) positive definite band matrix A and reduce its condition number (with respect to the two-norm). The output array s returns scale factors such that contains

$$s(i) = \frac{1}{\sqrt{a_{i,i}}}$$

These factors are chosen so that the scaled matrix B with elements $b_{ij}=s[i-1]*a_{ij}*s[j-1]$ has diagonal elements equal to 1. This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored in the array ab : If <i>uplo</i> = 'U', the array ab stores the upper triangular part of the matrix A . If <i>uplo</i> = 'L', the array ab stores the lower triangular part of the matrix A . |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>kd</i> | The number of superdiagonals or subdiagonals in the matrix A ; $kd \geq 0$. |
| <i>ab</i> | Array, size $\max(1, ldab*n)$. The array ap contains either the upper or the lower triangular part of the matrix A (as specified by <i>uplo</i>) in <i>band storage</i> (see Matrix Storage Schemes). |
| <i>ldab</i> | The leading dimension of the array ab ; $ldab \geq kd + 1$. |

Output Parameters

| | |
|--------------|---|
| <i>s</i> | Array, size (<i>n</i>). If <i>info</i> = 0, the array <i>s</i> contains the scale factors for <i>A</i> . |
| <i>scond</i> | If <i>info</i> = 0, <i>scond</i> contains the ratio of the smallest <i>s</i> [<i>i</i>] to the largest <i>s</i> [<i>i</i>]. |
| <i>amax</i> | Absolute value of the largest element of the matrix <i>A</i> . |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

Application Notes

If *scond* ≥ 0.1 and *amax* is neither too large nor too small, it is not worth scaling by *s*.

If *amax* is very close to SMLNUM or very close to BIGNUM, the matrix *A* should be scaled.

See Also

[Error Analysis](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?syequb

Computes row and column scaling factors intended to equilibrate a symmetric indefinite matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_ssyequb( int matrix_layout, char uplo, lapack_int n, const float* a,
lapack_int lda, float* s, float* sccond, float* amax );
```

```
lapack_int LAPACKE_dsyequb( int matrix_layout, char uplo, lapack_int n, const double*
a, lapack_int lda, double* s, double* sccond, double* amax );
```

```
lapack_int LAPACKE_csyequb( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float* s, float* sccond, float* amax );
```

```
lapack_int LAPACKE_zsyequb( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double* s, double* sccond, double* amax );
```

Include Files

- mkl.h

Description

The routine computes row and column scalings intended to equilibrate a symmetric indefinite matrix *A* and reduce its condition number (with respect to the two-norm).

The array *s* contains the scale factors, $s[i-1] = 1/\sqrt{A(i,i)}$. These factors are chosen so that the scaled matrix *B* with elements $b_{i,j}=s[i-1]*a_{i,j}*s[j-1]$ has ones on the diagonal.

This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <i>uplo</i> = 'U', the array a stores the upper triangular part of the matrix A . If <i>uplo</i> = 'L', the array a stores the lower triangular part of the matrix A . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array a : $\max(1, lda * n)$. Contains the n -by- n symmetric indefinite matrix A whose scaling factors are to be computed. Only the diagonal elements of A are referenced. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, m)$. |

Output Parameters

| | |
|--------------|--|
| <i>s</i> | Array, size (n). If <i>info</i> = 0, the array s contains the scale factors for A . |
| <i>scond</i> | If <i>info</i> = 0, <i>scond</i> contains the ratio of the smallest $s[i]$ to the largest $s[i]$. If <i>scond</i> ≥ 0.1 , and <i>amax</i> is neither too large nor too small, it is not worth scaling by s . |
| <i>amax</i> | Absolute value of the largest element of the matrix A . If <i>amax</i> is very close to SMLNUM or BIGNUM, the matrix should be scaled. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = $-i$, parameter i had an illegal value.

If *info* = i , the i -th diagonal element of A is nonpositive.

See Also

[Error Analysis](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?heequb

Computes row and column scaling factors intended to equilibrate a Hermitian indefinite matrix and reduce its condition number.

Syntax

```
lapack_int LAPACKE_cheequb( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* a, lapack_int lda, float* s, float* scond, float* amax );
```

```
lapack_int LAPACKE_zheequb( int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* a, lapack_int lda, double* s, double* scond, double* amax );
```

Include Files

- mkl.h

Description

The routine computes row and column scalings intended to equilibrate a Hermitian indefinite matrix A and reduce its condition number (with respect to the two-norm).

The array s contains the scale factors, $s[i-1] = 1/\sqrt{a_{i,i}}$. These factors are chosen so that the scaled matrix B with elements $b_{i,j}=s[i-1]*a_{i,j}*s[j-1]$ has ones on the diagonal.

This choice of s puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <i>uplo</i> = 'U', the array a stores the upper triangular part of the matrix A . If <i>uplo</i> = 'L', the array a stores the lower triangular part of the matrix A . |
| <i>n</i> | The order of the matrix A ; $n \geq 0$. |
| <i>a</i> | Array a : size $\max(1, lda*n)$. Contains the n -by- n symmetric indefinite matrix A whose scaling factors are to be computed. Only the diagonal elements of A are referenced. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, m)$. |

Output Parameters

| | |
|--------------|---|
| <i>s</i> | Array, size (n). If <i>info</i> = 0, the array s contains the scale factors for A . |
| <i>scond</i> | If <i>info</i> = 0, <i>scond</i> contains the ratio of the smallest $s[i]$ to the largest $s[i]$. If $scond \geq 0.1$, and <i>amax</i> is neither too large nor too small, it is not worth scaling by s . |
| <i>amax</i> | Absolute value of the largest element of the matrix A . If <i>amax</i> is very close to SMLNUM or BIGNUM, the matrix should be scaled. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the *i*-th diagonal element of *A* is nonpositive.

See Also

Error Analysis

Matrix Storage Schemes for LAPACK Routines

LAPACK Linear Equation Driver Routines

Table "Driver Routines for Solving Systems of Linear Equations" lists the LAPACK driver routines for solving systems of linear equations with real or complex matrices.

Driver Routines for Solving Systems of Linear Equations

| Matrix type, storage scheme | Simple Driver | Expert Driver | Expert Driver using Extra-Precise Iterative Refinement |
|--|---------------------------|---------------|--|
| general | ?gesv | ?gesvx | ?gesvxx |
| general band | ?gbsv | ?gbsvx | ?gbsvxx |
| general tridiagonal | ?gtsv | ?gtsvx | |
| diagonally dominant tridiagonal | ?dtsvb | | |
| symmetric/Hermitian positive-definite | ?posv | ?posvx | ?posvxx |
| symmetric/Hermitian positive-definite, storage | ?ppsv | ?ppsvx | |
| symmetric/Hermitian positive-definite, band | ?pbsv | ?pbsvx | |
| symmetric/Hermitian positive-definite, tridiagonal | ?ptsv | ?ptsvx | |
| symmetric/Hermitian indefinite | ?sysv/?hesv ?sysv_rook | ?sysvx/?hesvx | ?sysvxx/?hesvxx |
| symmetric/Hermitian indefinite, packed storage | ?spsv/?hpsv | ?spsvx/?hpsvx | |
| complex symmetric | ?sysv ?sysv_rook | ?sysvx | |
| complex symmetric, packed storage | ?spsv | ?spsvx | |

In this table ? stands for s (single precision real), d (double precision real), c (single precision complex), or z (double precision complex). In the description of ?gesv and ?posv routines, the ? sign stands for combined character codes ds and zc for the mixed precision subroutines.

?gesv

Computes the solution to the system of linear equations with a square coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_sgesv (int matrix_layout , lapack_int n , lapack_int nrhs , float *
a , lapack_int lda , lapack_int * ipiv , float * b , lapack_int ldb );

lapack_int LAPACKE_dgesv (int matrix_layout , lapack_int n , lapack_int nrhs , double *
a , lapack_int lda , lapack_int * ipiv , double * b , lapack_int ldb );

lapack_int LAPACKE_cgesv (int matrix_layout , lapack_int n , lapack_int nrhs ,
lapack_complex_float * a , lapack_int lda , lapack_int * ipiv , lapack_complex_float *
b , lapack_int ldb );

lapack_int LAPACKE_zgesv (int matrix_layout , lapack_int n , lapack_int nrhs ,
lapack_complex_double * a , lapack_int lda , lapack_int * ipiv , lapack_complex_double
* b , lapack_int ldb );

lapack_int LAPACKE_dsgesv (int matrix_layout, lapack_int n, lapack_int nrhs, double *
a, lapack_int lda, lapack_int * ipiv, double * b, lapack_int ldb, double * x,
lapack_int ldx, lapack_int * iter);

lapack_int LAPACKE_zcgesv (int matrix_layout, lapack_int n, lapack_int nrhs,
lapack_complex_double * a, lapack_int lda, lapack_int * ipiv, lapack_complex_double *
b, lapack_int ldb, lapack_complex_double * x, lapack_int ldx, lapack_int * iter);
```

Include Files

- mkl.h

Description

The routine solves for X the system of linear equations $A*X = B$, where A is an n -by- n matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The LU decomposition with partial pivoting and row interchanges is used to factor A as $A = P*L*U$, where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. The factored form of A is then used to solve the system of equations $A*X = B$.

The `dsgesv` and `zcgesv` are mixed precision iterative refinement subroutines for exploiting fast single precision hardware. They first attempt to factorize the matrix in single precision (`dsgesv`) or single complex precision (`zcgesv`) and use this factorization within an iterative refinement procedure to produce a solution with double precision (`dsgesv`) / double complex precision (`zcgesv`) normwise backward error quality (see below). If the approach fails, the method switches to a double precision or double complex precision factorization respectively and computes the solution.

The iterative refinement is not going to be a winning strategy if the ratio single precision performance over double precision performance is too small. A reasonable strategy should take the number of right-hand sides and the size of the matrix into account. This might be done with a call to `ilaenv` in the future. At present, iterative refinement is implemented.

The iterative refinement process is stopped if

```
iter > itermax
```

or for all the right-hand sides:

```
rnmr < sqrt(n)*xnrm*anrm*eps*bwdmax
```

where

- *iter* is the number of the current iteration in the iterative refinement process
- *rnmr* is the infinity-norm of the residual
- *xnrm* is the infinity-norm of the solution
- *anrm* is the infinity-operator-norm of the matrix *A*
- *eps* is the machine epsilon returned by `dlamch` ('Epsilon').

The values *itermax* and *bwdmax* are fixed to 30 and 1.0d+00 respectively.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The number of linear equations, that is, the order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, that is, the number of columns of the matrix <i>B</i> ; $nrhs \geq 0$. |
| <i>a</i> | The array <i>a</i> (size $\max(1, lda * n)$) contains the <i>n</i> -by- <i>n</i> coefficient matrix <i>A</i> . |
| <i>b</i> | The array <i>b</i> of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the <i>n</i> -by- <i>nrhs</i> matrix of right hand side matrix <i>B</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of the array <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of the array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | Overwritten by the factors <i>L</i> and <i>U</i> from the factorization of $A = P * L * U$; the unit diagonal elements of <i>L</i> are not stored. If iterative refinement has been successfully used (<i>info</i> = 0 and <i>iter</i> \geq 0), then <i>A</i> is unchanged. If double precision factorization has been used (<i>info</i> = 0 and <i>iter</i> < 0), then the array <i>A</i> contains the factors <i>L</i> and <i>U</i> from the factorization $A = P * L * U$; the unit diagonal elements of <i>L</i> are not stored. |
| <i>b</i> | Overwritten by the solution matrix <i>X</i> for <code>dgesv</code> , <code>sgesv</code> , <code>zgesv</code> , <code>zgesv</code> . Unchanged for <code>dsgesv</code> and <code>zcgsv</code> . |

| | |
|-------------|---|
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The pivot indices that define the permutation matrix P ; row i of the matrix was interchanged with row $ipiv[i-1]$. Corresponds to the single precision factorization (if $info=0$ and $iter \geq 0$) or the double precision factorization (if $info=0$ and $iter < 0$). |
| <i>x</i> | Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout. If $info = 0$, contains the n -by- $nrhs$ solution matrix X . |
| <i>iter</i> | <p>If $iter < 0$: iterative refinement has failed, double precision factorization has been performed</p> <ul style="list-style-type: none"> • If $iter = -1$: the routine fell back to full precision for implementation- or machine-specific reason • If $iter = -2$: narrowing the precision induced an overflow, the routine fell back to full precision • If $iter = -3$: failure of <code>sgetrf</code> for <code>dsgesv</code>, or <code>cgetrf</code> for <code>zcgsv</code> • If $iter = -31$: stop the iterative refinement after the 30th iteration. <p>If $iter > 0$: iterative refinement has been successfully used. Returns the number of iterations.</p> |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, $U_{i, i}$ (computed in double precision for mixed precision subroutines) is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

See Also

[?lamch](#)

[?getrf](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?gesvx

Computes the solution to the system of linear equations with a square coefficient matrix A and multiple right-hand sides, and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_sgesvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, float* r, float* c, float* b, lapack_int ldb, float* x, lapack_int
ldx, float* rcond, float* ferr, float* berr, float* rpivot );
```

```
lapack_int LAPACKE_dgesvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, double* r, double* c, double* b, lapack_int ldb, double* x,
lapack_int ldx, double* rcond, double* ferr, double* berr, double* rpivot );
```

```
lapack_int LAPACKE_cgesvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, float* r, float* c,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr, float* rpivot );
```

```
lapack_int LAPACKE_zgesvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, double* r, double* c,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* ferr, double* berr, double* rpivot );
```

Include Files

- mkl.h

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations $A \cdot X = B$, where A is an n -by- n matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?gesvx` performs the following steps:

1. If `fact = 'E'`, real scaling factors r and c are computed to equilibrate the system:

$$\text{trans} = \text{'N'}: \text{diag}(r) * A * \text{diag}(c) * \text{inv}(\text{diag}(c)) * X = \text{diag}(r) * B$$

$$\text{trans} = \text{'T'}: (\text{diag}(r) * A * \text{diag}(c))^T * \text{inv}(\text{diag}(r)) * X = \text{diag}(c) * B$$

$$\text{trans} = \text{'C'}: (\text{diag}(r) * A * \text{diag}(c))^H * \text{inv}(\text{diag}(r)) * X = \text{diag}(c) * B$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(r) * A * \text{diag}(c)$ and B by $\text{diag}(r) * B$ (if `trans='N'`) or $\text{diag}(c) * B$ (if `trans = 'T' or 'C'`).

2. If `fact = 'N' or 'E'`, the *LU* decomposition is used to factor the matrix A (after equilibration if `fact = 'E'`) as $A = P * L * U$, where P is a permutation matrix, L is a unit lower triangular matrix, and U is upper triangular.
3. If some $U_{i,i} = 0$, so that U is exactly singular, then the routine returns with `info = i`. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, `info = n + 1` is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
4. The system of equations is solved for X using the factored form of A .
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $\text{diag}(c)$ (if `trans = 'N'`) or $\text{diag}(r)$ (if `trans = 'T' or 'C'`) so that it solves the original system before equilibration.

Input Parameters

`matrix_layout` Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

`fact` Must be 'F', 'N', or 'E'.

| | |
|--------------|--|
| | <p>Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If $fact = 'F'$: on entry, af and $ipiv$ contain the factored form of A. If $equed$ is not $'N'$, the matrix A has been equilibrated with scaling factors given by r and c. a, af, and $ipiv$ are not modified.</p> <p>If $fact = 'N'$, the matrix A will be copied to af and factored.</p> <p>If $fact = 'E'$, the matrix A will be equilibrated if necessary, then copied to af and factored.</p> |
| <i>trans</i> | <p>Must be $'N'$, $'T'$, or $'C'$.</p> <p>Specifies the form of the system of equations:</p> <p>If $trans = 'N'$, the system has the form $A * X = B$ (No transpose). If $trans = 'T'$, the system has the form $A^T * X = B$ (Transpose). If $trans = 'C'$, the system has the form $A^H * X = B$ (Transpose for real flavors, conjugate transpose for complex flavors).</p> |
| <i>n</i> | The number of linear equations; the order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right hand sides; the number of columns of the matrices B and X ; $nrhs \geq 0$. |
| <i>a</i> | The array a (size $\max(1, lda * n)$) contains the matrix A . If $fact = 'F'$ and $equed$ is not $'N'$, then A must have been equilibrated by the scaling factors in r and/or c . |
| <i>af</i> | The array af (size $\max(1, ldaf * n)$) is an input argument if $fact = 'F'$. It contains the factored form of the matrix A , that is, the factors L and U from the factorization $A = P * L * U$ as computed by <code>?getrf</code> . If $equed$ is not $'N'$, then af is the factored form of the equilibrated matrix A . |
| <i>b</i> | The array b of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of af ; $ldaf \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The array $ipiv$ is an input argument if $fact = 'F'$. It contains the pivot indices from the factorization $A = P * L * U$ as computed by <code>?getrf</code> ; row i of the matrix was interchanged with row $ipiv[i-1]$. |
| <i>equed</i> | Must be $'N'$, $'R'$, $'C'$, or $'B'$. |

equed is an input argument if *fact* = 'F'. It specifies the form of equilibration that was done:

If *equed* = 'N', no equilibration was done (always true if *fact* = 'N').

If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by *diag(r)*.

If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by *diag(c)*.

If *equed* = 'B', both row and column equilibration was done, that is, *A* has been replaced by *diag(r)*A*diag(c)*.

r, c

Arrays: *r* (size *n*), *c* (size *n*). The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*. These arrays are input arguments if *fact* = 'F' only; otherwise they are output arguments.

If *equed* = 'R' or 'B', *A* is multiplied on the left by *diag(r)*; if *equed* = 'N' or 'C', *r* is not accessed.

If *fact* = 'F' and *equed* = 'R' or 'B', each element of *r* must be positive.

If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag(c)*; if *equed* = 'N' or 'R', *c* is not accessed.

If *fact* = 'F' and *equed* = 'C' or 'B', each element of *c* must be positive.

ldx

The leading dimension of the output array *x*; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

x

Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.

If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the *original* system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the *equilibrated* system is:

$diag(C)^{-1} * X$, if *trans* = 'N' and *equed* = 'C' or 'B';
 $diag(R)^{-1} * X$, if *trans* = 'T' or 'C' and *equed* = 'R' or 'B'. The second dimension of *x* must be at least $\max(1, nrhs)$.

a

Array *a* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'. If *equed* ≠ 'N', *A* is scaled on exit as follows:

equed = 'R': $A = diag(R) * A$

equed = 'C': $A = A * diag(c)$

equed = 'B': $A = diag(R) * A * diag(c)$.

| | |
|---------------|--|
| <i>af</i> | If <i>fact</i> = 'N' or 'E', then <i>af</i> is an output argument and on exit returns the factors <i>L</i> and <i>U</i> from the factorization $A = PLU$ of the original matrix <i>A</i> (if <i>fact</i> = 'N') or of the equilibrated matrix <i>A</i> (if <i>fact</i> = 'E'). See the description of <i>a</i> for the form of the equilibrated matrix. |
| <i>b</i> | Overwritten by $diag(r)*B$ if <i>trans</i> = 'N' and <i>equed</i> = 'R' or 'B'; overwritten by $diag(c)*B$ if <i>trans</i> = 'T' or 'C' and <i>equed</i> = 'C' or 'B'; not changed if <i>equed</i> = 'N'. |
| <i>r, c</i> | These arrays are output arguments if <i>fact</i> ≠ 'F'. See the description of <i>r, c</i> in <i>Input Arguments</i> section. |
| <i>rcond</i> | An estimate of the reciprocal condition number of the matrix <i>A</i> after equilibration (if done). If <i>rcond</i> is less than the machine precision, in particular, if <i>rcond</i> = 0, the matrix is singular to working precision. This condition is indicated by a return code of <i>info</i> > 0. |
| <i>ferr</i> | Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the <i>j</i> -th column of the solution matrix <i>X</i>). If <i>xtrue</i> is the true solution corresponding to x_j , <i>ferr</i> [<i>j</i> -1] is an estimated upper bound for the magnitude of the largest element in $(x_j - xtrue)$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for <i>rcond</i> , and is almost always a slight overestimate of the true error. |
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of <i>A</i> or <i>B</i> that makes x_j an exact solution. |
| <i>ipiv</i> | If <i>fact</i> = 'N' or 'E', then <i>ipiv</i> is an output argument and on exit contains the pivot indices from the factorization $A = P*L*U$ of the original matrix <i>A</i> (if <i>fact</i> = 'N') or of the equilibrated matrix <i>A</i> (if <i>fact</i> = 'E'). |
| <i>equed</i> | If <i>fact</i> ≠ 'F', then <i>equed</i> is an output argument. It specifies the form of equilibration that was done (see the description of <i>equed</i> in <i>Input Arguments</i> section). |
| <i>rpivot</i> | On exit, <i>rpivot</i> contains the reciprocal pivot growth factor: $\ A\ /\ U\ $ <p>If <i>rpivot</i> is much less than 1, then the stability of the <i>LU</i> factorization of the (equilibrated) matrix <i>A</i> could be poor. This also means that the solution <i>x</i>, condition estimator <i>rcond</i>, and forward error bound <i>ferr</i> could be unreliable. If factorization fails with $0 < info \leq n$, then <i>rpivot</i> contains the reciprocal pivot growth factor for the leading <i>info</i> columns of <i>A</i>.</p> |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, and $i \leq n$, then $U(i, i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; $rcond = 0$ is returned.

If *info* = $n + 1$, then U is nonsingular, but $rcond$ is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of $rcond$ would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gesvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a square coefficient matrix A and multiple right-hand sides

Syntax

```
lapack_int LAPACKE_sgesvxx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, float* r, float* c, float* b, lapack_int ldb, float* x, lapack_int
ldx, float* rcond, float* rpvgrw, float* berr, lapack_int n_err_bnds, float*
err_bnds_norm, float* err_bnds_comp, lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_dgesvxx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, double* r, double* c, double* b, lapack_int ldb, double* x,
lapack_int ldx, double* rcond, double* rpvgrw, double* berr, lapack_int n_err_bnds,
double* err_bnds_norm, double* err_bnds_comp, lapack_int nparams, const double*
params );
```

```
lapack_int LAPACKE_cgesvxx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, float* r, float* c,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* rpvgrw, float* berr, lapack_int n_err_bnds, float* err_bnds_norm,
float* err_bnds_comp, lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_zgesvxx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, double* r, double* c,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* rpvgrw, double* berr, lapack_int n_err_bnds, double*
err_bnds_norm, double* err_bnds_comp, lapack_int nparams, const double* params );
```

Include Files

- mkl.h

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations $A*X = B$, where A is an n -by- n matrix, the columns of the matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error ($O(\text{eps})$, where eps is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with $O(\text{eps})$ errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine `?gesvxx` performs the following steps:

1. If *fact* = 'E', scaling factors r and c are computed to equilibrate the system:

$$\text{trans} = \text{'N'}: \text{diag}(r) * A * \text{diag}(c) * \text{inv}(\text{diag}(c)) * X = \text{diag}(r) * B$$

$$\text{trans} = \text{'T'}: (\text{diag}(r) * A * \text{diag}(c))^T * \text{inv}(\text{diag}(r)) * X = \text{diag}(c) * B$$

$$\text{trans} = \text{'C'}: (\text{diag}(r) * A * \text{diag}(c))^H * \text{inv}(\text{diag}(r)) * X = \text{diag}(c) * B$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(r) * A * \text{diag}(c)$ and B by $\text{diag}(r) * B$ (if *trans*='N') or $\text{diag}(c) * B$ (if *trans* = 'T' or 'C').

2. If *fact* = 'N' or 'E', the *LU* decomposition is used to factor the matrix A (after equilibration if *fact* = 'E') as $A = P * L * U$, where P is a permutation matrix, L is a unit lower triangular matrix, and U is upper triangular.
3. If some $U_{i,i} = 0$, so that U is exactly singular, then the routine returns with *info* = i . Otherwise, the factored form of A is used to estimate the condition number of the matrix A (see the *rcond* parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for X and compute error bounds.
4. The system of equations is solved for X using the factored form of A .
5. By default, unless is set to zero, the routine applies iterative refinement to improve the computed solution matrix and calculate error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix X is premultiplied by $\text{diag}(c)$ (if *trans* = 'N') or $\text{diag}(r)$ (if *trans* = 'T' or 'C') so that it solves the original system before equilibration.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

fact Must be 'F', 'N', or 'E'.

Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.

If *fact* = 'F', on entry, *af* and *ipiv* contain the factored form of A . If *equed* is not 'N', the matrix A has been equilibrated with scaling factors given by r and c . Parameters a , *af*, and *ipiv* are not modified.

If *fact* = 'N', the matrix A will be copied to *af* and factored.

If *fact* = 'E', the matrix A will be equilibrated, if necessary, copied to *af* and factored.

| | |
|-----------------|---|
| <i>trans</i> | <p>Must be 'N', 'T', or 'C'.</p> <p>Specifies the form of the system of equations:</p> <p>If <i>trans</i> = 'N', the system has the form $A^*X = B$ (No transpose).</p> <p>If <i>trans</i> = 'T', the system has the form $A^T X = B$ (Transpose).</p> <p>If <i>trans</i> = 'C', the system has the form $A^H X = B$ (Conjugate Transpose = Transpose for real flavors, Conjugate Transpose for complex flavors).</p> |
| <i>n</i> | The number of linear equations; the order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right hand sides; the number of columns of the matrices <i>B</i> and <i>X</i> ; $nrhs \geq 0$. |
| <i>a, af, b</i> | <p>Arrays: <i>a</i>(size $\max(lda*n)$), <i>af</i>(size $\max(ldaf*n)$), <i>b</i>(size $\max(1, ldb*nrhs)$) for column major layout and $\max(1, ldb*n)$ for row major layout).</p> <p>The array <i>a</i> contains the matrix <i>A</i>. If <i>fact</i> = 'F' and <i>equed</i> is not 'N', then <i>A</i> must have been equilibrated by the scaling factors in <i>r</i> and/or <i>c</i>.</p> <p>The array <i>af</i> is an input argument if <i>fact</i> = 'F'. It contains the factored form of the matrix <i>A</i>, that is, the factors <i>L</i> and <i>U</i> from the factorization $A = P*L*U$ as computed by <code>?getrf</code>. If <i>equed</i> is not 'N', then <i>af</i> is the factored form of the equilibrated matrix <i>A</i>.</p> <p>The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations.</p> |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. It contains the pivot indices from the factorization $A = P*L*U$ as computed by <code>?getrf</code> ; row <i>i</i> of the matrix was interchanged with row <i>ipiv</i> [<i>i</i> -1]. |
| <i>equed</i> | <p>Must be 'N', 'R', 'C', or 'B'.</p> <p><i>equed</i> is an input argument if <i>fact</i> = 'F'. It specifies the form of equilibration that was done:</p> <p>If <i>equed</i> = 'N', no equilibration was done (always true if <i>fact</i> = 'N').</p> <p>If <i>equed</i> = 'R', row equilibration was done, that is, <i>A</i> has been premultiplied by <i>diag</i>(<i>r</i>).</p> <p>If <i>equed</i> = 'C', column equilibration was done, that is, <i>A</i> has been postmultiplied by <i>diag</i>(<i>c</i>).</p> <p>If <i>equed</i> = 'B', both row and column equilibration was done, that is, <i>A</i> has been replaced by <i>diag</i>(<i>r</i>)*<i>A</i>*<i>diag</i>(<i>c</i>).</p> |
| <i>r, c</i> | Arrays: <i>r</i> (size <i>n</i>), <i>c</i> (size <i>n</i>). The array <i>r</i> contains the row scale factors for <i>A</i> , and the array <i>c</i> contains the column scale factors for <i>A</i> . These arrays are input arguments if <i>fact</i> = 'F' only; otherwise they are output arguments. |

If *equed* = 'R' or 'B', *A* is multiplied on the left by *diag(r)*; if *equed* = 'N' or 'C', *r* is not accessed.

If *fact* = 'F' and *equed* = 'R' or 'B', each element of *r* must be positive.

If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag(c)*; if *equed* = 'N' or 'R', *c* is not accessed.

If *fact* = 'F' and *equed* = 'C' or 'B', each element of *c* must be positive.

Each element of *r* or *c* should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb The leading dimension of the array *b*; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

ldx The leading dimension of the output array *x*; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

n_err_bnds Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in *Output Arguments* section below.

nparams Specifies the number of parameters set in *params*. If ≤ 0 , the *params* array is never referenced and default values are used.

params Array, size $\max(1, nparams)$. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass *nparams* = 0, which prevents the source code from accessing the *params* argument.

params[0] : Whether to perform iterative refinement or not. Default: 1.0

=0.0 No refinement is performed and no error bounds are computed.

=1.0 Use the double-precision refinement algorithm, possibly with doubled-single computations if the compilation environment does not support double precision.

(Other values are reserved for future use.)

params[1] : Maximum number of residual computations allowed for refinement.

Default 10.0

Aggressive Set to 100.0 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a

technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

Output Parameters

- x* Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.
 If *info* = 0, the array *x* contains the solution *n*-by-*nrhs* matrix *X* to the *original* system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the *equilibrated* system is:

$$\text{inv}(\text{diag}(c)) * X, \text{ if } \text{trans} = \text{'N'} \text{ and } \text{equed} = \text{'C'} \text{ or } \text{'B'}; \text{ or}$$

$$\text{inv}(\text{diag}(r)) * X, \text{ if } \text{trans} = \text{'T'} \text{ or } \text{'C'} \text{ and } \text{equed} = \text{'R'} \text{ or } \text{'B'}.$$
- a* Array *a* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'.
 If *equed* ≠ 'N', *A* is scaled on exit as follows:

$$\text{equed} = \text{'R'}: A = \text{diag}(r) * A$$

$$\text{equed} = \text{'C'}: A = A * \text{diag}(c)$$

$$\text{equed} = \text{'B'}: A = \text{diag}(r) * A * \text{diag}(c).$$
- af* If *fact* = 'N' or 'E', then *af* is an output argument and on exit returns the factors *L* and *U* from the factorization $A = PLU$ of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *a* for the form of the equilibrated matrix.
- b* Overwritten by $\text{diag}(r) * B$ if *trans* = 'N' and *equed* = 'R' or 'B';
 overwritten by *trans* = 'T' or 'C' and *equed* = 'C' or 'B';
 not changed if *equed* = 'N'.
- r, c* These arrays are output arguments if *fact* ≠ 'F'. Each element of these arrays is a power of the radix. See the description of *r, c* in *Input Arguments* section.
- rcond* Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.
- rpvgrw* Contains the reciprocal pivot growth factor:

$$\|A\|/\|U\|$$

If this is much less than 1, the stability of the *LU* factorization of the (equilibrated) matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < info \leq n$, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of *A*. In *?gesvx*, this quantity is returned in *rpivot*.

berr Array, size at least $\max(1, nrhs)$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of *A* or *B* that makes x_j an exact solution.

err_bnds_norm Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:

Normwise relative error in the *i*-th solution vector

$$\frac{\max_j |X_{true_{ji}} - X_{ji}|}{\max_j |X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix *Z* are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * a$, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error err is stored in $err_bnds_norm[(err-1)*nrhs + i - 1]$.

err_bnds_comp

Array of size $nrhs*n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the i -th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested ($params[2] = 0.0$), then *err_bnds_comp* is not accessed.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error err is stored in $err_bnds_comp[(err-1)*nrhs + i - 1]$.

| | |
|---------------|---|
| <i>ipiv</i> | If <i>fact</i> = 'N' or 'E', then <i>ipiv</i> is an output argument and on exit contains the pivot indices from the factorization $A = P*L*U$ of the original matrix <i>A</i> (if <i>fact</i> = 'N') or of the equilibrated matrix <i>A</i> (if <i>fact</i> = 'E'). |
| <i>equed</i> | If <i>fact</i> ≠ 'F', then <i>equed</i> is an output argument. It specifies the form of equilibration that was done (see the description of <i>equed</i> in <i>Input Arguments</i> section). |
| <i>params</i> | If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.

If *info* = -*i*, parameter *i* had an illegal value.

If $0 < info \leq n$: $U_{info,info}$ is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.

If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params*[2] = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that for column major layout $err_bnds_norm[j - 1] = 0.0$ or $err_bnds_comp[j - 1] = 0.0$; or for row major layout $err_bnds_norm[(j - 1)*n_err_bnds] = 0.0$ or $err_bnds_comp[(j - 1)*n_err_bnds] = 0.0$). See the definition of *err_bnds_norm* and *err_bnds_comp* for *err* = 1. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gbsv

Computes the solution to the system of linear equations with a band coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_sgbsv (int matrix_layout , lapack_int n , lapack_int kl , lapack_int ku , lapack_int nrhs , float * ab , lapack_int ldab , lapack_int * ipiv , float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dgbsv (int matrix_layout , lapack_int n , lapack_int kl , lapack_int ku , lapack_int nrhs , double * ab , lapack_int ldab , lapack_int * ipiv , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_cgbsv (int matrix_layout , lapack_int n , lapack_int kl , lapack_int ku , lapack_int nrhs , lapack_complex_float * ab , lapack_int ldab , lapack_int * ipiv , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zgbsv (int matrix_layout , lapack_int n , lapack_int kl , lapack_int ku , lapack_int nrhs , lapack_complex_double * ab , lapack_int ldab , lapack_int * ipiv , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mk1.h`

Description

The routine solves for X the real or complex system of linear equations $A^*X = B$, where A is an n -by- n band matrix with kl subdiagonals and ku superdiagonals, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The LU decomposition with partial pivoting and row interchanges is used to factor A as $A = L^*U$, where L is a product of permutation and unit lower triangular matrices with kl subdiagonals, and U is upper triangular with $kl+ku$ superdiagonals. The factored form of A is then used to solve the system of equations $A^*X = B$.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of A . The number of rows in B ; $n \geq 0$. |
| <i>kl</i> | The number of subdiagonals within the band of A ; $kl \geq 0$. |
| <i>ku</i> | The number of superdiagonals within the band of A ; $ku \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides. The number of columns in B ; $nrhs \geq 0$. |
| <i>ab, b</i> | Arrays: <i>ab</i> (size $\max(1, ldab*n)$), <i>bof</i> size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. The array <i>ab</i> contains the matrix A in band storage (see Matrix Storage Schemes). The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> . ($ldab \geq 2kl + ku + 1$) |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-------------|--|
| <i>ab</i> | Overwritten by L and U . U is stored as an upper triangular band matrix with $kl + ku$ superdiagonals and L is stored as a lower triangular band matrix with kl subdiagonals. See Matrix Storage Schemes . |
| <i>b</i> | Overwritten by the solution matrix X . |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The pivot indices: row i was interchanged with row $ipiv[i-1]$. |

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, $U_{i,i}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gbsvx

Computes the solution to the real or complex system of linear equations with a band coefficient matrix A and multiple right-hand sides, and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_sgbsvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, float* ab, lapack_int ldab, float* afb,
lapack_int ldafb, lapack_int* ipiv, char* equed, float* r, float* c, float* b,
lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* ferr, float* berr,
float* rpivot );
```

```
lapack_int LAPACKE_dgbsvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, double* ab, lapack_int ldab, double*
afb, lapack_int ldafb, lapack_int* ipiv, char* equed, double* r, double* c, double* b,
lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr,
double* rpivot );
```

```
lapack_int LAPACKE_cgbsvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, lapack_complex_float* ab, lapack_int
ldab, lapack_complex_float* afb, lapack_int ldafb, lapack_int* ipiv, char* equed,
float* r, float* c, lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* ferr, float* berr, float* rpivot );
```

```
lapack_int LAPACKE_zgbsvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, lapack_complex_double* ab, lapack_int
ldab, lapack_complex_double* afb, lapack_int ldafb, lapack_int* ipiv, char* equed,
double* r, double* c, lapack_complex_double* b, lapack_int ldb, lapack_complex_double*
x, lapack_int ldx, double* rcond, double* ferr, double* berr, double* rpivot );
```

Include Files

- mkl.h

Description

The routine uses the LU factorization to compute the solution to a real or complex system of linear equations $A * X = B$, $A^T * X = B$, or $A^H * X = B$, where A is a band matrix of order n with kl subdiagonals and ku superdiagonals, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine ?gbsvx performs the following steps:

1. If $fact = 'E'$, real scaling factors r and c are computed to equilibrate the system:

$$trans = 'N': \text{diag}(r) * A * \text{diag}(c) * \text{inv}(\text{diag}(c)) * X = \text{diag}(r) * B$$

$$trans = 'T': (\text{diag}(r) * A * \text{diag}(c))^T * \text{inv}(\text{diag}(r)) * X = \text{diag}(c) * B$$

$$trans = 'C': (\text{diag}(r) * A * \text{diag}(c))^H * \text{inv}(\text{diag}(r)) * X = \text{diag}(c) * B$$

Whether the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(r)*A*\text{diag}(c)$ and B by $\text{diag}(r)*B$ (if $\text{trans} = 'N'$) or $\text{diag}(c)*B$ (if $\text{trans} = 'T'$ or $'C'$).

2. If $\text{fact} = 'N'$ or $'E'$, the LU decomposition is used to factor the matrix A (after equilibration if $\text{fact} = 'E'$) as $A = L*U$, where L is a product of permutation and unit lower triangular matrices with kl subdiagonals, and U is upper triangular with $kl+ku$ superdiagonals.
3. If some $U_{i,i} = 0$, so that U is exactly singular, then the routine returns with $\text{info} = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, $\text{info} = n + 1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
4. The system of equations is solved for X using the factored form of A .
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $\text{diag}(c)$ (if $\text{trans} = 'N'$) or $\text{diag}(r)$ (if $\text{trans} = 'T'$ or $'C'$) so that it solves the original system before equilibration.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>fact</i> | <p>Must be 'F', 'N', or 'E'.</p> <p>Specifies whether the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If $\text{fact} = 'F'$: on entry, afb and ipiv contain the factored form of A. If equed is not 'N', the matrix A is equilibrated with scaling factors given by r and c. ab, afb, and ipiv are not modified.</p> <p>If $\text{fact} = 'N'$, the matrix A will be copied to afb and factored.</p> <p>If $\text{fact} = 'E'$, the matrix A will be equilibrated if necessary, then copied to afb and factored.</p> |
| <i>trans</i> | <p>Must be 'N', 'T', or 'C'.</p> <p>Specifies the form of the system of equations:</p> <p>If $\text{trans} = 'N'$, the system has the form $A*X = B$ (No transpose).</p> <p>If $\text{trans} = 'T'$, the system has the form $A^T*X = B$ (Transpose).</p> <p>If $\text{trans} = 'C'$, the system has the form $A^H*X = B$ (Transpose for real flavors, conjugate transpose for complex flavors).</p> |
| <i>n</i> | The number of linear equations, the order of the matrix A ; $n \geq 0$. |
| <i>kl</i> | The number of subdiagonals within the band of A ; $kl \geq 0$. |
| <i>ku</i> | The number of superdiagonals within the band of A ; $ku \geq 0$. |
| <i>nrhs</i> | The number of right hand sides, the number of columns of the matrices B and X ; $\text{nrhs} \geq 0$. |

| | |
|-------------------|---|
| <i>ab, afb, b</i> | <p>Arrays: <i>ab</i> ($\max(\text{ldab} * n)$), <i>afb</i> ($\max(\text{ldafb} * n)$), <i>b</i> ($\max(1, \text{ldb} * \text{nrhs})$) for column major layout and $\max(1, \text{ldb} * n)$ for row major layout).</p> <p>The array <i>ab</i> contains the matrix <i>A</i> in band storage (see Matrix Storage Schemes). If <i>fact</i> = 'F' and <i>equed</i> is not 'N', then <i>A</i> must have been equilibrated by the scaling factors in <i>r</i> and/or <i>c</i>.</p> <p>The array <i>afb</i> is an input argument if <i>fact</i> = 'F'. It contains the factored form of the matrix <i>A</i>, that is, the factors <i>L</i> and <i>U</i> from the factorization $A = P * L * U$ as computed by <code>?gbtrf</code>. <i>U</i> is stored as an upper triangular band matrix with <i>kl</i> + <i>ku</i> superdiagonals. <i>L</i> is stored as lower triangular band matrix with <i>kl</i> subdiagonals. If <i>equed</i> is not 'N', then <i>afb</i> is the factored form of the equilibrated matrix <i>A</i>.</p> <p>The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations.</p> |
| <i>ldab</i> | The leading dimension of <i>ab</i> ; $\text{ldab} \geq \text{kl} + \text{ku} + 1$. |
| <i>ldafb</i> | The leading dimension of <i>afb</i> ; $\text{ldafb} \geq 2 * \text{kl} + \text{ku} + 1$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $\text{ldb} \geq \max(1, n)$ for column major layout and $\text{ldb} \geq \text{nrhs}$ for row major layout. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. It contains the pivot indices from the factorization $A = P * L * U$ as computed by <code>?gbtrf</code> ; row <i>i</i> of the matrix was interchanged with row <i>ipiv</i> [<i>i</i> -1]. |
| <i>equed</i> | <p>Must be 'N', 'R', 'C', or 'B'.</p> <p><i>equed</i> is an input argument if <i>fact</i> = 'F'. It specifies the form of equilibration that was done:</p> <p>If <i>equed</i> = 'N', no equilibration was done (always true if <i>fact</i> = 'N').</p> <p>If <i>equed</i> = 'R', row equilibration was done, that is, <i>A</i> has been premultiplied by <i>diag</i>(<i>r</i>).</p> <p>If <i>equed</i> = 'C', column equilibration was done, that is, <i>A</i> has been postmultiplied by <i>diag</i>(<i>c</i>).</p> <p>If <i>equed</i> = 'B', both row and column equilibration was done, that is, <i>A</i> has been replaced by <i>diag</i>(<i>r</i>) * <i>A</i> * <i>diag</i>(<i>c</i>).</p> |
| <i>r, c</i> | <p>Arrays: <i>r</i> (size <i>n</i>), <i>c</i> (size <i>n</i>).</p> <p>The array <i>r</i> contains the row scale factors for <i>A</i>, and the array <i>c</i> contains the column scale factors for <i>A</i>. These arrays are input arguments if <i>fact</i> = 'F' only; otherwise they are output arguments.</p> <p>If <i>equed</i> = 'R' or 'B', <i>A</i> is multiplied on the left by <i>diag</i>(<i>r</i>); if <i>equed</i> = 'N' or 'C', <i>r</i> is not accessed.</p> <p>If <i>fact</i> = 'F' and <i>equed</i> = 'R' or 'B', each element of <i>r</i> must be positive.</p> <p>If <i>equed</i> = 'C' or 'B', <i>A</i> is multiplied on the right by <i>diag</i>(<i>c</i>); if <i>equed</i> = 'N' or 'R', <i>c</i> is not accessed.</p> |

If $fact = 'F'$ and $equed = 'C'$ or $'B'$, each element of c must be positive.

ldx

The leading dimension of the output array x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

x

Array, size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout.

If $info = 0$ or $info = n+1$, the array x contains the solution matrix X to the *original* system of equations. Note that A and B are modified on exit if $equed \neq 'N'$, and the solution to the *equilibrated* system is: $inv(diag(c)) * X$, if $trans = 'N'$ and $equed = 'C'$ or $'B'$; $inv(diag(r)) * X$, if $trans = 'T'$ or $'C'$ and $equed = 'R'$ or $'B'$.

ab

Array ab is not modified on exit if $fact = 'F'$ or $'N'$, or if $fact = 'E'$ and $equed = 'N'$.

If $equed \neq 'N'$, A is scaled on exit as follows:

$equed = 'R'$: $A = diag(r) * A$

$equed = 'C'$: $A = A * diag(c)$

$equed = 'B'$: $A = diag(r) * A * diag(c)$.

afb

If $fact = 'N'$ or $'E'$, then afb is an output argument and on exit returns details of the *LU* factorization of the original matrix A (if $fact = 'N'$) or of the equilibrated matrix A (if $fact = 'E'$). See the description of ab for the form of the equilibrated matrix.

b

Overwritten by $diag(r) * b$ if $trans = 'N'$ and $equed = 'R'$ or $'B'$; overwritten by $diag(c) * b$ if $trans = 'T'$ or $'C'$ and $equed = 'C'$ or $'B'$; not changed if $equed = 'N'$.

r, c

These arrays are output arguments if $fact \neq 'F'$. See the description of r, c in *Input Arguments* section.

$rcond$

An estimate of the reciprocal condition number of the matrix A after equilibration (if done).

If $rcond$ is less than the machine precision (in particular, if $rcond = 0$), the matrix is singular to working precision. This condition is indicated by a return code of $info > 0$.

$ferr$

Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the j -th column of the solution matrix X). If $xtrue$ is the true solution corresponding to x_j , $ferr[j-1]$ is an estimated upper bound for the magnitude of the largest element in $(x_j - xtrue)$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for $rcond$, and is almost always a slight overestimate of the true error.

| | |
|---------------|---|
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution. |
| <i>ipiv</i> | If <i>fact</i> = 'N' or 'E', then <i>ipiv</i> is an output argument and on exit contains the pivot indices from the factorization $A = L*U$ of the original matrix A (if <i>fact</i> = 'N') or of the equilibrated matrix A (if <i>fact</i> = 'E'). |
| <i>equed</i> | If <i>fact</i> ≠ 'F', then <i>equed</i> is an output argument. It specifies the form of equilibration that was done (see the description of <i>equed</i> in <i>Input Arguments</i> section). |
| <i>rpivot</i> | On exit, <i>rpivot</i> contains the reciprocal pivot growth factor: <div style="text-align: center;"> $\ A\ /\ U\$ </div> <p>If <i>rpivot</i> is much less than 1, then the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution x, condition estimator <i>rcond</i>, and forward error bound <i>ferr</i> could be unreliable. If factorization fails with $0 < info \leq n$, then <i>rpivot</i> contains the reciprocal pivot growth factor for the leading <i>info</i> columns of A.</p> |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, and $i \leq n$, then $U_{i,i}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and $i = n+1$, then U is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gbsvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a banded coefficient matrix A and multiple right-hand sides

Syntax

```
lapack_int LAPACKE_sgbsvxx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, float* ab, lapack_int ldab, float* afb,
lapack_int ldafb, lapack_int* ipiv, char* equed, float* r, float* c, float* b,
lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* rpvgrw, float* berr,
lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams,
const float* params );
```

```
lapack_int LAPACKE_dgbsvxx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, double* ab, lapack_int ldab, double*
afb, lapack_int ldafb, lapack_int* ipiv, char* equed, double* r, double* c, double* b,
lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* rpvgrw, double*
berr, lapack_int n_err_bnds, double* err_bnds_norm, double* err_bnds_comp, lapack_int
nparams, const double* params );
```

```
lapack_int LAPACKE_cgbsvxx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, lapack_complex_float* ab, lapack_int
ldab, lapack_complex_float* afb, lapack_int ldafb, lapack_int* ipiv, char* equed,
float* r, float* c, lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* rpvgrw, float* berr, lapack_int n_err_bnds,
float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_zgbsvxx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int kl, lapack_int ku, lapack_int nrhs, lapack_complex_double* ab, lapack_int
ldab, lapack_complex_double* afb, lapack_int ldafb, lapack_int* ipiv, char* equed,
double* r, double* c, lapack_complex_double* b, lapack_int ldb, lapack_complex_double*
x, lapack_int ldx, double* rcond, double* rpvgrw, double* berr, lapack_int n_err_bnds,
double* err_bnds_norm, double* err_bnds_comp, lapack_int nparams, const double*
params );
```

Include Files

- mkl.h

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations $A*X = B$, $A^T*X = B$, or $A^H*X = B$, where A is an n -by- n banded matrix, the columns of the matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error ($O(\text{eps})$, where eps is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with $O(\text{eps})$ errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine `?gbsvxx` performs the following steps:

1. If *fact* = 'E', scaling factors r and c are computed to equilibrate the system:

$$\text{trans} = \text{'N'}: \text{diag}(r) * A * \text{diag}(c) * \text{inv}(\text{diag}(c)) * X = \text{diag}(r) * B$$

$$\text{trans} = \text{'T'}: (\text{diag}(r) * A * \text{diag}(c))^T * \text{inv}(\text{diag}(r)) * X = \text{diag}(c) * B$$

$$\text{trans} = \text{'C'}: (\text{diag}(r) * A * \text{diag}(c))^H * \text{inv}(\text{diag}(r)) * X = \text{diag}(c) * B$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(r) * A * \text{diag}(c)$ and B by $\text{diag}(r) * B$ (if *trans*='N') or $\text{diag}(c) * B$ (if *trans* = 'T' or 'C').

2. If *fact* = 'N' or 'E', the *LU* decomposition is used to factor the matrix A (after equilibration if *fact* = 'E') as $A = P * L * U$, where P is a permutation matrix, L is a unit lower triangular matrix, and U is upper triangular.

3. If some $U_{i,i} = 0$, so that U is exactly singular, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A (see the `rcond` parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for X and compute error bounds.
4. The system of equations is solved for X using the factored form of A .
5. By default, unless `params[0]` is set to zero, the routine applies iterative refinement to improve the computed solution matrix and calculate error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix X is premultiplied by $diag(c)$ (if `trans = 'N'`) or $diag(r)$ (if `trans = 'T'` or `'C'`) so that it solves the original system before equilibration.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether two-dimensional array storage is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>fact</code> | <p>Must be 'F', 'N', or 'E'.</p> <p>Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If <code>fact = 'F'</code>, on entry, <code>afb</code> and <code>ipiv</code> contain the factored form of A. If <code>equed</code> is not 'N', the matrix A has been equilibrated with scaling factors given by <code>r</code> and <code>c</code>. Parameters <code>ab</code>, <code>afb</code>, and <code>ipiv</code> are not modified.</p> <p>If <code>fact = 'N'</code>, the matrix A will be copied to <code>afb</code> and factored.</p> <p>If <code>fact = 'E'</code>, the matrix A will be equilibrated, if necessary, copied to <code>afb</code> and factored.</p> |
| <code>trans</code> | <p>Must be 'N', 'T', or 'C'.</p> <p>Specifies the form of the system of equations:</p> <p>If <code>trans = 'N'</code>, the system has the form $A * X = B$ (No transpose).</p> <p>If <code>trans = 'T'</code>, the system has the form $A^T * X = B$ (Transpose).</p> <p>If <code>trans = 'C'</code>, the system has the form $A^H * X = B$ (Conjugate Transpose = Transpose for real flavors, Conjugate Transpose for complex flavors).</p> |
| <code>n</code> | The number of linear equations; the order of the matrix A ; $n \geq 0$. |
| <code>kl</code> | The number of subdiagonals within the band of A ; $kl \geq 0$. |
| <code>ku</code> | The number of superdiagonals within the band of A ; $ku \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides; the number of columns of the matrices B and X ; $nrhs \geq 0$. |
| <code>ab, afb, b</code> | <p>Arrays: <code>ab</code> ($\max(ldab * n)$), <code>afb</code> ($\max(ldafb * n)$), <code>b</code> ($\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout).</p> <p>The array <code>ab</code> contains the matrix A in band storage.</p> <p>If <code>fact = 'F'</code> and <code>equed</code> is not 'N', then AB must have been equilibrated by the scaling factors in <code>r</code> and/or <code>c</code>.</p> |

The array *afb* is an input argument if *fact* = 'F'. It contains the factored form of the banded matrix *A*, that is, the factors *L* and *U* from the factorization $A = P * L * U$ as computed by `?gbtrf`. *U* is stored as an upper triangular banded matrix with *kl* + *ku* superdiagonals. *L* is stored as lower triangular band matrix with *kl* subdiagonals. If *equed* is not 'N', then *afb* is the factored form of the equilibrated matrix *A*.

The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.

| | |
|---------------------|---|
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; $ldab \geq kl + ku + 1$. |
| <i>ldafb</i> | The leading dimension of the array <i>afb</i> ; $ldafb \geq 2 * kl + ku + 1$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. It contains the pivot indices from the factorization $A = P * L * U$ as computed by <code>?gbtrf</code> ; row <i>i</i> of the matrix was interchanged with row <i>ipiv</i> [<i>i</i> -1]. |
| <i>equed</i> | Must be 'N', 'R', 'C', or 'B'. <i>equed</i> is an input argument if <i>fact</i> = 'F'. It specifies the form of equilibration that was done: If <i>equed</i> = 'N', no equilibration was done (always true if <i>fact</i> = 'N'). If <i>equed</i> = 'R', row equilibration was done, that is, <i>A</i> has been premultiplied by <i>diag</i> (<i>r</i>). If <i>equed</i> = 'C', column equilibration was done, that is, <i>A</i> has been postmultiplied by <i>diag</i> (<i>c</i>). If <i>equed</i> = 'B', both row and column equilibration was done, that is, <i>A</i> has been replaced by <i>diag</i> (<i>r</i>) * <i>A</i> * <i>diag</i> (<i>c</i>). |
| <i>r, c</i> | Arrays: <i>r</i> (size <i>n</i>), <i>c</i> (size <i>n</i>). The array <i>r</i> contains the row scale factors for <i>A</i> , and the array <i>c</i> contains the column scale factors for <i>A</i> . These arrays are input arguments if <i>fact</i> = 'F' only; otherwise they are output arguments. If <i>equed</i> = 'R' or 'B', <i>A</i> is multiplied on the left by <i>diag</i> (<i>r</i>); if <i>equed</i> = 'N' or 'C', <i>r</i> is not accessed. If <i>fact</i> = 'F' and <i>equed</i> = 'R' or 'B', each element of <i>r</i> must be positive. If <i>equed</i> = 'C' or 'B', <i>A</i> is multiplied on the right by <i>diag</i> (<i>c</i>); if <i>equed</i> = 'N' or 'R', <i>c</i> is not accessed. If <i>fact</i> = 'F' and <i>equed</i> = 'C' or 'B', each element of <i>c</i> must be positive. Each element of <i>r</i> or <i>c</i> should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable. |
| <i>ldb</i> | The leading dimension of the array <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

| | | | | | |
|-------------------|--|---------|------|------------|---|
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. | | | | |
| <i>n_err_bnds</i> | Number of error bounds to return for each right hand side and each type (normwise or componentwise). See <i>err_bnds_norm</i> and <i>err_bnds_comp</i> descriptions in <i>Output Arguments</i> section below. | | | | |
| <i>nparams</i> | Specifies the number of parameters set in <i>params</i> . If ≤ 0 , the <i>params</i> array is never referenced and default values are used. | | | | |
| <i>params</i> | <p>Array, size $\max(1, nparams)$. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to <i>nparams</i> are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass <i>nparams</i> = 0, which prevents the source code from accessing the <i>params</i> argument.</p> <p><i>params</i>[0] : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).</p> <p>=0.0 No refinement is performed and no error bounds are computed.</p> <p>=1.0 Use the extra-precise refinement algorithm.</p> <p>(Other values are reserved for future use.)</p> <p><i>params</i>[1] : Maximum number of residual computations allowed for refinement.</p> <table border="0" style="margin-left: 20px;"> <tr> <td style="padding-right: 20px;">Default</td> <td>10.0</td> </tr> <tr> <td>Aggressive</td> <td>Set to 100.0 to permit convergence using approximate factorizations or factorizations other than <i>LU</i>. If the factorization uses a technique other than Gaussian elimination, the guarantees in <i>err_bnds_norm</i> and <i>err_bnds_comp</i> may no longer be trustworthy.</td> </tr> </table> <p><i>params</i>[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).</p> | Default | 10.0 | Aggressive | Set to 100.0 to permit convergence using approximate factorizations or factorizations other than <i>LU</i> . If the factorization uses a technique other than Gaussian elimination, the guarantees in <i>err_bnds_norm</i> and <i>err_bnds_comp</i> may no longer be trustworthy. |
| Default | 10.0 | | | | |
| Aggressive | Set to 100.0 to permit convergence using approximate factorizations or factorizations other than <i>LU</i> . If the factorization uses a technique other than Gaussian elimination, the guarantees in <i>err_bnds_norm</i> and <i>err_bnds_comp</i> may no longer be trustworthy. | | | | |

Output Parameters

| | |
|----------|--|
| <i>x</i> | <p>Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.</p> <p>If <i>info</i> = 0, the array <i>x</i> contains the solution <i>n</i>-by-<i>nrhs</i> matrix <i>X</i> to the <i>original</i> system of equations. Note that <i>A</i> and <i>B</i> are modified on exit if <i>equed</i> ≠ 'N', and the solution to the <i>equilibrated</i> system is:</p> <p>$inv(diag(c)) * X$, if <i>trans</i> = 'N' and <i>equed</i> = 'C' or 'B'; or</p> <p>$inv(diag(r)) * X$, if <i>trans</i> = 'T' or 'C' and <i>equed</i> = 'R' or 'B'.</p> |
|----------|--|

- ab* Array *ab* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'.
If *equed* ≠ 'N', *A* is scaled on exit as follows:
equed = 'R': $A = \text{diag}(r) * A$
equed = 'C': $A = A * \text{diag}(c)$
equed = 'B': $A = \text{diag}(r) * A * \text{diag}(c)$.
- afb* If *fact* = 'N' or 'E', then *afb* is an output argument and on exit returns the factors *L* and *U* from the factorization $A = PLU$ of the original matrix *A* (if *fact* = 'N') or of the equilibrated matrix *A* (if *fact* = 'E').
- b* Overwritten by $\text{diag}(r) * B$ if *trans* = 'N' and *equed* = 'R' or 'B';
overwritten by *trans* = 'T' or 'C' and *equed* = 'C' or 'B';
not changed if *equed* = 'N'.
- r, c* These arrays are output arguments if *fact* ≠ 'F'. Each element of these arrays is a power of the radix. See the description of *r, c* in *Input Arguments* section.
- rcond* Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.
- rpvgrw* Contains the reciprocal pivot growth factor:
$$\|A\| / \|U\|$$

If this is much less than 1, the stability of the *LU* factorization of the (equilibrated) matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < \text{info} \leq n$, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of *A*. In ?gbsvx, this quantity is returned in *rpivot*.
- berr* Array, size at least $\max(1, \text{nrhs})$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of *A* or *B* that makes x_j an exact solution.
- err_bnds_norm* Array of size *nrhs***n_err_bnds*. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:
Normwise relative error in the *i*-th solution vector
$$\frac{\max_j |X_{\text{true}_{ji}} - X_{ji}|}{\max_j |X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix *Z* are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * a$, where *s* scales each row by a power of the radix so all absolute row sums of *z* are approximately 1.

The information for right-hand side *i*, where $1 \leq i \leq nrhs$, and type of error *err* is stored in *err_bnds_norm*[(*err*-1)**nrhs* + *i* - 1].

err_bnds_comp

Array of size *nrhs***n_err_bnds*. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the *i*-th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested (*params*[2] = 0.0), then *err_bnds_comp* is not accessed.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * \text{slamch}(\epsilon)$ for single precision flavors and $\sqrt{n} * \text{dlamch}(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq \text{nrhs}$, and type of error *err* is stored in `err_bnds_comp[(err-1)*nrhs + i - 1]`.

ipiv If *fact* = 'N' or 'E', then *ipiv* is an output argument and on exit contains the pivot indices from the factorization $A = P * L * U$ of the original matrix A (if *fact* = 'N') or of the equilibrated matrix A (if *fact* = 'E').

equed If *fact* ≠ 'F', then *equed* is an output argument. It specifies the form of equilibration that was done (see the description of *equed* in *Input Arguments* section).

params If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified.

info If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If $0 < \text{info} \leq n$: $U_{\text{info}, \text{info}}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.

If *info* = $n + j$: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides k with $k > j$ may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested `params[2] = 0.0`, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that `err_bnds_norm[j - 1] =`

0.0 or `err_bnds_comp[j - 1] = 0.0`. See the definition of `err_bnds_norm` and `err_bnds_comp` for `err = 1`. To get information about all of the right-hand sides, check `err_bnds_norm` or `err_bnds_comp`.

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful. The solution to every right-hand side is guaranteed.

If `info = -i`, parameter `i` had an illegal value.

If $0 < info \leq n$: $U_{info,info}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; `rcond = 0` is returned.

If `info = n+j`: The solution corresponding to the j -th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides k with $k > j$ may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested `params[2] = 0.0`, then the j -th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest j such that for column major layout `err_bnds_norm[j - 1] = 0.0` or `err_bnds_comp[j - 1] = 0.0`; or for row major layout `err_bnds_norm[(j - 1)*n_err_bnds] = 0.0` or `err_bnds_comp[(j - 1)*n_err_bnds] = 0.0`). See the definition of `err_bnds_norm` and `err_bnds_comp` for `err = 1`. To get information about all of the right-hand sides, check `err_bnds_norm` or `err_bnds_comp`.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gtsv

Computes the solution to the system of linear equations with a tridiagonal coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_sgtsv (int matrix_layout , lapack_int n , lapack_int nrhs , float * dl , float * d , float * du , float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dgtsv (int matrix_layout , lapack_int n , lapack_int nrhs , double * dl , double * d , double * du , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_cgtsv (int matrix_layout , lapack_int n , lapack_int nrhs , lapack_complex_float * dl , lapack_complex_float * d , lapack_complex_float * du , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zgtsv (int matrix_layout , lapack_int n , lapack_int nrhs , lapack_complex_double * dl , lapack_complex_double * d , lapack_complex_double * du , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mkl.h`

Description

The routine solves for X the system of linear equations $A * X = B$, where A is an n -by- n tridiagonal matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions. The routine uses Gaussian elimination with partial pivoting.

Note that the equation $A^T * X = B$ may be solved by interchanging the order of the arguments `du` and `dl`.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of <i>A</i> , the number of rows in <i>B</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in <i>B</i> ; $nrhs \geq 0$. |
| <i>dl</i> | The array <i>dl</i> (size $n - 1$) contains the $(n - 1)$ subdiagonal elements of <i>A</i> . |
| <i>d</i> | The array <i>d</i> (size n) contains the diagonal elements of <i>A</i> . |
| <i>du</i> | The array <i>du</i> (size $n - 1$) contains the $(n - 1)$ superdiagonal elements of <i>A</i> . |
| <i>b</i> | The array <i>b</i> of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-----------|--|
| <i>dl</i> | Overwritten by the $(n-2)$ elements of the second superdiagonal of the upper triangular matrix <i>U</i> from the <i>LU</i> factorization of <i>A</i> . These elements are stored in <i>dl</i> [0], ..., <i>dl</i> [$n - 3$]. |
| <i>d</i> | Overwritten by the n diagonal elements of <i>U</i> . |
| <i>du</i> | Overwritten by the $(n-1)$ elements of the first superdiagonal of <i>U</i> . |
| <i>b</i> | Overwritten by the solution matrix <i>X</i> . |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, $U_{i,i}$ is exactly zero, and the solution has not been computed. The factorization has not been completed unless $i = n$.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?gtsvx

*Computes the solution to the real or complex system of linear equations with a tridiagonal coefficient matrix *A* and multiple right-hand sides, and provides error bounds on the solution.*

Syntax

```
lapack_int LAPACKE_sgtsvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, const float* dl, const float* d, const float* du, float* dlf, float*
df, float* duf, float* du2, lapack_int* ipiv, const float* b, lapack_int ldb, float*
x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dgtsvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, const double* dl, const double* d, const double* du, double* dlf,
double* df, double* duf, double* du2, lapack_int* ipiv, const double* b, lapack_int
ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cgtsvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, const lapack_complex_float* dl, const lapack_complex_float* d, const
lapack_complex_float* du, lapack_complex_float* dlf, lapack_complex_float* df,
lapack_complex_float* duf, lapack_complex_float* du2, lapack_int* ipiv, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zgtsvx( int matrix_layout, char fact, char trans, lapack_int n,
lapack_int nrhs, const lapack_complex_double* dl, const lapack_complex_double* d, const
lapack_complex_double* du, lapack_complex_double* dlf, lapack_complex_double* df,
lapack_complex_double* duf, lapack_complex_double* du2, lapack_int* ipiv, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine uses the *LU* factorization to compute the solution to a real or complex system of linear equations $A \cdot X = B$, $A^T \cdot X = B$, or $A^H \cdot X = B$, where A is a tridiagonal matrix of order n , the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?gtsvx` performs the following steps:

1. If `fact = 'N'`, the *LU* decomposition is used to factor the matrix A as $A = L \cdot U$, where L is a product of permutation and unit lower bidiagonal matrices and U is an upper triangular matrix with nonzeros in only the main diagonal and first two superdiagonals.
2. If some $U_{i,i} = 0$, so that U is exactly singular, then the routine returns with `info = i`. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, `info = n + 1` is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A .
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>fact</code> | Must be 'F' or 'N'. |

| | |
|----------------------------|--|
| | Specifies whether or not the factored form of the matrix A has been supplied on entry. |
| | If $fact = 'F'$: on entry, dif , df , duf , $du2$, and $ipiv$ contain the factored form of A ; arrays dl , d , du , dif , df , duf , $du2$, and $ipiv$ will not be modified. |
| | If $fact = 'N'$, the matrix A will be copied to dif , df , and duf and factored. |
| $trans$ | Must be 'N', 'T', or 'C'. |
| | Specifies the form of the system of equations: |
| | If $trans = 'N'$, the system has the form $A^*X = B$ (No transpose). |
| | If $trans = 'T'$, the system has the form $A^T*X = B$ (Transpose). |
| | If $trans = 'C'$, the system has the form $A^H*X = B$ (Conjugate transpose). |
| n | The number of linear equations, the order of the matrix A ; $n \geq 0$. |
| $nrhs$ | The number of right hand sides, the number of columns of the matrices B and X ; $nrhs \geq 0$. |
| $dl,d,du,dif,df,duf,du2,b$ | Arrays: dl , size $(n - 1)$, contains the subdiagonal elements of A . d , size (n) , contains the diagonal elements of A . du , size $(n - 1)$, contains the superdiagonal elements of A . dif , size $(n - 1)$. If $fact = 'F'$, then dif is an input argument and on entry contains the $(n - 1)$ multipliers that define the matrix L from the LU factorization of A as computed by <code>?gttrf</code> . df , size (n) . If $fact = 'F'$, then df is an input argument and on entry contains the n diagonal elements of the upper triangular matrix U from the LU factorization of A . duf , size $(n - 1)$. If $fact = 'F'$, then duf is an input argument and on entry contains the $(n - 1)$ elements of the first superdiagonal of U . $du2$, size $(n - 2)$. If $fact = 'F'$, then $du2$ is an input argument and on entry contains the $(n - 2)$ elements of the second superdiagonal of U . b , size $\max(ldb*nrhs)$ for column major layout and $\max(ldb*n)$ for row major layout, contains the right-hand side matrix B . |
| ldb | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| ldx | The leading dimension of x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| $ipiv$ | Array, size at least $\max(1, n)$. If $fact = 'F'$, then $ipiv$ is an input argument and on entry contains the pivot indices, as returned by <code>?gttrf</code> . |

Output Parameters

| | |
|--------------|--|
| <i>x</i> | <p>Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.</p> <p>If <i>info</i> = 0 or <i>info</i> = <i>n</i>+1, the array <i>x</i> contains the solution matrix <i>X</i>.</p> |
| <i>dlf</i> | <p>If <i>fact</i> = 'N', then <i>dlf</i> is an output argument and on exit contains the (<i>n</i>-1) multipliers that define the matrix <i>L</i> from the <i>LU</i> factorization of <i>A</i>.</p> |
| <i>df</i> | <p>If <i>fact</i> = 'N', then <i>df</i> is an output argument and on exit contains the <i>n</i> diagonal elements of the upper triangular matrix <i>U</i> from the <i>LU</i> factorization of <i>A</i>.</p> |
| <i>duf</i> | <p>If <i>fact</i> = 'N', then <i>duf</i> is an output argument and on exit contains the (<i>n</i>-1) elements of the first superdiagonal of <i>U</i>.</p> |
| <i>du2</i> | <p>If <i>fact</i> = 'N', then <i>du2</i> is an output argument and on exit contains the (<i>n</i>-2) elements of the second superdiagonal of <i>U</i>.</p> |
| <i>ipiv</i> | <p>The array <i>ipiv</i> is an output argument if <i>fact</i> = 'N' and, on exit, contains the pivot indices from the factorization $A = L*U$; row <i>i</i> of the matrix was interchanged with row <i>ipiv</i>[<i>i</i>-1]. The value of <i>ipiv</i>[<i>i</i>-1] will always be <i>i</i> or <i>i</i>+1; <i>ipiv</i>[<i>i</i>-1]=<i>i</i> indicates a row interchange was not required.</p> |
| <i>rcond</i> | <p>An estimate of the reciprocal condition number of the matrix <i>A</i>. If <i>rcond</i> is less than the machine precision (in particular, if <i>rcond</i> = 0), the matrix is singular to working precision. This condition is indicated by a return code of <i>info</i>>0.</p> |
| <i>ferr</i> | <p>Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the <i>j</i>-th column of the solution matrix <i>X</i>). If <i>xtrue</i> is the true solution corresponding to x_j, <i>ferr</i>[<i>j</i>-1] is an estimated upper bound for the magnitude of the largest element in $x_j - xtrue$ divided by the magnitude of the largest element in x_j. The estimate is as reliable as the estimate for <i>rcond</i>, and is almost always a slight overestimate of the true error.</p> |
| <i>berr</i> | <p>Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j, that is, the smallest relative change in any element of <i>A</i> or <i>B</i> that makes x_j an exact solution.</p> |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, and $i \leq n$, then $U_{i,i}$ is exactly zero. The factorization has not been completed unless $i = n$, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned. If *info* = *i*, and $i = n + 1$, then *U* is nonsingular, but *rcond* is less than machine precision,

meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?dtsvb

Computes the solution to the system of linear equations with a diagonally dominant tridiagonal coefficient matrix A and multiple right-hand sides.

Syntax

```
void sdtsvb (const MKL_INT * n, const MKL_INT * nrhs, float * dl, float * d, const
float * du, float * b, const MKL_INT * ldb, MKL_INT * info );

void ddtsvb (const MKL_INT * n, const MKL_INT * nrhs, double * dl, double * d, const
double * du, double * b, const MKL_INT * ldb, MKL_INT * info );

void cdtsvb (const MKL_INT * n, const MKL_INT * nrhs, MKL_Complex8 * dl, MKL_Complex8 *
d, const MKL_Complex8 * du, MKL_Complex8 * b, const MKL_INT * ldb, MKL_INT * info );

void zdtsvb (const MKL_INT * n, const MKL_INT * nrhs, MKL_Complex16 * dl, MKL_Complex16
* d, const MKL_Complex16 * du, MKL_Complex16 * b, const MKL_INT * ldb, MKL_INT *
info );
```

Include Files

- `mkl.h`

Description

The ?dtsvb routine solves a system of linear equations $A^*X = B$ for X , where A is an n -by- n diagonally dominant tridiagonal matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions. The routine uses the BABE (Burning At Both Ends) algorithm.

Note that the equation $A^T X = B$ may be solved by interchanging the order of the arguments *du* and *dl*.

Input Parameters

| | |
|---------------------|---|
| <i>n</i> | The order of A , the number of rows in B ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <i>dl, d, du, b</i> | Arrays: <i>dl</i> (size $n - 1$), <i>d</i> (size n), <i>du</i> (size $n - 1$), <i>b</i> ($\max(ldb*nrhs)$ for column major layout and $\max(ldb*n)$ for row major layout). The array <i>dl</i> contains the $(n - 1)$ subdiagonal elements of A . The array <i>d</i> contains the diagonal elements of A . The array <i>du</i> contains the $(n - 1)$ superdiagonal elements of A . The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$. |

Output Parameters

| | |
|-------------|--|
| <i>dl</i> | Overwritten by the $(n-1)$ elements of the subdiagonal of the lower triangular matrices L_1, L_2 from the factorization of A (see dttrfb). |
| <i>d</i> | Overwritten by the n diagonal element reciprocals of U . |
| <i>b</i> | Overwritten by the solution matrix X . |
| <i>info</i> | <p>If $info = 0$, the execution is successful.</p> <p>If $info = -i$, the i-th parameter had an illegal value.</p> <p>If $info = i$, $u_{i,i}$ is exactly zero, and the solution has not been computed. The factorization has not been completed unless $i = n$.</p> |

Application Notes

A diagonally dominant tridiagonal system is defined such that $|d_i| > |dl_{i-1}| + |du_i|$ for any i :

$1 < i < n$, and $|d_1| > |du_1|$, $|d_n| > |dl_{n-1}|$

The underlying BABE algorithm is designed for diagonally dominant systems. Such systems have no numerical stability issue unlike the canonical systems that use elimination with partial pivoting (see [?gtsv](#)). The diagonally dominant systems are much faster than the canonical systems.

NOTE

- The current implementation of BABE has a potential accuracy issue on very small or large data close to the underflow or overflow threshold respectively. Scale the matrix before applying the solver in the case of such input data.
 - Applying the [?dtsvb](#) factorization to non-diagonally dominant systems may lead to an accuracy loss, or false singularity detected due to no pivoting.
-

?posv

Computes the solution to the system of linear equations with a symmetric or Hermitian positive-definite coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_sposv (int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
float * a, lapack_int lda, float * b, lapack_int ldb);

lapack_int LAPACKE_dposv (int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
double * a, lapack_int lda, double * b, lapack_int ldb);

lapack_int LAPACKE_cposv (int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
lapack_complex_float * a, lapack_int lda, lapack_complex_float * b, lapack_int ldb);

lapack_int LAPACKE_zposv (int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
lapack_complex_double * a, lapack_int lda, lapack_complex_double * b, lapack_int ldb);

lapack_int LAPACKE_dsposv (int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
double * a, lapack_int lda, double * b, lapack_int ldb, double * x, lapack_int ldx,
lapack_int * iter);
```

```
lapack_int LAPACKE_zcposv (int matrix_layout, char uplo, lapack_int n, lapack_int nrhs,
lapack_complex_double * a, lapack_int lda, lapack_complex_double * b, lapack_int ldb,
lapack_complex_double * x, lapack_int ldx, lapack_int * iter);
```

Include Files

- mkl.h

Description

The routine solves for X the real or complex system of linear equations $A^*X = B$, where A is an n -by- n symmetric/Hermitian positive-definite matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The Cholesky decomposition is used to factor A as

$A = U^T * U$ (real flavors) and $A = U^H * U$ (complex flavors), if $uplo = 'U'$

or $A = L * L^T$ (real flavors) and $A = L * L^H$ (complex flavors), if $uplo = 'L'$,

where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $A^*X = B$.

The `dsposv` and `zcposv` are mixed precision iterative refinement subroutines for exploiting fast single precision hardware. They first attempt to factorize the matrix in single precision (`dsposv`) or single complex precision (`zcposv`) and use this factorization within an iterative refinement procedure to produce a solution with double precision (`dsposv`) / double complex precision (`zcposv`) normwise backward error quality (see below). If the approach fails, the method switches to a double precision or double complex precision factorization respectively and computes the solution.

The iterative refinement is not going to be a winning strategy if the ratio single precision/complex performance over double precision/double complex performance is too small. A reasonable strategy should take the number of right-hand sides and the size of the matrix into account. This might be done with a call to `llaenv` in the future. At present, iterative refinement is implemented.

The iterative refinement process is stopped if

`iter > itermax`

or for all the right-hand sides:

`rnmr < sqrt(n) * xnmr * anrm * eps * bwdmax`,

where

- `iter` is the number of the current iteration in the iterative refinement process
- `rnmr` is the infinity-norm of the residual
- `xnmr` is the infinity-norm of the solution
- `anrm` is the infinity-operator-norm of the matrix A
- `eps` is the machine epsilon returned by `dlamch` ('Epsilon').

The values `itermax` and `bwdmax` are fixed to 30 and 1.0d+00 respectively.

Input Parameters

`matrix_layout` Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

`uplo` Must be 'U' or 'L'.

Indicates whether the upper or lower triangular part of A is stored:

If `uplo = 'U'`, the upper triangle of A is stored.

If `uplo = 'L'`, the lower triangle of A is stored.

| | |
|-------------|---|
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in <i>B</i> ; $nrhs \geq 0$. |
| <i>a, b</i> | Arrays: <i>a</i> (size $\max(1, lda)$), <i>b</i> , size $\max(ldb*nrhs)$ for column major layout and $\max(ldb*n)$ for row major layout,. The array <i>a</i> contains the upper or the lower triangular part of the matrix <i>A</i> (see <i>uplo</i>). Note that in the case of <i>zcposv</i> the imaginary parts of the diagonal elements need not be set and are assumed to be zero. The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of the array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | If <i>info</i> = 0, the upper or lower triangular part of <i>a</i> is overwritten by the Cholesky factor <i>U</i> or <i>L</i> , as specified by <i>uplo</i> . If iterative refinement has been successfully used (<i>info</i> = 0 and <i>iter</i> \geq 0), then <i>A</i> is unchanged. If double precision factorization has been used (<i>info</i> = 0 and <i>iter</i> < 0), then the array <i>A</i> contains the factors <i>L</i> or <i>U</i> from the Cholesky factorization. |
| <i>b</i> | Overwritten by the solution matrix <i>X</i> . |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. The pivot indices that define the permutation matrix <i>P</i> ; row <i>i</i> of the matrix was interchanged with row <i>ipiv</i> (<i>i</i>). Corresponds to the single precision factorization (if <i>info</i> = 0 and <i>iter</i> \geq 0) or the double precision factorization (if <i>info</i> = 0 and <i>iter</i> < 0). |
| <i>x</i> | Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout. If <i>info</i> = 0, contains the <i>n</i> -by- <i>nrhs</i> solution matrix <i>X</i> . |
| <i>iter</i> | If <i>iter</i> < 0: iterative refinement has failed, double precision factorization has been performed <ul style="list-style-type: none"> • If <i>iter</i> = -1: the routine fell back to full precision for implementation- or machine-specific reason • If <i>iter</i> = -2: narrowing the precision induced an overflow, the routine fell back to full precision • If <i>iter</i> = -3: failure of <i>spotrf</i> for <i>dsposv</i>, or <i>cpotrf</i> for <i>zcposv</i> |

- If $iter = -31$: stop the iterative refinement after the 30th iteration.

If $iter > 0$: iterative refinement has been successfully used. Returns the number of iterations.

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, the leading minor of order i (and therefore the matrix A itself) is not positive definite, so the factorization could not be completed, and the solution has not been computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?posvx

Uses the Cholesky factorization to compute the solution to the system of linear equations with a symmetric or Hermitian positive-definite coefficient matrix A , and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_sposvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, char* equed,
float* s, float* b, lapack_int ldb, float* x, lapack_int ldx, float* rcond, float*
ferr, float* berr );
```

```
lapack_int LAPACKE_dposvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, char* equed,
double* s, double* b, lapack_int ldb, double* x, lapack_int ldx, double* rcond,
double* ferr, double* berr );
```

```
lapack_int LAPACKE_cposvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, char* equed, float* s, lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zposvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, char* equed, double* s, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine uses the *Cholesky* factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n -by- n real symmetric/Hermitian positive definite matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?posvx` performs the following steps:

1. If `fact = 'E'`, real scaling factors s are computed to equilibrate the system:

$$\text{diag}(s) * A * \text{diag}(s) * \text{inv}(\text{diag}(s)) * X = \text{diag}(s) * B.$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(s) * A * \text{diag}(s)$ and B by $\text{diag}(s) * B$.

2. If `fact = 'N'` or `'E'`, the Cholesky decomposition is used to factor the matrix A (after equilibration if `fact = 'E'`) as

$$A = U^T * U \text{ (real)}, A = U^H * U \text{ (complex)}, \text{ if } \text{uplo} = \text{'U'},$$

$$\text{or } A = L * L^T \text{ (real)}, A = L * L^H \text{ (complex)}, \text{ if } \text{uplo} = \text{'L'},$$

where U is an upper triangular matrix and L is a lower triangular matrix.

3. If the leading i -by- i principal minor is not positive-definite, then the routine returns with `info = i`. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, `info = n + 1` is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
4. The system of equations is solved for X using the factored form of A .
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $\text{diag}(s)$ so that it solves the original system before equilibration.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>fact</code> | <p>Must be 'F', 'N', or 'E'.</p> <p>Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If <code>fact = 'F'</code>: on entry, <code>af</code> contains the factored form of A. If <code>equed = 'Y'</code>, the matrix A has been equilibrated with scaling factors given by s. <code>a</code> and <code>af</code> will not be modified.</p> <p>If <code>fact = 'N'</code>, the matrix A will be copied to <code>af</code> and factored.</p> <p>If <code>fact = 'E'</code>, the matrix A will be equilibrated if necessary, then copied to <code>af</code> and factored.</p> |
| <code>uplo</code> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of A is stored:</p> <p>If <code>uplo = 'U'</code>, the upper triangle of A is stored.</p> <p>If <code>uplo = 'L'</code>, the lower triangle of A is stored.</p> |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |

| | |
|-----------------|---|
| <i>a, af, b</i> | <p>Arrays: <i>a</i>(size $\max(1, lda*n)$), <i>af</i>(size $\max(1, ldaf*n)$), <i>b</i>, size $\max(ldb*nrhs)$ for column major layout and $\max(ldb*n)$ for row major layout, .</p> <p>The array <i>a</i> contains the matrix <i>A</i> as specified by <i>uplo</i>. If <i>fact</i> = 'F' and <i>equed</i> = 'Y', then <i>A</i> must have been equilibrated by the scaling factors in <i>s</i>, and <i>a</i> must contain the equilibrated matrix $diag(s)*A*diag(s)$.</p> <p>The array <i>af</i> is an input argument if <i>fact</i> = 'F'. It contains the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization of <i>A</i> in the same storage format as <i>A</i>. If <i>equed</i> is not 'N', then <i>af</i> is the factored form of the equilibrated matrix $diag(s)*A*diag(s)$.</p> <p>The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations.</p> |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>equed</i> | <p>Must be 'N' or 'Y'.</p> <p><i>equed</i> is an input argument if <i>fact</i> = 'F'. It specifies the form of equilibration that was done:</p> <p>if <i>equed</i> = 'N', no equilibration was done (always true if <i>fact</i> = 'N');</p> <p>if <i>equed</i> = 'Y', equilibration was done, that is, <i>A</i> has been replaced by $diag(s)*A*diag(s)$.</p> |
| <i>s</i> | <p>Array, size (<i>n</i>). The array <i>s</i> contains the scale factors for <i>A</i>. This array is an input argument if <i>fact</i> = 'F' only; otherwise it is an output argument.</p> <p>If <i>equed</i> = 'N', <i>s</i> is not accessed.</p> <p>If <i>fact</i> = 'F' and <i>equed</i> = 'Y', each element of <i>s</i> must be positive.</p> |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>x</i> | <p>Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.</p> <p>If <i>info</i> = 0 or <i>info</i> = <i>n</i>+1, the array <i>x</i> contains the solution matrix <i>X</i> to the <i>original</i> system of equations. Note that if <i>equed</i> = 'Y', <i>A</i> and <i>B</i> are modified on exit, and the solution to the equilibrated system is $inv(diag(s))*X$.</p> |
| <i>a</i> | Array <i>a</i> is not modified on exit if <i>fact</i> = 'F' or 'N', or if <i>fact</i> = 'E' and <i>equed</i> = 'N'. |

| | |
|---------|--|
| | If $fact = 'E'$ and $equed = 'Y'$, A is overwritten by $diag(s) * A * diag(s)$. |
| af | If $fact = 'N'$ or $'E'$, then af is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization $A = U^T * U$ or $A = L * L^T$ (real routines), $A = U^H * U$ or $A = L * L^H$ (complex routines) of the original matrix A (if $fact = 'N'$), or of the equilibrated matrix A (if $fact = 'E'$). See the description of a for the form of the equilibrated matrix. |
| b | Overwritten by $diag(s) * B$, if $equed = 'Y'$; not changed if $equed = 'N'$. |
| s | This array is an output argument if $fact \neq 'F'$. See the description of s in <i>Input Arguments</i> section. |
| $rcond$ | An estimate of the reciprocal condition number of the matrix A after equilibration (if done). If $rcond$ is less than the machine precision (in particular, if $rcond = 0$), the matrix is singular to working precision. This condition is indicated by a return code of $info > 0$. |
| $ferr$ | Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the j -th column of the solution matrix X). If $xtrue$ is the true solution corresponding to x_j , $ferr[j-1]$ is an estimated upper bound for the magnitude of the largest element in $(x_j - xtrue)$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for $rcond$, and is almost always a slight overestimate of the true error. |
| $berr$ | Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution. |
| $equed$ | If $fact \neq 'F'$, then $equed$ is an output argument. It specifies the form of equilibration that was done (see the description of $equed$ in <i>Input Arguments</i> section). |

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, and $i \leq n$, the leading minor of order i (and therefore the matrix A itself) is not positive-definite, so the factorization could not be completed, and the solution and error bounds could not be computed; $rcond = 0$ is returned.

If $info = i$, and $i = n + 1$, then U is nonsingular, but $rcond$ is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of $rcond$ would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?posvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a symmetric or Hermitian positive-definite coefficient matrix A applying the Cholesky factorization.

Syntax

```
lapack_int LAPACKE_sposvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, char* equed,
float* s, float* b, lapack_int ldb, float* x, lapack_int ldx, float* rcond, float*
rpvgrw, float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float*
err_bnds_comp, lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_dposvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, char* equed,
double* s, double* b, lapack_int ldb, double* x, lapack_int ldx, double* rcond,
double* rpvgrw, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, const double* params );
```

```
lapack_int LAPACKE_cposvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, char* equed, float* s, lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* rpvgrw, float* berr,
lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp, lapack_int nparams,
const float* params );
```

```
lapack_int LAPACKE_zposvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, char* equed, double* s, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* rpvgrw, double* berr,
lapack_int n_err_bnds, double* err_bnds_norm, double* err_bnds_comp, lapack_int
nparams, const double* params );
```

Include Files

- mkl.h

Description

The routine uses the *Cholesky* factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is an n -by- n real symmetric/Hermitian positive definite matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error ($O(\text{eps})$, where eps is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with $O(\text{eps})$ errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine ?posvxx performs the following steps:

1. If *fact* = 'E', scaling factors are computed to equilibrate the system:

$$\text{diag}(s)*A*\text{diag}(s) * \text{inv}(\text{diag}(s))*X = \text{diag}(s)*B$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $diag(s)*A*diag(s)$ and B by $diag(s)*B$.

2. If $fact = 'N'$ or $'E'$, the Cholesky decomposition is used to factor the matrix A (after equilibration if $fact = 'E'$) as

$A = U^T*U$ (real), $A = U^H*U$ (complex), if $uplo = 'U'$,

or $A = L*L^T$ (real), $A = L*L^H$ (complex), if $uplo = 'L'$,

where U is an upper triangular matrix and L is a lower triangular matrix.

3. If the leading i -by- i principal minor is not positive-definite, the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A (see the $rcond$ parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for X and compute error bounds.
4. The system of equations is solved for X using the factored form of A .
5. By default, unless $params[0]$ is set to zero, the routine applies iterative refinement to get a small error and error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix X is premultiplied by $diag(s)$ so that it solves the original system before equilibration.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>fact</i> | <p>Must be 'F', 'N', or 'E'.</p> <p>Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If $fact = 'F'$, on entry, af contains the factored form of A. If $equed$ is not 'N', the matrix A has been equilibrated with scaling factors given by s. Parameters a and af are not modified.</p> <p>If $fact = 'N'$, the matrix A will be copied to af and factored.</p> <p>If $fact = 'E'$, the matrix A will be equilibrated, if necessary, copied to af and factored.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of A is stored:</p> <p>If $uplo = 'U'$, the upper triangle of A is stored.</p> <p>If $uplo = 'L'$, the lower triangle of A is stored.</p> |
| <i>n</i> | The number of linear equations; the order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns of the matrices B and X ; $nrhs \geq 0$. |
| <i>a, af, b</i> | <p>Arrays: a(size $\max(lda*n)$), af(size $\max(ldaf*n)$), b(size $\max(1, ldb*nrhs)$) for column major layout and $\max(1, ldb*n)$ for row major layout).</p> <p>The array a contains the matrix A as specified by $uplo$. If $fact = 'F'$ and $equed = 'Y'$, then A must have been equilibrated by the scaling factors in s, and a must contain the equilibrated matrix $diag(s)*A*diag(s)$.</p> |

The array *af* is an input argument if *fact* = 'F'. It contains the triangular factor *U* or *L* from the Cholesky factorization of *A* in the same storage format as *A*. If *equed* is not 'N', then *af* is the factored form of the equilibrated matrix $diag(s) * A * diag(s)$.

The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.

| | |
|-------------------|---|
| <i>lda</i> | The leading dimension of the array <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of the array <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>equed</i> | Must be 'N' or 'Y'. <i>equed</i> is an input argument if <i>fact</i> = 'F'. It specifies the form of equilibration that was done: If <i>equed</i> = 'N', no equilibration was done (always true if <i>fact</i> = 'N'). if <i>equed</i> = 'Y', both row and column equilibration was done, that is, <i>A</i> has been replaced by $diag(s) * A * diag(s)$. |
| <i>s</i> | Array, size (<i>n</i>). The array <i>s</i> contains the scale factors for <i>A</i> . This array is an input argument if <i>fact</i> = 'F' only; otherwise it is an output argument. If <i>equed</i> = 'N', <i>s</i> is not accessed. If <i>fact</i> = 'F' and <i>equed</i> = 'Y', each element of <i>s</i> must be positive. Each element of <i>s</i> should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable. |
| <i>ldb</i> | The leading dimension of the array <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <i>n_err_bnds</i> | Number of error bounds to return for each right hand side and each type (normwise or componentwise). See <i>err_bnds_norm</i> and <i>err_bnds_comp</i> descriptions in the <i>Output Arguments</i> section below. |
| <i>nparams</i> | Specifies the number of parameters set in <i>params</i> . If ≤ 0 , the <i>params</i> array is never referenced and default values are used. |
| <i>params</i> | Array, size $\max(1, nparams)$. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to <i>nparams</i> are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass <i>nparams</i> = 0, which prevents the source code from accessing the <i>params</i> argument. <i>params</i> [0] : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors). |

=0.0 No refinement is performed and no error bounds are computed.

=1.0 Use the extra-precise refinement algorithm.

(Other values are reserved for future use.)

params[1] : Maximum number of residual computations allowed for refinement.

Default 10.0

Aggressive Set to 100.0 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

Output Parameters

x Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.

If *info* = 0, the array *x* contains the solution *n*-by-*nrhs* matrix *X* to the original system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the equilibrated system is:

$$\text{inv}(\text{diag}(s)) * X.$$

a Array *a* is not modified on exit if *fact* = 'F' or 'N', or if *fact* = 'E' and *equed* = 'N'.

If *fact* = 'E' and *equed* = 'Y', *A* is overwritten by $\text{diag}(s) * A * \text{diag}(s)$.

af If *fact* = 'N' or 'E', then *af* is an output argument and on exit returns the triangular factor *U* or *L* from the Cholesky factorization $A = U^T * U$ or $A = L * L^T$ (real routines), $A = U^H * U$ or $A = L * L^H$ (complex routines) of the original matrix *A* (if *fact* = 'N'), or of the equilibrated matrix *A* (if *fact* = 'E'). See the description of *a* for the form of the equilibrated matrix.

b If *equed* = 'N', *B* is not modified.

If *equed* = 'Y', *B* is overwritten by $\text{diag}(s) * B$.

s This array is an output argument if *fact* ≠ 'F'. Each element of this array is a power of the radix. See the description of *s* in *Input Arguments* section.

rcond Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix A after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

rpvgrw Contains the reciprocal pivot growth factor:

$$\|A\|/\|U\|$$

If this is much less than 1, the stability of the LU factorization of the (equilibrated) matrix A could be poor. This also means that the solution X , estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < info \leq n$, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of A .

berr Array, size at least $\max(1, nrhs)$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution.

err_bnds_norm Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:

Normwise relative error in the i -th solution vector

$$\frac{\max_j |X_{true_{ji}} - X_{ji}|}{\max_j |X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors to determine if the error estimate is

"guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * a$, where s scales each row by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error err is stored in `err_bnds_norm[(err-1)*nrhs + i - 1]`.

`err_bnds_comp`

Array of size `nrhs*n_err_bnds`. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the i -th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested (`params[2] = 0.0`), then `err_bnds_comp` is not accessed.

- `err=1` "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold `sqrt(n)*slamch(ε)` for single precision flavors and `sqrt(n)*dlamch(ε)` for double precision flavors.
- `err=2` "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold `sqrt(n)*slamch(ε)` for single precision flavors and `sqrt(n)*dlamch(ε)` for double precision flavors. This error bound should only be trusted if the previous boolean is true.
- `err=3` Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold `sqrt(n)*slamch(ε)` for single precision flavors and `sqrt(n)*dlamch(ε)` for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error err is stored in `err_bnds_comp[(err-1)*nrhs + i - 1]`.

`equed`

If `fact` is 'F', then `equed` is an output argument. It specifies the form of equilibration that was done (see the description of `equed` in *Input Arguments* section).

`params`

If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified.

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful. The solution to every right-hand side is guaranteed.

If `info = -i`, parameter i had an illegal value.

If $0 < info \leq n$: $U_{info,info}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; `rcond = 0` is returned.

If `info = n+j`: The solution corresponding to the j -th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides k with $k > j$ may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested `params[2] = 0.0`, then the j -th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest j such that for column major layout `err_bnds_norm[j - 1] = 0.0` or `err_bnds_comp[j - 1] = 0.0`; or for row major layout `err_bnds_norm[(j - 1)*n_err_bnds] = 0.0` or `err_bnds_comp[(j - 1)*n_err_bnds] = 0.0`). See the definition of `err_bnds_norm` and `err_bnds_comp` for `err = 1`. To get information about all of the right-hand sides, check `err_bnds_norm` or `err_bnds_comp`.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?ppsv

Computes the solution to the system of linear equations with a symmetric (Hermitian) positive definite packed coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_sppsv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , float * ap , float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dpssv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , double * ap , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_cpssv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_float * ap , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zpssv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_double * ap , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- `mkl.h`

Description

The routine solves for X the real or complex system of linear equations $A^*X = B$, where A is an n -by- n real symmetric/Hermitian positive-definite matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The Cholesky decomposition is used to factor A as

$A = U^T * U$ (real flavors) and $A = U^H * U$ (complex flavors), if $uplo = 'U'$

or $A = L * L^T$ (real flavors) and $A = L * L^H$ (complex flavors), if $uplo = 'L'$,

where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $A^*X = B$.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If $uplo = 'U'$, the upper triangle of A is stored. If $uplo = 'L'$, the lower triangle of A is stored. |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <code>ap, b</code> | Arrays: ap (size $\max(1, n * (n + 1) / 2)$), b , size $\max(ldb * nrhs)$ for column major layout and $\max(ldb * n)$ for row major layout,. The array ap contains the upper or the lower triangular part of the matrix A (as specified by $uplo$) in <i>packed storage</i> (see Matrix Storage Schemes). The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <code>ldb</code> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-----------------|---|
| <code>ap</code> | If $info = 0$, the upper or lower triangular part of A in packed storage is overwritten by the Cholesky factor U or L , as specified by $uplo$. |
| <code>b</code> | Overwritten by the solution matrix X . |

Return Values

This function returns a value `info`.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, the leading minor of order i (and therefore the matrix A itself) is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?ppsvx

Uses the Cholesky factorization to compute the solution to the system of linear equations with a symmetric (Hermitian) positive definite packed coefficient matrix A , and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_sppsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, float* ap, float* AFP, char* equed, float* s, float* b, lapack_int
ldb, float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dppsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, double* ap, double* AFP, char* equed, double* s, double* b,
lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cppsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* ap, lapack_complex_float* AFP, char* equed,
float* s, lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int
ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zppsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* ap, lapack_complex_double* AFP, char* equed,
double* s, lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x,
lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine uses the Cholesky factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n -by- n symmetric or Hermitian positive-definite matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine ?ppsvx performs the following steps:

1. If $fact = 'E'$, real scaling factors s are computed to equilibrate the system:

$$diag(s)*A*diag(s)*inv(diag(s))*X = diag(s)*B.$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $diag(s)*A*diag(s)$ and B by $diag(s)*B$.

2. If $fact = 'N'$ or $'E'$, the Cholesky decomposition is used to factor the matrix A (after equilibration if $fact = 'E'$) as

$$A = U^T*U \text{ (real)}, A = U^H*U \text{ (complex)}, \text{ if } uplo = 'U',$$

$$\text{or } A = L*L^T \text{ (real)}, A = L*L^H \text{ (complex)}, \text{ if } uplo = 'L',$$

where U is an upper triangular matrix and L is a lower triangular matrix.

3. If the leading i -by- i principal minor is not positive-definite, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, $info = n+1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
4. The system of equations is solved for X using the factored form of A .
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $diag(s)$ so that it solves the original system before equilibration.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>fact</i> | <p>Must be 'F', 'N', or 'E'.</p> <p>Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If <i>fact</i> = 'F': on entry, <i>afp</i> contains the factored form of A. If <i>equed</i> = 'Y', the matrix A has been equilibrated with scaling factors given by s. <i>ap</i> and <i>afp</i> will not be modified.</p> <p>If <i>fact</i> = 'N', the matrix A will be copied to <i>afp</i> and factored.</p> <p>If <i>fact</i> = 'E', the matrix A will be equilibrated if necessary, then copied to <i>afp</i> and factored.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of A is stored:</p> <p>If <i>uplo</i> = 'U', the upper triangle of A is stored.</p> <p>If <i>uplo</i> = 'L', the lower triangle of A is stored.</p> |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns in B ; $nrhs \geq 0$. |
| <i>ap, afp, b</i> | <p>Arrays: (size $\max(1, n*(n+1)/2)$, <i>afp</i> (size $\max(1, n*(n+1)/2)$, <i>bof</i> size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout.</p> <p>The array <i>ap</i> contains the upper or lower triangle of the original symmetric/Hermitian matrix A in <i>packed storage</i> (see Matrix Storage Schemes). In case when <i>fact</i> = 'F' and <i>equed</i> = 'Y', <i>ap</i> must contain the equilibrated matrix $diag(s)*A*diag(s)$.</p> <p>The array <i>afp</i> is an input argument if <i>fact</i> = 'F' and contains the triangular factor U or L from the Cholesky factorization of A in the same storage format as A. If <i>equed</i> is not 'N', then <i>afp</i> is the factored form of the equilibrated matrix A.</p> |

The array b contains the matrix B whose columns are the right-hand sides for the systems of equations.

ldb The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

$equed$ Must be 'N' or 'Y'.

$equed$ is an input argument if $fact = 'F'$. It specifies the form of equilibration that was done:

if $equed = 'N'$, no equilibration was done (always true if $fact = 'N'$);

if $equed = 'Y'$, equilibration was done, that is, A has been replaced by $diag(s)A*diag(s)$.

s Array, size (n) . The array s contains the scale factors for A . This array is an input argument if $fact = 'F'$ only; otherwise it is an output argument.

If $equed = 'N'$, s is not accessed.

If $fact = 'F'$ and $equed = 'Y'$, each element of s must be positive.

ldx The leading dimension of the output array x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

x Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.

If $info = 0$ or $info = n+1$, the array x contains the solution matrix X to the *original* system of equations. Note that if $equed = 'Y'$, A and B are modified on exit, and the solution to the equilibrated system is $inv(diag(s))*X$.

ap Array ap is not modified on exit if $fact = 'F'$ or 'N', or if $fact = 'E'$ and $equed = 'N'$.

If $fact = 'E'$ and $equed = 'Y'$, ap is overwritten by $diag(s)*A*diag(s)$.

afp If $fact = 'N'$ or 'E', then afp is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization $A=U^T*U$ or $A=L*L^T$ (real routines), $A=U^H*U$ or $A=L*L^H$ (complex routines) of the original matrix A (if $fact = 'N'$), or of the equilibrated matrix A (if $fact = 'E'$). See the description of ap for the form of the equilibrated matrix.

b Overwritten by $diag(s)*B$, if $equed = 'Y'$; not changed if $equed = 'N'$.

s This array is an output argument if $fact \neq 'F'$. See the description of s in *Input Arguments* section.

| | |
|--------------|---|
| <i>rcond</i> | An estimate of the reciprocal condition number of the matrix A after equilibration (if done). If <i>rcond</i> is less than the machine precision (in particular, if <i>rcond</i> = 0), the matrix is singular to working precision. This condition is indicated by a return code of <i>info</i> > 0. |
| <i>ferr</i> | Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the j -th column of the solution matrix X). If <i>xtrue</i> is the true solution corresponding to x_j , <i>ferr</i> [$j-1$] is an estimated upper bound for the magnitude of the largest element in $(x_j - x_{true})$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for <i>rcond</i> , and is almost always a slight overestimate of the true error. |
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution. |
| <i>equed</i> | If <i>fact</i> ≠ 'F', then <i>equed</i> is an output argument. It specifies the form of equilibration that was done (see the description of <i>equed</i> in <i>Input Arguments</i> section). |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, and $i \leq n$, the leading minor of order *i* (and therefore the matrix A itself) is not positive-definite, so the factorization could not be completed, and the solution and error bounds could not be computed; *rcond* = 0 is returned.

If *info* = *i*, and $i = n + 1$, then U is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pbsv

Computes the solution to the system of linear equations with a symmetric or Hermitian positive-definite band coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_spbsv (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_int nrhs , float * ab , lapack_int ldab , float * b , lapack_int ldb );
lapack_int LAPACKE_dpbsv (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_int nrhs , double * ab , lapack_int ldab , double * b , lapack_int ldb );
lapack_int LAPACKE_cpbsv (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_int nrhs , lapack_complex_float * ab , lapack_int ldab ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zpbsv (int matrix_layout , char uplo , lapack_int n , lapack_int
kd , lapack_int nrhs , lapack_complex_double * ab , lapack_int ldab ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the real or complex system of linear equations $A^*X = B$, where A is an n -by- n symmetric/Hermitian positive definite band matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The Cholesky decomposition is used to factor A as

$A = U^T * U$ (real flavors) and $A = U^H * U$ (complex flavors), if $uplo = 'U'$

or $A = L * L^T$ (real flavors) and $A = L * L^H$ (complex flavors), if $uplo = 'L'$,

where U is an upper triangular band matrix and L is a lower triangular band matrix, with the same number of superdiagonals or subdiagonals as A . The factored form of A is then used to solve the system of equations $A^*X = B$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If $uplo = 'U'$, the upper triangle of A is stored. If $uplo = 'L'$, the lower triangle of A is stored. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>kd</i> | The number of superdiagonals of the matrix A if $uplo = 'U'$, or the number of subdiagonals if $uplo = 'L'$; $kd \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <i>ab, b</i> | Arrays: ab (size $\max(1, ldab * n)$), b of size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * n)$ for row major layout. The array ab contains the upper or the lower triangular part of the matrix A (as specified by $uplo$) in <i>band storage</i> (see Matrix Storage Schemes). The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <i>ldab</i> | The leading dimension of the array ab ; $ldab \geq kd + 1$. |
| <i>ldb</i> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-----------|--|
| <i>ab</i> | The upper or lower triangular part of <i>A</i> (in band storage) is overwritten by the Cholesky factor <i>U</i> or <i>L</i> , as specified by <i>uplo</i> , in the same storage format as <i>A</i> . |
| <i>b</i> | Overwritten by the solution matrix <i>X</i> . |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?pbsvx

Uses the Cholesky factorization to compute the solution to the system of linear equations with a symmetric (Hermitian) positive-definite band coefficient matrix A, and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_spbsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int kd, lapack_int nrhs, float* ab, lapack_int ldab, float* afb, lapack_int
lda_fb, char* equed, float* s, float* b, lapack_int ldb, float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dpbsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int kd, lapack_int nrhs, double* ab, lapack_int ldab, double* afb, lapack_int
lda_fb, char* equed, double* s, double* b, lapack_int ldb, double* x, lapack_int ldx,
double* rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cpbsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int kd, lapack_int nrhs, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* afb, lapack_int lda_fb, char* equed, float* s,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zpbsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int kd, lapack_int nrhs, lapack_complex_double* ab, lapack_int ldab,
lapack_complex_double* afb, lapack_int lda_fb, char* equed, double* s,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int ldx,
double* rcond, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine uses the Cholesky factorization $A=U^T*U$ (real flavors) / $A=U^H*U$ (complex flavors) or $A=L*L^T$ (real flavors) / $A=L*L^H$ (complex flavors) to compute the solution to a real or complex system of linear equations $A*X = B$, where A is a n -by- n symmetric or Hermitian positive definite band matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?pbsvx` performs the following steps:

1. If `fact = 'E'`, real scaling factors s are computed to equilibrate the system:

$$\text{diag}(s)*A*\text{diag}(s)*\text{inv}(\text{diag}(s))*X = \text{diag}(s)*B.$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(s)*A*\text{diag}(s)$ and B by $\text{diag}(s)*B$.

2. If `fact = 'N'` or `'E'`, the Cholesky decomposition is used to factor the matrix A (after equilibration if `fact = 'E'`) as

$$A = U^T*U \text{ (real)}, A = U^H*U \text{ (complex)}, \text{ if } \text{uplo} = 'U',$$

$$\text{or } A = L*L^T \text{ (real)}, A = L*L^H \text{ (complex)}, \text{ if } \text{uplo} = 'L',$$

where U is an upper triangular band matrix and L is a lower triangular band matrix.

3. If the leading i -by- i principal minor is not positive definite, then the routine returns with `info = i`. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, `info = n+1` is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
4. The system of equations is solved for X using the factored form of A .
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $\text{diag}(s)$ so that it solves the original system before equilibration.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>fact</code> | <p>Must be 'F', 'N', or 'E'.</p> <p>Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If <code>fact = 'F'</code>: on entry, <code>afb</code> contains the factored form of A. If <code>equed = 'Y'</code>, the matrix A has been equilibrated with scaling factors given by s. <code>ab</code> and <code>afb</code> will not be modified.</p> <p>If <code>fact = 'N'</code>, the matrix A will be copied to <code>afb</code> and factored.</p> <p>If <code>fact = 'E'</code>, the matrix A will be equilibrated if necessary, then copied to <code>afb</code> and factored.</p> |
| <code>uplo</code> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of A is stored:</p> <p>If <code>uplo = 'U'</code>, the upper triangle of A is stored.</p> <p>If <code>uplo = 'L'</code>, the lower triangle of A is stored.</p> |

| | |
|-------------------|---|
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>kd</i> | The number of superdiagonals or subdiagonals in the matrix <i>A</i> ; $kd \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in <i>B</i> ; $nrhs \geq 0$. |
| <i>ab, afb, b</i> | <p>Arrays: <i>ab</i>(size $\max(1, ldab*n)$), <i>afb</i>(size $\max(1, ldafb*n)$), <i>b</i> of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout.</p> <p>The array <i>ab</i> contains the upper or lower triangle of the matrix <i>A</i> in <i>band storage</i> (see Matrix Storage Schemes).</p> <p>If <i>fact</i> = 'F' and <i>equed</i> = 'Y', then <i>ab</i> must contain the equilibrated matrix $diag(s) * A * diag(s)$.</p> <p>The array <i>afb</i> is an input argument if <i>fact</i> = 'F'. It contains the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization of the band matrix <i>A</i> in the same storage format as <i>A</i>. If <i>equed</i> = 'Y', then <i>afb</i> is the factored form of the equilibrated matrix <i>A</i>.</p> <p>The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations.</p> |
| <i>ldab</i> | The leading dimension of <i>ab</i> ; $ldab \geq kd + 1$. |
| <i>ldaafb</i> | The leading dimension of <i>afb</i> ; $ldaafb \geq kd + 1$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>equed</i> | <p>Must be 'N' or 'Y'.</p> <p><i>equed</i> is an input argument if <i>fact</i> = 'F'. It specifies the form of equilibration that was done:</p> <p>if <i>equed</i> = 'N', no equilibration was done (always true if <i>fact</i> = 'N')</p> <p>if <i>equed</i> = 'Y', equilibration was done, that is, <i>A</i> has been replaced by $diag(s) * A * diag(s)$.</p> |
| <i>s</i> | <p>Array, size (<i>n</i>). The array <i>s</i> contains the scale factors for <i>A</i>. This array is an input argument if <i>fact</i> = 'F' only; otherwise it is an output argument.</p> <p>If <i>equed</i> = 'N', <i>s</i> is not accessed.</p> <p>If <i>fact</i> = 'F' and <i>equed</i> = 'Y', each element of <i>s</i> must be positive.</p> |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>x</i> | Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout. |
|----------|--|

If $info = 0$ or $info = n+1$, the array x contains the solution matrix X to the *original* system of equations. Note that if $equed = 'Y'$, A and B are modified on exit, and the solution to the equilibrated system is $inv(diag(s)) * X$.

| | |
|--------------|---|
| <i>ab</i> | On exit, if $fact = 'E'$ and $equed = 'Y'$, A is overwritten by $diag(s) * A * diag(s)$. |
| <i>afb</i> | If $fact = 'N'$ or $'E'$, then <i>afb</i> is an output argument and on exit returns the triangular factor U or L from the Cholesky factorization $A = U^T * U$ or $A = L * L^T$ (real routines), $A = U^H * U$ or $A = L * L^H$ (complex routines) of the original matrix A (if $fact = 'N'$), or of the equilibrated matrix A (if $fact = 'E'$). See the description of <i>ab</i> for the form of the equilibrated matrix. |
| <i>b</i> | Overwritten by $diag(s) * B$, if $equed = 'Y'$; not changed if $equed = 'N'$. |
| <i>s</i> | This array is an output argument if $fact \neq 'F'$. See the description of s in <i>Input Arguments</i> section. |
| <i>rcond</i> | An estimate of the reciprocal condition number of the matrix A after equilibration (if done). If <i>rcond</i> is less than the machine precision (in particular, if $rcond = 0$), the matrix is singular to working precision. This condition is indicated by a return code of $info > 0$. |
| <i>ferr</i> | Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the j -th column of the solution matrix X). If x_{true} is the true solution corresponding to x_j , $ferr[j-1]$ is an estimated upper bound for the magnitude of the largest element in $(x_j - x_{true})$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for <i>rcond</i> , and is almost always a slight overestimate of the true error. |
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution. |
| <i>equed</i> | If $fact \neq 'F'$, then <i>equed</i> is an output argument. It specifies the form of equilibration that was done (see the description of <i>equed</i> in <i>Input Arguments</i> section). |

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, and $i \leq n$, the leading minor of order i (and therefore the matrix A itself) is not positive definite, so the factorization could not be completed, and the solution and error bounds could not be computed; $rcond = 0$ is returned. If $info = i$, and $i = n + 1$, then U is nonsingular, but *rcond* is less than machine precision,

meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?ptsv

Computes the solution to the system of linear equations with a symmetric or Hermitian positive definite tridiagonal coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_sptsv( int matrix_layout, lapack_int n, lapack_int nrhs, float* d,
float* e, float* b, lapack_int ldb );
```

```
lapack_int LAPACKE_dptsv( int matrix_layout, lapack_int n, lapack_int nrhs, double* d,
double* e, double* b, lapack_int ldb );
```

```
lapack_int LAPACKE_cptsv( int matrix_layout, lapack_int n, lapack_int nrhs, float* d,
lapack_complex_float* e, lapack_complex_float* b, lapack_int ldb );
```

```
lapack_int LAPACKE_zptsv( int matrix_layout, lapack_int n, lapack_int nrhs, double* d,
lapack_complex_double* e, lapack_complex_double* b, lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the real or complex system of linear equations $A \cdot X = B$, where A is an n -by- n symmetric/Hermitian positive-definite tridiagonal matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

A is factored as $A = L \cdot D \cdot L^T$ (real flavors) or $A = L \cdot D \cdot L^H$ (complex flavors), and the factored form of A is then used to solve the system of equations $A \cdot X = B$.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <i>d</i> | Array, dimension at least $\max(1, n)$. Contains the diagonal elements of the tridiagonal matrix A . |
| <i>e, b</i> | Arrays: <i>e</i> (size $n - 1$), <i>b</i> of size $\max(1, ldb \cdot nrhs)$ for column major layout and $\max(1, ldb \cdot n)$ for row major layout. The array <i>e</i> contains the $(n - 1)$ subdiagonal elements of A . The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations. |

ldb The leading dimension of *b*; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

Output Parameters

d Overwritten by the *n* diagonal elements of the diagonal matrix *D* from the $L^*D^*L^T$ (real)/ $L^*D^*L^H$ (complex) factorization of *A*.

e Overwritten by the (*n* - 1) subdiagonal elements of the unit bidiagonal factor *L* from the factorization of *A*.

b Overwritten by the solution matrix *X*.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, and the solution has not been computed. The factorization has not been completed unless *i* = *n*.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?ptsvx

Uses factorization to compute the solution to the system of linear equations with a symmetric (Hermitian) positive definite tridiagonal coefficient matrix A, and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_sptsvx( int matrix_layout, char fact, lapack_int n, lapack_int nrhs,
const float* d, const float* e, float* df, float* ef, const float* b, lapack_int ldb,
float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dptsvx( int matrix_layout, char fact, lapack_int n, lapack_int nrhs,
const double* d, const double* e, double* df, double* ef, const double* b, lapack_int
ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cptsvx( int matrix_layout, char fact, lapack_int n, lapack_int nrhs,
const float* d, const lapack_complex_float* e, float* df, lapack_complex_float* ef,
const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x, lapack_int ldx,
float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zptsvx( int matrix_layout, char fact, lapack_int n, lapack_int nrhs,
const double* d, const lapack_complex_double* e, double* df, lapack_complex_double* ef,
const lapack_complex_double* b, lapack_int ldb, lapack_complex_double* x, lapack_int
ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine uses the Cholesky factorization $A = L^*D^*L^T$ (real)/ $A = L^*D^*L^H$ (complex) to compute the solution to a real or complex system of linear equations $A^*X = B$, where A is a n -by- n symmetric or Hermitian positive definite tridiagonal matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?ptsvx` performs the following steps:

1. If `fact = 'N'`, the matrix A is factored as $A = L^*D^*L^T$ (real flavors)/ $A = L^*D^*L^H$ (complex flavors), where L is a unit lower bidiagonal matrix and D is diagonal. The factorization can also be regarded as having the form $A = U^T^*D^*U$ (real flavors)/ $A = U^H^*D^*U$ (complex flavors).
2. If the leading i -by- i principal minor is not positive-definite, then the routine returns with `info = i`. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, `info = n+1` is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A .
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>fact</code> | Must be 'F' or 'N'. Specifies whether or not the factored form of the matrix A is supplied on entry. If <code>fact = 'F'</code> : on entry, <code>df</code> and <code>ef</code> contain the factored form of A . Arrays <code>d</code> , <code>e</code> , <code>df</code> , and <code>ef</code> will not be modified. If <code>fact = 'N'</code> , the matrix A will be copied to <code>df</code> and <code>ef</code> , and factored. |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <code>d, df</code> | Arrays: <code>d</code> (size n), <code>df</code> (size n). The array <code>d</code> contains the n diagonal elements of the tridiagonal matrix A . The array <code>df</code> is an input argument if <code>fact = 'F'</code> and on entry contains the n diagonal elements of the diagonal matrix D from the $L^*D^*L^T$ (real)/ $L^*D^*L^H$ (complex) factorization of A . |
| <code>e,ef,b</code> | Arrays: <code>e</code> (size $n - 1$), <code>ef</code> (size $n - 1$), <code>b</code> , size $\max(ldb*nrhs)$ for column major layout and $\max(ldb*n)$ for row major layout. The array <code>e</code> contains the $(n - 1)$ subdiagonal elements of the tridiagonal matrix A . The array <code>ef</code> is an input argument if <code>fact = 'F'</code> and on entry contains the $(n - 1)$ subdiagonal elements of the unit bidiagonal factor L from the $L^*D^*L^T$ (real)/ $L^*D^*L^H$ (complex) factorization of A . The array <code>b</code> contains the matrix B whose columns are the right-hand sides for the systems of equations. |

| | |
|------------|---|
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|---------------|---|
| <i>x</i> | Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout. If $info = 0$ or $info = n+1$, the array <i>x</i> contains the solution matrix <i>X</i> to the system of equations. |
| <i>df, ef</i> | These arrays are output arguments if $fact = 'N'$. See the description of <i>df, ef</i> in <i>Input Arguments</i> section. |
| <i>rcond</i> | An estimate of the reciprocal condition number of the matrix <i>A</i> after equilibration (if done). If <i>rcond</i> is less than the machine precision (in particular, if $rcond = 0$), the matrix is singular to working precision. This condition is indicated by a return code of $info > 0$. |
| <i>ferr</i> | Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the <i>j</i> -th column of the solution matrix <i>X</i>). If <i>xtrue</i> is the true solution corresponding to x_j , $ferr_j$ is an estimated upper bound for the magnitude of the largest element in $(x_j - xtrue)$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for <i>rcond</i> , and is almost always a slight overestimate of the true error. |
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of <i>A</i> or <i>B</i> that makes x_j an exact solution. |

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, parameter *i* had an illegal value.

If $info = i$, and $i \leq n$, the leading minor of order *i* (and therefore the matrix *A* itself) is not positive-definite, so the factorization could not be completed, and the solution and error bounds could not be computed; $rcond = 0$ is returned.

If $info = i$, and $i = n + 1$, then *U* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?sysv

Computes the solution to the system of linear equations with a real or complex symmetric coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_ssysv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , float * a , lapack_int lda , lapack_int * ipiv , float * b , lapack_int ldb );
lapack_int LAPACKE_dsysv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , double * a , lapack_int lda , lapack_int * ipiv , double * b , lapack_int ldb );
lapack_int LAPACKE_csysv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_float * a , lapack_int lda , lapack_int * ipiv ,
lapack_complex_float * b , lapack_int ldb );
lapack_int LAPACKE_zsysv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_double * a , lapack_int lda , lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the real or complex system of linear equations $A*X = B$, where A is an n -by- n symmetric matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The diagonal pivoting method is used to factor A as $A = U*D*U^T$ or $A = L*D*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The factored form of A is then used to solve the system of equations $A*X = B$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns in B ; $nrhs \geq 0$. |
| <i>a, b</i> | Arrays: a (size $\max(1, lda*n)$), b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. The array a contains the upper or the lower triangular part of the symmetric matrix A (see <i>uplo</i>). |

The array b contains the matrix B whose columns are the right-hand sides for the systems of equations.

lda The leading dimension of a ; $lda \geq \max(1, n)$.

ldb The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

Output Parameters

a If $info = 0$, a is overwritten by the block-diagonal matrix D and the multipliers used to obtain the factor U (or L) from the factorization of A as computed by `?sytrf`.

b If $info = 0$, b is overwritten by the solution matrix X .

$ipiv$ Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D , as determined by `?sytrf`.

If $ipiv[i-1] = k > 0$, then d_{ii} is a 1-by-1 diagonal block, and the i -th row and column of A was interchanged with the k -th row and column.

If $uplo = 'U'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and (i)-th row and column of A was interchanged with the m -th row and column.

If $uplo = 'L'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and ($i+1$)-th row and column of A was interchanged with the m -th row and column.

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, d_{ii} is 0. The factorization has been completed, but D is exactly singular, so the solution could not be computed.

See Also

Matrix Storage Schemes for LAPACK Routines

?sysv_rook

Computes the solution to the system of linear equations with a real or complex symmetric coefficient matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_ssysv_rook (int matrix_layout , char uplo , lapack_int n ,
lapack_int nrhs , float * a , lapack_int lda , lapack_int * ipiv , float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_dsysv_rook (int matrix_layout , char uplo , lapack_int n ,
lapack_int nrhs , double * a , lapack_int lda , lapack_int * ipiv , double * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_csysv_rook (int matrix_layout , char uplo , lapack_int n ,
lapack_int nrhs , lapack_complex_float * a , lapack_int lda , lapack_int * ipiv ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zsysv_rook (int matrix_layout , char uplo , lapack_int n ,
lapack_int nrhs , lapack_complex_double * a , lapack_int lda , lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the real or complex system of linear equations $A^*X = B$, where A is an n -by- n symmetric matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The diagonal pivoting method is used to factor A as $A = U^*D^*U^T$ or $A = L^*D^*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The `?sysv_rook` routine is called to compute the factorization of a complex symmetric matrix A using the bounded Bunch-Kaufman ("rook") diagonal pivoting method.

The factored form of A is then used to solve the system of equations $A^*X = B$.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns in B ; $nrhs \geq 0$. |
| <i>a, b</i> | Arrays: a (size $\max(1, lda*n)$), b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. The array a contains the upper or the lower triangular part of the symmetric matrix A (see <i>uplo</i>). The second dimension of a must be at least $\max(1, n)$. The array b contains the matrix B whose columns are the right-hand sides for the systems of equations. The second dimension of b must be at least $\max(1, nrhs)$. |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | If <i>info</i> = 0, <i>a</i> is overwritten by the block-diagonal matrix <i>D</i> and the multipliers used to obtain the factor <i>U</i> (or <i>L</i>) from the factorization of <i>A</i> . |
| <i>b</i> | If <i>info</i> = 0, <i>b</i> is overwritten by the solution matrix <i>X</i> . |
| <i>ipiv</i> | <p>Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of <i>D</i>.</p> <p>If <i>ipiv</i>[<i>k</i> - 1] > 0, then rows and columns <i>k</i> and <i>ipiv</i>[<i>k</i> - 1] were interchanged and $D_{k,k}$ is a 1-by-1 diagonal block.</p> <p>If <i>uplo</i> = 'U' and <i>ipiv</i>[<i>k</i> - 1] < 0 and <i>ipiv</i>[<i>k</i> - 2] < 0, then rows and columns <i>k</i> and -<i>ipiv</i>[<i>k</i> - 1] were interchanged, rows and columns <i>k</i> - 1 and -<i>ipiv</i>[<i>k</i> - 2] were interchanged, and $D_{k-1:k, k-1:k}$ is a 2-by-2 diagonal block.</p> <p>If <i>uplo</i> = 'L' and <i>ipiv</i>[<i>k</i> - 1] < 0 and <i>ipiv</i>[<i>k</i>] < 0, then rows and columns <i>k</i> and -<i>ipiv</i>[<i>k</i> - 1] were interchanged, rows and columns <i>k</i> + 1 and -<i>ipiv</i>[<i>k</i>] were interchanged, and $D_{k:k+1, k:k+1}$ is a 2-by-2 diagonal block.</p> |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular, so the solution could not be computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?sysvx

*Uses the diagonal pivoting factorization to compute the solution to the system of linear equations with a real or complex symmetric coefficient matrix *A*, and provides error bounds on the solution.*

Syntax

```
lapack_int LAPACKE_ssysvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const float* a, lapack_int lda, float* af, lapack_int ldaf,
lapack_int* ipiv, const float* b, lapack_int ldb, float* x, lapack_int ldx, float*
rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dsysvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const double* a, lapack_int lda, double* af, lapack_int ldaf,
lapack_int* ipiv, const double* b, lapack_int ldb, double* x, lapack_int ldx, double*
rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_csysvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, lapack_complex_float*
af, lapack_int ldaf, lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zsysvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, lapack_complex_double*
af, lapack_int ldaf, lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine uses the diagonal pivoting factorization to compute the solution to a real or complex system of linear equations $A^*X = B$, where A is a n -by- n symmetric matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?sysvx` performs the following steps:

1. If $fact = 'N'$, the diagonal pivoting method is used to factor the matrix A . The form of the factorization is $A = U^*D^*U^T$ or $A = L^*D^*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
2. If some $d_{i,i} = 0$, so that D is exactly singular, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, $info = n+1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A .
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>fact</i> | Must be 'F' or 'N'. Specifies whether or not the factored form of the matrix A has been supplied on entry. If $fact = 'F'$: on entry, af and $ipiv$ contain the factored form of A . Arrays a , af , and $ipiv$ will not be modified. If $fact = 'N'$, the matrix A will be copied to af and factored. |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If $uplo = 'U'$, the upper triangle of A is stored. If $uplo = 'L'$, the lower triangle of A is stored. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |

| | |
|-----------------|--|
| <i>a, af, b</i> | <p>Arrays: <i>a</i>(size $\max(1, lda*n)$), <i>af</i>(size $\max(1, ldaf*n)$), <i>b</i> of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout .</p> <p>The array <i>a</i> contains the upper or the lower triangular part of the symmetric matrix <i>A</i> (see <i>uplo</i>).</p> <p>The array <i>af</i> is an input argument if <i>fact</i> = 'F'. It contains the block diagonal matrix <i>D</i> and the multipliers used to obtain the factor <i>U</i> or <i>L</i> from the factorization $A = U*D*U^T$ or $A = L*D*L^T$ as computed by ?sytrf.</p> <p>The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations.</p> |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ipiv</i> | <p>Array, size at least $\max(1, n)$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. It contains details of the interchanges and the block structure of <i>D</i>, as determined by ?sytrf.</p> <p>If $ipiv[i-1] = k > 0$, then d_{ii} is a 1-by-1 diagonal block, and the <i>i</i>-th row and column of <i>A</i> was interchanged with the <i>k</i>-th row and column.</p> <p>If <i>uplo</i> = 'U' and $ipiv[i] = ipiv[i-1] = -m < 0$, then <i>D</i> has a 2-by-2 block in rows/columns <i>i</i> and <i>i+1</i>, and (<i>i</i>)-th row and column of <i>A</i> was interchanged with the <i>m</i>-th row and column.</p> <p>If <i>uplo</i> = 'L' and $ipiv[i] = ipiv[i-1] = -m < 0$, then <i>D</i> has a 2-by-2 block in rows/columns <i>i</i> and <i>i+1</i>, and (<i>i+1</i>)-th row and column of <i>A</i> was interchanged with the <i>m</i>-th row and column.</p> |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-----------------|--|
| <i>x</i> | <p>Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.</p> <p>If <i>info</i> = 0 or <i>info</i> = <i>n</i>+1, the array <i>x</i> contains the solution matrix <i>X</i> to the system of equations.</p> |
| <i>af, ipiv</i> | <p>These arrays are output arguments if <i>fact</i> = 'N'.</p> <p>See the description of <i>af, ipiv</i> in <i>Input Arguments</i> section.</p> |
| <i>rcond</i> | An estimate of the reciprocal condition number of the matrix <i>A</i> . If <i>rcond</i> is less than the machine precision (in particular, if <i>rcond</i> = 0), the matrix is singular to working precision. This condition is indicated by a return code of <i>info</i> > 0. |

| | |
|-------------|---|
| <i>ferr</i> | Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the j -th column of the solution matrix X). If x_{true} is the true solution corresponding to x_j , $ferr[j-1]$ is an estimated upper bound for the magnitude of the largest element in $(x_j - x_{true})$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for <i>rcond</i> , and is almost always a slight overestimate of the true error. |
| <i>berr</i> | Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, and $i \leq n$, then d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.

If *info* = *i*, and $i = n + 1$, then D is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?sysvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a symmetric indefinite coefficient matrix A applying the diagonal pivoting factorization.

Syntax

```
lapack_int LAPACKE_ssysvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, float* s, float* b, lapack_int ldb, float* x, lapack_int ldx,
float* rcond, float* rpvgrw, float* berr, lapack_int n_err_bnds, float* err_bnds_norm,
float* err_bnds_comp, lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_dsysvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* af, lapack_int ldaf, lapack_int*
ipiv, char* equed, double* s, double* b, lapack_int ldb, double* x, lapack_int ldx,
double* rcond, double* rpvgrw, double* berr, lapack_int n_err_bnds, double*
err_bnds_norm, double* err_bnds_comp, lapack_int nparams, const double* params );
```

```
lapack_int LAPACKE_csysvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, float* s, lapack_complex_float* b,
lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* rcond, float* rpvgrw,
float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp,
lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_zsysvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, double* s, lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* rcond, double*
rpvgrw, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, const double* params );
```

Include Files

- mkl.h

Description

The routine uses the *diagonal pivoting* factorization to compute the solution to a real or complex system of linear equations $A^*X = B$, where A is an n -by- n real symmetric/Hermitian matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error ($O(\text{eps})$, where eps is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with $O(\text{eps})$ errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine `?sysvxx` performs the following steps:

1. If *fact* = 'E', scaling factors are computed to equilibrate the system:

$$\text{diag}(s) * A * \text{diag}(s) * \text{inv}(\text{diag}(s)) * X = \text{diag}(s) * B$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(s) * A * \text{diag}(s)$ and B by $\text{diag}(s) * B$.

2. If *fact* = 'N' or 'E', the LU decomposition is used to factor the matrix A (after equilibration if *fact* = 'E') as

$$A = U * D * U^T, \text{ if } \text{uplo} = \text{'U'},$$

$$\text{or } A = L * D * L^T, \text{ if } \text{uplo} = \text{'L'},$$

where U or L is a product of permutation and unit upper (lower) triangular matrices, and D is a symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

3. If some $D(i, i) = 0$, so that D is exactly singular, the routine returns with *info* = i . Otherwise, the factored form of A is used to estimate the condition number of the matrix A (see the *rcond* parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for X and compute error bounds.
4. The system of equations is solved for X using the factored form of A .
5. By default, unless `params[0]` is set to zero, the routine applies iterative refinement to get a small error and error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix X is premultiplied by $\text{diag}(r)$ so that it solves the original system before equilibration.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

| | |
|-----------------|---|
| <i>fact</i> | <p>Must be 'F', 'N', or 'E'.</p> <p>Specifies whether or not the factored form of the matrix <i>A</i> is supplied on entry, and if not, whether the matrix <i>A</i> should be equilibrated before it is factored.</p> <p>If <i>fact</i> = 'F', on entry, <i>af</i> and <i>ipiv</i> contain the factored form of <i>A</i>. If <i>equed</i> is not 'N', the matrix <i>A</i> has been equilibrated with scaling factors given by <i>s</i>. Parameters <i>a</i>, <i>af</i>, and <i>ipiv</i> are not modified.</p> <p>If <i>fact</i> = 'N', the matrix <i>A</i> will be copied to <i>af</i> and factored.</p> <p>If <i>fact</i> = 'E', the matrix <i>A</i> will be equilibrated, if necessary, copied to <i>af</i> and factored.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of <i>A</i> is stored:</p> <p>If <i>uplo</i> = 'U', the upper triangle of <i>A</i> is stored.</p> <p>If <i>uplo</i> = 'L', the lower triangle of <i>A</i> is stored.</p> |
| <i>n</i> | The number of linear equations; the order of the matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns of the matrices <i>B</i> and <i>X</i> ; $nrhs \geq 0$. |
| <i>a, af, b</i> | <p>Arrays: <i>a</i>(size $\max(1, lda * n)$), <i>af</i>(size $\max(1, ldaf * n)$), <i>b</i>, size $\max(ldb * nrhs)$ for column major layout and $\max(ldb * n)$ for row major layout,.</p> <p>The array <i>a</i> contains the symmetric matrix <i>A</i> as specified by <i>uplo</i>. If <i>uplo</i> = 'U', the leading <i>n</i>-by-<i>n</i> upper triangular part of <i>a</i> contains the upper triangular part of the matrix <i>A</i> and the strictly lower triangular part of <i>a</i> is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i>-by-<i>n</i> lower triangular part of <i>a</i> contains the lower triangular part of the matrix <i>A</i> and the strictly upper triangular part of <i>a</i> is not referenced.</p> <p>The array <i>af</i> is an input argument if <i>fact</i> = 'F'. It contains the block diagonal matrix <i>D</i> and the multipliers used to obtain the factor <i>U</i> and <i>L</i> from the factorization $A = U * D * U^T$ or $A = L * D * L^T$ as computed by ?sytrf.</p> <p>The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations.</p> |
| <i>lda</i> | The leading dimension of the array <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of the array <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ipiv</i> | <p>Array, size at least $\max(1, n)$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. It contains details of the interchanges and the block structure of <i>D</i> as determined by ?sytrf. If <i>ipiv</i>[<i>k</i>-1] > 0, rows and columns <i>k</i> and <i>ipiv</i>[<i>k</i>-1] were interchanged and <i>D</i>(<i>k</i>,<i>k</i>) is a 1-by-1 diagonal block.</p> <p>If <i>uplo</i> = 'U' and <i>ipiv</i>[<i>i</i>] = <i>ipiv</i>[<i>i</i> - 1] = <i>m</i> < 0, <i>D</i> has a 2-by-2 diagonal block in rows and columns <i>i</i> and <i>i</i> + 1, and the <i>i</i>-th row and column of <i>A</i> were interchanged with the <i>m</i>-th row and column.</p> |

If $uplo = 'L'$ and $ipiv[i] = ipiv[i - 1] = m < 0$, D has a 2-by-2 diagonal block in rows and columns i and $i + 1$, and the $(i + 1)$ -st row and column of A were interchanged with the m -th row and column.

eques

Must be 'N' or 'Y'.

eques is an input argument if $fact = 'F'$. It specifies the form of equilibration that was done:

If $eques = 'N'$, no equilibration was done (always true if $fact = 'N'$).

if $eques = 'Y'$, both row and column equilibration was done, that is, A has been replaced by $diag(s) * A * diag(s)$.

s

Array, size (n) . The array s contains the scale factors for A . If $eques = 'Y'$, A is multiplied on the left and right by $diag(s)$.

This array is an input argument if $fact = 'F'$ only; otherwise it is an output argument.

If $fact = 'F'$ and $eques = 'Y'$, each element of s must be positive.

Each element of s should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.

ldb

The leading dimension of the array b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

ldx

The leading dimension of the output array x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

n_err_bnds

Number of error bounds to return for each right hand side and each type (normwise or componentwise). See *err_bnds_norm* and *err_bnds_comp* descriptions in the *Output Arguments* section below.

nparams

Specifies the number of parameters set in *params*. If ≤ 0 , the *params* array is never referenced and default values are used.

params

Array, size $\max(1, nparams)$. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to *nparams* are accessed; defaults are used for higher-numbered parameters. If defaults are acceptable, you can pass $nparams = 0$, which prevents the source code from accessing the *params* argument.

params[0] : Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0 No refinement is performed and no error bounds are computed.

=1.0 Use the extra-precise refinement algorithm.

(Other values are reserved for future use.)

params[1] : Maximum number of residual computations allowed for refinement.

Default 10.0

Aggressive Set to 100.0 to permit convergence using approximate factorizations or factorizations other than *LU*. If the factorization uses a technique other than Gaussian elimination, the guarantees in *err_bnds_norm* and *err_bnds_comp* may no longer be trustworthy.

params[2] : Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

Output Parameters

x Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout).

If *info* = 0, the array *x* contains the solution *n*-by-*nrhs* matrix *X* to the original system of equations. Note that *A* and *B* are modified on exit if *equed* ≠ 'N', and the solution to the equilibrated system is:

$$\text{inv}(\text{diag}(s)) * X.$$

a If *fact* = 'E' and *equed* = 'Y', overwritten by $\text{diag}(s) * A * \text{diag}(s)$.

af If *fact* = 'N', *af* is an output argument and on exit returns the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization $A = U * D * U^T$ or $A = L * D * L^T$.

b If *equed* = 'N', *B* is not modified.

If *equed* = 'Y', *B* is overwritten by $\text{diag}(s) * B$.

s This array is an output argument if *fact* ≠ 'F'. Each element of this array is a power of the radix. See the description of *s* in *Input Arguments* section.

rcond Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix *A* after equilibration (if done). If *rcond* is less than the machine precision, in particular, if *rcond* = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.

rpvgrw Contains the reciprocal pivot growth factor:

$$\|A\| / \|U\|$$

If this is much less than 1, the stability of the *LU* factorization of the (equilibrated) matrix *A* could be poor. This also means that the solution *X*, estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < info \leq n$, this parameter contains the reciprocal pivot growth factor for the leading *info* columns of *A*.

berr Array, size at least $\max(1, nrhs)$. Contains the componentwise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution.

err_bnds_norm Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:

Normwise relative error in the i -th solution vector

$$\frac{\max_j |X_{true_{ji}} - X_{ji}|}{\max_j |X_{ji}|}$$

The array is indexed by the type of error information as described below. Up to three pieces of information are returned.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * a$, where s scales each row by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error *err* is stored in $err_bnds_norm[(err-1)*nrhs + i - 1]$.

err_bnds_comp Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the i -th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested ($params[2] = 0.0$), then `err_bnds_comp` is not accessed.

`err=1` "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors.

`err=2` "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors. This error bound should only be trusted if the previous boolean is true.

`err=3` Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for single precision flavors and $\sqrt{n} * dlamch(\epsilon)$ for double precision flavors to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error `err` is stored in `err_bnds_comp[(err-1)*nrhs + i - 1]`.

`ipiv` If `fact = 'N'`, `ipiv` is an output argument and on exit contains details of the interchanges and the block structure D , as determined by `ssytrf` for single precision flavors and `dsytrf` for double precision flavors.

`equed` If `fact != 'F'`, then `equed` is an output argument. It specifies the form of equilibration that was done (see the description of `equed` in *Input Arguments* section).

params If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful. The solution to every right-hand side is guaranteed.

If *info* = -*i*, parameter *i* had an illegal value.

If $0 < \text{info} \leq n$: $U_{\text{info}, \text{info}}$ is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.

If *info* = *n*+*j*: The solution corresponding to the *j*-th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides *k* with *k* > *j* may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested *params*[2] = 0.0, then the *j*-th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest *j* such that for column major layout $\text{err_bnds_norm}[j - 1] = 0.0$ or $\text{err_bnds_comp}[j - 1] = 0.0$; or for row major layout $\text{err_bnds_norm}[(j - 1)*n_err_bnds] = 0.0$ or $\text{err_bnds_comp}[(j - 1)*n_err_bnds] = 0.0$). See the definition of *err_bnds_norm* and *err_bnds_comp* for *err* = 1. To get information about all of the right-hand sides, check *err_bnds_norm* or *err_bnds_comp*.

See Also

Matrix Storage Schemes for LAPACK Routines

?hesv

Computes the solution to the system of linear equations with a Hermitian matrix A and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_chesv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_float * a , lapack_int lda , lapack_int * ipiv ,
lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_zhesv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_double * a , lapack_int lda , lapack_int * ipiv ,
lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for *X* the complex system of linear equations $A^*X = B$, where *A* is an *n*-by-*n* symmetric matrix, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

The diagonal pivoting method is used to factor *A* as $A = U^*D^*U^H$ or $A = L^*D^*L^H$, where *U* (or *L*) is a product of permutation and unit upper (lower) triangular matrices, and *D* is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The factored form of *A* is then used to solve the system of equations $A^*X = B$.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of A is stored and how A is factored:</p> <p>If $uplo = 'U'$, the array a stores the upper triangular part of the matrix A, and A is factored as U^*D*U^H.</p> <p>If $uplo = 'L'$, the array a stores the lower triangular part of the matrix A, and A is factored as L^*D*L^H.</p> |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <i>a, b</i> | <p>Arrays: a(size $\max(1, lda*n)$), b of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. The array a contains the upper or the lower triangular part of the Hermitian matrix A (see <i>uplo</i>).</p> <p>The array b contains the matrix B whose columns are the right-hand sides for the systems of equations.</p> |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of b ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | If <i>info</i> = 0, a is overwritten by the block-diagonal matrix D and the multipliers used to obtain the factor U (or L) from the factorization of A as computed by ?hetrf . |
| <i>b</i> | If <i>info</i> = 0, b is overwritten by the solution matrix X . |
| <i>ipiv</i> | <p>Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D, as determined by ?hetrf.</p> <p>If $ipiv[i-1] = k > 0$, then d_{ii} is a 1-by-1 diagonal block, and the i-th row and column of A was interchanged with the k-th row and column.</p> <p>If $uplo = 'U'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and (i)-th row and column of A was interchanged with the m-th row and column.</p> <p>If $uplo = 'L'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and $(i+1)$-th row and column of A was interchanged with the m-th row and column.</p> |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, d_{ii} is 0. The factorization has been completed, but *D* is exactly singular, so the solution could not be computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?hesvx

Uses the diagonal pivoting factorization to compute the solution to the complex system of linear equations with a Hermitian coefficient matrix A, and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_chesvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_float* a, lapack_int lda, lapack_complex_float*
af, lapack_int ldaf, lapack_int* ipiv, const lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zhesvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_double* a, lapack_int lda, lapack_complex_double*
af, lapack_int ldaf, lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- mkl.h

Description

The routine uses the diagonal pivoting factorization to compute the solution to a complex system of linear equations $A \cdot X = B$, where *A* is an *n*-by-*n* Hermitian matrix, the columns of matrix *B* are individual right-hand sides, and the columns of *X* are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine ?hesvx performs the following steps:

1. If *fact* = 'N', the diagonal pivoting method is used to factor the matrix *A*. The form of the factorization is $A = U \cdot D \cdot U^H$ or $A = L \cdot D \cdot L^H$, where *U* (or *L*) is a product of permutation and unit upper (lower) triangular matrices, and *D* is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
2. If some $d_{i,i} = 0$, so that *D* is exactly singular, then the routine returns with *info* = *i*. Otherwise, the factored form of *A* is used to estimate the condition number of the matrix *A*. If the reciprocal of the condition number is less than machine precision, *info* = *n*+1 is returned as a warning, but the routine still goes on to solve for *X* and compute error bounds as described below.
3. The system of equations is solved for *X* using the factored form of *A*.
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>fact</i> | <p>Must be 'F' or 'N'.</p> <p>Specifies whether or not the factored form of the matrix <i>A</i> has been supplied on entry.</p> <p>If <i>fact</i> = 'F': on entry, <i>af</i> and <i>ipiv</i> contain the factored form of <i>A</i>. Arrays <i>a</i>, <i>af</i>, and <i>ipiv</i> are not modified.</p> <p>If <i>fact</i> = 'N', the matrix <i>A</i> is copied to <i>af</i> and factored.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of <i>A</i> is stored and how <i>A</i> is factored:</p> <p>If <i>uplo</i> = 'U', the array <i>a</i> stores the upper triangular part of the Hermitian matrix <i>A</i>, and <i>A</i> is factored as $U^*D^*U^H$.</p> <p>If <i>uplo</i> = 'L', the array <i>a</i> stores the lower triangular part of the Hermitian matrix <i>A</i>; <i>A</i> is factored as $L^*D^*L^H$.</p> |
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in <i>B</i> ; $nrhs \geq 0$. |
| <i>a, af, b</i> | <p>Arrays: <i>a</i>(size $\max(1, lda*n)$), <i>af</i>(size $\max(1, ldaf*n)$), <i>b</i> of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout.</p> <p>The array <i>a</i> contains the upper or the lower triangular part of the Hermitian matrix <i>A</i> (see <i>uplo</i>).</p> <p>The array <i>af</i> is an input argument if <i>fact</i> = 'F'. It contains the block diagonal matrix <i>D</i> and the multipliers used to obtain the factor <i>U</i> or <i>L</i> from the factorization $A = U^*D^*U^H$ or $A = L^*D^*L^H$ as computed by ?hetrf.</p> <p>The array <i>b</i> contains the matrix <i>B</i> whose columns are the right-hand sides for the systems of equations.</p> |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ipiv</i> | <p>Array, size at least $\max(1, n)$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. It contains details of the interchanges and the block structure of <i>D</i>, as determined by ?hetrf.</p> <p>If <i>ipiv</i>[<i>i</i>-1] = <i>k</i> > 0, then d_{ii} is a 1-by-1 diagonal block, and the <i>i</i>-th row and column of <i>A</i> was interchanged with the <i>k</i>-th row and column.</p> |

If $uplo = 'U'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and (i) -th row and column of A was interchanged with the m -th row and column.

If $uplo = 'L'$ and $ipiv[i] = ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and $(i+1)$ -th row and column of A was interchanged with the m -th row and column.

ldx

The leading dimension of the output array x ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

x

Array, size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout.

If $info = 0$ or $info = n+1$, the array x contains the solution matrix X to the system of equations.

af, ipiv

These arrays are output arguments if $fact = 'N'$. See the description of *af, ipiv* in *Input Arguments* section.

rcond

An estimate of the reciprocal condition number of the matrix A . If *rcond* is less than the machine precision (in particular, if $rcond = 0$), the matrix is singular to working precision. This condition is indicated by a return code of $info > 0$.

ferr

Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the j -th column of the solution matrix X). If x_{true} is the true solution corresponding to x_j , $ferr[j-1]$ is an estimated upper bound for the magnitude of the largest element in $(x_j) - x_{true}$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.

berr

Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of A or B that makes x_j an exact solution.

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i$, and $i \leq n$, then d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix D is exactly singular, so the solution and error bounds could not be computed; $rcond = 0$ is returned.

If $info = i$, and $i = n + 1$, then D is nonsingular, but $rcond$ is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?hesvxx

Uses extra precise iterative refinement to compute the solution to the system of linear equations with a Hermitian indefinite coefficient matrix A applying the diagonal pivoting factorization.

Syntax

```
lapack_int LAPACKE_chesvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, float* s, lapack_complex_float* b,
lapack_int ldb, lapack_complex_float* x, lapack_int ldx, float* rcond, float* rpvgrw,
float* berr, lapack_int n_err_bnds, float* err_bnds_norm, float* err_bnds_comp,
lapack_int nparams, const float* params );
```

```
lapack_int LAPACKE_zhesvxx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* af,
lapack_int ldaf, lapack_int* ipiv, char* equed, double* s, lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* x, lapack_int ldx, double* rcond, double*
rpvgrw, double* berr, lapack_int n_err_bnds, double* err_bnds_norm, double*
err_bnds_comp, lapack_int nparams, const double* params );
```

Include Files

- mkl.h

Description

The routine uses the *diagonal pivoting* factorization to compute the solution to a complex/double complex system of linear equations $A \cdot X = B$, where A is an n -by- n Hermitian matrix, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Both normwise and maximum componentwise error bounds are also provided on request. The routine returns a solution with a small guaranteed error ($O(\text{eps})$, where eps is the working machine precision) unless the matrix is very ill-conditioned, in which case a warning is returned. Relevant condition numbers are also calculated and returned.

The routine accepts user-provided factorizations and equilibration factors; see definitions of the *fact* and *equed* options. Solving with refinement and using a factorization from a previous call of the routine also produces a solution with $O(\text{eps})$ errors or warnings but that may not be true for general user-provided factorizations and equilibration factors if they differ from what the routine would itself produce.

The routine ?hesvxx performs the following steps:

1. If *fact* = 'E', scaling factors are computed to equilibrate the system:

$$\text{diag}(s) * A * \text{diag}(s) * \text{inv}(\text{diag}(s)) * X = \text{diag}(s) * B$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(s) * A * \text{diag}(s)$ and B by $\text{diag}(s) * B$.

2. If *fact* = 'N' or 'E', the LU decomposition is used to factor the matrix A (after equilibration if *fact* = 'E') as

$$A = U * D * U^T, \text{ if } \text{uplo} = \text{'U'},$$

$$\text{or } A = L * D * L^T, \text{ if } \text{uplo} = \text{'L'},$$

where U or L is a product of permutation and unit upper (lower) triangular matrices, and D is a symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

3. If some $D(i, i) = 0$, so that D is exactly singular, the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A (see the *rcond* parameter). If the reciprocal of the condition number is less than machine precision, the routine still goes on to solve for X and compute error bounds.
4. The system of equations is solved for X using the factored form of A .
5. By default, unless `params[0]` is set to zero, the routine applies iterative refinement to get a small error and error bounds. Refinement calculates the residual to at least twice the working precision.
6. If equilibration was used, the matrix X is premultiplied by $diag(r)$ so that it solves the original system before equilibration.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>fact</i> | <p>Must be 'F', 'N', or 'E'.</p> <p>Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If <i>fact</i> = 'F', on entry, <i>af</i> and <i>ipiv</i> contain the factored form of A. If <i>equed</i> is not 'N', the matrix A has been equilibrated with scaling factors given by <i>s</i>. Parameters <i>a</i>, <i>af</i>, and <i>ipiv</i> are not modified.</p> <p>If <i>fact</i> = 'N', the matrix A will be copied to <i>af</i> and factored.</p> <p>If <i>fact</i> = 'E', the matrix A will be equilibrated, if necessary, copied to <i>af</i> and factored.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of A is stored:</p> <p>If <i>uplo</i> = 'U', the upper triangle of A is stored.</p> <p>If <i>uplo</i> = 'L', the lower triangle of A is stored.</p> |
| <i>n</i> | The number of linear equations; the order of the matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns of the matrices B and X ; $nrhs \geq 0$. |
| <i>a, af, b</i> | <p>Arrays: <i>a</i>(size $\max(lda*n)$), <i>af</i>(size $\max(ldaf*n)$), <i>b</i>, (size $\max(l db*nrhs)$ for column major layout and $\max(l db*n)$ for row major layout),.</p> <p>The array <i>a</i> contains the Hermitian matrix A as specified by <i>uplo</i>. If <i>uplo</i> = 'U', the leading n-by-n upper triangular part of <i>a</i> contains the upper triangular part of the matrix A and the strictly lower triangular part of <i>a</i> is not referenced. If <i>uplo</i> = 'L', the leading n-by-n lower triangular part of <i>a</i> contains the lower triangular part of the matrix A and the strictly upper triangular part of <i>a</i> is not referenced.</p> <p>The array <i>af</i> is an input argument if <i>fact</i> = 'F'. It contains the block diagonal matrix D and the multipliers used to obtain the factor U and L from the factorization $A = U*D*U^T$ or $A = L*D*L^T$ as computed by <code>?hetrf</code>.</p> <p>The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations.</p> |

| | |
|-------------------|---|
| <i>lda</i> | The leading dimension of the array <i>a</i> ; $lda \geq \max(1, n)$. |
| <i>ldaf</i> | The leading dimension of the array <i>af</i> ; $ldaf \geq \max(1, n)$. |
| <i>ipiv</i> | <p>Array, size at least $\max(1, n)$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. It contains details of the interchanges and the block structure of <i>D</i> as determined by ?sytrf.</p> <p>If $ipiv[k-1] > 0$, rows and columns <i>k</i> and $ipiv[k-1]$ were interchanged and $D_{k,k}$ is a 1-by-1 diagonal block.</p> <p>If <i>uplo</i> = 'U' and $ipiv[i] = ipiv[i - 1] = m < 0$, <i>D</i> has a 2-by-2 diagonal block in rows and columns <i>i</i> and <i>i + 1</i>, and the <i>i</i>-th row and column of <i>A</i> were interchanged with the <i>m</i>-th row and column.</p> <p>If <i>uplo</i> = 'L' and $ipiv[i] = ipiv[i - 1] = m < 0$, <i>D</i> has a 2-by-2 diagonal block in rows and columns <i>i</i> and <i>i + 1</i>, and the (<i>i + 1</i>)-st row and column of <i>A</i> were interchanged with the <i>m</i>-th row and column.</p> |
| <i>equed</i> | <p>Must be 'N' or 'Y'.</p> <p><i>equed</i> is an input argument if <i>fact</i> = 'F'. It specifies the form of equilibration that was done:</p> <p>If <i>equed</i> = 'N', no equilibration was done (always true if <i>fact</i> = 'N').</p> <p>if <i>equed</i> = 'Y', both row and column equilibration was done, that is, <i>A</i> has been replaced by $diag(s) * A * diag(s)$.</p> |
| <i>s</i> | <p>Array, size (<i>n</i>). The array <i>s</i> contains the scale factors for <i>A</i>. If <i>equed</i> = 'Y', <i>A</i> is multiplied on the left and right by $diag(s)$.</p> <p>This array is an input argument if <i>fact</i> = 'F' only; otherwise it is an output argument.</p> <p>If <i>fact</i> = 'F' and <i>equed</i> = 'Y', each element of <i>s</i> must be positive.</p> <p>Each element of <i>s</i> should be a power of the radix to ensure a reliable solution and error estimates. Scaling by powers of the radix does not cause rounding errors unless the result underflows or overflows. Rounding errors during scaling lead to refining with a matrix that is not equivalent to the input matrix, producing error estimates that may not be reliable.</p> |
| <i>ldb</i> | The leading dimension of the array <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |
| <i>n_err_bnds</i> | Number of error bounds to return for each right hand side and each type (normwise or componentwise). See err_bnds_norm and err_bnds_comp descriptions in the <i>Output Arguments</i> section below. |
| <i>nparams</i> | Specifies the number of parameters set in <i>params</i> . If ≤ 0 , the <i>params</i> array is never referenced and default values are used. |
| <i>params</i> | Array, size $\max(1, nparams)$. Specifies algorithm parameters. If an entry is less than 0.0, that entry is filled with the default value used for that parameter. Only positions up to <i>nparams</i> are accessed; defaults are used |

for higher-numbered parameters. If defaults are acceptable, you can pass $nparams = 0$, which prevents the source code from accessing the $params$ argument.

$params[0]$: Whether to perform iterative refinement or not. Default: 1.0 (for single precision flavors), 1.0D+0 (for double precision flavors).

=0.0 No refinement is performed and no error bounds are computed.

=1.0 Use the extra-precise refinement algorithm.

(Other values are reserved for future use.)

$params[1]$: Maximum number of residual computations allowed for refinement.

Default 10

Aggressive Set to 100 to permit convergence using approximate factorizations or factorizations other than LU . If the factorization uses a technique other than Gaussian elimination, the guarantees in err_bnds_norm and err_bnds_comp may no longer be trustworthy.

$params[2]$: Flag determining if the code will attempt to find a solution with a small componentwise relative error in the double-precision algorithm. Positive is true, 0.0 is false. Default: 1.0 (attempt componentwise convergence).

Output Parameters

- x Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.
If $info = 0$, the array x contains the solution n -by- $nrhs$ matrix X to the original system of equations. Note that A and B are modified on exit if $equed \neq 'N'$, and the solution to the equilibrated system is:
$$\text{inv}(\text{diag}(s)) * X.$$
- a If $fact = 'E'$ and $equed = 'Y'$, overwritten by $\text{diag}(s) * A * \text{diag}(s)$.
- af If $fact = 'N'$, af is an output argument and on exit returns the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization $A = U * D * U^T$ or $A = L * D * L^T$.
- b If $equed = 'N'$, B is not modified.
If $equed = 'Y'$, B is overwritten by $\text{diag}(s) * B$.
- s This array is an output argument if $fact \neq 'F'$. Each element of this array is a power of the radix. See the description of s in *Input Arguments* section.

| | | | | | | | |
|----------------------|--|---------------|---|---------------|--|---------------|--|
| <i>rcond</i> | <p>Reciprocal scaled condition number. An estimate of the reciprocal Skeel condition number of the matrix <i>A</i> after equilibration (if done). If <i>rcond</i> is less than the machine precision, in particular, if <i>rcond</i> = 0, the matrix is singular to working precision. Note that the error may still be small even if this number is very small and the matrix appears ill-conditioned.</p> | | | | | | |
| <i>rpvgrw</i> | <p>Contains the reciprocal pivot growth factor:</p> $\ A\ /\ U\ $ <p>If this is much less than 1, the stability of the <i>LU</i> factorization of the (equilibrated) matrix <i>A</i> could be poor. This also means that the solution <i>X</i>, estimated condition numbers, and error bounds could be unreliable. If factorization fails with $0 < info \leq n$, this parameter contains the reciprocal pivot growth factor for the leading <i>info</i> columns of <i>A</i>.</p> | | | | | | |
| <i>berr</i> | <p>Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j, that is, the smallest relative change in any element of <i>A</i> or <i>B</i> that makes x_j an exact solution.</p> | | | | | | |
| <i>err_bnds_norm</i> | <p>Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the normwise relative error, which is defined as follows:</p> <p>Normwise relative error in the <i>i</i>-th solution vector</p> $\frac{\max_j X_{true_{ji}} - X_{ji} }{\max_j X_{ji} }$ <p>The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned.</p> <table border="0" style="width: 100%;"> <tr> <td style="vertical-align: top;"><i>err</i>=1</td> <td>"Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i>.</td> </tr> <tr> <td style="vertical-align: top;"><i>err</i>=2</td> <td>"Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i>. This error bound should only be trusted if the previous boolean is true.</td> </tr> <tr> <td style="vertical-align: top;"><i>err</i>=3</td> <td>Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i> to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix <i>Z</i> are:</td> </tr> </table> | <i>err</i> =1 | "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i> . | <i>err</i> =2 | "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i> . This error bound should only be trusted if the previous boolean is true. | <i>err</i> =3 | Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i> to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix <i>Z</i> are: |
| <i>err</i> =1 | "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i> . | | | | | | |
| <i>err</i> =2 | "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i> . This error bound should only be trusted if the previous boolean is true. | | | | | | |
| <i>err</i> =3 | Reciprocal condition number. Estimated normwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for <i>chesvxx</i> and $\sqrt{n} * dlamch(\epsilon)$ for <i>zhesvxx</i> to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix <i>Z</i> are: | | | | | | |

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * a$, where s scales each row by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error err is stored in $err_bnds_norm[(err-1)*nrhs + i - 1]$.

err_bnds_comp

Array of size $nrhs * n_err_bnds$. For each right-hand side, contains information about various error bounds and condition numbers corresponding to the componentwise relative error, which is defined as follows:

Componentwise relative error in the i -th solution vector:

$$\max_j \frac{|X_{true_{ji}} - X_{ji}|}{|X_{ji}|}$$

The array is indexed by the type of error information as described below. There are currently up to three pieces of information returned for each right-hand side. If componentwise accuracy is not requested ($params[2] = 0.0$), then *err_bnds_comp* is not accessed.

err=1 "Trust/don't trust" boolean. Trust the answer if the reciprocal condition number is less than the threshold $\sqrt{n} * slamch(\epsilon)$ for *chesvxx* and $\sqrt{n} * dlamch(\epsilon)$ for *zhesvxx*.

err=2 "Guaranteed" error bound. The estimated forward error, almost certainly within a factor of 10 of the true error so long as the next entry is greater than the threshold $\sqrt{n} * slamch(\epsilon)$ for *chesvxx* and $\sqrt{n} * dlamch(\epsilon)$ for *zhesvxx*. This error bound should only be trusted if the previous boolean is true.

err=3 Reciprocal condition number. Estimated componentwise reciprocal condition number. Compared with the threshold $\sqrt{n} * slamch(\epsilon)$ for *chesvxx* and $\sqrt{n} * dlamch(\epsilon)$ for *zhesvxx* to determine if the error estimate is "guaranteed". These reciprocal condition numbers for some appropriately scaled matrix Z are:

$$\|Z\|_{\infty} \cdot \|Z^{-1}\|_{\infty}$$

Let $z = s * (a * \text{diag}(x))$, where x is the solution for the current right-hand side and s scales each row of $a * \text{diag}(x)$ by a power of the radix so all absolute row sums of z are approximately 1.

The information for right-hand side i , where $1 \leq i \leq nrhs$, and type of error err is stored in `err_bnds_comp[(err-1)*nrhs + i - 1]`.

`ipiv` If `fact = 'N'`, `ipiv` is an output argument and on exit contains details of the interchanges and the block structure D , as determined by `ssytrf` for single precision flavors and `dsytrf` for double precision flavors.

`equed` If `fact ≠ 'F'`, then `equed` is an output argument. It specifies the form of equilibration that was done (see the description of `equed` in *Input Arguments* section).

`params` If an entry is less than 0.0, that entry is filled with the default value used for that parameter, otherwise the entry is not modified.

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful. The solution to every right-hand side is guaranteed.

If `info = -i`, the i -th parameter had an illegal value.

If $0 < info \leq n$: $U_{info,info}$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed; `rcond = 0` is returned.

If `info = n+j`: The solution corresponding to the j -th right-hand side is not guaranteed. The solutions corresponding to other right-hand sides k with $k > j$ may not be guaranteed as well, but only the first such right-hand side is reported. If a small componentwise error is not requested `params[2] = 0.0`, then the j -th right-hand side is the first with a normwise error bound that is not guaranteed (the smallest j such that for column major layout `err_bnds_norm[j - 1] = 0.0` or `err_bnds_comp[j - 1] = 0.0`; or for row major layout `err_bnds_norm[(j - 1)*n_err_bnds] = 0.0` or `err_bnds_comp[(j - 1)*n_err_bnds] = 0.0`). See the definition of `err_bnds_norm` and `err_bnds_comp` for `err = 1`. To get information about all of the right-hand sides, check `err_bnds_norm` or `err_bnds_comp`.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?spsv

Computes the solution to the system of linear equations with a real or complex symmetric coefficient matrix A stored in packed format, and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_spsv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , float * ap , lapack_int * ipiv , float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dspv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , double * ap , lapack_int * ipiv , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_cspsv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_float * ap , lapack_int * ipiv , lapack_complex_float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_zspsv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_double * ap , lapack_int * ipiv , lapack_complex_double * b ,
lapack_int ldb );
```

Include Files

- `mk1.h`

Description

The routine solves for X the real or complex system of linear equations $A^*X = B$, where A is an n -by- n symmetric matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The diagonal pivoting method is used to factor A as $A = U^*D^*U^T$ or $A = L^*D^*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The factored form of A is then used to solve the system of equations $A^*X = B$.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <i>ap, b</i> | Arrays: <i>ap</i> (size $\max(1, n*(n+1)/2)$, <i>bof</i> size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. The array <i>ap</i> contains the factor U or L , as specified by <i>uplo</i> , in <i>packed storage</i> (see Matrix Storage Schemes). The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-------------|---|
| <i>ap</i> | The block-diagonal matrix D and the multipliers used to obtain the factor U (or L) from the factorization of A as computed by <code>?spturf</code> , stored as a packed triangular matrix in the same storage format as A . |
| <i>b</i> | If <i>info</i> = 0, <i>b</i> is overwritten by the solution matrix X . |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D , as determined by <code>?spturf</code> . If <i>ipiv</i> [<i>i</i> -1] = $k > 0$, then d_{ij} is a 1-by-1 block, and the i -th row and column of A was interchanged with the k -th row and column. |

If `uplo = 'U'` and `ipiv[i]=ipiv[i-1] = -m < 0`, then D has a 2-by-2 block in rows/columns i and $i+1$, and i -th row and column of A was interchanged with the m -th row and column.

If `uplo = 'L'` and `ipiv[i-1]=ipiv[i] = -m < 0`, then D has a 2-by-2 block in rows/columns i and $i+1$, and $(i+1)$ -th row and column of A was interchanged with the m -th row and column.

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful.

If `info = -i`, parameter i had an illegal value.

If `info = i`, `dii` is 0. The factorization has been completed, but D is exactly singular, so the solution could not be computed.

See Also

Matrix Storage Schemes for LAPACK Routines

?spsvx

Uses the diagonal pivoting factorization to compute the solution to the system of linear equations with a real or complex symmetric coefficient matrix A stored in packed format, and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_sspsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const float* ap, float* AFP, lapack_int* ipiv, const float* b,
lapack_int ldb, float* x, lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_dspsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const double* ap, double* AFP, lapack_int* ipiv, const double* b,
lapack_int ldb, double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

```
lapack_int LAPACKE_cspsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_float* ap, lapack_complex_float* AFP, lapack_int* ipiv,
const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zspsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_double* ap, lapack_complex_double* AFP,
lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- `mkl.h`

Description

The routine uses the diagonal pivoting factorization to compute the solution to a real or complex system of linear equations $A \cdot X = B$, where A is a n -by- n symmetric matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?spsvx` performs the following steps:

1. If $fact = 'N'$, the diagonal pivoting method is used to factor the matrix A . The form of the factorization is $A = U^*D^*U^T$ or $A = L^*D^*L^T$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is symmetric and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
2. If some $d_{i,i} = 0$, so that D is exactly singular, then the routine returns with $info = i$. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, $info = n+1$ is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A .
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>fact</i> | <p>Must be 'F' or 'N'.</p> <p>Specifies whether or not the factored form of the matrix A has been supplied on entry.</p> <p>If $fact = 'F'$: on entry, <i>ap</i> and <i>ipiv</i> contain the factored form of A. Arrays <i>ap</i>, <i>afp</i>, and <i>ipiv</i> are not modified.</p> <p>If $fact = 'N'$, the matrix A is copied to <i>afp</i> and factored.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>Indicates whether the upper or lower triangular part of A is stored and how A is factored:</p> <p>If $uplo = 'U'$, the array <i>ap</i> stores the upper triangular part of the symmetric matrix A, and A is factored as $U^*D^*U^T$.</p> <p>If $uplo = 'L'$, the array <i>ap</i> stores the lower triangular part of the symmetric matrix A; A is factored as $L^*D^*L^T$.</p> |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <i>ap, afp, b</i> | <p>Arrays: <i>ap</i> (size $\max(1, n*(n+1)/2)$), <i>afp</i> (size $\max(1, n*(n+1)/2)$), <i>b</i> of size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout.</p> <p>The array <i>ap</i> contains the upper or lower triangle of the symmetric matrix A in <i>packed storage</i> (see Matrix Storage Schemes).</p> <p>The array <i>afp</i> is an input argument if $fact = 'F'$. It contains the block diagonal matrix D and the multipliers used to obtain the factor U or L from the factorization $A = U^*D^*U^T$ or $A = L^*D^*L^T$ as computed by <code>?sptrf</code>, in the same storage format as A.</p> <p>The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations.</p> |

| | |
|-------------|--|
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |
| <i>ipiv</i> | <p>Array, size at least $\max(1, n)$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. It contains details of the interchanges and the block structure of <i>D</i>, as determined by <code>?sptrf</code>.</p> <p>If $ipiv[i-1] = k > 0$, then d_{ij} is a 1-by-1 block, and the <i>i</i>-th row and column of <i>A</i> was interchanged with the <i>k</i>-th row and column.</p> <p>If $uplo = 'U'$ and $ipiv[i]=ipiv[i-1] = -m < 0$, then <i>D</i> has a 2-by-2 block in rows/columns <i>i</i> and <i>i+1</i>, and <i>i</i>-th row and column of <i>A</i> was interchanged with the <i>m</i>-th row and column.</p> <p>If $uplo = 'L'$ and $ipiv[i-1] = ipiv[i] = -m < 0$, then <i>D</i> has a 2-by-2 block in rows/columns <i>i</i> and <i>i+1</i>, and (<i>i+1</i>)-th row and column of <i>A</i> was interchanged with the <i>m</i>-th row and column.</p> |
| <i>ldx</i> | The leading dimension of the output array <i>x</i> ; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-------------------|--|
| <i>x</i> | <p>Array, size $\max(1, ldx*nrhs)$ for column major layout and $\max(1, ldx*n)$ for row major layout.</p> <p>If <i>info</i> = 0 or <i>info</i> = <i>n</i>+1, the array <i>x</i> contains the solution matrix <i>X</i> to the system of equations.</p> |
| <i>afp, ipiv</i> | These arrays are output arguments if <i>fact</i> = 'N'. See the description of <i>afp, ipiv</i> in <i>Input Arguments</i> section. |
| <i>rcond</i> | An estimate of the reciprocal condition number of the matrix <i>A</i> . If <i>rcond</i> is less than the machine precision (in particular, if <i>rcond</i> = 0), the matrix is singular to working precision. This condition is indicated by a return code of <i>info</i> > 0. |
| <i>ferr, berr</i> | Arrays, size at least $\max(1, nrhs)$. Contain the component-wise forward and relative backward errors, respectively, for each solution vector. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, and $i \leq n$, then d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.

If *info* = *i*, and $i = n + 1$, then *D* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?hpsv

Computes the solution to the system of linear equations with a Hermitian coefficient matrix A stored in packed format, and multiple right-hand sides.

Syntax

```
lapack_int LAPACKE_chpsv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_float * ap , lapack_int * ipiv , lapack_complex_float * b ,
lapack_int ldb );
```

```
lapack_int LAPACKE_zhpsv (int matrix_layout , char uplo , lapack_int n , lapack_int
nrhs , lapack_complex_double * ap , lapack_int * ipiv , lapack_complex_double * b ,
lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine solves for X the system of linear equations $A^*X = B$, where A is an n -by- n Hermitian matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

The diagonal pivoting method is used to factor A as $A = U^*D^*U^H$ or $A = L^*D^*L^H$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.

The factored form of A is then used to solve the system of equations $A^*X = B$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored: If <i>uplo</i> = 'U', the upper triangle of A is stored. If <i>uplo</i> = 'L', the lower triangle of A is stored. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; the number of columns in B ; $nrhs \geq 0$. |
| <i>ap</i> , <i>b</i> | Arrays: <i>ap</i> (size $\max(1, n*(n+1)/2)$, <i>bof</i> size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. The array <i>ap</i> contains the factor U or L , as specified by <i>uplo</i> , in packed storage (see Matrix Storage Schemes). The array <i>b</i> contains the matrix B whose columns are the right-hand sides for the systems of equations. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout. |

Output Parameters

| | |
|-------------|--|
| <i>ap</i> | The block-diagonal matrix D and the multipliers used to obtain the factor U (or L) from the factorization of A as computed by <code>?hptrf</code> , stored as a packed triangular matrix in the same storage format as A . |
| <i>b</i> | If $info = 0$, b is overwritten by the solution matrix X . |
| <i>ipiv</i> | <p>Array, size at least $\max(1, n)$. Contains details of the interchanges and the block structure of D, as determined by <code>?hptrf</code>.</p> <p>If $ipiv[i-1] = k > 0$, then d_{ij} is a 1-by-1 block, and the i-th row and column of A was interchanged with the k-th row and column.</p> <p>If $uplo = 'U'$ and $ipiv[i]=ipiv[i-1] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and i-th row and column of A was interchanged with the m-th row and column.</p> <p>If $uplo = 'L'$ and $ipiv[i-1]=ipiv[i] = -m < 0$, then D has a 2-by-2 block in rows/columns i and $i+1$, and $(i+1)$-th row and column of A was interchanged with the m-th row and column.</p> |

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, parameter i had an illegal value.

If $info = i, d_{ii}$ is 0. The factorization has been completed, but D is exactly singular, so the solution could not be computed.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

?hpsvx

Uses the diagonal pivoting factorization to compute the solution to the system of linear equations with a Hermitian coefficient matrix A stored in packed format, and provides error bounds on the solution.

Syntax

```
lapack_int LAPACKE_chpsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_float* ap, lapack_complex_float* afp, lapack_int*
ipiv, const lapack_complex_float* b, lapack_int ldb, lapack_complex_float* x,
lapack_int ldx, float* rcond, float* ferr, float* berr );
```

```
lapack_int LAPACKE_zhpsvx( int matrix_layout, char fact, char uplo, lapack_int n,
lapack_int nrhs, const lapack_complex_double* ap, lapack_complex_double* afp,
lapack_int* ipiv, const lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* x, lapack_int ldx, double* rcond, double* ferr, double* berr );
```

Include Files

- `mkl.h`

Description

The routine uses the diagonal pivoting factorization to compute the solution to a complex system of linear equations $A*X = B$, where A is a n -by- n Hermitian matrix stored in packed format, the columns of matrix B are individual right-hand sides, and the columns of X are the corresponding solutions.

Error bounds on the solution and a condition estimate are also provided.

The routine `?hpsvx` performs the following steps:

1. If `fact = 'N'`, the diagonal pivoting method is used to factor the matrix A . The form of the factorization is $A = U*D*U^H$ or $A = L*D*L^H$, where U (or L) is a product of permutation and unit upper (lower) triangular matrices, and D is a Hermitian and block diagonal with 1-by-1 and 2-by-2 diagonal blocks.
2. If some $d_{i,i} = 0$, so that D is exactly singular, then the routine returns with `info = i`. Otherwise, the factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, `info = n+1` is returned as a warning, but the routine still goes on to solve for X and compute error bounds as described below.
3. The system of equations is solved for X using the factored form of A .
4. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>fact</code> | Must be 'F' or 'N'. Specifies whether or not the factored form of the matrix A has been supplied on entry. If <code>fact = 'F'</code> : on entry, <code>afp</code> and <code>ipiv</code> contain the factored form of A . Arrays <code>ap</code> , <code>afp</code> , and <code>ipiv</code> are not modified. If <code>fact = 'N'</code> , the matrix A is copied to <code>afp</code> and factored. |
| <code>uplo</code> | Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored and how A is factored: If <code>uplo = 'U'</code> , the array <code>ap</code> stores the upper triangular part of the Hermitian matrix A , and A is factored as $U*D*U^H$. If <code>uplo = 'L'</code> , the array <code>ap</code> stores the lower triangular part of the Hermitian matrix A , and A is factored as $L*D*L^H$. |
| <code>n</code> | The order of matrix A ; $n \geq 0$. |
| <code>nrhs</code> | The number of right-hand sides, the number of columns in B ; $nrhs \geq 0$. |
| <code>ap, afp, b</code> | Arrays: <code>ap</code> (size $\max(1, n*(n+1)/2)$), <code>afp</code> (size $\max(1, n*(n+1)/2)$), <code>bof</code> size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*n)$ for row major layout. The array <code>ap</code> contains the upper or lower triangle of the Hermitian matrix A in <i>packed storage</i> (see Matrix Storage Schemes). |

The array *afp* is an input argument if *fact* = 'F'. It contains the block diagonal matrix *D* and the multipliers used to obtain the factor *U* or *L* from the factorization $A = U * D * U^H$ or $A = L * D * L^H$ as computed by `?hptrfs`, in the same storage format as *A*.

The array *b* contains the matrix *B* whose columns are the right-hand sides for the systems of equations.

ldb The leading dimension of *b*; $ldb \geq \max(1, n)$ for column major layout and $ldb \geq nrhs$ for row major layout.

ipiv Array, size at least $\max(1, n)$. The array *ipiv* is an input argument if *fact* = 'F'. It contains details of the interchanges and the block structure of *D*, as determined by `?hptrfs`.

If $ipiv[i-1] = k > 0$, then d_{ii} is a 1-by-1 block, and the *i*-th row and column of *A* was interchanged with the *k*-th row and column.

If $uplo = 'U'$ and $ipiv[i]=ipiv[i-1] = -m < 0$, then *D* has a 2-by-2 block in rows/columns *i* and *i+1*, and *i*-th row and column of *A* was interchanged with the *m*-th row and column.

If $uplo = 'L'$ and $ipiv[i-1]=ipiv[i] = -m < 0$, then *D* has a 2-by-2 block in rows/columns *i* and *i+1*, and (*i+1*)-th row and column of *A* was interchanged with the *m*-th row and column.

ldx The leading dimension of the output array *x*; $ldx \geq \max(1, n)$ for column major layout and $ldx \geq nrhs$ for row major layout.

Output Parameters

x Array, size $\max(1, ldx * nrhs)$ for column major layout and $\max(1, ldx * n)$ for row major layout.

If *info* = 0 or *info* = *n*+1, the array *x* contains the solution matrix *X* to the system of equations.

afp, ipiv These arrays are output arguments if *fact* = 'N'. See the description of *afp, ipiv* in *Input Arguments* section.

rcond An estimate of the reciprocal condition number of the matrix *A*. If *rcond* is less than the machine precision (in particular, if *rcond* = 0), the matrix is singular to working precision. This condition is indicated by a return code of *info* > 0.

ferr Array, size at least $\max(1, nrhs)$. Contains the estimated forward error bound for each solution vector x_j (the *j*-th column of the solution matrix *X*). If *xtrue* is the true solution corresponding to x_j , $ferr[j-1]$ is an estimated upper bound for the magnitude of the largest element in $(x_j - xtrue)$ divided by the magnitude of the largest element in x_j . The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.

berr Array, size at least $\max(1, nrhs)$. Contains the component-wise relative backward error for each solution vector x_j , that is, the smallest relative change in any element of *A* or *B* that makes x_j an exact solution.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, parameter *i* had an illegal value.

If *info* = *i*, and $i \leq n$, then d_{ii} is exactly zero. The factorization has been completed, but the block diagonal matrix *D* is exactly singular, so the solution and error bounds could not be computed; *rcond* = 0 is returned.

If *info* = *i*, and $i = n + 1$, then *D* is nonsingular, but *rcond* is less than machine precision, meaning that the matrix is singular to working precision. Nevertheless, the solution and error bounds are computed because there are a number of situations where the computed solution can be more accurate than the value of *rcond* would suggest.

See Also

[Matrix Storage Schemes for LAPACK Routines](#)

LAPACK Least Squares and Eigenvalue Problem Routines

This section includes descriptions of LAPACK [computational routines](#) and [driver routines](#) for solving linear least squares problems, eigenvalue and singular value problems, and performing a number of related computational tasks. For a full reference on LAPACK routines and related information see [LUG].

Least Squares Problems. A typical *least squares problem* is as follows: given a matrix *A* and a vector *b*, find the vector *x* that minimizes the sum of squares $\sum_i ((Ax)_i - b_i)^2$ or, equivalently, find the vector *x* that minimizes the 2-norm $\|Ax - b\|_2$.

In the most usual case, *A* is an *m*-by-*n* matrix with $m \geq n$ and $\text{rank}(A) = n$. This problem is also referred to as finding the *least squares solution* to an *overdetermined* system of linear equations (here we have more equations than unknowns). To solve this problem, you can use the *QR* factorization of the matrix *A* (see [QR Factorization](#)).

If $m < n$ and $\text{rank}(A) = m$, there exist an infinite number of solutions *x* which exactly satisfy $Ax = b$, and thus minimize the norm $\|Ax - b\|_2$. In this case it is often useful to find the unique solution that minimizes $\|x\|_2$. This problem is referred to as finding the *minimum-norm solution* to an *underdetermined* system of linear equations (here we have more unknowns than equations). To solve this problem, you can use the *LQ* factorization of the matrix *A* (see [LQ Factorization](#)).

In the general case you may have a *rank-deficient least squares problem*, with $\text{rank}(A) < \min(m, n)$: find the *minimum-norm least squares solution* that minimizes both $\|x\|_2$ and $\|Ax - b\|_2^2$. In this case (or when the rank of *A* is in doubt) you can use the *QR* factorization with pivoting or *singular value decomposition* (see [Singular Value Decomposition](#)).

Eigenvalue Problems. The eigenvalue problems (from German *eigen* "own") are stated as follows: given a matrix *A*, find the *eigenvalues* λ and the corresponding *eigenvectors* *z* that satisfy the equation

$$Az = \lambda z \text{ (right eigenvectors } z)$$

or the equation

$$z^H A = \lambda z^H \text{ (left eigenvectors } z).$$

If *A* is a real symmetric or complex Hermitian matrix, the above two equations are equivalent, and the problem is called a *symmetric* eigenvalue problem. Routines for solving this type of problems are described in the section [Symmetric Eigenvalue Problems](#).

Routines for solving eigenvalue problems with nonsymmetric or non-Hermitian matrices are described in the section [Nonsymmetric Eigenvalue Problems](#).

The library also includes routines that handle *generalized symmetric-definite eigenvalue problems*: find the eigenvalues λ and the corresponding eigenvectors *x* that satisfy one of the following equations:

$$Az = \lambda Bz, \quad ABz = \lambda z, \quad \text{or} \quad BAz = \lambda z,$$

where A is symmetric or Hermitian, and B is symmetric positive-definite or Hermitian positive-definite. Routines for reducing these problems to standard symmetric eigenvalue problems are described in the section [Generalized Symmetric-Definite Eigenvalue Problems](#).

To solve a particular problem, you usually call several computational routines. Sometimes you need to combine the routines of this chapter with other LAPACK routines described in "LAPACK Routines: Linear Equations" as well as with BLAS routines described in "BLAS and Sparse BLAS Routines".

For example, to solve a set of least squares problems minimizing $\|Ax - b\|^2$ for all columns b of a given matrix B (where A and B are real matrices), you can call `?geqrf` to form the factorization $A = QR$, then call `?ormqr` to compute $C = Q^H B$ and finally call the BLAS routine `?trsm` to solve for X the system of equations $RX = C$.

Another way is to call an appropriate driver routine that performs several tasks in one call. For example, to solve the least squares problem the driver routine `?gels` can be used.

LAPACK Least Squares and Eigenvalue Problem Computational Routines

In the sections that follow, the descriptions of LAPACK computational routines are given. These routines perform distinct computational tasks that can be used for:

[Orthogonal Factorizations](#)

[Singular Value Decomposition](#)

[Symmetric Eigenvalue Problems](#)

[Generalized Symmetric-Definite Eigenvalue Problems](#)

[Nonsymmetric Eigenvalue Problems](#)

[Generalized Nonsymmetric Eigenvalue Problems](#)

[Generalized Singular Value Decomposition](#)

See also the respective [driver routines](#).

Orthogonal Factorizations: LAPACK Computational Routines

This section describes the LAPACK routines for the QR (RQ) and LQ (QL) factorization of matrices. Routines for the RZ factorization as well as for generalized QR and RQ factorizations are also included.

QR Factorization. Assume that A is an m -by- n matrix to be factored.

If $m \geq n$, the QR factorization is given by

$$A = Q \begin{pmatrix} R \\ 0 \end{pmatrix} = (Q_1, Q_2) \begin{pmatrix} R \\ 0 \end{pmatrix}$$

where R is an n -by- n upper triangular matrix with real diagonal elements, and Q is an m -by- m orthogonal (or unitary) matrix.

You can use the QR factorization for solving the following least squares problem: minimize $\|Ax - b\|^2$ where A is a full-rank m -by- n matrix ($m \geq n$). After factoring the matrix, compute the solution x by solving $Rx = (Q_1)^T b$.

If $m < n$, the QR factorization is given by

$$A = QR = Q(R_1 R_2)$$

where R is trapezoidal, R_1 is upper triangular and R_2 is rectangular.

Q is represented as a product of $\min(m, n)$ *elementary reflectors*. Routines are provided to work with Q in this representation.

LQ Factorization LQ factorization of an m -by- n matrix A is as follows. If $m \leq n$,

$$A = (L, 0) Q = (L, 0) \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = (LQ_1)$$

where L is an m -by- m lower triangular matrix with real diagonal elements, and Q is an n -by- n orthogonal (or unitary) matrix.

If $m > n$, the LQ factorization is

$$A = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} Q$$

where L_1 is an n -by- n lower triangular matrix, L_2 is rectangular, and Q is an n -by- n orthogonal (or unitary) matrix.

You can use the LQ factorization to find the minimum-norm solution of an underdetermined system of linear equations $Ax = b$ where A is an m -by- n matrix of rank m ($m < n$). After factoring the matrix, compute the solution vector x as follows: solve $L_1 y = b$ for y , and then compute $x = (Q_1)^H y$.

Table "Computational Routines for Orthogonal Factorization" lists LAPACK routines that perform orthogonal factorization of matrices.

Computational Routines for Orthogonal Factorization

| Matrix type, factorization | Factorize without pivoting | Factorize with pivoting | Generate matrix Q | Apply matrix Q |
|--|----------------------------|-------------------------|-------------------|----------------|
| general matrices, QR factorization | geqrf | geqpf | orgqr | ormqr |
| | geqrfp | geqp3 | ungqr | unmqr |
| general matrices, blocked QR factorization | geqrt | | | gemqrt |
| general matrices, RQ factorization | gerqf | | orgrq | ormrq |
| | | | ungrq | unmrq |
| general matrices, LQ factorization | gelqf | | orglq | ormlq |
| | | | unglq | unmlq |
| general matrices, QL factorization | geqlf | | orgql | ormql |
| | | | ungql | unmql |
| trapezoidal matrices, RZ factorization | tzzrf | | | ormrz |
| | | | | unmrz |
| pair of matrices, generalized QR factorization | ggqrf | | | |
| pair of matrices, generalized RQ factorization | ggrqf | | | |
| triangular-pentagonal matrices, blocked QR factorization | tpqrt | | | tpmqrt |

?sgeqrf

Computes the QR factorization of a general m -by- n matrix.

Syntax

```
lapack_int LAPACKE_sgeqrf (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* tau);
```

```
lapack_int LAPACKE_dgeqrf (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* tau);
```

```
lapack_int LAPACKE_cgeqrf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zgeqrf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine forms the QR factorization of a general m -by- n matrix A (see [Orthogonal Factorizations](#)). No pivoting is performed.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of $\min(m, n)$ elementary reflectors. Routines are provided to work with Q in this representation.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows in the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns in A ($n \geq 0$). |
| <i>a</i> | Array a of size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout contains the matrix A . |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | Overwritten by the factorization data as follows: The elements on and above the diagonal of the array contain the $\min(m, n)$ -by- n upper trapezoidal matrix R (R is upper triangular if $m \geq n$); the elements below the diagonal, with the array tau , present the orthogonal matrix Q as a product of $\min(m, n)$ elementary reflectors (see Orthogonal Factorizations). |
|----------|---|

tau Array, size at least $\max(1, \min(m, n))$. Contains scalars that define elementary reflectors for the matrix Q in its decomposition in a product of elementary reflectors (see [Orthogonal Factorizations](#)).

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed factorization is the exact factorization of a matrix $A + E$, where

$$\|E\|_2 = O(\epsilon) \|A\|_2.$$

The approximate number of floating-point operations for real flavors is

$$(4/3)n^3 \quad \text{if } m = n,$$

$$(2/3)n^2(3m-n) \quad \text{if } m > n,$$

$$(2/3)m^2(3n-m) \quad \text{if } m < n.$$

The number of operations for complex flavors is 4 times greater.

To solve a set of least squares problems minimizing $\|A^*x - b\|_2$ for all columns b of a given matrix B , you can call the following:

| | |
|------------------------------------|--|
| <code>?geqrf</code> (this routine) | to factorize $A = QR$; |
| <code>ormqr</code> | to compute $C = Q^T B$ (for real matrices); |
| <code>unmqr</code> | to compute $C = Q^H B$ (for complex matrices); |
| <code>trsm</code> (a BLAS routine) | to solve $R^*X = C$. |

(The columns of the computed X are the least squares solution vectors x .)

To compute the elements of Q explicitly, call

`orgqr` (for real matrices)

`ungqr` (for complex matrices).

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?geqrfp

Computes the QR factorization of a general m -by- n matrix with non-negative diagonal elements.

Syntax

```
lapack_int LAPACKE_sgeqrfp (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* tau);
```

```
lapack_int LAPACKE_dgeqrfp (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* tau);
```

```
lapack_int LAPACKE_cgeqrfp (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zgeqrfp (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine forms the QR factorization of a general m -by- n matrix A (see [Orthogonal Factorizations](#)). No pivoting is performed. The diagonal entries of R are real and nonnegative.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of $\min(m, n)$ *elementary reflectors*. Routines are provided to work with Q in this representation.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows in the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns in A ($n \geq 0$). |
| <i>a</i> | Array, size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout, containing the matrix A . |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|------------|--|
| <i>a</i> | Overwritten by the factorization data as follows: The elements on and above the diagonal of the array contain the $\min(m, n)$ -by- n upper trapezoidal matrix R (R is upper triangular if $m \geq n$); the elements below the diagonal, with the array <i>tau</i> , present the orthogonal matrix Q as a product of $\min(m, n)$ elementary reflectors (see Orthogonal Factorizations). The diagonal elements of the matrix R are real and non-negative. |
| <i>tau</i> | Array, size at least $\max(1, \min(m, n))$. Contains scalars that define elementary reflectors for the matrix Q in its decomposition in a product of elementary reflectors (see Orthogonal Factorizations). |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The computed factorization is the exact factorization of a matrix $A + E$, where

$$\|E\|_2 = O(\epsilon) \|A\|_2.$$

The approximate number of floating-point operations for real flavors is

$$(4/3)n^3 \quad \text{if } m = n,$$

$$(2/3)n^2(3m-n) \quad \text{if } m > n,$$

$$(2/3)m^2(3n-m) \quad \text{if } m < n.$$

The number of operations for complex flavors is 4 times greater.

To solve a set of least squares problems minimizing $\|A^*x - b\|_2$ for all columns b of a given matrix B , you can call the following:

| | |
|-------------------------------------|--|
| <code>?geqrfp</code> (this routine) | to factorize $A = QR$; |
| <code>ormqr</code> | to compute $C = Q^T * B$ (for real matrices); |
| <code>unmqr</code> | to compute $C = Q^H * B$ (for complex matrices); |
| <code>trsm</code> (a BLAS routine) | to solve $R * X = C$. |

(The columns of the computed X are the least squares solution vectors x .)

To compute the elements of Q explicitly, call

| | |
|--------------------|-------------------------|
| <code>orgqr</code> | (for real matrices) |
| <code>ungqr</code> | (for complex matrices). |

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?geqrt

Computes a blocked QR factorization of a general real or complex matrix using the compact WY representation of Q.

Syntax

```
lapack_int LAPACKE_sgeqrt (int matrix_layout, lapack_int m, lapack_int n, lapack_int nb, float* a, lapack_int lda, float* t, lapack_int ldt);
```

```
lapack_int LAPACKE_dgeqrt (int matrix_layout, lapack_int m, lapack_int n, lapack_int nb, double* a, lapack_int lda, double* t, lapack_int ldt);
```

```
lapack_int LAPACKE_cgeqrt (int matrix_layout, lapack_int m, lapack_int n, lapack_int nb, lapack_complex_float* a, lapack_int lda, lapack_complex_float* t, lapack_int ldt);
```

```
lapack_int LAPACKE_zgeqrt (int matrix_layout, lapack_int m, lapack_int n, lapack_int nb, lapack_complex_double* a, lapack_int lda, lapack_complex_double* t, lapack_int ldt);
```

Include Files

- `mkl.h`

Description

The strictly lower triangular matrix V contains the elementary reflectors $H(i)$ in the i th column below the diagonal. For example, if $m=5$ and $n=3$, the matrix V is

$$V = \begin{bmatrix} 1 & & & & \\ v_1 & 1 & & & \\ v_1 & v_2 & 1 & & \\ v_1 & v_2 & v_3 & & \\ v_1 & v_2 & v_3 & & \end{bmatrix}$$

where v_i represents one of the vectors that define $H(i)$. The vectors are returned in the lower triangular part of array a .

NOTE

The 1s along the diagonal of V are not stored in a .

Let $k = \min(m, n)$. The number of blocks is $b = \text{ceiling}(k/nb)$, where each block is of order nb except for the last block, which is of order $ib = k - (b-1)*nb$. For each of the b blocks, a upper triangular block reflector factor is computed: $t1, t2, \dots, tb$. The nb -by- nb (and ib -by- ib for the last block) ts are stored in the nb -by- n array t as

$$t = (t1t2 \dots tb).$$

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>m</code> | The number of rows in the matrix A ($m \geq 0$). |
| <code>n</code> | The number of columns in A ($n \geq 0$). |
| <code>nb</code> | The block size to be used in the blocked QR ($\min(m, n) \geq nb \geq 1$). |
| <code>a</code> | Array a of size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout contains the m -by- n matrix A . |
| <code>lda</code> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <code>ldt</code> | The leading dimension of t ; at least nb for column major layout and $\max(1, \min(m, n))$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | Overwritten by the factorization data as follows: The elements on and above the diagonal of the array contain the $\min(m,n)$ -by- n upper trapezoidal matrix R (R is upper triangular if $m \geq n$); the elements below the diagonal, with the array t , present the orthogonal matrix Q as a product of $\min(m,n)$ elementary reflectors (see Orthogonal Factorizations). |
| <i>t</i> | Array, size $\max(1, ldt * \min(m, n))$ for column major layout and $\max(1, ldt * nb)$ for row major layout. The upper triangular block reflector's factors stored as a sequence of upper triangular blocks. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* < 0 and *info* = -*i*, the *i*-th parameter had an illegal value.

?gemqrt

Multiplies a general matrix by the orthogonal/unitary matrix Q of the QR factorization formed by ?geqrt.

Syntax

```
lapack_int LAPACKE_sgemqrt (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int nb, const float* v, lapack_int ldv, const
float* t, lapack_int ldt, float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_dgemqrt (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int nb, const double* v, lapack_int ldv, const
double* t, lapack_int ldt, double* c, lapack_int ldc);
```

```
lapack_int LAPACKE_cgemqrt (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int nb, const lapack_complex_float* v, lapack_int
ldv, const lapack_complex_float* t, lapack_int ldt, lapack_complex_float* c, lapack_int
ldc);
```

```
lapack_int LAPACKE_zgemqrt (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int nb, const lapack_complex_double* v, lapack_int
ldv, const lapack_complex_double* t, lapack_int ldt, lapack_complex_double* c,
lapack_int ldc);
```

Include Files

- mkl.h

Description

The ?gemqrt routine overwrites the general real or complex m -by- n matrix C with

| | | |
|---------------------|-------------------|-------------------|
| | <i>side</i> = 'L' | <i>side</i> = 'R' |
| <i>trans</i> = 'N': | $Q * C$ | $C * Q$ |
| <i>trans</i> = 'T': | $Q^T * C$ | $C * Q^T$ |
| <i>trans</i> = 'C': | $Q^H * C$ | $C * Q^H$ |

where Q is a real orthogonal (complex unitary) matrix defined as the product of k elementary reflectors

$Q = H(1) H(2) \dots H(k) = I - V^* T^* V^T$ for real flavors, and

$Q = H(1) H(2) \dots H(k) = I - V^* T^* V^H$ for complex flavors,

generated using the compact WY representation as returned by [geqrt](#). Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | ='L': apply Q , Q^T , or Q^H from the left. ='R': apply Q , Q^T , or Q^H from the right. |
| <i>trans</i> | ='N', no transpose, apply Q . ='T', transpose, apply Q^T . ='C', transpose, apply Q^H . |
| <i>m</i> | The number of rows in the matrix C , ($m \geq 0$). |
| <i>n</i> | The number of columns in the matrix C , ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: If $side = 'L'$, $m \geq k \geq 0$ If $side = 'R'$, $n \geq k \geq 0$. |
| <i>nb</i> | The block size used for the storage of t , $k \geq nb \geq 1$. This must be the same value of nb used to generate t in geqrt . |
| <i>v</i> | Array of size $\max(1, ldv * k)$ for column major layout, $\max(1, ldv * m)$ for row major layout and $side = 'L'$, and $\max(1, ldv * n)$ for row major layout and $side = 'R'$. The i th column must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by geqrt in the first k columns of its array argument a . |
| <i>ldv</i> | The leading dimension of the array v . if $side = 'L'$, ldv must be at least $\max(1, m)$ for column major layout and $\max(1, k)$ for row major layout; if $side = 'R'$, ldv must be at least $\max(1, n)$ for column major layout and $\max(1, k)$ for row major layout. |
| <i>t</i> | Array, size $\max(1, ldt * \min(m, n))$ for column major layout and $\max(1, ldt * nb)$ for row major layout. The upper triangular factors of the block reflectors as returned by geqrt . |
| <i>ldt</i> | The leading dimension of the array t . ldt must be at least nb for column major layout and $\max(1, k)$ for row major layout. |
| <i>c</i> | The m -by- n matrix C . |

ldc The leading dimension of the array *c*. *ldc* must be at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.

Output Parameters

c Overwritten by the product Q^*C , C^*Q , Q^T*C , C^*Q^T , Q^H*C , or C^*Q^H as specified by *side* and *trans*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value.

?geqpf

Computes the QR factorization of a general *m*-by-*n* matrix with pivoting.

Syntax

```
lapack_int LAPACKE_sgeqpf (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, lapack_int* jpvt, float* tau);
```

```
lapack_int LAPACKE_dgeqpf (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, lapack_int* jpvt, double* tau);
```

```
lapack_int LAPACKE_cgeqpf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_int* jpvt, lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zgeqpf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_int* jpvt, lapack_complex_double*
tau);
```

Include Files

- mkl.h

Description

The routine is deprecated and has been replaced by routine [geqp3](#).

The routine `?geqpf` forms the QR factorization of a general *m*-by-*n* matrix *A* with column pivoting: $A^*P = Q^*R$ (see [Orthogonal Factorizations](#)). Here *P* denotes an *n*-by-*n* permutation matrix.

The routine does not form the matrix *Q* explicitly. Instead, *Q* is represented as a product of $\min(m, n)$ elementary reflectors. Routines are provided to work with *Q* in this representation.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

m The number of rows in the matrix *A* ($m \geq 0$).

n The number of columns in *A* ($n \geq 0$).

| | |
|-------------|---|
| <i>a</i> | Array <i>a</i> of size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout contains the matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>jpvt</i> | Array, size at least $\max(1, n)$. On entry, if $jpvt[i - 1] > 0$, the <i>i</i> -th column of <i>A</i> is moved to the beginning of <i>A*P</i> before the computation, and fixed in place during the computation. If $jpvt[i - 1] = 0$, the <i>i</i> th column of <i>A</i> is a free column (that is, it may be interchanged during the computation with any other free column). |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | Overwritten by the factorization data as follows: The elements on and above the diagonal of the array contain the $\min(m,n)$ -by- <i>n</i> upper trapezoidal matrix <i>R</i> (<i>R</i> is upper triangular if $m \geq n$); the elements below the diagonal, with the array <i>tau</i> , present the orthogonal matrix <i>Q</i> as a product of $\min(m,n)$ elementary reflectors (see Orthogonal Factorizations). |
| <i>tau</i> | Array, size at least $\max(1, \min(m, n))$. Contains additional information on the matrix <i>Q</i> . |
| <i>jpvt</i> | Overwritten by details of the permutation matrix <i>P</i> in the factorization $A*P = Q*R$. More precisely, the columns of <i>A*P</i> are the columns of <i>A</i> in the following order: $jpvt[0], jpvt[1], \dots, jpvt[n - 1]$. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed factorization is the exact factorization of a matrix $A + E$, where

$$\|E\|_2 = O(\epsilon) \|A\|_2.$$

The approximate number of floating-point operations for real flavors is

$$(4/3)n^3 \quad \text{if } m = n,$$

$$(2/3)n^2(3m-n) \quad \text{if } m > n,$$

$$(2/3)m^2(3n-m) \quad \text{if } m < n.$$

The number of operations for complex flavors is 4 times greater.

To solve a set of least squares problems minimizing $\|A*x - b\|_2$ for all columns *b* of a given matrix *B*, you can call the following:

`?geqpf` (this routine) to factorize $A*P = Q*R$;

| | |
|------------------------------------|--|
| <code>ormqr</code> | to compute $C = Q^T * B$ (for real matrices); |
| <code>unmqr</code> | to compute $C = Q^H * B$ (for complex matrices); |
| <code>trsm</code> (a BLAS routine) | to solve $R * X = C$. |

(The columns of the computed X are the permuted least squares solution vectors x ; the output array $jpvt$ specifies the permutation order.)

To compute the elements of Q explicitly, call

| | |
|--------------------|-------------------------|
| <code>orgqr</code> | (for real matrices) |
| <code>ungqr</code> | (for complex matrices). |

?geqp3

Computes the QR factorization of a general m -by- n matrix with column pivoting using level 3 BLAS.

Syntax

```
lapack_int LAPACKE_sgeqp3 (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, lapack_int* jpvt, float* tau);

lapack_int LAPACKE_dgeqp3 (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, lapack_int* jpvt, double* tau);

lapack_int LAPACKE_cgeqp3 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_int* jpvt, lapack_complex_float* tau);

lapack_int LAPACKE_zgeqp3 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_int* jpvt, lapack_complex_double*
tau);
```

Include Files

- `mk1.h`

Description

The routine forms the QR factorization of a general m -by- n matrix A with column pivoting: $A * P = Q * R$ (see [Orthogonal Factorizations](#)) using Level 3 BLAS. Here P denotes an n -by- n permutation matrix. Use this routine instead of `geqpf` for better performance.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of $\min(m, n)$ *elementary reflectors*. Routines are provided to work with Q in this representation.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>m</code> | The number of rows in the matrix A ($m \geq 0$). |
| <code>n</code> | The number of columns in A ($n \geq 0$). |
| <code>a</code> | Array a of size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout contains the matrix A . |

| | |
|-------------|---|
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>jpvt</i> | <p>Array, size at least $\max(1, n)$.</p> <p>On entry, if $jpvt[i - 1] \neq 0$, the <i>i</i>-th column of <i>A</i> is moved to the beginning of <i>AP</i> before the computation, and fixed in place during the computation.</p> <p>If $jpvt[i - 1] = 0$, the <i>i</i>-th column of <i>A</i> is a free column (that is, it may be interchanged during the computation with any other free column).</p> |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | <p>Overwritten by the factorization data as follows:</p> <p>The elements on and above the diagonal of the array contain the $\min(m, n)$-by-<i>n</i> upper trapezoidal matrix <i>R</i> (<i>R</i> is upper triangular if $m \geq n$); the elements below the diagonal, with the array <i>tau</i>, present the orthogonal matrix <i>Q</i> as a product of $\min(m, n)$ elementary reflectors (see Orthogonal Factorizations).</p> |
| <i>tau</i> | Array, size at least $\max(1, \min(m, n))$. Contains scalar factors of the elementary reflectors for the matrix <i>Q</i> . |
| <i>jpvt</i> | <p>Overwritten by details of the permutation matrix <i>P</i> in the factorization $A^*P = Q^*R$. More precisely, the columns of <i>AP</i> are the columns of <i>A</i> in the following order:</p> <p>$jpvt[0], jpvt[1], \dots, jpvt[n - 1]$.</p> |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

To solve a set of least squares problems minimizing $\|A^*x - b\|_2$ for all columns *b* of a given matrix *B*, you can call the following:

| | |
|------------------------------------|--|
| <code>?geqp3</code> (this routine) | to factorize $A^*P = Q^*R$; |
| <code>ormqr</code> | to compute $C = Q^T * B$ (for real matrices); |
| <code>unmqr</code> | to compute $C = Q^H * B$ (for complex matrices); |
| <code>trsm</code> (a BLAS routine) | to solve $R^*X = C$. |

(The columns of the computed *X* are the permuted least squares solution vectors *x*; the output array *jpvt* specifies the permutation order.)

To compute the elements of *Q* explicitly, call

| | |
|--------------------|-------------------------|
| <code>orgqr</code> | (for real matrices) |
| <code>ungqr</code> | (for complex matrices). |

?orgqr

Generates the real orthogonal matrix Q of the QR factorization formed by ?geqrf.

Syntax

```
lapack_int LAPACKE_sorgqr (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
float* a, lapack_int lda, const float* tau);
```

```
lapack_int LAPACKE_dorgqr (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
double* a, lapack_int lda, const double* tau);
```

Include Files

- mkl.h

Description

The routine generates the whole or part of m -by- m orthogonal matrix Q of the QR factorization formed by the routines [geqrf](#) or [geqpf](#). Use this routine after a call to [sgeqrf/dgeqrf](#) or [sgeqpf/dgeqpf](#).

Usually Q is determined from the QR factorization of an m by p matrix A with $m \geq p$. To compute the whole matrix Q , use:

```
LAPACKE_?orgqr(matrix_layout, m, m, p, a, lda, tau)
```

To compute the leading p columns of Q (which form an orthonormal basis in the space spanned by the columns of A):

```
LAPACKE_?orgqr(matrix_layout, m, p, p, a, lda)
```

To compute the matrix Q^k of the QR factorization of leading k columns of the matrix A :

```
LAPACKE_?orgqr(matrix_layout, m, m, k, a, lda, tau)
```

To compute the leading k columns of Q^k (which form an orthonormal basis in the space spanned by leading k columns of the matrix A):

```
LAPACKE_?orgqr(matrix_layout, m, k, k, a, lda, tau)
```

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The order of the orthogonal matrix Q ($m \geq 0$). |
| <i>n</i> | The number of columns of Q to be computed ($0 \leq n \leq m$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q ($0 \leq k \leq n$). |
| <i>a, tau</i> | Arrays: <i>a</i> and <i>tau</i> are the arrays returned by sgeqrf / dgeqrf or sgeqpf / dgeqpf . The size of <i>a</i> is $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout . The size of <i>tau</i> must be at least $\max(1, k)$. |

lda The leading dimension of *a*; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.

Output Parameters

a Overwritten by *n* leading columns of the *m*-by-*m* orthogonal matrix *Q*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value.

Application Notes

The computed *Q* differs from an exactly orthogonal matrix by a matrix *E* such that

$\|E\|_2 = O(\varepsilon) \|A\|_2$ where ε is the machine precision.

The total number of floating-point operations is approximately $4*m*n*k - 2*(m + n)*k^2 + (4/3)*k^3$.

If $n = k$, the number is approximately $(2/3)*n^2*(3m - n)$.

The complex counterpart of this routine is [ungqr](#).

?ormqr

Multiplies a real matrix by the orthogonal matrix Q of the QR factorization formed by ?geqrf or ?geqpf.

Syntax

```
lapack_int LAPACKE_sormqr (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const float* a, lapack_int lda, const float* tau, float*
c, lapack_int ldc);
```

```
lapack_int LAPACKE_dormqr (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const double* a, lapack_int lda, const double* tau,
double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a real matrix *C* by *Q* or Q^T , where *Q* is the orthogonal matrix *Q* of the *QR* factorization formed by the routines [geqrf](#) or [geqpf](#).

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^T*C , $C*Q$, or $C*Q^T$ (overwriting the result on *C*).

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

side Must be either 'L' or 'R'.

If *side*='L', *Q* or Q^T is applied to *C* from the left.

| | |
|------------------|---|
| | If $side='R'$, Q or Q^T is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'T'. If $trans='N'$, the routine multiplies C by Q . If $trans='T'$, the routine multiplies C by Q^T . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: $0 \leq k \leq m$ if $side='L'$; $0 \leq k \leq n$ if $side='R'$. |
| <i>a, tau, c</i> | Arrays: <i>a</i> and <i>tau</i> are the arrays returned by <i>sgeqrf</i> / <i>dgeqrf</i> or <i>sgeqpf</i> / <i>dgeqpf</i> . The size of <i>a</i> is $\max(1, lda*k)$ for column major layout, $\max(1, lda*m)$ for row major layout and $side = 'L'$, and $\max(1, lda*n)$ for row major layout and $side = 'R'$. The size of <i>tau</i> must be at least $\max(1, k)$. Array <i>c</i> of size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout contains the m -by- n matrix C . |
| <i>lda</i> | The leading dimension of <i>a</i> . Constraints: if $side = 'L'$, $lda \geq \max(1, m)$ for column major layout and $\max(1, k)$ for row major layout ; if $side = 'R'$, $lda \geq \max(1, n)$ for column major layout and $\max(1, k)$ for row major layout. |
| <i>ldc</i> | The leading dimension of <i>c</i> . Constraint: $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | Overwritten by the product Q^*C , Q^T*C , $C*Q$, or $C*Q^T$ (as specified by <i>side</i> and <i>trans</i>). |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The complex counterpart of this routine is *unmqr*.

?ungqr

Generates the complex unitary matrix Q of the QR factorization formed by ?geqrf.

Syntax

```
lapack_int LAPACKE_cungqr (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
lapack_complex_float* a, lapack_int lda, const lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zungqr (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
lapack_complex_double* a, lapack_int lda, const lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine generates the whole or part of m -by- m unitary matrix Q of the QR factorization formed by the routines `geqrf` or `geqpf`. Use this routine after a call to `cgeqrf/zgeqrf` or `cgeqpf/zgeqpf`.

Usually Q is determined from the QR factorization of an m by p matrix A with $m \geq p$. To compute the whole matrix Q , use:

```
LAPACKE_?ungqr(matrix_layout, m, m, p, a, lda, tau)
```

To compute the leading p columns of Q (which form an orthonormal basis in the space spanned by the columns of A):

```
LAPACKE_?ungqr(matrix_layout, m, p, p, a, lda, tau)
```

To compute the matrix Q^k of the QR factorization of the leading k columns of the matrix A :

```
LAPACKE_?ungqr(matrix_layout, m, m, k, a, lda, tau)
```

To compute the leading k columns of Q^k (which form an orthonormal basis in the space spanned by the leading k columns of the matrix A):

```
LAPACKE_?ungqr(matrix_layout, m, k, k, a, lda, tau)
```

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The order of the unitary matrix Q ($m \geq 0$). |
| <i>n</i> | The number of columns of Q to be computed ($0 \leq n \leq m$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q ($0 \leq k \leq n$). |
| <i>a, tau</i> | Arrays: <i>a</i> and <i>tau</i> are the arrays returned by <code>cgeqrf/zgeqrf</code> or <code>cgeqpf/zgeqpf</code> . The size of <i>a</i> is $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout . The size of <i>tau</i> must be at least $\max(1, k)$. |

lda The leading dimension of *a*; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.

Output Parameters

a Overwritten by *n* leading columns of the *m*-by-*m* unitary matrix *Q*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value.

Application Notes

The computed *Q* differs from an exactly unitary matrix by a matrix *E* such that $\|E\|_2 = O(\varepsilon) * \|A\|_2$, where ε is the machine precision.

The total number of floating-point operations is approximately $16*m*n*k - 8*(m+n)*k^2 + (16/3)*k^3$.

If *n* = *k*, the number is approximately $(8/3)*n^2*(3m - n)$.

The real counterpart of this routine is [orgqr](#).

?unmqr

Multiplies a complex matrix by the unitary matrix Q of the QR factorization formed by ?geqrf.

Syntax

```
lapack_int LAPACKE_cunmqr (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zunmqr (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* tau, lapack_complex_double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a rectangular complex matrix *C* by *Q* or *Q*^H, where *Q* is the unitary matrix *Q* of the *QR* factorization formed by the routines [geqrf](#) or [geqpf](#).

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products *Q***C*, *Q*^H**C*, *C***Q*, or *C***Q*^H (overwriting the result on *C*).

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

side Must be either 'L' or 'R'.

If *side* = 'L', *Q* or *Q*^H is applied to *C* from the left.

| | |
|------------------|---|
| | If <i>side</i> = 'R', Q or Q^H is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'C'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'C', the routine multiplies C by Q^H . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: $0 \leq k \leq m$ if <i>side</i> = 'L'; $0 \leq k \leq n$ if <i>side</i> = 'R'. |
| <i>a, c, tau</i> | Arrays: <i>a</i> size $\max(1, lda * k)$ for column major layout, $\max(1, lda * m)$ for row major layout when <i>side</i> = 'L', and $\max(1, lda * n)$ for row major layout when <i>side</i> = 'R' and <i>tau</i> are the arrays returned by <i>cgeqrf</i> / <i>zgeqrf</i> or <i>cgeqpf</i> / <i>zgeqpf</i> . The size of <i>tau</i> must be at least $\max(1, k)$. <i>c</i> (size $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout) contains the m -by- n matrix C . |
| <i>lda</i> | The leading dimension of <i>a</i> . Constraints: $lda \geq \max(1, m)$ for column major layout and $lda \geq \max(1, k)$ for row major layout if <i>side</i> = 'L'; $lda \geq \max(1, n)$ for column major layout and $lda \geq \max(1, k)$ for row major layout if <i>side</i> = 'R'. |
| <i>ldc</i> | The leading dimension of <i>c</i> . Constraint: $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Overwritten by the product $Q * C$, $Q^H * C$, $C * Q$, or $C * Q^H$ (as specified by <i>side</i> and <i>trans</i>). |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The real counterpart of this routine is [ormqr](#).

?gelqf

Computes the LQ factorization of a general m -by- n matrix.

Syntax

```
lapack_int LAPACKE_sgelqf (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* tau);

lapack_int LAPACKE_dgelqf (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* tau);

lapack_int LAPACKE_cgelqf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);

lapack_int LAPACKE_zgelqf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine forms the LQ factorization of a general m -by- n matrix A (see [Orthogonal Factorizations](#)). No pivoting is performed.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of $\min(m, n)$ elementary reflectors. Routines are provided to work with Q in this representation.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows in the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns in A ($n \geq 0$). |
| <i>a</i> | Array a of size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout contains the matrix A . |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|------------|--|
| <i>a</i> | Overwritten by the factorization data as follows: The elements on and below the diagonal of the array contain the m -by- $\min(m, n)$ lower trapezoidal matrix R (R is lower triangular if $m \leq n$); the elements above the diagonal, with the array τ , represent the orthogonal matrix Q as a product of elementary reflectors. |
| <i>tau</i> | Array, size at least $\max(1, \min(m, n))$. |

Contains scalars that define elementary reflectors for the matrix Q (see [Orthogonal Factorizations](#)).

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed factorization is the exact factorization of a matrix $A + E$, where

$$\|E\|_2 = O(\epsilon) \|A\|_2.$$

The approximate number of floating-point operations for real flavors is

$$(4/3)n^3 \quad \text{if } m = n,$$

$$(2/3)n^2(3m-n) \quad \text{if } m > n,$$

$$(2/3)m^2(3n-m) \quad \text{if } m < n.$$

The number of operations for complex flavors is 4 times greater.

To find the minimum-norm solution of an underdetermined least squares problem minimizing $\|A^*x - b\|_2$ for all columns b of a given matrix B , you can call the following:

| | |
|------------------------------------|--|
| <code>?gelqf</code> (this routine) | to factorize $A = L^*Q$; |
| <code>trsm</code> (a BLAS routine) | to solve $L^*Y = B$ for Y ; |
| <code>ormlq</code> | to compute $X = (Q_1)^T * Y$ (for real matrices); |
| <code>unmlq</code> | to compute $X = (Q_1)^H * Y$ (for complex matrices). |

(The columns of the computed X are the minimum-norm solution vectors x . Here A is an m -by- n matrix with $m < n$; Q_1 denotes the first m columns of Q).

To compute the elements of Q explicitly, call

| | |
|--------------------|-------------------------|
| <code>orglq</code> | (for real matrices) |
| <code>unglq</code> | (for complex matrices). |

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?orglq

Generates the real orthogonal matrix Q of the LQ factorization formed by `?gelqf`.

Syntax

```
lapack_int LAPACKE_sorglq (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
float* a, lapack_int lda, const float* tau);
```

```
lapack_int LAPACKE_dorglq (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
double* a, lapack_int lda, const double* tau);
```

Include Files

- `mkl.h`

Description

The routine generates the whole or part of n -by- n orthogonal matrix Q of the LQ factorization formed by the routines `gelqf`. Use this routine after a call to `sgelqf/dgelqf`.

Usually Q is determined from the LQ factorization of an p -by- n matrix A with $n \geq p$. To compute the whole matrix Q , use:

```
info = LAPACKE_?orglq(matrix_layout, n, n, p, a, lda, tau)
```

To compute the leading p rows of Q , which form an orthonormal basis in the space spanned by the rows of A , use:

```
info = LAPACKE_?orglq(matrix_layout, p, n, p, a, lda, tau)
```

To compute the matrix Q^k of the LQ factorization of the leading k rows of A , use:

```
info = LAPACKE_?orglq(matrix_layout, n, n, k, a, lda, tau)
```

To compute the leading k rows of Q^k , which form an orthonormal basis in the space spanned by the leading k rows of A , use:

```
info = LAPACKE_?orgqr(matrix_layout, k, n, k, a, lda, tau)
```

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>m</code> | The number of rows of Q to be computed ($0 \leq m \leq n$). |
| <code>n</code> | The order of the orthogonal matrix Q ($n \geq m$). |
| <code>k</code> | The number of elementary reflectors whose product defines the matrix Q ($0 \leq k \leq m$). |
| <code>a, tau</code> | Arrays: a (size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout) and tau are the arrays returned by <code>sgelqf/dgelqf</code> . The size of tau must be at least $\max(1, k)$. |
| <code>lda</code> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

`a` Overwritten by m leading rows of the n -by- n orthogonal matrix Q .

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

Application Notes

The computed Q differs from an exactly orthogonal matrix by a matrix E such that $\|E\|_2 = O(\epsilon) \|A\|_2$, where ϵ is the machine precision.

The total number of floating-point operations is approximately $4*m*n*k - 2*(m+n)*k^2 + (4/3)*k^3$.

If $m = k$, the number is approximately $(2/3)*m^2*(3n - m)$.

The complex counterpart of this routine is [unglq](#).

?ormlq

Multiplies a real matrix by the orthogonal matrix Q of the LQ factorization formed by [?gelqf](#).

Syntax

```
lapack_int LAPACKE_sormlq (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const float* a, lapack_int lda, const float* tau, float*
c, lapack_int ldc);
```

```
lapack_int LAPACKE_dormlq (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const double* a, lapack_int lda, const double* tau,
double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a real m -by- n matrix C by Q or Q^T , where Q is the orthogonal matrix Q of the LQ factorization formed by the routine [gelqf](#).

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^T*C , $C*Q$, or $C*Q^T$ (overwriting the result on C).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^T is applied to C from the left. If <i>side</i> = 'R', Q or Q^T is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'T'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'T', the routine multiplies C by Q^T . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: $0 \leq k \leq m$ if <i>side</i> = 'L'; |

$0 \leq k \leq n$ if *side* = 'R'.

a, *c*, *tau*

Arrays:

a and *tau* are arrays returned by `?gelqf`.

The size of *a* must be:

For *side* = 'L' and column major layout, $\max(1, lda * m)$.

For *side* = 'R' and column major layout, $\max(1, lda * n)$.

For row major layout regardless of *side*, $\max(1, lda * k)$.

The dimension of *tau* must be at least $\max(1, k)$.

c (size $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout) contains the *m*-by-*n* matrix *C*.

lda

The leading dimension of *a*. For column major layout, $lda \geq \max(1, k)$. For row major layout, if *side* = 'L', $lda \geq \max(1, m)$, or, if *side* = 'R', $lda \geq \max(1, n)$.

ldc

The leading dimension of *c*; $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.

Output Parameters

c

Overwritten by the product $Q * C$, $Q^T * C$, $C * Q$, or $C * Q^T$ (as specified by *side* and *trans*).

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value.

Application Notes

The complex counterpart of this routine is `unmlq`.

?unglq

Generates the complex unitary matrix *Q* of the LQ factorization formed by `?gelqf`.

Syntax

```
lapack_int LAPACKE_cunglq (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
lapack_complex_float* a, lapack_int lda, const lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zunglq (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
lapack_complex_double* a, lapack_int lda, const lapack_complex_double* tau);
```

Include Files

- `mk1.h`

Description

The routine generates the whole or part of *n*-by-*n* unitary matrix *Q* of the LQ factorization formed by the routines `gelqf`. Use this routine after a call to `cgelqf/zgelqf`.

Usually Q is determined from the LQ factorization of an p -by- n matrix A with $n < p$. To compute the whole matrix Q , use:

```
info = LAPACKE_?unglq(matrix_layout, n, n, p, a, lda, tau)
```

To compute the leading p rows of Q , which form an orthonormal basis in the space spanned by the rows of A , use:

```
info = LAPACKE_?unglq(matrix_layout, p, n, p, a, lda, tau)
```

To compute the matrix Q^k of the LQ factorization of the leading k rows of A , use:

```
info = LAPACKE_?unglq(matrix_layout, n, n, k, a, lda, tau)
```

To compute the leading k rows of Q^k , which form an orthonormal basis in the space spanned by the leading k rows of A , use:

```
info = LAPACKE_?ungqr(matrix_layout, k, n, k, a, lda, tau)
```

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of Q to be computed ($0 \leq m \leq n$). |
| <i>n</i> | The order of the unitary matrix Q ($n \geq m$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q ($0 \leq k \leq m$). |
| <i>a, tau</i> | Arrays: <i>a</i> (size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout) and <i>tau</i> are the arrays returned by <i>cgelqf/zgelqf</i> . The dimension of <i>tau</i> must be at least $\max(1, k)$. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | Overwritten by m leading rows of the n -by- n unitary matrix Q . |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The computed Q differs from an exactly unitary matrix by a matrix E such that $\|E\|_2 = O(\varepsilon) * \|A\|_2$, where ε is the machine precision.

The total number of floating-point operations is approximately $16 * m * n * k - 8 * (m + n) * k^2 + (16/3) * k^3$.

If $m = k$, the number is approximately $(8/3) * m^2 * (3n - m)$.

The real counterpart of this routine is [orglq](#).

?unmlq

Multiplies a complex matrix by the unitary matrix Q of the LQ factorization formed by ?gelqf.

Syntax

```
lapack_int LAPACKE_cunmlq (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zunmlq (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* tau, lapack_complex_double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a real m -by- n matrix C by Q or Q^H , where Q is the unitary matrix Q of the LQ factorization formed by the routine [gelqf](#).

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products Q^*C , Q^H*C , $C*Q$, or $C*Q^H$ (overwriting the result on C).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^H is applied to C from the left. If <i>side</i> = 'R', Q or Q^H is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'C'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'C', the routine multiplies C by Q^H . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: $0 \leq k \leq m$ if <i>side</i> = 'L'; $0 \leq k \leq n$ if <i>side</i> = 'R'. |
| <i>a, c, tau</i> | Arrays: <i>a</i> and <i>tau</i> are arrays returned by ?gelqf. The size of <i>a</i> must be: For <i>side</i> = 'L' and column major layout, $\max(1, lda*m)$. For <i>side</i> = 'R' and column major layout, $\max(1, lda*n)$. |

For row major layout regardless of *side*, $\max(1, lda*k)$.

The size of *tau* must be at least $\max(1, k)$.

c(size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m$ for row major layout) contains the *m*-by-*n* matrix *C*.

lda The leading dimension of *a*. For column major layout, $lda \geq \max(1, k)$. For row major layout, if *side* = 'L', $lda \geq \max(1, m)$, or, if *side* = 'R', $lda \geq \max(1, n)$.

ldc The leading dimension of *c*; $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.

Output Parameters

c Overwritten by the product $Q*C$, Q^H*C , $C*Q$, or $C*Q^H$ (as specified by *side* and *trans*).

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The real counterpart of this routine is [ormlq](#).

?geqlf

Computes the QL factorization of a general *m*-by-*n* matrix.

Syntax

```
lapack_int LAPACKE_sgeqlf (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* tau);
```

```
lapack_int LAPACKE_dgeqlf (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* tau);
```

```
lapack_int LAPACKE_cgeqlf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zgeqlf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine forms the QL factorization of a general *m*-by-*n* matrix *A* (see [Orthogonal Factorizations](#)). No pivoting is performed.

The routine does not form the matrix *Q* explicitly. Instead, *Q* is represented as a product of $\min(m, n)$ elementary reflectors. Routines are provided to work with *Q* in this representation.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>m</i> | The number of rows in the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns in A ($n \geq 0$). |
| <i>a</i> | Array a of size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout contains the matrix A . |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|------------|---|
| <i>a</i> | Overwritten on exit by the factorization data as follows: if $m \geq n$, the lower triangle of the subarray $a(m-n+1:m, 1:n)$ contains the n -by- n lower triangular matrix L ; if $m \leq n$, the elements on and below the $(n-m)$ -th superdiagonal contain the m -by- n lower trapezoidal matrix L ; in both cases, the remaining elements, with the array τ , represent the orthogonal/unitary matrix Q as a product of elementary reflectors. |
| <i>tau</i> | Array, size at least $\max(1, \min(m, n))$. Contains scalar factors of the elementary reflectors for the matrix Q (see Orthogonal Factorizations). |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

Related routines include:

| | |
|-----------------------|--|
| orgql | to generate matrix Q (for real matrices); |
| ungql | to generate matrix Q (for complex matrices); |
| ormql | to apply matrix Q (for real matrices); |
| unmql | to apply matrix Q (for complex matrices). |

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?orgql

Generates the real matrix Q of the QL factorization formed by ?geqlf.

Syntax

```
lapack_int LAPACKE_sorgql (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
float* a, lapack_int lda, const float* tau);
```

```
lapack_int LAPACKE_dorgql (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
double* a, lapack_int lda, const double* tau);
```

Include Files

- mkl.h

Description

The routine generates an m -by- n real matrix Q with orthonormal columns, which is defined as the last n columns of a product of k elementary reflectors $H(i)$ of order m : $Q = H(k) * \dots * H(2) * H(1)$ as returned by the routines [geqlf](#). Use this routine after a call to [sgeqlf/dgeqlf](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix Q ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrix Q ($m \geq n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q ($n \geq k \geq 0$). |
| <i>a, tau</i> | Arrays: <i>a</i> (size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout), <i>tau</i> . On entry, the $(n - k + i)$ th column of <i>a</i> must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by sgeqlf/dgeqlf in the last k columns of its array argument <i>a</i> ; <i>tau</i> [$i - 1$] must contain the scalar factor of the elementary reflector $H(i)$, as returned by sgeqlf/dgeqlf ; The size of <i>tau</i> must be at least $\max(1, k)$. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

a Overwritten by the last n columns of the m -by- m orthogonal matrix Q .

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The complex counterpart of this routine is [ungql](#).

?ungql

Generates the complex matrix Q of the QL factorization formed by ?geqlf.

Syntax

```
lapack_int LAPACKE_cungql (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
lapack_complex_float* a, lapack_int lda, const lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zungql (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
lapack_complex_double* a, lapack_int lda, const lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine generates an m -by- n complex matrix Q with orthonormal columns, which is defined as the last n columns of a product of k elementary reflectors $H(i)$ of order m : $Q = H(k) * \dots * H(2) * H(1)$ as returned by the routines [geqlf/geqlf](#). Use this routine after a call to [cgeqlf/zgeqlf](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix Q ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrix Q ($m \geq n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q ($n \geq k \geq 0$). |
| <i>a, tau</i> | Arrays: <i>a</i> (size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout), <i>tau</i> . On entry, the $(n - k + i)$ th column of <i>a</i> must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by cgeqlf/zgeqlf in the last k columns of its array argument <i>a</i> ; <i>tau</i> [<i>i</i> - 1] must contain the scalar factor of the elementary reflector $H(i)$, as returned by cgeqlf/zgeqlf ; The size of <i>tau</i> must be at least $\max(1, k)$. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

a Overwritten by the last n columns of the m -by- m unitary matrix Q .

Return Values

This function returns a value *info*.

If `info=0`, the execution is successful.

If `info = -i`, the *i*-th parameter had an illegal value.

Application Notes

The real counterpart of this routine is [orgql](#).

?ormql

Multiplies a real matrix by the orthogonal matrix Q of the QL factorization formed by ?geqlf.

Syntax

```
lapack_int LAPACKE_sormql (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const float* a, lapack_int lda, const float* tau, float*
c, lapack_int ldc);
```

```
lapack_int LAPACKE_dormql (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const double* a, lapack_int lda, const double* tau,
double* c, lapack_int ldc);
```

Include Files

- `mkl.h`

Description

The routine multiplies a real *m*-by-*n* matrix *C* by *Q* or Q^T , where *Q* is the orthogonal matrix *Q* of the *QL* factorization formed by the routine [geqlf](#).

Depending on the parameters *side* and *trans*, the routine [ormql](#) can form one of the matrix products $Q^T C$, $C^T Q$, or $C^* Q^T$ (overwriting the result over *C*).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', <i>Q</i> or Q^T is applied to <i>C</i> from the left. If <i>side</i> = 'R', <i>Q</i> or Q^T is applied to <i>C</i> from the right. |
| <i>trans</i> | Must be either 'N' or 'T'. If <i>trans</i> = 'N', the routine multiplies <i>C</i> by <i>Q</i> . If <i>trans</i> = 'T', the routine multiplies <i>C</i> by Q^T . |
| <i>m</i> | The number of rows in the matrix <i>C</i> ($m \geq 0$). |
| <i>n</i> | The number of columns in <i>C</i> ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix <i>Q</i> . Constraints: $0 \leq k \leq m$ if <i>side</i> = 'L'; $0 \leq k \leq n$ if <i>side</i> = 'R'. |

| | |
|----------------------------------|--|
| <i>a</i> , <i>tau</i> , <i>c</i> | <p>Arrays: <i>a</i>, <i>tau</i>, <i>c</i>.</p> <p>The size of <i>a</i> must be:</p> <p>For column major layout regardless of <i>side</i>, $\max(1, lda*k)$.</p> <p>For <i>side</i> = 'L' and row major layout, $\max(1, lda*m)$.</p> <p>For <i>side</i> = 'R' and row major layout, $\max(1, lda*n)$.</p> <p>On entry, the <i>i</i>th column of <i>a</i> must contain the vector which defines the elementary reflector H_i, for $i = 1, 2, \dots, k$, as returned by <i>sgeqlf/dgeqlf</i> in the last <i>k</i> columns of its array argument <i>a</i>.</p> <p><i>tau</i>[<i>i</i> - 1] must contain the scalar factor of the elementary reflector H_i, as returned by <i>sgeqlf/dgeqlf</i>.</p> <p>The size of <i>tau</i> must be at least $\max(1, k)$.</p> <p><i>c</i>(size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout) contains the <i>m</i>-by-<i>n</i> matrix <i>C</i>.</p> |
| <i>lda</i> | <p>The leading dimension of <i>a</i>;</p> <p>if <i>side</i> = 'L', $lda \geq \max(1, m)$ for column major layout and $\max(1, k)$ for row major layout ;</p> <p>if <i>side</i> = 'R', $lda \geq \max(1, n)$ for column major layout and $\max(1, k)$ for row major layout.</p> |
| <i>ldc</i> | <p>The leading dimension of <i>c</i>; $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.</p> |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Overwritten by the product $Q^T C$, $Q^T * C$, $C * Q$, or $C * Q^T$ (as specified by <i>side</i> and <i>trans</i>). |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The complex counterpart of this routine is [unmql](#).

?unmql

Multiplies a complex matrix by the unitary matrix Q of the QL factorization formed by ?geqlf.

Syntax

```
lapack_int LAPACKE_cunmql (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zunmql (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* tau, lapack_complex_double* c, lapack_int ldc);
```

Include Files

- `mkl.h`

Description

The routine multiplies a complex m -by- n matrix C by Q or Q^H , where Q is the unitary matrix Q of the QL factorization formed by the routine `geqlf`.

Depending on the parameters *side* and *trans*, the routine `unmql` can form one of the matrix products Q^*C , Q^H*C , $C*Q$, or $C*Q^H$ (overwriting the result over C).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^H is applied to C from the left. If <i>side</i> = 'R', Q or Q^H is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'C'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'C', the routine multiplies C by Q^H . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: $0 \leq k \leq m$ if <i>side</i> = 'L'; $0 \leq k \leq n$ if <i>side</i> = 'R'. |
| <i>a, tau, c</i> | Arrays: <i>a</i> , <i>tau</i> , <i>c</i> . The size of <i>a</i> must be: For column major layout regardless of <i>side</i> , $\max(1, lda*k)$. For <i>side</i> = 'L' and row major layout, $\max(1, lda*m)$. For <i>side</i> = 'R' and row major layout, $\max(1, lda*n)$. On entry, the i -th column of <i>a</i> must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by <code>cgeqlf/zgeqlf</code> in the last k columns of its array argument <i>a</i> . <i>tau</i> [$i - 1$] must contain the scalar factor of the elementary reflector $H(i)$, as returned by <code>cgeqlf/zgeqlf</code> . The size of <i>tau</i> must be at least $\max(1, k)$. <i>c</i> (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout) contains the m -by- n matrix C . |
| <i>lda</i> | The leading dimension of <i>a</i> . |

If $side = 'L'$, $lda \geq \max(1, m)$ for column major layout and $\max(1, k)$ for row major layout.

If $side = 'R'$, $lda \geq \max(1, n)$ for column major layout and $\max(1, k)$ for row major layout.

ldc The leading dimension of c ; $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.

Output Parameters

c Overwritten by the product Q^*C , Q^H*C , $C*Q$, or $C*Q^H$ (as specified by $side$ and $trans$).

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The real counterpart of this routine is [ormql](#).

?gerqf

Computes the RQ factorization of a general m -by- n matrix.

Syntax

```
lapack_int LAPACKE_sgerqf (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* tau);
```

```
lapack_int LAPACKE_dgerqf (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* tau);
```

```
lapack_int LAPACKE_cgerqf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zgerqf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- `mkl.h`

Description

The routine forms the RQ factorization of a general m -by- n matrix A (see [Orthogonal Factorizations](#)). No pivoting is performed.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of $\min(m, n)$ elementary reflectors. Routines are provided to work with Q in this representation.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>m</i> | The number of rows in the matrix <i>A</i> ($m \geq 0$). |
| <i>n</i> | The number of columns in <i>A</i> ($n \geq 0$). |
| <i>a</i> | Array <i>a</i> of size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout contains the <i>m</i> -by- <i>n</i> matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|------------|--|
| <i>a</i> | Overwritten on exit by the factorization data as follows: if $m \leq n$, the upper triangle of the subarray $a(1:m, n-m+1:n)$ contains the <i>m</i> -by- <i>m</i> upper triangular matrix <i>R</i> ; if $m \geq n$, the elements on and above the (<i>m</i> - <i>n</i>)th subdiagonal contain the <i>m</i> -by- <i>n</i> upper trapezoidal matrix <i>R</i> ; in both cases, the remaining elements, with the array <i>tau</i> , represent the orthogonal/unitary matrix <i>Q</i> as a product of $\min(m, n)$ elementary reflectors. |
| <i>tau</i> | Array, size at least $\max(1, \min(m, n))$. (See Orthogonal Factorizations .) Contains scalar factors of the elementary reflectors for the matrix <i>Q</i> . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

Related routines include:

| | |
|-----------------------|---|
| orgqr | to generate matrix <i>Q</i> (for real matrices); |
| ungqr | to generate matrix <i>Q</i> (for complex matrices); |
| ormqr | to apply matrix <i>Q</i> (for real matrices); |
| unmqr | to apply matrix <i>Q</i> (for complex matrices). |

See Also

[mkl_progress](#)

[Matrix Storage Schemes for LAPACK Routines](#)

?orgrq

Generates the real matrix Q of the RQ factorization formed by ?gerqf.

Syntax

```
lapack_int LAPACKE_sorgrq (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
float* a, lapack_int lda, const float* tau);
```

```
lapack_int LAPACKE_dorgrq (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
double* a, lapack_int lda, const double* tau);
```

Include Files

- mkl.h

Description

The routine generates an m -by- n real matrix with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors $H(i)$ of order n : $Q = H(1) * H(2) * \dots * H(k)$ as returned by the routines [gerqf](#). Use this routine after a call to [sgerqf/dgerqf](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix Q ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrix Q ($n \geq m$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q ($m \geq k \geq 0$). |
| <i>a, tau</i> | Arrays: <i>a</i> (size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout), <i>tau</i> . On entry, the $(m - k + i)$ -th row of <i>a</i> must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by sgerqf/dgerqf in the last k rows of its array argument <i>a</i> ; <i>tau</i> [$i - 1$] must contain the scalar factor of the elementary reflector $H(i)$, as returned by sgerqf/dgerqf ; The size of <i>tau</i> must be at least $\max(1, k)$. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

a Overwritten by the last m rows of the n -by- n orthogonal matrix Q .

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The complex counterpart of this routine is [ungrq](#).

?ungrq

Generates the complex matrix Q of the RQ factorization formed by ?gerqf.

Syntax

```
lapack_int LAPACKE_cungrq (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
lapack_complex_float* a, lapack_int lda, const lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zungrq (int matrix_layout, lapack_int m, lapack_int n, lapack_int k,
lapack_complex_double* a, lapack_int lda, const lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine generates an m -by- n complex matrix with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors $H(i)$ of order n : $Q = H(1)^{H*} H(2)^{H*} \dots H(k)^H$ as returned by the routines [gerqf](#). Use this routine after a call to [cgerqf/zgerqf](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix Q ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrix Q ($n \geq m$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q ($m \geq k \geq 0$). |
| <i>a, tau</i> | Arrays: <i>a</i> (size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout), <i>tau</i> . On entry, the $(m - k + i)$ th row of <i>a</i> must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by cgerqf/zgerqf in the last k rows of its array argument <i>a</i> ; <i>tau</i> [$i - 1$] must contain the scalar factor of the elementary reflector $H(i)$, as returned by cgerqf/zgerqf ; The size of <i>tau</i> must be at least $\max(1, k)$. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

a Overwritten by the m last rows of the n -by- n unitary matrix Q .

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The real counterpart of this routine is [orgqr](#).

?ormrq

Multiplies a real matrix by the orthogonal matrix Q of the RQ factorization formed by ?gerqf.

Syntax

```
lapack_int LAPACKE_sormrq (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const float* a, lapack_int lda, const float* tau, float*
c, lapack_int ldc);
```

```
lapack_int LAPACKE_dormrq (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const double* a, lapack_int lda, const double* tau,
double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a real m -by- n matrix C by Q or Q^T , where Q is the real orthogonal matrix defined as a product of k elementary reflectors H_i : $Q = H_1 H_2 \dots H_k$ as returned by the RQ factorization routine [gerqf](#).

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products Q^*C , $Q^{T*}C$, C^*Q , or C^*Q^T (overwriting the result over C).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^T is applied to C from the left. If <i>side</i> = 'R', Q or Q^T is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'T'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'T', the routine multiplies C by Q^T . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: $0 \leq k \leq m$, if <i>side</i> = 'L'; $0 \leq k \leq n$, if <i>side</i> = 'R'. |

a, *tau*, *c*

Arrays: *a*(size for *side* = 'L': $\max(1, lda*m)$ for column major layout and $\max(1, lda*k)$ for row major layout; for *side* = 'R': $\max(1, lda*n)$ for column major layout and $\max(1, lda*k)$ for row major layout), *tau*, *c* (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout).

On entry, the *i*th row of *a* must contain the vector which defines the elementary reflector H_i , for $i = 1, 2, \dots, k$, as returned by *sgerqf*/*dgerqf* in the last *k* rows of its array argument *a*.

tau[*i* - 1] must contain the scalar factor of the elementary reflector H_i , as returned by *sgerqf*/*dgerqf*.

The size of *tau* must be at least $\max(1, k)$.

c contains the *m*-by-*n* matrix *C*.

lda

The leading dimension of *a*; $lda \geq \max(1, k)$ for column major layout. For row major layout, $lda \geq \max(1, m)$ if *side* = 'L', and $lda \geq \max(1, n)$ if *side* = 'R'.

ldc

The leading dimension of *c*; $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.

Output Parameters

c

Overwritten by the product Q^*C , $Q^T C$, C^*Q , or C^*Q^T (as specified by *side* and *trans*).

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The complex counterpart of this routine is [unmrq](#).

?unmrq

Multiplies a complex matrix by the unitary matrix Q of the RQ factorization formed by ?gerqf.

Syntax

```
lapack_int LAPACKE_cunmrq (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zunmrq (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* tau, lapack_complex_double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a complex m -by- n matrix C by Q or Q^H , where Q is the complex unitary matrix defined as a product of k elementary reflectors $H(i)$ of order n : $Q = H(1)^{H*} H(2)^{H*} \dots H(k)^{H*}$ as returned by the RQ factorization routine [gerqf](#).

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products Q^*C , Q^H*C , $C*Q$, or $C*Q^H$ (overwriting the result over C).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^H is applied to C from the left. If <i>side</i> = 'R', Q or Q^H is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'C'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'C', the routine multiplies C by Q^H . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: $0 \leq k \leq m$, if <i>side</i> = 'L'; $0 \leq k \leq n$, if <i>side</i> = 'R'. |
| <i>a, tau, c</i> | Arrays: <i>a</i> (size for <i>side</i> = 'L': $\max(1, lda*m)$ for column major layout and $\max(1, lda*k)$ for row major layout; for <i>side</i> = 'R': $\max(1, lda*n)$ for column major layout and $\max(1, lda*k)$ for row major layout), <i>tau</i> , <i>c</i> (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout). On entry, the i th row of <i>a</i> must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by cgerqf/zgerqf in the last k rows of its array argument <i>a</i> . <i>tau</i> [$i - 1$] must contain the scalar factor of the elementary reflector $H(i)$, as returned by cgerqf/zgerqf . The size of <i>tau</i> must be at least $\max(1, k)$. <i>c</i> (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout) contains the m -by- n matrix C . |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, k)$ for column major layout. For row major layout, $lda \geq \max(1, m)$ if <i>side</i> = 'L', and $lda \geq \max(1, n)$ if <i>side</i> = 'R'. |
| <i>ldc</i> | The leading dimension of <i>c</i> ; $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

c Overwritten by the product Q^*C , Q^H*C , $C*Q$, or $C*Q^H$ (as specified by *side* and *trans*).

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The real counterpart of this routine is [ormrq](#).

?tzrzf

Reduces the upper trapezoidal matrix A to upper triangular form.

Syntax

```
lapack_int LAPACKE_stzrzf (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* tau);
```

```
lapack_int LAPACKE_dtzrzf (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* tau);
```

```
lapack_int LAPACKE_ctzrzf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);
```

```
lapack_int LAPACKE_ztzrzf (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine reduces the *m*-by-*n* ($m \leq n$) real/complex upper trapezoidal matrix *A* to upper triangular form by means of orthogonal/unitary transformations. The upper trapezoidal matrix $A = [A1 \ A2] = [A_{1:m, 1:m}, A_{1:m, m+1:n}]$ is factored as

$$A = [R \ 0] * Z,$$

where *Z* is an *n*-by-*n* orthogonal/unitary matrix, *R* is an *m*-by-*m* upper triangular matrix, and *O* is the *m*-by- $(n-m)$ zero matrix.

The ?tzrzf routine replaces the deprecated ?tzrqf routine.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

m The number of rows in the matrix *A* ($m \geq 0$).

n The number of columns in *A* ($n \geq m$).

| | |
|------------|---|
| <i>a</i> | Array <i>a</i> is of size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout. The leading <i>m</i> -by- <i>n</i> upper trapezoidal part of the array <i>a</i> contains the matrix <i>A</i> to be factorized. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|------------|---|
| <i>a</i> | Overwritten on exit by the factorization data as follows: the leading <i>m</i> -by- <i>m</i> upper triangular part of <i>a</i> contains the upper triangular matrix <i>R</i> , and elements <i>m</i> + 1 to <i>n</i> of the first <i>m</i> rows of <i>a</i> , with the array <i>tau</i> , represent the orthogonal matrix <i>Z</i> as a product of <i>m</i> elementary reflectors. |
| <i>tau</i> | Array, size at least $\max(1, m)$. Contains scalar factors of the elementary reflectors for the matrix <i>Z</i> . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The factorization is obtained by Householder's method. The *k*-th transformation matrix, *Z*(*k*), which is used to introduce zeros into the (*m* - *k* + 1)-th row of *A*, is given in the form

$$Z(k) = \begin{bmatrix} I & 0 \\ 0 & T(k) \end{bmatrix}$$

where for real flavors

$$T(k) = I - \tau u^* u^T, \quad u(k) = \begin{bmatrix} 1 \\ 0 \\ z(k) \end{bmatrix}$$

and for complex flavors

$$T(k) = I - \tau u^* u^H, \quad u(k) = \begin{bmatrix} 1 \\ 0 \\ z(k) \end{bmatrix}$$

τ is a scalar and $z(k)$ is an l -element vector. τ and $z(k)$ are chosen to annihilate the elements of the k -th row of $A2$.

The scalar τ is returned in the k -th element of τ and the vector $u(k)$ in the k -th row of A , such that the elements of $z(k)$ are stored in the last $m - n$ elements of the k -th row of array a .

The elements of R are returned in the upper triangular part of A .

The matrix Z is given by

$$Z = Z(1) * Z(2) * \dots * Z(m).$$

Related routines include:

[ormrz](#) to apply matrix Q (for real matrices)

[unmrz](#) to apply matrix Q (for complex matrices).

?ormrz

Multiplies a real matrix by the orthogonal matrix defined from the factorization formed by [?tzzrf](#).

Syntax

```
lapack_int LAPACKE_sormrz (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, const float* a, lapack_int lda, const float*
tau, float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_dormrz (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, const double* a, lapack_int lda, const
double* tau, double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The [?ormrz](#) routine multiplies a real m -by- n matrix C by Q or Q^T , where Q is the real orthogonal matrix defined as a product of k elementary reflectors $H(i)$ of order n : $Q = H(1) * H(2) * \dots * H(k)$ as returned by the factorization routine [tzrf](#).

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q * C$, $Q^T * C$, $C * Q$, or $C * Q^T$ (overwriting the result over C).

The matrix Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

The [?ormrz](#) routine replaces the deprecated [?latzm](#) routine.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^T is applied to C from the left. If <i>side</i> = 'R', Q or Q^T is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'T'. |

| | |
|----------------------------------|---|
| | If <i>trans</i> = 'N', the routine multiplies <i>C</i> by <i>Q</i> . If <i>trans</i> = 'T', the routine multiplies <i>C</i> by Q^T . |
| <i>m</i> | The number of rows in the matrix <i>C</i> ($m \geq 0$). |
| <i>n</i> | The number of columns in <i>C</i> ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix <i>Q</i> . Constraints: $0 \leq k \leq m$, if <i>side</i> = 'L'; $0 \leq k \leq n$, if <i>side</i> = 'R'. |
| <i>l</i> | The number of columns of the matrix <i>A</i> containing the meaningful part of the Householder reflectors. Constraints: $0 \leq l \leq m$, if <i>side</i> = 'L'; $0 \leq l \leq n$, if <i>side</i> = 'R'. |
| <i>a</i> , <i>tau</i> , <i>c</i> | Arrays: <i>a</i> (size for <i>side</i> = 'L': $\max(1, lda * m)$ for column major layout and $\max(1, lda * k)$ for row major layout; for <i>side</i> = 'R': $\max(1, lda * b)$ for column major layout and $\max(1, lda * k)$ for row major layout), <i>tau</i> , <i>c</i> (size $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout). On entry, the <i>i</i> th row of <i>a</i> must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by <i>stzrzf/dtzrzf</i> in the last <i>k</i> rows of its array argument <i>a</i> . <i>tau</i> [<i>i</i> - 1] must contain the scalar factor of the elementary reflector $H(i)$, as returned by <i>stzrzf/dtzrzf</i> . The size of <i>tau</i> must be at least $\max(1, k)$. <i>c</i> contains the <i>m</i> -by- <i>n</i> matrix <i>C</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, k)$ for column major layout. For row major layout, $lda \geq \max(1, m)$ if <i>side</i> = 'L', and $lda \geq \max(1, n)$ if <i>side</i> = 'R'. |
| <i>ldc</i> | The leading dimension of <i>c</i> ; $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Overwritten by the product $Q * C$, $Q^T * C$, $C * Q$, or $C * Q^T$ (as specified by <i>side</i> and <i>trans</i>). |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The complex counterpart of this routine is [unmrz](#).

?unmrz

Multiplies a complex matrix by the unitary matrix defined from the factorization formed by ?tzrzf.

Syntax

```
lapack_int LAPACKE_cunmrz (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, const lapack_complex_float* a, lapack_int
lda, const lapack_complex_float* tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zunmrz (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, const lapack_complex_double* a, lapack_int
lda, const lapack_complex_double* tau, lapack_complex_double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a complex m -by- n matrix C by Q or Q^H , where Q is the unitary matrix defined as a product of k elementary reflectors $H(i)$:

$Q = H(1)^{H*} H(2)^{H*} \dots H(k)^H$ as returned by the factorization routine [tzrzf](#).

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q*C$, Q^H*C , $C*Q$, or $C*Q^H$ (overwriting the result over C).

The matrix Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^H is applied to C from the left. If <i>side</i> = 'R', Q or Q^H is applied to C from the right. |
| <i>trans</i> | Must be either 'N' or 'C'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'C', the routine multiplies C by Q^H . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q . Constraints: $0 \leq k \leq m$, if <i>side</i> = 'L'; $0 \leq k \leq n$, if <i>side</i> = 'R'. |

| | |
|----------------------------------|---|
| <i>l</i> | The number of columns of the matrix <i>A</i> containing the meaningful part of the Householder reflectors. Constraints: $0 \leq l \leq m$, if <i>side</i> = 'L'; $0 \leq l \leq n$, if <i>side</i> = 'R'. |
| <i>a</i> , <i>tau</i> , <i>c</i> | Arrays: <i>a</i> (size for <i>side</i> = 'L': $\max(1, lda * m)$ for column major layout and $\max(1, lda * k)$ for row major layout; for <i>side</i> = 'R': $\max(1, lda * b)$ for column major layout and $\max(1, lda * k)$ for row major layout), <i>tau</i> , <i>c</i> (size $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout). On entry, the <i>i</i> th row of <i>a</i> must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by <code>ctzrzf/ztzrzf</code> in the last <i>k</i> rows of its array argument <i>a</i> . <i>tau</i> [<i>i</i> - 1] must contain the scalar factor of the elementary reflector $H(i)$, as returned by <code>ctzrzf/ztzrzf</code> . The size of <i>tau</i> must be at least $\max(1, k)$. <i>c</i> contains the <i>m</i> -by- <i>n</i> matrix <i>C</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, k)$ for column major layout. For row major layout, $lda \geq \max(1, m)$ if <i>side</i> = 'L', and $lda \geq \max(1, n)$ if <i>side</i> = 'R'. |
| <i>ldc</i> | The leading dimension of <i>c</i> ; $ldc \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Overwritten by the product $Q * C$, $Q^H * C$, $C * Q$, or $C * Q^H$ (as specified by <i>side</i> and <i>trans</i>). |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The real counterpart of this routine is [ormrz](#).

?ggqrf

Computes the generalized QR factorization of two matrices.

Syntax

```
lapack_int LAPACKE_sggqrf (int matrix_layout, lapack_int n, lapack_int m, lapack_int p,
float* a, lapack_int lda, float* taua, float* b, lapack_int ldb, float* taub);
```

```
lapack_int LAPACKE_dggqrf (int matrix_layout, lapack_int n, lapack_int m, lapack_int p,
double* a, lapack_int lda, double* taua, double* b, lapack_int ldb, double* taub);
```

```
lapack_int LAPACKE_cggqrf (int matrix_layout, lapack_int n, lapack_int m, lapack_int p,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* taua,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* taub);

lapack_int LAPACKE_zggqrf (int matrix_layout, lapack_int n, lapack_int m, lapack_int p,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* taua,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* taub);
```

Include Files

- mkl.h

Description

The routine forms the generalized *QR* factorization of an *n*-by-*m* matrix *A* and an *n*-by-*p* matrix *B* as $A = Q^*R$, $B = Q^*T^*Z$, where *Q* is an *n*-by-*n* orthogonal/unitary matrix, *Z* is a *p*-by-*p* orthogonal/unitary matrix, and *R* and *T* assume one of the forms:

$$R = \begin{matrix} & & m \\ & m & \begin{pmatrix} R_{11} \\ 0 \end{pmatrix} \\ n - m & & \end{matrix}, \quad \text{if } n \geq m$$

or

$$R = \begin{matrix} & n & m - n \\ & \begin{pmatrix} R_{11} & R_{12} \end{pmatrix} \\ n & & \end{matrix}, \quad \text{if } n < m$$

where R_{11} is upper triangular, and

$$T = \begin{matrix} & p - n & n \\ n & \begin{pmatrix} 0 & T_{12} \end{pmatrix} \\ & & \end{matrix}, \quad \text{if } n \leq p,$$

$$T = \begin{matrix} & & p \\ n - p & \begin{pmatrix} T_{11} \\ T_{21} \end{pmatrix} \\ p & & \end{matrix}, \quad \text{if } n > p,$$

where T_{12} or T_{21} is a *p*-by-*p* upper triangular matrix.

In particular, if *B* is square and nonsingular, the *GQR* factorization of *A* and *B* implicitly gives the *QR* factorization of $B^{-1}A$ as:

$B^{-1}A = Z^*T^*(T^{-1}R)$ (for real flavors) or $B^{-1}A = Z^{H*}(T^{-1}R)$ (for complex flavors).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The number of rows of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>m</i> | The number of columns in <i>A</i> ($m \geq 0$). |
| <i>p</i> | The number of columns in <i>B</i> ($p \geq 0$). |
| <i>a, b</i> | Array <i>a</i> of size $\max(1, lda*m)$ for column major layout and $\max(1, lda*n)$ for row major layout contains the matrix <i>A</i> . Array <i>b</i> of size $\max(1, ldb*p)$ for column major layout and $\max(1, ldb*n)$ for row major layout contains the matrix <i>B</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$ for column major layout and at least $\max(1, m)$ for row major layout. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, n)$ for column major layout and at least $\max(1, p)$ for row major layout. |

Output Parameters

| | |
|-------------------|---|
| <i>a, b</i> | Overwritten by the factorization data as follows: on exit, the elements on and above the diagonal of the array <i>a</i> contain the $\min(n,m)$ -by- <i>m</i> upper trapezoidal matrix <i>R</i> (<i>R</i> is upper triangular if $n \geq m$); the elements below the diagonal, with the array <i>taua</i> , represent the orthogonal/unitary matrix <i>Q</i> as a product of $\min(n,m)$ elementary reflectors ; if $n \leq p$, the upper triangle of the subarray <i>b</i> (1: <i>n</i> , <i>p</i> - <i>n</i> +1: <i>p</i>) contains the <i>n</i> -by- <i>n</i> upper triangular matrix <i>T</i> ; if $n > p$, the elements on and above the (<i>n</i> - <i>p</i>)th subdiagonal contain the <i>n</i> -by- <i>p</i> upper trapezoidal matrix <i>T</i> ; the remaining elements, with the array <i>taub</i> , represent the orthogonal/unitary matrix <i>Z</i> as a product of elementary reflectors. |
| <i>taua, taub</i> | Arrays, size at least $\max(1, \min(n, m))$ for <i>taua</i> and at least $\max(1, \min(n, p))$ for <i>taub</i> . The array <i>taua</i> contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix <i>Q</i> . The array <i>taub</i> contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix <i>Z</i> . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

$Q = H(1)H(2) \dots H(k)$, where $k = \min(n, m)$.

Each $H(i)$ has the form

$H(i) = I - \tau_a * v * v^T$ for real flavors, or

$H(i) = I - \tau_a * v * v^H$ for complex flavors,

where τ_a is a real/complex scalar, and v is a real/complex vector with $v_j = 0$ for $1 \leq j \leq i - 1$, $v_i = 1$.

On exit, for $i + 1 \leq j \leq n$, v_j is stored in $a[(j - 1) + (i - 1) * lda]$ for column major layout and in $a[(j - 1) * lda + (i - 1)]$ for row major layout and τ_a is stored in $taua[i - 1]$

The matrix Z is represented as a product of elementary reflectors

$Z = H(1)H(2) \dots H(k)$, where $k = \min(n, p)$.

Each $H(i)$ has the form

$H(i) = I - \tau_b * v * v^T$ for real flavors, or

$H(i) = I - \tau_b * v * v^H$ for complex flavors,

where τ_b is a real/complex scalar, and v is a real/complex vector with $v_{p - k + 1} = 1$, $v_j = 0$ for $p - k + 1 \leq j \leq p - 1$.

On exit, for $1 \leq j \leq p - k + i - 1$, v_j is stored in $b[(n - k + i - 1) + (j - 1) * ldb]$ for column major layout and in $b[(n - k + i - 1) * ldb + (j - 1)]$ for row major layout and τ_b is stored in $taub[i - 1]$.

?ggrqf

Computes the generalized RQ factorization of two matrices.

Syntax

```
lapack_int LAPACKE_sggrqf (int matrix_layout, lapack_int m, lapack_int p, lapack_int n,
float* a, lapack_int lda, float* taua, float* b, lapack_int ldb, float* taub);
```

```
lapack_int LAPACKE_dggrqf (int matrix_layout, lapack_int m, lapack_int p, lapack_int n,
double* a, lapack_int lda, double* taua, double* b, lapack_int ldb, double* taub);
```

```
lapack_int LAPACKE_cggrqf (int matrix_layout, lapack_int m, lapack_int p, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* taua,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* taub);
```

```
lapack_int LAPACKE_zggrqf (int matrix_layout, lapack_int m, lapack_int p, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* taua,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* taub);
```

Include Files

- mkl.h

Description

The routine forms the generalized RQ factorization of an m -by- n matrix A and an p -by- n matrix B as $A = R * Q$, $B = Z * T * Q$, where Q is an n -by- n orthogonal/unitary matrix, Z is a p -by- p orthogonal/unitary matrix, and R and T assume one of the forms:

$$R = \begin{matrix} n - m & m \\ m & (0 & R_{12}) \end{matrix}, \quad \text{if } m \leq n,$$

or

$$R = \begin{matrix} m - n & & n \\ & & \\ & & \end{matrix} \begin{pmatrix} R_{11} \\ R_{21} \end{pmatrix}, \text{ if } m > n,$$

where R_{11} or R_{21} is upper triangular, and

$$T = \begin{matrix} & & n \\ & n & \\ p - n & & \end{matrix} \begin{pmatrix} T_{11} \\ 0 \end{pmatrix}, \text{ if } p \geq n,$$

or

$$T = \begin{matrix} & p & n - p \\ p & (T_{11} & T_{12}) \end{matrix}, \text{ if } p < n,$$

where T_{11} is upper triangular.

In particular, if B is square and nonsingular, the GRQ factorization of A and B implicitly gives the RQ factorization of $A*B^{-1}$ as:

$$A*B^{-1} = (R*T^{-1}) * Z^T \text{ (for real flavors) or } A*B^{-1} = (R*T^{-1}) * Z^H \text{ (for complex flavors).}$$

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix A ($m \geq 0$). |
| <i>p</i> | The number of rows in B ($p \geq 0$). |
| <i>n</i> | The number of columns of the matrices A and B ($n \geq 0$). |
| <i>a, b</i> | Arrays: <i>a</i> (size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout) contains the m -by- n matrix A . <i>b</i> (size $\max(1, ldb*n)$ for column major layout and $\max(1, ldb*p)$ for row major layout) contains the p -by- n matrix B . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, p)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

a, b

Overwritten by the factorization data as follows:

on exit, if $m \leq n$, element R_{ij} ($1 \leq i \leq j \leq m$) of upper triangular matrix R is stored in $a[(i - 1) + (n - m + j - 1) * lda]$ for column major layout and in $a[(i - 1) * lda + (n - m + j - 1)]$ for row major layout.

if $m > n$, the elements on and above the $(m-n)$ th subdiagonal contain the m -by- n upper trapezoidal matrix R ;

the remaining elements, with the array *taua*, represent the orthogonal/unitary matrix Q as a product of elementary reflectors.

The elements on and above the diagonal of the array *b* contain the $\min(p, n)$ -by- n upper trapezoidal matrix T (T is upper triangular if $p \geq n$); the elements below the diagonal, with the array *taub*, represent the orthogonal/unitary matrix Z as a product of elementary reflectors.

taua, taub

Arrays, size at least $\max(1, \min(m, n))$ for *taua* and at least $\max(1, \min(p, n))$ for *taub*.

The array *taua* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix Q .

The array *taub* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix Z .

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The matrix Q is represented as a product of elementary reflectors

$$Q = H(1)H(2) \dots H(k), \text{ where } k = \min(m, n).$$

Each $H(i)$ has the form

$$H(i) = I - \tau_a v v^T \text{ for real flavors, or}$$

$$H(i) = I - \tau_a v v^H \text{ for complex flavors,}$$

where τ_a is a real/complex scalar, and v is a real/complex vector with $v_{n-k+i} = 1, v_{n-k+i+1:n} = 0$.

On exit, $v_{1:n-k+i-1}$ is stored in $a(m-k+i, 1:n-k+i-1)$ and τ_a is stored in $\tau_a[i - 1]$.

The matrix Z is represented as a product of elementary reflectors

$$Z = H(1)H(2) \dots H(k), \text{ where } k = \min(p, n).$$

Each $H(i)$ has the form

$$H(i) = I - \tau_b v v^T \text{ for real flavors, or}$$

$$H(i) = I - \tau_b v v^H \text{ for complex flavors,}$$

where τ_b is a real/complex scalar, and v is a real/complex vector with $v_{1:i-1} = 0, v_i = 1$.

On exit, $v_{i+1:p}$ is stored in $b(i+1:p, i)$ and τ_b is stored in $\tau_b[i - 1]$.

?tpqrt

Computes a blocked QR factorization of a real or complex "triangular-pentagonal" matrix, which is composed of a triangular block and a pentagonal block, using the compact WY representation for Q.

Syntax

```
lapack_int LAPACKE_stpqrt (int matrix_layout, lapack_int m, lapack_int n, lapack_int l,
lapack_int nb, float* a, lapack_int lda, float* b, lapack_int ldb, float* t,
lapack_int ldt);
```

```
lapack_int LAPACKE_dtpqrt (int matrix_layout, lapack_int m, lapack_int n, lapack_int l,
lapack_int nb, double* a, lapack_int lda, double* b, lapack_int ldb, double* t,
lapack_int ldt);
```

```
lapack_int LAPACKE_ctpqrt (int matrix_layout, lapack_int m, lapack_int n, lapack_int l,
lapack_int nb, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b,
lapack_int ldb, lapack_complex_float* t, lapack_int ldt);
```

```
lapack_int LAPACKE_ztpqrt (int matrix_layout, lapack_int m, lapack_int n, lapack_int l,
lapack_int nb, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b,
lapack_int ldb, lapack_complex_double* t, lapack_int ldt);
```

Include Files

- mkl.h

Description

The input matrix C is an $(n+m)$ -by- n matrix

$$C = \begin{bmatrix} A \\ B \end{bmatrix} \leftarrow \begin{array}{l} n \times n \text{ upper triangular} \\ m \times n \text{ pentagonal} \end{array}$$

where A is an n -by- n upper triangular matrix, and B is an m -by- n pentagonal matrix consisting of an $(m-1)$ -by- n rectangular matrix $B1$ on top of an l -by- n upper trapezoidal matrix $B2$:

$$B = \begin{bmatrix} B1 \\ B2 \end{bmatrix} \leftarrow \begin{array}{l} (m-1) \times n \text{ rectangular} \\ l \times n \text{ upper trapezoidal} \end{array}$$

The upper trapezoidal matrix $B2$ consists of the first l rows of an n -by- n upper triangular matrix, where $0 \leq l \leq \min(m, n)$. If $l=0$, B is an m -by- n rectangular matrix. If $m=1=n$, B is upper triangular. The elementary reflectors $H(i)$ are stored in the i th column below the diagonal in the $(n+m)$ -by- n input matrix C . The structure of vectors defining the elementary reflectors is illustrated by:

$$\begin{bmatrix} I \\ V \end{bmatrix} \leftarrow \begin{array}{l} n \times n \text{ identity} \\ m \times n \text{ pentagonal} \end{array}$$

The elements of the unit matrix I are not stored. Thus, V contains all of the necessary information, and is returned in array b .

NOTE

Note that V has the same form as B :

$$V = \begin{bmatrix} V1 \\ V2 \end{bmatrix} \leftarrow \begin{matrix} (m-l) \times n \text{ rectangular} \\ l \times n \text{ upper trapezoidal} \end{matrix}$$

The columns of V represent the vectors which define the $H(i)$ s.

The number of blocks is $k = \text{ceiling}(n/nb)$, where each block is of order nb except for the last block, which is of order $ib = n - (k-1)*nb$. For each of the k blocks, an upper triangular block reflector factor is computed: $T1, T2, \dots, Tk$. The nb -by- nb (ib -by- ib for the last block) T is are stored in the nb -by- n array t as

$$t = [T1T2 \dots Tk] .$$

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The total number of rows in the matrix B ($m \geq 0$). |
| <i>n</i> | The number of columns in B and the order of the triangular matrix A ($n \geq 0$). |
| <i>l</i> | The number of rows of the upper trapezoidal part of B ($\min(m, n) \geq l \geq 0$). |
| <i>nb</i> | The block size to use in the blocked QR factorization ($n \geq nb \geq 1$). |
| <i>a, b</i> | Arrays: <i>a</i> size $lda*n$ contains the n -by- n upper triangular matrix A . <i>b</i> size $\max(1, ldb*n)$ for column major layout and $\max(1, ldb*m)$ for row major layout, the pentagonal m -by- n matrix B . The first $(m-1)$ rows contain the rectangular $B1$ matrix, and the next l rows contain the upper trapezoidal $B2$ matrix. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>ldt</i> | The leading dimension of <i>t</i> ; at least nb for column major layout and at least $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | The elements on and above the diagonal of the array contain the upper triangular matrix R . |
| <i>b</i> | The pentagonal matrix V . |
| <i>t</i> | Array, size $ldt*n$ for column major layout and $ldt*nb$ for row major layout. |

The upper triangular block reflectors stored in compact form as a sequence of upper triangular blocks.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

?tpmqrt

Applies a real or complex orthogonal matrix obtained from a "triangular-pentagonal" complex block reflector to a general real or complex matrix, which consists of two blocks.

Syntax

```
lapack_int LAPACKE_stpmqrt (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, lapack_int nb, const float* v, lapack_int
ldv, const float* t, lapack_int ldt, float* a, lapack_int lda, float* b, lapack_int
ldb);
```

```
lapack_int LAPACKE_dtpmqrt (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, lapack_int nb, const double* v, lapack_int
ldv, const double* t, lapack_int ldt, double* a, lapack_int lda, double* b, lapack_int
ldb);
```

```
lapack_int LAPACKE_ctpmqrt (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, lapack_int nb, const lapack_complex_float*
v, lapack_int ldv, const lapack_complex_float* t, lapack_int ldt, lapack_complex_float*
a, lapack_int lda, lapack_complex_float* b, lapack_int ldb);
```

```
lapack_int LAPACKE_ztpmqrt (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int k, lapack_int l, lapack_int nb, const lapack_complex_double*
v, lapack_int ldv, const lapack_complex_double* t, lapack_int ldt,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb);
```

Include Files

- mkl.h

Description

The columns of the pentagonal matrix *V* contain the elementary reflectors $H(1), H(2), \dots, H(k)$; *V* is composed of a rectangular block *V1* and a trapezoidal block *V2*:

$$V = \begin{bmatrix} V1 \\ V2 \end{bmatrix}$$

The size of the trapezoidal block *V2* is determined by the parameter *l*, where $0 \leq l \leq k$. *V2* is upper trapezoidal, consisting of the first *l* rows of a *k*-by-*k* upper triangular matrix.

If $l=k$, *V2* is upper triangular;

If $l=0$, there is no trapezoidal block, so $V = V1$ is rectangular.

If $side = 'L'$:

$$C = \begin{bmatrix} A \\ B \end{bmatrix}$$

where A is k -by- n , B is m -by- n and V is m -by- k .

If $side = 'R'$:

$$C = [A \ B]$$

where A is m -by- k , B is m -by- n and V is n -by- k .

The real/complex orthogonal matrix Q is formed from V and T .

If $trans='N'$ and $side='L'$, c contains $Q * C$ on exit.

If $trans='T'$ and $side='L'$, c contains $Q^T * C$ on exit.

If $trans='C'$ and $side='L'$, c contains $Q^H * C$ on exit.

If $trans='N'$ and $side='R'$, c contains $C * Q$ on exit.

If $trans='T'$ and $side='R'$, c contains $C * Q^T$ on exit.

If $trans='C'$ and $side='R'$, c contains $C * Q^H$ on exit.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | ='L': apply Q , Q^T , or Q^H from the left. ='R': apply Q , Q^T , or Q^H from the right. |
| <i>trans</i> | ='N', no transpose, apply Q . ='T', transpose, apply Q^T . ='C', transpose, apply Q^H . |
| <i>m</i> | The number of rows in the matrix B , ($m \geq 0$). |
| <i>n</i> | The number of columns in the matrix B , ($n \geq 0$). |
| <i>k</i> | The number of elementary reflectors whose product defines the matrix Q , ($k \geq 0$). |
| <i>l</i> | The order of the trapezoidal part of V ($k \geq l \geq 0$). |
| <i>nb</i> | The block size used for the storage of t , $k \geq nb \geq 1$. This must be the same value of nb used to generate t in <code>tpqrt</code> . |
| <i>v</i> | Size $ldv*k$ for column major layout; $ldv*m$ for row major layout and $side = 'L'$, $ldv*n$ for row major layout and $side = 'R'$. |

| | |
|-------|--|
| | The i th column must contain the vector which defines the elementary reflector $H(i)$, for $i = 1, 2, \dots, k$, as returned by <code>tpqrt</code> in array argument b . |
| ldv | The leading dimension of the array v . If $side = 'L'$, ldv must be at least $\max(1, m)$ for column major layout and $\max(1, k)$ for row major layout; If $side = 'R'$, ldv must be at least $\max(1, n)$ for column major layout and $\max(1, k)$ for row major layout. |
| t | Array, size $ldt*k$ for column major layout and $ldt*nb$ for row major layout. The upper triangular factors of the block reflectors as returned by <code>tpqrt</code> |
| ldt | The leading dimension of the array t . ldt must be at least nb for column major layout and $\max(1, k)$ for row major layout. |
| a | If $side = 'L'$, size $lda*n$ for column major layout and $lda*k$ for row major layout .. If $side = 'R'$, size $lda*k$ for column major layout and $lda*m$ for row major layout .. The k -by- n or m -by- k matrix A . |
| lda | The leading dimension of the array a . If $side = 'L'$, lda must be at least $\max(1, k)$ for column major layout and $\max(1, n)$ for row major layout. If $side = 'R'$, lda must be at least $\max(1, m)$ for column major layout and $\max(1, k)$ for row major layout. |
| b | Size $ldb*n$ for column major layout and $ldb*m$ for row major layout. The m -by- n matrix B . |
| ldb | The leading dimension of the array b . ldb must be at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|-----|--|
| a | Overwritten by the corresponding block of the product $Q*C$, $C*Q$, Q^T*C , $C*Q^T$, Q^H*C , or $C*Q^H$. |
| b | Overwritten by the corresponding block of the product $Q*C$, $C*Q$, Q^T*C , $C*Q^T$, Q^H*C , or $C*Q^H$. |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Singular Value Decomposition: LAPACK Computational Routines

This section describes LAPACK routines for computing the *singular value decomposition* (SVD) of a general m -by- n matrix A :

$$A = U\Sigma V^H.$$

In this decomposition, U and V are unitary (for complex A) or orthogonal (for real A); Σ is an m -by- n diagonal matrix with real diagonal elements σ_i :

$$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_{\min(m, n)} \geq 0.$$

The diagonal elements σ_i are *singular values* of A . The first $\min(m, n)$ columns of the matrices U and V are, respectively, *left* and *right singular vectors* of A . The singular values and singular vectors satisfy

$$AV_i = \sigma_i u_i \text{ and } A^H u_i = \sigma_i v_i$$

where u_i and v_i are the i -th columns of U and V , respectively.

To find the SVD of a general matrix A , call the LAPACK routine `?gebrd` or `?gbbbrd` for reducing A to a bidiagonal matrix B by a unitary (orthogonal) transformation: $A = QBP^H$. Then call `?bdsqr`, which forms the SVD of a bidiagonal matrix: $B = U_1 \Sigma V_1^H$.

Thus, the sought-for SVD of A is given by $A = U\Sigma V^H = (QU_1)\Sigma(V_1^H P^H)$.

Table "Computational Routines for Singular Value Decomposition (SVD)" lists LAPACK routines that perform singular value decomposition of matrices.

Computational Routines for Singular Value Decomposition (SVD)

| Operation | Real matrices | Complex matrices |
|--|---|----------------------|
| Reduce A to a bidiagonal matrix B : $A = QBP^H$ (full storage) | <code>?gebrd</code> | <code>?gebrd</code> |
| Reduce A to a bidiagonal matrix B : $A = QBP^H$ (band storage) | <code>?gbbbrd</code> | <code>?gbbbrd</code> |
| Generate the orthogonal (unitary) matrix Q or P | <code>?orgbr</code> | <code>?ungbr</code> |
| Apply the orthogonal (unitary) matrix Q or P | <code>?ormbr</code> | <code>?unmbr</code> |
| Form singular value decomposition of the bidiagonal matrix B : $B = U_1 \Sigma V_1^H$ | <code>?bdsqr</code> <code>?bdsdc</code> | <code>?bdsqr</code> |

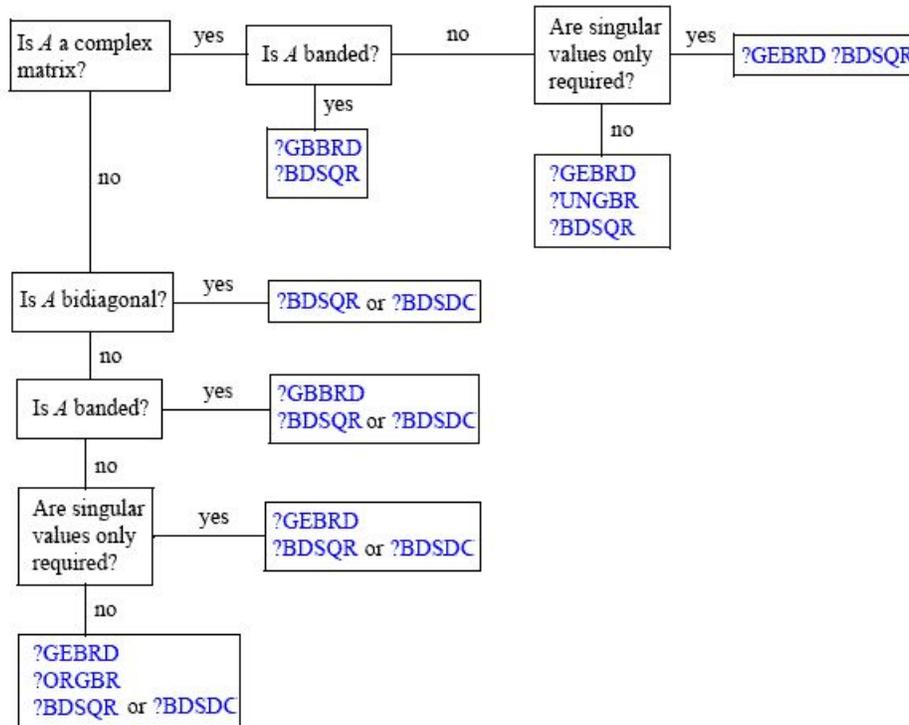
Decision Tree: Singular Value Decomposition

Figure "Decision Tree: Singular Value Decomposition" presents a decision tree that helps you choose the right sequence of routines for SVD, depending on whether you need singular values only or singular vectors as well, whether A is real or complex, and so on.

You can use the SVD to find a minimum-norm solution to a (possibly) rank-deficient least squares problem of minimizing $\|Ax - b\|^2$. The effective rank k of the matrix A can be determined as the number of singular values which exceed a suitable threshold. The minimum-norm solution is

$$x = V_k(\Sigma_k)^{-1}c$$

where Σ_k is the leading k -by- k submatrix of Σ , the matrix V_k consists of the first k columns of $V = PV_1$, and the vector c consists of the first k elements of $U^H b = U_1^H Q^H b$.

?gebrd

Reduces a general matrix to bidiagonal form.

Syntax

```
lapack_int LAPACKE_sgebrd( int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* d, float* e, float* tauq, float* tauq );
```

```
lapack_int LAPACKE_dgebrd( int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* d, double* e, double* tauq, double* tauq );
```

```
lapack_int LAPACKE_cgebrd( int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* d, float* e, lapack_complex_float*
tauq, lapack_complex_float* tauq );
```

```
lapack_int LAPACKE_zgebrd( int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* d, double* e, lapack_complex_double*
tauq, lapack_complex_double* tauq );
```

Include Files

- `mkl.h`

Description

The routine reduces a general m -by- n matrix A to a bidiagonal matrix B by an orthogonal (unitary) transformation.

$$A = QBP^H = Q \begin{pmatrix} B_1 \\ 0 \end{pmatrix} P^H = Q_1 B_1 P^H,$$

If $m \geq n$, the reduction is given by

where B_1 is an n -by- n upper diagonal matrix, Q and P are orthogonal or, for a complex A , unitary matrices; Q_1 consists of the first n columns of Q .

If $m < n$, the reduction is given by

$$A = Q^* B^* P^H = Q^* (B_1 0) P^H = Q_1^* B_1^* P_1^H,$$

where B_1 is an m -by- m lower diagonal matrix, Q and P are orthogonal or, for a complex A , unitary matrices; P_1 consists of the first m columns of P .

The routine does not form the matrices Q and P explicitly, but represents them as products of elementary reflectors. Routines are provided to work with the matrices Q and P in this representation:

If the matrix A is real,

- to compute Q and P explicitly, call [orgbr](#).
- to multiply a general matrix by Q or P , call [ormbr](#).

If the matrix A is complex,

- to compute Q and P explicitly, call [ungbr](#).
- to multiply a general matrix by Q or P , call [unmbr](#).

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>m</code> | The number of rows in the matrix A ($m \geq 0$). |
| <code>n</code> | The number of columns in A ($n \geq 0$). |
| <code>a</code> | Arrays: <code>a</code> (size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout) contains the matrix A . |
| <code>lda</code> | The leading dimension of <code>a</code> ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------------|---|
| <code>a</code> | If $m \geq n$, the diagonal and first super-diagonal of <code>a</code> are overwritten by the upper bidiagonal matrix B . The elements below the diagonal, with the array <code>tauq</code> , represent the orthogonal matrix Q as a product of elementary |
|----------------|---|

reflectors, and the elements above the first superdiagonal, with the array tau_p , represent the orthogonal matrix P as a product of elementary reflectors.

If $m < n$, the diagonal and first sub-diagonal of a are overwritten by the lower bidiagonal matrix B . The elements below the first subdiagonal, with the array tau_q , represent the orthogonal matrix Q as a product of elementary reflectors, and the elements above the diagonal, with the array tau_p , represent the orthogonal matrix P as a product of elementary reflectors.

| | |
|------------------------------|--|
| d | Array, size at least $\max(1, \min(m, n))$. Contains the diagonal elements of B . |
| e | Array, size at least $\max(1, \min(m, n) - 1)$. Contains the off-diagonal elements of B . |
| $\text{tau}_q, \text{tau}_p$ | Arrays, size at least $\max(1, \min(m, n))$. The scalar factors of the elementary reflectors which represent the orthogonal or unitary matrices P and Q . |

Return Values

This function returns a value info .

If $\text{info}=0$, the execution is successful.

If $\text{info} = -i$, the i -th parameter had an illegal value.

Application Notes

The computed matrices Q , B , and P satisfy $QB P^H = A + E$, where $\|E\|_2 = c(n)\varepsilon \|A\|_2$, $c(n)$ is a modestly increasing function of n , and ε is the machine precision.

The approximate number of floating-point operations for real flavors is

$$(4/3) * n^2 * (3 * m - n) \text{ for } m \geq n,$$

$$(4/3) * m^2 * (3 * n - m) \text{ for } m < n.$$

The number of operations for complex flavors is four times greater.

If n is much less than m , it can be more efficient to first form the QR factorization of A by calling [geqrf](#) and then reduce the factor R to bidiagonal form. This requires approximately $2 * n^2 * (m + n)$ floating-point operations.

If m is much less than n , it can be more efficient to first form the LQ factorization of A by calling [gelqf](#) and then reduce the factor L to bidiagonal form. This requires approximately $2 * m^2 * (m + n)$ floating-point operations.

?gbbbrd

Reduces a general band matrix to bidiagonal form.

Syntax

```
lapack_int LAPACKE_sgbbrd( int matrix_layout, char vect, lapack_int m, lapack_int n,
lapack_int ncc, lapack_int kl, lapack_int ku, float* ab, lapack_int ldab, float* d,
float* e, float* q, lapack_int ldq, float* pt, lapack_int ldpt, float* c, lapack_int
ldc );
```

```
lapack_int LAPACKE_dgbbrd( int matrix_layout, char vect, lapack_int m, lapack_int n,
lapack_int ncc, lapack_int kl, lapack_int ku, double* ab, lapack_int ldab, double* d,
double* e, double* q, lapack_int ldq, double* pt, lapack_int ldpt, double* c,
lapack_int ldc );
```

```
lapack_int LAPACKE_cgbbbrd( int matrix_layout, char vect, lapack_int m, lapack_int n,
lapack_int ncc, lapack_int kl, lapack_int ku, lapack_complex_float* ab, lapack_int
ldab, float* d, float* e, lapack_complex_float* q, lapack_int ldq,
lapack_complex_float* pt, lapack_int ldpt, lapack_complex_float* c, lapack_int ldc );
```

```
lapack_int LAPACKE_zgbbrd( int matrix_layout, char vect, lapack_int m, lapack_int n,
lapack_int ncc, lapack_int kl, lapack_int ku, lapack_complex_double* ab, lapack_int
ldab, double* d, double* e, lapack_complex_double* q, lapack_int ldq,
lapack_complex_double* pt, lapack_int ldpt, lapack_complex_double* c, lapack_int ldc );
```

Include Files

- mkl.h

Description

The routine reduces an m -by- n band matrix A to upper bidiagonal matrix B : $A = Q*B*P^H$. Here the matrices Q and P are orthogonal (for real A) or unitary (for complex A). They are determined as products of Givens rotation matrices, and may be formed explicitly by the routine if required. The routine can also update a matrix C as follows: $C = Q^H*C$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>vect</i> | Must be 'N' or 'Q' or 'P' or 'B'. If <i>vect</i> = 'N', neither Q nor P^H is generated. If <i>vect</i> = 'Q', the routine generates the matrix Q . If <i>vect</i> = 'P', the routine generates the matrix P^H . If <i>vect</i> = 'B', the routine generates both Q and P^H . |
| <i>m</i> | The number of rows in the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns in A ($n \geq 0$). |
| <i>ncc</i> | The number of columns in C ($ncc \geq 0$). |
| <i>kl</i> | The number of sub-diagonals within the band of A ($kl \geq 0$). |
| <i>ku</i> | The number of super-diagonals within the band of A ($ku \geq 0$). |
| <i>ab, c</i> | Arrays: <i>ab</i> (size $\max(1, ldab*n)$ for column major layout and $\max(1, ldab*m)$ for row major layout) contains the matrix A in band storage (see Matrix Storage Schemes). <i>c</i> (size $\max(1, ldc*ncc)$ for column major layout and $\max(1, ldc*m)$ for row major layout) contains an m -by- ncc matrix C . |

| | |
|--------|--|
| | If $ncc = 0$, the array c is not referenced. |
| $ldab$ | The leading dimension of the array ab ($ldab \geq kl + ku + 1$). |
| ldq | The leading dimension of the output array q . $ldq \geq \max(1, m)$ if $vect = 'Q'$ or $'B'$, $ldq \geq 1$ otherwise. |
| $ldpt$ | The leading dimension of the output array pt . $ldpt \geq \max(1, n)$ if $vect = 'P'$ or $'B'$, $ldpt \geq 1$ otherwise. |
| ldc | The leading dimension of the array c . $ldc \geq \max(1, m)$ if $ncc > 0$; $ldc \geq 1$ if $ncc = 0$. |

Output Parameters

| | |
|---------|--|
| ab | Overwritten by values generated during the reduction. |
| d | Array, size at least $\max(1, \min(m, n))$. Contains the diagonal elements of the matrix B . |
| e | Array, size at least $\max(1, \min(m, n) - 1)$. Contains the off-diagonal elements of B . |
| q, pt | Arrays: $qsize \max(1, ldq * m)$ contains the output m -by- m matrix Q . $psize \max(1, ldpt * n)$ contains the output n -by- n matrix P^T . |
| c | Overwritten by the product $Q^H * C$. c is not referenced if $ncc = 0$. |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed matrices Q , B , and P satisfy $Q * B * P^H = A + E$, where $\|E\|_2 = c(n)\epsilon \|A\|_2$, $c(n)$ is a modestly increasing function of n , and ϵ is the machine precision.

If $m = n$, the total number of floating-point operations for real flavors is approximately the sum of:

$6 * n^2 * (kl + ku)$ if $vect = 'N'$ and $ncc = 0$,

$3 * n^2 * ncc * (kl + ku - 1) / (kl + ku)$ if C is updated, and

$3 * n^3 * (kl + ku - 1) / (kl + ku)$ if either Q or P^H is generated (double this if both).

To estimate the number of operations for complex flavors, use the same formulas with the coefficients 20 and 10 (instead of 6 and 3).

?orgbr

Generates the real orthogonal matrix Q or P^T determined by ?gebrd.

Syntax

```
lapack_int LAPACKE_sorgbr (int matrix_layout, char vect, lapack_int m, lapack_int n,
lapack_int k, float* a, lapack_int lda, const float* tau);
```

```
lapack_int LAPACKE_dorgbr (int matrix_layout, char vect, lapack_int m, lapack_int n,
lapack_int k, double* a, lapack_int lda, const double* tau);
```

Include Files

- mkl.h

Description

The routine generates the whole or part of the orthogonal matrices Q and P^T formed by the routines [gebrd](#)/[gebrd](#). Use this routine after a call to [sgebrd](#)/[dgebrd](#). All valid combinations of arguments are described in *Input parameters*. In most cases you need the following:

To compute the whole m -by- m matrix Q :

```
LAPACKE_?orgbr(matrix_layout, 'Q', m, m, n, a, lda, tau )
```

(note that the array a must have at least m columns).

To form the n leading columns of Q if $m > n$:

```
LAPACKE_?orgbr(matrix_layout, 'Q', m, n, n, a, lda, tau )
```

To compute the whole n -by- n matrix P^T :

```
LAPACKE_?orgbr(matrix_layout, 'P', n, n, m, a, lda, tau )
```

(note that the array a must have at least n rows).

To form the m leading rows of P^T if $m < n$:

```
LAPACKE_?orgbr(matrix_layout, 'P', m, n, m, a, lda, tau )
```

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>vect</i> | Must be 'Q' or 'P'. If <i>vect</i> = 'Q', the routine generates the matrix Q . If <i>vect</i> = 'P', the routine generates the matrix P^T . |
| <i>m, n</i> | The number of rows (m) and columns (n) in the matrix Q or P^T to be returned ($m \geq 0, n \geq 0$). If <i>vect</i> = 'Q', $m \geq n \geq \min(m, k)$. If <i>vect</i> = 'P', $n \geq m \geq \min(n, k)$. |
| <i>k</i> | If <i>vect</i> = 'Q', the number of columns in the original m -by- k matrix reduced by gebrd . |

| | |
|------------|--|
| | If <i>vect</i> = 'P', the number of rows in the original <i>k</i> -by- <i>n</i> matrix reduced by <i>gebrd</i> . |
| <i>a</i> | Array, size at least <i>lda</i> * <i>n</i> for column major layout and <i>lda</i> * <i>m</i> for row major layout. The vectors which define the elementary reflectors, as returned by <i>gebrd</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout . |
| <i>tau</i> | Array, size $\min(m, k)$ if <i>vect</i> = 'Q', $\min(n, k)$ if <i>vect</i> = 'P'. Scalar factor of the elementary reflector <i>H</i> (<i>i</i>) or <i>G</i> (<i>i</i>), which determines <i>Q</i> and <i>P</i> ^T as returned by <i>gebrd</i> in the array <i>tauq</i> or <i>taup</i> . |

Output Parameters

| | |
|----------|---|
| <i>a</i> | Overwritten by the orthogonal matrix <i>Q</i> or <i>P</i> ^T (or the leading rows or columns thereof) as specified by <i>vect</i> , <i>m</i> , and <i>n</i> . |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed matrix *Q* differs from an exactly orthogonal matrix by a matrix *E* such that $\|E\|_2 = O(\epsilon)$.

The approximate numbers of floating-point operations for the cases listed in *Description* are as follows:

To form the whole of *Q*:

$(4/3) * n * (3m^2 - 3m * n + n^2)$ if $m > n$;

$(4/3) * m^3$ if $m \leq n$.

To form the *n* leading columns of *Q* when $m > n$:

$(2/3) * n^2 * (3m - n^2)$ if $m > n$.

To form the whole of *P*^T:

$(4/3) * n^3$ if $m \geq n$;

$(4/3) * m * (3n^2 - 3m * n + m^2)$ if $m < n$.

To form the *m* leading columns of *P*^T when $m < n$:

$(2/3) * n^2 * (3m - n^2)$ if $m > n$.

The complex counterpart of this routine is *ungbr*.

?ormbr

Multiplies an arbitrary real matrix by the real orthogonal matrix Q or P^T determined by ?gebrd.

Syntax

```
lapack_int LAPACKE_sormbr (int matrix_layout, char vect, char side, char trans,
lapack_int m, lapack_int n, lapack_int k, const float* a, lapack_int lda, const float*
tau, float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_dormbr (int matrix_layout, char vect, char side, char trans,
lapack_int m, lapack_int n, lapack_int k, const double* a, lapack_int lda, const
double* tau, double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

Given an arbitrary real matrix C , this routine forms one of the matrix products Q^*C , $Q^{T*}C$, C^*Q , C^*Q^T , P^*C , $P^{T*}C$, C^*P , C^*P^T , where Q and P are orthogonal matrices computed by a call to [gebrd](#). The routine overwrites the product on C .

Input Parameters

In the descriptions below, r denotes the order of Q or P^T :

If $side = 'L'$, $r = m$; if $side = 'R'$, $r = n$.

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>vect</i> | Must be 'Q' or 'P'. If <i>vect</i> = 'Q', then Q or Q^T is applied to C . If <i>vect</i> = 'P', then P or P^T is applied to C . |
| <i>side</i> | Must be 'L' or 'R'. If <i>side</i> = 'L', multipliers are applied to C from the left. If <i>side</i> = 'R', they are applied to C from the right. |
| <i>trans</i> | Must be 'N' or 'T'. If <i>trans</i> = 'N', then Q or P is applied to C . If <i>trans</i> = 'T', then Q^T or P^T is applied to C . |
| <i>m</i> | The number of rows in C . |
| <i>n</i> | The number of columns in C . |
| <i>k</i> | One of the dimensions of A in ?gebrd : If <i>vect</i> = 'Q', the number of columns in A ; If <i>vect</i> = 'P', the number of rows in A . Constraints: $m \geq 0$, $n \geq 0$, $k \geq 0$. |
| <i>a, c</i> | Arrays: <i>a</i> is the array <i>a</i> as returned by ?gebrd . The size of <i>a</i> depends on the value of the <i>matrix_layout</i> , <i>vect</i> , and <i>side</i> parameters: |

| <i>matrix_layout</i> | <i>vect</i> | <i>side</i> | size |
|----------------------|-------------|-------------|------------------|
| column major | 'Q' | - | $\max(1, lda*k)$ |

| <i>matrix_layout</i> | <i>vect</i> | <i>side</i> | size |
|----------------------|-------------|-------------|------------------|
| column major | 'P' | 'L' | $\max(1, lda*m)$ |
| column major | 'P' | 'R' | $\max(1, lda*n)$ |
| row major | 'Q' | 'L' | $\max(1, lda*m)$ |
| row major | 'Q' | 'R' | $\max(1, lda*n)$ |
| row major | 'P' | - | $\max(1, lda*k)$ |

c (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout) holds the matrix *C*.

lda

The leading dimension of *a*. Constraints:

$lda \geq \max(1, r)$ for column major layout and at least $\max(1, k)$ for row major layout if *vect* = 'Q';

$lda \geq \max(1, \min(r, k))$ for column major layout and at least $\max(1, r)$ for row major layout if *vect* = 'P'.

ldc

The leading dimension of *c*; $ldc \geq \max(1, m)$ for column major layout and $ldc \geq \max(1, n)$ for row major layout .

tau

Array, size at least $\max(1, \min(r, k))$.

For *vect* = 'Q', the array *tauq* as returned by ?gebrd. For *vect* = 'P', the array *taup* as returned by ?gebrd.

Output Parameters

c

Overwritten by the product Q^*C , Q^T*C , $C*Q$, $C*Q^T$, P^*C , P^T*C , C^*P , or C^*P^T , as specified by *vect*, *side*, and *trans*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed product differs from the exact product by a matrix *E* such that $\|E\|_2 = O(\epsilon) * \|C\|_2$.

The total number of floating-point operations is approximately

$2*n*k(2*m - k)$ if *side* = 'L' and $m \geq k$;

$2*m*k(2*n - k)$ if *side* = 'R' and $n \geq k$;

$2*m^2*n$ if *side* = 'L' and $m < k$;

$2*n^2*m$ if *side* = 'R' and $n < k$.

The complex counterpart of this routine is [unmbr](#).

?ungbr

Generates the complex unitary matrix Q or P^H determined by ?gebrd.

Syntax

```
lapack_int LAPACKE_cungbr (int matrix_layout, char vect, lapack_int m, lapack_int n,
lapack_int k, lapack_complex_float* a, lapack_int lda, const lapack_complex_float*
tau);
```

```
lapack_int LAPACKE_zungbr (int matrix_layout, char vect, lapack_int m, lapack_int n,
lapack_int k, lapack_complex_double* a, lapack_int lda, const lapack_complex_double*
tau);
```

Include Files

- mkl.h

Description

The routine generates the whole or part of the unitary matrices Q and P^H formed by the routines [gebrd/](#)[gebrd](#). Use this routine after a call to [cgebrd/zgebrd](#). All valid combinations of arguments are described in *Input Parameters*; in most cases you need the following:

To compute the whole m -by- m matrix Q , use:

```
LAPACKE_?ungbr(matrix_layout, 'Q', m, m, n, a, lda, tau)
```

(note that the array a must have at least m columns).

To form the n leading columns of Q if $m > n$, use:

```
LAPACKE_?ungbr(matrix_layout, 'Q', m, n, n, a, lda, tau)
```

To compute the whole n -by- n matrix P^H , use:

```
LAPACKE_?ungbr(matrix_layout, 'P', n, n, m, a, lda, tau)
```

(note that the array a must have at least n rows).

To form the m leading rows of P^H if $m < n$, use:

```
LAPACKE_?ungbr(matrix_layout, 'P', m, m, n, a, lda, tau)
```

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>vect</i> | Must be 'Q' or 'P'. If <i>vect</i> = 'Q', the routine generates the matrix Q . If <i>vect</i> = 'P', the routine generates the matrix P^H . |
| <i>m</i> | The number of required rows of Q or P^H . |
| <i>n</i> | The number of required columns of Q or P^H . |
| <i>k</i> | One of the dimensions of A in ?gebrd: If <i>vect</i> = 'Q', the number of columns in A ; If <i>vect</i> = 'P', the number of rows in A . |

Constraints: $m \geq 0$, $n \geq 0$, $k \geq 0$.

For $vect = 'Q'$: $k \leq n \leq m$ if $m > k$, or $m = n$ if $m \leq k$.

For $vect = 'P'$: $k \leq m \leq n$ if $n > k$, or $m = n$ if $n \leq k$.

a

Arrays:

a, size at least $lda*n$ for column major layout and $lda*m$ for row major layout, is the array *a* as returned by ?gebrd.

lda

The leading dimension of *a*; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout.

tau

For $vect = 'Q'$, the array *tauq* as returned by ?gebrd. For $vect = 'P'$, the array *taup* as returned by ?gebrd.

The dimension of *tau* must be at least $\max(1, \min(m, k))$ for $vect = 'Q'$, or $\max(1, \min(m, k))$ for $vect = 'P'$.

Output Parameters

a

Overwritten by the orthogonal matrix Q or P^T (or the leading rows or columns thereof) as specified by *vect*, *m*, and *n*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that $\|E\|_2 = O(\epsilon)$.

The approximate numbers of possible floating-point operations are listed below:

To compute the whole matrix Q :

$(16/3)n(3m^2 - 3m*n + n^2)$ if $m > n$;

$(16/3)m^3$ if $m \leq n$.

To form the n leading columns of Q when $m > n$:

$(8/3)n^2(3m - n^2)$.

To compute the whole matrix P^H :

$(16/3)n^3$ if $m \geq n$;

$(16/3)m(3n^2 - 3m*n + m^2)$ if $m < n$.

To form the m leading columns of P^H when $m < n$:

$(8/3)n^2(3m - n^2)$ if $m > n$.

The real counterpart of this routine is [orgbr](#).

?unmbr

Multiplies an arbitrary complex matrix by the unitary matrix Q or P determined by ?gebrd.

Syntax

```
lapack_int LAPACKE_cunmbr (int matrix_layout, char vect, char side, char trans,
lapack_int m, lapack_int n, lapack_int k, const lapack_complex_float* a, lapack_int
lda, const lapack_complex_float* tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zunmbr (int matrix_layout, char vect, char side, char trans,
lapack_int m, lapack_int n, lapack_int k, const lapack_complex_double* a, lapack_int
lda, const lapack_complex_double* tau, lapack_complex_double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

Given an arbitrary complex matrix C , this routine forms one of the matrix products Q^*C , Q^H*C , $C*Q$, $C*Q^H$, $P*C$, P^H*C , $C*P$, or $C*P^H$, where Q and P are unitary matrices computed by a call to [gebrd/gebrd](#). The routine overwrites the product on C .

Input Parameters

In the descriptions below, r denotes the order of Q or P^H :

If $side = 'L'$, $r = m$; if $side = 'R'$, $r = n$.

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>vect</i> | Must be 'Q' or 'P'. If $vect = 'Q'$, then Q or Q^H is applied to C . If $vect = 'P'$, then P or P^H is applied to C . |
| <i>side</i> | Must be 'L' or 'R'. If $side = 'L'$, multipliers are applied to C from the left. If $side = 'R'$, they are applied to C from the right. |
| <i>trans</i> | Must be 'N' or 'C'. If $trans = 'N'$, then Q or P is applied to C . If $trans = 'C'$, then Q^H or P^H is applied to C . |
| <i>m</i> | The number of rows in C . |
| <i>n</i> | The number of columns in C . |
| <i>k</i> | One of the dimensions of A in ?gebrd : If $vect = 'Q'$, the number of columns in A ; If $vect = 'P'$, the number of rows in A . Constraints: $m \geq 0$, $n \geq 0$, $k \geq 0$. |
| <i>a, c</i> | Arrays: a is the array a as returned by ?gebrd . |

The size of a depends on the value of the `matrix_layout`, `vect`, and `side` parameters:

| <code>matrix_layout</code> | <code>vect</code> | <code>side</code> | size |
|----------------------------|-------------------|-------------------|------------------|
| column major | 'Q' | - | $\max(1, lda*k)$ |
| column major | 'P' | 'L' | $\max(1, lda*m)$ |
| column major | 'P' | 'R' | $\max(1, lda*n)$ |
| row major | 'Q' | 'L' | $\max(1, lda*m)$ |
| row major | 'Q' | 'R' | $\max(1, lda*n)$ |
| row major | 'P' | - | $\max(1, lda*k)$ |
| | | | |
| | | | |

c (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m$ for row major layout) holds the matrix C .

`lda`

The leading dimension of a . Constraints:

$lda \geq \max(1, r)$ for column major layout and at least $\max(1, k)$ for row major layout if `vect` = 'Q';

$lda \geq \max(1, \min(r,k))$ for column major layout and at least $\max(1, r)$ for row major layout if `vect` = 'P'.

`ldc`

The leading dimension of c ; $ldc \geq \max(1, m)$.

`tau`

Array, size at least $\max(1, \min(r, k))$.

For `vect` = 'Q', the array `tauq` as returned by `?gebrd`. For `vect` = 'P', the array `taup` as returned by `?gebrd`.

Output Parameters

`c`

Overwritten by the product $Q*C$, Q^H*C , $C*Q$, $C*Q^H$, $P*C$, P^H*C , $C*P$, or $C*P^H$, as specified by `vect`, `side`, and `trans`.

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

Application Notes

The computed product differs from the exact product by a matrix E such that $\|E\|_2 = O(\epsilon) * \|C\|_2$.

The total number of floating-point operations is approximately

$8*n*k(2*m - k)$ if `side` = 'L' and $m \geq k$;

$8*m*k(2*n - k)$ if `side` = 'R' and $n \geq k$;

$8*m^2*n$ if `side` = 'L' and $m < k$;

$8*n^2*m$ if $side = 'R'$ and $n < k$.

The real counterpart of this routine is [ormbr](#).

?bdsqr

Computes the singular value decomposition of a general matrix that has been reduced to bidiagonal form.

Syntax

```
lapack_int LAPACKE_sbdsqr( int matrix_layout, char uplo, lapack_int n, lapack_int ncv,
lapack_int nru, lapack_int ncc, float* d, float* e, float* vt, lapack_int ldvt, float*
u, lapack_int ldu, float* c, lapack_int ldc );
```

```
lapack_int LAPACKE_dbdsqr( int matrix_layout, char uplo, lapack_int n, lapack_int ncv,
lapack_int nru, lapack_int ncc, double* d, double* e, double* vt, lapack_int ldvt,
double* u, lapack_int ldu, double* c, lapack_int ldc );
```

```
lapack_int LAPACKE_cbdsqr( int matrix_layout, char uplo, lapack_int n, lapack_int ncv,
lapack_int nru, lapack_int ncc, float* d, float* e, lapack_complex_float* vt,
lapack_int ldvt, lapack_complex_float* u, lapack_int ldu, lapack_complex_float* c,
lapack_int ldc );
```

```
lapack_int LAPACKE_zbdsqr( int matrix_layout, char uplo, lapack_int n, lapack_int ncv,
lapack_int nru, lapack_int ncc, double* d, double* e, lapack_complex_double* vt,
lapack_int ldvt, lapack_complex_double* u, lapack_int ldu, lapack_complex_double* c,
lapack_int ldc );
```

Include Files

- mkl.h

Description

The routine computes the singular values and, optionally, the right and/or left singular vectors from the [Singular Value Decomposition](#) (SVD) of a real n -by- n (upper or lower) bidiagonal matrix B using the implicit zero-shift QR algorithm. The SVD of B has the form $B = Q*S*P^H$ where S is the diagonal matrix of singular values, Q is an orthogonal matrix of left singular vectors, and P is an orthogonal matrix of right singular vectors. If left singular vectors are requested, this subroutine actually returns $U*Q$ instead of Q , and, if right singular vectors are requested, this subroutine returns P^H*VT instead of P^H , for given real/complex input matrices U and VT . When U and VT are the orthogonal/unitary matrices that reduce a general matrix A to bidiagonal form: $A = U*B*VT$, as computed by `?gebrd`, then

$$A = (U*Q) * S * (P^H*VT)$$

is the SVD of A . Optionally, the subroutine may also compute Q^H*C for a given real/complex input matrix C .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', B is an upper bidiagonal matrix. If <i>uplo</i> = 'L', B is a lower bidiagonal matrix. |
| <i>n</i> | The order of the matrix B ($n \geq 0$). |

| | |
|-----------------|--|
| <i>ncvt</i> | The number of columns of the matrix VT , that is, the number of right singular vectors ($ncvt \geq 0$). Set $ncvt = 0$ if no right singular vectors are required. |
| <i>nru</i> | The number of rows in U , that is, the number of left singular vectors ($nru \geq 0$). Set $nru = 0$ if no left singular vectors are required. |
| <i>ncc</i> | The number of columns in the matrix C used for computing the product $Q^H * C$ ($ncc \geq 0$). Set $ncc = 0$ if no matrix C is supplied. |
| <i>d, e</i> | Arrays: d contains the diagonal elements of B . The size of d must be at least $\max(1, n)$. e contains the $(n-1)$ off-diagonal elements of B . The size of e must be at least $\max(1, n - 1)$. |
| <i>vt, u, c</i> | Arrays: vt , size $\max(1, ldvt * ncvt)$ for column major layout and $\max(1, ldvt * n)$ for row major layout, contains an n -by- $ncvt$ matrix VT . vt is not referenced if $ncvt = 0$. u , size $\max(1, ldu * n)$ for column major layout and $\max(1, ldu * nru)$ for row major layout, contains an nru by n matrix U . u is not referenced if $nru = 0$. c , size $\max(1, ldc * ncc)$ for column major layout and $\max(1, ldc * n)$ for row major layout, contains the n -by- ncc matrix C for computing the product $Q^H * C$. |
| <i>ldvt</i> | The leading dimension of vt . Constraints: $ldvt \geq \max(1, n)$ if $ncvt > 0$ for column major layout and $ldvt \geq \max(1, ncvt)$ for row major layout; $ldvt \geq 1$ if $ncvt = 0$. |
| <i>ldu</i> | The leading dimension of u . Constraint: $ldu \geq \max(1, nru)$ for column major layout and $ldu \geq \max(1, n)$ for row major layout . |
| <i>ldc</i> | The leading dimension of c . Constraints: $ldc \geq \max(1, n)$ if $ncc > 0$ for column major layout and $ldc \geq \max(1, ncc)$ for row major layout; $ldc \geq 1$ otherwise. |

Output Parameters

| | |
|----------|---|
| <i>d</i> | On exit, if $info = 0$, overwritten by the singular values in decreasing order (see <i>info</i>). |
| <i>e</i> | On exit, if $info = 0$, e is destroyed. See also <i>info</i> below. |

| | |
|------|---|
| c | Overwritten by the product $Q^H * C$. |
| vt | On exit, this array is overwritten by $P^H * VT$. Not referenced if $ncvt = 0$. |
| u | On exit, this array is overwritten by $U * Q$. Not referenced if $nru = 0$. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0,

If *ncvt* = *nru* = *ncc* = 0,

- *info* = 1, a split was marked by a positive value in *e*
- *info* = 2, the current block of *z* not diagonalized after 100**n* iterations (in the inner `while` loop)
- *info* = 3, termination criterion of the outer `while` loop is not met (the program created more than *n* unreduced blocks).

In all other cases when *ncvt*, *nru*, or *ncc* > 0, the algorithm did not converge; *d* and *e* contain the elements of a bidiagonal matrix that is orthogonally similar to the input matrix *B*; if *info* = *i*, *i* elements of *e* have not converged to zero.

Application Notes

Each singular value and singular vector is computed to high relative accuracy. However, the reduction to bidiagonal form (prior to calling the routine) may decrease the relative accuracy in the small singular values of the original matrix if its singular values vary widely in magnitude.

If s_i is an exact singular value of *B*, and \hat{s}_i is the corresponding computed value, then

$$|s_i - \hat{s}_i| \leq p(m, n) * \varepsilon * s_i$$

where $p(m, n)$ is a modestly increasing function of *m* and *n*, and ε is the machine precision.

If only singular values are computed, they are computed more accurately than when some singular vectors are also computed (that is, the function $p(m, n)$ is smaller).

If u_i is the corresponding exact left singular vector of *B*, and w_i is the corresponding computed left singular vector, then the angle $\theta(u_i, w_i)$ between them is bounded as follows:

$$\theta(u_i, w_i) \leq p(m, n) * \varepsilon / \min_{i \neq j} (|\sigma_i - \sigma_j| / |\sigma_i + \sigma_j|).$$

Here $\min_{i \neq j} (|\sigma_i - \sigma_j| / |\sigma_i + \sigma_j|)$ is the *relative gap* between σ_i and the other singular values. A similar error bound holds for the right singular vectors.

The total number of real floating-point operations is roughly proportional to n^2 if only the singular values are computed. About $6n^2 * nru$ additional operations ($12n^2 * nru$ for complex flavors) are required to compute the left singular vectors and about $6n^2 * ncvt$ operations ($12n^2 * ncvt$ for complex flavors) to compute the right singular vectors.

?bdsdc

Computes the singular value decomposition of a real bidiagonal matrix using a divide and conquer method.

Syntax

```
lapack_int LAPACKE_sbdsdc (int matrix_layout, char uplo, char compq, lapack_int n,
float* d, float* e, float* u, lapack_int ldu, float* vt, lapack_int ldvt, float* q,
lapack_int* iq);
```

```
lapack_int LAPACKE_dbdsdc (int matrix_layout, char uplo, char compq, lapack_int n,
double* d, double* e, double* u, lapack_int ldu, double* vt, lapack_int ldvt, double*
q, lapack_int* iq);
```

Include Files

- mkl.h

Description

The routine computes the [Singular Value Decomposition](#) (SVD) of a real n -by- n (upper or lower) bidiagonal matrix B : $B = U \Sigma V^T$, using a divide and conquer method, where Σ is a diagonal matrix with non-negative diagonal elements (the singular values of B), and U and V are orthogonal matrices of left and right singular vectors, respectively. `?bdsdc` can be used to compute all singular values, and optionally, singular vectors or singular vectors in compact form.

This routine uses `?lasd0`, `?lasd1`, `?lasd2`, `?lasd3`, `?lasd4`, `?lasd5`, `?lasd6`, `?lasd7`, `?lasd8`, `?lasd9`, `?lasda`, `?lasdq`, `?lasdt`.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', B is an upper bidiagonal matrix. If <i>uplo</i> = 'L', B is a lower bidiagonal matrix. |
| <i>compq</i> | Must be 'N', 'P', or 'I'. If <i>compq</i> = 'N', compute singular values only. If <i>compq</i> = 'P', compute singular values and compute singular vectors in compact form. If <i>compq</i> = 'I', compute singular values and singular vectors. |
| <i>n</i> | The order of the matrix B ($n \geq 0$). |
| <i>d, e</i> | Arrays: d contains the n diagonal elements of the bidiagonal matrix B . The size of d must be at least $\max(1, n)$. e contains the off-diagonal elements of the bidiagonal matrix B . The size of e must be at least $\max(1, n)$. |
| <i>ldu</i> | The leading dimension of the output array u ; $ldu \geq 1$. If singular vectors are desired, then $ldu \geq \max(1, n)$, regardless of the value of <i>matrix_layout</i> . |
| <i>ldvt</i> | The leading dimension of the output array vt ; $ldvt \geq 1$. If singular vectors are desired, then $ldvt \geq \max(1, n)$, regardless of the value of <i>matrix_layout</i> . |

Output Parameters

| | |
|-----------------|--|
| <i>d</i> | If <i>info</i> = 0, overwritten by the singular values of <i>B</i> . |
| <i>e</i> | On exit, <i>e</i> is overwritten. |
| <i>u, vt, q</i> | <p>Arrays: <i>u</i>(size <i>ldu</i>*<i>n</i>), <i>vt</i>(size <i>ldvt</i>*<i>n</i>), <i>q</i>(size $\geq n \cdot (11 + 2 \cdot \text{smlsiz} + 8 \cdot \text{int}(\log_2(n/(\text{smlsiz}+1))))$) where <i>smlsiz</i> is returned by <i>ilaenv</i> and is equal to maximum size of the subproblems at the bottom of the computation tree).</p> <p>If <i>compq</i> = 'I', then on exit <i>u</i> contains the left singular vectors of the bidiagonal matrix <i>B</i>, unless <i>info</i> ≠ 0 (<i>seeinfo</i>). For other values of <i>compq</i>, <i>u</i> is not referenced.</p> <p>if <i>compq</i> = 'I', then on exit <i>vt</i>^T contains the right singular vectors of the bidiagonal matrix <i>B</i>, unless <i>info</i> ≠ 0 (<i>seeinfo</i>). For other values of <i>compq</i>, <i>vt</i> is not referenced.</p> <p>If <i>compq</i> = 'P', then on exit, if <i>info</i> = 0, <i>q</i> and <i>iq</i> contain the left and right singular vectors in a compact form. Specifically, <i>q</i> contains all the float (for <i>sbdsc</i>) or double (for <i>dbdsc</i>) data for singular vectors. For other values of <i>compq</i>, <i>q</i> is not referenced.</p> |
| <i>iq</i> | <p>Array: <i>iq</i>(size $\geq n \cdot (3 + 3 \cdot \text{int}(\log_2(n/(\text{smlsiz}+1))))$) where <i>smlsiz</i> is returned by <i>ilaenv</i> and is equal to maximum size of the subproblems at the bottom of the computation tree.).</p> <p>If <i>compq</i> = 'P', then on exit, if <i>info</i> = 0, <i>q</i> and <i>iq</i> contain the left and right singular vectors in a compact form. Specifically, <i>iq</i> contains all the <i>lapack_int</i> data for singular vectors. For other values of <i>compq</i>, <i>iq</i> is not referenced.</p> |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, the algorithm failed to compute a singular value. The update process of divide and conquer failed.

Symmetric Eigenvalue Problems: LAPACK Computational Routines

Symmetric eigenvalue problems are posed as follows: given an *n*-by-*n* real symmetric or complex Hermitian matrix *A*, find the *eigenvalues* λ and the corresponding *eigenvectors* *z* that satisfy the equation

$$Az = \lambda z \text{ (or, equivalently, } z^H A = \lambda z^H \text{)}.$$

In such eigenvalue problems, all *n* eigenvalues are real not only for real symmetric but also for complex Hermitian matrices *A*, and there exists an orthonormal system of *n* eigenvectors. If *A* is a symmetric or Hermitian positive-definite matrix, all eigenvalues are positive.

To solve a symmetric eigenvalue problem with LAPACK, you usually need to reduce the matrix to tridiagonal form and then solve the eigenvalue problem with the tridiagonal matrix obtained. LAPACK includes routines for reducing the matrix to a tridiagonal form by an orthogonal (or unitary) similarity transformation $A = QTQ^H$ as well as for solving tridiagonal symmetric eigenvalue problems. These routines are listed in [Table "Computational Routines for Solving Symmetric Eigenvalue Problems"](#).

There are different routines for symmetric eigenvalue problems, depending on whether you need all eigenvectors or only some of them or eigenvalues only, whether the matrix A is positive-definite or not, and so on.

These routines are based on three primary algorithms for computing eigenvalues and eigenvectors of symmetric problems: the divide and conquer algorithm, the QR algorithm, and bisection followed by inverse iteration. The divide and conquer algorithm is generally more efficient and is recommended for computing all eigenvalues and eigenvectors. Furthermore, to solve an eigenvalue problem using the divide and conquer algorithm, you need to call only one routine. In general, more than one routine has to be called if the QR algorithm or bisection followed by inverse iteration is used.

Computational Routines for Solving Symmetric Eigenvalue Problems

| Operation | Real symmetric matrices | Complex Hermitian matrices |
|---|-------------------------|----------------------------|
| Reduce to tridiagonal form $A = QTQ^H$ (full storage) | sytrd | hetrd |
| Reduce to tridiagonal form $A = QTQ^H$ (packed storage) | sptrd | hptrd |
| Reduce to tridiagonal form $A = QTQ^H$ (band storage). | sbtrd | hbtrd |
| Generate matrix Q (full storage) | orgtr | ungtr |
| Generate matrix Q (packed storage) | opgtr | upgtr |
| Apply matrix Q (full storage) | ormtr | unmtr |
| Apply matrix Q (packed storage) | opmtr | upmtr |
| Find all eigenvalues of a tridiagonal matrix T | sterf | |
| Find all eigenvalues and eigenvectors of a tridiagonal matrix T | steqr stedc | steqr stedc |
| Find all eigenvalues and eigenvectors of a tridiagonal positive-definite matrix T . | pteqr | pteqr |
| Find selected eigenvalues of a tridiagonal matrix T | stebz stegr | stegr |
| Find selected eigenvectors of a tridiagonal matrix T | stein stegr | stein stegr |
| Find selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix T | stemr | stemr |
| Compute the reciprocal condition numbers for the eigenvectors | disna | disna |

?sytrd

Reduces a real symmetric matrix to tridiagonal form.

Syntax

```
lapack_int LAPACKE_ssytrd (int matrix_layout, char uplo, lapack_int n, float* a,
lapack_int lda, float* d, float* e, float* tau);
```

```
lapack_int LAPACKE_dsytrd (int matrix_layout, char uplo, lapack_int n, double* a,
lapack_int lda, double* d, double* e, double* tau);
```

Include Files

- mkl.h

Description

The routine reduces a real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = Q^T Q$. The orthogonal matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors. Routines are provided for working with Q in this representation (see *Application Notes* below).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>a</i> | <i>a</i> (size $\max(1, lda * n)$) is an array containing either upper or lower triangular part of the matrix A , as specified by <i>uplo</i> . If <i>uplo</i> = 'U', the leading n -by- n upper triangular part of <i>a</i> contains the upper triangular part of the matrix A , and the strictly lower triangular part of A is not referenced. If <i>uplo</i> = 'L', the leading n -by- n lower triangular part of <i>a</i> contains the lower triangular part of the matrix A , and the strictly upper triangular part of A is not referenced. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|------------------|--|
| <i>a</i> | On exit, if <i>uplo</i> = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T , and the elements above the first superdiagonal, with the array <i>tau</i> , represent the orthogonal matrix Q as a product of elementary reflectors; if <i>uplo</i> = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T , and the elements below the first subdiagonal, with the array <i>tau</i> , represent the orthogonal matrix Q as a product of elementary reflectors. |
| <i>d, e, tau</i> | Arrays: <i>d</i> contains the diagonal elements of the matrix T . The size of <i>d</i> must be at least $\max(1, n)$. <i>e</i> contains the off-diagonal elements of T . The size of <i>e</i> must be at least $\max(1, n-1)$. <i>tau</i> stores ($n-1$) scalars that define elementary reflectors in decomposition of the orthogonal matrix Q in a product of $n-1$ elementary reflectors. <i>tau</i> (n) is used as workspace. |

The size of τ must be at least $\max(1, n)$.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed matrix T is exactly similar to a matrix $A+E$, where $\|E\|_2 = c(n) * \epsilon * \|A\|_2$, $c(n)$ is a modestly increasing function of n , and ϵ is the machine precision.

The approximate number of floating-point operations is $(4/3)n^3$.

After calling this routine, you can call the following:

`orgtr` to form the computed matrix Q explicitly

`ormtr` to multiply a real matrix by Q .

The complex counterpart of this routine is [?hetrd](#).

?orgtr

Generates the real orthogonal matrix Q determined by ?sytrd.

Syntax

```
lapack_int LAPACKE_sorgtr (int matrix_layout, char uplo, lapack_int n, float* a,
lapack_int lda, const float* tau);
```

```
lapack_int LAPACKE_dorgtr (int matrix_layout, char uplo, lapack_int n, double* a,
lapack_int lda, const double* tau);
```

Include Files

- mkl.h

Description

The routine explicitly generates the n -by- n orthogonal matrix Q formed by [sytrd](#) when reducing a real symmetric matrix A to tridiagonal form: $A = Q * T * Q^T$. Use this routine after a call to [?sytrd](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Use the same <i>uplo</i> as supplied to ?sytrd . |
| <i>n</i> | The order of the matrix Q ($n \geq 0$). |
| <i>a, tau</i> | Arrays: <i>a</i> (size $\max(1, lda * n)$) is the array <i>a</i> as returned by ?sytrd . |

tau is the array *tau* as returned by `?sytrd`.

The size of *tau* must be at least $\max(1, n-1)$.

lda The leading dimension of *a*; at least $\max(1, n)$.

Output Parameters

a Overwritten by the orthogonal matrix *Q*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the *i*-th parameter had an illegal value.

Application Notes

The computed matrix *Q* differs from an exactly orthogonal matrix by a matrix *E* such that $\|E\|_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of floating-point operations is $(4/3)n^3$.

The complex counterpart of this routine is [ungtr](#).

?ormtr

Multiplies a real matrix by the real orthogonal matrix Q determined by ?sytrd.

Syntax

```
lapack_int LAPACKE_sormtr (int matrix_layout, char side, char uplo, char trans,
lapack_int m, lapack_int n, const float* a, lapack_int lda, const float* tau, float*
c, lapack_int ldc);
```

```
lapack_int LAPACKE_dormtr (int matrix_layout, char side, char uplo, char trans,
lapack_int m, lapack_int n, const double* a, lapack_int lda, const double* tau,
double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a real matrix *C* by *Q* or Q^T , where *Q* is the orthogonal matrix *Q* formed by [sytrd](#) when reducing a real symmetric matrix *A* to tridiagonal form: $A = Q^T Q$. Use this routine after a call to `?sytrd`.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q^T C$, $Q^T C$, $C^T Q$, or $C^T Q$ (overwriting the result on *C*).

Input Parameters

In the descriptions below, *r* denotes the order of *Q*:

If *side* = 'L', $r = m$; if *side* = 'R', $r = n$.

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

| | |
|------------------|---|
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^T is applied to C from the left. If <i>side</i> = 'R', Q or Q^T is applied to C from the right. |
| <i>uplo</i> | Must be 'U' or 'L'. Use the same <i>uplo</i> as supplied to <code>?sytrd</code> . |
| <i>trans</i> | Must be either 'N' or 'T'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'T', the routine multiplies C by Q^T . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>a, c, tau</i> | <i>a</i> (size $\max(1, lda * r)$) and <i>tau</i> are the arrays returned by <code>?sytrd</code> . The size of <i>tau</i> must be at least $\max(1, r-1)$. <i>c</i> (size $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout) contains the matrix C . |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, r)$. |
| <i>ldc</i> | The leading dimension of <i>c</i> ; $ldc \geq \max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout . |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Overwritten by the product $Q * C$, $Q^T * C$, $C * Q$, or $C * Q^T$ (as specified by <i>side</i> and <i>trans</i>). |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the *i*-th parameter had an illegal value.

Application Notes

The computed product differs from the exact product by a matrix E such that $\|E\|_2 = O(\epsilon) * \|C\|_2$.

The total number of floating-point operations is approximately $2 * m^2 * n$, if *side* = 'L', or $2 * n^2 * m$, if *side* = 'R'.

The complex counterpart of this routine is `unmtr`.

?hetrd

Reduces a complex Hermitian matrix to tridiagonal form.

Syntax

```
lapack_int LAPACKE_chetrd( int matrix_layout, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* d, float* e, lapack_complex_float*
tau );
```

```
lapack_int LAPACKE_zhetrd( int matrix_layout, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* d, double* e, lapack_complex_double*
tau );
```

Include Files

- mkl.h

Description

The routine reduces a complex Hermitian matrix A to symmetric tridiagonal form T by a unitary similarity transformation: $A = Q^*T^*Q^H$. The unitary matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors. Routines are provided to work with Q in this representation. (They are described later in this section.)

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>a</i> | <i>a</i> (size $\max(1, lda*n)$) is an array containing either upper or lower triangular part of the matrix A , as specified by <i>uplo</i> . If <i>uplo</i> = 'U', the leading n -by- n upper triangular part of <i>a</i> contains the upper triangular part of the matrix A , and the strictly lower triangular part of A is not referenced. If <i>uplo</i> = 'L', the leading n -by- n lower triangular part of <i>a</i> contains the lower triangular part of the matrix A , and the strictly upper triangular part of A is not referenced. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | On exit, if <i>uplo</i> = 'U', the diagonal and first superdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T , and the elements above the first superdiagonal, with the array <i>tau</i> , represent the orthogonal matrix Q as a product of elementary reflectors; if <i>uplo</i> = 'L', the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T , and the elements below the first subdiagonal, with the array <i>tau</i> , represent the orthogonal matrix Q as a product of elementary reflectors. |
| <i>d, e</i> | Arrays: |

d contains the diagonal elements of the matrix T .

The dimension of d must be at least $\max(1, n)$.

e contains the off-diagonal elements of T .

The dimension of e must be at least $\max(1, n-1)$.

τ

Array, size at least $\max(1, n-1)$. Stores $(n-1)$ scalars that define elementary reflectors in decomposition of the unitary matrix Q in a product of $n-1$ elementary reflectors.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed matrix T is exactly similar to a matrix $A + E$, where $\|E\|_2 = c(n) * \epsilon * \|A\|_2$, $c(n)$ is a modestly increasing function of n , and ϵ is the machine precision.

The approximate number of floating-point operations is $(16/3)n^3$.

After calling this routine, you can call the following:

`ungtr` to form the computed matrix Q explicitly

`unmtr` to multiply a complex matrix by Q .

The real counterpart of this routine is [?sytrd](#).

?ungtr

Generates the complex unitary matrix Q determined by ?hetrd.

Syntax

```
lapack_int LAPACKE_cungtr (int matrix_layout, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, const lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zungtr (int matrix_layout, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, const lapack_complex_double* tau);
```

Include Files

- `mkl.h`

Description

The routine explicitly generates the n -by- n unitary matrix Q formed by [hetrd](#) when reducing a complex Hermitian matrix A to tridiagonal form: $A = Q * T * Q^H$. Use this routine after a call to [?hetrd](#).

Input Parameters

$matrix_layout$

Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

| | |
|---------------|---|
| <i>uplo</i> | Must be 'U' or 'L'. Use the same <i>uplo</i> as supplied to <code>?hetrd</code> . |
| <i>n</i> | The order of the matrix Q ($n \geq 0$). |
| <i>a, tau</i> | Arrays: <i>a</i> (size $\max(1, lda*n)$) is the array <i>a</i> as returned by <code>?hetrd</code> . <i>tau</i> is the array <i>tau</i> as returned by <code>?hetrd</code> . The dimension of <i>tau</i> must be at least $\max(1, n-1)$. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | Overwritten by the unitary matrix Q . |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed matrix Q differs from an exactly unitary matrix by a matrix E such that $\|E\|_2 = O(\epsilon)$, where ϵ is the machine precision.

The approximate number of floating-point operations is $(16/3)n^3$.

The real counterpart of this routine is [orgtr](#).

?unmtr

Multiplies a complex matrix by the complex unitary matrix Q determined by ?hetrd.

Syntax

```
lapack_int LAPACKE_cunmtr (int matrix_layout, char side, char uplo, char trans,
lapack_int m, lapack_int n, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zunmtr (int matrix_layout, char side, char uplo, char trans,
lapack_int m, lapack_int n, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* tau, lapack_complex_double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a complex matrix C by Q or Q^H , where Q is the unitary matrix Q formed by [hetrd](#) when reducing a complex Hermitian matrix A to tridiagonal form: $A = Q^T Q^H$. Use this routine after a call to `?hetrd`.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q^H C$, $Q^H C Q$, $C Q$, or $C Q^H$ (overwriting the result on C).

Input Parameters

In the descriptions below, r denotes the order of Q :

If $side = 'L'$, $r = m$; if $side = 'R'$, $r = n$.

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If $side = 'L'$, Q or Q^H is applied to C from the left. If $side = 'R'$, Q or Q^H is applied to C from the right. |
| <i>uplo</i> | Must be 'U' or 'L'. Use the same <i>uplo</i> as supplied to <code>?hetrd</code> . |
| <i>trans</i> | Must be either 'N' or 'T'. If $trans = 'N'$, the routine multiplies C by Q . If $trans = 'C'$, the routine multiplies C by Q^H . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>a, c, tau</i> | a (size $\max(1, lda*r)$) and tau are the arrays returned by <code>?hetrd</code> . The dimension of tau must be at least $\max(1, r-1)$. c (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout) contains the matrix C . |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, r)$. |
| <i>ldc</i> | The leading dimension of c ; $ldc \geq \max(1, n)$ for column major layout and $ldc \geq \max(1, m)$ for row major layout. |

Output Parameters

c Overwritten by the product $Q*C$, Q^H*C , $C*Q$, or $C*Q^H$ (as specified by *side* and *trans*).

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed product differs from the exact product by a matrix E such that $\|E\|_2 = O(\epsilon) * \|C\|_2$, where ϵ is the machine precision.

The total number of floating-point operations is approximately $8*m^2*n$ if $side = 'L'$ or $8*n^2*m$ if $side = 'R'$.

The real counterpart of this routine is [ormtr](#).

?sptrd

Reduces a real symmetric matrix to tridiagonal form using packed storage.

Syntax

```
lapack_int LAPACKE_ssptrd (int matrix_layout, char uplo, lapack_int n, float* ap,
float* d, float* e, float* tau);
```

```
lapack_int LAPACKE_dsptrd (int matrix_layout, char uplo, lapack_int n, double* ap,
double* d, double* e, double* tau);
```

Include Files

- mkl.h

Description

The routine reduces a packed real symmetric matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = Q^*T^*Q^T$. The orthogonal matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors. Routines are provided for working with Q in this representation. See *Application Notes* below for details.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ap</i> stores the packed upper triangle of A . If <i>uplo</i> = 'L', <i>ap</i> stores the packed lower triangle of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>ap</i> | Array, size at least $\max(1, n(n+1)/2)$. Contains either upper or lower triangle of A (as specified by <i>uplo</i>) in the packed form described in Matrix Storage Schemes . |

Output Parameters

| | |
|------------------|---|
| <i>ap</i> | Overwritten by the tridiagonal matrix T and details of the orthogonal matrix Q , as specified by <i>uplo</i> . |
| <i>d, e, tau</i> | Arrays: <i>d</i> contains the diagonal elements of the matrix T . The dimension of <i>d</i> must be at least $\max(1, n)$. <i>e</i> contains the off-diagonal elements of T . The dimension of <i>e</i> must be at least $\max(1, n-1)$. <i>tau</i> Stores $(n-1)$ scalars that define elementary reflectors in decomposition of the matrix Q in a product of $n-1$ reflectors. The dimension of <i>tau</i> must be at least $\max(1, n-1)$. |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The matrix Q is represented as a product of $n-1$ *elementary reflectors*, as follows :

- If $uplo = 'U'$, $Q = H(n-1) \dots H(2)H(1)$

Each $H(i)$ has the form

$$H(i) = I - \tau v v^T$$

where τ is a real scalar and v is a real vector with $v(i+1:n) = 0$ and $v(i) = 1$.

On exit, τ is stored in $\tau[i - 1]$, and $v(1:i-1)$ is stored in AP , overwriting $A(1:i-1, i+1)$.

- If $uplo = 'L'$, $Q = H(1)H(2) \dots H(n-1)$

Each $H(i)$ has the form

$$H(i) = I - \tau v v^T$$

where τ is a real scalar and v is a real vector with $v(1:i) = 0$ and $v(i+1) = 1$.

On exit, τ is stored in $\tau[i - 1]$, and $v(i+2:n)$ is stored in AP , overwriting $A(i+2:n, i)$.

The computed matrix T is exactly similar to a matrix $A+E$, where $\|E\|_2 = c(n) * \epsilon * \|A\|_2$, $c(n)$ is a modestly increasing function of n , and ϵ is the machine precision. The approximate number of floating-point operations is $(4/3)n^3$.

After calling this routine, you can call the following:

`opgtr` to form the computed matrix Q explicitly

`opmtr` to multiply a real matrix by Q .

The complex counterpart of this routine is `hptrd`.

?opgtr

Generates the real orthogonal matrix Q determined by ?sptrd.

Syntax

```
lapack_int LAPACKE_sopgtr (int matrix_layout, char uplo, lapack_int n, const float* ap,
const float* tau, float* q, lapack_int ldq);
```

```
lapack_int LAPACKE_dopgtr (int matrix_layout, char uplo, lapack_int n, const double*
ap, const double* tau, double* q, lapack_int ldq);
```

Include Files

- `mkl.h`

Description

The routine explicitly generates the n -by- n orthogonal matrix Q formed by `sptrd` when reducing a packed real symmetric matrix A to tridiagonal form: $A = Q^T * Q^T$. Use this routine after a call to `?sptrd`.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Use the same <i>uplo</i> as supplied to ?sptrd. |
| <i>n</i> | The order of the matrix Q ($n \geq 0$). |
| <i>ap, tau</i> | Arrays <i>ap</i> and <i>tau</i> , as returned by ?sptrd. The size of <i>ap</i> must be at least $\max(1, n(n+1)/2)$. The size of <i>tau</i> must be at least $\max(1, n-1)$. |
| <i>ldq</i> | The leading dimension of the output array q ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>q</i> | Array, size (size $\max(1, ldq*n)$) . Contains the computed matrix Q . |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that $\|E\|_2 = O(\epsilon)$, where ϵ is the machine precision.

The approximate number of floating-point operations is $(4/3)n^3$.

The complex counterpart of this routine is [upgtr](#).

?opmtr

Multiplies a real matrix by the real orthogonal matrix Q determined by ?sptrd.

Syntax

```
lapack_int LAPACKE_sopmtr (int matrix_layout, char side, char uplo, char trans,
lapack_int m, lapack_int n, const float* ap, const float* tau, float* c, lapack_int
ldc);
```

```
lapack_int LAPACKE_dopmtr (int matrix_layout, char side, char uplo, char trans,
lapack_int m, lapack_int n, const double* ap, const double* tau, double* c, lapack_int
ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a real matrix C by Q or Q^T , where Q is the orthogonal matrix Q formed by `sptrd` when reducing a packed real symmetric matrix A to tridiagonal form: $A = Q^T Q$. Use this routine after a call to `sptrd`.

Depending on the parameters *side* and *trans*, the routine can form one of the matrix products $Q^T C$, $Q^T C$, $C^T Q$, or $C^T Q^T$ (overwriting the result on C).

Input Parameters

In the descriptions below, r denotes the order of Q :

If *side* = 'L', $r = m$; if *side* = 'R', $r = n$.

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If <i>side</i> = 'L', Q or Q^T is applied to C from the left. If <i>side</i> = 'R', Q or Q^T is applied to C from the right. |
| <i>uplo</i> | Must be 'U' or 'L'. Use the same <i>uplo</i> as supplied to <code>?sptrd</code> . |
| <i>trans</i> | Must be either 'N' or 'T'. If <i>trans</i> = 'N', the routine multiplies C by Q . If <i>trans</i> = 'T', the routine multiplies C by Q^T . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>ap, tau, c</i> | <i>ap</i> and <i>tau</i> are the arrays returned by <code>?sptrd</code> . The dimension of <i>ap</i> must be at least $\max(1, r(r+1)/2)$. The dimension of <i>tau</i> must be at least $\max(1, r-1)$. <i>c</i> (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout) contains the matrix C . |
| <i>ldc</i> | The leading dimension of <i>c</i> ; $ldc \geq \max(1, n)$ for column major layout and $ldc \geq \max(1, m)$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Overwritten by the product $Q^T C$, $Q^T C$, $C^T Q$, or $C^T Q^T$ (as specified by <i>side</i> and <i>trans</i>). |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The computed product differs from the exact product by a matrix E such that $\|E\|_2 = O(\epsilon) \|C\|_2$, where ϵ is the machine precision.

The total number of floating-point operations is approximately $2*m^2*n$ if $side = 'L'$, or $2*n^2*m$ if $side = 'R'$.

The complex counterpart of this routine is [upmtr](#).

?hptrd

Reduces a complex Hermitian matrix to tridiagonal form using packed storage.

Syntax

```
lapack_int LAPACKE_chptrd( int matrix_layout, char uplo, lapack_int n,
lapack_complex_float* ap, float* d, float* e, lapack_complex_float* tau );

lapack_int LAPACKE_zhptrd( int matrix_layout, char uplo, lapack_int n,
lapack_complex_double* ap, double* d, double* e, lapack_complex_double* tau );
```

Include Files

- `mkl.h`

Description

The routine reduces a packed complex Hermitian matrix A to symmetric tridiagonal form T by a unitary similarity transformation: $A = Q^*T^*Q^H$. The unitary matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors. Routines are provided for working with Q in this representation (see *Application Notes* below).

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>uplo</code> | Must be 'U' or 'L'. If <code>uplo = 'U'</code> , <code>ap</code> stores the packed upper triangle of A . If <code>uplo = 'L'</code> , <code>ap</code> stores the packed lower triangle of A . |
| <code>n</code> | The order of the matrix A ($n \geq 0$). |
| <code>ap</code> | Array, size at least $\max(1, n(n+1)/2)$. Contains either upper or lower triangle of A (as specified by <code>uplo</code>) in the packed form described in " Matrix Storage Schemes ". |

Output Parameters

| | |
|-------------------|--|
| <code>ap</code> | Overwritten by the tridiagonal matrix T and details of the unitary matrix Q , as specified by <code>uplo</code> . |
| <code>d, e</code> | Arrays: <code>d</code> contains the diagonal elements of the matrix T . The size of <code>d</code> must be at least $\max(1, n)$. |

e contains the off-diagonal elements of T .

The size of e must be at least $\max(1, n-1)$.

tau Array, size at least $\max(1, n-1)$. Stores $(n-1)$ scalars that define elementary reflectors in decomposition of the unitary matrix Q in a product of reflectors.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The computed matrix T is exactly similar to a matrix $A + E$, where $\|E\|_2 = c(n) * \epsilon * \|A\|_2$, $c(n)$ is a modestly increasing function of n , and ϵ is the machine precision.

The approximate number of floating-point operations is $(16/3)n^3$.

After calling this routine, you can call the following:

[upgtr](#) to form the computed matrix Q explicitly

[upmtr](#) to multiply a complex matrix by Q .

The real counterpart of this routine is [sptrd](#).

?upgtr

Generates the complex unitary matrix Q determined by ?hptrd.

Syntax

```
lapack_int LAPACKE_cupgtr (int matrix_layout, char uplo, lapack_int n, const
lapack_complex_float* ap, const lapack_complex_float* tau, lapack_complex_float* q,
lapack_int ldq);
```

```
lapack_int LAPACKE_zupgtr (int matrix_layout, char uplo, lapack_int n, const
lapack_complex_double* ap, const lapack_complex_double* tau, lapack_complex_double* q,
lapack_int ldq);
```

Include Files

- `mkl.h`

Description

The routine explicitly generates the n -by- n unitary matrix Q formed by [hptrd](#) when reducing a packed complex Hermitian matrix A to tridiagonal form: $A = Q^*T^*Q^H$. Use this routine after a call to [?hptrd](#).

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

uplo Must be 'U' or 'L'. Use the same *uplo* as supplied to [?hptrd](#).

| | |
|------------|--|
| n | The order of the matrix Q ($n \geq 0$). |
| ap, τ | Arrays ap and τ , as returned by <code>?hptrd</code> . The dimension of ap must be at least $\max(1, n(n+1)/2)$. The dimension of τ must be at least $\max(1, n-1)$. |
| ldq | The leading dimension of the output array q ; at least $\max(1, n)$. |

Output Parameters

| | |
|-----|--|
| q | Array, size (size $\max(1, ldq*n)$) . Contains the computed matrix Q . |
|-----|--|

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed matrix Q differs from an exactly orthogonal matrix by a matrix E such that $\|E\|_2 = O(\epsilon)$, where ϵ is the machine precision.

The approximate number of floating-point operations is $(16/3)n^3$.

The real counterpart of this routine is [opgtr](#).

?upmtr

Multiplies a complex matrix by the unitary matrix Q determined by ?hptrd.

Syntax

```
lapack_int LAPACKE_cupmtr (int matrix_layout, char side, char uplo, char trans,
lapack_int m, lapack_int n, const lapack_complex_float* ap, const lapack_complex_float*
tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zupmtr (int matrix_layout, char side, char uplo, char trans,
lapack_int m, lapack_int n, const lapack_complex_double* ap, const
lapack_complex_double* tau, lapack_complex_double* c, lapack_int ldc);
```

Include Files

- `mkl.h`

Description

The routine multiplies a complex matrix C by Q or Q^H , where Q is the unitary matrix formed by [hptrd](#) when reducing a packed complex Hermitian matrix A to tridiagonal form: $A = Q^*T^*Q^H$. Use this routine after a call to `?hptrd`.

Depending on the parameters $side$ and $trans$, the routine can form one of the matrix products Q^*C , Q^H^*C , C^*Q , or C^*Q^H (overwriting the result on C).

Input Parameters

In the descriptions below, r denotes the order of Q :

If $side = 'L'$, $r = m$; if $side = 'R'$, $r = n$.

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be either 'L' or 'R'. If $side = 'L'$, Q or Q^H is applied to C from the left. If $side = 'R'$, Q or Q^H is applied to C from the right. |
| <i>uplo</i> | Must be 'U' or 'L'. Use the same <i>uplo</i> as supplied to ?hptrd. |
| <i>trans</i> | Must be either 'N' or 'T'. If $trans = 'N'$, the routine multiplies C by Q . If $trans = 'T'$, the routine multiplies C by Q^H . |
| <i>m</i> | The number of rows in the matrix C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>ap, tau, c,</i> | <i>ap</i> and <i>tau</i> are the arrays returned by ?hptrd. The size of <i>ap</i> must be at least $\max(1, r(r+1)/2)$. The size of <i>tau</i> must be at least $\max(1, r-1)$. <i>c</i> (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout) contains the matrix C . |
| <i>ldc</i> | The leading dimension of <i>c</i> ; $ldc \geq \max(1, m)$ for column major layout and $ldc \geq \max(1, n)$ for row major layout . |

Output Parameters

| | |
|----------|--|
| <i>c</i> | Overwritten by the product $Q^H C$, $Q^H C$, $C^H Q$, or $C^H Q^H$ (as specified by <i>side</i> and <i>trans</i>). |
|----------|--|

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed product differs from the exact product by a matrix E such that $\|E\|_2 = O(\epsilon) * \|C\|_2$, where ϵ is the machine precision.

The total number of floating-point operations is approximately $8*m^2*n$ if $side = 'L'$ or $8*n^2*m$ if $side = 'R'$.

The real counterpart of this routine is [opmtr](#).

?sbtrd

Reduces a real symmetric band matrix to tridiagonal form.

Syntax

```
lapack_int LAPACKE_ssbtrd (int matrix_layout, char vect, char uplo, lapack_int n,
lapack_int kd, float* ab, lapack_int ldab, float* d, float* e, float* q, lapack_int
ldq);

lapack_int LAPACKE_dsbtrd (int matrix_layout, char vect, char uplo, lapack_int n,
lapack_int kd, double* ab, lapack_int ldab, double* d, double* e, double* q,
lapack_int ldq);
```

Include Files

- mkl.h

Description

The routine reduces a real symmetric band matrix A to symmetric tridiagonal form T by an orthogonal similarity transformation: $A = Q^*T^*Q^T$. The orthogonal matrix Q is determined as a product of Givens rotations.

If required, the routine can also form the matrix Q explicitly.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>vect</i> | Must be 'V', 'N', or 'U'. If <i>vect</i> = 'V', the routine returns the explicit matrix Q . If <i>vect</i> = 'N', the routine does not return Q . If <i>vect</i> = 'U', the routine updates matrix X by forming X^*Q . |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ab</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>ab</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>kd</i> | The number of super- or sub-diagonals in A ($kd \geq 0$). |
| <i>ab, q</i> | <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and at least $\max(1, ldab*(kd+1))$ for row major layout) is an array containing either upper or lower triangular part of the matrix A (as specified by <i>uplo</i>) in band storage format. <i>q</i> (size $\max(1, ldq*n)$) is an array. If <i>vect</i> = 'U', the <i>q</i> array must contain an n -by- n matrix X . If <i>vect</i> = 'N' or 'V', the <i>q</i> parameter need not be set. |

| | |
|-------------|--|
| <i>ldab</i> | The leading dimension of <i>ab</i> ; at least $kd+1$ for column major layout and n for row major layout . |
| <i>ldq</i> | The leading dimension of <i>q</i> . Constraints: $ldq \geq \max(1, n)$ if <i>vect</i> = 'V' or 'U'; $ldq \geq 1$ if <i>vect</i> = 'N'. |

Output Parameters

| | |
|----------------|---|
| <i>ab</i> | On exit, the diagonal elements of the array <i>ab</i> are overwritten by the diagonal elements of the tridiagonal matrix <i>T</i> . If $kd > 0$, the elements on the first superdiagonal (if <i>uplo</i> = 'U') or the first subdiagonal (if <i>uplo</i> = 'L') are overwritten by the off-diagonal elements of <i>T</i> . The rest of <i>ab</i> is overwritten by values generated during the reduction. |
| <i>d, e, q</i> | Arrays: <i>d</i> contains the diagonal elements of the matrix <i>T</i> . The size of <i>d</i> must be at least $\max(1, n)$. <i>e</i> contains the off-diagonal elements of <i>T</i> . The size of <i>e</i> must be at least $\max(1, n-1)$. <i>q</i> is not referenced if <i>vect</i> = 'N'. If <i>vect</i> = 'V', <i>q</i> contains the n -by- n matrix <i>Q</i> . If <i>vect</i> = 'U', <i>q</i> contains the product $X^* Q$. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The computed matrix *T* is exactly similar to a matrix $A+E$, where $\|E\|_2 = c(n)*\epsilon*\|A\|_2$, $c(n)$ is a modestly increasing function of n , and ϵ is the machine precision. The computed matrix *Q* differs from an exactly orthogonal matrix by a matrix *E* such that $\|E\|_2 = O(\epsilon)$.

The total number of floating-point operations is approximately $6n^2*kd$ if *vect* = 'N', with $3n^3*(kd-1)/kd$ additional operations if *vect* = 'V'.

The complex counterpart of this routine is [hbtrd](#).

?hbtrd

Reduces a complex Hermitian band matrix to tridiagonal form.

Syntax

```
lapack_int LAPACKE_chbtrd( int matrix_layout, char vect, char uplo, lapack_int n,
lapack_int kd, lapack_complex_float* ab, lapack_int ldab, float* d, float* e,
lapack_complex_float* q, lapack_int ldq );
```

```
lapack_int LAPACKE_zhbtrd( int matrix_layout, char vect, char uplo, lapack_int n,
lapack_int kd, lapack_complex_double* ab, lapack_int ldab, double* d, double* e,
lapack_complex_double* q, lapack_int ldq );
```

Include Files

- mkl.h

Description

The routine reduces a complex Hermitian band matrix A to symmetric tridiagonal form T by a unitary similarity transformation: $A = Q^*T^*Q^H$. The unitary matrix Q is determined as a product of Givens rotations.

If required, the routine can also form the matrix Q explicitly.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>vect</i> | Must be 'V', 'N', or 'U'. If <i>vect</i> = 'V', the routine returns the explicit matrix Q . If <i>vect</i> = 'N', the routine does not return Q . If <i>vect</i> = 'U', the routine updates matrix X by forming Q^*X . |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ab</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>ab</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>kd</i> | The number of super- or sub-diagonals in A ($kd \geq 0$). |
| <i>ab</i> | <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and at least $\max(1, ldab*(kd+1))$ for row major layout) is an array containing either upper or lower triangular part of the matrix A (as specified by <i>uplo</i>) in band storage format. |
| <i>q</i> | <i>q</i> (size $\max(1, ldq*n)$) is an array. If <i>vect</i> = 'U', the <i>q</i> array must contain an n -by- n matrix X . If <i>vect</i> = 'N' or 'V', the <i>q</i> parameter need not be set.' |
| <i>ldab</i> | The leading dimension of <i>ab</i> ; at least $kd+1$ for column major layout and n for row major layout. |
| <i>ldq</i> | The leading dimension of <i>q</i> . Constraints: $ldq \geq \max(1, n)$ if <i>vect</i> = 'V' or 'U'; $ldq \geq 1$ if <i>vect</i> = 'N'. |

Output Parameters

| | |
|-------------|--|
| <i>ab</i> | On exit, the diagonal elements of the array <i>ab</i> are overwritten by the diagonal elements of the tridiagonal matrix <i>T</i> . If $kd > 0$, the elements on the first superdiagonal (if <i>uplo</i> = 'U') or the first subdiagonal (if <i>uplo</i> = 'L') are overwritten by the off-diagonal elements of <i>T</i> . The rest of <i>ab</i> is overwritten by values generated during the reduction. |
| <i>d, e</i> | Arrays: <i>d</i> contains the diagonal elements of the matrix <i>T</i> . The dimension of <i>d</i> must be at least $\max(1, n)$. <i>e</i> contains the off-diagonal elements of <i>T</i> . The dimension of <i>e</i> must be at least $\max(1, n-1)$. |
| <i>q</i> | If <i>vect</i> = 'N', <i>q</i> is not referenced. If <i>vect</i> = 'V', <i>q</i> contains the <i>n</i> -by- <i>n</i> matrix <i>Q</i> . If <i>vect</i> = 'U', <i>q</i> contains the product $X^* Q$. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed matrix *T* is exactly similar to a matrix $A + E$, where $\|E\|_2 = c(n) * \epsilon * \|A\|_2$, $c(n)$ is a modestly increasing function of *n*, and ϵ is the machine precision. The computed matrix *Q* differs from an exactly unitary matrix by a matrix *E* such that $\|E\|_2 = O(\epsilon)$.

The total number of floating-point operations is approximately $20n^2 * kd$ if *vect* = 'N', with $10n^3 * (kd-1) / kd$ additional operations if *vect* = 'V'.

The real counterpart of this routine is [sbtrd](#).

?sterf

Computes all eigenvalues of a real symmetric tridiagonal matrix using QR algorithm.

Syntax

```
lapack_int LAPACKE_ssterf (lapack_int n, float* d, float* e);
lapack_int LAPACKE_dsterf (lapack_int n, double* d, double* e);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues of a real symmetric tridiagonal matrix *T* (which can be obtained by reducing a symmetric or Hermitian matrix to tridiagonal form). The routine uses a square-root-free variant of the *QR* algorithm.

If you need not only the eigenvalues but also the eigenvectors, call [steqr](#).

Input Parameters

| | |
|--------|---|
| n | The order of the matrix T ($n \geq 0$). |
| d, e | Arrays: d contains the diagonal elements of T . The dimension of d must be at least $\max(1, n)$. e contains the off-diagonal elements of T . The dimension of e must be at least $\max(1, n-1)$. |

Output Parameters

| | |
|-----|---|
| d | The n eigenvalues in ascending order, unless $info > 0$. See also <i>info</i> . |
| e | On exit, the array is overwritten; see <i>info</i> . |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = i$, the algorithm failed to find all the eigenvalues after $30n$ iterations:

i off-diagonal elements have not converged to zero. On exit, d and e contain, respectively, the diagonal and off-diagonal elements of a tridiagonal matrix orthogonally similar to T .

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T+E$ such that $\|E\|_2 = O(\epsilon) * \|T\|_2$, where ϵ is the machine precision.

If λ_i is an exact eigenvalue, and m_i is the corresponding computed value, then

$$|\mu_i - \lambda_i| \leq c(n) * \epsilon * \|T\|_2$$

where $c(n)$ is a modestly increasing function of n .

The total number of floating-point operations depends on how rapidly the algorithm converges. Typically, it is about $14n^2$.

?steqr

Computes all eigenvalues and eigenvectors of a symmetric or Hermitian matrix reduced to tridiagonal form (QR algorithm).

Syntax

```
lapack_int LAPACKE_ssteqr( int matrix_layout, char compz, lapack_int n, float* d,
float* e, float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_dsteqr( int matrix_layout, char compz, lapack_int n, double* d,
double* e, double* z, lapack_int ldz );
```

```
lapack_int LAPACKE_csteqr( int matrix_layout, char compz, lapack_int n, float* d,
float* e, lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zsteqr( int matrix_layout, char compz, lapack_int n, double* d,
double* e, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues and (optionally) all the eigenvectors of a real symmetric tridiagonal matrix T . In other words, the routine can compute the spectral factorization: $T = Z\Lambda Z^T$. Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i ; Z is an orthogonal matrix whose columns are eigenvectors. Thus,

$$T^*z_i = \lambda_i^*z_i \text{ for } i = 1, 2, \dots, n.$$

The routine normalizes the eigenvectors so that $\|z_i\|_2 = 1$.

You can also use the routine for computing the eigenvalues and eigenvectors of an arbitrary real symmetric (or complex Hermitian) matrix A reduced to tridiagonal form T : $A = Q^*T^*Q^H$. In this case, the spectral factorization is as follows: $A = Q^*T^*Q^H = (Q^*Z)^*\Lambda^*(Q^*Z)^H$. Before calling `?steqr`, you must reduce A to tridiagonal form and generate the explicit matrix Q by calling the following routines:

| | for real matrices: | for complex matrices: |
|-----------------------|---------------------------|---------------------------|
| full storage | ?sytrd, ?orgtr | ?hetrd, ?ungtr |
| packed storage | ?sptrd, ?opgtr | ?hptrd, ?upgtr |
| band storage | ?sbtrd(<i>vect</i> ='V') | ?hbtrd(<i>vect</i> ='V') |

If you need eigenvalues only, it's more efficient to call [sterf](#). If T is positive-definite, [pteqr](#) can compute small eigenvalues more accurately than `?steqr`.

To solve the problem by a single call, use one of the divide and conquer routines [stevd](#), [syevd](#), [spevd](#), or [sbevd](#) for real symmetric matrices or [heevd](#), [hpevd](#), or [hbevd](#) for complex Hermitian matrices.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>compz</i> | Must be 'N' or 'I' or 'V'. If <i>compz</i> = 'N', the routine computes eigenvalues only. If <i>compz</i> = 'I', the routine computes the eigenvalues and eigenvectors of the tridiagonal matrix T . If <i>compz</i> = 'V', the routine computes the eigenvalues and eigenvectors of the original symmetric matrix. On entry, z must contain the orthogonal matrix used to reduce the original matrix to tridiagonal form. |
| <i>n</i> | The order of the matrix T ($n \geq 0$). |
| <i>d, e</i> | Arrays: d contains the diagonal elements of T . The size of d must be at least $\max(1, n)$. e contains the off-diagonal elements of T . |

The size of e must be at least $\max(1, n-1)$.

z Array, size $\max(1, ldz*n)$.

If $compz = 'N'$ or $'I'$, z need not be set.

If $vect = 'V'$, z must contain the orthogonal matrix used in the reduction to tridiagonal form.

ldz The leading dimension of z . Constraints:

$ldz \geq 1$ if $compz = 'N'$;

$ldz \geq \max(1, n)$ if $compz = 'V'$ or $'I'$.

Output Parameters

d The n eigenvalues in ascending order, unless $info > 0$.

See also *info*.

e On exit, the array is overwritten; see *info*.

z If $info = 0$, contains the n -by- n matrix the columns of which are orthonormal eigenvectors (the i -th column corresponds to the i -th eigenvalue).

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = i$, the algorithm failed to find all the eigenvalues after $30n$ iterations: i off-diagonal elements have not converged to zero. On exit, d and e contain, respectively, the diagonal and off-diagonal elements of a tridiagonal matrix orthogonally similar to T .

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T+E$ such that $\|E\|_2 = O(\epsilon) * \|T\|_2$, where ϵ is the machine precision.

If λ_j is an exact eigenvalue, and μ_j is the corresponding computed value, then

$$|\mu_i - \lambda_i| \leq c(n) * \epsilon * \|T\|_2$$

where $c(n)$ is a modestly increasing function of n .

If z_i is the corresponding exact eigenvector, and w_i is the corresponding computed vector, then the angle $\theta(z_i, w_i)$ between them is bounded as follows:

$$\theta(z_i, w_i) \leq c(n) * \epsilon * \|T\|_2 / \min_{i \neq j} |\lambda_i - \lambda_j|.$$

The total number of floating-point operations depends on how rapidly the algorithm converges. Typically, it is about

$$24n^2 \text{ if } compz = 'N';$$

$$7n^3 \text{ (for complex flavors, } 14n^3) \text{ if } compz = 'V' \text{ or } 'I'.$$

?stemr

Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_sstemr( int matrix_layout, char jobz, char range, lapack_int n,
const float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu,
lapack_int* m, float* w, float* z, lapack_int ldz, lapack_int nzc, lapack_int* isuppz,
lapack_logical* tryrac );
```

```
lapack_int LAPACKE_dstemr( int matrix_layout, char jobz, char range, lapack_int n,
const double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu,
lapack_int* m, double* w, double* z, lapack_int ldz, lapack_int nzc, lapack_int*
isuppz, lapack_logical* tryrac );
```

```
lapack_int LAPACKE_cstemr( int matrix_layout, char jobz, char range, lapack_int n,
const float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu,
lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz, lapack_int nzc,
lapack_int* isuppz, lapack_logical* tryrac );
```

```
lapack_int LAPACKE_zstemr( int matrix_layout, char jobz, char range, lapack_int n,
const double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu,
lapack_int* m, double* w, lapack_complex_double* z, lapack_int ldz, lapack_int nzc,
lapack_int* isuppz, lapack_logical* tryrac );
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T . Any such unreduced matrix has a well defined set of pairwise different real eigenvalues, the corresponding real eigenvectors are pairwise orthogonal.

The spectrum may be computed either completely or partially by specifying either an interval $(vl, vu]$ or a range of indices $il:iu$ for the desired eigenvalues.

Depending on the number of desired eigenvalues, these are computed either by bisection or the *dqds* algorithm. Numerically orthogonal eigenvectors are computed by the use of various suitable $L^*D^*L^T$ factorizations near clusters of close eigenvalues (referred to as RRRs, Relatively Robust Representations). An informal sketch of the algorithm follows.

For each unreduced block (submatrix) of T ,

- Compute $T - \sigma I = L^*D^*L^T$, so that L and D define all the wanted eigenvalues to high relative accuracy. This means that small relative changes in the entries of L and D cause only small relative changes in the eigenvalues and eigenvectors. The standard (unfactored) representation of the tridiagonal matrix T does not have this property in general.
- Compute the eigenvalues to suitable accuracy. If the eigenvectors are desired, the algorithm attains full accuracy of the computed eigenvalues only right before the corresponding vectors have to be computed, see steps c and d.
- For each cluster of close eigenvalues, select a new shift close to the cluster, find a new factorization, and refine the shifted eigenvalues to suitable accuracy.
- For each eigenvalue with a large enough relative separation compute the corresponding eigenvector by forming a rank revealing twisted factorization. Go back to step c for any clusters that remain.

For more details, see: [Dhillon04], [Dhillon04-02], [Dhillon97]

Input Parameters

| | |
|-----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | <p>Must be 'N' or 'V'.</p> <p>If <i>jobz</i> = 'N', then only eigenvalues are computed.</p> <p>If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed.</p> |
| <i>range</i> | <p>Must be 'A' or 'V' or 'I'.</p> <p>If <i>range</i> = 'A', the routine computes all eigenvalues.</p> <p>If <i>range</i> = 'V', the routine computes all eigenvalues in the half-open interval: (<i>vl</i>, <i>vu</i>].</p> <p>If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i>.</p> |
| <i>n</i> | The order of the matrix <i>T</i> ($n \geq 0$). |
| <i>d</i> | <p>Array, size <i>n</i>.</p> <p>Contains <i>n</i> diagonal elements of the tridiagonal matrix <i>T</i>.</p> |
| <i>e</i> | <p>Array, size <i>n</i>.</p> <p>Contains (<i>n</i>-1) off-diagonal elements of the tridiagonal matrix <i>T</i> in elements 0 to <i>n</i>-2 of <i>e</i>. <i>e</i>[<i>n</i> - 1] need not be set on input, but is used internally as workspace.</p> |
| <i>vl</i> , <i>vu</i> | <p>If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$.</p> <p>If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced.</p> |
| <i>il</i> , <i>iu</i> | <p>If <i>range</i> = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.</p> <p>Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$.</p> <p>If <i>range</i> = 'A' or 'V', <i>il</i> and <i>iu</i> are not referenced.</p> |
| <i>ldz</i> | <p>The leading dimension of the output array <i>z</i>.</p> <p>if <i>jobz</i> = 'V', then $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout ;</p> <p>$ldz \geq 1$ otherwise.</p> |
| <i>nzc</i> | <p>The number of eigenvectors to be held in the array <i>z</i>.</p> <p>If <i>range</i> = 'A', then $nzc \geq \max(1, n)$;</p> <p>If <i>range</i> = 'V', then <i>nzc</i> is greater than or equal to the number of eigenvalues in the half-open interval: (<i>vl</i>, <i>vu</i>].</p> <p>If <i>range</i> = 'I', then $nzc \geq iu - il + 1$.</p> <p>If <i>nzc</i> = -1, then a workspace query is assumed; the routine calculates the number of columns of the array <i>z</i> that are needed to hold the eigenvectors.</p> |

This value is returned as the first entry of the array z , and no error message related to nzc is issued by the routine `xerbla`.

tryrac

If *tryrac* is true, it indicates that the code should check whether the tridiagonal matrix defines its eigenvalues to high relative accuracy. If so, the code uses relative-accuracy preserving algorithms that might be (a bit) slower depending on the matrix. If the matrix does not define its eigenvalues to high relative accuracy, the code can use possibly faster algorithms.

If *tryrac* is not true, the code is not required to guarantee relatively accurate eigenvalues and can use the fastest possible techniques.

Output Parameters

d

On exit, the array d is overwritten.

e

On exit, the array e is overwritten.

m

The total number of eigenvalues found, $0 \leq m \leq n$.

If *range* = 'A', then $m=n$, and if *range* = 'I', then $m=i_u-i_l+1$.

w

Array, size n .

The first m elements contain the selected eigenvalues in ascending order.

z

Array z (size $\max(1, ldz*m)$ for column major layout and $\max(1, ldz*n)$ for row major layout).

If *jobz* = 'V', and *info* = 0, then the first m columns of z contain the orthonormal eigenvectors of the matrix T corresponding to the selected eigenvalues, with the i -th column of z holding the eigenvector associated with $w(i)$.

If *jobz* = 'N', then z is not referenced.

Note: the exact value of m is not known in advance and can be computed with a workspace query by setting $nzc=-1$, see description of the parameter *nzc*.

isuppz

Array, size $(2*\max(1, m))$.

The support of the eigenvectors in z , that is the indices indicating the nonzero elements in z . The i -th computed eigenvector is nonzero only in elements $isuppz[2*i - 2]$ through $isuppz[2*i - 1]$. This is relevant in the case when the matrix is split. *isuppz* is only accessed when *jobz* = 'V' and $n>0$.

tryrac

On exit, , set to true. *tryrac* is set to false if the matrix does not define its eigenvalues to high relative accuracy.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info > 0$, an internal error occurred.

?stedc

Computes all eigenvalues and eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method.

Syntax

```
lapack_int LAPACKE_sstedc( int matrix_layout, char compz, lapack_int n, float* d,
float* e, float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_dstedc( int matrix_layout, char compz, lapack_int n, double* d,
double* e, double* z, lapack_int ldz );
```

```
lapack_int LAPACKE_cstedc( int matrix_layout, char compz, lapack_int n, float* d,
float* e, lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zstedc( int matrix_layout, char compz, lapack_int n, double* d,
double* e, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues and (optionally) all the eigenvectors of a symmetric tridiagonal matrix using the divide and conquer method. The eigenvectors of a full or band real symmetric or complex Hermitian matrix can also be found if [sytrd/hetrd](#) or [sptrd/hptrd](#) or [sbtrd/hbtrd](#) has been used to reduce this matrix to tridiagonal form.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>compz</i> | <p>Must be 'N' or 'I' or 'V'.</p> <p>If <i>compz</i> = 'N', the routine computes eigenvalues only.</p> <p>If <i>compz</i> = 'I', the routine computes the eigenvalues and eigenvectors of the tridiagonal matrix.</p> <p>If <i>compz</i> = 'V', the routine computes the eigenvalues and eigenvectors of original symmetric/Hermitian matrix. On entry, the array <i>z</i> must contain the orthogonal/unitary matrix used to reduce the original matrix to tridiagonal form.</p> |
| <i>n</i> | The order of the symmetric tridiagonal matrix ($n \geq 0$). |
| <i>d, e</i> | <p>Arrays:</p> <p><i>d</i> contains the diagonal elements of the tridiagonal matrix.</p> <p>The dimension of <i>d</i> must be at least $\max(1, n)$.</p> <p><i>e</i> contains the subdiagonal elements of the tridiagonal matrix.</p> <p>The dimension of <i>e</i> must be at least $\max(1, n-1)$.</p> |

| | |
|------------|---|
| <i>z</i> | Array <i>z</i> is of size $\max(1, \text{ldz} * n)$. If <i>compz</i> = 'V', then, on entry, <i>z</i> must contain the orthogonal/unitary matrix used to reduce the original matrix to tridiagonal form. |
| <i>ldz</i> | The leading dimension of <i>z</i> . Constraints: <i>ldz</i> \geq 1 if <i>compz</i> = 'N'; <i>ldz</i> \geq $\max(1, n)$ if <i>compz</i> = 'V' or 'I'. |

Output Parameters

| | |
|----------|--|
| <i>d</i> | The <i>n</i> eigenvalues in ascending order, unless <i>info</i> \neq 0. See also <i>info</i> . |
| <i>e</i> | On exit, the array is overwritten; see <i>info</i> . |
| <i>z</i> | If <i>info</i> = 0, then if <i>compz</i> = 'V', <i>z</i> contains the orthonormal eigenvectors of the original symmetric/Hermitian matrix, and if <i>compz</i> = 'I', <i>z</i> contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If <i>compz</i> = 'N', <i>z</i> is not referenced. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns *i*/(*n*+1) through $\text{mod}(i, n+1)$.

?stegr

Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_sstegr( int matrix_layout, char jobz, char range, lapack_int n,
float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu, float abstol,
lapack_int* m, float* w, float* z, lapack_int ldz, lapack_int* isuppz );
```

```
lapack_int LAPACKE_dstegr( int matrix_layout, char jobz, char range, lapack_int n,
double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu, double
abstol, lapack_int* m, double* w, double* z, lapack_int ldz, lapack_int* isuppz );
```

```
lapack_int LAPACKE_cstegr( int matrix_layout, char jobz, char range, lapack_int n,
float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu, float abstol,
lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz, lapack_int* isuppz );
```

```
lapack_int LAPACKE_zstegr( int matrix_layout, char jobz, char range, lapack_int n,
double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu, double
abstol, lapack_int* m, double* w, lapack_complex_double* z, lapack_int ldz,
lapack_int* isuppz );
```

Include Files

- `mkl.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T .

The spectrum may be computed either completely or partially by specifying either an interval $(vl, vu]$ or a range of indices $il:iu$ for the desired eigenvalues.

`?stegr` is a compatibility wrapper around the improved `stemr` routine. See its description for further details.

Note that the `abstol` parameter no longer provides any benefit and hence is no longer used.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>jobz</code> | Must be 'N' or 'V'. If <code>jobz = 'N'</code> , then only eigenvalues are computed. If <code>jobz = 'V'</code> , then eigenvalues and eigenvectors are computed. |
| <code>range</code> | Must be 'A' or 'V' or 'I'. If <code>range = 'A'</code> , the routine computes all eigenvalues. If <code>range = 'V'</code> , the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$. If <code>range = 'I'</code> , the routine computes eigenvalues with indices il to iu . |
| <code>n</code> | The order of the matrix T ($n \geq 0$). |
| <code>d, e</code> | Arrays: <code>d</code> contains the diagonal elements of T . The dimension of <code>d</code> must be at least $\max(1, n)$. <code>e</code> contains the subdiagonal elements of T in elements 1 to $n-1$; <code>e(n)</code> need not be set on input, but it is used as a workspace. The dimension of <code>e</code> must be at least $\max(1, n)$. |
| <code>vl, vu</code> | If <code>range = 'V'</code> , the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If <code>range = 'A'</code> or 'I', <code>vl</code> and <code>vu</code> are not referenced. |
| <code>il, iu</code> | If <code>range = 'I'</code> , the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$. If <code>range = 'A'</code> or 'V', <code>il</code> and <code>iu</code> are not referenced. |

| | |
|---------------|--|
| <i>abstol</i> | Unused. Was the absolute error tolerance for the eigenvalues/eigenvectors in previous versions. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> . Constraints: $ldz \geq 1$ if <i>jobz</i> = 'N'; $ldz \geq \max(1, n)$ if <i>jobz</i> = 'V'. |

Output Parameters

| | |
|---------------|---|
| <i>d, e</i> | On exit, <i>d</i> and <i>e</i> are overwritten. |
| <i>m</i> | The total number of eigenvalues found, $0 \leq m \leq n$. If <i>range</i> = 'A', $m = n$, and if <i>range</i> = 'I', $m = iu - il + 1$. |
| <i>w</i> | Array, size at least $\max(1, n)$. The selected eigenvalues in ascending order, stored in <i>w</i> [0] to <i>w</i> [<i>m</i> - 1]. |
| <i>z</i> | Array <i>z</i> (size $\max(1, ldz * m)$). If <i>jobz</i> = 'V', and if <i>info</i> = 0, the first <i>m</i> columns of <i>z</i> contain the orthonormal eigenvectors of the matrix <i>T</i> corresponding to the selected eigenvalues, with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with <i>w</i> [<i>i</i> - 1]. If <i>jobz</i> = 'N', then <i>z</i> is not referenced. Note: if <i>range</i> = 'V', the exact value of <i>m</i> is not known in advance and an upper bound must be used. Using $n = m$ is always safe. |
| <i>isuppz</i> | Array, size at least $(2 * \max(1, m))$. The support of the eigenvectors in <i>z</i> , that is the indices indicating the nonzero elements in <i>z</i> . The <i>i</i> -th computed eigenvector is nonzero only in elements <i>isuppz</i> [2 * <i>i</i> - 2] through <i>isuppz</i> [2 * <i>i</i> - 1]. This is relevant in the case when the matrix is split. <i>isuppz</i> is only accessed when <i>jobz</i> = 'V', and $n > 0$. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, an internal error occurred.

?pteqr

Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric positive-definite tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_spteqr( int matrix_layout, char compz, lapack_int n, float* d,
float* e, float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_dpteqr( int matrix_layout, char compz, lapack_int n, double* d,
double* e, double* z, lapack_int ldz );
```

```
lapack_int LAPACKE_cpteqr( int matrix_layout, char compz, lapack_int n, float* d,
float* e, lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zpteqr( int matrix_layout, char compz, lapack_int n, double* d,
double* e, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues and (optionally) all the eigenvectors of a real symmetric positive-definite tridiagonal matrix T . In other words, the routine can compute the spectral factorization: $T = Z^* \Lambda^* Z^T$.

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i ; Z is an orthogonal matrix whose columns are eigenvectors. Thus,

$$T^* z_i = \lambda_i^* z_i \text{ for } i = 1, 2, \dots, n.$$

(The routine normalizes the eigenvectors so that $\|z_i\|_2 = 1$.)

You can also use the routine for computing the eigenvalues and eigenvectors of real symmetric (or complex Hermitian) positive-definite matrices A reduced to tridiagonal form T : $A = Q^* T^* Q^H$. In this case, the spectral factorization is as follows: $A = Q^* T^* Q^H = (QZ)^* \Lambda^* (QZ)^H$. Before calling `?pteqr`, you must reduce A to tridiagonal form and generate the explicit matrix Q by calling the following routines:

| | for real matrices: | for complex matrices: |
|-----------------------|---------------------------|---------------------------|
| full storage | ?sytrd, ?orgtr | ?hetrd, ?ungtr |
| packed storage | ?sptrd, ?opgtr | ?hptrd, ?upgtr |
| band storage | ?sbtrd(<i>vect</i> ='V') | ?hbtrd(<i>vect</i> ='V') |

The routine first factorizes T as $L^* D^* L^H$ where L is a unit lower bidiagonal matrix, and D is a diagonal matrix. Then it forms the bidiagonal matrix $B = L^* D^{1/2}$ and calls `?bdsqr` to compute the singular values of B , which are the square roots of the eigenvalues of T .

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

compz Must be 'N' or 'I' or 'V'.
 If *compz* = 'N', the routine computes eigenvalues only.
 If *compz* = 'I', the routine computes the eigenvalues and eigenvectors of the tridiagonal matrix T .
 If *compz* = 'V', the routine computes the eigenvalues and eigenvectors of A (and the array z must contain the matrix Q on entry).

n The order of the matrix T ($n \geq 0$).

d, e Arrays:

d contains the diagonal elements of T .

The size of d must be at least $\max(1, n)$.

e contains the off-diagonal elements of T .

The size of e must be at least $\max(1, n-1)$.

| | |
|-------|--|
| z | <p>Array, size $\max(1, ldz*n)$</p> <p>If $compz = 'N'$ or $'I'$, z need not be set.</p> <p>If $compz = 'V'$, z must contain the orthogonal matrix used in the reduction to tridiagonal form..</p> |
| ldz | <p>The leading dimension of z. Constraints:</p> <p>$ldz \geq 1$ if $compz = 'N'$;</p> <p>$ldz \geq \max(1, n)$ if $compz = 'V'$ or $'I'$.</p> |

Output Parameters

| | |
|-----|---|
| d | <p>The n eigenvalues in descending order, unless $info > 0$.</p> <p>See also <i>info</i>.</p> |
| e | <p>On exit, the array is overwritten.</p> |
| z | <p>If $info = 0$, contains an n-by-n matrix the columns of which are orthonormal eigenvectors. (The i-th column corresponds to the i-th eigenvalue.)</p> |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = i$, the leading minor of order i (and hence T itself) is not positive-definite.

If $info = n + i$, the algorithm for computing singular values failed to converge; i off-diagonal elements have not converged to zero.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

If λ_j is an exact eigenvalue, and μ_j is the corresponding computed value, then

$$|\mu_j - \lambda_j| \leq c(n) * \varepsilon * K * \lambda_j$$

where $c(n)$ is a modestly increasing function of n , ε is the machine precision, and $K = \|(DTD)\|_2 * \|(DTD)^{-1}\|_2$, D is diagonal with $d_{ii} = t_{ii}^{-1/2}$.

If z_j is the corresponding exact eigenvector, and w_j is the corresponding computed vector, then the angle $\theta(z_j, w_j)$ between them is bounded as follows:

$$\theta(z_j, w_j) \leq c(n) \varepsilon K / \min_{i \neq j} (|\lambda_i - \lambda_j| / |\lambda_i + \lambda_j|).$$

Here $\min_{i \neq j} (|\lambda_i - \lambda_j| / |\lambda_i + \lambda_j|)$ is the *relative gap* between λ_j and the other eigenvalues.

The total number of floating-point operations depends on how rapidly the algorithm converges.

Typically, it is about

$30n^2$ if `compz = 'N'`;

$6n^3$ (for complex flavors, $12n^3$) if `compz = 'V' or 'I'`.

?stebz

Computes selected eigenvalues of a real symmetric tridiagonal matrix by bisection.

Syntax

```
lapack_int LAPACKE_sstebz (char range, char order, lapack_int n, float vl, float vu,
lapack_int il, lapack_int iu, float abstol, const float* d, const float* e,
lapack_int* m, lapack_int* nsplit, float* w, lapack_int* iblock, lapack_int* isplit);

lapack_int LAPACKE_dstebz (char range, char order, lapack_int n, double vl, double vu,
lapack_int il, lapack_int iu, double abstol, const double* d, const double* e,
lapack_int* m, lapack_int* nsplit, double* w, lapack_int* iblock, lapack_int* isplit);
```

Include Files

- mkl.h

Description

The routine computes some (or all) of the eigenvalues of a real symmetric tridiagonal matrix T by bisection. The routine searches for zero or negligible off-diagonal elements to see if T splits into block-diagonal form $T = \text{diag}(T_1, T_2, \dots)$. Then it performs bisection on each of the blocks T_i and returns the block index of each computed eigenvalue, so that a subsequent call to [stein](#) can also take advantage of the block structure.

Input Parameters

| | |
|---------------|--|
| <i>range</i> | <p>Must be 'A' or 'V' or 'I'.</p> <p>If <i>range</i> = 'A', the routine computes all eigenvalues.</p> <p>If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$.</p> <p>If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i>.</p> |
| <i>order</i> | <p>Must be 'B' or 'E'.</p> <p>If <i>order</i> = 'B', the eigenvalues are to be ordered from smallest to largest within each split-off block.</p> <p>If <i>order</i> = 'E', the eigenvalues for the entire matrix are to be ordered from smallest to largest.</p> |
| <i>n</i> | The order of the matrix T ($n \geq 0$). |
| <i>vl, vu</i> | <p>If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval:</p> $vl < w[i] \leq vu.$ <p>If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced.</p> |
| <i>il, iu</i> | <p>Constraint: $1 \leq il \leq iu \leq n$.</p> <p>If <i>range</i> = 'I', the routine computes eigenvalues $w[i]$ such that $il \leq i \leq iu$ (assuming that the eigenvalues $w[i]$ are in ascending order).</p> |

If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol The absolute tolerance to which each eigenvalue is required. An eigenvalue (or cluster) is considered to have converged if it lies in an interval of width *abstol*.

If $abstol \leq 0.0$, then the tolerance is taken as $\epsilon_{ps} * |T|$, where ϵ_{ps} is the machine precision, and $|T|$ is the 1-norm of the matrix *T*.

d, e

Arrays:

d contains the diagonal elements of *T*.

The size of *d* must be at least $\max(1, n)$.

e contains the off-diagonal elements of *T*.

The size of *e* must be at least $\max(1, n-1)$.

Output Parameters

m The actual number of eigenvalues found.

nsplit The number of diagonal blocks detected in *T*.

w Array, size at least $\max(1, n)$. The computed eigenvalues, stored in *w*[0] to *w*[*m* - 1].

iblock, isplit

Arrays, size at least $\max(1, n)$.

A positive value *iblock*[*i*] is the block number of the eigenvalue stored in *w*[*i*] (see also *info*).

The leading *nsplit* elements of *isplit* contain points at which *T* splits into blocks *T_i*, as follows: the block *T₁* contains rows/columns 1 to *isplit*[0]; the block *T₂* contains rows/columns *isplit*[0]+1 to *isplit*[1], and so on.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = 1, for *range* = 'A' or 'V', the algorithm failed to compute some of the required eigenvalues to the desired accuracy; *iblock*[*i*] < 0 indicates that the eigenvalue stored in *w*[*i*] failed to converge.

If *info* = 2, for *range* = 'I', the algorithm failed to compute some of the required eigenvalues. Try calling the routine again with *range* = 'A'.

If *info* = 3:

for *range* = 'A' or 'V', same as *info* = 1;

for *range* = 'I', same as *info* = 2.

If *info* = 4, no eigenvalues have been computed. The floating-point arithmetic on the computer is not behaving as expected.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The eigenvalues of T are computed to high relative accuracy which means that if they vary widely in magnitude, then any small eigenvalues will be computed more accurately than, for example, with the standard QR method. However, the reduction to tridiagonal form (prior to calling the routine) may exclude the possibility of obtaining high relative accuracy in the small eigenvalues of the original matrix if its eigenvalues vary widely in magnitude.

?stein

Computes the eigenvectors corresponding to specified eigenvalues of a real symmetric tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_sstein( int matrix_layout, lapack_int n, const float* d, const
float* e, lapack_int m, const float* w, const lapack_int* iblock, const lapack_int*
isplit, float* z, lapack_int ldz, lapack_int* ifailv );
```

```
lapack_int LAPACKE_dstein( int matrix_layout, lapack_int n, const double* d, const
double* e, lapack_int m, const double* w, const lapack_int* iblock, const lapack_int*
isplit, double* z, lapack_int ldz, lapack_int* ifailv );
```

```
lapack_int LAPACKE_cstein( int matrix_layout, lapack_int n, const float* d, const
float* e, lapack_int m, const float* w, const lapack_int* iblock, const lapack_int*
isplit, lapack_complex_float* z, lapack_int ldz, lapack_int* ifailv );
```

```
lapack_int LAPACKE_zstein( int matrix_layout, lapack_int n, const double* d, const
double* e, lapack_int m, const double* w, const lapack_int* iblock, const lapack_int*
isplit, lapack_complex_double* z, lapack_int ldz, lapack_int* ifailv );
```

Include Files

- mkl.h

Description

The routine computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, by inverse iteration. It is designed to be used in particular after the specified eigenvalues have been computed by ?stebz with *order* = 'B', but may also be used when the eigenvalues have been computed by other routines.

If you use this routine after ?stebz, it can take advantage of the block structure by performing inverse iteration on each block T_i separately, which is more efficient than using the whole matrix T .

If T has been formed by reduction of a full symmetric or Hermitian matrix A to tridiagonal form, you can transform eigenvectors of T to eigenvectors of A by calling ?ormtr or ?opmtr (for real flavors) or by calling ?unmtr or ?upmtr (for complex flavors).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of the matrix T ($n \geq 0$). |
| <i>m</i> | The number of eigenvectors to be returned. |
| <i>d, e, w</i> | Arrays: |

d contains the diagonal elements of T .

The size of d must be at least $\max(1, n)$.

e contains the sub-diagonal elements of T stored in elements 1 to $n-1$

The size of e must be at least $\max(1, n-1)$.

w contains the eigenvalues of T , stored in $w[0]$ to $w[m - 1]$ (as returned by `stebz`). Eigenvalues of T_1 must be supplied first, in non-decreasing order; then those of T_2 , again in non-decreasing order, and so on.

Constraint:

if $iblock[i] = iblock[i+1]$, $w[i] \leq w[i+1]$.

The size of w must be at least $\max(1, n)$.

iblock, isplit

Arrays, size at least $\max(1, n)$. The arrays *iblock* and *isplit*, as returned by `?stebz` with *order* = 'B'.

If you did not call `?stebz` with *order* = 'B', set all elements of *iblock* to 1, and *isplit*[0] to n .)

ldz

The leading dimension of the output array z ; $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout.

Output Parameters

z

Array, size at least $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout.

If $info = 0$, z contains an n -by- n matrix the columns of which are orthonormal eigenvectors. (The i -th column corresponds to the i th eigenvalue.)

ifailv

Array, size at least $\max(1, m)$.

If $info = i > 0$, the first i elements of *ifailv* contain the indices of any eigenvectors that failed to converge.

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = i$, then i eigenvectors (as indicated by the parameter *ifailv*) each failed to converge in 5 iterations. The current iterates are stored in the corresponding columns/rows of the array z .

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

Each computed eigenvector z_i is an exact eigenvector of a matrix $T+E_i$, where $\|E_i\|_2 = O(\epsilon) * \|T\|_2$. However, a set of eigenvectors computed by this routine may not be orthogonal to so high a degree of accuracy as those computed by `?steqr`.

?disna

Computes the reciprocal condition numbers for the eigenvectors of a symmetric/ Hermitian matrix or for the left or right singular vectors of a general matrix.

Syntax

```
lapack_int LAPACKE_sdisna (char job, lapack_int m, lapack_int n, const float* d, float* sep);
```

```
lapack_int LAPACKE_ddisna (char job, lapack_int m, lapack_int n, const double* d, double* sep);
```

Include Files

- mkl.h

Description

The routine computes the reciprocal condition numbers for the eigenvectors of a real symmetric or complex Hermitian matrix or for the left or right singular vectors of a general m -by- n matrix.

The reciprocal condition number is the 'gap' between the corresponding eigenvalue or singular value and the nearest other one.

The bound on the error, measured by angle in radians, in the i -th computed vector is given by

$$?lamch('E') * (anorm / sep(i))$$

where $anorm = ||A||_2 = \max(|d(j)|)$. $sep(i)$ is not allowed to be smaller than $slamch('E') * anorm$ in order to limit the size of the error bound.

$?disna$ may also be used to compute error bounds for eigenvectors of the generalized symmetric definite eigenproblem.

Input Parameters

| | |
|------------|--|
| <i>job</i> | Must be 'E', 'L', or 'R'. Specifies for which problem the reciprocal condition numbers should be computed: <i>job</i> = 'E': for the eigenvectors of a symmetric/Hermitian matrix; <i>job</i> = 'L': for the left singular vectors of a general matrix; <i>job</i> = 'R': for the right singular vectors of a general matrix. |
| <i>m</i> | The number of rows of the matrix ($m \geq 0$). |
| <i>n</i> | If <i>job</i> = 'L', or 'R', the number of columns of the matrix ($n \geq 0$). Ignored if <i>job</i> = 'E'. |
| <i>d</i> | Array, dimension at least $\max(1, m)$ if <i>job</i> = 'E', and at least $\max(1, \min(m, n))$ if <i>job</i> = 'L' or 'R'. This array must contain the eigenvalues (if <i>job</i> = 'E') or singular values (if <i>job</i> = 'L' or 'R') of the matrix, in either increasing or decreasing order. If singular values, they must be non-negative. |

Output Parameters

| | |
|------------|---|
| <i>sep</i> | Array, dimension at least $\max(1, m)$ if <i>job</i> = 'E', and at least $\max(1, \min(m, n))$ if <i>job</i> = 'L' or 'R'. The reciprocal condition numbers of the vectors. |
|------------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = *-i*, the *i*-th parameter had an illegal value.

Generalized Symmetric-Definite Eigenvalue Problems: LAPACK Computational Routines

Generalized symmetric-definite eigenvalue problems are as follows: find the eigenvalues λ and the corresponding eigenvectors z that satisfy one of these equations:

$$Az = \lambda Bz, ABz = \lambda z, \text{ or } BAz = \lambda z,$$

where A is an n -by- n symmetric or Hermitian matrix, and B is an n -by- n symmetric positive-definite or Hermitian positive-definite matrix.

In these problems, there exist n real eigenvectors corresponding to real eigenvalues (even for complex Hermitian matrices A and B).

Routines described in this section allow you to reduce the above generalized problems to standard symmetric eigenvalue problem $Cy = \lambda y$, which you can solve by calling LAPACK routines described earlier in this chapter (see [Symmetric Eigenvalue Problems](#)).

Different routines allow the matrices to be stored either conventionally or in packed storage. Prior to reduction, the positive-definite matrix B must first be factorized using either [potrf](#) or [pptrf](#).

The reduction routine for the banded matrices A and B uses a split Cholesky factorization for which a specific routine [pbstf](#) is provided. This refinement halves the amount of work required to form matrix C .

[Table "Computational Routines for Reducing Generalized Eigenproblems to Standard Problems"](#) lists LAPACK routines that can be used to solve generalized symmetric-definite eigenvalue problems.

Computational Routines for Reducing Generalized Eigenproblems to Standard Problems

| Matrix type | Reduce to standard problems (full storage) | Reduce to standard problems (packed storage) | Reduce to standard problems (band matrices) | Factorize band matrix |
|----------------------------|--|--|---|-----------------------|
| real symmetric matrices | sygst | spgst | sbgst | pbstf |
| complex Hermitian matrices | hegst | hpgst | hbgst | pbstf |

?sygst

Reduces a real symmetric-definite generalized eigenvalue problem to the standard form.

Syntax

```
lapack_int LAPACKE_ssygst (int matrix_layout, lapack_int itype, char uplo, lapack_int n, float* a, lapack_int lda, const float* b, lapack_int ldb);
```

```
lapack_int LAPACKE_dsygst (int matrix_layout, lapack_int itype, char uplo, lapack_int n, double* a, lapack_int lda, const double* b, lapack_int ldb);
```

Include Files

- `mkl.h`

Description

The routine reduces real symmetric-definite generalized eigenproblems

$$A^*z = \lambda^*B^*z, \quad A^*B^*z = \lambda^*z, \quad \text{or} \quad B^*A^*z = \lambda^*z$$

to the standard form $C^*y = \lambda^*y$. Here A is a real symmetric matrix, and B is a real symmetric positive-definite matrix. Before calling this routine, call `?potrf` to compute the Cholesky factorization: $B = U^T*U$ or $B = L*L^T$.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>itype</code> | Must be 1 or 2 or 3. If <code>itype = 1</code> , the generalized eigenproblem is $A^*z = \lambda^*B^*z$ for <code>uplo = 'U'</code> : $C = \text{inv}(U^T)*A*\text{inv}(U)$, $z = \text{inv}(U)*y$; for <code>uplo = 'L'</code> : $C = \text{inv}(L)*A*\text{inv}(L^T)$, $z = \text{inv}(L^T)*y$. If <code>itype = 2</code> , the generalized eigenproblem is $A^*B^*z = \lambda^*z$ for <code>uplo = 'U'</code> : $C = U^*A^*U^T$, $z = \text{inv}(U)*y$; for <code>uplo = 'L'</code> : $C = L^T*A*L$, $z = \text{inv}(L^T)*y$. If <code>itype = 3</code> , the generalized eigenproblem is $B^*A^*z = \lambda^*z$ for <code>uplo = 'U'</code> : $C = U^*A^*U^T$, $z = U^T*y$; for <code>uplo = 'L'</code> : $C = L^T*A*L$, $z = L*y$. |
| <code>uplo</code> | Must be 'U' or 'L'. If <code>uplo = 'U'</code> , the array <code>a</code> stores the upper triangle of A ; you must supply B in the factored form $B = U^T*U$. If <code>uplo = 'L'</code> , the array <code>a</code> stores the lower triangle of A ; you must supply B in the factored form $B = L*L^T$. |
| <code>n</code> | The order of the matrices A and B ($n \geq 0$). |
| <code>a, b</code> | Arrays: <code>a</code> (size $\max(1, lda*n)$) contains the upper or lower triangle of A . <code>b</code> (size $\max(1, ldb*n)$) contains the Cholesky-factored matrix B : $B = U^T*U$ or $B = L*L^T$ (as returned by <code>?potrf</code>). |
| <code>lda</code> | The leading dimension of <code>a</code> ; at least $\max(1, n)$. |
| <code>ldb</code> | The leading dimension of <code>b</code> ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------------|--|
| <code>a</code> | The upper or lower triangle of A is overwritten by the upper or lower triangle of C , as specified by the arguments <code>itype</code> and <code>uplo</code> . |
|----------------|--|

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

Forming the reduced matrix C is a stable procedure. However, it involves implicit multiplication by $\text{inv}(B)$ (if $itype = 1$) or B (if $itype = 2$ or 3). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion.

The approximate number of floating-point operations is n^3 .

?hegst

Reduces a complex Hermitian positive-definite generalized eigenvalue problem to the standard form.

Syntax

```
lapack_int LAPACKE_chegst (int matrix_layout, lapack_int itype, char uplo, lapack_int
n, lapack_complex_float* a, lapack_int lda, const lapack_complex_float* b, lapack_int
ldb);
```

```
lapack_int LAPACKE_zhegst (int matrix_layout, lapack_int itype, char uplo, lapack_int
n, lapack_complex_double* a, lapack_int lda, const lapack_complex_double* b, lapack_int
ldb);
```

Include Files

- mkl.h

Description

The routine reduces a complex Hermitian positive-definite generalized eigenvalue problem to standard form.

| <i>itype</i> | Problem | Result |
|--------------|---------------------|---|
| 1 | $A*x = \lambda*B*x$ | A overwritten by $\text{inv}(U^H)*A*\text{inv}(U)$ or $\text{inv}(L)*A*\text{inv}(L^H)$ |
| 2 | $A*B*x = \lambda*x$ | A overwritten by $U*A*U^H$ or L^H*A*L |
| 3 | $B*A*x = \lambda*x$ | |

Before calling this routine, you must call ?potrf to compute the Cholesky factorization: $B = U^H*U$ or $B = L*L^H$.

Input Parameters

itype Must be 1 or 2 or 3.
 If $itype = 1$, the generalized eigenproblem is $A*z = \text{lambda}*B*z$
 for $uplo = 'U'$: $C = (U^H)^{-1}*A*U^{-1}$;
 for $uplo = 'L'$: $C = L^{-1}*A*(L^H)^{-1}$.
 If $itype = 2$, the generalized eigenproblem is $A*B*z = \text{lambda}*z$
 for $uplo = 'U'$: $C = U*A*U^H$;
 for $uplo = 'L'$: $C = L^H*A*L$.
 If $itype = 3$, the generalized eigenproblem is $B*A*z = \text{lambda}*z$

| | |
|--------|--|
| | for $uplo = 'U'$: $C = U^*A^*U^H$; |
| | for $uplo = 'L'$: $C = L^H^*A^*L$. |
| $uplo$ | Must be 'U' or 'L'. |
| | If $uplo = 'U'$, the array a stores the upper triangle of A ; you must supply B in the factored form $B = U^H^*U$. |
| | If $uplo = 'L'$, the array a stores the lower triangle of A ; you must supply B in the factored form $B = L^*L^H$. |
| n | The order of the matrices A and B ($n \geq 0$). |
| a, b | Arrays: a (size $\max(1, lda*n)$) contains the upper or lower triangle of A . b (size $\max(1, ldb*n)$) contains the Cholesky-factored matrix B : $B = U^H^*U$ or $B = L^*L^H$ (as returned by <code>?potrf</code>). |
| lda | The leading dimension of a ; at least $\max(1, n)$. |
| ldb | The leading dimension of b ; at least $\max(1, n)$. |

Output Parameters

| | |
|-----|--|
| a | The upper or lower triangle of A is overwritten by the upper or lower triangle of C , as specified by the arguments $itype$ and $uplo$. |
|-----|--|

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

Forming the reduced matrix C is a stable procedure. However, it involves implicit multiplication by B^{-1} (if $itype = 1$) or B (if $itype = 2$ or 3). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion.

The approximate number of floating-point operations is n^3 .

?spgst

Reduces a real symmetric-definite generalized eigenvalue problem to the standard form using packed storage.

Syntax

```
lapack_int LAPACK_esspgst (int matrix_layout, lapack_int itype, char uplo, lapack_int n, float* ap, const float* bp);
```

```
lapack_int LAPACK_dspgst (int matrix_layout, lapack_int itype, char uplo, lapack_int n, double* ap, const double* bp);
```

Include Files

- `mk1.h`

Description

The routine reduces real symmetric-definite generalized eigenproblems

$$A^*x = \lambda^*B^*x, A^*B^*x = \lambda^*x, \text{ or } B^*A^*x = \lambda^*x$$

to the standard form $C^*y = \lambda^*y$, using packed matrix storage. Here A is a real symmetric matrix, and B is a real symmetric positive-definite matrix. Before calling this routine, call `?pptrf` to compute the Cholesky factorization: $B = U^T*U$ or $B = L*L^T$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>itype</i> | <p>Must be 1 or 2 or 3.</p> <p>If <i>itype</i> = 1, the generalized eigenproblem is $A^*z = \lambda^*B^*z$ for <i>uplo</i> = 'U': $C = \text{inv}(U^T)*A*\text{inv}(U)$, $z = \text{inv}(U)*y$; for <i>uplo</i> = 'L': $C = \text{inv}(L)*A*\text{inv}(L^T)$, $z = \text{inv}(L^T)*y$.</p> <p>If <i>itype</i> = 2, the generalized eigenproblem is $A^*B^*z = \lambda^*z$ for <i>uplo</i> = 'U': $C = U^*A^*U^T$, $z = \text{inv}(U)*y$; for <i>uplo</i> = 'L': $C = L^T*A^*L$, $z = \text{inv}(L^T)*y$.</p> <p>If <i>itype</i> = 3, the generalized eigenproblem is $B^*A^*z = \lambda^*z$ for <i>uplo</i> = 'U': $C = U^*A^*U^T$, $z = U^T*y$; for <i>uplo</i> = 'L': $C = L^T*A^*L$, $z = L*y$.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>If <i>uplo</i> = 'U', <i>ap</i> stores the packed upper triangle of A; you must supply B in the factored form $B = U^T*U$.</p> <p>If <i>uplo</i> = 'L', <i>ap</i> stores the packed lower triangle of A; you must supply B in the factored form $B = L*L^T$.</p> |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ap, bp</i> | <p>Arrays:</p> <p><i>ap</i> contains the packed upper or lower triangle of A. The dimension of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$.</p> <p><i>bp</i> contains the packed Cholesky factor of B (as returned by <code>?pptrf</code> with the same <i>uplo</i> value). The dimension of <i>bp</i> must be at least $\max(1, n*(n+1)/2)$.</p> |

Output Parameters

ap The upper or lower triangle of A is overwritten by the upper or lower triangle of C , as specified by the arguments *itype* and *uplo*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

Forming the reduced matrix C is a stable procedure. However, it involves implicit multiplication by $\text{inv}(B)$ (if *itype* = 1) or B (if *itype* = 2 or 3). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion.

The approximate number of floating-point operations is n^3 .

?hpgst

Reduces a generalized eigenvalue problem with a Hermitian matrix to a standard eigenvalue problem using packed storage.

Syntax

```
lapack_int LAPACKE_chpgst (int matrix_layout, lapack_int itype, char uplo, lapack_int n, lapack_complex_float* ap, const lapack_complex_float* bp);
```

```
lapack_int LAPACKE_zhpgst (int matrix_layout, lapack_int itype, char uplo, lapack_int n, lapack_complex_double* ap, const lapack_complex_double* bp);
```

Include Files

- mkl.h

Description

The routine reduces generalized eigenproblems with Hermitian matrices

$$A^*z = \lambda^*B^*z, A^*B^*z = \lambda^*z, \text{ or } B^*A^*z = \lambda^*z.$$

to standard eigenproblems $C^*y = \lambda^*y$, using packed matrix storage. Here A is a complex Hermitian matrix, and B is a complex Hermitian positive-definite matrix. Before calling this routine, you must call ?pptrf to compute the Cholesky factorization: $B = U^H*U$ or $B = L*L^H$.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

itype Must be 1 or 2 or 3.

If *itype* = 1, the generalized eigenproblem is $A^*z = \text{lambda}^*B^*z$

for *uplo* = 'U': $C = \text{inv}(U^H)*A*\text{inv}(U)$, $z = \text{inv}(U)*y$;

for *uplo* = 'L': $C = \text{inv}(L)*A*\text{inv}(L^H)$, $z = \text{inv}(L^H)*y$.

If $itype = 2$, the generalized eigenproblem is $A^*B^*z = \lambda z$

for $uplo = 'U'$: $C = U^*A^*U^H$, $z = \text{inv}(U)^*y$;

for $uplo = 'L'$: $C = L^H*A^*L$, $z = \text{inv}(L^H)^*y$.

If $itype = 3$, the generalized eigenproblem is $B^*A^*z = \lambda z$

for $uplo = 'U'$: $C = U^*A^*U^H$, $z = U^H*y$;

for $uplo = 'L'$: $C = L^H*A^*L$, $z = L*y$.

$uplo$

Must be 'U' or 'L'.

If $uplo = 'U'$, ap stores the packed upper triangle of A ; you must supply B in the factored form $B = U^H*U$.

If $uplo = 'L'$, ap stores the packed lower triangle of A ; you must supply B in the factored form $B = L*L^H$.

n

The order of the matrices A and B ($n \geq 0$).

ap, bp

Arrays:

ap contains the packed upper or lower triangle of A .

The dimension of a must be at least $\max(1, n*(n+1)/2)$.

bp contains the packed Cholesky factor of B (as returned by `?pptrf` with the same $uplo$ value).

The dimension of b must be at least $\max(1, n*(n+1)/2)$.

Output Parameters

ap

The upper or lower triangle of A is overwritten by the upper or lower triangle of C , as specified by the arguments $itype$ and $uplo$.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

Forming the reduced matrix C is a stable procedure. However, it involves implicit multiplication by $\text{inv}(B)$ (if $itype = 1$) or B (if $itype = 2$ or 3). When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion.

The approximate number of floating-point operations is n^3 .

?sbgst

Reduces a real symmetric-definite generalized eigenproblem for banded matrices to the standard form using the factorization performed by ?pbstf.

Syntax

```
lapack_int LAPACKE_ssbgst (int matrix_layout, char vect, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, float* ab, lapack_int ldab, const float* bb, lapack_int
ldbb, float* x, lapack_int ldx);
```

```
lapack_int LAPACKE_dsbgst (int matrix_layout, char vect, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, double* ab, lapack_int ldab, const double* bb,
lapack_int ldbb, double* x, lapack_int ldx);
```

Include Files

- mkl.h

Description

To reduce the real symmetric-definite generalized eigenproblem $A*z = \lambda*B*z$ to the standard form $C*y = \lambda*y$, where A , B and C are banded, this routine must be preceded by a call to [pbstf](#), which computes the split Cholesky factorization of the positive-definite matrix B : $B=S^T*S$. The split Cholesky factorization, compared with the ordinary Cholesky factorization, allows the work to be approximately halved.

This routine overwrites A with $C = X^T*A*X$, where $X = \text{inv}(S)*Q$ and Q is an orthogonal matrix chosen (implicitly) to preserve the bandwidth of A . The routine also has an option to allow the accumulation of X , and then, if z is an eigenvector of C , $X*z$ is an eigenvector of the original system.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>vect</i> | Must be 'N' or 'V'. If <i>vect</i> = 'N', then matrix X is not returned; If <i>vect</i> = 'V', then matrix X is returned. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ab</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>ab</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ka</i> | The number of super- or sub-diagonals in A ($ka \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in B ($ka \geq kb \geq 0$). |
| <i>ab, bb</i> | <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and at least $\max(1, ldab*(ka + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix A (as specified by <i>uplo</i>) in band storage format. |

bb (size at least $\max(1, l_{dbb} * n)$ for column major layout and at least $\max(1, l_{dbb} * (kb + 1))$ for row major layout) is an array containing the banded split Cholesky factor of B as specified by $uplo$, n and kb and returned by `pbstf/pbstf`.

| | |
|-----------|--|
| $ldab$ | The leading dimension of the array ab ; must be at least $ka+1$ for column major layout and $\max(1, n)$ for row major layout. |
| l_{dbb} | The leading dimension of the array bb ; must be at least $kb+1$ for column major layout and $\max(1, n)$ for row major layout. |
| ldx | The leading dimension of the output array x . Constraints: if $vect = 'N'$, then $ldx \geq 1$; if $vect = 'V'$, then $ldx \geq \max(1, n)$. |

Output Parameters

| | |
|------|---|
| ab | On exit, this array is overwritten by the upper or lower triangle of C as specified by $uplo$. |
| x | Array. If $vect = 'V'$, then x (size at least $\max(1, l_{dx} * n)$) contains the n -by- n matrix $X = \text{inv}(S) * Q$. If $vect = 'N'$, then x is not referenced. |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

Forming the reduced matrix C involves implicit multiplication by $\text{inv}(B)$. When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion.

If ka and kb are much less than n then the total number of floating-point operations is approximately $6n^2 * kb$, when $vect = 'N'$. Additional $(3/2)n^3 * (kb/ka)$ operations are required when $vect = 'V'$.

?hbgst

Reduces a complex Hermitian positive-definite generalized eigenproblem for banded matrices to the standard form using the factorization performed by ?pbstf.

Syntax

```
lapack_int LAPACKE_chbgst (int matrix_layout, char vect, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, lapack_complex_float* ab, lapack_int ldab, const
lapack_complex_float* bb, lapack_int l_{dbb}, lapack_complex_float* x, lapack_int l_{dx});
```

```
lapack_int LAPACKE_zhbgst (int matrix_layout, char vect, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, lapack_complex_double* ab, lapack_int ldab, const
lapack_complex_double* bb, lapack_int l_{dbb}, lapack_complex_double* x, lapack_int l_{dx});
```

Include Files

- `mkl.h`

Description

To reduce the complex Hermitian positive-definite generalized eigenproblem $A^*z = \lambda^*B^*z$ to the standard form $C^*x = \lambda^*y$, where A , B and C are banded, this routine must be preceded by a call to [pbstf/pbstf](#), which computes the split Cholesky factorization of the positive-definite matrix B : $B = S^H * S$. The split Cholesky factorization, compared with the ordinary Cholesky factorization, allows the work to be approximately halved.

This routine overwrites A with $C = X^H * A * X$, where $X = \text{inv}(S) * Q$, and Q is a unitary matrix chosen (implicitly) to preserve the bandwidth of A . The routine also has an option to allow the accumulation of X , and then, if z is an eigenvector of C , X^*z is an eigenvector of the original system.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>vect</i> | Must be 'N' or 'V'. If <i>vect</i> = 'N', then matrix X is not returned; If <i>vect</i> = 'V', then matrix X is returned. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ab</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>ab</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ka</i> | The number of super- or sub-diagonals in A ($ka \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in B ($ka \geq kb \geq 0$). |
| <i>ab, bb</i> | <i>ab</i> (size at least $\max(1, ldab * n)$ for column major layout and at least $\max(1, ldab * (ka + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix A (as specified by <i>uplo</i>) in band storage format. <i>bb</i> (size at least $\max(1, ldbb * n)$ for column major layout and at least $\max(1, ldbb * (kb + 1))$ for row major layout) is an array containing the banded split Cholesky factor of B as specified by <i>uplo</i> , <i>n</i> and <i>kb</i> and returned by pbstf/pbstf . |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; must be at least $ka+1$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldbb</i> | The leading dimension of the array <i>bb</i> ; must be at least $kb+1$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldx</i> | The leading dimension of the output array x . Constraints: if <i>vect</i> = 'N', then $ldx \geq 1$; |

if $vect = 'V'$, then $ldx \geq \max(1, n)$.

Output Parameters

| | |
|------|--|
| ab | On exit, this array is overwritten by the upper or lower triangle of C as specified by $uplo$. |
| x | Array. If $vect = 'V'$, then x (size at least $\max(1, ldx*n)$) contains the n -by- n matrix $X = \text{inv}(S)*Q$. If $vect = 'N'$, then x is not referenced. |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

Forming the reduced matrix C involves implicit multiplication by $\text{inv}(B)$. When the routine is used as a step in the computation of eigenvalues and eigenvectors of the original problem, there may be a significant loss of accuracy if B is ill-conditioned with respect to inversion. The total number of floating-point operations is approximately $20n^2*kb$, when $vect = 'N'$. Additional $5n^3*(kb/ka)$ operations are required when $vect = 'V'$. All these estimates assume that both ka and kb are much less than n .

?pbstf

Computes a split Cholesky factorization of a real symmetric or complex Hermitian positive-definite banded matrix used in ?sbgst/?hbgst.

Syntax

```
lapack_int LAPACKE_spbstf (int matrix_layout, char uplo, lapack_int n, lapack_int kb,
float* bb, lapack_int ldbb);
```

```
lapack_int LAPACKE_dpbstf (int matrix_layout, char uplo, lapack_int n, lapack_int kb,
double* bb, lapack_int ldbb);
```

```
lapack_int LAPACKE_cpbstf (int matrix_layout, char uplo, lapack_int n, lapack_int kb,
lapack_complex_float* bb, lapack_int ldbb);
```

```
lapack_int LAPACKE_zpbstf (int matrix_layout, char uplo, lapack_int n, lapack_int kb,
lapack_complex_double* bb, lapack_int ldbb);
```

Include Files

- mkl.h

Description

The routine computes a split Cholesky factorization of a real symmetric or complex Hermitian positive-definite band matrix B . It is to be used in conjunction with [sbgst/hbgst](#).

The factorization has the form $B = S^T*S$ (or $B = S^H*S$ for complex flavors), where S is a band matrix of the same bandwidth as B and the following structure: S is upper triangular in the first $(n+kb)/2$ rows and lower triangular in the remaining rows.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>bb</i> stores the upper triangular part of <i>B</i> . If <i>uplo</i> = 'L', <i>bb</i> stores the lower triangular part of <i>B</i> . |
| <i>n</i> | The order of the matrix <i>B</i> ($n \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in <i>B</i> ($kb \geq 0$). |
| <i>bb</i> | <i>bb</i> (size at least $\max(1, lddb*n)$ for column major layout and at least $\max(1, lddb*(kb + 1))$ for row major layout) is an array containing either upper or lower triangular part of the matrix <i>B</i> (as specified by <i>uplo</i>) in band storage format. |
| <i>ldbb</i> | The leading dimension of <i>bb</i> ; must be at least $kb+1$ for column major and at least $\max(1, n)$ for row major. |

Output Parameters

| | |
|-----------|--|
| <i>bb</i> | On exit, this array is overwritten by the elements of the split Cholesky factor <i>S</i> . |
|-----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = *i*, then the factorization could not be completed, because the updated element b_{ii} would be the square root of a negative number; hence the matrix *B* is not positive-definite.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed factor *S* is the exact factor of a perturbed matrix $B + E$, where

$$|E| \leq c(kb + 1)\epsilon \|S^H\| |S|, \quad |e_{ij}| \leq c(kb + 1)\epsilon \sqrt{b_{ii}b_{jj}}$$

$c(n)$ is a modest linear function of *n*, and ϵ is the machine precision.

The total number of floating-point operations for real flavors is approximately $n(kb+1)^2$. The number of operations for complex flavors is 4 times greater. All these estimates assume that *kb* is much less than *n*.

After calling this routine, you can call [sbgst/hbgst](#) to solve the generalized eigenproblem $Az = \lambda Bz$, where *A* and *B* are banded and *B* is positive-definite.

Nonsymmetric Eigenvalue Problems: LAPACK Computational Routines

This section describes LAPACK routines for solving nonsymmetric eigenvalue problems, computing the Schur factorization of general matrices, as well as performing a number of related computational tasks.

A nonsymmetric eigenvalue problem is as follows: given a nonsymmetric (or non-Hermitian) matrix A , find the eigenvalues λ and the corresponding eigenvectors z that satisfy the equation

$$Az = \lambda z \text{ (right eigenvectors } z)$$

or the equation

$$z^H A = \lambda z^H \text{ (left eigenvectors } z).$$

Nonsymmetric eigenvalue problems have the following properties:

- The number of eigenvectors may be less than the matrix order (but is not less than the number of *distinct eigenvalues* of A).
- Eigenvalues may be complex even for a real matrix A .
- If a real nonsymmetric matrix has a complex eigenvalue $a+bi$ corresponding to an eigenvector z , then $a-bi$ is also an eigenvalue. The eigenvalue $a-bi$ corresponds to the eigenvector whose elements are complex conjugate to the elements of z .

To solve a nonsymmetric eigenvalue problem with LAPACK, you usually need to reduce the matrix to the upper Hessenberg form and then solve the eigenvalue problem with the Hessenberg matrix obtained. [Table "Computational Routines for Solving Nonsymmetric Eigenvalue Problems"](#) lists LAPACK routines to reduce the matrix to the upper Hessenberg form by an orthogonal (or unitary) similarity transformation $A = QHQ^H$ as well as routines to solve eigenvalue problems with Hessenberg matrices, forming the Schur factorization of such matrices and computing the corresponding condition numbers.

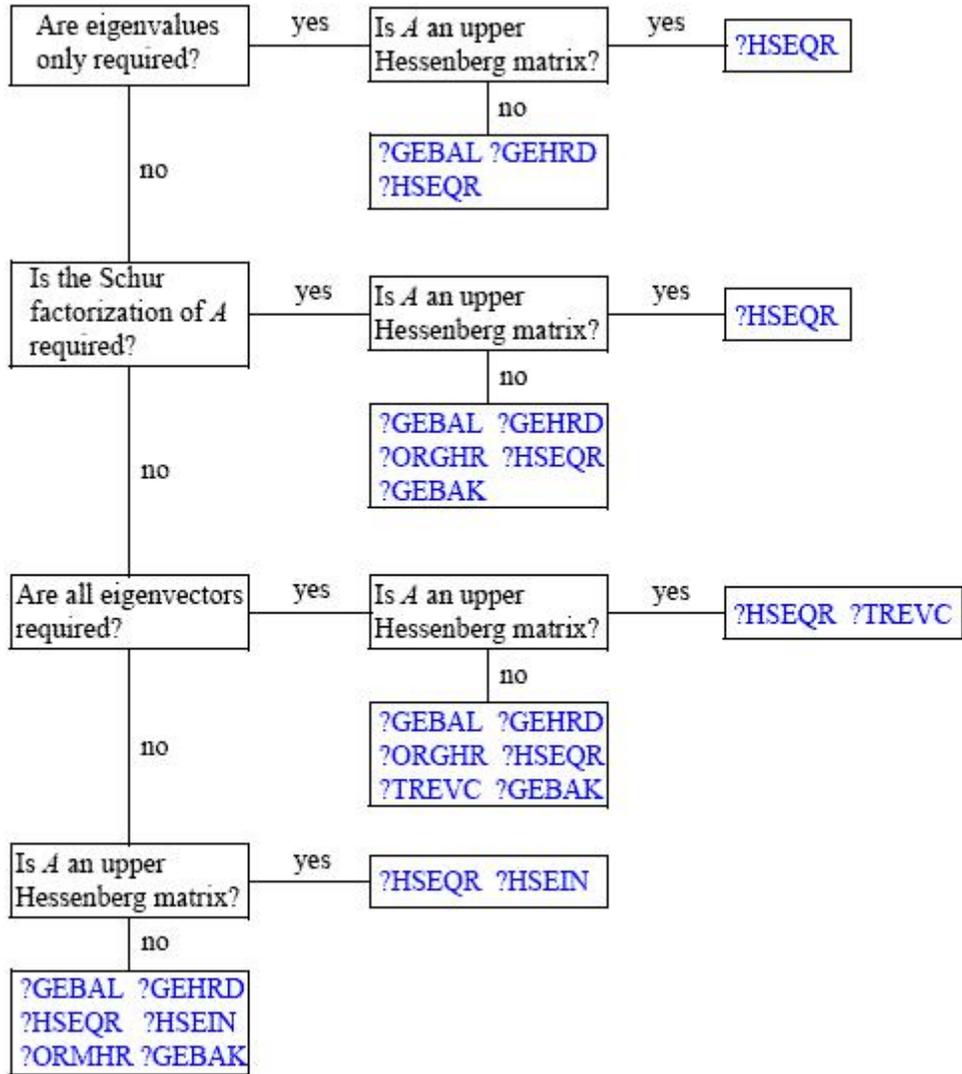
The decision tree in [Figure "Decision Tree: Real Nonsymmetric Eigenvalue Problems"](#) helps you choose the right routine or sequence of routines for an eigenvalue problem with a real nonsymmetric matrix. If you need to solve an eigenvalue problem with a complex non-Hermitian matrix, use the decision tree shown in [Figure "Decision Tree: Complex Non-Hermitian Eigenvalue Problems"](#).

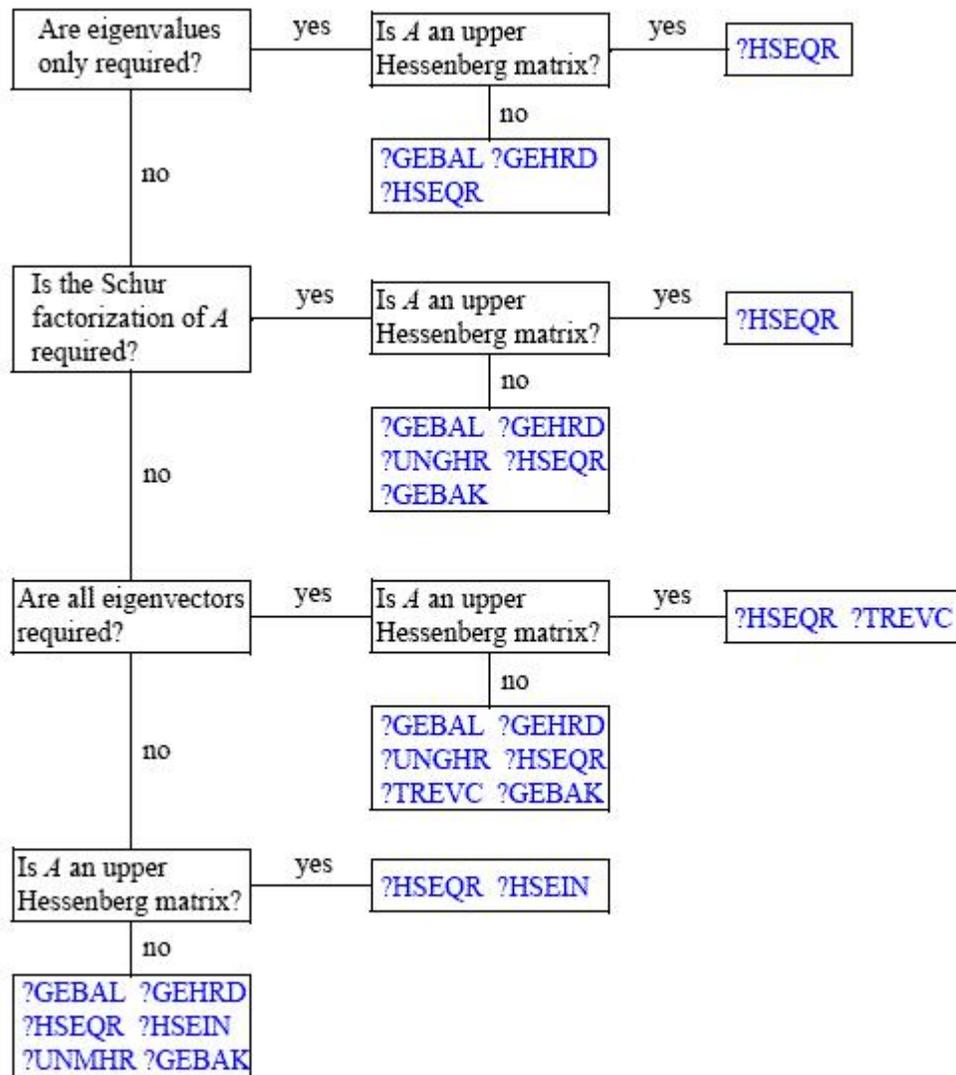
Computational Routines for Solving Nonsymmetric Eigenvalue Problems

| Operation performed | Routines for real matrices | Routines for complex matrices |
|---|----------------------------|-------------------------------|
| Reduce to Hessenberg form $A = QHQ^H$ | ?gehrd, | ?gehrd |
| Generate the matrix Q | ?orghr | ?unghr |
| Apply the matrix Q | ?ormhr | ?unmhr |
| Balance matrix | ?gebal | ?gebal |
| Transform eigenvectors of balanced matrix to those of the original matrix | ?gebak | ?gebak |
| Find eigenvalues and Schur factorization (QR algorithm) | ?hseqr | ?hseqr |
| Find eigenvectors from Hessenberg form (inverse iteration) | ?hsein | ?hsein |
| Find eigenvectors from Schur factorization | ?trevc | ?trevc |
| Estimate sensitivities of eigenvalues and eigenvectors | ?trsna | ?trsna |
| Reorder Schur factorization | ?trexc | ?trexc |
| Reorder Schur factorization, find the invariant subspace and estimate sensitivities | ?trsen | ?trsen |

| Operation performed | Routines for real matrices | Routines for complex matrices |
|------------------------------|----------------------------|-------------------------------|
| Solves Sylvester's equation. | ?trsyl | ?trsyl |

Decision Tree: Real Nonsymmetric Eigenvalue Problems



Decision Tree: Complex Non-Hermitian Eigenvalue Problems**?gehrd**

Reduces a general matrix to upper Hessenberg form.

Syntax

```
lapack_int LAPACKE_sgehrd (int matrix_layout, lapack_int n, lapack_int ilo, lapack_int
ihi, float* a, lapack_int lda, float* tau);
```

```
lapack_int LAPACKE_dgehrd (int matrix_layout, lapack_int n, lapack_int ilo, lapack_int
ihi, double* a, lapack_int lda, double* tau);
```

```
lapack_int LAPACKE_cgehrd (int matrix_layout, lapack_int n, lapack_int ilo, lapack_int
ihi, lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zgehrd (int matrix_layout, lapack_int n, lapack_int ilo, lapack_int
ihi, lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- `mkl.h`

Description

The routine reduces a general matrix A to upper Hessenberg form H by an orthogonal or unitary similarity transformation $A = Q^*H^*Q^H$. Here H has real subdiagonal elements.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of *elementary reflectors*. Routines are provided to work with Q in this representation.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>n</code> | The order of the matrix A ($n \geq 0$). |
| <code>ilo, ihi</code> | If A is an output by <code>?gebal</code> , then <code>ilo</code> and <code>ihi</code> must contain the values returned by that routine. Otherwise <code>ilo = 1</code> and <code>ihi = n</code> . (If $n > 0$, then $1 \leq ilo \leq ihi \leq n$; if $n = 0$, <code>ilo = 1</code> and <code>ihi = 0</code> .) |
| <code>a</code> | Arrays: <code>a</code> (size $\max(1, lda * n)$) contains the matrix A . |
| <code>lda</code> | The leading dimension of <code>a</code> ; at least $\max(1, n)$. |

Output Parameters

| | |
|------------------|--|
| <code>a</code> | The elements on and above the subdiagonal contain the upper Hessenberg matrix H . The subdiagonal elements of H are real. The elements below the subdiagonal, with the array <code>tau</code> , represent the orthogonal matrix Q as a product of n elementary reflectors. |
| <code>tau</code> | Array, size at least $\max(1, n-1)$. Contains scalars that define elementary reflectors for the matrix Q . |

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

Application Notes

The computed Hessenberg matrix H is exactly similar to a nearby matrix $A + E$, where $\|E\|_2 < c(n)\epsilon \|A\|_2$, $c(n)$ is a modestly increasing function of n , and ϵ is the machine precision.

The approximate number of floating-point operations for real flavors is $(2/3) * (ihi - ilo)^2 (2ihi + 2ilo + 3n)$; for complex flavors it is 4 times greater.

?orghr

Generates the real orthogonal matrix Q determined by `?gehrd`.

Syntax

```
lapack_int LAPACKE_sorghr (int matrix_layout, lapack_int n, lapack_int ilo, lapack_int
ihi, float* a, lapack_int lda, const float* tau);
```

```
lapack_int LAPACKE_dorghr (int matrix_layout, lapack_int n, lapack_int ilo, lapack_int
ihi, double* a, lapack_int lda, const double* tau);
```

Include Files

- mkl.h

Description

The routine explicitly generates the orthogonal matrix Q that has been determined by a preceding call to `sgehrd/dgehrd`. (The routine `?gehrd` reduces a real general matrix A to upper Hessenberg form H by an orthogonal similarity transformation, $A = Q^*H^*Q^T$, and represents the matrix Q as a product of ihi - ilo elementary reflectors. Here ilo and ihi are values determined by `sgebal/dgebal` when balancing the matrix; if the matrix has not been balanced, $ilo = 1$ and $ihi = n$.)

The matrix Q generated by `?orghr` has the structure:

$$Q = \begin{bmatrix} I & 0 & 0 \\ 0 & Q_{22} & 0 \\ 0 & 0 & I \end{bmatrix}$$

where Q_{22} occupies rows and columns ilo to ihi .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of the matrix Q ($n \geq 0$). |
| <i>ilo, ihi</i> | These must be the same parameters ilo and ihi , respectively, as supplied to <code>?gehrd</code> . (If $n > 0$, then $1 \leq ilo \leq ihi \leq n$; if $n = 0$, $ilo = 1$ and $ihi = 0$.) |
| <i>a, tau</i> | Arrays: a (size $\max(1, lda*n)$) contains details of the vectors which define the elementary reflectors, as returned by <code>?gehrd</code> . tau contains further details of the elementary reflectors, as returned by <code>?gehrd</code> . The dimension of tau must be at least $\max(1, n-1)$. |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | Overwritten by the n -by- n orthogonal matrix Q . |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed matrix *Q* differs from the exact result by a matrix *E* such that $\|E\|_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of floating-point operations is $(4/3)(ihi-ilo)^3$.

The complex counterpart of this routine is [unghr](#).

?ormhr

Multiplies an arbitrary real matrix C by the real orthogonal matrix Q determined by ?gehrd.

Syntax

```
lapack_int LAPACKE_sormhr (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int ilo, lapack_int ihi, const float* a, lapack_int lda, const
float* tau, float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_dormhr (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int ilo, lapack_int ihi, const double* a, lapack_int lda, const
double* tau, double* c, lapack_int ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a matrix *C* by the orthogonal matrix *Q* that has been determined by a preceding call to *sgehrd/dgehrd*. (The routine *?gehrd* reduces a real general matrix *A* to upper Hessenberg form *H* by an orthogonal similarity transformation, $A = Q^T H Q$, and represents the matrix *Q* as a product of *ihi-ilo* elementary reflectors. Here *ilo* and *ihi* are values determined by *sgebal/dgebal* when balancing the matrix; if the matrix has not been balanced, *ilo* = 1 and *ihi* = *n*.)

With *?ormhr*, you can form one of the matrix products $Q^T C$, $Q^T C^T$, $C^T Q$, or $C^T Q^T$, overwriting the result on *C* (which may be any real rectangular matrix).

A common application of *?ormhr* is to transform a matrix *V* of eigenvectors of *H* to the matrix *QV* of eigenvectors of *A*.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be 'L' or 'R'. If <i>side</i> = 'L', then the routine forms $Q^T C$ or $Q^T C^T$. If <i>side</i> = 'R', then the routine forms $C^T Q$ or $C^T Q^T$. |
| <i>trans</i> | Must be 'N' or 'T'. |

| | |
|------------------|--|
| | If <i>trans</i> = 'N', then <i>Q</i> is applied to <i>C</i> . |
| | If <i>trans</i> = 'T', then Q^T is applied to <i>C</i> . |
| <i>m</i> | The number of rows in <i>C</i> ($m \geq 0$). |
| <i>n</i> | The number of columns in <i>C</i> ($n \geq 0$). |
| <i>ilo, ihi</i> | These must be the same parameters <i>ilo</i> and <i>ihi</i> , respectively, as supplied to ?gehrd. If $m > 0$ and <i>side</i> = 'L', then $1 \leq ilo \leq ihi \leq m$. If $m = 0$ and <i>side</i> = 'L', then <i>ilo</i> = 1 and <i>ihi</i> = 0. If $n > 0$ and <i>side</i> = 'R', then $1 \leq ilo \leq ihi \leq n$. If $n = 0$ and <i>side</i> = 'R', then <i>ilo</i> = 1 and <i>ihi</i> = 0. |
| <i>a, tau, c</i> | Arrays: <i>a</i> (size $\max(1, lda*n)$ for <i>side</i> ='R' and size $\max(1, lda*m)$ for <i>side</i> ='L') contains details of the vectors which define the <i>elementary reflectors</i> , as returned by ?gehrd. <i>tau</i> contains further details of the <i>elementary reflectors</i> , as returned by ?gehrd. The dimension of <i>tau</i> must be at least $\max(1, m-1)$ if <i>side</i> = 'L' and at least $\max(1, n-1)$ if <i>side</i> = 'R'. <i>c</i> (size $\max(1, ldc*n)$ for column major layout and $\max(1, ldc*m)$ for row major layout) contains the <i>m</i> by <i>n</i> matrix <i>C</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ if <i>side</i> = 'L' and at least $\max(1, n)$ if <i>side</i> = 'R'. |
| <i>ldc</i> | The leading dimension of <i>c</i> ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout . |

Output Parameters

| | |
|----------|---|
| <i>c</i> | <i>C</i> is overwritten by product Q^*C , Q^T*C , $C*Q$, or $C*Q^T$ as specified by <i>side</i> and <i>trans</i> . |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the *i*-th parameter had an illegal value.

Application Notes

The computed matrix *Q* differs from the exact result by a matrix *E* such that $\|E\|_2 = O(\epsilon) \|C\|_2$, where ϵ is the machine precision.

The approximate number of floating-point operations is

$2n(ihi-ilo)^2$ if *side* = 'L';

$2m(ihi-ilo)^2$ if *side* = 'R'.

The complex counterpart of this routine is [unmhr](#).

?unghr

Generates the complex unitary matrix Q determined by ?gehrd.

Syntax

```
lapack_int LAPACKE_cunghr (int matrix_layout, lapack_int n, lapack_int ilo, lapack_int
ihi, lapack_complex_float* a, lapack_int lda, const lapack_complex_float* tau);
lapack_int LAPACKE_zunghr (int matrix_layout, lapack_int n, lapack_int ilo, lapack_int
ihi, lapack_complex_double* a, lapack_int lda, const lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine is intended to be used following a call to `cgehrd/zgehrd`, which reduces a complex matrix A to upper Hessenberg form H by a unitary similarity transformation: $A = Q^*H^*Q^H$. ?gehrd represents the matrix Q as a product of *ihi-ilo* elementary reflectors. Here *ilo* and *ihi* are values determined by `cgebal/zgebal` when balancing the matrix; if the matrix has not been balanced, $ilo = 1$ and $ihi = n$.

Use the routine [unghr](#) to generate Q explicitly as a square matrix. The matrix Q has the structure:

$$Q = \begin{bmatrix} I & 0 & 0 \\ 0 & Q_{22} & 0 \\ 0 & 0 & I \end{bmatrix}$$

where Q_{22} occupies rows and columns *ilo* to *ihi*.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of the matrix Q ($n \geq 0$). |
| <i>ilo, ihi</i> | These must be the same parameters <i>ilo</i> and <i>ihi</i> , respectively, as supplied to ?gehrd. (If $n > 0$, then $1 \leq ilo \leq ihi \leq n$. If $n = 0$, then $ilo = 1$ and $ihi = 0$.) |
| <i>a, tau</i> | <p>Arrays:</p> <p><i>a</i> (size $\max(1, lda * n)$) contains details of the vectors which define the <i>elementary reflectors</i>, as returned by ?gehrd.</p> <p><i>tau</i> contains further details of the <i>elementary reflectors</i>, as returned by ?gehrd.</p> <p>The dimension of <i>tau</i> must be at least $\max(1, n-1)$.</p> |

lda The leading dimension of *a*; at least $\max(1, n)$.

Output Parameters

a Overwritten by the *n*-by-*n* unitary matrix *Q*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed matrix *Q* differs from the exact result by a matrix *E* such that $\|E\|_2 = O(\varepsilon)$, where ε is the machine precision.

The approximate number of real floating-point operations is $(16/3)(ihi-ilo)^3$.

The real counterpart of this routine is [orghr](#).

?unmhr

Multiplies an arbitrary complex matrix C by the complex unitary matrix Q determined by ?gehrd.

Syntax

```
lapack_int LAPACKE_cunmhr (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int ilo, lapack_int ihi, const lapack_complex_float* a, lapack_int
lda, const lapack_complex_float* tau, lapack_complex_float* c, lapack_int ldc);
```

```
lapack_int LAPACKE_zunmhr (int matrix_layout, char side, char trans, lapack_int m,
lapack_int n, lapack_int ilo, lapack_int ihi, const lapack_complex_double* a,
lapack_int lda, const lapack_complex_double* tau, lapack_complex_double* c, lapack_int
ldc);
```

Include Files

- mkl.h

Description

The routine multiplies a matrix *C* by the unitary matrix *Q* that has been determined by a preceding call to *cgehrd/zgehrd*. (The routine *?gehrd* reduces a real general matrix *A* to upper Hessenberg form *H* by an orthogonal similarity transformation, $A = Q^H H Q$, and represents the matrix *Q* as a product of *ihi-ilo* elementary reflectors. Here *ilo* and *ihi* are values determined by *cgebal/zgebal* when balancing the matrix; if the matrix has not been balanced, *ilo* = 1 and *ihi* = *n*.)

With *?unmhr*, you can form one of the matrix products $Q^H C$, $Q C$, $C Q$, or $C Q^H$, overwriting the result on *C* (which may be any complex rectangular matrix). A common application of this routine is to transform a matrix *V* of eigenvectors of *H* to the matrix *QV* of eigenvectors of *A*.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

| | |
|------------------|--|
| <i>side</i> | <p>Must be 'L' or 'R'.</p> <p>If <i>side</i> = 'L', then the routine forms $Q^H * C$ or $Q^H * C$.</p> <p>If <i>side</i> = 'R', then the routine forms $C * Q$ or $C * Q^H$.</p> |
| <i>trans</i> | <p>Must be 'N' or 'C'.</p> <p>If <i>trans</i> = 'N', then Q is applied to C.</p> <p>If <i>trans</i> = 'T', then Q^H is applied to C.</p> |
| <i>m</i> | The number of rows in C ($m \geq 0$). |
| <i>n</i> | The number of columns in C ($n \geq 0$). |
| <i>ilo, ihi</i> | <p>These must be the same parameters <i>ilo</i> and <i>ihi</i>, respectively, as supplied to <code>?gehrd</code>.</p> <p>If $m > 0$ and <i>side</i> = 'L', then $1 \leq ilo \leq ihi \leq m$.</p> <p>If $m = 0$ and <i>side</i> = 'L', then $ilo = 1$ and $ihi = 0$.</p> <p>If $n > 0$ and <i>side</i> = 'R', then $1 \leq ilo \leq ihi \leq n$.</p> <p>If $n = 0$ and <i>side</i> = 'R', then $ilo = 1$ and $ihi = 0$.</p> |
| <i>a, tau, c</i> | <p>Arrays:</p> <p><i>a</i>(size $\max(1, lda * n)$ for <i>side</i>='R' and size $\max(1, lda * m)$ for <i>side</i>='L') contains details of the vectors which define the elementary reflectors, as returned by <code>?gehrd</code>.</p> <p><i>tau</i> contains further details of the elementary reflectors, as returned by <code>?gehrd</code>.</p> <p>The dimension of <i>tau</i> must be at least $\max(1, m-1)$ if <i>side</i> = 'L' and at least $\max(1, n-1)$ if <i>side</i> = 'R'.</p> <p><i>c</i>(size $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout) contains the m-by-n matrix C.</p> |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ if <i>side</i> = 'L' and at least $\max(1, n)$ if <i>side</i> = 'R'. |
| <i>ldc</i> | The leading dimension of <i>c</i> ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | C is overwritten by $Q^H * C$, or $Q^H * C$, or $C * Q^H$, or $C * Q$ as specified by <i>side</i> and <i>trans</i> . |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

Application Notes

The computed matrix Q differs from the exact result by a matrix E such that $\|E\|_2 = O(\varepsilon) * \|C\|_2$, where ε is the machine precision.

The approximate number of floating-point operations is

$8n(ihi-ilo)^2$ if $side = 'L'$;

$8m(ihi-ilo)^2$ if $side = 'R'$.

The real counterpart of this routine is [ormhr](#).

?gebal

Balances a general matrix to improve the accuracy of computed eigenvalues and eigenvectors.

Syntax

```
lapack_int LAPACKE_sgebal( int matrix_layout, char job, lapack_int n, float* a,
lapack_int lda, lapack_int* ilo, lapack_int* ihi, float* scale );
```

```
lapack_int LAPACKE_dgebal( int matrix_layout, char job, lapack_int n, double* a,
lapack_int lda, lapack_int* ilo, lapack_int* ihi, double* scale );
```

```
lapack_int LAPACKE_cgebal( int matrix_layout, char job, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_int* ilo, lapack_int* ihi, float*
scale );
```

```
lapack_int LAPACKE_zgebal( int matrix_layout, char job, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_int* ilo, lapack_int* ihi, double*
scale );
```

Include Files

- mkl.h

Description

The routine *balances* a matrix A by performing either or both of the following two similarity transformations:

(1) The routine first attempts to permute A to block upper triangular form:

$$PAP^T = A' = \begin{bmatrix} A'_{11} & A'_{12} & A'_{13} \\ 0 & A'_{22} & A'_{23} \\ 0 & 0 & A'_{33} \end{bmatrix}$$

where P is a permutation matrix, and A'_{11} and A'_{33} are upper triangular. The diagonal elements of A'_{11} and A'_{33} are eigenvalues of A . The rest of the eigenvalues of A are the eigenvalues of the central diagonal block A'_{22} , in rows and columns ilo to ihi . Subsequent operations to compute the eigenvalues of A (or its Schur factorization) need only be applied to these rows and columns; this can save a significant amount of work if $ilo > 1$ and $ihi < n$.

If no suitable permutation exists (as is often the case), the routine sets $ilo = 1$ and $ihi = n$, and A'_{22} is the whole of A .

(2) The routine applies a diagonal similarity transformation to A' , to make the rows and columns of A'_{22} as close in norm as possible:

$$A'' = DA'D^{-1} = \begin{bmatrix} I & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & I \end{bmatrix} \times \begin{bmatrix} A'_{11} & A'_{12} & A'_{13} \\ 0 & A'_{22} & A'_{23} \\ 0 & 0 & A'_{33} \end{bmatrix} \times \begin{bmatrix} I & 0 & 0 \\ 0 & D_{22}^{-1} & 0 \\ 0 & 0 & I \end{bmatrix}$$

This scaling can reduce the norm of the matrix (that is, $\|A''_{22}\| < \|A'_{22}\|$), and hence reduce the effect of rounding errors on the accuracy of computed eigenvalues and eigenvectors.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>job</i> | Must be 'N' or 'P' or 'S' or 'B'. If <i>job</i> = 'N', then <i>A</i> is neither permuted nor scaled (but <i>ilo</i> , <i>ihi</i> , and <i>scale</i> get their values). If <i>job</i> = 'P', then <i>A</i> is permuted but not scaled. If <i>job</i> = 'S', then <i>A</i> is scaled but not permuted. If <i>job</i> = 'B', then <i>A</i> is both scaled and permuted. |
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>a</i> | Array <i>a</i> (size $\max(1, lda*n)$) contains the matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|-------------------------|--|
| <i>a</i> | Overwritten by the balanced matrix (<i>a</i> is not referenced if <i>job</i> = 'N'). |
| <i>ilo</i> , <i>ihi</i> | The values <i>ilo</i> and <i>ihi</i> such that on exit <i>a</i> (<i>i</i> , <i>j</i>) is zero if $i > j$ and $1 \leq j < ilo$ or $ihi < j \leq n$. If <i>job</i> = 'N' or 'S', then <i>ilo</i> = 1 and <i>ihi</i> = <i>n</i> . |
| <i>scale</i> | Array, size at least $\max(1, n)$. Contains details of the permutations and scaling factors. More precisely, if p_j is the index of the row and column interchanged with row and column <i>j</i> , and d_j is the scaling factor used to balance row and column <i>j</i> , then $scale[j - 1] = p_j$ for $j = 1, 2, \dots, ilo-1, ihi+1, \dots, n$; $scale[j - 1] = d_j$ for $j = ilo, ilo + 1, \dots, ihi$. The order in which the interchanges are made is <i>n</i> to <i>ihi</i> +1, then 1 to <i>ilo</i> -1. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The errors are negligible, compared with those in subsequent computations.

If the matrix A is balanced by this routine, then any eigenvectors computed subsequently are eigenvectors of the matrix A'' and hence you must call [gebak](#) to transform them back to eigenvectors of A .

If the Schur vectors of A are required, do not call this routine with $job = 'S'$ or $'B'$, because then the balancing transformation is not orthogonal (not unitary for complex flavors).

If you call this routine with $job = 'P'$, then any Schur vectors computed subsequently are Schur vectors of the matrix A'' , and you need to call [gebak](#) (with $side = 'R'$) to transform them back to Schur vectors of A .

The total number of floating-point operations is proportional to n^2 .

?gebak

Transforms eigenvectors of a balanced matrix to those of the original nonsymmetric matrix.

Syntax

```
lapack_int LAPACKE_sgebak( int matrix_layout, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const float* scale, lapack_int m, float* v, lapack_int
ldv );

lapack_int LAPACKE_dgebak( int matrix_layout, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const double* scale, lapack_int m, double* v,
lapack_int ldv );

lapack_int LAPACKE_cgebak( int matrix_layout, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const float* scale, lapack_int m, lapack_complex_float*
v, lapack_int ldv );

lapack_int LAPACKE_zgebak( int matrix_layout, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const double* scale, lapack_int m,
lapack_complex_double* v, lapack_int ldv );
```

Include Files

- mkl.h

Description

The routine is intended to be used after a matrix A has been balanced by a call to [?gebal](#), and eigenvectors of the balanced matrix A''_{22} have subsequently been computed. For a description of balancing, see [gebal](#). The balanced matrix A'' is obtained as $A'' = D*P*A*P^T*inv(D)$, where P is a permutation matrix and D is a diagonal scaling matrix. This routine transforms the eigenvectors as follows:

if x is a right eigenvector of A'' , then $P^T*inv(D)*x$ is a right eigenvector of A ; if y is a left eigenvector of A'' , then P^T*D*y is a left eigenvector of A .

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>job</i> | Must be 'N' or 'P' or 'S' or 'B'. The same parameter <i>job</i> as supplied to ?gebal . |

| | |
|-----------------|--|
| <i>side</i> | Must be 'L' or 'R'. If <i>side</i> = 'L', then left eigenvectors are transformed. If <i>side</i> = 'R', then right eigenvectors are transformed. |
| <i>n</i> | The number of rows of the matrix of eigenvectors ($n \geq 0$). |
| <i>ilo, ihi</i> | The values <i>ilo</i> and <i>ihi</i> , as returned by ?gebal. (If $n > 0$, then $1 \leq ilo \leq ihi \leq n$; if $n = 0$, then $ilo = 1$ and $ihi = 0$.) |
| <i>scale</i> | Array, size at least $\max(1, n)$. Contains details of the permutations and/or the scaling factors used to balance the original general matrix, as returned by ?gebal. |
| <i>m</i> | The number of columns of the matrix of eigenvectors ($m \geq 0$). |
| <i>v</i> | Arrays: <i>v</i> (size $\max(1, ldv * n)$ for column major layout and $\max(1, ldv * m)$ for row major layout) contains the matrix of left or right eigenvectors to be transformed. |
| <i>ldv</i> | The leading dimension of <i>v</i> ; at least $\max(1, n)$ for column major layout and at least $\max(1, m)$ for row major layout . |

Output Parameters

| | |
|----------|--|
| <i>v</i> | Overwritten by the transformed eigenvectors. |
|----------|--|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The errors in this routine are negligible.

The approximate number of floating-point operations is approximately proportional to $m * n$.

?hseqr

Computes all eigenvalues and (optionally) the Schur factorization of a matrix reduced to Hessenberg form.

Syntax

```
lapack_int LAPACKE_shseqr( int matrix_layout, char job, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, float* h, lapack_int ldh, float* wr, float* wi, float*
z, lapack_int ldz );
```

```
lapack_int LAPACKE_dhseqr( int matrix_layout, char job, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, double* h, lapack_int ldh, double* wr, double* wi,
double* z, lapack_int ldz );
```


| | |
|-----------------|--|
| <i>compz</i> | <p>Must be 'N' or 'I' or 'V'.</p> <p>If <i>compz</i> = 'N', then no Schur vectors are computed (and the array <i>z</i> is not referenced).</p> <p>If <i>compz</i> = 'I', then the Schur vectors of <i>H</i> are computed (and the array <i>z</i> is initialized by the routine).</p> <p>If <i>compz</i> = 'V', then the Schur vectors of <i>A</i> are computed (and the array <i>z</i> must contain the matrix <i>Q</i> on entry).</p> |
| <i>n</i> | The order of the matrix <i>H</i> ($n \geq 0$). |
| <i>ilo, ihi</i> | If <i>A</i> has been balanced by <code>?gebal</code> , then <i>ilo</i> and <i>ihi</i> must contain the values returned by <code>?gebal</code> . Otherwise, <i>ilo</i> must be set to 1 and <i>ihi</i> to <i>n</i> . |
| <i>h, z</i> | <p>Arrays:</p> <p><i>h</i> (size $\max(1, ldh*n)$) The <i>n</i>-by-<i>n</i> upper Hessenberg matrix <i>H</i>.</p> <p><i>z</i> (size $\max(1, ldz*n)$)</p> <p>If <i>compz</i> = 'V', then <i>z</i> must contain the matrix <i>Q</i> from the reduction to Hessenberg form.</p> <p>If <i>compz</i> = 'I', then <i>z</i> need not be set.</p> <p>If <i>compz</i> = 'N', then <i>z</i> is not referenced.</p> |
| <i>ldh</i> | The leading dimension of <i>h</i> ; at least $\max(1, n)$. |
| <i>ldz</i> | <p>The leading dimension of <i>z</i>;</p> <p>If <i>compz</i> = 'N', then $ldz \geq 1$.</p> <p>If <i>compz</i> = 'V' or 'I', then $ldz \geq \max(1, n)$.</p> |

Output Parameters

| | |
|---------------|---|
| <i>w</i> | Array, size at least $\max(1, n)$. Contains the computed eigenvalues, unless <i>info</i> > 0. The eigenvalues are stored in the same order as on the diagonal of the Schur form <i>T</i> (if computed). |
| <i>wr, wi</i> | <p>Arrays, size at least $\max(1, n)$ each.</p> <p>Contain the real and imaginary parts, respectively, of the computed eigenvalues, unless <i>info</i> > 0. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first. The eigenvalues are stored in the same order as on the diagonal of the Schur form <i>T</i> (if computed).</p> |
| <i>h</i> | <p>If <i>info</i> = 0 and <i>job</i> = 'S', <i>h</i> contains the upper quasi-triangular matrix <i>T</i> from the Schur decomposition (the Schur form).</p> <p>If <i>info</i> = 0 and <i>job</i> = 'E', the contents of <i>h</i> are unspecified on exit. (The output value of <i>h</i> when <i>info</i> > 0 is given under the description of <i>info</i> below.)</p> |
| <i>z</i> | <p>If <i>compz</i> = 'V' and <i>info</i> = 0, then <i>z</i> contains <i>Q</i>*<i>Z</i>.</p> <p>If <i>compz</i> = 'I' and <i>info</i> = 0, then <i>z</i> contains the unitary or orthogonal matrix <i>Z</i> of the Schur vectors of <i>H</i>.</p> |

If `compz = 'N'`, then `z` is not referenced.

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

If `info = i`, `?hseqr` failed to compute all of the eigenvalues. Elements $1, 2, \dots, ilo-1$ and $i+1, i+2, \dots, n$ of the eigenvalue arrays (`wr` and `wi` for real flavors and `w` for complex flavors) contain the real and imaginary parts of those eigenvalues that have been successfully found.

If `info > 0`, and `job = 'E'`, then on exit, the remaining unconverged eigenvalues are the eigenvalues of the upper Hessenberg matrix rows and columns `ilo` through `info` of the final output value of `H`.

If `info > 0`, and `job = 'S'`, then on exit (initial value of `H`)*`U` = `U`*(final value of `H`), where `U` is a unitary matrix. The final value of `H` is upper Hessenberg and triangular in rows and columns `info+1` through `ihi`.

If `info > 0`, and `compz = 'V'`, then on exit (final value of `Z`) = (initial value of `Z`)*`U`, where `U` is the unitary matrix (regardless of the value of `job`).

If `info > 0`, and `compz = 'I'`, then on exit (final value of `Z`) = `U`, where `U` is the unitary matrix (regardless of the value of `job`).

If `info > 0`, and `compz = 'N'`, then `Z` is not accessed.

Application Notes

The computed Schur factorization is the exact factorization of a nearby matrix $H + E$, where $\|E\|_2 < O(\epsilon) \|H\|_2/s_i$, and ϵ is the machine precision.

If λ_j is an exact eigenvalue, and μ_j is the corresponding computed value, then $|\lambda_j - \mu_j| \leq c(n) * \epsilon * \|H\|_2/s_j$, where $c(n)$ is a modestly increasing function of n , and s_j is the reciprocal condition number of λ_j . The condition numbers s_j may be computed by calling `trsna`.

The total number of floating-point operations depends on how rapidly the algorithm converges; typical numbers are as follows.

| | |
|--|--|
| If only eigenvalues are computed: | $7n^3$ for real flavors $25n^3$ for complex flavors. |
| If the Schur form is computed: | $10n^3$ for real flavors $35n^3$ for complex flavors. |
| If the full Schur factorization is computed: | $20n^3$ for real flavors $70n^3$ for complex flavors. |

?hsein

Computes selected eigenvectors of an upper Hessenberg matrix that correspond to specified eigenvalues.

Syntax

```
lapack_int LAPACKE_shsein( int matrix_layout, char side, char eigsrc, char initv,
lapack_logical* select, lapack_int n, const float* h, lapack_int ldh, float* wr, const
float* wi, float* vl, lapack_int ldvl, float* vr, lapack_int ldvr, lapack_int mm,
lapack_int* m, lapack_int* ifaill, lapack_int* ifailr );
```

```
lapack_int LAPACKE_dhsein( int matrix_layout, char side, char eigsrc, char initv,
lapack_logical* select, lapack_int n, const double* h, lapack_int ldh, double* wr,
const double* wi, double* vl, lapack_int ldvl, double* vr, lapack_int ldvr, lapack_int
mm, lapack_int* m, lapack_int* ifaill, lapack_int* ifailr );
```

```
lapack_int LAPACKE_chsein( int matrix_layout, char side, char eigsrc, char initv, const
lapack_logical* select, lapack_int n, const lapack_complex_float* h, lapack_int ldh,
lapack_complex_float* w, lapack_complex_float* vl, lapack_int ldvl,
lapack_complex_float* vr, lapack_int ldvr, lapack_int mm, lapack_int* m, lapack_int*
ifaill, lapack_int* ifailr );
```

```
lapack_int LAPACKE_zhsein( int matrix_layout, char side, char eigsrc, char initv, const
lapack_logical* select, lapack_int n, const lapack_complex_double* h, lapack_int ldh,
lapack_complex_double* w, lapack_complex_double* vl, lapack_int ldvl,
lapack_complex_double* vr, lapack_int ldvr, lapack_int mm, lapack_int* m, lapack_int*
ifaill, lapack_int* ifailr );
```

Include Files

- mkl.h

Description

The routine computes left and/or right eigenvectors of an upper Hessenberg matrix H , corresponding to selected eigenvalues.

The right eigenvector x and the left eigenvector y , corresponding to an eigenvalue λ , are defined by: $H^*x = \lambda^*x$ and $y^{H*}H = \lambda^*y^H$ (or $H^{H*}y = \lambda^*y^*$). Here λ^* denotes the conjugate of λ .

The eigenvectors are computed by inverse iteration. They are scaled so that, for a real eigenvector x , $\max |x_i| = 1$, and for a complex eigenvector, $\max(|\text{Re}x_i| + |\text{Im}x_i|) = 1$.

If H has been formed by reduction of a general matrix A to upper Hessenberg form, then eigenvectors of H may be transformed to eigenvectors of A by [ormhr](#) or [unmhr](#).

Input Parameters

| | |
|---------------|--|
| <i>side</i> | Must be 'R' or 'L' or 'B'. If <i>side</i> = 'R', then only right eigenvectors are computed. If <i>side</i> = 'L', then only left eigenvectors are computed. If <i>side</i> = 'B', then all eigenvectors are computed. |
| <i>eigsrc</i> | Must be 'Q' or 'N'. If <i>eigsrc</i> = 'Q', then the eigenvalues of H were found using hseqr ; thus if H has any zero sub-diagonal elements (and so is block triangular), then the j -th eigenvalue can be assumed to be an eigenvalue of the block containing the j -th row/column. This property allows the routine to perform inverse iteration on just one diagonal block. If <i>eigsrc</i> = 'N', then no such assumption is made and the routine performs inverse iteration using the whole matrix. |
| <i>initv</i> | Must be 'N' or 'U'. If <i>initv</i> = 'N', then no initial estimates for the selected eigenvectors are supplied. |

| | |
|------------------|---|
| | If <i>initv</i> = 'U', then initial estimates for the selected eigenvectors are supplied in <i>vl</i> and/or <i>vr</i> . |
| <i>select</i> | <p>Array, size at least $\max(1, n)$. Specifies which eigenvectors are to be computed.</p> <p><i>For real flavors:</i></p> <p>To obtain the real eigenvector corresponding to the real eigenvalue <i>wr</i>[<i>j</i>], set <i>select</i>[<i>j</i>] to 1</p> <p>To select the complex eigenvector corresponding to the complex eigenvalue (<i>wr</i>[<i>j</i> - 1], <i>wi</i>[<i>j</i> - 1]) with complex conjugate (<i>wr</i>[<i>j</i>], <i>wi</i>[<i>j</i>]), set <i>select</i>[<i>j</i> - 1] and/or <i>select</i>[<i>j</i>] to 1; the eigenvector corresponding to the first eigenvalue in the pair is computed.</p> <p><i>For complex flavors:</i></p> <p>To select the eigenvector corresponding to the eigenvalue <i>w</i>[<i>j</i>], set <i>select</i>[<i>j</i>] to 1</p> |
| <i>n</i> | The order of the matrix <i>H</i> ($n \geq 0$). |
| <i>h, vl, vr</i> | <p>Arrays:</p> <p><i>h</i> (size $\max(1, ldh*n)$) The <i>n</i>-by-<i>n</i> upper Hessenberg matrix <i>H</i>. If an NAN value is detected in <i>h</i>, the routine returns with <i>info</i> = -6.</p> <p><i>vl</i>(size $\max(1, ldvl*nm)$ for column major layout and $\max(1, ldvl*n)$ for row major layout)</p> <p>If <i>initv</i> = 'V' and <i>side</i> = 'L' or 'B', then <i>vl</i> must contain starting vectors for inverse iteration for the left eigenvectors. Each starting vector must be stored in the same column or columns as will be used to store the corresponding eigenvector.</p> <p>If <i>initv</i> = 'N', then <i>vl</i> need not be set.</p> <p>The array <i>vl</i> is not referenced if <i>side</i> = 'R'.</p> <p><i>vr</i>(size $\max(1, ldvr*nm)$ for column major layout and $\max(1, ldvr*n)$ for row major layout)</p> <p>If <i>initv</i> = 'V' and <i>side</i> = 'R' or 'B', then <i>vr</i> must contain starting vectors for inverse iteration for the right eigenvectors. Each starting vector must be stored in the same column or columns as will be used to store the corresponding eigenvector.</p> <p>If <i>initv</i> = 'N', then <i>vr</i> need not be set.</p> <p>The array <i>vr</i> is not referenced if <i>side</i> = 'L'.</p> |
| <i>ldh</i> | The leading dimension of <i>h</i> ; at least $\max(1, n)$. |
| <i>w</i> | <p>Array, size at least $\max(1, n)$.</p> <p>Contains the eigenvalues of the matrix <i>H</i>.</p> <p>If <i>eigsrc</i> = 'Q', the array must be exactly as returned by ?hseqr.</p> |
| <i>wr, wi</i> | Arrays, size at least $\max(1, n)$ each. |

Contain the real and imaginary parts, respectively, of the eigenvalues of the matrix H . Complex conjugate pairs of values must be stored in consecutive elements of the arrays. If $eigsrc = 'Q'$, the arrays must be exactly as returned by `?hseqr`.

ldvl

The leading dimension of vl .

If $side = 'L'$ or $'B'$, $ldvl \geq \max(1, n)$ for column major layout and $ldvl \geq \max(1, mm)$ for row major layout .

If $side = 'R'$, $ldvl \geq 1$.

ldvr

The leading dimension of vr .

If $side = 'R'$ or $'B'$, $ldvr \geq \max(1, n)$ for column major layout and $ldvr \geq \max(1, mm)$ for row major layout .

If $side = 'L'$, $ldvr \geq 1$.

mm

The number of columns in vl and/or vr .

Must be at least m , the actual number of columns required (see *Output Parameters* below).

For real flavors, m is obtained by counting 1 for each selected real eigenvector and 2 for each selected complex eigenvector (see *select*).

For complex flavors, m is the number of selected eigenvectors (see *select*).

Constraint:

$$0 \leq mm \leq n.$$

Output Parameters

select

Overwritten for real flavors only.

If a complex eigenvector was selected as specified above, then $select[j - 1]$ is set to 1 and $select[j]$ to 0

w

The real parts of some elements of w may be modified, as close eigenvalues are perturbed slightly in searching for independent eigenvectors.

wr

Some elements of wr may be modified, as close eigenvalues are perturbed slightly in searching for independent eigenvectors.

vl, vr

If $side = 'L'$ or $'B'$, vl contains the computed left eigenvectors (as specified by *select*).

If $side = 'R'$ or $'B'$, vr contains the computed right eigenvectors (as specified by *select*).

The eigenvectors treated column-wise form a rectangular n -by- mm matrix.

For real flavors: a real eigenvector corresponding to a real eigenvalue occupies one column of the matrix; a complex eigenvector corresponding to a complex eigenvalue occupies two columns: the first column holds the real part of the eigenvector and the second column holds the imaginary part of the eigenvector. The matrix is stored in a one-dimensional array as described by *matrix_layout* (using either column major or row major layout).

m For real flavors: the number of columns of *vl* and/or *vr* required to store the selected eigenvectors.
 For complex flavors: the number of selected eigenvectors.

ifaill, ifailr Arrays, size at least $\max(1, mm)$ each.
 $ifaill[i - 1] = 0$ if the *i*th column of *vl* converged;
 $ifaill[i - 1] = j > 0$ if the eigenvector stored in the *i*-th column of *vl* (corresponding to the *j*th eigenvalue) failed to converge.
 $ifailr[i - 1] = 0$ if the *i*th column of *vr* converged;
 $ifailr[i - 1] = j > 0$ if the eigenvector stored in the *i*-th column of *vr* (corresponding to the *j*th eigenvalue) failed to converge.
 For real flavors: if the *i*th and (*i*+1)th columns of *vl* contain a selected complex eigenvector, then $ifaill[i - 1]$ and $ifaill[i]$ are set to the same value. A similar rule holds for *vr* and *ifailr*.
 The array *ifaill* is not referenced if *side* = 'R'. The array *ifailr* is not referenced if *side* = 'L'.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, then *i* eigenvectors (as indicated by the parameters *ifaill* and/or *ifailr* above) failed to converge. The corresponding columns of *vl* and/or *vr* contain no useful information.

Application Notes

Each computed right eigenvector x_i is the exact eigenvector of a nearby matrix $A + E_i$, such that $\|E_i\| < O(\epsilon) \|A\|$. Hence the residual is small:

$$\|Ax_i - \lambda_i x_i\| = O(\epsilon) \|A\|.$$

However, eigenvectors corresponding to close or coincident eigenvalues may not accurately span the relevant subspaces.

Similar remarks apply to computed left eigenvectors.

?trevc

Computes selected eigenvectors of an upper (quasi-) triangular matrix computed by ?hseqr.

Syntax

```
lapack_int LAPACKE_strevc( int matrix_layout, char side, char howmny, lapack_logical*
select, lapack_int n, const float* t, lapack_int ldt, float* vl, lapack_int ldvl,
float* vr, lapack_int ldvr, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_dtrevc( int matrix_layout, char side, char howmny, lapack_logical*
select, lapack_int n, const double* t, lapack_int ldt, double* vl, lapack_int ldvl,
double* vr, lapack_int ldvr, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ctrevc( int matrix_layout, char side, char howmny, const
lapack_logical* select, lapack_int n, lapack_complex_float* t, lapack_int ldt,
lapack_complex_float* vl, lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr,
lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ztrevc( int matrix_layout, char side, char howmny, const
lapack_logical* select, lapack_int n, lapack_complex_double* t, lapack_int ldt,
lapack_complex_double* vl, lapack_int ldvl, lapack_complex_double* vr, lapack_int ldvr,
lapack_int mm, lapack_int* m );
```

Include Files

- mkl.h

Description

The routine computes some or all of the right and/or left eigenvectors of an upper triangular matrix T (or, for real flavors, an upper quasi-triangular matrix T). Matrices of this type are produced by the Schur factorization of a general matrix: $A = Q^* T^* Q^H$, as computed by [hseqr](#).

The right eigenvector x and the left eigenvector y of T corresponding to an eigenvalue w , are defined by:

$$T^* x = w^* x, y^H T = w^* y^H, \text{ where } y^H \text{ denotes the conjugate transpose of } y.$$

The eigenvalues are not input to this routine, but are read directly from the diagonal blocks of T .

This routine returns the matrices X and/or Y of right and left eigenvectors of T , or the products $Q^* X$ and/or $Q^* Y$, where Q is an input matrix.

If Q is the orthogonal/unitary factor that reduces a matrix A to Schur form T , then $Q^* X$ and $Q^* Y$ are the matrices of right and left eigenvectors of A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | Must be 'R' or 'L' or 'B'. If <i>side</i> = 'R', then only right eigenvectors are computed. If <i>side</i> = 'L', then only left eigenvectors are computed. If <i>side</i> = 'B', then all eigenvectors are computed. |
| <i>howmny</i> | Must be 'A' or 'B' or 'S'. If <i>howmny</i> = 'A', then all eigenvectors (as specified by <i>side</i>) are computed. If <i>howmny</i> = 'B', then all eigenvectors (as specified by <i>side</i>) are computed and backtransformed by the matrices supplied in <i>vl</i> and <i>vr</i> . If <i>howmny</i> = 'S', then selected eigenvectors (as specified by <i>side</i> and <i>select</i>) are computed. |
| <i>select</i> | Array, size at least $\max(1, n)$. If <i>howmny</i> = 'S', <i>select</i> specifies which eigenvectors are to be computed. If <i>howmny</i> = 'A' or 'B', <i>select</i> is not referenced. <i>For real flavors:</i> |

If $\omega[j]$ is a real eigenvalue, the corresponding real eigenvector is computed if $\text{select}[j]$ is 1.

If $\omega[j - 1]$ and $\omega[j]$ are the real and imaginary parts of a complex eigenvalue, the corresponding complex eigenvector is computed if either $\text{select}[j - 1]$ or $\text{select}[j]$ is 1, and on exit $\text{select}[j - 1]$ is set to 1 and $\text{select}[j]$ is set to 0.

For complex flavors:

The eigenvector corresponding to the j -th eigenvalue is computed if $\text{select}[j - 1]$ is 1.

n The order of the matrix T ($n \geq 0$).

t, vl, vr Arrays:

t (size $\max(1, ldt * n)$) contains the n -by- n matrix T in Schur canonical form. For complex flavors `ctrevc` and `ztrevc`, contains the upper triangular matrix T .

vl (size $\max(1, ldvl * mm)$ for column major layout and $\max(1, ldvl * n)$ for row major layout)

If $\text{howmny} = 'B'$ and $\text{side} = 'L'$ or $'B'$, then vl must contain an n -by- n matrix Q (usually the matrix of Schur vectors returned by `?hseqr`).

If $\text{howmny} = 'A'$ or $'S'$, then vl need not be set.

The array vl is not referenced if $\text{side} = 'R'$.

vr (size $\max(1, ldvr * mm)$ for column major layout and $\max(1, ldvr * n)$ for row major layout)

If $\text{howmny} = 'B'$ and $\text{side} = 'R'$ or $'B'$, then vr must contain an n -by- n matrix Q (usually the matrix of Schur vectors returned by `?hseqr`).

If $\text{howmny} = 'A'$ or $'S'$, then vr need not be set.

The array vr is not referenced if $\text{side} = 'L'$.

ldt The leading dimension of t ; at least $\max(1, n)$.

$ldvl$ The leading dimension of vl .

If $\text{side} = 'L'$ or $'B'$, $ldvl \geq n$.

If $\text{side} = 'R'$, $ldvl \geq 1$.

$ldvr$ The leading dimension of vr .

If $\text{side} = 'R'$ or $'B'$, $ldvr \geq n$.

If $\text{side} = 'L'$, $ldvr \geq 1$.

mm The number of columns in the arrays vl and/or vr . Must be at least m (the precise number of columns required).

If $\text{howmny} = 'A'$ or $'B'$, $mm = n$.

If $\text{howmny} = 'S'$: for real flavors, mm is obtained by counting 1 for each selected real eigenvector and 2 for each selected complex eigenvector;

for complex flavors, mm is the number of selected eigenvectors (see `select`).

Constraint: $0 \leq mm \leq n$.

Output Parameters

| | |
|---------------|---|
| <i>select</i> | If a complex eigenvector of a real matrix was selected as specified above, then <i>select</i> [<i>j</i>] is set to 1 and <i>select</i> [<i>j</i> + 1] to 0 |
| <i>t</i> | <i>ctrevc</i> / <i>ztrevc</i> modify the <i>t</i> array, which is restored on exit. |
| <i>vl, vr</i> | <p>If <i>side</i> = 'L' or 'B', <i>vl</i> contains the computed left eigenvectors (as specified by <i>howmny</i> and <i>select</i>).</p> <p>If <i>side</i> = 'R' or 'B', <i>vr</i> contains the computed right eigenvectors (as specified by <i>howmny</i> and <i>select</i>).</p> <p>The eigenvectors treated column-wise form a rectangular <i>n</i>-by-<i>mm</i> matrix.</p> <p><i>For real flavors:</i> a real eigenvector corresponding to a real eigenvalue occupies one column of the matrix; a complex eigenvector corresponding to a complex eigenvalue occupies two columns: the first column holds the real part of the eigenvector and the second column holds the imaginary part of the eigenvector. The matrix is stored in a one-dimensional array as described by <i>matrix_layout</i> (using either column major or row major layout).</p> |
| <i>m</i> | <p><i>For complex flavors:</i> the number of selected eigenvectors.</p> <p>If <i>howmny</i> = 'A' or 'B', <i>m</i> is set to <i>n</i>.</p> <p><i>For real flavors:</i> the number of columns of <i>vl</i> and/or <i>vr</i> actually used to store the selected eigenvectors.</p> <p>If <i>howmny</i> = 'A' or 'B', <i>m</i> is set to <i>n</i>.</p> |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

If x_i is an exact right eigenvector and y_i is the corresponding computed eigenvector, then the angle $\theta(y_i, x_i)$ between them is bounded as follows: $\theta(y_i, x_i) \leq (c(n)\epsilon ||T||_2) / \text{sep}_i$ where sep_i is the reciprocal condition number of x_i . The condition number sep_i may be computed by calling `?trsna`.

?trsna

Estimates condition numbers for specified eigenvalues and right eigenvectors of an upper (quasi-) triangular matrix.

Syntax

```
lapack_int LAPACKE_strsna( int matrix_layout, char job, char howmny, const
lapack_logical* select, lapack_int n, const float* t, lapack_int ldt, const float* vl,
lapack_int ldvl, const float* vr, lapack_int ldvr, float* s, float* sep, lapack_int
mm, lapack_int* m );
```

```
lapack_int LAPACKE_dtrsna( int matrix_layout, char job, char howmny, const
lapack_logical* select, lapack_int n, const double* t, lapack_int ldt, const double*
vl, lapack_int ldvl, const double* vr, lapack_int ldvr, double* s, double* sep,
lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ctrnsa( int matrix_layout, char job, char howmny, const
lapack_logical* select, lapack_int n, const lapack_complex_float* t, lapack_int ldt,
const lapack_complex_float* vl, lapack_int ldvl, const lapack_complex_float* vr,
lapack_int ldvr, float* s, float* sep, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ztrsna( int matrix_layout, char job, char howmny, const
lapack_logical* select, lapack_int n, const lapack_complex_double* t, lapack_int ldt,
const lapack_complex_double* vl, lapack_int ldvl, const lapack_complex_double* vr,
lapack_int ldvr, double* s, double* sep, lapack_int mm, lapack_int* m );
```

Include Files

- mkl.h

Description

The routine estimates condition numbers for specified eigenvalues and/or right eigenvectors of an upper triangular matrix T (or, for real flavors, upper quasi-triangular matrix T in canonical Schur form). These are the same as the condition numbers of the eigenvalues and right eigenvectors of an original matrix $A = Z^* T^* Z^H$ (with unitary or, for real flavors, orthogonal Z), from which T may have been derived.

The routine computes the reciprocal of the condition number of an eigenvalue λ_i as $s_i = |v^T * u| / (||u||_E ||v||_E)$ for real flavors and $s_i = |v^H * u| / (||u||_E ||v||_E)$ for complex flavors,

where:

- u and v are the right and left eigenvectors of T , respectively, corresponding to λ_i .
- v^T/v^H denote transpose/conjugate transpose of v , respectively.

This reciprocal condition number always lies between zero (ill-conditioned) and one (well-conditioned).

An approximate error estimate for a computed eigenvalue λ_i is then given by $\epsilon^* ||T|| / s_i$, where ϵ is the *machine precision*.

To estimate the reciprocal of the condition number of the right eigenvector corresponding to λ_i , the routine first calls [trexc](#) to reorder the diagonal elements of matrix T so that λ_i is in the leading position:

$$T = Q \begin{bmatrix} \lambda_i & C^H \\ \mathbf{0} & T_{22} \end{bmatrix} Q^H$$

The reciprocal condition number of the eigenvector is then estimated as sep_i , the smallest singular value of the matrix $T_{22} - \lambda_i I$.

An approximate error estimate for a computed right eigenvector u corresponding to λ_i is then given by $\epsilon^* ||T|| / sep_i$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>job</i> | Must be 'E' or 'V' or 'B'. |

If $job = 'E'$, then condition numbers for eigenvalues only are computed.
 If $job = 'V'$, then condition numbers for eigenvectors only are computed.
 If $job = 'B'$, then condition numbers for both eigenvalues and eigenvectors are computed.

howmny

Must be 'A' or 'S'.
 If $howmny = 'A'$, then the condition numbers for all eigenpairs are computed.
 If $howmny = 'S'$, then condition numbers for selected eigenpairs (as specified by *select*) are computed.

select

Array, size at least $\max(1, n)$ if $howmny = 'S'$ and at least 1 otherwise.
 Specifies the eigenpairs for which condition numbers are to be computed if $howmny = 'S'$.
For real flavors:
 To select condition numbers for the eigenpair corresponding to the real eigenvalue λ_j , $select[j]$ must be set 1;
 to select condition numbers for the eigenpair corresponding to a complex conjugate pair of eigenvalues λ_j and λ_{j+1} , $select[j - 1]$ and/or $select[j]$ must be set 1
For complex flavors
 To select condition numbers for the eigenpair corresponding to the eigenvalue λ_j , $select[j]$ must be set 1 *select* is not referenced if $howmny = 'A'$.

n

The order of the matrix T ($n \geq 0$).

t, vl, vr

Arrays:
t (size $\max(1, ldt * n)$) contains the n -by- n matrix T .
vl (size $\max(1, ldvl * mm)$ for column major layout and $\max(1, ldvl * n)$ for row major layout)
 If $job = 'E'$ or $'B'$, then *vl* must contain the left eigenvectors of T (or of any matrix $Q * T * Q^H$ with Q unitary or orthogonal) corresponding to the eigenpairs specified by *howmny* and *select*. The eigenvectors must be stored in consecutive columns of *vl*, as returned by [trevc](#) or [hsein](#).
 The array *vl* is not referenced if $job = 'V'$.
vr (size $\max(1, ldvr * mm)$ for column major layout and $\max(1, ldvr * n)$ for row major layout)
 If $job = 'E'$ or $'B'$, then *vr* must contain the right eigenvectors of T (or of any matrix $Q * T * Q^H$ with Q unitary or orthogonal) corresponding to the eigenpairs specified by *howmny* and *select*. The eigenvectors must be stored in consecutive columns of *vr*, as returned by [trevc](#) or [hsein](#).
 The array *vr* is not referenced if $job = 'V'$.

ldt

The leading dimension of *t*; at least $\max(1, n)$.

ldvl

The leading dimension of *vl*.

If $job = 'E'$ or $'B'$, $ldvl \geq \max(1, n)$ for column major layout and $ldvl \geq \max(1, mm)$ for row major layout .

If $job = 'V'$, $ldvl \geq 1$.

ldvr

The leading dimension of *vr*.

If $job = 'E'$ or $'B'$, $ldvr \geq \max(1, n)$ for column major layout and $ldvr \geq \max(1, mm)$ for row major layout .

If $job = 'R'$, $ldvr \geq 1$.

mm

The number of elements in the arrays *s* and *sep*, and the number of columns in *vl* and *vr* (if used). Must be at least *m* (the precise number required).

If *howmny* = 'A', $mm = n$;

if *howmny* = 'S', for *real flavors* *mm* is obtained by counting 1 for each selected real eigenvalue and 2 for each selected complex conjugate pair of eigenvalues.

for *complex flavors* *mm* is the number of selected eigenpairs (see *select*).

Constraint:

$0 \leq mm \leq n$.

Output Parameters

s

Array, size at least $\max(1, mm)$ if $job = 'E'$ or $'B'$ and at least 1 if $job = 'V'$.

Contains the reciprocal condition numbers of the selected eigenvalues if $job = 'E'$ or $'B'$, stored in consecutive elements of the array. Thus $s[j - 1]$, $sep[j - 1]$ and the *j*-th columns of *vl* and *vr* all correspond to the same eigenpair (but not in general the *j*th eigenpair unless all eigenpairs have been selected).

For *real flavors*: for a complex conjugate pair of eigenvalues, two consecutive elements of *s* are set to the same value. The array *s* is not referenced if $job = 'V'$.

sep

Array, size at least $\max(1, mm)$ if $job = 'V'$ or $'B'$ and at least 1 if $job = 'E'$. Contains the estimated reciprocal condition numbers of the selected right eigenvectors if $job = 'V'$ or $'B'$, stored in consecutive elements of the array.

For *real flavors*: for a complex eigenvector, two consecutive elements of *sep* are set to the same value; if the eigenvalues cannot be reordered to compute $sep[j - 1]$, then $sep[j - 1]$ is set to zero; this can only occur when the true value would be very small anyway. The array *sep* is not referenced if $job = 'E'$.

m

For *complex flavors*: the number of selected eigenpairs.

If *howmny* = 'A', *m* is set to *n*.

For *real flavors*: the number of elements of *s* and/or *sep* actually used to store the estimated condition numbers.

If $howmny = 'A'$, m is set to n .

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed values sep_i may overestimate the true value, but seldom by a factor of more than 3.

?trexc

Reorders the Schur factorization of a general matrix.

Syntax

```
lapack_int LAPACKE_strexc( int matrix_layout, char compq, lapack_int n, float* t,
lapack_int ldt, float* q, lapack_int ldq, lapack_int* ifst, lapack_int* ilst );
```

```
lapack_int LAPACKE_dtrexc( int matrix_layout, char compq, lapack_int n, double* t,
lapack_int ldt, double* q, lapack_int ldq, lapack_int* ifst, lapack_int* ilst );
```

```
lapack_int LAPACKE_ctrexc( int matrix_layout, char compq, lapack_int n,
lapack_complex_float* t, lapack_int ldt, lapack_complex_float* q, lapack_int ldq,
lapack_int ifst, lapack_int ilst );
```

```
lapack_int LAPACKE_ztrexc( int matrix_layout, char compq, lapack_int n,
lapack_complex_double* t, lapack_int ldt, lapack_complex_double* q, lapack_int ldq,
lapack_int ifst, lapack_int ilst );
```

Include Files

- mkl.h

Description

The routine reorders the Schur factorization of a general matrix $A = Q^*T^*Q^H$, so that the diagonal element or block of T with row index $ifst$ is moved to row $ilst$.

The reordered Schur form S is computed by an unitary (or, for real flavors, orthogonal) similarity transformation: $S = Z^H * T * Z$. Optionally the updated matrix P of Schur vectors is computed as $P = Q * Z$, giving $A = P * S * P^H$.

Input Parameters

| | |
|------------------|--|
| $matrix_layout$ | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| $compq$ | Must be 'V' or 'N'. If $compq = 'V'$, then the Schur vectors (Q) are updated. If $compq = 'N'$, then no Schur vectors are updated. |
| n | The order of the matrix T ($n \geq 0$). |
| t, q | Arrays: |

t (size $\max(1, ldt*n)$) contains the n -by- n matrix T .

q (size $\max(1, ldq*n)$)

If $compq = 'V'$, then q must contain Q (Schur vectors).

If $compq = 'N'$, then q is not referenced.

ldt The leading dimension of t ; at least $\max(1, n)$.

ldq The leading dimension of q ;

If $compq = 'N'$, then $ldq \geq 1$.

If $compq = 'V'$, then $ldq \geq \max(1, n)$.

$ifst, ilst$ $1 \leq ifst \leq n$; $1 \leq ilst \leq n$.

Must specify the reordering of the diagonal elements (or blocks, which is possible for real flavors) of the matrix T . The element (or block) with row index $ifst$ is moved to row $ilst$ by a sequence of exchanges between adjacent elements (or blocks).

Output Parameters

t Overwritten by the updated matrix S .

q If $compq = 'V'$, q contains the updated matrix of Schur vectors.

$ifst, ilst$ Overwritten for real flavors only.

If $ifst$ pointed to the second row of a 2 by 2 block on entry, it is changed to point to the first row; $ilst$ always points to the first row of the block in its final position (which may differ from its input value by ± 1).

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed matrix S is exactly similar to a matrix $T+E$, where $\|E\|_2 = O(\epsilon) * \|T\|_2$, and ϵ is the machine precision.

Note that if a 2 by 2 diagonal block is involved in the re-ordering, its off-diagonal elements are in general changed; the diagonal elements and the eigenvalues of the block are unchanged unless the block is sufficiently ill-conditioned, in which case they may be noticeably altered. It is possible for a 2 by 2 block to break into two 1 by 1 blocks, that is, for a pair of complex eigenvalues to become purely real.

The approximate number of floating-point operations is

for real flavors: $6n(ifst-ilst)$ if $compq = 'N'$;
 $12n(ifst-ilst)$ if $compq = 'V'$;

for complex flavors: $20n(ifst-ilst)$ if $compq = 'N'$;
 $40n(ifst-ilst)$ if $compq = 'V'$.

?trsen

Reorders the Schur factorization of a matrix and (optionally) computes the reciprocal condition numbers for the selected cluster of eigenvalues and respective invariant subspace.

Syntax

```
lapack_int LAPACKE_strsen( int matrix_layout, char job, char compq, const
lapack_logical* select, lapack_int n, float* t, lapack_int ldt, float* q, lapack_int
ldq, float* wr, float* wi, lapack_int* m, float* s, float* sep );

lapack_int LAPACKE_dtrsen( int matrix_layout, char job, char compq, const
lapack_logical* select, lapack_int n, double* t, lapack_int ldt, double* q, lapack_int
ldq, double* wr, double* wi, lapack_int* m, double* s, double* sep );

lapack_int LAPACKE_ctrsen( int matrix_layout, char job, char compq, const
lapack_logical* select, lapack_int n, lapack_complex_float* t, lapack_int ldt,
lapack_complex_float* q, lapack_int ldq, lapack_complex_float* w, lapack_int* m, float*
s, float* sep );

lapack_int LAPACKE_ztrsen( int matrix_layout, char job, char compq, const
lapack_logical* select, lapack_int n, lapack_complex_double* t, lapack_int ldt,
lapack_complex_double* q, lapack_int ldq, lapack_complex_double* w, lapack_int* m,
double* s, double* sep );
```

Include Files

- mkl.h

Description

The routine reorders the Schur factorization of a general matrix $A = Q^*T^*Q^T$ (for real flavors) or $A = Q^*T^*Q^H$ (for complex flavors) so that a selected cluster of eigenvalues appears in the leading diagonal elements (or, for real flavors, diagonal blocks) of the Schur form. The reordered Schur form R is computed by a unitary (orthogonal) similarity transformation: $R = Z^H * T * Z$. Optionally the updated matrix P of Schur vectors is computed as $P = Q * Z$, giving $A = P * R * P^H$.

Let

$$R = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{13} \end{bmatrix}$$

where the selected eigenvalues are precisely the eigenvalues of the leading m -by- m submatrix T_{11} . Let P be correspondingly partitioned as $(Q_1 Q_2)$ where Q_1 consists of the first m columns of Q . Then $A^* Q_1 = Q_1^* T_{11}$, and so the m columns of Q_1 form an orthonormal basis for the invariant subspace corresponding to the selected cluster of eigenvalues.

Optionally the routine also computes estimates of the reciprocal condition numbers of the average of the cluster of eigenvalues and of the invariant subspace.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

| | |
|---------------|---|
| <i>job</i> | <p>Must be 'N' or 'E' or 'V' or 'B'.</p> <p>If <i>job</i> = 'N', then no condition numbers are required.</p> <p>If <i>job</i> = 'E', then only the condition number for the cluster of eigenvalues is computed.</p> <p>If <i>job</i> = 'V', then only the condition number for the invariant subspace is computed.</p> <p>If <i>job</i> = 'B', then condition numbers for both the cluster and the invariant subspace are computed.</p> |
| <i>compq</i> | <p>Must be 'V' or 'N'.</p> <p>If <i>compq</i> = 'V', then <i>Q</i> of the Schur vectors is updated.</p> <p>If <i>compq</i> = 'N', then no Schur vectors are updated.</p> |
| <i>select</i> | <p>Array, size at least $\max(1, n)$.</p> <p>Specifies the eigenvalues in the selected cluster. To select an eigenvalue λ_j, <i>select</i>[<i>j</i>] must be 1</p> <p><i>For real flavors:</i> to select a complex conjugate pair of eigenvalues λ_j and λ_{j+1} (corresponding 2 by 2 diagonal block), <i>select</i>[<i>j</i> - 1] and/or <i>select</i>[<i>j</i>] must be 1; the complex conjugate λ_j and λ_{j+1} must be either both included in the cluster or both excluded.</p> |
| <i>n</i> | The order of the matrix <i>T</i> ($n \geq 0$). |
| <i>t, q</i> | <p>Arrays:</p> <p><i>t</i> (size $\max(1, ldt * n)$) The upper quasi-triangular <i>n</i>-by-<i>n</i> matrix <i>T</i>, in Schur canonical form.</p> <p><i>q</i> (size $\max(1, ldq * n)$)</p> <p>If <i>compq</i> = 'V', then <i>q</i> must contain the matrix <i>Q</i> of Schur vectors.</p> <p>If <i>compq</i> = 'N', then <i>q</i> is not referenced.</p> |
| <i>ldt</i> | The leading dimension of <i>t</i> ; at least $\max(1, n)$. |
| <i>ldq</i> | <p>The leading dimension of <i>q</i>;</p> <p>If <i>compq</i> = 'N', then $ldq \geq 1$.</p> <p>If <i>compq</i> = 'V', then $ldq \geq \max(1, n)$.</p> |

Output Parameters

| | |
|----------|---|
| <i>t</i> | Overwritten by the reordered matrix <i>R</i> in Schur canonical form with the selected eigenvalues in the leading diagonal blocks. |
| <i>q</i> | If <i>compq</i> = 'V', <i>q</i> contains the updated matrix of Schur vectors; the first <i>m</i> columns of the <i>Q</i> form an orthogonal basis for the specified invariant subspace. |
| <i>w</i> | Array, size at least $\max(1, n)$. The recorded eigenvalues of <i>R</i> . The eigenvalues are stored in the same order as on the diagonal of <i>R</i> . |

| | |
|---------------|--|
| <i>wr, wi</i> | Arrays, size at least $\max(1, n)$. Contain the real and imaginary parts, respectively, of the reordered eigenvalues of R . The eigenvalues are stored in the same order as on the diagonal of R . Note that if a complex eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering. |
| <i>m</i> | <p><i>For complex flavors:</i> the dimension of the specified invariant subspaces, which is the same as the number of selected eigenvalues (see <i>select</i>).</p> <p><i>For real flavors:</i> the dimension of the specified invariant subspace. The value of m is obtained by counting 1 for each selected real eigenvalue and 2 for each selected complex conjugate pair of eigenvalues (see <i>select</i>).</p> <p>Constraint: $0 \leq m \leq n$.</p> |
| <i>s</i> | <p>If $job = 'E'$ or $'B'$, s is a lower bound on the reciprocal condition number of the average of the selected cluster of eigenvalues.</p> <p>If $m = 0$ or n, then $s = 1$.</p> <p><i>For real flavors:</i> if $info = 1$, then s is set to zero. s is not referenced if $job = 'N'$ or $'V'$.</p> |
| <i>sep</i> | <p>If $job = 'V'$ or $'B'$, sep is the estimated reciprocal condition number of the specified invariant subspace.</p> <p>If $m = 0$ or n, then $sep = T$.</p> <p><i>For real flavors:</i> if $info = 1$, then sep is set to zero.</p> <p>sep is not referenced if $job = 'N'$ or $'E'$.</p> |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = 1$, the reordering of T failed because some eigenvalues are too close to separate (the problem is very ill-conditioned); T may have been partially reordered, and wr and wi contain the eigenvalues in the same order as in T ; s and sep (if requested) are set to zero.

Application Notes

The computed matrix R is exactly similar to a matrix $T+E$, where $\|E\|_2 = O(\epsilon) * \|T\|_2$, and ϵ is the machine precision. The computed s cannot underestimate the true reciprocal condition number by more than a factor of $(\min(m, n-m))_{1/2}$; sep may differ from the true value by $(m*n-m^2)_{1/2}$. The angle between the computed invariant subspace and the true subspace is $O(\epsilon) * \|A\|_2 / sep$. Note that if a 2-by-2 diagonal block is involved in the re-ordering, its off-diagonal elements are in general changed; the diagonal elements and the eigenvalues of the block are unchanged unless the block is sufficiently ill-conditioned, in which case they may be noticeably altered. It is possible for a 2-by-2 block to break into two 1-by-1 blocks, that is, for a pair of complex eigenvalues to become purely real.

?trsyl

Solves Sylvester equation for real quasi-triangular or complex triangular matrices.

Syntax

```
lapack_int LAPACKE_strsyl( int matrix_layout, char trana, char tranb, lapack_int isgn,
lapack_int m, lapack_int n, const float* a, lapack_int lda, const float* b, lapack_int
ldb, float* c, lapack_int ldc, float* scale );
```

```
lapack_int LAPACKE_dtrsyl( int matrix_layout, char trana, char tranb, lapack_int isgn,
lapack_int m, lapack_int n, const double* a, lapack_int lda, const double* b,
lapack_int ldb, double* c, lapack_int ldc, double* scale );
```

```
lapack_int LAPACKE_ctrsyl( int matrix_layout, char trana, char tranb, lapack_int isgn,
lapack_int m, lapack_int n, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* c, lapack_int ldc,
float* scale );
```

```
lapack_int LAPACKE_ztrsyl( int matrix_layout, char trana, char tranb, lapack_int isgn,
lapack_int m, lapack_int n, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* c, lapack_int ldc,
double* scale );
```

Include Files

- mkl.h

Description

The routine solves the Sylvester matrix equation $\text{op}(A) * X \pm X * \text{op}(B) = \alpha * C$, where $\text{op}(A) = A$ or A^H , and the matrices A and B are upper triangular (or, for real flavors, upper quasi-triangular in canonical Schur form); $\alpha \leq 1$ is a scale factor determined by the routine to avoid overflow in X ; A is m -by- m , B is n -by- n , and C and X are both m -by- n . The matrix X is obtained by a straightforward process of back substitution.

The equation has a unique solution if and only if $\alpha_i \pm \beta_i \neq 0$, where $\{\alpha_i\}$ and $\{\beta_i\}$ are the eigenvalues of A and B , respectively, and the sign (+ or -) is the same as that used in the equation to be solved.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>trana</i> | Must be 'N' or 'T' or 'C'. If <i>trana</i> = 'N', then $\text{op}(A) = A$. If <i>trana</i> = 'T', then $\text{op}(A) = A^T$ (real flavors only). If <i>trana</i> = 'C' then $\text{op}(A) = A^H$. |
| <i>tranb</i> | Must be 'N' or 'T' or 'C'. If <i>tranb</i> = 'N', then $\text{op}(B) = B$. If <i>tranb</i> = 'T', then $\text{op}(B) = B^T$ (real flavors only). If <i>tranb</i> = 'C', then $\text{op}(B) = B^H$. |
| <i>isgn</i> | Indicates the form of the Sylvester equation. If <i>isgn</i> = +1, $\text{op}(A) * X + X * \text{op}(B) = \text{alpha} * C$. If <i>isgn</i> = -1, $\text{op}(A) * X - X * \text{op}(B) = \text{alpha} * C$. |
| <i>m</i> | The order of A , and the number of rows in X and C ($m \geq 0$). |

| | |
|-----------|--|
| n | The order of B , and the number of columns in X and C ($n \geq 0$). |
| a, b, c | Arrays: a (size $\max(1, lda * m)$) contains the matrix A . b (size $\max(1, ldb * n)$) contains the matrix B . c (size $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout) contains the matrix C . |
| lda | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| ldb | The leading dimension of b ; at least $\max(1, n)$. |
| ldc | The leading dimension of c ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout . |

Output Parameters

| | |
|---------|--|
| c | Overwritten by the solution matrix X . |
| $scale$ | The value of the scale factor α . |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = 1$, A and B have common or close eigenvalues; perturbed values were used to solve the equation.

Application Notes

Let X be the exact, Y the corresponding computed solution, and R the residual matrix: $R = C - (AY \pm YB)$. Then the residual is always small:

$$\|R\|_F = O(\epsilon) * (\|A\|_F + \|B\|_F) * \|Y\|_F.$$

However, Y is not necessarily the exact solution of a slightly perturbed equation; in other words, the solution is not backwards stable.

For the forward error, the following bound holds:

$$\|Y - X\|_F \leq \|R\|_F / \text{sep}(A, B)$$

but this may be a considerable overestimate. See [Golub96] for a definition of $\text{sep}(A, B)$.

The approximate number of floating-point operations for real flavors is $m * n * (m + n)$. For complex flavors it is 4 times greater.

Generalized Nonsymmetric Eigenvalue Problems: LAPACK Computational Routines

This section describes LAPACK routines for solving generalized nonsymmetric eigenvalue problems, reordering the generalized Schur factorization of a pair of matrices, as well as performing a number of related computational tasks.

A *generalized nonsymmetric eigenvalue problem* is as follows: given a pair of nonsymmetric (or non-Hermitian) n -by- n matrices A and B , find the *generalized eigenvalues* λ and the corresponding *generalized eigenvectors* x and y that satisfy the equations

$$Ax = \lambda Bx \text{ (right generalized eigenvectors } x)$$

and

$y^H A = \lambda y^H B$ (left generalized eigenvectors y).

Table "Computational Routines for Solving Generalized Nonsymmetric Eigenvalue Problems" lists LAPACK routines used to solve the generalized nonsymmetric eigenvalue problems and the generalized Sylvester equation.

Computational Routines for Solving Generalized Nonsymmetric Eigenvalue Problems

| Routine name | Operation performed |
|------------------------|--|
| ggghrd | Reduces a pair of matrices to generalized upper Hessenberg form using orthogonal/unitary transformations. |
| ggbal | Balances a pair of general real or complex matrices. |
| ggbak | Forms the right or left eigenvectors of a generalized eigenvalue problem. |
| ggghd3 | Reduces a pair of matrices to generalized upper Hessenberg form. |
| hgeqz | Implements the QZ method for finding the generalized eigenvalues of the matrix pair (H,T). |
| tgevc | Computes some or all of the right and/or left generalized eigenvectors of a pair of upper triangular matrices |
| tgexc | Reorders the generalized Schur decomposition of a pair of matrices (A,B) so that one diagonal block of (A,B) moves to another row index. |
| tgsen | Reorders the <i>generalized</i> Schur decomposition of a pair of matrices (A,B) so that a selected cluster of eigenvalues appears in the leading diagonal blocks of (A,B). |
| tgsyl | Solves the generalized Sylvester equation. |
| tgsyl | Estimates reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a pair of matrices in generalized real Schur canonical form. |

?ggghrd

Reduces a pair of matrices to generalized upper Hessenberg form using orthogonal/unitary transformations.

Syntax

```
lapack_int LAPACKE_sgghrd (int matrix_layout, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, float* a, lapack_int lda, float* b, lapack_int ldb,
float* q, lapack_int ldq, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dgghrd (int matrix_layout, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, double* a, lapack_int lda, double* b, lapack_int ldb,
double* q, lapack_int ldq, double* z, lapack_int ldz);
```

```
lapack_int LAPACKE_cgghrd (int matrix_layout, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, lapack_complex_float* a, lapack_int lda,
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* q, lapack_int ldq,
lapack_complex_float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_zgghrd (int matrix_layout, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, lapack_complex_double* a, lapack_int lda,
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* q, lapack_int ldq,
lapack_complex_double* z, lapack_int ldz);
```

Include Files

- `mkl.h`

Description

The routine reduces a pair of real/complex matrices (A, B) to generalized upper Hessenberg form using orthogonal/unitary transformations, where A is a general matrix and B is upper triangular. The form of the generalized eigenvalue problem is $A^*x = \lambda^*B^*x$, and B is typically made upper triangular by computing its QR factorization and moving the orthogonal matrix Q to the left side of the equation.

This routine simultaneously reduces A to a Hessenberg matrix H :

$$Q^H A Z = H$$

and transforms B to another upper triangular matrix T :

$$Q^H B Z = T$$

in order to reduce the problem to its standard form $H^*y = \lambda^*T^*y$, where $y = Z^H x$.

The orthogonal/unitary matrices Q and Z are determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1 , so that

$$Q_1^* A Z_1^H = (Q_1^* Q)^* H^* (Z_1^* Z)^H$$

$$Q_1^* B Z_1^H = (Q_1^* Q)^* T^* (Z_1^* Z)^H$$

If Q_1 is the orthogonal/unitary matrix from the QR factorization of B in the original equation $A^*x = \lambda^*B^*x$, then the routine `?gghrd` reduces the original problem to generalized Hessenberg form.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>compq</code> | <p>Must be 'N', 'I', or 'V'.</p> <p>If <code>compq = 'N'</code>, matrix Q is not computed.</p> <p>If <code>compq = 'I'</code>, Q is initialized to the unit matrix, and the orthogonal/unitary matrix Q is returned;</p> <p>If <code>compq = 'V'</code>, Q must contain an orthogonal/unitary matrix Q_1 on entry, and the product Q_1^*Q is returned.</p> |
| <code>compz</code> | <p>Must be 'N', 'I', or 'V'.</p> <p>If <code>compz = 'N'</code>, matrix Z is not computed.</p> <p>If <code>compz = 'I'</code>, Z is initialized to the unit matrix, and the orthogonal/unitary matrix Z is returned;</p> <p>If <code>compz = 'V'</code>, Z must contain an orthogonal/unitary matrix Z_1 on entry, and the product Z_1^*Z is returned.</p> |
| <code>n</code> | The order of the matrices A and B ($n \geq 0$). |
| <code>ilo, ihi</code> | <p><code>ilo</code> and <code>ihi</code> mark the rows and columns of A which are to be reduced. It is assumed that A is already upper triangular in rows and columns $1:ilo-1$ and $ihi+1:n$. Values of <code>ilo</code> and <code>ihi</code> are normally set by a previous call to <code>ggbal</code>; otherwise they should be set to 1 and n respectively.</p> <p>Constraint:</p> |

| | |
|--------------|---|
| | <p>If $n > 0$, then $1 \leq ilo \leq ihi \leq n$;</p> <p>if $n = 0$, then $ilo = 1$ and $ihi = 0$.</p> |
| a, b, q, z | <p>Arrays:</p> <p>a (size $\max(1, lda*n)$) contains the n-by-n general matrix A.</p> <p>b (size $\max(1, ldb*n)$) contains the n-by-n upper triangular matrix B.</p> <p>q (size $\max(1, ldq*n)$)</p> <p>If $compq = 'N'$, then q is not referenced.</p> <p>If $compq = 'V'$, then q must contain the orthogonal/unitary matrix Q_1, typically from the QR factorization of B.</p> <p>z (size $\max(1, ldz*n)$)</p> <p>If $compz = 'N'$, then z is not referenced.</p> <p>If $compz = 'V'$, then z must contain the orthogonal/unitary matrix Z_1.</p> |
| lda | The leading dimension of a ; at least $\max(1, n)$. |
| ldb | The leading dimension of b ; at least $\max(1, n)$. |
| ldq | <p>The leading dimension of q;</p> <p>If $compq = 'N'$, then $ldq \geq 1$.</p> <p>If $compq = 'I'$ or $'V'$, then $ldq \geq \max(1, n)$.</p> |
| ldz | <p>The leading dimension of z;</p> <p>If $compz = 'N'$, then $ldz \geq 1$.</p> <p>If $compz = 'I'$ or $'V'$, then $ldz \geq \max(1, n)$.</p> |

Output Parameters

| | |
|-----|--|
| a | On exit, the upper triangle and the first subdiagonal of A are overwritten with the upper Hessenberg matrix H , and the rest is set to zero. |
| b | On exit, overwritten by the upper triangular matrix $T = Q^H * B * Z$. The elements below the diagonal are set to zero. |
| q | <p>If $compq = 'I'$, then q contains the orthogonal/unitary matrix Q;</p> <p>If $compq = 'V'$, then q is overwritten by the product $Q_1 * Q$.</p> |
| z | <p>If $compz = 'I'$, then z contains the orthogonal/unitary matrix Z;</p> <p>If $compz = 'V'$, then z is overwritten by the product $Z_1 * Z$.</p> |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

?ggbal

Balances a pair of general real or complex matrices.

Syntax

```
lapack_int LAPACKE_sggbal( int matrix_layout, char job, lapack_int n, float* a,
lapack_int lda, float* b, lapack_int ldb, lapack_int* ilo, lapack_int* ihi, float*
lscale, float* rscale );
```

```
lapack_int LAPACKE_dggbal( int matrix_layout, char job, lapack_int n, double* a,
lapack_int lda, double* b, lapack_int ldb, lapack_int* ilo, lapack_int* ihi, double*
lscale, double* rscale );
```

```
lapack_int LAPACKE_cggbal( int matrix_layout, char job, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb,
lapack_int* ilo, lapack_int* ihi, float* lscale, float* rscale );
```

```
lapack_int LAPACKE_zggbal( int matrix_layout, char job, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
lapack_int* ilo, lapack_int* ihi, double* lscale, double* rscale );
```

Include Files

- mkl.h

Description

The routine balances a pair of general real/complex matrices (A, B). This involves, first, permuting A and B by similarity transformations to isolate eigenvalues in the first 1 to $ilo-1$ and last $ihi+1$ to n elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns ilo to ihi to make the rows and columns as close in norm as possible. Both steps are optional. Balancing may reduce the 1-norm of the matrices, and improve the accuracy of the computed eigenvalues and/or eigenvectors in the generalized eigenvalue problem $A*x = \lambda*B*x$.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>job</i> | Specifies the operations to be performed on A and B . Must be 'N' or 'P' or 'S' or 'B'. If <i>job</i> = 'N', then no operations are done; simply set $ilo = 1$, $ihi = n$, $lscale[i] = 1.0$ and $rscale[i] = 1.0$ for $i = 0, \dots, n - 1$. If <i>job</i> = 'P', then permute only. If <i>job</i> = 'S', then scale only. If <i>job</i> = 'B', then both permute and scale. |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>a, b</i> | Arrays: a (size $\max(1, lda*n)$) contains the matrix A . b (size $\max(1, ldb*n)$) contains the matrix B . If <i>job</i> = 'N', a and b are not referenced. |

lda The leading dimension of *a*; at least $\max(1, n)$.

ldb The leading dimension of *b*; at least $\max(1, n)$.

Output Parameters

a, b Overwritten by the balanced matrices *A* and *B*, respectively.

ilo, ihi *ilo* and *ihi* are set to integers such that on exit $A_{i,j} = 0$ and $B_{i,j} = 0$ if $i > j$ and $j=1, \dots, ilo-1$ or $i=ihi+1, \dots, n$.
If *job* = 'N' or 'S', then *ilo* = 1 and *ihi* = *n*.

lscale, rscale Arrays, size at least $\max(1, n)$.
lscale contains details of the permutations and scaling factors applied to the left side of *A* and *B*.
If P_j is the index of the row interchanged with row *j*, and D_j is the scaling factor applied to row *j*, then

$$lscale[j - 1] = P_j, \text{ for } j = 1, \dots, ilo-1$$

$$= D_j, \text{ for } j = ilo, \dots, ihi$$

$$= P_j, \text{ for } j = ihi+1, \dots, n.$$
rscale contains details of the permutations and scaling factors applied to the right side of *A* and *B*.
If P_j is the index of the column interchanged with column *j*, and D_j is the scaling factor applied to column *j*, then

$$rscale[j - 1] = P_j, \text{ for } j = 1, \dots, ilo-1$$

$$= D_j, \text{ for } j = ilo, \dots, ihi$$

$$= P_j, \text{ for } j = ihi+1, \dots, n$$
The order in which the interchanges are made is *n* to *ihi+1*, then 1 to *ilo-1*.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

?ggbak

Forms the right or left eigenvectors of a generalized eigenvalue problem.

Syntax

```
lapack_int LAPACKE_sggbak( int matrix_layout, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const float* lscale, const float* rscale, lapack_int
m, float* v, lapack_int ldv );
```

```
lapack_int LAPACKE_dggbak( int matrix_layout, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const double* lscale, const double* rscale, lapack_int
m, double* v, lapack_int ldv );
```

```
lapack_int LAPACKE_cggbak( int matrix_layout, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const float* lscale, const float* rscale, lapack_int
m, lapack_complex_float* v, lapack_int ldv );
```

```
lapack_int LAPACKE_zggbak( int matrix_layout, char job, char side, lapack_int n,
lapack_int ilo, lapack_int ihi, const double* lscale, const double* rscale, lapack_int
m, lapack_complex_double* v, lapack_int ldv );
```

Include Files

- mkl.h

Description

The routine forms the right or left eigenvectors of a real/complex generalized eigenvalue problem

$$A*x = \lambda*B*x$$

by backward transformation on the computed eigenvectors of the balanced pair of matrices output by [ggbal](#).

Input Parameters

| | |
|-----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>job</i> | Specifies the type of backward transformation required. Must be 'N', 'P', 'S', or 'B'. If <i>job</i> = 'N', then no operations are done; return. If <i>job</i> = 'P', then do backward transformation for permutation only. If <i>job</i> = 'S', then do backward transformation for scaling only. If <i>job</i> = 'B', then do backward transformation for both permutation and scaling. This argument must be the same as the argument <i>job</i> supplied to ? ggbal . |
| <i>side</i> | Must be 'L' or 'R'. If <i>side</i> = 'L', then <i>v</i> contains left eigenvectors. If <i>side</i> = 'R', then <i>v</i> contains right eigenvectors. |
| <i>n</i> | The number of rows of the matrix <i>V</i> ($n \geq 0$). |
| <i>ilo, ihi</i> | The integers <i>ilo</i> and <i>ihi</i> determined by ? ggbal . Constraint: If $n > 0$, then $1 \leq ilo \leq ihi \leq n$; if $n = 0$, then $ilo = 1$ and $ihi = 0$. |
| <i>lscale, rscale</i> | Arrays, size at least $\max(1, n)$. The array <i>lscale</i> contains details of the permutations and/or scaling factors applied to the left side of <i>A</i> and <i>B</i> , as returned by ? ggbal . The array <i>rscale</i> contains details of the permutations and/or scaling factors applied to the right side of <i>A</i> and <i>B</i> , as returned by ? ggbal . |
| <i>m</i> | The number of columns of the matrix <i>V</i> ($m \geq 0$). |

| | |
|------------|---|
| <i>v</i> | Array <i>v</i> (size $\max(1, ldv*m)$ for column major layout and $\max(1, ldv*n)$ for row major layout) . Contains the matrix of right or left eigenvectors to be transformed, as returned by <code>tgevc</code> . |
| <i>ldv</i> | The leading dimension of <i>v</i> ; at least $\max(1, n)$ for column major layout and at least $\max(1, m)$ for row major layout . |

Output Parameters

| | |
|----------|---|
| <i>v</i> | Overwritten by the transformed eigenvectors |
|----------|---|

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

?gghd3

Reduces a pair of matrices to generalized upper Hessenberg form.

Syntax

```
lapack_int LAPACKE_sgghd3 (int matrix_layout, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, float * a, lapack_int lda, float * b, lapack_int ldb,
float * q, lapack_int ldq, float * z, lapack_int ldz);
```

```
lapack_int LAPACKE_dgghd3 (int matrix_layout, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, double * a, lapack_int lda, double * b, lapack_int
ldb, double * q, lapack_int ldq, double * z, lapack_int ldz);
```

```
lapack_int LAPACKE_cgghd3 (int matrix_layout, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, lapack_complex_float * a, lapack_int lda,
lapack_complex_float * b, lapack_int ldb, lapack_complex_float * q, lapack_int ldq,
lapack_complex_float * z, lapack_int ldz);
```

```
lapack_int LAPACKE_zgghd3 (int matrix_layout, char compq, char compz, lapack_int n,
lapack_int ilo, lapack_int ihi, lapack_complex_double * a, lapack_int lda,
lapack_complex_double * b, lapack_int ldb, lapack_complex_double * q, lapack_int ldq,
lapack_complex_double * z, lapack_int ldz);
```

Include Files

- mkl.h

Description

?gghd3 reduces a pair of real or complex matrices (*A*, *B*) to generalized upper Hessenberg form using orthogonal/unitary transformations, where *A* is a general matrix and *B* is upper triangular. The form of the generalized eigenvalue problem is

$$A*x = \lambda*B*x,$$

and *B* is typically made upper triangular by computing its QR factorization and moving the orthogonal/unitary matrix *Q* to the left side of the equation.

This subroutine simultaneously reduces *A* to a Hessenberg matrix *H*:

$$Q^T*A*Z = H \text{ for real flavors}$$

or

$$Q^T * A * Z = H \text{ for complex flavors}$$

and transforms B to another upper triangular matrix T :

$$Q^T * B * Z = T \text{ for real flavors}$$

or

$$Q^T * B * Z = T \text{ for complex flavors}$$

in order to reduce the problem to its standard form

$$H * y = \lambda * T * y$$

where $y = Z^T * x$ for real flavors

or

$$y = Z^T * x \text{ for complex flavors.}$$

The orthogonal/unitary matrices Q and Z are determined as products of Givens rotations. They may either be formed explicitly, or they may be postmultiplied into input matrices Q_1 and Z_1 , so that

for real flavors:

$$Q_1 * A * Z_1^T = (Q_1 * Q) * H * (Z_1 * Z)^T$$

$$Q_1 * B * Z_1^T = (Q_1 * Q) * T * (Z_1 * Z)^T$$

for complex flavors:

$$Q_1 * A * Z_1^H = (Q_1 * Q) * H * (Z_1 * Z)^T$$

$$Q_1 * B * Z_1^T = (Q_1 * Q) * T * (Z_1 * Z)^T$$

If Q_1 is the orthogonal/unitary matrix from the QR factorization of B in the original equation $A * x = \lambda * B * x$, then `?ggghd3` reduces the original problem to generalized Hessenberg form.

This is a blocked variant of `?ggghrd`, using matrix-matrix multiplications for parts of the computation to enhance performance.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>compq</code> | <ul style="list-style-type: none"> = 'N': do not compute q; = 'I': q is initialized to the unit matrix, and the orthogonal/unitary matrix Q is returned; = 'V': q must contain an orthogonal/unitary matrix Q_1 on entry, and the product $Q_1 * q$ is returned. |
| <code>compz</code> | <ul style="list-style-type: none"> = 'N': do not compute z; = 'I': z is initialized to the unit matrix, and the orthogonal/unitary matrix Z is returned; = 'V': z must contain an orthogonal/unitary matrix Z_1 on entry, and the product $Z_1 * z$ is returned. |
| <code>n</code> | <p>The order of the matrices A and B.</p> <p>$n \geq 0$.</p> |

| | |
|-----------------|---|
| <i>ilo, ihi</i> | <i>ilo</i> and <i>ihi</i> mark the rows and columns of <i>a</i> which are to be reduced. It is assumed that <i>a</i> is already upper triangular in rows and columns 1: <i>ilo</i> - 1 and <i>ihi</i> + 1: <i>n</i> . <i>ilo</i> and <i>ihi</i> are normally set by a previous call to ?ggbal; otherwise they should be set to 1 and <i>n</i> , respectively. $1 \leq ilo \leq ihi \leq n$, if $n > 0$; $ilo=1$ and $ihi=0$, if $n=0$. |
| <i>a</i> | Array, size (<i>lda</i> * <i>n</i>). On entry, the <i>n</i> -by- <i>n</i> general matrix to be reduced. |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1, n)$. |
| <i>b</i> | Array, (<i>ldb</i> * <i>n</i>). On entry, the <i>n</i> -by- <i>n</i> upper triangular matrix <i>B</i> . |
| <i>ldb</i> | The leading dimension of the array <i>b</i> . $ldb \geq \max(1, n)$. |
| <i>q</i> | Array, size (<i>ldq</i> * <i>n</i>). On entry, if <i>compq</i> = 'V', the orthogonal/unitary matrix Q_1 , typically from the QR factorization of <i>b</i> . |
| <i>ldq</i> | The leading dimension of the array <i>q</i> . $ldq \geq n$ if <i>compq</i> ='V' or 'I'; $ldq \geq 1$ otherwise. |
| <i>z</i> | Array, size (<i>ldz</i> * <i>n</i>). On entry, if <i>compz</i> = 'V', the orthogonal/unitary matrix Z_1 . Not referenced if <i>compz</i> ='N'. |
| <i>ldz</i> | The leading dimension of the array <i>z</i> . $ldz \geq n$ if <i>compz</i> ='V' or 'I'; $ldz \geq 1$ otherwise. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | On exit, the upper triangle and the first subdiagonal of <i>a</i> are overwritten with the upper Hessenberg matrix <i>H</i> , and the rest is set to zero. |
| <i>b</i> | On exit, the upper triangular matrix $T = Q^T B Z$ for real flavors or $T = Q^H B Z$ for complex flavors. The elements below the diagonal are set to zero. |
| <i>q</i> | On exit, if <i>compq</i> ='I', the orthogonal/unitary matrix <i>Q</i> , and if <i>compq</i> = 'V', the product $Q_1 * Q$. Not referenced if <i>compq</i> ='N'. |
| <i>z</i> | On exit, if <i>compz</i> ='I', the orthogonal/unitary matrix <i>Z</i> , and if <i>compz</i> = 'V', the product $Z_1 * Z$. Not referenced if <i>compz</i> ='N'. |

Return Values

This function returns a value *info*.

= 0: successful exit.

< 0: if *info* = -*i*, the *i*-th argument had an illegal value.

Application Notes

This routine reduces *A* to Hessenberg form and maintains *B* in using a blocked variant of Moler and Stewart's original algorithm, as described by Kagstrom, Kressner, Quintana-Orti, and Quintana-Orti (BIT 2008).

?hgeqz

Implements the QZ method for finding the generalized eigenvalues of the matrix pair (H,T).

Syntax

```
lapack_int LAPACKE_shgeqz( int matrix_layout, char job, char compq, char compz,
lapack_int n, lapack_int ilo, lapack_int ihi, float* h, lapack_int ldh, float* t,
lapack_int ldt, float* alphas, float* alphai, float* beta, float* q, lapack_int ldq,
float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_dhgeqz( int matrix_layout, char job, char compq, char compz,
lapack_int n, lapack_int ilo, lapack_int ihi, double* h, lapack_int ldh, double* t,
lapack_int ldt, double* alphas, double* alphai, double* beta, double* q, lapack_int
ldq, double* z, lapack_int ldz );
```

```
lapack_int LAPACKE_chgeqz( int matrix_layout, char job, char compq, char compz,
lapack_int n, lapack_int ilo, lapack_int ihi, lapack_complex_float* h, lapack_int ldh,
lapack_complex_float* t, lapack_int ldt, lapack_complex_float* alpha,
lapack_complex_float* beta, lapack_complex_float* q, lapack_int ldq,
lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zhgeqz( int matrix_layout, char job, char compq, char compz,
lapack_int n, lapack_int ilo, lapack_int ihi, lapack_complex_double* h, lapack_int
ldh, lapack_complex_double* t, lapack_int ldt, lapack_complex_double* alpha,
lapack_complex_double* beta, lapack_complex_double* q, lapack_int ldq,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes the eigenvalues of a real/complex matrix pair (*H,T*), where *H* is an upper Hessenberg matrix and *T* is upper triangular, using the double-shift version (for real flavors) or single-shift version (for complex flavors) of the QZ method. Matrix pairs of this type are produced by the reduction to generalized upper Hessenberg form of a real/complex matrix pair (*A,B*):

$$A = Q_1 * H * Z_1^H, B = Q_1 * T * Z_1^H,$$

as computed by ?gghrd.

For real flavors:

If *job* = 'S', then the Hessenberg-triangular pair (*H,T*) is reduced to generalized Schur form,

$$H = Q * S * Z^T, T = Q * P * Z^T,$$

where Q and Z are orthogonal matrices, P is an upper triangular matrix, and S is a quasi-triangular matrix with 1-by-1 and 2-by-2 diagonal blocks. The 1-by-1 blocks correspond to real eigenvalues of the matrix pair (H,T) and the 2-by-2 blocks correspond to complex conjugate pairs of eigenvalues.

Additionally, the 2-by-2 upper triangular diagonal blocks of P corresponding to 2-by-2 blocks of S are reduced to positive diagonal form, that is, if $S_{j+1,j}$ is non-zero, then $P_{j+1,j} = P_{j,j+1} = 0$, $P_{j,j} > 0$, and $P_{j+1,j+1} > 0$.

For complex flavors:

If $job = 'S'$, then the Hessenberg-triangular pair (H,T) is reduced to generalized Schur form,

$$H = Q^* S^* Z^H, T = Q^* P^* Z^H,$$

where Q and Z are unitary matrices, and S and P are upper triangular.

For all function flavors:

Optionally, the orthogonal/unitary matrix Q from the generalized Schur factorization may be post-multiplied by an input matrix Q_1 , and the orthogonal/unitary matrix Z may be post-multiplied by an input matrix Z_1 .

If Q_1 and Z_1 are the orthogonal/unitary matrices from `?gghrd` that reduced the matrix pair (A,B) to generalized upper Hessenberg form, then the output matrices Q_1Q and Z_1Z are the orthogonal/unitary factors from the generalized Schur factorization of (A,B) :

$$A = (Q_1Q)^* S^* (Z_1Z)^H, B = (Q_1Q)^* P^* (Z_1Z)^H.$$

To avoid overflow, eigenvalues of the matrix pair (H,T) (equivalently, of (A,B)) are computed as a pair of values $(alpha,beta)$. For `chgeqz/zhgeqz`, $alpha$ and $beta$ are complex, and for `shgeqz/dhgeqz`, $alpha$ is complex and $beta$ real. If $beta$ is nonzero, $\lambda = alpha/beta$ is an eigenvalue of the generalized nonsymmetric eigenvalue problem (GNEP)

$$A^*x = \lambda^* B^*x$$

and if $alpha$ is nonzero, $\mu = beta/alpha$ is an eigenvalue of the alternate form of the GNEP

$$\mu^* A^*y = B^*y.$$

Real eigenvalues (for real flavors) or the values of $alpha$ and $beta$ for the i -th eigenvalue (for complex flavors) can be read directly from the generalized Schur form:

$$alpha = S_{i,i}, beta = P_{i,i}.$$

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>job</code> | Specifies the operations to be performed. Must be 'E' or 'S'. If <code>job = 'E'</code> , then compute eigenvalues only; If <code>job = 'S'</code> , then compute eigenvalues and the Schur form. |
| <code>compq</code> | Must be 'N', 'I', or 'V'. If <code>compq = 'N'</code> , left Schur vectors (q) are not computed; If <code>compq = 'I'</code> , q is initialized to the unit matrix and the matrix of left Schur vectors of (H,T) is returned; If <code>compq = 'V'</code> , q must contain an orthogonal/unitary matrix Q_1 on entry and the product Q_1^*Q is returned. |
| <code>compz</code> | Must be 'N', 'I', or 'V'. |

If *compz* = 'N', right Schur vectors (*z*) are not computed;
 If *compz* = 'I', *z* is initialized to the unit matrix and the matrix of right Schur vectors of (*H*,*T*) is returned;
 If *compz* = 'V', *z* must contain an orthogonal/unitary matrix Z_1 on entry and the product $Z_1 * Z$ is returned.

n The order of the matrices *H*, *T*, *Q*, and *Z*
 ($n \geq 0$).

ilo, *ihi* *ilo* and *ihi* mark the rows and columns of *H* which are in Hessenberg form. It is assumed that *H* is already upper triangular in rows and columns 1:*ilo*-1 and *ihi*+1:*n*.

Constraint:

If $n > 0$, then $1 \leq ilo \leq ihi \leq n$;
 if $n = 0$, then $ilo = 1$ and $ihi = 0$.

h, *t*, *q*, *z*

Arrays:

On entry, *h* (size $\max(1, ldh * n)$) contains the *n*-by-*n* upper Hessenberg matrix *H*.

On entry, *t* (size $\max(1, ldt * n)$) contains the *n*-by-*n* upper triangular matrix *T*.

q (size $\max(1, ldq * n)$) :

On entry, if *compq* = 'V', this array contains the orthogonal/unitary matrix Q_1 used in the reduction of (*A*,*B*) to generalized Hessenberg form.

If *compq* = 'N', then *q* is not referenced.

z (size $\max(1, ldz * n)$) :

On entry, if *compz* = 'V', this array contains the orthogonal/unitary matrix Z_1 used in the reduction of (*A*,*B*) to generalized Hessenberg form.

If *compz* = 'N', then *z* is not referenced.

ldh The leading dimension of *h*; at least $\max(1, n)$.

ldt The leading dimension of *t*; at least $\max(1, n)$.

ldq The leading dimension of *q*;
 If *compq* = 'N', then $ldq \geq 1$.
 If *compq* = 'I' or 'V', then $ldq \geq \max(1, n)$.

ldz The leading dimension of *z*;
 If *compz* = 'N', then $ldz \geq 1$.
 If *compz* = 'I' or 'V', then $ldz \geq \max(1, n)$.

Output Parameters

h For real flavors:

If $job = 'S'$, then on exit h contains the upper quasi-triangular matrix S from the generalized Schur factorization.

If $job = 'E'$, then on exit the diagonal blocks of h match those of S , but the rest of h is unspecified.

For complex flavors:

If $job = 'S'$, then, on exit, h contains the upper triangular matrix S from the generalized Schur factorization.

If $job = 'E'$, then on exit the diagonal of h matches that of S , but the rest of h is unspecified.

t If $job = 'S'$, then, on exit, t contains the upper triangular matrix P from the generalized Schur factorization.

For real flavors:

2-by-2 diagonal blocks of P corresponding to 2-by-2 blocks of S are reduced to positive diagonal form, that is, if $h(j+1,j)$ is non-zero, then $t(j+1,j) = t(j,j+1) = 0$ and $t(j,j)$ and $t(j+1,j+1)$ will be positive.

If $job = 'E'$, then on exit the diagonal blocks of t match those of P , but the rest of t is unspecified.

For complex flavors:

if $job = 'E'$, then on exit the diagonal of t matches that of P , but the rest of t is unspecified.

$alphan, alphan$

Arrays, size at least $\max(1, n)$. The real and imaginary parts, respectively, of each scalar $alpha$ defining an eigenvalue of GNEP.

If $alphan[j - 1]$ is zero, then the j -th eigenvalue is real; if positive, then the j -th and $(j+1)$ -th eigenvalues are a complex conjugate pair, with

$$alphan[j] = -alphan[j - 1].$$

$alpha$

Array, size at least $\max(1, n)$.

The complex scalars $alpha$ that define the eigenvalues of GNEP. $alphan[i - 1] = S_i, i$ in the generalized Schur factorization.

$beta$

Array, size at least $\max(1, n)$.

For real flavors:

The scalars $beta$ that define the eigenvalues of GNEP.

Together, the quantities $alpha = (alphan[j - 1], alphan[j - 1])$ and $beta = beta[j - 1]$ represent the j -th eigenvalue of the matrix pair (A,B) , in one of the forms $lambda = alpha/beta$ or $mu = beta/alpha$. Since either $lambda$ or mu may overflow, they should not, in general, be computed.

For complex flavors:

The real non-negative scalars $beta$ that define the eigenvalues of GNEP.

$\beta[i - 1] = P_{i, i}$ in the generalized Schur factorization. Together, the quantities $\alpha = \alpha[j - 1]$ and $\beta = \beta[j - 1]$ represent the j -th eigenvalue of the matrix pair (A, B) , in one of the forms $\lambda = \alpha/\beta$ or $\mu = \beta/\alpha$. Since either λ or μ may overflow, they should not, in general, be computed.

- q On exit, if $compq = 'I'$, q is overwritten by the orthogonal/unitary matrix of left Schur vectors of the pair (H, T) , and if $compq = 'V'$, q is overwritten by the orthogonal/unitary matrix of left Schur vectors of (A, B) .
- z On exit, if $compz = 'I'$, z is overwritten by the orthogonal/unitary matrix of right Schur vectors of the pair (H, T) , and if $compz = 'V'$, z is overwritten by the orthogonal/unitary matrix of right Schur vectors of (A, B) .

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = 1, \dots, n$, the QZ iteration did not converge.

(H, T) is not in Schur form, but α_{i-1} , β_{i-1} (for real flavors), $\alpha[i - 1]$ (for complex flavors), and $\beta[i - 1]$, $i=info+1, \dots, n$ should be correct.

If $info = n+1, \dots, 2n$, the shift calculation failed.

(H, T) is not in Schur form, but α_{i-1} , β_{i-1} (for real flavors), $\alpha[i - 1]$ (for complex flavors), and $\beta[i - 1]$, $i = info-n+1, \dots, n$ should be correct.

?tgevc

Computes some or all of the right and/or left generalized eigenvectors of a pair of upper triangular matrices.

Syntax

```
lapack_int LAPACKE_stgevc (int matrix_layout, char side, char howmny, const
lapack_logical* select, lapack_int n, const float* s, lapack_int lds, const float* p,
lapack_int ldp, float* vl, lapack_int ldvl, float* vr, lapack_int ldvr, lapack_int mm,
lapack_int* m);
```

```
lapack_int LAPACKE_dtgevc (int matrix_layout, char side, char howmny, const
lapack_logical* select, lapack_int n, const double* s, lapack_int lds, const double*
p, lapack_int ldp, double* vl, lapack_int ldvl, double* vr, lapack_int ldvr,
lapack_int mm, lapack_int* m);
```

```
lapack_int LAPACKE_ctgevc (int matrix_layout, char side, char howmny, const
lapack_logical* select, lapack_int n, const lapack_complex_float* s, lapack_int lds,
const lapack_complex_float* p, lapack_int ldp, lapack_complex_float* vl, lapack_int
ldvl, lapack_complex_float* vr, lapack_int ldvr, lapack_int mm, lapack_int* m);
```

```
lapack_int LAPACKE_ztgevc (int matrix_layout, char side, char howmny, const
lapack_logical* select, lapack_int n, const lapack_complex_double* s, lapack_int lds,
const lapack_complex_double* p, lapack_int ldp, lapack_complex_double* vl, lapack_int
ldvl, lapack_complex_double* vr, lapack_int ldvr, lapack_int mm, lapack_int* m);
```

Include Files

- `mkl.h`

Description

The routine computes some or all of the right and/or left eigenvectors of a pair of real/complex matrices (S,P) , where S is quasi-triangular (for real flavors) or upper triangular (for complex flavors) and P is upper triangular.

Matrix pairs of this type are produced by the generalized Schur factorization of a real/complex matrix pair (A,B) :

$$A = Q^* S^* Z^H, B = Q^* P^* Z^H$$

as computed by `?gghrd` plus `?hgeqz`.

The right eigenvector x and the left eigenvector y of (S,P) corresponding to an eigenvalue w are defined by:

$$S^* x = w^* P^* x, y^H S = w^* y^H P$$

The eigenvalues are not input to this routine, but are computed directly from the diagonal blocks or diagonal elements of S and P .

This routine returns the matrices X and/or Y of right and left eigenvectors of (S,P) , or the products $Z^* X$ and/or $Q^* Y$, where Z and Q are input matrices.

If Q and Z are the orthogonal/unitary factors from the generalized Schur factorization of a matrix pair (A,B) , then $Z^* X$ and $Q^* Y$ are the matrices of right and left eigenvectors of (A,B) .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>side</i> | Must be 'R', 'L', or 'B'. If <i>side</i> = 'R', compute right eigenvectors only. If <i>side</i> = 'L', compute left eigenvectors only. If <i>side</i> = 'B', compute both right and left eigenvectors. |
| <i>howmny</i> | Must be 'A', 'B', or 'S'. If <i>howmny</i> = 'A', compute all right and/or left eigenvectors. If <i>howmny</i> = 'B', compute all right and/or left eigenvectors, backtransformed by the matrices in <i>vr</i> and/or <i>vl</i> . If <i>howmny</i> = 'S', compute selected right and/or left eigenvectors, specified by the logical array <i>select</i> . |
| <i>select</i> | Array, size at least $\max(1, n)$. If <i>howmny</i> = 'S', <i>select</i> specifies the eigenvectors to be computed. If <i>howmny</i> = 'A' or 'B', <i>select</i> is not referenced. <i>For real flavors:</i> If $w[j]$ is a real eigenvalue, the corresponding real eigenvector is computed if <i>select</i> [j] is 1. |

If $w[j]$ and $omega[j + 1]$ are the real and imaginary parts of a complex eigenvalue, the corresponding complex eigenvector is computed if either $select[j]$ or $select[j + 1]$ is 1, and on exit $select[j]$ is set to 1 and $select[j + 1]$ is set to 0.

For complex flavors:

The eigenvector corresponding to the j -th eigenvalue is computed if $select[j]$ is 1.

n

The order of the matrices S and P ($n \geq 0$).

s, p, vl, vr

Arrays:

s (size $\max(1, lds*n)$) contains the matrix S from a generalized Schur factorization as computed by `?hgeqz`. This matrix is upper quasi-triangular for real flavors, and upper triangular for complex flavors.

p (size $\max(1, ldp*n)$) contains the upper triangular matrix P from a generalized Schur factorization as computed by `?hgeqz`.

For real flavors, 2-by-2 diagonal blocks of P corresponding to 2-by-2 blocks of S must be in positive diagonal form.

For complex flavors, P must have real diagonal elements.

If $side = 'L'$ or $'B'$ and $howmny = 'B'$, vl (size $\max(1, ldvl*mm)$ for column major layout and $\max(1, ldvl*n)$ for row major layout) must contain an n -by- n matrix Q (usually the orthogonal/unitary matrix Q of left Schur vectors returned by `?hgeqz`).

If $side = 'R'$, vl is not referenced.

If $side = 'R'$ or $'B'$ and $howmny = 'B'$, vr (size $\max(1, ldvr*mm)$ for column major layout and $\max(1, ldvr*n)$ for row major layout) must contain an n -by- n matrix Z (usually the orthogonal/unitary matrix Z of right Schur vectors returned by `?hgeqz`).

If $side = 'L'$, vr is not referenced.

lds

The leading dimension of s ; at least $\max(1, n)$.

ldp

The leading dimension of p ; at least $\max(1, n)$.

$ldvl$

The leading dimension of vl ;

If $side = 'L'$ or $'B'$, then $ldvl \geq n$ for column major layout and $ldvl \geq \max(1, mm)$ for row major layout.

If $side = 'R'$, then $ldvl \geq 1$.

$ldvr$

The leading dimension of vr ;

If $side = 'R'$ or $'B'$, then $ldvr \geq n$ for column major layout and $ldvr \geq \max(1, mm)$ for row major layout.

If $side = 'L'$, then $ldvr \geq 1$.

mm

The number of columns in the arrays vl and/or vr ($mm \geq m$).

Output Parameters

- vl* On exit, if *side* = 'L' or 'B', *vl* contains:
 if *howmny* = 'A', the matrix *Y* of left eigenvectors of (*S,P*);
 if *howmny* = 'B', the matrix *Q*Y*;
 if *howmny* = 'S', the left eigenvectors of (*S,P*) specified by *select*, stored consecutively in the columns of *vl*, in the same order as their eigenvalues.
For real flavors:
 A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part, and the second the imaginary part.
- vr* On exit, if *side* = 'R' or 'B', *vr* contains:
 if *howmny* = 'A', the matrix *X* of right eigenvectors of (*S,P*);
 if *howmny* = 'B', the matrix *Z*X*;
 if *howmny* = 'S', the right eigenvectors of (*S,P*) specified by *select*, stored consecutively in the columns of *vr*, in the same order as their eigenvalues.
For real flavors:
 A complex eigenvector corresponding to a complex eigenvalue is stored in two consecutive columns, the first holding the real part, and the second the imaginary part.
- m* The number of columns in the arrays *vl* and/or *vr* actually used to store the eigenvectors.
 If *howmny* = 'A' or 'B', *m* is set to *n*.
For real flavors:
 Each selected real eigenvector occupies one column and each selected complex eigenvector occupies two columns.
For complex flavors:
 Each selected eigenvector occupies one column.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

For real flavors:

if *info* = *i*>0, the 2-by-2 block (*i*:*i*+1) does not have a complex eigenvalue.

?tgexc

Reorders the generalized Schur decomposition of a pair of matrices (*A,B*) so that one diagonal block of (*A,B*) moves to another row index.

Syntax

```
lapack_int LAPACKE_stgexc (int matrix_layout, lapack_logical wantq, lapack_logical wantz, lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, float* q, lapack_int ldq, float* z, lapack_int ldz, lapack_int* ifst, lapack_int* ilst);
```

```
lapack_int LAPACKE_dtgexc (int matrix_layout, lapack_logical wantq, lapack_logical wantz, lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, double* q, lapack_int ldq, double* z, lapack_int ldz, lapack_int* ifst, lapack_int* ilst);
```

```
lapack_int LAPACKE_ctgexc (int matrix_layout, lapack_logical wantq, lapack_logical wantz, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, lapack_complex_float* q, lapack_int ldq, lapack_complex_float* z, lapack_int ldz, lapack_int ifst, lapack_int ilst);
```

```
lapack_int LAPACKE_ztgexc (int matrix_layout, lapack_logical wantq, lapack_logical wantz, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb, lapack_complex_double* q, lapack_int ldq, lapack_complex_double* z, lapack_int ldz, lapack_int ifst, lapack_int ilst);
```

Include Files

- mkl.h

Description

The routine reorders the generalized real-Schur/Schur decomposition of a real/complex matrix pair (A, B) using an orthogonal/unitary equivalence transformation

$$(A, B) = Q^* (A, B) Z^H,$$

so that the diagonal block of (A, B) with row index *ifst* is moved to row *ilst*. Matrix pair (A, B) must be in a generalized real-Schur/Schur canonical form (as returned by [gges](#)), that is, A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks and B is upper triangular. Optionally, the matrices Q and Z of generalized Schur vectors are updated.

$$Q_{in} * A_{in} * Z_{in}^T = Q_{out} * A_{out} * Z_{out}^T$$

$$Q_{in} * B_{in} * Z_{in}^T = Q_{out} * B_{out} * Z_{out}^T.$$

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

wantq, wantz If *wantq* = 1, update the left transformation matrix Q ;
 If *wantq* = 0, do not update Q ;
 If *wantz* = 1, update the right transformation matrix Z ;
 If *wantz* = 0, do not update Z .

n The order of the matrices A and B ($n \geq 0$).

a, b, q, z Arrays:
a (size $\max(1, lda * n)$) contains the matrix A .
b (size $\max(1, ldb * n)$) contains the matrix B .
q (size at least 1 if *wantq* = 0 and at least $\max(1, ldq * n)$ if *wantq* = 1)

If $wantq = 0$, then q is not referenced.

If $wantq = 1$, then q must contain the orthogonal/unitary matrix Q .

z (size at least 1 if $wantz = 0$ and at least $\max(1, ldz*n)$ if $wantz = 1$)

If $wantz = 0$, then z is not referenced.

If $wantz = 1$, then z must contain the orthogonal/unitary matrix Z .

lda The leading dimension of a ; at least $\max(1, n)$.

ldb The leading dimension of b ; at least $\max(1, n)$.

ldq The leading dimension of q ;

If $wantq = 0$, then $ldq \geq 1$.

If $wantq = 1$, then $ldq \geq \max(1, n)$.

ldz The leading dimension of z ;

If $wantz = 0$, then $ldz \geq 1$.

If $wantz = 1$, then $ldz \geq \max(1, n)$.

$ifst, ilst$ Specify the reordering of the diagonal blocks of (A, B) . The block with row index $ifst$ is moved to row $ilst$, by a sequence of swapping between adjacent blocks. Constraint: $1 \leq ifst, ilst \leq n$.

Output Parameters

a, b, q, z Overwritten by the updated matrices A, B, Q , and Z respectively.

$ifst, ilst$ Overwritten for real flavors only.

If $ifst$ pointed to the second row of a 2 by 2 block on entry, it is changed to point to the first row; $ilst$ always points to the first row of the block in its final position (which may differ from its input value by ± 1).

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = 1$, the transformed matrix pair (A, B) would be too far from generalized Schur form; the problem is ill-conditioned. (A, B) may have been partially reordered, and $ilst$ points to the first row of the current position of the block being moved.

?tgsen

Reorders the generalized Schur decomposition of a pair of matrices (A, B) so that a selected cluster of eigenvalues appears in the leading diagonal blocks of (A, B) .

Syntax

```
lapack_int LAPACKE_stgsen( int matrix_layout, lapack_int ijob, lapack_logical wantq,
lapack_logical wantz, const lapack_logical* select, lapack_int n, float* a, lapack_int
lda, float* b, lapack_int ldb, float* alphas, float* alphas, float* beta, float* q,
lapack_int ldq, float* z, lapack_int ldz, lapack_int* m, float* pl, float* pr, float*
dif );
```

```
lapack_int LAPACKE_dtgsen( int matrix_layout, lapack_int ijob, lapack_logical wantq,
lapack_logical wantz, const lapack_logical* select, lapack_int n, double* a, lapack_int
lda, double* b, lapack_int ldb, double* alphas, double* alphas, double* beta, double*
q, lapack_int ldq, double* z, lapack_int ldz, lapack_int* m, double* pl, double* pr,
double* dif );
```

```
lapack_int LAPACKE_ctgsen( int matrix_layout, lapack_int ijob, lapack_logical wantq,
lapack_logical wantz, const lapack_logical* select, lapack_int n, lapack_complex_float*
a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, lapack_complex_float*
alpha, lapack_complex_float* beta, lapack_complex_float* q, lapack_int ldq,
lapack_complex_float* z, lapack_int ldz, lapack_int* m, float* pl, float* pr, float*
dif );
```

```
lapack_int LAPACKE_ztgsen( int matrix_layout, lapack_int ijob, lapack_logical wantq,
lapack_logical wantz, const lapack_logical* select, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* alpha, lapack_complex_double* beta, lapack_complex_double* q,
lapack_int ldq, lapack_complex_double* z, lapack_int ldz, lapack_int* m, double* pl,
double* pr, double* dif );
```

Include Files

- mkl.h

Description

The routine reorders the generalized real-Schur/Schur decomposition of a real/complex matrix pair (A, B) (in terms of an orthogonal/unitary equivalence transformation $Q^T(A, B) * Z$ for real flavors or $Q^H(A, B) * Z$ for complex flavors), so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the pair (A, B) . The leading columns of Q and Z form orthonormal/unitary bases of the corresponding left and right eigenspaces (deflating subspaces).

(A, B) must be in generalized real-Schur/Schur canonical form (as returned by [gges](#)), that is, A and B are both upper triangular.

?tgsen also computes the generalized eigenvalues

$$\omega_j = (\text{alphar}(j) + \text{alphai}(j)*i)/\text{beta}(j) \text{ (for real flavors)}$$

$$\omega_j = \text{alpha}(j)/\text{beta}(j) \text{ (for complex flavors)}$$

of the reordered matrix pair (A, B) .

Optionally, the routine computes the estimates of reciprocal condition numbers for eigenvalues and eigenspaces. These are $\text{Difu}[(A_{11}, B_{11}), (A_{22}, B_{22})]$ and $\text{Difl}[(A_{11}, B_{11}), (A_{22}, B_{22})]$, that is, the separation(s) between the matrix pairs (A_{11}, B_{11}) and (A_{22}, B_{22}) that correspond to the selected cluster and the eigenvalues outside the cluster, respectively, and norms of "projections" onto left and right eigenspaces with respect to the selected cluster in the (1,1)-block.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>ijob</i> | <p>Specifies whether condition numbers are required for the cluster of eigenvalues (<i>pl</i> and <i>pr</i>) or the deflating subspaces <i>Difu</i> and <i>Difl</i>.</p> <p>If <i>ijob</i> = 0, only reorder with respect to <i>select</i>;</p> <p>If <i>ijob</i> = 1, reciprocal of norms of "projections" onto left and right eigenspaces with respect to the selected cluster (<i>pl</i> and <i>pr</i>);</p> <p>If <i>ijob</i> = 2, compute upper bounds on <i>Difu</i> and <i>Difl</i>, using F-norm-based estimate (<i>dif</i> (1:2));</p> <p>If <i>ijob</i> = 3, compute estimate of <i>Difu</i> and <i>Difl</i>, using 1-norm-based estimate (<i>dif</i> (1:2)). This option is about 5 times as expensive as <i>ijob</i> = 2;</p> <p>If <i>ijob</i> = 4, compute <i>pl</i>, <i>pr</i> and <i>dif</i> (i.e., options 0, 1 and 2 above). This is an economic version to get it all;</p> <p>If <i>ijob</i> = 5, compute <i>pl</i>, <i>pr</i> and <i>dif</i> (i.e., options 0, 1 and 3 above).</p> |
| <i>wantq, wantz</i> | <p>If <i>wantq</i> = 1, update the left transformation matrix <i>Q</i>;</p> <p>If <i>wantq</i> = 0, do not update <i>Q</i>;</p> <p>If <i>wantz</i> = 1, update the right transformation matrix <i>Z</i>;</p> <p>If <i>wantz</i> = 0, do not update <i>Z</i>.</p> |
| <i>select</i> | <p>Array, size at least $\max(1, n)$. Specifies the eigenvalues in the selected cluster.</p> <p>To select an eigenvalue ω_j, <i>select</i>[<i>j</i> - 1] must be 1. For real flavors: to select a complex conjugate pair of eigenvalues ω_j and ω_{j+1} (corresponding 2 by 2 diagonal block), <i>select</i>[<i>j</i> - 1] and/or <i>select</i>[<i>j</i>] must be set to 1; the complex conjugate ω_j and ω_{j+1} must be either both included in the cluster or both excluded.</p> |
| <i>n</i> | The order of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>a, b, q, z</i> | <p>Arrays:</p> <p><i>a</i> (size $\max(1, lda * n)$) contains the matrix <i>A</i>.</p> <p>For real flavors: <i>A</i> is upper quasi-triangular, with (<i>A</i>, <i>B</i>) in generalized real Schur canonical form.</p> <p>For complex flavors: <i>A</i> is upper triangular, in generalized Schur canonical form.</p> <p><i>b</i> (size $\max(1, ldb * n)$) contains the matrix <i>B</i>.</p> <p>For real flavors: <i>B</i> is upper triangular, with (<i>A</i>, <i>B</i>) in generalized real Schur canonical form.</p> <p>For complex flavors: <i>B</i> is upper triangular, in generalized Schur canonical form.</p> <p><i>q</i> (size at least 1 if <i>wantq</i> = 0 and at least $\max(1, ldq * n)$ if <i>wantq</i> = 1)</p> <p>If <i>wantq</i> = 1, then <i>q</i> is an <i>n</i>-by-<i>n</i> matrix;</p> |

If $wantq = 0$, then q is not referenced.
 z (size at least 1 if $wantz = 0$ and at least $\max(1, ldz*n)$ if $wantz = 1$)
 If $wantz = 1$, then z is an n -by- n matrix;
 If $wantz = 0$, then z is not referenced.

lda The leading dimension of a ; at least $\max(1, n)$.

ldb The leading dimension of b ; at least $\max(1, n)$.

ldq The leading dimension of q ; $ldq \geq 1$.
 If $wantq = 1$, then $ldq \geq \max(1, n)$.

ldz The leading dimension of z ; $ldz \geq 1$.
 If $wantz = 1$, then $ldz \geq \max(1, n)$.

Output Parameters

a, b Overwritten by the reordered matrices A and B , respectively.

$alphar, alphai$ Arrays, size at least $\max(1, n)$. Contain values that form generalized eigenvalues in real flavors.
 See *beta*.

$alpha$ Array, size at least $\max(1, n)$. Contain values that form generalized eigenvalues in complex flavors.
 See *beta*.

$beta$ Array, size at least $\max(1, n)$.
For real flavors:
 On exit, $(alphar[j] + alphai[j]*i)/beta[j], j=0, \dots, n-1$, will be the generalized eigenvalues.

$alphar[j] + alphai[j]*i$ and $beta[j], j=0, \dots, n-1$ are the diagonals of the complex Schur form (S, T) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (A, B) were further reduced to triangular form using complex unitary transformations.

If $alphai[j-1]$ is zero, then the j -th eigenvalue is real; if positive, then the j -th and $(j+1)$ -st eigenvalues are a complex conjugate pair, with $alphai[j]$ negative.

For complex flavors:

The diagonal elements of A and B , respectively, when the pair (A, B) has been reduced to generalized Schur form. $alpha[i]/beta[i], i=0, \dots, n-1$ are the generalized eigenvalues.

q If $wantq = 1$, then, on exit, Q has been postmultiplied by the left orthogonal transformation matrix which reorder (A, B) . The leading m columns of Q form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces).

| | |
|-----------------------|--|
| <i>z</i> | If <i>wantz</i> = 1, then, on exit, <i>Z</i> has been postmultiplied by the left orthogonal transformation matrix which reorder (<i>A</i> , <i>B</i>). The leading <i>m</i> columns of <i>Z</i> form orthonormal bases for the specified pair of left eigenspaces (deflating subspaces). |
| <i>m</i> | The dimension of the specified pair of left and right eigen-spaces (deflating subspaces); $0 \leq m \leq n$. |
| <i>pl</i> , <i>pr</i> | If <i>ijob</i> = 1, 4, or 5, <i>pl</i> and <i>pr</i> are lower bounds on the reciprocal of the norm of "projections" onto left and right eigenspaces with respect to the selected cluster. $0 < pl, pr \leq 1$. If $m = 0$ or $m = n$, $pl = pr = 1$. If <i>ijob</i> = 0, 2 or 3, <i>pl</i> and <i>pr</i> are not referenced |
| <i>dif</i> | Array, size (2). If <i>ijob</i> ≥ 2, <i>dif</i> (1:2) store the estimates of Difu and Difl. If <i>ijob</i> = 2 or 4, <i>dif</i> (1:2) are F-norm-based upper bounds on Difu and Difl. If <i>ijob</i> = 3 or 5, <i>dif</i> (1:2) are 1-norm-based estimates of Difu and Difl. If $m = 0$ or $m = n$, <i>dif</i> (1:2) = F-norm([<i>A</i> , <i>B</i>]). If <i>ijob</i> = 0 or 1, <i>dif</i> is not referenced. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = 1, Reordering of (*A*, *B*) failed because the transformed matrix pair (*A*, *B*) would be too far from generalized Schur form; the problem is very ill-conditioned. (*A*, *B*) may have been partially reordered.

If *ijob* > 0, 0 is returned in *dif*, *pl* and *pr*.

?tgsyl

Solves the generalized Sylvester equation.

Syntax

```
lapack_int LAPACKE_stgsyl( int matrix_layout, char trans, lapack_int ijob, lapack_int
m, lapack_int n, const float* a, lapack_int lda, const float* b, lapack_int ldb,
float* c, lapack_int ldc, const float* d, lapack_int ldd, const float* e, lapack_int
lde, float* f, lapack_int ldf, float* scale, float* dif );
```

```
lapack_int LAPACKE_dtgsyl( int matrix_layout, char trans, lapack_int ijob, lapack_int
m, lapack_int n, const double* a, lapack_int lda, const double* b, lapack_int ldb,
double* c, lapack_int ldc, const double* d, lapack_int ldd, const double* e,
lapack_int lde, double* f, lapack_int ldf, double* scale, double* dif );
```

```
lapack_int LAPACKE_ctgsyl( int matrix_layout, char trans, lapack_int ijob, lapack_int
m, lapack_int n, const lapack_complex_float* a, lapack_int lda, const
lapack_complex_float* b, lapack_int ldb, lapack_complex_float* c, lapack_int ldc, const
lapack_complex_float* d, lapack_int ldd, const lapack_complex_float* e, lapack_int lde,
lapack_complex_float* f, lapack_int ldf, float* scale, float* dif );
```

```
lapack_int LAPACKE_ztgsyl( int matrix_layout, char trans, lapack_int ijob, lapack_int
m, lapack_int n, const lapack_complex_double* a, lapack_int lda, const
lapack_complex_double* b, lapack_int ldb, lapack_complex_double* c, lapack_int ldc,
const lapack_complex_double* d, lapack_int ldd, const lapack_complex_double* e,
lapack_int lde, lapack_complex_double* f, lapack_int ldf, double* scale, double* dif );
```

Include Files

- mkl.h

Description

The routine solves the generalized Sylvester equation:

$$A^*R - L^*B = scale^*C$$

$$D^*R - L^*E = scale^*F$$

where R and L are unknown m -by- n matrices, (A, D) , (B, E) and (C, F) are given matrix pairs of size m -by- m , n -by- n and m -by- n , respectively, with real/complex entries. (A, D) and (B, E) must be in generalized real-Schur/Schur canonical form, that is, A, B are upper quasi-triangular/triangular and D, E are upper triangular.

The solution (R, L) overwrites (C, F) . The factor $scale$, $0 \leq scale \leq 1$, is an output scaling factor chosen to avoid overflow.

In matrix notation the above equation is equivalent to the following: solve $Z^*x = scale^*b$, where Z is defined as

$$Z = \begin{pmatrix} \text{kron}(I_n, A) & -\text{kron}(B^T, I_m) \\ \text{kron}(I_n, D) & -\text{kron}(E^T, I_m) \end{pmatrix}$$

Here I_k is the identity matrix of size k and X^T is the transpose/conjugate-transpose of X . $\text{kron}(X, Y)$ is the Kronecker product between the matrices X and Y .

If $trans = 'T'$ (for real flavors), or $trans = 'C'$ (for complex flavors), the routine `?tgsyl` solves the transposed/conjugate-transposed system $Z^T*y = scale^*b$, which is equivalent to solve for R and L in

$$A^T*R + D^T*L = scale^*C$$

$$R*B^T + L*E^T = scale^*(-F)$$

This case ($trans = 'T'$ for `stgsyl/dtgsyl` or $trans = 'C'$ for `ctgsyl/ztgsyl`) is used to compute an one-norm-based estimate of $\text{Dif}[(A, D), (B, E)]$, the separation between the matrix pairs (A, D) and (B, E) .

If $ijob \geq 1$, `?tgsyl` computes a Frobenius norm-based estimate of $\text{Dif}[(A, D), (B, E)]$. That is, the reciprocal of a lower bound on the reciprocal of the smallest singular value of Z . This is a level 3 BLAS algorithm.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>trans</code> | Must be <code>'N'</code> , <code>'T'</code> , or <code>'C'</code> . If $trans = 'N'$, solve the generalized Sylvester equation. If $trans = 'T'$, solve the 'transposed' system (for real flavors only). |

| | |
|-------------------------|--|
| | If <i>trans</i> = 'C', solve the 'conjugate transposed' system (for complex flavors only). |
| <i>ijob</i> | Specifies what kind of functionality to be performed: If <i>ijob</i> =0, solve the generalized Sylvester equation only; If <i>ijob</i> =1, perform the functionality of <i>ijob</i> =0 and <i>ijob</i> =3; If <i>ijob</i> =2, perform the functionality of <i>ijob</i> =0 and <i>ijob</i> =4; If <i>ijob</i> =3, only an estimate of $\text{Dif}[(A, D), (B, E)]$ is computed (look ahead strategy is used); If <i>ijob</i> =4, only an estimate of $\text{Dif}[(A, D), (B, E)]$ is computed (?gecon on sub-systems is used). If <i>trans</i> = 'T' or 'C', <i>ijob</i> is not referenced. |
| <i>m</i> | The order of the matrices <i>A</i> and <i>D</i> , and the row dimension of the matrices <i>C</i> , <i>F</i> , <i>R</i> and <i>L</i> . |
| <i>n</i> | The order of the matrices <i>B</i> and <i>E</i> , and the column dimension of the matrices <i>C</i> , <i>F</i> , <i>R</i> and <i>L</i> . |
| <i>a, b, c, d, e, f</i> | Arrays: <i>a</i> (size $\max(1, lda * m)$) contains the upper quasi-triangular (for real flavors) or upper triangular (for complex flavors) matrix <i>A</i> . <i>b</i> (size $\max(1, ldb * n)$) contains the upper quasi-triangular (for real flavors) or upper triangular (for complex flavors) matrix <i>B</i> . <i>c</i> (size $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout) contains the right-hand-side of the first matrix equation in the generalized Sylvester equation (as defined by <i>trans</i>) <i>d</i> (size $\max(1, ldd * m)$) contains the upper triangular matrix <i>D</i> . <i>e</i> (size $\max(1, lde * n)$) contains the upper triangular matrix <i>E</i> . <i>f</i> (size $\max(1, ldf * n)$ for column major layout and $\max(1, ldf * m)$ for row major layout) contains the right-hand-side of the second matrix equation in the generalized Sylvester equation (as defined by <i>trans</i>) |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, n)$. |
| <i>ldc</i> | The leading dimension of <i>c</i> ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout . |
| <i>ldd</i> | The leading dimension of <i>d</i> ; at least $\max(1, m)$. |
| <i>lde</i> | The leading dimension of <i>e</i> ; at least $\max(1, n)$. |
| <i>ldf</i> | The leading dimension of <i>f</i> ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout . |

Output Parameters

| | |
|----------|--|
| <i>c</i> | If <i>ijob</i> =0, 1, or 2, overwritten by the solution <i>R</i> . |
|----------|--|

| | |
|---------|---|
| | If $ijob=3$ or 4 and $trans = 'N'$, c holds R , the solution achieved during the computation of the Dif-estimate. |
| f | If $ijob=0, 1,$ or 2 , overwritten by the solution L . If $ijob=3$ or 4 and $trans = 'N'$, f holds L , the solution achieved during the computation of the Dif-estimate. |
| dif | On exit, dif is the reciprocal of a lower bound of the reciprocal of the Dif-function, that is, dif is an upper bound of $\text{Dif}[(A, D), (B, E)] = \text{sigma_min}(Z)$, where Z as defined in the description. If $ijob = 0$, or $trans = 'T'$ (for real flavors), or $trans = 'C'$ (for complex flavors), dif is not touched. |
| $scale$ | On exit, $scale$ is the scaling factor in the generalized Sylvester equation. If $0 < scale < 1$, c and f hold the solutions R and L , respectively, to a slightly perturbed system but the input matrices A, B, D and E have not been changed. If $scale = 0$, c and f hold the solutions R and L , respectively, to the homogeneous system with $C = F = 0$. Normally, $scale = 1$. |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info > 0$, (A, D) and (B, E) have common or close eigenvalues.

?tgsna

Estimates reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a pair of matrices in generalized real Schur canonical form.

Syntax

```
lapack_int LAPACKE_stgsna( int matrix_layout, char job, char howmny, const
lapack_logical* select, lapack_int n, const float* a, lapack_int lda, const float* b,
lapack_int ldb, const float* vl, lapack_int ldvl, const float* vr, lapack_int ldvr,
float* s, float* dif, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_dtgsna( int matrix_layout, char job, char howmny, const
lapack_logical* select, lapack_int n, const double* a, lapack_int lda, const double*
b, lapack_int ldb, const double* vl, lapack_int ldvl, const double* vr, lapack_int
ldvr, double* s, double* dif, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ctgsna( int matrix_layout, char job, char howmny, const
lapack_logical* select, lapack_int n, const lapack_complex_float* a, lapack_int lda,
const lapack_complex_float* b, lapack_int ldb, const lapack_complex_float* vl,
lapack_int ldvl, const lapack_complex_float* vr, lapack_int ldvr, float* s, float*
dif, lapack_int mm, lapack_int* m );
```

```
lapack_int LAPACKE_ztgsna( int matrix_layout, char job, char howmny, const
lapack_logical* select, lapack_int n, const lapack_complex_double* a, lapack_int lda,
const lapack_complex_double* b, lapack_int ldb, const lapack_complex_double* vl,
lapack_int ldvl, const lapack_complex_double* vr, lapack_int ldvr, double* s, double*
dif, lapack_int mm, lapack_int* m );
```

Include Files

- mkl.h

Description

The real flavors `stgsna/dtgsna` of this routine estimate reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B) in generalized real Schur canonical form (or of any matrix pair $(Q^*A^*Z^T, Q^*B^*Z^T)$ with orthogonal matrices Q and Z).

(A, B) must be in generalized real Schur form (as returned by `gges/gges`), that is, A is block upper triangular with 1-by-1 and 2-by-2 diagonal blocks. B is upper triangular.

The complex flavors `ctgsna/ztgsna` estimate reciprocal condition numbers for specified eigenvalues and/or eigenvectors of a matrix pair (A, B) . (A, B) must be in generalized Schur canonical form, that is, A and B are both upper triangular.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>job</code> | Specifies whether condition numbers are required for eigenvalues or eigenvectors. Must be 'E' or 'V' or 'B'. If <code>job = 'E'</code> , for eigenvalues only (compute s). If <code>job = 'V'</code> , for eigenvectors only (compute dif). If <code>job = 'B'</code> , for both eigenvalues and eigenvectors (compute both s and dif). |
| <code>howmny</code> | Must be 'A' or 'S'. If <code>howmny = 'A'</code> , compute condition numbers for all eigenpairs. If <code>howmny = 'S'</code> , compute condition numbers for selected eigenpairs specified by the logical array <code>select</code> . |
| <code>select</code> | Array, size at least $\max(1, n)$. If <code>howmny = 'S'</code> , <code>select</code> specifies the eigenpairs for which condition numbers are required. If <code>howmny = 'A'</code> , <code>select</code> is not referenced. <i>For real flavors:</i> To select condition numbers for the eigenpair corresponding to a real eigenvalue ω_j , <code>select[j - 1]</code> must be set to 1; to select condition numbers corresponding to a complex conjugate pair of eigenvalues ω_j and ω_{j+1} , either <code>select[j - 1]</code> or <code>select[j]</code> must be set to 1. <i>For complex flavors:</i> |

| | |
|---------------------------|--|
| | To select condition numbers for the corresponding j -th eigenvalue and/or eigenvector, <code>select[j - 1]</code> must be set to 1. |
| <code>n</code> | The order of the square matrix pair (A, B) ($n \geq 0$). |
| <code>a, b, vl, vr</code> | <p>Arrays:</p> <p><code>a</code> (size $\max(1, lda * n)$) contains the upper quasi-triangular (for real flavors) or upper triangular (for complex flavors) matrix A in the pair (A, B).</p> <p><code>b</code> (size $\max(1, ldb * n)$) contains the upper triangular matrix B in the pair (A, B).</p> <p>If <code>job = 'E'</code> or <code>'B'</code>, <code>vl</code> (size $\max(1, ldvl * m)$ for column major layout and $\max(1, ldvl * n)$ for row major layout) must contain left eigenvectors of (A, B), corresponding to the eigenpairs specified by <code>howmny</code> and <code>select</code>. The eigenvectors must be stored in consecutive columns of <code>vl</code>, as returned by <code>?tgevc</code>.</p> <p>If <code>job = 'V'</code>, <code>vl</code> is not referenced.</p> <p>If <code>job = 'E'</code> or <code>'B'</code>, <code>vr</code> (size $\max(1, ldvr * m)$ for column major layout and $\max(1, ldvr * n)$ for row major layout) must contain right eigenvectors of (A, B), corresponding to the eigenpairs specified by <code>howmny</code> and <code>select</code>. The eigenvectors must be stored in consecutive columns of <code>vr</code>, as returned by <code>?tgevc</code>.</p> <p>If <code>job = 'V'</code>, <code>vr</code> is not referenced.</p> |
| <code>lda</code> | The leading dimension of <code>a</code> ; at least $\max(1, n)$. |
| <code>ldb</code> | The leading dimension of <code>b</code> ; at least $\max(1, n)$. |
| <code>ldvl</code> | <p>The leading dimension of <code>vl</code>; $ldvl \geq 1$.</p> <p>If <code>job = 'E'</code> or <code>'B'</code>, then $ldvl \geq \max(1, n)$ for column major layout and $ldvl \geq \max(1, m)$ for row major layout .</p> |
| <code>ldvr</code> | <p>The leading dimension of <code>vr</code>; $ldvr \geq 1$.</p> <p>If <code>job = 'E'</code> or <code>'B'</code>, then $ldvr \geq \max(1, n)$ for column major layout and $ldvr \geq \max(1, m)$ for row major layout.</p> |
| <code>mm</code> | The number of elements in the arrays <code>s</code> and <code>dif</code> ($mm \geq m$). |

Output Parameters

| | |
|----------------|---|
| <code>s</code> | <p>Array, size <code>mm</code>.</p> <p>If <code>job = 'E'</code> or <code>'B'</code>, contains the reciprocal condition numbers of the selected eigenvalues, stored in consecutive elements of the array.</p> <p>If <code>job = 'V'</code>, <code>s</code> is not referenced.</p> <p><i>For real flavors:</i></p> |
|----------------|---|

For a complex conjugate pair of eigenvalues two consecutive elements of s are set to the same value. Thus, $s[j - 1]$, $dif[j - 1]$, and the j -th columns of vl and vr all correspond to the same eigenpair (but not in general the j -th eigenpair, unless all eigenpairs are selected).

dif

Array, size mm .

If $job = 'V'$ or $'B'$, contains the estimated reciprocal condition numbers of the selected eigenvectors, stored in consecutive elements of the array.

If the eigenvalues cannot be reordered to compute $dif[j]$, $dif[j]$ is set to 0; this can only occur when the true value would be very small anyway.

If $job = 'E'$, dif is not referenced.

For real flavors:

For a complex eigenvector, two consecutive elements of dif are set to the same value.

For complex flavors:

For each eigenvalue/vector specified by $select$, dif stores a Frobenius norm-based estimate of $Difl$.

m

The number of elements in the arrays s and dif used to store the specified condition numbers; for each selected eigenvalue one element is used.

If $howmny = 'A'$, m is set to n .

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

Generalized Singular Value Decomposition: LAPACK Computational Routines

This section describes LAPACK computational routines used for finding the generalized singular value decomposition (GSVD) of two matrices A and B as

$$U^H A Q = D_1 * (0 \ R),$$

$$V^H B Q = D_2 * (0 \ R),$$

where U , V , and Q are orthogonal/unitary matrices, R is a nonsingular upper triangular matrix, and D_1 , D_2 are "diagonal" matrices of the structure detailed in the routines description section.

Table "Computational Routines for Generalized Singular Value Decomposition" lists LAPACK routines that perform generalized singular value decomposition of matrices.

Computational Routines for Generalized Singular Value Decomposition

| Routine name | Operation performed |
|------------------------|--|
| ggsvp | Computes the preprocessing decomposition for the generalized SVD |
| ggsvp3 | Performs preprocessing for a generalized SVD. |
| ggsvd3 | Computes generalized SVD. |
| tgsja | Computes the generalized SVD of two upper triangular or trapezoidal matrices |

You can use routines listed in the above table as well as the driver routine [ggsvd](#) to find the GSVD of a pair of general rectangular matrices.

?ggsvp

Computes the preprocessing decomposition for the generalized SVD (deprecated).

Syntax

```
lapack_int LAPACKE_sggsvp( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, float* a, lapack_int lda, float* b,
lapack_int ldb, float tola, float tolb, lapack_int* k, lapack_int* l, float* u,
lapack_int ldu, float* v, lapack_int ldv, float* q, lapack_int ldq );
```

```
lapack_int LAPACKE_dggsvp( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, double* a, lapack_int lda, double* b,
lapack_int ldb, double tola, double tolb, lapack_int* k, lapack_int* l, double* u,
lapack_int ldu, double* v, lapack_int ldv, double* q, lapack_int ldq );
```

```
lapack_int LAPACKE_cggsvp( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_complex_float* a, lapack_int lda,
lapack_complex_float* b, lapack_int ldb, float tola, float tolb, lapack_int* k,
lapack_int* l, lapack_complex_float* u, lapack_int ldu, lapack_complex_float* v,
lapack_int ldv, lapack_complex_float* q, lapack_int ldq );
```

```
lapack_int LAPACKE_zggsvp( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_complex_double* a, lapack_int lda,
lapack_complex_double* b, lapack_int ldb, double tola, double tolb, lapack_int* k,
lapack_int* l, lapack_complex_double* u, lapack_int ldu, lapack_complex_double* v,
lapack_int ldv, lapack_complex_double* q, lapack_int ldq );
```

Include Files

- mkl.h

Description

This routine is deprecated; use [ggsvp3](#).

The routine computes orthogonal matrices U , V and Q such that

$$U^H A Q = \begin{matrix} & n - k - l & k & l \\ & k & & \\ & l & & \\ m - k - l & & & \end{matrix} \begin{pmatrix} 0 & A_{12} & A_{13} \\ 0 & 0 & A_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{if } m - k - l \geq 0$$

$$= \begin{matrix} & n - k - l & k & l \\ & k & & \\ m - k & & & \end{matrix} \begin{pmatrix} 0 & A_{12} & A_{13} \\ 0 & 0 & A_{23} \end{pmatrix}, \quad \text{if } m - k - l < 0$$

$$V^H B Q = \begin{matrix} & n-k-l & k & l \\ & l & \begin{pmatrix} 0 & 0 & B_{13} \\ 0 & 0 & 0 \end{pmatrix} \\ p-l & & & \end{matrix}$$

where the k -by- k matrix A_{12} and l -by- l matrix B_{13} are nonsingular upper triangular; A_{23} is l -by- l upper triangular if $m-k-l \geq 0$, otherwise A_{23} is $(m-k)$ -by- l upper trapezoidal. The sum $k+l$ is equal to the effective numerical rank of the $(m+p)$ -by- n matrix $(A^H, B^H)^H$.

This decomposition is the preprocessing step for computing the Generalized Singular Value Decomposition (GSVD), see subroutine [?tgsja](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobu</i> | Must be 'U' or 'N'. If <i>jobu</i> = 'U', orthogonal/unitary matrix U is computed. If <i>jobu</i> = 'N', U is not computed. |
| <i>jobv</i> | Must be 'V' or 'N'. If <i>jobv</i> = 'V', orthogonal/unitary matrix V is computed. If <i>jobv</i> = 'N', V is not computed. |
| <i>jobq</i> | Must be 'Q' or 'N'. If <i>jobq</i> = 'Q', orthogonal/unitary matrix Q is computed. If <i>jobq</i> = 'N', Q is not computed. |
| <i>m</i> | The number of rows of the matrix A ($m \geq 0$). |
| <i>p</i> | The number of rows of the matrix B ($p \geq 0$). |
| <i>n</i> | The number of columns of the matrices A and B ($n \geq 0$). |
| <i>a, b</i> | Arrays: a (size at least $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout) contains the m -by- n matrix A . b (size at least $\max(1, ldb*n)$ for column major layout and $\max(1, ldb*p)$ for row major layout) contains the p -by- n matrix B . |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of b ; at least $\max(1, p)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>tola, tol b</i> | <i>tola</i> and <i>tol b</i> are the thresholds to determine the effective numerical rank of matrix B and a subblock of A . Generally, they are set to $tola = \max(m, n) * A * \text{MACHEPS}$, |

$tolb = \max(p, n) * ||B|| * \text{MACHEPS}$.

The size of $tola$ and $tolb$ may affect the size of backward errors of the decomposition.

ldu The leading dimension of the output array u . $ldu \geq \max(1, m)$ if $jobu = 'U'$; $ldu \geq 1$ otherwise.

ldv The leading dimension of the output array v . $ldv \geq \max(1, p)$ if $jobv = 'V'$; $ldv \geq 1$ otherwise.

ldq The leading dimension of the output array q . $ldq \geq \max(1, n)$ if $jobq = 'Q'$; $ldq \geq 1$ otherwise.

Output Parameters

a Overwritten by the triangular (or trapezoidal) matrix described in the *Description* section.

b Overwritten by the triangular matrix described in the *Description* section.

k, l On exit, k and l specify the dimension of subblocks. The sum $k + l$ is equal to effective numerical rank of $(A^H, B^H)^H$.

u, v, q Arrays:

If $jobu = 'U'$, u (size $\max(1, ldu * m)$) contains the orthogonal/unitary matrix U .

If $jobu = 'N'$, u is not referenced.

If $jobv = 'V'$, v (size $\max(1, ldv * p)$) contains the orthogonal/unitary matrix V .

If $jobv = 'N'$, v is not referenced.

If $jobq = 'Q'$, q (size $\max(1, ldq * n)$) contains the orthogonal/unitary matrix Q .

If $jobq = 'N'$, q is not referenced.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

?ggsvp3

Performs preprocessing for a generalized SVD.

Syntax

```
lapack_int LAPACKE_sggsvp3 (int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, float * a, lapack_int lda, float * b,
lapack_int ldb, float tola, float tolb, lapack_int * k, lapack_int * l, float * u,
lapack_int ldu, float * v, lapack_int ldv, float * q, lapack_int ldq);
```

```
lapack_int LAPACKE_dggsvp3 (int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, double * a, lapack_int lda, double * b,
lapack_int ldb, double tola, double tolb, lapack_int * k, lapack_int * l, double * u,
lapack_int ldu, double * v, lapack_int ldv, double * q, lapack_int ldq);
```

```
lapack_int LAPACKE_cggsvp3 (int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_complex_float * a, lapack_int lda,
lapack_complex_float * b, lapack_int ldb, float tola, float tolb, lapack_int * k,
lapack_int * l, lapack_complex_float * u, lapack_int ldu, lapack_complex_float * v,
lapack_int ldv, lapack_complex_float * q, lapack_int ldq);
```

```
lapack_int LAPACKE_zggsvp3 (int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_complex_double * a, lapack_int lda,
lapack_complex_double * b, lapack_int ldb, double tola, double tolb, lapack_int * k,
lapack_int * l, lapack_complex_double * u, lapack_int ldu, lapack_complex_double * v,
lapack_int ldv, lapack_complex_double * q, lapack_int ldq);
```

Include Files

- mkl_lapack.h

Include Files

- mkl.h

Description

?ggsvp3 computes orthogonal or unitary matrices U , V , and Q such that

for real flavors:

$$U^T A Q = \begin{matrix} & & n-k-l & k & l \\ & k & & & \\ & l & & & \\ m-k-l & & & & \end{matrix} \begin{pmatrix} 0 & A12 & A13 \\ 0 & 0 & A23 \\ 0 & 0 & 0 \end{pmatrix} \text{ if } m-k-l \geq 0;$$

$$U^T A Q = \begin{matrix} & & n-k-l & k & l \\ & k & & & \\ m-k & & & & \end{matrix} \begin{pmatrix} 0 & A12 & A13 \\ 0 & 0 & A23 \end{pmatrix} \text{ if } m-k-l < 0;$$

$$V^T B Q = \begin{matrix} & & n-k-l & k & l \\ l & & & & \\ p-l & & & & \end{matrix} \begin{pmatrix} 0 & 0 & B13 \\ 0 & 0 & 0 \end{pmatrix}$$

for complex flavors:

$$U^H A Q = \begin{matrix} & & n-k-l & k & l \\ & k & & & \\ & l & & & \\ m-k-l & & & & \end{matrix} \begin{pmatrix} 0 & A12 & A13 \\ 0 & 0 & A23 \\ 0 & 0 & 0 \end{pmatrix} \text{ if } m-k-l \geq 0;$$

$$U^H A Q = \begin{matrix} & & n-k-l & k & l \\ & k & & & \\ m-k & & & & \end{matrix} \begin{pmatrix} 0 & A12 & A13 \\ 0 & 0 & A23 \end{pmatrix} \text{ if } m-k-l < 0;$$

$$V^H B Q = \begin{matrix} & & n-k-l & k & l \\ l & & & & \\ p-l & & & & \end{matrix} \begin{pmatrix} 0 & 0 & B13 \\ 0 & 0 & 0 \end{pmatrix}$$

where the k -by- k matrix $A12$ and l -by- l matrix $B13$ are nonsingular upper triangular; $A23$ is l -by- l upper triangular if $m-k-l \geq 0$, otherwise $A23$ is $(m-k)$ -by- l upper trapezoidal. $k + l =$ the effective numerical rank of the $(m + p)$ -by- n matrix $(A^T, B^T)^T$ for real flavors or $(A^H, B^H)^H$ for complex flavors.

This decomposition is the preprocessing step for computing the Generalized Singular Value Decomposition (GSVD), see `?ggsvd3`.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>jobu</code> | = 'U': Orthogonal/unitary matrix U is computed; = 'N': U is not computed. |
| <code>jobv</code> | = 'V': Orthogonal/unitary matrix V is computed; = 'N': V is not computed. |
| <code>jobq</code> | = 'Q': Orthogonal/unitary matrix Q is computed; = 'N': Q is not computed. |
| <code>m</code> | The number of rows of the matrix A . $m \geq 0$. |
| <code>p</code> | The number of rows of the matrix B . $p \geq 0$. |
| <code>n</code> | The number of columns of the matrices A and B . $n \geq 0$. |
| <code>a</code> | Array, size $(lda * n)$. On entry, the m -by- n matrix A . |
| <code>lda</code> | The leading dimension of the array a . $lda \geq \max(1, m)$. |
| <code>b</code> | Array, size $(ldb * n)$. On entry, the p -by- n matrix B . |
| <code>ldb</code> | The leading dimension of the array b . $ldb \geq \max(1, p)$. |
| <code>tolu, tol</code> | $tolu$ and tol are the thresholds to determine the effective numerical rank of matrix B and a subblock of A . Generally, they are set to $tolu = \max(m, n) * \text{norm}(a) * \text{MACHEPS}$, $tol = \max(p, n) * \text{norm}(b) * \text{MACHEPS}$. The size of $tolu$ and tol may affect the size of backward errors of the decomposition. |
| <code>ldu</code> | The leading dimension of the array u . |

| | |
|-------|---|
| | $ldu \geq \max(1, m)$ if $jobu = 'U'$; $ldu \geq 1$ otherwise. |
| ldv | The leading dimension of the array v . $ldv \geq \max(1, p)$ if $jobv = 'V'$; $ldv \geq 1$ otherwise. |
| ldq | The leading dimension of the array q . $ldq \geq \max(1, n)$ if $jobq = 'Q'$; $ldq \geq 1$ otherwise. |

Output Parameters

| | |
|--------|---|
| a | On exit, a contains the triangular (or trapezoidal) matrix described in the Description section. |
| b | On exit, b contains the triangular matrix described in the Description section. |
| k, l | On exit, k and l specify the dimension of the subblocks described in Description section. $k + l =$ effective numerical rank of $(A^T, B^T)^T$ for real flavors or $(A^H, B^H)^H$ for complex flavors. |
| u | Array, size $(ldu * m)$. If $jobu = 'U'$, u contains the orthogonal/unitary matrix U . If $jobu = 'N'$, u is not referenced. |
| v | Array, size $(ldv * p)$. If $jobv = 'V'$, v contains the orthogonal/unitary matrix V . If $jobv = 'N'$, v is not referenced. |
| q | Array, size $(ldq * n)$. If $jobq = 'Q'$, q contains the orthogonal/unitary matrix Q . If $jobq = 'N'$, q is not referenced. |

Return Values

This function returns a value $info$.

= 0: successful exit.

< 0: if $info = -i$, the i -th argument had an illegal value.

Application Notes

The subroutine uses LAPACK subroutine `?geqp3` for the QR factorization with column pivoting to detect the effective numerical rank of the A matrix. It may be replaced by a better rank determination strategy.

`?ggsvp3` replaces the deprecated subroutine `?ggsvp`.

?ggsvd3

Computes generalized SVD.

Syntax

```
lapack_int LAPACKE_sggsvd3 (int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int * k, lapack_int * l, float * a,
lapack_int lda, float * b, lapack_int ldb, float * alpha, float * beta, float * u,
lapack_int ldu, float * v, lapack_int ldv, float * q, lapack_int ldq, lapack_int *
iwork);
```

```
lapack_int LAPACKE_dggsvd3 (int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int * k, lapack_int * l, double * a,
lapack_int lda, double * b, lapack_int ldb, double * alpha, double * beta, double * u,
lapack_int ldu, double * v, lapack_int ldv, double * q, lapack_int ldq, lapack_int *
iwork);
```

```
lapack_int LAPACKE_cggsvd3 (int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int * k, lapack_int * l,
lapack_complex_float * a, lapack_int lda, lapack_complex_float * b, lapack_int ldb,
float * alpha, float * beta, lapack_complex_float * u, lapack_int ldu,
lapack_complex_float * v, lapack_int ldv, lapack_complex_float * q, lapack_int ldq,
lapack_int * iwork);
```

```
lapack_int LAPACKE_zggsvd3 (int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int * k, lapack_int * l,
lapack_complex_double * a, lapack_int lda, lapack_complex_double * b, lapack_int ldb,
double * alpha, double * beta, lapack_complex_double * u, lapack_int ldu,
lapack_complex_double * v, lapack_int ldv, lapack_complex_double * q, lapack_int ldq,
lapack_int * iwork);
```

Include Files

- mkl.h

Description

?ggsvd3 computes the generalized singular value decomposition (GSVD) of an m -by- n real or complex matrix A and p -by- n real or complex matrix B :

$$U^T A Q = D_1 (0 R), V^T B Q = D_2 (0 R) \text{ for real flavors}$$

or

$$U^H A Q = D_1 (0 R), V^H B Q = D_2 (0 R) \text{ for complex flavors}$$

where U , V and Q are orthogonal/unitary matrices.

Let $k+1$ = the effective numerical rank of the matrix $(A^T B^T)^T$ for real flavors or the matrix $(A^H B^H)^H$ for complex flavors, then R is a $(k+1)$ -by- $(k+1)$ nonsingular upper triangular matrix, D_1 and D_2 are m -by- $(k+1)$ and p -by- $(k+1)$ "diagonal" matrices and of the following structures, respectively:

If $m-k-1 \geq 0$,

$$D_1 = \begin{matrix} & & k & l \\ & & \begin{pmatrix} I & 0 \\ 0 & C \end{pmatrix} \\ m-k-l & \begin{pmatrix} 0 & 0 \end{pmatrix} \end{matrix}$$

$$D_2 = \begin{matrix} & & k & l \\ & & \begin{pmatrix} 0 & S \end{pmatrix} \\ p-l & \begin{pmatrix} 0 & 0 \end{pmatrix} \end{matrix}$$

$$\begin{pmatrix} 0 & R \end{pmatrix} = \begin{matrix} & n-k-l & k & l \\ k & \begin{pmatrix} 0 & R11 & R12 \\ 0 & 0 & R22 \end{pmatrix} \end{matrix}$$

where

$$C = \text{diag}(\alpha(k+1), \dots, \alpha(k+l)),$$

$$S = \text{diag}(\beta(k+1), \dots, \beta(k+l)),$$

$$C^2 + S^2 = I.$$

If $m - k - l < 0$,

$$D_1 = \begin{matrix} & k & m-k & k+l-m \\ m-k & \begin{pmatrix} I & 0 & 0 \\ 0 & C & 0 \end{pmatrix} \end{matrix}$$

$$D_2 = \begin{matrix} & m-k & k+l-m & p-l \\ k+l-m & \begin{pmatrix} 0 & S & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$\begin{pmatrix} 0 & R \end{pmatrix} = \begin{matrix} & n-k-l & k & m-k & k+l-m \\ k & \begin{pmatrix} 0 & R11 & R12 & R13 \\ m-k & 0 & R22 & R23 \\ k+l-m & 0 & 0 & R33 \end{pmatrix} \end{matrix}$$

where

$$C = \text{diag}(\alpha(k+1), \dots, \alpha(m)),$$

$$S = \text{diag}(\beta(k+1), \dots, \beta(m)),$$

$$C^2 + S^2 = I.$$

The routine computes C , S , R , and optionally the orthogonal/unitary transformation matrices U , V and Q .

In particular, if B is an n -by- n nonsingular matrix, then the GSVD of A and B implicitly gives the SVD of $A \cdot \text{inv}(B)$:

$$A \cdot \text{inv}(B) = U \cdot (D_1 \cdot \text{inv}(D_2)) \cdot V^T \text{ for real flavors}$$

or

$$A \cdot \text{inv}(B) = U \cdot (D_1 \cdot \text{inv}(D_2)) \cdot V^H \text{ for complex flavors.}$$

If $(A^T, B^T)^T$ for real flavors or $(A^H, B^H)^H$ for complex flavors has orthonormal columns, then the GSVD of A and B is also equal to the CS decomposition of A and B . Furthermore, the GSVD can be used to derive the solution of the eigenvalue problem:

$$A^T \cdot A \cdot X = \lambda \cdot B^T \cdot B \cdot X \text{ for real flavors}$$

or

$$A^H \cdot A \cdot X = \lambda \cdot B^H \cdot B \cdot X \text{ for complex flavors}$$

In some literature, the GSVD of A and B is presented in the form

$$U^T \cdot A \cdot X = (0 \ D_1), \quad V^T \cdot B \cdot X = (0 \ D_2) \text{ for real } (A, B)$$

or

$$U^H \cdot A \cdot X = (0 \ D_1), \quad V^H \cdot B \cdot X = (0 \ D_2) \text{ for complex } (A, B)$$

where U and V are orthogonal and X is nonsingular, D_1 and D_2 are "diagonal". The former GSVD form can be converted to the latter form by taking the nonsingular matrix X as

$$X = Q * \begin{pmatrix} I & 0 \\ 0 & inv(R) \end{pmatrix}$$

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobu</i> | = 'U': Orthogonal/unitary matrix <i>U</i> is computed; = 'N': <i>U</i> is not computed. |
| <i>jobv</i> | = 'V': Orthogonal/unitary matrix <i>V</i> is computed; = 'N': <i>V</i> is not computed. |
| <i>jobq</i> | = 'Q': Orthogonal/unitary matrix <i>Q</i> is computed; = 'N': <i>Q</i> is not computed. |
| <i>m</i> | The number of rows of the matrix <i>A</i> . $m \geq 0$. |
| <i>n</i> | The number of columns of the matrices <i>A</i> and <i>B</i> . $n \geq 0$. |
| <i>p</i> | The number of rows of the matrix <i>B</i> . $p \geq 0$. |
| <i>a</i> | Array, size (<i>lda</i> * <i>n</i>). On entry, the <i>m</i> -by- <i>n</i> matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1, m)$. |
| <i>b</i> | Array, size (<i>ldb</i> * <i>n</i>). On entry, the <i>p</i> -by- <i>n</i> matrix <i>B</i> . |
| <i>ldb</i> | The leading dimension of the array <i>b</i> . $ldb \geq \max(1, p)$. |
| <i>ldu</i> | The leading dimension of the array <i>u</i> . $ldu \geq \max(1, m)$ if <i>jobu</i> = 'U'; $ldu \geq 1$ otherwise. |
| <i>ldv</i> | The leading dimension of the array <i>v</i> . $ldv \geq \max(1, p)$ if <i>jobv</i> = 'V'; $ldv \geq 1$ otherwise. |
| <i>ldq</i> | The leading dimension of the array <i>q</i> . $ldq \geq \max(1, n)$ if <i>jobq</i> = 'Q'; $ldq \geq 1$ otherwise. |
| <i>iwork</i> | Array, size (<i>n</i>). |

Output Parameters

| | |
|--------------|--|
| <i>k, l</i> | <p>On exit, <i>k</i> and <i>l</i> specify the dimension of the subblocks described in the Description section.</p> <p>$k + l =$ effective numerical rank of $(A^T, B^T)^T$ for real flavors or $(A^H, B^H)^H$ for complex flavors.</p> |
| <i>a</i> | <p>On exit, <i>a</i> contains the triangular matrix <i>R</i>, or part of <i>R</i>.</p> <p>If $m - k - l \geq 0$, <i>R</i> is stored in the elements of array <i>a</i> corresponding to $A_{1:k+l, n-k-l+1:n}$.</p> <p>If $m - k - l < 0$, $\begin{pmatrix} R_{11} & R_{12} & R_{13} \\ 0 & R_{22} & R_{23} \end{pmatrix}$ is stored in the elements of array <i>a</i> corresponding to $A_{(1:m, n-k-l+1:n)}$, and <i>R33</i> is stored in the elements of array <i>a</i> corresponding to $A_{m-k+1:l, n+m-k-l+1:n}$ on exit.</p> |
| <i>b</i> | <p>On exit, <i>b</i> contains part of the triangular matrix <i>R</i> if $m - k - l < 0$.</p> <p>See Description for details.</p> |
| <i>alpha</i> | Array, size (<i>n</i>) |
| <i>beta</i> | <p>Array, size (<i>n</i>)</p> <p>On exit, <i>alpha</i> and <i>beta</i> contain the generalized singular value pairs of <i>a</i> and <i>b</i>;</p> <p>$alpha[0:k-1] = 1$,</p> <p>$beta[0:k-1] = 0$,</p> <p>and if $m - k - l \geq 0$,</p> <p>$alpha[k:k+l-1] = C$,</p> <p>$beta[k:k+l-1] = S$,</p> <p>or if $m - k - l < 0$,</p> <p>$alpha[k:m-1] = C$, $alpha[m:k+l-1] = 0$</p> <p>$beta[k:m-1] = S$, $beta[m:k+l-1] = 1$</p> <p>and</p> <p>$alpha[k+l:n-1] = 0$</p> <p>$beta[k+l:n-1] = 0$</p> |
| <i>u</i> | <p>Array, size (<i>ldu</i>*<i>m</i>).</p> <p>If <i>jobu</i> = 'U', <i>u</i> contains the <i>m</i>-by-<i>m</i> orthogonal/unitary matrix <i>U</i>.</p> <p>If <i>jobu</i> = 'N', <i>u</i> is not referenced.</p> |
| <i>v</i> | <p>Array, size (<i>ldv</i>*<i>p</i>).</p> <p>If <i>jobv</i> = 'V', <i>v</i> contains the <i>p</i>-by-<i>p</i> orthogonal/unitary matrix <i>V</i>.</p> <p>If <i>jobv</i> = 'N', <i>v</i> is not referenced.</p> |
| <i>q</i> | <p>Array, size (<i>ldq</i>*<i>n</i>).</p> <p>If <i>jobq</i> = 'Q', <i>q</i> contains the <i>n</i>-by-<i>n</i> orthogonal/unitary matrix <i>Q</i>.</p> |

If $jobq = 'N'$, q is not referenced.

iwork

On exit, *iwork* stores the sorting information. More precisely, the following loop uses *iwork* to sort *alpha*:

```
for (i = k; i < min(m, k + 1); i++) {
    swap (alpha[i], alpha[iwork[i] - 1]);
}
```

such that $alpha[0] \geq alpha[1] \geq \dots \geq alpha[n - 1]$.

Return Values

This function returns a value *info*.

= 0: successful exit.

< 0: if *info* = -*i*, the *i*-th argument had an illegal value.

> 0: if *info* = 1, the Jacobi-type procedure failed to converge.

For further details, see subroutine ?tgsja.

Application Notes

?ggsvd3 replaces the deprecated subroutine ?ggsvd.

?tgsja

Computes the generalized SVD of two upper triangular or trapezoidal matrices.

Syntax

```
lapack_int LAPACKE_stgsja( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_int k, lapack_int l, float* a,
lapack_int lda, float* b, lapack_int ldb, float tola, float tolb, float* alpha, float*
beta, float* u, lapack_int ldu, float* v, lapack_int ldv, float* q, lapack_int ldq,
lapack_int* ncycle );
```

```
lapack_int LAPACKE_dtgsja( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_int k, lapack_int l, double* a,
lapack_int lda, double* b, lapack_int ldb, double tola, double tolb, double* alpha,
double* beta, double* u, lapack_int ldu, double* v, lapack_int ldv, double* q,
lapack_int ldq, lapack_int* ncycle );
```

```
lapack_int LAPACKE_ctgsja( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_int k, lapack_int l,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb, float
tola, float tolb, float* alpha, float* beta, lapack_complex_float* u, lapack_int ldu,
lapack_complex_float* v, lapack_int ldv, lapack_complex_float* q, lapack_int ldq,
lapack_int* ncycle );
```

```
lapack_int LAPACKE_ztgsja( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int p, lapack_int n, lapack_int k, lapack_int l,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
double tola, double tolb, double* alpha, double* beta, lapack_complex_double* u,
lapack_int ldu, lapack_complex_double* v, lapack_int ldv, lapack_complex_double* q,
lapack_int ldq, lapack_int* ncycle );
```


$$D_1 = \begin{matrix} & & k & l \\ & & & \\ & l & & \\ p-l & \begin{pmatrix} 0 & S \\ 0 & 0 \end{pmatrix} \end{matrix}$$

$$(0 R) = \begin{matrix} & n-k-l & k & l \\ & & & \\ k & \begin{pmatrix} 0 & R_{11} & R_{12} \\ 0 & 0 & R_{22} \end{pmatrix} \\ l & \end{matrix}$$

where

$$C = \text{diag}(\alpha[k], \dots, \alpha[k+l-1])$$

$$S = \text{diag}(\beta[k], \dots, \beta[k+l-1])$$

$$C^2 + S^2 = I$$

R is stored in $a(1:k+l, n-k-l+1:n)$ on exit.

If $m-k-l < 0$,

$$\begin{matrix} & & k & m-k & k+l-m \\ & & & & \\ k & \begin{pmatrix} I & 0 & 0 \\ 0 & C & 0 \end{pmatrix} \\ m-k & \end{matrix}$$

$$D_2 = \begin{matrix} & & k & m-k & k+l-m \\ & & & & \\ m-k & \begin{pmatrix} 0 & S & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{pmatrix} \\ k+l-m & \\ p-l & \end{matrix}$$

$$(0 R) = \begin{matrix} & & n-k-l & k & m-k & k+l-m \\ & & & & & \\ & & k & \begin{pmatrix} 0 & R_{11} & R_{12} & R_{13} \\ 0 & 0 & R_{22} & R_{23} \\ 0 & 0 & 0 & R_{33} \end{pmatrix} \\ m-k & \\ k+l-m & \end{matrix}$$

where

$$C = \text{diag}(\alpha[k], \dots, \alpha[m-1]),$$

$$S = \text{diag}(\beta[k], \dots, \beta[m-1]),$$

$$C^2 + S^2 = I$$

On exit, $\begin{pmatrix} R_{11} & R_{12} & R_{13} \\ \mathbf{0} & R_{22} & R_{23} \end{pmatrix}$ is stored in $a(1:m, n-k-l+1:n)$ and R_{33} is stored in $b(m-k+1:l, n+m-k-l+1:n)$.

The computation of the orthogonal/unitary transformation matrices U , V or Q is optional. These matrices may either be formed explicitly, or they *may* be postmultiplied into input matrices U_1 , V_1 , or Q_1 .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobu</i> | <p>Must be 'U', 'I', or 'N'.</p> <p>If <i>jobu</i> = 'U', <i>u</i> must contain an orthogonal/unitary matrix U_1 on entry.</p> <p>If <i>jobu</i> = 'I', <i>u</i> is initialized to the unit matrix.</p> <p>If <i>jobu</i> = 'N', <i>u</i> is not computed.</p> |
| <i>jobv</i> | <p>Must be 'V', 'I', or 'N'.</p> <p>If <i>jobv</i> = 'V', <i>v</i> must contain an orthogonal/unitary matrix V_1 on entry.</p> <p>If <i>jobv</i> = 'I', <i>v</i> is initialized to the unit matrix.</p> <p>If <i>jobv</i> = 'N', <i>v</i> is not computed.</p> |
| <i>jobq</i> | <p>Must be 'Q', 'I', or 'N'.</p> <p>If <i>jobq</i> = 'Q', <i>q</i> must contain an orthogonal/unitary matrix Q_1 on entry.</p> <p>If <i>jobq</i> = 'I', <i>q</i> is initialized to the unit matrix.</p> <p>If <i>jobq</i> = 'N', <i>q</i> is not computed.</p> |
| <i>m</i> | The number of rows of the matrix A ($m \geq 0$). |
| <i>p</i> | The number of rows of the matrix B ($p \geq 0$). |
| <i>n</i> | The number of columns of the matrices A and B ($n \geq 0$). |
| <i>k, l</i> | Specify the subblocks in the input matrices A and B , whose GSVD is computed. |
| <i>a, b, u, v, q</i> | <p>Arrays:</p> <p><i>a</i> (size at least $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout) contains the m-by-n matrix A.</p> <p><i>b</i> (size at least $\max(1, ldb*n)$ for column major layout and $\max(1, ldb*p)$ for row major layout) contains the p-by-n matrix B.</p> <p>If <i>jobu</i> = 'U', <i>u</i> (size $\max(1, ldu*m)$) must contain a matrix U_1 (usually the orthogonal/unitary matrix returned by ?ggsvp).</p> <p>If <i>jobv</i> = 'V', <i>v</i> (size at least $\max(1, ldv*p)$) must contain a matrix V_1 (usually the orthogonal/unitary matrix returned by ?ggsvp).</p> <p>If <i>jobq</i> = 'Q', <i>q</i> (size at least $\max(1, ldq*n)$) must contain a matrix Q_1 (usually the orthogonal/unitary matrix returned by ?ggsvp).</p> |

| | |
|---|---|
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, p)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldu</i> | The leading dimension of the array <i>u</i> . <i>ldu</i> $\geq \max(1, m)$ if <i>jobu</i> = 'U'; <i>ldu</i> ≥ 1 otherwise. |
| <i>ldv</i> | The leading dimension of the array <i>v</i> . <i>ldv</i> $\geq \max(1, p)$ if <i>jobv</i> = 'V'; <i>ldv</i> ≥ 1 otherwise. |
| <i>ldq</i> | The leading dimension of the array <i>q</i> . <i>ldq</i> $\geq \max(1, n)$ if <i>jobq</i> = 'Q'; <i>ldq</i> ≥ 1 otherwise. |
| <i>tol</i> <i>a</i> , <i>tol</i> <i>b</i> | <i>tol</i> <i>a</i> and <i>tol</i> <i>b</i> are the convergence criteria for the Jacobi-Kogbetliantz iteration procedure. Generally, they are the same as used in ?ggsvp: <i>tol</i> <i>a</i> = $\max(m, n) * A * \text{MACHEPS}$, <i>tol</i> <i>b</i> = $\max(p, n) * B * \text{MACHEPS}$. |

Output Parameters

| | |
|----------------------------|---|
| <i>a</i> | On exit, <i>a</i> (<i>n</i> - <i>k</i> +1: <i>n</i> , 1:min(<i>k</i> +1, <i>m</i>)) contains the triangular matrix <i>R</i> or part of <i>R</i> . |
| <i>b</i> | On exit, if necessary, <i>b</i> (<i>m</i> - <i>k</i> +1: <i>l</i> , <i>n</i> + <i>m</i> - <i>k</i> - <i>l</i> +1: <i>n</i>)) contains a part of <i>R</i> . |
| <i>alpha</i> , <i>beta</i> | Arrays, size at least $\max(1, n)$. Contain the generalized singular value pairs of <i>A</i> and <i>B</i> : <i>alpha</i> (1: <i>k</i>) = 1, <i>beta</i> (1: <i>k</i>) = 0, and if <i>m</i> - <i>k</i> -1 ≥ 0 , <i>alpha</i> (<i>k</i> +1: <i>k</i> +1) = diag(<i>C</i>), <i>beta</i> (<i>k</i> +1: <i>k</i> +1) = diag(<i>S</i>), or if <i>m</i> - <i>k</i> -1 < 0, <i>alpha</i> (<i>k</i> +1: <i>m</i>) = diag(<i>C</i>), <i>alpha</i> (<i>m</i> +1: <i>k</i> +1) = 0 <i>beta</i> (<i>k</i> +1: <i>m</i>) = diag(<i>S</i>), <i>beta</i> (<i>m</i> +1: <i>k</i> +1) = 1. Furthermore, if <i>k</i> +1 < <i>n</i> , <i>alpha</i> (<i>k</i> +1+1: <i>n</i>) = 0 and <i>beta</i> (<i>k</i> +1+1: <i>n</i>) = 0. |
| <i>u</i> | If <i>jobu</i> = 'I', <i>u</i> contains the orthogonal/unitary matrix <i>U</i> . If <i>jobu</i> = 'U', <i>u</i> contains the product <i>U</i> ₁ * <i>U</i> . If <i>jobu</i> = 'N', <i>u</i> is not referenced. |

| | |
|----------|--|
| v | <p>If $jobv = 'I'$, v contains the orthogonal/unitary matrix U.</p> <p>If $jobv = 'V'$, v contains the product V_1*V.</p> <p>If $jobv = 'N'$, v is not referenced.</p> |
| q | <p>If $jobq = 'I'$, q contains the orthogonal/unitary matrix U.</p> <p>If $jobq = 'Q'$, q contains the product Q_1*Q.</p> <p>If $jobq = 'N'$, q is not referenced.</p> |
| $ncycle$ | The number of cycles required for convergence. |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = 1$, the procedure does not converge after MAXIT cycles.

Cosine-Sine Decomposition: LAPACK Computational Routines

This section describes LAPACK computational routines for computing the *cosine-sine decomposition* (CS decomposition) of a partitioned unitary/orthogonal matrix. The algorithm computes a complete 2-by-2 CS decomposition, which requires simultaneous diagonalization of all the four blocks of a unitary/orthogonal matrix partitioned into a 2-by-2 block structure.

The computation has the following phases:

1. The matrix is reduced to a bidiagonal block form.
2. The blocks are simultaneously diagonalized using techniques from the bidiagonal SVD algorithms.

Table "Computational Routines for Cosine-Sine Decomposition (CSD)" lists LAPACK routines that perform CS decomposition of matrices.

Computational Routines for Cosine-Sine Decomposition (CSD)

| Operation | Real matrices | Complex matrices |
|---|-----------------------------|-----------------------------|
| Compute the CS decomposition of an orthogonal/unitary matrix in bidiagonal-block form | bbcsd/bbcsd | bbcsd/bbcsd |
| Simultaneously bidiagonalize the blocks of a partitioned orthogonal matrix | orbdb unbdb | |
| Simultaneously bidiagonalize the blocks of a partitioned unitary matrix | | orbdb unbdb |

See Also

[Cosine-Sine Decomposition: LAPACK Driver Routines](#)

?bbcsd

Computes the CS decomposition of an orthogonal/unitary matrix in bidiagonal-block form.

Syntax

```
lapack_int LAPACKE_sbbcsd( int matrix_layout, char jobu1, char jobu2, char jobvt, char
jobv2t, char trans, lapack_int m, lapack_int p, lapack_int q, float* theta, float*
phi, float* u1, lapack_int ldu1, float* u2, lapack_int ldu2, float* v1t, lapack_int
ldv1t, float* v2t, lapack_int ldv2t, float* b11d, float* b11e, float* b12d, float*
b12e, float* b21d, float* b21e, float* b22d, float* b22e );
```

```
lapack_int LAPACKE_dbbcsd( int matrix_layout, char jobu1, char jobu2, char jobvt, char
jobv2t, char trans, lapack_int m, lapack_int p, lapack_int q, double* theta, double*
phi, double* u1, lapack_int ldu1, double* u2, lapack_int ldu2, double* v1t, lapack_int
ldv1t, double* v2t, lapack_int ldv2t, double* b11d, double* b11e, double* b12d,
double* b12e, double* b21d, double* b21e, double* b22d, double* b22e );
```

```
lapack_int LAPACKE_cbbcsd( int matrix_layout, char jobu1, char jobu2, char jobvt, char
jobv2t, char trans, lapack_int m, lapack_int p, lapack_int q, float* theta, float*
phi, lapack_complex_float* u1, lapack_int ldu1, lapack_complex_float* u2, lapack_int
ldu2, lapack_complex_float* v1t, lapack_int ldv1t, lapack_complex_float* v2t,
lapack_int ldv2t, float* b11d, float* b11e, float* b12d, float* b12e, float* b21d,
float* b21e, float* b22d, float* b22e );
```

```
lapack_int LAPACKE_zbbcsd( int matrix_layout, char jobu1, char jobu2, char jobvt, char
jobv2t, char trans, lapack_int m, lapack_int p, lapack_int q, double* theta, double*
phi, lapack_complex_double* u1, lapack_int ldu1, lapack_complex_double* u2, lapack_int
ldu2, lapack_complex_double* v1t, lapack_int ldv1t, lapack_complex_double* v2t,
lapack_int ldv2t, double* b11d, double* b11e, double* b12d, double* b12e, double*
b21d, double* b21e, double* b22d, double* b22e );
```

Include Files

- mkl.h

Description

mkl_lapack.fi The routine ?bbcsd computes the CS decomposition of an orthogonal or unitary matrix in bidiagonal-block form:

$$X = \begin{pmatrix} b_{11}|b_{12} & 0 & 0 \\ 0 & 0 & -I & 0 \\ b_{21}|b_{22} & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix} = \begin{pmatrix} u_1 & | & \\ & & u_2 \end{pmatrix} \begin{pmatrix} C & | & -S & 0 & 0 \\ 0 & 0 & -I & 0 \\ S & | & C & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix} \begin{pmatrix} v_1 & | & \\ & & v_2 \end{pmatrix}^T$$

or

$$X = \begin{pmatrix} b_{11}|b_{12} & 0 & 0 \\ 0 & 0 & -I & 0 \\ b_{21}|b_{22} & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix} = \begin{pmatrix} u_1 & | & \\ & & u_2 \end{pmatrix} \begin{pmatrix} C & | & -S & 0 & 0 \\ 0 & 0 & -I & 0 \\ S & | & C & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix} \begin{pmatrix} v_1 & | & \\ & & v_2 \end{pmatrix}^H$$

respectively.

x is m -by- m with the top-left block p -by- q . Note that q must not be larger than p , $m-p$, or $m-q$. If q is not the smallest index, x must be transposed and/or permuted in constant time using the *trans* option. See ?
orcsd/?uncsd for details.

The bidiagonal matrices b_{11} , b_{12} , b_{21} , and b_{22} are represented implicitly by angles $theta(1:q)$ and $phi(1:q-1)$.

The orthogonal/unitary matrices u_1 , u_2 , v_1^t , and v_2^t are input/output. The input matrices are pre- or post-multiplied by the appropriate singular vector matrices.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobu1</i> | If equals <i>Y</i> , then u_1 is updated. Otherwise, u_1 is not updated. |
| <i>jobu2</i> | If equals <i>Y</i> , then u_2 is updated. Otherwise, u_2 is not updated. |
| <i>jobv1t</i> | If equals <i>Y</i> , then v_1^t is updated. Otherwise, v_1^t is not updated. |
| <i>jobv2t</i> | If equals <i>Y</i> , then v_2^t is updated. Otherwise, v_2^t is not updated. |
| <i>trans</i> | = 'T': X , u_1 , u_2 , v_1^t , v_2^t are stored in row-major order. otherwise X , u_1 , u_2 , v_1^t , v_2^t are stored in column-major order. |
| <i>m</i> | The number of rows and columns of the orthogonal/unitary matrix X in bidiagonal-block form. |
| <i>p</i> | The number of rows in the top-left block of x . $0 \leq p \leq m$. |
| <i>q</i> | The number of columns in the top-left block of x . $0 \leq q \leq \min(p, m-p, m-q)$. |
| <i>theta</i> | Array, size q . On entry, the angles $theta[0], \dots, theta[q-1]$ that, along with $phi[0], \dots, phi[q-2]$, define the matrix in bidiagonal-block form as returned by orbdb/unbdb . |
| <i>phi</i> | Array, size $q-1$. The angles $phi[0], \dots, phi[q-2]$ that, along with $theta[0], \dots, theta[q-1]$, define the matrix in bidiagonal-block form as returned by orbdb/unbdb . |
| <i>u1</i> | Array, size at least $\max(1, ldu1*p)$. On entry, a p -by- p matrix. |
| <i>ldu1</i> | The leading dimension of the array u_1 , $ldu1 \leq \max(1, p)$. |
| <i>u2</i> | Array, size $\max(1, ldu2*(m-p))$. On entry, an $(m-p)$ -by- $(m-p)$ matrix. |
| <i>ldu2</i> | The leading dimension of the array u_2 , $ldu2 \leq \max(1, m-p)$. |

| | |
|--------------|---|
| <i>v1t</i> | Array, size $\max(1, ldv1t * q)$. On entry, a q -by- q matrix. |
| <i>ldv1t</i> | The leading dimension of the array <i>v1t</i> , $ldv1t \leq \max(1, q)$. |
| <i>v2t</i> | Array, size. On entry, an $(m-q)$ -by- $(m-q)$ matrix. |
| <i>ldv2t</i> | The leading dimension of the array <i>v2t</i> , $ldv2t \leq \max(1, m-q)$. |

Output Parameters

| | |
|--------------|---|
| <i>theta</i> | On exit, the angles whose cosines and sines define the diagonal blocks in the CS decomposition. |
| <i>u1</i> | On exit, <i>u1</i> is postmultiplied by the left singular vector matrix common to $\begin{bmatrix} b11 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} b12 & 0 & 0 \\ 0 & -I & 0 \end{bmatrix}$. |
| <i>u2</i> | On exit, <i>u2</i> is postmultiplied by the left singular vector matrix common to $\begin{bmatrix} b21 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} b22 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$. |
| <i>v1t</i> | Array, size q . On exit, <i>v1t</i> is premultiplied by the transpose of the right singular vector matrix common to $\begin{bmatrix} b11 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} b21 & 0 \\ 0 & 0 \end{bmatrix}$. |
| <i>v2t</i> | On exit, <i>v2t</i> is premultiplied by the transpose of the right singular vector matrix common to $\begin{bmatrix} b12 & 0 & 0 \\ 0 & -I & 0 \end{bmatrix}$ and $\begin{bmatrix} b22 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$. |
| <i>b11d</i> | Array, size q . When <code>?bbcsd</code> converges, <i>b11d</i> contains the cosines of $\theta[0], \dots, \theta[q - 1]$. If <code>?bbcsd</code> fails to converge, <i>b11d</i> contains the diagonal of the partially reduced top left block. |
| <i>b11e</i> | Array, size $q-1$. When <code>?bbcsd</code> converges, <i>b11e</i> contains zeros. If <code>?bbcsd</code> fails to converge, <i>b11e</i> contains the superdiagonal of the partially reduced top left block. |
| <i>b12d</i> | Array, size q . When <code>?bbcsd</code> converges, <i>b12d</i> contains the negative sines of $\theta[0], \dots, \theta[q - 1]$. If <code>?bbcsd</code> fails to converge, <i>b12d</i> contains the diagonal of the partially reduced top right block. |
| <i>b12e</i> | Array, size $q-1$. When <code>?bbcsd</code> converges, <i>b12e</i> contains zeros. If <code>?bbcsd</code> fails to converge, <i>b12e</i> contains the superdiagonal of the partially reduced top right block. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

If $info > 0$ and if ?bbcsc did not converge, $info$ specifies the number of nonzero entries in phi , and $b11d$, $b11e$, etc. contain the partially reduced matrix.

See Also

?orcsc/?uncsc

xerbla

?orbdb/?unbdb

Simultaneously bidiagonalizes the blocks of a partitioned orthogonal/unitary matrix.

Syntax

```
lapack_int LAPACKE_sorbdb( int matrix_layout, char trans, char signs, lapack_int m,
lapack_int p, lapack_int q, float* x11, lapack_int ldx11, float* x12, lapack_int
ldx12, float* x21, lapack_int ldx21, float* x22, lapack_int ldx22, float* theta,
float* phi, float* taup1, float* taup2, float* tauq1, float* tauq2 );
```

```
lapack_int LAPACKE_dorbdb( int matrix_layout, char trans, char signs, lapack_int m,
lapack_int p, lapack_int q, double* x11, lapack_int ldx11, double* x12, lapack_int
ldx12, double* x21, lapack_int ldx21, double* x22, lapack_int ldx22, double* theta,
double* phi, double* taup1, double* taup2, double* tauq1, double* tauq );
```

```
lapack_int LAPACKE_cunbdb( int matrix_layout, char trans, char signs, lapack_int m,
lapack_int p, lapack_int q, lapack_complex_float* x11, lapack_int ldx11,
lapack_complex_float* x12, lapack_int ldx12, lapack_complex_float* x21, lapack_int
ldx21, lapack_complex_float* x22, lapack_int ldx22, float* theta, float* phi,
lapack_complex_float* taup1, lapack_complex_float* taup2, lapack_complex_float* tauq1,
lapack_complex_float* tauq2 );
```

```
lapack_int LAPACKE_zunbdb( int matrix_layout, char trans, char signs, lapack_int m,
lapack_int p, lapack_int q, lapack_complex_double* x11, lapack_int ldx11,
lapack_complex_double* x12, lapack_int ldx12, lapack_complex_double* x21, lapack_int
ldx21, lapack_complex_double* x22, lapack_int ldx22, double* theta, double* phi,
lapack_complex_double* taup1, lapack_complex_double* taup2, lapack_complex_double*
tauq1, lapack_complex_double* tauq2 );
```

Include Files

- mkl.h

Description

The routines ?orbdb/?unbdb simultaneously bidiagonalizes the blocks of an m -by- m partitioned orthogonal matrix X :

$$X = \begin{pmatrix} x_{11} & | & x_{12} \\ x_{21} & | & x_{22} \end{pmatrix} = \begin{pmatrix} p_1 & | & \\ & & p_2 \end{pmatrix} \begin{pmatrix} b_{11}|b_{12} & 0 & 0 \\ 0 & | & 0 & -I & 0 \\ b_{21}|b_{22} & 0 & 0 \\ 0 & | & 0 & 0 & I \end{pmatrix} \begin{pmatrix} q_1 & | & \\ & & q_2 \end{pmatrix}^T$$

or unitary matrix:

$$X = \left(\begin{array}{c|c} x_{11} & x_{12} \\ \hline x_{21} & x_{22} \end{array} \right) = \left(\begin{array}{c|c} p_1 & \\ \hline & p_2 \end{array} \right) \begin{pmatrix} b_{11} & b_{12} & 0 & 0 \\ 0 & 0 & -I & 0 \\ \hline b_{21} & b_{22} & 0 & 0 \\ 0 & 0 & 0 & I \end{pmatrix} \left(\begin{array}{c|c} q_1 & \\ \hline & q_2 \end{array} \right)^H$$

x_{11} is p -by- q . q must not be larger than p , $m-p$, or $m-q$. Otherwise, x must be transposed and/or permuted in constant time using the *trans* and *signs* options.

The orthogonal/unitary matrices p_1 , p_2 , q_1 , and q_2 are p -by- p , $(m-p)$ -by- $(m-p)$, q -by- q , $(m-q)$ -by- $(m-q)$, respectively. They are represented implicitly by Housholder vectors.

The bidiagonal matrices b_{11} , b_{12} , b_{21} , and b_{22} are q -by- q bidiagonal matrices represented implicitly by angles $\theta[0], \dots, \theta[q - 1]$ and $\phi[0], \dots, \phi[q - 2]$. b_{11} and b_{12} are upper bidiagonal, while b_{21} and b_{22} are lower bidiagonal. Every entry in each bidiagonal band is a product of a sine or cosine of θ with a sine or cosine of ϕ . See [Sutton09] for details.

p_1 , p_2 , q_1 , and q_2 are represented as products of elementary reflectors. .

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>trans</i> | = 'T': x , u_1 , u_2 , v_1^t , v_2^t are stored in row-major order. otherwise x , u_1 , u_2 , v_1^t , v_2^t are stored in column-major order. |
| <i>signs</i> | = 'O': The lower-left block is made nonpositive (the "other" convention). otherwise The upper-right block is made nonpositive (the "default" convention). |
| <i>m</i> | The number of rows and columns of the matrix X . |
| <i>p</i> | The number of rows in x_{11} and x_{12} . $0 \leq p \leq m$. |
| <i>q</i> | The number of columns in x_{11} and x_{21} . $0 \leq q \leq \min(p, m-p, m-q)$. |
| <i>x11</i> | Array, size (size $\max(1, ldx11 * q)$ for column major layout and $\max(1, ldx11 * p)$ for row major layout) . On entry, the top-left block of the orthogonal/unitary matrix to be reduced. |
| <i>ldx11</i> | The leading dimension of the array X_{11} . If <i>trans</i> = 'T', $ldx11 \geq p$ for column major layout and $ldx11 \geq q$ for row major layout. Otherwise, $ldx11 \geq q$. |
| <i>x12</i> | Array, size (size $\max(1, ldx12 * (m-q))$ for column major layout and $\max(1, ldx12 * p)$ for row major layout). On entry, the top-right block of the orthogonal/unitary matrix to be reduced. |

| | |
|--------------|---|
| <i>ldx12</i> | The leading dimension of the array X_{12} . If <i>trans</i> = 'N', $ldx12 \geq p$ for column major layout and $ldx12 \geq m - q$ for row major layout. . Otherwise, $ldx12 \geq m - q$. |
| <i>x21</i> | Array, size (size $\max(1, ldx21 * q)$ for column major layout and $\max(1, ldx21 * (m - p))$ for row major layout). On entry, the bottom-left block of the orthogonal/unitary matrix to be reduced. |
| <i>ldx21</i> | The leading dimension of the array X_{21} . If <i>trans</i> = 'N', $ldx21 \geq m - p$ for column major layout and $ldx21 \geq q$ for row major layout. . Otherwise, $ldx21 \geq q$. |
| <i>x22</i> | Array, size ((size $\max(1, ldx22 * (m - q))$ for column major layout and $\max(1, ldx22 * (m - p))$ for row major layout). On entry, the bottom-right block of the orthogonal/unitary matrix to be reduced. |
| <i>ldx22</i> | The leading dimension of the array X_{21} . If <i>trans</i> = 'N', $ldx22 \geq m - p$ for column major layout and $ldx22 \geq m - q$ for row major layout. . Otherwise, $ldx22 \geq m - q$. |

Output Parameters

| | |
|------------|---|
| <i>x11</i> | On exit, the form depends on <i>trans</i> : If <i>trans</i> ='N', the columns of the lower triangle of <i>x11</i> specify reflectors for p_1 , the rows of the upper triangle of $x11(1:q - 1, q:q - 1)$ specify reflectors for q_1 otherwise <i>trans</i> ='T', the rows of the upper triangle of <i>x11</i> specify reflectors for p_1 , the columns of the lower triangle of $x11(1:q - 1, q:q - 1)$ specify reflectors for q_1 |
| <i>x12</i> | On exit, the form depends on <i>trans</i> : If <i>trans</i> ='N', the columns of the upper triangle of <i>x12</i> specify the first p reflectors for q_2 otherwise <i>trans</i> ='T', the columns of the lower triangle of <i>x12</i> specify the first p reflectors for q_2 |
| <i>x21</i> | On exit, the form depends on <i>trans</i> : If <i>trans</i> ='N', the columns of the lower triangle of <i>x21</i> specify the reflectors for p_2 otherwise <i>trans</i> ='T', the columns of the upper triangle of <i>x21</i> specify the reflectors for p_2 |
| <i>x22</i> | On exit, the form depends on <i>trans</i> : If <i>trans</i> ='N', the rows of the upper triangle of $x22(q+1:m-p, p+1:m-q)$ specify the last $m-p-q$ reflectors for q_2 otherwise <i>trans</i> ='T', the columns of the lower triangle of $x22(p+1:m-q, q+1:m-p)$ specify the last $m-p-q$ reflectors for p_2 |

| | |
|--------------|---|
| <i>theta</i> | Array, size q . The entries of bidiagonal blocks b_{11} , b_{12} , b_{21} , and b_{22} can be computed from the angles <i>theta</i> and <i>phi</i> . See the Description section for details. |
| <i>phi</i> | Array, size $q-1$. The entries of bidiagonal blocks b_{11} , b_{12} , b_{21} , and b_{22} can be computed from the angles <i>theta</i> and <i>phi</i> . See the Description section for details. |
| <i>taup1</i> | Array, size p . Scalar factors of the elementary reflectors that define p_1 . |
| <i>taup2</i> | Array, size $m-p$. Scalar factors of the elementary reflectors that define p_2 . |
| <i>tauq1</i> | Array, size q . Scalar factors of the elementary reflectors that define q_1 . |
| <i>tauq2</i> | Array, size $m-q$. Scalar factors of the elementary reflectors that define q_2 . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

See Also

[?orcscd/?uncscd](#)
[?orgqr](#)
[?ungqr](#)
[?orglq](#)
[?unglq](#)
[xerbla](#)

LAPACK Least Squares and Eigenvalue Problem Driver Routines

Each of the LAPACK driver routines solves a complete problem. To arrive at the solution, driver routines typically call a sequence of appropriate [computational routines](#).

Driver routines are described in the following sections :

[Linear Least Squares \(LLS\) Problems](#)
[Generalized LLS Problems](#)
[Symmetric Eigenproblems](#)
[Nonsymmetric Eigenproblems](#)
[Singular Value Decomposition](#)
[Cosine-Sine Decomposition](#)
[Generalized Symmetric Definite Eigenproblems](#)
[Generalized Nonsymmetric Eigenproblems](#)

Linear Least Squares (LLS) Problems: LAPACK Driver Routines

This section describes LAPACK driver routines used for solving linear least squares problems. [Table "Driver Routines for Solving LLS Problems"](#) lists all such routines.

Driver Routines for Solving LLS Problems

| Routine Name | Operation performed |
|-----------------------|---|
| sgels | Uses QR or LQ factorization to solve a overdetermined or underdetermined linear system with full rank matrix. |
| gelsy | Computes the minimum-norm solution to a linear least squares problem using a complete orthogonal factorization of A. |
| gels | Computes the minimum-norm solution to a linear least squares problem using the singular value decomposition of A. |
| gelsd | Computes the minimum-norm solution to a linear least squares problem using the singular value decomposition of A and a divide and conquer method. |

[?sgels](#)

Uses QR or LQ factorization to solve a overdetermined or underdetermined linear system with full rank matrix.

Syntax

```
lapack_int LAPACKE_sgels (int matrix_layout, char trans, lapack_int m, lapack_int n,
lapack_int nrhs, float* a, lapack_int lda, float* b, lapack_int ldb);
```

```
lapack_int LAPACKE_dgels (int matrix_layout, char trans, lapack_int m, lapack_int n,
lapack_int nrhs, double* a, lapack_int lda, double* b, lapack_int ldb);
```

```
lapack_int LAPACKE_cgels (int matrix_layout, char trans, lapack_int m, lapack_int n,
lapack_int nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b,
lapack_int ldb);
```

```
lapack_int LAPACKE_zgels (int matrix_layout, char trans, lapack_int m, lapack_int n,
lapack_int nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b,
lapack_int ldb);
```

Include Files

- `mkl.h`

Description

The routine solves overdetermined or underdetermined real/ complex linear systems involving an m -by- n matrix A , or its transpose/ conjugate-transpose, using a QR or LQ factorization of A . It is assumed that A has full rank.

The following options are provided:

1. If $trans = 'N'$ and $m \geq n$: find the least squares solution of an overdetermined system, that is, solve the least squares problem

$$\text{minimize } \|b - A*x\|_2$$

2. If $trans = 'N'$ and $m < n$: find the minimum norm solution of an underdetermined system $A*X = B$.
3. If $trans = 'T'$ or $'C'$ and $m \geq n$: find the minimum norm solution of an undetermined system $A^H*X = B$.
4. If $trans = 'T'$ or $'C'$ and $m < n$: find the least squares solution of an overdetermined system, that is, solve the least squares problem

minimize $\|b - A^H x\|_2$

Several right hand side vectors b and solution vectors x can be handled in a single call; they are formed by the columns of the right hand side matrix B and the solution matrix X (when coefficient matrix is A , B is m -by- $nrhs$ and X is n -by- $nrhs$; if the coefficient matrix is A^T or A^H , B is n -by- $nrhs$ and X is m -by- $nrhs$).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>trans</i> | Must be 'N', 'T', or 'C'. If <i>trans</i> = 'N', the linear system involves matrix A ; If <i>trans</i> = 'T', the linear system involves the transposed matrix A^T (for real flavors only); If <i>trans</i> = 'C', the linear system involves the conjugate-transposed matrix A^H (for complex flavors only). |
| <i>m</i> | The number of rows of the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrix A ($n \geq 0$). |
| <i>nrhs</i> | The number of right-hand sides; the number of columns in B ($nrhs \geq 0$). |
| <i>a, b</i> | Arrays: a (size $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout) contains the m -by- n matrix A . b (size $\max(1, ldb * nrhs)$ for column major layout and $\max(1, ldb * \max(m, n))$ for row major layout) contains the matrix B of right hand side vectors. |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of b ; must be at least $\max(1, m, n)$ for column major layout if <i>trans</i> ='N' and at least $\max(1, n)$ if <i>trans</i> ='T' and at least $\max(1, nrhs)$ for row major layout regardless of the value of <i>trans</i> . |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, overwritten by the factorization data as follows: if $m \geq n$, array a contains the details of the QR factorization of the matrix A as returned by <code>?geqrf</code> ; if $m < n$, array a contains the details of the LQ factorization of the matrix A as returned by <code>?gelqf</code> . |
| <i>b</i> | If <i>info</i> = 0, b overwritten by the solution vectors, stored columnwise: if <i>trans</i> = 'N' and $m \geq n$, rows 1 to n of b contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of modulus of elements $n+1$ to m in that column; |

if $trans = 'N'$ and $m < n$, rows 1 to n of b contain the minimum norm solution vectors;

if $trans = 'T'$ or $'C'$ and $m \geq n$, rows 1 to m of b contain the minimum norm solution vectors;

if $trans = 'T'$ or $'C'$ and $m < n$, rows 1 to m of b contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of modulus of elements $m+1$ to n in that column.

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = i$, the i -th diagonal element of the triangular factor of A is zero, so that A does not have full rank; the least squares solution could not be computed.

?gelsy

Computes the minimum-norm solution to a linear least squares problem using a complete orthogonal factorization of A .

Syntax

```
lapack_int LAPACKE_sgelsy( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, float* a, lapack_int lda, float* b, lapack_int ldb, lapack_int* jpvt, float
rcond, lapack_int* rank );
```

```
lapack_int LAPACKE_dgelsy( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, double* a, lapack_int lda, double* b, lapack_int ldb, lapack_int* jpvt, double
rcond, lapack_int* rank );
```

```
lapack_int LAPACKE_cgelsy( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int
ldb, lapack_int* jpvt, float rcond, lapack_int* rank );
```

```
lapack_int LAPACKE_zgelsy( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int
ldb, lapack_int* jpvt, double rcond, lapack_int* rank );
```

Include Files

- mkl.h

Description

The ?gelsy routine computes the minimum-norm solution to a real/complex linear least squares problem:

minimize $\|b - A*x\|_2$

using a complete orthogonal factorization of A . A is an m -by- n matrix which may be rank-deficient. Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m -by- $nrhs$ right hand side matrix B and the n -by- $nrhs$ solution matrix X .

The routine first computes a QR factorization with column pivoting:

$$AP = Q \begin{pmatrix} R_{11} & R_{12} \\ \mathbf{0} & R_{22} \end{pmatrix}$$

with R_{11} defined as the largest leading submatrix whose estimated condition number is less than $1/rcond$. The order of R_{11} , $rank$, is the effective rank of A . Then, R_{22} is considered to be negligible, and R_{12} is annihilated by orthogonal/unitary transformations from the right, arriving at the complete orthogonal factorization:

$$AP = Q \begin{pmatrix} T_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} Z$$

The minimum-norm solution is then

$$X = PZ^T \begin{pmatrix} T_{11}^{-1} Q_1^T B \\ \mathbf{0} \end{pmatrix} \text{ for real flavors and}$$

$$X = PZ^H \begin{pmatrix} T_{11}^{-1} Q_1^H B \\ \mathbf{0} \end{pmatrix} \text{ for complex flavors,}$$

where Q_1 consists of the first $rank$ columns of Q .

The `?gelsy` routine is identical to the original deprecated `?gelsx` routine except for the following differences:

- The call to the subroutine `?geqpf` has been substituted by the call to the subroutine `?geqp3`, which is a BLAS-3 version of the QR factorization with column pivoting.
- The matrix B (the right hand side) is updated with BLAS-3.
- The permutation of the matrix B (the right hand side) is faster and more simple.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>m</code> | The number of rows of the matrix A ($m \geq 0$). |
| <code>n</code> | The number of columns of the matrix A ($n \geq 0$). |
| <code>nrhs</code> | The number of right-hand sides; the number of columns in B ($nrhs \geq 0$). |
| <code>a, b</code> | Arrays: a (size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout) contains the m -by- n matrix A . b (size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*\max(m, n))$ for row major layout) contains the m -by- $nrhs$ right hand side matrix B . |

| | |
|--------------|---|
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; must be at least $\max(1, m, n)$ for column major layout and at least $\max(1, nrhs)$ for row major layout. |
| <i>jpvt</i> | Array, size at least $\max(1, n)$. On entry, if <i>jpvt</i> [<i>i</i> - 1] $\neq 0$, the <i>i</i> -th column of <i>A</i> is permuted to the front of <i>AP</i> , otherwise the <i>i</i> -th column of <i>A</i> is a free column. |
| <i>rcond</i> | <i>rcond</i> is used to determine the effective rank of <i>A</i> , which is defined as the order of the largest leading triangular submatrix R_{11} in the <i>QR</i> factorization with pivoting of <i>A</i> , whose estimated condition number $< 1/rcond$. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | On exit, overwritten by the details of the complete orthogonal factorization of <i>A</i> . |
| <i>b</i> | Overwritten by the <i>n</i> -by- <i>nrhs</i> solution matrix <i>X</i> . |
| <i>jpvt</i> | On exit, if <i>jpvt</i> [<i>i</i> - 1] = <i>k</i> , then the <i>i</i> -th column of <i>AP</i> was the <i>k</i> -th column of <i>A</i> . |
| <i>rank</i> | The effective rank of <i>A</i> , that is, the order of the submatrix R_{11} . This is the same as the order of the submatrix T_{11} in the complete orthogonal factorization of <i>A</i> . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

?gelss

Computes the minimum-norm solution to a linear least squares problem using the singular value decomposition of *A*.

Syntax

```
lapack_int LAPACKE_sgelss( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, float* a, lapack_int lda, float* b, lapack_int ldb, float* s, float rcond,
lapack_int* rank );
```

```
lapack_int LAPACKE_dgelss( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, double* a, lapack_int lda, double* b, lapack_int ldb, double* s, double rcond,
lapack_int* rank );
```

```
lapack_int LAPACKE_cgelss( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int
ldb, float* s, float rcond, lapack_int* rank );
```

```
lapack_int LAPACKE_zgelss( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int
ldb, double* s, double rcond, lapack_int* rank );
```

Include Files

- `mkl.h`

Description

The routine computes the minimum norm solution to a real linear least squares problem:

$$\text{minimize } \|b - A*x\|_2$$

using the singular value decomposition (SVD) of A . A is an m -by- n matrix which may be rank-deficient. Several right hand side vectors b and solution vectors x can be handled in a single call; they are stored as the columns of the m -by- $nrhs$ right hand side matrix B and the n -by- $nrhs$ solution matrix X . The effective rank of A is determined by treating as zero those singular values which are less than $rcond$ times the largest singular value.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>m</i> | The number of rows of the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrix A ($n \geq 0$). |
| <i>nrhs</i> | The number of right-hand sides; the number of columns in B ($nrhs \geq 0$). |
| <i>a, b</i> | Arrays: a (size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout) contains the m -by- n matrix A . b (size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*\max(m, n))$ for row major layout) contains the m -by- $nrhs$ right hand side matrix B . |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of b ; must be at least $\max(1, m, n)$ for column major layout and at least $\max(1, nrhs)$ for row major layout. |
| <i>rcond</i> | $rcond$ is used to determine the effective rank of A . Singular values $s(i) \leq rcond * s(1)$ are treated as zero. If $rcond < 0$, machine precision is used instead. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, the first $\min(m, n)$ rows of a are overwritten with the matrix of right singular vectors of A , stored row-wise. |
| <i>b</i> | Overwritten by the n -by- $nrhs$ solution matrix X . If $m \geq n$ and $rank = n$, the residual sum-of-squares for the solution in the i -th column is given by the sum of squares of modulus of elements $n+1:m$ in that column. |

| | |
|-------------|--|
| <i>s</i> | Array, size at least $\max(1, \min(m, n))$. The singular values of <i>A</i> in decreasing order. The condition number of <i>A</i> in the 2-norm is $k_2(A) = s(1) / s(\min(m, n))$. |
| <i>rank</i> | The effective rank of <i>A</i> , that is, the number of singular values which are greater than $rcond * s(1)$. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, then the algorithm for computing the SVD failed to converge; *i* indicates the number of off-diagonal elements of an intermediate bidiagonal form which did not converge to zero.

?gelsd

Computes the minimum-norm solution to a linear least squares problem using the singular value decomposition of A and a divide and conquer method.

Syntax

```
lapack_int LAPACKE_sgelsd( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, float* a, lapack_int lda, float* b, lapack_int ldb, float* s, float rcond,
lapack_int* rank );
```

```
lapack_int LAPACKE_dgelsd( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, double* a, lapack_int lda, double* b, lapack_int ldb, double* s, double rcond,
lapack_int* rank );
```

```
lapack_int LAPACKE_cgelsd( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int
ldb, float* s, float rcond, lapack_int* rank );
```

```
lapack_int LAPACKE_zgelsd( int matrix_layout, lapack_int m, lapack_int n, lapack_int
nrhs, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int
ldb, double* s, double rcond, lapack_int* rank );
```

Include Files

- mkl.h

Description

The routine computes the minimum-norm solution to a real linear least squares problem:

$$\text{minimize } \|b - A*x\|_2$$

using the singular value decomposition (SVD) of *A*. *A* is an *m*-by-*n* matrix which may be rank-deficient.

Several right hand side vectors *b* and solution vectors *x* can be handled in a single call; they are stored as the columns of the *m*-by-*nrhs* right hand side matrix *B* and the *n*-by-*nrhs* solution matrix *X*.

The problem is solved in three steps:

1. Reduce the coefficient matrix *A* to bidiagonal form with Householder transformations, reducing the original problem into a "bidiagonal least squares problem" (BLS).
2. Solve the BLS using a divide and conquer approach.

3. Apply back all the Householder transformations to solve the original least squares problem.

The effective rank of A is determined by treating as zero those singular values which are less than $rcond$ times the largest singular value.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrix A ($n \geq 0$). |
| <i>nrhs</i> | The number of right-hand sides; the number of columns in B ($nrhs \geq 0$). |
| <i>a, b</i> | Arrays: a (size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout) contains the m -by- n matrix A . b (size $\max(1, ldb*nrhs)$ for column major layout and $\max(1, ldb*\max(m, n))$ for row major layout) contains the m -by- $nrhs$ right hand side matrix B . |
| <i>lda</i> | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of b ; must be at least $\max(1, m, n)$ for column major layout and at least $\max(1, nrhs)$ for row major layout. |
| <i>rcond</i> | $rcond$ is used to determine the effective rank of A . Singular values $s(i) \leq rcond * s(1)$ are treated as zero. If $rcond \leq 0$, machine precision is used instead. |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | On exit, A has been overwritten. |
| <i>b</i> | Overwritten by the n -by- $nrhs$ solution matrix X . If $m \geq n$ and $rank = n$, the residual sum-of-squares for the solution in the i -th column is given by the sum of squares of modulus of elements $n+1:m$ in that column. |
| <i>s</i> | Array, size at least $\max(1, \min(m, n))$. The singular values of A in decreasing order. The condition number of A in the 2-norm is $k_2(A) = s(1) / s(\min(m, n))$. |
| <i>rank</i> | The effective rank of A , that is, the number of singular values which are greater than $rcond * s(1)$. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

If $info = i$, then the algorithm for computing the SVD failed to converge; i indicates the number of off-diagonal elements of an intermediate bidiagonal form that did not converge to zero.

Generalized Linear Least Squares (LLS) Problems: LAPACK Driver Routines

This section describes LAPACK driver routines used for solving generalized linear least squares problems. Table "Driver Routines for Solving Generalized LLS Problems" lists all such routines.

Driver Routines for Solving Generalized LLS Problems

| Routine Name | Operation performed |
|-----------------------|--|
| gglse | Solves the linear equality-constrained least squares problem using a generalized RQ factorization. |
| ggglm | Solves a general Gauss-Markov linear model problem using a generalized QR factorization. |

?gglse

Solves the linear equality-constrained least squares problem using a generalized RQ factorization.

Syntax

```
lapack_int LAPACKE_sgglse (int matrix_layout, lapack_int m, lapack_int n, lapack_int p,
float* a, lapack_int lda, float* b, lapack_int ldb, float* c, float* d, float* x);
```

```
lapack_int LAPACKE_dgglse (int matrix_layout, lapack_int m, lapack_int n, lapack_int p,
double* a, lapack_int lda, double* b, lapack_int ldb, double* c, double* d, double*
x);
```

```
lapack_int LAPACKE_cgglse (int matrix_layout, lapack_int m, lapack_int n, lapack_int p,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* c, lapack_complex_float* d, lapack_complex_float* x);
```

```
lapack_int LAPACKE_zgglse (int matrix_layout, lapack_int m, lapack_int n, lapack_int p,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* c, lapack_complex_double* d, lapack_complex_double* x);
```

Include Files

- mkl.h

Description

The routine solves the linear equality-constrained least squares (LSE) problem:

minimize $\|c - A*x\|^2$ subject to $B*x = d$

where A is an m -by- n matrix, B is a p -by- n matrix, c is a given m -vector, and d is a given p -vector. It is assumed that $p \leq n \leq m+p$, and

$$\text{rank}(B) = p \text{ and } \text{rank} \begin{pmatrix} A \\ B \end{pmatrix} = n.$$

These conditions ensure that the LSE problem has a unique solution, which is obtained using a generalized RQ factorization of the matrices (B, A) given by

$$B = \begin{pmatrix} 0 & R \end{pmatrix} * Q, \quad A = Z * T * Q$$

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>m</i> | The number of rows of the matrix <i>A</i> ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>p</i> | The number of rows of the matrix <i>B</i> ($0 \leq p \leq n \leq m+p$). |
| <i>a, b, c, d</i> | Arrays: <i>a</i> (size $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout) contains the <i>m</i> -by- <i>n</i> matrix <i>A</i> . <i>b</i> (size $\max(1, ldb*n)$ for column major layout and $\max(1, ldb*p)$ for row major layout) contains the <i>p</i> -by- <i>n</i> matrix <i>B</i> . <i>c</i> size at least $\max(1, m)$, contains the right hand side vector for the least squares part of the LSE problem. <i>d</i> , size at least $\max(1, p)$, contains the right hand side vector for the constrained equation. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, p)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | The elements on and above the diagonal contain the $\min(m, n)$ -by- <i>n</i> upper trapezoidal matrix <i>T</i> as returned by <code>?ggrqf</code> . |
| <i>x</i> | The solution of the LSE problem. |
| <i>b</i> | On exit, the upper right triangle contains the <i>p</i> -by- <i>p</i> upper triangular matrix <i>R</i> as returned by <code>?ggrqf</code> . |
| <i>d</i> | On exit, <i>d</i> is destroyed. |
| <i>c</i> | On exit, the residual sum-of-squares for the solution is given by the sum of squares of elements <i>n-p+1</i> to <i>m</i> of vector <i>c</i> . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = 1, the upper triangular factor *R* associated with *B* in the generalized RQ factorization of the pair (*B*, *A*) is singular, so that $\text{rank}(B) < p$; the least squares solution could not be computed.

If $info = 2$, the $(n-p)$ -by- $(n-p)$ part of the upper trapezoidal factor T associated with A in the generalized RQ factorization of the pair (B, A) is singular, so that

$$\text{rank} \begin{pmatrix} A \\ B \end{pmatrix} < n$$

; the least squares solution could not be computed.

sggglm

Solves a general Gauss-Markov linear model problem using a generalized QR factorization.

Syntax

```
lapack_int LAPACKE_sggglm (int matrix_layout, lapack_int n, lapack_int m, lapack_int p,
float* a, lapack_int lda, float* b, lapack_int ldb, float* d, float* x, float* y);
```

```
lapack_int LAPACKE_dggglm (int matrix_layout, lapack_int n, lapack_int m, lapack_int p,
double* a, lapack_int lda, double* b, lapack_int ldb, double* d, double* x, double*
y);
```

```
lapack_int LAPACKE_cggglm (int matrix_layout, lapack_int n, lapack_int m, lapack_int p,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb,
lapack_complex_float* d, lapack_complex_float* x, lapack_complex_float* y);
```

```
lapack_int LAPACKE_zggglm (int matrix_layout, lapack_int n, lapack_int m, lapack_int p,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int ldb,
lapack_complex_double* d, lapack_complex_double* x, lapack_complex_double* y);
```

Include Files

- mkl.h

Description

The routine solves a general Gauss-Markov linear model (GLM) problem:

minimize_x ||y||₂ subject to $d = A*x + B*y$

where A is an n -by- m matrix, B is an n -by- p matrix, and d is a given n -vector. It is assumed that $m \leq n \leq m+p$, and $\text{rank}(A) = m$ and $\text{rank}(AB) = n$.

Under these assumptions, the constrained equation is always consistent, and there is a unique solution x and a minimal 2-norm solution y , which is obtained using a generalized QR factorization of the matrices (A, B) given by

$$A = Q \begin{pmatrix} R \\ 0 \end{pmatrix}; \quad B = Q * T * Z.$$

In particular, if matrix B is square nonsingular, then the problem GLM is equivalent to the following weighted linear least squares problem

minimize_x ||B⁻¹(d-A*x)||₂.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>n</i> | The number of rows of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>m</i> | The number of columns in <i>A</i> ($m \geq 0$). |
| <i>p</i> | The number of columns in <i>B</i> ($p \geq n - m$). |
| <i>a, b, d</i> | Arrays: <i>a</i> (size $\max(1, lda * m)$ for column major layout and $\max(1, lda * n)$ for row major layout) contains the <i>n</i> -by- <i>m</i> matrix <i>A</i> . <i>b</i> (size $\max(1, ldb * p)$ for column major layout and $\max(1, ldb * n)$ for row major layout) contains the <i>n</i> -by- <i>p</i> matrix <i>B</i> . <i>d</i> , size at least $\max(1, n)$, contains the left hand side of the GLM equation. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$ for column major layout and $\max(1, m)$ for row major layout. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, n)$ for column major layout and $\max(1, p)$ for row major layout. |

Output Parameters

| | |
|-------------|---|
| <i>x, y</i> | Arrays <i>x, y</i> . size at least $\max(1, m)$ for <i>x</i> and at least $\max(1, p)$ for <i>y</i> . On exit, <i>x</i> and <i>y</i> are the solutions of the GLM problem. |
| <i>a</i> | On exit, the upper triangular part of the array <i>a</i> contains the <i>m</i> -by- <i>m</i> upper triangular matrix <i>R</i> . |
| <i>b</i> | On exit, if $n \leq p$, the upper right triangle contains the <i>n</i> -by- <i>n</i> upper triangular matrix <i>T</i> as returned by <code>?ggrqf</code> ; if $n > p$, the elements on and above the $(n-p)$ -th subdiagonal contain the <i>n</i> -by- <i>p</i> upper trapezoidal matrix <i>T</i> . |
| <i>d</i> | On exit, <i>d</i> is destroyed |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = 1, the upper triangular factor *R* associated with *A* in the generalized QR factorization of the pair (*A, B*) is singular, so that $\text{rank}(A) < m$; the least squares solution could not be computed.

If *info* = 2, the bottom $(n-m)$ -by- $(n-m)$ part of the upper trapezoidal factor *T* associated with *B* in the generalized QR factorization of the pair (*A, B*) is singular, so that $\text{rank}(AB) < n$; the least squares solution could not be computed.

Symmetric Eigenvalue Problems: LAPACK Driver Routines

This section describes LAPACK driver routines used for solving symmetric eigenvalue problems. See also [computational routines](#) that can be called to solve these problems. [Table "Driver Routines for Solving Symmetric Eigenproblems"](#) lists all such driver routines.

Driver Routines for Solving Symmetric Eigenproblems

| Routine Name | Operation performed |
|-----------------------------|---|
| syev/heev | Computes all eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian matrix. |
| syevd/heevd | Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric / Hermitian matrix using divide and conquer algorithm. |
| syevx/heevx | Computes selected eigenvalues and, optionally, eigenvectors of a symmetric / Hermitian matrix. |
| syevr/heevr | Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian matrix using the Relatively Robust Representations. |
| spev/hpev | Computes all eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian matrix in packed storage. |
| spevd/hpevd | Uses divide and conquer algorithm to compute all eigenvalues and (optionally) all eigenvectors of a real symmetric / Hermitian matrix held in packed storage. |
| spevx/hpevx | Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian matrix in packed storage. |
| sbev /hbev | Computes all eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian band matrix. |
| sbevd/hbevd | Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric / Hermitian band matrix using divide and conquer algorithm. |
| sbevz/hbevz | Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric / Hermitian band matrix. |
| stev | Computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix. |
| stevd | Computes all eigenvalues and (optionally) all eigenvectors of a real symmetric tridiagonal matrix using divide and conquer algorithm. |
| stevx | Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix. |
| stevr | Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix using the Relatively Robust Representations. |

?syev

Computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix.

Syntax

```
lapack_int LAPACKE_ssyev (int matrix_layout, char jobz, char uplo, lapack_int n, float* a, lapack_int lda, float* w);
```

```
lapack_int LAPACKE_dsyev (int matrix_layout, char jobz, char uplo, lapack_int n, double* a, lapack_int lda, double* w);
```

Include Files

- `mkl.h`

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A .

Note that for most cases of real symmetric eigenvalue problems the default choice should be `syevr` function as its underlying algorithm is faster and uses less workspace.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>jobz</code> | Must be 'N' or 'V'. If <code>jobz = 'N'</code> , then only eigenvalues are computed. If <code>jobz = 'V'</code> , then eigenvalues and eigenvectors are computed. |
| <code>uplo</code> | Must be 'U' or 'L'. If <code>uplo = 'U'</code> , a stores the upper triangular part of A . If <code>uplo = 'L'</code> , a stores the lower triangular part of A . |
| <code>n</code> | The order of the matrix A ($n \geq 0$). |
| <code>a</code> | a (size $\max(1, lda * n)$) is an array containing either upper or lower triangular part of the symmetric matrix A , as specified by <code>uplo</code> . |
| <code>lda</code> | The leading dimension of the array a . Must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------------|--|
| <code>a</code> | On exit, if <code>jobz = 'V'</code> , then if <code>info = 0</code> , array a contains the orthonormal eigenvectors of the matrix A . If <code>jobz = 'N'</code> , then on exit the lower triangle (if <code>uplo = 'L'</code>) or the upper triangle (if <code>uplo = 'U'</code>) of A , including the diagonal, is overwritten. |
| <code>w</code> | Array, size at least $\max(1, n)$. If <code>info = 0</code> , contains the eigenvalues of the matrix A in ascending order. |

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

If `info = i`, then the algorithm failed to converge; i indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

?heev

Computes all eigenvalues and, optionally, eigenvectors of a Hermitian matrix.

Syntax

```
lapack_int LAPACKE_cheev( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* w );
```

```
lapack_int LAPACKE_zheev( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* w );
```

Include Files

- mkl.h

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A .

Note that for most cases of complex Hermitian eigenvalue problems the default choice should be [heevr](#) function as its underlying algorithm is faster and uses less workspace.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>a</i> | <i>a</i> (size $\max(1, lda*n)$) is an array containing either upper or lower triangular part of the Hermitian matrix A , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | On exit, if <i>jobz</i> = 'V', then if <i>info</i> = 0, array <i>a</i> contains the orthonormal eigenvectors of the matrix A . If <i>jobz</i> = 'N', then on exit the lower triangle (if <i>uplo</i> = 'L') or the upper triangle (if <i>uplo</i> = 'U') of A , including the diagonal, is overwritten. |
| <i>w</i> | Array, size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues of the matrix A in ascending order. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

?syevd

Computes all eigenvalues and, optionally, all eigenvectors of a real symmetric matrix using divide and conquer algorithm.

Syntax

```
lapack_int LAPACKE_ssyevd (int matrix_layout, char jobz, char uplo, lapack_int n,
float* a, lapack_int lda, float* w);
```

```
lapack_int LAPACKE_dsyevd (int matrix_layout, char jobz, char uplo, lapack_int n,
double* a, lapack_int lda, double* w);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a real symmetric matrix *A*. In other words, it can compute the spectral factorization of *A* as: $A = Z*\Lambda*Z^T$.

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i , and *Z* is the orthogonal matrix whose columns are the eigenvectors z_i . Thus,

$$A*z_i = \lambda_i*z_i \text{ for } i = 1, 2, \dots, n.$$

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the *QL* or *QR* algorithm.

Note that for most cases of real symmetric eigenvalue problems the default choice should be [syevr](#) function as its underlying algorithm is faster and uses less workspace. `?syevd` requires more workspace but is faster in some cases, especially for large matrices.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular part of <i>A</i> . If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular part of <i>A</i> . |

| | |
|------------|--|
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>a</i> | Array, size (<i>lda</i> , *). <i>a</i> (size $\max(1, lda * n)$) is an array containing either upper or lower triangular part of the symmetric matrix A , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>w</i> | Array, size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues of the matrix A in ascending order. See also <i>info</i> . |
| <i>a</i> | If <i>jobz</i> = 'V', then on exit this array is overwritten by the orthogonal matrix Z which contains the eigenvectors of A . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = *i*, and *jobz* = 'N', then the algorithm failed to converge; *i* indicates the number of off-diagonal elements of an intermediate tridiagonal form which did not converge to zero.

If *info* = *i*, and *jobz* = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns *info*/(*n*+1) through $\text{mod}(\text{info}, n+1)$.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $A+E$ such that $\|E\|_2 = O(\epsilon) * \|A\|_2$, where ϵ is the machine precision.

The complex analogue of this routine is [heevd](#)

?heevd

Computes all eigenvalues and, optionally, all eigenvectors of a complex Hermitian matrix using divide and conquer algorithm.

Syntax

```
lapack_int LAPACKE_cheevd( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* w );
```

```
lapack_int LAPACKE_zheevd( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* w );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a complex Hermitian matrix A . In other words, it can compute the spectral factorization of A as: $A = Z^* \Lambda Z^H$.

Here Λ is a real diagonal matrix whose diagonal elements are the eigenvalues λ_i , and Z is the (complex) unitary matrix whose columns are the eigenvectors z_i . Thus,

$$A z_i = \lambda_i z_i \text{ for } i = 1, 2, \dots, n.$$

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

Note that for most cases of complex Hermitian eigenvalue problems the default choice should be `heevr` function as its underlying algorithm is faster and uses less workspace. `?heevd` requires more workspace but is faster in some cases, especially for large matrices.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>jobz</code> | Must be 'N' or 'V'. If <code>jobz = 'N'</code> , then only eigenvalues are computed. If <code>jobz = 'V'</code> , then eigenvalues and eigenvectors are computed. |
| <code>uplo</code> | Must be 'U' or 'L'. If <code>uplo = 'U'</code> , <code>a</code> stores the upper triangular part of A . If <code>uplo = 'L'</code> , <code>a</code> stores the lower triangular part of A . |
| <code>n</code> | The order of the matrix A ($n \geq 0$). |
| <code>a</code> | <code>a</code> (size $\max(1, lda * n)$) is an array containing either upper or lower triangular part of the Hermitian matrix A , as specified by <code>uplo</code> . |
| <code>lda</code> | The leading dimension of the array <code>a</code> . Must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------------|--|
| <code>w</code> | Array, size at least $\max(1, n)$. If <code>info = 0</code> , contains the eigenvalues of the matrix A in ascending order. See also <code>info</code> . |
| <code>a</code> | If <code>jobz = 'V'</code> , then on exit this array is overwritten by the unitary matrix Z which contains the eigenvectors of A . |

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = i`, and `jobz = 'N'`, then the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero;

if `info = i`, and `jobz = 'V'`, then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns `info/(n+1)` through `mod(info, n+1)`.

If `info = -i`, the i -th parameter had an illegal value.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $A + E$ such that $\|E\|_2 = O(\epsilon) * \|A\|_2$, where ϵ is the machine precision.

The real analogue of this routine is [syevd](#). See also [hpevd](#) for matrices held in packed storage, and [hbevd](#) for banded matrices.

?syevx

Computes selected eigenvalues and, optionally, eigenvectors of a symmetric matrix.

Syntax

```
lapack_int LAPACKE_ssyevx (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, float* a, lapack_int lda, float vl, float vu, lapack_int il, lapack_int
iu, float abstol, lapack_int* m, float* w, float* z, lapack_int ldz, lapack_int*
ifail);
```

```
lapack_int LAPACKE_dsyevx (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, double* a, lapack_int lda, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, double* z, lapack_int ldz,
lapack_int* ifail);
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Note that for most cases of real symmetric eigenvalue problems the default choice should be [syevr](#) function as its underlying algorithm is faster and uses less workspace. [?syevx](#) is faster for a few selected eigenvalues.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>range</i> | Must be 'A', 'V', or 'I'. If <i>range</i> = 'A', all eigenvalues will be found. If <i>range</i> = 'V', all eigenvalues in the half-open interval $(vl, vu]$ will be found. If <i>range</i> = 'I', the eigenvalues with indices <i>il</i> through <i>iu</i> will be found. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular part of A . |

| | |
|---------------|--|
| | If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular part of <i>A</i> . |
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>a</i> | <i>a</i> (size $\max(1, lda * n)$) is an array containing either upper or lower triangular part of the symmetric matrix <i>A</i> , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$. |
| <i>vl, vu</i> | If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues; $vl \leq vu$. Not referenced if <i>range</i> = 'A' or 'I'. |
| <i>il, iu</i> | If <i>range</i> = 'I', the indices of the smallest and largest eigenvalues to be returned. Constraints: $1 \leq il \leq iu \leq n$, if $n > 0$; $il = 1$ and $iu = 0$, if $n = 0$. Not referenced if <i>range</i> = 'A' or 'V'. |
| <i>abstol</i> | The absolute error tolerance for the eigenvalues. See <i>Application Notes</i> for more information. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> ; $ldz \geq 1$. If <i>jobz</i> = 'V', then $ldz \geq \max(1, n)$ for column major layout and $lda \geq \max(1, m)$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, the lower triangle (if <i>uplo</i> = 'L') or the upper triangle (if <i>uplo</i> = 'U') of <i>A</i> , including the diagonal, is overwritten. |
| <i>m</i> | The total number of eigenvalues found; $0 \leq m \leq n$. If <i>range</i> = 'A', $m = n$, and if <i>range</i> = 'I', $m = iu - il + 1$. |
| <i>w</i> | Array, size at least $\max(1, n)$. The first <i>m</i> elements contain the selected eigenvalues of the matrix <i>A</i> in ascending order. |
| <i>z</i> | Array <i>z</i> (size $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout) contains eigenvectors. If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> columns of <i>z</i> contain the orthonormal eigenvectors of the matrix <i>A</i> corresponding to the selected eigenvalues, with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with <i>w</i> (<i>i</i>). If an eigenvector fails to converge, then that column of <i>z</i> contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in <i>ifail</i> . If <i>jobz</i> = 'N', then <i>z</i> is not referenced. |

Note: you must ensure that at least $\max(1, m)$ columns are supplied in the array z ; if $range = 'V'$, the exact value of m is not known in advance and an upper bound must be used.

ifail

Array, size at least $\max(1, n)$.

If $jobz = 'V'$, then if $info = 0$, the first m elements of *ifail* are zero; if $info > 0$, then *ifail* contains the indices of the eigenvectors that failed to converge.

If $jobz = 'V'$, then *ifail* is not referenced.

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = i$, then i eigenvectors failed to converge; their indices are stored in the array *ifail*.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \epsilon \cdot \max(|a|, |b|)$, where ϵ is the machine precision.

If $abstol$ is less than or equal to zero, then $\epsilon \cdot ||T||$ is used as tolerance, where $||T||$ is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues are computed most accurately when $abstol$ is set to twice the underflow threshold $2 * \lambda_{\text{lamch}}('S')$, not zero.

If this routine returns with $info > 0$, indicating that some eigenvectors did not converge, try setting $abstol$ to $2 * \lambda_{\text{lamch}}('S')$.

?heevx

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix.

Syntax

```
lapack_int LAPACKC_cheevx( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_complex_float* a, lapack_int lda, float vl, float vu, lapack_int
il, lapack_int iu, float abstol, lapack_int* m, float* w, lapack_complex_float* z,
lapack_int ldz, lapack_int* ifail );
```

```
lapack_int LAPACKC_zheevx( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_complex_double* a, lapack_int lda, double vl, double vu,
lapack_int il, lapack_int iu, double abstol, lapack_int* m, double* w,
lapack_complex_double* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Note that for most cases of complex Hermitian eigenvalue problems the default choice should be `heevr` function as its underlying algorithm is faster and uses less workspace. `?heevx` is faster for a few selected eigenvalues.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>jobz</code> | Must be 'N' or 'V'. If <code>jobz = 'N'</code> , then only eigenvalues are computed. If <code>jobz = 'V'</code> , then eigenvalues and eigenvectors are computed. |
| <code>range</code> | Must be 'A', 'V', or 'I'. If <code>range = 'A'</code> , all eigenvalues will be found. If <code>range = 'V'</code> , all eigenvalues in the half-open interval $(vl, vu]$ will be found. If <code>range = 'I'</code> , the eigenvalues with indices <code>il</code> through <code>iu</code> will be found. |
| <code>uplo</code> | Must be 'U' or 'L'. If <code>uplo = 'U'</code> , <code>a</code> stores the upper triangular part of <code>A</code> . If <code>uplo = 'L'</code> , <code>a</code> stores the lower triangular part of <code>A</code> . |
| <code>n</code> | The order of the matrix <code>A</code> ($n \geq 0$). |
| <code>a</code> | <code>a</code> (size $\max(1, lda * n)$) is an array containing either upper or lower triangular part of the Hermitian matrix <code>A</code> , as specified by <code>uplo</code> . |
| <code>lda</code> | The leading dimension of the array <code>a</code> . Must be at least $\max(1, n)$. |
| <code>vl, vu</code> | If <code>range = 'V'</code> , the lower and upper bounds of the interval to be searched for eigenvalues; $vl \leq vu$. Not referenced if <code>range = 'A'</code> or 'I'. |
| <code>il, iu</code> | If <code>range = 'I'</code> , the indices of the smallest and largest eigenvalues to be returned. Constraints: $1 \leq il \leq iu \leq n$, if $n > 0$; $il = 1$ and $iu = 0$, if $n = 0$. Not referenced if <code>range = 'A'</code> or 'V'. |
| <code>abstol</code> | |
| <code>ldz</code> | The leading dimension of the output array <code>z</code> ; $ldz \geq 1$. If <code>jobz = 'V'</code> , then $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout. |

Output Parameters

| | |
|----------------|---|
| <code>a</code> | On exit, the lower triangle (if <code>uplo = 'L'</code>) or the upper triangle (if <code>uplo = 'U'</code>) of <code>A</code> , including the diagonal, is overwritten. |
| <code>m</code> | The total number of eigenvalues found; $0 \leq m \leq n$. |

If *range* = 'A', $m = n$, and if *range* = 'I', $m = iu-il+1$.

w Array, size $\max(1, n)$. The first m elements contain the selected eigenvalues of the matrix A in ascending order.

z Array z (size $\max(1, ldz*m)$ for column major layout and $\max(1, ldz*n)$ for row major layout) contains eigenvectors.

If *jobz* = 'V', then if *info* = 0, the first m columns of z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i -th column of z holding the eigenvector associated with $w(i)$.

If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

If *jobz* = 'N', then z is not referenced.

ifail Array, size at least $\max(1, n)$.

If *jobz* = 'V', then if *info* = 0, the first m elements of *ifail* are zero; if *info* > 0, then *ifail* contains the indices of the eigenvectors that failed to converge.

If *jobz* = 'V', then *ifail* is not referenced.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

If *info* = i , then i eigenvectors failed to converge; their indices are stored in the array *ifail*.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \epsilon \cdot \max(|a|, |b|)$, where ϵ is the machine precision.

If *abstol* is less than or equal to zero, then $\epsilon \cdot ||T||$ will be used in its place, where $||T||$ is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold $2 * \text{?lamch}('S')$, not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to $2 * \text{?lamch}('S')$.

?syevr

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix using the Relatively Robust Representations.

Syntax

```
lapack_int LAPACKE_ssyevr (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, float* a, lapack_int lda, float vl, float vu, lapack_int il, lapack_int
iu, float abstol, lapack_int* m, float* w, float* z, lapack_int ldz, lapack_int*
isuppz);
```

```
lapack_int LAPACKE_dsyevr (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, double* a, lapack_int lda, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, double* z, lapack_int ldz,
lapack_int* isuppz);
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

The routine first reduces the matrix A to tridiagonal form T . Then, whenever possible, `?syevr` calls `stemr` to compute the eigenspectrum using Relatively Robust Representations. `stemr` computes eigenvalues by the *dqds* algorithm, while orthogonal eigenvectors are computed from various "good" L^*D*L^T representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the each unreduced block of T :

- Compute $T - \sigma I = L^*D*L^T$, so that L and D define all the wanted eigenvalues to high relative accuracy. This means that small relative changes in the entries of D and L cause only small relative changes in the eigenvalues and eigenvectors. The standard (unfactored) representation of the tridiagonal matrix T does not have this property in general.
- Compute the eigenvalues to suitable accuracy. If the eigenvectors are desired, the algorithm attains full accuracy of the computed eigenvalues only right before the corresponding vectors have to be computed, see Steps c) and d).
- For each cluster of close eigenvalues, select a new shift close to the cluster, find a new factorization, and refine the shifted eigenvalues to suitable accuracy.
- For each eigenvalue with a large enough relative separation, compute the corresponding eigenvector by forming a rank revealing twisted factorization. Go back to Step c) for any clusters that remain.

The desired accuracy of the output can be specified by the input parameter *abstol*.

The routine `?syevr` calls `stemr` when the full spectrum is requested on machines that conform to the IEEE-754 floating point standard. `?syevr` calls `stebz` and `stein` on non-IEEE machines and when partial spectrum requests are made.

Note that `?syevr` is preferable for most cases of real symmetric eigenvalue problems as its underlying algorithm is fast and uses less workspace.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the routine computes all eigenvalues. If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval: |

| | |
|---------------|--|
| | $vl < w[i] \leq vu$. |
| | If $range = 'I'$, the routine computes eigenvalues with indices il to iu . |
| | For $range = 'V'$ or $'I'$ and $iu-il < n-1$, <code>sstebz/dstebz</code> and <code>sstein/dstein</code> are called. |
| <i>uplo</i> | Must be $'U'$ or $'L'$. |
| | If $uplo = 'U'$, a stores the upper triangular part of A . |
| | If $uplo = 'L'$, a stores the lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>a</i> | a (size $\max(1, lda*n)$) is an array containing either upper or lower triangular part of the symmetric matrix A , as specified by $uplo$. |
| <i>lda</i> | The leading dimension of the array a . Must be at least $\max(1, n)$. |
| <i>vl, vu</i> | If $range = 'V'$, the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If $range = 'A'$ or $'I'$, vl and vu are not referenced. |
| <i>il, iu</i> | If $range = 'I'$, the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$, if $n = 0$. If $range = 'A'$ or $'V'$, il and iu are not referenced. |
| <i>abstol</i> | If $jobz = 'V'$, the eigenvalues and eigenvectors output have residual norms bounded by $abstol$, and the dot products between different eigenvectors are bounded by $abstol$. If $abstol < n * eps * T $, then $n * eps * T $ is used instead, where eps is the machine precision, and $ T $ is the 1-norm of the matrix T . The eigenvalues are computed to an accuracy of $eps * T $ irrespective of $abstol$. If high relative accuracy is important, set $abstol$ to <code>?lamch('S')</code> . |
| <i>ldz</i> | The leading dimension of the output array z . Constraints: $ldz \geq 1$ if $jobz = 'N'$ and $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout if $jobz = 'V'$. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | On exit, the lower triangle (if $uplo = 'L'$) or the upper triangle (if $uplo = 'U'$) of A , including the diagonal, is overwritten. |
|----------|--|

| | |
|---------------|--|
| <i>m</i> | <p>The total number of eigenvalues found, $0 \leq m \leq n$.</p> <p>If <i>range</i> = 'A', $m = n$, if <i>range</i> = 'I', $m = iu-il+1$, and if <i>range</i> = 'V' the exact value of <i>m</i> is not known in advance.</p> |
| <i>w, z</i> | <p>Arrays:</p> <p><i>w</i>, size at least $\max(1, n)$, contains the selected eigenvalues in ascending order, stored in <i>w</i>[0] to <i>w</i>[<i>m</i> - 1];</p> <p><i>z</i>(size $\max(1, ldz*m)$ for column major layout and $\max(1, ldz*n)$ for row major layout) .</p> <p>If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> columns of <i>z</i> contain the orthonormal eigenvectors of the matrix <i>A</i> corresponding to the selected eigenvalues, with the <i>i</i>-th column of <i>z</i> holding the eigenvector associated with <i>w</i>[<i>i</i> - 1].</p> <p>If <i>jobz</i> = 'N', then <i>z</i> is not referenced.</p> |
| <i>isuppz</i> | <p>Array, size at least $2 * \max(1, m)$.</p> <p>The support of the eigenvectors in <i>z</i>, i.e., the indices indicating the nonzero elements in <i>z</i>. The <i>i</i>-th eigenvector is nonzero only in elements <i>isuppz</i>[2<i>i</i> - 2] through <i>isuppz</i>[2<i>i</i> - 1]. Referenced only if eigenvectors are needed (<i>jobz</i> = 'V') and all eigenvalues are needed, that is, <i>range</i> = 'A' or <i>range</i> = 'I' and <i>il</i> = 1 and <i>iu</i> = <i>n</i>.</p> |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, an internal error has occurred.

Application Notes

?heevr

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix using the Relatively Robust Representations.

Syntax

```
lapack_int LAPACKE_cheevr( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_complex_float* a, lapack_int lda, float vl, float vu, lapack_int
il, lapack_int iu, float abstol, lapack_int* m, float* w, lapack_complex_float* z,
lapack_int ldz, lapack_int* isuppz );
```

```
lapack_int LAPACKE_zheevr( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_complex_double* a, lapack_int lda, double vl, double vu,
lapack_int il, lapack_int iu, double abstol, lapack_int* m, double* w,
lapack_complex_double* z, lapack_int ldz, lapack_int* isuppz );
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

The routine first reduces the matrix A to tridiagonal form T with a call to [hetrd](#). Then, whenever possible, `?heevr` calls [stegr](#) to compute the eigenspectrum using Relatively Robust Representations. `?stegr` computes eigenvalues by the *dqds* algorithm, while orthogonal eigenvectors are computed from various "good" $L^*D^*L^T$ representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For each unreduced block (submatrix) of T :

- a.** Compute $T - \sigma I = L^*D^*L^T$, so that L and D define all the wanted eigenvalues to high relative accuracy. This means that small relative changes in the entries of D and L cause only small relative changes in the eigenvalues and eigenvectors. The standard (unfactored) representation of the tridiagonal matrix T does not have this property in general.
- b.** Compute the eigenvalues to suitable accuracy. If the eigenvectors are desired, the algorithm attains full accuracy of the computed eigenvalues only right before the corresponding vectors have to be computed, see Steps c) and d).
- c.** For each cluster of close eigenvalues, select a new shift close to the cluster, find a new factorization, and refine the shifted eigenvalues to suitable accuracy.
- d.** For each eigenvalue with a large enough relative separation, compute the corresponding eigenvector by forming a rank revealing twisted factorization. Go back to Step c) for any clusters that remain.

The desired accuracy of the output can be specified by the input parameter *abstol*.

The routine `?heevr` calls [stemr](#) when the full spectrum is requested on machines which conform to the IEEE-754 floating point standard, or [stebz](#) and [stein](#) on non-IEEE machines and when partial spectrum requests are made.

Note that the routine `?heevr` is preferable for most cases of complex Hermitian eigenvalue problems as its underlying algorithm is fast and uses less workspace.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the routine computes all eigenvalues. If <i>range</i> = 'V', the routine computes eigenvalues $\lambda(i)$ in the half-open interval: $v_l < \lambda(i) \leq v_u$. If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i> . For <i>range</i> = 'V' or 'I', <code>sstebz/dstebz</code> and <code>cstein/zstein</code> are called. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular part of A . |

| | |
|---------------|--|
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>a</i> | <i>a</i> (size $\max(1, lda * n)$) is an array containing either upper or lower triangular part of the Hermitian matrix <i>A</i> , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$. |
| <i>vl, vu</i> | If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced. |
| <i>il, iu</i> | If <i>range</i> = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$. If <i>range</i> = 'A' or 'V', <i>il</i> and <i>iu</i> are not referenced. |
| <i>abstol</i> | The absolute error tolerance to which each eigenvalue/eigenvector is required. If <i>jobz</i> = 'V', the eigenvalues and eigenvectors output have residual norms bounded by <i>abstol</i> , and the dot products between different eigenvectors are bounded by <i>abstol</i> . If $abstol < n * eps * T $, then $n * eps * T $ is used instead, where <i>eps</i> is the machine precision, and $ T $ is the 1-norm of the matrix <i>T</i> . The eigenvalues are computed to an accuracy of $eps * T $ irrespective of <i>abstol</i> . If high relative accuracy is important, set <i>abstol</i> to <code>?lamch('S')</code> . |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> . Constraints: $ldz \geq 1$ if <i>jobz</i> = 'N'; $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout if <i>jobz</i> = 'V'. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, the lower triangle (if <i>uplo</i> = 'L') or the upper triangle (if <i>uplo</i> = 'U') of <i>A</i> , including the diagonal, is overwritten. |
| <i>m</i> | The total number of eigenvalues found, $0 \leq m \leq n$. If <i>range</i> = 'A', $m = n$, if <i>range</i> = 'I', $m = iu - il + 1$, and if <i>range</i> = 'V' the exact value of <i>m</i> is not known in advance. |
| <i>w</i> | Array, size at least $\max(1, n)$, contains the selected eigenvalues in ascending order, stored in <i>w</i> [0] to <i>w</i> [<i>m</i> - 1]. |
| <i>z</i> | Array <i>z</i> (size $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout) . |

If `jobz = 'V'`, then if `info = 0`, the first m columns of z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i -th column of z holding the eigenvector associated with $w[i - 1]$.

If `jobz = 'N'`, then z is not referenced.

`isuppz`

Array, size at least $2 * \max(1, m)$.

The support of the eigenvectors in z , i.e., the indices indicating the nonzero elements in z . The i -th eigenvector is nonzero only in elements `isuppz[2i - 2]` through `isuppz[2i - 1]`. Referenced only if eigenvectors are needed (`jobz = 'V'`) and all eigenvalues are needed, that is, `range = 'A'` or `range = 'I'` and `il = 1` and `iu = n`.

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

If `info = i`, an internal error has occurred.

Application Notes

Normal execution of `?stemr` may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the IEEE standard default manner.

For more details, see `?stemr` and these references:

- Inderjit S. Dhillon and Beresford N. Parlett: "Multiple representations to compute orthogonal eigenvectors of symmetric tridiagonal matrices," *Linear Algebra and its Applications*, 387(1), pp. 1-28, August 2004.
- Inderjit Dhillon and Beresford Parlett: "Orthogonal Eigenvectors and Relative Gaps," *SIAM Journal on Matrix Analysis and Applications*, Vol. 25, 2004. Also LAPACK Working Note 154.
- Inderjit Dhillon: "A new $O(n^2)$ algorithm for the symmetric tridiagonal eigenvalue/eigenvector problem", Computer Science Division Technical Report No. UCB/CSD-97-971, UC Berkeley, May 1997.

?spev

Computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix in packed storage.

Syntax

```
lapack_int LAPACKE_sspev (int matrix_layout, char jobz, char uplo, lapack_int n, float*
ap, float* w, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dspev (int matrix_layout, char jobz, char uplo, lapack_int n,
double* ap, double* w, double* z, lapack_int ldz);
```

Include Files

- `mk1.h`

Description

The routine computes all the eigenvalues and, optionally, eigenvectors of a real symmetric matrix A in packed storage.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ap</i> stores the packed upper triangular part of <i>A</i> . If <i>uplo</i> = 'L', <i>ap</i> stores the packed lower triangular part of <i>A</i> . |
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>ap</i> | Array <i>ap</i> contains the packed upper or lower triangle of symmetric matrix <i>A</i> , as specified by <i>uplo</i> . The size of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> . Constraints: if <i>jobz</i> = 'N', then $ldz \geq 1$; if <i>jobz</i> = 'V', then $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-------------|--|
| <i>w, z</i> | Arrays: <i>w</i> , size at least $\max(1, n)$. If <i>info</i> = 0, <i>w</i> contains the eigenvalues of the matrix <i>A</i> in ascending order. <i>z</i> (size $\max(1, ldz*n)$). If <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>z</i> contains the orthonormal eigenvectors of the matrix <i>A</i> , with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with $w[i - 1]$. If <i>jobz</i> = 'N', then <i>z</i> is not referenced. |
| <i>ap</i> | On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of <i>A</i> . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

?hpev

Computes all eigenvalues and, optionally, eigenvectors of a Hermitian matrix in packed storage.

Syntax

```
lapack_int LAPACKE_chpev( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_complex_float* ap, float* w, lapack_complex_float* z, lapack_int ldz );

lapack_int LAPACKE_zhpev( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_complex_double* ap, double* w, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ap</i> stores the packed upper triangular part of A . If <i>uplo</i> = 'L', <i>ap</i> stores the packed lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>ap</i> | Array <i>ap</i> contains the packed upper or lower triangle of Hermitian matrix A , as specified by <i>uplo</i> . The size of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> . Constraints: if <i>jobz</i> = 'N', then $ldz \geq 1$; if <i>jobz</i> = 'V', then $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>w</i> | Array, size at least $\max(1, n)$. If <i>info</i> = 0, <i>w</i> contains the eigenvalues of the matrix A in ascending order. |
| <i>z</i> | Array <i>z</i> (size at least $\max(1, ldz*n)$). |

If `jobz = 'V'`, then if `info = 0`, `z` contains the orthonormal eigenvectors of the matrix `A`, with the `i`-th column of `z` holding the eigenvector associated with `w[i - 1]`.

If `jobz = 'N'`, then `z` is not referenced.

`ap`

On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of `A`.

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the `i`-th parameter had an illegal value.

If `info = i`, then the algorithm failed to converge; `i` indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

?spevd

Uses divide and conquer algorithm to compute all eigenvalues and (optionally) all eigenvectors of a real symmetric matrix held in packed storage.

Syntax

```
lapack_int LAPACKE_sspevd (int matrix_layout, char jobz, char uplo, lapack_int n,
float* ap, float* w, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dspevd (int matrix_layout, char jobz, char uplo, lapack_int n,
double* ap, double* w, double* z, lapack_int ldz);
```

Include Files

- `mk1.h`

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a real symmetric matrix `A` (held in packed storage). In other words, it can compute the spectral factorization of `A` as:

$$A = Z \Lambda Z^T.$$

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i , and Z is the orthogonal matrix whose columns are the eigenvectors z_i . Thus,

$$A z_i = \lambda_i z_i \text{ for } i = 1, 2, \dots, n.$$

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the `QL` or `QR` algorithm.

Input Parameters

`matrix_layout`

Specifies whether matrix storage layout is row major (`LAPACK_ROW_MAJOR`) or column major (`LAPACK_COL_MAJOR`).

`jobz`

Must be `'N'` or `'V'`.

| | |
|--------|---|
| | If $jobz = 'N'$, then only eigenvalues are computed. |
| | If $jobz = 'V'$, then eigenvalues and eigenvectors are computed. |
| $uplo$ | Must be $'U'$ or $'L'$. |
| | If $uplo = 'U'$, ap stores the packed upper triangular part of A . |
| | If $uplo = 'L'$, ap stores the packed lower triangular part of A . |
| n | The order of the matrix A ($n \geq 0$). |
| ap | ap contains the packed upper or lower triangle of symmetric matrix A , as specified by $uplo$. |
| | The dimension of ap must be $\max(1, n*(n+1)/2)$ |
| ldz | The leading dimension of the output array z . |
| | Constraints: |
| | if $jobz = 'N'$, then $ldz \geq 1$; |
| | if $jobz = 'V'$, then $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|--------|--|
| w, z | Arrays: |
| | w , size at least $\max(1, n)$. |
| | If $info = 0$, contains the eigenvalues of the matrix A in ascending order. See also $info$. |
| | z (size $\max(1, ldz*n)$). |
| | If $jobz = 'V'$, then this array is overwritten by the orthogonal matrix Z which contains the eigenvectors of A . If $jobz = 'N'$, then z is not referenced. |
| ap | On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A . |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = i$, then the algorithm failed to converge; i indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $A+E$ such that $\|E\|_2 = O(\epsilon) * \|A\|_2$, where ϵ is the machine precision.

The complex analogue of this routine is [hpevd](#).

See also [syevd](#) for matrices held in full storage, and [sbevd](#) for banded matrices.

?hpevd

Uses divide and conquer algorithm to compute all eigenvalues and, optionally, all eigenvectors of a complex Hermitian matrix held in packed storage.

Syntax

```
lapack_int LAPACKE_chpevd( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_complex_float* ap, float* w, lapack_complex_float* z, lapack_int ldz );

lapack_int LAPACKE_zhpevd( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_complex_double* ap, double* w, lapack_complex_double* z, lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a complex Hermitian matrix A (held in packed storage). In other words, it can compute the spectral factorization of A as: $A = Z^* \Lambda Z^H$.

Here Λ is a real diagonal matrix whose diagonal elements are the eigenvalues λ_i , and Z is the (complex) unitary matrix whose columns are the eigenvectors z_i . Thus,

$$A z_i = \lambda_i z_i \text{ for } i = 1, 2, \dots, n.$$

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ap</i> stores the packed upper triangular part of A . If <i>uplo</i> = 'L', <i>ap</i> stores the packed lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>ap</i> | <i>ap</i> contains the packed upper or lower triangle of Hermitian matrix A , as specified by <i>uplo</i> . The dimension of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$. |
| <i>ldz</i> | The leading dimension of the output array z . Constraints: if <i>jobz</i> = 'N', then $ldz \geq 1$; if <i>jobz</i> = 'V', then $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-----------|--|
| <i>w</i> | Array, size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues of the matrix <i>A</i> in ascending order. See also <i>info</i> . |
| <i>z</i> | Array, size 1 if <i>jobz</i> = 'N' and $\max(1, ldz*n)$ if <i>jobz</i> = 'V'. If <i>jobz</i> = 'V', then this array is overwritten by the unitary matrix <i>Z</i> which contains the eigenvectors of <i>A</i> . If <i>jobz</i> = 'N', then <i>z</i> is not referenced. |
| <i>ap</i> | On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of <i>A</i> . |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, then the algorithm failed to converge; *i* indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $A + E$ such that $\|E\|_2 = O(\epsilon) * \|A\|_2$, where ϵ is the machine precision.

The real analogue of this routine is [spevd](#).

See also [heevd](#) for matrices held in full storage, and [hbevd](#) for banded matrices.

?spevx

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix in packed storage.

Syntax

```
lapack_int LAPACKE_sspevx (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, float* ap, float vl, float vu, lapack_int il, lapack_int iu, float
abstol, lapack_int* m, float* w, float* z, lapack_int ldz, lapack_int* ifail);
```

```
lapack_int LAPACKE_dspevx (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, double* ap, double vl, double vu, lapack_int il, lapack_int iu, double
abstol, lapack_int* m, double* w, double* z, lapack_int ldz, lapack_int* ifail);
```

Include Files

- `mkl.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix *A* in packed storage. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the routine computes all eigenvalues. If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$. If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i> . |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ap</i> stores the packed upper triangular part of <i>A</i> . If <i>uplo</i> = 'L', <i>ap</i> stores the packed lower triangular part of <i>A</i> . |
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>ap</i> | Array <i>ap</i> contains the packed upper or lower triangle of the symmetric matrix <i>A</i> , as specified by <i>uplo</i> . The size of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$. |
| <i>vl, vu</i> | If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced. |
| <i>il, iu</i> | If <i>range</i> = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$. If <i>range</i> = 'A' or 'V', <i>il</i> and <i>iu</i> are not referenced. |
| <i>abstol</i> | The absolute error tolerance to which each eigenvalue is required. See <i>Application notes</i> for details on error tolerance. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> . Constraints: if <i>jobz</i> = 'N', then $ldz \geq 1$; if <i>jobz</i> = 'V', then $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout. |

Output Parameters

| | |
|--------------|---|
| <i>ap</i> | On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of <i>A</i> . |
| <i>m</i> | The total number of eigenvalues found, $0 \leq m \leq n$. If <i>range</i> = 'A', $m = n$, if <i>range</i> = 'I', $m = i_u - i_l + 1$, and if <i>range</i> = 'V' the exact value of <i>m</i> is not known in advance.. |
| <i>w, z</i> | Arrays: <i>w</i> , size at least $\max(1, n)$. If <i>info</i> = 0, contains the selected eigenvalues of the matrix <i>A</i> in ascending order. <i>z</i> (size $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout). If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> columns of <i>z</i> contain the orthonormal eigenvectors of the matrix <i>A</i> corresponding to the selected eigenvalues, with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with $w[i - 1]$. If an eigenvector fails to converge, then that column of <i>z</i> contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in <i>ifail</i> . If <i>jobz</i> = 'N', then <i>z</i> is not referenced. |
| <i>ifail</i> | Array, size at least $\max(1, n)$. If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> elements of <i>ifail</i> are zero; if <i>info</i> > 0, the <i>ifail</i> contains the indices the eigenvectors that failed to converge. If <i>jobz</i> = 'N', then <i>ifail</i> is not referenced. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to $abstol + \epsilon * \max(|a|, |b|)$, where ϵ is the machine precision.

If *abstol* is less than or equal to zero, then $\epsilon * ||T||_1$ will be used in its place, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold $2 * ?lamch('S')$, not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to $2 * ?lamch('S')$.

?hpevx

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix in packed storage.

Syntax

```
lapack_int LAPACKE_chpevx( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_complex_float* ap, float vl, float vu, lapack_int il, lapack_int
iu, float abstol, lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz,
lapack_int* ifail );
```

```
lapack_int LAPACKE_zhpevx( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_complex_double* ap, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, lapack_complex_double* z,
lapack_int ldz, lapack_int* ifail );
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A in packed storage. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>job</i> = 'N', then only eigenvalues are computed. If <i>job</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the routine computes all eigenvalues. If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$. If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i> . |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ap</i> stores the packed upper triangular part of A . If <i>uplo</i> = 'L', <i>ap</i> stores the packed lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>ap</i> | Array <i>ap</i> contains the packed upper or lower triangle of the Hermitian matrix A , as specified by <i>uplo</i> . The size of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$. |
| <i>vl, vu</i> | If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. |

Constraint: $vl < vu$.

If $range = 'A'$ or $'I'$, vl and vu are not referenced.

il, iu

If $range = 'I'$, the indices in ascending order of the smallest and largest eigenvalues to be returned.

Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$.

If $range = 'A'$ or $'V'$, il and iu are not referenced.

$abstol$

The absolute error tolerance to which each eigenvalue is required. See *Application notes* for details on error tolerance.

ldz

The leading dimension of the output array z .

Constraints:

if $jobz = 'N'$, then $ldz \geq 1$;

if $jobz = 'V'$, then $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout.

Output Parameters

ap

On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. The elements of the diagonal and the off-diagonal of the tridiagonal matrix overwrite the corresponding elements of A .

m

The total number of eigenvalues found, $0 \leq m \leq n$.

$0 \leq m \leq n$. If $range = 'A'$, $m = n$, if $range = 'I'$, $m = iu - il + 1$, and if $range = 'V'$ the exact value of m is not known in advance..

w

Array, size at least $\max(1, n)$.

If $info = 0$, contains the selected eigenvalues of the matrix A in ascending order.

z

Array z (size $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout).

If $jobz = 'V'$, then if $info = 0$, the first m columns of z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i -th column of z holding the eigenvector associated with $w(i)$.

If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

If $jobz = 'N'$, then z is not referenced.

$ifail$

Array, size at least $\max(1, n)$.

If $jobz = 'V'$, then if $info = 0$, the first m elements of *ifail* are zero; if $info > 0$, the *ifail* contains the indices the eigenvectors that failed to converge.

If $jobz = 'N'$, then *ifail* is not referenced.

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the *i*-th parameter had an illegal value.

If $info = i$, then *i* eigenvectors failed to converge; their indices are stored in the array *ifail*.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \epsilon \max(|a|, |b|)$, where ϵ is the machine precision.

If *abstol* is less than or equal to zero, then $\epsilon * ||T||_1$ will be used in its place, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold $2 * ?lamch('S')$, not zero.

If this routine returns with $info > 0$, indicating that some eigenvectors did not converge, try setting *abstol* to $2 * ?lamch('S')$.

?sbev

Computes all eigenvalues and, optionally, eigenvectors of a real symmetric band matrix.

Syntax

```
lapack_int LAPACKE_sbev (int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int kd, float* ab, lapack_int ldab, float* w, float* z, lapack_int ldz);
lapack_int LAPACKE_dsbev (int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int kd, double* ab, lapack_int ldab, double* w, double* z, lapack_int ldz);
```

Include Files

- mkl.h

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a real symmetric band matrix *A*.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If $jobz = 'N'$, then only eigenvalues are computed. If $jobz = 'V'$, then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | Must be 'U' or 'L'. If $uplo = 'U'$, <i>ab</i> stores the upper triangular part of <i>A</i> . If $uplo = 'L'$, <i>ab</i> stores the lower triangular part of <i>A</i> . |

| | |
|-------------|---|
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>kd</i> | The number of super- or sub-diagonals in <i>A</i> ($kd \geq 0$). |
| <i>ab</i> | <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and at least $\max(1, ldab*(kd + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix <i>A</i> (as specified by <i>uplo</i>) in band storage format. |
| <i>ldab</i> | The leading dimension of <i>ab</i> ; must be at least $kd + 1$ for column major layout and <i>n</i> for row major layout. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> . Constraints: if <i>jobz</i> = 'N', then $ldz \geq 1$; if <i>jobz</i> = 'V', then $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-------------|---|
| <i>w, z</i> | Arrays: <i>w</i> , size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues of the matrix <i>A</i> in ascending order. <i>z</i> (size $\max(1, ldz*n)$). If <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>z</i> contains the orthonormal eigenvectors of the matrix <i>A</i> , with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with $w[i - 1]$. If <i>jobz</i> = 'N', then <i>z</i> is not referenced. |
| <i>ab</i> | On exit, this array is overwritten by the values generated during the reduction to tridiagonal form (see the description of ?sbtrd). |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

?hbev

Computes all eigenvalues and, optionally, eigenvectors of a Hermitian band matrix.

Syntax

```
lapack_int LAPACKE_chbev( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int kd, lapack_complex_float* ab, lapack_int ldab, float* w,
lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zhbev( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int kd, lapack_complex_double* ab, lapack_int ldab, double* w,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- `mkl.h`

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A .

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>jobz</code> | Must be 'N' or 'V'. If <code>jobz = 'N'</code> , then only eigenvalues are computed. If <code>jobz = 'V'</code> , then eigenvalues and eigenvectors are computed. |
| <code>uplo</code> | Must be 'U' or 'L'. If <code>uplo = 'U'</code> , <code>ab</code> stores the upper triangular part of A . If <code>uplo = 'L'</code> , <code>ab</code> stores the lower triangular part of A . |
| <code>n</code> | The order of the matrix A ($n \geq 0$). |
| <code>kd</code> | The number of super- or sub-diagonals in A ($kd \geq 0$). |
| <code>ab</code> | <code>ab</code> (size at least $\max(1, ldab*n)$ for column major layout and at least $\max(1, ldab*(kd + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix A (as specified by <code>uplo</code>) in band storage format. |
| <code>ldab</code> | The leading dimension of <code>ab</code> ; must be at least $kd + 1$ for column major layout and n for row major layout. |
| <code>ldz</code> | The leading dimension of the output array <code>z</code> . Constraints: if <code>jobz = 'N'</code> , then $ldz \geq 1$; if <code>jobz = 'V'</code> , then $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|----------------|--|
| <code>w</code> | Array, size at least $\max(1, n)$. If <code>info = 0</code> , contains the eigenvalues in ascending order. |
| <code>z</code> | Array <code>z</code> (size $\max(1, ldz*n)$). If <code>jobz = 'V'</code> , then if <code>info = 0</code> , <code>z</code> contains the orthonormal eigenvectors of the matrix A , with the i -th column of <code>z</code> holding the eigenvector associated with <code>w[i - 1]</code> . If <code>jobz = 'N'</code> , then <code>z</code> is not referenced. |

ab On exit, this array is overwritten by the values generated during the reduction to tridiagonal form (see the description of [hbtrd](#)).

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, then the algorithm failed to converge;

i indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

?sbevd

Computes all eigenvalues and, optionally, all eigenvectors of a real symmetric band matrix using divide and conquer algorithm.

Syntax

```
lapack_int LAPACKE_ssbevd (int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int kd, float* ab, lapack_int ldab, float* w, float* z, lapack_int ldz);
lapack_int LAPACKE_dsbevd (int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int kd, double* ab, lapack_int ldab, double* w, double* z, lapack_int ldz);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a real symmetric band matrix *A*. In other words, it can compute the spectral factorization of *A* as:

$$A = Z \Lambda Z^T$$

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i , and *Z* is the orthogonal matrix whose columns are the eigenvectors z_i . Thus,

$$A z_i = \lambda_i z_i \text{ for } i = 1, 2, \dots, n.$$

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the *QL* or *QR* algorithm.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ab</i> stores the upper triangular part of <i>A</i> . |

| | |
|--------|--|
| | If $uplo = 'L'$, ab stores the lower triangular part of A . |
| n | The order of the matrix A ($n \geq 0$). |
| kd | The number of super- or sub-diagonals in A ($kd \geq 0$). |
| ab | ab (size at least $\max(1, ldab*n)$ for column major layout and at least $\max(1, ldab*(kd + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix A (as specified by $uplo$) in band storage format. |
| $ldab$ | The leading dimension of ab ; must be at least $kd+1$ for column major layout and n for row major layout. |
| ldz | The leading dimension of the output array z . Constraints: if $jobz = 'N'$, then $ldz \geq 1$; if $jobz = 'V'$, then $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|--------|---|
| w, z | Arrays: w , size at least $\max(1, n)$. If $info = 0$, contains the eigenvalues of the matrix A in ascending order. See also $info$. z (size $\max(1, ldz*n$ if $job = 'V'$ and at least 1 if $job = 'N'$). If $job = 'V'$, then this array is overwritten by the orthogonal matrix Z which contains the eigenvectors of A . The i -th column of Z contains the eigenvector which corresponds to the eigenvalue $w[i - 1]$. If $job = 'N'$, then z is not referenced. |
| ab | On exit, this array is overwritten by the values generated during the reduction to tridiagonal form. |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = i$, then the algorithm failed to converge; i indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $A+E$ such that $\|E\|_2 = O(\epsilon) * \|A\|_2$, where ϵ is the machine precision.

The complex analogue of this routine is [hbevd](#).

See also [syevd](#) for matrices held in full storage, and [spevd](#) for matrices held in packed storage.

?hbevd

Computes all eigenvalues and, optionally, all eigenvectors of a complex Hermitian band matrix using divide and conquer algorithm.

Syntax

```
lapack_int LAPACKE_chbevd( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int kd, lapack_complex_float* ab, lapack_int ldab, float* w,
lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zhbevd( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int kd, lapack_complex_double* ab, lapack_int ldab, double* w,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a complex Hermitian band matrix A . In other words, it can compute the spectral factorization of A as: $A = Z*\Lambda*Z^H$.

Here Λ is a real diagonal matrix whose diagonal elements are the eigenvalues λ_i , and Z is the (complex) unitary matrix whose columns are the eigenvectors z_i . Thus,

$$A*z_i = \lambda_i*z_i \text{ for } i = 1, 2, \dots, n.$$

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ab</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>ab</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>kd</i> | The number of super- or sub-diagonals in A ($kd \geq 0$). |
| <i>ab</i> | <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and at least $\max(1, ldab*(kd + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix A (as specified by <i>uplo</i>) in band storage format. |

ldab The leading dimension of *ab*; must be at least $kd+1$ for column major layout and n for row major layout.

ldz The leading dimension of the output array *z*.

Constraints:

if *jobz* = 'N', then $ldz \geq 1$;

if *jobz* = 'V', then $ldz \geq \max(1, n)$.

Output Parameters

w Array, size at least $\max(1, n)$.

If *info* = 0, contains the eigenvalues of the matrix *A* in ascending order. See also *info*.

z Array, size $\max(1, ldz*n)$ if *job* = 'V' and at least 1 if *job* = 'N'.

If *jobz* = 'V', then this array is overwritten by the unitary matrix *Z* which contains the eigenvectors of *A*. The *i*-th column of *Z* contains the eigenvector which corresponds to the eigenvalue $w[i - 1]$.

If *jobz* = 'N', then *z* is not referenced.

ab On exit, this array is overwritten by the values generated during the reduction to tridiagonal form.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $A + E$ such that $\|E\|_2 = O(\epsilon) \|A\|_2$, where ϵ is the machine precision.

The real analogue of this routine is [sbevd](#).

See also [heevd](#) for matrices held in full storage, and [hpevd](#) for matrices held in packed storage.

?sbevz

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix.

Syntax

```
lapack_int LAPACKE_ssbevz (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_int kd, float* ab, lapack_int ldab, float* q, lapack_int ldq,
float vl, float vu, lapack_int il, lapack_int iu, float abstol, lapack_int* m, float*
w, float* z, lapack_int ldz, lapack_int* ifail);
```

```
lapack_int LAPACKE_dsbevz (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_int kd, double* ab, lapack_int ldab, double* q, lapack_int ldq,
double vl, double vu, lapack_int il, lapack_int iu, double abstol, lapack_int* m,
double* w, double* z, lapack_int ldz, lapack_int* ifail);
```

Include Files

- `mk1.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric band matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the routine computes all eigenvalues. If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$. If <i>range</i> = 'I', the routine computes eigenvalues with indices in range <i>il</i> to <i>iu</i> . |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', <i>ab</i> stores the upper triangular part of A . If <i>uplo</i> = 'L', <i>ab</i> stores the lower triangular part of A . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>kd</i> | The number of super- or sub-diagonals in A ($kd \geq 0$). |
| <i>ab</i> | Arrays: Array <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and at least $\max(1, ldab*(kd + 1))$ for row major layout) contains either upper or lower triangular part of the symmetric matrix A (as specified by <i>uplo</i>) in band storage format. |
| <i>ldab</i> | The leading dimension of <i>ab</i> ; must be at least $kd + 1$ for column major layout and n for row major layout. |
| <i>vl, vu</i> | If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced. |
| <i>il, iu</i> | |

If $range = 'I'$, the indices in ascending order of the smallest and largest eigenvalues to be returned.

Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$

if $n = 0$.

If $range = 'A'$ or $'V'$, il and iu are not referenced.

abstol

The absolute error tolerance to which each eigenvalue is required. See *Application notes* for details on error tolerance.

ldq, ldz

The leading dimensions of the output arrays q and z , respectively.

Constraints:

$ldq \geq 1$, $ldz \geq 1$;

If $jobz = 'V'$, then $ldq \geq \max(1, n)$ and $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout .

Output Parameters

q

Array, size $\max(1, ldz*n)$.

If $jobz = 'V'$, the n -by- n orthogonal matrix is used in the reduction to tridiagonal form.

If $jobz = 'N'$, the array q is not referenced.

m

The total number of eigenvalues found, $0 \leq m \leq n$.

If $range = 'A'$, $m = n$, if $range = 'I'$, $m = iu - il + 1$, and if $range = 'V'$, the exact value of m is not known in advance.

w, z

Arrays:

w , size at least $\max(1, n)$. The first m elements of w contain the selected eigenvalues of the matrix A in ascending order.

z (size at least $\max(1, ldz*m)$ for column major layout and $\max(1, ldz*n)$ for row major layout).

If $jobz = 'V'$, then if $info = 0$, the first m columns of z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i -th column of z holding the eigenvector associated with $w[i - 1]$.

If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

If $jobz = 'N'$, then z is not referenced.

ab

On exit, this array is overwritten by the values generated during the reduction to tridiagonal form.

ifail

Array, size at least $\max(1, n)$.

If $jobz = 'V'$, then if $info = 0$, the first m elements of *ifail* are zero; if $info > 0$, the *ifail* contains the indices the eigenvectors that failed to converge.

If `jobz = 'N'`, then `ifail` is not referenced.

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

If `info = i`, then i eigenvectors failed to converge; their indices are stored in the array `ifail`.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \epsilon \max(|a|, |b|)$, where ϵ is the machine precision.

If `abstol` is less than or equal to zero, then $\epsilon \|T\|_1$ is used as tolerance, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues will be computed most accurately when `abstol` is set to twice the underflow threshold $2 * \lambda_{\text{mach}}('S')$, not zero.

If this routine returns with `info > 0`, indicating that some eigenvectors did not converge, try setting `abstol` to $2 * \lambda_{\text{mach}}('S')$.

?hbev

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian band matrix.

Syntax

```
lapack_int LAPACKE_chbev( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_int kd, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* q, lapack_int ldq, float vl, float vu, lapack_int il, lapack_int
iu, float abstol, lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz,
lapack_int* ifail );
```

```
lapack_int LAPACKE_zhbev( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_int kd, lapack_complex_double* ab, lapack_int ldab,
lapack_complex_double* q, lapack_int ldq, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, lapack_complex_double* z,
lapack_int ldz, lapack_int* ifail );
```

Include Files

- `mkl.h`

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian band matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

`matrix_layout` Specifies whether matrix storage layout is row major (`LAPACK_ROW_MAJOR`) or column major (`LAPACK_COL_MAJOR`).

`jobz` Must be 'N' or 'V'.

If `job = 'N'`, then only eigenvalues are computed.

| | |
|-----------------|--|
| | If $job = 'V'$, then eigenvalues and eigenvectors are computed. |
| <i>range</i> | <p>Must be 'A' or 'V' or 'I'.</p> <p>If $range = 'A'$, the routine computes all eigenvalues.</p> <p>If $range = 'V'$, the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$.</p> <p>If $range = 'I'$, the routine computes eigenvalues with indices il to iu.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>If $uplo = 'U'$, <i>ab</i> stores the upper triangular part of <i>A</i>.</p> <p>If $uplo = 'L'$, <i>ab</i> stores the lower triangular part of <i>A</i>.</p> |
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>kd</i> | The number of super- or sub-diagonals in <i>A</i> ($kd \geq 0$). |
| <i>ab</i> | <i>ab</i> (size at least $\max(1, ldab * n)$ for column major layout and at least $\max(1, ldab * (kd + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix <i>A</i> (as specified by <i>uplo</i>) in band storage format. |
| <i>ldab</i> | The leading dimension of <i>ab</i> ; must be at least $kd + 1$ for column major layout and n for row major layout. |
| <i>vl, vu</i> | <p>If $range = 'V'$, the lower and upper bounds of the interval to be searched for eigenvalues.</p> <p>Constraint: $vl < vu$.</p> <p>If $range = 'A'$ or 'I', <i>vl</i> and <i>vu</i> are not referenced.</p> |
| <i>il, iu</i> | <p>If $range = 'I'$, the indices in ascending order of the smallest and largest eigenvalues to be returned.</p> <p>Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$.</p> <p>If $range = 'A'$ or 'V', <i>il</i> and <i>iu</i> are not referenced.</p> |
| <i>abstol</i> | The absolute error tolerance to which each eigenvalue is required. See <i>Application notes</i> for details on error tolerance. |
| <i>ldq, ldz</i> | <p>The leading dimensions of the output arrays <i>q</i> and <i>z</i>, respectively.</p> <p>Constraints:</p> <p>$ldq \geq 1, ldz \geq 1$;</p> <p>If $jobz = 'V'$, then $ldq \geq \max(1, n)$ and $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout.</p> |

Output Parameters

q Array, size $\max(1, ldz * n)$.

| | |
|--------------|---|
| | <p>If $jobz = 'V'$, the n-by-n unitary matrix is used in the reduction to tridiagonal form.</p> <p>If $jobz = 'N'$, the array q is not referenced.</p> |
| m | <p>The total number of eigenvalues found,</p> <p>$0 \leq m \leq n$.</p> <p>If $range = 'A'$, $m = n$, if $range = 'I'$, $m = iu-il+1$, and if $range = 'V'$, the exact value of m is not known in advance..</p> |
| w | <p>Array, size at least $\max(1, n)$. The first m elements contain the selected eigenvalues of the matrix A in ascending order.</p> |
| z | <p>Array z (size at least $\max(1, ldz*m)$ for column major layout and $\max(1, ldz*n)$ for row major layout).</p> <p>If $jobz = 'V'$, then if $info = 0$, the first m columns of z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i-th column of z holding the eigenvector associated with $w[i - 1]$.</p> <p>If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in <i>ifail</i>.</p> <p>If $jobz = 'N'$, then z is not referenced.</p> |
| ab | <p>On exit, this array is overwritten by the values generated during the reduction to tridiagonal form.</p> <p>If $uplo = 'U'$, the first superdiagonal and the diagonal of the tridiagonal matrix T are returned in rows kd and $kd+1$ of ab, and if $uplo = 'L'$, the diagonal and first subdiagonal of T are returned in the first two rows of ab.</p> |
| <i>ifail</i> | <p>Array, size at least $\max(1, n)$.</p> <p>If $jobz = 'V'$, then if $info = 0$, the first m elements of <i>ifail</i> are zero; if $info > 0$, the <i>ifail</i> contains the indices of the eigenvectors that failed to converge.</p> <p>If $jobz = 'N'$, then <i>ifail</i> is not referenced.</p> |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = i$, then i eigenvectors failed to converge; their indices are stored in the array *ifail*.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \epsilon * \max(|a|, |b|)$, where ϵ is the machine precision.

If $abstol$ is less than or equal to zero, then $\epsilon * ||T||_1$ will be used in its place, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues will be computed most accurately when $abstol$ is set to twice the underflow threshold $2 * \text{lamch}('S')$, not zero.

If this routine returns with $info > 0$, indicating that some eigenvectors did not converge, try setting $abstol$ to $2 * \epsilon_{mach}('S')$.

?stev

Computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_sstev (int matrix_layout, char jobz, lapack_int n, float* d, float* e, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dstev (int matrix_layout, char jobz, lapack_int n, double* d, double* e, double* z, lapack_int ldz);
```

Include Files

- mkl.h

Description

The routine computes all eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>d, e</i> | Arrays: Array <i>d</i> contains the n diagonal elements of the tridiagonal matrix A . The size of <i>d</i> must be at least $\max(1, n)$. Array <i>e</i> contains the $n-1$ subdiagonal elements of the tridiagonal matrix A . The size of <i>e</i> must be at least $\max(1, n)$. The n -th element of this array is used as workspace. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> ; $ldz \geq 1$. If <i>jobz</i> = 'V' then $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>d</i> | On exit, if $info = 0$, contains the eigenvalues of the matrix A in ascending order. |
| <i>z</i> | Array, size (size $\max(1, ldz * n)$). If <i>jobz</i> = 'V', then if $info = 0$, <i>z</i> contains the orthonormal eigenvectors of the matrix A , with the i -th column of <i>z</i> holding the eigenvector associated with the eigenvalue returned in $d[i - 1]$. |

If $jobz = 'N'$, then z is not referenced.

e On exit, this array is overwritten with intermediate results.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = i$, then the algorithm failed to converge;

i elements of e did not converge to zero.

?stevd

Computes all eigenvalues and, optionally, all eigenvectors of a real symmetric tridiagonal matrix using divide and conquer algorithm.

Syntax

```
lapack_int LAPACKE_sstevd (int matrix_layout, char jobz, lapack_int n, float* d, float*
e, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dstevd (int matrix_layout, char jobz, lapack_int n, double* d,
double* e, double* z, lapack_int ldz);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally all the eigenvectors, of a real symmetric tridiagonal matrix T . In other words, the routine can compute the spectral factorization of T as: $T = Z\Lambda Z^T$.

Here Λ is a diagonal matrix whose diagonal elements are the eigenvalues λ_i , and Z is the orthogonal matrix whose columns are the eigenvectors z_i . Thus,

$$T^*z_i = \lambda_i^*z_i \text{ for } i = 1, 2, \dots, n.$$

If the eigenvectors are requested, then this routine uses a divide and conquer algorithm to compute eigenvalues and eigenvectors. However, if only eigenvalues are required, then it uses the Pal-Walker-Kahan variant of the QL or QR algorithm.

There is no complex analogue of this routine.

Input Parameters

$matrix_layout$ Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

$jobz$ Must be 'N' or 'V'.

If $jobz = 'N'$, then only eigenvalues are computed.

If $jobz = 'V'$, then eigenvalues and eigenvectors are computed.

n The order of the matrix T ($n \geq 0$).

| | |
|--------|---|
| d, e | <p>Arrays:</p> <p>d contains the n diagonal elements of the tridiagonal matrix T.</p> <p>The dimension of d must be at least $\max(1, n)$.</p> <p>e contains the $n-1$ off-diagonal elements of T.</p> <p>The dimension of e must be at least $\max(1, n)$. The n-th element of this array is used as workspace.</p> |
| ldz | <p>The leading dimension of the output array z. Constraints:</p> <p>$ldz \geq 1$ if $job = 'N'$;</p> <p>$ldz \geq \max(1, n)$ if $job = 'V'$.</p> |

Output Parameters

| | |
|-----|--|
| d | <p>On exit, if $info = 0$, contains the eigenvalues of the matrix T in ascending order.</p> <p>See also <i>info</i>.</p> |
| z | <p>Array, size $\max(1, ldz*n)$ if $jobz = 'V'$ and 1 if $jobz = 'N'$.</p> <p>If $jobz = 'V'$, then this array is overwritten by the orthogonal matrix Z which contains the eigenvectors of T.</p> <p>If $jobz = 'N'$, then z is not referenced.</p> |
| e | <p>On exit, this array is overwritten with intermediate results.</p> |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = i$, then the algorithm failed to converge; i indicates the number of elements of an intermediate tridiagonal form which did not converge to zero.

If $info = -i$, the i -th parameter had an illegal value.

Application Notes

The computed eigenvalues and eigenvectors are exact for a matrix $T+E$ such that $\|E\|_2 = O(\epsilon) * \|T\|_2$, where ϵ is the machine precision.

If λ_j is an exact eigenvalue, and μ_j is the corresponding computed value, then

$$|\mu_j - \lambda_j| \leq c(n) * \epsilon * \|T\|_2$$

where $c(n)$ is a modestly increasing function of n .

If z_i is the corresponding exact eigenvector, and w_i is the corresponding computed vector, then the angle $\theta(z_i, w_i)$ between them is bounded as follows:

$$\theta(z_i, w_i) \leq c(n) * \epsilon * \|T\|_2 / \min_{i \neq j} |\lambda_i - \lambda_j|.$$

Thus the accuracy of a computed eigenvector depends on the gap between its eigenvalue and all the other eigenvalues.

?stevx

Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix.

Syntax

```
lapack_int LAPACKE_sstevx (int matrix_layout, char jobz, char range, lapack_int n,
float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu, float abstol,
lapack_int* m, float* w, float* z, lapack_int ldz, lapack_int* ifail);

lapack_int LAPACKE_dstevx (int matrix_layout, char jobz, char range, lapack_int n,
double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu, double
abstol, lapack_int* m, double* w, double* z, lapack_int ldz, lapack_int* ifail);
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix A . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>job</i> = 'N', then only eigenvalues are computed. If <i>job</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the routine computes all eigenvalues. If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$. If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i> . |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>d, e</i> | Arrays: <i>d</i> contains the n diagonal elements of the tridiagonal matrix A . The dimension of <i>d</i> must be at least $\max(1, n)$. <i>e</i> contains the $n-1$ subdiagonal elements of A . The dimension of <i>e</i> must be at least $\max(1, n-1)$. The n -th element of this array is used as workspace. |
| <i>vl, vu</i> | If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced. |

il, iu If *range* = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.
 Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$.
 If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol

ldz The leading dimensions of the output array *z*; $ldz \geq 1$. If *jobz* = 'V', then $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout.

Output Parameters

m The total number of eigenvalues found,
 $0 \leq m \leq n$.
 If *range* = 'A', $m = n$, if *range* = 'I', $m = iu - il + 1$, and if *range* = 'V' the exact value of *m* is unknown.

w, z Arrays:
w, size at least $\max(1, n)$.
 The first *m* elements of *w* contain the selected eigenvalues of the matrix *A* in ascending order.
z (size at least $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout) .
 If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with $w[i - 1]$.
 If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.
 If *jobz* = 'N', then *z* is not referenced.

d, e On exit, these arrays may be multiplied by a constant factor chosen to avoid overflow or underflow in computing the eigenvalues.

ifail Array, size at least $\max(1, n)$.
 If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge.
 If *jobz* = 'N', then *ifail* is not referenced.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If $info = i$, then i eigenvectors failed to converge; their indices are stored in the array *ifail*.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \epsilon \max(|a|, |b|)$, where ϵ is the machine precision.

If $abstol$ is less than or equal to zero, then $\epsilon * |A|_1$ is used instead. Eigenvalues are computed most accurately when $abstol$ is set to twice the underflow threshold $2 * \lambda_{\text{mach}}('S')$, not zero.

If this routine returns with $info > 0$, indicating that some eigenvectors did not converge, set $abstol$ to $2 * \lambda_{\text{mach}}('S')$.

?stevr

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix using the Relatively Robust Representations.

Syntax

```
lapack_int LAPACKE_sstevr (int matrix_layout, char jobz, char range, lapack_int n,
float* d, float* e, float vl, float vu, lapack_int il, lapack_int iu, float abstol,
lapack_int* m, float* w, float* z, lapack_int ldz, lapack_int* isuppz);
```

```
lapack_int LAPACKE_dstevr (int matrix_layout, char jobz, char range, lapack_int n,
double* d, double* e, double vl, double vu, lapack_int il, lapack_int iu, double
abstol, lapack_int* m, double* w, double* z, lapack_int ldz, lapack_int* isuppz);
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T . Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Whenever possible, the routine calls [stemr](#) to compute the eigenspectrum using Relatively Robust Representations. [stegr](#) computes eigenvalues by the *dqds* algorithm, while orthogonal eigenvectors are computed from various "good" $L * D * L^T$ representations (also known as Relatively Robust Representations). Gram-Schmidt orthogonalization is avoided as far as possible. More specifically, the various steps of the algorithm are as follows. For the i -th unreduced block of T :

- Compute $T - \sigma_i = L_i * D_i * L_i^T$, such that $L_i * D_i * L_i^T$ is a relatively robust representation.
- Compute the eigenvalues, λ_j , of $L_i * D_i * L_i^T$ to high relative accuracy by the *dqds* algorithm.
- If there is a cluster of close eigenvalues, "choose" σ_i close to the cluster, and go to Step (a).
- Given the approximate eigenvalue λ_j of $L_i * D_i * L_i^T$, compute the corresponding eigenvector by forming a rank-revealing twisted factorization.

The desired accuracy of the output can be specified by the input parameter *abstol*.

The routine `?stevr` calls [stemr](#) when the full spectrum is requested on machines which conform to the IEEE-754 floating point standard. `?stevr` calls [stebz](#) and [stein](#) on non-IEEE machines and when partial spectrum requests are made.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | <p>Must be 'N' or 'V'.</p> <p>If <i>jobz</i> = 'N', then only eigenvalues are computed.</p> <p>If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed.</p> |
| <i>range</i> | <p>Must be 'A' or 'V' or 'I'.</p> <p>If <i>range</i> = 'A', the routine computes all eigenvalues.</p> <p>If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval:</p> $vl < w[i] \leq vu.$ <p>If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i>.</p> <p>For <i>range</i> = 'V' or 'I' and $iu - il < n - 1$, <i>sstebz/dstebz</i> and <i>sstein/dstein</i> are called.</p> |
| <i>n</i> | The order of the matrix T ($n \geq 0$). |
| <i>d, e</i> | <p>Arrays:</p> <p><i>d</i> contains the n diagonal elements of the tridiagonal matrix T.</p> <p>The dimension of <i>d</i> must be at least $\max(1, n)$.</p> <p><i>e</i> contains the $n-1$ subdiagonal elements of A.</p> <p>The dimension of <i>e</i> must be at least $\max(1, n-1)$. The n-th element of this array is used as workspace.</p> |
| <i>vl, vu</i> | <p>If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.</p> <p>Constraint: $vl < vu$.</p> <p>If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced.</p> |
| <i>il, iu</i> | <p>If <i>range</i> = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.</p> <p>Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$.</p> <p>If <i>range</i> = 'A' or 'V', <i>il</i> and <i>iu</i> are not referenced.</p> |
| <i>abstol</i> | <p>The absolute error tolerance to which each eigenvalue/eigenvector is required.</p> <p>If <i>jobz</i> = 'V', the eigenvalues and eigenvectors output have residual norms bounded by <i>abstol</i>, and the dot products between different eigenvectors are bounded by <i>abstol</i>. If $abstol < n * \text{eps} * T$, then $n * \text{eps} * T$ will be used in its place, where <i>eps</i> is the machine precision, and T is the 1-norm of the matrix T. The eigenvalues are computed to an accuracy of $\text{eps} * T$ irrespective of <i>abstol</i>.</p> <p>If high relative accuracy is important, set <i>abstol</i> to <code>?lamch('S')</code>.</p> |

ldz The leading dimension of the output array *z*.
 Constraints:
 $ldz \geq 1$ if *jobz* = 'N';
 $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout if *jobz* = 'V'.

Output Parameters

m The total number of eigenvalues found,
 $0 \leq m \leq n$. If *range* = 'A', $m = n$, if *range* = 'I', $m = iu - il + 1$, and if *range* = 'V' the exact value of *m* is unknown..

w, z Arrays:
w, size at least $\max(1, n)$.
 The first *m* elements of *w* contain the selected eigenvalues of the matrix *T* in ascending order.
z (size at least $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout).
 If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *T* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with $w[i - 1]$.
 If *jobz* = 'N', then *z* is not referenced.

d, e On exit, these arrays may be multiplied by a constant factor chosen to avoid overflow or underflow in computing the eigenvalues.

isuppz Array, size at least $2 * \max(1, m)$.
 The support of the eigenvectors in *z*, i.e., the indices indicating the nonzero elements in *z*. The *i*-th eigenvector is nonzero only in elements *isuppz*[2*i* - 2] through *isuppz*[2*i* - 1].
 Implemented only for *range* = 'A' or 'I' and $iu - il = n - 1$.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, an internal error has occurred.

Application Notes

Normal execution of the routine `?stegr` may create NaNs and infinities and hence may abort due to a floating point exception in environments which do not handle NaNs and infinities in the IEEE standard default manner.

Nonsymmetric Eigenvalue Problems: LAPACK Driver Routines

This section describes LAPACK driver routines used for solving nonsymmetric eigenproblems. See also [computational routines](#) that can be called to solve these problems.

Table "Driver Routines for Solving Nonsymmetric Eigenproblems" lists all such driver routines.

Driver Routines for Solving Nonsymmetric Eigenproblems

| Routine Name | Operation performed |
|-----------------------|--|
| gees | Computes the eigenvalues and Schur factorization of a general matrix, and orders the factorization so that selected eigenvalues are at the top left of the Schur form. |
| geesx | Computes the eigenvalues and Schur factorization of a general matrix, orders the factorization and computes reciprocal condition numbers. |
| geev | Computes the eigenvalues and left and right eigenvectors of a general matrix. |
| geevx | Computes the eigenvalues and left and right eigenvectors of a general matrix, with preliminary matrix balancing, and computes reciprocal condition numbers for the eigenvalues and right eigenvectors. |

?gees

Computes the eigenvalues and Schur factorization of a general matrix, and orders the factorization so that selected eigenvalues are at the top left of the Schur form.

Syntax

```
lapack_int LAPACKE_sgees( int matrix_layout, char jobvs, char sort, LAPACK_S_SELECT2
select, lapack_int n, float* a, lapack_int lda, lapack_int* sdim, float* wr, float*
wi, float* vs, lapack_int ldvs );
```

```
lapack_int LAPACKE_dgees( int matrix_layout, char jobvs, char sort, LAPACK_D_SELECT2
select, lapack_int n, double* a, lapack_int lda, lapack_int* sdim, double* wr, double*
wi, double* vs, lapack_int ldvs );
```

```
lapack_int LAPACKE_cgees( int matrix_layout, char jobvs, char sort, LAPACK_C_SELECT1
select, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_int* sdim,
lapack_complex_float* w, lapack_complex_float* vs, lapack_int ldvs );
```

```
lapack_int LAPACKE_zgees( int matrix_layout, char jobvs, char sort, LAPACK_Z_SELECT1
select, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_int* sdim,
lapack_complex_double* w, lapack_complex_double* vs, lapack_int ldvs );
```

Include Files

- mkl.h

Description

The routine computes for an n -by- n real/complex nonsymmetric matrix A , the eigenvalues, the real Schur form T , and, optionally, the matrix of Schur vectors Z . This gives the Schur factorization $A = Z^* T^* Z^H$.

Optionally, it also orders the eigenvalues on the diagonal of the real-Schur/Schur form so that selected eigenvalues are at the top left. The leading columns of Z then form an orthonormal basis for the invariant subspace corresponding to the selected eigenvalues.

A real matrix is in real-Schur form if it is upper quasi-triangular with 1-by-1 and 2-by-2 blocks. 2-by-2 blocks will be standardized in the form

$$\begin{pmatrix} a & b \\ c & a \end{pmatrix}$$

where $b*c < 0$. The eigenvalues of such a block are $a \pm i\sqrt{bc}$

A complex matrix is in Schur form if it is upper triangular.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobvs</i> | Must be 'N' or 'V'. If <i>jobvs</i> = 'N', then Schur vectors are not computed. If <i>jobvs</i> = 'V', then Schur vectors are computed. |
| <i>sort</i> | Must be 'N' or 'S'. Specifies whether or not to order the eigenvalues on the diagonal of the Schur form. If <i>sort</i> = 'N', then eigenvalues are not ordered. If <i>sort</i> = 'S', eigenvalues are ordered (see <i>select</i>). |
| <i>select</i> | If <i>sort</i> = 'S', <i>select</i> is used to select eigenvalues to sort to the top left of the Schur form. If <i>sort</i> = 'N', <i>select</i> is not referenced. <i>For real flavors:</i> An eigenvalue $wr[j] + \sqrt{-1} * wi[j]$ is selected if <i>select</i> (<i>wr</i> [<i>j</i>], <i>wi</i> [<i>j</i>]) is true; that is, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected. <i>For complex flavors:</i> An eigenvalue <i>w</i> [<i>j</i>] is selected if <i>select</i> (<i>w</i> [<i>j</i>]) is true. Note that a selected complex eigenvalue may no longer satisfy <i>select</i> (<i>wr</i> [<i>j</i>], <i>wi</i> [<i>j</i>]) = 1 after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case <i>info</i> may be set to <i>n</i> +2 (see <i>info</i> below). |
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>a</i> | Arrays: <i>a</i> (size at least $\max(1, lda*n)$) is an array containing the <i>n</i> -by- <i>n</i> matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$. |
| <i>ldvs</i> | The leading dimension of the output array <i>vs</i> . Constraints: $ldvs \geq 1$; $ldvs \geq \max(1, n)$ if <i>jobvs</i> = 'V'. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | On exit, this array is overwritten by the real-Schur/Schur form <i>T</i> . |
| <i>sdim</i> | |

If *sort* = 'N', *sdim*= 0.

If *sort* = 'S', *sdim* is equal to the number of eigenvalues (after sorting) for which *select* is true.

Note that for real flavors complex conjugate pairs for which *select* is true for either eigenvalue count as 2.

wr, *wi*

Arrays, size at least $\max(1, n)$ each. Contain the real and imaginary parts, respectively, of the computed eigenvalues, in the same order that they appear on the diagonal of the output real-Schur form *T*. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first.

w

Array, size at least $\max(1, n)$. Contains the computed eigenvalues. The eigenvalues are stored in the same order as they appear on the diagonal of the output Schur form *T*.

vs

Array *vs* (size at least $\max(1, ldvs*n)$).

If *jobvs* = 'V', *vs* contains the orthogonal/unitary matrix *Z* of Schur vectors.

If *jobvs* = 'N', *vs* is not referenced.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, and

$i \leq n$:

the QR algorithm failed to compute all the eigenvalues; elements 1:*ilo*-1 and *i*+1:*n* of *wr* and *wi* (for real flavors) or *w* (for complex flavors) contain those eigenvalues which have converged; if *jobvs* = 'V', *vs* contains the matrix which reduces *A* to its partially converged Schur form;

$i = n+1$:

the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned);

$i = n+2$:

after reordering, round-off changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy *select* = 1. This could also be caused by underflow due to scaling.

?geesx

Computes the eigenvalues and Schur factorization of a general matrix, orders the factorization and computes reciprocal condition numbers.

Syntax

```
lapack_int LAPACKE_sgeesx( int matrix_layout, char jobvs, char sort, LAPACK_S_SELECT2
select, char sense, lapack_int n, float* a, lapack_int lda, lapack_int* sdim, float*
wr, float* wi, float* vs, lapack_int ldvs, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_dgeesx( int matrix_layout, char jobvs, char sort, LAPACK_D_SELECT2
select, char sense, lapack_int n, double* a, lapack_int lda, lapack_int* sdim, double*
wr, double* wi, double* vs, lapack_int ldvs, double* rconde, double* rcondv );
```

```
lapack_int LAPACKE_cgeesx( int matrix_layout, char jobvs, char sort, LAPACK_C_SELECT1
select, char sense, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_int*
sdim, lapack_complex_float* w, lapack_complex_float* vs, lapack_int ldvs, float*
rconde, float* rcondv );
```

```
lapack_int LAPACKE_zgeesx( int matrix_layout, char jobvs, char sort, LAPACK_Z_SELECT1
select, char sense, lapack_int n, lapack_complex_double* a, lapack_int lda,
lapack_int* sdim, lapack_complex_double* w, lapack_complex_double* vs, lapack_int ldvs,
double* rconde, double* rcondv );
```

Include Files

- mkl.h

Description

The routine computes for an n -by- n real/complex nonsymmetric matrix A , the eigenvalues, the real-Schur/Schur form T , and, optionally, the matrix of Schur vectors Z . This gives the Schur factorization $A = Z^* T^* Z^H$.

Optionally, it also orders the eigenvalues on the diagonal of the real-Schur/Schur form so that selected eigenvalues are at the top left; computes a reciprocal condition number for the average of the selected eigenvalues ($rconde$); and computes a reciprocal condition number for the right invariant subspace corresponding to the selected eigenvalues ($rcondv$). The leading columns of Z form an orthonormal basis for this invariant subspace.

For further explanation of the reciprocal condition numbers $rconde$ and $rcondv$, see [LUG], Section 4.10 (where these quantities are called s and sep respectively).

A real matrix is in real-Schur form if it is upper quasi-triangular with 1-by-1 and 2-by-2 blocks. 2-by-2 blocks will be standardized in the form

$$\begin{pmatrix} a & b \\ c & a \end{pmatrix}$$

where $b^*c < 0$. The eigenvalues of such a block are $a \pm i\sqrt{bc}$

A complex matrix is in Schur form if it is upper triangular.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobvs</i> | Must be 'N' or 'V'. If <i>jobvs</i> = 'N', then Schur vectors are not computed. If <i>jobvs</i> = 'V', then Schur vectors are computed. |
| <i>sort</i> | Must be 'N' or 'S'. Specifies whether or not to order the eigenvalues on the diagonal of the Schur form. If <i>sort</i> = 'N', then eigenvalues are not ordered. |

| | |
|---------------|---|
| | If <i>sort</i> = 'S', eigenvalues are ordered (see <i>select</i>). |
| <i>select</i> | <p>If <i>sort</i> = 'S', <i>select</i> is used to select eigenvalues to sort to the top left of the Schur form.</p> <p>If <i>sort</i> = 'N', <i>select</i> is not referenced.</p> <p><i>For real flavors:</i></p> <p>An eigenvalue $wr[j] + \sqrt{-1} * wi[j]$ is selected if <i>select</i>(<i>wr</i>[<i>j</i>], <i>wi</i>[<i>j</i>]) is true; that is, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected.</p> <p><i>For complex flavors:</i></p> <p>An eigenvalue <i>w</i>[<i>j</i>] is selected if <i>select</i>(<i>w</i>[<i>j</i>]) is true.</p> <p>Note that a selected complex eigenvalue may no longer satisfy <i>select</i>(<i>wr</i>[<i>j</i>], <i>wi</i>[<i>j</i>]) = 1 after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case <i>info</i> may be set to <i>n</i>+2 (see <i>info</i> below).</p> |
| <i>sense</i> | <p>Must be 'N', 'E', 'V', or 'B'. Determines which reciprocal condition number are computed.</p> <p>If <i>sense</i> = 'N', none are computed;</p> <p>If <i>sense</i> = 'E', computed for average of selected eigenvalues only;</p> <p>If <i>sense</i> = 'V', computed for selected right invariant subspace only;</p> <p>If <i>sense</i> = 'B', computed for both.</p> <p>If <i>sense</i> is 'E', 'V', or 'B', then <i>sort</i> must equal 'S'.</p> |
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>a</i> | <p>Arrays:</p> <p><i>a</i> (size at least $\max(1, lda * n)$) is an array containing the <i>n</i>-by-<i>n</i> matrix <i>A</i>.</p> |
| <i>lda</i> | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$. |
| <i>ldvs</i> | <p>The leading dimension of the output array <i>vs</i>. Constraints:</p> <p>$ldvs \geq 1$;</p> <p>$ldvs \geq \max(1, n)$ if <i>jobvs</i> = 'V'.</p> |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | On exit, this array is overwritten by the real-Schur/Schur form <i>T</i> . |
| <i>sdim</i> | <p>If <i>sort</i> = 'N', <i>sdim</i> = 0.</p> <p>If <i>sort</i> = 'S', <i>sdim</i> is equal to the number of eigenvalues (after sorting) for which <i>select</i> is true.</p> <p>Note that for real flavors complex conjugate pairs for which <i>select</i> is true for either eigenvalue count as 2.</p> |

| | |
|-----------------------|--|
| <i>wr, wi</i> | Arrays, size at least $\max(1, n)$ each. Contain the real and imaginary parts, respectively, of the computed eigenvalues, in the same order that they appear on the diagonal of the output real-Schur form <i>T</i> . Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first. |
| <i>w</i> | Array, size at least $\max(1, n)$. Contains the computed eigenvalues. The eigenvalues are stored in the same order as they appear on the diagonal of the output Schur form <i>T</i> . |
| <i>vs</i> | Array <i>vs</i> (size at least $\max(1, ldvs*n)$) If <i>jobvs</i> = 'V', <i>vs</i> contains the orthogonal/unitary matrix <i>Z</i> of Schur vectors. If <i>jobvs</i> = 'N', <i>vs</i> is not referenced. |
| <i>rconde, rcondv</i> | If <i>sense</i> = 'E' or 'B', <i>rconde</i> contains the reciprocal condition number for the average of the selected eigenvalues. If <i>sense</i> = 'N' or 'V', <i>rconde</i> is not referenced. If <i>sense</i> = 'V' or 'B', <i>rcondv</i> contains the reciprocal condition number for the selected right invariant subspace. If <i>sense</i> = 'N' or 'E', <i>rcondv</i> is not referenced. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, and

i ≤ *n*:

the QR algorithm failed to compute all the eigenvalues; elements 1:*ilo*-1 and *i*+1:*n* of *wr* and *wi* (for real flavors) or *w* (for complex flavors) contain those eigenvalues which have converged; if *jobvs* = 'V', *vs* contains the transformation which reduces *A* to its partially converged Schur form;

i = *n*+1:

the eigenvalues could not be reordered because some eigenvalues were too close to separate (the problem is very ill-conditioned);

i = *n*+2:

after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the Schur form no longer satisfy *select* = 1. This could also be caused by underflow due to scaling.

?ggev

Computes the eigenvalues and left and right eigenvectors of a general matrix.

Syntax

```
lapack_int LAPACKE_sgeev( int matrix_layout, char jobvl, char jobvr, lapack_int n,
float* a, lapack_int lda, float* wr, float* wi, float* vl, lapack_int ldvl, float* vr,
lapack_int ldvr );
```

```
lapack_int LAPACKE_dgeev( int matrix_layout, char jobvl, char jobvr, lapack_int n,
double* a, lapack_int lda, double* wr, double* wi, double* vl, lapack_int ldvl,
double* vr, lapack_int ldvr );
```

```
lapack_int LAPACKE_cgeev( int matrix_layout, char jobvl, char jobvr, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* w, lapack_complex_float*
vl, lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr );
```

```
lapack_int LAPACKE_zgeev( int matrix_layout, char jobvl, char jobvr, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* w,
lapack_complex_double* vl, lapack_int ldvl, lapack_complex_double* vr, lapack_int
ldvr );
```

Include Files

- mkl.h

Description

The routine computes for an n -by- n real/complex nonsymmetric matrix A , the eigenvalues and, optionally, the left and/or right eigenvectors. The right eigenvector v of A satisfies

$$A*v = \lambda*v$$

where λ is its eigenvalue.

The left eigenvector u of A satisfies

$$u^H*A = \lambda*u^H$$

where u^H denotes the conjugate transpose of u . The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobvl</i> | Must be 'N' or 'V'. If <i>jobvl</i> = 'N', then left eigenvectors of A are not computed. If <i>jobvl</i> = 'V', then left eigenvectors of A are computed. |
| <i>jobvr</i> | Must be 'N' or 'V'. If <i>jobvr</i> = 'N', then right eigenvectors of A are not computed. If <i>jobvr</i> = 'V', then right eigenvectors of A are computed. |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>a</i> | a (size at least $\max(1, lda*n)$) is an array containing the n -by- n matrix A . |
| <i>lda</i> | The leading dimension of the array a . Must be at least $\max(1, n)$. |
| <i>ldvl, ldvr</i> | The leading dimensions of the output arrays vl and vr , respectively. Constraints: $ldvl \geq 1; ldvr \geq 1$. If <i>jobvl</i> = 'V', $ldvl \geq \max(1, n)$; |

If $jobvr = 'V'$, $ldvr \geq \max(1, n)$.

Output Parameters

| | |
|----------|--|
| a | On exit, this array is overwritten. |
| wr, wi | Arrays, size at least $\max(1, n)$ each. Contain the real and imaginary parts, respectively, of the computed eigenvalues. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first. |
| w | Array, size at least $\max(1, n)$. Contains the computed eigenvalues. |
| vl, vr | Arrays: vl (size at least $\max(1, ldvl*n)$). If $jobvl = 'N'$, vl is not referenced. <i>For real flavors:</i> If the j -th eigenvalue is real, the i -th component of the j -th eigenvector u_j is stored in $vl[(i - 1) + (j - 1)*ldvl]$ for column major layout and in $vl[(i - 1)*ldvl + (j - 1)]$ for row major layout.. If the j -th and $(j+1)$ -st eigenvalues form a complex conjugate pair, then for $i = \text{sqrt}(-1)$, the k -th component of the j -th eigenvector u_j is $vl[(k - 1) + (j - 1)*ldvl] + i*vl[(k - 1) + j*ldvl]$ for column major layout and as $vl[(k - 1)*ldvl + (j - 1)] + i*vl[(k-1)*ldvl + j]$ for row major layout. Similarly, the k -th component of vector $(j+1)$ u_{j+1} is $vl[(k - 1) + (j - 1)*ldvl] - i*vl[(k - 1) + j*ldvl]$ for column major layout and as $vl[(k - 1)*ldvl + (j - 1)] - i*vl[(k - 1)*ldvl + j]$ for row major layout. . <i>For complex flavors:</i> The i -th component of the j -th eigenvector u_j is stored in $vl[(i - 1) + (j - 1)*ldvl]$ for column major layout and in $vl[(i - 1)*ldvl + (j - 1)]$ for row major layout. vr (size at least $\max(1, ldvr*n)$). If $jobvr = 'N'$, vr is not referenced. <i>For real flavors:</i> If the j -th eigenvalue is real, then the i -th component of j -th eigenvector v_j is stored in $vr[(i - 1) + (j - 1)*ldvr]$ for column major layout and in $vr[(i - 1)*ldvr + (j - 1)]$ for row major layout.. If the j -th and $(j+1)$ -st eigenvalues form a complex conjugate pair, then for $i = \text{sqrt}(-1)$, the k -th component of the j -th eigenvector v_j is $vr[(k - 1) + (j - 1)*ldvr] + i*vr[(k - 1) + j*ldvr]$ for column major layout and as $vr[(k - 1)*ldvr + (j - 1)] + i*vr[(k - 1)*ldvr + j]$ for row major layout. Similarly, the k -th component of vector $j + 1$ v_{j+1} is $vr[(k - 1) + (j - 1)*ldvr] - i*vr[(k - 1) + j*ldvr]$ for column major layout and as $vr[(k - 1)*ldvr + (j - 1)] - i*vr[(k - 1)*ldvr + j]$ for row major layout. <i>For complex flavors:</i> |

The i -th component of the j -th eigenvector v_j is stored in $vr[(i - 1) + (j - 1)*ldvr]$ for column major layout and in $vr[(i - 1)*ldvr + (j - 1)]$ for row major layout.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = $-i$, the i -th parameter had an illegal value.

If *info* = i , the QR algorithm failed to compute all the eigenvalues, and no eigenvectors have been computed; elements $i+1:n$ of wr and wi (for real flavors) or w (for complex flavors) contain those eigenvalues which have converged.

?geevx

Computes the eigenvalues and left and right eigenvectors of a general matrix, with preliminary matrix balancing, and computes reciprocal condition numbers for the eigenvalues and right eigenvectors.

Syntax

```
lapack_int LAPACKE_sgeevx( int matrix_layout, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, float* a, lapack_int lda, float* wr, float* wi, float* vl,
lapack_int ldvl, float* vr, lapack_int ldvr, lapack_int* ilo, lapack_int* ihi, float*
scale, float* abnrm, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_dgeevx( int matrix_layout, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, double* a, lapack_int lda, double* wr, double* wi, double* vl,
lapack_int ldvl, double* vr, lapack_int ldvr, lapack_int* ilo, lapack_int* ihi,
double* scale, double* abnrm, double* rconde, double* rcondv );
```

```
lapack_int LAPACKE_cgeevx( int matrix_layout, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* w,
lapack_complex_float* vl, lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr,
lapack_int* ilo, lapack_int* ihi, float* scale, float* abnrm, float* rconde, float*
rcondv );
```

```
lapack_int LAPACKE_zgeevx( int matrix_layout, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double*
w, lapack_complex_double* vl, lapack_int ldvl, lapack_complex_double* vr, lapack_int
ldvr, lapack_int* ilo, lapack_int* ihi, double* scale, double* abnrm, double* rconde,
double* rcondv );
```

Include Files

- mkl.h

Description

The routine computes for an n -by- n real/complex nonsymmetric matrix A , the eigenvalues and, optionally, the left and/or right eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (*ilo*, *ihi*, *scale*, and *abnrm*), reciprocal condition numbers for the eigenvalues (*rconde*), and reciprocal condition numbers for the right eigenvectors (*rcondv*).

The right eigenvector v of A satisfies

$$A \cdot v = \lambda \cdot v$$

where λ is its eigenvalue.

The left eigenvector u of A satisfies

$$u^H A = \lambda u^H$$

where u^H denotes the conjugate transpose of u . The computed eigenvectors are normalized to have Euclidean norm equal to 1 and largest component real.

Balancing a matrix means permuting the rows and columns to make it more nearly upper triangular, and applying a diagonal similarity transformation $D \cdot A \cdot \text{inv}(D)$, where D is a diagonal matrix, to make its rows and columns closer in norm and the condition numbers of its eigenvalues and eigenvectors smaller. The computed reciprocal condition numbers correspond to the balanced matrix. Permuting rows and columns will not change the condition numbers in exact arithmetic) but diagonal scaling will. For further explanation of balancing, see [LUG], Section 4.10.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>balanc</i> | <p>Must be 'N', 'P', 'S', or 'B'. Indicates how the input matrix should be diagonally scaled and/or permuted to improve the conditioning of its eigenvalues.</p> <p>If <i>balanc</i> = 'N', do not diagonally scale or permute;</p> <p>If <i>balanc</i> = 'P', perform permutations to make the matrix more nearly upper triangular. Do not diagonally scale;</p> <p>If <i>balanc</i> = 'S', diagonally scale the matrix, i.e. replace A by $D \cdot A \cdot \text{inv}(D)$, where D is a diagonal matrix chosen to make the rows and columns of A more equal in norm. Do not permute;</p> <p>If <i>balanc</i> = 'B', both diagonally scale and permute A.</p> <p>Computed reciprocal condition numbers will be for the matrix after balancing and/or permuting. Permuting does not change condition numbers (in exact arithmetic), but balancing does.</p> |
| <i>jobvl</i> | <p>Must be 'N' or 'V'.</p> <p>If <i>jobvl</i> = 'N', left eigenvectors of A are not computed;</p> <p>If <i>jobvl</i> = 'V', left eigenvectors of A are computed.</p> <p>If <i>sense</i> = 'E' or 'B', then <i>jobvl</i> must be 'V'.</p> |
| <i>jobvr</i> | <p>Must be 'N' or 'V'.</p> <p>If <i>jobvr</i> = 'N', right eigenvectors of A are not computed;</p> <p>If <i>jobvr</i> = 'V', right eigenvectors of A are computed.</p> <p>If <i>sense</i> = 'E' or 'B', then <i>jobvr</i> must be 'V'.</p> |
| <i>sense</i> | <p>Must be 'N', 'E', 'V', or 'B'. Determines which reciprocal condition number are computed.</p> <p>If <i>sense</i> = 'N', none are computed;</p> <p>If <i>sense</i> = 'E', computed for eigenvalues only;</p> |

If *sense* = 'V', computed for right eigenvectors only;
 If *sense* = 'B', computed for eigenvalues and right eigenvectors.
 If *sense* is 'E' or 'B', both left and right eigenvectors must also be computed (*jobvl* = 'V' and *jobvr* = 'V').

n The order of the matrix *A* ($n \geq 0$).

a Arrays:
a (size at least $\max(1, lda*n)$) is an array containing the *n*-by-*n* matrix *A*.

lda The leading dimension of the array *a*. Must be at least $\max(1, n)$.

ldvl, ldvr The leading dimensions of the output arrays *vl* and *vr*, respectively.
 Constraints:
 $ldvl \geq 1; ldvr \geq 1$.
 If *jobvl* = 'V', $ldvl \geq \max(1, n)$;
 If *jobvr* = 'V', $ldvr \geq \max(1, n)$.

Output Parameters

a On exit, this array is overwritten.
 If *jobvl* = 'V' or *jobvr* = 'V', it contains the real-Schur/Schur form of the balanced version of the input matrix *A*.

wr, wi Arrays, size at least $\max(1, n)$ each. Contain the real and imaginary parts, respectively, of the computed eigenvalues. Complex conjugate pairs of eigenvalues appear consecutively with the eigenvalue having positive imaginary part first.

w Array, size at least $\max(1, n)$. Contains the computed eigenvalues.

vl, vr Arrays:
vl (size at least $\max(1, ldvl*n)$).
 If *jobvl* = 'N', *vl* is not referenced.
For real flavors:
 If the *j*-th eigenvalue is real, the *i*-th component of the *j*-th eigenvector u_j is stored in $vl[(i - 1) + (j - 1)*ldvl]$ for column major layout and in $vl[(i - 1)*ldvl + (j - 1)]$ for row major layout..
 If the *j*-th and (*j*+1)-st eigenvalues form a complex conjugate pair, then for $i = \text{sqrt}(-1)$, the *k*-th component of the *j*-th eigenvector u_j is $vl[(k - 1) + (j - 1)*ldvl] + i*vl[(k - 1) + j*ldvl]$ for column major layout and as $vl[(k - 1)*ldvl + (j - 1)] + i*vl[(k-1)*ldvl + j]$ for row major layout.
 Similarly, the *k*-th component of vector (*j*+1) u_{j+1} is $vl[(k - 1) + (j - 1)*ldvl] - i*vl[(k - 1) + j*ldvl]$ for column major layout and as $vl[(k - 1)*ldvl + (j - 1)] - i*vl[(k - 1)*ldvl + j]$ for row major layout. .
For complex flavors:

The i -th component of the j -th eigenvector u_j is stored in $vl[(i - 1) + (j - 1)*ldvl]$ for column major layout and in $vl[(i - 1)*ldvl + (j - 1)]$ for row major layout.

vr (size at least $\max(1, ldvr*n)$).

If $jobvr = 'N'$, vr is not referenced.

For real flavors:

If the j -th eigenvalue is real, then the i -th component of j -th eigenvector v_j is stored in $vr[(i - 1) + (j - 1)*ldvr]$ for column major layout and in $vr[(i - 1)*ldvr + (j - 1)]$ for row major layout..

If the j -th and $(j+1)$ -st eigenvalues form a complex conjugate pair, then for $i = \text{sqrt}(-1)$, the k -th component of the j -th eigenvector v_j is $vr[(k - 1) + (j - 1)*ldvr] + i*vr[(k - 1) + j*ldvr]$ for column major layout and as $vr[(k - 1)*ldvr + (j - 1)] + i*vr[(k - 1)*ldvr + j]$ for row major layout. Similarly, the k -th component of vector $j + 1$ v_{j+1} is $vr[(k - 1) + (j - 1)*ldvr] - i*vr[(k - 1) + j*ldvr]$ for column major layout and as $vr[(k - 1)*ldvr + (j - 1)] - i*vr[(k - 1)*ldvr + j]$ for row major layout.

For complex flavors:

The i -th component of the j -th eigenvector v_j is stored in $vr[(i - 1) + (j - 1)*ldvr]$ for column major layout and in $vr[(i - 1)*ldvr + (j - 1)]$ for row major layout.

ilo, ihi

ilo and *ihi* are integer values determined when A was balanced.

The balanced $A(i, j) = 0$ if $i > j$ and $j = 1, \dots, ilo-1$ or $i = ihi+1, \dots, n$.

If $balanc = 'N'$ or $'S'$, $ilo = 1$ and $ihi = n$.

scale

Array, size at least $\max(1, n)$. Details of the permutations and scaling factors applied when balancing A .

If $P[j - 1]$ is the index of the row and column interchanged with row and column j , and $D[j - 1]$ is the scaling factor applied to row and column j , then

$scale[j - 1] = P[j - 1]$, for $j = 1, \dots, ilo-1$

$= D[j - 1]$, for $j = ilo, \dots, ihi$

$= P[j - 1]$ for $j = ihi+1, \dots, n$.

The order in which the interchanges are made is n to $ihi+1$, then 1 to $ilo-1$.

abnrm

The one-norm of the balanced matrix (the maximum of the sum of absolute values of elements of any column).

rconde, rcondv

Arrays, size at least $\max(1, n)$ each.

$rconde[j - 1]$ is the reciprocal condition number of the j -th eigenvalue.

$rcondv[j - 1]$ is the reciprocal condition number of the j -th right eigenvector.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, the QR algorithm failed to compute all the eigenvalues, and no eigenvectors or condition numbers have been computed; elements 1:*ilo*-1 and *i*+1:*n* of *wr* and *wi* (for real flavors) or *w* (for complex flavors) contain eigenvalues which have converged.

Singular Value Decomposition: LAPACK Driver Routines

Table "Driver Routines for Singular Value Decomposition" lists the LAPACK driver routines that perform singular value decomposition .

Driver Routines for Singular Value Decomposition

| Routine Name | Operation performed |
|--------------|--|
| ?gesvd | Computes the singular value decomposition of a general rectangular matrix. |
| ?gesdd | Computes the singular value decomposition of a general rectangular matrix using a divide and conquer method. |
| ?gejsv | Computes the singular value decomposition of a real matrix using a preconditioned Jacobi SVD method. |
| ?gesvj | Computes the singular value decomposition of a real matrix using Jacobi plane rotations. |
| ?ggsvd | Computes the generalized singular value decomposition of a pair of general rectangular matrices. |
| ?gesvdx | Computes the SVD and left and right singular vectors for a matrix. |
| ?bdsvdx | Computes the SVD of a bidiagonal matrix. |

Singular Value Decomposition: LAPACK Computational Routines

?gesvd

Computes the singular value decomposition of a general rectangular matrix.

Syntax

```
lapack_int LAPACKE_sgesvd( int matrix_layout, char jobu, char jobvt, lapack_int m,
lapack_int n, float* a, lapack_int lda, float* s, float* u, lapack_int ldu, float* vt,
lapack_int ldvt, float* superb );
```

```
lapack_int LAPACKE_dgesvd( int matrix_layout, char jobu, char jobvt, lapack_int m,
lapack_int n, double* a, lapack_int lda, double* s, double* u, lapack_int ldu, double*
vt, lapack_int ldvt, double* superb );
```

```
lapack_int LAPACKE_cgesvd( int matrix_layout, char jobu, char jobvt, lapack_int m,
lapack_int n, lapack_complex_float* a, lapack_int lda, float* s, lapack_complex_float*
u, lapack_int ldu, lapack_complex_float* vt, lapack_int ldvt, float* superb );
```

```
lapack_int LAPACKE_zgesvd( int matrix_layout, char jobu, char jobvt, lapack_int m,
lapack_int n, lapack_complex_double* a, lapack_int lda, double* s,
lapack_complex_double* u, lapack_int ldu, lapack_complex_double* vt, lapack_int ldvt,
double* superb );
```

Include Files

- mkl.h

Description

The routine computes the singular value decomposition (SVD) of a real/complex m -by- n matrix A , optionally computing the left and/or right singular vectors. The SVD is written as

$$A = U^* \Sigma^* V^T \text{ for real routines}$$

$$A = U^* \Sigma^* V^H \text{ for complex routines}$$

where Σ is an m -by- n matrix which is zero except for its $\min(m, n)$ diagonal elements, U is an m -by- m orthogonal/unitary matrix, and V is an n -by- n orthogonal/unitary matrix. The diagonal elements of Σ are the singular values of A ; they are real and non-negative, and are returned in descending order. The first $\min(m, n)$ columns of U and V are the left and right singular vectors of A .

Note that the routine returns V^T (for real flavors) or V^H (for complex flavors), not V .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobu</i> | <p>Must be 'A', 'S', 'O', or 'N'. Specifies options for computing all or part of the matrix U.</p> <p>If <i>jobu</i> = 'A', all m columns of U are returned in the array u;</p> <p>if <i>jobu</i> = 'S', the first $\min(m, n)$ columns of U (the left singular vectors) are returned in the array u;</p> <p>if <i>jobu</i> = 'O', the first $\min(m, n)$ columns of U (the left singular vectors) are overwritten on the array a;</p> <p>if <i>jobu</i> = 'N', no columns of U (no left singular vectors) are computed.</p> |
| <i>jobvt</i> | <p>Must be 'A', 'S', 'O', or 'N'. Specifies options for computing all or part of the matrix V^T/V^H.</p> <p>If <i>jobvt</i> = 'A', all n rows of V^T/V^H are returned in the array vt;</p> <p>if <i>jobvt</i> = 'S', the first $\min(m, n)$ rows of V^T/V^H (the right singular vectors) are returned in the array vt;</p> <p>if <i>jobvt</i> = 'O', the first $\min(m, n)$ rows of V^T/V^H (the right singular vectors) are overwritten on the array a;</p> <p>if <i>jobvt</i> = 'N', no rows of V^T/V^H (no right singular vectors) are computed.</p> <p><i>jobvt</i> and <i>jobu</i> cannot both be 'O'.</p> |
| <i>m</i> | The number of rows of the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns in A ($n \geq 0$). |
| <i>a</i> | <p>Arrays:</p> <p>a(size at least $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout) is an array containing the m-by-n matrix A.</p> |
| <i>lda</i> | <p>The leading dimension of the array a.</p> <p>Must be at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout .</p> |

ldu, ldvt

The leading dimensions of the output arrays *u* and *vt*, respectively.

Constraints:

$ldu \geq 1$; $ldvt \geq 1$.

If *jobu* = 'A', $ldu \geq m$;

If *jobu* = 'S', $ldu \geq m$ for column major layout and $ldu \geq \min(m, n)$ for row major layout;

If *jobvt* = 'A', $ldvt \geq n$;

If *jobvt* = 'S', $ldvt \geq \min(m, n)$ for column major layout and $ldvt \geq n$ for row major layout .

Output Parameters

a

On exit,

If *jobu* = 'O', *a* is overwritten with the first $\min(m, n)$ columns of *U* (the left singular vectors stored columnwise);

If *jobvt* = 'O', *a* is overwritten with the first $\min(m, n)$ rows of V^T/V^H (the right singular vectors stored rowwise);

If *jobu* ≠ 'O' and *jobvt* ≠ 'O', the contents of *a* are destroyed.

s

Array, size at least $\max(1, \min(m, n))$. Contains the singular values of *A* sorted so that $s[i] \geq s[i + 1]$.

u, vt

Arrays:

Array *u* minimum size:

| | Column major layout | Row major layout |
|-------------------|-----------------------------|--------------------|
| <i>jobu</i> = 'A' | $\max(1, ldu * m)$ | $\max(1, ldu * m)$ |
| <i>jobu</i> = 'S' | $\max(1, ldu * \min(m, n))$ | $\max(1, ldu * m)$ |

If *jobu* = 'A', *u* contains the *m*-by-*m* orthogonal/unitary matrix *U*.

If *jobu* = 'S', *u* contains the first $\min(m, n)$ columns of *U* (the left singular vectors stored column-wise).

If *jobu* = 'N' or 'O', *u* is not referenced.

Array *v* minimum size:

| | Column major layout | Row major layout |
|--------------------|------------------------------|---------------------|
| <i>jobvt</i> = 'A' | $\max(1, ldvt * n)$ | $\max(1, ldvt * n)$ |
| <i>jobvt</i> = 'S' | $\max(1, ldvt * \min(m, n))$ | $\max(1, ldvt * n)$ |

If *jobvt* = 'A', *vt* contains the *n*-by-*n* orthogonal/unitary matrix V^T/V^H .

If *jobvt* = 'S', *vt* contains the first $\min(m, n)$ rows of V^T/V^H (the right singular vectors stored row-wise).

If `jobvt = 'N'` or `'O'`, `vt` is not referenced.

superb

If `?bdsqr` does not converge (indicated by the return value `info > 0`), on exit `superb(0:min(m,n)-2)` contains the unconverged superdiagonal elements of an upper bidiagonal matrix B whose diagonal is in `s` (not necessarily sorted). B satisfies $A = u^*B^*v^T$ (real flavors) or $A = u^*B^*v^H$ (complex flavors), so it has the same singular values as A , and singular vectors related by u and vt .

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

If `info = i`, then if `?bdsqr` did not converge, i specifies how many superdiagonals of the intermediate bidiagonal form B did not converge to zero (see the description of the `superb` parameter for details).

?gesdd

Computes the singular value decomposition of a general rectangular matrix using a divide and conquer method.

Syntax

```
lapack_int LAPACKE_sgesdd( int matrix_layout, char jobz, lapack_int m, lapack_int n,
float* a, lapack_int lda, float* s, float* u, lapack_int ldu, float* vt, lapack_int
ldvt );
```

```
lapack_int LAPACKE_dgesdd( int matrix_layout, char jobz, lapack_int m, lapack_int n,
double* a, lapack_int lda, double* s, double* u, lapack_int ldu, double* vt,
lapack_int ldvt );
```

```
lapack_int LAPACKE_cgesdd( int matrix_layout, char jobz, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, float* s, lapack_complex_float* u, lapack_int
ldu, lapack_complex_float* vt, lapack_int ldvt );
```

```
lapack_int LAPACKE_zgesdd( int matrix_layout, char jobz, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, double* s, lapack_complex_double* u,
lapack_int ldu, lapack_complex_double* vt, lapack_int ldvt );
```

Include Files

- `mkl.h`

Description

The routine computes the singular value decomposition (SVD) of a real/complex m -by- n matrix A , optionally computing the left and/or right singular vectors.

If singular vectors are desired, it uses a divide-and-conquer algorithm. The SVD is written

$$A = U^* \Sigma^* V^T \text{ for real routines,}$$

$$A = U^* \Sigma^* V^H \text{ for complex routines,}$$

where Σ is an m -by- n matrix which is zero except for its $\min(m,n)$ diagonal elements, U is an m -by- m orthogonal/unitary matrix, and V is an n -by- n orthogonal/unitary matrix. The diagonal elements of Σ are the singular values of A ; they are real and non-negative, and are returned in descending order. The first $\min(m, n)$ columns of U and V are the left and right singular vectors of A .

Note that the routine returns $vt = V^T$ (for real flavors) or $vt = V^H$ (for complex flavors), not V .

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

jobz Must be 'A', 'S', 'O', or 'N'.
 Specifies options for computing all or part of the matrices U and V .
 If $jobz = 'A'$, all m columns of U and all n rows of V^T or V^H are returned in the arrays u and vt ;
 if $jobz = 'S'$, the first $\min(m, n)$ columns of U and the first $\min(m, n)$ rows of V^T or V^H are returned in the arrays u and vt ;
 if $jobz = 'O'$, then
 if $m \geq n$, the first n columns of U are overwritten in the array a and all rows of V^T or V^H are returned in the array vt ;
 if $m < n$, all columns of U are returned in the array u and the first m rows of V^T or V^H are overwritten in the array a ;
 if $jobz = 'N'$, no columns of U or rows of V^T or V^H are computed.

m The number of rows of the matrix A ($m \geq 0$).

n The number of columns in A ($n \geq 0$).

a $a(\text{size max}(1, lda*n)$ for column major layout and $\text{max}(1, lda*m)$ for row major layout) is an array containing the m -by- n matrix A .

lda The leading dimension of the array a . Must be at least $\text{max}(1, m)$ for column major layout and at least $\text{max}(1, n)$ for row major layout.

ldu, ldvt The leading dimensions of the output arrays u and vt , respectively.

The minimum size of *ldu* is

| <i>jobz</i> | $m \geq n$ | $m < n$ |
|-------------|---|---------|
| 'N' | 1 | 1 |
| 'A' | m | m |
| 'S' | m for column major layout; n for row major layout | m |
| 'O' | 1 | m |

The minimum size of *ldvt* is

| <i>jobz</i> | $m \geq n$ | $m < n$ |
|-------------|------------|---------|
| 'N' | 1 | 1 |
| 'A' | n | n |

| <i>jobz</i> | $m \geq n$ | $m < n$ |
|-------------|------------|---|
| 'S' | n | m for column major layout; n for row major layout |
| 'O' | n | 1 |

Output Parameters

a

On exit:

If $jobz = 'O'$, then if $m \geq n$, a is overwritten with the first n columns of U (the left singular vectors, stored columnwise). If $m < n$, a is overwritten with the first m rows of V^T (the right singular vectors, stored rowwise);

If $jobz \neq 'O'$, the contents of a are destroyed.

s

Array, size at least $\max(1, \min(m, n))$. Contains the singular values of A sorted so that $s(i) \geq s(i+1)$.

u, vt

Arrays:

Array u is of size:

| <i>jobz</i> | $m \geq n$ | $m < n$ |
|-------------|--|--------------------|
| 'N' | 1 | 1 |
| 'A' | $\max(1, ldu * m)$ | $\max(1, ldu * m)$ |
| 'S' | $\max(1, ldu * n)$ for column major layout; $\max(1, ldu * m)$ for row major layout | $\max(1, ldu * m)$ |
| 'O' | 1 | $\max(1, ldu * m)$ |

If $jobz = 'A'$ or $jobz = 'O'$ and $m < n$, u contains the m -by- m orthogonal/unitary matrix U .

If $jobz = 'S'$, u contains the first $\min(m, n)$ columns of U (the left singular vectors, stored columnwise).

If $jobz = 'O'$ and $m \geq n$, or $jobz = 'N'$, u is not referenced.

Array vt is of size:

| <i>jobz</i> | $m \geq n$ | $m < n$ |
|-------------|---------------------|---|
| 'N' | 1 | 1 |
| 'A' | $\max(1, ldvt * n)$ | $\max(1, ldvt * n)$ |
| 'S' | $\max(1, ldvt * n)$ | $\max(1, ldvt * n)$ for column major layout; $\max(1, ldvt * m)$ for row major layout; |

| <i>jobz</i> | $m \geq n$ | $m < n$ |
|-------------|-------------------|---------|
| 'O' | $\max(1, ldvt*n)$ | 1 |

If *jobz* = 'A' or *jobz* = 'O' and $m \geq n$, *vt* contains the *n*-by-*n* orthogonal/unitary matrix V^T .

If *jobz* = 'S', *vt* contains the first $\min(m, n)$ rows of V^T (the right singular vectors, stored rowwise).

If *jobz* = 'O' and $m < n$, or *jobz* = 'N', *vt* is not referenced.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, then ?bdsdc did not converge, updating process failed.

?gejsv

Computes the singular value decomposition using a preconditioned Jacobi SVD method.

Syntax

```
lapack_int LAPACKE_sgejsv (int matrix_layout, char joba, char jobu, char jobv, char jobr, char jobt, char jobp, lapack_int m, lapack_int n, float * a, lapack_int lda, float * sva, float * u, lapack_int ldu, float * v, lapack_int ldv, float * stat, lapack_int * istat);
```

```
lapack_int LAPACKE_dgejsv (int matrix_layout, char joba, char jobu, char jobv, char jobr, char jobt, char jobp, lapack_int m, lapack_int n, double * a, lapack_int lda, double * sva, double * u, lapack_int ldu, double * v, lapack_int ldv, double * stat, lapack_int * istat);
```

```
lapack_int LAPACKE_cgejsv (int matrix_layout, char joba, char jobu, char jobv, char jobr, char jobt, char jobp, lapack_int m, lapack_int n, lapack_complex_float * a, lapack_int lda, float * sva, lapack_complex_float * u, lapack_int ldu, lapack_complex_float * v, lapack_int ldv, float * stat, lapack_int * istat);
```

```
lapack_int LAPACKE_zgejsv (int matrix_layout, char joba, char jobu, char jobv, char jobr, char jobt, char jobp, lapack_int m, lapack_int n, lapack_complex_double * a, lapack_int lda, double * sva, lapack_complex_double * u, lapack_int ldu, lapack_complex_double * v, lapack_int ldv, double * stat, lapack_int * istat);
```

Include Files

- mkl.h

Description

The routine computes the singular value decomposition (SVD) of a real/complex *m*-by-*n* matrix *A*, where $m \geq n$.

The SVD is written as

$$A = U * \Sigma * V^T, \text{ for real routines}$$

$$A = U * \Sigma * V^H, \text{ for complex routines}$$

where Σ is an m -by- n matrix which is zero except for its n diagonal elements, U is an m -by- n (or m -by- m) orthonormal matrix, and V is an n -by- n orthogonal matrix. The diagonal elements of Σ are the singular values of A ; the columns of U and V are the left and right singular vectors of A , respectively. The matrices U and V are computed and stored in the arrays u and v , respectively. The diagonal of Σ is computed and stored in the array sva .

The `?gejsv` routine can sometimes compute tiny singular values and their singular vectors much more accurately than other SVD routines.

The routine implements a preconditioned Jacobi SVD algorithm. It uses `?geqp3`, `?geqrf`, and `?gelqf` as preprocessors and preconditioners. Optionally, an additional row pivoting can be used as a preprocessor, which in some cases results in much higher accuracy. An example is matrix A with the structure $A = D_1 * C * D_2$, where D_1 , D_2 are arbitrarily ill-conditioned diagonal matrices and C is a well-conditioned matrix. In that case, complete pivoting in the first QR factorizations provides accuracy dependent on the condition number of C , and independent of D_1 , D_2 . Such higher accuracy is not completely understood theoretically, but it works well in practice.

If A can be written as $A = B * D$, with well-conditioned B and some diagonal D , then the high accuracy is guaranteed, both theoretically and in software, independent of D . For more details see [[Drmac08-1](#)], [[Drmac08-2](#)].

The computational range for the singular values can be the full range (`UNDERFLOW,OVERFLOW`), provided that the machine arithmetic and the BLAS and LAPACK routines called by `?gejsv` are implemented to work in that range. If that is not the case, the restriction for safe computation with the singular values in the range of normalized IEEE numbers is that the spectral condition number $\kappa(A) = \sigma_{\max}(A) / \sigma_{\min}(A)$ does not overflow. This code (`?gejsv`) is best used in this restricted range, meaning that singular values of magnitude below $\|A\|_2 / \text{slamch}('O')$ (for single precision) or $\|A\|_2 / \text{dlamch}('O')$ (for double precision) are returned as zeros. See `jobr` for details on this.

This implementation is slower than the one described in [[Drmac08-1](#)], [[Drmac08-2](#)] due to replacement of some non-LAPACK components, and because the choice of some tuning parameters in the iterative part (`?gesvj`) is left to the implementer on a particular machine.

The rank revealing QR factorization (in this code: `?geqp3`) should be implemented as in [[Drmac08-3](#)].

If m is much larger than n , it is obvious that the initial QRF with column pivoting can be preprocessed by the QRF without pivoting. That well known trick is not used in `?gejsv` because in some cases heavy row weighting can be treated with complete pivoting. The overhead in cases m much larger than n is then only due to pivoting, but the benefits in accuracy have prevailed. You can incorporate this extra QRF step easily and also improve data movement (matrix transpose, matrix copy, matrix transposed copy) - this implementation of `?gejsv` uses only the simplest, naive data movement.

Optimization Notice

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Notice revision #20110804

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | <p>Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).</p> |
| <i>joba</i> | <p>Must be 'C', 'E', 'F', 'G', 'A', or 'R'.</p> <p>Specifies the level of accuracy:</p> <p>If <i>joba</i> = 'C', high relative accuracy is achieved if $A = B \cdot D$ with well-conditioned B and arbitrary diagonal matrix D. The accuracy cannot be spoiled by column scaling. The accuracy of the computed output depends on the condition of B, and the procedure aims at the best theoretical accuracy. The relative error $\max_{i=1:N} \delta \sigma_i / \sigma_i$ is bounded by $f(M,N) \cdot \epsilon \cdot \text{cond}(B)$, independent of D. The input matrix is preprocessed with the QRF with column pivoting. This initial preprocessing and preconditioning by a rank revealing QR factorization is common for all values of <i>joba</i>. Additional actions are specified as follows:</p> <p>If <i>joba</i> = 'E', computation as with 'C' with an additional estimate of the condition number of B. It provides a realistic error bound.</p> <p>If <i>joba</i> = 'F', accuracy higher than in the 'C' option is achieved, if $A = D1 \cdot C \cdot D2$ with ill-conditioned diagonal scalings $D1$, $D2$, and a well-conditioned matrix C. This option is advisable, if the structure of the input matrix is not known and relative accuracy is desirable. The input matrix A is preprocessed with QR factorization with full (row and column) pivoting.</p> <p>If <i>joba</i> = 'G', computation as with 'F' with an additional estimate of the condition number of B, where $A = B \cdot D$. If A has heavily weighted rows, using this condition number gives too pessimistic error bound.</p> <p>If <i>joba</i> = 'A', small singular values are the noise and the matrix is treated as numerically rank deficient. The error in the computed singular values is bounded by $f(m,n) \cdot \epsilon \cdot \ A\$. The computed SVD $A = U \cdot S \cdot V^T$ (for real flavors) or $A = U \cdot S \cdot V^H$ (for complex flavors) restores A up to $f(m,n) \cdot \epsilon \cdot \ A\$. This enables the procedure to set all singular values below $n \cdot \epsilon \cdot \ A\$ to zero.</p> <p>If <i>joba</i> = 'R', the procedure is similar to the 'A' option. Rank revealing property of the initial QR factorization is used to reveal (using triangular factor) a gap $\sigma_{r+1} < \epsilon \cdot \sigma_r$, in which case the numerical rank is declared to be r. The SVD is computed with absolute error bounds, but more accurately than with 'A'.</p> |
| <i>jobu</i> | <p>Must be 'U', 'F', 'W', or 'N'.</p> <p>Specifies whether to compute the columns of the matrix U:</p> <p>If <i>jobu</i> = 'U', n columns of U are returned in the array u</p> <p>If <i>jobu</i> = 'F', a full set of m left singular vectors is returned in the array u.</p> <p>If <i>jobu</i> = 'W', u may be used as workspace of length $m \cdot n$. See the description of u.</p> <p>If <i>jobu</i> = 'N', u is not computed.</p> |
| <i>jobv</i> | <p>Must be 'V', 'J', 'W', or 'N'.</p> <p>Specifies whether to compute the matrix V:</p> |

If $jobv = 'V'$, n columns of V are returned in the array v ; Jacobi rotations are not explicitly accumulated.

If $jobv = 'J'$, n columns of V are returned in the array v but they are computed as the product of Jacobi rotations. This option is allowed only if $jobu \neq 'N'$

If $jobv = 'W'$, v may be used as workspace of length $n*n$. See the description of v .

If $jobv = 'N'$, v is not computed.

jobr

Must be 'N' or 'R'.

Specifies the range for the singular values. If small positive singular values are outside the specified range, they may be set to zero. If A is scaled so that the largest singular value of the scaled matrix is around $\sqrt{\text{big}}$, $\text{big} = ?\text{lamch}('O')$, the function can remove columns of A whose norm in the scaled matrix is less than $\sqrt{?\text{lamch}('S')}$ (for $jobr = 'R'$), or less than $\text{small} = ?\text{lamch}('S')/?\text{lamch}('E')$.

If $jobr = 'N'$, the function does not remove small columns of the scaled matrix. This option assumes that BLAS and QR factorizations and triangular solvers are implemented to work in that range. If the condition of A is greater than big , use $?\text{gesvj}$.

If $jobr = 'R'$, restricted range for singular values of the scaled matrix A is $[\sqrt{?\text{lamch}('S')}, \sqrt{\text{big}}]$, roughly as described above. This option is recommended.

For computing the singular values in the full range $[?\text{lamch}('S'), \text{big}]$, use $?\text{gesvj}$.

jobt

Must be 'T' or 'N'.

If the matrix is square, the procedure may determine to use a transposed A if A^T (for real flavors) or A^H (for complex flavors) seems to be better with respect to convergence. If the matrix is not square, $jobt$ is ignored. This is subject to changes in the future.

The decision is based on two values of entropy over the adjoint orbit of $A^T * A$ (for real flavors) or $A^H * A$ (for complex flavors). See the descriptions of $stat[5]$ and $stat[6]$.

If $jobt = 'T'$, the function performs transposition if the entropy test indicates possibly faster convergence of the Jacobi process, if A is taken as input. If A is replaced with A^T or A^H , the row pivoting is included automatically.

If $jobt = 'N'$, the functions attempts no speculations. This option can be used to compute only the singular values, or the full SVD (u , σ , and v). For only one set of singular vectors (u or v), the caller should provide both u and v , as one of the arrays is used as workspace if the matrix A is transposed. The implementer can easily remove this constraint and make the code more complicated. See the descriptions of u and v .

jobp

Must be 'P' or 'N'.

Enables structured perturbations of denormalized numbers. This option should be active if the denormals are poorly implemented, causing slow computation, especially in cases of fast convergence. For details, see [Drmac08-1], [Drmac08-2]. For simplicity, such perturbations are included only when the full SVD or only the singular values are requested. You can add the perturbation for the cases of computing one set of singular vectors.

If $jobp = 'P'$, the function introduces perturbation.

If $jobp = 'N'$, the function introduces no perturbation.

| | |
|-----------|---|
| m | The number of rows of the input matrix A ; $m \geq 0$. |
| n | The number of columns in the input matrix A ; $m \geq n \geq 0$. |
| a, u, v | <p>Array a (size $lda*n$ for column major layout and $lda*m$ for row major layout) is an array containing the m-by-n matrix A.</p> <p>u is a workspace array, its size for column major layout is $ldu*n$ for $jobu='U'$ or $'W'$ and $ldu*m$ for $jobu='F'$; for row major layout its size is at least $ldu*m$. When $jobt = 'T'$ and $m = n$, u must be provided even though $jobu = 'N'$.</p> <p>v is a workspace array, its size is $ldv*n$. When $jobt = 'T'$ and $m = n$, v must be provided even though $jobv = 'N'$.</p> |
| lda | The leading dimension of the array a . Must be at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout. |
| sva | sva is a workspace array, its size is n . |
| ldu | <p>The leading dimension of the array u; $ldu \geq 1$.</p> <p>$jobu = 'U'$ or $'F'$ or $'W'$, $ldu \geq m$ for column major layout; for row major layout if $jobu = 'U'$ or $jobu = 'W'$ $ldu \geq n$ and if $jobu = 'F'$ $ldu \geq m$.</p> |
| ldv | <p>The leading dimension of the array v; $ldv \geq 1$.</p> <p>$jobv = 'V'$ or $'J'$ or $'W'$, $ldv \geq n$.</p> |
| $cwork$ | $cwork$ is a workspace array, dimension is at least $lwork$. |
| $rwork$ | $rwork$ is an array, dimension is at least $\max(7, lrwork)$. |

Output Parameters

| | |
|-------|--|
| sva | <p>On exit:</p> <p>For $stat[0]/stat[1] = 1$: the singular values of A. During the computation sva contains Euclidean column norms of the iterated matrices in the array a.</p> <p>For $stat[0] \neq stat[1]$: the singular values of A are $(stat[0]/stat[1]) * sva[0:n - 1]$. This factored form is used if $\sigma_{max}(A)$ overflows or if small singular values have been saved from underflow by scaling the input matrix A.</p> |
|-------|--|

$jobr = 'R'$, some of the singular values may be returned as exact zeros obtained by 'setting to zero' because they are below the numerical rank threshold or are denormalized numbers.

u

On exit:

If $jobu = 'U'$, contains the m -by- n matrix of the left singular vectors.

If $jobu = 'F'$, contains the m -by- m matrix of the left singular vectors, including an orthonormal basis of the orthogonal complement of the range of A .

If $jobu = 'W'$ and $jobv = 'V'$, $jobt = 'T'$, and $m = n$, then u is used as workspace if the procedure replaces A with A^T (for real flavors) or A^H (for complex flavors). In that case, v is computed in u as left singular vectors of A^T or A^H and copied back to the v array. This 'W' option is just a reminder to the caller that in this case u is reserved as workspace of length $n*n$.

If $jobu = 'N'$, u is not referenced.

v

On exit:

If $jobv = 'V'$ or $'J'$, contains the n -by- n matrix of the right singular vectors.

If $jobv = 'W'$ and $jobu = 'U'$, $jobt = 'T'$, and $m = n$, then v is used as workspace if the procedure replaces A with A^T (for real flavors) or A^H (for complex flavors). In that case, u is computed in v as right singular vectors of A^T or A^H and copied back to the u array. This 'W' option is just a reminder to the caller that in this case v is reserved as workspace of length $n*n$.

If $jobv = 'N'$, v is not referenced.

$stat$

On exit,

$stat[0] = scale = stat[1]/stat[0]$ is the scaling factor such that $scale*sva(1:n)$ are the computed singular values of A . See the description of sva .

$stat[1] =$ see the description of $stat[0]$.

$stat[2] = sconda$ is an estimate for the condition number of column equilibrated A . If $joba = 'E'$ or $'G'$, $sconda$ is an estimate of $\sqrt{|| (R^T * R)^{-1} ||_1}$. It is computed using `?pocon`. It holds $n^{-1/4} * sconda \leq || R^{-1} ||_2 \leq n^{1/4} * sconda$, where R is the triangular factor from the QRF of A . However, if R is truncated and the numerical rank is determined to be strictly smaller than n , $sconda$ is returned as -1, indicating that the smallest singular values might be lost.

If full SVD is needed, the following two condition numbers are useful for the analysis of the algorithm. They are provided for a user who is familiar with the details of the method.

$stat[3] =$ an estimate of the scaled condition number of the triangular factor in the first QR factorization.

$stat[4] =$ an estimate of the scaled condition number of the triangular factor in the second QR factorization.

The following two parameters are computed if *jobt* = 'T'. They are provided for a user who is familiar with the details of the method.

stat[5] = the entropy of $A^T * A$: : this is the Shannon entropy of $\text{diag}(A^T * A) / \text{Trace}(A^T * A)$ taken as point in the probability simplex.

stat[6] = the entropy of $A * A^T$.

istat

On exit,

istat[0] = the numerical rank determined after the initial QR factorization with pivoting. See the descriptions of *joba* and *jobr*.

istat[1] = the number of the computed nonzero singular value.

istat[2] = if nonzero, a warning message. If *istat*[2]=1, some of the column norms of *A* were denormalized floats. The requested high accuracy is not warranted by the data.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, the function did not converge in the maximal number of sweeps. The computed values may be inaccurate.

See Also

[?geqp3](#)

[?geqrf](#)

[?gelqf](#)

[?gesvj](#)

[?lamch](#)

[?pocon](#)

[?ormlq](#)

[?gesvj](#)

Computes the singular value decomposition of a real matrix using Jacobi plane rotations.

Syntax

```
lapack_int LAPACKE_sgesvj (int matrix_layout, char joba, char jobu, char jobv,
lapack_int m, lapack_int n, float * a, lapack_int lda, float * sva, lapack_int mv,
float * v, lapack_int ldv, float * stat);
```

```
lapack_int LAPACKE_dgesvj (int matrix_layout, char joba, char jobu, char jobv,
lapack_int m, lapack_int n, double * a, lapack_int lda, double * sva, lapack_int mv,
double * v, lapack_int ldv, double * stat);
```

```
lapack_int LAPACKE_cgesvj (int matrix_layout, char joba, char jobu, char jobv,
lapack_int m, lapack_int n, lapack_complex_float * a, lapack_int lda, float * sva,
lapack_int mv, lapack_complex_float * v, lapack_int ldv, float * stat);
```

```
lapack_int LAPACKE_zgesvj (int matrix_layout, char joba, char jobu, char jobv,
lapack_int m, lapack_int n, lapack_complex_double * a, lapack_int lda, double * sva,
lapack_int mv, lapack_complex_double * v, lapack_int ldv, double * stat);
```

Include Files

- `mk1.h`

Description

The routine computes the singular value decomposition (SVD) of a real or complex m -by- n matrix A , where $m \geq n$.

The SVD of A is written as

$$A = U \Sigma^* V^T \text{ for real flavors, or}$$

$$A = U \Sigma^* V^H \text{ for complex flavors,}$$

where Σ is an m -by- n diagonal matrix, U is an m -by- n orthonormal matrix, and V is an n -by- n orthogonal/unitary matrix. The diagonal elements of Σ are the singular values of A ; the columns of U and V are the left and right singular vectors of A , respectively. The matrices U and V are computed and stored in the arrays u and v , respectively. The diagonal of Σ is computed and stored in the array `sva`.

The `?gesvj` routine can sometimes compute tiny singular values and their singular vectors much more accurately than other SVD routines.

The n -by- n orthogonal matrix V is obtained as a product of Jacobi plane rotations. The rotations are implemented as fast scaled rotations of Anda and Park [[AndaPark94](#)]. In the case of underflow of the Jacobi angle, a modified Jacobi transformation of Drmac ([[Drmac08-4](#)]) is used. Pivot strategy uses column interchanges of de Rijk ([[deRijk98](#)]). The relative accuracy of the computed singular values and the accuracy of the computed singular vectors (in angle metric) is as guaranteed by the theory of Demmel and Veselic [[Demmel92](#)]. The condition number that determines the accuracy in the full rank case is essentially

$$\left(\min_i d_{ii} \right) \cdot \kappa(A \cdot D)$$

where $\kappa(\cdot)$ is the spectral condition number. The best performance of this Jacobi SVD procedure is achieved if used in an accelerated version of Drmac and Veselic [[Drmac08-1](#)], [[Drmac08-2](#)].

The computational range for the nonzero singular values is the machine number interval (`UNDERFLOW,OVERFLOW`). In extreme cases, even denormalized singular values can be computed with the corresponding gradual loss of accurate digit.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>joba</code> | Must be 'L', 'U' or 'G'. Specifies the structure of A : If <code>joba</code> = 'L', the input matrix A is lower triangular. If <code>joba</code> = 'U', the input matrix A is upper triangular. If <code>joba</code> = 'G', the input matrix A is a general m -by- n , $m \geq n$. |
| <code>jobu</code> | Must be 'U', 'C' or 'N'. Specifies whether to compute the left singular vectors (columns of U): |

If $jobu = 'U'$, the left singular vectors corresponding to the nonzero singular values are computed and returned in the leading columns of A . See more details in the description of a . The default numerical orthogonality threshold is set to approximately $TOL=CTOL*EPS$, $CTOL=\sqrt{m}$, $EPS = ?lamch('E')$

If $jobu = 'C'$, analogous to $jobu = 'U'$, except that you can control the level of numerical orthogonality of the computed left singular vectors. TOL can be set to $TOL=CTOL*EPS$, where $CTOL$ is given on input in the array $stat$. No $CTOL$ smaller than ONE is allowed. $CTOL$ greater than $1 / EPS$ is meaningless. The option 'C' can be used if $m*EPS$ is satisfactory orthogonality of the computed left singular vectors, so $CTOL=m$ could save a few sweeps of Jacobi rotations. See the descriptions of a and $stat[0]$.

If $jobu = 'N'$, u is not computed. However, see the description of a .

$jobv$

Must be 'V', 'A' or 'N'.

Specifies whether to compute the right singular vectors, that is, the matrix V :

If $jobv = 'V'$, the matrix V is computed and returned in the array v .

If $jobv = 'A'$, the Jacobi rotations are applied to the mv -by- n array v . In other words, the right singular vector matrix V is not computed explicitly, instead it is applied to an mv -by- n matrix initially stored in the first mv rows of V .

If $jobv = 'N'$, the matrix V is not computed and the array v is not referenced.

m

The number of rows of the input matrix A .

$1/\text{slamch}('E') > m \geq 0$ for $sgesvj$.

$1/\text{dlamch}('E') > m \geq 0$ for $dgesvj$.

n

The number of columns in the input matrix A ; $m \geq n \geq 0$.

a, v

Array a (size at least $lda*n$ for column major layout and $lda*m$ for row major layout) is an array containing the m -by- n matrix A .

Array v (size at least $\max(1, ldv*n)$) contains, if $jobv = 'A'$ the mv -by- n matrix to be post-multiplied by Jacobi rotations.

lda

The leading dimension of the array a . Must be at least $\max(1, m)$ for column major layout and at least $\max(1, n)$ for row major layout.

mv

If $jobv = 'A'$, the product of Jacobi rotations in $?gesvj$ is applied to the first mv rows of v . See the description of $jobv$. $0 \leq mv \leq ldv$.

ldv

The leading dimension of the array v ; $ldv \geq 1$.

$jobv = 'V', ldv \geq \max(1, n)$.

$jobv = 'A', ldv \geq \max(1, mv)$ for column major layout and $ldv \geq \max(1, n)$ for row major layout.

stat

Array size 6. If *jobu* = 'C', *stat*[0] = *CTOL*, where *CTOL* defines the threshold for convergence. The process stops if all columns of *A* are mutually orthogonal up to *CTOL***EPS*, where *EPS* = $\text{?lamch}('E')$. It is required that $CTOL \geq 1$ - that is, it is not allowed to force the routine to obtain orthogonality below ϵ .

Output Parameters

a

On exit:

If *jobu* = 'U' or *jobu* = 'C':

- if *info* = 0, the leading columns of *A* contain left singular vectors corresponding to the computed singular values of *a* that are above the underflow threshold $\text{?lamch}('S')$, that is, non-zero singular values. The number of the computed non-zero singular values is returned in *stat*[1]. Also see the descriptions of *sva* and *stat*. The computed columns of *u* are mutually numerically orthogonal up to approximately $TOL = \sqrt{m} * EPS$ (default); or $TOL = CTOL * EPS$ *jobu* = 'C', see the description of *jobu*.
- if *info* > 0, the procedure *?gesvj* did not converge in the given number of iterations (sweeps). In that case, the computed columns of *u* may not be orthogonal up to *TOL*. The output *u* (stored in *a*), *sigma* (given by the computed singular values in *sva*(1:n)) and *v* is still a decomposition of the input matrix *A* in the sense that the residual $\|A - scale * U * sigma * V^T\|_2 / \|A\|_2$ for real flavors or $\|A - scale * U * sigma * V^H\|_2 / \|A\|_2$ for complex flavors (where *scale* = *stat*[0]) is small.

If *jobu* = 'N':

- if *info* = 0, note that the left singular vectors are 'for free' in the one-sided Jacobi SVD algorithm. However, if only the singular values are needed, the level of numerical orthogonality of *u* is not an issue and iterations are stopped when the columns of the iterated matrix are numerically orthogonal up to approximately $m * EPS$. Thus, on exit, *a* contains the columns of *u* scaled with the corresponding singular values.
- if *info* > 0, the procedure *?gesvj* did not converge in the given number of iterations (sweeps).

*sva*Array size *n*.

If *info* = 0, depending on the value *scale* = *stat*[0], where *scale* is the scaling factor:

- if *scale* = 1, *sva*[0:n - 1] contains the computed singular values of *a*.
- if *scale* ≠ 1, the singular values of *a* are *scale***sva*(1:n), and this factored representation is due to the fact that some of the singular values of *a* might underflow or overflow.

If *info* > 0, the procedure *?gesvj* did not converge in the given number of iterations (sweeps) and *scale***sva*(1:n) may not be accurate.

v

On exit:

If *jobv* = 'V', contains the *n*-by-*n* matrix of the right singular vectors.

If *jobv* = 'A', then *v* contains the product of the computed right singular vector matrix and the initial matrix in the array *v*.

If *jobv* = 'N', *v* is not referenced.

stat

On exit,

stat[0] = *scale* is the scaling factor such that *scale***sva*(1:*n*) are the computed singular values of *A*. See the description of *sva*.

stat[1] is the number of the computed nonzero singular values.

stat[2] is the number of the computed singular values that are larger than the underflow threshold.

stat[3] is the number of sweeps of Jacobi rotations needed for numerical convergence.

stat[4] = $\max_{\{i \neq j\}} |\cos(A(:,i), A(:,j))|$ in the last sweep. This is useful information in cases when *?gesvj* did not converge, as it can be used to estimate whether the output is still useful and for post festum analysis.

stat[5] is the largest absolute value over all sines of the Jacobi rotation angles in the last sweep. It can be useful in a post festum analysis.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, the function did not converge in the maximal number (30) of sweeps. The output may still be useful. See the description of *stat*.

?ggsvd

Computes the generalized singular value decomposition of a pair of general rectangular matrices (deprecated).

Syntax

```
lapack_int LAPACKE_sggsvd( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int* k, lapack_int* l, float* a,
lapack_int lda, float* b, lapack_int ldb, float* alpha, float* beta, float* u,
lapack_int ldu, float* v, lapack_int ldv, float* q, lapack_int ldq, lapack_int*
iwork );
```

```
lapack_int LAPACKE_dggsvd( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int* k, lapack_int* l, double* a,
lapack_int lda, double* b, lapack_int ldb, double* alpha, double* beta, double* u,
lapack_int ldu, double* v, lapack_int ldv, double* q, lapack_int ldq, lapack_int*
iwork );
```

```
lapack_int LAPACKE_cggsvd( int matrix_layout, char jobu, char jobv, char jobq,
lapack_int m, lapack_int n, lapack_int p, lapack_int* k, lapack_int* l,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int ldb,
```


$$C = \text{diag}(\alpha[k], \dots, \alpha[k + l - 1])$$

$$S = \text{diag}(\beta[k], \dots, \beta[k + l - 1])$$

$$C^2 + S^2 = I$$

Nonzero element r_{ij} ($1 \leq j \leq k + l$) of R is stored in $a[(i - 1) + (n - k - l + j - 1) * lda]$ for column major layout and in $a[(i - 1) * lda + (n - k - l + j - 1)]$ for row major layout.

If $m - k - l < 0$,

$$D_1 = \begin{matrix} & k & m - k & k + l - m \\ m - k & \begin{pmatrix} I & 0 & 0 \\ 0 & C & 0 \end{pmatrix} \end{matrix}$$

$$D_2 = \begin{matrix} & k & m - k & k + l - m \\ m - k & \begin{pmatrix} 0 & S & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{pmatrix} \\ k + l - m & \end{matrix}$$

$$(0 \ R) = \begin{matrix} & n - k - l & k & m - k & k + l - m \\ m - k & \begin{pmatrix} 0 & R_{11} & R_{12} & R_{13} \\ 0 & 0 & R_{22} & R_{23} \\ 0 & 0 & 0 & R_{33} \end{pmatrix} \\ k + l - m & \end{matrix}$$

where

$$C = \text{diag}(\alpha[k], \dots, \alpha[m]),$$

$$S = \text{diag}(\beta[k], \dots, \beta[m - 1]),$$

$$C_2 + S_2 = I$$

On exit, the location of nonzero element r_{ij} ($1 \leq j \leq k + l$) of R depends on the value of i . For $i \leq m$ this element is stored in $a[(i - 1) + (n - k - l + j - 1) * lda]$ for column major layout and in $a[(i - 1) * lda + (n - k - l + j - 1)]$ for row major layout. For $m < i \leq k + l$ it is stored in $b[(i - k - 1) + (n - k - l + j - 1) * ldb]$ for column major layout and in $b[(i - k - 1) * ldb + (n - k - l + j - 1)]$ for row major layout.

The routine computes C , S , R , and optionally the orthogonal/unitary transformation matrices U , V and Q .

In particular, if B is an n -by- n nonsingular matrix, then the GSVD of A and B implicitly gives the SVD of $A * B^{-1}$:

$$A * B^{-1} = U * (D_1 * D_2^{-1}) * V'$$

If (A', B') has orthonormal columns, then the GSVD of A and B is also equal to the CS decomposition of A and B . Furthermore, the GSVD can be used to derive the solution of the eigenvalue problem:

$$A' * A * x = \lambda * B' * B * x.$$

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobu</i> | Must be 'U' or 'N'. If <i>jobu</i> = 'U', orthogonal/unitary matrix <i>U</i> is computed. If <i>jobu</i> = 'N', <i>U</i> is not computed. |
| <i>jobv</i> | Must be 'V' or 'N'. If <i>jobv</i> = 'V', orthogonal/unitary matrix <i>V</i> is computed. If <i>jobv</i> = 'N', <i>V</i> is not computed. |
| <i>jobq</i> | Must be 'Q' or 'N'. If <i>jobq</i> = 'Q', orthogonal/unitary matrix <i>Q</i> is computed. If <i>jobq</i> = 'N', <i>Q</i> is not computed. |
| <i>m</i> | The number of rows of the matrix <i>A</i> ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>p</i> | The number of rows of the matrix <i>B</i> ($p \geq 0$). |
| <i>a, b</i> | Arrays: <i>a</i> (size at least $\max(1, lda * n)$ for column major layout and $\max(1, lda * m)$ for row major layout) contains the <i>m</i> -by- <i>n</i> matrix <i>A</i> . <i>b</i> (size at least $\max(1, ldb * n)$ for column major layout and $\max(1, ldb * p)$ for row major layout) contains the <i>p</i> -by- <i>n</i> matrix <i>B</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, p)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldu</i> | The leading dimension of the array <i>u</i> . $ldu \geq \max(1, m)$ if <i>jobu</i> = 'U'; $ldu \geq 1$ otherwise. |
| <i>ldv</i> | The leading dimension of the array <i>v</i> . $ldv \geq \max(1, p)$ if <i>jobv</i> = 'V'; $ldv \geq 1$ otherwise. |
| <i>ldq</i> | The leading dimension of the array <i>q</i> . $ldq \geq \max(1, n)$ if <i>jobq</i> = 'Q'; $ldq \geq 1$ otherwise. |

Output Parameters

| | |
|-------------|--|
| <i>k, l</i> | On exit, <i>k</i> and <i>l</i> specify the dimension of the subblocks. The sum <i>k+l</i> is equal to the effective numerical rank of (<i>A</i> , <i>B</i>). |
| <i>a</i> | On exit, <i>a</i> contains the triangular matrix <i>R</i> or part of <i>R</i> . |

b On exit, *b* contains part of the triangular matrix R if $m-k-l < 0$.

alpha, beta Arrays, size at least $\max(1, n)$ each.
 Contain the generalized singular value pairs of *A* and *B*:
 $alpha(1:k) = 1,$
 $beta(1:k) = 0,$
 and if $m-k-l \geq 0,$
 $alpha(k+1:k+l) = C,$
 $beta(k+1:k+l) = S,$
 or if $m-k-l < 0,$
 $alpha(k+1:m) = C, alpha(m+1:k+l) = 0$
 $beta(k+1:m) = S, beta(m+1:k+l) = 1$
 and
 $alpha(k+l+1:n) = 0$
 $beta(k+l+1:n) = 0.$

u, v, q Arrays:
u, size at least $\max(1, ldu*m)$.
 If *jobu* = 'U', *u* contains the *m*-by-*m* orthogonal/unitary matrix *U*.
 If *jobu* = 'N', *u* is not referenced.
v, size at least $\max(1, ldv*p)$.
 If *jobv* = 'V', *v* contains the *p*-by-*p* orthogonal/unitary matrix *V*.
 If *jobv* = 'N', *v* is not referenced.
q, size at least $\max(1, ldq*n)$.
 If *jobq* = 'Q', *q* contains the *n*-by-*n* orthogonal/unitary matrix *Q*.
 If *jobq* = 'N', *q* is not referenced.

iwork On exit, *iwork* stores the sorting information.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = 1, the Jacobi-type procedure failed to converge. For further details, see subroutine [tgsja](#).

?gesvdx

Computes the SVD and left and right singular vectors for a matrix.

Syntax

```
lapack_int LAPACKE_sgesvdx (int matrix_layout, char jobu, char jobvt, char range,
lapack_int m, lapack_int n, float * a, lapack_int lda, float vl, float vu, lapack_int
il, lapack_int iu, lapack_int * ns, float * s, float * u, lapack_int ldu, float * vt,
lapack_int ldvt, lapack_int * superb);
```

```
lapack_int LAPACKE_dgesvdx (int matrix_layout, char jobu, char jobvt, char range,
lapack_int m, lapack_int n, double * a, lapack_int lda, double vl, double vu,
lapack_int il, lapack_int iu, lapack_int * ns, double * s, double * u, lapack_int ldu,
double * vt, lapack_int ldvt, lapack_int * superb);
```

```
lapack_int LAPACKE_cgesvdx (int matrix_layout, char jobu, char jobvt, char range,
lapack_int m, lapack_int n, lapack_complex_float * a, lapack_int lda, float vl, float
vu, lapack_int il, lapack_int iu, lapack_int * ns, float * s, lapack_complex_float *
u, lapack_int ldu, lapack_complex_float * vt, lapack_int ldvt, lapack_int * superb);
```

```
lapack_int LAPACKE_zgesvdx (int matrix_layout, char jobu, char jobvt, char range,
lapack_int m, lapack_int n, lapack_complex_double * a, lapack_int lda, double vl,
double vu, lapack_int il, lapack_int iu, lapack_int * ns, double * s,
lapack_complex_double * u, lapack_int ldu, lapack_complex_double * vt, lapack_int ldvt,
lapack_int * superb);
```

Include Files

- mkl.h

Description

`?gesvdx` computes the singular value decomposition (SVD) of a real or complex m -by- n matrix A , optionally computing the left and right singular vectors. The SVD is written

$$A = U * \Sigma * \text{transpose}(V)$$

where Σ is an m -by- n matrix which is zero except for its $\min(m,n)$ diagonal elements, U is an m -by- m matrix, and V is an n -by- n matrix. The matrices U and V are orthogonal for real A , and unitary for complex A . The diagonal elements of Σ are the singular values of A ; they are real and non-negative, and are returned in descending order. The first $\min(m,n)$ columns of U and V are the left and right singular vectors of A .

`?gesvdx` uses an eigenvalue problem for obtaining the SVD, which allows for the computation of a subset of singular values and vectors. See `?bdsvdx` for details.

Note that the routine returns V^T , not V .

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

jobu Specifies options for computing all or part of the matrix U :
 = 'V': the first $\min(m,n)$ columns of U (the left singular vectors) or as specified by *range* are returned in the array *u*;
 = 'N': no columns of U (no left singular vectors) are computed.

jobvt Specifies options for computing all or part of the matrix V^T :
 = 'V': the first $\min(m,n)$ rows of V^T (the right singular vectors) or as specified by *range* are returned in the array *vt*;

| | |
|--------------|---|
| | = 'N': no rows of V^T (no right singular vectors) are computed. |
| <i>range</i> | = 'A': find all singular values. = 'V': all singular values in the half-open interval $(v_l, v_u]$ are found. = 'I': the <i>il</i> -th through <i>iu</i> -th singular values are found. |
| <i>m</i> | The number of rows of the input matrix <i>A</i> . $m \geq 0$. |
| <i>n</i> | The number of columns of the input matrix <i>A</i> . $n \geq 0$. |
| <i>a</i> | Array, size $lda * n$ On entry, the <i>m</i> -by- <i>n</i> matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1, m)$. |
| <i>v_l</i> | $v_l \geq 0$. |
| <i>v_u</i> | If <i>range</i> ='V', the lower and upper bounds of the interval to be searched for singular values. $v_u > v_l$. Not referenced if <i>range</i> = 'A' or 'I'. |
| <i>il</i> | |
| <i>iu</i> | If <i>range</i> ='I', the indices (in ascending order) of the smallest and largest singular values to be returned. $1 \leq il \leq iu \leq \min(m, n)$, if $\min(m, n) > 0$. Not referenced if <i>range</i> = 'A' or 'V'. |
| <i>ldu</i> | The leading dimension of the array <i>u</i> . $ldu \geq 1$; if <i>jobu</i> = 'V', $ldu \geq m$. |
| <i>ldvt</i> | The leading dimension of the array <i>vt</i> . $ldvt \geq 1$; if <i>jobvt</i> = 'V', $ldvt \geq ns$ (see above). |

Output Parameters

| | |
|-----------|---|
| <i>a</i> | On exit, the contents of <i>a</i> are destroyed. |
| <i>ns</i> | The total number of singular values found, $0 \leq ns \leq \min(m, n)$. If <i>range</i> = 'A', $ns = \min(m, n)$; if <i>range</i> = 'I', $ns = iu - il + 1$. |
| <i>s</i> | Array, size $(\min(m, n))$ The singular values of <i>A</i> , sorted so that $s[i] \geq s[i + 1]$. |
| <i>u</i> | Array, size $ldu * ucol$ If <i>jobu</i> = 'V', <i>u</i> contains columns of <i>U</i> (the left singular vectors, stored columnwise) as specified by <i>range</i> ; if <i>jobu</i> = 'N', <i>u</i> is not referenced. |

NOTE

Make sure that $ucol \geq ns$; if *range* = 'V', the exact value of *ns* is not known in advance and an upper bound must be used.

vt Array, size $ldvt*n$
 If *jobvt* = 'V', *vt* contains the rows of V^T (the right singular vectors, stored rowwise) as specified by *range*; if *jobvt* = 'N', *vt* is not referenced.

NOTE

Make sure that $ldvt \geq ns$; if *range* = 'V', the exact value of *ns* is not known in advance and an upper bound must be used.

superb Array, size $(12*\min(m, n))$.
 If *info* = 0, the first *ns* elements of *iwork* are zero. If *info* > 0, then *iwork* contains the indices of the eigenvectors that failed to converge in ?bdsvdx/?stevx.

Return Values

This function returns a value *info*.

= 0: successful exit.

< 0: if *info* = -*i*, the *i*-th argument had an illegal value.

> 0: if *info* = *i*, then *i* eigenvectors failed to converge in ?bdsvdx/?stevx. if *info* = $n*2 + 1$, an internal error occurred in ?bdsvdx.

?bdsvdx

Computes the SVD of a bidiagonal matrix.

Syntax

```
lapack_int LAPACKE_sbdsvdx (int matrix_layout, char uplo, char jobz, char range,
lapack_int n, float * d, float * e, float vl, float vu, lapack_int il, lapack_int iu,
lapack_int * ns, float * s, float * z, lapack_int ldz, lapack_int * superb);
```

```
lapack_int LAPACKE_dbdsvdx (int matrix_layout, char uplo, char jobz, char range,
lapack_int n, double * d, double * e, double vl, double vu, lapack_int il, lapack_int
iu, lapack_int * ns, double * s, double * z, lapack_int ldz, lapack_int * superb);
```

Include Files

- mkl.h

Description

?bdsvdx computes the singular value decomposition (SVD) of a real n -by- n (upper or lower) bidiagonal matrix B , $B = U * S * VT$, where S is a diagonal matrix with non-negative diagonal elements (the singular values of B), and U and VT are orthogonal matrices of left and right singular vectors, respectively.

Given an upper bidiagonal B with diagonal $d = [d_1 d_2 \dots d_n]$ and superdiagonal $e = [e_1 e_2 \dots e_{n-1}]$, ?bdsvdx computes the singular value decomposition of B through the eigenvalues and eigenvectors of the $n*2$ -by- $n*2$ tridiagonal matrix

$$T G K = \begin{pmatrix} 0 & d_1 & & & \\ d_1 & 0 & e_1 & & \\ & e_1 & 0 & d_2 & \\ & & & d_2 & \ddots & \ddots \\ & & & & \ddots & \ddots \end{pmatrix}$$

If (s, u, v) is a singular triplet of B with $\|u\| = \|v\| = 1$, then $(\pm s, q)$, $\|q\| = 1$, are eigenpairs of TGK, with $q = P * \frac{(u \pm v)}{\sqrt{2}} = \frac{(v_1 \ u_1 \ v_2 \ u_2 \ \dots \ v_n \ u_n)}{\sqrt{2}}$, and $P = (e_{n+1} \ e_1 \ e_{n+2} \ e_2 \ \dots)$.

Given a TGK matrix, one can either

1. compute $-s$, $-v$ and change signs so that the singular values (and corresponding vectors) are already in descending order (as in `?gesvd/?gesdd`) or
2. compute s , v and reorder the values (and corresponding vectors).

`?bdsvd` implements (1) by calling `?stevx` (bisection plus inverse iteration, to be replaced with a version of the Multiple Relative Robust Representation algorithm. (See P. Willems and B. Lang, A framework for the MR³ algorithm: theory and implementation, SIAM J. Sci. Comput., 35:740-766, 2013.)

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>uplo</code> | = 'U': B is upper bidiagonal; = 'L': B is lower bidiagonal. |
| <code>jobz</code> | = 'N': Compute singular values only; = 'V': Compute singular values and singular vectors. |
| <code>range</code> | = 'A': Find all singular values. = 'V': all singular values in the half-open interval $[vl, vu)$ are found. = 'I': the il -th through iu -th singular values are found. |
| <code>n</code> | The order of the bidiagonal matrix. $n \geq 0$. |
| <code>d</code> | Array, size n . The n diagonal elements of the bidiagonal matrix B . |
| <code>e</code> | Array, size $(\max(1, n - 1))$ The $(n - 1)$ superdiagonal elements of the bidiagonal matrix B in elements 1 to $n - 1$. |
| <code>vl</code> | $vl \geq 0$. |
| <code>vu</code> | If <code>range='V'</code> , the lower and upper bounds of the interval to be searched for singular values. $vu > vl$. Not referenced if <code>range = 'A'</code> or <code>'I'</code> . |
| <code>il, iu</code> | If <code>range='I'</code> , the indices (in ascending order) of the smallest and largest singular values to be returned. $1 \leq il \leq iu \leq \min(m, n)$, if $\min(m, n) > 0$. Not referenced if <code>range = 'A'</code> or <code>'V'</code> . |
| <code>ldz</code> | The leading dimension of the array z . |

$ldz \geq 1$, and if $jobz = 'V'$, $ldz \geq \max(2, n*2)$.

Output Parameters

| | |
|-----------|---|
| <i>ns</i> | The total number of singular values found. $0 \leq ns \leq n$. If $range = 'A'$, $ns = n$, and if $range = 'I'$, $ns = iu - il + 1$. |
| <i>s</i> | Array, size (n) The first ns elements contain the selected singular values in ascending order. |
| <i>z</i> | Array, size $2*n*k$ If $jobz = 'V'$, then if $info = 0$ the first ns columns of z contain the singular vectors of the matrix B corresponding to the selected singular values, with U in rows 1 to n and V in rows $n+1$ to $n*2$, i.e. $z = \begin{pmatrix} U \\ V \end{pmatrix}$ If $jobz = 'N'$, then z is not referenced. |

NOTE

Make sure that at least $k = ns+1$ columns are supplied in the array z ; if $range = 'V'$, the exact value of ns is not known in advance and an upper bound must be used.

| | |
|---------------|--|
| <i>superb</i> | Array, size ($12*n$). If $jobz = 'V'$, then if $info = 0$, the first ns elements of $iwork$ are zero. If $info > 0$, then $iwork$ contains the indices of the eigenvectors that failed to converge in $?stevx$. |
|---------------|--|

Return Values

This function returns a value *info*.

= 0: successful exit.

< 0: if $info = -i$, the i -th argument had an illegal value.

> 0:

if $info = i$, then i eigenvectors failed to converge in $?stevx$. The indices of the eigenvectors (as returned by $?stevx$) are stored in the array *iwork*.

if $info = n*2 + 1$, an internal error occurred.

Cosine-Sine Decomposition: LAPACK Driver Routines

This section describes LAPACK driver routines for computing the *cosine-sine decomposition* (CS decomposition). You can also call the corresponding computational routines to perform the same task.

The computation has the following phases:

1. The matrix is reduced to a bidiagonal block form.
2. The blocks are simultaneously diagonalized using techniques from the bidiagonal SVD algorithms.

Table "Driver Routines for Cosine-Sine Decomposition (CSD)" lists LAPACK routines that perform CS decomposition of matrices.

Computational Routines for Cosine-Sine Decomposition (CSD)

| Operation | Real matrices | Complex matrices |
|---|---|---|
| Compute the CS decomposition of a block-partitioned orthogonal matrix | orcsd uncsd | |
| Compute the CS decomposition of a block-partitioned unitary matrix | | orcsd uncsd |

See Also

[Cosine-Sine Decomposition: LAPACK Computational Routines](#)

?orcsd/?uncsd

Computes the CS decomposition of a block-partitioned orthogonal/unitary matrix.

Syntax

```
lapack_int LAPACKE_sorcsd( int matrix_layout, char jobu1, char jobu2, char jobvt, char
jobv2t, char trans, char signs, lapack_int m, lapack_int p, lapack_int q, float* x11,
lapack_int ldx11, float* x12, lapack_int ldx12, float* x21, lapack_int ldx21, float*
x22, lapack_int ldx22, float* theta, float* u1, lapack_int ldu1, float* u2, lapack_int
ldu2, float* v1t, lapack_int ldv1t, float* v2t, lapack_int ldv2t );
```

```
lapack_int LAPACKE_dorcsd( int matrix_layout, char jobu1, char jobu2, char jobvt, char
jobv2t, char trans, char signs, lapack_int m, lapack_int p, lapack_int q, double* x11,
lapack_int ldx11, double* x12, lapack_int ldx12, double* x21, lapack_int ldx21,
double* x22, lapack_int ldx22, double* theta, double* u1, lapack_int ldu1, double* u2,
lapack_int ldu2, double* v1t, lapack_int ldv1t, double* v2t, lapack_int ldv2t );
```

```
lapack_int LAPACKE_cuncsd( int matrix_layout, char jobu1, char jobu2, char jobvt, char
jobv2t, char trans, char signs, lapack_int m, lapack_int p, lapack_int q,
lapack_complex_float* x11, lapack_int ldx11, lapack_complex_float* x12, lapack_int
ldx12, lapack_complex_float* x21, lapack_int ldx21, lapack_complex_float* x22,
lapack_int ldx22, float* theta, lapack_complex_float* u1, lapack_int ldu1,
lapack_complex_float* u2, lapack_int ldu2, lapack_complex_float* v1t, lapack_int ldv1t,
lapack_complex_float* v2t, lapack_int ldv2t );
```

```
lapack_int LAPACKE_zuncsd( int matrix_layout, char jobu1, char jobu2, char jobvt, char
jobv2t, char trans, char signs, lapack_int m, lapack_int p, lapack_int q,
lapack_complex_double* x11, lapack_int ldx11, lapack_complex_double* x12, lapack_int
ldx12, lapack_complex_double* x21, lapack_int ldx21, lapack_complex_double* x22,
lapack_int ldx22, double* theta, lapack_complex_double* u1, lapack_int ldu1,
lapack_complex_double* u2, lapack_int ldu2, lapack_complex_double* v1t, lapack_int
ldv1t, lapack_complex_double* v2t, lapack_int ldv2t );
```

Include Files

- `mkl.h`

Description

The routines `?orcsd/?uncsd` compute the CS decomposition of an m -by- m partitioned orthogonal matrix X :

$$X = \begin{pmatrix} x_{11} & | & x_{12} \\ x_{21} & | & x_{22} \end{pmatrix} = \begin{pmatrix} u_1 & | \\ & | & u_2 \end{pmatrix} \left(\begin{array}{ccc|ccc} I & 0 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & -S & 0 \\ \hline 0 & 0 & 0 & I & 0 & 0 \\ 0 & S & 0 & 0 & C & 0 \\ 0 & 0 & I & 0 & 0 & 0 \end{array} \right) \begin{pmatrix} v_1 & | \\ & | & v_2 \end{pmatrix}^T$$

or unitary matrix:

$$X = \begin{pmatrix} x_{11} & | & x_{12} \\ x_{21} & | & x_{22} \end{pmatrix} = \begin{pmatrix} u_1 & | \\ & | & u_2 \end{pmatrix} \left(\begin{array}{ccc|ccc} I & 0 & 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 & -S & 0 \\ \hline 0 & 0 & 0 & I & 0 & 0 \\ 0 & S & 0 & 0 & C & 0 \\ 0 & 0 & I & 0 & 0 & 0 \end{array} \right) \begin{pmatrix} v_1 & | \\ & | & v_2 \end{pmatrix}^H$$

x_{11} is p -by- q . The orthogonal/unitary matrices u_1 , u_2 , v_1 , and v_2 are p -by- p , $(m-p)$ -by- $(m-p)$, q -by- q , $(m-q)$ -by- $(m-q)$, respectively. C and S are r -by- r nonnegative diagonal matrices satisfying $C^2 + S^2 = I$, in which $r = \min(p, m-p, q, m-q)$.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobu1</i> | If equals Y, then u_1 is computed. Otherwise, u_1 is not computed. |
| <i>jobu2</i> | If equals Y, then u_2 is computed. Otherwise, u_2 is not computed. |
| <i>jobv1t</i> | If equals Y, then v_1^t is computed. Otherwise, v_1^t is not computed. |
| <i>jobv2t</i> | If equals Y, then v_2^t is computed. Otherwise, v_2^t is not computed. |
| <i>trans</i> | = 'T': X , u_1 , u_2 , v_1^t , v_2^t are stored in row-major order. otherwise X , u_1 , u_2 , v_1^t , v_2^t are stored in column-major order. |
| <i>signs</i> | = 'O': The lower-left block is made nonpositive (the "other" convention). otherwise The upper-right block is made nonpositive (the "default" convention). |
| <i>m</i> | The number of rows and columns of the matrix X . |
| <i>p</i> | The number of rows in x_{11} and x_{12} . $0 \leq p \leq m$. |

| | |
|--|--|
| q | The number of columns in x_{11} and x_{21} . $0 \leq q \leq m$. |
| $x_{11}, x_{12}, x_{21}, x_{22}$ | Arrays of size x_{11} (ldx_{11}, q), x_{12} ($ldx_{12}, m - q$), x_{21} (ldx_{21}, q), and x_{22} ($ldx_{22}, m - q$). Contain the parts of the orthogonal/unitary matrix whose CSD is desired. |
| $ldx_{11}, ldx_{12}, ldx_{21}, ldx_{22}$ | The leading dimensions of the parts of array X . $ldx_{11} \geq \max(1, p)$, $ldx_{12} \geq \max(1, p)$, $ldx_{21} \geq \max(1, m - p)$, $ldx_{22} \geq \max(1, m - p)$. |
| ldu_1 | The leading dimension of the array u_1 . If $jobu_1 = 'Y'$, $ldu_1 \geq \max(1, p)$. |
| ldu_2 | The leading dimension of the array u_2 . If $jobu_2 = 'Y'$, $ldu_2 \geq \max(1, m - p)$. |
| ldv_{1t} | The leading dimension of the array v_{1t} . If $jobv_{1t} = 'Y'$, $ldv_{1t} \geq \max(1, q)$. |
| ldv_{2t} | The leading dimension of the array v_{2t} . If $jobv_{2t} = 'Y'$, $ldv_{2t} \geq \max(1, m - q)$. |

Output Parameters

| | |
|----------|---|
| $theta$ | Array, size r , in which $r = \min(p, m - p, q, m - q)$. $C = \text{diag}(\cos(theta[0]), \dots, \cos(theta[r - 1]))$, and $S = \text{diag}(\sin(theta[0]), \dots, \sin(theta[r - 1]))$. |
| u_1 | Array, size at least $\max(1, ldu_1 * p)$. If $jobu_1 = 'Y'$, u_1 contains the p -by- p orthogonal/unitary matrix u_1 . |
| u_2 | Array, size at least $\max(1, ldu_2 * (m - p))$. If $jobu_2 = 'Y'$, u_2 contains the $(m - p)$ -by- $(m - p)$ orthogonal/unitary matrix u_2 . |
| v_{1t} | Array, size at least $\max(1, ldv_{1t} * q)$. If $jobv_{1t} = 'Y'$, v_{1t} contains the q -by- q orthogonal matrix v_{1t}^T or unitary matrix v_{1t}^H . |
| v_{2t} | Array, size at least $\max(1, ldv_{2t} * (m - q))$. If $jobv_{2t} = 'Y'$, v_{2t} contains the $(m - q)$ -by- $(m - q)$ orthogonal matrix v_{2t}^T or unitary matrix v_{2t}^H . |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

> 0 : ?orcsd/?uncsd did not converge.

See Also

[?bbcscd](#)

[xerbla](#)

?orcsd2by1/?uncsd2by1

Computes the CS decomposition of a block-partitioned orthogonal/unitary matrix.

Syntax

```
lapack_int LAPACKE_sorcsd2by1 (int matrix_layout, char jobu1, char jobu2, char jobvt,
lapack_int m, lapack_int p, lapack_int q, float * x11, lapack_int ldx11, float * x21,
lapack_int ldx21, float * theta, float * u1, lapack_int ldu1, float * u2, lapack_int
ldu2, float * v1t, lapack_int ldvt);
```

```
lapack_int LAPACKE_dorcsd2by1 (int matrix_layout, char jobu1, char jobu2, char jobvt,
lapack_int m, lapack_int p, lapack_int q, double * x11, lapack_int ldx11, double *
x21, lapack_int ldx21, double * theta, double * u1, lapack_int ldu1, double * u2,
lapack_int ldu2, double * v1t, lapack_int ldvt);
```

```
lapack_int LAPACKE_cuncsd2by1 (int matrix_layout, char jobu1, char jobu2, char jobvt,
lapack_int m, lapack_int p, lapack_int q, lapack_complex_float * x11, lapack_int
ldx11, lapack_complex_float * x21, lapack_int ldx21, float * theta,
lapack_complex_float * u1, lapack_int ldu1, lapack_complex_float * u2, lapack_int ldu2,
lapack_complex_float * v1t, lapack_int ldvt);
```

```
lapack_int LAPACKE_zuncsd2by1 (int matrix_layout, char jobu1, char jobu2, char jobvt,
lapack_int m, lapack_int p, lapack_int q, lapack_complex_double * x11, lapack_int
ldx11, lapack_complex_double * x21, lapack_int ldx21, double * theta,
lapack_complex_double * u1, lapack_int ldu1, lapack_complex_double * u2, lapack_int
ldu2, lapack_complex_double * v1t, lapack_int ldvt);
```

Include Files

- mkl.h

Description

The routines ?orcsd2by1/?uncsd2by1 compute the CS decomposition of an m -by- q matrix X with orthonormal columns that has been partitioned into a 2-by-1 block structure:

$$X = \begin{bmatrix} X_{11} \\ X_{21} \end{bmatrix} = \begin{bmatrix} U_1 & | & \\ \hline & & U_2 \end{bmatrix} \frac{\begin{bmatrix} I & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & I \end{bmatrix}}{V_1^H}$$

x_{11} is p -by- q . The orthogonal/unitary matrices u_1 , u_2 , v_1 , and v_2 are p -by- p , $(m-p)$ -by- $(m-p)$, q -by- q , $(m-q)$ -by- $(m-q)$, respectively. C and S are r -by- r nonnegative diagonal matrices satisfying $C^2 + S^2 = I$, in which $r = \min(p, m-p, q, m-q)$.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

jobu1 If equal to 'Y', then u_1 is computed. Otherwise, u_1 is not computed.

| | |
|---------------|---|
| <i>jobu2</i> | If equal to 'Y', then u_2 is computed. Otherwise, u_2 is not computed. |
| <i>jobv1t</i> | If equal to 'Y', then v_1^t is computed. Otherwise, v_1^t is not computed. |
| <i>m</i> | The number of rows and columns of the matrix X . |
| <i>p</i> | The number of rows in x_{11} . $0 \leq p \leq m$. |
| <i>q</i> | The number of columns in x_{11} . $0 \leq q \leq m$. |
| <i>x11</i> | Array, size ($ldx11 * q$). On entry, the part of the orthogonal matrix whose CSD is desired. |
| <i>ldx11</i> | The leading dimension of the array $x11$. $ldx11 \geq \max(1, p)$. |
| <i>x21</i> | Array, size ($ldx21 * q$). On entry, the part of the orthogonal matrix whose CSD is desired. |
| <i>ldx21</i> | The leading dimension of the array X . $ldx21 \geq \max(1, m - p)$. |
| <i>ldu1</i> | The leading dimension of the array u_1 . If <i>jobu1</i> = 'Y', $ldu1 \geq \max(1, p)$. |
| <i>ldu2</i> | The leading dimension of the array u_2 . If <i>jobu2</i> = 'Y', $ldu2 \geq \max(1, m - p)$. |
| <i>ldv1t</i> | The leading dimension of the array $v1t$. If <i>jobv1t</i> = 'Y', $ldv1t \geq \max(1, q)$. |

Output Parameters

| | |
|--------------|---|
| <i>theta</i> | Array, size r , in which $r = \min(p, m - p, q, m - q)$. $C = \text{diag}(\cos(\text{theta}(1)), \dots, \cos(\text{theta}(r)))$, and $S = \text{diag}(\sin(\text{theta}(1)), \dots, \sin(\text{theta}(r)))$. |
| <i>u1</i> | Array, size ($ldu1 * p$). If <i>jobu1</i> = 'Y', <i>u1</i> contains the p -by- p orthogonal/unitary matrix u_1 . |
| <i>u2</i> | Array, size ($ldu2 * (m - p)$). If <i>jobu2</i> = 'Y', <i>u2</i> contains the $(m - p)$ -by- $(m - p)$ orthogonal/unitary matrix u_2 . |
| <i>v1t</i> | Array, size ($ldv1t * q$). If <i>jobv1t</i> = 'Y', <i>v1t</i> contains the q -by- q orthogonal matrix v_1^T or unitary matrix v_1^H . |

Return Values

This function returns a value *info*.

= 0: successful exit

< 0: if *info* = $-i$, the i -th argument has an illegal value

> 0: ?orcsd2by1/?uncsd2by1 did not converge.

See Also

?bbcscd
xerbla

Generalized Symmetric Definite Eigenvalue Problems: LAPACK Driver Routines

This section describes LAPACK driver routines used for solving generalized symmetric definite eigenproblems. See also [computational routines](#) that can be called to solve these problems. [Table "Driver Routines for Solving Generalized Symmetric Definite Eigenproblems"](#) lists all such driver routines.

Driver Routines for Solving Generalized Symmetric Definite Eigenproblems

| Routine Name | Operation performed |
|-----------------------------|---|
| sygv/hegv | Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem. |
| sygvd/hegvd | Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem. If eigenvectors are desired, it uses a divide and conquer method. |
| sygvx/hegvx | Computes selected eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem. |
| spgv/hpgv | Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem with matrices in packed storage. |
| spgvd/hpgvd | Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem with matrices in packed storage. If eigenvectors are desired, it uses a divide and conquer method. |
| spgvx/hpgvx | Computes selected eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem with matrices in packed storage. |
| sbgv/hbgv | Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem with banded matrices. |
| sbgvd/hbgvd | Computes all eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem with banded matrices. If eigenvectors are desired, it uses a divide and conquer method. |
| sbgvx/hbgvx | Computes selected eigenvalues and, optionally, eigenvectors of a real / complex generalized symmetric /Hermitian positive-definite eigenproblem with banded matrices. |

?sygv

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem.

Syntax

```
lapack_int LAPACKE_ssygv (int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, float* w);
```

```
lapack_int LAPACKE_dsygv (int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, double* w);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$$A*x = \lambda*B*x, A*B*x = \lambda*x, \text{ or } B*A*x = \lambda*x.$$

Here A and B are assumed to be symmetric and B is also positive definite.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>itype</i> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if <i>itype</i> = 1, the problem type is $A*x = \lambda*B*x$; if <i>itype</i> = 2, the problem type is $A*B*x = \lambda*x$; if <i>itype</i> = 3, the problem type is $B*A*x = \lambda*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>a</i> and <i>b</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>a</i> and <i>b</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>a, b</i> | Arrays: <i>a</i> (size at least $\max(1, lda*n)$) contains the upper or lower triangle of the symmetric matrix A , as specified by <i>uplo</i> . <i>b</i> (size at least $\max(1, ldb*n)$) contains the upper or lower triangle of the symmetric positive definite matrix B , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, if <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>a</i> contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if <i>itype</i> = 1 or 2, $Z^T*B*Z = I$; if <i>itype</i> = 3, $Z^T*inv(B)*Z = I$; If <i>jobz</i> = 'N', then on exit the upper triangle (if <i>uplo</i> = 'U') or the lower triangle (if <i>uplo</i> = 'L') of A , including the diagonal, is destroyed. |
|----------|---|

| | |
|----------|--|
| <i>b</i> | On exit, if $info \leq n$, the part of <i>b</i> containing the matrix is overwritten by the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization $B = U^T * U$ or $B = L * L^T$. |
| <i>w</i> | Array, size at least $\max(1, n)$. If $info = 0$, contains the eigenvalues in ascending order. |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the *i*-th parameter had an illegal value.

If $info > 0$, `spotrf/dpotrf` or `ssyev/dsyev` returned an error code:

If $info = i \leq n$, `ssyev/dsyev` failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero;

If $info = n + i$, for $1 \leq i \leq n$, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

?hegv

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem.

Syntax

```
lapack_int LAPACKE_chegv( int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b,
lapack_int ldb, float* w );
```

```
lapack_int LAPACKE_zhegv( int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b,
lapack_int ldb, double* w );
```

Include Files

- `mkl.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite eigenproblem, of the form

$$A * x = \lambda * B * x, \quad A * B * x = \lambda * x, \quad \text{or} \quad B * A * x = \lambda * x.$$

Here *A* and *B* are assumed to be Hermitian and *B* is also positive definite.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>itype</i> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if $itype = 1$, the problem type is $A * x = \lambda * B * x$; if $itype = 2$, the problem type is $A * B * x = \lambda * x$; |

| | |
|-------------|---|
| | if $itype = 3$, the problem type is $B^*A*x = lambda*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If $jobz = 'N'$, then compute eigenvalues only. If $jobz = 'V'$, then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If $uplo = 'U'$, arrays <i>a</i> and <i>b</i> store the upper triangles of <i>A</i> and <i>B</i> ; If $uplo = 'L'$, arrays <i>a</i> and <i>b</i> store the lower triangles of <i>A</i> and <i>B</i> . |
| <i>n</i> | The order of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>a, b</i> | Arrays: <i>a</i> (size at least $\max(1, lda*n)$) contains the upper or lower triangle of the Hermitian matrix <i>A</i> , as specified by <i>uplo</i> . <i>b</i> (size at least $\max(1, ldb*n)$) contains the upper or lower triangle of the Hermitian positive definite matrix <i>B</i> , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, if $jobz = 'V'$, then if $info = 0$, <i>a</i> contains the matrix <i>Z</i> of eigenvectors. The eigenvectors are normalized as follows: if $itype = 1$ or 2 , $Z^H*B*Z = I$; if $itype = 3$, $Z^H*inv(B)*Z = I$; If $jobz = 'N'$, then on exit the upper triangle (if $uplo = 'U'$) or the lower triangle (if $uplo = 'L'$) of <i>A</i> , including the diagonal, is destroyed. |
| <i>b</i> | On exit, if $info \leq n$, the part of <i>b</i> containing the matrix is overwritten by the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization $B = U^H*U$ or $B = L*L^H$. |
| <i>w</i> | Array, size at least $\max(1, n)$. If $info = 0$, contains the eigenvalues in ascending order. |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the *i*-th parameter had an illegal value.

If $info > 0$, *cpotrf/zpotrf* or *cheev/zheev* return an error code:

If $info = i \leq n$, *cheev/zheev* fails to converge, and *i* off-diagonal elements of an intermediate tridiagonal do not converge to zero;

If $info = n + i$, for $1 \leq i \leq n$, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* can not be completed and no eigenvalues or eigenvectors are computed.

?sygvd

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem using a divide and conquer method.

Syntax

```
lapack_int LAPACKE_ssygvd (int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, float* w);
```

```
lapack_int LAPACKE_dsygvd (int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, double* w);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$$A*x = \lambda*B*x, \quad A*B*x = \lambda*x, \quad \text{or} \quad B*A*x = \lambda*x.$$

Here A and B are assumed to be symmetric and B is also positive definite.

It uses a divide and conquer algorithm.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>itype</i> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if <i>itype</i> = 1, the problem type is $A*x = \lambda*B*x$; if <i>itype</i> = 2, the problem type is $A*B*x = \lambda*x$; if <i>itype</i> = 3, the problem type is $B*A*x = \lambda*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>a</i> and <i>b</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>a</i> and <i>b</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>a, b</i> | Arrays: <i>a</i> (size at least $lda*n$) contains the upper or lower triangle of the symmetric matrix A , as specified by <i>uplo</i> . <i>b</i> (size at least $ldb*n$) contains the upper or lower triangle of the symmetric positive definite matrix B , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |

ldb The leading dimension of *b*; at least $\max(1, n)$.

Output Parameters

a On exit, if *jobz* = 'V', then if *info* = 0, *a* contains the matrix *Z* of eigenvectors. The eigenvectors are normalized as follows:
 if *itype* = 1 or 2, $Z^T * B * Z = I$;
 if *itype* = 3, $Z^T * \text{inv}(B) * Z = I$;
 If *jobz* = 'N', then on exit the upper triangle (if *uplo* = 'U') or the lower triangle (if *uplo* = 'L') of *A*, including the diagonal, is destroyed.

b On exit, if $\text{info} \leq n$, the part of *b* containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization $B = U^T * U$ or $B = L * L^T$.

w Array, size at least $\max(1, n)$.
 If *info* = 0, contains the eigenvalues in ascending order.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, an error code is returned as specified below.

- For $\text{info} \leq n$:
 - If *info* = *i* and *jobz* = 'N', then the algorithm failed to converge; *i* off-diagonal elements of an intermediate tridiagonal form did not converge to zero.
 - If *jobz* = 'V', then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns $\text{info}/(n+1)$ through $\text{mod}(\text{info}, n+1)$.
- For *info* > *n*:
 - If *info* = *n* + *i*, for $1 \leq i \leq n$, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

?hegvd

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem using a divide and conquer method.

Syntax

```
lapack_int LAPACKE_chegvd( int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b,
lapack_int ldb, float* w );
```

```
lapack_int LAPACKE_zhegvd( int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double* b,
lapack_int ldb, double* w );
```

Include Files

- `mkl.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite eigenproblem, of the form

$$A^*x = \lambda^*B^*x, \quad A^*B^*x = \lambda^*x, \quad \text{or} \quad B^*A^*x = \lambda^*x.$$

Here A and B are assumed to be Hermitian and B is also positive definite.

It uses a divide and conquer algorithm.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>itype</i> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if <i>itype</i> = 1, the problem type is $A^*x = \lambda^*B^*x$; if <i>itype</i> = 2, the problem type is $A^*B^*x = \lambda^*x$; if <i>itype</i> = 3, the problem type is $B^*A^*x = \lambda^*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>a</i> and <i>b</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>a</i> and <i>b</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>a, b</i> | Arrays: <i>a</i> (size at least $\max(1, lda*n)$) contains the upper or lower triangle of the Hermitian matrix A , as specified by <i>uplo</i> . <i>b</i> (size at least $\max(1, ldb*n)$) contains the upper or lower triangle of the Hermitian positive definite matrix B , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | On exit, if <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>a</i> contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if <i>itype</i> = 1 or 2, $Z^H * B * Z = I$; if <i>itype</i> = 3, $Z^H * \text{inv}(B) * Z = I$; |
|----------|--|

If $jobz = 'N'$, then on exit the upper triangle (if $uplo = 'U'$) or the lower triangle (if $uplo = 'L'$) of A , including the diagonal, is destroyed.

b On exit, if $info \leq n$, the part of b containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization $B = U^H * U$ or $B = L * L^H$.

w Array, size at least $\max(1, n)$.
If $info = 0$, contains the eigenvalues in ascending order.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = i$, and $jobz = 'N'$, then the algorithm failed to converge; i off-diagonal elements of an intermediate tridiagonal form did not converge to zero;

if $info = i$, and $jobz = 'V'$, then the algorithm failed to compute an eigenvalue while working on the submatrix lying in rows and columns $info/(n+1)$ through $\text{mod}(info, n+1)$.

If $info = n + i$, for $1 \leq i \leq n$, then the leading minor of order i of B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

?sygvx

Computes selected eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem.

Syntax

```
lapack_int LAPACKE_ssygvx (int matrix_layout, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, float vl,
float vu, lapack_int il, lapack_int iu, float abstol, lapack_int* m, float* w, float*
z, lapack_int ldz, lapack_int* ifail);
```

```
lapack_int LAPACKE_dsygvx (int matrix_layout, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, double
vl, double vu, lapack_int il, lapack_int iu, double abstol, lapack_int* m, double* w,
double* z, lapack_int ldz, lapack_int* ifail);
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$$A * x = \lambda * B * x, A * B * x = \lambda * x, \text{ or } B * A * x = \lambda * x.$$

Here A and B are assumed to be symmetric and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>itype</i> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if <i>itype</i> = 1, the problem type is $A*x = \lambda*B*x$; if <i>itype</i> = 2, the problem type is $A*B*x = \lambda*x$; if <i>itype</i> = 3, the problem type is $B*A*x = \lambda*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the routine computes all eigenvalues. If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$. If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i> . |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>a</i> and <i>b</i> store the upper triangles of <i>A</i> and <i>B</i> ; If <i>uplo</i> = 'L', arrays <i>a</i> and <i>b</i> store the lower triangles of <i>A</i> and <i>B</i> . |
| <i>n</i> | The order of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>a, b</i> | Arrays: <i>a</i> (size at least $\max(1, lda*n)$) contains the upper or lower triangle of the symmetric matrix <i>A</i> , as specified by <i>uplo</i> . <i>b</i> (size at least $\max(1, ldb*n)$) contains the upper or lower triangle of the symmetric positive definite matrix <i>B</i> , as specified by <i>uplo</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, n)$. |
| <i>vl, vu</i> | If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced. |
| <i>il, iu</i> | If <i>range</i> = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$. |

If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol

ldz

The leading dimension of the output array *z*. Constraints:

$ldz \geq 1$; if *jobz* = 'V', $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout .

Output Parameters

a

On exit, the upper triangle (if *uplo* = 'U') or the lower triangle (if *uplo* = 'L') of *A*, including the diagonal, is overwritten.

b

On exit, if $info \leq n$, the part of *b* containing the matrix is overwritten by the triangular factor *U* or *L* from the Cholesky factorization $B = U^T * U$ or $B = L * L^T$.

m

The total number of eigenvalues found,

$0 \leq m \leq n$. If *range* = 'A', $m = n$, and if *range* = 'I',
 $m = iu - il + 1$.

w, z

Arrays:

w, size at least $\max(1, n)$.

The first *m* elements of *w* contain the selected eigenvalues in ascending order.

z (size at least $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout) .

If *jobz* = 'V', then if *info* = 0, the first *m* columns of *z* contain the orthonormal eigenvectors of the matrix *A* corresponding to the selected eigenvalues, with the *i*-th column of *z* holding the eigenvector associated with $w[i - 1]$. The eigenvectors are normalized as follows:

if *itype* = 1 or 2, $Z^T * B * Z = I$;

if *itype* = 3, $Z^T * \text{inv}(B) * Z = I$;

If *jobz* = 'N', then *z* is not referenced.

If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

Note: you must ensure that at least $\max(1, m)$ columns are supplied in the array *z*; if *range* = 'V', the exact value of *m* is not known in advance and an upper bound must be used.

ifail

Array, size at least $\max(1, n)$.

If *jobz* = 'V', then if *info* = 0, the first *m* elements of *ifail* are zero; if *info* > 0, the *ifail* contains the indices of the eigenvectors that failed to converge.

If *jobz* = 'N', then *ifail* is not referenced.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, *spotrf/dpotrf* and *ssyevx/dsyevx* returned an error code:

If *info* = $i \leq n$, *ssyevx/dsyevx* failed to converge, and *i* eigenvectors failed to converge. Their indices are stored in the array *ifail*;

If *info* = $n + i$, for $1 \leq i \leq n$, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to $abstol + \epsilon \max(|a|, |b|)$, where ϵ is the machine precision.

If *abstol* is less than or equal to zero, then $\epsilon \|T\|_1$ is used as tolerance, where *T* is the tridiagonal matrix obtained by reducing *C* to tridiagonal form, where *C* is the symmetric matrix of the standard symmetric problem to which the generalized problem is transformed. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold $2 * \text{lamch}('S')$, not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, set *abstol* to $2 * \text{lamch}('S')$.

?hegvx

Computes selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem.

Syntax

```
lapack_int LAPACKC_chegvx( int matrix_layout, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float*
b, lapack_int ldb, float vl, float vu, lapack_int il, lapack_int iu, float abstol,
lapack_int* m, float* w, lapack_complex_float* z, lapack_int ldz, lapack_int* ifail );

lapack_int LAPACKC_zhegvx( int matrix_layout, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, lapack_complex_double* a, lapack_int lda,
lapack_complex_double* b, lapack_int ldb, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, lapack_complex_double* z,
lapack_int ldz, lapack_int* ifail );
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite eigenproblem, of the form

$$A^*x = \lambda B^*x, \quad A^*B^*x = \lambda^*x, \quad \text{or} \quad B^*A^*x = \lambda^*x.$$

Here *A* and *B* are assumed to be Hermitian and *B* is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>itype</i> | <p>Must be 1 or 2 or 3. Specifies the problem type to be solved:</p> <p>if <i>itype</i> = 1, the problem type is $A*x = \lambda*B*x$;</p> <p>if <i>itype</i> = 2, the problem type is $A*B*x = \lambda*x$;</p> <p>if <i>itype</i> = 3, the problem type is $B*A*x = \lambda*x$.</p> |
| <i>jobz</i> | <p>Must be 'N' or 'V'.</p> <p>If <i>jobz</i> = 'N', then compute eigenvalues only.</p> <p>If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors.</p> |
| <i>range</i> | <p>Must be 'A' or 'V' or 'I'.</p> <p>If <i>range</i> = 'A', the routine computes all eigenvalues.</p> <p>If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval:</p> $vl < w[i] \leq vu.$ <p>If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i>.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>If <i>uplo</i> = 'U', arrays <i>a</i> and <i>b</i> store the upper triangles of <i>A</i> and <i>B</i>;</p> <p>If <i>uplo</i> = 'L', arrays <i>a</i> and <i>b</i> store the lower triangles of <i>A</i> and <i>B</i>.</p> |
| <i>n</i> | The order of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>a, b</i> | <p>Arrays:</p> <p><i>a</i> (size at least $\max(1, lda*n)$) contains the upper or lower triangle of the Hermitian matrix <i>A</i>, as specified by <i>uplo</i>.</p> <p><i>b</i> (size at least $\max(1, ldb*n)$) contains the upper or lower triangle of the Hermitian positive definite matrix <i>B</i>, as specified by <i>uplo</i>.</p> |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, n)$. |
| <i>vl, vu</i> | <p>If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.</p> <p>Constraint: $vl < vu$.</p> <p>If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced.</p> |
| <i>il, iu</i> | <p>If <i>range</i> = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.</p> <p>Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$.</p> |

If $range = 'A'$ or $'V'$, il and iu are not referenced.

abstol

The absolute error tolerance for the eigenvalues. See *Application Notes* for more information.

ldz

The leading dimension of the output array z . Constraints:

$ldz \geq 1$; if $jobz = 'V'$, $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout.

Output Parameters

a

On exit, the upper triangle (if $uplo = 'U'$) or the lower triangle (if $uplo = 'L'$) of A , including the diagonal, is overwritten.

b

On exit, if $info \leq n$, the part of b containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization $B = U^H * U$ or $B = L * L^H$.

m

The total number of eigenvalues found,

$0 \leq m \leq n$. If $range = 'A'$, $m = n$, and if $range = 'I'$,
 $m = iu - il + 1$.

w

Array, size at least $\max(1, n)$.

The first m elements of w contain the selected eigenvalues in ascending order.

z

Array z (size at least $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout).

If $jobz = 'V'$, then if $info = 0$, the first m columns of z contain the orthonormal eigenvectors of the matrix A corresponding to the selected eigenvalues, with the i -th column of z holding the eigenvector associated with $w[i - 1]$. The eigenvectors are normalized as follows:

if $itype = 1$ or 2 , $Z^H * B * Z = I$;

if $itype = 3$, $Z^H * \text{inv}(B) * Z = I$;

If $jobz = 'N'$, then z is not referenced.

If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

Note: you must ensure that at least $\max(1, m)$ columns are supplied in the array z ; if $range = 'V'$, the exact value of m is not known in advance and an upper bound must be used.

ifail

Array, size at least $\max(1, n)$.

If $jobz = 'V'$, then if $info = 0$, the first m elements of *ifail* are zero; if $info > 0$, the *ifail* contains the indices of the eigenvectors that failed to converge.

If $jobz = 'N'$, then *ifail* is not referenced.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, `cpotrf/zpotrf` and `cheevx/zheevx` returned an error code:

If *info* = *i* ≤ *n*, `cheevx/zheevx` failed to converge, and *i* eigenvectors failed to converge. Their indices are stored in the array *ifail*;

If *info* = *n* + *i*, for 1 ≤ *i* ≤ *n*, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to $abstol + \epsilon \cdot \max(|a|, |b|)$, where ϵ is the machine precision.

If *abstol* is less than or equal to zero, then $\epsilon \cdot \|T\|_1$ will be used in its place, where *T* is the tridiagonal matrix obtained by reducing *C* to tridiagonal form, where *C* is the symmetric matrix of the standard symmetric problem to which the generalized problem is transformed. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold $2 * \text{lamch}('S')$, not zero.

If this routine returns with *info* > 0, indicating that some eigenvectors did not converge, try setting *abstol* to $2 * \text{lamch}('S')$.

?spgv

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with matrices in packed storage.

Syntax

```
lapack_int LAPACKE_sspgv (int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, float* ap, float* bp, float* w, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dspgv (int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, double* ap, double* bp, double* w, double* z, lapack_int ldz);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$$A*x = \lambda*B*x, A*B*x = \lambda*x, \text{ or } B*A*x = \lambda*x.$$

Here *A* and *B* are assumed to be symmetric, stored in packed format, and *B* is also positive definite.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

itype Must be 1 or 2 or 3. Specifies the problem type to be solved:
if *itype* = 1, the problem type is $A*x = \lambda*B*x$;

| | |
|---------------|---|
| | if $itype = 2$, the problem type is $A*B*x = lambda*x$; if $itype = 3$, the problem type is $B*A*x = lambda*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If $jobz = 'N'$, then compute eigenvalues only. If $jobz = 'V'$, then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If $uplo = 'U'$, arrays <i>ap</i> and <i>bp</i> store the upper triangles of <i>A</i> and <i>B</i> ; If $uplo = 'L'$, arrays <i>ap</i> and <i>bp</i> store the lower triangles of <i>A</i> and <i>B</i> . |
| <i>n</i> | The order of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>ap, bp</i> | Arrays: <i>ap</i> contains the packed upper or lower triangle of the symmetric matrix <i>A</i> , as specified by <i>uplo</i> . The dimension of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$. <i>bp</i> contains the packed upper or lower triangle of the symmetric matrix <i>B</i> , as specified by <i>uplo</i> . The dimension of <i>bp</i> must be at least $\max(1, n*(n+1)/2)$. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> ; $ldz \geq 1$. If $jobz = 'V'$, $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-------------|--|
| <i>ap</i> | On exit, the contents of <i>ap</i> are overwritten. |
| <i>bp</i> | On exit, contains the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization $B = U^T*U$ or $B = L*L^T$, in the same storage format as <i>B</i> . |
| <i>w, z</i> | Arrays: <i>w</i> , size at least $\max(1, n)$. If $info = 0$, contains the eigenvalues in ascending order. <i>z</i> (size $\max(1, ldz*n)$) . If $jobz = 'V'$, then if $info = 0$, <i>z</i> contains the matrix <i>Z</i> of eigenvectors. The eigenvectors are normalized as follows: if $itype = 1$ or 2 , $Z^T*B*Z = I$; if $itype = 3$, $Z^T*inv(B)*Z = I$; If $jobz = 'N'$, then <i>z</i> is not referenced. |

Return Values

This function returns a value *info*.

If $info=0$, the execution is successful.

If $info = -i$, the *i*-th parameter had an illegal value.

If $info > 0$, `sppstrf/dppstrf` and `sspev/dspev` returned an error code:

If $info = i \leq n$, `sspev/dspev` failed to converge, and i off-diagonal elements of an intermediate tridiagonal did not converge to zero;

If $info = n + i$, for $1 \leq i \leq n$, then the leading minor of order i of B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

?hpgv

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem with matrices in packed storage.

Syntax

```
lapack_int LAPACKE_chpgv( int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_float* ap, lapack_complex_float* bp, float* w,
lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zhpgv( int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_double* ap, lapack_complex_double* bp, double* w,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- `mkl.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite eigenproblem, of the form

$$A*x = \lambda*B*x, A*B*x = \lambda*x, \text{ or } B*A*x = \lambda*x.$$

Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>itype</code> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if <code>itype = 1</code> , the problem type is $A*x = \lambda*B*x$; if <code>itype = 2</code> , the problem type is $A*B*x = \lambda*x$; if <code>itype = 3</code> , the problem type is $B*A*x = \lambda*x$. |
| <code>jobz</code> | Must be 'N' or 'V'. If <code>jobz = 'N'</code> , then compute eigenvalues only. If <code>jobz = 'V'</code> , then compute eigenvalues and eigenvectors. |
| <code>uplo</code> | Must be 'U' or 'L'. If <code>uplo = 'U'</code> , arrays <code>ap</code> and <code>bp</code> store the upper triangles of A and B ; If <code>uplo = 'L'</code> , arrays <code>ap</code> and <code>bp</code> store the lower triangles of A and B . |

| | |
|----------|---|
| n | The order of the matrices A and B ($n \geq 0$). |
| ap, bp | Arrays: ap contains the packed upper or lower triangle of the Hermitian matrix A , as specified by $uplo$. The dimension of ap must be at least $\max(1, n*(n+1)/2)$. bp contains the packed upper or lower triangle of the Hermitian matrix B , as specified by $uplo$. The dimension of bp must be at least $\max(1, n*(n+1)/2)$. |
| ldz | The leading dimension of the output array z ; $ldz \geq 1$. If $jobz = 'V'$, $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|------|--|
| ap | On exit, the contents of ap are overwritten. |
| bp | On exit, contains the triangular factor U or L from the Cholesky factorization $B = U^H * U$ or $B = L * L^H$, in the same storage format as B . |
| w | Array, size at least $\max(1, n)$. If $info = 0$, contains the eigenvalues in ascending order. |
| z | Array z (size $\max(1, ldz*n)$). If $jobz = 'V'$, then if $info = 0$, z contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: if $itype = 1$ or 2 , $Z^H * B * Z = I$; if $itype = 3$, $Z^H * \text{inv}(B) * Z = I$; If $jobz = 'N'$, then z is not referenced. |

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info > 0$, `cpptrf/zpptrf` and `chpev/zhpev` returned an error code:

If $info = i \leq n$, `chpev/zhpev` failed to converge, and i off-diagonal elements of an intermediate tridiagonal did not converge to zero;

If $info = n + i$, for $1 \leq i \leq n$, then the leading minor of order i of B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

?spgvd

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with matrices in packed storage using a divide and conquer method.

Syntax

```
lapack_int LAPACKE_sspgvd (int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, float* ap, float* bp, float* w, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dspgvd (int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, double* ap, double* bp, double* w, double* z, lapack_int ldz);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$$A*x = \lambda*B*x, \quad A*B*x = \lambda*x, \quad \text{or} \quad B*A*x = \lambda*x.$$

Here A and B are assumed to be symmetric, stored in packed format, and B is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>itype</i> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if <i>itype</i> = 1, the problem type is $A*x = \lambda*B*x$; if <i>itype</i> = 2, the problem type is $A*B*x = \lambda*x$; if <i>itype</i> = 3, the problem type is $B*A*x = \lambda*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>ap</i> and <i>bp</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>ap</i> and <i>bp</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ap, bp</i> | Arrays: <i>ap</i> contains the packed upper or lower triangle of the symmetric matrix A , as specified by <i>uplo</i> . The dimension of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$. <i>bp</i> contains the packed upper or lower triangle of the symmetric matrix B , as specified by <i>uplo</i> . The dimension of <i>bp</i> must be at least $\max(1, n*(n+1)/2)$. |
| <i>ldz</i> | The leading dimension of the output array z ; $ldz \geq 1$. If <i>jobz</i> = 'V', $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-------------|---|
| <i>ap</i> | On exit, the contents of <i>ap</i> are overwritten. |
| <i>bp</i> | On exit, contains the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization $B = U^T * U$ or $B = L * L^T$, in the same storage format as <i>B</i> . |
| <i>w, z</i> | <p>Arrays:</p> <p><i>w</i>, size at least $\max(1, n)$.</p> <p>If <i>info</i> = 0, contains the eigenvalues in ascending order.</p> <p><i>z</i> (size at least $\max(1, ldz * n)$).</p> <p>If <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>z</i> contains the matrix <i>Z</i> of eigenvectors. The eigenvectors are normalized as follows:</p> <p>if <i>itype</i> = 1 or 2, $Z^T * B * Z = I$;</p> <p>if <i>itype</i> = 3, $Z^T * \text{inv}(B) * Z = I$;</p> <p>If <i>jobz</i> = 'N', then <i>z</i> is not referenced.</p> |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, *sppttrf*/*dppttrf* and *sspevd*/*dspevd* returned an error code:

If *info* = *i* ≤ *n*, *sspevd*/*dspevd* failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero;

If *info* = *n* + *i*, for $1 \leq i \leq n$, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

?hpgvd

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem with matrices in packed storage using a divide and conquer method.

Syntax

```
lapack_int LAPACKE_chpgvd( int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_float* ap, lapack_complex_float* bp, float* w,
lapack_complex_float* z, lapack_int ldz );
```

```
lapack_int LAPACKE_zhpgvd( int matrix_layout, lapack_int itype, char jobz, char uplo,
lapack_int n, lapack_complex_double* ap, lapack_complex_double* bp, double* w,
lapack_complex_double* z, lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite eigenproblem, of the form

$A^*x = \lambda^*B^*x$, $A^*B^*x = \lambda^*x$, or $B^*A^*x = \lambda^*x$.

Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>itype</i> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if <i>itype</i> = 1, the problem type is $A^*x = \lambda^*B^*x$; if <i>itype</i> = 2, the problem type is $A^*B^*x = \lambda^*x$; if <i>itype</i> = 3, the problem type is $B^*A^*x = \lambda^*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>ap</i> and <i>bp</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>ap</i> and <i>bp</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ap, bp</i> | Arrays: <i>ap</i> contains the packed upper or lower triangle of the Hermitian matrix A , as specified by <i>uplo</i> . The dimension of <i>ap</i> must be at least $\max(1, n*(n+1)/2)$. <i>bp</i> contains the packed upper or lower triangle of the Hermitian matrix B , as specified by <i>uplo</i> . The dimension of <i>bp</i> must be at least $\max(1, n*(n+1)/2)$. |
| <i>ldz</i> | The leading dimension of the output array z ; $ldz \geq 1$. If <i>jobz</i> = 'V', $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-----------|---|
| <i>ap</i> | On exit, the contents of <i>ap</i> are overwritten. |
| <i>bp</i> | On exit, contains the triangular factor U or L from the Cholesky factorization $B = U^H*U$ or $B = L*L^H$, in the same storage format as B . |
| <i>w</i> | Array, size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues in ascending order. |
| <i>z</i> | Array z (size at least $\max(1, ldz*n)$). If <i>jobz</i> = 'V', then if <i>info</i> = 0, z contains the matrix Z of eigenvectors. The eigenvectors are normalized as follows: |

if $itype = 1$ or 2 , $Z^H * B * Z = I$;
 if $itype = 3$, $Z^H * \text{inv}(B) * Z = I$;
 If $jobz = 'N'$, then z is not referenced.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info > 0$, `cpptrf/zpptrf` and `chpevd/zhpevd` returned an error code:

If $info = i \leq n$, `chpevd/zhpevd` failed to converge, and i off-diagonal elements of an intermediate tridiagonal did not converge to zero;

If $info = n + i$, for $1 \leq i \leq n$, then the leading minor of order i of B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

?spgvx

Computes selected eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with matrices in packed storage.

Syntax

```
lapack_int LAPACKE_sspgvx (int matrix_layout, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, float* ap, float* bp, float vl, float vu, lapack_int il,
lapack_int iu, float abstol, lapack_int* m, float* w, float* z, lapack_int ldz,
lapack_int* ifail);
```

```
lapack_int LAPACKE_dspgvx (int matrix_layout, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, double* ap, double* bp, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, double* z, lapack_int ldz,
lapack_int* ifail);
```

Include Files

- `mkl.h`

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$$A * x = \lambda * B * x, A * B * x = \lambda * x, \text{ or } B * A * x = \lambda * x.$$

Here A and B are assumed to be symmetric, stored in packed format, and B is also positive definite.

Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (`LAPACK_ROW_MAJOR`) or column major (`LAPACK_COL_MAJOR`).

itype Must be 1 or 2 or 3. Specifies the problem type to be solved:
 if $itype = 1$, the problem type is $A * x = \lambda * B * x$;

| | |
|---------------|--|
| | <p>if $itype = 2$, the problem type is $A*B*x = lambda*x$;</p> <p>if $itype = 3$, the problem type is $B*A*x = lambda*x$.</p> |
| <i>jobz</i> | <p>Must be 'N' or 'V'.</p> <p>If $jobz = 'N'$, then compute eigenvalues only.</p> <p>If $jobz = 'V'$, then compute eigenvalues and eigenvectors.</p> |
| <i>range</i> | <p>Must be 'A' or 'V' or 'I'.</p> <p>If $range = 'A'$, the routine computes all eigenvalues.</p> <p>If $range = 'V'$, the routine computes eigenvalues $w[i]$ in the half-open interval:</p> $vl < w[i] \leq vu.$ <p>If $range = 'I'$, the routine computes eigenvalues with indices il to iu.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>If $uplo = 'U'$, arrays ap and bp store the upper triangles of A and B;</p> <p>If $uplo = 'L'$, arrays ap and bp store the lower triangles of A and B.</p> |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ap, bp</i> | <p>Arrays:</p> <p>ap contains the packed upper or lower triangle of the symmetric matrix A, as specified by $uplo$.</p> <p>The size of ap must be at least $\max(1, n*(n+1)/2)$.</p> <p>bp contains the packed upper or lower triangle of the symmetric matrix B, as specified by $uplo$.</p> <p>The size of bp must be at least $\max(1, n*(n+1)/2)$.</p> |
| <i>vl, vu</i> | <p>If $range = 'V'$, the lower and upper bounds of the interval to be searched for eigenvalues.</p> <p>Constraint: $vl < vu$.</p> <p>If $range = 'A'$ or 'I', vl and vu are not referenced.</p> |
| <i>il, iu</i> | <p>If $range = 'I'$, the indices in ascending order of the smallest and largest eigenvalues to be returned.</p> <p>Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$.</p> <p>If $range = 'A'$ or 'V', il and iu are not referenced.</p> |
| <i>abstol</i> | The absolute error tolerance for the eigenvalues. See <i>Application Notes</i> for more information. |
| <i>ldz</i> | <p>The leading dimension of the output array z. Constraints:</p> <p>$ldz \geq 1$; if $jobz = 'V'$, $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout .</p> |

Output Parameters

| | |
|--------------|---|
| <i>ap</i> | On exit, the contents of <i>ap</i> are overwritten. |
| <i>bp</i> | On exit, contains the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization $B = U^T * U$ or $B = L * L^T$, in the same storage format as <i>B</i> . |
| <i>m</i> | The total number of eigenvalues found, $0 \leq m \leq n$. If <i>range</i> = 'A', $m = n$, and if <i>range</i> = 'I', $m = i_u - i_l + 1$. |
| <i>w, z</i> | Arrays: <i>w</i> , size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues in ascending order. <i>z</i> (size at least $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout) . If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> columns of <i>z</i> contain the orthonormal eigenvectors of the matrix <i>A</i> corresponding to the selected eigenvalues, with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with <i>w</i> (<i>i</i>). The eigenvectors are normalized as follows: if <i>itype</i> = 1 or 2, $Z^T * B * Z = I$; if <i>itype</i> = 3, $Z^T * \text{inv}(B) * Z = I$; If <i>jobz</i> = 'N', then <i>z</i> is not referenced. If an eigenvector fails to converge, then that column of <i>z</i> contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in <i>ifail</i> . Note: you must ensure that at least $\max(1, m)$ columns are supplied in the array <i>z</i> ; if <i>range</i> = 'V', the exact value of <i>m</i> is not known in advance and an upper bound must be used. |
| <i>ifail</i> | Array, size at least $\max(1, n)$. If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> elements of <i>ifail</i> are zero; if <i>info</i> > 0, the <i>ifail</i> contains the indices of the eigenvectors that failed to converge. If <i>jobz</i> = 'N', then <i>ifail</i> is not referenced. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, *sppstrf/dppstrf* and *sspevx/dspevx* returned an error code:

If *info* = $i \leq n$, *sspevx/dspevx* failed to converge, and *i* eigenvectors failed to converge. Their indices are stored in the array *ifail*;

If *info* = $n + i$, for $1 \leq i \leq n$, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \epsilon \cdot \max(|a|, |b|)$, where ϵ is the machine precision.

If $abstol$ is less than or equal to zero, then $\epsilon \cdot \|T\|_1$ is used instead, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues are computed most accurately when $abstol$ is set to twice the underflow threshold $2 \cdot \text{?lamch}('S')$, not zero.

If this routine returns with $info > 0$, indicating that some eigenvectors did not converge, set $abstol$ to $2 \cdot \text{?lamch}('S')$.

?hpgvx

Computes selected eigenvalues and, optionally, eigenvectors of a generalized Hermitian positive-definite eigenproblem with matrices in packed storage.

Syntax

```
lapack_int LAPACKE_chpgvx( int matrix_layout, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, lapack_complex_float* ap, lapack_complex_float* bp, float vl,
float vu, lapack_int il, lapack_int iu, float abstol, lapack_int* m, float* w,
lapack_complex_float* z, lapack_int ldz, lapack_int* ifail );
```

```
lapack_int LAPACKE_zhpgvx( int matrix_layout, lapack_int itype, char jobz, char range,
char uplo, lapack_int n, lapack_complex_double* ap, lapack_complex_double* bp, double vl,
double vu, lapack_int il, lapack_int iu, double abstol, lapack_int* m, double* w,
lapack_complex_double* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite eigenproblem, of the form

$$A^*x = \lambda B^*x, A^*B^*x = \lambda^*x, \text{ or } B^*A^*x = \lambda^*x.$$

Here A and B are assumed to be Hermitian, stored in packed format, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>itype</i> | Must be 1 or 2 or 3. Specifies the problem type to be solved: if <i>itype</i> = 1, the problem type is $A^*x = \lambda B^*x$; if <i>itype</i> = 2, the problem type is $A^*B^*x = \lambda^*x$; if <i>itype</i> = 3, the problem type is $B^*A^*x = \lambda^*x$. |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. |

| | |
|---------------|---|
| | If $jobz = 'V'$, then compute eigenvalues and eigenvectors. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. If $range = 'A'$, the routine computes all eigenvalues. If $range = 'V'$, the routine computes eigenvalues $w[i]$ in the half-open interval: $v_l < w[i] \leq v_u$. If $range = 'I'$, the routine computes eigenvalues with indices il to iu . |
| <i>uplo</i> | Must be 'U' or 'L'. If $uplo = 'U'$, arrays ap and bp store the upper triangles of A and B ; If $uplo = 'L'$, arrays ap and bp store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ap, bp</i> | Arrays: ap contains the packed upper or lower triangle of the Hermitian matrix A , as specified by $uplo$. The dimension of ap must be at least $\max(1, n*(n+1)/2)$. bp contains the packed upper or lower triangle of the Hermitian matrix B , as specified by $uplo$. The dimension of bp must be at least $\max(1, n*(n+1)/2)$. |
| <i>vl, vu</i> | If $range = 'V'$, the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If $range = 'A'$ or 'I', vl and vu are not referenced. |
| <i>il, iu</i> | If $range = 'I'$, the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$. If $range = 'A'$ or 'V', il and iu are not referenced. |
| <i>abstol</i> | The absolute error tolerance for the eigenvalues. See <i>Application Notes</i> for more information. |
| <i>ldz</i> | The leading dimension of the output array z ; $ldz \geq 1$. If $jobz = 'V'$, $ldz \geq \max(1, n)$ for column major layout and $ldz \geq \max(1, m)$ for row major layout. |

Output Parameters

| | |
|-----------|---|
| <i>ap</i> | On exit, the contents of ap are overwritten. |
| <i>bp</i> | On exit, contains the triangular factor U or L from the Cholesky factorization $B = U^H * U$ or $B = L * L^H$, in the same storage format as B . |

| | |
|--------------|--|
| <i>m</i> | <p>The total number of eigenvalues found, $0 \leq m \leq n$. If <i>range</i> = 'A', $m = n$, and if <i>range</i> = 'I', $m = i_u - i_l + 1$.</p> |
| <i>w</i> | <p>Array, size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues in ascending order.</p> |
| <i>z</i> | <p>Array <i>z</i> (size at least $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout). If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> columns of <i>z</i> contain the orthonormal eigenvectors of the matrix <i>A</i> corresponding to the selected eigenvalues, with the <i>i</i>-th column of <i>z</i> holding the eigenvector associated with <i>w</i>(<i>i</i>). The eigenvectors are normalized as follows: if <i>itype</i> = 1 or 2, $Z^H * B * Z = I$; if <i>itype</i> = 3, $Z^H * \text{inv}(B) * Z = I$; If <i>jobz</i> = 'N', then <i>z</i> is not referenced. If an eigenvector fails to converge, then that column of <i>z</i> contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in <i>ifail</i>. Note: you must ensure that at least $\max(1, m)$ columns are supplied in the array <i>z</i>; if <i>range</i> = 'V', the exact value of <i>m</i> is not known in advance and an upper bound must be used.</p> |
| <i>ifail</i> | <p>Array, size at least $\max(1, n)$. If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> elements of <i>ifail</i> are zero; if <i>info</i> > 0, the <i>ifail</i> contains the indices of the eigenvectors that failed to converge. If <i>jobz</i> = 'N', then <i>ifail</i> is not referenced.</p> |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, *cpptrf/zpptrf* and *chpevx/zhpevx* returned an error code:

If *info* = $i \leq n$, *chpevx/zhpevx* failed to converge, and *i* eigenvectors failed to converge. Their indices are stored in the array *ifail*;

If *info* = $n + i$, for $1 \leq i \leq n$, then the leading minor of order *i* of *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval [a,b] of width less than or equal to $abstol + \epsilon * \max(|a|, |b|)$, where ϵ is the machine precision.

If *abstol* is less than or equal to zero, then $\epsilon * ||T||_1$ is used as tolerance, where *T* is the tridiagonal matrix obtained by reducing *A* to tridiagonal form. Eigenvalues will be computed most accurately when *abstol* is set to twice the underflow threshold $2 * ?lamch('S')$, not zero.

If this routine returns with $info > 0$, indicating that some eigenvectors did not converge, try setting $abstol$ to $2*?lamch('S')$.

?sbgv

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with banded matrices.

Syntax

```
lapack_int LAPACKE_ssbgv (int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, float* ab, lapack_int ldab, float* bb, lapack_int ldbb,
float* w, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dsbgv (int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, double* ab, lapack_int ldab, double* bb, lapack_int
ldbb, double* w, double* z, lapack_int ldz);
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form $A*x = \lambda*B*x$. Here A and B are assumed to be symmetric and banded, and B is also positive definite.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>ab</i> and <i>bb</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>ab</i> and <i>bb</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ka</i> | The number of super- or sub-diagonals in A ($ka \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in B ($kb \geq 0$). |
| <i>ab, bb</i> | Arrays: <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and $\max(1, ldab*(ka + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix A (as specified by <i>uplo</i>) in band storage format. |

bb (size at least $\max(1, ldbb*n)$ for column major layout and $\max(1, ldbb*(kb + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix B (as specified by $uplo$) in band storage format.

$ldab$ The leading dimension of the array ab ; must be at least $ka+1$ for column major layout and at least $\max(1, n)$ for row major layout .

$ldbb$ The leading dimension of the array bb ; must be at least $kb+1$ for column major layout and at least $\max(1, n)$ for row major layout.

ldz The leading dimension of the output array z ; $ldz \geq 1$. If $jobz = 'V'$, $ldz \geq \max(1, n)$.

Output Parameters

ab On exit, the contents of ab are overwritten.

bb On exit, contains the factor S from the split Cholesky factorization $B = S^T * S$, as returned by [pbstf/pbstf](#).

w, z Arrays:
 w , size at least $\max(1, n)$.
 If $info = 0$, contains the eigenvalues in ascending order.
 z (size at least $\max(1, ldz*n)$) .
 If $jobz = 'V'$, then if $info = 0$, z contains the matrix Z of eigenvectors, with the i -th column of z holding the eigenvector associated with $w(i)$. The eigenvectors are normalized so that $Z^T * B * Z = I$.
 If $jobz = 'N'$, then z is not referenced.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info > 0$, and

if $i \leq n$, the algorithm failed to converge, and i off-diagonal elements of an intermediate tridiagonal did not converge to zero;

if $info = n + i$, for $1 \leq i \leq n$, then [pbstf/pbstf](#) returned $info = i$ and B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

?hbgv

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem with banded matrices.

Syntax

```
lapack_int LAPACKE_chbgv( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* bb, lapack_int ldbb, float* w, lapack_complex_float* z,
lapack_int ldz );
```

```
lapack_int LAPACKE_zhbgv( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, lapack_complex_double* ab, lapack_int ldab,
lapack_complex_double* bb, lapack_int ldbb, double* w, lapack_complex_double* z,
lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite banded eigenproblem, of the form $A^*x = \lambda B^*x$. Here A and B are Hermitian and banded matrices, and matrix B is also positive definite.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>ab</i> and <i>bb</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>ab</i> and <i>bb</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ka</i> | The number of super- or sub-diagonals in A ($ka \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in B ($kb \geq 0$). |
| <i>ab, bb</i> | Arrays: <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and $\max(1, ldab*(ka + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix A (as specified by <i>uplo</i>) in band storage format. <i>bb</i> (size at least $\max(1, ldbb*n)$ for column major layout and $\max(1, ldbb*(kb + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix B (as specified by <i>uplo</i>) in band storage format. |

| | |
|-------------|--|
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; must be at least $ka+1$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>ldbb</i> | The leading dimension of the array <i>bb</i> ; must be at least $kb+1$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> ; $ldz \geq 1$. If <i>jobz</i> = 'V', $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-----------|--|
| <i>ab</i> | On exit, the contents of <i>ab</i> are overwritten. |
| <i>bb</i> | On exit, contains the factor <i>S</i> from the split Cholesky factorization $B = S^H * S$, as returned by pbstf/pbstf . |
| <i>w</i> | Array, size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues in ascending order. |
| <i>z</i> | Array <i>z</i> (size at least $\max(1, ldz * n)$). If <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>z</i> contains the matrix <i>Z</i> of eigenvectors, with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with $w(i)$. The eigenvectors are normalized so that $Z^H * B * Z = I$. If <i>jobz</i> = 'N', then <i>z</i> is not referenced. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, and

if $i \leq n$, the algorithm failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero;

if $info = n + i$, for $1 \leq i \leq n$, then [pbstf/pbstf](#) returned *info* = *i* and *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

?sbgvd

Computes all eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with banded matrices. If eigenvectors are desired, it uses a divide and conquer method.

Syntax

```
lapack_int LAPACKE_ssbgvd (int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, float* ab, lapack_int ldab, float* bb, lapack_int ldbb,
float* w, float* z, lapack_int ldz);
```

```
lapack_int LAPACKE_dsbgvd (int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, double* ab, lapack_int ldab, double* bb, lapack_int
ldbb, double* w, double* z, lapack_int ldz);
```

Include Files

- `mk1.h`

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form $A^*x = \lambda^*B^*x$. Here A and B are assumed to be symmetric and banded, and B is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>ab</i> and <i>bb</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>ab</i> and <i>bb</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ka</i> | The number of super- or sub-diagonals in A ($ka \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in B ($kb \geq 0$). |
| <i>ab, bb</i> | Arrays: <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and $\max(1, ldab*(ka + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix A (as specified by <i>uplo</i>) in band storage format. <i>bb</i> (size at least $\max(1, ldbb*n)$ for column major layout and $\max(1, ldbb*(kb + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix B (as specified by <i>uplo</i>) in band storage format. |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; must be at least $ka+1$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>ldbb</i> | The leading dimension of the array <i>bb</i> ; must be at least $kb+1$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> ; $ldz \geq 1$. If <i>jobz</i> = 'V', $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-------------|---|
| <i>ab</i> | On exit, the contents of <i>ab</i> are overwritten. |
| <i>bb</i> | On exit, contains the factor <i>S</i> from the split Cholesky factorization $B = S^T * S$, as returned by pbstf/pbstf . |
| <i>w, z</i> | <p>Arrays:</p> <p><i>w</i>, size at least $\max(1, n)$.</p> <p>If <i>info</i> = 0, contains the eigenvalues in ascending order.</p> <p><i>z</i> (size at least $\max(1, ldz * n)$).</p> <p>If <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>z</i> contains the matrix <i>Z</i> of eigenvectors, with the <i>i</i>-th column of <i>z</i> holding the eigenvector associated with $w[i - 1]$. The eigenvectors are normalized so that $Z^T * B * Z = I$.</p> <p>If <i>jobz</i> = 'N', then <i>z</i> is not referenced.</p> |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, and

if $i \leq n$, the algorithm failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero;

if $info = n + i$, for $1 \leq i \leq n$, then [pbstf/pbstf](#) returned *info* = *i* and *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

?hbgvd

Computes all eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem with banded matrices. If eigenvectors are desired, it uses a divide and conquer method.

Syntax

```
lapack_int LAPACKE_chbgvd( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* bb, lapack_int ldbb, float* w, lapack_complex_float* z,
lapack_int ldz );
```

```
lapack_int LAPACKE_zhbgvd( int matrix_layout, char jobz, char uplo, lapack_int n,
lapack_int ka, lapack_int kb, lapack_complex_double* ab, lapack_int ldab,
lapack_complex_double* bb, lapack_int ldbb, double* w, lapack_complex_double* z,
lapack_int ldz );
```

Include Files

- mkl.h

Description

The routine computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite banded eigenproblem, of the form $A^*x = \lambda^*B^*x$. Here A and B are assumed to be Hermitian and banded, and B is also positive definite.

If eigenvectors are desired, it uses a divide and conquer algorithm.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>uplo</i> | Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>ab</i> and <i>bb</i> store the upper triangles of A and B ; If <i>uplo</i> = 'L', arrays <i>ab</i> and <i>bb</i> store the lower triangles of A and B . |
| <i>n</i> | The order of the matrices A and B ($n \geq 0$). |
| <i>ka</i> | The number of super- or sub-diagonals in A ($ka \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in B ($kb \geq 0$). |
| <i>ab, bb</i> | Arrays: <i>ab</i> (size at least $\max(1, ldab*n)$ for column major layout and $\max(1, ldab*(ka + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix A (as specified by <i>uplo</i>) in band storage format. <i>bb</i> (size at least $\max(1, ldbb*n)$ for column major layout and $\max(1, ldbb*(kb + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix B (as specified by <i>uplo</i>) in band storage format. |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; must be at least $ka+1$. |
| <i>ldbb</i> | The leading dimension of the array <i>bb</i> ; must be at least $kb+1$. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> ; $ldz \geq 1$. If <i>jobz</i> = 'V', $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|-----------|---|
| <i>ab</i> | On exit, the contents of <i>ab</i> are overwritten. |
| <i>bb</i> | On exit, contains the factor S from the split Cholesky factorization $B = S^H * S$, as returned by pbstf/pbstf . |
| <i>w</i> | Array, size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues in ascending order. |

z Array *z* (size at least $\max(1, ldz*n)$).

If *jobz* = 'V', then if *info* = 0, *z* contains the matrix *Z* of eigenvectors, with the *i*-th column of *z* holding the eigenvector associated with *w*(*i*). The eigenvectors are normalized so that $Z^H*B*Z = I$.

If *jobz* = 'N', then *z* is not referenced.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, and

if $i \leq n$, the algorithm failed to converge, and *i* off-diagonal elements of an intermediate tridiagonal did not converge to zero;

if $info = n + i$, for $1 \leq i \leq n$, then `pbstf/pbstf` returned $info = i$ and *B* is not positive-definite. The factorization of *B* could not be completed and no eigenvalues or eigenvectors were computed.

?sbgvx

Computes selected eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem with banded matrices.

Syntax

```
lapack_int LAPACKE_ssbgvx (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_int ka, lapack_int kb, float* ab, lapack_int ldab, float* bb,
lapack_int ldbb, float* q, lapack_int ldq, float vl, float vu, lapack_int il,
lapack_int iu, float abstol, lapack_int* m, float* w, float* z, lapack_int ldz,
lapack_int* ifail);
```

```
lapack_int LAPACKE_dsbgvx (int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_int ka, lapack_int kb, double* ab, lapack_int ldab, double* bb,
lapack_int ldbb, double* q, lapack_int ldq, double vl, double vu, lapack_int il,
lapack_int iu, double abstol, lapack_int* m, double* w, double* z, lapack_int ldz,
lapack_int* ifail);
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite banded eigenproblem, of the form $A*x = \lambda*B*x$. Here *A* and *B* are assumed to be symmetric and banded, and *B* is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either all eigenvalues, a range of values or a range of indices for the desired eigenvalues.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

jobz Must be 'N' or 'V'.

| | |
|---------------|---|
| | If $jobz = 'N'$, then compute eigenvalues only. |
| | If $jobz = 'V'$, then compute eigenvalues and eigenvectors. |
| <i>range</i> | Must be 'A' or 'V' or 'I'. |
| | If $range = 'A'$, the routine computes all eigenvalues. |
| | If $range = 'V'$, the routine computes eigenvalues $w[i]$ in the half-open interval: $vl < w[i] \leq vu$. |
| | If $range = 'I'$, the routine computes eigenvalues in range il to iu . |
| <i>uplo</i> | Must be 'U' or 'L'. |
| | If $uplo = 'U'$, arrays <i>ab</i> and <i>bb</i> store the upper triangles of <i>A</i> and <i>B</i> ; |
| | If $uplo = 'L'$, arrays <i>ab</i> and <i>bb</i> store the lower triangles of <i>A</i> and <i>B</i> . |
| <i>n</i> | The order of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>ka</i> | The number of super- or sub-diagonals in <i>A</i> ($ka \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in <i>B</i> ($kb \geq 0$). |
| <i>ab, bb</i> | Arrays: <i>ab</i> (size at least $\max(1, ldab * n)$ for column major layout and $\max(1, ldab * (ka + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix <i>A</i> (as specified by <i>uplo</i>) in band storage format. <i>bb</i> (size at least $\max(1, ldbb * n)$ for column major layout and $\max(1, ldbb * (kb + 1))$ for row major layout) is an array containing either upper or lower triangular part of the symmetric matrix <i>B</i> (as specified by <i>uplo</i>) in band storage format. |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; must be at least $ka+1$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>ldbb</i> | The leading dimension of the array <i>bb</i> ; must be at least $kb+1$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>vl, vu</i> | If $range = 'V'$, the lower and upper bounds of the interval to be searched for eigenvalues. Constraint: $vl < vu$. If $range = 'A'$ or 'I', <i>vl</i> and <i>vu</i> are not referenced. |
| <i>il, iu</i> | If $range = 'I'$, the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$. If $range = 'A'$ or 'V', <i>il</i> and <i>iu</i> are not referenced. |

| | |
|---------------|---|
| <i>abstol</i> | The absolute error tolerance for the eigenvalues. See <i>Application Notes</i> for more information. |
| <i>ldz</i> | The leading dimension of the output array <i>z</i> ; $ldz \geq 1$. If <i>jobz</i> = 'V', $ldz \geq \max(1, n)$. |
| <i>ldq</i> | The leading dimension of the output array <i>q</i> ; $ldq < 1$. If <i>jobz</i> = 'V', $ldq < \max(1, n)$. |

Output Parameters

| | |
|----------------|---|
| <i>ab</i> | On exit, the contents of <i>ab</i> are overwritten. |
| <i>bb</i> | On exit, contains the factor <i>S</i> from the split Cholesky factorization $B = S^T * S$, as returned by pbstf/pbstf . |
| <i>m</i> | The total number of eigenvalues found, $0 \leq m \leq n$. If <i>range</i> = 'A', $m = n$, and if <i>range</i> = 'I', $m = iu - il + 1$. |
| <i>w, z, q</i> | Arrays: <i>w</i> , size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues in ascending order. <i>z</i> (size $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout) . If <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>z</i> contains the matrix <i>Z</i> of eigenvectors, with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with <i>w</i> (<i>i</i>). The eigenvectors are normalized so that $Z^T * B * Z = I$. If <i>jobz</i> = 'N', then <i>z</i> is not referenced. <i>q</i> (size $\max(1, ldq * n)$) . If <i>jobz</i> = 'V', then <i>q</i> contains the <i>n</i> -by- <i>n</i> matrix used in the reduction of $A * x = \lambda * B * x$ to standard form, that is, $C * x = \lambda * x$ and consequently <i>C</i> to tridiagonal form. If <i>jobz</i> = 'N', then <i>q</i> is not referenced. |
| <i>ifail</i> | Array, size <i>m</i> . If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> elements of <i>ifail</i> are zero; if <i>info</i> > 0, the <i>ifail</i> contains the indices of the eigenvectors that failed to converge. If <i>jobz</i> = 'N', then <i>ifail</i> is not referenced. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, and

if $i \leq n$, the algorithm failed to converge, and i off-diagonal elements of an intermediate tridiagonal did not converge to zero;

if $info = n + i$, for $1 \leq i \leq n$, then `pbstf/pbstf` returned $info = i$ and B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \varepsilon \max(|a|, |b|)$, where ε is the machine precision.

If $abstol$ is less than or equal to zero, then $\varepsilon * ||T||_1$ is used as tolerance, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues will be computed most accurately when $abstol$ is set to twice the underflow threshold $2 * ?lamch('S')$, not zero.

If this routine returns with $info > 0$, indicating that some eigenvectors did not converge, try setting $abstol$ to $2 * ?lamch('S')$.

?hbgvx

Computes selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem with banded matrices.

Syntax

```
lapack_int LAPACKE_chbgvx( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_int ka, lapack_int kb, lapack_complex_float* ab, lapack_int ldab,
lapack_complex_float* bb, lapack_int ldbb, lapack_complex_float* q, lapack_int ldq,
float vl, float vu, lapack_int il, lapack_int iu, float abstol, lapack_int* m, float*
w, lapack_complex_float* z, lapack_int ldz, lapack_int* ifail );
```

```
lapack_int LAPACKE_zhbgvx( int matrix_layout, char jobz, char range, char uplo,
lapack_int n, lapack_int ka, lapack_int kb, lapack_complex_double* ab, lapack_int
ldab, lapack_complex_double* bb, lapack_int ldbb, lapack_complex_double* q, lapack_int
ldq, double vl, double vu, lapack_int il, lapack_int iu, double abstol, lapack_int* m,
double* w, lapack_complex_double* z, lapack_int ldz, lapack_int* ifail );
```

Include Files

- mkl.h

Description

The routine computes selected eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite banded eigenproblem, of the form $A*x = \lambda*B*x$. Here A and B are assumed to be Hermitian and banded, and B is also positive definite. Eigenvalues and eigenvectors can be selected by specifying either all eigenvalues, a range of values or a range of indices for the desired eigenvalues.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobz</i> | Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |

| | |
|---------------|--|
| <i>range</i> | <p>Must be 'A' or 'V' or 'I'.</p> <p>If <i>range</i> = 'A', the routine computes all eigenvalues.</p> <p>If <i>range</i> = 'V', the routine computes eigenvalues $w[i]$ in the half-open interval:</p> $vl < w[i] \leq vu.$ <p>If <i>range</i> = 'I', the routine computes eigenvalues with indices <i>il</i> to <i>iu</i>.</p> |
| <i>uplo</i> | <p>Must be 'U' or 'L'.</p> <p>If <i>uplo</i> = 'U', arrays <i>ab</i> and <i>bb</i> store the upper triangles of <i>A</i> and <i>B</i>;</p> <p>If <i>uplo</i> = 'L', arrays <i>ab</i> and <i>bb</i> store the lower triangles of <i>A</i> and <i>B</i>.</p> |
| <i>n</i> | The order of the matrices <i>A</i> and <i>B</i> ($n \geq 0$). |
| <i>ka</i> | The number of super- or sub-diagonals in <i>A</i> ($ka \geq 0$). |
| <i>kb</i> | The number of super- or sub-diagonals in <i>B</i> ($kb \geq 0$). |
| <i>ab, bb</i> | <p>Arrays:</p> <p><i>ab</i>(size at least $\max(1, ldab * n)$ for column major layout and $\max(1, ldab * (ka + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix <i>A</i> (as specified by <i>uplo</i>) in band storage format.</p> <p><i>bb</i>(size at least $\max(1, ldbb * n)$ for column major layout and $\max(1, ldbb * (kb + 1))$ for row major layout) is an array containing either upper or lower triangular part of the Hermitian matrix <i>B</i> (as specified by <i>uplo</i>) in band storage format.</p> |
| <i>ldab</i> | The leading dimension of the array <i>ab</i> ; must be at least $ka+1$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>ldbb</i> | The leading dimension of the array <i>bb</i> ; must be at least $kb+1$ for column major layout and at least $\max(1, n)$ for row major layout. |
| <i>vl, vu</i> | <p>If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues.</p> <p>Constraint: $vl < vu$.</p> <p>If <i>range</i> = 'A' or 'I', <i>vl</i> and <i>vu</i> are not referenced.</p> |
| <i>il, iu</i> | <p>If <i>range</i> = 'I', the indices in ascending order of the smallest and largest eigenvalues to be returned.</p> <p>Constraint: $1 \leq il \leq iu \leq n$, if $n > 0$; $il=1$ and $iu=0$ if $n = 0$.</p> <p>If <i>range</i> = 'A' or 'V', <i>il</i> and <i>iu</i> are not referenced.</p> |
| <i>abstol</i> | The absolute error tolerance for the eigenvalues. See <i>Application Notes</i> for more information. |

| | |
|------------|---|
| <i>ldz</i> | The leading dimension of the output array <i>z</i> ; $ldz \geq 1$. If <i>jobz</i> = 'V', $ldz \geq \max(1, n)$ for column major layout and at least $\max(1, m)$ for row major layout. |
| <i>ldq</i> | The leading dimension of the output array <i>q</i> ; $ldq \geq 1$. If <i>jobz</i> = 'V', $ldq \geq \max(1, n)$. |

Output Parameters

| | |
|--------------|---|
| <i>ab</i> | On exit, the contents of <i>ab</i> are overwritten. |
| <i>bb</i> | On exit, contains the factor <i>S</i> from the split Cholesky factorization $B = S^H * S$, as returned by pbstf/pbstf . |
| <i>m</i> | The total number of eigenvalues found, $0 \leq m \leq n$. If <i>range</i> = 'A', $m = n$, and if <i>range</i> = 'I', $m = iu - il + 1$. |
| <i>w</i> | Array <i>w</i> , size at least $\max(1, n)$. If <i>info</i> = 0, contains the eigenvalues in ascending order. |
| <i>z, q</i> | Arrays: <i>z</i> (size $\max(1, ldz * m)$ for column major layout and $\max(1, ldz * n)$ for row major layout). If <i>jobz</i> = 'V', then if <i>info</i> = 0, <i>z</i> contains the matrix <i>Z</i> of eigenvectors, with the <i>i</i> -th column of <i>z</i> holding the eigenvector associated with $w[i - 1]$. The eigenvectors are normalized so that $Z^H * B * Z = I$. If <i>jobz</i> = 'N', then <i>z</i> is not referenced. <i>q</i> (size $\max(1, ldq * n)$). If <i>jobz</i> = 'V', then <i>q</i> contains the <i>n</i> -by- <i>n</i> matrix used in the reduction of $Ax = \lambda Bx$ to standard form, that is, $Cx = \lambda x$ and consequently <i>C</i> to tridiagonal form. If <i>jobz</i> = 'N', then <i>q</i> is not referenced. |
| <i>ifail</i> | Array, size at least $\max(1, n)$. If <i>jobz</i> = 'V', then if <i>info</i> = 0, the first <i>m</i> elements of <i>ifail</i> are zero; if <i>info</i> > 0, the <i>ifail</i> contains the indices of the eigenvectors that failed to converge. If <i>jobz</i> = 'N', then <i>ifail</i> is not referenced. |

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* > 0, and

if $i \leq n$, the algorithm failed to converge, and i off-diagonal elements of an intermediate tridiagonal did not converge to zero;

if $info = n + i$, for $1 \leq i \leq n$, then `pbstf/pbstf` returned $info = i$ and B is not positive-definite. The factorization of B could not be completed and no eigenvalues or eigenvectors were computed.

Application Notes

An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a,b]$ of width less than or equal to $abstol + \epsilon \max(|a|, |b|)$, where ϵ is the machine precision.

If $abstol$ is less than or equal to zero, then $\epsilon * ||T||_1$ will be used in its place, where T is the tridiagonal matrix obtained by reducing A to tridiagonal form. Eigenvalues will be computed most accurately when $abstol$ is set to twice the underflow threshold $2 * ?lamch('S')$, not zero.

If this routine returns with $info > 0$, indicating that some eigenvectors did not converge, try setting $abstol$ to $2 * ?lamch('S')$.

Generalized Nonsymmetric Eigenvalue Problems: LAPACK Driver Routines

This section describes LAPACK driver routines used for solving generalized nonsymmetric eigenproblems. See also [computational routines](#) that can be called to solve these problems. [Table "Driver Routines for Solving Generalized Nonsymmetric Eigenproblems"](#) lists all such driver routines.

Driver Routines for Solving Generalized Nonsymmetric Eigenproblems

| Routine Name | Operation performed |
|-----------------------|--|
| gges | Computes the generalized eigenvalues, Schur form, and the left and/or right Schur vectors for a pair of nonsymmetric matrices. |
| ggesx | Computes the generalized eigenvalues, Schur form, and, optionally, the left and/or right matrices of Schur vectors. |
| gges3 | Computes generalized Schur factorization for a pair of matrices. |
| ggev | Computes the generalized eigenvalues, and the left and/or right generalized eigenvectors for a pair of nonsymmetric matrices. |
| ggev3 | Computes the generalized eigenvalues, and, optionally, the left and/or right generalized eigenvectors. |
| ggev3 | Computes generalized Schur factorization for a pair of matrices. |

?gges

Computes the generalized eigenvalues, Schur form, and the left and/or right Schur vectors for a pair of nonsymmetric matrices.

Syntax

```
lapack_int LAPACKE_sgges( int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_S_SELECT3 select, lapack_int n, float* a, lapack_int lda, float* b, lapack_int
ldb, lapack_int* sdim, float* alphas, float* alphai, float* beta, float* vs1,
lapack_int ldvs1, float* vsr, lapack_int ldvsr );
```

```
lapack_int LAPACKE_dgges( int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_D_SELECT3 select, lapack_int n, double* a, lapack_int lda, double* b,
lapack_int ldb, lapack_int* sdim, double* alphas, double* alphai, double* beta,
double* vs1, lapack_int ldvs1, double* vsr, lapack_int ldvsr );
```

```

lapack_int LAPACKE_cgges( int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_C_SELECT2 select, lapack_int n, lapack_complex_float* a, lapack_int lda,
lapack_complex_float* b, lapack_int ldb, lapack_int* sdim, lapack_complex_float* alpha,
lapack_complex_float* beta, lapack_complex_float* vsl, lapack_int ldvsl,
lapack_complex_float* vsr, lapack_int ldvsr );

lapack_int LAPACKE_zgges( int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_Z_SELECT2 select, lapack_int n, lapack_complex_double* a, lapack_int lda,
lapack_complex_double* b, lapack_int ldb, lapack_int* sdim, lapack_complex_double*
alpha, lapack_complex_double* beta, lapack_complex_double* vsl, lapack_int ldvsl,
lapack_complex_double* vsr, lapack_int ldvsr );

```

Include Files

- mkl.h

Description

The `?gges` routine computes the generalized eigenvalues, the generalized real/complex Schur form (S,T) , optionally, the left and/or right matrices of Schur vectors (vsl and vsr) for a pair of n -by- n real/complex nonsymmetric matrices (A,B) . This gives the generalized Schur factorization

$$(A,B) = (vsl * S * vsr^H, vsl * T * vsr^H)$$

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix S and the upper triangular matrix T . The leading columns of vsl and vsr then form an orthonormal/unitary basis for the corresponding left and right eigenspaces (deflating subspaces).

If only the generalized eigenvalues are needed, use the driver `ggev` instead, which is faster.

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio $alpha / beta = w$, such that $A - w * B$ is singular. It is usually represented as the pair $(alpha, beta)$, as there is a reasonable interpretation for $beta=0$ or for both being zero. A pair of matrices (S,T) is in the generalized real Schur form if T is upper triangular with non-negative diagonal and S is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of S are "standardized" by making the corresponding elements of T have the form:

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

and the pair of corresponding 2-by-2 blocks in S and T will have a complex conjugate pair of generalized eigenvalues. A pair of matrices (S,T) is in generalized complex Schur form if S and T are upper triangular and, in addition, the diagonal of T are non-negative real numbers.

The `?gges` routine replaces the deprecated `?gegs` routine.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>jobvsl</code> | Must be 'N' or 'V'. If <code>jobvsl = 'N'</code> , then the left Schur vectors are not computed. |

| | |
|---------------------|--|
| | If <i>jobvsl</i> = 'V', then the left Schur vectors are computed. |
| <i>jobvsr</i> | Must be 'N' or 'V'. If <i>jobvsr</i> = 'N', then the right Schur vectors are not computed. If <i>jobvsr</i> = 'V', then the right Schur vectors are computed. |
| <i>sort</i> | Must be 'N' or 'S'. Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. If <i>sort</i> = 'N', then eigenvalues are not ordered. If <i>sort</i> = 'S', eigenvalues are ordered (see <i>select</i>). |
| <i>select</i> | The <i>select</i> parameter is a pointer to a function returning a value of <code>lapack_logical</code> type. For different flavors the function has different arguments: <pre>LAPACKE_sggcs: lapack_logical (*LAPACK_S_SELECT3) (const float*, const float*, const float*);</pre> <pre>LAPACKE_dggcs: lapack_logical (*LAPACK_D_SELECT3) (const double*, const double*, const double*);</pre> <pre>LAPACKE_cggcs: lapack_logical (*LAPACK_C_SELECT2) (const lapack_complex_float*, const lapack_complex_float*);</pre> <pre>LAPACKE_zggcs: lapack_logical (*LAPACK_Z_SELECT2) (const lapack_complex_double*, const lapack_complex_double*);</pre> If <i>sort</i> = 'S', <i>select</i> is used to select eigenvalues to sort to the top left of the Schur form. If <i>sort</i> = 'N', <i>select</i> is not referenced. <i>For real flavors:</i> An eigenvalue $(\text{alphar}[j] + \text{alphai}[j])/beta[j]$ is selected if <i>select</i> (<i>alphar</i> [<i>j</i>], <i>alphai</i> [<i>j</i>], <i>beta</i> [<i>j</i>]) is true; that is, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected. Note that in the ill-conditioned case, a selected complex eigenvalue may no longer satisfy <i>select</i> (<i>alphar</i> [<i>j</i>], <i>alphai</i> [<i>j</i>], <i>beta</i> [<i>j</i>]) = 1 after ordering. In this case <i>info</i> is set to <i>n</i> +2 . <i>For complex flavors:</i> An eigenvalue $\text{alpha}[j] / \text{beta}[j]$ is selected if <i>select</i> (<i>alpha</i> [<i>j</i>], <i>beta</i> [<i>j</i>]) is true. Note that a selected complex eigenvalue may no longer satisfy <i>select</i> (<i>alpha</i> [<i>j</i>], <i>beta</i> [<i>j</i>]) = 1 after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case <i>info</i> is set to <i>n</i> +2 (see <i>info</i> below). |
| <i>n</i> | The order of the matrices <i>A</i> , <i>B</i> , <i>vsl</i> , and <i>vsr</i> ($n \geq 0$). |
| <i>a</i> , <i>b</i> | Arrays: <i>a</i> (size at least $\max(1, lda*n)$) is an array containing the <i>n</i> -by- <i>n</i> matrix <i>A</i> (first of the pair of matrices). |

b (size at least $\max(1, l_{db}*n)$) is an array containing the n -by- n matrix B (second of the pair of matrices).

lda The leading dimension of the array a . Must be at least $\max(1, n)$.

ldb The leading dimension of the array b . Must be at least $\max(1, n)$.

$ldvsl, ldvsr$ The leading dimensions of the output matrices vsl and vsr , respectively. Constraints:

$ldvsl \geq 1$. If $jobvsl = 'V'$, $ldvsl \geq \max(1, n)$.

$ldvsr \geq 1$. If $jobvsr = 'V'$, $ldvsr \geq \max(1, n)$.

Output Parameters

a On exit, this array has been overwritten by its generalized Schur form S .

b On exit, this array has been overwritten by its generalized Schur form T .

$sdim$ If $sort = 'N'$, $sdim = 0$.

If $sort = 'S'$, $sdim$ is equal to the number of eigenvalues (after sorting) for which $select$ is true.

Note that for real flavors complex conjugate pairs for which $select$ is true for either eigenvalue count as 2.

$alphar, alphai$ Arrays, size at least $\max(1, n)$ each. Contain values that form generalized eigenvalues in real flavors.

See $beta$.

$alpha$ Array, size at least $\max(1, n)$. Contain values that form generalized eigenvalues in complex flavors. See $beta$.

$beta$ Array, size at least $\max(1, n)$.

For real flavors:

On exit, $(alphar[j] + alphai[j]*i)/beta[j]$, $j=0, \dots, n-1$, will be the generalized eigenvalues.

$alphar[j] + alphai[j]*i$ and $beta[j]$, $j=0, \dots, n-1$ are the diagonals of the complex Schur form (S, T) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (A, B) were further reduced to triangular form using complex unitary transformations. If $alphai[j]$ is zero, then the j -th eigenvalue is real; if positive, then the j -th and $(j+1)$ -st eigenvalues are a complex conjugate pair, with $alphai[j+1]$ negative.

For complex flavors:

On exit, $alpha[j]/beta[j]$, $j=0, \dots, n-1$, will be the generalized eigenvalues. $alpha[j]$ and $beta[j]$, $j=0, \dots, n-1$ are the diagonals of the complex Schur form (S, T) output by $cggges/zggges$. The $beta[j]$ will be non-negative real.

See also *Application Notes* below.

vsl, vsr

Arrays:

vsl (size at least $\max(1, ldvsl*n)$).

If $jobvsl = 'V'$, this array will contain the left Schur vectors.

If `jobvsl = 'N'`, `vsl` is not referenced.

`vsr` (size at least $\max(1, ldvsr * n)$).

If `jobvsr = 'V'`, this array will contain the right Schur vectors.

If `jobvsr = 'N'`, `vsr` is not referenced.

Return Values

This function returns a value `info`.

If `info=0`, the execution is successful.

If `info = -i`, the *i*-th parameter had an illegal value.

If `info = i`, and

$i \leq n$:

the QZ iteration failed. (*A*, *B*) is not in Schur form, but `alphar[j]`, `alphai[j]` (for real flavors), or `alpha[j]` (for complex flavors), and `beta[j]`, $j = info, \dots, n - 1$ should be correct.

$i > n$: errors that usually indicate LAPACK problems:

$i = n+1$: other than QZ iteration failed in `hgeqz`;

$i = n+2$: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the generalized Schur form no longer satisfy `select = 1`. This could also be caused due to scaling;

$i = n+3$: reordering failed in `tgsen`.

Application Notes

The quotients `alphar[j]/beta[j]` and `alphai[j]/beta[j]` may easily over- or underflow, and `beta[j]` may even be zero. Thus, you should avoid simply computing the ratio. However, `alphar` and `alphai` will be always less than and usually comparable with `norm(A)` in magnitude, and `beta` always less than and usually comparable with `norm(B)`.

?ggesx

Computes the generalized eigenvalues, Schur form, and, optionally, the left and/or right matrices of Schur vectors.

Syntax

```
lapack_int LAPACKE_sggesx( int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_S_SELECT3 select, char sense, lapack_int n, float* a, lapack_int lda, float* b,
lapack_int ldb, lapack_int* sdim, float* alphar, float* alphai, float* beta, float*
vsl, lapack_int ldvsl, float* vsr, lapack_int ldvsr, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_dggesx( int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_D_SELECT3 select, char sense, lapack_int n, double* a, lapack_int lda, double*
b, lapack_int ldb, lapack_int* sdim, double* alphar, double* alphai, double* beta,
double* vsl, lapack_int ldvsl, double* vsr, lapack_int ldvsr, double* rconde, double*
rcondv );
```

```
lapack_int LAPACKE_cggesx( int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_C_SELECT2 select, char sense, lapack_int n, lapack_complex_float* a, lapack_int
lda, lapack_complex_float* b, lapack_int ldb, lapack_int* sdim, lapack_complex_float*
alpha, lapack_complex_float* beta, lapack_complex_float* vsl, lapack_int ldvsl,
lapack_complex_float* vsr, lapack_int ldvsr, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_zggesx( int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_Z_SELECT2 select, char sense, lapack_int n, lapack_complex_double* a, lapack_int
lda, lapack_complex_double* b, lapack_int ldb, lapack_int* sdim, lapack_complex_double*
alpha, lapack_complex_double* beta, lapack_complex_double* vs1, lapack_int ldvs1,
lapack_complex_double* vsr, lapack_int ldvsr, double* rconde, double* rcondv );
```

Include Files

- mkl.h

Description

The routine computes for a pair of n -by- n real/complex nonsymmetric matrices (A,B) , the generalized eigenvalues, the generalized real/complex Schur form (S,T) , optionally, the left and/or right matrices of Schur vectors ($vs1$ and vsr). This gives the generalized Schur factorization

$$(A,B) = (vs1 * S * vsr^H, vs1 * T * vsr^H)$$

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix S and the upper triangular matrix T ; computes a reciprocal condition number for the average of the selected eigenvalues ($rconde$); and computes a reciprocal condition number for the right and left deflating subspaces corresponding to the selected eigenvalues ($rcondv$). The leading columns of $vs1$ and vsr then form an orthonormal/unitary basis for the corresponding left and right eigenspaces (deflating subspaces).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio $alpha / beta = w$, such that $A - w*B$ is singular. It is usually represented as the pair $(alpha, beta)$, as there is a reasonable interpretation for $beta=0$ or for both being zero. A pair of matrices (S,T) is in generalized real Schur form if T is upper triangular with non-negative diagonal and S is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of S will be "standardized" by making the corresponding elements of T have the form:

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

and the pair of corresponding 2-by-2 blocks in S and T will have a complex conjugate pair of generalized eigenvalues. A pair of matrices (S,T) is in generalized complex Schur form if S and T are upper triangular and, in addition, the diagonal of T are non-negative real numbers.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobvsl</i> | Must be 'N' or 'V'. If <i>jobvsl</i> = 'N', then the left Schur vectors are not computed. If <i>jobvsl</i> = 'V', then the left Schur vectors are computed. |
| <i>jobvsr</i> | Must be 'N' or 'V'. If <i>jobvsr</i> = 'N', then the right Schur vectors are not computed. If <i>jobvsr</i> = 'V', then the right Schur vectors are computed. |

| | |
|---------------------|---|
| <i>sort</i> | <p>Must be 'N' or 'S'. Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form.</p> <p>If <i>sort</i> = 'N', then eigenvalues are not ordered.</p> <p>If <i>sort</i> = 'S', eigenvalues are ordered (see <i>select</i>).</p> |
| <i>select</i> | <p>The <i>select</i> parameter is a pointer to a function returning a value of <code>lapack_logical</code> type. For different flavors the function has different arguments:</p> <pre>LAPACKE_sggesx: lapack_logical (*LAPACK_S_SELECT3) (const float*, const float*, const float*);</pre> <pre>LAPACKE_dggesx: lapack_logical (*LAPACK_D_SELECT3) (const double*, const double*, const double*);</pre> <pre>LAPACKE_cggesx: lapack_logical (*LAPACK_C_SELECT2) (const lapack_complex_float*, const lapack_complex_float*);</pre> <pre>LAPACKE_zggesx: lapack_logical (*LAPACK_Z_SELECT2) (const lapack_complex_double*, const lapack_complex_double*);</pre> <p>If <i>sort</i> = 'S', <i>select</i> is used to select eigenvalues to sort to the top left of the Schur form.</p> <p>If <i>sort</i> = 'N', <i>select</i> is not referenced.</p> <p><i>For real flavors:</i></p> <p>An eigenvalue $(\text{alphar}[j] + \text{alphai}[j])/beta[j]$ is selected if <i>select</i>(<i>alphar</i>[<i>j</i>], <i>alphai</i>[<i>j</i>], <i>beta</i>[<i>j</i>]) is true; that is, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected.</p> <p>Note that in the ill-conditioned case, a selected complex eigenvalue may no longer satisfy <i>select</i>(<i>alphar</i>[<i>j</i>], <i>alphai</i>[<i>j</i>], <i>beta</i>[<i>j</i>]) = 1 after ordering. In this case <i>info</i> is set to <i>n</i>+2.</p> <p><i>For complex flavors:</i></p> <p>An eigenvalue $\text{alpha}[j] / \text{beta}[j]$ is selected if <i>select</i>(<i>alpha</i>[<i>j</i>], <i>beta</i>[<i>j</i>]) is true.</p> <p>Note that a selected complex eigenvalue may no longer satisfy <i>select</i>(<i>alpha</i>[<i>j</i>], <i>beta</i>[<i>j</i>]) = 1 after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned); in this case <i>info</i> is set to <i>n</i>+2 (see <i>info</i> below).</p> |
| <i>sense</i> | <p>Must be 'N', 'E', 'V', or 'B'. Determines which reciprocal condition number are computed.</p> <p>If <i>sense</i> = 'N', none are computed;</p> <p>If <i>sense</i> = 'E', computed for average of selected eigenvalues only;</p> <p>If <i>sense</i> = 'V', computed for selected deflating subspaces only;</p> <p>If <i>sense</i> = 'B', computed for both.</p> <p>If <i>sense</i> is 'E', 'V', or 'B', then <i>sort</i> must equal 'S'.</p> |
| <i>n</i> | The order of the matrices <i>A</i> , <i>B</i> , <i>vsl</i> , and <i>vsr</i> ($n \geq 0$). |
| <i>a</i> , <i>b</i> | Arrays: |

a (size at least $\max(1, lda*n)$) is an array containing the n -by- n matrix A (first of the pair of matrices).

b (size at least $\max(1, ldb*n)$) is an array containing the n -by- n matrix B (second of the pair of matrices).

lda The leading dimension of the array a .
Must be at least $\max(1, n)$.

ldb The leading dimension of the array b .
Must be at least $\max(1, n)$.

$ldvsl, ldvsr$ The leading dimensions of the output matrices vs_l and vs_r , respectively.
Constraints:

$ldvsl \geq 1$. If $jobvsl = 'V'$, $ldvsl \geq \max(1, n)$.

$ldvsr \geq 1$. If $jobvsr = 'V'$, $ldvsr \geq \max(1, n)$.

Output Parameters

a On exit, this array has been overwritten by its generalized Schur form S .

b On exit, this array has been overwritten by its generalized Schur form T .

$sdim$ If $sort = 'N'$, $sdim = 0$.
If $sort = 'S'$, $sdim$ is equal to the number of eigenvalues (after sorting) for which $select$ is true.

Note that for real flavors complex conjugate pairs for which $select$ is true for either eigenvalue count as 2.

$alphan, alphan_i$ Arrays, size at least $\max(1, n)$ each. Contain values that form generalized eigenvalues in real flavors.

See $beta$.

$alpha$ Array, size at least $\max(1, n)$. Contain values that form generalized eigenvalues in complex flavors. See $beta$.

$beta$ Array, size at least $\max(1, n)$.

For real flavors:

On exit, $(alphan[j] + alphan_i[j]*i)/beta[j]$, $j=0, \dots, n-1$ will be the generalized eigenvalues.

$alphan[j] + alphan_i[j]*i$ and $beta[j]$, $j=0, \dots, n-1$ are the diagonals of the complex Schur form (S, T) that would result if the 2-by-2 diagonal blocks of the real generalized Schur form of (A, B) were further reduced to triangular form using complex unitary transformations. If $alphan_i[j]$ is zero, then the j -th eigenvalue is real; if positive, then the j -th and $(j+1)$ -st eigenvalues are a complex conjugate pair, with $alphan_i[j+1]$ negative.

For complex flavors:

On exit, $alpha[j]/beta[j]$, $j=0, \dots, n-1$ will be the generalized eigenvalues. $alpha[j]$ and $beta[j]$, $j=0, \dots, n-1$ are the diagonals of the complex Schur form (S, T) output by `cggexs/zggexs`. The $beta[j]$ will be non-negative real.

See also *Application Notes* below.

vsl, *vsr*

Arrays:

vsl (size at least $\max(1, ldvsl*n)$).

If *jobvsl* = 'V', this array will contain the left Schur vectors.

If *jobvsl* = 'N', *vsl* is not referenced.

vsr (size at least $\max(1, ldvsr*n)$).

If *jobvsr* = 'V', this array will contain the right Schur vectors.

If *jobvsr* = 'N', *vsr* is not referenced.

rconde, *rcondv*

Arrays, size 2 each

If *sense* = 'E' or 'B', *rconde*(1) and *rconde*(2) contain the reciprocal condition numbers for the average of the selected eigenvalues.

Not referenced if *sense* = 'N' or 'V'.

If *sense* = 'V' or 'B', *rcondv*[0] and *rcondv*[1] contain the reciprocal condition numbers for the selected deflating subspaces.

Not referenced if *sense* = 'N' or 'E'.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, and

$i \leq n$:

the QZ iteration failed. (*A*, *B*) is not in Schur form, but *alphar*[*j*], *alphai*[*j*] (for real flavors), or *alpha*[*j*] (for complex flavors), and *beta*[*j*], $j = info, \dots, n - 1$ should be correct.

$i > n$: errors that usually indicate LAPACK problems:

$i = n+1$: other than QZ iteration failed in [hgeqz](#);

$i = n+2$: after reordering, roundoff changed values of some complex eigenvalues so that leading eigenvalues in the generalized Schur form no longer satisfy *select* = 1. This could also be caused due to scaling;

$i = n+3$: reordering failed in [tgsen](#).

Application Notes

The quotients *alphar*[*j*]/*beta*[*j*] and *alphai*[*j*]/*beta*[*j*] may easily over- or underflow, and *beta*[*j*] may even be zero. Thus, you should avoid simply computing the ratio. However, *alphar* and *alphai* will be always less than and usually comparable with $\text{norm}(A)$ in magnitude, and *beta* always less than and usually comparable with $\text{norm}(B)$.

?gges3

Computes generalized Schur factorization for a pair of matrices.

Syntax

```
lapack_int LAPACKE_sgges3 (int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_S_SELECT3 selctg, lapack_int n, float * a, lapack_int lda, float * b,
lapack_int ldb, lapack_int * sdim, float * alphas, float * alphai, float * beta, float
* vsl, lapack_int ldvsl, float * vsr, lapack_int ldvsr);
```

```
lapack_int LAPACKE_dgges3 (int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_D_SELECT3 selctg, lapack_int n, double * a, lapack_int lda, double * b,
lapack_int ldb, lapack_int * sdim, double * alphas, double * alphai, double * beta,
double * vsl, lapack_int ldvsl, double * vsr, lapack_int ldvsr);
```

```
lapack_int LAPACKE_cgges3 (int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_C_SELECT2 selctg, lapack_int n, lapack_complex_float * a, lapack_int lda,
lapack_complex_float * b, lapack_int ldb, lapack_int * sdim, lapack_complex_float *
alpha, lapack_complex_float * beta, lapack_complex_float * vsl, lapack_int ldvsl,
lapack_complex_float * vsr, lapack_int ldvsr);
```

```
lapack_int LAPACKE_zgges3 (int matrix_layout, char jobvsl, char jobvsr, char sort,
LAPACK_Z_SELECT2 selctg, lapack_int n, lapack_complex_double * a, lapack_int lda,
lapack_complex_double * b, lapack_int ldb, lapack_int * sdim, lapack_complex_double *
alpha, lapack_complex_double * beta, lapack_complex_double * vsl, lapack_int ldvsl,
lapack_complex_double * vsr, lapack_int ldvsr);
```

Include Files

- mkl.h

Description

For a pair of n -by- n real or complex nonsymmetric matrices (A,B) , ?gges3 computes the generalized eigenvalues, the generalized real or complex Schur form (S,T) , and optionally the left or right matrices of Schur vectors (VSL and VSR). This gives the generalized Schur factorization

$$(A,B) = ((VSL)*S*(VSR)^T, (VSL)*T*(VSR)^T) \text{ for real } (A,B)$$

or

$$(A,B) = ((VSL)*S*(VSR)^H, (VSL)*T*(VSR)^H) \text{ for complex } (A,B)$$

where $(VSR)^H$ is the conjugate-transpose of VSR .

Optionally, it also orders the eigenvalues so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix S and the upper triangular matrix T . The leading columns of VSL and VSR then form an orthonormal basis for the corresponding left and right eigenspaces (deflating subspaces).

NOTE

If only the generalized eigenvalues are needed, use the driver ?ggeev instead, which is faster.

A generalized eigenvalue for a pair of matrices (A,B) is a scalar w or a ratio $alpha/beta = w$, such that $A - w*B$ is singular. It is usually represented as the pair $(alpha,beta)$, as there is a reasonable interpretation for $beta=0$ or both being zero.

For real flavors:

A pair of matrices (S,T) is in generalized real Schur form if T is upper triangular with non-negative diagonal and S is block upper triangular with 1-by-1 and 2-by-2 blocks. 1-by-1 blocks correspond to real generalized eigenvalues, while 2-by-2 blocks of S will be "standardized" by making the corresponding elements of T have the form:

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

and the pair of corresponding 2-by-2 blocks in S and T have a complex conjugate pair of generalized eigenvalues.

For complex flavors:

A pair of matrices (S, T) is in generalized complex Schur form if S and T are upper triangular and, in addition, the diagonal elements of T are non-negative real numbers.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobvsl</i> | = 'N': do not compute the left Schur vectors; |
| <i>jobvsr</i> | = 'N': do not compute the right Schur vectors; = 'V': compute the right Schur vectors. |
| <i>sort</i> | Specifies whether or not to order the eigenvalues on the diagonal of the generalized Schur form. = 'N': Eigenvalues are not ordered; = 'S': Eigenvalues are ordered (see <i>selctg</i>). |
| <i>selctg</i> | <i>selctg</i> is a function of three arguments for real flavors or two arguments for complex flavors. <i>selctg</i> must be declared EXTERNAL in the calling subroutine. If <i>sort</i> = 'N', <i>selctg</i> is not referenced. If <i>sort</i> = 'S', <i>selctg</i> is used to select eigenvalues to sort to the top left of the Schur form. For real flavors: An eigenvalue $(\text{alphan}[j - 1] + \text{alphai}[j - 1])/\text{betan}[j - 1]$ is selected if $\text{selctg}(\text{alphan}[j - 1], \text{alphai}[j - 1], \text{betan}[j - 1])$ is true. In other words, if either one of a complex conjugate pair of eigenvalues is selected, then both complex eigenvalues are selected. Note that in the ill-conditioned case, a selected complex eigenvalue may no longer satisfy $\text{selctg}(\text{alphan}[j - 1], \text{alphai}[j - 1], \text{betan}[j - 1]) \neq 0$ after ordering. <i>info</i> is to be set to $n+2$ in this case. For complex flavors: An eigenvalue $\text{alpha}[j - 1]/\text{betan}[j - 1]$ is selected if $\text{selctg}(\text{alpha}[j - 1], \text{betan}[j - 1])$ is true. Note that a selected complex eigenvalue may no longer satisfy $\text{selctg}(\text{alpha}[j - 1], \text{betan}[j - 1]) \neq 0$ after ordering, since ordering may change the value of complex eigenvalues (especially if the eigenvalue is ill-conditioned), in this case <i>info</i> returns $n + 2$. |
| <i>n</i> | The order of the matrices A , B , VSL , and VSR . $n \geq 0$. |
| <i>a</i> | Array, size $(lda * n)$. On entry, the first of the pair of matrices. |
| <i>lda</i> | The leading dimension of a . $lda \geq \max(1, n)$. |
| <i>b</i> | Array, size $(ldb * n)$. On entry, the second of the pair of matrices. |

| | |
|--------------|--|
| <i>ldb</i> | The leading dimension of <i>b</i> . $ldb \geq \max(1, n)$. |
| <i>ldvsl</i> | The leading dimension of the matrix <i>VSL</i> . $ldvsl \geq 1$, and if <i>jobvsl</i> = 'V', $ldvsl \geq n$. |
| <i>ldvsr</i> | The leading dimension of the matrix <i>VSR</i> . $ldvsr \geq 1$, and if <i>jobvsr</i> = 'V', $ldvsr \geq n$. |

Output Parameters

| | |
|----------------|---|
| <i>a</i> | On exit, <i>a</i> is overwritten by its generalized Schur form <i>S</i> . |
| <i>b</i> | On exit, <i>b</i> is overwritten by its generalized Schur form <i>T</i> . |
| <i>sdim</i> | If <i>sort</i> = 'N', <i>sdim</i> = 0. If <i>sort</i> = 'S', <i>sdim</i> = number of eigenvalues (after sorting) for which <i>selctg</i> is true. |
| <i>alpha</i> | Array, size (<i>n</i>). |
| <i>alphar</i> | Array, size (<i>n</i>). |
| <i>alpha_i</i> | Array, size (<i>n</i>). |
| <i>beta</i> | Array, size (<i>n</i>). |

For real flavors:

On exit, $(\text{alphar}[j - 1] + \text{alpha}_i[j - 1]*i)/\text{beta}[j - 1]$, $j=1, \dots, n$, are the generalized eigenvalues. $\text{alphar}[j - 1] + \text{alpha}_i[j - 1]*i$, and $\text{beta}[j - 1]$, $j=1, \dots, n$ are the diagonals of the complex Schur form (*S*, *T*) that would result if the 2-by-2 diagonal blocks of the real Schur form of (*a*, *b*) were further reduced to triangular form using 2-by-2 complex unitary transformations. If $\text{alpha}_i[j - 1]$ is zero, then the *j*-th eigenvalue is real; if positive, then the *j*-th and (*j*+1)-st eigenvalues are a complex conjugate pair, with $\text{alpha}_i[j]$ negative.

Note: the quotients $\text{alphar}[j - 1]/\text{beta}[j - 1]$ and $\text{alpha}_i[j - 1]/\text{beta}[j - 1]$ can easily over- or underflow, and $\text{beta}[j - 1]$ might even be zero. Thus, you should avoid computing the ratio alpha/beta by simply dividing *alpha* by *beta*. However, *alphar* and *alpha_i* is always less than and usually comparable with $\text{norm}(a)$ in magnitude, and *beta* is always less than and usually comparable with $\text{norm}(b)$.

For complex flavors:

On exit, $\text{alpha}[j - 1][j - 1]/\text{beta}[j - 1]$, $j=1, \dots, n$, are the generalized eigenvalues. $\text{alpha}[j - 1]$, $j=1, \dots, n$ and $\text{beta}[j - 1]$, $j=1, \dots, n$ are the diagonals of the complex Schur form (*a*, *b*) output by *gges3*. The $\text{beta}[j - 1]$ is non-negative real.

Note: the quotient $\text{alpha}[j - 1][j - 1]/\text{beta}[j - 1]$ can easily over- or underflow, and $\text{beta}[j - 1]$ might even be zero. Thus, you should avoid computing the ratio alpha/beta by simply dividing *alpha* by *beta*. However, *alpha* is always less than and usually comparable with $\text{norm}(a)$ in magnitude, and *beta* is always less than and usually comparable with $\text{norm}(b)$.

Include Files

- `mk1.h`

Description

The `?ggeev` routine computes the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors for a pair of n -by- n real/complex nonsymmetric matrices (A,B) .

A generalized eigenvalue for a pair of matrices (A,B) is a scalar λ or a ratio $alpha / beta = \lambda$, such that $A - \lambda*B$ is singular. It is usually represented as the pair $(alpha, beta)$, as there is a reasonable interpretation for $beta = 0$ and even for both being zero.

The right generalized eigenvector $v(j)$ corresponding to the generalized eigenvalue $\lambda(j)$ of (A,B) satisfies $A*v(j) = \lambda(j)*B*v(j)$.

The left generalized eigenvector $u(j)$ corresponding to the generalized eigenvalue $\lambda(j)$ of (A,B) satisfies $u(j)^H*A = \lambda(j)*u(j)^H*B$

where $u(j)^H$ denotes the conjugate transpose of $u(j)$.

The `?ggeev` routine replaces the deprecated `?gegev` routine.

Input Parameters

| | |
|----------------------------|--|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <code>jobvl</code> | Must be 'N' or 'V'. If <code>jobvl = 'N'</code> , the left generalized eigenvectors are not computed; If <code>jobvl = 'V'</code> , the left generalized eigenvectors are computed. |
| <code>jobvr</code> | Must be 'N' or 'V'. If <code>jobvr = 'N'</code> , the right generalized eigenvectors are not computed; If <code>jobvr = 'V'</code> , the right generalized eigenvectors are computed. |
| <code>n</code> | The order of the matrices A , B , vl , and vr ($n \geq 0$). |
| <code>a, b</code> | Arrays: a (size at least $\max(1, lda*n)$) is an array containing the n -by- n matrix A (first of the pair of matrices). b (size at least $\max(1, ldb*n)$) is an array containing the n -by- n matrix B (second of the pair of matrices). |
| <code>lda</code> | The leading dimension of the array a . Must be at least $\max(1, n)$. |
| <code>ldb</code> | The leading dimension of the array b . Must be at least $\max(1, n)$. |
| <code>ldvl, ldvr</code> | The leading dimensions of the output matrices vl and vr , respectively. Constraints: $ldvl \geq 1$. If <code>jobvl = 'V'</code> , $ldvl \geq \max(1, n)$. $ldvr \geq 1$. If <code>jobvr = 'V'</code> , $ldvr \geq \max(1, n)$. |

Output Parameters

| | |
|-----------------------|---|
| <i>a, b</i> | On exit, these arrays have been overwritten. |
| <i>alphar, alphai</i> | Arrays, size at least $\max(1, n)$ each. Contain values that form generalized eigenvalues in real flavors. See <i>beta</i> . |
| <i>alpha</i> | Array, size at least $\max(1, n)$. Contain values that form generalized eigenvalues in complex flavors. See <i>beta</i> . |
| <i>beta</i> | Array, size at least $\max(1, n)$. <i>For real flavors:</i> On exit, $(\text{alphar}[j] + \text{alphai}[j]*i)/\text{beta}[j]$, $j=0, \dots, n - 1$, are the generalized eigenvalues. If <i>alphai</i> [<i>j</i>] is zero, then the <i>j</i> -th eigenvalue is real; if positive, then the <i>j</i> -th and (<i>j</i> +1)-st eigenvalues are a complex conjugate pair, with <i>alphai</i> [<i>j</i> +1] negative. <i>For complex flavors:</i> On exit, $\text{alpha}[j]/\text{beta}[j]$, $j=0, \dots, n - 1$, are the generalized eigenvalues. See also <i>Application Notes</i> below. |
| <i>vl, vr</i> | Arrays: <i>vl</i> (size at least $\max(1, \text{ldvl}*n)$). Contains the matrix of left generalized eigenvectors <i>VL</i> . If <i>jobvl</i> = 'V', the left generalized eigenvectors u_j are stored one after another in the columns of <i>VL</i> , in the same order as their eigenvalues. Each eigenvector is scaled so the largest component has $\text{abs}(\text{Re}) + \text{abs}(\text{Im}) = 1$. If <i>jobvl</i> = 'N', <i>vl</i> is not referenced. <i>For real flavors:</i> If the <i>j</i> -th eigenvalue is real, then the <i>k</i> -th component of the <i>j</i> -th left eigenvector u_j is stored in $\text{vl}[(k - 1) + (j - 1)*\text{ldvl}]$ for column major layout and in $\text{vl}[(k - 1)*\text{ldvl} + (j - 1)]$ for row major layout.. If the <i>j</i> -th and (<i>j</i> +1)-st eigenvalues form a complex conjugate pair, then for $i = \text{sqrt}(-1)$, the <i>k</i> -th components of the <i>j</i> -th left eigenvector u_j are $\text{vl}[(k - 1) + (j - 1)*\text{ldvl}] + i*\text{vl}[(k - 1) + j*\text{ldvl}]$ for column major layout and $\text{vl}[(k - 1)*\text{ldvl} + (j - 1)] + i*\text{vl}[(k - 1)*\text{ldvl} + j]$ for row major layout. Similarly, the <i>k</i> -th components of left eigenvector <i>j</i> +1 u_{j+1} are $\text{vl}[(k - 1) + (j - 1)*\text{ldvl}] - i*\text{vl}[(k - 1) + j*\text{ldvl}]$ for column major layout and $\text{vl}[(k - 1)*\text{ldvl} + (j - 1)] - i*\text{vl}[(k - 1)*\text{ldvl} + j]$ for row major layout.. <i>For complex flavors:</i> The <i>k</i> -th component of the <i>j</i> -th left eigenvector u_j is stored in $\text{vl}[(k - 1) + (j - 1)*\text{ldvl}]$ for column major layout and in $\text{vl}[(k - 1)*\text{ldvl} + (j - 1)]$ for row major layout. <i>vr</i> (size at least $\max(1, \text{ldvr}*n)$). Contains the matrix of right generalized eigenvectors <i>VR</i> . |

If $jobvr = 'V'$, the right generalized eigenvectors v_j are stored one after another in the columns of VR , in the same order as their eigenvalues. Each eigenvector is scaled so the largest component has $\text{abs}(\text{Re}) + \text{abs}(\text{Im}) = 1$.

If $jobvr = 'N'$, vr is not referenced.

For real flavors:

If the j -th eigenvalue is real, then The k -th component of the j -th right eigenvector v_j is stored in $vr[(k - 1) + (j - 1)*ldvr]$ for column major layout and in $vr[(k - 1)*ldvr + (j - 1)]$ for row major layout..

If the j -th and $(j+1)$ -st eigenvalues form a complex conjugate pair, then the k -th components of the j -th right eigenvector v_j can be computed as $vr[(k - 1) + (j - 1)*ldvr] + i*vr[(k - 1) + j*ldvr]$ for column major layout and $vr[(k - 1)*ldvr + (j - 1)] + i*vr[(k - 1)*ldvr + j]$ for row major layout. Similarly, the k -th components of the right eigenvector $j+1$ $v_{\{j+1\}}$ can be computed as $vr[(k - 1) + (j - 1)*ldvr] - i*vr[(k - 1) + j*ldvr]$ for column major layout and $vr[(k - 1)*ldvr + (j - 1)] - i*vr[(k - 1)*ldvr + j]$ for row major layout..

For complex flavors:

The k -th component of the j -th right eigenvector v_j is stored in $vr[(k - 1) + (j - 1)*ldvr]$ for column major layout and in $vr[(k - 1)*ldvr + (j - 1)]$ for row major layout.

Return Values

This function returns a value $info$.

If $info=0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = i$, and

$i \leq n$: the QZ iteration failed. No eigenvectors have been calculated, but $alphan[j]$, $alphai[j]$ (for real flavors), or $alpha[j]$ (for complex flavors), and $beta[j]$, $j=info, \dots, n - 1$ should be correct.

$i > n$: errors that usually indicate LAPACK problems:

$i = n+1$: other than QZ iteration failed in [hgeqz](#);

$i = n+2$: error return from [tgevc](#).

Application Notes

The quotients $alphan[j]/beta[j]$ and $alphai[j]/beta[j]$ may easily over- or underflow, and $beta[j]$ may even be zero. Thus, you should avoid simply computing the ratio. However, $alphan$ and $alphai$ (for real flavors) or $alpha$ (for complex flavors) will be always less than and usually comparable with $\text{norm}(A)$ in magnitude, and $beta$ always less than and usually comparable with $\text{norm}(B)$.

?ggev

Computes the generalized eigenvalues, and, optionally, the left and/or right generalized eigenvectors.

Syntax

```
lapack_int LAPACKE_sggev( int matrix_layout, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, float* a, lapack_int lda, float* b, lapack_int ldb, float*
alphar, float* alphas, float* beta, float* vl, lapack_int ldvl, float* vr, lapack_int
ldvr, lapack_int* ilo, lapack_int* ihi, float* lscale, float* rscale, float* abnrm,
float* bbnrm, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_dggev( int matrix_layout, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, double* a, lapack_int lda, double* b, lapack_int ldb, double*
alphar, double* alphas, double* beta, double* vl, lapack_int ldvl, double* vr,
lapack_int ldvr, lapack_int* ilo, lapack_int* ihi, double* lscale, double* rscale,
double* abnrm, double* bbnrm, double* rconde, double* rcondv );
```

```
lapack_int LAPACKE_cggev( int matrix_layout, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, lapack_complex_float* a, lapack_int lda, lapack_complex_float* b,
lapack_int ldb, lapack_complex_float* alpha, lapack_complex_float* beta,
lapack_complex_float* vl, lapack_int ldvl, lapack_complex_float* vr, lapack_int ldvr,
lapack_int* ilo, lapack_int* ihi, float* lscale, float* rscale, float* abnrm, float*
bbnrm, float* rconde, float* rcondv );
```

```
lapack_int LAPACKE_zggev( int matrix_layout, char balanc, char jobvl, char jobvr, char
sense, lapack_int n, lapack_complex_double* a, lapack_int lda, lapack_complex_double*
b, lapack_int ldb, lapack_complex_double* alpha, lapack_complex_double* beta,
lapack_complex_double* vl, lapack_int ldvl, lapack_complex_double* vr, lapack_int ldvr,
lapack_int* ilo, lapack_int* ihi, double* lscale, double* rscale, double* abnrm,
double* bbnrm, double* rconde, double* rcondv );
```

Include Files

- mkl.h

Description

The routine computes for a pair of n -by- n real/complex nonsymmetric matrices (A,B) , the generalized eigenvalues, and optionally, the left and/or right generalized eigenvectors.

Optionally also, it computes a balancing transformation to improve the conditioning of the eigenvalues and eigenvectors (ilo , ihi , $lscale$, $rscale$, $abnrm$, and $bbnrm$), reciprocal condition numbers for the eigenvalues ($rconde$), and reciprocal condition numbers for the right eigenvectors ($rcondv$).

A generalized eigenvalue for a pair of matrices (A,B) is a scalar λ or a ratio $alpha / beta = \lambda$, such that $A - \lambda*B$ is singular. It is usually represented as the pair $(alpha, beta)$, as there is a reasonable interpretation for $beta=0$ and even for both being zero. The right generalized eigenvector $v(j)$ corresponding to the generalized eigenvalue $\lambda(j)$ of (A,B) satisfies

$$A*v(j) = \lambda(j)*B*v(j).$$

The left generalized eigenvector $u(j)$ corresponding to the generalized eigenvalue $\lambda(j)$ of (A,B) satisfies

$$u(j)^H*A = \lambda(j)*u(j)^H*B$$

where $u(j)^H$ denotes the conjugate transpose of $u(j)$.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

| | |
|-------------------|--|
| <i>balanc</i> | <p>Must be 'N', 'P', 'S', or 'B'. Specifies the balance option to be performed.</p> <p>If <i>balanc</i> = 'N', do not diagonally scale or permute;</p> <p>If <i>balanc</i> = 'P', permute only;</p> <p>If <i>balanc</i> = 'S', scale only;</p> <p>If <i>balanc</i> = 'B', both permute and scale.</p> <p>Computed reciprocal condition numbers will be for the matrices after balancing and/or permuting. Permuting does not change condition numbers (in exact arithmetic), but balancing does.</p> |
| <i>jobvl</i> | <p>Must be 'N' or 'V'.</p> <p>If <i>jobvl</i> = 'N', the left generalized eigenvectors are not computed;</p> <p>If <i>jobvl</i> = 'V', the left generalized eigenvectors are computed.</p> |
| <i>jobvr</i> | <p>Must be 'N' or 'V'.</p> <p>If <i>jobvr</i> = 'N', the right generalized eigenvectors are not computed;</p> <p>If <i>jobvr</i> = 'V', the right generalized eigenvectors are computed.</p> |
| <i>sense</i> | <p>Must be 'N', 'E', 'V', or 'B'. Determines which reciprocal condition number are computed.</p> <p>If <i>sense</i> = 'N', none are computed;</p> <p>If <i>sense</i> = 'E', computed for eigenvalues only;</p> <p>If <i>sense</i> = 'V', computed for eigenvectors only;</p> <p>If <i>sense</i> = 'B', computed for eigenvalues and eigenvectors.</p> |
| <i>n</i> | The order of the matrices <i>A</i> , <i>B</i> , <i>vl</i> , and <i>vr</i> ($n \geq 0$). |
| <i>a, b</i> | <p>Arrays:</p> <p><i>a</i> (size at least $\max(1, lda*n)$) is an array containing the <i>n</i>-by-<i>n</i> matrix <i>A</i> (first of the pair of matrices).</p> <p><i>b</i> (size at least $\max(1, ldb*n)$) is an array containing the <i>n</i>-by-<i>n</i> matrix <i>B</i> (second of the pair of matrices).</p> |
| <i>lda</i> | <p>The leading dimension of the array <i>a</i>.</p> <p>Must be at least $\max(1, n)$.</p> |
| <i>ldb</i> | <p>The leading dimension of the array <i>b</i>.</p> <p>Must be at least $\max(1, n)$.</p> |
| <i>ldvl, ldvr</i> | <p>The leading dimensions of the output matrices <i>vl</i> and <i>vr</i>, respectively.</p> <p>Constraints:</p> <p><i>ldvl</i> ≥ 1. If <i>jobvl</i> = 'V', <i>ldvl</i> $\geq \max(1, n)$.</p> <p><i>ldvr</i> ≥ 1. If <i>jobvr</i> = 'V', <i>ldvr</i> $\geq \max(1, n)$.</p> |

Output Parameters

| | |
|-----------------------|--|
| <i>a, b</i> | <p>On exit, these arrays have been overwritten.</p> <p>If <i>jobvl</i> = 'V' or <i>jobvr</i> = 'V' or both, then <i>a</i> contains the first part of the real Schur form of the "balanced" versions of the input <i>A</i> and <i>B</i>, and <i>b</i> contains its second part.</p> |
| <i>alphar, alphas</i> | <p>Arrays, size at least $\max(1, n)$ each. Contain values that form generalized eigenvalues in real flavors.</p> <p>See <i>beta</i>.</p> |
| <i>alpha</i> | <p>Array, size at least $\max(1, n)$. Contain values that form generalized eigenvalues in complex flavors. See <i>beta</i>.</p> |
| <i>beta</i> | <p>Array, size at least $\max(1, n)$.</p> <p><i>For real flavors:</i></p> <p>On exit, $(\text{alphar}[j] + \text{alphas}[j]*i)/\text{beta}[j]$, $j=0, \dots, n - 1$, will be the generalized eigenvalues.</p> <p>If <i>alphas</i>[<i>j</i>] is zero, then the <i>j</i>-th eigenvalue is real; if positive, then the <i>j</i>-th and (<i>j</i>+1)-st eigenvalues are a complex conjugate pair, with <i>alphas</i>[<i>j</i>+1] negative.</p> <p><i>For complex flavors:</i></p> <p>On exit, $\text{alpha}[j]/\text{beta}[j]$, $j=0, \dots, n - 1$, will be the generalized eigenvalues.</p> <p>See also <i>Application Notes</i> below.</p> |
| <i>vl, vr</i> | <p>Arrays:</p> <p><i>vl</i> (size at least $\max(1, \text{ldvl}*n)$).</p> <p>If <i>jobvl</i> = 'V', the left generalized eigenvectors $u(j)$ are stored one after another in the columns of <i>vl</i>, in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component have $\text{abs}(\text{Re}) + \text{abs}(\text{Im}) = 1$.</p> <p>If <i>jobvl</i> = 'N', <i>vl</i> is not referenced.</p> <p><i>For real flavors:</i></p> <p>If the <i>j</i>-th eigenvalue is real, then <i>k</i>-th component of <i>j</i>-th left eigenvector u_j is stored in $\text{vl}[(k - 1) + (j - 1)*\text{ldvl}]$ for column major layout and in $\text{vl}[(k - 1)*\text{ldvl} + (j - 1)]$ for row major layout..</p> <p>If the <i>j</i>-th and (<i>j</i>+1)-st eigenvalues form a complex conjugate pair, then for $i = \text{sqrt}(-1)$, the <i>k</i>-th components of the <i>j</i>-th left eigenvector u_j can be computed as $\text{vl}[(k - 1) + (j - 1)*\text{ldvl}] + i*\text{vl}[(k - 1) + j*\text{ldvl}]$ for column major layout and $\text{vl}[(k - 1)*\text{ldvl} + (j - 1)] + i*\text{vl}[(k - 1)*\text{ldvl} + j]$ for row major layout. Similarly, the <i>k</i>-th components of the left eigenvector <i>j</i>+1 u_{j+1} can be computed as $\text{vl}[(k - 1) + (j - 1)*\text{ldvl}] - i*\text{vl}[(k - 1) + j*\text{ldvl}]$ for column major layout and $\text{vl}[(k - 1)*\text{ldvl} + (j - 1)] - i*\text{vl}[(k - 1)*\text{ldvl} + j]$ for row major layout..</p> <p><i>For complex flavors:</i></p> |

The k -th component of the j -th left eigenvector u_j is stored in $vl[(k - 1) + (j - 1)*ldvl]$ for column major layout and in $vl[(k - 1)*ldvl + (j - 1)]$ for row major layout.

vr (size at least $\max(1, ldvr*n)$).

If $jobvr = 'V'$, the right generalized eigenvectors $v(j)$ are stored one after another in the columns of vr , in the same order as their eigenvalues. Each eigenvector will be scaled so the largest component have $\text{abs}(\text{Re}) + \text{abs}(\text{Im}) = 1$.

If $jobvr = 'N'$, vr is not referenced.

For real flavors:

If the j -th eigenvalue is real, then the k -th component of the j -th right eigenvector v_j is stored in $vr[(k - 1) + (j - 1)*ldvr]$ for column major layout and in $vr[(k - 1)*ldvr + (j - 1)]$ for row major layout..

If the j -th and $(j+1)$ -st eigenvalues form a complex conjugate pair, then The k -th components of the j -th right eigenvector v_j can be computed as $vr[(k - 1) + (j - 1)*ldvr] + i*vr[(k - 1) + j*ldvr]$ for column major layout and $vr[(k - 1)*ldvr + (j - 1)] + i*vr[(k - 1)*ldvr + j]$ for row major layout. Respectively, the k -th components of right eigenvector $j+1$ v_{j+1} can be computed as $vr[(k - 1) + (j - 1)*ldvr] - i*vr[(k - 1) + j*ldvr]$ for column major layout and $vr[(k - 1)*ldvr + (j - 1)] - i*vr[(k - 1)*ldvr + j]$ for row major layout..

For complex flavors:

The k -th component of the j -th right eigenvector v_j is stored in $vr[(k - 1) + (j - 1)*ldvr]$ for column major layout and in $vr[(k - 1)*ldvr + (j - 1)]$ for row major layout.

ilo, ihi

ilo and *ihi* are integer values such that on exit $A_{i j} = 0$ and $B_{i j} = 0$ if $i > j$ and $j = 1, \dots, ilo-1$ or $i = ihi+1, \dots, n$.

If *balanc* = 'N' or 'S', *ilo* = 1 and *ihi* = *n*.

lscale, rscale

Arrays, size at least $\max(1, n)$ each.

lscale contains details of the permutations and scaling factors applied to the left side of *A* and *B*.

If $PL(j)$ is the index of the row interchanged with row j , and $DL(j)$ is the scaling factor applied to row j , then

$lscale[j - 1] = PL(j)$, for $j = 1, \dots, ilo-1$

$= DL(j)$, for $j = ilo, \dots, ihi$

$= PL(j)$ for $j = ihi+1, \dots, n$.

The order in which the interchanges are made is n to $ihl+1$, then 1 to $ilo-1$.

rscale contains details of the permutations and scaling factors applied to the right side of *A* and *B*.

If $PR(j)$ is the index of the column interchanged with column j , and $DR(j)$ is the scaling factor applied to column j , then

$rscale[j - 1] = PR(j)$, for $j = 1, \dots, ilo-1$

$= DR(j)$, for $j = ilo, \dots, ihi$

= $PR(j)$ for $j = ihi+1, \dots, n$.

The order in which the interchanges are made is n to $ihi+1$, then 1 to $ilo-1$.

abnrm, bbnrm

The one-norms of the balanced matrices A and B , respectively.

rconde, rcondv

Arrays, size at least $\max(1, n)$ each.

If *sense* = 'E', or 'B', *rconde* contains the reciprocal condition numbers of the eigenvalues, stored in consecutive elements of the array. For a complex conjugate pair of eigenvalues two consecutive elements of *rconde* are set to the same value. Thus *rconde*[*j*], *rcondv*[*j*], and the *j*-th columns of *v*l and *v*r all correspond to the same eigenpair (but not in general the *j*-th eigenpair, unless all eigenpairs are selected).

If *sense* = 'N', or 'V', *rconde* is not referenced.

If *sense* = 'V', or 'B', *rcondv* contains the estimated reciprocal condition numbers of the eigenvectors, stored in consecutive elements of the array. For a complex eigenvector two consecutive elements of *rcondv* are set to the same value.

If the eigenvalues cannot be reordered to compute , *rcondv*[*j*] is set to 0; this can only occur when the true value would be very small anyway.

If *sense* = 'N', or 'E', *rcondv* is not referenced.

Return Values

This function returns a value *info*.

If *info*=0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = *i*, and

$i \leq n$: the QZ iteration failed. No eigenvectors have been calculated, but *alphar*[*j*], *alphai*[*j*] (for real flavors), or *alpha*[*j*] (for complex flavors), and *beta*[*j*], $j=info, \dots, n - 1$ should be correct.

$i > n$: errors that usually indicate LAPACK problems:

$i = n+1$: other than QZ iteration failed in [hgeqz](#);

$i = n+2$: error return from [tgevc](#).

Application Notes

The quotients *alphar*[*j*]/*beta*[*j*] and *alphai*[*j*]/*beta*[*j*] may easily over- or underflow, and *beta*[*j*] may even be zero. Thus, you should avoid simply computing the ratio. However, *alphar* and *alphai* (for real flavors) or *alpha* (for complex flavors) will be always less than and usually comparable with $\text{norm}(A)$ in magnitude, and *beta* always less than and usually comparable with $\text{norm}(B)$.

?ggeev3

Computes the generalized eigenvalues and the left and right generalized eigenvectors for a pair of matrices.

Syntax

```
lapack_int LAPACKE_sggev3 (int matrix_layout, char jobvl, char jobvr, lapack_int n,
float * a, lapack_int lda, float * b, lapack_int ldb, float * alphar, float * alphai,
float * beta, float * vl, lapack_int ldvl, float * vr, lapack_int ldvr);
```

```

lapack_int LAPACKE_dggeev3 (int matrix_layout, char jobvl, char jobvr, lapack_int n,
double * a, lapack_int lda, double * b, lapack_int ldb, double * alphas, double *
alphai, double * beta, double * vl, lapack_int ldvl, double * vr, lapack_int ldvr);

lapack_int LAPACKE_cggeev3 (int matrix_layout, char jobvl, char jobvr, lapack_int n,
lapack_complex_float * a, lapack_int lda, lapack_complex_float * b, lapack_int ldb,
lapack_complex_float * alpha, lapack_complex_float * beta, lapack_complex_float * vl,
lapack_int ldvl, lapack_complex_float * vr, lapack_int ldvr);

lapack_int LAPACKE_zggeev3 (int matrix_layout, char jobvl, char jobvr, lapack_int n,
lapack_complex_double * a, lapack_int lda, lapack_complex_double * b, lapack_int ldb,
lapack_complex_double * alpha, lapack_complex_double * beta, lapack_complex_double *
vl, lapack_int ldvl, lapack_complex_double * vr, lapack_int ldvr);

```

Include Files

- mkl.h

Description

For a pair of n -by- n real or complex nonsymmetric matrices (A, B) , `?ggeev3` computes the generalized eigenvalues, and optionally, the left and right generalized eigenvectors.

A generalized eigenvalue for a pair of matrices (A, B) is a scalar λ or a ratio $alpha/beta = \lambda$, such that $A - \lambda*B$ is singular. It is usually represented as the pair $(alpha, beta)$, as there is a reasonable interpretation for $beta=0$, and even for both being zero.

For real flavors:

The right eigenvector v_j corresponding to the eigenvalue λ_j of (A, B) satisfies

$$A * v_j = \lambda_j * B * v_j.$$

The left eigenvector u_j corresponding to the eigenvalue λ_j of (A, B) satisfies

$$u_j^H * A = \lambda_j * u_j^H * B$$

where u_j^H is the conjugate-transpose of u_j .

For complex flavors:

The right generalized eigenvector v_j corresponding to the generalized eigenvalue λ_j of (A, B) satisfies

$$A * v_j = \lambda_j * B * v_j.$$

The left generalized eigenvector u_j corresponding to the generalized eigenvalues λ_j of (A, B) satisfies

$$u_j^H * A = \lambda_j * u_j^H * B$$

where u_j^H is the conjugate-transpose of u_j .

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>jobvl</i> | = 'N': do not compute the left generalized eigenvectors; = 'V': compute the left generalized eigenvectors. |
| <i>jobvr</i> | = 'N': do not compute the right generalized eigenvectors; = 'V': compute the right generalized eigenvectors. |
| <i>n</i> | The order of the matrices A, B, VL , and VR . |

| | |
|-------------|---|
| | $n \geq 0$. |
| <i>a</i> | Array, size ($lda * n$). On entry, the matrix <i>A</i> in the pair (<i>A</i> , <i>B</i>). |
| <i>lda</i> | The leading dimension of <i>a</i> . $lda \geq \max(1, n)$. |
| <i>b</i> | Array, size ($ldb * n$). On entry, the matrix <i>B</i> in the pair (<i>A</i> , <i>B</i>). |
| <i>ldb</i> | The leading dimension of <i>b</i> . $ldb \geq \max(1, n)$. |
| <i>ldvl</i> | The leading dimension of the matrix <i>VL</i> . $ldvl \geq 1$, and if <i>jobvl</i> = 'V', $ldvl \geq n$. |
| <i>ldvr</i> | The leading dimension of the matrix <i>VR</i> . $ldvr \geq 1$, and if <i>jobvr</i> = 'V', $ldvr \geq n$. |

Output Parameters

| | |
|---------------|-----------------------------------|
| <i>a</i> | On exit, <i>a</i> is overwritten. |
| <i>b</i> | On exit, <i>b</i> is overwritten. |
| <i>alphar</i> | Array, size (<i>n</i>). |
| <i>alphai</i> | Array, size (<i>n</i>). |
| <i>alpha</i> | Array, size (<i>n</i>). |
| <i>beta</i> | Array, size (<i>n</i>). |

For real flavors:

On exit, $(\text{alphar}[j] + \text{alphai}[j]*i)/\text{beta}[j]$, $j=0, \dots, n-1$, are the generalized eigenvalues. If $\text{alphai}[j-1]$ is zero, then the *j*-th eigenvalue is real; if positive, then the *j*-th and (*j*+1)-st eigenvalues are a complex conjugate pair, with $\text{alphai}[j]$ negative.

Note: the quotients $\text{alphar}[j-1]/\text{beta}[j-1]$ and $\text{alphai}[j-1]/\text{beta}[j-1]$ can easily over- or underflow, and $\text{beta}(j)$ might even be zero. Thus, you should avoid computing the ratio alpha/beta by simply dividing *alpha* by *beta*. However, *alphar* and *alphai* are always less than and usually comparable with $\text{norm}(A)$ in magnitude, and *beta* is always less than and usually comparable with $\text{norm}(B)$.

For complex flavors:

On exit, $\text{alpha}[j]/\text{beta}[j]$, $j=0, \dots, n-1$, are the generalized eigenvalues.

Note: the quotients $\alpha[j - 1]/\beta[j - 1]$ may easily over- or underflow, and $\beta(j)$ can even be zero. Thus, you should avoid computing the ratio α/β by simply dividing α by β . However, α is always less than and usually comparable with $\text{norm}(A)$ in magnitude, and β is always less than and usually comparable with $\text{norm}(B)$.

vl

Array, size ($ldvl * n$).

For real flavors:

If $jobvl = 'V'$, the left eigenvectors u_j are stored one after another in the columns of *vl*, in the same order as their eigenvalues. If the j -th eigenvalue is real, then $u_j =$ the j -th column of *vl*. If the j -th and $(j+1)$ -st eigenvalues form a complex conjugate pair, then the real part of $u_j =$ the j -th column of *vl* and the imaginary part of $v_j =$ the $(j + 1)$ -st column of *vl*.

Each eigenvector is scaled so the largest component has $\text{abs}(\text{real part}) + \text{abs}(\text{imag. part}) = 1$.

Not referenced if $jobvl = 'N'$.

For complex flavors:

If $jobvl = 'V'$, the left generalized eigenvectors u_j are stored one after another in the columns of *vl*, in the same order as their eigenvalues.

Each eigenvector is scaled so the largest component has $\text{abs}(\text{real part}) + \text{abs}(\text{imag. part}) = 1$.

Not referenced if $jobvl = 'N'$.

vr

Array, size ($ldvr * n$).

For real flavors:

If $jobvr = 'V'$, the right eigenvectors v_j are stored one after another in the columns of *vr*, in the same order as their eigenvalues. If the j -th eigenvalue is real, then $v_j =$ the j -th column of *vr*. If the j -th and $(j + 1)$ -st eigenvalues form a complex conjugate pair, then the real part of $v_j =$ the j -th column of *vr* and the imaginary part of $v_j =$ the $(j + 1)$ -st column of *vr*.

Each eigenvector is scaled so the largest component has $\text{abs}(\text{real part}) + \text{abs}(\text{imag. part}) = 1$.

Not referenced if $jobvr = 'N'$.

For complex flavors:

If $jobvr = 'V'$, the right generalized eigenvectors v_j are stored one after another in the columns of *vr*, in the same order as their eigenvalues. Each eigenvector is scaled so the largest component has $\text{abs}(\text{real part}) + \text{abs}(\text{imag. part}) = 1$.

Not referenced if $jobvr = 'N'$.

Return Values

This function returns a value *info*.

= 0: successful exit

< 0: if $info = -i$, the i -th argument had an illegal value.

= 1, ..., n:

for real flavors:

The QZ iteration failed. No eigenvectors have been calculated, but $\alpha[j]$, $\beta[j]$ and $\gamma[j]$ should be correct for $j = info, \dots, n - 1$.

for complex flavors:

The QZ iteration failed. No eigenvectors have been calculated, but $\alpha[j]$ and $\beta[j]$ should be correct for $j = info, \dots, n - 1$.

$> n$:

$= n + 1$: other than QZ iteration failed in `?hgeqz`,

$= n + 2$: error return from `?tgevc`.

LAPACK Auxiliary Routines

Routine naming conventions, mathematical notation, and matrix storage schemes used for LAPACK auxiliary routines are the same as for the driver and computational routines described in previous chapters.

?lacgv

Conjugates a complex vector.

Syntax

```
lapack_int LAPACKE_clacgv (lapack_int n, lapack_complex_float* x, lapack_int incx);
lapack_int LAPACKE_zlacgv (lapack_int n, lapack_complex_double* x, lapack_int incx);
```

Include Files

- `mkl.h`

Description

The routine conjugates a complex vector x of length n and increment $incx$ (see "[Vector Arguments in BLAS](#)" in Appendix B).

Input Parameters

| | |
|--------|--|
| n | The length of the vector x ($n \geq 0$). |
| x | Array, dimension $(1 + (n-1) * incx)$. Contains the vector of length n to be conjugated. |
| $incx$ | The spacing between successive elements of x . |

Output Parameters

| | |
|-----|---|
| x | On exit, overwritten with <code>conjg(x)</code> . |
|-----|---|

?syconv

Converts a symmetric matrix given by a triangular matrix factorization into two matrices and vice versa.

Syntax

```
lapack_int LAPACKE_ssyconv (int matrix_layout, char uplo, char way, lapack_int n, float
* a, lapack_int lda, const lapack_int * ipiv, float * e);
```

```
lapack_int LAPACKE_dsyconv (int matrix_layout, char uplo, char way, lapack_int n,
double* a, lapack_int lda, const lapack_int * ipiv, double * e);
```

```
lapack_int LAPACKE_csyconv (int matrix_layout, char uplo, char way, lapack_int n,
lapack_complex_float * a, lapack_int lda, const lapack_int * ipiv, lapack_complex_float
* e);
```

```
lapack_int LAPACKE_zsyconv (int matrix_layout, char uplo, char way, lapack_int n,
lapack_complex_double* a, lapack_int lda, const lapack_int * ipiv,
lapack_complex_double * e);
```

Include Files

- mkl.h

Description

The routine converts matrix A , which results from a triangular matrix factorization, into matrices L and D and vice versa. The routine returns non-diagonalized elements of D and applies or reverses permutation done with the triangular matrix factorization.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates whether the details of the factorization are stored as an upper or lower triangular matrix: If <i>uplo</i> = 'U': the upper triangular, $A = U * D * U^T$. If <i>uplo</i> = 'L': the lower triangular, $A = L * D * L^T$. |
| <i>way</i> | Must be 'C' or 'R'. |
| <i>n</i> | The order of matrix A ; $n \geq 0$. |
| <i>a</i> | Array of size $\max(1, lda * n)$. The block diagonal matrix D and the multipliers used to obtain the factor U or L as computed by <code>?sytrf</code> . |
| <i>lda</i> | The leading dimension of a ; $lda \geq \max(1, n)$. |
| <i>ipiv</i> | Array, size at least $\max(1, n)$. Details of the interchanges and the block structure of D , as returned by <code>?sytrf</code> . |

Output Parameters

| | |
|----------|--|
| <i>e</i> | Array of size $\max(1, n)$ containing the superdiagonal/subdiagonal of the symmetric 1-by-1 or 2-by-2 block diagonal matrix D in $L * D * L^T$. |
|----------|--|

Return Values

info If *info* = 0, the execution is successful.
 If *info* < 0, the *i*-th parameter had an illegal value.
 If *info* = -1011, memory allocation error occurred.

See Also

[?sytrf](#)

?syr

Performs the symmetric rank-1 update of a complex symmetric matrix.

Syntax

```
lapack_int LAPACKE_csyr (int matrix_layout, char uplo, lapack_int n,
lapack_complex_float alpha, const lapack_complex_float * x, lapack_int incx,
lapack_complex_float * a, lapack_int lda);
```

```
lapack_int LAPACKE_zsyr (int matrix_layout, char uplo, lapack_int n,
lapack_complex_double alpha, const lapack_complex_double * x, lapack_int incx,
lapack_complex_double * a, lapack_int lda);
```

Include Files

- mkl.h

Description

The routine performs the symmetric rank 1 operation defined as

$$a := \alpha * x * x^H + a,$$

where:

- *alpha* is a complex scalar.
- *x* is an *n*-element complex vector.
- *a* is an *n*-by-*n* complex symmetric matrix.

These routines have their real equivalents in BLAS (see [?syr](#) in Chapter 2).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Specifies whether the upper or lower triangular part of the array <i>a</i> is used: If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the array <i>a</i> is used. If <i>uplo</i> = 'L' or 'l', then the lower triangular part of the array <i>a</i> is used. |
| <i>n</i> | Specifies the order of the matrix <i>a</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | Specifies the scalar <i>alpha</i> . |

| | |
|-------------|---|
| <i>x</i> | Array, size at least $(1 + (n - 1) * \text{abs}(incx))$. Before entry, the incremented array <i>x</i> must contain the <i>n</i> -element vector <i>x</i> . |
| <i>incx</i> | Specifies the increment for the elements of <i>x</i> . The value of <i>incx</i> must not be zero. |
| <i>a</i> | Array, size $\max(1, lda * n)$. Before entry with <i>uplo</i> = 'U' or 'u', the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> must contain the upper triangular part of the symmetric matrix and the strictly lower triangular part of <i>a</i> is not referenced. Before entry with <i>uplo</i> = 'L' or 'l', the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> must contain the lower triangular part of the symmetric matrix and the strictly upper triangular part of <i>a</i> is not referenced. |
| <i>lda</i> | Specifies the leading dimension of <i>a</i> as declared in the calling (sub)program. The value of <i>lda</i> must be at least $\max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | With <i>uplo</i> = 'U' or 'u', the upper triangular part of the array <i>a</i> is overwritten by the upper triangular part of the updated matrix. With <i>uplo</i> = 'L' or 'l', the lower triangular part of the array <i>a</i> is overwritten by the lower triangular part of the updated matrix. |
|----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

i?max1

Finds the index of the vector element whose real part has maximum absolute value.

Syntax

```
MKL_INT icmax1(const MKL_INT*n, const MKL_Complex8*cx, const MKL_INT*incx)
```

```
MKL_INT izmax1(const MKL_INT*n, const MKL_Complex16*cx, const MKL_INT*incx)
```

Include Files

- mkl.h

Description

Given a complex vector *cx*, the *i?max1* functions return the index of the first vector element of maximum absolute value. These functions are based on the BLAS functions *icamax/izamax*, but using the absolute value of components. They are designed for use with *clacon/zlacon*.

Input Parameters

| | |
|-------------|--|
| <i>n</i> | Specifies the number of elements in the vector <i>cx</i> . |
| <i>cx</i> | Array, size at least $(1+(n-1)*abs(incx))$. Contains the input vector. |
| <i>incx</i> | Specifies the spacing between successive elements of <i>cx</i> . |

Return Values

Index of the vector element of maximum absolute value.

?sum1

Forms the 1-norm of the complex vector using the true absolute value.

Syntax

```
float ssum1(const MKL_INT*n, const MKL_Complex8*cx, const MKL_INT*incx)
double dsum1(const MKL_INT*n, const MKL_Complex16*cx, const MKL_INT*incx)
```

Include Files

- mkl.h

Description

Given a complex vector *cx*, *ssum1*/*dsum1* functions take the sum of the absolute values of vector elements and return a single/double precision result, respectively. These functions are based on *scasum*/*dzasum* from Level 1 BLAS, but use the true absolute value and were designed for use with *clacon*/*zlacon*.

Input Parameters

| | |
|-------------|--|
| <i>n</i> | Specifies the number of elements in the vector <i>cx</i> . |
| <i>cx</i> | Array, size at least $(1+(n-1)*abs(incx))$. Contains the input vector whose elements will be summed. |
| <i>incx</i> | Specifies the spacing between successive elements of <i>cx</i> (<i>incx</i> > 0). |

Return Values

Sum of absolute values.

?gelq2

Computes the LQ factorization of a general rectangular matrix using an unblocked algorithm.

Syntax

```
lapack_int LAPACKE_sgelq2 (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* tau);
lapack_int LAPACKE_dgelq2 (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double * tau);
```

```
lapack_int LAPACKE_cgelq2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);

lapack_int LAPACKE_zgelq2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine computes an LQ factorization of a real/complex m -by- n matrix A as $A = L^*Q$.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of $\min(m, n)$ elementary reflectors :

$Q = H(k) \dots H(2) H(1)$ (or $Q = H(k)^H \dots H(2)^H H(1)^H$ for complex flavors), where $k = \min(m, n)$

Each $H(i)$ has the form

$H(i) = I - \tau v v^T$ for real flavors, or

$H(i) = I - \tau v v^H$ for complex flavors,

where τ is a real/complex scalar stored in $\tau(i)$, and v is a real/complex vector with $v_{1:i-1} = 0$ and $v_i = 1$.

On exit, the j -th ($i+1 \leq j \leq n$) component of vector v (for real functions) or its conjugate (for complex functions) is stored in $a[i - 1 + lda*(j - 1)]$ for column major layout or in $a[j - 1 + lda*(i - 1)]$ for row major layout.

Input Parameters

| | |
|-------|--|
| m | The number of rows in the matrix A ($m \geq 0$). |
| n | The number of columns in A ($n \geq 0$). |
| a | Array, size at least $\max(1, lda*n)$ for column major and $\max(1, lda*m)$ for row major layout. Array a contains the m -by- n matrix A . |
| lda | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|--------|--|
| a | Overwritten by the factorization data as follows: on exit, the elements on and below the diagonal of the array a contain the m -by- $\min(n, m)$ lower trapezoidal matrix L (L is lower triangular if $n \geq m$); the elements above the diagonal, with the array τ , represent the orthogonal/unitary matrix Q as a product of $\min(n, m)$ elementary reflectors. |
| τ | Array, size at least $\max(1, \min(m, n))$. Contains scalar factors of the elementary reflectors. |

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

If $info = -1011$, memory allocation error occurred.

?geqr2

Computes the QR factorization of a general rectangular matrix using an unblocked algorithm.

Syntax

```
lapack_int LAPACKE_sgeqr2 (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, float* tau);
```

```
lapack_int LAPACKE_dgeqr2 (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, double* tau);
```

```
lapack_int LAPACKE_cgeqr2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_complex_float* tau);
```

```
lapack_int LAPACKE_zgeqr2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_complex_double* tau);
```

Include Files

- mkl.h

Description

The routine computes a QR factorization of a real/complex m -by- n matrix A as $A = Q^*R$.

The routine does not form the matrix Q explicitly. Instead, Q is represented as a product of $\min(m, n)$ elementary reflectors :

$$Q = H(1) * H(2) * \dots * H(k), \text{ where } k = \min(m, n)$$

Each $H(i)$ has the form

$$H(i) = I - \tau v^* v^T \text{ for real flavors, or}$$

$$H(i) = I - \tau v^* v^H \text{ for complex flavors}$$

where τ is a real/complex scalar stored in $\tau[i]$, and v is a real/complex vector with $v_{1:i-1} = 0$ and $v_i = 1$.

On exit, $v_{i+1:m}$ is stored in $a(i+1:m, i)$.

Input Parameters

| | |
|-------|--|
| m | The number of rows in the matrix A ($m \geq 0$). |
| n | The number of columns in A ($n \geq 0$). |
| a | Array, size at least $\max(1, lda * n)$ for column major and $\max(1, lda * m)$ for row major layout. Array a contains the m -by- n matrix A . |
| lda | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|------------|---|
| <i>a</i> | Overwritten by the factorization data as follows: on exit, the elements on and above the diagonal of the array <i>a</i> contain the $\min(n,m)$ -by- <i>n</i> upper trapezoidal matrix <i>R</i> (<i>R</i> is upper triangular if $m \geq n$); the elements below the diagonal, with the array <i>tau</i> , represent the orthogonal/unitary matrix <i>Q</i> as a product of elementary reflectors. |
| <i>tau</i> | Array, size at least $\max(1, \min(m, n))$. Contains scalar factors of the elementary reflectors. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?geqrt2

Computes a QR factorization of a general real or complex matrix using the compact WY representation of *Q*.

Syntax

```
lapack_int LAPACKE_sgeqrt2 (int matrix_layout, lapack_int m, lapack_int n, float * a,
lapack_int lda, float * t, lapack_int ldt );

lapack_int LAPACKE_dgeqrt2 (int matrix_layout, lapack_int m, lapack_int n, double * a,
lapack_int lda, double * t, lapack_int ldt );

lapack_int LAPACKE_cgeqrt2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float * a, lapack_int lda, lapack_complex_float * t, lapack_int ldt );

lapack_int LAPACKE_zgeqrt2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double * a, lapack_int lda, lapack_complex_double * t, lapack_int ldt );
```

Include Files

- mkl.h

Description

The strictly lower triangular matrix *V* contains the elementary reflectors *H*(*i*) in the *i*th column below the diagonal. For example, if $m=5$ and $n=3$, the matrix *V* is

$$V = \begin{bmatrix} 1 & & & & \\ v_1 & 1 & & & \\ v_1 & v_2 & 1 & & \\ v_1 & v_2 & v_3 & & \\ v_1 & v_2 & v_3 & & \end{bmatrix}$$

where v_i represents the vector that defines $H(i)$. The vectors are returned in the lower triangular part of array a .

NOTE

The 1s along the diagonal of V are not stored in a .

The block reflector H is then given by

$$H = I - V^* T^* V^T \text{ for real flavors, and}$$

$$H = I - V^* T^* V^H \text{ for complex flavors,}$$

where V^T is the transpose and V^H is the conjugate transpose of V .

Input Parameters

| | |
|-------|--|
| m | The number of rows in the matrix A ($m \geq n$). |
| n | The number of columns in A ($n \geq 0$). |
| a | Array, size at least $\max(1, lda * n)$ for column major and $\max(1, lda * m)$ for row major layout. Array a contains the m -by- n matrix A . |
| lda | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| ldt | The leading dimension of t ; at least $\max(1, n)$. |

Output Parameters

| | |
|-----|--|
| a | Overwritten by the factorization data as follows: The elements on and above the diagonal of the array contain the n -by- n upper triangular matrix R . The elements below the diagonal are the columns of V . |
| t | Array, size at least $\max(1, ldt * n)$. The n -by- n upper triangular factor of the block reflector. The elements on and above the diagonal contain the block reflector T . The elements below the diagonal are not used. |

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info < 0$ and $info = -i$, the i th argument had an illegal value.

If $info = -1011$, memory allocation error occurred.

?geqrt3

Recursively computes a QR factorization of a general real or complex matrix using the compact WY representation of Q .

Syntax

```
lapack_int LAPACKE_sgeqrt3 (int matrix_layout , lapack_int m , lapack_int n , float *
a , lapack_int lda , float * t , lapack_int ldt );
```

```
lapack_int LAPACKE_dgeqrt3 (int matrix_layout , lapack_int m , lapack_int n , double *
a , lapack_int lda , double * t , lapack_int ldt );
```

```
lapack_int LAPACKE_cgeqrt3 (int matrix_layout , lapack_int m , lapack_int n ,
lapack_complex_float * a , lapack_int lda , lapack_complex_float * t , lapack_int
ldt );
```

```
lapack_int LAPACKE_zgeqrt3 (int matrix_layout , lapack_int m , lapack_int n ,
lapack_complex_double * a , lapack_int lda , lapack_complex_double * t , lapack_int
ldt );
```

Include Files

- mkl.h

Description

The strictly lower triangular matrix V contains the elementary reflectors $H(i)$ in the i th column below the diagonal. For example, if $m=5$ and $n=3$, the matrix V is

$$V = \begin{bmatrix} 1 & & & & \\ v_1 & 1 & & & \\ v_1 & v_2 & 1 & & \\ v_1 & v_2 & v_3 & & \\ v_1 & v_2 & v_3 & & \end{bmatrix}$$

where v_i represents one of the vectors that define $H(i)$. The vectors are returned in the lower part of triangular array a .

NOTE

The 1s along the diagonal of V are not stored in a .

The block reflector H is then given by

$H = I - V^* T^* V^T$ for real flavors, and

$H = I - V^* T^* V^H$ for complex flavors,

where V^T is the transpose and V^H is the conjugate transpose of V .

Input Parameters

m The number of rows in the matrix A ($m \geq n$).

n The number of columns in A ($n \geq 0$).

| | |
|------------|--|
| <i>a</i> | Array, size at least $\max(1, lda*n)$ for column major and $\max(1, lda*m)$ for row major layout. Array <i>a</i> contains the <i>m</i> -by- <i>n</i> matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldt</i> | The leading dimension of <i>t</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | The elements on and above the diagonal of the array contain the <i>n</i> -by- <i>n</i> upper triangular matrix <i>R</i> . The elements below the diagonal are the columns of <i>V</i> . |
| <i>t</i> | Array, size <i>ldt</i> by <i>n</i> . The <i>n</i> -by- <i>n</i> upper triangular factor of the block reflector. The elements on and above the diagonal contain the block reflector <i>T</i> . The elements below the diagonal are not used. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0 and *info* = -*i*, the *i*th argument had an illegal value.

If *info* = -1011, memory allocation error occurred.

?getf2

Computes the LU factorization of a general *m*-by-*n* matrix using partial pivoting with row interchanges (unblocked algorithm).

Syntax

```
lapack_int LAPACKE_sgetf2 (int matrix_layout, lapack_int m, lapack_int n, float* a,
lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_dgetf2 (int matrix_layout, lapack_int m, lapack_int n, double* a,
lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_cgetf2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_float* a, lapack_int lda, lapack_int * ipiv);
```

```
lapack_int LAPACKE_zgetf2 (int matrix_layout, lapack_int m, lapack_int n,
lapack_complex_double* a, lapack_int lda, lapack_int * ipiv);
```

Include Files

- mkl.h

Description

The routine computes the LU factorization of a general *m*-by-*n* matrix *A* using partial pivoting with row interchanges. The factorization has the form

$$A = P*LU$$

where p is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$).

Input Parameters

| | |
|-------|--|
| m | The number of rows in the matrix A ($m \geq 0$). |
| n | The number of columns in A ($n \geq 0$). |
| a | Array, size at least $\max(1, lda*n)$ for column major and $\max(1, lda*m)$ for row major layout. Array a contains the m -by- n matrix A . |
| lda | The leading dimension of a ; at least $\max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|--------|--|
| a | Overwritten by L and U . The unit diagonal elements of L are not stored. |
| $ipiv$ | Array, size at least $\max(1, \min(m, n))$. The pivot indices: for $1 \leq i \leq n$, row i was interchanged with row $ipiv(i)$. |

Return Values

This function returns a value $info$.

If $info = -i$, the i -th parameter had an illegal value.

If $info = i > 0$, u_{ij} is 0. The factorization has been completed, but U is exactly singular. Division by 0 will occur if you use the factor U for solving a system of linear equations.

If $info = -1011$, memory allocation error occurred.

?lacn2

Estimates the 1-norm of a square matrix, using reverse communication for evaluating matrix-vector products.

Syntax

C:

```
lapack_int LAPACKE_slacn2 (lapack_int n, float * v, float * x, lapack_int * isgn, float * est, lapack_int * kase, lapack_int * isave);
```

```
lapack_int LAPACKE_clacn2 (lapack_int n, lapack_complex_float * v, lapack_complex_float * x, float * est, lapack_int * kase, lapack_int * isave);
```

```
lapack_int LAPACKE_dlacn2 (lapack_int n, double * v, double * x, lapack_int * isgn, double * est, lapack_int * kase, lapack_int * isave);
```

```
lapack_int LAPACKE_zlacn2 (lapack_int n, lapack_complex_double * v, lapack_complex_double * x, double * est, lapack_int * kase, lapack_int * isave);
```

Include Files

- mkl.h

Description

The routine estimates the 1-norm of a square, real or complex matrix A . Reverse communication is used for evaluating matrix-vector products.

Input Parameters

| | |
|---------|---|
| n | The order of the matrix A ($n \geq 1$). |
| v, x | Arrays, size (n) each. v is a workspace array. x is used as input after an intermediate return. |
| $isgn$ | Workspace array, size (n), used with real flavors only. |
| est | On entry with $kase$ set to 1 or 2, and $isave(1) = 1$, est must be unchanged from the previous call to the routine. |
| $kase$ | On the initial call to the routine, $kase$ must be set to 0. |
| $isave$ | Array, size (3). Contains variables from the previous call to the routine. |

Output Parameters

| | |
|---------|--|
| est | An estimate (a lower bound) for $\text{norm}(A)$. |
| $kase$ | On an intermediate return, $kase$ is set to 1 or 2, indicating whether x is overwritten by $A*x$ or A^T*x for real flavors and $A*x$ or A^H*x for complex flavors. On the final return, $kase$ is set to 0. |
| v | On the final return, $v = A*w$, where $est = \text{norm}(v) / \text{norm}(w)$ (w is not returned). |
| x | On an intermediate return, x is overwritten by $A*x$, if $kase = 1$, A^T*x , if $kase = 2$ (for real flavors), A^H*x , if $kase = 2$ (for complex flavors), and the routine must be re-called with all the other parameters unchanged. |
| $isave$ | This parameter is used to save variables between calls to the routine. |

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info = -i$, the i -th parameter had an illegal value.

?lacpy

Copies all or part of one two-dimensional array to another.

Syntax

```
lapack_int LAPACKE_slacpy (int matrix_layout, char uplo, lapack_int m, lapack_int n,
const float* a, lapack_int lda, float* b, lapack_int ldb);
```

```
lapack_int LAPACKE_dlacpy (int matrix_layout, char uplo, lapack_int m, lapack_int n,
const double* a, lapack_int lda, double* b, lapack_int ldb);
```

```
lapack_int LAPACKE_clacpy (int matrix_layout, char uplo, lapack_int m, lapack_int n,
const lapack_complex_float* a, lapack_int lda, lapack_complex_float* b, lapack_int
ldb);
```

```
lapack_int LAPACKE_zlacpy (int matrix_layout, char uplo, lapack_int m, lapack_int n,
const lapack_complex_double* a, lapack_int lda, lapack_complex_double* b, lapack_int
ldb);
```

Include Files

- mkl.h

Description

The routine copies all or part of a two-dimensional matrix *A* to another matrix *B*.

Input Parameters

| | |
|-------------|---|
| <i>uplo</i> | Specifies the part of the matrix <i>A</i> to be copied to <i>B</i> . If <i>uplo</i> = 'U', the upper triangular part of <i>A</i> ; if <i>uplo</i> = 'L', the lower triangular part of <i>A</i> . Otherwise, all of the matrix <i>A</i> is copied. |
| <i>m</i> | The number of rows in the matrix <i>A</i> ($m \geq 0$). |
| <i>n</i> | The number of columns in <i>A</i> ($n \geq 0$). |
| <i>a</i> | Array, size at least $\max(1, lda \cdot n)$ for column major and $\max(1, lda \cdot m)$ for row major layout. <i>A</i> contains the <i>m</i> -by- <i>n</i> matrix <i>A</i> . If <i>uplo</i> = 'U', only the upper triangle or trapezoid is accessed; if <i>uplo</i> = 'L', only the lower triangle or trapezoid is accessed. |
| <i>lda</i> | The leading dimension of <i>a</i> ; $lda \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |
| <i>ldb</i> | The leading dimension of the output array <i>b</i> ; $ldb \geq \max(1, m)$ for column major layout and $\max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|--|
| <i>b</i> | Array, size at least $\max(1, ldb \cdot n)$ for column major and $\max(1, ldb \cdot m)$ for row major layout. Array <i>a</i> contains the <i>m</i> -by- <i>n</i> matrix <i>B</i> . |
|----------|--|

On exit, $B = A$ in the locations specified by *uplo*.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

?lakf2

Forms a matrix containing Kronecker products between the given matrices.

Syntax

```
void slakf2 (lapack_int *m, lapack_int *n, float *a, lapack_int *lda, float *b, float
*d, float *e, float *z, lapack_int *ldz);
```

```
void dlakf2 (lapack_int *m, lapack_int *n, double *a, lapack_int *lda, double *b,
double *d, double *e, double *z, lapack_int *ldz);
```

```
void clakf2 (lapack_int *m, lapack_int *n, lapack_complex *a, lapack_int *lda,
lapack_complex *b, lapack_complex *d, lapack_complex *e, lapack_complex *z, lapack_int
*ldz);
```

```
void zlakf2 (lapack_int *m, lapack_int *n, lapack_complex_double *a, lapack_int *lda,
lapack_complex_double *b, lapack_complex_double *d, lapack_complex_double *e,
lapack_complex_double *z, lapack_int *ldz);
```

Include Files

- mkl.h

Description

The routine ?lakf2 forms the $2*m*n$ by $2*m*n$ matrix Z .

$$Z = \begin{bmatrix} \text{kron}(In, A) & -\text{kron}(B^T, Im) \\ \text{kron}(In, D) & -\text{kron}(E^T, Im) \end{bmatrix},$$

where In is the identity matrix of size n and X^T is the transpose of X . $\text{kron}(X, Y)$ is the Kronecker product between the matrices X and Y .

Input Parameters

| | |
|------------|---|
| <i>m</i> | Size of matrix, $m \geq 1$ |
| <i>n</i> | Size of matrix, $n \geq 1$ |
| <i>a</i> | Array, size <i>lda</i> -by- <i>n</i> . The matrix A in the output matrix Z . |
| <i>lda</i> | The leading dimension of <i>a</i> , <i>b</i> , <i>d</i> , and <i>e</i> . $lda \geq m+n$. |
| <i>b</i> | Array, size <i>lda</i> by <i>n</i> . Matrix used in forming the output matrix Z . |
| <i>d</i> | Array, size <i>lda</i> by <i>m</i> . Matrix used in forming the output matrix Z . |

| | |
|------------|--|
| <i>e</i> | Array, size <i>lda</i> by <i>n</i> . Matrix used in forming the output matrix <i>Z</i> . |
| <i>ldz</i> | The leading dimension of <i>Z</i> . $ldz \geq 2 * m * n$. |

Output Parameters

| | |
|----------|--|
| <i>z</i> | Array, size <i>ldz</i> -by- $2 * m * n$. The resultant Kronecker $m * n * 2$ -by- $m * n * 2$ matrix. |
|----------|--|

?lange

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a general rectangular matrix.

Syntax

```
float LAPACKE_slange (int matrix_layout, char norm, lapack_int m, lapack_int n, const float * a, lapack_int lda);
```

```
double LAPACKE_dlange (int matrix_layout, char norm, lapack_int m, lapack_int n, const double * a, lapack_int lda);
```

```
float LAPACKE_clange (int matrix_layout, char norm, lapack_int m, lapack_int n, const lapack_complex_float * a, lapack_int lda);
```

```
double LAPACKE_zlange (int matrix_layout, char norm, lapack_int m, lapack_int n, const lapack_complex_double * a, lapack_int lda);
```

Include Files

- mkl.h

Description

The function ?lange returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/complex matrix *A*.

Input Parameters

| | |
|-------------|---|
| <i>norm</i> | Specifies the value to be returned by the routine: = 'M' or 'm': $val = \max(\text{abs}(A_{ij}))$, largest absolute value of the matrix <i>A</i> . = '1' or 'O' or 'o': $val = \text{norm1}(A)$, 1-norm of the matrix <i>A</i> (maximum column sum), = 'I' or 'i': $val = \text{normI}(A)$, infinity norm of the matrix <i>A</i> (maximum row sum), = 'F', 'f', 'E' or 'e': $val = \text{normF}(A)$, Frobenius norm of the matrix <i>A</i> (square root of sum of squares). |
| <i>m</i> | The number of rows of the matrix <i>A</i> . $m \geq 0$. When $m = 0$, ?lange is set to zero. |
| <i>n</i> | The number of columns of the matrix <i>A</i> . |

$n \geq 0$. When $n = 0$, `?lange` is set to zero.

`a` Array, size at least $\max(1, lda * n)$ for column major and $\max(1, lda * m)$ for row major layout. Array `a` contains the m -by- n matrix A .

`lda` The leading dimension of the array `a`.
 $lda \geq \max(n, 1)$ for column major layout and $\max(1, n)$ for row major layout.

?lansy

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/complex symmetric matrix.

Syntax

```
float LAPACKE_slansy (int matrix_layout, char norm, char uplo, lapack_int n, const float * a, lapack_int lda);
```

```
double LAPACKE_dlansy (int matrix_layout, char norm, char uplo, lapack_int n, const double * a, lapack_int lda);
```

```
float LAPACKE_clansy (int matrix_layout, char norm, char uplo, lapack_int n, const lapack_complex_float * a, lapack_int lda);
```

```
double LAPACKE_zlansy (int matrix_layout, char norm, char uplo, lapack_int n, const lapack_complex_double * a, lapack_int lda);
```

Include Files

- `mkl.h`

Description

The function `?lansy` returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a real/complex symmetric matrix A .

Input Parameters

`norm` Specifies the value to be returned by the routine:

- = 'M' or 'm': $val = \max(\text{abs}(A_{ij}))$, largest absolute value of the matrix A .
- = '1' or 'O' or 'o': $val = \text{norm1}(A)$, 1-norm of the matrix A (maximum column sum),
- = 'I' or 'i': $val = \text{normI}(A)$, infinity norm of the matrix A (maximum row sum),
- = 'F', 'f', 'E' or 'e': $val = \text{normF}(A)$, Frobenius norm of the matrix A (square root of sum of squares).

`uplo` Specifies whether the upper or lower triangular part of the symmetric matrix A is to be referenced.

- = 'U': Upper triangular part of A is referenced.

| | |
|-------|--|
| | = 'L': Lower triangular part of A is referenced |
| n | The order of the matrix A . $n \geq 0$. When $n = 0$, <code>?lanhsy</code> is set to zero. |
| a | Array, size at least $\max(1, lda * n)$. The symmetric matrix A . If <code>uplo = 'U'</code> , the leading n -by- n upper triangular part of a contains the upper triangular part of the matrix A , and the strictly lower triangular part of a is not referenced. If <code>uplo = 'L'</code> , the leading n -by- n lower triangular part of a contains the lower triangular part of the matrix A , and the strictly upper triangular part of a is not referenced. |
| lda | The leading dimension of the array a . $lda \geq \max(n, 1)$. |

?lanhe

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a complex Hermitian matrix.

Syntax

```
float LAPACKE_clanhe (int matrix_layout, char norm, char uplo, lapack_int n, const
lapack_complex_float * a, lapack_int lda);
```

```
double LAPACKE_zlanhe (int matrix_layout, char norm, char uplo, lapack_int n, const
lapack_complex_double * a, lapack_int lda);
```

Include Files

- `mk1.h`

Description

The function `?lanhe` returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a complex Hermitian matrix A .

Input Parameters

| | |
|--------|---|
| $norm$ | Specifies the value to be returned by the routine: = 'M' or 'm': $val = \max(\text{abs}(A_{ij}))$, largest absolute value of the matrix A . = '1' or 'O' or 'o': $val = \text{norm1}(A)$, 1-norm of the matrix A (maximum column sum), = 'I' or 'i': $val = \text{normI}(A)$, infinity norm of the matrix A (maximum row sum), = 'F', 'f', 'E' or 'e': $val = \text{normF}(A)$, Frobenius norm of the matrix A (square root of sum of squares). |
| $uplo$ | Specifies whether the upper or lower triangular part of the Hermitian matrix A is to be referenced. = 'U': Upper triangular part of A is referenced. |

= 'L': Lower triangular part of A is referenced

n The order of the matrix A . $n \geq 0$. When $n = 0$, `?lanhe` is set to zero.

a Array, size at least $\max(1, lda*n)$. The Hermitian matrix A .
 If `uplo = 'U'`, the leading n -by- n upper triangular part of a contains the upper triangular part of the matrix A , and the strictly lower triangular part of a is not referenced.

If `uplo = 'L'`, the leading n -by- n lower triangular part of a contains the lower triangular part of the matrix A , and the strictly upper triangular part of a is not referenced.

lda The leading dimension of the array a .

$lda \geq \max(n, 1)$.

?lantr

Returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a trapezoidal or triangular matrix.

Syntax

```
float LAPACKE_slantr (char * norm, char * uplo, char * diag, lapack_int * m, lapack_int * n, const float * a, lapack_int * lda, float * work);
```

```
double LAPACKE_dlantr (char * norm, char * uplo, char * diag, lapack_int * m, lapack_int * n, const double * a, lapack_int * lda, double * work);
```

```
float LAPACKE_clantr (char * norm, char * uplo, char * diag, lapack_int * m, lapack_int * n, const lapack_complex_float * a, lapack_int * lda, float * work);
```

```
double LAPACKE_zlantr (char * norm, char * uplo, char * diag, lapack_int * m, lapack_int * n, const lapack_complex_double * a, lapack_int * lda, double * work);
```

Include Files

- `mkl.h`

Description

The function `?lantr` returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a trapezoidal or triangular matrix A .

Input Parameters

$norm$ Specifies the value to be returned by the routine:

- = 'M' or 'm': $val = \max(\text{abs}(A_{ij}))$, largest absolute value of the matrix A .
- = '1' or 'O' or 'o': $val = \text{norm1}(A)$, 1-norm of the matrix A (maximum column sum),
- = 'I' or 'i': $val = \text{normI}(A)$, infinity norm of the matrix A (maximum row sum),

| | |
|-------------|--|
| | = 'F', 'f', 'E' or 'e': $val = \text{normF}(A)$, Frobenius norm of the matrix A (square root of sum of squares). |
| <i>uplo</i> | Specifies whether the matrix A is upper or lower trapezoidal. = 'U': Upper trapezoidal = 'L': Lower trapezoidal. Note that A is triangular instead of trapezoidal if $m = n$. |
| <i>diag</i> | Specifies whether or not the matrix A has unit diagonal. = 'N': Non-unit diagonal = 'U': Unit diagonal. |
| <i>m</i> | The number of rows of the matrix A . $m \geq 0$, and if <i>uplo</i> = 'U', $m \leq n$. When $m = 0$, <i>?lantr</i> is set to zero. |
| <i>n</i> | The number of columns of the matrix A . $n \geq 0$, and if <i>uplo</i> = 'L', $n \leq m$. When $n = 0$, <i>?lantr</i> is set to zero. |
| <i>a</i> | Array, size at least $\max(1, lda * n)$ for column major and $\max(1, lda * m)$ for row major layout. The trapezoidal matrix A (A is triangular if $m = n$). If <i>uplo</i> = 'U', the leading m -by- n upper trapezoidal part of the array a contains the upper trapezoidal matrix, and the strictly lower triangular part of A is not referenced. If <i>uplo</i> = 'L', the leading m -by- n lower trapezoidal part of the array a contains the lower trapezoidal matrix, and the strictly upper triangular part of A is not referenced. Note that when <i>diag</i> = 'U', the diagonal elements of A are not referenced and are assumed to be one. |
| <i>lda</i> | The leading dimension of the array a . $lda \geq \max(m, 1)$ for column major layout and $\geq \max(1, n)$ for row major layout. |

?lapmr

Rearranges rows of a matrix as specified by a permutation vector.

Syntax

```
lapack_int LAPACKE_slapmr (int matrix_layout, lapack_logical forwr, lapack_int m,
lapack_int n, float* x, lapack_int ldx, lapack_int * k);
```

```
lapack_int LAPACKE_dlapmr (int matrix_layout, lapack_logical forwr, lapack_int m,
lapack_int n, double* x, lapack_int ldx, lapack_int * k);
```

```
lapack_int LAPACKE_clapmr (int matrix_layout, lapack_logical forwr, lapack_int m,
lapack_int n, lapack_complex_float* x, lapack_int ldx, lapack_int * k);
```

```
lapack_int LAPACKE_zlapmr (int matrix_layout, lapack_logical forwr, lapack_int m,
lapack_int n, lapack_complex_double* x, lapack_int ldx, lapack_int * k);
```

Include Files

- `mkl.h`

Description

The `?lapmr` routine rearranges the rows of the m -by- n matrix X as specified by the permutation $k[0], k[1], \dots, k[m-1]$ of the integers $1, \dots, m$.

If `forwrd` is true, forward permutation:

$X(k[i-1], :)$ is moved to $X(i, :)$ for $i = 1, 2, \dots, m$.

If `forwrd` is false, backward permutation:

$X(i, :)$ is moved to $X(k[i-1], :)$ for $i = 1, 2, \dots, m$.

Input Parameters

| | |
|---------------------|---|
| <code>forwrd</code> | If <code>forwrd</code> is true, forward permutation. If <code>forwrd</code> is false, backward permutation. |
| <code>m</code> | The number of rows of the matrix X . $m \geq 0$. |
| <code>n</code> | The number of columns of the matrix X . $n \geq 0$. |
| <code>x</code> | Array, size at least $\max(1, ldx * n)$ for column major and $\max(1, ldx * m)$ for row major layout. On entry, the m -by- n matrix X . |
| <code>ldx</code> | The leading dimension of the array X , $ldx \geq \max(1, m)$ for column major layout and $ldx \geq \max(1, n)$ for row major layout. |
| <code>k</code> | Array, size (m). On entry, k contains the permutation vector and is used as internal workspace. |

Output Parameters

| | |
|----------------|---|
| <code>x</code> | On exit, x contains the permuted matrix X . |
| <code>k</code> | On exit, k is reset to its original value. |

Return Values

This function returns a value `info`.

If `info = 0`, the execution is successful.

If `info = -i`, the i -th parameter had an illegal value.

If `info = -1011`, memory allocation error occurred.

See Also

?lapmt

Performs a forward or backward permutation of the columns of a matrix.

Syntax

```
lapack_int LAPACKE_slapmt (int matrix_layout, lapack_logical forwr, lapack_int m,
lapack_int n, float * x, lapack_int ldx, lapack_int * k);
```

```
lapack_int LAPACKE_dlapmt (int matrix_layout, lapack_logical forwr, lapack_int m,
lapack_int n, double * x, lapack_int ldx, lapack_int * k);
```

```
lapack_int LAPACKE_clapmt (int matrix_layout, lapack_logical forwr, lapack_int m,
lapack_int n, lapack_complex_float * x, lapack_int ldx, lapack_int * k);
```

```
lapack_int LAPACKE_zlapmt (int matrix_layout, lapack_logical forwr, lapack_int m,
lapack_int n, lapack_complex_double * x, lapack_int ldx, lapack_int * k);
```

Include Files

- mkl.h

Description

The routine `?lapmt` rearranges the columns of the m -by- n matrix X as specified by the permutation $k[i - 1]$ for $i = 1, \dots, n$.

If $forwr \neq 0$, forward permutation:

$X(*, k(j))$ is moved to $X(*, j)$ for $j=1, 2, \dots, n$.

If $forwr = 0$, backward permutation:

$X(*, j)$ is moved to $X(*, k(j))$ for $j = 1, 2, \dots, n$.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>forwr</i> | If $forwr \neq 0$, forward permutation If $forwr = 0$, backward permutation |
| <i>m</i> | The number of rows of the matrix X . $m \geq 0$. |
| <i>n</i> | The number of columns of the matrix X . $n \geq 0$. |
| <i>x</i> | Array, size $ldx*n$. On entry, the m -by- n matrix X . |
| <i>ldx</i> | The leading dimension of the array x , $ldx \geq \max(1, m)$. |
| <i>k</i> | Array, size (n). On entry, k contains the permutation vector and is used as internal workspace. |

Output Parameters

| | |
|----------|---|
| <i>x</i> | On exit, x contains the permuted matrix X . |
| <i>k</i> | On exit, k is reset to its original value. |

See Also

[?lapmr](#)

?lapy2

Returns $\text{sqrt}(x^2+y^2)$.

Syntax

```
float LAPACKE_slapy2 (floatx, floaty);  
double LAPACKE_dlapy2 (doublex, doubley);
```

Include Files

- mkl.h

Description

The function ?lapy2 returns $\text{sqrt}(x^2+y^2)$, avoiding unnecessary overflow or harmful underflow.

Input Parameters

x, y Specify the input values x and y .

Return Values

The function returns a value val .

If $val=-1D0$, the first argument was NaN.

If $val=-2D0$, the second argument was NaN.

?lapy3

Returns $\text{sqrt}(x^2+y^2+z^2)$.

Syntax

```
float LAPACKE_slapy3 (floatx, floaty, floatz);  
double LAPACKE_dlapy3 (double x, doubley, doublez);
```

Include Files

- mkl.h

Description

The function ?lapy3 returns $\text{sqrt}(x^2+y^2+z^2)$, avoiding unnecessary overflow or harmful underflow.

Input Parameters

x, y, z Specify the input values x, y and z .

Return Values

This function returns a value val .

If $val = -1D0$, the first argument was NaN.

If $val = -2D0$, the second argument was NaN.

If $val = -3D0$, the third argument was NaN.

Description

The real flavors of the routine `?larfb` apply a real block reflector H or its transpose H^T to a real m -by- n matrix C from either left or right.

The complex flavors of the routine `?larfb` apply a complex block reflector H or its conjugate transpose H^H to a complex m -by- n matrix C from either left or right.

Input Parameters

| | |
|---------------|--|
| <i>side</i> | <p>If <i>side</i> = 'L': apply H or H^T for real flavors and H or H^H for complex flavors from the left.</p> <p>If <i>side</i> = 'R': apply H or H^T for real flavors and H or H^H for complex flavors from the right.</p> |
| <i>trans</i> | <p>If <i>trans</i> = 'N': apply H (No transpose).</p> <p>If <i>trans</i> = 'C': apply H^H (Conjugate transpose).</p> <p>If <i>trans</i> = 'T': apply H^T (Transpose).</p> |
| <i>direct</i> | <p>Indicates how H is formed from a product of elementary reflectors</p> <p>If <i>direct</i> = 'F': $H = H(1) * H(2) * \dots * H(k)$ (forward)</p> <p>If <i>direct</i> = 'B': $H = H(k) * \dots * H(2) * H(1)$ (backward)</p> |
| <i>storev</i> | <p>Indicates how the vectors which define the elementary reflectors are stored:</p> <p>If <i>storev</i> = 'C': Column-wise</p> <p>If <i>storev</i> = 'R': Row-wise</p> |
| <i>m</i> | The number of rows of the matrix C . |
| <i>n</i> | The number of columns of the matrix C . |
| <i>k</i> | The order of the matrix T (equal to the number of elementary reflectors whose product defines the block reflector). |
| <i>v</i> | The size limitations depend on values of parameters <i>storev</i> and <i>side</i> as described in the following table: |

| | <i>storev</i> = C | | <i>storev</i> = R | |
|--------------|--------------------|--------------------|--------------------|--------------------|
| | <i>side</i> = L | <i>side</i> = R | <i>side</i> = L | <i>side</i> = R |
| Column major | $\max(1, ldv * k)$ | $\max(1, ldv * k)$ | $\max(1, ldv * m)$ | $\max(1, ldv * n)$ |
| Row major | $\max(1, ldv * m)$ | $\max(1, ldv * n)$ | $\max(1, ldv * k)$ | $\max(1, ldv * k)$ |

The matrix v . See *Application Notes* below.

ldv The leading dimension of the array v . It should satisfy the following conditions:

| | <i>storev</i> = C | | <i>storev</i> = R | |
|--------------|-------------------|-----------------|-------------------|-----------------|
| | <i>side</i> = L | <i>side</i> = R | <i>side</i> = L | <i>side</i> = R |
| Column major | $\max(1,m)$ | $\max(1,n)$ | $\max(1,k)$ | $\max(1,k)$ |
| Row major | $\max(1,k)$ | $\max(1,k)$ | $\max(1,m)$ | $\max(1,n)$ |

| | |
|------------|---|
| <i>t</i> | Array, size at least $\max(1, ldt * k)$. Contains the triangular k -by- k matrix T in the representation of the block reflector. |
| <i>ldt</i> | The leading dimension of the array <i>t</i> . $ldt \geq k$. |
| <i>c</i> | Array, size at least $\max(1, ldc * n)$ for column major layout and $\max(1, ldc * m)$ for row major layout. On entry, the m -by- n matrix C . |
| <i>ldc</i> | The leading dimension of the array <i>c</i> . $ldc \geq \max(1, m)$ for column major layout and $ldc \geq \max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | On exit, <i>c</i> is overwritten by the product of the following: <ul style="list-style-type: none"> $H * C$, or $H^T * C$, or $C * H$, or $C * H^T$ for real flavors $H * C$, or $H^H * C$, or $C * H$, or $C * H^H$ for complex flavors |
|----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

Application Notes

The shape of the matrix V and the storage of the vectors which define the $H(i)$ is best illustrated by the following example with $n = 5$ and $k = 3$. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

direct = 'F' and *storev* = 'C': *direct* = 'F' and *storev* = 'R':

$$\begin{bmatrix} 1 & & & & \\ v_1 & 1 & & & \\ v_1 & v_2 & 1 & & \\ v_1 & v_2 & v_3 & & \\ v_1 & v_2 & v_3 & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & v_1 & v_1 & v_1 & v_1 \\ & 1 & v_2 & v_2 & v_2 \\ & & 1 & v_3 & v_3 \end{bmatrix}$$

direct = 'B' and *storev* = 'C': *direct* = 'B' and *storev* = 'R':

$$\begin{bmatrix} v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \\ 1 & v_2 & v_3 \\ & 1 & v_3 \\ & & 1 \end{bmatrix}$$

$$\begin{bmatrix} v_1 & v_1 & 1 & & \\ v_2 & v_2 & v_2 & 1 & \\ v_3 & v_3 & v_3 & v_3 & 1 \end{bmatrix}$$

?larfg

Generates an elementary reflector (Householder matrix).

Syntax

```
lapack_int LAPACKE_slarfg (lapack_int n , float * alpha , float * x , lapack_int incx , float * tau );
```

```
lapack_int LAPACKE_dlarfg (lapack_int n , double * alpha , double * x , lapack_int incx , double * tau );
```

```
lapack_int LAPACKE_clarfg (lapack_int n , lapack_complex_float * alpha , lapack_complex_float * x , lapack_int incx , lapack_complex_float * tau );
```

```
lapack_int LAPACKE_zlarfg (lapack_int n , lapack_complex_double * alpha , lapack_complex_double * x , lapack_int incx , lapack_complex_double * tau );
```

Include Files

- mkl.h

Description

The routine ?larfg generates a real/complex elementary reflector H of order n , such that

$$H^* \begin{bmatrix} \alpha \\ x \end{bmatrix} = \begin{bmatrix} \beta \\ 0 \end{bmatrix}, \quad H^T H = I, \quad \text{for real flavors and}$$

$$H^X * \begin{bmatrix} \text{alpha} \\ x \end{bmatrix} = \begin{bmatrix} \text{beta} \\ 0 \end{bmatrix}, \quad H^X * H = I, \quad \text{for complex flavors,}$$

where *alpha* and *beta* are scalars (with *beta* real for all flavors), and *x* is an (*n*-1)-element real/complex vector. *H* is represented in the form

$$H = I - \text{tau} * \begin{bmatrix} 1 \\ v \end{bmatrix} * \begin{bmatrix} 1 & v^T \end{bmatrix} \quad \text{for real flavors and}$$

$$H = I - \text{tau} * \begin{bmatrix} 1 \\ v \end{bmatrix} * \begin{bmatrix} 1 & v^H \end{bmatrix} \quad \text{for complex flavors,}$$

where *tau* is a real/complex scalar and *v* is a real/complex (*n*-1)-element vector, respectively. Note that for `clarfg/zlarfg`, *H* is not Hermitian.

If the elements of *x* are all zero (and, for complex flavors, *alpha* is real), then *tau* = 0 and *H* is taken to be the unit matrix.

Otherwise, $1 \leq \text{tau} \leq 2$ (for real flavors), or

$1 \leq \text{Re}(\text{tau}) \leq 2$ and $\text{abs}(\text{tau}-1) \leq 1$ (for complex flavors).

Input Parameters

| | |
|--------------|--|
| <i>n</i> | The order of the elementary reflector. |
| <i>alpha</i> | |
| <i>x</i> | Array, size $(1+(n-2)*\text{abs}(\text{incx}))$. On entry, the vector <i>x</i> . |
| <i>incx</i> | The increment between elements of <i>x</i> . <i>incx</i> > 0. |

Output Parameters

| | |
|--------------|---|
| <i>alpha</i> | On exit, it is overwritten with the value <i>beta</i> . |
| <i>x</i> | On exit, it is overwritten with the vector <i>v</i> . |
| <i>tau</i> | |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -2, *alpha* is NaN

If *info* = -3, array *x* contains NaN components.

?larft

Forms the triangular factor *T* of a block reflector $H = I - V * T * V^* * H$.

Syntax

```
lapack_int LAPACKE_slarft (int matrix_layout , char direct , char storev , lapack_int
n , lapack_int k , const float * v , lapack_int ldv , const float * tau , float * t ,
lapack_int ldt );
```

```
lapack_int LAPACKE_dlarft (int matrix_layout , char direct , char storev , lapack_int
n , lapack_int k , const double * v , lapack_int ldv , const double * tau , double *
t , lapack_int ldt );
```

```
lapack_int LAPACKE_clarft (int matrix_layout , char direct , char storev , lapack_int
n , lapack_int k , const lapack_complex_float * v , lapack_int ldv , const
lapack_complex_float * tau , lapack_complex_float * t , lapack_int ldt );
```

```
lapack_int LAPACKE_zlarft (int matrix_layout , char direct , char storev , lapack_int
n , lapack_int k , const lapack_complex_double * v , lapack_int ldv , const
lapack_complex_double * tau , lapack_complex_double * t , lapack_int ldt );
```

Include Files

- mkl.h

Description

The routine `?larft` forms the triangular factor T of a real/complex block reflector H of order n , which is defined as a product of k elementary reflectors.

If $direct = 'F'$, $H = H(1)*H(2)* \dots *H(k)$ and T is upper triangular;

If $direct = 'B'$, $H = H(k)* \dots *H(2)*H(1)$ and T is lower triangular.

If $storev = 'C'$, the vector which defines the elementary reflector $H(i)$ is stored in the i -th column of the array v , and $H = I - V*T*V^T$ (for real flavors) or $H = I - V*T*V^H$ (for complex flavors) .

If $storev = 'R'$, the vector which defines the elementary reflector $H(i)$ is stored in the i -th row of the array v , and $H = I - V^T*T*V$ (for real flavors) or $H = I - V^H*T*V$ (for complex flavors).

Input Parameters

| | |
|---------------|---|
| <i>direct</i> | Specifies the order in which the elementary reflectors are multiplied to form the block reflector: = 'F': $H = H(1)*H(2)* \dots *H(k)$ (forward) = 'B': $H = H(k)* \dots *H(2)*H(1)$ (backward) |
| <i>storev</i> | Specifies how the vectors which define the elementary reflectors are stored (see also <i>Application Notes</i> below): = 'C': column-wise = 'R': row-wise. |
| <i>n</i> | The order of the block reflector H . $n \geq 0$. |
| <i>k</i> | The order of the triangular factor T (equal to the number of elementary reflectors). $k \geq 1$. |
| <i>v</i> | The size limitations depend on values of parameters <i>storev</i> and <i>side</i> as described in the following table: |

| | <i>storev</i> = C | <i>storev</i> = R |
|--------------|-------------------|-------------------|
| Column major | $\max(1, ldv*k)$ | $\max(1, ldv*n)$ |
| Row major | $\max(1, ldv*n)$ | $\max(1, ldv*k)$ |

The matrix *v*. See *Application Notes* below.

ldv

The leading dimension of the array *v*.

If *storev* = 'C', $ldv \geq \max(1, n)$ for column major and $ldv \geq \max(1, k)$ for row major;

if *storev* = 'R', $ldv \geq k$ for column major and $ldv \geq \max(1, n)$ for row major.

tau

Array, size (*k*). *tau*[*i*-1] must contain the scalar factor of the elementary reflector *H*(*i*).

ldt

The leading dimension of the output array *t*. $ldt \geq k$.

Output Parameters

t

Array, size *ldt* * *k*. The *k*-by-*k* triangular factor *T* of the block reflector. If *direct* = 'F', *T* is upper triangular; if *direct* = 'B', *T* is lower triangular. The rest of the array is not used.

v

The matrix *V*.

Application Notes

The shape of the matrix *V* and the storage of the vectors which define the *H*(*i*) is best illustrated by the following example with $n = 5$ and $k = 3$. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

direct = 'F' and *storev* = 'C': *direct* = 'F' and *storev* = 'R':

$$\begin{bmatrix} 1 & & & & \\ v_1 & 1 & & & \\ v_1 & v_2 & 1 & & \\ v_1 & v_2 & v_3 & & \\ v_1 & v_2 & v_3 & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & v_1 & v_1 & v_1 & v_1 \\ & 1 & v_2 & v_2 & v_2 \\ & & 1 & v_3 & v_3 \end{bmatrix}$$

| | |
|-------------|--|
| <i>n</i> | The number of columns of the matrix <i>C</i> . |
| <i>v</i> | Array, size (<i>m</i>) if <i>side</i> = 'L' or (<i>n</i>) if <i>side</i> = 'R'. The vector <i>v</i> in the representation of <i>H</i> . |
| <i>tau</i> | The value <i>tau</i> in the representation of <i>H</i> . |
| <i>c</i> | Array, size at least max(1, <i>ldc</i> * <i>n</i>) for column major layout and max (1, <i>ldc</i> * <i>m</i>) for row major layout. On entry, the <i>m</i> -by- <i>n</i> matrix <i>C</i> . |
| <i>ldc</i> | The leading dimension of the array <i>c</i> . <i>lda</i> ≥ (1, <i>m</i>). |
| <i>work</i> | Workspace array, size (<i>n</i>) if <i>side</i> = 'L' or (<i>m</i>) if <i>side</i> = 'R'. <i>work</i> is not referenced if <i>H</i> has order < 11. |

Output Parameters

| | |
|----------|---|
| <i>c</i> | On exit, <i>C</i> is overwritten by the matrix H^*C if <i>side</i> = 'L', or C^*H if <i>side</i> = 'R'. |
|----------|---|

?large

Pre- and post-multiplies a real general matrix with a random orthogonal matrix.

Syntax

```
void slarge (lapack_int *n, float *a, lapack_int *lda, lapack_int *iseed, float * work,
lapack_int *info);
```

```
void dlarge (lapack_int *n, double *a, lapack_int *lda, lapack_int *iseed, double *
work, lapack_int *info);
```

```
void clarge (lapack_int *n, lapack_complex *a, lapack_int *lda, lapack_int *iseed,
lapack_complex * work, lapack_int *info);
```

```
void zlarge (lapack_int *n, lapack_complex_double *a, lapack_int *lda, lapack_int
*iseed, lapack_complex_double * work, lapack_int *info);
```

Include Files

- mkl.h

Description

The routine ?large pre- and post-multiplies a general *n*-by-*n* matrix *A* with a random orthogonal or unitary matrix: $A = U^*D^*U^T$.

Input Parameters

| | |
|--------------|---|
| <i>n</i> | The order of the matrix <i>A</i> . $n \geq 0$ |
| <i>a</i> | Array, size <i>lda</i> by <i>n</i> . On entry, the original <i>n</i> -by- <i>n</i> matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq n$. |
| <i>iseed</i> | Array, size 4. On entry, the seed of the random number generator. The array elements must be between 0 and 4095, and <i>iseed</i> [3] must be odd. |
| <i>work</i> | Workspace array, size $2*n$. |

Output Parameters

| | |
|--------------|--|
| <i>a</i> | On exit, <i>A</i> is overwritten by $U*A*U'$ for some random orthogonal matrix <i>U</i> . |
| <i>iseed</i> | On exit, the seed is updated. |
| <i>info</i> | If <i>info</i> = 0, the execution is successful. If <i>info</i> < 0, the <i>i</i> -th parameter had an illegal value. |

?larnd

Returns a random real number from a uniform or normal distribution.

Syntax

```
float slarnd (lapack_int *idist, lapack_int *iseed);
double dlarnd (lapack_int *idist, lapack_int *iseed);
```

The data types for complex variations depend on whether or not the application links with Gnu Fortran (gfortran) libraries.

For non-gfortran (libmkl_intel_*) interface libraries:

```
void clarnd (lapack_complex_float *res, lapack_int *idist, lapack_int *iseed);
void zlarnd (lapack_complex_double *res, lapack_int *idist, lapack_int *iseed);
```

For gfortran (libmkl_gf_*) interface libraries:

```
lapack_complex_float clarnd (lapack_int *idist, lapack_int *iseed);
lapack_complex_double zlarnd (lapack_int *idist, lapack_int *iseed);
```

To understand the difference between the non-gfortran and gfortran interfaces and when to use each of them, see Dynamic Libraries in the lib/intel64 Directory in the *Intel MKL Developer Guide*.

Include Files

- mkl.h

Description

The routine ?larnd returns a random number from a uniform or normal distribution.

Input Parameters

| | |
|--------------|---|
| <i>idist</i> | <p>Specifies the distribution of the random numbers. For <code>slarnv</code> and <code>dlarnv</code>:</p> <ul style="list-style-type: none"> = 1: uniform (0,1) = 2: uniform (-1,1) = 3: normal (0,1). <p>For <code>clarnv</code> and <code>zlarnv</code>:</p> <ul style="list-style-type: none"> = 1: real and imaginary parts each uniform (0,1) = 2: real and imaginary parts each uniform (-1,1) = 3: real and imaginary parts each normal (0,1) = 4: uniformly distributed on the disc $\text{abs}(z) \leq 1$ = 5: uniformly distributed on the circle $\text{abs}(z) = 1$ |
| <i>iseed</i> | <p>Array, size 4.</p> <p>On entry, the seed of the random number generator. The array elements must be between 0 and 4095, and <code>iseed[3]</code> must be odd.</p> |

Output Parameters

| | |
|--------------|-------------------------------|
| <i>iseed</i> | On exit, the seed is updated. |
|--------------|-------------------------------|

Return Values

The function returns a random number (for complex variations `libmkl_gf_*` interface layer/libraries return the result as the parameter `res`).

?larnv

Returns a vector of random numbers from a uniform or normal distribution.

Syntax

```
lapack_int LAPACKE_slarnv (lapack_int idist , lapack_int * iseed , lapack_int n , float
* x );

lapack_int LAPACKE_dlarnv (lapack_int idist , lapack_int * iseed , lapack_int n ,
double * x );

lapack_int LAPACKE_clarnv (lapack_int idist , lapack_int * iseed , lapack_int n ,
lapack_complex_float * x );

lapack_int LAPACKE_zlarnv (lapack_int idist , lapack_int * iseed , lapack_int n ,
lapack_complex_double * x );
```

Include Files

- `mkl.h`

Description

The routine `?larnv` returns a vector of n random real/complex numbers from a uniform or normal distribution.

This routine calls the auxiliary routine `?laruv` to generate random real numbers from a uniform (0,1) distribution, in batches of up to 128 using vectorisable code. The Box-Muller method is used to transform numbers from a uniform to a normal distribution.

Input Parameters

| | |
|--------------|--|
| <i>idist</i> | Specifies the distribution of the random numbers: for <code>slarnv</code> and <code>dlarnv</code> : = 1: uniform (0,1) = 2: uniform (-1,1) = 3: normal (0,1). for <code>clarv</code> and <code>zlarnv</code> : = 1: real and imaginary parts each uniform (0,1) = 2: real and imaginary parts each uniform (-1,1) = 3: real and imaginary parts each normal (0,1) = 4: uniformly distributed on the disc $abs(z) < 1$ = 5: uniformly distributed on the circle $abs(z) = 1$ |
| <i>iseed</i> | Array, size (4). On entry, the seed of the random number generator; the array elements must be between 0 and 4095, and <i>iseed</i> (4) must be odd. |
| <i>n</i> | The number of random numbers to be generated. |

Output Parameters

| | |
|--------------|---|
| <i>x</i> | Array, size (<i>n</i>). The generated random numbers. |
| <i>iseed</i> | On exit, the seed is updated. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

?laror

Pre- or post-multiplies an m-by-n matrix by a random orthogonal/unitary matrix.

Syntax

```
void slaror (char *side, char *init, lapack_int *m, lapack_int *n, float *a, lapack_int
*lda, lapack_int *iseed, float *x, lapack_int *info);
void dlaror (char *side, char *init, lapack_int *m, lapack_int *n, double *a,
lapack_int *lda, lapack_int *iseed, double *x, lapack_int *info);
void claror (char *side, char *init, lapack_int *m, lapack_int *n, lapack_complex *a,
lapack_int *lda, lapack_int *iseed, lapack_complex *x, lapack_int *info);
void zlaror (char *side, char *init, lapack_int *m, lapack_int *n,
lapack_complex_double *a, lapack_int *lda, lapack_int *iseed, lapack_complex_double *x,
lapack_int *info);
```

Include Files

- `mkl.h`

Description

The routine `?laror` pre- or post-multiplies an m -by- n matrix A by a random orthogonal or unitary matrix U , overwriting A . A may optionally be initialized to the identity matrix before multiplying by U . U is generated using the method of G.W. Stewart (SIAM J. Numer. Anal. 17, 1980, 403-409).

Input Parameters

| | |
|-------------|--|
| <i>side</i> | <p>Specifies whether A is multiplied by U on the left or right.</p> <p>for <code>slaror</code> and <code>dlaror</code>:</p> <p>If <i>side</i> = 'L', multiply A on the left (premultiply) by U.</p> <p>If <i>side</i> = 'R', multiply A on the right (postmultiply) by U^T.</p> <p>If <i>side</i> = 'C' or 'T', multiply A on the left by U and the right by U^T.</p> <p>for <code>claror</code> and <code>zlaror</code>:</p> <p>If <i>side</i> = 'L', multiply A on the left (premultiply) by U.</p> <p>If <i>side</i> = 'R', multiply A on the right (postmultiply) by $UC^>$.</p> <p>If <i>side</i> = 'C', multiply A on the left by U and the right by $UC^>$.</p> <p>If <i>side</i> = 'T', multiply A on the left by U and the right by U^T.</p> |
| <i>init</i> | <p>Specifies whether or not a should be initialized to the identity matrix.</p> <p>If <i>init</i> = 'I', initialize a to (a section of) the identity matrix before applying U.</p> <p>If <i>init</i> = 'N', no initialization. Apply U to the input matrix A.</p> <p><i>init</i> = 'I' generates square or rectangular orthogonal matrices:</p> <p>For $m = n$ and <i>side</i> = 'L' or 'R', the rows and the columns are orthogonal to each other.</p> <p>For rectangular matrices where $m < n$:</p> <ul style="list-style-type: none"> • If <i>side</i> = 'R', <code>?laror</code> produces a dense matrix in which rows are orthogonal and columns are not. • If <i>side</i> = 'L', <code>?laror</code> produces a matrix in which rows are orthogonal, first m columns are orthogonal, and remaining columns are zero. <p>For rectangular matrices where $m > n$:</p> <ul style="list-style-type: none"> • If <i>side</i> = 'L', <code>?laror</code> produces a dense matrix in which columns are orthogonal and rows are not. • If <i>side</i> = 'R', <code>?laror</code> produces a matrix in which columns are orthogonal, first m rows are orthogonal, and remaining rows are zero. |
| <i>m</i> | The number of rows of A . |
| <i>n</i> | The number of columns of A . |
| <i>a</i> | Array, size <i>lda</i> by n . |

lda The leading dimension of the array *a*.

$$lda \geq \max(1, m).$$

iseed Array, size (4).

On entry, specifies the seed of the random number generator. The array elements must be between 0 and 4095; if not they are reduced mod 4096. Also, *iseed*[3] must be odd.

x Workspace array, size $(3 * \max(m, n))$.

| Value of <i>side</i> | Length of workspace |
|----------------------|---------------------|
| 'L' | $2 * m + n$ |
| 'R' | $2 * n + m$ |
| 'C' or 'T' | $3 * n$ |

Output Parameters

a On exit, overwritten
 by *UA* (if *side* = 'L'),
 by *AU* (if *side* = 'R'),
 by *UAU^T* (if *side* = 'C' or 'T').

iseed The values of *iseed* are changed on exit, and can be used in the next call to continue the same random number sequence.

info Array, size (4).
 For *slaror* and *dlaror*:
 If *info* = 0, the execution is successful.
 If *info* < 0, the *i*-th parameter had an illegal value.
 If *info* = 1, the random numbers generated by *?laror* are bad.
 For *claror* and *zlaror*:
 If *info* = 0, the execution is successful.
 If *info* = -1, *side* is not 'L', 'R', 'C', or 'T'.
 If *info* = -3, if *m* is negative.
 If *info* = -4, if *m* is negative or if *side* is 'C' or 'T' and *n* is not equal to *m*.
 If *info* = -6, if *lda* is less than *m*.

?larot

Applies a Givens rotation to two adjacent rows or columns.

If *lrows* = 0, *?larot* rotates two columns.

lleft

If *lleft* = 1, *xleft* is used instead of the corresponding element of *a* for the first element in the second row (if *lrows* = 0) or column (if *lrows*=1).

If *lleft* = 0, the corresponding element of *a* is used.

lright

If *lleft* = 1, *xright* is used instead of the corresponding element of *a* for the first element in the second row (if *lrows* = 0) or column (if *lrows*=1).

If *lright* = 0, the corresponding element of *a* is used.

nl

The length of the rows (if *lrows*=1) or columns (if *lrows*=1) to be rotated.

If *xleft* or *xright* are used, the columns or rows they are in should be included in *nl*, e.g., if *lleft* = *lright* = 1, then *nl* must be at least 2.

The number of rows or columns to be rotated exclusive of those involving *xleft* and/or *xright* may not be negative, i.e., *nl* minus how many of *lleft* and *lright* are 1 must be at least zero; if not, *xerbla* is called.

c, s

Specify the Givens rotation to be applied.

If *lrows* = 1, then the matrix

$$\begin{bmatrix} c & s \\ -s & c \end{bmatrix}$$

is applied from the left.

If *lrows* = 0, then the transpose thereof is applied from the right.

a

The array containing the rows or columns to be rotated. The first element of *a* should be the upper left element to be rotated.

lda

The "effective" leading dimension of *a*.

If *a* contains a matrix stored in GE or SY format, then this is just the leading dimension of *A*.

If *a* contains a matrix stored in band (GB or SB) format, then this should be one less than the leading dimension used in the calling routine. Thus, if *a* in *?larot* is of size *lda***n*, then *a[(j - 1)*lda]* would be the *j*-th element in the first of the two rows to be rotated, and *a[(j - 1)*lda + 1]* would be the *j*-th in the second, regardless of how the array may be stored in the calling routine. *a* cannot be dimensioned, because for band format the row number may exceed *lda*, which is not legal FORTRAN.

If *lrows* = 1, then *lda* must be at least 1, otherwise it must be at least *nl* minus the number of 1 values in *xleft* and *xright*.

xleft

If *lrows* = 1, *xleft* is used and modified instead of *a[1]* (if *lrows* = 1) or *a[lda + 1]* (if *lrows* = 0).

xright

If *lright* = 1, *xright* is used and modified instead of *a[(nl - 1)*lda]* (if *lrows* = 1) or *a[nl - 1]* (if *lrows* = 0).

Output Parameters

a On exit, modified array *A*.

?lartgp

Generates a plane rotation.

Syntax

```
lapack_int LAPACKE_slartgp (float f, float g, float* cs, float* sn, float* r);
lapack_int LAPACKE_dlartgp (double f, double g, double* cs, double* sn, double* r);
```

Include Files

- mkl.h

Description

The routine generates a plane rotation so that

$$\begin{bmatrix} cs & sn \\ -sn & cs \end{bmatrix} \cdot \begin{bmatrix} f \\ g \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

where $cs^2 + sn^2 = 1$

This is a slower, more accurate version of the BLAS Level 1 routine ?rotg, except for the following differences:

- *f* and *g* are unchanged on return.
- If *g*=0, then *cs*=(+/-)1 and *sn*=0.
- If *f*=0 and *g*≠ 0, then *cs*=0 and *sn*=(+/-)1.

The sign is chosen so that $r \geq 0$.

Input Parameters

f, g The first and second component of the vector to be rotated.

Output Parameters

cs The cosine of the rotation.

sn The sine of the rotation.

r The nonzero component of the rotated vector.

Return Values

If *info* = 0, the execution is successful.

If *info* = -1, *f* is NaN.

If *info* = -2, *g* is NaN.

See Also

[cblas_?rotg](#)
[?lartgs](#)

?lartgs

Generates a plane rotation designed to introduce a bulge in implicit QR iteration for the bidiagonal SVD problem.

Syntax

```
lapack_int LAPACKE_slartgs (floatx, floaty, floatsigma, float* cs, float* sn);
lapack_int LAPACKE_dlartgs (doublex, doubley, doublesigma, double* cs, double* sn);
```

Include Files

- `mkl.h`

Description

The routine generates a plane rotation designed to introduce a bulge in Golub-Reinsch-style implicit QR iteration for the bidiagonal SVD problem. x and y are the top-row entries, and $sigma$ is the shift. The computed cs and sn define a plane rotation that satisfies the following:

$$\begin{bmatrix} cs & sn \\ -sn & cs \end{bmatrix} \cdot \begin{bmatrix} x^2 - sigma \\ x * y \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

with r nonnegative.

If $x^2 - sigma$ and $x * y$ are 0, the rotation is by $\pi/2$

Input Parameters

| | |
|---------|--|
| x, y | The (1,1) and (1,2) entries of an upper bidiagonal matrix, respectively. |
| $sigma$ | Shift |

Output Parameters

| | |
|------|-----------------------------|
| cs | The cosine of the rotation. |
| sn | The sine of the rotation. |

Return Values

If $info = 0$, the execution is successful.

If $info = - 1$, x is NaN.

If $info = - 2$, y is NaN.

If $info = - 3$, $sigma$ is NaN.

See Also

[?lartgp](#)

?lascl

Multiplies a general rectangular matrix by a real scalar defined as c_{to}/c_{from} .

Syntax

```
lapack_int LAPACKE_slascl (int matrix_layout, char type, lapack_int kl, lapack_int ku, float cfrom, float cto, lapack_int m, lapack_int n, float * a, lapack_int lda);
```

```
lapack_int LAPACKE_dlascl (int matrix_layout, char type, lapack_int kl, lapack_int ku, double cfrom, double cto, lapack_int m, lapack_int n, double * a, lapack_int lda);
```

```
lapack_int LAPACKE_clascl (int matrix_layout, char type, lapack_int kl, lapack_int ku, float cfrom, float cto, lapack_int m, lapack_int n, lapack_complex_float * a, lapack_int lda);
```

```
lapack_int LAPACKE_zlascl (int matrix_layout, char type, lapack_int kl, lapack_int ku, double cfrom, double cto, lapack_int m, lapack_int n, lapack_complex_double * a, lapack_int lda);
```

Include Files

- mkl.h

Description

The routine ?lascl multiplies the m -by- n real/complex matrix A by the real scalar c_{to}/c_{from} . The operation is performed without over/underflow as long as the final result $c_{to} * A(i, j) / c_{from}$ does not over/underflow.

type specifies that A may be full, upper triangular, lower triangular, upper Hessenberg, or banded.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>type</i> | This parameter specifies the storage type of the input matrix. = 'G': A is a full matrix. = 'L': A is a lower triangular matrix. = 'U': A is an upper triangular matrix. = 'H': A is an upper Hessenberg matrix. = 'B': A is a symmetric band matrix with lower bandwidth kl and upper bandwidth ku and with the only the lower half stored = 'Q': A is a symmetric band matrix with lower bandwidth kl and upper bandwidth ku and with the only the upper half stored. = 'Z': A is a band matrix with lower bandwidth kl and upper bandwidth ku . See description of the ?gbtrf function for storage details. |
| <i>kl</i> | The lower bandwidth of A . Referenced only if <i>type</i> = 'B', 'Q' or 'Z'. |
| <i>ku</i> | The upper bandwidth of A . Referenced only if <i>type</i> = 'B', 'Q' or 'Z'. |

| | |
|-------------------|--|
| <i>cfrom, cto</i> | The matrix A is multiplied by $cto/cfrom$. $A(i, j)$ is computed without over/underflow if the final result $cto * A(i, j) / cfrom$ can be represented without over/underflow. $cfrom$ must be nonzero. |
| <i>m</i> | The number of rows of the matrix A . $m \geq 0$. |
| <i>n</i> | The number of columns of the matrix A . $n \geq 0$. |
| <i>a</i> | Array, size $(lda * n)$. The matrix to be multiplied by $cto/cfrom$. See <i>type</i> for the storage type. |
| <i>lda</i> | The leading dimension of the array a . $lda \geq \max(1, m)$. |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | The multiplied matrix A . |
| <i>info</i> | If $info = 0$ - successful exit If $info = -i < 0$, the i -th argument had an illegal value. |

See Also

[?gbtrf](#)

?laset

Initializes the off-diagonal elements and the diagonal elements of a matrix to given values.

Syntax

```
lapack_int LAPACKE_slaset (int matrix_layout , char uplo , lapack_int m , lapack_int n , float alpha , float beta , float * a , lapack_int lda );
```

```
lapack_int LAPACKE_dlaset (int matrix_layout , char uplo , lapack_int m , lapack_int n , double alpha , double beta , double * a , lapack_int lda );
```

```
lapack_int LAPACKE_claset (int matrix_layout , char uplo , lapack_int m , lapack_int n , lapack_complex_float alpha , lapack_complex_float beta , lapack_complex_float * a , lapack_int lda );
```

```
lapack_int LAPACKE_zlaset (int matrix_layout , char uplo , lapack_int m , lapack_int n , lapack_complex_double alpha , lapack_complex_double beta , lapack_complex_double * a , lapack_int lda );
```

Include Files

- `mkl.h`

Description

The routine initializes an m -by- n matrix A to $beta$ on the diagonal and $alpha$ on the off-diagonals.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Specifies the part of the matrix <i>A</i> to be set. If <i>uplo</i> = 'U', upper triangular part is set; the strictly lower triangular part of <i>A</i> is not changed. If <i>uplo</i> = 'L': lower triangular part is set; the strictly upper triangular part of <i>A</i> is not changed. Otherwise: All of the matrix <i>A</i> is set. |
| <i>m</i> | The number of rows of the matrix <i>A</i> . $m \geq 0$. |
| <i>n</i> | The number of columns of the matrix <i>A</i> . $n \geq 0$. |
| <i>alpha, beta</i> | The constants to which the off-diagonal and diagonal elements are to be set, respectively. |
| <i>a</i> | Array, size at least $\max(1, lda*n)$ for column major and $\max(1, lda*m)$ for row major layout. The array <i>a</i> contains the <i>m</i> -by- <i>n</i> matrix <i>A</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1, m)$ for column major layout and $lda \geq \max(1, n)$ for row major layout. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, the leading <i>m</i> -by- <i>n</i> submatrix of <i>A</i> is set as follows: if <i>uplo</i> = 'U', $A_{ij} = \alpha, 1 \leq i \leq j-1, 1 \leq j \leq n$, if <i>uplo</i> = 'L', $A_{ij} = \alpha, j+1 \leq i \leq m, 1 \leq j \leq n$, otherwise, $A_{ij} = \alpha, 1 \leq i \leq m, 1 \leq j \leq n, i \neq j$, and, for all <i>uplo</i> , $A_{ij} = \beta, 1 \leq i \leq \min(m, n)$. |
|----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = *i* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?lasrt

Sorts numbers in increasing or decreasing order.

Syntax

```
lapack_int LAPACKE_slasrt (char id , lapack_int n , float * d );
lapack_int LAPACKE_dlasrt (char id , lapack_int n , double * d );
```

Include Files

- `mkl.h`

Description

The routine `?lasrt` sorts the numbers in d in increasing order (if $id = 'I'$) or in decreasing order (if $id = 'D'$). It uses Quick Sort, reverting to Insertion Sort on arrays of size ≤ 20 . Dimension of `stack` limits n to about 2^{32} .

Input Parameters

| | |
|------|--|
| id | = 'I': sort d in increasing order; = 'D': sort d in decreasing order. |
| n | The length of the array d . |
| d | On entry, the array to be sorted. |

Output Parameters

| | |
|-----|--|
| d | On exit, d has been sorted into increasing order ($d[0] \leq d[1] \leq \dots \leq d[n-1]$) or into decreasing order ($d[0] \geq d[1] \geq \dots \geq d[n-1]$), depending on id . |
|-----|--|

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info < 0$, the i -th parameter had an illegal value.

?laswp

Performs a series of row interchanges on a general rectangular matrix.

Syntax

```
lapack_int LAPACKE_slaswp (int matrix_layout , lapack_int n , float * a , lapack_int
lda , lapack_int k1 , lapack_int k2 , const lapack_int * ipiv , lapack_int incx );
lapack_int LAPACKE_dlaswp (int matrix_layout , lapack_int n , double * a , lapack_int
lda , lapack_int k1 , lapack_int k2 , const lapack_int * ipiv , lapack_int incx );
lapack_int LAPACKE_claswp (int matrix_layout , lapack_int n , lapack_complex_float *
a , lapack_int lda , lapack_int k1 , lapack_int k2 , const lapack_int * ipiv ,
lapack_int incx );
lapack_int LAPACKE_zlaswp (int matrix_layout , lapack_int n , lapack_complex_double *
a , lapack_int lda , lapack_int k1 , lapack_int k2 , const lapack_int * ipiv ,
lapack_int incx );
```

Include Files

- `mkl.h`

Description

The routine performs a series of row interchanges on the matrix A . One row interchange is initiated for each of rows $k1$ through $k2$ of A .

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The number of columns of the matrix A . |
| <i>a</i> | Array, size $\max(1, lda*n)$ for column major and $\max(1, lda*mm)$ for row major layout. Here mm is not less than maximum of values $ipiv[k1-1+j* incx]$, $0 \leq j < k2-k1$. Array a contains the m -by- n matrix A . |
| <i>lda</i> | The leading dimension of the array a . |
| <i>k1</i> | The first element of $ipiv$ for which a row interchange will be done. |
| <i>k2</i> | The last element of $ipiv$ for which a row interchange will be done. |
| <i>ipiv</i> | Array, size $k1+(k2-k1)* incx $. The vector of pivot indices. Only the elements in positions $k1$ through $k2$ of $ipiv$ are accessed. $ipiv(k) = l$ implies rows k and l are to be interchanged. |
| <i>incx</i> | The increment between successive values of $ipiv$. If $ipiv$ is negative, the pivots are applied in reverse order. |

Output Parameters

| | |
|----------|-------------------------------|
| <i>a</i> | On exit, the permuted matrix. |
|----------|-------------------------------|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?latm1

Computes the entries of a matrix as specified.

Syntax

```
void slatm1 (lapack_int *mode, *cond, lapack_int *irsign, lapack_int *idist, lapack_int *iseed, float *d, lapack_int *n, lapack_int *info);
```

```
void dlatm1 (lapack_int *mode, *cond, lapack_int *irsign, lapack_int *idist, lapack_int *iseed, double *d, lapack_int *n, lapack_int *info);
```

```
void clatm1 (lapack_int *mode, *cond, lapack_int *irsign, lapack_int *idist, lapack_int
*iseed, lapack_complex *d, lapack_int *n, lapack_int *info);

void zlatm1 (lapack_int *mode, *cond, lapack_int *irsign, lapack_int *idist, lapack_int
*iseed, lapack_complex_double *d, lapack_int *n, lapack_int *info);
```

Include Files

- mkl.h

Description

The ?latm1 routine computes the entries of $D(1..n)$ as specified by *mode*, *cond* and *irsign*. *idist* and *iseed* determine the generation of random numbers.

?latm1 is called by slatmr (for slatm1 and dlatm1), and by clatmr (for clatm1 and zlatm1) to generate random test matrices for LAPACK programs.

Input Parameters

| | |
|---------------|---|
| <i>mode</i> | <p>On entry describes how <i>d</i> is to be computed:</p> <p><i>mode</i> = 0 means do not change <i>d</i>.</p> <p><i>mode</i> = 1 sets $d[0] = 1$ and $d[1:n - 1] = 1.0/cond$</p> <p><i>mode</i> = 2 sets $d[0:n - 2] = 1$ and $d[n - 1] = 1.0/cond$</p> <p><i>mode</i> = 3 sets $d[i - 1] = cond^{-(i-1)/(n-1)}$</p> <p><i>mode</i> = 4 sets $d[i - 1] = 1 - (i-1)/(n-1) * (1 - 1/cond)$</p> <p><i>mode</i> = 5 sets <i>d</i> to random numbers in the range ($1/cond$, 1) such that their logarithms are uniformly distributed.</p> <p><i>mode</i> = 6 sets <i>d</i> to random numbers from same distribution as the rest of the matrix.</p> <p><i>mode</i> < 0 has the same meaning as $abs(mode)$, except that the order of the elements of <i>d</i> is reversed.</p> <p>Thus if <i>mode</i> is positive, <i>d</i> has entries ranging from 1 to $1/cond$, if negative, from $1/cond$ to 1.</p> |
| <i>cond</i> | On entry, used as described under <i>mode</i> above. If used, it must be ≥ 1 . |
| <i>irsign</i> | <p>On entry, if <i>mode</i> is not -6, 0, or 6, determines sign of entries of <i>d</i>.</p> <p>If <i>irsign</i> = 0, entries of <i>d</i> are unchanged.</p> <p>If <i>irsign</i> = 1, each entry of <i>d</i> is multiplied by a random complex number uniformly distributed with absolute value 1.</p> |
| <i>idist</i> | <p>Specifies the distribution of the random numbers.</p> <p>For slatm1 and dlatm1:</p> <p>= 1: uniform (0,1)</p> <p>= 2: uniform (-1,1)</p> <p>= 3: normal (0,1)</p> <p>For clatm1 and zlatm1:</p> |

- = 1: real and imaginary parts each uniform (0,1)
- = 2: real and imaginary parts each uniform (-1,1)
- = 3: real and imaginary parts each normal (0,1)
- = 4: complex number uniform in disk(0, 1)

| | |
|--------------|---|
| <i>iseed</i> | Array, size (4). Specifies the seed of the random number generator. The random number generator uses a linear congruential sequence limited to small integers, and so should produce machine independent random numbers. The values of <i>iseed</i> [3] are changed on exit, and can be used in the next call to ?latm1 to continue the same random number sequence. |
| <i>d</i> | Array, size <i>n</i> . |
| <i>n</i> | Number of entries of <i>d</i> . |

Output Parameters

| | |
|--------------|--|
| <i>iseed</i> | On exit, the seed is updated. |
| <i>d</i> | On exit, <i>d</i> is updated, unless <i>mode</i> = 0. |
| <i>info</i> | If <i>info</i> = 0, the execution is successful. If <i>info</i> = -1, <i>mode</i> is not in range -6 to 6. If <i>info</i> = -2, <i>mode</i> is neither -6, 0 nor 6, and <i>irsign</i> is neither 0 nor 1. If <i>info</i> = -3, <i>mode</i> is neither -6, 0 nor 6 and <i>cond</i> is less than 1. If <i>info</i> = -4, <i>mode</i> equals 6 or -6 and <i>idist</i> is not in range 1 to 4. If <i>info</i> = -7, <i>n</i> is negative. |

?latm2

Returns an entry of a random matrix.

Syntax

```
float slatm2 (lapack_int *m, lapack_int *n, lapack_int *i, lapack_int *j, lapack_int
*kl, lapack_int *ku, lapack_int *idist, lapack_int *iseed, float *d, lapack_int
*igrade, float *dl, float *dr, lapack_int *ipvtng, lapack_int *iwork, float *sparse);

double dlatm2 (lapack_int *m, lapack_int *n, lapack_int *i, lapack_int *j, lapack_int
*kl, lapack_int *ku, lapack_int *idist, lapack_int *iseed, double *d, lapack_int
*igrade, double *dl, double *dr, lapack_int *ipvtng, lapack_int *iwork, double
*sparse);
```

The data types for complex variations depend on whether or not the application links with Gnu Fortran (gfortran) libraries.

For non-gfortran (libmkl_intel_*) interface libraries:

```
void clatm2 (lapack_complex_float *res, lapack_int *m, lapack_int *n, lapack_int *i,
lapack_int *j, lapack_int *kl, lapack_int *ku, lapack_int *idist, lapack_int *iseed,
lapack_complex_float *d, lapack_int *igrade, lapack_complex_float *dl,
lapack_complex_float *dr, lapack_int *ipvtng, lapack_int *iwork, float *sparse);
```

```
void zlatm2 (lapack_complex_double *res, lapack_int *m, lapack_int *n, lapack_int *i,
lapack_int *j, lapack_int *kl, lapack_int *ku, lapack_int *idist, lapack_int *iseed,
lapack_complex_double *d, lapack_int *igrade, lapack_complex_double *dl,
lapack_complex_double *dr, lapack_int *ipvtng, lapack_int *iwork, double *sparse);
```

For gfortran (libmkl_gf_*) interface libraries:

```
lapack_complex_float clatm2 (lapack_int *m, lapack_int *n, lapack_int *i, lapack_int
*j, lapack_int *kl, lapack_int *ku, lapack_int *idist, lapack_int *iseed,
lapack_complex_float *d, lapack_int *igrade, lapack_complex_float *dl,
lapack_complex_float *dr, lapack_int *ipvtng, lapack_int *iwork, float *sparse);

lapack_complex_double zlatm2 (lapack_int *m, lapack_int *n, lapack_int *i, lapack_int
*j, lapack_int *kl, lapack_int *ku, lapack_int *idist, lapack_int *iseed,
lapack_complex_double *d, lapack_int *igrade, lapack_complex_double *dl,
lapack_complex_double *dr, lapack_int *ipvtng, lapack_int *iwork, double *sparse);
```

To understand the difference between the non-gfortran and gfortran interfaces and when to use each of them, see Dynamic Libraries in the lib/intel64 Directory in the *Intel MKL Developer Guide*.

Include Files

- mkl.h

Description

The `?latm2` routine returns entry (i, j) of a random matrix of dimension (m, n) . It is called by the `?latmr` routine in order to build random test matrices. No error checking on parameters is done, because this routine is called in a tight loop by `?latmr` which has already checked the parameters.

Use of `?latm2` differs from `?latm3` in the order in which the random number generator is called to fill in random matrix entries. With `?latm2`, the generator is called to fill in the pivoted matrix columnwise. With `?latm3`, the generator is called to fill in the matrix columnwise, after which it is pivoted. Thus, `?latm3` can be used to construct random matrices which differ only in their order of rows and/or columns. `?latm2` is used to construct band matrices while avoiding calling the random number generator for entries outside the band (and therefore generating random numbers).

The matrix whose (i, j) entry is returned is constructed as follows (this routine only computes one entry):

- If i is outside $(1..m)$ or j is outside $(1..n)$, returns zero (this is convenient for generating matrices in band format).
- Generate a matrix A with random entries of distribution $idist$.
- Set the diagonal to D .
- Grade the matrix, if desired, from the left (by dl) and/or from the right (by dr or dl) as specified by $igrade$.
- Permute, if desired, the rows and/or columns as specified by $ipvtng$ and $iwork$.
- Band the matrix to have lower bandwidth kl and upper bandwidth ku .
- Set random entries to zero as specified by $sparse$.

Input Parameters

| | |
|-----|-------------------------------------|
| m | Number of rows of the matrix. |
| n | Number of columns of the matrix. |
| i | Row of the entry to be returned. |
| j | Column of the entry to be returned. |

| | |
|---------------|---|
| <i>kl</i> | Lower bandwidth. |
| <i>ku</i> | Upper bandwidth. |
| <i>idist</i> | On entry, <i>idist</i> specifies the type of distribution to be used to generate a random matrix . for <i>slatm2</i> and <i>dlatm2</i> : = 1: uniform (0,1) = 2: uniform (-1,1) = 3: normal (0,1) for <i>clatm2</i> and <i>zlatm2</i> : = 1: real and imaginary parts each uniform (0,1) = 2: real and imaginary parts each uniform (-1,1) = 3: real and imaginary parts each normal (0,1) = 4: complex number uniform in disk (0, 1) |
| <i>iseed</i> | Array, size 4. Seed for the random number generator. |
| <i>d</i> | Array, size $(\min(i, j))$. Diagonal entries of matrix. |
| <i>igrade</i> | Specifies grading of matrix as follows: = 0: no grading = 1: matrix premultiplied by $\text{diag}(dl)$ = 2: matrix postmultiplied by $\text{diag}(dr)$ = 3: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(dr)$ = 4: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{inv}(\text{diag}(dl))$ For <i>slatm2</i> and <i>slatm2</i> : = 5: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(dl)$ For <i>clatm2</i> and <i>zlatm2</i> : = 5: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(\text{conjg}(dl))$ = 6: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(dl)$ |
| <i>dl</i> | Array, size $(i \text{ or } j)$, as appropriate. Left scale factors for grading matrix. |
| <i>dr</i> | Array, size $(i \text{ or } j)$, as appropriate. Right scale factors for grading matrix. |
| <i>ipvtng</i> | On entry specifies pivoting permutations as follows: = 0: none = 1: row pivoting |

- = 2: column pivoting
- = 3: full pivoting, i.e., on both sides

iwork Array, size (*i* or *j*), as appropriate. This array specifies the permutation used. The row (or column) in position *k* was originally in position *iwork*[*k* - 1]. This differs from *iwork* for ?latm3.

sparse Specifies the sparsity of the matrix. If sparse matrix is to be generated, *sparse* should lie between 0 and 1. A uniform (0, 1) random number *x* is generated and compared to *sparse*. If *x* is larger the matrix entry is unchanged and if *x* is smaller the entry is set to zero. Thus on the average a fraction *sparse* of the entries will be set to zero.

Output Parameters

iseed On exit, the seed is updated.

Return Values

The function returns an entry of a random matrix (for complex variations libmkl_gf_* interface layer/ libraries return the result as the parameter *res*).

?latm3

Returns set entry of a random matrix.

Syntax

```
float slatm3 (lapack_int *m, lapack_int *n, lapack_int *i, lapack_int *j, lapack_int
*isub, lapack_int *jsub, lapack_int *kl, lapack_int *ku, lapack_int *idist, lapack_int
*iseed, float *d, lapack_int *igrade, float *dl, float *dr, lapack_int *ipvtng,
lapack_int *iwork, float *sparse);
```

```
double dlatm3 (lapack_int *m, lapack_int *n, lapack_int *i, lapack_int *j, lapack_int
*isub, lapack_int *jsub, lapack_int *kl, lapack_int *ku, lapack_int *idist, lapack_int
*iseed, double *d, lapack_int *igrade, double *dl, double *dr, lapack_int *ipvtng,
lapack_int *iwork, double *sparse);
```

The data types for complex variations depend on whether or not the application links with Gnu Fortran (gfortran) libraries.

For non-gfortran (libmkl_intel_*) interface libraries:

```
void clatm3 (lapack_complex_float *res, lapack_int *m, lapack_int *n, lapack_int *i,
lapack_int *j, lapack_int *isub, lapack_int *jsub, lapack_int *kl, lapack_int *ku,
lapack_int *idist, lapack_int *iseed, lapack_complex_float *d, lapack_int *igrade,
lapack_complex_float *dl, lapack_complex_float *dr, lapack_int *ipvtng, lapack_int
*iwork, float *sparse);
```

```
void zlatm3 (lapack_complex_double *res, lapack_int *m, lapack_int *n, lapack_int *i,
lapack_int *j, lapack_int *isub, lapack_int *jsub, lapack_int *kl, lapack_int *ku,
lapack_int *idist, lapack_int *iseed, lapack_complex_double *d, lapack_int *igrade,
lapack_complex_double *dl, lapack_complex_double *dr, lapack_int *ipvtng, lapack_int
*iwork, double *sparse);
```

For gfortran (libmkl_gf_*) interface libraries:

```
lapack_complex_float clatm3 (lapack_int *m, lapack_int *n, lapack_int *i, lapack_int
*j, lapack_int *isub, lapack_int *jsub, lapack_int *kl, lapack_int *ku, lapack_int
*idist, lapack_int *iseed, lapack_complex_float *d, lapack_int *igrade,
lapack_complex_float *dl, lapack_complex_float *dr, lapack_int *ipvtng, lapack_int
*iwork, float *sparse);
```

```
lapack_complex_double zlatm3 (lapack_int *m, lapack_int *n, lapack_int *i, lapack_int
*j, lapack_int *isub, lapack_int *jsub, lapack_int *kl, lapack_int *ku, lapack_int
*idist, lapack_int *iseed, lapack_complex_double *d, lapack_int *igrade,
lapack_complex_double *dl, lapack_complex_double *dr, lapack_int *ipvtng, lapack_int
*iwork, double *sparse);
```

To understand the difference between the non-gfortran and gfortran interfaces and when to use each of them, see Dynamic Libraries in the lib/intel64 Directory in the *Intel MKL Developer Guide*.

Include Files

- mkl.h

Description

The `?latm3` routine returns the $(isub, jsub)$ entry of a random matrix of dimension (m, n) described by the other parameters. $(isub, jsub)$ is the final position of the (i, j) entry after pivoting according to *ipvtng* and *iwork*. `?latm3` is called by the `?latmr` routine in order to build random test matrices. No error checking on parameters is done, because this routine is called in a tight loop by `?latmr` which has already checked the parameters.

Use of `?latm3` differs from `?latm2` in the order in which the random number generator is called to fill in random matrix entries. With `?latm2`, the generator is called to fill in the pivoted matrix columnwise. With `?latm3`, the generator is called to fill in the matrix columnwise, after which it is pivoted. Thus, `?latm3` can be used to construct random matrices which differ only in their order of rows and/or columns. `?latm2` is used to construct band matrices while avoiding calling the random number generator for entries outside the band (and therefore generating random numbers in different orders for different pivot orders).

The matrix whose $(isub, jsub)$ entry is returned is constructed as follows (this routine only computes one entry):

- If *isub* is outside $(1..m)$ or *jsub* is outside $(1..n)$, returns zero (this is convenient for generating matrices in band format).
- Generate a matrix *A* with random entries of distribution *idist*.
- Set the diagonal to *D*.
- Grade the matrix, if desired, from the left (by *dl*) and/or from the right (by *dr* or *dl*) as specified by *igrade*.
- Permute, if desired, the rows and/or columns as specified by *ipvtng* and *iwork*.
- Band the matrix to have lower bandwidth *kl* and upper bandwidth *ku*.
- Set random entries to zero as specified by *sparse*.

Input Parameters

| | |
|----------|---|
| <i>m</i> | Number of rows of matrix. |
| <i>n</i> | Number of columns of matrix. |
| <i>i</i> | Row of unpivoted entry to be returned. |
| <i>j</i> | Column of unpivoted entry to be returned. |

| | |
|---------------|---|
| <i>isub</i> | Row of pivoted entry to be returned. |
| <i>jsub</i> | Column of pivoted entry to be returned. |
| <i>kl</i> | Lower bandwidth. |
| <i>ku</i> | Upper bandwidth. |
| <i>idist</i> | <p>On entry, <i>idist</i> specifies the type of distribution to be used to generate a random matrix.</p> <p>for <i>slatm2</i> and <i>dlatm2</i>:</p> <ul style="list-style-type: none"> = 1: uniform (0,1) = 2: uniform (-1,1) = 3: normal (0,1) <p>for <i>clatm2</i> and <i>zlatm2</i>:</p> <ul style="list-style-type: none"> = 1: real and imaginary parts each uniform (0,1) = 2: real and imaginary parts each uniform (-1,1) = 3: real and imaginary parts each normal (0,1) = 4: complex number uniform in disk(0, 1) |
| <i>iseed</i> | <p>Array, size 4.</p> <p>Seed for random number generator.</p> |
| <i>d</i> | Array, size $(\min(i, j))$. Diagonal entries of matrix. |
| <i>igrade</i> | <p>Specifies grading of matrix as follows:</p> <ul style="list-style-type: none"> = 0: no grading = 1: matrix premultiplied by $\text{diag}(dl)$ = 2: matrix postmultiplied by $\text{diag}(dr)$ = 3: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(dr)$ = 4: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{inv}(\text{diag}(dl))$ <p>For <i>slatm2</i> and <i>slatm2</i>:</p> <ul style="list-style-type: none"> = 5: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(dl)$ <p>For <i>clatm2</i> and <i>zlatm2</i>:</p> <ul style="list-style-type: none"> = 5: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(\text{conjg}(dl))$ = 6: matrix premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(dl)$ |
| <i>dl</i> | <p>Array, size $(i$ or j, as appropriate).</p> <p>Left scale factors for grading matrix.</p> |
| <i>dr</i> | <p>Array, size $(i$ or j, as appropriate).</p> <p>Right scale factors for grading matrix.</p> |

| | |
|---------------|---|
| <i>ipvtng</i> | On entry specifies pivoting permutations as follows: If <i>ipvtng</i> = 0: none. If <i>ipvtng</i> = 1: row pivoting. If <i>ipvtng</i> = 2: column pivoting. If <i>ipvtng</i> = 3: full pivoting, i.e., on both sides. |
| <i>sparse</i> | On entry, specifies the sparsity of the matrix if sparse matrix is to be generated. <i>sparse</i> should lie between 0 and 1. A uniform(0, 1) random number <i>x</i> is generated and compared to <i>sparse</i> ; if <i>x</i> is larger the matrix entry is unchanged and if <i>x</i> is smaller the entry is set to zero. Thus on the average a fraction <i>sparse</i> of the entries will be set to zero. |
| <i>iwork</i> | Array, size (<i>i</i> or <i>j</i> , as appropriate). This array specifies the permutation used. The row (or column) originally in position <i>k</i> is in position <i>iwork</i> [<i>k</i> - 1] after pivoting. This differs from <i>iwork</i> for ?latm2. |

Output Parameters

| | |
|--------------|--|
| <i>isub</i> | On exit, row of pivoted entry is updated. |
| <i>jsub</i> | On exit, column of pivoted entry is updated. |
| <i>iseed</i> | On exit, the seed is updated. |

Return Values

The function returns an entry of a random matrix (for complex variations libmkl_gf_* interface layer/libraries return the result as the parameter *res*).

?latm5

Generates matrices involved in the Generalized Sylvester equation.

Syntax

```
void slatm5 (*prtype, lapack_int *m, lapack_int *n, float *a, lapack_int *lda, float
*b, lapack_int *ldb, float *c, lapack_int *ldc, float *d, lapack_int *ldd, float *e,
lapack_int *lde, float *f, lapack_int *ldf, float *r, lapack_int *ldr, float *l,
lapack_int *ldl, float *alpha, lapack_int *qblcka, lapack_int *qblckb);
```

```
void dlatm5 (*prtype, lapack_int *m, lapack_int *n, double *a, lapack_int *lda, double
*b, lapack_int *ldb, double *c, lapack_int *ldc, double *d, lapack_int *ldd, double
*e, lapack_int *lde, double *f, lapack_int *ldf, double *r, lapack_int *ldr, double
*l, lapack_int *ldl, double *alpha, lapack_int *qblcka, lapack_int *qblckb);
```

```
void clatm5 (*prtype, lapack_int *m, lapack_int *n, lapack_complex_float *a, lapack_int
*lda, lapack_complex_float *b, lapack_int *ldb, lapack_complex_float *c, lapack_int
*ldc, lapack_complex_float *d, lapack_int *ldd, lapack_complex_float *e, lapack_int
*lde, lapack_complex_float *f, lapack_int *ldf, lapack_complex_float *r, lapack_int
*ldr, lapack_complex_float *l, lapack_int *ldl, float *alpha, lapack_int *qblcka,
lapack_int *qblckb);
```

```
void zlatm5 (*prtype, lapack_int *m, lapack_int *n, lapack_complex_double *a,
lapack_int *lda, lapack_complex_double *b, lapack_int *ldb, lapack_complex_double *c,
lapack_int *ldc, lapack_complex_double *d, lapack_int *ldd, lapack_complex_double *e,
lapack_int *lde, lapack_complex_double *f, lapack_int *ldf, lapack_complex_double *r,
lapack_int *ldr, lapack_complex_double *l, lapack_int *ldl, float *alpha, lapack_int
*qblkca, lapack_int *qblkcb);
```

Include Files

- mkl.h

Description

The `zlatm5` routine generates matrices involved in the Generalized Sylvester equation:

$$A * R - L * B = C$$

$$D * R - L * E = F$$

They also satisfy the diagonalization condition:

$$\begin{bmatrix} I & -L \\ & I \end{bmatrix} \begin{bmatrix} A & -C \\ & B \end{bmatrix} \begin{bmatrix} I & R \\ & I \end{bmatrix} = \begin{bmatrix} A & \\ & B \end{bmatrix}$$

$$\begin{bmatrix} I & -L \\ & I \end{bmatrix} \begin{bmatrix} D & -F \\ & E \end{bmatrix} \begin{bmatrix} I & R \\ & I \end{bmatrix} = \begin{bmatrix} D & \\ & E \end{bmatrix}$$

Input Parameters

prtype

Specifies the type of matrices to generate.

- If *prtype* = 1, *A* and *B* are Jordan blocks, *D* and *E* are identity matrices.

A:

If (*i* == *j*) then $A_{i,j} = 1.0$.

If (*j* == *i* + 1) then $A_{i,j} = -1.0$.

Otherwise $A_{i,j} = 0.0$, *i*, *j* = 1...*m*

B:

If (*i* == *j*) then $B_{i,j} = 1.0 - \alpha$.

If (*j* == *i* + 1) then $B_{i,j} = 1.0$.

Otherwise $B_{i,j} = 0.0$, *i*, *j* = 1...*n*.

D:

If (*i* == *j*) then $D_{i,j} = 1.0$.

Otherwise $D_{i,j} = 0.0$, *i*, *j* = 1...*m*.

E:

If (*i* == *j*) then $E_{i,j} = 1.0$

Otherwise $E_{i,j} = 0.0$, *i*, *j* = 1...*n*.

L = *R* are chosen from [-10...10], which specifies the right hand sides (*C*, *F*).

- If *prtype* = 2 or 3: Triangular and/or quasi- triangular.

A:

If $(i \leq j)$ then $A_{i,j} = [-1 \dots 1]$.

Otherwise $A_{i,j} = 0.0$, $i, j = 1 \dots M$.

If $(prtype = 3)$ then $A_{k+1,k+1} = A_{k,k}$;

$A_{k+1,k} = [-1 \dots 1]$;

$\text{sign}(A_{k,k+1}) = -(\text{sign}(A_{k+1,k}))$.

$k = 1, m-1, qblcka$

B :

If $(i \leq j)$ then $B_{i,j} = [-1 \dots 1]$.

Otherwise $B_{i,j} = 0.0$, $i, j = 1 \dots n$.

If $(prtype = 3)$ then $B_{k+1,k+1} = B_{k,k}$

$B_{k+1,k} = [-1 \dots 1]$

$\text{sign}(B_{k,k+1}) = -(\text{sign}(B_{k+1,k}))$

$k = 1, n-1, qblckb$.

D:

If $(i \leq j)$ then $D_{i,j} = [-1 \dots 1]$.

Otherwise $D_{i,j} = 0.0$, $i, j = 1 \dots m$.

E:

If $(i \leq j)$ then $E_{i,j} = [-1 \dots 1]$.

Otherwise $E_{i,j} = 0.0$, $i, j = 1 \dots N$.

L, R are chosen from $[-10 \dots 10]$, which specifies the right hand sides (C, F).

- If $prtype = 4$ Full

$A_{i,j} = [-10 \dots 10]$

$D_{i,j} = [-1 \dots 1]$ $i, j = 1 \dots m$

$B_{i,j} = [-10 \dots 10]$

$E_{i,j} = [-1 \dots 1]$ $i, j = 1 \dots n$

$R_{i,j} = [-10 \dots 10]$

$L_{i,j} = [-1 \dots 1]$ $i = 1 \dots m, j = 1 \dots n$

L and R specifies the right hand sides (C, F).

- If $prtype = 5$ special case common and/or close eigs.

m Specifies the order of A and D and the number of rows in C, F, R and L .

n Specifies the order of B and E and the number of columns in C, F, R and L .

lda The leading dimension of a .

ldb The leading dimension of b .

ldc The leading dimension of c .

| | |
|--------------------|---|
| <i>l</i> <i>dd</i> | The leading dimension of <i>d</i> . |
| <i>l</i> <i>de</i> | The leading dimension of <i>e</i> . |
| <i>l</i> <i>df</i> | The leading dimension of <i>f</i> . |
| <i>l</i> <i>dr</i> | The leading dimension of <i>r</i> . |
| <i>l</i> <i>dl</i> | The leading dimension of <i>l</i> . |
| <i>alpha</i> | Parameter used in generating <i>prtype</i> = 1 and 5 matrices. |
| <i>qblcka</i> | When <i>prtype</i> = 3, specifies the distance between 2-by-2 blocks on the diagonal in <i>A</i> . Otherwise, <i>qblcka</i> is not referenced. <i>qblcka</i> > 1. |
| <i>qblckb</i> | When <i>prtype</i> = 3, specifies the distance between 2-by-2 blocks on the diagonal in <i>B</i> . Otherwise, <i>qblckb</i> is not referenced. <i>qblckb</i> > 1. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | Array, size <i>lda</i> * <i>m</i> . On exit <i>a</i> contains them-by- <i>m</i> array <i>A</i> initialized according to <i>prtype</i> . |
| <i>b</i> | Array, size <i>ldb</i> * <i>n</i> . On exit <i>b</i> contains the <i>n</i> -by- <i>n</i> array <i>B</i> initialized according to <i>prtype</i> . |
| <i>c</i> | Array, size <i>ldc</i> * <i>n</i> . On exit <i>c</i> contains the <i>m</i> -by- <i>n</i> array <i>C</i> initialized according to <i>prtype</i> . |
| <i>d</i> | Array, size <i>l</i> <i>dd</i> * <i>m</i> . On exit <i>d</i> contains the <i>m</i> -by- <i>m</i> array <i>D</i> initialized according to <i>prtype</i> . |
| <i>e</i> | Array, size <i>l</i> <i>de</i> * <i>n</i> . On exit <i>e</i> contains the <i>n</i> -by- <i>n</i> array <i>E</i> initialized according to <i>prtype</i> . |
| <i>f</i> | Array, size <i>l</i> <i>df</i> * <i>n</i> . On exit <i>f</i> contains the <i>m</i> -by- <i>n</i> array <i>F</i> initialized according to <i>prtype</i> . |
| <i>r</i> | Array, size <i>l</i> <i>dr</i> * <i>n</i> . On exit <i>R</i> contains the <i>m</i> -by- <i>n</i> array <i>R</i> initialized according to <i>prtype</i> . |
| <i>l</i> | Array, size <i>l</i> <i>dl</i> * <i>n</i> . On exit <i>l</i> contains the <i>m</i> -by- <i>n</i> array <i>L</i> initialized according to <i>prtype</i> . |

?latm6

Generates test matrices for the generalized eigenvalue problem, their corresponding right and left eigenvector matrices, and also reciprocal condition numbers for all eigenvalues and the reciprocal condition numbers of eigenvectors corresponding to the 1th and 5th eigenvalues.

Syntax

```
void slatm6 (lapack_int *type, lapack_int *n, float *a, lapack_int *lda, float *b,
float *x, lapack_int *ldx, float *y, lapack_int *ldy, float *alpha, float *beta, float
*wx, float *wy, float *s, float *dif);
```

```
void dlatm6 (lapack_int *type, lapack_int *n, double *a, lapack_int *lda, double *b,
double *x, lapack_int *ldx, double *y, lapack_int *ldy, double *alpha, double *beta,
double *wx, double *wy, double *s, double *dif);
```

```
void clatm6 (lapack_int *type, lapack_int *n, lapack_complex_float *a, lapack_int *lda,
lapack_complex_float *b, lapack_complex_float *x, lapack_int *ldx, lapack_complex_float
*y, lapack_int *ldy, lapack_complex_float *alpha, lapack_complex_float *beta,
lapack_complex_float *wx, lapack_complex_float *wy, float *s, float *dif);
```

```
void zlatm6 (lapack_int *type, lapack_int *n, lapack_complex_double *a, lapack_int
*lda, lapack_complex_double *b, lapack_complex_double *x, lapack_int *ldx,
lapack_complex_double *y, lapack_int *ldy, lapack_complex_double *alpha,
lapack_complex_double *beta, lapack_complex_double *wx, lapack_complex_double *wy,
double *s, double *dif);
```

Include Files

- mkl.h

Description

The `?latm6` routine generates test matrices for the generalized eigenvalue problem, their corresponding right and left eigenvector matrices, and also reciprocal condition numbers for all eigenvalues and the reciprocal condition numbers of eigenvectors corresponding to the 1th and 5th eigenvalues.

There two kinds of test matrix pairs:

$$(A, B) = \text{inverse}(YH) * (Da, Db) * \text{inverse}(X)$$

Type 1:

$$Da = \begin{bmatrix} 1+a & 0 & 0 & 0 & 0 \\ 0 & 2+a & 0 & 0 & 0 \\ 0 & 0 & 3+a & 0 & 0 \\ 0 & 0 & 0 & 4+a & 0 \\ 0 & 0 & 0 & 0 & 5+2 \end{bmatrix} \quad Db = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Type 2:

$$Da = \begin{bmatrix} 1+i & 0 & 0 & 0 & 0 \\ 0 & 1-i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & (1+a)+(1+b)i & 0 \\ 0 & 0 & 0 & 0 & (1+a)-(1+b)i \end{bmatrix} \quad Db = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In both cases the same `inverse(YH)` and `inverse(X)` are used to compute (A, B) , giving the exact eigenvectors to (A, B) as (YH, X) :

$$YH = \begin{bmatrix} 1 & 0 & -y & y & -y \\ 0 & 1 & -y & y & -y \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} 1 & 0 & -x & -x & x \\ 0 & 1 & x & -x & -x \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

where a , b , x and y will have all values independently of each other.

Input Parameters

| | |
|--------------------|---|
| <i>type</i> | Specifies the problem type. |
| <i>n</i> | Size of the matrices A and B . |
| <i>lda</i> | The leading dimension of a and of b . |
| <i>ldx</i> | The leading dimension of x . |
| <i>ldy</i> | The leading dimension of y . |
| <i>alpha, beta</i> | Weighting constants for matrix A . |
| <i>wx</i> | Constant for right eigenvector matrix. |
| <i>wy</i> | Constant for left eigenvector matrix. |

Output Parameters

| | |
|------------|--|
| <i>a</i> | Array, size $lda*n$. On exit, a contains the n -by- n matrix initialized according to <i>type</i> . |
| <i>b</i> | Array, size $lda*n$. On exit, b contains the n -by- n matrix initialized according to <i>type</i> . |
| <i>x</i> | Array, size $ldx*n$. On exit, x contains the n -by- n matrix of right eigenvectors. |
| <i>y</i> | Array, size $ldy*n$. On exit, y is the n -by- n matrix of left eigenvectors. |
| <i>s</i> | Array, size (n) . $s[i - 1]$ is the reciprocal condition number for eigenvalue i . |
| <i>dif</i> | Array, size (n) . $dif[i - 1]$ is the reciprocal condition number for eigenvector i . |

?latme

Generates random non-symmetric square matrices with specified eigenvalues.

Syntax

```
void slatme (lapack_int *n, char *dist, lapack_int *iseed, float *d, lapack_int *mode,
float *cond, float *dmax, char *ei, char *rsign, char *upper, char *sim, float *ds,
lapack_int *modes, float *conds, lapack_int *kl, lapack_int *ku, float *anorm, float
*a, lapack_int *lda, float *work, lapack_int *info);void dlatme (lapack_int *n, char
*dist, lapack_int *iseed, double *d, lapack_int *mode, double *cond, double *dmax,
char *ei, char *rsign, char *upper, char *sim, double *ds, lapack_int *modes, double
*conds, lapack_int *kl, lapack_int *ku, double *anorm, double *a, lapack_int *lda,
double *work, lapack_int *info);void clatme (lapack_int *n, char *dist, lapack_int
*iseed, lapack_complex_float *d, lapack_int *mode, float *cond, lapack_complex_float
*dmax, char *ei, char *rsign, char *upper, char *sim, float *ds, lapack_int *modes,
float *conds, lapack_int *kl, lapack_int *ku, float *anorm, lapack_complex_float *a,
lapack_int *lda, lapack_complex_float *work, lapack_int *info);void zlatme (lapack_int
*n, char *dist, lapack_int *iseed, lapack_complex_double *d, lapack_int *mode, double
*cond, lapack_complex_double *dmax, char *ei, char *rsign, char *upper, char *sim,
double *ds, lapack_int *modes, double *conds, lapack_int *kl, lapack_int *ku, double
*anorm, lapack_complex_double *a, lapack_int *lda, lapack_complex_double *work,
lapack_int *info);
```

Include Files

- mkl.h

Description

The `?latme` routine generates random non-symmetric square matrices with specified eigenvalues. `?latme` operates by applying the following sequence of operations:

1. Set the diagonal to d , where d may be input or computed according to $mode$, $cond$, $dmax$, and $rsign$ as described below.
2. If $upper = 'T'$, the upper triangle of a is set to random values out of distribution $dist$.
3. If $sim='T'$, a is multiplied on the left by a random matrix X , whose singular values are specified by ds , $modes$, and $conds$, and on the right by X inverse.
4. If $kl < n-1$, the lower bandwidth is reduced to kl using Householder transformations. If $ku < n-1$, the upper bandwidth is reduced to ku .
5. If $anorm$ is not negative, the matrix is scaled to have maximum-element-norm $anorm$.

NOTE

Since the matrix cannot be reduced beyond Hessenberg form, no packing options are available.

Input Parameters

| | |
|--------|---|
| n | The number of columns (or rows) of A . |
| $dist$ | On entry, $dist$ specifies the type of distribution to be used to generate the random eigen-/singular values, and on the upper triangle (see $upper$). If $dist = 'U'$: uniform(0, 1) If $dist = 'S'$: uniform(-1, 1) If $dist = 'N'$: normal(0, 1) If $dist = 'D'$: uniform on the complex disc $ z < 1$. |

| | |
|--------------|---|
| <i>iseed</i> | <p>Array, size 4.</p> <p>On entry <i>iseed</i> specifies the seed of the random number generator. The elements should lie between 0 and 4095 inclusive, and <i>iseed</i>[3] should be odd. The random number generator uses a linear congruential sequence limited to small integers, and so should produce machine independent random numbers.</p> |
| <i>d</i> | <p>Array, size (<i>n</i>). This array is used to specify the eigenvalues of <i>A</i>.</p> <p>If <i>mode</i> = 0, then <i>d</i> is assumed to contain the eigenvalues. Otherwise they are computed according to <i>mode</i>, <i>cond</i>, <i>dmax</i>, and <i>rsign</i> and placed in <i>d</i>.</p> |
| <i>mode</i> | <p>On entry <i>mode</i> describes how the eigenvalues are to be specified:</p> <p><i>mode</i> = 0 means use <i>d</i> (with <i>ei</i> for <i>slatme</i> and <i>dlatme</i>) as input.</p> <p><i>mode</i> = 1 sets <i>d</i>[0] = 1 and <i>d</i>(2:<i>n</i>)=1.0/<i>cond</i>.</p> <p><i>mode</i> = 2 sets <i>d</i>[0:<i>n</i> - 2] = 1 and <i>d</i>[<i>n</i> - 1]=1.0/<i>cond</i>.</p> <p><i>mode</i> = 3 sets <i>d</i>[<i>i</i> - 1] = <i>cond</i>**(-(<i>i</i>-1)/(<i>n</i>-1)).</p> <p><i>mode</i> = 4 sets <i>d</i>[<i>i</i> - 1] = 1 - (<i>i</i>-1)/(<i>n</i>-1)*(1 - 1/<i>cond</i>).</p> <p><i>mode</i> = 5 sets <i>d</i> to random numbers in the range (1/<i>cond</i> , 1) such that their logarithms are uniformly distributed.</p> <p><i>mode</i> = 6 sets <i>d</i> to random numbers from same distribution as the rest of the matrix.</p> <p><i>mode</i> < 0 has the same meaning as <i>abs(mode)</i>, except that the order of the elements of <i>d</i> is reversed.</p> <p>Thus if <i>mode</i> is between 1 and 4, <i>d</i> has entries ranging from 1 to 1/<i>cond</i>, if between -1 and -4, <i>d</i> has entries ranging from 1/<i>cond</i> to 1.</p> |
| <i>cond</i> | <p>On entry, this is used as described under <i>mode</i> above. If used, it must be ≥ 1.</p> |
| <i>dmax</i> | <p>If <i>mode</i> is not -6, 0 or 6, the contents of <i>d</i> as computed according to <i>mode</i> and <i>cond</i> are scaled by <i>dmax</i> / max(abs(<i>d</i>[<i>i</i> - 1])). Note that <i>dmax</i> needs not be positive or real: if <i>dmax</i> is negative or complex (or zero), <i>d</i> will be scaled by a negative or complex number (or zero). If <i>rsign</i>='F' then the largest (absolute) eigenvalue will be equal to <i>dmax</i>.</p> |
| <i>ei</i> | <p>Used by <i>slatme</i> and <i>dlatme</i> only.</p> <p>Array, size (<i>n</i>).</p> <p>If <i>mode</i> = 0, and <i>ei</i>[0] is not ' ' (space character), this array specifies which elements of <i>d</i> (on input) are real eigenvalues and which are the real and imaginary parts of a complex conjugate pair of eigenvalues. The elements of <i>ei</i> may then only have the values 'R' and 'I'.</p> <p>If <i>ei</i>[<i>j</i> - 1] = 'R' and <i>ei</i>[<i>j</i>] = 'I', then the <i>j</i>-th eigenvalue is <i>cmplx(d[j - 1] , d[j])</i>, and the (<i>j</i>+1)-th is the complex conjugate thereof.</p> <p>If <i>ei</i>[<i>j</i> - 1] = <i>ei</i>[<i>j</i>]='R', then the <i>j</i>-th eigenvalue is <i>d</i>[<i>j</i> - 1] (i.e., real). <i>ei</i>[0] may not be 'I', nor may two adjacent elements of <i>ei</i> both have the value 'I'.</p> |

| | |
|--------------|--|
| | If <i>mode</i> is not 0, then <i>ei</i> is ignored. If <i>mode</i> is 0 and <i>ei</i> [0] = ' ', then the eigenvalues will all be real. |
| <i>rsign</i> | <p>If <i>mode</i> is not 0, 6, or -6, and <i>rsign</i> = 'T', then the elements of <i>d</i>, as computed according to <i>mode</i> and <i>cond</i>, are multiplied by a random sign (+1 or -1) for <i>slatme</i> and <i>dlatme</i> or by a complex number from the unit circle $z = 1$ for <i>clatme</i> and <i>zlatme</i>.</p> <p>If <i>rsign</i> = 'F', the elements of <i>d</i> are not multiplied. <i>rsign</i> may only have the values 'T' or 'F'.</p> |
| <i>upper</i> | <p>If <i>upper</i> = 'T', then the elements of <i>a</i> above the diagonal will be set to random numbers out of <i>dist</i>.</p> <p>If <i>upper</i> = 'F', they will not. <i>upper</i> may only have the values 'T' or 'F'.</p> |
| <i>sim</i> | <p>If <i>sim</i> = 'T', then <i>a</i> will be operated on by a "similarity transform", i.e., multiplied on the left by a matrix <i>X</i> and on the right by <i>X</i> inverse. $X = USV$, where <i>U</i> and <i>V</i> are random unitary matrices and <i>S</i> is a (diagonal) matrix of singular values specified by <i>ds</i>, <i>modes</i>, and <i>conds</i>.</p> <p>If <i>sim</i> = 'F', then <i>a</i> will not be transformed.</p> |
| <i>ds</i> | This array is used to specify the singular values of <i>X</i> , in the same way that <i>d</i> specifies the eigenvalues of <i>a</i> . If <i>mode</i> = 0, the <i>ds</i> contains the singular values, which may not be zero. |
| <i>modes</i> | Similar to <i>mode</i> , but for specifying the diagonal of <i>S</i> . <i>modes</i> = -6 and +6 are not allowed (since they would result in randomly ill-conditioned eigenvalues.) |
| <i>conds</i> | Similar to <i>cond</i> , but for specifying the diagonal of <i>S</i> . |
| <i>kl</i> | This specifies the lower bandwidth of the matrix. <i>kl</i> = 1 specifies upper Hessenberg form. If <i>kl</i> is at least <i>n</i> -1, then <i>A</i> will have full lower bandwidth. |
| <i>ku</i> | <p>This specifies the upper bandwidth of the matrix. <i>ku</i> = 1 specifies lower Hessenberg form.</p> <p>If <i>ku</i> is at least <i>n</i>-1, then <i>a</i> will have full upper bandwidth.</p> <p>If <i>ku</i> and <i>kl</i> are both at least <i>n</i>-1, then <i>a</i> will be dense. Only one of <i>ku</i> and <i>kl</i> may be less than <i>n</i>-1.</p> |
| <i>anorm</i> | If <i>anorm</i> is not negative, then <i>a</i> is scaled by a non-negative real number to make the maximum-element-norm of <i>a</i> to be <i>anorm</i> . |
| <i>lda</i> | Number of rows of matrix <i>A</i> . |
| <i>work</i> | Array, size (3*n) . Workspace. |

Output Parameters

| | |
|--------------|-------------------------------------|
| <i>iseed</i> | On exit, the seed is updated. |
| <i>d</i> | Modified if <i>mode</i> is nonzero. |

| | |
|-------------|---|
| <i>ds</i> | Modified if <i>mode</i> is nonzero. |
| <i>a</i> | Array, size <i>lda*n</i> . On exit, <i>a</i> is the desired test matrix. |
| <i>info</i> | <p>If <i>info</i> = 0, execution is successful.</p> <p>If <i>info</i> = -1, <i>n</i> is negative .</p> <p>If <i>info</i> = -2, <i>dist</i> is an illegal string.</p> <p>If <i>info</i> = -5, <i>mode</i> is not in range -6 to 6.</p> <p>If <i>info</i> = -6, <i>cond</i> is less than 1.0, and <i>mode</i> is not -6, 0, or 6 .</p> <p>If <i>info</i> = -9, <i>rsign</i> is not 'T' or 'F' .</p> <p>If <i>info</i> = -10, <i>upper</i> is not 'T' or 'F'.</p> <p>If <i>info</i> = -11, <i>sim</i> is not 'T' or 'F'.</p> <p>If <i>info</i> = -12, <i>modes</i> = 0 and <i>ds</i> has a zero singular value.</p> <p>If <i>info</i> = -13, <i>modes</i> is not in the range -5 to 5.</p> <p>If <i>info</i> = -14, <i>modes</i> is nonzero and <i>conds</i> is less than 1. .</p> <p>If <i>info</i> = -15, <i>kl</i> is less than 1.</p> <p>If <i>info</i> = -16, <i>ku</i> is less than 1, or <i>kl</i> and <i>ku</i> are both less than <i>n</i>-1.</p> <p>If <i>info</i> = -19, <i>lda</i> is less than <i>m</i>.</p> <p>If <i>info</i> = 1, error return from ?latm1 (computing <i>d</i>) .</p> <p>If <i>info</i> = 2, cannot scale to <i>dmax</i> (max. eigenvalue is 0) .</p> <p>If <i>info</i> = 3, error return from slatm1(for slatme and clatme), dlatm1 (for dlatme and zlatme) .</p> <p>If <i>info</i> = 4, error return from ?large.</p> <p>If <i>info</i> = 5, zero singular value from slatm1(for slatme and clatme), dlatm1(for dlatme and zlatme).</p> |

?latmr

Generates random matrices of various types.

Syntax

```
void slatmr (lapack_int *m, lapack_int *n, char *dist, lapack_int *iseed, char *sym,
float *d, lapack_int *mode, float *cond, float *dmax, char *rsign, char *grade, float
*d1, lapack_int *model, float *cond1, float *dr, lapack_int *moder, float *condr, char
*pivtng, lapack_int *ipivot, lapack_int *kl, lapack_int *ku, float *sparse, float
*anorm, char *pack, float *a, lapack_int *lda, lapack_int *iwork, lapack_int *info);

void dlatmr (lapack_int *m, lapack_int *n, char *dist, lapack_int *iseed, char *sym,
double *d, lapack_int *mode, double *cond, double *dmax, char *rsign, char *grade,
double *d1, lapack_int *model, double *cond1, double *dr, lapack_int *moder, double
*condr, char *pivtng, lapack_int *ipivot, lapack_int *kl, lapack_int *ku, double
*sparse, double *anorm, char *pack, double *a, lapack_int *lda, lapack_int *iwork,
lapack_int *info);
```

```
void clatmr (lapack_int *m, lapack_int *n, char *dist, lapack_int *iseed, char *sym,
lapack_complex *d, lapack_int *mode, float *cond, lapack_complex *dmax, char *rsign,
char *grade, lapack_complex *dl, lapack_int *model, float *concl, lapack_complex *dr,
lapack_int *moder, float *condr, char *pivtng, lapack_int *ipivot, lapack_int *kl,
lapack_int *ku, float *sparse, float *anorm, char *pack, float *a, lapack_int *lda,
lapack_int *iwork, lapack_int *info);
```

```
void zlatmr (lapack_int *m, lapack_int *n, char *dist, lapack_int *iseed, char *sym,
lapack_complex_double *d, lapack_int *mode, float *cond, lapack_complex_double *dmax,
char *rsign, char *grade, lapack_complex_double *dl, lapack_int *model, float *concl,
lapack_complex_double *dr, lapack_int *moder, float *condr, char *pivtng, lapack_int
*ipivot, lapack_int *kl, lapack_int *ku, float *sparse, float *anorm, char *pack,
float *a, lapack_int *lda, lapack_int *iwork, lapack_int *info);
```

Description

The ?latmr routine operates by applying the following sequence of operations:

1. Generate a matrix A with random entries of distribution $dist$:
 - If $sym = 'S'$, the matrix is symmetric,
 - If $sym = 'H'$, the matrix is Hermitian,
 - If $sym = 'N'$, the matrix is nonsymmetric.
2. Set the diagonal to D , where D may be input or computed according to $mode$, $cond$, $dmax$ and $rsign$ as described below.
3. Grade the matrix, if desired, from the left or right as specified by $grade$. The inputs dl , $model$, $concl$, dr , $moder$ and $condr$ also determine the grading as described below.
4. Permute, if desired, the rows and/or columns as specified by $pivtng$ and $ipivot$.
5. Set random entries to zero, if desired, to get a random sparse matrix as specified by $sparse$.
6. Make A a band matrix, if desired, by zeroing out the matrix outside a band of lower bandwidth kl and upper bandwidth ku .
7. Scale A , if desired, to have maximum entry $anorm$.
8. Pack the matrix if desired. See options specified by the $pack$ parameter.

NOTE

If two calls to ?latmr differ only in the $pack$ parameter, they generate mathematically equivalent matrices. If two calls to ?latmr both have full bandwidth ($kl = m-1$ and $ku = n-1$), and differ only in the $pivtng$ and $pack$ parameters, then the matrices generated differ only in the order of the rows and columns, and otherwise contain the same data. This consistency cannot be and is not maintained with less than full bandwidth.

Input Parameters

| | |
|--------|---|
| m | Number of rows of A . |
| n | Number of columns of A . |
| $dist$ | On entry, $dist$ specifies the type of distribution to be used to generate a random matrix . If $dist = 'U'$, real and imaginary parts are independent uniform(0, 1). If $dist = 'S'$, real and imaginary parts are independent uniform(-1, 1). |

If $dist = 'N'$, real and imaginary parts are independent normal(0, 1).

If $dist = 'D'$, distribution is uniform on interior of unit disk.

iseed

Array, size 4.

On entry, *iseed* specifies the seed of the random number generator. They should lie between 0 and 4095 inclusive, and *iseed*[3] should be odd. The random number generator uses a linear congruential sequence limited to small integers, and so should produce machine independent random numbers.

sym

If $sym = 'S'$, generated matrix is symmetric.

If $sym = 'H'$, generated matrix is Hermitian.

If $sym = 'N'$, generated matrix is nonsymmetric.

d

On entry this array specifies the diagonal entries of the diagonal of *A*. *d* may either be specified on entry, or set according to *mode* and *cond* as described below. If the matrix is Hermitian, the real part of *d* is taken. May be changed on exit if *mode* is nonzero.

mode

On entry describes how *d* is to be used:

mode = 0 means use *d* as input.

mode = 1 sets $d[0]=1$ and $d[1:n - 1]=1.0/cond$.

mode = 2 sets $d[0:n - 2]=1$ and $d[n - 1]=1.0/cond$.

mode = 3 sets $d[i - 1]=cond^{*(-(i-1)/(n-1))}$.

mode = 4 sets $d[i - 1]=1 - (i-1)/(n-1)*(1 - 1/cond)$.

mode = 5 sets *d* to random numbers in the range ($1/cond$, 1) such that their logarithms are uniformly distributed.

mode = 6 sets *d* to random numbers from same distribution as the rest of the matrix.

mode < 0 has the same meaning as $abs(mode)$, except that the order of the elements of *d* is reversed.

Thus if *mode* is between 1 and 4, *d* has entries ranging from 1 to $1/cond$, if between -1 and -4, *D* has entries ranging from $1/cond$ to 1.

cond

On entry, used as described under *mode* above. If used, *cond* must be ≥ 1 .

dmax

If *mode* is not -6, 0, or 6, the diagonal is scaled by $dmax / \max(abs(d[i]))$, so that maximum absolute entry of diagonal is $abs(dmax)$. If *dmax* is complex (or zero), the diagonal is scaled by a complex number (or zero).

rsign

If *mode* is not -6, 0, or 6, specifies the sign of the diagonal as follows:

For *slatmr* and *dlatmr*, if $rsign = 'T'$, diagonal entries are multiplied 1 or -1 with a probability of 0.5.

For *clatmr* and *zlatmr*, if $rsign = 'T'$, diagonal entries are multiplied by a random complex number uniformly distributed with absolute value 1.

If *rsign* = 'F', diagonal entries are unchanged.

grade

Specifies grading of matrix as follows:

If *grade* = 'N', there is no grading

If *grade* = 'L', matrix is premultiplied by $\text{diag}(dl)$ (only if matrix is nonsymmetric)

If *grade* = 'R', matrix is postmultiplied by $\text{diag}(dr)$ (only if matrix is nonsymmetric)

If *grade* = 'B', matrix is premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(dr)$ (only if matrix is nonsymmetric)

If *grade* = 'H', matrix is premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(\text{conjg}(dl))$ (only if matrix is Hermitian or nonsymmetric)

If *grade* = 'S', matrix is premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{diag}(dl)$ (only if matrix is symmetric or nonsymmetric)

If *grade* = 'E', matrix is premultiplied by $\text{diag}(dl)$ and postmultiplied by $\text{inv}(\text{diag}(dl))$ (only if matrix is nonsymmetric)

NOTE

if *grade* = 'E', then *m* must equal *n*.

dl

Array, size (*m*).

If *model* = 0, then on entry this array specifies the diagonal entries of a diagonal matrix used as described under *grade* above.

If *model* is not zero, then *dl* is set according to *model* and *condl*, analogous to the way *D* is set according to *mode* and *cond* (except there is no *dmax* parameter for *dl*).

If *grade* = 'E', then *dl* cannot have zero entries.

Not referenced if *grade* = 'N' or 'R'. Changed on exit.

model

This specifies how the diagonal array *dl* is computed, just as *mode* specifies how *D* is computed.

condl

When *model* is not zero, this specifies the condition number of the computed *dl*.

dr

If *moder* = 0, then on entry this array specifies the diagonal entries of a diagonal matrix used as described under *grade* above.

If *moder* is not zero, then *dr* is set according to *moder* and *condr*, analogous to the way *d* is set according to *mode* and *cond* (except there is no *dmax* parameter for *dr*).

Not referenced if *grade* = 'N', 'L', 'H', 'S' or 'E'.

moder

This specifies how the diagonal array *dr* is to be computed, just as *mode* specifies how *d* is to be computed.

| | |
|-------------------------|--|
| <i>con_{dr}</i> | When <i>moder</i> is not zero, this specifies the condition number of the computed <i>dr</i> . |
| <i>piv_{tn}</i> | <p>On entry specifies pivoting permutations as follows:</p> <p>If <i>piv_{tn}</i> = 'N' or ' ': no pivoting permutation.</p> <p>If <i>piv_{tn}</i> = 'L': left or row pivoting (matrix must be nonsymmetric).</p> <p>If <i>piv_{tn}</i> = 'R': right or column pivoting (matrix must be nonsymmetric).</p> <p>If <i>piv_{tn}</i> = 'B' or 'F': both or full pivoting, i.e., on both sides. In this case, <i>m</i> must equal <i>n</i>.</p> <p>If two calls to ?<i>latmr</i> both have full bandwidth ($k_l = m - 1$ and $k_u = n - 1$), and differ only in the <i>piv_{tn}</i> and <i>pack</i> parameters, then the matrices generated differs only in the order of the rows and columns, and otherwise contain the same data. This consistency cannot be maintained with less than full bandwidth.</p> |
| <i>ipivot</i> | <p>Array, size (<i>n</i> or <i>m</i>) This array specifies the permutation used. After the basic matrix is generated, the rows, columns, or both are permuted.</p> <p>If row pivoting is selected, ?<i>latmr</i> starts with the last row and interchanges row <i>m</i> and row <i>ipivot</i>[<i>m</i> - 1], then moves to the next-to-last row, interchanging rows [<i>m</i> - 2] and row <i>ipivot</i>[<i>m</i> - 2], and so on. In terms of "2-cycles", the permutation is (1 <i>ipivot</i>[0]) (2 <i>ipivot</i>[1]) ... (<i>mipivot</i>[<i>m</i> - 1]) where the rightmost cycle is applied first. This is the inverse of the effect of pivoting in LINPACK. The idea is that factoring (with pivoting) an identity matrix which has been inverse-pivoted in this way should result in a pivot vector identical to <i>ipivot</i>. Not referenced if <i>piv_{tn}</i> = 'N'.</p> |
| <i>sparse</i> | On entry, specifies the sparsity of the matrix if a sparse matrix is to be generated. <i>sparse</i> should lie between 0 and 1. To generate a sparse matrix, for each matrix entry a uniform (0, 1) random number <i>x</i> is generated and compared to <i>sparse</i> ; if <i>x</i> is larger the matrix entry is unchanged and if <i>x</i> is smaller the entry is set to zero. Thus on the average a fraction <i>sparse</i> of the entries is set to zero. |
| <i>k_l</i> | On entry, specifies the lower bandwidth of the matrix. For example, $k_l = 0$ implies upper triangular, $k_l = 1$ implies upper Hessenberg, and k_l at least <i>m</i> -1 implies the matrix is not banded. Must equal <i>k_u</i> if matrix is symmetric or Hermitian. |
| <i>k_u</i> | On entry, specifies the upper bandwidth of the matrix. For example, $k_u = 0$ implies lower triangular, $k_u = 1$ implies lower Hessenberg, and k_u at least <i>n</i> -1 implies the matrix is not banded. Must equal <i>k_l</i> if matrix is symmetric or Hermitian. |
| <i>anorm</i> | On entry, specifies maximum entry of output matrix (output matrix is multiplied by a constant so that its largest absolute entry equal <i>anorm</i>) if <i>anorm</i> is nonnegative. If <i>anorm</i> is negative no scaling is done. |
| <i>pack</i> | <p>On entry, specifies packing of matrix as follows:</p> <p>If <i>pack</i> = 'N': no packing</p> <p>If <i>pack</i> = 'U': zero out all subdiagonal entries (if symmetric or Hermitian)</p> |

If $pack = 'L'$: zero out all superdiagonal entries (if symmetric or Hermitian)

If $pack = 'C'$: store the upper triangle columnwise (only if matrix symmetric or Hermitian or square upper triangular)

If $pack = 'R'$: store the lower triangle columnwise (only if matrix symmetric or Hermitian or square lower triangular) (same as upper half rowwise if symmetric) (same as conjugate upper half rowwise if Hermitian)

If $pack = 'B'$: store the lower triangle in band storage scheme (only if matrix symmetric or Hermitian)

If $pack = 'Q'$: store the upper triangle in band storage scheme (only if matrix symmetric or Hermitian)

If $pack = 'Z'$: store the entire matrix in band storage scheme (pivoting can be provided for by using this option to store A in the trailing rows of the allocated storage)

Using these options, the various LAPACK packed and banded storage schemes can be obtained:

| LAPACK storage scheme | Value of $pack$ |
|-----------------------|-----------------|
| GB | 'Z' |
| PB, HB or TB | 'B' or 'Q' |
| PP, HP or TP | 'C' or 'R' |

If two calls to `?latmr` differ only in the $pack$ parameter, they generate mathematically equivalent matrices.

lda

On entry, lda specifies the first dimension of a as declared in the calling program.

If $pack = 'N', 'U'$ or $'L'$, lda must be at least $\max(1, m)$.

If $pack = 'C'$ or $'R'$, lda must be at least 1.

If $pack = 'B',$ or $'Q'$, lda must be $\min(ku + 1, n)$.

If $pack = 'Z'$, lda must be at least $k_{uu} + k_{ll} + 1$, where $k_{uu} = \min(ku, n-1)$ and $k_{ll} = \min(kl, n-1)$.

$iwork$

Array, size $(n$ or $m)$. Workspace. Not referenced if $pivtnq = 'N'$. Changed on exit.

Output Parameters

$iseed$

On exit, the seed is changed.

d

May be changed on exit if $mode$ is nonzero.

dl

On exit, array is changed.

dr

On exit, array is changed.

a

On exit, a is the desired test matrix. Only those entries of a which are significant on output is referenced (even if a is in packed or band storage format). The unoccupied corners of a in band format are zeroed out.

info

- If *info* = 0, the execution is successful.
- If *info* = -1, *m* is negative or unequal to *n* and *sym* = 'S' or 'H'.
- If *info* = -2, *n* is negative .
- If *info* = -3, *dist* is an illegal string.
- If *info* = -5, *sym* is an illegal string..
- If *info* = -7, *mode* is not in range -6 to 6.
- If *info* = -8, *cond* is less than 1.0, and *mode* is neither -6, 0 nor 6.
- If *info* = -10, *mode* is neither -6, 0 nor 6 and *rsign* is an illegal string.
- If *info* = -11, *grade* is an illegal string, or *grade* = 'E' and *m* is not equal to *n*, or *grade*='L', 'R', 'B', 'S' or 'E' and *sym* = 'H', or *grade* = 'L', 'R', 'B', 'H' or 'E' and *sym* = 'S'
- If *info* = -12, *grade* = 'E' and *dl* contains zero .
- If *info* = -13, *model* is not in range -6 to 6 and *grade* = 'L', 'B', 'H', 'S' or 'E' .
- If *info* = -14, *concl* is less than 1.0, *grade* = 'L', 'B', 'H', 'S' or 'E', and *model* is neither -6, 0 nor 6.
- If *info* = -16, *moder* is not in range -6 to 6 and *grade* = 'R' or 'B' .
- If *info* = -17, *condr* is less than 1.0, *grade* = 'R' or 'B', and *moder* is neither -6, 0 nor 6 .
- If *info* = -18, *pivtnng* is an illegal string, or *pivtnng* = 'B' or 'F' and *m* is not equal to *n*, or *pivtnng* = 'L' or 'R' and *sym* = 'S' or 'H'.
- If *info* = -19, *ipivot* contains out of range number and *pivtnng* is not equal to 'N' .
- If *info* = -20, *kl* is negative.
- If *info* = -21, *ku* is negative, or *sym* = 'S' or 'H' and *ku* not equal to *kl* .
- If *info* = -22, *sparse* is not in range 0 to 1.
- If *info* = -24, *pack* is an illegal string, or *pack* = 'U', 'L', 'B' or 'Q' and *sym* = 'N', or *pack* = 'C' and *sym* = 'N' and either *kl* is not equal to 0 or *n* is not equal to *m*, or *pack* = 'R' and *sym* = 'N', and either *ku* is not equal to 0 or *n* is not equal to *m* .
- If *info* = -26, *lda* is too small .
- If *info* = 1, error return from ?latm1 (computing *D*) .
- If *info* = 2, cannot scale to *dmax* (max. entry is 0) .
- If *info* = 3, error return from ?latm1(computing *dl*) .
- If *info* = 4, error return from ?latm1(computing *dr*) .
- If *info* = 5, *anorm* is positive, but matrix constructed prior to attempting to scale it to have norm *anorm*, is zero .

?lauum

Computes the product $U^*U^T(U^*U^H)$ or $L^T*L(L^H*L)$, where U and L are upper or lower triangular matrices (blocked algorithm).

Syntax

```
lapack_int LAPACKE_slauum (int matrix_layout , char uplo , lapack_int n , float * a ,
lapack_int lda );

lapack_int LAPACKE_dlauum (int matrix_layout , char uplo , lapack_int n , double * a ,
lapack_int lda );

lapack_int LAPACKE_clauum (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int lda );

lapack_int LAPACKE_zlauum (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int lda );
```

Include Files

- mkl.h

Description

The routine ?lauum computes the product U^*U^T or L^T*L for real flavors, and U^*U^H or L^H*L for complex flavors. Here the triangular factor U or L is stored in the upper or lower triangular part of the array a .

If $uplo = 'U'$ or $'u'$, then the upper triangle of the result is stored, overwriting the factor U in A .

If $uplo = 'L'$ or $'l'$, then the lower triangle of the result is stored, overwriting the factor L in A .

This is the blocked form of the algorithm, calling [BLAS Level 3 Routines](#).

Input Parameters

| | |
|--------|---|
| $uplo$ | Specifies whether the triangular factor stored in the array a is upper or lower triangular: = 'U': Upper triangular = 'L': Lower triangular |
| n | The order of the triangular factor U or L . $n \geq 0$. |
| a | Array of size $\max(1, lda * n)$. On entry, the triangular factor U or L . |
| lda | The leading dimension of the array a . $lda \geq \max(1, n)$. |

Output Parameters

| | |
|-----|--|
| a | On exit, if $uplo = 'U'$, then the upper triangle of a is overwritten with the upper triangle of the product $U^*U^T(U^*U^H)$; if $uplo = 'L'$, then the lower triangle of a is overwritten with the lower triangle of the product $L^T*L(L^H*L)$. |
|-----|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*k*, the *k*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?syswapr

Applies an elementary permutation on the rows and columns of a symmetric matrix.

Syntax

```
lapack_int LAPACKE_ssyswapr (int matrix_layout , char uplo , lapack_int n , float * a ,
lapack_int i1 , lapack_int i2 );
```

```
lapack_int LAPACKE_dsyswapr (int matrix_layout , char uplo , lapack_int n , double *
a , lapack_int i1 , lapack_int i2 );
```

```
lapack_int LAPACKE_csyswapr (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_float * a , lapack_int i1 , lapack_int i2 );
```

```
lapack_int LAPACKE_zsyswapr (int matrix_layout , char uplo , lapack_int n ,
lapack_complex_double * a , lapack_int i1 , lapack_int i2 );
```

Include Files

- mkl.h

Description

The routine applies an elementary permutation on the rows and columns of a symmetric matrix.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Must be 'U' or 'L'. Indicates how the input matrix <i>A</i> has been factored: If <i>uplo</i> = 'U', the array <i>a</i> stores the upper triangular factor <i>U</i> of the factorization $A = U * D * U^T$. If <i>uplo</i> = 'L', the array <i>a</i> stores the lower triangular factor <i>L</i> of the factorization $A = L * D * L^T$. |
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>a</i> | Array of size at least $\max(1, lda * n)$. The array <i>a</i> contains the block diagonal matrix <i>D</i> and the multipliers used to obtain the factor <i>U</i> or <i>L</i> as computed by ?sytrf. |
| <i>i1</i> | Index of the first row to swap. |

i2 Index of the second row to swap.

Output Parameters

a If *info* = 0, the symmetric inverse of the original matrix.
 If *info* = 'U', the upper triangular part of the inverse is formed and the part of *A* below the diagonal is not referenced.
 If *info* = 'L', the lower triangular part of the inverse is formed and the part of *A* above the diagonal is not referenced.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

See Also

?sytrf

?heswapr

Applies an elementary permutation on the rows and columns of a Hermitian matrix.

Syntax

```
lapack_int LAPACKE_cheswapr (int matrix_layout, char uplo, lapack_int n,
lapack_complex_float* a, lapack_int i1, lapack_int i2);
```

```
lapack_int LAPACKE_zheswapr (int matrix_layout, char uplo, lapack_int n,
lapack_complex_double* a, lapack_int i1, lapack_int i2);
```

Include Files

- mkl.h

Description

The routine applies an elementary permutation on the rows and columns of a Hermitian matrix.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

uplo Must be 'U' or 'L'.
 Indicates how the input matrix *A* has been factored:
 If *uplo* = 'U', the array *a* stores the upper triangular factor *U* of the factorization $A = U^*D^*U^H$.
 If *uplo* = 'L', the array *a* stores the lower triangular factor *L* of the factorization $A = L^*D^*L^H$.

| | |
|-------------|---|
| <i>n</i> | The order of matrix <i>A</i> ; $n \geq 0$. |
| <i>nrhs</i> | The number of right-hand sides; $nrhs \geq 0$. |
| <i>a</i> | Array of size at least $\max(1, lda * n)$. The array <i>a</i> contains the block diagonal matrix <i>D</i> and the multipliers used to obtain the factor <i>U</i> or <i>L</i> as computed by ?hetrf. |
| <i>i1</i> | Index of the first row to swap. |
| <i>i2</i> | Index of the second row to swap. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | If <i>info</i> = 0, the inverse of the original matrix. If <i>info</i> = 'U', the upper triangular part of the inverse is formed and the part of <i>A</i> below the diagonal is not referenced. If <i>info</i> = 'L', the lower triangular part of the inverse is formed and the part of <i>A</i> above the diagonal is not referenced. |
|----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

See Also

[?hetrf](#)

?sfrk

Performs a symmetric rank-k operation for matrix in RFP format.

Syntax

```
lapack_int LAPACKE_ssfrk (int matrix_layout , char transr , char uplo , char trans ,
lapack_int n , lapack_int k , float alpha , const float * a , lapack_int lda , float
beta , float * c );
```

```
lapack_int LAPACKE_dsfrk (int matrix_layout , char transr , char uplo , char trans ,
lapack_int n , lapack_int k , double alpha , const double * a , lapack_int lda ,
double beta , double * c );
```

Include Files

- mkl.h

Description

The ?sfrk routines perform a matrix-matrix operation using symmetric matrices. The operation is defined as

$$C := \alpha * A * A^T + \beta * C,$$

or

$$C := \alpha * A^T * A + \beta * C,$$

where:

α and β are scalars,

C is an n -by- n symmetric matrix in rectangular full packed (RFP) format,

A is an n -by- k matrix in the first case and a k -by- n matrix in the second case.

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

transr if *transr* = 'N' or 'n', the normal form of RFP C is stored;
if *transr* = 'T' or 't', the transpose form of RFP C is stored.

uplo Specifies whether the upper or lower triangular part of the array c is used.
If *uplo* = 'U' or 'u', then the upper triangular part of the array c is used.
If *uplo* = 'L' or 'l', then the low triangular part of the array c is used.

trans Specifies the operation:
if *trans* = 'N' or 'n', then $C := \alpha * A * A^T + \beta * C$;
if *trans* = 'T' or 't', then $C := \alpha * A^T * A + \beta * C$;

n Specifies the order of the matrix C . The value of n must be at least zero.

k On entry with *trans* = 'N' or 'n', k specifies the number of columns of the matrix A , and on entry with *trans* = 'T' or 't', k specifies the number of rows of the matrix A .
The value of k must be at least zero.

alpha Specifies the scalar α .

a Array, size $\max(1, lda * ka)$, where ka is in the following table:

| | Col_major | Row_major |
|--------------------|-----------|-----------|
| <i>trans</i> = 'N' | k | n |
| <i>trans</i> = 'T' | n | k |

Before entry with *trans* = 'N' or 'n', the leading n -by- k part of the array a must contain the matrix A , otherwise the leading k -by- n part of the array a must contain the matrix A .

lda Specifies the leading dimension of a as declared in the calling (sub)program. lda is defined by the following table:

| | Col_major | Row_major |
|--------------------|--------------|--------------|
| <i>trans</i> = 'N' | $\max(1, n)$ | $\max(1, k)$ |
| <i>trans</i> = 'T' | $\max(1, k)$ | $\max(1, n)$ |

| | |
|-------------|--|
| <i>beta</i> | Specifies the scalar <i>beta</i> . |
| <i>c</i> | Array, size $(n*(n+1)/2)$. Before entry contains the symmetric matrix <i>C</i> in RFP format. |

Output Parameters

| | |
|----------|--|
| <i>c</i> | If <i>trans</i> = 'N' or 'n', then <i>c</i> contains $C := \alpha*A*A' + \beta*C$; if <i>trans</i> = 'T' or 't', then <i>c</i> contains $C := \alpha*A'*A + \beta*C$; |
|----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?hfrk

Performs a Hermitian rank-*k* operation for matrix in RFP format.

Syntax

```
lapack_int LAPACKE_chfrk( int matrix_layout, char transr, char uplo, char trans,
lapack_int n, lapack_int k, float alpha, const lapack_complex_float* a, lapack_int
lda, float beta, lapack_complex_float* c );
```

```
lapack_int LAPACKE_zhfrk( int matrix_layout, char transr, char uplo, char trans,
lapack_int n, lapack_int k, double alpha, const lapack_complex_double* a, lapack_int
lda, double beta, lapack_complex_double* c );
```

Include Files

- mkl.h

Description

The ?hfrk routines perform a matrix-matrix operation using Hermitian matrices. The operation is defined as

$$C := \alpha*A*A^H + \beta*C,$$

or

$$C := \alpha*A^H*A + \beta*C,$$

where:

alpha and *beta* are real scalars,

C is an *n*-by-*n* Hermitian matrix in RFP format,

A is an *n*-by-*k* matrix in the first case and a *k*-by-*n* matrix in the second case.

Input Parameters

- matrix_layout* Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).
- transr* if *transr* = 'N' or 'n', the normal form of RFP *C* is stored;
if *transr* = 'C' or 'c', the conjugate-transpose form of RFP *C* is stored.
- uplo* Specifies whether the upper or lower triangular part of the array *c* is used.
If *uplo* = 'U' or 'u', then the upper triangular part of the array *c* is used.
If *uplo* = 'L' or 'l', then the low triangular part of the array *c* is used.
- trans* Specifies the operation:
if *trans* = 'N' or 'n', then $C := \alpha * A * A^H + \beta * C$;
if *trans* = 'C' or 'c', then $C := \alpha * A^H * A + \beta * C$.
- n* Specifies the order of the matrix *C*. The value of *n* must be at least zero.
- k* On entry with *trans* = 'N' or 'n', *k* specifies the number of columns of the matrix *a*, and on entry with *trans* = 'T' or 't' or 'C' or 'c', *k* specifies the number of rows of the matrix *a*.
The value of *k* must be at least zero.
- alpha* Specifies the scalar *alpha*.
- a* Array, size $\max(1, lda * ka)$, where *ka* is in the following table:
- | | Col_major | Row_major |
|--------------------|-----------|-----------|
| <i>trans</i> = 'N' | <i>k</i> | <i>n</i> |
| <i>trans</i> = 'T' | <i>n</i> | <i>k</i> |
- Before entry with *trans* = 'N' or 'n', the leading *n*-by-*k* part of the array *a* must contain the matrix *A*, otherwise the leading *k*-by-*n* part of the array *a* must contain the matrix *A*.
- lda* Specifies the leading dimension of *a* as declared in the calling (sub)program. *lda* is defined by the following table:
- | | Col_major | Row_major |
|--------------------|--------------|--------------|
| <i>trans</i> = 'N' | $\max(1, n)$ | $\max(1, k)$ |
| <i>trans</i> = 'T' | $\max(1, k)$ | $\max(1, n)$ |
- beta* Specifies the scalar *beta*.
- c* Array, size $(n * (n + 1) / 2)$. Before entry contains the Hermitian matrix *C* in [RFP format](#).

Output Parameters

- c* If *trans* = 'N' or 'n', then *c* contains $C := \alpha * A * A^H + \beta * C$;

if $trans = 'C'$ or $'c'$, then c contains $C := \alpha * A^H * A + \beta * C$;

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info < 0$, the i -th parameter had an illegal value.

If $info = -1011$, memory allocation error occurred.

?tfsm

Solves a matrix equation (one operand is a triangular matrix in RFP format).

Syntax

```
lapack_int LAPACKE_stfsm (int matrix_layout , char transr , char side , char uplo ,
char trans , char diag , lapack_int m , lapack_int n , float alpha , const float * a ,
float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_dtfsm (int matrix_layout , char transr , char side , char uplo ,
char trans , char diag , lapack_int m , lapack_int n , double alpha , const double *
a , double * b , lapack_int ldb );
```

```
lapack_int LAPACKE_ctfsm (int matrix_layout , char transr , char side , char uplo ,
char trans , char diag , lapack_int m , lapack_int n , lapack_complex_float alpha ,
const lapack_complex_float * a , lapack_complex_float * b , lapack_int ldb );
```

```
lapack_int LAPACKE_ztfsm (int matrix_layout , char transr , char side , char uplo ,
char trans , char diag , lapack_int m , lapack_int n , lapack_complex_double alpha ,
const lapack_complex_double * a , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The ?tfsm routines solve one of the following matrix equations:

$$\text{op}(A) * X = \alpha * B,$$

or

$$X * \text{op}(A) = \alpha * B,$$

where:

α is a scalar,

X and B are m -by- n matrices,

A is a unit, or non-unit, upper or lower triangular matrix in [rectangular full packed \(RFP\) format](#).

$\text{op}(A)$ can be one of the following:

- $\text{op}(A) = A$ or $\text{op}(A) = A^T$ for real flavors
- $\text{op}(A) = A$ or $\text{op}(A) = A^H$ for complex flavors

The matrix B is overwritten by the solution matrix X .

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>transr</i> | <p>if <i>transr</i> = 'N' or 'n', the normal form of RFP <i>A</i> is stored;</p> <p>if <i>transr</i> = 'T' or 't', the transpose form of RFP <i>A</i> is stored;</p> <p>if <i>transr</i> = 'C' or 'c', the conjugate-transpose form of RFP <i>A</i> is stored.</p> |
| <i>side</i> | <p>Specifies whether $\text{op}(A)$ appears on the left or right of <i>X</i> in the equation:</p> <p>if <i>side</i> = 'L' or 'l', then $\text{op}(A) * X = \alpha * B$;</p> <p>if <i>side</i> = 'R' or 'r', then $X * \text{op}(A) = \alpha * B$.</p> |
| <i>uplo</i> | <p>Specifies whether the RFP matrix <i>A</i> is upper or lower triangular:</p> <p>if <i>uplo</i> = 'U' or 'u', then the matrix is upper triangular;</p> <p>if <i>uplo</i> = 'L' or 'l', then the matrix is low triangular.</p> |
| <i>trans</i> | <p>Specifies the form of $\text{op}(A)$ used in the matrix multiplication:</p> <p>if <i>trans</i> = 'N' or 'n', then $\text{op}(A) = A$;</p> <p>if <i>trans</i> = 'T' or 't', then $\text{op}(A) = A'$;</p> <p>if <i>trans</i> = 'C' or 'c', then $\text{op}(A) = \text{conjg}(A')$.</p> |
| <i>diag</i> | <p>Specifies whether the RFP matrix <i>A</i> is unit triangular:</p> <p>if <i>diag</i> = 'U' or 'u' then the matrix is unit triangular;</p> <p>if <i>diag</i> = 'N' or 'n', then the matrix is not unit triangular.</p> |
| <i>m</i> | Specifies the number of rows of <i>B</i> . The value of <i>m</i> must be at least zero. |
| <i>n</i> | Specifies the number of columns of <i>B</i> . The value of <i>n</i> must be at least zero. |
| <i>alpha</i> | <p>Specifies the scalar <i>alpha</i>.</p> <p>When <i>alpha</i> is zero, then <i>a</i> is not referenced and <i>b</i> need not be set before entry.</p> |
| <i>a</i> | Array, size $(n * (n + 1) / 2)$. Contains the matrix <i>A</i> in RFP format. |
| <i>b</i> | <p>Array, size $\max(1, ldb * n)$ for column major and $\max(1, ldb * m)$ for row major.</p> <p>Before entry, the leading <i>m</i>-by-<i>n</i> part of the array <i>b</i> must contain the right-hand side matrix <i>B</i>.</p> |
| <i>ldb</i> | Specifies the leading dimension of <i>b</i> as declared in the calling (sub)program. The value of <i>ldb</i> must be at least $\max(1, m)$ for column major and $\max(1, n)$ for row major. |

Output Parameters

| | |
|----------|---|
| <i>b</i> | Overwritten by the solution matrix <i>X</i> . |
|----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?tfttp

Copies a triangular matrix from the rectangular full packed format (TF) to the standard packed format (TP).

Syntax

```
lapack_int LAPACKE_stfttp (int matrix_layout , char transr , char uplo , lapack_int n ,
const float * arf , float * ap );
```

```
lapack_int LAPACKE_dtfttp (int matrix_layout , char transr , char uplo , lapack_int n ,
const double * arf , double * ap );
```

```
lapack_int LAPACKE_ctfttp (int matrix_layout , char transr , char uplo , lapack_int n ,
const lapack_complex_float * arf , lapack_complex_float * ap );
```

```
lapack_int LAPACKE_ztfttp (int matrix_layout , char transr , char uplo , lapack_int n ,
const lapack_complex_double * arf , lapack_complex_double * ap );
```

Include Files

- mkl.h

Description

The routine copies a triangular matrix *A* from the Rectangular Full Packed (RFP) format to the standard packed format. For the description of the RFP format, see [Matrix Storage Schemes](#).

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>transr</i> | = 'N': <i>arf</i> is in the Normal format, = 'T': <i>arf</i> is in the Transpose format (for stfttp and dtfttp), = 'C': <i>arf</i> is in the Conjugate-transpose format (for ctfttp and ztfttp). |
| <i>uplo</i> | Specifies whether <i>A</i> is upper or lower triangular: = 'U': <i>A</i> is upper triangular, = 'L': <i>A</i> is lower triangular. |
| <i>n</i> | The order of the matrix <i>A</i> . $n \geq 0$. |
| <i>arf</i> | Array, size at least $\max(1, n*(n+1)/2)$. On entry, the upper or lower triangular matrix <i>A</i> stored in the RFP format. |

Output Parameters

ap Array, size at least $\max(1, n*(n+1)/2)$.
On exit, the upper or lower triangular matrix *A*, packed columnwise in a linear array.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?tfttr

Copies a triangular matrix from the rectangular full packed format (TF) to the standard full format (TR) .

Syntax

```
lapack_int LAPACKE_stfttr (int matrix_layout , char transr , char uplo , lapack_int n ,
const float * arf , float * a , lapack_int lda );
```

```
lapack_int LAPACKE_dftttr (int matrix_layout , char transr , char uplo , lapack_int n ,
const double * arf , double * a , lapack_int lda );
```

```
lapack_int LAPACKE_ctfttr (int matrix_layout , char transr , char uplo , lapack_int n ,
const lapack_complex_float * arf , lapack_complex_float * a , lapack_int lda );
```

```
lapack_int LAPACKE_zftttr (int matrix_layout , char transr , char uplo , lapack_int n ,
const lapack_complex_double * arf , lapack_complex_double * a , lapack_int lda );
```

Include Files

- mkl.h

Description

The routine copies a triangular matrix *A* from the Rectangular Full Packed (RFP) format to the standard full format. For the description of the RFP format, see [Matrix Storage Schemes](#).

Input Parameters

matrix_layout Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR).

transr = 'N': *arf* is in the Normal format,
= 'T': *arf* is in the Transpose format (for stfttr and dtfttr),
= 'C': *arf* is in the Conjugate-transpose format (for ctfttr and zftttr).

uplo Specifies whether *A* is upper or lower triangular:
= 'U': *A* is upper triangular,
= 'L': *A* is lower triangular.

| | |
|------------|--|
| <i>n</i> | The order of the matrices <i>arf</i> and <i>a</i> . $n \geq 0$. |
| <i>arf</i> | Array, size at least $\max(1, n*(n+1)/2)$. On entry, the upper or lower triangular matrix <i>A</i> stored in the RFP format. |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | Array, size $\max(1, lda * n)$. On exit, the triangular matrix <i>A</i> . If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> contains the upper triangular matrix, and the strictly lower triangular part of <i>a</i> is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> contains the lower triangular matrix, and the strictly upper triangular part of <i>a</i> is not referenced. |
|----------|---|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?tpqrt2

Computes a QR factorization of a real or complex "triangular-pentagonal" matrix, which is composed of a triangular block and a pentagonal block, using the compact WY representation for *Q*.

Syntax

```
lapack_int LAPACKE_stpqrt2 (int matrix_layout, lapack_int m, lapack_int n, lapack_int l, float * a, lapack_int lda, float * b, lapack_int ldb, float * t, lapack_int ldt);
```

```
lapack_int LAPACKE_dtpqrt2 (int matrix_layout, lapack_int m, lapack_int n, lapack_int l, double * a, lapack_int lda, double * b, lapack_int ldb, double * t, lapack_int ldt);
```

```
lapack_int LAPACKE_ctpqrt2 (int matrix_layout, lapack_int m, lapack_int n, lapack_int l, lapack_complex_float * a, lapack_int lda, lapack_complex_float * b, lapack_int ldb, lapack_complex_float * t, lapack_int ldt );
```

```
lapack_int LAPACKE_ztpqrt2 (int matrix_layout, lapack_int m, lapack_int n, lapack_int l, lapack_complex_double * a, lapack_int lda, lapack_complex_double * b, lapack_int ldb, lapack_complex_double * t, lapack_int ldt );
```

Include Files

- mkl.h

Description

The input matrix *C* is an $(n+m)$ -by-*n* matrix

$$C = \begin{bmatrix} A \\ B \end{bmatrix} \left\{ \begin{array}{l} \leftarrow n \times n \text{ upper triangular} \\ \leftarrow m \times n \text{ pentagonal} \end{array} \right.$$

where A is an n -by- n upper triangular matrix, and B is an m -by- n pentagonal matrix consisting of an $(m-1)$ -by- n rectangular matrix $B1$ on top of an 1 -by- n upper trapezoidal matrix $B2$:

$$B = \begin{bmatrix} B1 \\ B2 \end{bmatrix} \left\{ \begin{array}{l} \leftarrow (m-1) \times n \text{ rectangular} \\ \leftarrow 1 \times n \text{ upper trapezoidal} \end{array} \right.$$

The upper trapezoidal matrix $B2$ consists of the first l rows of an n -by- n upper triangular matrix, where $0 \leq l \leq \min(m, n)$. If $l=0$, B is an m -by- n rectangular matrix. If $m=l=n$, B is upper triangular. The matrix W contains the elementary reflectors $H(i)$ in the i th column below the diagonal (of A) in the $(n+m)$ -by- n input matrix C so that W can be represented as

$$W = \begin{bmatrix} I \\ V \end{bmatrix} \left\{ \begin{array}{l} \leftarrow n \times n \text{ identity} \\ \leftarrow m \times n \text{ pentagonal} \end{array} \right.$$

Thus, V contains all of the information needed for W , and is returned in array b .

NOTE

V has the same form as B :

$$V = \begin{bmatrix} V1 \\ V2 \end{bmatrix} \left\{ \begin{array}{l} \leftarrow (m-1) \times n \text{ rectangular} \\ \leftarrow l \times n \text{ upper trapezoidal} \end{array} \right.$$

The columns of V represent the vectors which define the $H(i)$ s.

The $(m+n)$ -by- $(m+n)$ block reflector H is then given by

$$H = I - W^* T^* W^T \text{ for real flavors, and}$$

$$H = I - W^* T^* W^H \text{ for complex flavors}$$

where W^T is the transpose of W , W^H is the conjugate transpose of W , and T is the upper triangular factor of the block reflector.

Input Parameters

| | |
|----------------------------|---|
| <code>matrix_layout</code> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <code>m</code> | The total number of rows in the matrix B ($m \geq 0$). |
| <code>n</code> | The number of columns in B and the order of the triangular matrix A ($n \geq 0$). |

| | |
|-------------|---|
| <i>l</i> | The number of rows of the upper trapezoidal part of <i>B</i> ($\min(m, n) \geq l \geq 0$). |
| <i>a, b</i> | Arrays: <i>a</i> , size $\max(1, lda * n)$ contains the <i>n</i> -by- <i>n</i> upper triangular matrix <i>A</i> . <i>b</i> , size $\max(1, ldb * n)$ for column major and $\max(1, ldb * m)$ for row major, the pentagonal <i>m</i> -by- <i>n</i> matrix <i>B</i> . The first (<i>m</i> -1) rows contain the rectangular <i>B1</i> matrix, and the next <i>l</i> rows contain the upper trapezoidal <i>B2</i> matrix. |
| <i>lda</i> | The leading dimension of <i>a</i> ; at least $\max(1, n)$. |
| <i>ldb</i> | The leading dimension of <i>b</i> ; at least $\max(1, m)$ for column major and $\max(1, n)$ for row major. |
| <i>ldt</i> | The leading dimension of <i>t</i> ; at least $\max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | The elements on and above the diagonal of the array contain the upper triangular matrix <i>R</i> . |
| <i>b</i> | The pentagonal matrix <i>V</i> . |
| <i>t</i> | Array, size $\max(1, ldt * n)$. The upper <i>n</i> -by- <i>n</i> upper triangular factor <i>T</i> of the block reflector. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0 and *info* = -*i*, the *i*th argument had an illegal value.

If *info* = -1011, memory allocation error occurred.

?tprfb

Applies a real or complex "triangular-pentagonal" blocked reflector to a real or complex matrix, which is composed of two blocks.

Syntax

```
lapack_int LAPACKE_stprfb (int matrix_layout, char side, char trans, char direct, char storev, lapack_int m, lapack_int n, lapack_int k, lapack_int l, const float * v, lapack_int ldv, const float * t, lapack_int ldt, float * a, lapack_int lda, float * b, lapack_int ldb);
```

```
lapack_int LAPACKE_dtprfb (int matrix_layout, char side, char trans, char direct, char storev, lapack_int m, lapack_int n, lapack_int k, lapack_int l, const double * v, lapack_int ldv, const double * t, lapack_int ldt, double * a, lapack_int lda, double * b, lapack_int ldb);
```

```
lapack_int LAPACKE_ctprfb (int matrix_layout, char side, char trans, char direct, char
storev, lapack_int m, lapack_int n, lapack_int k, lapack_int l, const
lapack_complex_float * v, lapack_int ldv, const lapack_complex_float * t, lapack_int
ldt, lapack_complex_float * a, lapack_int lda, lapack_complex_float * b, lapack_int
ldb);
```

```
lapack_int LAPACKE_ztprfb (int matrix_layout, char side, char trans, char direct, char
storev, lapack_int m, lapack_int n, lapack_int k, lapack_int l, const
lapack_complex_double * v, lapack_int ldv, const lapack_complex_double * t, lapack_int
ldt, lapack_complex_double * a, lapack_int lda, lapack_complex_double * b, lapack_int
ldb);
```

Include Files

- mkl.h

Description

The ?tprfb routine applies a real or complex "triangular-pentagonal" block reflector H , H^T , or H^H from either the left or the right to a real or complex matrix C , which is composed of two blocks A and B .

The block B is m -by- n . If $side = 'R'$, A is m -by- k , and if $side = 'L'$, A is of size k -by- n .

$$\begin{array}{l}
 \text{direct} = 'F' \quad \text{direct} = 'B' \\
 \text{side} = 'R' \quad C = \begin{bmatrix} A & B \end{bmatrix} \quad C = \begin{bmatrix} B & A \end{bmatrix} \\
 \text{side} = 'L' \quad C = \begin{bmatrix} A \\ B \end{bmatrix} \quad C = \begin{bmatrix} B \\ A \end{bmatrix}
 \end{array}$$

The pentagonal matrix V is composed of a rectangular block $V1$ and a trapezoidal block $V2$. The size of the trapezoidal block is determined by the parameter l , where $0 \leq l \leq k$. if $l=k$, the $V2$ block of V is triangular; if $l=0$, there is no trapezoidal block, thus $V = V1$ is rectangular.

| | |
|---|--|
| <i>direct</i> ='F' | <i>direct</i> ='B' |
| <i>storev</i> ='C' | <i>storev</i> ='C' |
| $V = \begin{bmatrix} V1 \\ V2 \end{bmatrix}$ | $V = \begin{bmatrix} V2 \\ V1 \end{bmatrix}$ |
| V2 is upper trapezoidal (first l rows of k -by- k upper triangular) | V2 is lower trapezoidal (last l rows of k -by- k lower triangular matrix) |
| <i>storev</i> ='R' | <i>storev</i> ='R' |
| $V = \begin{bmatrix} V1 & V2 \end{bmatrix}$ | $V = \begin{bmatrix} V2 & V1 \end{bmatrix}$ |
| V2 is lower trapezoidal (first l columns of k -by- k lower triangular matrix) | V2 is upper trapezoidal (last l columns of k -by- k upper triangular matrix) |

| | |
|---------------------|---------------------|
| <i>side</i> ='L' | <i>side</i> ='R' |
| <i>storev</i> ='C' | <i>storev</i> ='C' |
| V is m -by- k | V is n -by- k |

| | | |
|--------------------|--------------|--------------|
| <i>storev</i> ='R' | V2 is 1-by-k | V2 is 1-by-k |
| | V is k-by-m | V is k-by-n |
| | V2 is k-by-l | V2 is k-by-l |

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>side</i> | = 'L': apply H , H^T , or H^H from the left, = 'R': apply H , H^T , or H^H from the right. |
| <i>trans</i> | = 'N': apply H (no transpose), = 'T': apply H^T (transpose), = 'C': apply H^H (conjugate transpose). |
| <i>direct</i> | Indicates how H is formed from a product of elementary reflectors: = 'F': $H = H(1) H(2) \dots H(k)$ (Forward), = 'B': $H = H(k) \dots H(2) H(1)$ (Backward). |
| <i>storev</i> | Indicates how the vectors that define the elementary reflectors are stored: = 'C': Columns, = 'R': Rows. |
| <i>m</i> | The total number of rows in the matrix B ($m \geq 0$). |
| <i>n</i> | The number of columns in B ($n \geq 0$). |
| <i>k</i> | The order of the matrix T , which is the number of elementary reflectors whose product defines the block reflector. ($k \geq 0$) |
| <i>l</i> | The order of the trapezoidal part of V . ($k \geq l \geq 0$). |
| <i>v</i> | An array containing the pentagonal matrix V (the elementary reflectors $H(1), H(2), \dots, H(k)$). The size limitations depend on values of parameters <i>storev</i> and <i>side</i> as described in the following table |

| | <i>storev</i> = C | | <i>storev</i> = R | |
|--------------|--------------------|--------------------|--------------------|--------------------|
| | <i>side</i> = L | <i>side</i> = R | <i>side</i> = L | <i>side</i> = R |
| Column major | $\max(1, ldv * k)$ | $\max(1, ldv * k)$ | $\max(1, ldv * m)$ | $\max(1, ldv * n)$ |
| Row major | $\max(1, ldv * m)$ | $\max(1, ldv * n)$ | $\max(1, ldv * k)$ | $\max(1, ldv * k)$ |

ldv The leading dimension of the array *v*. It should satisfy the following conditions:

| | |
|-------------------|-------------------|
| <i>storev</i> = C | <i>storev</i> = R |
|-------------------|-------------------|

| | $side = L$ | $side = R$ | $side = L$ | $side = R$ |
|--------------|--------------|--------------|--------------|--------------|
| Column major | $\max(1, m)$ | $\max(1, n)$ | $\max(1, k)$ | $\max(1, k)$ |
| Row major | $\max(1, k)$ | $\max(1, k)$ | $\max(1, m)$ | $\max(1, n)$ |

t Array size $\max(1, ldt * k)$. The triangular k -by- k matrix T in the representation of the block reflector.

ldt The leading dimension of the array t ($ldt \geq k$).

a size should satisfy the following conditions:
 k if $side = 'R'$.

| | $side = L$ | $side = R$ |
|--------------|--------------------|--------------------|
| Column major | $\max(1, lda * n)$ | $\max(1, lda * k)$ |
| Row major | $\max(1, lda * k)$ | $\max(1, lda * m)$ |

The k -by- n or m -by- k matrix A .

lda The leading dimension of the array a should satisfy the following conditions:

| | $side = L$ | $side = R$ |
|--------------|--------------|--------------|
| Column major | $\max(1, k)$ | $\max(1, m)$ |
| Row major | $\max(1, n)$ | $\max(1, k)$ |

b Array size at least $\max(1, ldb * n)$ for column major layout and $\max(1, ldb * m)$ for row major layout, the m -by- n matrix B .

ldb The leading dimension of the array b ($ldb \geq \max(1, m)$ for column major layout and $ldb \geq \max(1, n)$ for row major layout).

Output Parameters

a Contains the corresponding block of H^*C , $H^T * C$, $H^H * C$, C^*H , C^*H^T , or C^*H^H .

b Contains the corresponding block of H^*C , $H^T * C$, $H^H * C$, C^*H , C^*H^T , or C^*H^H .

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info < 0$, the i -th parameter had an illegal value.

If $info = -1011$, memory allocation error occurred.

?tpdff

Copies a triangular matrix from the standard packed format (TP) to the rectangular full packed format (TF).

Syntax

```
lapack_int LAPACKE_stpttf (int matrix_layout , char transr , char uplo , lapack_int n ,
const float * ap , float * arf );
```

```
lapack_int LAPACKE_dtpttf (int matrix_layout , char transr , char uplo , lapack_int n ,
const double * ap , double * arf );
```

```
lapack_int LAPACKE_ctpttf (int matrix_layout , char transr , char uplo , lapack_int n ,
const lapack_complex_float * ap , lapack_complex_float * arf );
```

```
lapack_int LAPACKE_ztpttf (int matrix_layout , char transr , char uplo , lapack_int n ,
const lapack_complex_double * ap , lapack_complex_double * arf );
```

Include Files

- mkl.h

Description

The routine copies a triangular matrix A from the standard packed format to the Rectangular Full Packed (RFP) format. For the description of the RFP format, see [Matrix Storage Schemes](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>transr</i> | = 'N': <i>arf</i> must be in the Normal format, = 'T': <i>arf</i> must be in the Transpose format (for <i>stpttf</i> and <i>dtpttf</i>), = 'C': <i>arf</i> must be in the Conjugate-transpose format (for <i>ctpttf</i> and <i>ztpttf</i>). |
| <i>uplo</i> | Specifies whether A is upper or lower triangular: = 'U': A is upper triangular, = 'L': A is lower triangular. |
| <i>n</i> | The order of the matrix A . $n \geq 0$. |
| <i>ap</i> | Array, size at least $\max(1, n*(n+1)/2)$. On entry, the upper or lower triangular matrix A , packed in a linear array. See Matrix Storage Schemes for more information. |

Output Parameters

| | |
|------------|--|
| <i>arf</i> | Array, size at least $\max(1, n*(n+1)/2)$. On exit, the upper or lower triangular matrix A stored in the RFP format. |
|------------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

< 0: if *info* = -*i*, the *i*-th parameter had an illegal value.

If `info = -1011`, memory allocation error occurred.

?tptr

Copies a triangular matrix from the standard packed format (TP) to the standard full format (TR) .

Syntax

```
lapack_int LAPACKE_stptr (int matrix_layout , char uplo , lapack_int n , const float *
ap , float * a , lapack_int lda );
```

```
lapack_int LAPACKE_dtptr (int matrix_layout , char uplo , lapack_int n , const double
* ap , double * a , lapack_int lda );
```

```
lapack_int LAPACKE_ctptr (int matrix_layout , char uplo , lapack_int n , const
lapack_complex_float * ap , lapack_complex_float * a , lapack_int lda );
```

```
lapack_int LAPACKE_ztptr (int matrix_layout , char uplo , lapack_int n , const
lapack_complex_double * ap , lapack_complex_double * a , lapack_int lda );
```

Include Files

- `mkl.h`

Description

The routine copies a triangular matrix *A* from the standard packed format to the standard full format.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (<code>LAPACK_ROW_MAJOR</code>) or column major (<code>LAPACK_COL_MAJOR</code>). |
| <i>uplo</i> | Specifies whether <i>A</i> is upper or lower triangular: = 'U': <i>A</i> is upper triangular, = 'L': <i>A</i> is lower triangular. |
| <i>n</i> | The order of the matrices <i>ap</i> and <i>a</i> . $n \geq 0$. |
| <i>ap</i> | Array, size at least $\max(1, n*(n+1)/2)$. (see Matrix Storage Schemes). |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1,n)$. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | Array, size $\max(1,lda*n)$. On exit, the triangular matrix <i>A</i> . If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> contains the upper triangular part of the matrix <i>A</i> , and the strictly lower triangular part of <i>a</i> is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> contains the lower triangular part of the matrix <i>A</i> , and the strictly upper triangular part of <i>a</i> is not referenced. |
|----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* = -*i*, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?trttf

Copies a triangular matrix from the standard full format (TR) to the rectangular full packed format (TF).

Syntax

```
lapack_int LAPACKE_strttf (int matrix_layout , char transr , char uplo , lapack_int n ,
const float * a , lapack_int lda , float * arf );
```

```
lapack_int LAPACKE_dtrttf (int matrix_layout , char transr , char uplo , lapack_int n ,
const double * a , lapack_int lda , double * arf );
```

```
lapack_int LAPACKE_ctrttf (int matrix_layout , char transr , char uplo , lapack_int n ,
const lapack_complex_float * a , lapack_int lda , lapack_complex_float * arf );
```

```
lapack_int LAPACKE_ztrttf (int matrix_layout , char transr , char uplo , lapack_int n ,
const lapack_complex_double * a , lapack_int lda , lapack_complex_double * arf );
```

Include Files

- mkl.h

Description

The routine copies a triangular matrix *A* from the standard full format to the Rectangular Full Packed (RFP) format. For the description of the RFP format, see [Matrix Storage Schemes](#).

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>transr</i> | = 'N': <i>arf</i> must be in the Normal format, = 'T': <i>arf</i> must be in the Transpose format (for <i>strttf</i> and <i>dtrttf</i>), = 'C': <i>arf</i> must be in the Conjugate-transpose format (for <i>ctrttf</i> and <i>ztrttf</i>). |
| <i>uplo</i> | Specifies whether <i>A</i> is upper or lower triangular: = 'U': <i>A</i> is upper triangular, = 'L': <i>A</i> is lower triangular. |
| <i>n</i> | The order of the matrix <i>A</i> . $n \geq 0$. |
| <i>a</i> | Array, size $\max(1, (lda * n))$. |

On entry, the triangular matrix A . If $uplo = 'U'$, the leading n -by- n upper triangular part of the array a contains the upper triangular matrix, and the strictly lower triangular part of a is not referenced. If $uplo = 'L'$, the leading n -by- n lower triangular part of the array a contains the lower triangular matrix, and the strictly upper triangular part of a is not referenced.

lda The leading dimension of the array a . $lda \geq \max(1, n)$.

Output Parameters

arf Array, size at least $\max(1, n*(n+1)/2)$.
On exit, the upper or lower triangular matrix A stored in the RFP format.

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info < 0$, the i -th parameter had an illegal value.

If $info = -1011$, memory allocation error occurred.

?trttp

Copies a triangular matrix from the standard full format (TR) to the standard packed format (TP).

Syntax

```
lapack_int LAPACKE_strttp (int matrix_layout , char uplo , lapack_int n , const float *
a , lapack_int lda , float * ap );
```

```
lapack_int LAPACKE_dtrttp (int matrix_layout , char uplo , lapack_int n , const double
* a , lapack_int lda , double * ap );
```

```
lapack_int LAPACKE_ctrttp (int matrix_layout , char uplo , lapack_int n , const
lapack_complex_float * a , lapack_int lda , lapack_complex_float * ap );
```

```
lapack_int LAPACKE_ztrttp (int matrix_layout , char uplo , lapack_int n , const
lapack_complex_double * a , lapack_int lda , lapack_complex_double * ap );
```

Include Files

- mkl.h

Description

The routine copies a triangular matrix A from the standard full format to the standard packed format.

Input Parameters

$uplo$ Specifies whether A is upper or lower triangular:
= 'U': A is upper triangular,
= 'L': A is lower triangular.

| | |
|------------|--|
| <i>n</i> | The order of the matrix <i>A</i> , $n \geq 0$. |
| <i>a</i> | Array, size $\max(1, lda * n)$. On entry, the triangular matrix <i>A</i> . If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of the array <i>a</i> contains the upper triangular matrix, and the strictly lower triangular part of <i>a</i> is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i> -by- <i>n</i> lower triangular part of the array <i>a</i> contains the lower triangular matrix, and the strictly upper triangular part of <i>a</i> is not referenced. |
| <i>lda</i> | The leading dimension of the array <i>a</i> . $lda \geq \max(1, n)$. |

Output Parameters

| | |
|-----------|--|
| <i>ap</i> | Array, size at least $\max(1, n*(n+1)/2)$. On exit, the upper or lower triangular matrix <i>A</i> , packed columnwise in a linear array. (see Matrix Storage Schemes) |
|-----------|--|

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?zlacp2

Copies all or part of a real two-dimensional array to a complex array.

Syntax

```
lapack_int LAPACKE_clacp2 (int matrix_layout , char uplo , lapack_int m , lapack_int
n , const float * a , lapack_int lda , lapack_complex_float * b , lapack_int ldb );
lapack_int LAPACKE_zlacp2 (int matrix_layout , char uplo , lapack_int m , lapack_int
n , const double * a , lapack_int lda , lapack_complex_double * b , lapack_int ldb );
```

Include Files

- mkl.h

Description

The routine copies all or part of a real matrix *A* to another matrix *B*.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>uplo</i> | Specifies the part of the matrix <i>A</i> to be copied to <i>B</i> . If <i>uplo</i> = 'U', the upper triangular part of <i>A</i> ; if <i>uplo</i> = 'L', the lower triangular part of <i>A</i> . |

| | |
|-------|--|
| | Otherwise, all of the matrix A is copied. |
| m | The number of rows in the matrix A ($m \geq 0$). |
| n | The number of columns in A ($n \geq 0$). |
| a | Array, size at least $\max(1, lda*n)$ for column major and $\max(1, lda*m)$ for row major, contains the m -by- n matrix A . If $uplo = 'U'$, only the upper triangle or trapezoid is accessed; if $uplo = 'L'$, only the lower triangle or trapezoid is accessed. |
| lda | The leading dimension of a ; $lda \geq \max(1, m)$ for column major and $lda \geq \max(1, n)$ for row major. |
| ldb | The leading dimension of the output array b ; $ldb \geq \max(1, m)$ for column major and $ldb \geq \max(1, n)$ for row major. |

Output Parameters

| | |
|-----|--|
| b | Array, size at least $\max(1, ldb*n)$ for column major layout and $\max(1, ldb*m)$ for row major layout, contains the m -by- n matrix B . On exit, $B = A$ in the locations specified by $uplo$. |
|-----|--|

Return Values

This function returns a value *info*.

If $info = 0$, the execution is successful.

If $info < 0$, the i -th parameter had an illegal value.

If $info = -1011$, memory allocation error occurred.

mkl_?tppack

Copies a triangular/symmetric matrix or submatrix from standard full format to standard packed format.

Syntax

```
lapack_int LAPACKE_mkl_stppack (int matrix_layout, char uplo, char trans, lapack_int n,
float* ap, lapack_int i, lapack_int j, lapack_int rows, lapack_int cols, const float*
a, lapack_int lda);
```

```
lapack_int LAPACKE_mkl_dtpack (int matrix_layout, char uplo, char trans, lapack_int n,
double* ap, lapack_int i, lapack_int j, lapack_int rows, lapack_int cols, const
double* a, lapack_int lda);
```

```
lapack_int LAPACKE_mkl_ctppack (int matrix_layout, char uplo, char trans, lapack_int n,
MKL_Complex8* ap, lapack_int i, lapack_int j, lapack_int rows, lapack_int cols, const
MKL_Complex8* a, lapack_int lda);
```

```
lapack_int LAPACKE_mkl_ztpack (int matrix_layout, char uplo, char trans, lapack_int n,
MKL_Complex16* ap, lapack_int i, lapack_int j, lapack_int rows, lapack_int cols, const
MKL_Complex16* a, lapack_int lda);
```

Include Files

- `mk1.h`

Description

The routine copies a triangular or symmetric matrix or its submatrix from standard full format to packed format

$$AP_{i:i+rows-1, j:j+cols-1} := \text{op}(A)$$

Standard packed formats include:

- TP: triangular packed storage
- SP: symmetric indefinite packed storage
- HP: Hermitian indefinite packed storage
- PP: symmetric or Hermitian positive definite packed storage

Full formats include:

- GE: general
- TR: triangular
- SY: symmetric indefinite
- HE: Hermitian indefinite
- PO: symmetric or Hermitian positive definite

NOTE

Any elements of the copied submatrix rectangular outside of the triangular part of the matrix AP are skipped.

Input Parameters

| | |
|--------------|---|
| <i>uplo</i> | Specifies whether the matrix AP is upper or lower triangular. If <i>uplo</i> = 'U', AP is upper triangular. If <i>uplo</i> = 'L': AP is lower triangular. |
| <i>trans</i> | Specifies whether or not the copied block of A is transposed or not. If <i>trans</i> = 'N', no transpose: $\text{op}(A) = A$. If <i>trans</i> = 'T', transpose: $\text{op}(A) = A^T$. If <i>trans</i> = 'C', conjugate transpose: $\text{op}(A) = A^H$. For real data this is the same as <i>trans</i> = 'T'. |
| <i>n</i> | The order of the matrix AP ; $n \geq 0$ |
| <i>i, j</i> | Coordinates of the left upper corner of the destination submatrix in AP . If <i>uplo</i> = 'U', $1 \leq i \leq j \leq n$. If <i>uplo</i> = 'L', $1 \leq j \leq i \leq n$. |
| <i>rows</i> | Number of rows in the destination submatrix. $0 \leq \text{rows} \leq n - i + 1$. |
| <i>cols</i> | Number of columns in the destination submatrix. $0 \leq \text{cols} \leq n - j + 1$. |
| <i>a</i> | Pointer to the source submatrix. |

Array *a* contains the *rows*-by-*cols* submatrix stored as unpacked rows-by-columns if *trans* = 'N', or unpacked columns-by-rows if *trans* = 'T' or *trans* = 'C'.

The size of *a* is

| | <i>trans</i> = 'N' | <i>trans</i> ='T' or <i>trans</i> ='C' |
|--|--------------------------|---|
| <i>matrix_layout</i> = LAPACK_COL_MAJOR | <i>lda</i> * <i>cols</i> | <i>lda</i> * <i>rows</i> |
| <i>matrix_layout</i> = LAPACK_ROW_MAJOR | <i>lda</i> * <i>rows</i> | <i>lda</i> * <i>cols</i> |

NOTE

If there are elements outside of the triangular part of *AP*, they are skipped and are not copied from *a*.

lda

The leading dimension of the array *a*.

| | <i>trans</i> = 'N' | <i>trans</i> ='T' or <i>trans</i> ='C' |
|--|--------------------------------------|---|
| <i>matrix_layout</i> = LAPACK_COL_MAJOR | <i>lda</i> ≥ max(1, <i>rows</i>) | <i>lda</i> ≥ max(1, <i>cols</i>) |
| <i>matrix_layout</i> = LAPACK_ROW_MAJOR | <i>lda</i> ≥ max(1, <i>cols</i>) | <i>lda</i> ≥ max(1, <i>rows</i>) |

Output Parameters

ap

Array of size at least max(1, *n*(*n*+1)/2). The array *ap* contains either the upper or the lower triangular part of the matrix *AP* (as specified by *uplo*) in packed storage (see [Matrix Storage Schemes](#)). The submatrix of *ap* from row *i* to row *i* + *rows* - 1 and column *j* to column *j* + *cols* - 1 is overwritten with a copy of the source matrix.

Return Values

This function returns a value *info*. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

mkl_?tpunpack

Copies a triangular/symmetric matrix or submatrix from standard packed format to full format.

Syntax

```
lapack_int LAPACKE_mkl_stpunpack ( int matrix_layout, char uplo, char trans,
    lapack_int n, const float* ap, lapack_int i, lapack_int j, lapack_int rows,
    lapack_int cols, float* a, lapack_int lda );
```

```
lapack_int LAPACKE_mkl_dtpunpack ( int matrix_layout, char uplo, char trans,
    lapack_int n, const double* ap, lapack_int i, lapack_int j, lapack_int rows,
    lapack_int cols, double* a, lapack_int lda );

lapack_int LAPACKE_mkl_ctpunpack ( int matrix_layout, char uplo, char trans,
    lapack_int n, const MKL_Complex8* ap, lapack_int i, lapack_int j, lapack_int rows,
    lapack_int cols, MKL_Complex8* a, lapack_int lda );

lapack_int LAPACKE_mkl_ztpunpack ( int matrix_layout, char uplo, char trans,
    lapack_int n, const MKL_Complex16* ap, lapack_int i, lapack_int j, lapack_int rows,
    lapack_int cols, MKL_Complex16* a, lapack_int lda );
```

Include Files

- mkl.h

Description

The routine copies a triangular or symmetric matrix or its submatrix from standard packed format to full format.

$A := \text{op}(AP_{i:i+\text{rows}-1, j:j+\text{cols}-1})$

Standard packed formats include:

- TP: triangular packed storage
- SP: symmetric indefinite packed storage
- HP: Hermitian indefinite packed storage
- PP: symmetric or Hermitian positive definite packed storage

Full formats include:

- GE: general
- TR: triangular
- SY: symmetric indefinite
- HE: Hermitian indefinite
- PO: symmetric or Hermitian positive definite

NOTE

Any elements of the copied submatrix rectangular outside of the triangular part of AP are skipped.

Input Parameters

| | |
|--------------|---|
| <i>uplo</i> | Specifies whether matrix AP is upper or lower triangular. If <i>uplo</i> = 'U', AP is upper triangular. If <i>uplo</i> = 'L': AP is lower triangular. |
| <i>trans</i> | Specifies whether or not the copied block of AP is transposed. If <i>trans</i> = 'N', no transpose: $\text{op}(AP) = AP$. If <i>trans</i> = 'T', transpose: $\text{op}(AP) = AP^T$. If <i>trans</i> = 'C', conjugate transpose: $\text{op}(AP) = AP^H$. For real data this is the same as <i>trans</i> = 'T'. |
| <i>n</i> | The order of the matrix AP ; $n \geq 0$. |

ap Array, size at least $\max(1, n(n+1)/2)$. The array *ap* contains either the upper or the lower triangular part of the matrix *AP* (as specified by *uplo*) in packed storage (see [Matrix Storage Schemes](#)). It is the source for the submatrix of *AP* from row *i* to row *i + rows - 1* and column *j* to column *j + cols - 1* to be copied.

i, j Coordinates of left upper corner of the submatrix in *AP* to copy.
 If *uplo*='U', $1 \leq i \leq j \leq n$.
 If *uplo*='L', $1 \leq j \leq i \leq n$.

rows Number of rows to copy. $0 \leq rows \leq n - i + 1$.

cols Number of columns to copy. $0 \leq cols \leq n - j + 1$.

lda The leading dimension of array *a*.

| | <i>trans</i> = 'N' | <i>trans</i> ='T' or <i>trans</i> ='C' |
|--|-----------------------------------|---|
| <i>matrix_layout</i> = LAPACK_COL_MAJOR | <i>lda</i> ≥ max(1, <i>rows</i>) | <i>lda</i> ≥ max(1, <i>cols</i>) |
| <i>matrix_layout</i> = LAPACK_ROW_MAJOR | <i>lda</i> ≥ max(1, <i>cols</i>) | <i>lda</i> ≥ max(1, <i>rows</i>) |

Output Parameters

a Pointer to the destination matrix. On exit, array *a* is overwritten with a copy of the unpacked *rows*-by-*cols* submatrix of *ap* unpacked rows-by-columns if *trans* = 'N', or unpacked columns-by-rows if *trans* = 'T' or *trans* = 'C'.
 The size of *a* is

| | <i>trans</i> = 'N' | <i>trans</i> ='T' or <i>trans</i> ='C' |
|--|--------------------------|---|
| <i>matrix_layout</i> = LAPACK_COL_MAJOR | <i>lda</i> * <i>cols</i> | <i>lda</i> * <i>rows</i> |
| <i>matrix_layout</i> = LAPACK_ROW_MAJOR | <i>lda</i> * <i>rows</i> | <i>lda</i> * <i>cols</i> |

NOTE

If there are elements outside of the triangular part of *ap* indicated by *uplo*, they are skipped and are not copied to *a*.

Return Values

This function returns a value *info*. If *info*=0, the execution is successful. If *info* = -*i*, the *i*-th parameter had an illegal value.

LAPACK Utility Functions and Routines

This section describes LAPACK utility functions and routines.

See Also

[lsame](#) Tests two characters for equality regardless of the case.

[lsamen](#) Tests two character strings for equality regardless of the case.

[second/dsecond](#) Returns elapsed time in seconds. Use to estimate real time between two calls to this function.

[xerbla](#) Error handling function called by BLAS, LAPACK, Vector Math, and Vector Statistics functions.

ilaver

Returns the version of the LAPACK library.

Syntax

```
void LAPACKE_ilaver (lapack_int * vers_major, lapack_int * vers_minor, lapack_int * vers_patch);
```

Include Files

- mkl.h

Description

This routine returns the version of the LAPACK library.

Output Parameters

| | |
|-------------------|---|
| <i>vers_major</i> | Returns the major version of the LAPACK library. |
| <i>vers_minor</i> | Returns the minor version from the major version of the LAPACK library. |
| <i>vers_patch</i> | Returns the patch version from the minor version of the LAPACK library. |

?lamch

Determines machine parameters for floating-point arithmetic.

Syntax

```
float LAPACKE_slamch (char cmach );  
double LAPACKE_dlamch (char cmach );
```

Include Files

- mkl.h

Description

The function ?lamch determines single precision and double precision machine parameters.

Input Parameters

cmach

Specifies the value to be returned by `?lamch`:

- = 'E' or 'e', *val* = *eps*
- = 'S' or 's', *val* = *sfmin*
- = 'B' or 'b', *val* = *base*
- = 'P' or 'p', *val* = *eps*base*
- = 'n' or 'n', *val* = *t*
- = 'R' or 'r', *val* = *rnd*
- = 'M' or 'm', *val* = *emin*
- = 'U' or 'u', *val* = *rmin*
- = 'L' or 'l', *val* = *emax*
- = 'O' or 'o', *val* = *rmax*

where

eps = relative machine precision;

sfmin = safe minimum, such that $1/sfmin$ does not overflow;

base = base of the machine;

prec = $eps*base$;

t = number of (base) digits in the mantissa;

rnd = 1.0 when rounding occurs in addition, 0.0 otherwise;

emin = minimum exponent before (gradual) underflow;

rmin = $underflow_threshold - base**(emin-1)$;

emax = largest exponent before overflow;

rmax = $overflow_threshold - (base**emax)*(1-eps)$.

NOTE

You can use a character string for *cmach* instead of a single character in order to make your code more readable. The first character of the string determines the value to be returned. For example, 'Precision' is interpreted as 'p'.

Output Parameters

val

Value returned by the function.

LAPACK Test Functions and Routines

This section describes LAPACK test functions and routines.

?lagge

Generates a general m-by-n matrix .

Syntax

```
lapack_int LAPACKE_slagge (int matrix_layout , lapack_int m , lapack_int n , lapack_int
kl , lapack_int ku , const float * d , float * a , lapack_int lda , lapack_int *
iseed );
```

```
lapack_int LAPACKE_dlagge (int matrix_layout , lapack_int m , lapack_int n , lapack_int
kl , lapack_int ku , const double * d , double * a , lapack_int lda , lapack_int *
iseed );
```

```
lapack_int LAPACKE_clagge (int matrix_layout , lapack_int m , lapack_int n , lapack_int
kl , lapack_int ku , const float * d , lapack_complex_float * a , lapack_int lda ,
lapack_int * iseed );
```

```
lapack_int LAPACKE_zlagge (int matrix_layout , lapack_int m , lapack_int n , lapack_int
kl , lapack_int ku , const double * d , lapack_complex_double * a , lapack_int lda ,
lapack_int * iseed );
```

Include Files

- mkl.h

Description

The routine generates a general m -by- n matrix A , by pre- and post- multiplying a real diagonal matrix D with random matrices U and V :

$$A := U * D * V,$$

where U and V are orthogonal for real flavors and unitary for complex flavors. The lower and upper bandwidths may then be reduced to kl and ku by additional orthogonal transformations.

Input Parameters

| | |
|---------|--|
| m | The number of rows of the matrix A ($m \geq 0$). |
| n | The number of columns of the matrix A ($n \geq 0$). |
| kl | The number of nonzero subdiagonals within the band of A ($0 \leq kl \leq m-1$). |
| ku | The number of nonzero superdiagonals within the band of A ($0 \leq ku \leq n-1$). |
| d | The array d with the dimension of $(\min(m, n))$ contains the diagonal elements of the diagonal matrix D . |
| lda | The leading dimension of the array a ($lda \geq m$) for column major layout and ($lda \geq n$) for row major layout. |
| $iseed$ | The array $iseed$ with the dimension of 4 contains the seed of the random number generator. The elements must be between 0 and 4095 and $iseed$ must be odd. |

Output Parameters

| | |
|--------------|--|
| <i>a</i> | The array <i>a</i> with size at least $\max(1, lda*n)$ for column major layout and $\max(1, lda*m)$ for row major layout contains the generated <i>m</i> -by- <i>n</i> matrix <i>A</i> . |
| <i>iseed</i> | The array <i>iseed</i> contains the updated seed on exit. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?laghe

Generates a complex Hermitian matrix .

Syntax

```
lapack_int LAPACKE_claghe (int matrix_layout , lapack_int n , lapack_int k , const
float * d , lapack_complex_float * a , lapack_int lda , lapack_int * iseed );
```

```
lapack_int LAPACKE_zlaghe (int matrix_layout , lapack_int n , lapack_int k , const
double * d , lapack_complex_double * a , lapack_int lda , lapack_int * iseed );
```

Include Files

- mkl.h

Description

The routine generates a complex Hermitian matrix *A*, by pre- and post- multiplying a real diagonal matrix *D* with random unitary matrix:

$$A := U * D * U^H$$

The semi-bandwidth may then be reduced to *k* by additional unitary transformations.

Input Parameters

| | |
|----------------------|--|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>n</i> | The order of the matrix <i>A</i> ($n \geq 0$). |
| <i>k</i> | The number of nonzero subdiagonals within the band of <i>A</i> ($0 \leq k \leq n-1$). |
| <i>d</i> | The array <i>d</i> with the dimension of (<i>n</i>) contains the diagonal elements of the diagonal matrix <i>D</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> ($lda \geq n$). |

iseed The array *iseed* with the dimension of 4 contains the seed of the random number generator. The elements must be between 0 and 4095 and *iseed*[3] must be odd.

Output Parameters

a The array *a* of size at least $\max(1, lda*n)$ contains the generated *n*-by-*n* Hermitian matrix *D*.

iseed The array *iseed* contains the updated seed on exit.

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?lagsy

Generates a symmetric matrix by pre- and post-multiplying a real diagonal matrix with a random unitary matrix .

Syntax

```
lapack_int LAPACKE_slagsy (int matrix_layout , lapack_int n , lapack_int k , const float * d , float * a , lapack_int lda , lapack_int * iseed );
```

```
lapack_int LAPACKE_dlagsy (int matrix_layout , lapack_int n , lapack_int k , const double * d , double * a , lapack_int lda , lapack_int * iseed );
```

```
lapack_int LAPACKE_clagsy (int matrix_layout , lapack_int n , lapack_int k , const float * d , lapack_complex_float * a , lapack_int lda , lapack_int * iseed );
```

```
lapack_int LAPACKE_zlagsy (int matrix_layout , lapack_int n , lapack_int k , const double * d , lapack_complex_double * a , lapack_int lda , lapack_int * iseed );
```

Include Files

- mkl.h

Description

The ?lagsy routine generates a symmetric matrix *A* by pre- and post- multiplying a real diagonal matrix *D* with a random matrix *U*:

$$A := U * D * U^T,$$

where *U* is orthogonal for real flavors and unitary for complex flavors. The semi-bandwidth may then be reduced to *k* by additional unitary transformations.

Input Parameters

n The order of the matrix *A* ($n \geq 0$).

k The number of nonzero subdiagonals within the band of *A* ($0 \leq k \leq n-1$).

| | |
|--------------|--|
| <i>d</i> | The array <i>d</i> with the dimension of (<i>n</i>) contains the diagonal elements of the diagonal matrix <i>D</i> . |
| <i>lda</i> | The leading dimension of the array <i>a</i> ($lda \geq n$). |
| <i>iseed</i> | The array <i>iseed</i> with the dimension of 4 contains the seed of the random number generator. The elements must be between 0 and 4095 and <i>iseed</i> [3] must be odd. |

Output Parameters

| | |
|--------------|--|
| <i>a</i> | The array <i>a</i> of size max (1, <i>lda</i> * <i>n</i>) contains the generated symmetric <i>n</i> -by- <i>n</i> matrix <i>D</i> . |
| <i>iseed</i> | The array <i>iseed</i> contains the updated seed on exit. |

Return Values

This function returns a value *info*.

If *info* = 0, the execution is successful.

If *info* < 0, the *i*-th parameter had an illegal value.

If *info* = -1011, memory allocation error occurred.

?latms

Generates a general m-by-n matrix with specific singular values.

Syntax

```
lapack_int LAPACKE_slatms (int matrix_layout, lapack_int m, lapack_int n, char dist,
lapack_int * iseed, char sym, float * d, lapack_int mode, float cond, float dmax,
lapack_int kl, lapack_int ku, char pack, float * a, lapack_int lda);

lapack_int LAPACKE_dlatms (int matrix_layout, lapack_int m, lapack_int n, char dist,
lapack_int * iseed, char sym, double * d, lapack_int mode, double cond, double dmax,
lapack_int kl, lapack_int ku, char pack, double * a, lapack_int lda);

lapack_int LAPACKE_clatms (int matrix_layout, lapack_int m, lapack_int n, char dist,
lapack_int * iseed, char sym, float * d, lapack_int mode, float cond, float dmax,
lapack_int kl, lapack_int ku, char pack, lapack_complex_float * a, lapack_int lda);

lapack_int LAPACKE_zlatms (int matrix_layout, lapack_int m, lapack_int n, char dist,
lapack_int * iseed, char sym, double * d, lapack_int mode, double cond, double dmax,
lapack_int kl, lapack_int ku, char pack, lapack_complex_double * a, lapack_int lda);
```

Include Files

- mkl.h

Description

The ?latms routine generates random matrices with specified singular values, or symmetric/Hermitian matrices with specified eigenvalues for testing LAPACK programs.

It applies this sequence of operations:

1. Set the diagonal to d , where d is input or computed according to $mode$, $cond$, $dmax$, and sym as described in Input Parameters.
2. Generate a matrix with the appropriate band structure, by one of two methods:

Method A

1. Generate a dense m -by- n matrix by multiplying d on the left and the right by random unitary matrices, then:
2. Reduce the bandwidth according to kl and ku , using Householder transformations.

Method B:

Convert the bandwidth-0 (i.e., diagonal) matrix to a bandwidth-1 matrix using Givens rotations, "chasing" out-of-band elements back, much as in QR; then convert the bandwidth-1 to a bandwidth-2 matrix, etc.

Note that for reasonably small bandwidths (relative to m and n) this requires less storage, as a dense matrix is not generated. Also, for symmetric or Hermitian matrices, only one triangle is generated.

Method A is chosen if the bandwidth is a large fraction of the order of the matrix, and lda is at least m (so a dense matrix can be stored.) Method B is chosen if the bandwidth is small (less than $(1/2)*n$ for symmetric or Hermitian or less than $.3*n+m$ for nonsymmetric), or lda is less than m and not less than the bandwidth.

Pack the matrix if desired, using one of the methods specified by the $pack$ parameter.

If Method B is chosen and band format is specified, then the matrix is generated in the band format and no repacking is necessary.

Input Parameters

| | |
|----------------------|---|
| <i>matrix_layout</i> | Specifies whether matrix storage layout is row major (LAPACK_ROW_MAJOR) or column major (LAPACK_COL_MAJOR). |
| <i>m</i> | The number of rows of the matrix A ($m \geq 0$). |
| <i>n</i> | The number of columns of the matrix A ($n \geq 0$). |
| <i>dist</i> | Specifies the type of distribution to be used to generate the random singular values or eigenvalues: <ul style="list-style-type: none"> • 'U': uniform distribution (0, 1) • 'S': symmetric uniform distribution (-1, 1) • 'N': normal distribution (0, 1) |
| <i>iseed</i> | Array with size 4. Specifies the seed of the random number generator. Values should lie between 0 and 4095 inclusive, and <i>iseed</i> [3] should be odd. The random number generator uses a linear congruential sequence limited to small integers, and so should produce machine independent random numbers. The values of the array are modified, and can be used in the next call to <code>latms</code> to continue the same random number sequence. |
| <i>sym</i> | If <i>sym</i> ='S' or 'H', the generated matrix is symmetric or Hermitian, with eigenvalues specified by d , $cond$, $mode$, and $dmax$; they can be positive, negative, or zero. |

If *sym*='P', the generated matrix is symmetric or Hermitian, with eigenvalues (which are singular, non-negative values) specified by *d*, *cond*, *mode*, and *dmax*.

If *sym*='N', the generated matrix is nonsymmetric, with singular, non-negative values specified by *d*, *cond*, *mode*, and *dmax*.

| | |
|---|--|
| <i>d</i> | <p>Array, size (MIN(<i>m</i>, <i>n</i>))</p> <p>This array is used to specify the singular values or eigenvalues of <i>A</i> (see the description of <i>sym</i>). If <i>mode</i>=0, then <i>d</i> is assumed to contain the eigenvalues or singular values, otherwise elements of <i>d</i> are computed according to <i>mode</i>, <i>cond</i>, and <i>dmax</i>.</p> |
| <i>mode</i> | <p>Describes how the singular/eigenvalues are specified.</p> <ul style="list-style-type: none"> • <i>mode</i> = 0: use <i>d</i> as input • <i>mode</i> = 1: set $d[0] = 1$ and $d[1:n - 1] = 1.0/cond$ • <i>mode</i> = 2: set $d[0:n - 2] = 1$ and $d[n - 1] = 1.0/cond$ • <i>mode</i> = 3: set $d[i] = cond^{-i/(n - 1)}$ • <i>mode</i> = 4: set $d[i] = 1 - i/(n - 1) * (1 - 1/cond)$ • <i>mode</i> = 5: set elements of <i>d</i> to random numbers in the range (1/<i>cond</i>, 1) such that their logarithms are uniformly distributed. • <i>mode</i> = 6: set elements of <i>d</i> to random numbers from same distribution as the rest of the matrix. <p><i>mode</i> < 0 has the same meaning as ABS(<i>mode</i>), except that the order of the elements of <i>d</i> is reversed. Thus, if <i>mode</i> is positive, <i>d</i> has entries ranging from 1 to 1/<i>cond</i>, if negative, from 1/<i>cond</i> to 1.</p> <p>If <i>sym</i>='S' or 'H', and <i>mode</i> is not 0, 6, nor -6, then the elements of <i>d</i> are also given a random sign (multiplied by +1 or -1).</p> |
| <i>cond</i> | Used in setting <i>d</i> as described for the <i>mode</i> parameter. If used, $cond \geq 1$. |
| <i>dmax</i> | If <i>mode</i> is not -6, 0 nor 6, the contents of <i>d</i> , as computed according to <i>mode</i> and <i>cond</i> , are scaled by $dmax / \max(\text{abs}(d[i-1]))$; thus, the maximum absolute eigenvalue or singular value (the norm) is $\text{abs}(dmax)$. |
| <p>NOTE</p> <p><i>dmax</i> need not be positive: if <i>dmax</i> is negative (or zero), <i>d</i> will be scaled by a negative number (or zero).</p> | |
| <i>kl</i> | Specifies the lower bandwidth of the matrix. For example, <i>kl</i> =0 implies upper triangular, <i>kl</i> =1 implies upper Hessenberg, and <i>kl</i> being at least <i>m</i> - 1 means that the matrix has full lower bandwidth. <i>kl</i> must equal <i>ku</i> if the matrix is symmetric or Hermitian. |
| <i>ku</i> | Specifies the upper bandwidth of the matrix. For example, <i>ku</i> =0 implies lower triangular, <i>ku</i> =1 implies lower Hessenberg, and <i>ku</i> being at least <i>n</i> - 1 means that the matrix has full upper bandwidth. <i>kl</i> must equal <i>ku</i> if the matrix is symmetric or Hermitian. |
| <i>pack</i> | Specifies packing of matrix: |

- 'N': no packing
- 'U': zero out all subdiagonal entries (if symmetric or Hermitian)
- 'L': zero out all superdiagonal entries (if symmetric or Hermitian)
- 'B': store the lower triangle in band storage scheme (only if matrix symmetric, Hermitian, or lower triangular)
- 'Q': store the upper triangle in band storage scheme (only if matrix symmetric, Hermitian, or upper triangular)
- 'Z': store the entire matrix in band storage scheme (pivoting can be provided for by using this option to store *A* in the trailing rows of the allocated storage)

Using these options, the various LAPACK packed and banded storage schemes can be obtained:

| | 'Z' | 'B' | 'Q' | 'C' | 'R' |
|--|-----|-----|-----|-----|-----|
| GB: general band | x | | | | |
| PB: symmetric positive definite band | | x | x | | |
| SB: symmetric band | | x | x | | |
| HB: Hermitian band | | x | x | | |
| TB: triangular band | | x | x | | |
| PP: symmetric positive definite packed | | | | x | x |
| SP: symmetric packed | | | | x | x |
| HP: Hermitian packed | | | | x | x |
| TP: triangular packed | | | | x | x |

If two calls to `?latms` differ only in the `pack` parameter, they generate mathematically equivalent matrices.

lda

lda specifies the first dimension of *a* as declared in the calling program.

If `pack='N', 'U', 'L', 'C', or 'R'`, then *lda* must be at least *m* for column major or at least *n* for row major.

If `pack='B' or 'Q'`, then *lda* must be at least $\text{MIN}(kl, m - 1)$ (which is equal to $\text{MIN}(ku, n - 1)$).

If `pack='Z'`, *lda* must be large enough to hold the packed array: $\text{MIN}(ku, n - 1) + \text{MIN}(kl, m - 1) + 1$.

Output Parameters

iseed

The array *iseed* contains the updated seed.

d

The array *d* contains the updated seed.

NOTE

The array *d* is not modified if `mode = 0`.

a

Array of size *lda* by *n*.

The array a contains the generated m -by- n matrix A .

a is first generated in full (unpacked) form, and then packed, if so specified by $pack$. Thus, the first m elements of the first n columns are always modified. If $pack$ specifies a packed or banded storage scheme, all lda elements of the first n columns are modified; the elements of the array which do not correspond to elements of the generated matrix are set to zero.

Return Values

This function returns a value $info$.

If $info = 0$, the execution is successful.

If $info < 0$, the i -th parameter had an illegal value.

If $info = -1011$, memory allocation error occurred.

If $info = 2$, cannot scale to $dmax$ (maximum singular value is 0).

If $info = 3$, error return from [lagge](#), [?laghe](#), or [lagsy](#).

ScaLAPACK Routines

Intel® Math Kernel Library implements routines from the ScaLAPACK package for distributed-memory architectures. Routines are supported for both real and complex dense and band matrices to perform the tasks of solving systems of linear equations, solving linear least-squares problems, eigenvalue and singular value problems, as well as performing a number of related computational tasks.

Intel MKL ScaLAPACK routines are written in FORTRAN 77 with exception of a few utility routines written in C to exploit the IEEE arithmetic. All routines are available in all precision types: single precision, double precision, complexm, and double complex precision. See the `mkl_scalapack.h` header file for C declarations of ScaLAPACK routines.

NOTE

ScaLAPACK routines are provided only for Intel® 64 or Intel® Many Integrated Core architectures.

See descriptions of ScaLAPACK [computational routines](#) that perform distinct computational tasks, as well as [driver routines](#) for solving standard types of problems in one call. Additionally, Intel® Math Kernel Library implements ScaLAPACK [Auxiliary Routines](#), [Utility Functions and Routines](#), and [Matrix Redistribution/Copy Routines](#). The library includes routines for both real and complex data.

The `<install_directory>/examples/scalapackf` directory contains sample code demonstrating the use of ScaLAPACK routines.

Generally, ScaLAPACK runs on a network of computers using MPI as a message-passing layer and a set of prebuilt communication subprograms (BLACS), as well as a set of BLAS optimized for the target architecture. Intel MKL version of ScaLAPACK is optimized for Intel® processors. For the detailed system and environment requirements, see *Intel® MKL Release Notes* and *Intel® MKL Developer Guide*.

For full reference on ScaLAPACK routines and related information, see [\[SLUG\]](#).

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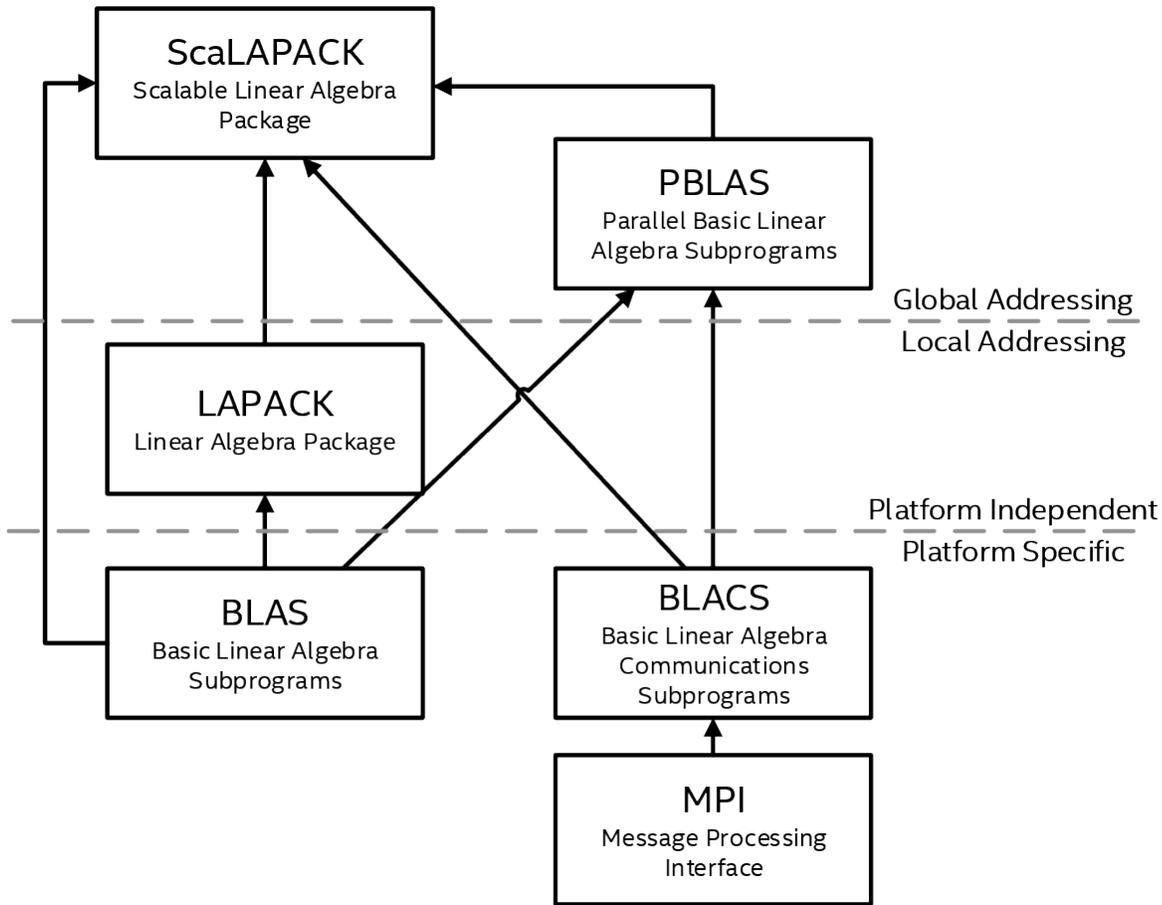
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Overview of ScaLAPACK Routines

The model of the computing environment for ScaLAPACK is represented as a one-dimensional array of processes (for operations on band or tridiagonal matrices) or also a two-dimensional process grid (for operations on dense matrices). To use ScaLAPACK, all global matrices or vectors should be distributed on this array or grid prior to calling the ScaLAPACK routines.

ScaLAPACK is closely tied to other components, including BLAS, BLACS, LAPACK, and PBLAS.



ScaLAPACK Array Descriptors

ScaLAPACK uses two-dimensional block-cyclic data distribution as a layout for dense matrix computations. This distribution provides good work balance between available processors, and also allows use of BLAS Level 3 routines for optimal local computations. Information about the data distribution that is required to establish the mapping between each global matrix and its corresponding process and memory location is contained in the array called the *array descriptor* associated with each global matrix. The size of the array descriptor is denoted as *dlen_*.

Let A be a two-dimensional block cyclicly distributed matrix with the array descriptor array *desca*. The meaning of each array descriptor element depends on the type of the matrix A . The tables "Array descriptor for dense matrices" and "Array descriptor for narrow-band and tridiagonal matrices" describe the meaning of each element for the different types of matrices.

Array descriptor for dense matrices (*dlen_=9*)

| Element Name | Stored in | Description | Element Index Number |
|----------------|----------------------|--|----------------------|
| <i>dtype_a</i> | <i>desca[dtype_]</i> | Descriptor type (=1 for dense matrices). | 0 |
| <i>ctxt_a</i> | <i>desca[ctxt_]</i> | BLACS context handle for the process grid. | 1 |
| <i>m_a</i> | <i>desca[m_]</i> | Number of rows in the global matrix A . | 2 |
| <i>n_a</i> | <i>desca[n_]</i> | Number of columns in the global matrix A . | 3 |
| <i>mb_a</i> | <i>desca[mb_]</i> | Row blocking factor. | 4 |
| <i>nb_a</i> | <i>desca[nb_]</i> | Column blocking factor. | 5 |

| Element Name | Stored in | Description | Element Index Number |
|---------------|---------------------|--|----------------------|
| <i>rsrc_a</i> | <i>desca[rsrc_]</i> | Process row over which the first row of the global matrix <i>A</i> is distributed. | 6 |
| <i>csrc_a</i> | <i>desca[csrc_]</i> | Process column over which the first column of the global matrix <i>A</i> is distributed. | 7 |
| <i>lld_a</i> | <i>desca[lld_]</i> | Leading dimension of the local matrix <i>A</i> . | 8 |

Array descriptor for narrow-band and tridiagonal matrices (*dlen_=7*)

| Element Name | Stored in | Description | Element Index Number |
|----------------|----------------------|--|----------------------|
| <i>dtype_a</i> | <i>desca[dtype_]</i> | Descriptor type <ul style="list-style-type: none"> <i>dtype_a</i>=501: 1-by-<i>P</i> grid, <i>dtype_a</i>=502: <i>P</i>-by-1 grid. | 0 |
| <i>ctxt_a</i> | <i>desca[ctxt_]</i> | BLACS context handle indicating the BLACS process grid over which the global matrix <i>A</i> is distributed. The context itself is global, but the handle (the integer value) can vary. | 1 |
| <i>n_a</i> | <i>desca[n_]</i> | The size of the matrix dimension being distributed. | 2 |
| <i>nb_a</i> | <i>desca[nb_]</i> | The blocking factor used to distribute the distributed dimension of the matrix <i>A</i> . | 3 |
| <i>src_a</i> | <i>desca[src_]</i> | The process row or column over which the first row or column of the matrix <i>A</i> is distributed. | 4 |
| <i>lld_a</i> | <i>desca[lld_]</i> | The leading dimension of the local matrix storing the local blocks of the distributed matrix <i>A</i> . The minimum value of <i>lld_a</i> depends on <i>dtype_a</i> . <ul style="list-style-type: none"> <i>dtype_a</i>=501: $lld_a \geq \max(\text{size of undistributed dimension}, 1)$, <i>dtype_a</i>=502: $lld_a \geq \max(nb_a, 1)$. | 5 |
| Not applicable | | Reserved for future use. | 6 |

Similar notations are used for different matrices. For example: *lld_b* is the leading dimension of the local matrix storing the local blocks of the distributed matrix *B* and *dtype_z* is the type of the global matrix *Z*.

The number of rows and columns of a global dense matrix that a particular process in a grid receives after data distributing is denoted by $LOC_r()$ and $LOC_c()$, respectively. To compute these numbers, you can use the ScaLAPACK tool routine `numroc`.

After the block-cyclic distribution of global data is done, you may choose to perform an operation on a submatrix $sub(A)$ of the global matrix *A* defined by the following 6 values (for dense matrices):

| | |
|-----------|--|
| <i>m</i> | The number of rows of $sub(A)$ |
| <i>n</i> | The number of columns of $sub(A)$ |
| <i>a</i> | A pointer to the local matrix containing the entire global matrix <i>A</i> |
| <i>ia</i> | The row index of $sub(A)$ in the global matrix <i>A</i> |
| <i>ja</i> | The column index of $sub(A)$ in the global matrix <i>A</i> |

*desca*The array descriptor for the global matrix *A***Optimization Notice**

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Naming Conventions for ScaLAPACK Routines

For each routine introduced in this chapter, you can use the ScaLAPACK name. The naming convention for ScaLAPACK routines is similar to that used for LAPACK routines. A general rule is that each routine name in ScaLAPACK, which has an LAPACK equivalent, is simply the LAPACK name prefixed by initial letter *p*.

ScaLAPACK names have the structure *p?yyzzz* or *p?yyzz*, which is described below.

The initial letter *p* is a distinctive prefix of ScaLAPACK routines and is present in each such routine.

The second symbol *?* indicates the data type:

| | |
|----------|---------------------------|
| <i>s</i> | real, single precision |
| <i>d</i> | real, double precision |
| <i>c</i> | complex, single precision |
| <i>z</i> | complex, double precision |

The second and third letters *yy* indicate the matrix type as:

| | |
|-----------|--|
| <i>ge</i> | general |
| <i>gb</i> | general band |
| <i>gg</i> | a pair of general matrices (for a generalized problem) |
| <i>dt</i> | general tridiagonal (diagonally dominant-like) |
| <i>db</i> | general band (diagonally dominant-like) |
| <i>po</i> | symmetric or Hermitian positive-definite |
| <i>pb</i> | symmetric or Hermitian positive-definite band |
| <i>pt</i> | symmetric or Hermitian positive-definite tridiagonal |
| <i>sy</i> | symmetric |
| <i>st</i> | symmetric tridiagonal (real) |
| <i>he</i> | Hermitian |
| <i>or</i> | orthogonal |

| | |
|----|----------------------------------|
| tr | triangular (or quasi-triangular) |
| tz | trapezoidal |
| un | unitary |

For computational routines, the last three letters **zzz** indicate the computation performed and have the same meaning as for LAPACK routines.

For driver routines, the last two letters **zz** or three letters **zzz** have the following meaning:

| | |
|-----|--|
| sv | a <i>simple</i> driver for solving a linear system |
| svx | an <i>expert</i> driver for solving a linear system |
| ls | a driver for solving a linear least squares problem |
| ev | a simple driver for solving a symmetric eigenvalue problem |
| evd | a simple driver for solving an eigenvalue problem using a divide and conquer algorithm |
| evx | an expert driver for solving a symmetric eigenvalue problem |
| svd | a driver for computing a singular value decomposition |
| gvx | an expert driver for solving a generalized symmetric definite eigenvalue problem |

Simple driver here means that the driver just solves the general problem, whereas an *expert* driver is more versatile and can also optionally perform some related computations (such, for example, as refining the solution and computing error bounds after the linear system is solved).

ScaLAPACK Computational Routines

In the sections that follow, the descriptions of ScaLAPACK computational routines are given. These routines perform distinct computational tasks that can be used for:

- [Solving Systems of Linear Equations](#)
- [Orthogonal Factorizations and LLS Problems](#)
- [Symmetric Eigenproblems](#)
- [Nonsymmetric Eigenproblems](#)
- [Singular Value Decomposition](#)
- [Generalized Symmetric-Definite Eigenproblems](#)

See also the respective [driver routines](#).

Systems of Linear Equations: ScaLAPACK Computational Routines

ScaLAPACK supports routines for the systems of equations with the following types of matrices:

- general
- general banded
- general diagonally dominant-like banded (including general tridiagonal)
- symmetric or Hermitian positive-definite
- symmetric or Hermitian positive-definite banded
- symmetric or Hermitian positive-definite tridiagonal

A *diagonally dominant-like* matrix is defined as a matrix for which it is known in advance that pivoting is not required in the *LU* factorization of this matrix.

For the above matrix types, the library includes routines for performing the following computations: *factoring* the matrix; *equilibrating* the matrix; *solving* a system of linear equations; *estimating the condition number* of a matrix; *refining* the solution of linear equations and computing its error bounds; *inverting* the matrix. Note that for some of the listed matrix types only part of the computational routines are provided (for example, routines that refine the solution are not provided for band or tridiagonal matrices). See [Table “Computational Routines for Systems of Linear Equations”](#) for full list of available routines.

To solve a particular problem, you can either call two or more computational routines or call a corresponding [driver routine](#) that combines several tasks in one call. Thus, to solve a system of linear equations with a general matrix, you can first call `p?getrf` (LU factorization) and then `p?getrs` (computing the solution). Then, you might wish to call `p?gerfs` to refine the solution and get the error bounds. Alternatively, you can just use the driver routine `p?gesvx` which performs all these tasks in one call.

[Table “Computational Routines for Systems of Linear Equations”](#) lists the ScaLAPACK computational routines for factorizing, equilibrating, and inverting matrices, estimating their condition numbers, solving systems of equations with real matrices, refining the solution, and estimating its error.

Computational Routines for Systems of Linear Equations

| Matrix type, storage scheme | Factorize matrix | Equilibrate matrix | Solve system | Condition number | Estimate error | Invert matrix |
|--|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| general (partial pivoting) | <code>p?getrf</code> | <code>p?geequ</code> | <code>p?getrs</code> | <code>p?gecon</code> | <code>p?gerfs</code> | <code>p?getri</code> |
| general band (partial pivoting) | <code>p?gbtrf</code> | | <code>p?gbtrs</code> | | | |
| general band (no pivoting) | <code>p?dbtrf</code> | | <code>p?dbtrs</code> | | | |
| general tridiagonal (no pivoting) | <code>p?dttrf</code> | | <code>p?dttrs</code> | | | |
| symmetric/Hermitian positive-definite | <code>p?potrf</code> | <code>p?poequ</code> | <code>p?potrs</code> | <code>p?pocon</code> | <code>p?porfs</code> | <code>p?potri</code> |
| symmetric/Hermitian positive-definite, band | <code>p?pbtrf</code> | | <code>p?pbtrs</code> | | | |
| symmetric/Hermitian positive-definite, tridiagonal | <code>p?pttrf</code> | | <code>p?pttrs</code> | | | |
| triangular | | | <code>p?trtrs</code> | <code>p?trcon</code> | <code>p?trrfs</code> | <code>p?trtri</code> |

In this table ? stands for s (single precision real), d (double precision real), c (single precision complex), or z (double precision complex).

Matrix Factorization: ScaLAPACK Computational Routines

This section describes the ScaLAPACK routines for matrix factorization. The following factorizations are supported:

- LU factorization of general matrices
- LU factorization of diagonally dominant-like matrices
- Cholesky factorization of real symmetric or complex Hermitian positive-definite matrices

You can compute the factorizations using full and band storage of matrices.

`p?getrf`

Computes the LU factorization of a general m-by-n distributed matrix.

Syntax

```
void psgetrf (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_INT *info );
```

```
void pdgetrf (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_INT *info );
```

```
void pcgetrf (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , MKL_INT *info );

void pzgetrf (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?getrf` function forms the LU factorization of a general m -by- n distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia} + m - 1, \text{ja}:\text{ja} + n - 1)$ as

$$A = P * L * U$$

where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$) and U is upper triangular (upper trapezoidal if $m < n$). L and U are stored in $\text{sub}(A)$.

The function uses partial pivoting, with row interchanges.

NOTE

This function supports the Progress Routine feature. See [mkl_progress](#) for details.

Input Parameters

| | |
|---------------|---|
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$; $m \geq 0$. |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$; $n \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja + n - 1)$. Contains the local pieces of the distributed matrix $\text{sub}(A)$ to be factored. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | Overwritten by local pieces of the factors L and U from the factorization $A = P * L * U$. The unit diagonal elements of L are not stored. |
| <i>ipiv</i> | (local) Array of size $LOCr(m_a) + mb_a$. Contains the pivoting information: local row i was interchanged with global row $ipiv[i - 1]$. This array is tied to the distributed matrix A . |
| <i>info</i> | (global) If $info = 0$, the execution is successful. |

$info < 0$: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

If $info = i > 0$, $u_{ia+i, ja+j-1}$ is 0. The factorization has been completed, but the factor U is exactly singular. Division by zero will occur if you use the factor U for solving a system of linear equations.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gbtrf

Computes the LU factorization of a general n -by- n banded distributed matrix.

Syntax

```
void psgbtrf (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , float *a , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , float *af , MKL_INT *laf , float *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pdgbtrf (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , double *a , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , double *af , MKL_INT *laf , double *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pcgbtrf (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_Complex8 *a , MKL_INT
*ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8
*work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzgbtrf (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_Complex16 *a , MKL_INT
*ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex16 *af , MKL_INT *laf ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?gbtrf` function computes the LU factorization of a general n -by- n real/complex banded distributed matrix $A(1:n, ja:ja+n-1)$ using partial pivoting with row interchanges.

The resulting factorization is not the same factorization as returned from the LAPACK function `?gbtrf`. Additional permutations are performed on the matrix for the sake of parallelism.

The factorization has the form

$$A(1:n, ja:ja+n-1) = P*L*U*Q$$

where P and Q are permutation matrices, and L and U are banded lower and upper triangular matrices, respectively. The matrix Q represents reordering of columns for the sake of parallelism, while P represents reordering of rows for numerical stability using classic partial pivoting.

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Input Parameters

| | |
|--------------|---|
| <i>n</i> | (global) The number of rows and columns in the distributed submatrix $A(1:n, ja:ja+n-1)$; $n \geq 0$. |
| <i>bwl</i> | (global) The number of sub-diagonals within the band of A ($0 \leq bwl \leq n-1$). |
| <i>bwu</i> | (global) The number of super-diagonals within the band of A ($0 \leq bwu \leq n-1$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCc(ja+n-1)$ where $lld_a \geq 2*bwl + 2*bwu + 1$. Contains the local pieces of the n -by- n distributed banded matrix $A(1:n, ja:ja+n-1)$ to be factored. |
| <i>ja</i> | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . If $dtype_a = 501$, then $dlen_ \geq 7$; else if $dtype_a = 1$, then $dlen_ \geq 9$. |
| <i>laf</i> | (local) The size of the array <i>af</i> . Must be $laf \geq (nb_a + bwu) * (bwl + bwu) + 6 * (bwl + bwu) * (bwl + 2 * bwu)$. If <i>laf</i> is not large enough, an error code will be returned and the minimum acceptable size will be returned in <i>af</i> [0]. |
| <i>work</i> | (local) Same type as <i>a</i> . Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the <i>work</i> array ($lwork \geq 1$). If <i>lwork</i> is too small, the minimal acceptable size will be returned in <i>work</i> [0] and an error code is returned. |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, this array contains details of the factorization. Note that additional permutations are performed on the matrix, so that the factors returned are different from those returned by LAPACK. |
|----------|---|

| | |
|----------------|---|
| <i>ipiv</i> | <p>(local) array.</p> <p>The size of <i>ipiv</i> must be $\geq nb_a$.</p> <p>Contains pivot indices for local factorizations. Note that you <i>should not alter</i> the contents of this array between factorization and solve.</p> |
| <i>af</i> | <p>(local)</p> <p>Array of size <i>laf</i>.</p> <p>Auxiliary fill-in space. The fill-in space is created in a call to the factorization function <code>p?gbtrf</code> and is stored in <i>af</i>.</p> <p>Note that if a linear system is to be solved using <code>p?gbtrs</code> after the factorization function, <i>af</i> must not be altered after the factorization.</p> |
| <i>work[0]</i> | <p>On exit, <i>work[0]</i> contains the minimum value of <i>lwork</i> required.</p> |
| <i>info</i> | <p>(global)</p> <p>If <i>info</i>=0, the execution is successful.</p> <p><i>info</i> < 0:</p> <p>If the <i>i</i>-th argument is an array and the <i>j</i>-th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>j</i>*100+<i>j</i>); if the <i>i</i>-th argument is a scalar and had an illegal value, then <i>info</i> = -<i>i</i>.</p> <p><i>info</i> > 0:</p> <p>If <i>info</i> = <i>k</i> ≤ NPROCS, the submatrix stored on processor <i>info</i> and factored locally was not nonsingular, and the factorization was not completed.</p> <p>If <i>info</i> = <i>k</i> > NPROCS, the submatrix stored on processor <i>info</i>-NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed.</p> |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?dbtrf

Computes the LU factorization of a n-by-n diagonally dominant-like banded distributed matrix.

Syntax

```
void psdbtrf (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , float *a , MKL_INT *ja ,
MKL_INT *desca , float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT
*info );

void pddbtrf (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , double *a , MKL_INT *ja ,
MKL_INT *desca , double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT
*info );

void pcdbrf (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_Complex8 *a , MKL_INT
*ja , MKL_INT *desca , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );

void pzdbtrf (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_Complex16 *a , MKL_INT
*ja , MKL_INT *desca , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?dbtrff` function computes the LU factorization of a n -by- n real/complex diagonally dominant-like banded distributed matrix $A(1:n, ja:ja+n-1)$ without pivoting.

NOTE

A matrix is called *diagonally dominant-like* if pivoting is not required for LU to be numerically stable.

Note that the resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

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Input Parameters

| | |
|---------|---|
| n | (global) The number of rows and columns in the distributed submatrix $A(1:n, ja:ja+n-1)$; $n \geq 0$. |
| bwl | (global) The number of sub-diagonals within the band of A ($0 \leq bwl \leq n-1$). |
| bwu | (global) The number of super-diagonals within the band of A ($0 \leq bwu \leq n-1$). |
| a | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja+n-1)$. Contains the local pieces of the n -by- n distributed banded matrix $A(1:n, ja:ja+n-1)$ to be factored. |
| ja | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . If $dtype_a = 501$, then $dlen_ \geq 7$; else if $dtype_a = 1$, then $dlen_ \geq 9$. |
| laf | (local) The size of the array af . |

Must be $laf \geq NB * (bwl + bwu) + 6 * (\max(bwl, bwu))^2$.

If laf is not large enough, an error code will be returned and the minimum acceptable size will be returned in $af[0]$.

work (local) Workspace array of size *lwork*.

lwork (local or global) The size of the *work* array, must be $lwork \geq (\max(bwl, bwu))^2$. If *lwork* is too small, the minimal acceptable size will be returned in $work[0]$ and an error code is returned.

Output Parameters

a On exit, this array contains details of the factorization. Note that additional permutations are performed on the matrix, so that the factors returned are different from those returned by LAPACK.

af (local)
 Array of size *laf*.
 Auxiliary fill-in space. The fill-in space is created in a call to the factorization function `p?dbtrf` and is stored in *af*.
 Note that if a linear system is to be solved using `p?dbtrs` after the factorization function, *af* must not be altered after the factorization.

work[0] On exit, *work[0]* contains the minimum value of *lwork* required for optimum performance.

info (global)
 If *info*=0, the execution is successful.
info < 0:
 If the *i*-th argument is an array and the *j*-th entry, indexed *j* - 1, had an illegal value, then $info = -(i*100+j)$; if the *i*-th argument is a scalar and had an illegal value, then $info = -i$.
info > 0:
 If $info = k \leq NPROCS$, the submatrix stored on processor *info* and factored locally was not diagonally dominant-like, and the factorization was not completed.
 If $info = k > NPROCS$, the submatrix stored on processor $info - NPROCS$ representing interactions with other processors was not nonsingular, and the factorization was not completed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?dttrf

Computes the LU factorization of a diagonally dominant-like tridiagonal distributed matrix.

Syntax

```
void psdttrf (MKL_INT *n , float *dl , float *d , float *du , MKL_INT *ja , MKL_INT
*desca , float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );

void pddttrf (MKL_INT *n , double *dl , double *d , double *du , MKL_INT *ja , MKL_INT
*desca , double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );

void pcdttrf (MKL_INT *n , MKL_Complex8 *dl , MKL_Complex8 *d , MKL_Complex8 *du ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );

void pzdtttrf (MKL_INT *n , MKL_Complex16 *dl , MKL_Complex16 *d , MKL_Complex16 *du ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16
*work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?dttrf` function computes the LU factorization of an n -by- n real/complex diagonally dominant-like tridiagonal distributed matrix $A(1:n, ja:ja+n-1)$ without pivoting for stability.

The resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

The factorization has the form:

$$A(1:n, ja:ja+n-1) = P * L * U * P^T,$$

where P is a permutation matrix, and L and U are banded lower and upper triangular matrices, respectively.

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Input Parameters

- n (global) The number of rows and columns to be operated on, that is, the order of the distributed submatrix $A(1:n, ja:ja+n-1)$ ($n \geq 0$).
- dl, d, du (local)
Pointers to the local arrays of size nb_a each.
On entry, the array dl contains the local part of the global vector storing the subdiagonal elements of the matrix. Globally, $dl[0]$ is not referenced, and dl must be aligned with d .
On entry, the array d contains the local part of the global vector storing the diagonal elements of the matrix.

On entry, the array *du* contains the local part of the global vector storing the super-diagonal elements of the matrix. *du*[*n*-1] is not referenced, and *du* must be aligned with *d*.

| | |
|--------------|--|
| <i>ja</i> | (global) The index in the global matrix <i>A</i> indicating the start of the matrix to be operated on (which may be either all of <i>A</i> or a submatrix of <i>A</i>). |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . If <i>dtype_a</i> = 501, then <i>dlen_</i> ≥ 7; else if <i>dtype_a</i> = 1, then <i>dlen_</i> ≥ 9. |
| <i>laf</i> | (local) The size of the array <i>af</i> . Must be <i>laf</i> ≥ 2*(NB+2) . If <i>laf</i> is not large enough, an error code will be returned and the minimum acceptable size will be returned in <i>af</i> [0]. |
| <i>work</i> | (local) Same type as <i>d</i> . Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the <i>work</i> array, must be at least <i>lwork</i> ≥ 8*NPCOL. |

Output Parameters

| | |
|------------------|--|
| <i>dl, d, du</i> | On exit, overwritten by the information containing the factors of the matrix. |
| <i>af</i> | (local) Array of size <i>laf</i> . Auxiliary fill-in space. The fill-in space is created in a call to the factorization function <code>p?dttrf</code> and is stored in <i>af</i> . Note that if a linear system is to be solved using <code>p?dttrs</code> after the factorization function, <i>af</i> must not be altered. |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>j</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . <i>info</i> > 0: If <i>info</i> = <i>k</i> ≤ NPROCS, the submatrix stored on processor <i>info</i> and factored locally was not diagonally dominant-like, and the factorization was not completed. |

If $info = k > NPROCS$, the submatrix stored on processor $info - NPROCS$ representing interactions with other processors was not nonsingular, and the factorization was not completed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?potrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite distributed matrix.

Syntax

```
void pspotrf (char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , MKL_INT *info );

void pdpotrf (char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *info );

void pcpotrf (char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *info );

void pzpotrf (char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?potrf` function computes the Cholesky factorization of a real symmetric or complex Hermitian positive-definite distributed n -by- n matrix $A(ia:ia+n-1, ja:ja+n-1)$, denoted below as $sub(A)$.

The factorization has the form

$sub(A) = U^H * U$ if $uplo = 'U'$, or

$sub(A) = L * L^H$ if $uplo = 'L'$

where L is a lower triangular matrix and U is upper triangular.

Input Parameters

| | |
|-------------------|---|
| <code>uplo</code> | (global) |
| | Indicates whether the upper or lower triangular part of $sub(A)$ is stored. Must be 'U' or 'L'. |
| | If $uplo = 'U'$, the array a stores the upper triangular part of the matrix $sub(A)$ that is factored as $U^H * U$. |
| | If $uplo = 'L'$, the array a stores the lower triangular part of the matrix $sub(A)$ that is factored as $L * L^H$. |
| <code>n</code> | (global) The order of the distributed matrix $sub(A)$ ($n \geq 0$). |
| <code>a</code> | (local) |
| | Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. |

On entry, this array contains the local pieces of the n -by- n symmetric/Hermitian distributed matrix $\text{sub}(A)$ to be factored.

Depending on *uplo*, the array *a* contains either the upper or the lower triangular part of the matrix $\text{sub}(A)$ (see *uplo*).

| | |
|---------------|---|
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | The upper or lower triangular part of <i>a</i> is overwritten by the Cholesky factor <i>U</i> or <i>L</i> , as specified by <i>uplo</i> . |
| <i>info</i> | (global) . If <i>info</i> =0, the execution is successful; <i>info</i> < 0: if the <i>i</i> -th argument is an array, and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . If <i>info</i> = <i>k</i> > 0, the leading minor of order <i>k</i> , $A(\text{ia}:\text{ia}+\text{k}-1, \text{ja}:\text{ja}+\text{k}-1)$, is not positive-definite, and the factorization could not be completed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?pbtrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite banded distributed matrix.

Syntax

```
void pspbtrf (char *uplo , MKL_INT *n , MKL_INT *bw , float *a , MKL_INT *ja , MKL_INT *desca , float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );
void pdpbtrf (char *uplo , MKL_INT *n , MKL_INT *bw , double *a , MKL_INT *ja , MKL_INT *desca , double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );
void pcpbtrf (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_Complex8 *a , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
void pzpbtfrf (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_Complex16 *a , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?pbtrf` function computes the Cholesky factorization of an n -by- n real symmetric or complex Hermitian positive-definite banded distributed matrix $A(1:n, ja:ja+n-1)$.

The resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

The factorization has the form:

$$A(1:n, ja:ja+n-1) = P*U^H*U*P^T, \text{ if } uplo='U', \text{ or}$$

$$A(1:n, ja:ja+n-1) = P*L*L^H*P^T, \text{ if } uplo='L',$$

where P is a permutation matrix and U and L are banded upper and lower triangular matrices, respectively.

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Input Parameters

| | |
|--------------|--|
| <i>uplo</i> | (global) Must be 'U' or 'L'. If <i>uplo</i> = 'U', upper triangle of $A(1:n, ja:ja+n-1)$ is stored; If <i>uplo</i> = 'L', lower triangle of $A(1:n, ja:ja+n-1)$ is stored. |
| <i>n</i> | (global) The order of the distributed submatrix $A(1:n, ja:ja+n-1)$. ($n \geq 0$). |
| <i>bw</i> | (global) The number of superdiagonals of the distributed matrix if <i>uplo</i> = 'U', or the number of subdiagonals if <i>uplo</i> = 'L' ($bw \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a*LOCc(ja+n-1)$. On entry, this array contains the local pieces of the upper or lower triangle of the symmetric/Hermitian band distributed matrix $A(1:n, ja:ja+n-1)$ to be factored. |
| <i>ja</i> | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . If <i>dtype_a</i> = 501, then $dlen_ \geq 7$; else if <i>dtype_a</i> = 1, then $dlen_ \geq 9$. |

| | |
|--------------|--|
| <i>laf</i> | (local) The size of the array <i>af</i> . Must be $laf \geq (NB+2*bw) * bw$. If <i>laf</i> is not large enough, an error code will be returned and the minimum acceptable size will be returned in <i>af</i> [0]. |
| <i>work</i> | (local) Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the <i>work</i> array, must be $lwork \geq bw^2$. |

Output Parameters

| | |
|-----------------|---|
| <i>a</i> | On exit, if <i>info</i> =0, contains the permuted triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization of the band matrix $A(1:n, ja:ja+n-1)$, as specified by <i>uplo</i> . |
| <i>af</i> | (local) Array of size <i>laf</i> . Auxiliary fill-in space. The fill-in space is created in a call to the factorization function <i>p?pbtrf</i> and stored in <i>af</i> . Note that if a linear system is to be solved using <i>p?pbtrs</i> after the factorization function, <i>af</i> must not be altered. |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then $info = -(i*100+j)$; if the <i>i</i> -th argument is a scalar and had an illegal value, then $info = -i$. <i>info</i> >0: If $info = k \leq NPROCS$, the submatrix stored on processor <i>info</i> and factored locally was not positive definite, and the factorization was not completed. If $info = k > NPROCS$, the submatrix stored on processor <i>info</i> -NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?pttrf

Computes the Cholesky factorization of a symmetric (Hermitian) positive-definite tridiagonal distributed matrix.

Syntax

```
void psppttrf (MKL_INT *n , float *d , float *e , MKL_INT *ja , MKL_INT *desca , float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );
```

```

void pdpstrf (MKL_INT *n , double *d , double *e , MKL_INT *ja , MKL_INT *desca ,
double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );

void pcptstrf (MKL_INT *n , float *d , MKL_Complex8 *e , MKL_INT *ja , MKL_INT *desca ,
MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT
*info );

void pzptstrf (MKL_INT *n , double *d , MKL_Complex16 *e , MKL_INT *ja , MKL_INT
*desca , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16 *work , MKL_INT *lwork ,
MKL_INT *info );

```

Include Files

- mkl_scalapack.h

Description

The `p?ptstrf` function computes the Cholesky factorization of an n -by- n real symmetric or complex hermitian positive-definite tridiagonal distributed matrix $A(1:n, ja:ja+n-1)$.

The resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

The factorization has the form:

$$A(1:n, ja:ja+n-1) = P*L*D*L^H*P^T, \text{ or}$$

$$A(1:n, ja:ja+n-1) = P*U^H*D*U*P^T,$$

where P is a permutation matrix, and U and L are tridiagonal upper and lower triangular matrices, respectively.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|--------|--|
| n | (global) The order of the distributed submatrix $A(1:n, ja:ja+n-1)$ ($n \geq 0$). |
| d, e | (local) Pointers into the local memory to arrays of size nb_a each. On entry, the array d contains the local part of the global vector storing the main diagonal of the distributed matrix A . On entry, the array e contains the local part of the global vector storing the upper diagonal of the distributed matrix A . |
| ja | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |

| | |
|--------------|--|
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . If <i>dtype_a</i> = 501, then <i>dlen_</i> ≥ 7; else if <i>dtype_a</i> = 1, then <i>dlen_</i> ≥ 9. |
| <i>laf</i> | (local) The size of the array <i>af</i> . Must be <i>laf</i> ≥ <i>nb_a</i> + 2. If <i>laf</i> is not large enough, an error code will be returned and the minimum acceptable size will be returned in <i>af</i> [0]. |
| <i>work</i> | (local) Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the <i>work</i> array, must be at least <i>lwork</i> ≥ 8 * NPCOL. |

Output Parameters

| | |
|-----------------|---|
| <i>d, e</i> | On exit, overwritten by the details of the factorization. |
| <i>af</i> | (local) Array of size <i>laf</i> . Auxiliary fill-in space. The fill-in space is created in a call to the factorization function <code>p?pttrf</code> and stored in <i>af</i> . Note that if a linear system is to be solved using <code>p?pttrs</code> after the factorization function, <i>af</i> must not be altered. |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>j</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . <i>info</i> > 0: If <i>info</i> = <i>k</i> ≤ NPCROCS, the submatrix stored on processor <i>info</i> and factored locally was not positive definite, and the factorization was not completed. If <i>info</i> = <i>k</i> > NPCROCS, the submatrix stored on processor <i>info</i> -NPCROCS representing interactions with other processors was not nonsingular, and the factorization was not completed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Solving Systems of Linear Equations: ScaLAPACK Computational Routines

This section describes the ScaLAPACK routines for solving systems of linear equations. Before calling most of these routines, you need to factorize the matrix of your system of equations (see [Routines for Matrix Factorization](#) in this chapter). However, the factorization is not necessary if your system of equations has a triangular matrix.

p?getrs

Solves a system of distributed linear equations with a general square matrix, using the LU factorization computed by p?getrf.

Syntax

```
void psgetrs (char *trans , MKL_INT *n , MKL_INT *nrhs , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , float *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_INT *info );
```

```
void pdgetrs (char *trans , MKL_INT *n , MKL_INT *nrhs , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , double *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_INT *info );
```

```
void pcgetrs (char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex8 *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , MKL_INT *info );
```

```
void pzgetrs (char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex16 *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?getrs function solves a system of distributed linear equations with a general n -by- n distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ using the LU factorization computed by p?getrf.

The system has one of the following forms specified by *trans*:

$\text{sub}(A)*X = \text{sub}(B)$ (no transpose),

$\text{sub}(A)^T*X = \text{sub}(B)$ (transpose),

$\text{sub}(A)^H*X = \text{sub}(B)$ (conjugate transpose),

where $\text{sub}(B) = B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+nrhs-1)$.

Before calling this function, you must call p?getrf to compute the LU factorization of $\text{sub}(A)$.

Input Parameters

trans (global) Must be 'N' or 'T' or 'C'.
Indicates the form of the equations:
If *trans* = 'N', then $\text{sub}(A)*X = \text{sub}(B)$ is solved for X .
If *trans* = 'T', then $\text{sub}(A)^T*X = \text{sub}(B)$ is solved for X .
If *trans* = 'C', then $\text{sub}(A)^H *X = \text{sub}(B)$ is solved for X .

| | |
|---------------|---|
| <i>n</i> | (global) The number of linear equations; the order of the matrix sub(<i>A</i>) ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right hand sides; the number of columns of the distributed matrix sub(<i>B</i>) ($nrhs \geq 0$). |
| <i>a, b</i> | (local) Pointers into the local memory to arrays of local sizes $lld_a * LOCC(ja+n-1)$ and $lld_b * LOCC(jb+nrhs-1)$, respectively. On entry, the array <i>a</i> contains the local pieces of the factors <i>L</i> and <i>U</i> from the factorization $sub(A) = P * L * U$; the unit diagonal elements of <i>L</i> are not stored. On entry, the array <i>b</i> contains the right hand sides sub(<i>B</i>). |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix sub(<i>A</i>), respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>ipiv</i> | (local) Array of size of $LOCr(m_a) + mb_a$. Contains the pivoting information: local row <i>i</i> of the matrix was interchanged with the global row <i>ipiv</i> [<i>i</i> -1]. This array is tied to the distributed matrix <i>A</i> . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix <i>B</i> indicating the first row and the first column of the matrix sub(<i>B</i>), respectively. |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>B</i> . |

Output Parameters

| | |
|-------------|--|
| <i>b</i> | On exit, overwritten by the solution distributed matrix <i>X</i> . |
| <i>info</i> | If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gbtrs

Solves a system of distributed linear equations with a general band matrix, using the LU factorization computed by [p?gbtrf](#).

Syntax

```
void psgbtrs (char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs ,
float *a , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , float *b , MKL_INT *ib ,
MKL_INT *descb , float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pdgbtrs (char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs ,
double *a , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , double *b , MKL_INT *ib ,
MKL_INT *descb , double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pcgbtrs (char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs ,
MKL_Complex8 *a , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex8 *b ,
MKL_INT *ib , MKL_INT *descb , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pzgbtrs (char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs ,
MKL_Complex16 *a , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex16 *b ,
MKL_INT *ib , MKL_INT *descb , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16
*work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?gbtrs` function solves a system of distributed linear equations with a general band distributed matrix $\text{sub}(A) = A(1:n, ja:ja+n-1)$ using the LU factorization computed by `p?gbtrf`.

The system has one of the following forms specified by `trans`:

$\text{sub}(A)*X = \text{sub}(B)$ (no transpose),

$\text{sub}(A)^T*X = \text{sub}(B)$ (transpose),

$\text{sub}(A)^H*X = \text{sub}(B)$ (conjugate transpose),

where $\text{sub}(B) = B(ib:ib+n-1, 1:nrhs)$.

Before calling this function, you must call `p?gbtrf` to compute the LU factorization of $\text{sub}(A)$.

Input Parameters

| | |
|--------------------|---|
| <code>trans</code> | (global) Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <code>trans</code> = 'N', then $\text{sub}(A)*X = \text{sub}(B)$ is solved for X . If <code>trans</code> = 'T', then $\text{sub}(A)^T*X = \text{sub}(B)$ is solved for X . If <code>trans</code> = 'C', then $\text{sub}(A)^H *X = \text{sub}(B)$ is solved for X . |
| <code>n</code> | (global) The number of linear equations; the order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <code>bwl</code> | (global) The number of sub-diagonals within the band of A ($0 \leq bwl \leq n-1$). |
| <code>bwu</code> | (global) The number of super-diagonals within the band of A ($0 \leq bwu \leq n-1$). |
| <code>nrhs</code> | (global) The number of right hand sides; the number of columns of the distributed matrix $\text{sub}(B)$ ($nrhs \geq 0$). |
| <code>a, b</code> | (local) |

Pointers into the local memory to arrays of local sizes $lld_a*LOCc(j_a+n-1)$ and $lld_b*LOCc(nrhs)$, respectively.

The array a contains details of the LU factorization of the distributed band matrix A .

On entry, the array b contains the local pieces of the right hand sides $B(ib:ib+n-1, 1:nrhs)$.

| | |
|---------|--|
| ja | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . If $dtype_a = 501$, then $dlen_ \geq 7$; else if $dtype_a = 1$, then $dlen_ \geq 9$. |
| ib | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| $descb$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . If $dtype_b = 502$, then $dlen_ \geq 7$; else if $dtype_b = 1$, then $dlen_ \geq 9$. |
| laf | (local) The size of the array af . Must be $laf \geq nb_a*(bwl+bwu)+6*(bwl+bwu)*(bwl+2*bwu)$. If laf is not large enough, an error code will be returned and the minimum acceptable size will be returned in $af[0]$. |
| $work$ | (local) Same type as a . Workspace array of size $lwork$. |
| $lwork$ | (local or global) The size of the $work$ array, must be at least $lwork \geq nrhs*(nb_a+2*bwl+4*bwu)$. |

Output Parameters

| | |
|--------|---|
| $ipiv$ | (local) array. The size of $ipiv$ must be $\geq nb_a$. Contains pivot indices for local factorizations. Note that you should not alter the contents of this array between factorization and solve. |
| b | On exit, overwritten by the local pieces of the solution distributed matrix X . |
| af | (local) Array of size laf . Auxiliary Fill-in space. The fill-in space is created in a call to the factorization function $p?gbtrf$ and is stored in af . Note that if a linear system is to be solved using $p?gbtrs$ after the factorization function, af must not be altered after the factorization. |

`work[0]` On exit, `work[0]` contains the minimum value of `lwork` required for optimum performance.

`info` If `info=0`, the execution is successful.
`info < 0`:
 If the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then `info = -(i*100+j)`; if the i -th argument is a scalar and had an illegal value, then `info = -i`.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?dbtrs

Solves a system of linear equations with a diagonally dominant-like banded distributed matrix using the factorization computed by p?dbtrf.

Syntax

```
void psdbtrs (char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs ,
float *a , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb ,
float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pddbtrs (char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs ,
double *a , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb ,
double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcdsbs (char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs ,
MKL_Complex8 *a , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pzdbtrs (char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs ,
MKL_Complex16 *a , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?dbtrs` function solves for X one of the systems of equations:

$$\text{sub}(A) * X = \text{sub}(B),$$

$$(\text{sub}(A))^T * X = \text{sub}(B), \text{ or}$$

$$(\text{sub}(A))^H * X = \text{sub}(B),$$

where $\text{sub}(A) = A(1:n, ja:ja+n-1)$ is a diagonally dominant-like banded distributed matrix, and $\text{sub}(B)$ denotes the distributed matrix $B(ib:ib+n-1, 1:nrhs)$.

This function uses the LU factorization computed by `p?dbtrf`.

Input Parameters

| | |
|-----------------|--|
| <i>trans</i> | (global) Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', then $\text{sub}(A)*X = \text{sub}(B)$ is solved for X . If <i>trans</i> = 'T', then $(\text{sub}(A))^T*X = \text{sub}(B)$ is solved for X . If <i>trans</i> = 'C', then $(\text{sub}(A))^H*X = \text{sub}(B)$ is solved for X . |
| <i>n</i> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>bwl</i> | (global) The number of subdiagonals within the band of A ($0 \leq bwl \leq n-1$). |
| <i>bwu</i> | (global) The number of superdiagonals within the band of A ($0 \leq bwu \leq n-1$). |
| <i>nrhs</i> | (global) The number of right hand sides; the number of columns of the distributed matrix $\text{sub}(B)$ ($nrhs \geq 0$). |
| <i>a, b</i> | (local) Pointers into the local memory to arrays of local sizes $lld_a*LOCc(ja+n-1)$ and $lld_b*LOCc(nrhs)$, respectively. On entry, the array <i>a</i> contains details of the LU factorization of the band matrix A , as computed by <code>p?dbtrf</code> . On entry, the array <i>b</i> contains the local pieces of the right hand side distributed matrix $\text{sub}(B)$. |
| <i>ja</i> | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . If <i>dtype_a</i> = 501, then $dlen_ \geq 7$; else if <i>dtype_a</i> = 1, then $dlen_ \geq 9$. |
| <i>ib</i> | (global) The row index in the global matrix B indicating the first row of the matrix to be operated on (which may be either all of B or a submatrix of B). |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix B . If <i>dtype_b</i> = 502, then $dlen_ \geq 7$; else if <i>dtype_b</i> = 1, then $dlen_ \geq 9$. |
| <i>af, work</i> | (local) Arrays of size <i>laf</i> and <i>lwork</i> , respectively The array <i>af</i> contains auxiliary fill-in space. The fill-in space is created in a call to the factorization function <code>p?dbtrf</code> and is stored in <i>af</i> . The array <i>work</i> is a workspace array. |

| | |
|--------------|---|
| <i>laf</i> | (local) The size of the array <i>af</i> . Must be $laf \geq NB * (bwl + bwu) + 6 * (\max(bwl, bwu))^2$. If <i>laf</i> is not large enough, an error code will be returned and the minimum acceptable size will be returned in <i>af</i> [0]. |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> , must be at least $lwork \geq (\max(bwl, bwu))^2$. |

Output Parameters

| | |
|-----------------|--|
| <i>b</i> | On exit, this array contains the local pieces of the solution distributed matrix <i>X</i> . |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>j</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?dttrs

Solves a system of linear equations with a diagonally dominant-like tridiagonal distributed matrix using the factorization computed by p?dttrf.

Syntax

```
void psdttrs (char *trans , MKL_INT *n , MKL_INT *nrhs , float *dl , float *d , float
*du , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb , float
*af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pddttrs (char *trans , MKL_INT *n , MKL_INT *nrhs , double *dl , double *d ,
double *du , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb ,
double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcdttrs (char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *dl ,
MKL_Complex8 *d , MKL_Complex8 *du , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b ,
MKL_INT *ib , MKL_INT *descb , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pzdtttrs (char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *dl ,
MKL_Complex16 *d , MKL_Complex16 *du , MKL_INT *ja , MKL_INT *desca , MKL_Complex16
*b , MKL_INT *ib , MKL_INT *descb , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16
*work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?dttrs` function solves for X one of the systems of equations:

$$\text{sub}(A)*X = \text{sub}(B),$$

$$(\text{sub}(A))^T*X = \text{sub}(B), \text{ or}$$

$$(\text{sub}(A))^H*X = \text{sub}(B),$$

where $\text{sub}(A) = A(1:n, ja:ja+n-1)$ is a diagonally dominant-like tridiagonal distributed matrix, and $\text{sub}(B)$ denotes the distributed matrix $B(ib:ib+n-1, 1:nrhs)$.

This function uses the LU factorization computed by `p?dttrf`.

Input Parameters

| | |
|------------------|--|
| <i>trans</i> | (global) Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', then $\text{sub}(A)*X = \text{sub}(B)$ is solved for X . If <i>trans</i> = 'T', then $(\text{sub}(A))^T*X = \text{sub}(B)$ is solved for X . If <i>trans</i> = 'C', then $(\text{sub}(A))^H*X = \text{sub}(B)$ is solved for X . |
| <i>n</i> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right hand sides; the number of columns of the distributed matrix $\text{sub}(B)$ ($nrhs \geq 0$). |
| <i>d1, d, du</i> | (local) Pointers to the local arrays of size <i>nb_a</i> each. On entry, these arrays contain details of the factorization. Globally, <i>d1</i> [0] and <i>du</i> [<i>n</i> -1] are not referenced; <i>d1</i> and <i>du</i> must be aligned with <i>d</i> . |
| <i>ja</i> | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . If <i>dtype_a</i> = 501 or <i>dtype_a</i> = 502, then $dlen_ \geq 7$; else if <i>dtype_a</i> = 1, then $dlen_ \geq 9$. |
| <i>b</i> | (local) Same type as <i>d</i> . Pointer into the local memory to an array of local size $lld_b*LOCc(nrhs)$ On entry, the array <i>b</i> contains the local pieces of the n -by- <i>nrhs</i> right hand side distributed matrix $\text{sub}(B)$. |
| <i>ib</i> | (global) The row index in the global matrix B indicating the first row of the matrix to be operated on (which may be either all of B or a submatrix of B). |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix B . If <i>dtype_b</i> = 502, then $dlen_ \geq 7$; else if <i>dtype_b</i> = 1, then $dlen_ \geq 9$. |

| | |
|-----------------|--|
| <i>af, work</i> | (local) Arrays of size <i>laf</i> and (<i>lwork</i>), respectively. The array <i>af</i> contains auxiliary fill-in space. The fill-in space is created in a call to the factorization function <i>p?dttrf</i> and is stored in <i>af</i> . If a linear system is to be solved using <i>p?dttrs</i> after the factorization function, <i>af</i> must not be altered. The array <i>work</i> is a workspace array. |
| <i>laf</i> | (local) The size of the array <i>af</i> . Must be $laf \geq NB * (bwl + bwu) + 6 * (bwl + bwu) * (bwl + 2 * bwu)$. If <i>laf</i> is not large enough, an error code will be returned and the minimum acceptable size will be returned in <i>af</i> [0]. |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> , must be at least $lwork \geq 10 * NPCOL + 4 * nrhs$. |

Output Parameters

| | |
|-----------------|--|
| <i>b</i> | On exit, this array contains the local pieces of the solution distributed matrix <i>X</i> . |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>j</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?potrs

Solves a system of linear equations with a Cholesky-factored symmetric/Hermitian distributed positive-definite matrix.

Syntax

```
void pspotrs (char *uplo , MKL_INT *n , MKL_INT *nrhs , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
MKL_INT *info );

void pdpotrs (char *uplo , MKL_INT *n , MKL_INT *nrhs , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );

void pcpotrs (char *uplo , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );
```

```
void pzpotrs (char *uplo , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The pzpotrs function solves for X a system of distributed linear equations in the form:

$$\text{sub}(A)*X = \text{sub}(B) ,$$

where $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ is an n -by- n real symmetric or complex Hermitian positive definite distributed matrix, and $\text{sub}(B)$ denotes the distributed matrix $B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+nrhs-1)$.

This function uses Cholesky factorization

$$\text{sub}(A) = U^H*U, \text{ or } \text{sub}(A) = L*L^H$$

computed by pzpotrf.

Input Parameters

| | |
|---------------|---|
| <i>uplo</i> | (global) Must be 'U' or 'L'. If <i>uplo</i> = 'U', upper triangle of sub(A) is stored; If <i>uplo</i> = 'L', lower triangle of sub(A) is stored. |
| <i>n</i> | (global) The order of the distributed matrix sub(A) ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right hand sides; the number of columns of the distributed matrix sub(B) ($nrhs \geq 0$). |
| <i>a, b</i> | (local) Pointers into the local memory to arrays of local sizes $lld_a*LOCc(ja+n-1)$ and $lld_b*LOCc(jb+nrhs-1)$, respectively. The array <i>a</i> contains the factors L or U from the Cholesky factorization $\text{sub}(A) = L*L^H$ or $\text{sub}(A) = U^H*U$, as computed by pzpotrf. On entry, the array <i>b</i> contains the local pieces of the right hand sides sub(B). |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix sub(A), respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of the matrix sub(B), respectively. |
| <i>descb</i> | (local) array of size $dlen_$. The array descriptor for the distributed matrix B . |

Output Parameters

b Overwritten by the local pieces of the solution matrix X .

info If *info*=0, the execution is successful.
info < 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j* - 1, had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?pbtrs

Solves a system of linear equations with a Cholesky-factored symmetric/Hermitian positive-definite band matrix.

Syntax

```
void pspbtrs (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs , float *a ,
MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb , float *af ,
MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );

void pdpbtrs (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs , double *a ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb , double *af ,
MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );

void pcpbtrs (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs , MKL_Complex8 *a ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *descb ,
MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT
*info );

void pzpbttrs (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs , MKL_Complex16
*a , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *descb ,
MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT
*info );
```

Include Files

- mkl_scalapack.h

Description

The p?pbtrs function solves for X a system of distributed linear equations in the form:

$$\text{sub}(A) * X = \text{sub}(B) ,$$

where $\text{sub}(A) = A(1:n, ja:ja+n-1)$ is an n -by- n real symmetric or complex Hermitian positive definite distributed band matrix, and $\text{sub}(B)$ denotes the distributed matrix $B(ib:ib+n-1, 1:nrhs)$.

This function uses Cholesky factorization

$$\text{sub}(A) = P * U^H * U * P^T, \text{ or } \text{sub}(A) = P * L * L^H * P^T$$

computed by p?pbtrf.

Input Parameters

uplo (global) Must be 'U' or 'L'.
 If *uplo* = 'U', upper triangle of $\text{sub}(A)$ is stored;
 If *uplo* = 'L', lower triangle of $\text{sub}(A)$ is stored.

| | |
|-----------------|--|
| <i>n</i> | (global) The order of the distributed matrix sub(<i>A</i>) ($n \geq 0$). |
| <i>bw</i> | (global) The number of superdiagonals of the distributed matrix if <i>uplo</i> = 'U', or the number of subdiagonals if <i>uplo</i> = 'L' ($bw \geq 0$). |
| <i>nrhs</i> | (global) The number of right hand sides; the number of columns of the distributed matrix sub(<i>B</i>) ($nrhs \geq 0$). |
| <i>a, b</i> | (local) Pointers into the local memory to arrays of local sizes $lld_a * LOCC(j_a + n - 1)$ and $lld_b * LOCC(nrhs - 1)$, respectively. The array <i>a</i> contains the permuted triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization $sub(A) = P * U^H * U * P^T$, or $sub(A) = P * L * L^H * P^T$ of the band matrix <i>A</i> , as returned by <code>p?pbtrf</code> . On entry, the array <i>b</i> contains the local pieces of the <i>n</i> -by- <i>nrhs</i> right hand side distributed matrix sub(<i>B</i>). |
| <i>ja</i> | (global) The index in the global matrix <i>A</i> indicating the start of the matrix to be operated on (which may be either all of <i>A</i> or a submatrix of <i>A</i>). |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . If <i>dtype_a</i> = 501, then $dlen_ \geq 7$; else if <i>dtype_a</i> = 1, then $dlen_ \geq 9$. |
| <i>ib</i> | (global) The row index in the global matrix <i>B</i> indicating the first row of the matrix sub(<i>B</i>). |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>B</i> . If <i>dtype_b</i> = 502, then $dlen_ \geq 7$; else if <i>dtype_b</i> = 1, then $dlen_ \geq 9$. |
| <i>af, work</i> | (local) Arrays, same type as <i>a</i> . The array <i>af</i> is of size <i>laf</i> . It contains auxiliary fill-in space. The fill-in space is created in a call to the factorization function <code>p?dbtrf</code> and is stored in <i>af</i> . The array <i>work</i> is a workspace array of size <i>lwork</i> . |
| <i>laf</i> | (local) The size of the array <i>af</i> . Must be $laf \geq nrhs * bw$. If <i>laf</i> is not large enough, an error code will be returned and the minimum acceptable size will be returned in <i>af</i> [0]. |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> , must be at least $lwork \geq bw^2$. |

Output Parameters

| | |
|-----------------|---|
| <i>b</i> | On exit, if <i>info</i> =0, this array contains the local pieces of the <i>n</i> -by- <i>nrhs</i> solution distributed matrix <i>X</i> . |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?pttrs

Solves a system of linear equations with a symmetric (Hermitian) positive-definite tridiagonal distributed matrix using the factorization computed by p?pttrf.

Syntax

```
void pspttrs (MKL_INT *n , MKL_INT *nrhs , float *d , float *e , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb , float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdpttrs (MKL_INT *n , MKL_INT *nrhs , double *d , double *e , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb , double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcpttrs (char *uplo , MKL_INT *n , MKL_INT *nrhs , float *d , MKL_Complex8 *e , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *descb , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzpttrs (char *uplo , MKL_INT *n , MKL_INT *nrhs , double *d , MKL_Complex16 *e , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *descb , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?pttrs function solves for *X* a system of distributed linear equations in the form:

$$\text{sub}(A)*X = \text{sub}(B) ,$$

where $\text{sub}(A) = A(1:n, ja:ja+n-1)$ is an *n*-by-*n* real symmetric or complex Hermitian positive definite tridiagonal distributed matrix, and $\text{sub}(B)$ denotes the distributed matrix $B(ib:ib+n-1, 1:nrhs)$.

This function uses the factorization

$$\text{sub}(A) = P*L*D*L^H*P^T, \text{ or } \text{sub}(A) = P*U^H*D*U*P^T$$

computed by `p?pttrf`.

Input Parameters

| | |
|-----------------------|--|
| <code>uplo</code> | (global, used in complex flavors only) Must be 'U' or 'L'. If <code>uplo = 'U'</code> , upper triangle of <code>sub(A)</code> is stored; If <code>uplo = 'L'</code> , lower triangle of <code>sub(A)</code> is stored. |
| <code>n</code> | (global) The order of the distributed matrix <code>sub(A)</code> ($n \geq 0$). |
| <code>nrhs</code> | (global) The number of right hand sides; the number of columns of the distributed matrix <code>sub(B)</code> ($nrhs \geq 0$). |
| <code>d, e</code> | (local) Pointers into the local memory to arrays of size <code>nb_a</code> each. These arrays contain details of the factorization as returned by <code>p?pttrf</code> |
| <code>ja</code> | (global) The index in the global matrix <code>A</code> indicating the start of the matrix to be operated on (which may be either all of <code>A</code> or a submatrix of <code>A</code>). |
| <code>desca</code> | (global and local) array of size <code>dlen_</code> . The array descriptor for the distributed matrix <code>A</code> . If <code>dtype_a = 501</code> or <code>dtype_a = 502</code> , then <code>dlen_ ≥ 7</code> ; else if <code>dtype_a = 1</code> , then <code>dlen_ ≥ 9</code> . |
| <code>b</code> | (local) Same type as <code>d, e</code> . Pointer into the local memory to an array of local size <code>lld_b*LOCc(nrhs)</code> . On entry, the array <code>b</code> contains the local pieces of the <code>n</code> -by- <code>nrhs</code> right hand side distributed matrix <code>sub(B)</code> . |
| <code>ib</code> | (global) The row index in the global matrix <code>B</code> indicating the first row of the matrix to be operated on (which may be either all of <code>B</code> or a submatrix of <code>B</code>). |
| <code>descb</code> | (global and local) array of size <code>dlen_</code> . The array descriptor for the distributed matrix <code>B</code> . If <code>dtype_b = 502</code> , then <code>dlen_ ≥ 7</code> ; else if <code>dtype_b = 1</code> , then <code>dlen_ ≥ 9</code> . |
| <code>af, work</code> | (local) Arrays of size <code>laf</code> and (<code>lwork</code>), respectively. The array <code>af</code> contains auxiliary fill-in space. The fill-in space is created in a call to the factorization function <code>p?pttrf</code> and is stored in <code>af</code> . The array <code>work</code> is a workspace array. |
| <code>laf</code> | (local) The size of the array <code>af</code> . Must be <code>laf ≥ nb_a + 2</code> . |

If *laf* is not large enough, an error code is returned and the minimum acceptable size will be returned in *af*[0].

lwork (local or global) The size of the array *work*, must be at least
 $lwork \geq (10+2*\min(100,nrhs))*NPCOL+4*nrhs$.

Output Parameters

b On exit, this array contains the local pieces of the solution distributed matrix *X*.

work[0]) On exit, *work*[0] contains the minimum value of *lwork* required for optimum performance.

info If *info*=0, the execution is successful.
info < 0:
 if the *i*-th argument is an array and the *j*-th entry, indexed *j* - 1, had an illegal value, then *info* = $-(i*100+j)$; if the *i*-th argument is a scalar and had an illegal value, then *info* = $-i$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trtrs

Solves a system of linear equations with a triangular distributed matrix.

Syntax

```
void pstrtrs (char *uplo , char *trans , char *diag , MKL_INT *n , MKL_INT *nrhs ,
float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , MKL_INT *info );
```

```
void pdtrtrs (char *uplo , char *trans , char *diag , MKL_INT *n , MKL_INT *nrhs ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , MKL_INT *info );
```

```
void pctrtrs (char *uplo , char *trans , char *diag , MKL_INT *n , MKL_INT *nrhs ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_INT *info );
```

```
void pztrtrs (char *uplo , char *trans , char *diag , MKL_INT *n , MKL_INT *nrhs ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?trtrs` function solves for *X* one of the following systems of linear equations:

$$\text{sub}(A)*X = \text{sub}(B),$$

$$(\text{sub}(A))^T*X = \text{sub}(B), \text{ or}$$

$$(\text{sub}(A))^H*X = \text{sub}(B),$$

where $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ is a triangular distributed matrix of order n , and $\text{sub}(B)$ denotes the distributed matrix $B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+nrhs-1)$.

A check is made to verify that $\text{sub}(A)$ is nonsingular.

Input Parameters

| | |
|---------------|---|
| <i>uplo</i> | (global) Must be 'U' or 'L'. Indicates whether $\text{sub}(A)$ is upper or lower triangular: If <i>uplo</i> = 'U', then $\text{sub}(A)$ is upper triangular. If <i>uplo</i> = 'L', then $\text{sub}(A)$ is lower triangular. |
| <i>trans</i> | (global) Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', then $\text{sub}(A)*X = \text{sub}(B)$ is solved for X . If <i>trans</i> = 'T', then $\text{sub}(A)^T*X = \text{sub}(B)$ is solved for X . If <i>trans</i> = 'C', then $\text{sub}(A)^H*X = \text{sub}(B)$ is solved for X . |
| <i>diag</i> | (global) Must be 'N' or 'U'. If <i>diag</i> = 'N', then $\text{sub}(A)$ is not a unit triangular matrix. If <i>diag</i> = 'U', then $\text{sub}(A)$ is unit triangular. |
| <i>n</i> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right-hand sides; i.e., the number of columns of the distributed matrix $\text{sub}(B)$ ($nrhs \geq 0$). |
| <i>a, b</i> | (local) Pointers into the local memory to arrays of local sizes $ld_a*LOCc(\text{ja}+n-1)$ and $ld_b*LOCc(\text{jb}+nrhs-1)$, respectively. The array <i>a</i> contains the local pieces of the distributed triangular matrix $\text{sub}(A)$. If <i>uplo</i> = 'U', the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular matrix, and the strictly lower triangular part of $\text{sub}(A)$ is not referenced. If <i>uplo</i> = 'L', the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular matrix, and the strictly upper triangular part of $\text{sub}(A)$ is not referenced. If <i>diag</i> = 'U', the diagonal elements of $\text{sub}(A)$ are also not referenced and are assumed to be 1. On entry, the array <i>b</i> contains the local pieces of the right hand side distributed matrix $\text{sub}(B)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |

| | |
|---------------|--|
| <i>ib, jb</i> | (global) The row and column indices in the global matrix <i>B</i> indicating the first row and the first column of the matrix sub(<i>B</i>), respectively. |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>B</i> . |

Output Parameters

| | |
|-------------|---|
| <i>b</i> | On exit, if <i>info</i> =0, sub(<i>B</i>) is overwritten by the solution matrix <i>X</i> . |
| <i>info</i> | If <i>info</i> =0, the execution is successful. <i>info</i> < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . <i>info</i> > 0: if <i>info</i> = <i>i</i> , the <i>i</i> -th diagonal element of sub(<i>A</i>) is zero, indicating that the submatrix is singular and the solutions <i>X</i> have not been computed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Estimating the Condition Number: ScaLAPACK Computational Routines

This section describes the ScaLAPACK routines for estimating the condition number of a matrix. The condition number is used for analyzing the errors in the solution of a system of linear equations. Since the condition number may be arbitrarily large when the matrix is nearly singular, the routines actually compute the *reciprocal* condition number.

p?gecon

Estimates the reciprocal of the condition number of a general distributed matrix in either the 1-norm or the infinity-norm.

Syntax

```
void psgecon (char *norm , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *anorm , float *rcond , float *work , MKL_INT *lwork , MKL_INT *iwork ,
MKL_INT *liwork , MKL_INT *info );
```

```
void pdgecon (char *norm , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *anorm , double *rcond , double *work , MKL_INT *lwork ,
MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pcgecon (char *norm , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *anorm , float *rcond , MKL_Complex8 *work , MKL_INT *lwork ,
float *rwork , MKL_INT *lrwork , MKL_INT *info );
```

```
void pzgecon (char *norm , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *anorm , double *rcond , MKL_Complex16 *work , MKL_INT
*lwork , double *rwork , MKL_INT *lrwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?gecon` function estimates the reciprocal of the condition number of a general distributed real/complex matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ in either the 1-norm or infinity-norm, using the *LU* factorization computed by `p?getrf`.

An estimate is obtained for $\|(\text{sub}(A))^{-1}\|$, and the reciprocal of the condition number is computed as

$$rcond = \frac{1}{\|\text{sub}(A)\| \times \|(\text{sub}(A))^{-1}\|}$$

Input Parameters

| | |
|---------------------|---|
| <code>norm</code> | (global) Must be '1' or 'O' or 'I'. Specifies whether the 1-norm condition number or the infinity-norm condition number is required. If <code>norm = '1'</code> or 'O', then the 1-norm is used; If <code>norm = 'I'</code> , then the infinity-norm is used. |
| <code>n</code> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <code>a</code> | (local) Pointer into the local memory to an array of size <code>lld_a*LOCc(ja+n-1)</code> . The array <code>a</code> contains the local pieces of the factors <i>L</i> and <i>U</i> from the factorization $\text{sub}(A) = P*L*U$; the unit diagonal elements of <i>L</i> are not stored. |
| <code>ia, ja</code> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <code>desca</code> | (global and local) array of size <code>dlen_</code> . The array descriptor for the distributed matrix <i>A</i> . |
| <code>anorm</code> | (global) If <code>norm = '1'</code> or 'O', the 1-norm of the original distributed matrix $\text{sub}(A)$; If <code>norm = 'I'</code> , the infinity-norm of the original distributed matrix $\text{sub}(A)$. |
| <code>work</code> | (local) The array <code>work</code> of size <code>lwork</code> is a workspace array. |
| <code>lwork</code> | (local or global) The size of the array <code>work</code> . <i>For real flavors:</i> <code>lwork</code> must be at least |

$$lwork \geq 2*LOCr(n+\text{mod}(ia-1,mb_a))+2*LOCc(n+\text{mod}(ja-1,nb_a)) + \max(2, \max(nb_a*\max(1, \text{iceil}(NPROW-1, NPCOL)), LOCc(n+\text{mod}(ja-1,nb_a)) + nb_a*\max(1, \text{iceil}(NPCOL-1, NPROW)))).$$

For complex flavors:

lwork must be at least

$$lwork \geq 2*LOCr(n+\text{mod}(ia-1,mb_a))+\max(2, \max(nb_a*\text{iceil}(NPROW-1, NPCOL), LOCc(n+\text{mod}(ja-1,nb_a))+ nb_a*\text{iceil}(NPCOL-1, NPROW))).$$

LOCr and *LOCc* values can be computed using the ScaLAPACK tool function `numroc`; *NPROW* and *NPCOL* can be determined by calling the function `blacs_gridinfo`.

NOTE

`iceil(x,y)` is the ceiling of x/y , and `mod(x,y)` is the integer remainder of x/y .

| | |
|---------------|---|
| <i>iwork</i> | (local) Workspace array of size <i>liwork</i> . Used in real flavors only. |
| <i>liwork</i> | (local or global) The size of the array <i>iwork</i> ; used in real flavors only. Must be at least $liwork \geq LOCr(n+\text{mod}(ia-1,mb_a)).$ |
| <i>rwork</i> | (local) Workspace array of size <i>lrwork</i> . Used in complex flavors only. |
| <i>lrwork</i> | (local or global) The size of the array <i>rwork</i> ; used in complex flavors only. Must be at least $lrwork \geq \max(1, 2*LOCc(n+\text{mod}(ja-1,nb_a))).$ |

Output Parameters

| | |
|------------------|---|
| <i>rcond</i> | (global) The reciprocal of the condition number of the distributed matrix <code>sub(A)</code> . See Description. |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>iwork</i> [0] | On exit, <i>iwork</i> [0] contains the minimum value of <i>liwork</i> required for optimum performance (for real flavors). |
| <i>rwork</i> [0] | On exit, <i>rwork</i> [0] contains the minimum value of <i>lrwork</i> required for optimum performance (for complex flavors). |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. <i>info</i> < 0: |

If the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?pocon

Estimates the reciprocal of the condition number (in the 1 - norm) of a symmetric / Hermitian positive-definite distributed matrix.

Syntax

```
void pspocon (char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *anorm , float *rcond , float *work , MKL_INT *lwork , MKL_INT *iwork ,
MKL_INT *liwork , MKL_INT *info );
```

```
void pdpocon (char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *anorm , double *rcond , double *work , MKL_INT *lwork ,
MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pcpocon (char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *anorm , float *rcond , MKL_Complex8 *work , MKL_INT *lwork ,
float *rwork , MKL_INT *lrwork , MKL_INT *info );
```

```
void pzpocon (char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *anorm , double *rcond , MKL_Complex16 *work , MKL_INT
*iwork , double *rwork , MKL_INT *lrwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?pocon function estimates the reciprocal of the condition number (in the 1 - norm) of a real symmetric or complex Hermitian positive definite distributed matrix $sub(A) = A(ia:ia+n-1, ja:ja+n-1)$, using the Cholesky factorization $sub(A) = U^H * U$ or $sub(A) = L * L^H$ computed by p?potrf.

An estimate is obtained for $\|(sub(A))^{-1}\|$, and the reciprocal of the condition number is computed as

$$rcond = \frac{1}{\|sub(A)\| \times \|(sub(A))^{-1}\|}$$

Input Parameters

uplo

(global) Must be 'U' or 'L'.

Specifies whether the factor stored in $sub(A)$ is upper or lower triangular.

If *uplo* = 'U', $sub(A)$ stores the upper triangular factor U of the Cholesky factorization $sub(A) = U^H * U$.

If $uplo = 'L'$, $sub(A)$ stores the lower triangular factor L of the Cholesky factorization $sub(A) = L*L^H$.

| | |
|----------|--|
| n | (global) The order of the distributed matrix $sub(A)$ ($n \geq 0$). |
| a | (local) Pointer into the local memory to an array of size $lld_a*LOCc(ja+n-1)$. The array a contains the local pieces of the factors L or U from the Cholesky factorization $sub(A) = U^H*U$, or $sub(A) = L*L^H$, as computed by <code>p?potrf</code> . |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $sub(A)$, respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| $anorm$ | (global) The 1-norm of the symmetric/Hermitian distributed matrix $sub(A)$. |
| $work$ | (local) The array $work$ of size $lwork$ is a workspace array. |
| $lwork$ | (local or global) The size of the array $work$. |

For real flavors:

$lwork$ must be at least

$$lwork \geq 2*LOCr(n+\text{mod}(ia-1, mb_a)) + 2*LOCc(n+\text{mod}(ja-1, nb_a)) + \max(2, \max(nb_a*\text{ceil}(NPROW-1, NPCOL), LOCc(n+\text{mod}(ja-1, nb_a)) + nb_a*\text{ceil}(NPCOL-1, NPROW))).$$

For complex flavors:

$lwork$ must be at least

$$lwork \geq 2*LOCr(n+\text{mod}(ia-1, mb_a)) + \max(2, \max(nb_a*\max(1, \text{ceil}(NPROW-1, NPCOL)), LOCc(n+\text{mod}(ja-1, nb_a)) + nb_a*\max(1, \text{ceil}(NPCOL-1, NPROW)))).$$

NOTE

$\text{ceil}(x, y)$ is the ceiling of x/y , and $\text{mod}(x, y)$ is the integer remainder of x/y .

| | |
|----------|--|
| $iwork$ | (local) Workspace array of size $liwork$. Used in real flavors only. |
| $liwork$ | (local or global) The size of the array $iwork$; used in real flavors only. Must be at least $liwork \geq LOCr(n+\text{mod}(ia-1, mb_a))$. |
| $rwork$ | (local) Workspace array of size $lrwork$. Used in complex flavors only. |
| $lrwork$ | (local or global) The size of the array $rwork$; used in complex flavors only. Must be at least $lrwork \geq 2*LOCc(n+\text{mod}(ja-1, nb_a))$. |

Output Parameters

| | |
|------------------|--|
| <i>rcond</i> | (global) The reciprocal of the condition number of the distributed matrix sub(A). |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>iwork</i> [0] | On exit, <i>iwork</i> [0] contains the minimum value of <i>liwork</i> required for optimum performance (for real flavors). |
| <i>rwork</i> [0] | On exit, <i>rwork</i> [0] contains the minimum value of <i>lrwork</i> required for optimum performance (for complex flavors). |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trcon

Estimates the reciprocal of the condition number of a triangular distributed matrix in either 1-norm or infinity-norm.

Syntax

```
void pstrcon (char *norm , char *uplo , char *diag , MKL_INT *n , float *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , float *rcond , float *work , MKL_INT *lwork ,
MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pdtrcon (char *norm , char *uplo , char *diag , MKL_INT *n , double *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , double *rcond , double *work , MKL_INT *lwork ,
MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pctrcon (char *norm , char *uplo , char *diag , MKL_INT *n , MKL_Complex8 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *rcond , MKL_Complex8 *work ,
MKL_INT *lwork , float *rwork , MKL_INT *lrwork , MKL_INT *info );
```

```
void pztrcon (char *norm , char *uplo , char *diag , MKL_INT *n , MKL_Complex16 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *rcond , MKL_Complex16 *work ,
MKL_INT *lwork , double *rwork , MKL_INT *lrwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?trcon` function estimates the reciprocal of the condition number of a triangular distributed matrix sub(A) = A(*ia:ia+n-1, ja:ja+n-1*), in either the 1-norm or the infinity-norm.

The norm of sub(A) is computed and an estimate is obtained for $||(\text{sub}(A))^{-1}||$, then the reciprocal of the condition number is computed as

$$rcond = \frac{1}{\|sub(A)\| \times \|(sub(A))^{-1}\|}$$

Input Parameters

| | |
|---------------|--|
| <i>norm</i> | <p>(global) Must be '1' or 'O' or 'I'.</p> <p>Specifies whether the 1-norm condition number or the infinity-norm condition number is required.</p> <p>If <i>norm</i> = '1' or 'O', then the 1-norm is used;</p> <p>If <i>norm</i> = 'I', then the infinity-norm is used.</p> |
| <i>uplo</i> | <p>(global) Must be 'U' or 'L'.</p> <p>If <i>uplo</i> = 'U', sub(A) is upper triangular. If <i>uplo</i> = 'L', sub(A) is lower triangular.</p> |
| <i>diag</i> | <p>(global) Must be 'N' or 'U'.</p> <p>If <i>diag</i> = 'N', sub(A) is non-unit triangular. If <i>diag</i> = 'U', sub(A) is unit triangular.</p> |
| <i>n</i> | (global) The order of the distributed matrix sub(A), ($n \geq 0$). |
| <i>a</i> | <p>(local)</p> <p>Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$.</p> <p>The array <i>a</i> contains the local pieces of the triangular distributed matrix sub(A).</p> <p>If <i>uplo</i> = 'U', the leading <i>n</i>-by-<i>n</i> upper triangular part of this distributed matrix contains the upper triangular matrix, and its strictly lower triangular part is not referenced.</p> <p>If <i>uplo</i> = 'L', the leading <i>n</i>-by-<i>n</i> lower triangular part of this distributed matrix contains the lower triangular matrix, and its strictly upper triangular part is not referenced.</p> <p>If <i>diag</i> = 'U', the diagonal elements of sub(A) are also not referenced and are assumed to be 1.</p> |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix sub(A), respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | <p>(local)</p> <p>The array <i>work</i> of size <i>lwork</i> is a workspace array.</p> |
| <i>lwork</i> | <p>(local or global) The size of the array <i>work</i>.</p> <p><i>For real flavors:</i></p> <p><i>lwork</i> must be at least</p> |

$$lwork \geq 2 * LOCr(n + \text{mod}(ia - 1, mb_a)) + LOcc(n + \text{mod}(ja - 1, nb_a)) + \max(2, \max(nb_a * \max(1, \text{ceil}(\text{NPROW} - 1, \text{NPCOL})), LOcc(n + \text{mod}(ja - 1, nb_a)) + nb_a * \max(1, \text{ceil}(\text{NPCOL} - 1, \text{NPROW}))))).$$

For complex flavors:

lwork must be at least

$$lwork \geq 2 * LOCr(n + \text{mod}(ia - 1, mb_a)) + \max(2, \max(nb_a * \text{ceil}(\text{NPROW} - 1, \text{NPCOL}), LOcc(n + \text{mod}(ja - 1, nb_a)) + nb_a * \text{ceil}(\text{NPCOL} - 1, \text{NPROW}))).$$

NOTE

$\text{ceil}(x, y)$ is the ceiling of x/y , and $\text{mod}(x, y)$ is the integer remainder of x/y .

| | |
|---------------|--|
| <i>iwork</i> | (local) Workspace array of size <i>liwork</i> . Used in real flavors only. |
| <i>liwork</i> | (local or global) The size of the array <i>iwork</i> ; used in real flavors only. Must be at least $liwork \geq LOCr(n + \text{mod}(ia - 1, mb_a))$. |
| <i>rwork</i> | (local) Workspace array of size <i>lrwork</i> . Used in complex flavors only. |
| <i>lrwork</i> | (local or global) The size of the array <i>rwork</i> ; used in complex flavors only. Must be at least $lrwork \geq LOcc(n + \text{mod}(ja - 1, nb_a))$. |

Output Parameters

| | |
|------------------|--|
| <i>rcond</i> | (global) The reciprocal of the condition number of the distributed matrix sub(A). |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>iwork</i> [0] | On exit, <i>iwork</i> [0] contains the minimum value of <i>liwork</i> required for optimum performance (for real flavors). |
| <i>rwork</i> [0] | On exit, <i>rwork</i> [0] contains the minimum value of <i>lrwork</i> required for optimum performance (for complex flavors). |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>j</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

Overview of [ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Refining the Solution and Estimating Its Error: ScaLAPACK Computational Routines

This section describes the ScaLAPACK routines for refining the computed solution of a system of linear equations and estimating the solution error. You can call these routines after factorizing the matrix of the system of equations and computing the solution (see [Routines for Matrix Factorization](#) and [Solving Systems of Linear Equations](#)).

p?gerfs

Improves the computed solution to a system of linear equations and provides error bounds and backward error estimates for the solution.

Syntax

```
void psgerfs (char *trans , MKL_INT *n , MKL_INT *nrhs , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *af , MKL_INT *iaf , MKL_INT *jaf , MKL_INT
*descaf , MKL_INT *ipiv , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
float *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , float *ferr , float *berr ,
float *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pdgerfs (char *trans , MKL_INT *n , MKL_INT *nrhs , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *af , MKL_INT *iaf , MKL_INT *jaf , MKL_INT
*descaf , MKL_INT *ipiv , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
double *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , double *ferr , double *berr ,
double *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pcgerfs (char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *af , MKL_INT *iaf , MKL_INT *jaf ,
MKL_INT *descaf , MKL_INT *ipiv , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_Complex8 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , float
*ferr , float *berr , MKL_Complex8 *work , MKL_INT *lwork , float *rwork , MKL_INT
*lrwork , MKL_INT *info );
```

```
void pzgerfs (char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *af , MKL_INT *iaf , MKL_INT *jaf ,
MKL_INT *descaf , MKL_INT *ipiv , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_Complex16 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx ,
double *ferr , double *berr , MKL_Complex16 *work , MKL_INT *lwork , double *rwork ,
MKL_INT *lrwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?gerfs function improves the computed solution to one of the systems of linear equations

$$\text{sub}(A) \cdot \text{sub}(X) = \text{sub}(B),$$

$$\text{sub}(A)^T \cdot \text{sub}(X) = \text{sub}(B), \text{ or}$$

$$\text{sub}(A)^H \cdot \text{sub}(X) = \text{sub}(B) \text{ and provides error bounds and backward error estimates for the solution.}$$

Here $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$, $\text{sub}(B) = B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+nrhs-1)$, and $\text{sub}(X) = X(\text{ix}:\text{ix}+n-1, \text{jx}:\text{jx}+nrhs-1)$.

Input Parameters

| | |
|--------------------|---|
| <i>trans</i> | <p>(global) Must be 'N' or 'T' or 'C'.</p> <p>Specifies the form of the system of equations:</p> <p>If <i>trans</i> = 'N', the system has the form $\text{sub}(A) * \text{sub}(X) = \text{sub}(B)$ (No transpose);</p> <p>If <i>trans</i> = 'T', the system has the form $\text{sub}(A)^T * \text{sub}(X) = \text{sub}(B)$ (Transpose);</p> <p>If <i>trans</i> = 'C', the system has the form $\text{sub}(A)^H * \text{sub}(X) = \text{sub}(B)$ (Conjugate transpose).</p> |
| <i>n</i> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right-hand sides, i.e., the number of columns of the matrices $\text{sub}(B)$ and $\text{sub}(X)$ ($nrhs \geq 0$). |
| <i>a, af, b, x</i> | <p>(local)</p> <p>Pointers into the local memory to arrays of local sizes</p> <p><i>a</i>: $lld_a * LOCC(ja+n-1)$,</p> <p><i>af</i>: $lld_af * LOCC(jaf+n-1)$,</p> <p><i>b</i>: $lld_b * LOCC(jb+nrhs-1)$,</p> <p><i>x</i>: $lld_x * LOCC(jx+nrhs-1)$.</p> <p>The array <i>a</i> contains the local pieces of the distributed matrix $\text{sub}(A)$.</p> <p>The array <i>af</i> contains the local pieces of the distributed factors of the matrix $\text{sub}(A) = P * L * U$ as computed by <code>p?getrf</code>.</p> <p>The array <i>b</i> contains the local pieces of the distributed matrix of right hand sides $\text{sub}(B)$.</p> <p>On entry, the array <i>x</i> contains the local pieces of the distributed solution matrix $\text{sub}(X)$.</p> |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>iaf, jaf</i> | (global) The row and column indices in the global matrix <i>AF</i> indicating the first row and the first column of the matrix $\text{sub}(AF)$, respectively. |
| <i>descaf</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>AF</i> . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix <i>B</i> indicating the first row and the first column of the matrix $\text{sub}(B)$, respectively. |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>B</i> . |

| | |
|---|--|
| <i>ix, jx</i> | (global) The row and column indices in the global matrix X indicating the first row and the first column of the matrix sub(X), respectively. |
| <i>descx</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix X . |
| <i>ipiv</i> | (local) Array of size $LOCr(m_af) + mb_af$. This array contains pivoting information as computed by <code>p?getrf</code> . If $ipiv[i]=j$, then the local row $i+1$ was swapped with the global row j where $i=0, \dots, LOCr(m_af) + mb_af - 1$. This array is tied to the distributed matrix A . |
| <i>work</i> | (local) The array <i>work</i> of size <i>lwork</i> is a workspace array. |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> . <i>For real flavors:</i> <i>lwork</i> must be at least $lwork \geq 3 * LOCr(n + \text{mod}(ia-1, mb_a))$ <i>For complex flavors:</i> <i>lwork</i> must be at least $lwork \geq 2 * LOCr(n + \text{mod}(ia-1, mb_a))$ |
| <hr/> NOTE $\text{mod}(x, y)$ is the integer remainder of x/y . <hr/> | |
| <i>iwork</i> | (local) Workspace array, size <i>liwork</i> . Used in real flavors only. |
| <i>liwork</i> | (local or global) The size of the array <i>iwork</i> ; used in real flavors only. Must be at least $liwork \geq LOCr(n + \text{mod}(ib-1, mb_b))$. |
| <i>rwork</i> | (local) Workspace array, size <i>lrwork</i> . Used in complex flavors only. |
| <i>lrwork</i> | (local or global) The size of the array <i>rwork</i> ; used in complex flavors only. Must be at least $lrwork \geq LOCr(n + \text{mod}(ib-1, mb_b))$. |

Output Parameters

| | |
|-------------------|--|
| <i>x</i> | On exit, contains the improved solution vectors. |
| <i>ferr, berr</i> | Arrays of size $LOCc(jb + nrhs - 1)$ each. The array <i>ferr</i> contains the estimated forward error bound for each solution vector of sub(X). |

If `XTRUE` is the true solution corresponding to `sub(X)`, `ferr` is an estimated upper bound for the magnitude of the largest element in `(sub(X) - XTRUE)` divided by the magnitude of the largest element in `sub(X)`. The estimate is as reliable as the estimate for `rcond`, and is almost always a slight overestimate of the true error.

This array is tied to the distributed matrix `X`.

The array `berr` contains the component-wise relative backward error of each solution vector (that is, the smallest relative change in any entry of `sub(A)` or `sub(B)` that makes `sub(X)` an exact solution). This array is tied to the distributed matrix `X`.

`work[0]` On exit, `work[0]` contains the minimum value of `lwork` required for optimum performance.

`iwork[0]` On exit, `iwork[0]` contains the minimum value of `liwork` required for optimum performance (for real flavors).

`rwork[0]` On exit, `rwork[0]` contains the minimum value of `lrwork` required for optimum performance (for complex flavors).

`info` (global) If `info=0`, the execution is successful.

`info < 0`:

If the *i*-th argument is an array and the *j*-th entry, indexed *j* - 1, had an illegal value, then `info = -(i*100+j)`; if the *i*-th argument is a scalar and had an illegal value, then `info = -i`.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?porfs

Improves the computed solution to a system of linear equations with symmetric/Hermitian positive definite distributed matrix and provides error bounds and backward error estimates for the solution.

Syntax

```
void psporfs (char *uplo , MKL_INT *n , MKL_INT *nrhs , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *af , MKL_INT *iaf , MKL_INT *jaf , MKL_INT
*descaf , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , float *x , MKL_INT
*ix , MKL_INT *jx , MKL_INT *descx , float *ferr , float *berr , float *work , MKL_INT
*lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pdporfs (char *uplo , MKL_INT *n , MKL_INT *nrhs , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *af , MKL_INT *iaf , MKL_INT *jaf , MKL_INT
*descaf , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , double *x , MKL_INT
*ix , MKL_INT *jx , MKL_INT *descx , double *ferr , double *berr , double *work ,
MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pcporfs (char *uplo , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *af , MKL_INT *iaf , MKL_INT *jaf ,
MKL_INT *descaf , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
```

```
MKL_Complex8 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , float *ferr , float
*berr , MKL_Complex8 *work , MKL_INT *lwork , float *rwork , MKL_INT *lrwork , MKL_INT
*info );
```

```
void pzporfs (char *uplo , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *af , MKL_INT *iaf , MKL_INT *jaf ,
MKL_INT *descaf , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
MKL_Complex16 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , double *ferr , double
*berr , MKL_Complex16 *work , MKL_INT *lwork , double *rwork , MKL_INT *lrwork ,
MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `pzporfs` function improves the computed solution to the system of linear equations

$$\text{sub}(A) \cdot \text{sub}(X) = \text{sub}(B),$$

where $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ is a real symmetric or complex Hermitian positive definite distributed matrix and

$$\text{sub}(B) = B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+nrhs-1),$$

$$\text{sub}(X) = X(\text{ix}:\text{ix}+n-1, \text{jx}:\text{jx}+nrhs-1)$$

are right-hand side and solution submatrices, respectively. This function also provides error bounds and backward error estimates for the solution.

Input Parameters

| | |
|--------------------|---|
| <i>uplo</i> | (global) Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix $\text{sub}(A)$ is stored. If <i>uplo</i> = 'U', $\text{sub}(A)$ is upper triangular. If <i>uplo</i> = 'L', $\text{sub}(A)$ is lower triangular. |
| <i>n</i> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right-hand sides, i.e., the number of columns of the matrices $\text{sub}(B)$ and $\text{sub}(X)$ ($nrhs \geq 0$). |
| <i>a, af, b, x</i> | (local) Pointers into the local memory to arrays of local sizes <i>a</i> : $lld_a * LOCC(\text{ja}+n-1)$, <i>af</i> : $lld_af * LOCC(\text{jaf}+n-1)$, <i>b</i> : $lld_b * LOCC(\text{jb}+nrhs-1)$, <i>x</i> : $lld_x * LOCC(\text{jx}+nrhs-1)$. The array <i>a</i> contains the local pieces of the <i>n</i> -by- <i>n</i> symmetric/Hermitian distributed matrix $\text{sub}(A)$. If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. |

If $uplo = 'L'$, the leading n -by- n lower triangular part of $sub(A)$ contains the lower triangular part of the distributed matrix, and its strictly upper triangular part is not referenced.

The array af contains the factors L or U from the Cholesky factorization $sub(A) = L * L^H$ or $sub(A) = U^H * U$, as computed by `p?potrf`.

On entry, the array b contains the local pieces of the distributed matrix of right hand sides $sub(B)$.

On entry, the array x contains the local pieces of the solution vectors $sub(X)$.

| | |
|------------|---|
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $sub(A)$, respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| iaf, jaf | (global) The row and column indices in the global matrix AF indicating the first row and the first column of the matrix $sub(AF)$, respectively. |
| $descaf$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix AF . |
| ib, jb | (global) The row and column indices in the global matrix B indicating the first row and the first column of the matrix $sub(B)$, respectively. |
| $descb$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |
| ix, jx | (global) The row and column indices in the global matrix X indicating the first row and the first column of the matrix $sub(X)$, respectively. |
| $descx$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix X . |
| $work$ | (local) The array $work$ of size $lwork$ is a workspace array. |
| $lwork$ | (local) The size of the array $work$. |

For real flavors:

$lwork$ must be at least

$$lwork \geq 3 * LOCr(n + \text{mod}(ia-1, mb_a))$$

For complex flavors:

$lwork$ must be at least

$$lwork \geq 2 * LOCr(n + \text{mod}(ia-1, mb_a))$$

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

| | |
|---------|---|
| $iwork$ | (local) Workspace array of size $liwork$. Used in real flavors only. |
|---------|---|

| | |
|---------------|--|
| <i>liwork</i> | (local or global) The size of the array <i>liwork</i> ; used in real flavors only. Must be at least $liwork \geq LOCr(n + \text{mod}(ib-1, mb_b))$. |
| <i>rwork</i> | (local) Workspace array of size <i>lrwork</i> . Used in complex flavors only. |
| <i>lrwork</i> | (local or global) The size of the array <i>rwork</i> ; used in complex flavors only. Must be at least $lrwork \geq LOCr(n + \text{mod}(ib-1, mb_b))$. |

Output Parameters

| | |
|-------------------|---|
| <i>x</i> | On exit, contains the improved solution vectors. |
| <i>ferr, berr</i> | Arrays of size $LOCc(jb + nrhs - 1)$ each. The array <i>ferr</i> contains the estimated forward error bound for each solution vector of sub(<i>X</i>). If <i>XTRUE</i> is the true solution corresponding to sub(<i>X</i>), <i>ferr</i> is an estimated upper bound for the magnitude of the largest element in (sub(<i>X</i>) - <i>XTRUE</i>) divided by the magnitude of the largest element in sub(<i>X</i>). The estimate is as reliable as the estimate for <i>rcond</i> , and is almost always a slight overestimate of the true error. This array is tied to the distributed matrix <i>X</i> . The array <i>berr</i> contains the component-wise relative backward error of each solution vector (that is, the smallest relative change in any entry of sub(<i>A</i>) or sub(<i>B</i>) that makes sub(<i>X</i>) an exact solution). This array is tied to the distributed matrix <i>X</i> . |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>iwork</i> [0] | On exit, <i>iwork</i> [0] contains the minimum value of <i>liwork</i> required for optimum performance (for real flavors). |
| <i>rwork</i> [0] | On exit, <i>rwork</i> [0] contains the minimum value of <i>lrwork</i> required for optimum performance (for complex flavors). |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. $info < 0$: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then $info = -(j*100+j)$; if the <i>i</i> -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trfs

Provides error bounds and backward error estimates for the solution to a system of linear equations with a distributed triangular coefficient matrix.

Syntax

```
void pstrrfs (char *uplo , char *trans , char *diag , MKL_INT *n , MKL_INT *nrhs ,
float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , float *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx ,
float *ferr , float *berr , float *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT
*liwork , MKL_INT *info );
```

```
void pdtrrfs (char *uplo , char *trans , char *diag , MKL_INT *n , MKL_INT *nrhs ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , double *x , MKL_INT *ix , MKL_INT *jx , MKL_INT
*descx , double *ferr , double *berr , double *work , MKL_INT *lwork , MKL_INT
*iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pctrfs (char *uplo , char *trans , char *diag , MKL_INT *n , MKL_INT *nrhs ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_Complex8 *x , MKL_INT *ix , MKL_INT
*jx , MKL_INT *descx , float *ferr , float *berr , MKL_Complex8 *work , MKL_INT
*lwork , float *rwork , MKL_INT *lrwork , MKL_INT *info );
```

```
void pztrrfs (char *uplo , char *trans , char *diag , MKL_INT *n , MKL_INT *nrhs ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_Complex16 *x , MKL_INT *ix , MKL_INT
*jx , MKL_INT *descx , double *ferr , double *berr , MKL_Complex16 *work , MKL_INT
*lwork , double *rwork , MKL_INT *lrwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?trrfs` function provides error bounds and backward error estimates for the solution to one of the systems of linear equations

$$\text{sub}(A) * \text{sub}(X) = \text{sub}(B),$$

$$\text{sub}(A)^T * \text{sub}(X) = \text{sub}(B), \text{ or}$$

$$\text{sub}(A)^H * \text{sub}(X) = \text{sub}(B),$$

where $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ is a triangular matrix,

$\text{sub}(B) = B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+nrhs-1)$, and

$\text{sub}(X) = X(\text{ix}:\text{ix}+n-1, \text{jx}:\text{jx}+nrhs-1)$.

The solution matrix X must be computed by `p?trtrs` or some other means before entering this function. The function `p?trrfs` does not do iterative refinement because doing so cannot improve the backward error.

Input Parameters

`uplo` (global) Must be 'U' or 'L'.
If `uplo = 'U'`, $\text{sub}(A)$ is upper triangular. If `uplo = 'L'`, $\text{sub}(A)$ is lower triangular.

`trans` (global) Must be 'N' or 'T' or 'C'.
Specifies the form of the system of equations:
If `trans = 'N'`, the system has the form $\text{sub}(A) * \text{sub}(X) = \text{sub}(B)$ (No transpose);

| | |
|----------------|---|
| | <p>If $trans = 'T'$, the system has the form $sub(A)^T * sub(X) = sub(B)$ (Transpose);</p> <p>If $trans = 'C'$, the system has the form $sub(A)^H * sub(X) = sub(B)$ (Conjugate transpose).</p> |
| <i>diag</i> | <p>Must be $'N'$ or $'U'$.</p> <p>If $diag = 'N'$, then $sub(A)$ is non-unit triangular.</p> <p>If $diag = 'U'$, then $sub(A)$ is unit triangular.</p> |
| <i>n</i> | (global) The order of the distributed matrix $sub(A)$ ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right-hand sides, that is, the number of columns of the matrices $sub(B)$ and $sub(X)$ ($nrhs \geq 0$). |
| <i>a, b, x</i> | <p>(local)</p> <p>Pointers into the local memory to arrays of local sizes</p> <p>$a: lld_a * LOCc(ja+n-1)$,</p> <p>$b: lld_b * LOCc(jb+nrhs-1)$,</p> <p>$x: lld_x * LOCc(jx+nrhs-1)$.</p> <p>The array a contains the local pieces of the original triangular distributed matrix $sub(A)$.</p> <p>If $uplo = 'U'$, the leading n-by-n upper triangular part of $sub(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced.</p> <p>If $uplo = 'L'$, the leading n-by-n lower triangular part of $sub(A)$ contains the lower triangular part of the distributed matrix, and its strictly upper triangular part is not referenced.</p> <p>If $diag = 'U'$, the diagonal elements of $sub(A)$ are also not referenced and are assumed to be 1.</p> <p>On entry, the array b contains the local pieces of the distributed matrix of right hand sides $sub(B)$.</p> <p>On entry, the array x contains the local pieces of the solution vectors $sub(X)$.</p> |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $sub(A)$, respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of the matrix $sub(B)$, respectively. |
| <i>descb</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |
| <i>ix, jx</i> | (global) The row and column indices in the global matrix X indicating the first row and the first column of the matrix $sub(X)$, respectively. |

descx (global and local) array of size *dlen_*. The array descriptor for the distributed matrix *X*.

work (local)
The array *work* of size *lwork* is a workspace array.

lwork (local) The size of the array *work*.
For real flavors:
lwork must be at least $lwork \geq 3 * LOCr(n + \text{mod}(ia-1, mb_a))$
For complex flavors:
lwork must be at least $lwork \geq 2 * LOCr(n + \text{mod}(ia-1, mb_a))$

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

iwork (local) Workspace array of size *liwork*. Used in real flavors only.

liwork (local or global) The size of the array *iwork*; used in real flavors only. Must be at least $liwork \geq LOCr(n + \text{mod}(ib-1, mb_b))$.

rwork (local)
Workspace array of size *lrwork*. Used in complex flavors only.

lrwork (local or global) The size of the array *rwork*; used in complex flavors only. Must be at least $lrwork \geq LOCr(n + \text{mod}(ib-1, mb_b))$.

Output Parameters

ferr, berr Arrays of size $LOCc(jb + nrhs - 1)$ each.

The array *ferr* contains the estimated forward error bound for each solution vector of $\text{sub}(X)$.

If *XTRUE* is the true solution corresponding to $\text{sub}(X)$, *ferr* is an estimated upper bound for the magnitude of the largest element in $(\text{sub}(X) - XTRUE)$ divided by the magnitude of the largest element in $\text{sub}(X)$. The estimate is as reliable as the estimate for *rcond*, and is almost always a slight overestimate of the true error.

This array is tied to the distributed matrix *X*.

The array *berr* contains the component-wise relative backward error of each solution vector (that is, the smallest relative change in any entry of $\text{sub}(A)$ or $\text{sub}(B)$ that makes $\text{sub}(X)$ an exact solution). This array is tied to the distributed matrix *X*.

work[0] On exit, *work[0]* contains the minimum value of *lwork* required for optimum performance.

| | |
|-----------------------|--|
| <code>iwork[0]</code> | On exit, <code>iwork[0]</code> contains the minimum value of <code>liwork</code> required for optimum performance (for real flavors). |
| <code>rwork[0]</code> | On exit, <code>rwork[0]</code> contains the minimum value of <code>lrwork</code> required for optimum performance (for complex flavors). |
| <code>info</code> | (global) If <code>info=0</code> , the execution is successful. <code>info < 0</code> : If the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Matrix Inversion: ScaLAPACK Computational Routines

This sections describes ScaLAPACK routines that compute the inverse of a matrix based on the previously obtained factorization. Note that it is not recommended to solve a system of equations $Ax = b$ by first computing A^{-1} and then forming the matrix-vector product $x = A^{-1}b$. Call a solver routine instead (see [Solving Systems of Linear Equations](#)); this is more efficient and more accurate.

p?getri

Computes the inverse of a LU-factored distributed matrix.

Syntax

```
void psgetri (MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
MKL_INT *ipiv , float *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork ,
MKL_INT *info );
```

```
void pdgetri (MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
MKL_INT *ipiv , double *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork ,
MKL_INT *info );
```

```
void pcgetri (MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , MKL_INT *ipiv , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *iwork ,
MKL_INT *liwork , MKL_INT *info );
```

```
void pzgetri (MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , MKL_INT *ipiv , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *iwork ,
MKL_INT *liwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?getri` function computes the inverse of a general distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ using the LU factorization computed by `p?getrf`. This method inverts U and then computes the inverse of $\text{sub}(A)$ by solving the system

$$\text{inv}(\text{sub}(A)) * L = \text{inv}(U)$$

for $\text{inv}(\text{sub}(A))$.

Input Parameters

| | |
|---------------|--|
| <i>n</i> | (global) The number of rows and columns to be operated on, that is, the order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja+n-1)$. On entry, the array <i>a</i> contains the local pieces of the <i>L</i> and <i>U</i> obtained by the factorization $\text{sub}(A) = P * L * U$ computed by <code>p?getrf</code> . |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local) The array <i>work</i> of size <i>lwork</i> is a workspace array. |
| <i>lwork</i> | (local) The size of the array <i>work</i> . <i>lwork</i> must be at least $lwork \geq LOCr(n + \text{mod}(ia-1, mb_a)) * nb_a$. |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

The array *work* is used to keep at most an entire column block of $\text{sub}(A)$.

| | |
|--------------|--|
| <i>iwork</i> | (local) Workspace array used for physically transposing the pivots, size <i>liwork</i> . |
|--------------|--|

| | |
|---------------|---|
| <i>liwork</i> | (local or global) The size of the array <i>iwork</i> . The minimal value <i>liwork</i> of is determined by the following code: |
|---------------|---|

```

if NPROW == NPCOL then
    liwork = LOCC(n_a + mod(ja-1, nb_a)) + nb_a
else
    liwork = LOCC(n_a + mod(ja-1, nb_a)) +
max(ceil(ceil(LOCr(m_a)/mb_a)/(lcm/NPROW)), nb_a)
end if

```

where *lcm* is the least common multiple of process rows and columns (NPROW and NPCOL).

Output Parameters

| | |
|-------------|--|
| <i>ipiv</i> | (local) Array of size $LOCr(m_a) + mb_a$. This array contains the pivoting information. |
|-------------|--|

If $ipiv[i]=j$, then the local row $i+1$ was swapped with the global row j where $i=0, \dots, LOCr(m_a) + mb_a - 1$.

This array is tied to the distributed matrix A .

`work[0]`

On exit, `work[0]` contains the minimum value of `lwork` required for optimum performance.

`iwork[0]`

On exit, `iwork[0]` contains the minimum value of `liwork` required for optimum performance.

`info`

(global) If `info=0`, the execution is successful.

`info < 0`:

If the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then `info = -(i*100+j)`; if the i -th argument is a scalar and had an illegal value, then `info = -i`.

`info > 0`:

If `info = i`, the matrix element $U(i,i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, and division by zero will occur if it is used to solve a system of equations.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?potri

Computes the inverse of a symmetric/Hermitian positive definite distributed matrix.

Syntax

```
void pspotri (char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
```

```
void pdpotri (char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
```

```
void pcpotri (char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
```

```
void pzpotri (char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?potri` function computes the inverse of a real symmetric or complex Hermitian positive definite distributed matrix $sub(A) = A(ia:ia+n-1, ja:ja+n-1)$ using the Cholesky factorization $sub(A) = U^H * U$ or $sub(A) = L * L^H$ computed by `p?potrf`.

Input Parameters

`uplo`

(global) Must be 'U' or 'L'.

| | |
|----------|--|
| | Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix $\text{sub}(A)$ is stored. If $\text{uplo} = 'U'$, upper triangle of $\text{sub}(A)$ is stored. If $\text{uplo} = 'L'$, lower triangle of $\text{sub}(A)$ is stored. |
| n | (global) The number of rows and columns to be operated on, that is, the order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| a | (local) Pointer into the local memory to an array of local size $\text{lld}_a * \text{LOCc}(j_a + n - 1)$. On entry, the array a contains the local pieces of the triangular factor U or L from the Cholesky factorization $\text{sub}(A) = U^H * U$, or $\text{sub}(A) = L * L^H$, as computed by <code>p?potrf</code> . |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |

Output Parameters

| | |
|--------|--|
| a | On exit, overwritten by the local pieces of the upper or lower triangle of the (symmetric/Hermitian) inverse of $\text{sub}(A)$. |
| $info$ | (global) If $info=0$, the execution is successful. $info < 0$: If the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. $info > 0$: If $info = i$, the element (i, i) of the factor U or L is zero, and the inverse could not be computed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trtri

Computes the inverse of a triangular distributed matrix.

Syntax

```
void pstrtri (char *uplo , char *diag , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
void pdtrtri (char *uplo , char *diag , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
void pctrtri (char *uplo , char *diag , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
```

```
void pztrtri (char *uplo , char *diag , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `pztrtri` function computes the inverse of a real or complex upper or lower triangular distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$.

Input Parameters

| | |
|---------------|---|
| <i>uplo</i> | (global) Must be 'U' or 'L'. Specifies whether the distributed matrix $\text{sub}(A)$ is upper or lower triangular. If <i>uplo</i> = 'U', $\text{sub}(A)$ is upper triangular. If <i>uplo</i> = 'L', $\text{sub}(A)$ is lower triangular. |
| <i>diag</i> | Must be 'N' or 'U'. Specifies whether or not the distributed matrix $\text{sub}(A)$ is unit triangular. If <i>diag</i> = 'N', then $\text{sub}(A)$ is non-unit triangular. If <i>diag</i> = 'U', then $\text{sub}(A)$ is unit triangular. |
| <i>n</i> | (global) The number of rows and columns to be operated on, that is, the order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCc(\text{ja}+n-1)$. The array <i>a</i> contains the local pieces of the triangular distributed matrix $\text{sub}(A)$. If <i>uplo</i> = 'U', the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular matrix to be inverted, and the strictly lower triangular part of $\text{sub}(A)$ is not referenced. If <i>uplo</i> = 'L', the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular matrix, and the strictly upper triangular part of $\text{sub}(A)$ is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | On exit, overwritten by the (triangular) inverse of the original matrix. |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. <i>info</i> < 0: |

If the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

$info > 0$:

If $info = k$, the matrix element $A(ia+k-1, ja+k-1)$ is exactly zero. The triangular matrix $sub(A)$ is singular and its inverse cannot be computed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Matrix Equilibration: ScaLAPACK Computational Routines

ScaLAPACK routines described in this section are used to compute scaling factors needed to equilibrate a matrix. Note that these routines do not actually scale the matrices.

p?geequ

Computes row and column scaling factors intended to equilibrate a general rectangular distributed matrix and reduce its condition number.

Syntax

```
void psgeequ (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *r , float *c , float *rowcnd , float *colcnd , float *amax , MKL_INT
*info );
```

```
void pdgeequ (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *r , double *c , double *rowcnd , double *colcnd , double
*amax , MKL_INT *info );
```

```
void pcgeequ (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *r , float *c , float *rowcnd , float *colcnd , float *amax ,
MKL_INT *info );
```

```
void pzgeequ (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *r , double *c , double *rowcnd , double *colcnd , double
*amax , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?geequ` function computes row and column scalings intended to equilibrate an m -by- n distributed matrix $sub(A) = A(ia:ia+m-1, ja:ja+n-1)$ and reduce its condition number. The output array r returns the row scale factors r_i , and the array c returns the column scale factors c_j . These factors are chosen to try to make the largest element in each row and column of the matrix B with elements $b_{ij}=r_i*a_{ij}*c_j$ have absolute value 1.

r_i and c_j are restricted to be between $SMLNUM =$ smallest safe number and $BIGNUM =$ largest safe number. Use of these scaling factors is not guaranteed to reduce the condition number of $sub(A)$ but works well in practice.

SMLNUM and *BIGNUM* are parameters representing machine precision. You can use the `?slamch` routines to compute them. For example, compute single precision values of *SMLNUM* and *BIGNUM* as follows:

```
SMLNUM = slamch ('s')
BIGNUM = 1 / SMLNUM
```

The auxiliary function `p?laqge` uses scaling factors computed by `p?geequ` to scale a general rectangular matrix.

Input Parameters

| | |
|---------------|---|
| <i>m</i> | (global) The number of rows to be operated on, that is, the number of rows of the distributed matrix $\text{sub}(A)$ ($m \geq 0$). |
| <i>n</i> | (global) The number of columns to be operated on, that is, the number of columns of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja+n-1)$. The array <i>a</i> contains the local pieces of the <i>m</i> -by- <i>n</i> distributed matrix whose equilibration factors are to be computed. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |

Output Parameters

| | |
|-----------------------|--|
| <i>r, c</i> | (local) Arrays of sizes $LOCr(m_a)$ and $LOCC(n_a)$, respectively. If <i>info</i> = 0, or <i>info</i> > <i>ia</i> + <i>m</i> -1, <i>r</i> [<i>i</i>] contain the row scale factors for $\text{sub}(A)$ for $ia-1 \leq i < ia+m-1$. <i>r</i> is aligned with the distributed matrix <i>A</i> , and replicated across every process column. <i>r</i> is tied to the distributed matrix <i>A</i> . If <i>info</i> = 0, <i>c</i> [<i>i</i>] contain the column scale factors for $\text{sub}(A)$ for $ja-1 \leq i < ja+n-1$. <i>c</i> is aligned with the distributed matrix <i>A</i> , and replicated down every process row. <i>c</i> is tied to the distributed matrix <i>A</i> . |
| <i>rowcnd, colcnd</i> | (global) If <i>info</i> = 0 or <i>info</i> > <i>ia</i> + <i>m</i> -1, <i>rowcnd</i> contains the ratio of the smallest r_i to the largest r_i ($ia \leq i \leq ia+m-1$). If <i>rowcnd</i> ≥ 0.1 and <i>amax</i> is neither too large nor too small, it is not worth scaling by r_i . If <i>info</i> = 0, <i>colcnd</i> contains the ratio of the smallest c_j to the largest c_j ($ja \leq j \leq ja+n-1$). If <i>colcnd</i> ≥ 0.1 , it is not worth scaling by c_j . |
| <i>amax</i> | (global) Absolute value of the largest matrix element. If <i>amax</i> is very close to overflow or very close to underflow, the matrix should be scaled. |

info (global) If *info*=0, the execution is successful.

info < 0:

If the *i*-th argument is an array and the *j*-th entry, indexed *j* - 1, had an illegal value, then *info* = -(*i**100+*j*); if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

info > 0:

If *info* = *i* and

i ≤ *m*, the *i*-th row of the distributed matrix sub(*A*) is exactly zero;

i > *m*, the (*i* - *m*)-th column of the distributed matrix sub(*A*) is exactly zero.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?poequ

Computes row and column scaling factors intended to equilibrate a symmetric (Hermitian) positive definite distributed matrix and reduce its condition number.

Syntax

```
void pspoequ (MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
float *sr , float *sc , float *scond , float *amax , MKL_INT *info );
void pdpoequ (MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
double *sr , double *sc , double *scond , double *amax , MKL_INT *info );
void pcspoequ (MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *sr , float *sc , float *scond , float *amax , MKL_INT *info );
void pzspoequ (MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , double *sr , double *sc , double *scond , double *amax , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?poequ function computes row and column scalings intended to equilibrate a real symmetric or complex Hermitian positive definite distributed matrix sub(*A*) = *A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1) and reduce its condition number (with respect to the two-norm). The output arrays *sr* and *sc* return the row and column scale factors

$$s(i) = \frac{1}{\sqrt{a_{i,i}}}$$

These factors are chosen so that the scaled distributed matrix *B* with elements *b*_{*ij*}=*s*(*i*)**a*_{*ij*}**s*(*j*) has ones on the diagonal.

This choice of sr and sc puts the condition number of B within a factor n of the smallest possible condition number over all possible diagonal scalings.

The auxiliary function `p?laqsy` uses scaling factors computed by `p?geequ` to scale a general rectangular matrix.

Input Parameters

| | |
|----------|--|
| n | (global) The number of rows and columns to be operated on, that is, the order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| a | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja+n-1)$. The array a contains the n -by- n symmetric/Hermitian positive definite distributed matrix $\text{sub}(A)$ whose scaling factors are to be computed. Only the diagonal elements of $\text{sub}(A)$ are referenced. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |

Output Parameters

| | |
|----------|--|
| sr, sc | (local) Arrays of sizes $LOCr(m_a)$ and $LOCC(n_a)$, respectively. If $info = 0$, the array $sr(ia:ia+n-1)$ contains the row scale factors for $\text{sub}(A)$. sr is aligned with the distributed matrix A , and replicated across every process column. sr is tied to the distributed matrix A . If $info = 0$, the array $sc(ja:ja+n-1)$ contains the column scale factors for $\text{sub}(A)$. sc is aligned with the distributed matrix A , and replicated down every process row. sc is tied to the distributed matrix A . |
| $scond$ | (global) If $info = 0$, $scond$ contains the ratio of the smallest $sr[i]$ (or $sc[j]$) to the largest $sr[i]$ (or $sc[j]$), with $ia-1 \leq i < ia+n-1$ and $ja-1 \leq j < ja+n-1$. If $scond \geq 0.1$ and $amax$ is neither too large nor too small, it is not worth scaling by sr (or sc). |
| $amax$ | (global) Absolute value of the largest matrix element. If $amax$ is very close to overflow or very close to underflow, the matrix should be scaled. |
| $info$ | (global) If $info=0$, the execution is successful. $info < 0$: |

If the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

$info > 0$:

If $info = k$, the k -th diagonal entry of $sub(A)$ is nonpositive.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Orthogonal Factorizations: ScaLAPACK Computational Routines

This section describes the ScaLAPACK routines for the $QR(RQ)$ and $LQ(QL)$ factorization of matrices. Routines for the RZ factorization as well as for generalized QR and RQ factorizations are also included. For the mathematical definition of the factorizations, see the respective LAPACK sections or refer to [SLUG].

Table "Computational Routines for Orthogonal Factorizations" lists ScaLAPACK routines that perform orthogonal factorization of matrices.

Computational Routines for Orthogonal Factorizations

| Matrix type, factorization | Factorize without pivoting | Factorize with pivoting | Generate matrix Q | Apply matrix Q |
|--|----------------------------|-------------------------|--|--|
| general matrices, QR factorization | p?geqrf | p?geqpf | p?orgqr p?ungqr | p?ormqr p?unmqr |
| general matrices, RQ factorization | p?gerqf | | p?orgrq p?ungrq | p?ormrq p?unmrq |
| general matrices, LQ factorization | p?gelqf | | p?orglq p?unglq | p?ormlq p?unmlq |
| general matrices, QL factorization | p?geqlf | | p?orgql p?ungql | p?ormql p?unmql |
| trapezoidal matrices, RZ factorization | p?tzrzf | | | p?ormrz p?unmrz |
| pair of matrices, generalized QR factorization | p?ggqrf | | | |
| pair of matrices, generalized RQ factorization | p?ggrqf | | | |

p?geqrf

Computes the QR factorization of a general m-by-n matrix.

Syntax

```
void psgeqrf (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdgeqrf (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcgeqrf (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pzgeqrf (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT
*info );
```

Include Files

- mkl_scalapack.h

Description

The `p?geqrf` function forms the QR factorization of a general m -by- n distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia} + m - 1, \text{ja}:\text{ja} + n - 1)$ as

$$A = Q * R.$$

Input Parameters

| | |
|---------------|--|
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$; ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$; ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja + n - 1)$. Contains the local pieces of the distributed matrix $\text{sub}(A)$ to be factored. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix $A(\text{ia}:\text{ia} + m - 1, \text{ja}:\text{ja} + n - 1)$, respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A |
| <i>work</i> | (local). Workspace array of size $lwork$. |
| <i>lwork</i> | (local or global) size of $work$, must be at least $lwork \geq nb_a * (mp0 + nq0 + nb_a)$, where $iroff = \text{mod}(ia - 1, mb_a)$, $icoff = \text{mod}(ja - 1, nb_a)$, $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW)$, $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$, $mp0 = \text{numroc}(m + iroff, mb_a, MYROW, iarow, NPROW)$, $nq0 = \text{numroc}(n + icoff, nb_a, MYCOL, iacol, NPCOL)$, and numroc , indxg2p are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the function <code>blacs_gridinfo</code> . |

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|---|
| <code>a</code> | The elements on and above the diagonal of $\text{sub}(A)$ contain the $\min(m,n)$ -by- n upper trapezoidal matrix R (R is upper triangular if $m \geq n$); the elements below the diagonal, with the array <code>tau</code> , represent the orthogonal/unitary matrix Q as a product of elementary reflectors (see <i>Application Notes</i> below). |
| <code>tau</code> | (local) Array of size $LOCc(ja + \min(m, n) - 1)$. Contains the scalar factor of elementary reflectors. <code>tau</code> is tied to the distributed matrix A . |
| <code>work[0]</code> | On exit, <code>work[0]</code> contains the minimum value of $lwork$ required for optimum performance. |
| <code>info</code> | (global) = 0, the execution is successful. < 0, if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i * 100 + j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

Application Notes

The matrix Q is represented as a product of elementary reflectors

$$Q = H(ja) * H(ja+1) * \dots * H(ja+k-1),$$

where $k = \min(m, n)$.

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v'$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$; $v(i+1:m)$ is stored on exit in $A(ia+i:ia+m-1, ja+i-1)$, and τ in $\tau[ja+i-2]$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?geqpf

Computes the QR factorization of a general m -by- n matrix with pivoting.

Syntax

```
void psgeqpf (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , MKL_INT *ipiv , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );

void pdgeqpf (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , double *tau , double *work , MKL_INT *lwork , MKL_INT
*info );
```

```

void pcgeqpf (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , float *rwork , MKL_INT *lrwork , MKL_INT *info );

void pzgeqpf (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT
*lwork , double *rwork , MKL_INT *lrwork , MKL_INT *info );

```

Include Files

- mkl_scalapack.h

Description

The `p?geqpf` function forms the *QR* factorization with column pivoting of a general m -by- n distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$ as

$\text{sub}(A)*P=Q*R$.

Input Parameters

| | |
|---------------|--|
| <i>m</i> | (global) The number of rows in the matrix $\text{sub}(A)$ ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a*LOCc(ja+n-1)$. Contains the local pieces of the distributed matrix $\text{sub}(A)$ to be factored. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix $A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local). Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least <i>For real flavors:</i> $lwork \geq \max(3, mp0+nq0) + LOCc(ja+n-1) + nq0$. <i>For complex flavors:</i> $lwork \geq \max(3, mp0+nq0)$. |

Here

```

iroff = mod(ia-1, mb_a), icoff = mod(ja-1, nb_a),
iarow = indxg2p(ia, mb_a, MYROW, rsrc_a, NPROW),
iacol = indxg2p(ja, nb_a, MYCOL, csrc_a, NPCOL),
mp0 = numroc(m+iroff, mb_a, MYROW, iarow, NPROW),
nq0 = numroc(n+icoff, nb_a, MYCOL, iacol, NPCOL),

```

$LOCc(ja+n-1) = \text{numroc}(ja+n-1, nb_a, MYCOL, csrc_a, NPCOL)$, and numroc , indxg2p are ScaLAPACK tool functions.

You can determine $MYROW$, $MYCOL$, $NPROW$ and $NPCOL$ by calling the `blacs_gridinfo` function.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

$rwork$

(local).

Workspace array of size $lrwork$ (complex flavors only).

$lrwork$

(local or global) size of $rwork$ (complex flavors only). The value of $lrwork$ must be at least

$$lwork \geq LOCc(ja+n-1) + nq0.$$

Here

$$iroff = \text{mod}(ia-1, mb_a), \quad icoff = \text{mod}(ja-1, nb_a),$$

$$iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$$

$$iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL),$$

$$mp0 = \text{numroc}(m+iroff, mb_a, MYROW, iarow, NPROW),$$

$$nq0 = \text{numroc}(n+icoff, nb_a, MYCOL, iacol, NPCOL),$$

$LOCc(ja+n-1) = \text{numroc}(ja+n-1, nb_a, MYCOL, csrc_a, NPCOL)$, and numroc , indxg2p are ScaLAPACK tool functions.

You can determine $MYROW$, $MYCOL$, $NPROW$ and $NPCOL$ by calling the `blacs_gridinfo` function.

If $lrwork = -1$, then $lrwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

a

The elements on and above the diagonal of $\text{sub}(A)$ contain the $\min(m,n)$ -by- n upper trapezoidal matrix R (R is upper triangular if $m \geq n$); the elements below the diagonal, with the array τ , represent the orthogonal/unitary matrix Q as a product of elementary reflectors (see *Application Notes* below).

$ipiv$

(local) Array of size $LOCc(ja+n-1)$.

$ipiv[i] = k$, the local $(i+1)$ -th column of $\text{sub}(A)*P$ was the global k -th column of $\text{sub}(A)$ ($0 \leq i < LOCc(ja+n-1)$). $ipiv$ is tied to the distributed matrix A .

τ

(local)

Array of size $LOCc(ja+\min(m, n)-1)$.

| | |
|-----------------------|--|
| | Contains the scalar factor τ of elementary reflectors. τ is tied to the distributed matrix A . |
| <code>work[0]</code> | On exit, <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>rwork[0]</code> | On exit, <code>rwork[0]</code> contains the minimum value of <code>lrwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0, the execution is successful. < 0, if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

Application Notes

The matrix Q is represented as a product of elementary reflectors

$$Q = H(1)*H(2)*...*H(k)$$

where $k = \min(m,n)$.

Each $H(i)$ has the form

$$H = I - \tau*v*v'$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$; $v(i+1:m)$ is stored on exit in $A(ia+i:ia+m-1, ja+i-1)$.

The matrix P is represented in `ipiv` as follows: if `ipiv[j]= i` then the $(j+1)$ -th column of P is the i -th canonical unit vector ($0 \leq j < LOCC(ja+n-1)$).

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?orgqr

Generates the orthogonal matrix Q of the QR factorization formed by p?geqrf.

Syntax

```
void psorgqr (MKL_INT *m , MKL_INT *n , MKL_INT *k , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
void pdorgqr (MKL_INT *m , MKL_INT *n , MKL_INT *k , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?orgqr` function generates the whole or part of m -by- n real distributed matrix Q denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal columns, which is defined as the first n columns of a product of k elementary reflectors of order m

$$Q = H(1)*H(2)*...*H(k)$$

as returned by `p?geqrf`.

Input Parameters

| | |
|---------------|---|
| <i>m</i> | (global) The number of rows in the matrix $\text{sub}(Q)$ ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the matrix $\text{sub}(Q)$ ($m \geq n \geq 0$). |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q ($n \geq k \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja+n-1)$. The j -th column of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by <code>p?geqrf</code> in the k columns of its distributed matrix argument $A(ia:*, ja:ja+k-1)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix $A(ia:ia+m-1, ja:ja+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size $LOCC(ja+k-1)$. Contains the scalar factor $\tau[j]$ of elementary reflectors $H(j+1)$ as returned by <code>p?geqrf</code> ($0 \leq j < LOCC(ja+k-1)$). <i>tau</i> is tied to the distributed matrix A . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> . Must be at least $lwork \geq nb_a * (nqa0 + mpa0 + nb_a)$, where $irowfa = \text{mod}(ia-1, mb_a)$, $icoffa = \text{mod}(ja-1, nb_a)$, $irow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW)$, $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$, $mpa0 = \text{numroc}(m+irowfa, mb_a, MYROW, irow, NPROW)$, $nqa0 = \text{numroc}(n+icoffa, nb_a, MYCOL, iacol, NPCOL)$; indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the function <code>blacs_gridinfo</code> . If $lwork = -1$, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <code>pxerbla</code> . |

Output Parameters

| | |
|-----------------|--|
| <i>a</i> | Contains the local pieces of the <i>m</i> -by- <i>n</i> distributed matrix <i>Q</i> . |
| <i>work</i> [0] | On exit, [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | (global) = 0: the execution is successful. < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ungqr

Generates the complex unitary matrix Q of the QR factorization formed by p?geqrf.

Syntax

```
void pcungqr (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pzungqr (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

This function generates the whole or part of *m*-by-*n* complex distributed matrix *Q* denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal columns, which is defined as the first *n* columns of a product of *k* elementary reflectors of order *m*

$$Q = H(1)*H(2)*...*H(k)$$

as returned by [p?geqrf](#).

Input Parameters

| | |
|----------|---|
| <i>m</i> | (global) The number of rows in the matrix sub(<i>Q</i>); ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the matrix sub(<i>Q</i>) ($m \geq n \geq 0$). |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix <i>Q</i> ($n \geq k \geq 0$). |
| <i>a</i> | (local) |

Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. The j -th column of the matrix stored in a must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by `p?geqrf` in the k columns of its distributed matrix argument $A(ia:*, ja:ja+k-1)$.

ia, ja

(global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively.

desca

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix A .

tau

(local)

Array of size $LOCC(ja+k-1)$.

Contains the scalar factor $tau[j]$ of elementary reflectors $H(j+1)$ as returned by `p?geqrf` ($0 \leq j < LOCC(ja+k-1)$). tau is tied to the distributed matrix A .

work

(local)

Workspace array of size of $lwork$.

lwork

(local or global) size of $work$, must be at least $lwork \geq nb_a * (nqa0 + mpa0 + nb_a)$, where

$iroffa = \text{mod}(ia-1, mb_a)$,

$icoffa = \text{mod}(ja-1, nb_a)$,

$iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW)$,

$iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$,

$mpa0 = \text{numroc}(m+iroffa, mb_a, MYROW, iarow, NPROW)$,

$nqa0 = \text{numroc}(n+icoffa, nb_a, MYCOL, iacol, NPCOL)$

`indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

a

Contains the local pieces of the m -by- n distributed matrix Q .

work[0]

On exit $work[0]$ contains the minimum value of $lwork$ required for optimum performance.

info

(global)

= 0: the execution is successful.

< 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

See Also

Overview of ScaLAPACK Routines for details of ScaLAPACK array descriptor structures and related notations.

p?ormqr

Multiplies a general matrix by the orthogonal matrix Q of the QR factorization formed by p?geqrf.

Syntax

```
void psormqr (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );
void pdormqr (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?ormqr function overwrites the general real m -by- n distributed matrix sub(C) = C(ic:ic+m-1,jc:jc+n-1) with

| | | |
|---------------------|---------------------------|---------------------------|
| | <i>side</i> = 'L' | <i>side</i> = 'R' |
| <i>trans</i> = 'N': | $Q \cdot \text{sub}(C)$ | $\text{sub}(C) \cdot Q$ |
| <i>trans</i> = 'T': | $Q^T \cdot \text{sub}(C)$ | $\text{sub}(C) \cdot Q^T$ |

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$$Q = H(1) H(2) \dots H(k)$$

as returned by p?geqrf. Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

| | |
|--------------|--|
| <i>side</i> | (global) = 'L' : Q or Q^T is applied from the left. = 'R' : Q or Q^T is applied from the right. |
| <i>trans</i> | (global) = 'N', no transpose, Q is applied. = 'T', transpose, Q^T is applied. |
| <i>m</i> | (global) The number of rows in the distributed matrix sub(C) ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix sub(C) ($n \geq 0$). |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If <i>side</i> = 'L', $m \geq k \geq 0$ If <i>side</i> = 'R', $n \geq k \geq 0$. |

| | |
|---------------|--|
| <i>a</i> | <p>(local)</p> <p>Pointer into the local memory to an array of size $lld_a * LOCc(ja+n-1)$. The j-th column of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by <code>p?geqrf</code> in the k columns of its distributed matrix argument $A(ia:*, ja:ja+k-1)$. $A(ia:*, ja:ja+k-1)$ is modified by the function but restored on exit.</p> <p>If <code>side = 'L'</code>, $lld_a \geq \max(1, LOCr(ia+m-1))$</p> <p>If <code>side = 'R'</code>, $lld_a \geq \max(1, LOCr(ia+n-1))$</p> |
| <i>ia, ja</i> | <p>(global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A, respectively.</p> |
| <i>desca</i> | <p>(global and local) array of size <i>dlen_</i>. The array descriptor for the distributed matrix A.</p> |
| <i>tau</i> | <p>(local)</p> <p>Array of size $LOCc(ja+k-1)$.</p> <p>Contains the scalar factor $\tau[j]$ of elementary reflectors $H(j+1)$ as returned by <code>p?geqrf</code> ($0 \leq j < LOCc(ja+k-1)$). <i>tau</i> is tied to the distributed matrix A.</p> |
| <i>c</i> | <p>(local)</p> <p>Pointer into the local memory to an array of local size $lld_c * LOCc(jc+n-1)$.</p> <p>Contains the local pieces of the distributed matrix sub(C) to be factored.</p> |
| <i>ic, jc</i> | <p>(global) The row and column indices in the global matrix C indicating the first row and the first column of the matrix sub(C), respectively.</p> |
| <i>desc</i> | <p>(global and local) array of size <i>dlen_</i>. The array descriptor for the distributed matrix C.</p> |
| <i>work</i> | <p>(local)</p> <p>Workspace array of size of <i>lwork</i>.</p> |
| <i>lwork</i> | <p>(local or global) size of <i>work</i>, must be at least:</p> <p>if <code>side = 'L'</code>,</p> $lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + mpc0) * nb_a) + nb_a * nb_a$ <p>else if <code>side = 'R'</code>,</p> $lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + \max(npa0 + \text{numroc}(\text{numroc}(n + icoffc, nb_a, 0, 0, NPCOL), nb_a, 0, 0, lcmq), mpc0)) * nb_a) + nb_a * nb_a$ <p>end if</p> <p>where</p> $lcmq = lcm / NPCOL \text{ with } lcm = ilcm(NPROW, NPCOL),$ $iroffa = \text{mod}(ia - 1, mb_a),$ $icoffa = \text{mod}(ja - 1, nb_a),$ |

```

iarow = indxg2p(ia, mb_a, MYROW, rsrc_a, NPROW),
npa0= numroc(n+iroffa, mb_a, MYROW, iarow, NPROW),
iroffc = mod(ic-1, mb_c),
icoffc = mod(jc-1, nb_c),
icrow = indxg2p(ic, mb_c, MYROW, rsrc_c, NPROW),
iccol = indxg2p(jc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0= numroc(m+iroffc, mb_c, MYROW, icrow, NPROW),
nqc0= numroc(n+icoffc, nb_c, MYCOL, iccol, NPCOL),

ilcm, indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL,
NPROW and NPCOL can be determined by calling the function
blacs_gridinfo.

```

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q \cdot \text{sub}(C)$, or $Q^T \cdot \text{sub}(C)$, or $\text{sub}(C) \cdot Q^T$, or $\text{sub}(C) \cdot Q$. |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of $lwork$ required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i \cdot 100 + j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unmqr

Multiplies a complex matrix by the unitary matrix Q of the QR factorization formed by `p?geqrf`.

Syntax

```

void pcunmqr (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );

```

```

void pzunmqr (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );

```

Include Files

- `mkl_scalapack.h`

Description

This function overwrites the general complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|---------------------|---------------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q*\text{sub}(C)$ | $\text{sub}(C)*Q$ |
| $trans = 'T':$ | $Q^H*\text{sub}(C)$ | $\text{sub}(C)*Q^H$ |

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(1) H(2) \dots H(k)$ as returned by `p?geqrf`. Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|----------|--|
| $side$ | (global) $= 'L'$: Q or Q^H is applied from the left. $= 'R'$: Q or Q^H is applied from the right. |
| $trans$ | (global) $= 'N'$, no transpose, Q is applied. $= 'C'$, conjugate transpose, Q^H is applied. |
| m | (global) The number of rows in the distributed matrix $\text{sub}(C)$ ($m \geq 0$). |
| n | (global) The number of columns in the distributed matrix $\text{sub}(C)$ ($n \geq 0$). |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If $side = 'L'$, $m \geq k \geq 0$ If $side = 'R'$, $n \geq k \geq 0$. |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCr(ja+k-1)$. The j -th column of the matrix stored in a must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by <code>p?geqrf</code> in the k columns of its distributed matrix argument $A(ia:*, ja:ja+k-1)$. $A(ia:*, ja:ja+k-1)$ is modified by the function but restored on exit. If $side = 'L'$, $lld_a \geq \max(1, LOCr(ia+m-1))$ If $side = 'R'$, $lld_a \geq \max(1, LOCr(ia+n-1))$ |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| tau | (local) |

Array of size $LOCc(ja+k-1)$.

Contains the scalar factor $\tau[j]$ of elementary reflectors $H(j+1)$ as returned by `p?geqrf` ($0 \leq j < LOCc(ja+k-1)$). τ is tied to the distributed matrix A .

c

(local)

Pointer into the local memory to an array of local size $lld_c * LOCc(jc+n-1)$. Contains the local pieces of the distributed matrix sub(C) to be factored.

ic, jc

(global) The row and column indices in the global matrix C indicating the first row and the first column of the submatrix C , respectively.

desc

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix C .

work

(local)

Workspace array of size of *lwork*.

lwork

(local or global) size of *work*, must be at least:

If *side* = 'L',

$$lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + mpc0) * nb_a) + nb_a * nb_a$$

else if *side* = 'R',

$$lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + \max(npa0 + \text{numroc}(\text{numroc}(n + icoffc, nb_a, 0, 0, NPCOL), nb_a, 0, 0, lcmq), mpc0)) * nb_a) + nb_a * nb_a$$

end if

where

$$lcmq = lcm / NPCOL \text{ with } lcm = ilcm(NPROW, NPCOL),$$

$$iroffa = \text{mod}(ia - 1, mb_a),$$

$$icoffa = \text{mod}(ja - 1, nb_a),$$

$$iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$$

$$npa0 = \text{numroc}(n + iroffa, mb_a, MYROW, iarow, NPROW),$$

$$iroffc = \text{mod}(ic - 1, mb_c),$$

$$icoffc = \text{mod}(jc - 1, nb_c),$$

$$icrow = \text{indxg2p}(ic, mb_c, MYROW, rsrc_c, NPROW),$$

$$iccol = \text{indxg2p}(jc, nb_c, MYCOL, csrc_c, NPCOL),$$

$$mpc0 = \text{numroc}(m + iroffc, mb_c, MYROW, icrow, NPROW),$$

$$nqc0 = \text{numroc}(n + icoffc, nb_c, MYCOL, iccol, NPCOL),$$

ilcm, *indxg2p* and *numroc* are ScaLAPACK tool functions; *MYROW*, *MYCOL*, *NPROW* and *NPCOL* can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q*\text{sub}(C)$, or $Q^H*\text{sub}(C)$, or $\text{sub}(C)*Q^H$, or $\text{sub}(C)*Q$. |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of $lwork$ required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gelqf

Computes the LQ factorization of a general rectangular matrix.

Syntax

```
void psgelqf (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
void pdgelqf (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
void pcgelqf (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
void pzgelqf (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?gelqf` function computes the LQ factorization of a real/complex distributed m -by- n matrix $\text{sub}(A) = A(ia:ia+m-1,ja:ja+n-1) = L*Q$.

Input Parameters

`m` (global) The number of rows in the distributed submatrix $\text{sub}(A)$ ($m \geq 0$).

| | |
|---------------|---|
| <i>n</i> | (global) The number of columns in the distributed submatrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja+n-1)$. Contains the local pieces of the distributed matrix $\text{sub}(A)$ to be factored. |
| <i>ia, ja</i> | (global) The row and column indices in the global array <i>A</i> indicating the first row and the first column of the submatrix $A(ia:ia+m-1, ja:ja+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least $lwork \geq mb_a * (mp0 + nq0 + mb_a)$, where $iroff = \text{mod}(ia-1, mb_a),$ $icoff = \text{mod}(ja-1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$ $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL),$ $mp0 = \text{numroc}(m+iroff, mb_a, MYROW, iarow, NPROW),$ $nq0 = \text{numroc}(n+icoff, nb_a, MYCOL, iacol, NPCOL)$ indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the function <code>blacs_gridinfo</code> . |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

If $lwork = -1$, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|------------|--|
| <i>a</i> | The elements on and below the diagonal of $\text{sub}(A)$ contain the m -by- $\min(m, n)$ lower trapezoidal matrix <i>L</i> (<i>L</i> is lower trapezoidal if $m \leq n$); the elements above the diagonal, with the array <i>tau</i> , represent the orthogonal/unitary matrix <i>Q</i> as a product of elementary reflectors (see <i>Application Notes</i> below). |
| <i>tau</i> | (local) Array of size $LOCr(ia+\min(m, n)-1)$. |

Contains the scalar factors of elementary reflectors. *tau* is tied to the distributed matrix *A*.

work[0]

On exit, *work*[0] contains the minimum value of *lwork* required for optimum performance.

info

(global)

= 0: the execution is successful.

< 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j* - 1, had an illegal value, then *info* = $-(i*100+j)$; if the *i*-th argument is a scalar and had an illegal value, then *info* = $-i$.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

$$Q = H(ia+k-1)*H(ia+k-2)*...*H(ia),$$

where $k = \min(m, n)$

Each *H*(*i*) has the form

$$H(i) = I - \tau * v * v'$$

where *tau* is a real/complex scalar, and *v* is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$; $v(i+1:n)$ is stored on exit in $A(ia+i-1, ja+i:ja+n-1)$, and *tau* in $\tau[ia+i-2]$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?orglq

Generates the real orthogonal matrix Q of the LQ factorization formed by p?gelqf.

Syntax

```
void psorglq (MKL_INT *m , MKL_INT *n , MKL_INT *k , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdorglq (MKL_INT *m , MKL_INT *n , MKL_INT *k , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?orglq function generates the whole or part of *m*-by-*n* real distributed matrix *Q* denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal rows, which is defined as the first *m* rows of a product of *k* elementary reflectors of order *n*

$$Q = H(k)*...*H(2)*H(1)$$

as returned by p?gelqf.

Input Parameters

m

(global) The number of rows in the matrix sub(*Q*); ($m \geq 0$).

| | |
|---------------|---|
| <i>n</i> | (global) The number of columns in the matrix sub(<i>Q</i>) ($n \geq m \geq 0$). |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix <i>Q</i> ($m \geq k \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja+n-1)$. On entry, the <i>i</i> -th row of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by p?gelqf in the <i>k</i> rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix $A(ia:ia+m-1, ja:ja+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least $lwork \geq mb_a * (mpa0 + nqa0 + mb_a)$, where $iroffa = \text{mod}(ia-1, mb_a),$ $icoffa = \text{mod}(ja-1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$ $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL),$ $mpa0 = \text{numroc}(m + iroffa, mb_a, MYROW, iarow, NPROW),$ $nqa0 = \text{numroc}(n + icoffa, nb_a, MYCOL, iacol, NPCOL)$ |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by [pxerbla](#).

Output Parameters

| | |
|------------|--|
| <i>a</i> | Contains the local pieces of the <i>m</i> -by- <i>n</i> distributed matrix <i>Q</i> to be factored. |
| <i>tau</i> | (local) Array of size $LOCr(ia+k-1)$. Contains the scalar factors $\tau[j]$ of elementary reflectors $H(j+1)$, $0 \leq j < LOCr(ia+k-1)$. <i>tau</i> is tied to the distributed matrix <i>A</i> . |

`work[0]` On exit, `work[0]` contains the minimum value of `lwork` required for optimum performance.

`info` (global)

= 0: the execution is successful.

< 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then `info = -(i*100+j)`; if the i -th argument is a scalar and had an illegal value, then `info = -i`.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unglq

Generates the unitary matrix Q of the LQ factorization formed by `p?gelqf`.

Syntax

```
void pcunglq (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pzunglq (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

This function generates the whole or part of m -by- n complex distributed matrix Q denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal rows, which is defined as the first m rows of a product of k elementary reflectors of order n

$Q = (H(k))^H \dots (H(2))^H (H(1))^H$ as returned by `p?gelqf`.

Input Parameters

m (global) The number of rows in the matrix `sub(Q)` ($m \geq 0$).

n (global) The number of columns in the matrix `sub(Q)` ($n \geq m \geq 0$).

k (global) The number of elementary reflectors whose product defines the matrix `Q` ($m \geq k \geq 0$).

a (local)

Pointer into the local memory to an array of local size `ld_a*LOCc(ja+n-1)`. On entry, the i -th row of the matrix stored in `a` must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by `p?gelqf` in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$.

| | |
|---------------|---|
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix $A(ia:ia+m-1, ja:ja+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array of size $LOCr(ia+k-1)$. Contains the scalar factors $\tau[j]$ of elementary reflectors $H(j+1)$, $0 \leq j < LOCr(ia+k-1)$. τ is tied to the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least $lwork \geq mb_a * (mpa0 + nqa0 + mb_a)$, where $iroffa = \text{mod}(ia-1, mb_a),$ $icoffa = \text{mod}(ja-1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$ $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL),$ $mpa0 = \text{numroc}(m + iroffa, mb_a, MYROW, iarow, NPROW),$ $nqa0 = \text{numroc}(n + icoffa, nb_a, MYCOL, iacol, NPCOL)$ indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the function <code>blacs_gridinfo</code> . |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

If $lwork = -1$, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|-----------------|---|
| <i>a</i> | Contains the local pieces of the m -by- n distributed matrix <i>Q</i> to be factored. |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | (global) = 0: the execution is successful. < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the <i>i</i> -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

Overview of ScaLAPACK Routines for details of ScaLAPACK array descriptor structures and related notations.

p?ormlq

Multiplies a general matrix by the orthogonal matrix Q of the LQ factorization formed by p?gelqf.

Syntax

```
void psormlq (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );
void pdormlq (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?ormlq function overwrites the general real m -by- n distributed matrix $\text{sub}(C) = C(\text{ic}:\text{ic}+m-1, \text{jc}:\text{jc}+n-1)$ with

| | | |
|---------------------|---------------------------|---------------------------|
| | <i>side</i> = 'L' | <i>side</i> = 'R' |
| <i>trans</i> = 'N': | $Q \cdot \text{sub}(C)$ | $\text{sub}(C) \cdot Q$ |
| <i>trans</i> = 'T': | $Q^T \cdot \text{sub}(C)$ | $\text{sub}(C) \cdot Q^T$ |

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$$Q = H(k) \dots H(2) H(1)$$

as returned by p?gelqf. Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

| | |
|--------------|--|
| <i>side</i> | (global) = 'L': Q or Q^T is applied from the left. = 'R': Q or Q^T is applied from the right. |
| <i>trans</i> | (global) = 'N', no transpose, Q is applied. = 'T', transpose, Q^T is applied. |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(C)$ ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(C)$ ($n \geq 0$). |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If <i>side</i> = 'L', $m \geq k \geq 0$ If <i>side</i> = 'R', $n \geq k \geq 0$. |

| | |
|---------------|---|
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a*LOCc(ja+m-1)$, if $side = 'L'$ and $lld_a*LOCc(ja+n-1)$, if $side = 'R'$. The i -th row of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by <code>p?gelqf</code> in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the function but restored on exit. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size $LOCc(ja+k-1)$. Contains the scalar factor $tau[j]$ of elementary reflectors $H(j+1)$ as returned by <code>p?gelqf</code> ($0 \leq j < LOCc(ja+k-1)$). <i>tau</i> is tied to the distributed matrix A . |
| <i>c</i> | (local) Pointer into the local memory to an array of local size $lld_c*LOCc(jc+n-1)$. Contains the local pieces of the distributed matrix $sub(C)$ to be factored. |
| <i>ic, jc</i> | (global) The row and column indices in the global matrix C indicating the first row and the first column of the submatrix C , respectively. |
| <i>desc</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix C . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of the array <i>work</i> ; must be at least: If $side = 'L'$, $lwork \geq \max((mb_a*(mb_a-1))/2, (mpc0+\max(mqa0) + \text{numroc}(\text{numroc}(m + iroffa, mb_a, 0, 0, NPROW), mb_a, 0, 0, lcm), nqc0)) * mb_a) + mb_a*mb_a$ else if $side = 'R'$, $lwork \geq \max((mb_a*(mb_a-1))/2, (mpc0+nqc0)*mb_a + mb_a*mb_a$ end if where $lcm = lcm/NPROW$ with $lcm = ilcm(NPROW, NPCOL)$, $iroffa = \text{mod}(ia-1, mb_a)$, $icoffa = \text{mod}(ja-1, nb_a)$, $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$, $mqa0 = \text{numroc}(m+icoffa, nb_a, MYCOL, iacol, NPCOL)$, |

```

iroffc = mod(ic-1, mb_c),
icoffc = mod(jc-1, nb_c),
icrow = indxg2p(ic, mb_c, MYROW, rsrc_c, NPROW),
iccol = indxg2p(jc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0 = numroc(m+iroffc, mb_c, MYROW, icrow, NPROW),
nqc0 = numroc(n+icoffc, nb_c, MYCOL, iccol, NPCOL),

```

NOTE

`mod(x, y)` is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q \cdot \text{sub}(C)$, or $Q' \cdot \text{sub}(C)$, or $\text{sub}(C) \cdot Q'$, or $\text{sub}(C) \cdot Q$ |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unmlq

Multiplies a general matrix by the unitary matrix Q of the LQ factorization formed by `p?gelqf`.

Syntax

```

void pcunmlq (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );

```

```
void pzunmlq (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *desc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

This function overwrites the general complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|-----------------------|-----------------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| $trans = 'T':$ | $Q^H * \text{sub}(C)$ | $\text{sub}(C) * Q^H$ |

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$$Q = H(k)' \dots H(2)' H(1)'$$

as returned by [p?gelqf](#). Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|---------------|--|
| <i>side</i> | (global) $= 'L'$: Q or Q^H is applied from the left. $= 'R'$: Q or Q^H is applied from the right. |
| <i>trans</i> | (global) $= 'N'$, no transpose, Q is applied. $= 'C'$, conjugate transpose, Q^H is applied. |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(C)$ ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(C)$ ($n \geq 0$). |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If $side = 'L'$, $m \geq k \geq 0$ If $side = 'R'$, $n \geq k \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+m-1)$, if $side = 'L'$ and $lld_a * LOCC(ja+n-1)$, if $side = 'R'$, where $lld_a \geq \max(1, LOCr(ia+k-1))$. The i -th column of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by p?gelqf in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the function but restored on exit. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |

| | |
|---------------|--|
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array of size $LOCc(ia+k-1)$. Contains the scalar factor $\tau[j]$ of elementary reflectors $H(j+1)$ as returned by <code>p?gelqf</code> ($0 \leq j < LOCc(ia+k-1)$). <i>tau</i> is tied to the distributed matrix <i>A</i> . |
| <i>c</i> | (local) Pointer into the local memory to an array of local size $lld_c * LOCc(jc+n-1)$. Contains the local pieces of the distributed matrix sub(<i>C</i>) to be factored. |
| <i>ic, jc</i> | (global) The row and column indices in the global matrix <i>C</i> indicating the first row and the first column of the submatrix <i>C</i> , respectively. |
| <i>desc</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>C</i> . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of the array <i>work</i> ; must be at least: If <i>side</i> = 'L', $lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + \max(mqa0) + \text{numroc}(\text{numroc}(m + iroffc, mb_a, 0, 0, NPROW), mb_a, 0, 0, lcm), nqc0)) * mb_a) + mb_a * mb_a$ else if <i>side</i> = 'R', $lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + nqc0) * mb_a + mb_a * mb_a$ end if where $lcmp = lcm / NPROW$ with $lcm = ilcm(NPROW, NPCOL)$, $iroffa = \text{mod}(ia-1, mb_a)$, $icoffa = \text{mod}(ja-1, nb_a)$, $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$, $mqa0 = \text{numroc}(m + icoffa, nb_a, MYCOL, iacol, NPCOL)$, $iroffc = \text{mod}(ic-1, mb_c)$, $icoffc = \text{mod}(jc-1, nb_c)$, $icrow = \text{indxg2p}(ic, mb_c, MYROW, rsrc_c, NPROW)$, $iccol = \text{indxg2p}(jc, nb_c, MYCOL, csrc_c, NPCOL)$, $mpc0 = \text{numroc}(m + iroffc, mb_c, MYROW, icrow, NPROW)$, $nqc0 = \text{numroc}(n + icoffc, nb_c, MYCOL, iccol, NPCOL)$, |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `p?xerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q \cdot \text{sub}(C)$, or $Q' \cdot \text{sub}(C)$, or $\text{sub}(C) \cdot Q'$, or $\text{sub}(C) \cdot Q$ |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gqqlf

Computes the QL factorization of a general matrix.

Syntax

```
void psgeqlf (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdgeqlf (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcgeqlf (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzgeqlf (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?geqlf` function forms the QL factorization of a real/complex distributed m -by- n matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1) = Q * L$.

Input Parameters

| | |
|---------------|--|
| <i>m</i> | (global) The number of rows in the matrix $\text{sub}(Q)$; ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the matrix $\text{sub}(Q)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $\text{ld}_a * \text{LOCc}(\text{ja}+n-1)$. Contains the local pieces of the distributed matrix $\text{sub}(A)$ to be factored. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix $A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least $\text{lwork} \geq \text{nb}_a * (\text{mp0} + \text{nq0} + \text{nb}_a)$, where $\text{iroff} = \text{mod}(\text{ia}-1, \text{mb}_a),$ $\text{icoff} = \text{mod}(\text{ja}-1, \text{nb}_a),$ $\text{iarow} = \text{indxg2p}(\text{ia}, \text{mb}_a, \text{MYROW}, \text{rsrc}_a, \text{NPROW}),$ $\text{iacol} = \text{indxg2p}(\text{ja}, \text{nb}_a, \text{MYCOL}, \text{csrc}_a, \text{NPCOL}),$ $\text{mp0} = \text{numroc}(\text{m}+\text{iroff}, \text{mb}_a, \text{MYROW}, \text{iarow}, \text{NPROW}),$ $\text{nq0} = \text{numroc}(\text{n}+\text{icoff}, \text{nb}_a, \text{MYCOL}, \text{iacol}, \text{NPCOL})$ |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`numroc` and `indxg2p` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If $\text{lwork} = -1$, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, if $m \geq n$, the lower triangle of the distributed submatrix $A(\text{ia}+m-n:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$ contains the n -by- n lower triangular matrix L ; if $m \leq n$, the elements on and below the $(n - m)$ -th superdiagonal contain the m -by- n |
|----------|---|

lower trapezoidal matrix L ; the remaining elements, with the array τ , represent the orthogonal/unitary matrix Q as a product of elementary reflectors (see *Application Notes* below).

| | |
|-----------|--|
| τ | (local) Array of size $LOC_c(ja+n-1)$. Contains the scalar factors of elementary reflectors. τ is tied to the distributed matrix A . |
| $work[0]$ | On exit, $work[0]$ contains the minimum value of $lwork$ required for optimum performance. |
| $info$ | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

Application Notes

The matrix Q is represented as a product of elementary reflectors

$$Q = H(ja+k-1)*\dots*H(ja+1)*H(ja)$$

where $k = \min(m, n)$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v'$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(m-k+i+1:m) = 0$ and $v(m-k+i) = 1$; $v(1:m-k+i-1)$ is stored on exit in $A(ia:ia+m-k+i-2, ja+n-k+i-1)$, and τ in $\tau[ja+n-k+i-2]$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?orgql

Generates the orthogonal matrix Q of the QL factorization formed by p?geqlf.

Syntax

```
void psorgql (MKL_INT *m , MKL_INT *n , MKL_INT *k , float *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
void pdorgql (MKL_INT *m , MKL_INT *n , MKL_INT *k , double *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?orgql` function generates the whole or part of m -by- n real distributed matrix Q denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal rows, which is defined as the first m rows of a product of k elementary reflectors of order n

$Q = H(k)*...*H(2)*H(1)$

as returned by `p?geqlf`.

Input Parameters

| | |
|---------------|---|
| <i>m</i> | (global) The number of rows in the matrix $\text{sub}(Q)$, ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the matrix $\text{sub}(Q)$, ($m \geq n \geq 0$). |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q ($n \geq k \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $ld_a * LOCC(ja+n-1)$. On entry, the j -th column of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(j)$, $ja+n-k \leq j \leq ja+n-1$, as returned by <code>p?geqlf</code> in the k columns of its distributed matrix argument $A(ia:*, ja+n-k:ja+n-1)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix $A(ia:ia+m-1, ja:ja+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size $LOCc(ja+n-1)$. Contains the scalar factors $\tau[j]$ of elementary reflectors $H(j+1)$, $0 \leq j < LOCc(ia+n-1)$. <i>tau</i> is tied to the distributed matrix A . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least $lwork \geq nb_a * (nqa0 + mpa0 + nb_a)$, where $iroffa = \text{mod}(ia-1, mb_a),$ $icoffa = \text{mod}(ja-1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$ $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL),$ $mpa0 = \text{numroc}(m + iroffa, mb_a, MYROW, iarow, NPROW),$ $nqa0 = \text{numroc}(n + icoffa, nb_a, MYCOL, iacol, NPCOL)$ |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>a</code> | Contains the local pieces of the m -by- n distributed matrix Q to be factored. |
| <code>work[0]</code> | On exit, <code>work[0]</code> contains the minimum value of $lwork$ required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ungql

Generates the unitary matrix Q of the QL factorization formed by `p?geqlf`.

Syntax

```
void pcungql (const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , MKL_Complex8
*a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8
*tau , MKL_Complex8 *work , const MKL_INT *lwork , MKL_INT *info );

void pzungql (const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , MKL_Complex16
*a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16
*tau , MKL_Complex16 *work , const MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

This function generates the whole or part of m -by- n complex distributed matrix Q denoting $A(ia:ia+m-1,ja:ja+n-1)$ with orthonormal rows, which is defined as the first n columns of a product of k elementary reflectors of order m

$Q = (H(k))^H \dots (H(2))^H (H(1))^H$ as returned by `p?geqlf`.

Input Parameters

| | |
|----------------|--|
| <code>m</code> | (global) The number of rows in the matrix sub(Q) ($m \geq 0$). |
| <code>n</code> | (global) The number of columns in the matrix sub(Q) ($m \geq n \geq 0$). |

| | |
|---------------|--|
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q ($n \geq k \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja + n - 1)$. On entry, the j -th column of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(j)$, $ja + n - k \leq j \leq ja + n - 1$, as returned by <code>p?geqlf</code> in the k columns of its distributed matrix argument $A(ia:*, ja+n-k: ja+n-1)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix $A(ia:ia+m-1, ja:ja+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size $LOCr(ia+n-1)$. Contains the scalar factors $tau[j]$ of elementary reflectors $H(j+1)$, $0 \leq j < LOCr(ia+n-1)$. <i>tau</i> is tied to the distributed matrix A . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least $lwork \geq nb_a * (nqa0 + mpa0 + nb_a)$, where $iroffa = \text{mod}(ia-1, mb_a),$ $icoffa = \text{mod}(ja-1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$ $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL),$ $mpa0 = \text{numroc}(m+iroffa, mb_a, MYROW, iarow, NPROW),$ $nqa0 = \text{numroc}(n+icoffa, nb_a, MYCOL, iacol, NPCOL)$ <code>indxg2p</code> and <code>numroc</code> are ScaLAPACK tool functions; <code>MYROW</code> , <code>MYCOL</code> , <code>NPROW</code> and <code>NPCOL</code> can be determined by calling the function <code>blacs_gridinfo</code> . If $lwork = -1$, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <code>p?xerbla</code> . |

Output Parameters

| | |
|-----------------|---|
| <i>a</i> | Contains the local pieces of the m -by- n distributed matrix Q to be factored. |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>info</i> | (global) = 0: the execution is successful. |

< 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

See Also

Overview of ScaLAPACK Routines for details of ScaLAPACK array descriptor structures and related notations.

p?ormql

Multiplies a general matrix by the orthogonal matrix Q of the QL factorization formed by p?geqlf.

Syntax

```
void psormql (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );
void pdormql (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?ormql function overwrites the general real m -by- n distributed matrix $sub(C) = C(ic:ic+m-1,jc:jc+n-1)$ with

| | | |
|----------------|--------------|--------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q*sub(C)$ | $sub(C)*Q$ |
| $trans = 'T':$ | $Q^T*sub(C)$ | $sub(C)*Q^T$ |

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$Q = H(k)' \dots H(2)' H(1)'$

as returned by p?geqlf. Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|---------|---|
| $side$ | (global) = $'L'$: Q or Q^T is applied from the left. = $'R'$: Q or Q^T is applied from the right. |
| $trans$ | (global) = $'N'$, no transpose, Q is applied. = $'T'$, transpose, Q^T is applied. |
| m | (global) The number of rows in the distributed matrix $sub(C)$, ($m \geq 0$). |
| n | (global) The number of columns in the distributed matrix $sub(C)$, ($n \geq 0$). |

| | |
|---------------|--|
| <i>k</i> | <p>(global) The number of elementary reflectors whose product defines the matrix <i>Q</i>. Constraints:</p> <p>If <i>side</i> = 'L', $m \geq k \geq 0$</p> <p>If <i>side</i> = 'R', $n \geq k \geq 0$.</p> |
| <i>a</i> | <p>(local)</p> <p>Pointer into the local memory to an array of size $lld_a * LOCC(ja+k-1)$. The <i>j</i>-th column of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by <code>p?ge1qf</code> in the <i>k</i> columns of its distributed matrix argument $A(ia:*, ja:ja+k-1)$. $A(ia:*, ja:ja+k-1)$ is modified by the function but restored on exit.</p> <p>If <i>side</i> = 'L', $lld_a \geq \max(1, LOCr(ia+m-1))$,</p> <p>If <i>side</i> = 'R', $lld_a \geq \max(1, LOCr(ia+n-1))$.</p> |
| <i>ia, ja</i> | <p>(global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i>, respectively.</p> |
| <i>desca</i> | <p>(global and local) array of size <i>dlen_</i>. The array descriptor for the distributed matrix <i>A</i>.</p> |
| <i>tau</i> | <p>(local)</p> <p>Array of size $LOCC(ja+n-1)$.</p> <p>Contains the scalar factor $\tau[j]$ of elementary reflectors $H(j+1)$ as returned by <code>p?geqlf</code> ($0 \leq j < LOCC(ja+k-1)$). <i>tau</i> is tied to the distributed matrix <i>A</i>.</p> |
| <i>c</i> | <p>(local)</p> <p>Pointer into the local memory to an array of local size $lld_c * LOCC(jc+n-1)$.</p> <p>Contains the local pieces of the distributed matrix sub(<i>C</i>) to be factored.</p> |
| <i>ic, jc</i> | <p>(global) The row and column indices in the global matrix <i>C</i> indicating the first row and the first column of the submatrix <i>C</i>, respectively.</p> |
| <i>descc</i> | <p>(global and local) array of size <i>dlen_</i>. The array descriptor for the distributed matrix <i>C</i>.</p> |
| <i>work</i> | <p>(local)</p> <p>Workspace array of size of <i>lwork</i>.</p> |
| <i>lwork</i> | <p>(local or global) dimension of <i>work</i>, must be at least:</p> <p>If <i>side</i> = 'L',</p> $lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + mpc0) * nb_a + nb_a * nb_a)$ <p>else if <i>side</i> = 'R',</p> $lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + \max(npa0 + \text{numroc}(\text{numroc}(n + icoffc, nb_a, 0, 0, NPCOL), nb_a, 0, 0, lcmq), mpc0)) * nb_a + nb_a * nb_a)$ <p>end if</p> <p>where</p> |

```

lcmq = lcm/NPCOL with lcm = ilcm (NPROW, NPCOL),
iroffa = mod(ia-1, mb_a),
icoffa = mod(ja-1, nb_a),
iarow = indxg2p(ia, mb_a, MYROW, rsrc_a, NPROW),
npa0 = numroc(n + iroffa, mb_a, MYROW, iarow, NPROW),
iroffc = mod(ic-1, mb_c),
icoffc = mod(jc-1, nb_c),
icrow = indxg2p(ic, mb_c, MYROW, rsrc_c, NPROW),
iccol = indxg2p(jc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0 = numroc(m+iroffc, mb_c, MYROW, icrow, NPROW),
nqc0 = numroc(n+icoffc, nb_c, MYCOL, iccol, NPCOL),

```

NOTE

`mod(x,y)` is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `perbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q^* \text{sub}(C)$, or $Q^* \text{sub}(C)$, or $\text{sub}(C)^* Q'$, or $\text{sub}(C)^* Q$ |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $\text{info} = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $\text{info} = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unmql

Multiplies a general matrix by the unitary matrix Q of the QL factorization formed by p?geqlf.

Syntax

```
void pcunmql (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pzunmql (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

This function overwrites the general complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|-----------------------|-----------------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| $trans = 'C':$ | $Q^H * \text{sub}(C)$ | $\text{sub}(C) * Q^H$ |

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$$Q = H(k)' \dots H(2)' H(1)'$$

as returned by `p?geqlf`. Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|---------|--|
| $side$ | (global) = $'L'$: Q or Q^H is applied from the left. = $'R'$: Q or Q^H is applied from the right. |
| $trans$ | (global) = $'N'$, no transpose, Q is applied. = $'C'$, conjugate transpose, Q^H is applied. |
| m | (global) The number of rows in the distributed matrix $\text{sub}(C)$ ($m \geq 0$). |
| n | (global) The number of columns in the distributed matrix $\text{sub}(C)$ ($n \geq 0$). |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If $side = 'L'$, $m \geq k \geq 0$ If $side = 'R'$, $n \geq k \geq 0$. |
| a | (local) |

Pointer into the local memory to an array of size $lld_a * LOCC(ja+k-1)$. The j -th column of the matrix stored in a must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by `p?geqlf` in the k columns of its distributed matrix argument $A(ia:*, ja:ja+k-1)$. $A(ia:*, ja:ja+k-1)$ is modified by the function but restored on exit.

If $side = 'L'$, $lld_a \geq \max(1, LOCr(ia+m-1))$,

If $side = 'R'$, $lld_a \geq \max(1, LOCr(ia+n-1))$.

ia, ja

(global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively.

$desca$

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix A .

tau

(local)

Array of size $LOCC(ia+n-1)$.

Contains the scalar factor $tau[j]$ of elementary reflectors $H(j+1)$ as returned by `p?geqlf` ($0 \leq j < LOCC(ia+n-1)$). tau is tied to the distributed matrix A .

c

(local)

Pointer into the local memory to an array of local size $lld_c * LOCC(jc+n-1)$.

Contains the local pieces of the distributed matrix $sub(C)$ to be factored.

ic, jc

(global) The row and column indices in the global matrix C indicating the first row and the first column of the submatrix C , respectively.

$descC$

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix C .

$work$

(local)

Workspace array of size of $lwork$.

$lwork$

(local or global) size of $work$, must be at least:

If $side = 'L'$,

$lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + mpc0) * nb_a + nb_a * nb_a)$

else if $side = 'R'$,

$lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + \max(npa0) + \text{numroc}(\text{numroc}(n + icoffc, nb_a, 0, 0, NPCOL), nb_a, 0, 0, lcmq), mpc0)) * nb_a + nb_a * nb_a)$

end if

where

$lcmp = lcm / NPCOL$ with $lcm = \text{ilcm}(NPROW, NPCOL)$,

$iroffa = \text{mod}(ia - 1, mb_a)$,

$icoffa = \text{mod}(ja - 1, nb_a)$,

$iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW)$,

```

npa0 = numroc ( n + iroffa, mb_a, MYROW, iarow, NPROW ),
iroffc = mod(ic-1, mb_c),
icoffc = mod(jc-1, nb_c),
icrow = indxg2p(ic, mb_c, MYROW, rsrc_c, NPROW),
iccol = indxg2p(jc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0 = numroc(m+iroffc, mb_c, MYROW, icrow, NPROW),
nqc0 = numroc(n+icoffc, nb_c, MYCOL, iccol, NPCOL),

```

NOTE

`mod(x, y)` is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

NOTE

`mod(x, y)` is the integer remainder of x/y .

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q^* \text{sub}(C)$, or $Q' \text{sub}(C)$, or $\text{sub}(C)^* Q'$, or $\text{sub}(C)^* Q$ |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gerqf

Computes the RQ factorization of a general rectangular matrix.

Syntax

```

void psggerqf (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );

```

```

void pdgerqf (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );

void pcgerqf (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT
*info );

void pzgerqf (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT
*info );

```

Include Files

- mkl_scalapack.h

Description

The `p?gerqf` function forms the QR factorization of a general m -by- n distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia} + m - 1, \text{ja}:\text{ja} + n - 1)$ as

$$A = R * Q$$

Input Parameters

| | |
|---------------|--|
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$; ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$; ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja + n - 1)$. Contains the local pieces of the distributed matrix $\text{sub}(A)$ to be factored. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix $A(\text{ia}:\text{ia} + m - 1, \text{ja}:\text{ja} + n - 1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A |
| <i>work</i> | (local). Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least $lwork \geq mb_a * (mp0 + nq0 + mb_a)$, where $irow = \text{mod}(ia - 1, mb_a)$, $icol = \text{mod}(ja - 1, nb_a)$, $irow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW)$, $icol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$, $mp0 = \text{numroc}(m + irow, mb_a, MYROW, irow, NPROW)$, |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`ng0 = numroc(n+icoff, nb_a, MYCOL, iacol, NPCOL)` and `numroc, indxg2p` are ScaLAPACK tool functions; `MYROW, MYCOL, NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>a</code> | On exit, if $m \leq n$, the upper triangle of $A(ia:ia+m-1, ja:ja+n-1)$ contains the m -by- m upper triangular matrix R ; if $m \geq n$, the elements on and above the $(m - n)$ -th subdiagonal contain the m -by- n upper trapezoidal matrix R ; the remaining elements, with the array <code>tau</code> , represent the orthogonal/unitary matrix Q as a product of elementary reflectors (see <i>Application Notes</i> below). |
| <code>tau</code> | (local) Array of size <code>LOCr(ia+m-1)</code> . Contains the scalar factor of elementary reflectors. <code>tau</code> is tied to the distributed matrix A . |
| <code>work[0]</code> | On exit, <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0, the execution is successful. < 0, if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

Application Notes

The matrix Q is represented as a product of elementary reflectors

$$Q = H(ia) * H(ia+1) * \dots * H(ia+k-1),$$

where $k = \min(m, n)$.

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v'$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(n-k+i+1:n) = 0$ and $v(n-k+i) = 1$; $v(1:n-k+i-1)$ is stored on exit in $A(ia+m-k+i-1, ja:ja+n-k+i-2)$, and τ in `tau[ia+m-k+i-2]`.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?orgrq

Generates the orthogonal matrix Q of the RQ factorization formed by `p?gerqf`.

Syntax

```
void psorgqr (MKL_INT *m , MKL_INT *n , MKL_INT *k , float *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
void pdorgqr (MKL_INT *m , MKL_INT *n , MKL_INT *k , double *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?orgqr` function generates the whole or part of m -by- n real distributed matrix Q denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal rows that is defined as the last m rows of a product of k elementary reflectors of order n

$$Q = H(1) * H(2) * \dots * H(k)$$

as returned by `p?gerqf`.

Input Parameters

| | |
|----------|---|
| m | (global) The number of rows in the matrix $\text{sub}(Q)$, ($m \geq 0$). |
| n | (global) The number of columns in the matrix $\text{sub}(Q)$, ($n \geq m \geq 0$). |
| k | (global) The number of elementary reflectors whose product defines the matrix Q ($m \geq k \geq 0$). |
| a | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja+n-1)$. The i -th row of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+m-1$, as returned by <code>p?gerqf</code> in the k rows of its distributed matrix argument $A(ia+m-k:ia+m-1, ja:*)$. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| tau | (local) Array of size $LOCC(ja+k-1)$. Contains the scalar factor $tau[i]$ of elementary reflectors $H(i+1)$ as returned by <code>p?gerqf</code> , $0 \leq i < LOCC(ja+k-1)$. tau is tied to the distributed matrix A . |
| $work$ | (local) Workspace array of size of $lwork$. |
| $lwork$ | (local or global) size of $work$, must be at least $lwork \geq mb_a * (mpa0 + nqa0 + mb_a)$, where $iroffa = \text{mod}(ia-1, mb_a)$, $icoffa = \text{mod}(ja-1, nb_a)$, |

```

iarow = indxg2p(ia, mb_a, MYROW, rsrc_a, NPROW),
iacol = indxg2p(ja, nb_a, MYCOL, csrc_a, NPCOL),
mpa0 = numroc(m+iroffa, mb_a, MYROW, iarow, NPROW),
nqa0 = numroc(n+icoffa, nb_a, MYCOL, iacol, NPCOL)
indxg2p and numroc are ScaLAPACK tool functions; MYROW, MYCOL, NPROW
and NPCOL can be determined by calling the function blacs_gridinfo.

```

NOTE

`mod(x, y)` is the integer remainder of x/y .

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by [pxerbla](#).

Output Parameters

| | |
|----------------------|--|
| <code>a</code> | Contains the local pieces of the m -by- n distributed matrix Q . |
| <code>work[0]</code> | On exit, <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ungrq

Generates the unitary matrix Q of the RQ factorization formed by [p?gerqf](#).

Syntax

```

void pcungrq (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );

```

```

void pzungrq (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );

```

Include Files

- `mkl_scalapack.h`

Description

This function generates the m -by- n complex distributed matrix Q denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors of order n

$Q = (H(1))^H * (H(2))^H * \dots * (H(k))^H$ as returned by `p?gerqf`.

Input Parameters

| | |
|----------|--|
| m | (global) The number of rows in the matrix sub(Q); ($m \geq 0$). |
| n | (global) The number of columns in the matrix sub(Q) ($n \geq m \geq 0$). |
| k | (global) The number of elementary reflectors whose product defines the matrix Q ($m \geq k \geq 0$). |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. The i -th row of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia+m-k \leq i \leq ia+m-1$, as returned by <code>p?gerqf</code> in the k rows of its distributed matrix argument $A(ia+m-k:ia+m-1, ja:*)$. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| tau | (local) Array of size $LOCr(ia+m-1)$. Contains the scalar factor $tau[i]$ of elementary reflectors $H(i+1)$ as returned by <code>p?gerqf</code> , $0 \leq i < LOCr(ia+m-1)$. tau is tied to the distributed matrix A . |
| $work$ | (local) Workspace array of size of $lwork$. |
| $lwork$ | (local or global) size of $work$, must be at least $lwork \geq mb_a * (mpa0 + nqa0 + mb_a)$, where $iroffa = \text{mod}(ia-1, mb_a),$ $icoffa = \text{mod}(ja-1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$ $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL),$ $mpa0 = \text{numroc}(m + iroffa, mb_a, MYROW, iarow, NPROW),$ $nqa0 = \text{numroc}(n + icoffa, nb_a, MYCOL, iacol, NPCOL)$ |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>a</code> | Contains the local pieces of the m -by- n distributed matrix Q . |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ormr3

Applies an orthogonal distributed matrix to a general m -by- n distributed matrix.

Syntax

```
void psormr3 (const char* side, const char* trans, const MKL_INT* m, const MKL_INT* n,
const MKL_INT* k, const MKL_INT* l, const float* a, const MKL_INT* ia, const MKL_INT*
ja, const MKL_INT* desca, const float* tau, float* c, const MKL_INT* ic, const
MKL_INT* jc, const MKL_INT* descc, float* work, const MKL_INT* lwork, MKL_INT* info);

void pdormr3 (const char* side, const char* trans, const MKL_INT* m, const MKL_INT* n,
const MKL_INT* k, const MKL_INT* l, const double* a, const MKL_INT* ia, const MKL_INT*
ja, const MKL_INT* desca, const double* tau, double* c, const MKL_INT* ic, const
MKL_INT* jc, const MKL_INT* descc, double* work, const MKL_INT* lwork, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

`p?ormr3` overwrites the general real m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|--------------------------|-------------------------|-------------------------|
| | <code>side = 'L'</code> | <code>side = 'R'</code> |
| <code>trans = 'N'</code> | $Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| <code>trans = 'T'</code> | $Q^T * \text{sub}(C)$ | $\text{sub}(C) * Q^T$ |
| | $Q * \text{sub}(C)$ | |

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$$Q = H(1) H(2) \dots H(k)$$

as returned by `p?tzrzf`. Q is of order m if `side = 'L'` and of order n if `side = 'R'`.

Input Parameters

| | |
|--------------------|--|
| <code>side</code> | (global) = 'L': apply Q or Q^T from the Left; = 'R': apply Q or Q^T from the Right. |
| <code>trans</code> | (global) = 'N': No transpose, apply Q ; = 'T': Transpose, apply Q^T . |
| <code>m</code> | (global) The number of rows to be operated on i.e the number of rows of the distributed submatrix <code>sub(C)</code> . $m \geq 0$. |
| <code>n</code> | (global) The number of columns to be operated on i.e the number of columns of the distributed submatrix <code>sub(C)</code> . $n \geq 0$. |
| <code>k</code> | (global) The number of elementary reflectors whose product defines the matrix Q . If <code>side = 'L'</code> , $m \geq k \geq 0$, if <code>side = 'R'</code> , $n \geq k \geq 0$. |
| <code>l</code> | (global) The columns of the distributed submatrix <code>sub(A)</code> containing the meaningful part of the Householder reflectors. If <code>side = 'L'</code> , $m \geq l \geq 0$, if <code>side = 'R'</code> , $n \geq l \geq 0$. |
| <code>a</code> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+m-1)$ if <code>side='L'</code> , and $lld_a * LOCC(ja+n-1)$ if <code>side='R'</code> , where $lld_a \geq \text{MAX}(1, LOCr(ia+k-1))$; On entry, the i -th row must contain the vector which defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by <code>p?tzrzf</code> in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the routine but restored on exit. |
| <code>ia</code> | (global) The row index in the global array <code>a</code> indicating the first row of <code>sub(A)</code> . |
| <code>ja</code> | (global) |

| | |
|--------------|---|
| | The column index in the global array <i>a</i> indicating the first column of sub(<i>A</i>). |
| <i>desca</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array, size $LOCc(ia+k-1)$. This array contains the scalar factors <i>tau</i> (<i>i</i>) of the elementary reflectors <i>H</i> (<i>i</i>) as returned by <i>p?tzrzf</i> . <i>tau</i> is tied to the distributed matrix <i>A</i> . |
| <i>c</i> | (local) Pointer into the local memory to an array of size $lld_c*LOCc(jc+n-1)$. On entry, the local pieces of the distributed matrix sub(<i>C</i>). |
| <i>ic</i> | (global) The row index in the global array <i>c</i> indicating the first row of sub(<i>C</i>). |
| <i>jc</i> | (global) The column index in the global array <i>c</i> indicating the first column of sub(<i>C</i>). |
| <i>desc</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>C</i> . |
| <i>work</i> | (local) Array, size (<i>lwork</i>) |
| <i>lwork</i> | (local) The size of the array <i>work</i> . <i>lwork</i> is local input and must be at least If <i>side</i> = 'L', $lwork \geq MpCO + \text{MAX}(\text{MAX}(1, NqCO), \text{numroc}(\text{numroc}(m + IROFFC, mb_a, 0, 0, NPROW), mb_a, 0, 0, NqCO));$ if <i>side</i> = 'R', $lwork \geq NqCO + \text{MAX}(1, MpCO);$ where $LCMP = LCM / NPROW$ $LCM = \text{icl}m(NPROW, NPCOL),$ $IROFFC = \text{MOD}(ic-1, mb_c),$ $ICOFFC = \text{MOD}(jc-1, nb_c),$ $ICROW = \text{indxg2p}(ic, mb_c, MYROW, rsrc_c, NPROW),$ $ICCOL = \text{indxg2p}(jc, nb_c, MYCOL, csrc_c, NPCOL),$ $MpCO = \text{numroc}(m + IROFFC, mb_c, MYROW, ICROW, NPROW),$ $NqCO = \text{numroc}(n + ICOFFC, nb_c, MYCOL, ICCOL, NPCOL),$ |

`ilcm`, `indxg2p`, and `numroc` are ScaLAPACK tool functions;

`MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the subroutine `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|-------------------|--|
| <code>c</code> | On exit, <code>sub(C)</code> is overwritten by $Q \cdot \text{sub}(C)$ or $Q' \cdot \text{sub}(C)$ or $\text{sub}(C) \cdot Q'$ or $\text{sub}(C) \cdot Q$. |
| <code>work</code> | On exit, <code>work[0]</code> returns the minimal and optimal <code>lwork</code> . |
| <code>info</code> | (local) = 0: successful exit < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry had an illegal value, then <code>info = -(i*100+j)</code> , if the <i>i</i> -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

Application Notes

Alignment requirements

The distributed submatrices $A(ia:*, ja:*)$ and $C(ic:ic+m-1, jc:jc+n-1)$ must verify some alignment properties, namely the following expressions should be true:

If `side = 'L'`,

$(nb_a = mb_c \text{ .AND. } ICOFFA = IROFFC)$

If `side = 'R'`,

$(nb_a = nb_c \text{ .AND. } ICOFFA = ICOFFC \text{ .AND. } IACOL = ICCOL)$

p?unmr3

Applies an orthogonal distributed matrix to a general m-by-n distributed matrix.

Syntax

```
void pcunmr3 (const char* side, const char* trans, const MKL_INT* m, const MKL_INT* n,
const MKL_INT* k, const MKL_INT* l, const MKL_Complex8* a, const MKL_INT* ia, const
MKL_INT* ja, const MKL_INT* desca, const MKL_Complex8* tau, MKL_Complex8* c, const
MKL_INT* ic, const MKL_INT* jc, const MKL_INT* descc, MKL_Complex8* work, const
MKL_INT* lwork, MKL_INT* info);
```

```
void pzunmr3 (const char* side, const char* trans, const MKL_INT* m, const MKL_INT* n,
const MKL_INT* k, const MKL_INT* l, const MKL_Complex16* a, const MKL_INT* ia, const
MKL_INT* ja, const MKL_INT* desca, const MKL_Complex16* tau, MKL_Complex16* c, const
MKL_INT* ic, const MKL_INT* jc, const MKL_INT* descc, MKL_Complex16* work, const
MKL_INT* lwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

p?unmr3 overwrites the general complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | |
|------------------------------------|-----------------------|
| $side = 'L'$ | $side = 'R'$ |
| $trans = 'N': Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| $trans = 'C': Q^H * \text{sub}(C)$ | $\text{sub}(C) * Q^H$ |

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(1)' H(2)' \dots H(k)'$

as returned by p?tzzrf. Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|---------|--|
| $side$ | (global) = 'L': apply Q or Q^H from the Left; = 'R': apply Q or Q^H from the Right. |
| $trans$ | (global) = 'N': No transpose, apply Q ; = 'C': Conjugate transpose, apply Q^H . |
| m | (global) The number of rows to be operated on i.e the number of rows of the distributed submatrix $\text{sub}(C)$. $m \geq 0$. |
| n | (global) The number of columns to be operated on i.e the number of columns of the distributed submatrix $\text{sub}(C)$. $n \geq 0$. |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . If $side = 'L'$, $m \geq k \geq 0$, if $side = 'R'$, $n \geq k \geq 0$. |
| l | (global) The columns of the distributed submatrix $\text{sub}(A)$ containing the meaningful part of the Householder reflectors. If $side = 'L'$, $m \geq l \geq 0$, if $side = 'R'$, $n \geq l \geq 0$. |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+m-1)$ if $side='L'$, and $lld_a * LOCC(ja+n-1)$ if $side='R'$, where $lld_a \geq \text{MAX}(1, LOCr(ia+k-1))$; On entry, the i -th row must contain the vector which defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by p?tzzrf in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. |

$A(ia:ia+k-1,ja:*)$ is modified by the routine but restored on exit.

ia (global)
The row index in the global array *a* indicating the first row of *sub(A)*.

ja (global)
The column index in the global array *a* indicating the first column of *sub(A)*.

desca (global and local)
Array of size *dlen_*.
The array descriptor for the distributed matrix *A*.

tau (local)
Array, size $LOCc(ia+k-1)$.
This array contains the scalar factors *tau*(*i*) of the elementary reflectors *H*(*i*) as returned by `p?tzrzf`. *tau* is tied to the distributed matrix *A*.

c (local)
Pointer into the local memory to an array of size $lld_c*LOCc(jc+n-1)$.
On entry, the local pieces of the distributed matrix *sub(C)*.

ic (global)
The row index in the global array *c* indicating the first row of *sub(C)*.

jc (global)
The column index in the global array *c* indicating the first column of *sub(C)*.

descc (global and local)
Array of size *dlen_*.
The array descriptor for the distributed matrix *C*.

work (local)
Array, size (*lwork*)
On exit, *work*(1) returns the minimal and optimal *lwork*.

lwork (local or global)
The size of the array *work*.
lwork is local input and must be at least
If *side* = 'L', $lwork \geq MpC0 + \text{MAX}(\text{MAX}(1, NqC0), \text{numroc}(\text{numroc}(m + IROFFC, mb_a, 0, 0, NPROW), mb_a, 0, 0, LCMP))$;
if *side* = 'R', $lwork \geq NqC0 + \text{MAX}(1, MpC0)$;
where $LCMP = LCM / NPROW$ with $LCM = \text{ICLM}(NPROW, NPCOL)$,
 $IROFFC = \text{MOD}(ic-1, MB_C)$, $ICOFFC = \text{MOD}(jc-1, nb_c)$,
 $ICROW = \text{indxg2p}(ic, MB_C, MYROW, rsrc_c, NPROW)$,

```

ICCOL = indxg2p( jc, nb_c, MYCOL, csrc_c, NPCOL ),
MpC0 = numroc( m+IROFFC, MB_C, MYROW, ICROW, NPROW ),
NqC0 = numroc( n+ICOFFC, nb_c, MYCOL, ICCOL, NPCOL ),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions;
MYROW, MYCOL, NPROW and NPCOL can be determined by calling the
subroutine blacs_gridinfo.

```

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the routine only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|-------------|--|
| <i>c</i> | On exit, <code>sub(C)</code> is overwritten by <code>Q*sub(C)</code> or <code>Q'*sub(C)</code> or <code>sub(C)*Q'</code> or <code>sub(C)*Q</code> . |
| <i>work</i> | (local) Array, size (<i>lwork</i>) On exit, <code>work[0]</code> returns the minimal and optimal <i>lwork</i> . |
| <i>info</i> | (local) = 0: successful exit < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry had an illegal value, then <code>info = -(i*100+j)</code> , if the <i>i</i> -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

Application Notes

Alignment requirements

The distributed submatrices $A(ia:*, ja:*)$ and $C(ic:ic+m-1, jc:jc+n-1)$ must verify some alignment properties, namely the following expressions should be true:

If *side* = 'L', (*nb_a* = MB_C and ICOFFA = IROFFC)

If *side* = 'R', (*nb_a* = *nb_c* and ICOFFA = ICOFFC and IACOL = ICCOL)

p?ormrq

Multiplies a general matrix by the orthogonal matrix *Q* of the RQ factorization formed by `p?gerqf`.

Syntax

```

void psormrq (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );

void pdormrq (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );

```

Include Files

- `mkl_scalapack.h`

Description

The `p?ormrq` function overwrites the general real m -by- n distributed matrix sub(C) = $C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|----------------|----------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q * sub(C)$ | $sub(C) * Q$ |
| $trans = 'T':$ | $Q^T * sub(C)$ | $sub(C) * Q^T$ |

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$$Q = H(1) H(2) \dots H(k)$$

as returned by `p?gerqf`. Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|----------|---|
| $side$ | (global) = $'L'$: Q or Q^T is applied from the left. = $'R'$: Q or Q^T is applied from the right. |
| $trans$ | (global) = $'N'$, no transpose, Q is applied. = $'T'$, transpose, Q^T is applied. |
| m | (global) The number of rows in the distributed matrix sub(C) ($m \geq 0$). |
| n | (global) The number of columns in the distributed matrix sub(C) ($n \geq 0$). |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If $side = 'L'$, $m \geq k \geq 0$ If $side = 'R'$, $n \geq k \geq 0$. |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+m-1)$ if $side = 'L'$, and $lld_a * LOCC(ja+n-1)$ if $side = 'R'$. The i -th row of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by <code>p?gerqf</code> in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the function but restored on exit. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| tau | (local) |

Array of size $LOCc(ja+k-1)$.

Contains the scalar factor $\tau[i]$ of elementary reflectors $H(i+1)$ as returned by `p?gerqf` ($0 \leq i < LOCc(ja+k-1)$). τ is tied to the distributed matrix A .

c

(local)

Pointer into the local memory to an array of local size $lld_c * LOCc(jc+n-1)$.

Contains the local pieces of the distributed matrix sub(C) to be factored.

ic, jc

(global) The row and column indices in the global matrix C indicating the first row and the first column of the matrix sub(C), respectively.

desc

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix C .

work

(local)

Workspace array of size of *lwork*.

lwork

(local or global) size of *work*, must be at least:

If *side* = 'L',

$$lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + \max(mqa0 + \text{numroc}(\text{numroc}(n + iroffc, mb_a, 0, 0, NPROW), mb_a, 0, 0, lcm), nqc0)) * mb_a) + mb_a * mb_a)$$

else if *side* = 'R',

$$lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + nqc0) * mb_a) + mb_a * mb_a$$

end if

where

$lcmp = lcm / NPROW$ with $lcm = ilcm(NPROW, NPCOL)$,

$iroffa = \text{mod}(ia - 1, mb_a)$,

$icoffa = \text{mod}(ja - 1, nb_a)$,

$iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$,

$mqa0 = \text{numroc}(n + icoffa, nb_a, MYCOL, iacol, NPCOL)$,

$iroffc = \text{mod}(ic - 1, mb_c)$,

$icoffc = \text{mod}(jc - 1, nb_c)$,

$icrow = \text{indxg2p}(ic, mb_c, MYROW, rsrc_c, NPROW)$,

$iccol = \text{indxg2p}(jc, nb_c, MYCOL, csrc_c, NPCOL)$,

$mpc0 = \text{numroc}(m + iroffc, mb_c, MYROW, icrow, NPROW)$,

$nqc0 = \text{numroc}(n + icoffc, nb_c, MYCOL, iccol, NPCOL)$,

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NP COL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q * \text{sub}(C)$, or $Q * \text{sub}(C)$, or $\text{sub}(C) * Q'$, or $\text{sub}(C) * Q$ |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i * 100 + j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unmrq

Multiplies a general matrix by the unitary matrix Q of the RQ factorization formed by `p?gerqf`.

Syntax

```
void pcunmrq (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pzunmrq (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

This function overwrites the general complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|---------------------|-----------------------|-----------------------|
| | <i>side</i> = 'L' | <i>side</i> = 'R' |
| <i>trans</i> = 'N': | $Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| <i>trans</i> = 'C': | $Q^H * \text{sub}(C)$ | $\text{sub}(C) * Q^H$ |

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$$Q = H(1)' H(2)' \dots H(k)'$$

as returned by `p?gerqf`. Q is of order m if `side = 'L'` and of order n if `side = 'R'`.

Input Parameters

| | |
|---------------------|---|
| <code>side</code> | (global) = 'L': Q or Q^H is applied from the left. = 'R': Q or Q^H is applied from the right. |
| <code>trans</code> | (global) = 'N', no transpose, Q is applied. = 'C', conjugate transpose, Q^H is applied. |
| <code>m</code> | (global) The number of rows in the distributed matrix <code>sub(C)</code> , ($m \geq 0$). |
| <code>n</code> | (global) The number of columns in the distributed matrix <code>sub(C)</code> , ($n \geq 0$). |
| <code>k</code> | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If <code>side = 'L'</code> , $m \geq k \geq 0$ If <code>side = 'R'</code> , $n \geq k \geq 0$. |
| <code>a</code> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+m-1)$ if <code>side = 'L'</code> , and $lld_a * LOCC(ja+n-1)$ if <code>side = 'R'</code> . The i -th row of the matrix stored in <code>a</code> must contain the vector that defines the elementary reflector $H(i)$, $ia \leq j \leq ia+k-1$, as returned by <code>p?gerqf</code> in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the function but restored on exit. |
| <code>ia, ja</code> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <code>desca</code> | (global and local) array of size <code>dlen_</code> . The array descriptor for the distributed matrix A . |
| <code>tau</code> | (local) Array of size $LOCC(ja+k-1)$. Contains the scalar factor $tau[i]$ of elementary reflectors $H(i+1)$ as returned by <code>p?gerqf</code> ($0 \leq i < LOCC(ja+k-1)$). <code>tau</code> is tied to the distributed matrix A . |
| <code>c</code> | (local) Pointer into the local memory to an array of local size $lld_c * LOCC(jc+n-1)$. Contains the local pieces of the distributed matrix <code>sub(C)</code> to be factored. |
| <code>ic, jc</code> | (global) The row and column indices in the global matrix C indicating the first row and the first column of the submatrix C , respectively. |

desc (global and local) array of size *dlen_*. The array descriptor for the distributed matrix *C*.

work (local)
Workspace array of size of *lwork*.

lwork (local or global) size of *work*, must be at least:
If *side* = 'L',
 $lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + \max(mqa0 + \text{numroc}(\text{numroc}(n + iroffc, mb_a, 0, 0, NPROW), mb_a, 0, 0, lcmp), nqc0)) * mb_a) + mb_a * mb_a)$
else if *side* = 'R',
 $lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + nqc0) * mb_a) + mb_a * mb_a$
end if
where
 $lcmp = lcm / NPROW$ with $lcm = ilcm(NPROW, NPCOL)$,
 $iroffa = \text{mod}(ia - 1, mb_a)$,
 $icoffa = \text{mod}(ja - 1, nb_a)$,
 $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$,
 $mqa0 = \text{numroc}(m + icoffa, nb_a, MYCOL, iacol, NPCOL)$,
 $iroffc = \text{mod}(ic - 1, mb_c)$,
 $icoffc = \text{mod}(jc - 1, nb_c)$,
 $icrow = \text{indxg2p}(ic, mb_c, MYROW, rsrc_c, NPROW)$,
 $iccol = \text{indxg2p}(jc, nb_c, MYCOL, csrc_c, NPCOL)$,
 $mpc0 = \text{numroc}(m + iroffc, mb_c, MYROW, icrow, NPROW)$,
 $nqc0 = \text{numroc}(n + icoffc, nb_c, MYCOL, iccol, NPCOL)$,

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

$ilcm$, $indxg2p$ and $numroc$ are ScaLAPACK tool functions; $MYROW$, $MYCOL$, $NPROW$ and $NPCOL$ can be determined by calling the function $blacs_gridinfo$.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by $p\text{xerbla}$.

Output Parameters

c Overwritten by the product $Q * \text{sub}(C)$ or $Q' * \text{sub}(C)$, or $\text{sub}(C) * Q'$, or $\text{sub}(C) * Q$

`work[0]` On exit `work[0]` contains the minimum value of `lwork` required for optimum performance.

`info` (global)

= 0: the execution is successful.

< 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j* - 1, had an illegal value, then `info = -(i*100+j)`; if the *i*-th argument is a scalar and had an illegal value, then `info = -i`.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?tzrzf

Reduces the upper trapezoidal matrix A to upper triangular form.

Syntax

```
void pstzrzf (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdtzrzf (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pctzrzf (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pztzrzf (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?tzrzf` function reduces the *m*-by-*n* ($m \leq n$) real/complex upper trapezoidal matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$ to upper triangular form by means of orthogonal/unitary transformations. The upper trapezoidal matrix *A* is factored as

$$A = (R \ 0) * Z,$$

where *Z* is an *n*-by-*n* orthogonal/unitary matrix and *R* is an *m*-by-*m* upper triangular matrix.

Input Parameters

m (global) The number of rows in the matrix $\text{sub}(A)$; ($m \geq 0$).

n (global) The number of columns in the matrix $\text{sub}(A)$ ($n \geq 0$).

a (local)

Pointer into the local memory to an array of size `lld_a*LOCc(ja+n-1)`. Contains the local pieces of the *m*-by-*n* distributed matrix $\text{sub}(A)$ to be factored.

| | |
|---------------|--|
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least $lwork \geq mb_a * (mp0 + nq0 + mb_a)$, where $iroff = \text{mod}(ia-1, mb_a)$, $icoff = \text{mod}(ja-1, nb_a)$, $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW)$, $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$, $mp0 = \text{numroc}(m + iroff, mb_a, MYROW, iarow, NPROW)$, $nq0 = \text{numroc}(n + icoff, nb_a, MYCOL, iacol, NPCOL)$ |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

indxg2p and *numroc* are ScaLAPACK tool functions; *MYROW*, *MYCOL*, *NPROW* and *NPCOL* can be determined by calling the function *blacs_gridinfo*.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by *p_xerbla*.

Output Parameters

| | |
|-----------------|--|
| <i>a</i> | On exit, the leading <i>m</i> -by- <i>m</i> upper triangular part of <i>sub(A)</i> contains the upper triangular matrix <i>R</i> , and elements <i>m</i> +1 to <i>n</i> of the first <i>m</i> rows of <i>sub(A)</i> , with the array <i>tau</i> , represent the orthogonal/unitary matrix <i>Z</i> as a product of <i>m</i> elementary reflectors. |
| <i>work</i> [0] | On exit <i>work</i> [0] contains the minimum value of <i>lwork</i> required for optimum performance. |
| <i>tau</i> | (local) Array of size $LOCr(ia+m-1)$. Contains the scalar factor of elementary reflectors. <i>tau</i> is tied to the distributed matrix <i>A</i> . |
| <i>info</i> | (global) = 0: the execution is successful. |

< 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

Application Notes

The factorization is obtained by the Householder's method. The k -th transformation matrix, $Z(k)$, which is or whose conjugate transpose is used to introduce zeros into the $(m - k + 1)$ -th row of $\text{sub}(A)$, is given in the form

$$Z(k) = \begin{bmatrix} i & 0 \\ 0 & T(k) \end{bmatrix}$$

where

$$T(k) = i - \tau * u(k) * u(k)',$$

$$u(k) = \begin{bmatrix} 1 \\ 0 \\ z(k) \end{bmatrix}$$

τ is a scalar and $Z(k)$ is an $(n - m)$ element vector. τ and $Z(k)$ are chosen to annihilate the elements of the k -th row of $\text{sub}(A)$. The scalar τ is returned in the k -th element of τ , indexed $k-1$, and the vector $u(k)$ in the k -th row of $\text{sub}(A)$, such that the elements of $Z(k)$ are in $a(k, m + 1), \dots, a(k, n)$. The elements of R are returned in the upper triangular part of $\text{sub}(A)$. Z is given by

$$Z = Z(1) * Z(2) * \dots * Z(m).$$

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ormrz

Multiplies a general matrix by the orthogonal matrix from a reduction to upper triangular form formed by p?tzzrf.

Syntax

```
void psormrz (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_INT *l , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau ,
float *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork ,
MKL_INT *info );
```

```
void pdormrz (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_INT *l , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau ,
double *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

This function overwrites the general real m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|-----------------------|-----------------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| $trans = 'T':$ | $Q^T * \text{sub}(C)$ | $\text{sub}(C) * Q^T$ |

where Q is a real orthogonal distributed matrix defined as the product of k elementary reflectors

$$Q = H(1) H(2) \dots H(k)$$

as returned by `p?tzrzf`. Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|---------|---|
| $side$ | (global) $= 'L'$: Q or Q^T is applied from the left. $= 'R'$: Q or Q^T is applied from the right. |
| $trans$ | (global) $= 'N'$, no transpose, Q is applied. $= 'T'$, transpose, Q^T is applied. |
| m | (global) The number of rows in the distributed matrix $\text{sub}(C)$ ($m \geq 0$). |
| n | (global) The number of columns in the distributed matrix $\text{sub}(C)$ ($n \geq 0$). |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If $side = 'L'$, $m \geq k \geq 0$ If $side = 'R'$, $n \geq k \geq 0$. |
| l | (global) The columns of the distributed matrix $\text{sub}(A)$ containing the meaningful part of the Householder reflectors. If $side = 'L'$, $m \geq l \geq 0$ If $side = 'R'$, $n \geq l \geq 0$. |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+m-1)$ if $side = 'L'$, and $lld_a * LOCC(ja+n-1)$ if $side = 'R'$, where $lld_a \geq \max(1, LOCr(ia+k-1))$. The i -th row of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by <code>p?tzrzf</code> in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the function but restored on exit. |

| | |
|---------------|---|
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array of size <i>LOCc(ia+k-1)</i> . Contains the scalar factor <i>tau[i]</i> of elementary reflectors <i>H(i+1)</i> as returned by <i>p?tzrzf</i> ($0 \leq i < LOCc(ia+k-1)$). <i>tau</i> is tied to the distributed matrix <i>A</i> . |
| <i>c</i> | (local) Pointer into the local memory to an array of local size <i>lld_c*LOCc(jc+n-1)</i> . Contains the local pieces of the distributed matrix sub(<i>C</i>) to be factored. |
| <i>ic, jc</i> | (global) The row and column indices in the global matrix <i>C</i> indicating the first row and the first column of the submatrix <i>C</i> , respectively. |
| <i>desc</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>C</i> . |
| <i>work</i> | (local) Workspace array of size of <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least: If <i>side</i> = 'L', $lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + \max(mqa0 + \text{numroc}(\text{numroc}(n + iroffc, mb_a, 0, 0, NPROW), mb_a, 0, 0, lcmp), nqc0)) * mb_a) + mb_a * mb_a)$ else if <i>side</i> = 'R', $lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + nqc0) * mb_a) + mb_a * mb_a$ end if where $lcmp = lcm / NPROW$ with $lcm = ilcm(NPROW, NPCOL)$, $iroffa = \text{mod}(ia - 1, mb_a)$, $icoffa = \text{mod}(ja - 1, nb_a)$, $iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL)$, $mqa0 = \text{numroc}(n + icoffa, nb_a, MYCOL, iacol, NPCOL)$, $iroffc = \text{mod}(ic - 1, mb_c)$, $icoffc = \text{mod}(jc - 1, nb_c)$, $icrow = \text{indxg2p}(ic, mb_c, MYROW, rsrc_c, NPROW)$, $iccol = \text{indxg2p}(jc, nb_c, MYCOL, csrc_c, NPCOL)$, $mpc0 = \text{numroc}(m + iroffc, mb_c, MYROW, icrow, NPROW)$, $nqc0 = \text{numroc}(n + icoffc, nb_c, MYCOL, iccol, NPCOL)$, |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q \cdot \text{sub}(C)$, or $Q' \cdot \text{sub}(C)$, or $\text{sub}(C) \cdot Q'$, or $\text{sub}(C) \cdot Q$ |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unmrz

Multiplies a general matrix by the unitary transformation matrix from a reduction to upper triangular form determined by p?tzzrf.

Syntax

```
void pcunmrz (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_INT *l , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
MKL_Complex8 *tau , MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc ,
MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzunmrz (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_INT *l , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
MKL_Complex16 *tau , MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

This function overwrites the general complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|---------------------|-----------------------|-----------------------|
| | <i>side</i> = 'L' | <i>side</i> = 'R' |
| <i>trans</i> = 'N': | $Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| <i>trans</i> = 'C': | $Q^H * \text{sub}(C)$ | $\text{sub}(C) * Q^H$ |

where Q is a complex unitary distributed matrix defined as the product of k elementary reflectors

$$Q = H(1)' H(2)' \dots H(k)'$$

as returned by `pctzrzf/pztzrzf`. Q is of order m if *side* = 'L' and of order n if *side* = 'R'.

Input Parameters

| | |
|---------------|---|
| <i>side</i> | (global) = 'L': Q or Q^H is applied from the left. = 'R': Q or Q^H is applied from the right. |
| <i>trans</i> | (global) = 'N', no transpose, Q is applied. = 'C', conjugate transpose, Q^H is applied. |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(C)$, ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(C)$, ($n \geq 0$). |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q . Constraints: If <i>side</i> = 'L', $m \geq k \geq 0$ If <i>side</i> = 'R', $n \geq k \geq 0$. |
| <i>l</i> | (global) The columns of the distributed matrix $\text{sub}(A)$ containing the meaningful part of the Householder reflectors. If <i>side</i> = 'L', $m \geq l \geq 0$ If <i>side</i> = 'R', $n \geq l \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCc(ja+m-1)$ if <i>side</i> = 'L', and $lld_a * LOCc(ja+n-1)$ if <i>side</i> = 'R', where $lld_a \geq \max(1, LOCr(ja+k-1))$. The i -th row of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by <code>p?gerqf</code> in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. $A(ia:ia+k-1, ja:*)$ is modified by the function but restored on exit. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size $LOCc(ia+k-1)$. |

Contains the scalar factor $\tau[i]$ of elementary reflectors $H(i+1)$ as returned by `p?gerqf` ($0 \leq i < LOCC(ia+k-1)$). τ is tied to the distributed matrix A .

`c`

(local)

Pointer into the local memory to an array of local size $lld_c * LOCC(jc+n-1)$.

Contains the local pieces of the distributed matrix $\text{sub}(C)$ to be factored.

`ic, jc`

(global) The row and column indices in the global matrix C indicating the first row and the first column of the submatrix C , respectively.

`desc`

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix C .

`work`

(local)

Workspace array of size $lwork$.

`lwork`

(local or global) size of $work$, must be at least:

If $side = 'L'$,

$$lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + \max(mqa0 + \text{numroc}(\text{numroc}(n + iroffc, mb_a, 0, 0, NPROW), mb_a, 0, 0, lcmp), nqc0)) * mb_a + mb_a * mb_a)$$

else if $side = 'R'$,

$$lwork \geq \max((mb_a * (mb_a - 1)) / 2, (mpc0 + nqc0) * mb_a + mb_a * mb_a)$$

end if

where

$$lcmp = lcm / NPROW \text{ with } lcm = ilcm(NPROW, NPCOL),$$

$$iroffa = \text{mod}(ia - 1, mb_a),$$

$$icoffa = \text{mod}(ja - 1, nb_a),$$

$$iacol = \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL),$$

$$mqa0 = \text{numroc}(m + icoffa, nb_a, MYCOL, iacol, NPCOL),$$

$$iroffc = \text{mod}(ic - 1, mb_c),$$

$$icoffc = \text{mod}(jc - 1, nb_c),$$

$$icrow = \text{indxg2p}(ic, mb_c, MYROW, rsrc_c, NPROW),$$

$$iccol = \text{indxg2p}(jc, nb_c, MYCOL, csrc_c, NPCOL),$$

$$mpc0 = \text{numroc}(m + iroffc, mb_c, MYROW, icrow, NPROW),$$

$$nqc0 = \text{numroc}(n + icoffc, nb_c, MYCOL, iccol, NPCOL),$$

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q^* \text{sub}(C)$, or $Q'^* \text{sub}(C)$, or $\text{sub}(C)^*Q'$, or $\text{sub}(C)^*Q$ |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of $lwork$ required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ggqrf

Computes the generalized QR factorization.

Syntax

```
void psggqrf (MKL_INT *n , MKL_INT *m , MKL_INT *p , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *taua , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , float *taub , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdggqrf (MKL_INT *n , MKL_INT *m , MKL_INT *p , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *taua , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , double *taub , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcggqrf (MKL_INT *n , MKL_INT *m , MKL_INT *p , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *taua , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_Complex8 *taub , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzggqrf (MKL_INT *n , MKL_INT *m , MKL_INT *p , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *taua , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_Complex16 *taub , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?ggqrf` function forms the generalized QR factorization of an n -by- m matrix

$$\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+m-1)$$

and an n -by- p matrix

| | |
|---------------|---|
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(j_{a+m}-1)$. Contains the local pieces of the n -by- m matrix sub(A) to be factored. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>b</i> | (local) Pointer into the local memory to an array of size $lld_b * LOCC(j_{b+p}-1)$. Contains the local pieces of the n -by- p matrix sub(B) to be factored. |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of the submatrix B , respectively. |
| <i>descb</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |
| <i>work</i> | (local) Workspace array of size of $lwork$. |
| <i>lwork</i> | (local or global) Size of $work$, must be at least |

$$lwork \geq \max(nb_a * (npa0 + mqa0 + nb_a), \max((nb_a * (nb_a - 1)) / 2, (pqb0 + npb0) * nb_a) + nb_a * nb_a, mb_b * (npb0 + pqb0 + mb_b)),$$

where

$$\begin{aligned} iroffa &= \text{mod}(ia-1, mb_A), \\ icoffa &= \text{mod}(ja-1, nb_a), \\ iarow &= \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW), \\ iacol &= \text{indxg2p}(ja, nb_a, MYCOL, csrc_a, NPCOL), \\ npa0 &= \text{numroc}(n + iroffa, mb_a, MYROW, iarow, NPROW), \\ mqa0 &= \text{numroc}(m + icoffa, nb_a, MYCOL, iacol, NPCOL), \\ iroffb &= \text{mod}(ib-1, mb_b), \\ icoffb &= \text{mod}(jb-1, nb_b), \\ ibrow &= \text{indxg2p}(ib, mb_b, MYROW, rsrc_b, NPROW), \\ ibcol &= \text{indxg2p}(jb, nb_b, MYCOL, csrc_b, NPCOL), \\ npb0 &= \text{numroc}(n + iroffb, mb_b, MYROW, ibrow, NPROW), \\ pqb0 &= \text{numroc}(m + icoffb, nb_b, MYCOL, ibcol, NPCOL) \end{aligned}$$

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

and `numroc`, `indxg2p` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|-------------------------|--|
| <code>a</code> | On exit, the elements on and above the diagonal of sub (A) contain the $\min(n, m)$ -by- m upper trapezoidal matrix R (R is upper triangular if $n \geq m$); the elements below the diagonal, with the array <code>taua</code> , represent the orthogonal/unitary matrix Q as a product of $\min(n, m)$ elementary reflectors. (See Application Notes below). |
| <code>taua, taub</code> | (local) Arrays of size <code>LOCc(ja+min(n,m)-1)</code> for <code>taua</code> and <code>LOCr(ib+n-1)</code> for <code>taub</code> . The array <code>taua</code> contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix Q . <code>taua</code> is tied to the distributed matrix A . (See Application Notes below). The array <code>taub</code> contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix Z . <code>taub</code> is tied to the distributed matrix B . (See Application Notes below). |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of $lwork$ required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info</code> = $-(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then <code>info</code> = $-i$. |

Application Notes

The matrix Q is represented as a product of elementary reflectors

$$Q = H(j_a) * H(j_a+1) * \dots * H(j_a+k-1),$$

where $k = \min(n, m)$.

Each $H(i)$ has the form

$$H(i) = I - \tau_{au} * v * v'$$

where τ_{au} is a real/complex scalar, and v is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$; $v(i+1:n)$ is stored on exit in $A(ia+i:ia+n-1, ja+i-1)$, and τ_{au} in `taua[ja+i-2]`. To form Q explicitly, use ScaLAPACK function `p?orgqr/p?ungqr`. To use Q to update another matrix, use ScaLAPACK function `p?ormqr/p?unmqr`.

The matrix Z is represented as a product of elementary reflectors

$$Z = H(i_b) * H(i_b+1) * \dots * H(i_b+k-1), \text{ where } k = \min(n, p).$$

Each $H(i)$ has the form

$$H(i) = I - \tau_{bu} * v * v'$$

where $taub$ is a real/complex scalar, and v is a real/complex vector with $v_{(p-k+i+1:p)} = 0$ and $v_{(p-k+i)} = 1$; $v_{(1:p-k+i-1)}$ is stored on exit in $B(ib+n-k+i-1, jb:jb+p-k+i-2)$, and $taub$ in $taub[ib+n-k+i-2]$. To form Z explicitly, use ScaLAPACK function `p?orgqr/p?ungrq`. To use Z to update another matrix, use ScaLAPACK function `p?ormrq/p?unmrq`.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ggrqf

Computes the generalized RQ factorization.

Syntax

```
void psggrqf (MKL_INT *m , MKL_INT *p , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , float *taua , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , float *taub , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdggrqf (MKL_INT *m , MKL_INT *p , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , double *taua , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , double *taub , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcggrqf (MKL_INT *m , MKL_INT *p , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *taua , MKL_Complex8 *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , MKL_Complex8 *taub , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pzggrqf (MKL_INT *m , MKL_INT *p , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *taua , MKL_Complex16 *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , MKL_Complex16 *taub , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?ggrqf` function forms the generalized RQ factorization of an m -by- n matrix $sub(A) = A(ia:ia+m-1, ja:ja+n-1)$ and a p -by- n matrix $sub(B) = B(ib:ib+p-1, jb:jb+n-1)$:

$sub(A) = R*Q$, $sub(B) = Z*T*Q$,

where Q is an n -by- n orthogonal/unitary matrix, Z is a p -by- p orthogonal/unitary matrix, and R and T assume one of the forms:

$$R = \begin{matrix} m & (0 & R_{12}) \\ & n - m & m \end{matrix}, \text{ if } m \leq n,$$

or

$$R = \begin{matrix} (R_{11}) & m - n \\ & n \\ & & n \end{matrix}, \text{ if } m > n$$

where R_{11} or R_{21} is upper triangular, and

$$T = \begin{pmatrix} T_{11} & & n \\ & & p-n \\ 0 & & n \end{pmatrix}, \text{ if } p \geq n$$

or

$$T = \begin{pmatrix} p & (T_{11} & T_{12}) \\ & p & \text{if } p < n, \\ & p & n-p \end{pmatrix}$$

where T_{11} is upper triangular.

In particular, if $\text{sub}(B)$ is square and nonsingular, the GRQ factorization of $\text{sub}(A)$ and $\text{sub}(B)$ implicitly gives the RQ factorization of $\text{sub}(A) \cdot \text{inv}(\text{sub}(B))$:

$$\text{sub}(A) \cdot \text{inv}(\text{sub}(B)) = (R \cdot \text{inv}(T)) \cdot Z'$$

where $\text{inv}(\text{sub}(B))$ denotes the inverse of the matrix $\text{sub}(B)$, and Z' denotes the transpose (conjugate transpose) of matrix Z .

Input Parameters

| | |
|----------|---|
| m | (global) The number of rows in the distributed matrices $\text{sub}(A)$ ($m \geq 0$). |
| p | The number of rows in the distributed matrix $\text{sub}(B)$ ($p \geq 0$). |
| n | (global) The number of columns in the distributed matrices $\text{sub}(A)$ and $\text{sub}(B)$ ($n \geq 0$). |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCc(ja+n-1)$. Contains the local pieces of the m -by- n distributed matrix $\text{sub}(A)$ to be factored. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| b | (local) Pointer into the local memory to an array of size $lld_b * LOCc(jb+n-1)$. Contains the local pieces of the p -by- n matrix $\text{sub}(B)$ to be factored. |
| ib, jb | (global) The row and column indices in the global matrix B indicating the first row and the first column of the submatrix B , respectively. |

descb (global and local) array of size *dlen_*. The array descriptor for the distributed matrix *B*.

work (local)
Workspace array of size of *lwork*.

lwork (local or global)
Size of *work*, must be at least $lwork \geq \max(mb_a * (mpa0 + nqa0 + mb_a), \max((mb_a * (mb_a - 1)) / 2, (ppb0 + nqb0) * mb_a) + mb_a * mb_a, nb_b * (ppb0 + nqb0 + nb_b))$, where

```

iroffa = mod(ia-1, mb_A),
icoffa = mod(ja-1, nb_a),
iarow = indxg2p(ia, mb_a, MYROW, rsrc_a, NPROW),
iacol = indxg2p(ja, nb_a, MYCOL, csrc_a, NPCOL),
mpa0 = numroc(m+iroffa, mb_a, MYROW, iarow, NPROW),
nqa0 = numroc(m+icoffa, nb_a, MYCOL, iacol, NPCOL)
iroffb = mod(ib-1, mb_b),
icoffb = mod(jb-1, nb_b),
ibrow = indxg2p(ib, mb_b, MYROW, rsrc_b, NPROW),
ibcol = indxg2p(jb, nb_b, MYCOL, csrc_b, NPCOL),
ppb0 = numroc(p+iroffb, mb_b, MYROW, ibrow, NPROW),
nqb0 = numroc(n+icoffb, nb_b, MYCOL, ibcol, NPCOL)

```

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

and `numroc`, `indxg2p` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

a On exit, if $m \leq n$, the upper triangle of $A(ia:ia+m-1, ja+n-m:ja+n-1)$ contains the m -by- m upper triangular matrix R ; if $m \geq n$, the elements on and above the $(m-n)$ -th subdiagonal contain the m -by- n upper trapezoidal matrix R ; the remaining elements, with the array *taua*, represent the orthogonal/unitary matrix Q as a product of $\min(n, m)$ elementary reflectors (see *Application Notes* below).

taua, taub (local)
Arrays of size $LOCr(ia+m-1)$ for *taua* and $LOCc(jb+\min(p, n)-1)$ for *taub*.

The array *taua* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Q*. *taua* is tied to the distributed matrix *A*. (See *Application Notes* below).

The array *taub* contains the scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix *Z*. *taub* is tied to the distributed matrix *B*. (See *Application Notes* below).

work[0] On exit *work*[0] contains the minimum value of *lwork* required for optimum performance.

info (global)

= 0: the execution is successful.

< 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j* - 1, had an illegal value, then *info* = $-(i*100+j)$; if the *i*-th argument is a scalar and had an illegal value, then *info* = $-i$.

Application Notes

The matrix *Q* is represented as a product of elementary reflectors

$$Q = H(i_a) * H(i_a+1) * \dots * H(i_a+k-1),$$

where $k = \min(m, n)$.

Each *H*(*i*) has the form

$$H(i) = I - \tau_{ua} * v * v'$$

where *taua* is a real/complex scalar, and *v* is a real/complex vector with $v(n-k+i+1:n) = 0$ and $v(n-k+i) = 1$; $v(1:n-k+i-1)$ is stored on exit in $A(i_a+m-k+i-1, j_a:j_a+n-k+i-2)$, and *taua* in $\tau_{ua}[i_a+m-k+i-2]$. To form *Q* explicitly, use ScaLAPACK function [p?orgqr/p?ungrq](#). To use *Q* to update another matrix, use ScaLAPACK function [p?ormrq/p?unmrq](#).

The matrix *Z* is represented as a product of elementary reflectors

$$Z = H(j_b) * H(j_b+1) * \dots * H(j_b+k-1), \text{ where } k = \min(p, n).$$

Each *H*(*i*) has the form

$$H(i) = I - \tau_{ub} * v * v'$$

where *taub* is a real/complex scalar, and *v* is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$; $v(i+1:p)$ is stored on exit in $B(i_b+i:i_b+p-1, j_b+i-1)$, and *taub* in $\tau_{ub}[j_b+i-2]$. To form *Z* explicitly, use ScaLAPACK function [p?orgqr/p?ungqr](#). To use *Z* to update another matrix, use ScaLAPACK function [p?ormqr/p?unmqr](#).

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Symmetric Eigenvalue Problems: ScaLAPACK Computational Routines

To solve a symmetric eigenproblem with ScaLAPACK, you usually need to reduce the matrix to real tridiagonal form *T* and then find the eigenvalues and eigenvectors of the tridiagonal matrix *T*. ScaLAPACK includes routines for reducing the matrix to a tridiagonal form by an orthogonal (or unitary) similarity transformation $A = QTQ^H$ as well as for solving tridiagonal symmetric eigenvalue problems. These routines are listed in [Table "Computational Routines for Solving Symmetric Eigenproblems"](#).

There are different routines for symmetric eigenproblems, depending on whether you need eigenvalues only or eigenvectors as well, and on the algorithm used (either the *QTQ* algorithm, or bisection followed by inverse iteration).

Computational Routines for Solving Symmetric Eigenproblems

| Operation | Dense symmetric/ Hermitian matrix | Orthogonal/unitary matrix | Symmetric tridiagonal matrix |
|--|--------------------------------------|------------------------------|------------------------------------|
| Reduce to tridiagonal form $A = QTQ^H$ | <code>p?sytrd/p?hetrd</code> | | |
| Multiply matrix after reduction | | <code>p?ormtr/p?unmtr</code> | |
| Find all eigenvalues and eigenvectors of a tridiagonal matrix T by a <i>QTQ</i> method | | | <code>steqr2*</code> |
| Find selected eigenvalues of a tridiagonal matrix T via bisection | | | <code>p?stebz</code> |
| Find selected eigenvectors of a tridiagonal matrix T by inverse iteration | | | <code>p?stein</code> |

* This routine is described as part of auxiliary ScaLAPACK routines.

p?syngst

Reduces a complex Hermitian-definite generalized eigenproblem to standard form.

Syntax

```
void pssyngst (const MKL_INT* ibtype, const char* uplo, const MKL_INT* n, float* a,
const MKL_INT* ia, const MKL_INT* ja, const MKL_INT* desca, const float* b, const
MKL_INT* ib, const MKL_INT* jb, const MKL_INT* descb, float* scale, float* work, const
MKL_INT* lwork, MKL_INT* info);
```

```
void pdsyngst (const MKL_INT* ibtype, const char* uplo, const MKL_INT* n, double* a,
const MKL_INT* ia, const MKL_INT* ja, const MKL_INT* desca, const double* b, const
MKL_INT* ib, const MKL_INT* jb, const MKL_INT* descb, double* scale, double* work,
const MKL_INT* lwork, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

`p?syngst` reduces a complex Hermitian-definite generalized eigenproblem to standard form.

`p?syngst` performs the same function as `p?hegst`, but is based on rank 2K updates, which are faster and more scalable than triangular solves (the basis of `p?syngst`).

`p?syngst` calls `p?hegst` when `uplo='U'`, hence `p?hengst` provides improved performance only when `uplo='L'`, `ibtype=1`.

`p?syngst` also calls `p?hegst` when insufficient workspace is provided, hence `p?syngst` provides improved performance only when `lwork >= 2 * NPO * NB + NQO * NB + NB * NB`

In the following `sub(A)` denotes $A(ia:ia+n-1, ja:ja+n-1)$ and `sub(B)` denotes $B(ib:ib+n-1, jb:jb+n-1)$.

If `ibtype = 1`, the problem is $sub(A) * x = \lambda * sub(B) * x$, and `sub(A)` is overwritten by $inv(U^H) * sub(A) * inv(U)$ or $inv(L) * sub(A) * inv(L^H)$

If `ibtype = 2` or `3`, the problem is $sub(A) * sub(B) * x = \lambda * x$ or $sub(B) * sub(A) * x = \lambda * x$, and `sub(A)` is overwritten by $U * sub(A) * U^H$ or $L^H * sub(A) * L$.

`sub(B)` must have been previously factorized as $U^H * U$ or $L * L^H$ by `p?potrf`.

Input Parameters

| | |
|---------------|---|
| <i>ibtype</i> | (global) = 1: compute $\text{inv}(U^H) * \text{sub}(A) * \text{inv}(U)$ or $\text{inv}(L) * \text{sub}(A) * \text{inv}(L^H)$; = 2 or 3: compute $U * \text{sub}(A) * U^H$ or $L^H * \text{sub}(A) * L$. |
| <i>uplo</i> | (global) = 'U': Upper triangle of $\text{sub}(A)$ is stored and $\text{sub}(B)$ is factored as $U^H * U$; = 'L': Lower triangle of $\text{sub}(A)$ is stored and $\text{sub}(B)$ is factored as $L * L^H$. |
| <i>n</i> | (global) The order of the matrices $\text{sub}(A)$ and $\text{sub}(B)$. $n \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the n -by- n Hermitian distributed matrix $\text{sub}(A)$. If <i>uplo</i> = 'U', the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If <i>uplo</i> = 'L', the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. |
| <i>ia</i> | (global) A's global row index, which points to the beginning of the submatrix which is to be operated on. |
| <i>ja</i> | (global) A's global column index, which points to the beginning of the submatrix which is to be operated on. |
| <i>desca</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix A. |
| <i>b</i> | (local) Pointer into the local memory to an array of size $lld_b * LOCC(jb+n-1)$. On entry, this array contains the local pieces of the triangular factor from the Cholesky factorization of $\text{sub}(B)$, as returned by <code>p?potrf</code> . |
| <i>ib</i> | (global) B's global row index, which points to the beginning of the submatrix which is to be operated on. |
| <i>jb</i> | (global) B's global column index, which points to the beginning of the submatrix which is to be operated on. |
| <i>descb</i> | (global and local) |

Array of size $dlen_$.
 The array descriptor for the distributed matrix B .

work (local)
 Array, size ($lwork$)

lwork (local or global)
 The size of the array *work*.
lwork is local input and must be at least $lwork \geq \text{MAX}(NB * (NPO + 1), 3 * NB)$
 When $ibtype = 1$ and $uplo = 'L'$, $p?syngst$ provides improved performance when $lwork \geq 2 * NPO * NB + NQ0 * NB + NB * NB$, where $NB = mb_a = nb_a$,
 $NPO = \text{numroc}(n, NB, 0, 0, NPROW)$,
 $NQ0 = \text{numroc}(n, NB, 0, 0, NPROW)$,
 numroc is a ScaLAPACK tool functions
 $MYROW, MYCOL, NPROW$ and $NPOL$ can be determined by calling the subroutine `blacs_gridinfo`.
 If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the routine only calculates the optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `p?erbla`.

Output Parameters

a On exit, if $info = 0$, the transformed matrix, stored in the same format as `sub(A)`.

scale (global)
 Amount by which the eigenvalues should be scaled to compensate for the scaling performed in this routine. At present, *scale* is always returned as 1.0, it is returned here to allow for future enhancement.

work (local)
 Array, size ($lwork$)
 On exit, $work[0]$ returns the minimal and optimal $lwork$.

info (global)
 = 0: successful exit
 < 0: If the i -th argument is an array and the j -th entry had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

p?syntrd

Reduces a real symmetric matrix to symmetric tridiagonal form.

Syntax

```
void pssytrd (const char* uplo, const MKL_INT* n, float* a, const MKL_INT* ia, const
MKL_INT* ja, const MKL_INT* desca, float* d, float* e, float* tau, float* work, const
MKL_INT* lwork, MKL_INT* info);
```

```
void pdsytrd (const char* uplo, const MKL_INT* n, double* a, const MKL_INT* ia, const
MKL_INT* ja, const MKL_INT* desca, double* d, double* e, double* tau, double* work,
const MKL_INT* lwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

`p?sytrd` is a prototype version of `p?sytrd` which uses tailored codes (either the serial, `?sytrd`, or the parallel code, `p?sytrd`) when the workspace provided by the user is adequate.

`p?sytrd` reduces a real symmetric matrix $\text{sub}(A)$ to symmetric tridiagonal form T by an orthogonal similarity transformation:

$$Q' * \text{sub}(A) * Q = T, \text{ where } \text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1).$$

Features

`p?sytrd` is faster than `p?sytrd` on almost all matrices, particularly small ones (i.e. $n < 500 * \text{sqrt}(P)$), provided that enough workspace is available to use the tailored codes.

The tailored codes provide performance that is essentially independent of the input data layout.

The tailored codes place no restrictions on `ia`, `ja`, `MB` or `NB`. At present, `ia`, `ja`, `MB` and `NB` are restricted to those values allowed by `p?hetrd` to keep the interface simple (see the Application Notes section for more information about the restrictions).

Input Parameters

| | |
|-------------------|--|
| <code>uplo</code> | (global) |
| | Specifies whether the upper or lower triangular part of the symmetric matrix $\text{sub}(A)$ is stored: |
| | = 'U': Upper triangular |
| | = 'L': Lower triangular |
| <code>n</code> | (global) |
| | The number of rows and columns to be operated on, i.e. the order of the distributed submatrix $\text{sub}(A)$. $n \geq 0$. |
| <code>a</code> | (local) |
| | Pointer into the local memory to an array of size $ld_a * LOCc(\text{ja}+n-1)$. |
| | On entry, this array contains the local pieces of the symmetric distributed matrix $\text{sub}(A)$. If <code>uplo = 'U'</code> , the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If <code>uplo = 'L'</code> , the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. |
| <code>ia</code> | (global) |

| | |
|--------------|---|
| | The row index in the global array <i>a</i> indicating the first row of sub(<i>A</i>). |
| <i>ja</i> | (global) The column index in the global array <i>a</i> indicating the first column of sub(<i>A</i>). |
| <i>desca</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Array, size (<i>lwork</i>) |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> . <i>lwork</i> is local input and must be at least $lwork \geq \text{MAX}(NB * (NP + 1), 3 * NB)$ For optimal performance, greater workspace is needed, i.e. $lwork \geq 2 * (ANB + 1) * (4 * NPS + 2) + (NPS + 4) * NPS$ $ANB = \text{pjlaenv}(ICTXT, 3, 'p?sytttrd', 'L', 0, 0, 0, 0)$ $ICTXT = \text{desca}(ctxt_)$ $SQNPC = \text{INT}(\text{sqrt}(\text{REAL}(NPROW * NPCOL)))$ <i>numroc</i> is a ScaLAPACK tool function. <i>pjlaenv</i> is a ScaLAPACK environmental inquiry function. <i>NPROW</i> and <i>NPCOL</i> can be determined by calling the subroutine <i>blacs_gridinfo</i> . |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, if <i>uplo</i> = 'U', the diagonal and first superdiagonal of sub(<i>A</i>) are overwritten by the corresponding elements of the tridiagonal matrix <i>T</i> , and the elements above the first superdiagonal, with the array <i>tau</i> , represent the orthogonal matrix <i>Q</i> as a product of elementary reflectors; if <i>uplo</i> = 'L', the diagonal and first subdiagonal of sub(<i>A</i>) are overwritten by the corresponding elements of the tridiagonal matrix <i>T</i> , and the elements below the first subdiagonal, with the array <i>tau</i> , represent the orthogonal matrix <i>Q</i> as a product of elementary reflectors. See Further Details . |
| <i>d</i> | (local) Array, size $\text{LOCc}(ja+n-1)$ The diagonal elements of the tridiagonal matrix <i>T</i> : $d(i) = A(i,i)$. <i>d</i> is tied to the distributed matrix <i>A</i> . |
| <i>e</i> | (local) Array, size $\text{LOCc}(ja+n-1)$ if <i>uplo</i> = 'U', $\text{LOCc}(ja+n-2)$ otherwise. The off-diagonal elements of the tridiagonal matrix <i>T</i> : $e(i) = A(i,i+1)$ if <i>uplo</i> = 'U', $e(i) = A(i+1,i)$ if <i>uplo</i> = 'L'. <i>e</i> is tied to the distributed matrix <i>A</i> . |

| | |
|-------------|---|
| <i>tau</i> | (local) Array, size $LOCc(ja+n-1)$. This array contains the scalar factors <i>tau</i> of the elementary reflectors. <i>tau</i> is tied to the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Array, size (<i>lwork</i>) On exit, <i>work</i> [0] returns the optimal <i>lwork</i> . |
| <i>info</i> | (global) = 0: successful exit < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry had an illegal value, then <i>info</i> = $-(i*100+j)$, if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = $-i$. |

Application Notes

If *uplo* = 'U', the matrix *Q* is represented as a product of elementary reflectors

$$Q = H(n-1) \dots H(2) H(1).$$

Each *H*(*i*) has the form

$H(i) = I - \tau * v * v'$, where *tau* is a complex scalar, and *v* is a complex vector with $v(i+1:n) = 0$ and $v(i) = 1$; $v(1:i-1)$ is stored on exit in $A(ia:ia+i-2, ja+i)$, and *tau* in $\tau(ja+i-1)$.

If *uplo* = 'L', the matrix *Q* is represented as a product of elementary reflectors

$$Q = H(1) H(2) \dots H(n-1).$$

Each *H*(*i*) has the form

$H(i) = I - \tau * v * v'$, where *tau* is a complex scalar, and *v* is a complex vector with $v(1:i) = 0$ and $v(i+1) = 1$; $v(i+2:n)$ is stored on exit in $A(ia+i+1:ia+n-1, ja+i-1)$, and *tau* in $\tau(ja+i-1)$.

The contents of sub(*A*) on exit are illustrated by the following examples with *n* = 5:

if *uplo* = 'U':

$$\begin{pmatrix} d & e & v2 & v3 & v4 \\ & d & e & v3 & v4 \\ & & d & e & v3 \\ & & & d & e \\ & & & & d \end{pmatrix}$$

if *uplo* = 'L':

$$\begin{pmatrix} d & & & & \\ e & d & & & \\ v1 & e & d & & \\ v1 & v2 & e & d & \\ v1 & v2 & v3 & e & d \end{pmatrix}$$

where *d* and *e* denote diagonal and off-diagonal elements of *T*, and *v_i* denotes an element of the vector defining *H*(*i*).

Alignment requirements

The distributed submatrix $\text{sub}(A)$ must verify some alignment properties, namely the following expression should be true:

($mb_a = nb_a$ and $IROFFA = ICOFFA$ and $IROFFA = 0$) with $IROFFA = \text{mod}(ia-1, mb_a)$, and $ICOFFA = \text{mod}(ja-1, nb_a)$.

p?sytrd

Reduces a symmetric matrix to real symmetric tridiagonal form by an orthogonal similarity transformation.

Syntax

```
void pssytrd (char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *d , float *e , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdsytrd (char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *d , double *e , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?sytrd` function reduces a real symmetric matrix $\text{sub}(A)$ to symmetric tridiagonal form T by an orthogonal similarity transformation:

$$Q^* \text{sub}(A) Q = T,$$

where $\text{sub}(A) = A(ia:ia+n-1, ja:ja+n-1)$.

Input Parameters

| | |
|-------------------|---|
| <code>uplo</code> | (global) Specifies whether the upper or lower triangular part of the symmetric matrix $\text{sub}(A)$ is stored: If <code>uplo = 'U'</code> , upper triangular If <code>uplo = 'L'</code> , lower triangular |
| <code>n</code> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <code>a</code> | (local) Pointer into the local memory to an array of size <code>lld_a*LOCc(ja+n-1)</code> . On entry, this array contains the local pieces of the symmetric distributed matrix $\text{sub}(A)$. If <code>uplo = 'U'</code> , the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If <code>uplo = 'L'</code> , the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. See <i>Application Notes</i> below. |

| | |
|---------------|--|
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least: $lwork \geq \max(NB * (np + 1), 3 * NB)$, where $NB = mb_a = nb_a$, $np = \text{numroc}(n, NB, MYROW, iarow, NPROW)$, $iarow = \text{indxg2p}(ia, NB, MYROW, rsrc_a, NPROW)$. <i>indxg2p</i> and <i>numroc</i> are ScaLAPACK tool functions; <i>MYROW</i> , <i>MYCOL</i> , <i>NPROW</i> and <i>NPCOL</i> can be determined by calling the function <i>blacs_gridinfo</i> . If <i>lwork</i> = -1, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <i>pxerbla</i> . |

Output Parameters

| | |
|------------|--|
| <i>a</i> | On exit, if <i>uplo</i> = 'U', the diagonal and first superdiagonal of <i>sub(A)</i> are overwritten by the corresponding elements of the tridiagonal matrix <i>T</i> , and the elements above the first superdiagonal, with the array <i>tau</i> , represent the orthogonal matrix <i>Q</i> as a product of elementary reflectors; if <i>uplo</i> = 'L', the diagonal and first subdiagonal of <i>sub(A)</i> are overwritten by the corresponding elements of the tridiagonal matrix <i>T</i> , and the elements below the first subdiagonal, with the array <i>tau</i> , represent the orthogonal matrix <i>Q</i> as a product of elementary reflectors. See <i>Application Notes</i> below. |
| <i>d</i> | (local) Arrays of size $LOCc(ja+n-1)$. The diagonal elements of the tridiagonal matrix <i>T</i> : $d[i] = A(i+1, i+1), 0 \leq i < LOCc(ja+n-1)$. <i>d</i> is tied to the distributed matrix <i>A</i> . |
| <i>e</i> | (local) Arrays of size $LOCc(ja+n-1)$ if <i>uplo</i> = 'U', $LOCc(ja+n-2)$ otherwise. The off-diagonal elements of the tridiagonal matrix <i>T</i> : $e[i] = A(i+1, i+2), 0 \leq i < LOCc(ja+n-1)$ if <i>uplo</i> = 'U', $e[i] = A(i+2, i+1)$ if <i>uplo</i> = 'L'. <i>e</i> is tied to the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) |

Arrays of size $LOCc(ja+n-1)$. This array contains the scalar factors of the elementary reflectors. tau is tied to the distributed matrix A .

$work[0]$

On exit $work[0]$ contains the minimum value of $lwork$ required for optimum performance.

$info$

(global)

= 0: the execution is successful.

< 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

Application Notes

If $uplo = 'U'$, the matrix Q is represented as a product of elementary reflectors

$$Q = H(n-1) \dots H(2) H(1).$$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v',$$

where τ is a real scalar, and v is a real vector with $v(i+1:n) = 0$ and $v(i) = 1$; $v(1:i-1)$ is stored on exit in $A(ia:ia+i-2, ja+i)$, and τ in $tau[ja+i-2]$.

If $uplo = 'L'$, the matrix Q is represented as a product of elementary reflectors

$$Q = H(1) H(2) \dots H(n-1).$$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v',$$

where τ is a real scalar, and v is a real vector with $v(1:i) = 0$ and $v(i+1) = 1$; $v(i+2:n)$ is stored on exit in $A(ia+i+1:ia+n-1, ja+i-1)$, and τ in $tau[ja+i-2]$.

The contents of $sub(A)$ on exit are illustrated by the following examples with $n = 5$:

If $uplo = 'U'$:

$$\begin{bmatrix} d & e & v2 & v3 & v4 \\ & d & e & v3 & v4 \\ & & d & e & v4 \\ & & & d & e \\ & & & & d \end{bmatrix}$$

If $uplo = 'L'$:

$$\begin{bmatrix} d & & & & \\ e & d & & & \\ v_1 & e & d & & \\ v_1 & v_2 & e & d & \\ v_1 & v_2 & v_3 & e & d \end{bmatrix}$$

where d and e denote diagonal and off-diagonal elements of T , and v_i denotes an element of the vector defining $H(i)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ormtr

Multiplies a general matrix by the orthogonal transformation matrix from a reduction to tridiagonal form determined by p?sytrd.

Syntax

```
void psormtr (char *side , char *uplo , char *trans , MKL_INT *m , MKL_INT *n , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );
void pdormtr (char *side , char *uplo , char *trans , MKL_INT *m , MKL_INT *n , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

This function overwrites the general real distributed m -by- n matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|---------------------|---------------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q*\text{sub}(C)$ | $\text{sub}(C)*Q$ |
| $trans = 'T':$ | $Q^T*\text{sub}(C)$ | $\text{sub}(C)*Q^T$ |

where Q is a real orthogonal distributed matrix of order nq , with $nq = m$ if $side = 'L'$ and $nq = n$ if $side = 'R'$.

Q is defined as the product of nq elementary reflectors, as returned by [p?sytrd](#).

If $uplo = 'U'$, $Q = H(nq-1) \dots H(2) H(1)$;

If $uplo = 'L'$, $Q = H(1) H(2) \dots H(nq-1)$.

Input Parameters

$side$ (global)
 $= 'L'$: Q or Q^T is applied from the left.

| | |
|---------------|--|
| | = 'R': Q or Q^T is applied from the right. |
| <i>trans</i> | (global) = 'N', no transpose, Q is applied. = 'T', transpose, Q^T is applied. |
| <i>uplo</i> | (global) = 'U': Upper triangle of $A(ia:*, ja:*)$ contains elementary reflectors from $p?sytrd$; = 'L': Lower triangle of $A(ia:*, ja:*)$ contains elementary reflectors from $p?sytrd$ |
| <i>m</i> | (global) The number of rows in the distributed matrix $sub(C)$ ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $sub(C)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOcc(ja+m-1)$ if $side = 'L'$, and $lld_a * LOcc(ja+n-1)$ if $side = 'R'$. Contains the vectors that define the elementary reflectors, as returned by $p?sytrd$. If $side='L'$, $lld_a \geq \max(1, LOCr(ia+m-1))$; If $side = 'R'$, $lld_a \geq \max(1, LOCr(ia+n-1))$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size of $ltau$ where if $side = 'L'$ and $uplo = 'U'$, $ltau = LOcc(m_a)$, if $side = 'L'$ and $uplo = 'L'$, $ltau = LOcc(ja+m-2)$, if $side = 'R'$ and $uplo = 'U'$, $ltau = LOcc(n_a)$, if $side = 'R'$ and $uplo = 'L'$, $ltau = LOcc(ja+n-2)$. $tau[i]$ must contain the scalar factor of the elementary reflector $H(i+1)$, as returned by $p?sytrd$ ($0 \leq i < ltau$). tau is tied to the distributed matrix A . |
| <i>c</i> | (local) Pointer into the local memory to an array of size $lld_c * LOcc(jc+n-1)$. Contains the local pieces of the distributed matrix $sub(C)$. |
| <i>ic, jc</i> | (global) The row and column indices in the global matrix C indicating the first row and the first column of the submatrix C , respectively. |
| <i>desc</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix C . |

work (local)
Workspace array of size *lwork*.

lwork (local or global) size of *work*, must be at least:

```

if uplo = 'U',
  iaa= ia; jaa= ja+1, icc= ic; jcc= jc;
else uplo = 'L',
  iaa= ia+1, jaa= ja;
If side = 'L',
  icc= ic+1; jcc= jc;
else icc= ic; jcc= jc+1;
end if
end if
If side = 'L',
mi= m-1; ni= n
lwork ≥ max((nb_a*(nb_a-1))/2, (nqc0 + mpc0)*nb_a) + nb_a*nb_a
else
If side = 'R',
mi= m; ni = n-1;
lwork ≥ max((nb_a*(nb_a-1))/2, (nqc0 +
max(npa0+numroc(numroc(ni+icoffc, nb_a, 0, 0, NPCOL), nb_a,
0, 0, lcmq), mpc0))*nb_a)+ nb_a*nb_a
end if
where lcmq = lcm/NPCOL with lcm = ilcm(NPROW, NPCOL),
iroffa = mod(iaa-1, mb_a),
icoffa = mod(jaa-1, nb_a),
iarow = indxg2p(iaa, mb_a, MYROW, rsrc_a, NPROW),
npa0 = numroc(ni+iroffa, mb_a, MYROW, iarow, NPROW),
iroffc = mod(icc-1, mb_c),
icoffc = mod(jcc-1, nb_c),
icrow = indxg2p(icc, mb_c, MYROW, rsrc_c, NPROW),
iccol = indxg2p(jcc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0 = numroc(mi+iroffc, mb_c, MYROW, icrow, NPROW),
nqc0 = numroc(ni+icoffc, nb_c, MYCOL, iccol, NPCOL),

```

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`ilcm`, `indxcg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPOL` can be determined by calling the function `blacs_gridinfo`. If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q \cdot \text{sub}(C)$, or $Q' \cdot \text{sub}(C)$, or $\text{sub}(C) \cdot Q'$, or $\text{sub}(C) \cdot Q$. |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $\text{info} = -(i \cdot 100 + j)$; if the i -th argument is a scalar and had an illegal value, then $\text{info} = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?hengst

Reduces a complex Hermitian-definite generalized eigenproblem to standard form.

Syntax

```
void pchengst (const MKL_INT* ibtype, const char* uplo, const MKL_INT* n, MKL_Complex8*
a, const MKL_INT* ia, const MKL_INT* ja, const MKL_INT* desca, const MKL_Complex8* b,
const MKL_INT* ib, const MKL_INT* jb, const MKL_INT* descb, float* scale, MKL_Complex8*
work, const MKL_INT* lwork, MKL_INT* info);
```

```
void pzhengst (const MKL_INT* ibtype, const char* uplo, const MKL_INT* n,
MKL_Complex16* a, const MKL_INT* ia, const MKL_INT* ja, const MKL_INT* desca, const
MKL_Complex16* b, const MKL_INT* ib, const MKL_INT* jb, const MKL_INT* descb, double*
scale, MKL_Complex16* work, const MKL_INT* lwork, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

`p?hengst` reduces a complex Hermitian-definite generalized eigenproblem to standard form.

`p?hengst` performs the same function as `p?hegst`, but is based on rank 2K updates, which are faster and more scalable than triangular solves (the basis of `p?hengst`).

`p?hengst` calls `p?hegst` when `uplo='U'`, hence `p?hengst` provides improved performance only when `uplo='L'` and `ibtype=1`.

`p?hengst` also calls `p?hegst` when insufficient workspace is provided, hence `p?hengst` provides improved performance only when `lwork` is sufficient (as described in the parameter descriptions).

In the following `sub(A)` denotes the submatrix $A(ia:ia+n-1, ja:ja+n-1)$ and `sub(B)` denotes the submatrix $B(ib:ib+n-1, jb:jb+n-1)$.

If `ibtype = 1`, the problem is $sub(A) * x = lambda * sub(B) * x$, and `sub(A)` is overwritten by $inv(U^H) * sub(A) * inv(U)$ or $inv(L) * sub(A) * inv(L^H)$

If `ibtype = 2` or `3`, the problem is $sub(A) * sub(B) * x = lambda * x$ or $sub(B) * sub(A) * x = lambda * x$, and `sub(A)` is overwritten by $U * sub(A) * U^H$ or $L^H * sub(A) * L$.

`sub(B)` must have been previously factorized as $U^H * U$ or $L * L^H$ by `p?potrf`.

Input Parameters

| | |
|---------------------|---|
| <code>ibtype</code> | (global) = 1: compute $inv(U^H) * sub(A) * inv(U)$ or $inv(L) * sub(A) * inv(L^H)$; = 2 or 3: compute $U * sub(A) * U^H$ or $L^H * sub(A) * L$. |
| <code>uplo</code> | (global) = 'U': Upper triangle of <code>sub(A)</code> is stored and <code>sub(B)</code> is factored as $U^H * U$; = 'L': Lower triangle of <code>sub(A)</code> is stored and <code>sub(B)</code> is factored as $L * L^H$. |
| <code>n</code> | (global) The order of the matrices <code>sub(A)</code> and <code>sub(B)</code> . $n \geq 0$. |
| <code>a</code> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the n -by- n Hermitian distributed matrix <code>sub(A)</code> . If <code>uplo = 'U'</code> , the leading n -by- n upper triangular part of <code>sub(A)</code> contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If <code>uplo = 'L'</code> , the leading n -by- n lower triangular part of <code>sub(A)</code> contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. |
| <code>ia</code> | (global) Global row index of matrix A , which points to the beginning of the submatrix on which to operate. |
| <code>ja</code> | (global) Global column index of matrix A , which points to the beginning of the submatrix on which to operate. |
| <code>desca</code> | (global and local) Array of size <code>dlen_</code> . The array descriptor for the distributed matrix A . |
| <code>b</code> | (local) Pointer into the local memory to an array of size $lld_b * LOCC(jb+n-1)$. |
| <code>ib</code> | (global) |

| | |
|--------------|---|
| | Global row index of matrix B , which points to the beginning of the submatrix on which to operate. |
| <i>jb</i> | (global) Global column index of matrix B , which points to the beginning of the submatrix on which to operate. |
| <i>descb</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix B . |
| <i>work</i> | (local) Array, size (<i>lwork</i>) On exit, <i>work</i> (1) returns the minimal and optimal <i>lwork</i> . |
| <i>lwork</i> | (local) The size of the array <i>work</i> . <i>lwork</i> is local input and must be at least $lwork \geq \text{MAX}(NB * (NPO + 1), 3 * NB)$. When <i>ibtype</i> = 1 and <i>uplo</i> = 'L', p?hengst provides improved performance when $lwork \geq 2 * NPO * NB + NQ0 * NB + NB * NB$, where $NB = mb_a = nb_a$, $NPO = \text{numroc}(n, NB, 0, 0, NPROW)$, $NQ0 = \text{numroc}(n, NB, 0, 0, NPROW)$, and <i>numroc</i> is a ScaLAPACK tool function. MYROW, MYCOL, NPROW and NPCOL can be determined by calling the subroutine <i>blacs_gridinfo</i> . If <i>lwork</i> = -1, then <i>lwork</i> is global input and a workspace query is assumed; the routine only calculates the optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <i>p?erbla</i> . |

Output Parameters

| | |
|--------------|---|
| <i>a</i> | On exit, if <i>info</i> = 0, the transformed matrix, stored in the same format as sub(A). |
| <i>scale</i> | (global) Amount by which the eigenvalues should be scaled to compensate for the scaling performed in this routine. <i>scale</i> is always returned as 1.0. |
| <i>work</i> | On exit, <i>work</i> [0] returns the minimal and optimal <i>lwork</i> . |
| <i>info</i> | (global) = 0: successful exit < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -entry had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

p?hentr

Reduces a complex Hermitian matrix to Hermitian tridiagonal form.

Syntax

```
void pchentrd (const char* uplo, const MKL_INT* n, MKL_Complex8* a, const MKL_INT* ia,
const MKL_INT* ja, const MKL_INT* desca, float* d, float* e, MKL_Complex8* tau,
MKL_Complex8* work, const MKL_INT* lwork, float* rwork, const MKL_INT* lrwork, MKL_INT*
info);
```

```
void pzchentrd (const char* uplo, const MKL_INT* n, MKL_Complex16* a, const MKL_INT* ia,
const MKL_INT* ja, const MKL_INT* desca, double* d, double* e, MKL_Complex16* tau,
MKL_Complex16* work, const MKL_INT* lwork, double* rwork, const MKL_INT* lrwork,
MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

p?hentr is a prototype version of p?hetrd which uses tailored codes (either the serial, ?hetrd, or the parallel code, p?hettrd) when adequate workspace is provided.

p?hentr reduces a complex Hermitian matrix $\text{sub}(A)$ to Hermitian tridiagonal form T by an unitary similarity transformation:

$$Q^* \text{sub}(A) Q = T, \text{ where } \text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1).$$

p?hentr is faster than p?hetrd on almost all matrices, particularly small ones (i.e. $n < 500 * \text{sqrt}(P)$), provided that enough workspace is available to use the tailored codes.

The tailored codes provide performance that is essentially independent of the input data layout.

The tailored codes place no restrictions on ia , ja , MB or NB. At present, ia , ja , MB and NB are restricted to those values allowed by p?hetrd to keep the interface simple (see the Application Notes section for more information about the restrictions).

Input Parameters

| | |
|-------------|--|
| <i>uplo</i> | (global) Specifies whether the upper or lower triangular part of the Hermitian matrix $\text{sub}(A)$ is stored: = 'U': Upper triangular = 'L': Lower triangular |
| <i>n</i> | (global) The number of rows and columns to be operated on, i.e. the order of the distributed submatrix $\text{sub}(A)$. $n \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $\text{lld}_a * \text{LOCc}(\text{ja}+n-1)$. On entry, this array contains the local pieces of the Hermitian distributed matrix $\text{sub}(A)$. If $\text{uplo} = 'U'$, the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and its strictly |

lower triangular part is not referenced. If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of sub(*A*) contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced.

| | |
|---------------|---|
| <i>ia</i> | (global) The row index in the global array <i>a</i> indicating the first row of sub(<i>A</i>). |
| <i>ja</i> | (global) The column index in the global array <i>a</i> indicating the first column of sub(<i>A</i>). |
| <i>desca</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Array, size (<i>lwork</i>) |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> . <i>lwork</i> is local input and must be at least $lwork \geq \text{MAX}(NB * (NP + 1), 3 * NB)$. For optimal performance, greater workspace is needed: $lwork \geq 2 * (ANB + 1) * (4 * NPS + 2) + (NPS + 4) * NPS$ $ANB = \text{pjlaenv}(ICTXT, 3, 'p?hettrd', 'L', 0, 0, 0, 0)$ $ICTXT = \text{desca}(ctxt_)$ $SQNPC = \text{INT}(\text{sqrt}(\text{REAL}(NPROW * NPCOL)))$ $NPS = \text{MAX}(\text{numroc}(n, 1, 0, 0, SQNPC), 2 * ANB)$ <i>numroc</i> is a ScaLAPACK tool function. <i>pjlaenv</i> is a ScaLAPACK environmental inquiry function. <i>NPROW</i> and <i>NPCOL</i> can be determined by calling the subroutine <i>blacs_gridinfo</i> . |
| <i>rwork</i> | (local) Array, size (<i>lrwork</i>) |
| <i>lrwork</i> | (local or global) The size of the array <i>rwork</i> . <i>lrwork</i> is local input and must be at least $lrwork \geq 1$. For optimal performance, greater workspace is needed, i.e. $lrwork \geq \text{MAX}(2 * n)$ |

Output Parameters

| | |
|--------------|---|
| <i>a</i> | On exit, if <i>uplo</i> = 'U', the diagonal and first superdiagonal of sub(<i>A</i>) are overwritten by the corresponding elements of the tridiagonal matrix <i>T</i> , and the elements above the first superdiagonal, with the array <i>tau</i> , represent the unitary matrix <i>Q</i> as a product of elementary reflectors; if <i>uplo</i> = 'L', the diagonal and first subdiagonal of sub(<i>A</i>) are overwritten by the corresponding elements of the tridiagonal matrix <i>T</i> , and the elements below the first subdiagonal, with the array <i>tau</i> , represent the unitary matrix <i>Q</i> as a product of elementary reflectors. See Application Notes. |
| <i>d</i> | (local) Array, size LOCC(<i>ja+n-1</i>) The diagonal elements of the tridiagonal matrix <i>T</i> : $d[i - 1] = A(i,i)$. <i>d</i> is tied to the distributed matrix <i>A</i> . |
| <i>e</i> | (local) Array, size LOCC(<i>ja+n-1</i>) if <i>uplo</i> = 'U', LOCC(<i>ja+n-2</i>) otherwise. The off-diagonal elements of the tridiagonal matrix <i>T</i> : $e[i - 1] = A(i,i+1)$ if <i>uplo</i> = 'U', $e[i - 1] = A(i+1,i)$ if <i>uplo</i> = 'L'. <i>e</i> is tied to the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array, size LOCC(<i>ja+n-1</i>). This array contains the scalar factors <i>tau</i> of the elementary reflectors. <i>tau</i> is tied to the distributed matrix <i>A</i> . |
| <i>work</i> | On exit, <i>work</i> [0] returns the optimal <i>lwork</i> . |
| <i>rwork</i> | On exit, <i>rwork</i> [0] returns the optimal <i>lrwork</i> . |
| <i>info</i> | (global) = 0: successful exit < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry had an illegal value, then <i>info</i> = -(<i>j</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

Application Notes

If *uplo* = 'U', the matrix *Q* is represented as a product of elementary reflectors

$$Q = H(n-1) \dots H(2) H(1).$$

Each *H*(*i*) has the form

$H(i) = I - \tau * v * v'$, where *tau* is a complex scalar, and *v* is a complex vector with $v(i+1:n) = 0$ and $v(i) = 1$; $v(1:i-1)$ is stored on exit in $A(ia:ia+i-2,ja+i)$, and *tau* in $\tau(ja+i-1)$.

If *uplo* = 'L', the matrix *Q* is represented as a product of elementary reflectors

$$Q = H(1) H(2) \dots H(n-1).$$

Each *H*(*i*) has the form

$H(i) = I - \tau * v * v'$, where τ is a complex scalar, and v is a complex vector with $v(1:i) = 0$ and $v(i+1) = 1$; $v(i+2:n)$ is stored on exit in $A(ia+i+1:ia+n-1, ja+i-1)$, and τ in $\tau(ja+i-1)$.

The contents of sub(A) on exit are illustrated by the following examples with $n = 5$:

if $uplo = 'U'$:

$$\begin{pmatrix} d & e & v2 & v3 & v4 \\ & d & e & v3 & v4 \\ & & d & e & v3 \\ & & & d & e \\ & & & & d \end{pmatrix}$$

if $uplo = 'L'$:

$$\begin{pmatrix} d & & & & \\ e & d & & & \\ v1 & e & d & & \\ v1 & v2 & e & d & \\ v1 & v2 & v3 & e & d \end{pmatrix}$$

where d and e denote diagonal and off-diagonal elements of T , and v_i denotes an element of the vector defining $H(i)$.

Alignment requirements

The distributed submatrix sub(A) must verify some alignment properties, namely the following expression should be true:

($mb_a = nb_a$ and $IROFFA = ICOFFA$ and $IROFFA = 0$) with $IROFFA = \text{mod}(ia-1, mb_a)$, and $ICOFFA = \text{mod}(ja-1, nb_a)$.

p?hetrd

Reduces a Hermitian matrix to Hermitian tridiagonal form by a unitary similarity transformation.

Syntax

```
void pchetrd (char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *d , float *e , MKL_Complex8 *tau , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pzhetrd (char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *d , double *e , MKL_Complex16 *tau , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?hetrd function reduces a complex Hermitian matrix sub(A) to Hermitian tridiagonal form T by a unitary similarity transformation:

$$Q^* \text{sub}(A) Q = T$$

where $\text{sub}(A) = A(ia:ia+n-1, ja:ja+n-1)$.

Input Parameters

| | |
|---------------|--|
| <i>uplo</i> | (global) Specifies whether the upper or lower triangular part of the Hermitian matrix sub(A) is stored: If <i>uplo</i> = 'U', upper triangular If <i>uplo</i> = 'L', lower triangular |
| <i>n</i> | (global) The order of the distributed matrix sub(A) ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(j_a + n - 1)$. On entry, this array contains the local pieces of the Hermitian distributed matrix sub(A). If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of sub(A) contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i> -by- <i>n</i> lower triangular part of sub(A) contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. (see <i>Application Notes</i> below). |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A. |
| <i>work</i> | (local) Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least: $lwork \geq \max(NB * (np + 1), 3 * NB)$ where $NB = mb_a = nb_a$, $np = \text{numroc}(n, NB, MYROW, iarow, NPROW)$, $iarow = \text{indxg2p}(ia, NB, MYROW, rsrc_a, NPROW)$. <i>indxg2p</i> and <i>numroc</i> are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the function <i>blacs_gridinfo</i> . If <i>lwork</i> = -1, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <i>pxerbla</i> . |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, If <i>uplo</i> = 'U', the diagonal and first superdiagonal of sub(A) are overwritten by the corresponding elements of the tridiagonal matrix <i>T</i> , and the elements above the first superdiagonal, with the array <i>tau</i> , represent |
|----------|---|

the unitary matrix Q as a product of elementary reflectors; if $uplo = 'L'$, the diagonal and first subdiagonal of $sub(A)$ are overwritten by the corresponding elements of the tridiagonal matrix T , and the elements below the first subdiagonal, with the array tau , represent the unitary matrix Q as a product of elementary reflectors (see *Application Notes* below).

| | |
|-----------|--|
| d | (local) Arrays of size $LOCc(ja+n-1)$. The diagonal elements of the tridiagonal matrix T : $d[i] = A(i+1, i+1)$, $0 \leq i < LOCc(ja+n-1)$. d is tied to the distributed matrix A . |
| e | (local) Arrays of size $LOCc(ja+n-1)$ if $uplo = 'U'$; $LOCc(ja+n-2)$ - otherwise. The off-diagonal elements of the tridiagonal matrix T : $e[i] = A(i+1, i+2)$, $0 \leq i < LOCc(ja+n-1)$ if $uplo = 'U'$, $e[i] = A(i+2, i+1)$ if $uplo = 'L'$. e is tied to the distributed matrix A . |
| tau | (local) Array of size $LOCc(ja+n-1)$. This array contains the scalar factors of the elementary reflectors. tau is tied to the distributed matrix A . |
| $work[0]$ | On exit $work[0]$ contains the minimum value of $lwork$ required for optimum performance. |
| $info$ | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

Application Notes

If $uplo = 'U'$, the matrix Q is represented as a product of elementary reflectors

$$Q = H(n-1) * \dots * H(2) * H(1).$$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v',$$

where τ is a complex scalar, and v is a complex vector with $v(i+1:n) = 0$ and $v(i) = 1$; $v(1:i-1)$ is stored on exit in $A(ia:ia+i-2, ja+i)$, and τ in $tau[ja+i-2]$.

If $uplo = 'L'$, the matrix Q is represented as a product of elementary reflectors

$$Q = H(1) * H(2) * \dots * H(n-1).$$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v',$$

where τ is a complex scalar, and v is a complex vector with $v(1:i) = 0$ and $v(i+1) = 1$; $v(i+2:n)$ is stored on exit in $A(ia+i+1:ia+n-1, ja+i-1)$, and τ in $tau[ja+i-2]$.

The contents of $\text{sub}(A)$ on exit are illustrated by the following examples with $n = 5$:

If $\text{uplo} = 'U'$:

$$\begin{bmatrix} d & e & v2 & v3 & v4 \\ & d & e & v3 & v4 \\ & & d & e & v4 \\ & & & d & e \\ & & & & d \end{bmatrix}$$

If $\text{uplo} = 'L'$:

$$\begin{bmatrix} d & & & & \\ e & d & & & \\ v1 & e & d & & \\ v1 & v2 & e & d & \\ v1 & v2 & v3 & e & d \end{bmatrix}$$

where d and e denote diagonal and off-diagonal elements of T , and v_i denotes an element of the vector defining $H(i)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unmtr

Multiplies a general matrix by the unitary transformation matrix from a reduction to tridiagonal form determined by p?hetrd.

Syntax

```
void pcunmtr (char *side , char *uplo , char *trans , MKL_INT *m , MKL_INT *n ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pzunmtr (char *side , char *uplo , char *trans , MKL_INT *m , MKL_INT *n ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

This function overwrites the general complex distributed m -by- n matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|-----------------------|-----------------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| $trans = 'C':$ | $Q^H * \text{sub}(C)$ | $\text{sub}(C) * Q^H$ |

where Q is a complex unitary distributed matrix of order nq , with $nq = m$ if $side = 'L'$ and $nq = n$ if $side = 'R'$.

Q is defined as the product of $nq-1$ elementary reflectors, as returned by [p?hetrd](#).

If $uplo = 'U'$, $Q = H(nq-1) \dots H(2) H(1)$;

If $uplo = 'L'$, $Q = H(1) H(2) \dots H(nq-1)$.

Input Parameters

| | |
|---------------|---|
| <i>side</i> | (global) $= 'L'$: Q or Q^H is applied from the left. $= 'R'$: Q or Q^H is applied from the right. |
| <i>trans</i> | (global) $= 'N'$, no transpose, Q is applied. $= 'C'$, conjugate transpose, Q^H is applied. |
| <i>uplo</i> | (global) $= 'U'$: Upper triangle of $A(ia:*, ja:*)$ contains elementary reflectors from p?hetrd ; $= 'L'$: Lower triangle of $A(ia:*, ja:*)$ contains elementary reflectors from p?hetrd |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(C)$ ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(C)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCc(ja+m-1)$ if $side = 'L'$, and $lld_a * LOCc(ja+n-1)$ if $side = 'R'$. Contains the vectors which define the elementary reflectors, as returned by p?hetrd . If $side = 'L'$, $lld_a \geq \max(1, LOCr(ia+m-1))$; If $side = 'R'$, $lld_a \geq \max(1, LOCr(ia+n-1))$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) |

Array of size of *ltau* where

```
If side = 'L' and uplo = 'U', ltau = LOCc(m_a),
if side = 'L' and uplo = 'L', ltau = LOCc(ja+m-2),
if side = 'R' and uplo = 'U', ltau = LOCc(n_a),
if side = 'R' and uplo = 'L', ltau = LOCc(ja+n-2).
```

tau[*i*] must contain the scalar factor of the elementary reflector $H(i+1)$, as returned by *p?hetrd* ($0 \leq j < ltau$). *tau* is tied to the distributed matrix *A*.

c

(local)

Pointer into the local memory to an array of size *lld_c***LOCc*(*jc+n-1*). Contains the local pieces of the distributed matrix sub (*C*).

ic, jc

(global) The row and column indices in the global matrix *C* indicating the first row and the first column of the submatrix *C*, respectively.

descc

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix *C*.

work

(local)

Workspace array of size *lwork*.

lwork

(local or global) size of *work*, must be at least:

```
If uplo = 'U',
  iaa= ia; jaa= ja+1, icc= ic; jcc= jc;
else uplo = 'L',
  iaa= ia+1, jaa= ja;
If side = 'L',
  icc= ic+1; jcc= jc;
else icc= ic; jcc= jc+1;
end if
end if
If side = 'L',
  mi= m-1; ni= n
  lwork ≥ max((nb_a*(nb_a-1))/2, (nqc0 + mpc0)*nb_a) + nb_a*nb_a
else
  If side = 'R',
    mi= m; ni = n-1;
    lwork ≥ max((nb_a*(nb_a-1))/2, (nqc0 +
max(npa0+numroc(numroc(ni+icoffc, nb_a, 0, 0, NPCOL), nb_a,
0, 0, lcmq), mpc0))*nb_a) + nb_a*nb_a
end if
where lcmq = lcm/NPCOL with lcm = ilcm(NPROW, NPCOL),
```

```

iroffa = mod(iaa-1, mb_a),
icoffa = mod(jaa-1, nb_a),
iarow = indxg2p(iaa, mb_a, MYROW, rsrc_a, NPROW),
npa0 = numroc(ni+iroffa, mb_a, MYROW, iarow, NPROW),
iroffc = mod(icc-1, mb_c),
icoffc = mod(jcc-1, nb_c),
icrow = indxg2p(icc, mb_c, MYROW, rsrc_c, NPROW),
iccol = indxg2p(jcc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0 = numroc(mi+iroffc, mb_c, MYROW, icrow, NPROW),
nqc0 = numroc(ni+icoffc, nb_c, MYCOL, iccol, NPCOL),

```

NOTE

`mod(x, y)` is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`. If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `p?erbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | Overwritten by the product $Q \cdot \text{sub}(C)$, or $Q' \cdot \text{sub}(C)$, or $\text{sub}(C) \cdot Q'$, or $\text{sub}(C) \cdot Q$. |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?stebz

Computes the eigenvalues of a symmetric tridiagonal matrix by bisection.

Syntax

```
void psstebz (MKL_INT *ictxt , char *range , char *order , MKL_INT *n , float *vl ,
float *vu , MKL_INT *il , MKL_INT *iu , float *abstol , float *d , float *e , MKL_INT
*m , MKL_INT *nsplit , float *w , MKL_INT *iblock , MKL_INT *isplit , float *work ,
MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pdstebz (MKL_INT *ictxt , char *range , char *order , MKL_INT *n , double *vl ,
double *vu , MKL_INT *il , MKL_INT *iu , double *abstol , double *d , double *e ,
MKL_INT *m , MKL_INT *nsplit , double *w , MKL_INT *iblock , MKL_INT *isplit , double
*work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?stebz` function computes the eigenvalues of a symmetric tridiagonal matrix in parallel. These may be all eigenvalues, all eigenvalues in the interval $[vl, vu]$, or the eigenvalues il through iu . A static partitioning of work is done at the beginning of `p?stebz` which results in all processes finding an (almost) equal number of eigenvalues.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|---------------|---|
| <i>ictxt</i> | (global) The BLACS context handle. |
| <i>range</i> | (global) Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the function computes all eigenvalues. If <i>range</i> = 'V', the function computes eigenvalues in the interval $[vl, vu]$. If <i>range</i> = 'I', the function computes eigenvalues il through iu . |
| <i>order</i> | (global) Must be 'B' or 'E'. If <i>order</i> = 'B', the eigenvalues are to be ordered from smallest to largest within each split-off block. If <i>order</i> = 'E', the eigenvalues for the entire matrix are to be ordered from smallest to largest. |
| <i>n</i> | (global) The order of the tridiagonal matrix T ($n \geq 0$). |
| <i>vl, vu</i> | (global) |

If *range* = 'V', the function computes the lower and the upper bounds for the eigenvalues on the interval [*l*, *vu*].

If *range* = 'A' or 'I', *vl* and *vu* are not referenced.

il, iu

(global)

Constraint: $1 \leq il \leq iu \leq n$.

If *range* = 'I', the index of the smallest eigenvalue is returned for *il* and of the largest eigenvalue for *iu* (assuming that the eigenvalues are in ascending order) must be returned.

If *range* = 'A' or 'V', *il* and *iu* are not referenced.

abstol

(global)

The absolute tolerance to which each eigenvalue is required. An eigenvalue (or cluster) is considered to have converged if it lies in an interval of width *abstol*. If *abstol* ≤ 0, then the tolerance is taken as $ulp ||T||$, where *ulp* is the machine precision, and $||T||$ means the 1-norm of *T*

Eigenvalues will be computed most accurately when *abstol* is set to the underflow threshold `slamch('U')`, not 0. Note that if eigenvectors are desired later by inverse iteration (`p?stein`), *abstol* should be set to $2 * p?lamch('S')$.

d

(global)

Array of size *n*.

Contains *n* diagonal elements of the tridiagonal matrix *T*. To avoid overflow, the matrix must be scaled so that its largest entry is no greater than $overflow^{(1/2)} * underflow^{(1/4)}$ in absolute value, and for greatest accuracy, it should not be much smaller than that.

e

(global)

Array of size *n* - 1.

Contains (*n*-1) off-diagonal elements of the tridiagonal matrix *T*. To avoid overflow, the matrix must be scaled so that its largest entry is no greater than $overflow^{(1/2)} * underflow^{(1/4)}$ in absolute value, and for greatest accuracy, it should not be much smaller than that.

work

(local)

Array of size $\max(5n, 7)$. This is a workspace array.

lwork

(local) The size of the *work* array must be $\geq \max(5n, 7)$.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

iwork

(local) Array of size $\max(4n, 14)$. This is a workspace array.

liwork

(local) The size of the *iwork* array must $\geq \max(4n, 14, NPROCS)$.

If $liwork = -1$, then $liwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------|--|
| m | (global) The actual number of eigenvalues found. $0 \leq m \leq n$ |
| $nsplit$ | (global) The number of diagonal blocks detected in T . $1 \leq nsplit \leq n$ |
| w | (global) Array of size n . On exit, the first m elements of w contain the eigenvalues on all processes. |
| $iblock$ | (global) Array of size n . At each row/column j where $e[j-1]$ is zero or small, the matrix T is considered to split into a block diagonal matrix. On exit $iblock[j]$ specifies which block (from 1 to the number of blocks) the eigenvalue $w[j]$ belongs to. |

NOTE

In the (theoretically impossible) event that bisection does not converge for some or all eigenvalues, $info$ is set to 1 and the ones for which it did not are identified by a negative block number.

| | |
|----------|--|
| $isplit$ | (global) Array of size n . Contains the splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to $isplit[0]$, the second of rows/columns $isplit[0]+1$ through $isplit[1]$, and so on, and the $nsplit$ -th submatrix consists of rows/columns $isplit[nsplit-2]+1$ through $isplit[nsplit-1]=n$. (Only the first $nsplit$ elements are used, but since the $nsplit$ values are not known, n words must be reserved for $isplit$.) |
| $info$ | (global) If $info = 0$, the execution is successful. If $info < 0$, if $info = -i$, the i -th argument has an illegal value. If $info > 0$, some or all of the eigenvalues fail to converge or are not computed. If $info = 1$, bisection fails to converge for some eigenvalues; these eigenvalues are flagged by a negative block number. The effect is that the eigenvalues may not be as accurate as the absolute and relative tolerances. If $info = 2$, mismatch between the number of eigenvalues output and the number desired. |

If *info* = 3: *range*='I', and the Gershgorin interval initially used is incorrect. No eigenvalues are computed. Probable cause: the machine has a sloppy floating-point arithmetic. Increase the *fudge* parameter, recompile, and try again.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?stedc

Computes all eigenvalues and eigenvectors of a symmetric tridiagonal matrix in parallel.

Syntax

```
void psstedc (const char* compz, const MKL_INT* n, float* d, float* e, float* q, const
MKL_INT* iq, const MKL_INT* jq, const MKL_INT* descq, float* work, MKL_INT* lwork,
MKL_INT* iwork, const MKL_INT* liwork, MKL_INT* info);
```

```
void pdstedc (const char* compz, const MKL_INT* n, double* d, double* e, double* q,
const MKL_INT* iq, const MKL_INT* jq, const MKL_INT* descq, double* work, MKL_INT*
lwork, MKL_INT* iwork, const MKL_INT* liwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

p?stedc computes all eigenvalues and eigenvectors of a symmetric tridiagonal matrix in parallel, using the divide and conquer algorithm.

Input Parameters

| | |
|--------------|---|
| <i>compz</i> | = 'N': Compute eigenvalues only. (NOT IMPLEMENTED YET) = 'I': Compute eigenvectors of tridiagonal matrix also. = 'V': Compute eigenvectors of original dense symmetric matrix also. On entry, <i>Z</i> contains the orthogonal matrix used to reduce the original matrix to tridiagonal form. (NOT IMPLEMENTED YET) |
| <i>n</i> | (global) The order of the tridiagonal matrix <i>T</i> . $n \geq 0$. |
| <i>d</i> | (global) Array, size (<i>n</i>) On entry, the diagonal elements of the tridiagonal matrix. |
| <i>e</i> | (global) Array, size ($n-1$). On entry, the subdiagonal elements of the tridiagonal matrix. |
| <i>iq</i> | (global) |

| | |
|---------------|--|
| | Q's global row index, which points to the beginning of the submatrix which is to be operated on. |
| <i>jq</i> | (global) Q's global column index, which points to the beginning of the submatrix which is to be operated on. |
| <i>descq</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix Q. |
| <i>work</i> | (local) Array, size (<i>lwork</i>) |
| <i>lwork</i> | (local) The size of the array <i>work</i> . $lwork = 6*n + 2*NP*NQ$ $NP = \text{numroc}(n, NB, MYROW, \text{DESCQ}(rsrc_), NPROW)$ $NQ = \text{numroc}(n, NB, MYCOL, \text{DESCQ}(csrc_), NPCOL)$ <i>numroc</i> is a ScaLAPACK tool function. If <i>lwork</i> = -1, the <i>lwork</i> is global input and a workspace query is assumed; the routine only calculates the minimum size for the <i>work</i> array. The required workspace is returned as the first element of <i>work</i> and no error message is issued by <i>pxerbla</i> . |
| <i>iwork</i> | (local) Array, size (<i>liwork</i>) |
| <i>liwork</i> | The size of the array <i>iwork</i> . $liwork = 2 + 7*n + 8*NPCOL$ |

Output Parameters

| | |
|--------------|--|
| <i>d</i> | On exit, if <i>info</i> = 0, the eigenvalues in descending order. |
| <i>q</i> | (local) Array, local size (<i>lld_q</i> , $LOCc(jq+n-1)$) <i>q</i> contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. On output, <i>q</i> is distributed across the P processes in block cyclic format. |
| <i>work</i> | On output, <i>work</i> [0] returns the workspace needed. |
| <i>iwork</i> | On exit, if <i>liwork</i> > 0, <i>iwork</i> [0] returns the optimal <i>liwork</i> . |
| <i>info</i> | (global) = 0: successful exit. |

< 0: If the i -th argument is an array and the j -th entry had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

> 0: The algorithm failed to compute the $info/(n+1)$ -th eigenvalue while working on the submatrix lying in global rows and columns $\text{mod}(info, n+1)$.

p?stein

Computes the eigenvectors of a tridiagonal matrix using inverse iteration.

Syntax

```
void psstein (MKL_INT *n , float *d , float *e , MKL_INT *m , float *w , MKL_INT
 *iblock , MKL_INT *isplit , float *orfac , float *z , MKL_INT *iz , MKL_INT *jz ,
 MKL_INT *descz , float *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork ,
 MKL_INT *ifail , MKL_INT *iclustr , float *gap , MKL_INT *info );
```

```
void pdstein (MKL_INT *n , double *d , double *e , MKL_INT *m , double *w , MKL_INT
 *iblock , MKL_INT *isplit , double *orfac , double *z , MKL_INT *iz , MKL_INT *jz ,
 MKL_INT *descz , double *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork ,
 MKL_INT *ifail , MKL_INT *iclustr , double *gap , MKL_INT *info );
```

```
void pcstein (MKL_INT *n , float *d , float *e , MKL_INT *m , float *w , MKL_INT
 *iblock , MKL_INT *isplit , float *orfac , MKL_Complex8 *z , MKL_INT *iz , MKL_INT
 *jz , MKL_INT *descz , float *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT
 *liwork , MKL_INT *ifail , MKL_INT *iclustr , float *gap , MKL_INT *info );
```

```
void pzstein (MKL_INT *n , double *d , double *e , MKL_INT *m , double *w , MKL_INT
 *iblock , MKL_INT *isplit , double *orfac , MKL_Complex16 *z , MKL_INT *iz , MKL_INT
 *jz , MKL_INT *descz , double *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT
 *liwork , MKL_INT *ifail , MKL_INT *iclustr , double *gap , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?stein` function computes the eigenvectors of a symmetric tridiagonal matrix T corresponding to specified eigenvalues, by inverse iteration. `p?stein` does not orthogonalize vectors that are on different processes. The extent of orthogonalization is controlled by the input parameter `lwork`. Eigenvectors that are to be orthogonalized are computed by the same process. `p?stein` decides on the allocation of work among the processes and then calls `?stein2` (modified LAPACK function) on each individual process. If insufficient workspace is allocated, the expected orthogonalization may not be done.

NOTE

If the eigenvectors obtained are not orthogonal, increase `lwork` and run the code again.

$p = \text{NPROW} * \text{NPCOL}$ is the total number of processes.

Input Parameters

n (global) The order of the matrix T ($n \geq 0$).

| | |
|----------------|--|
| <i>m</i> | (global) The number of eigenvectors to be returned. |
| <i>d, e, w</i> | (global) Arrays: <i>d</i> of size <i>n</i> contains the diagonal elements of <i>T</i> . <i>e</i> of size <i>n-1</i> contains the off-diagonal elements of <i>T</i> . <i>w</i> of size <i>m</i> contains all the eigenvalues grouped by split-off block. The eigenvalues are supplied from smallest to largest within the block. (Here the output array <i>w</i> from <code>p?stebz</code> with order = 'B' is expected. The array should be replicated in all processes.) |
| <i>iblock</i> | (global) Array of size <i>n</i> . The submatrix indices associated with the corresponding eigenvalues in <i>w</i> : 1 for eigenvalues belonging to the first submatrix from the top, 2 for those belonging to the second submatrix, etc. (The output array <i>iblock</i> from <code>p?stebz</code> is expected here). |
| <i>isplit</i> | (global) Array of size <i>n</i> . The splitting points at which <i>T</i> breaks up into submatrices. The first submatrix consists of rows/columns 1 to <i>isplit</i> [0], the second of rows/columns <i>isplit</i> [0]+1 through <i>isplit</i> [1], and so on, and the <i>nsplit</i> -th submatrix consists of rows/columns <i>isplit</i> [<i>nsplit</i> -2]+1 through <i>isplit</i> [<i>nsplit</i> -1]= <i>n</i> . (The output array <i>isplit</i> from <code>p?stebz</code> is expected here.) |
| <i>orfac</i> | (global) <i>orfac</i> specifies which eigenvectors should be orthogonalized. Eigenvectors that correspond to eigenvalues within <i>orfac</i> * T of each other are to be orthogonalized. However, if the workspace is insufficient (see <i>lwork</i>), this tolerance may be decreased until all eigenvectors can be stored in one process. No orthogonalization is done if <i>orfac</i> is equal to zero. A default value of 1000 is used if <i>orfac</i> is negative. <i>orfac</i> should be identical on all processes |
| <i>iz, jz</i> | (global) The row and column indices in the global matrix <i>Z</i> indicating the first row and the first column of the submatrix <i>Z</i> , respectively. |
| <i>descz</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>Z</i> . |
| <i>work</i> | (local). Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local) <i>lwork</i> controls the extent of orthogonalization which can be done. The number of eigenvectors for which storage is allocated on each process is $nvec = \text{floor}((lwork - \max(5*n, np00*mq00))/n)$. Eigenvectors corresponding to eigenvalue clusters of size $(nvec - \text{ceil}(m/p) + 1)$ are guaranteed to be orthogonal (the orthogonality is similar to that obtained from <code>?stein2</code>). |

NOTE

$lwork$ must be no smaller than $\max(5*n, np00*mq00) + \text{ceil}(m/p)*n$ and should have the same input value on all processes.

It is the minimum value of $lwork$ input on different processes that is significant.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

$iwork$ (local)
Workspace array of size $3n+p+1$.

$liwork$ (local) The size of the array $iwork$. It must be greater than $3*n+p+1$.
If $liwork = -1$, then $liwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

z (local)
Array of size $descz[dlen_-1], n/NPCOL + NB$. z contains the computed eigenvectors associated with the specified eigenvalues. Any vector which fails to converge is set to its current iterate after MAXIT iterations (See ? [stein2](#)). On output, z is distributed across the p processes in block cyclic format.

$work$ On exit, $work[0]$ gives a lower bound on the workspace ($lwork$) that guarantees the user desired orthogonalization (see $orfac$). Note that this may overestimate the minimum workspace needed.

$iwork$ On exit, $iwork[0]$ contains the amount of integer workspace required.
On exit, the $iwork[1]$ through $iwork[p+1]$ indicate the eigenvectors computed by each process. Process i computes eigenvectors indexed $iwork[i+1]+1$ through $iwork[i+2]$.

$ifail$ (global) Array of size m . On normal exit, all elements of $ifail$ are zero. If one or more eigenvectors fail to converge after MAXIT iterations (as in ? [stein](#)), then $info > 0$ is returned. If $\text{mod}(info, m+1) > 0$, then for $i=1$ to $\text{mod}(info, m+1)$, the eigenvector corresponding to the eigenvalue $w[ifail[i-1]-1]$ failed to converge (w refers to the array of eigenvalues on output).

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

$iclustr$ (global) Array of size $2*p$.

This output array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be orthogonalized due to insufficient workspace (see *lwork*, *orfac* and *info*). Eigenvectors corresponding to clusters of eigenvalues indexed *iclustr*(2*I-1) to *iclustr*(2*I), *i* = 1 to *info*/(*m*+1), could not be orthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal. *iclustr* is a zero terminated array: *iclustr*[2*k-1] ≠ 0 and *iclustr*[2*k] = 0 if and only if *k* is the number of clusters.

gap

(global)

This output array contains the gap between eigenvalues whose eigenvectors could not be orthogonalized. The *info*/*m* output values in this array correspond to the *info*/(*m*+1) clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the *i*-th cluster may be as high as $(O(n) * macheps) / gap[i-1]$.

info

(global)

If *info* = 0, the execution is successful.

If *info* < 0: If the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value, then *info* = -(*i**100+*j*),

If the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

If *info* < 0: if *info* = -*i*, the *i*-th argument had an illegal value.

If *info* > 0: if mod(*info*, *m*+1) = *i*, then *i* eigenvectors failed to converge in MAXIT iterations. Their indices are stored in the array *ifail*. If *info*/(*m*+1) = *i*, then eigenvectors corresponding to *i* clusters of eigenvalues could not be orthogonalized due to insufficient workspace. The indices of the clusters are stored in the array *iclustr*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Nonsymmetric Eigenvalue Problems: ScaLAPACK Computational Routines

This section describes ScaLAPACK routines for solving nonsymmetric eigenvalue problems, computing the Schur factorization of general matrices, as well as performing a number of related computational tasks.

To solve a nonsymmetric eigenvalue problem with ScaLAPACK, you usually need to reduce the matrix to the upper Hessenberg form and then solve the eigenvalue problem with the Hessenberg matrix obtained.

[Table "Computational Routines for Solving Nonsymmetric Eigenproblems"](#) lists ScaLAPACK routines for reducing the matrix to the upper Hessenberg form by an orthogonal (or unitary) similarity transformation $A = QHQ^H$, as well as routines for solving eigenproblems with Hessenberg matrices, and multiplying the matrix after reduction.

Computational Routines for Solving Nonsymmetric Eigenproblems

| Operation performed | General matrix | Orthogonal/Unitary matrix | Hessenberg matrix |
|--|-------------------------|---|-------------------------|
| Reduce to Hessenberg form $A = QHQ^H$ | p?gehrd | | |
| Multiply the matrix after reduction | | p?ormhr / p?unmhr | |
| Find eigenvalues and Schur factorization | | | p?lahqr |

p?gehrd

Reduces a general matrix to upper Hessenberg form.

Syntax

```
void psgehrd (MKL_INT *n , MKL_INT *ilo , MKL_INT *ihi , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pdgehrd (MKL_INT *n , MKL_INT *ilo , MKL_INT *ihi , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pcgehrd (MKL_INT *n , MKL_INT *ilo , MKL_INT *ihi , MKL_Complex8 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pzgehrd (MKL_INT *n , MKL_INT *ilo , MKL_INT *ihi , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?gehrd function reduces a real/complex general distributed matrix sub(A) to upper Hessenberg form H by an orthogonal or unitary similarity transformation

$$Q^* \text{sub}(A) Q = H,$$

where $\text{sub}(A) = A(ia:ia+n-1, ja:ja+n-1)$.

Input Parameters

| | |
|------------|---|
| n | (global). The order of the distributed matrix sub(A) ($n \geq 0$). |
| ilo, ihi | (global). It is assumed that sub(A) is already upper triangular in rows $ia:ia+ilo-2$ and $ia+ihi:ia+n-1$ and columns $ja:ja+ilo-2$ and $ja+ihi:ja+n-1$. (See <i>Application Notes</i> below). If $n > 0$, $1 \leq ilo \leq ihi \leq n$; otherwise set $ilo = 1$, $ihi = n$. |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the n -by- n general distributed matrix sub(A) to be reduced. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| $work$ | (local) Workspace array of size $lwork$. |

lwork (local or global) size of the array *work*. *lwork* is local input and must be at least

$$lwork \geq NB * NB + NB * \max(ihip+1, ihlp+inlq)$$

where $NB = mb_a = nb_a$,

$iroffa = \text{mod}(ia-1, NB)$,

$icoffa = \text{mod}(ja-1, NB)$,

$ioff = \text{mod}(ia+ilo-2, NB)$, $iarow = \text{indxg2p}(ia, NB, MYROW, rsrc_a, NPROW)$, $ihip = \text{numroc}(ihi+iroffa, NB, MYROW, iarow, NPROW)$,

$ilrow = \text{indxg2p}(ia+ilo-1, NB, MYROW, rsrc_a, NPROW)$,

$ihlp = \text{numroc}(ihi-ilo+ioff+1, NB, MYROW, ilrow, NPROW)$,

$ilcol = \text{indxg2p}(ja+ilo-1, NB, MYCOL, csrc_a, NPCOL)$,

$inlq = \text{numroc}(n-ilo+ioff+1, NB, MYCOL, ilcol, NPCOL)$,

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

a On exit, the upper triangle and the first subdiagonal of $\text{sub}(A)$ are overwritten with the upper Hessenberg matrix H , and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal/unitary matrix Q as a product of elementary reflectors (see *Application Notes* below).

tau (local).
Array of size at least $\max(ja+n-2)$.
The scalar factors of the elementary reflectors (see *Application Notes* below). Elements $ja:ja+ilo-2$ and $ja+ihi:ja+n-2$ of the global vector *tau* are set to zero. *tau* is tied to the distributed matrix A .

work[0] On exit *work*[0] contains the minimum value of *lwork* required for optimum performance.

info (global)
= 0: the execution is successful.
< 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

Application Notes

The matrix Q is represented as a product of $(ihi-ilo)$ elementary reflectors

$$Q = H(ilo)*H(ilo+1)*...*H(ihi-1).$$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v'$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1:i) = 0$, $v(i+1) = 1$ and $v(ihi+1:n) = 0$; $v(i+2:ihi)$ is stored on exit in $A(ia+ilo+i:ia+ihi-1, ja+ilo+i-2)$, and τ in $\tau[ja+ilo+i-3]$. The contents of A

$(ia:ia+n-1, ja:ja+n-1)$ are illustrated by the following example, with $n = 7$, $ilo = 2$ and $ihi = 6$:
on entry

$$\begin{bmatrix} a & a & a & a & a & a & a \\ & a & a & a & a & a & a \\ & & a & a & a & a & a \\ & & & a & a & a & a \\ & & & & a & a & a \\ & & & & & a & a \\ & & & & & & a \end{bmatrix}$$

on exit

$$\begin{bmatrix} a & a & a & h & h & h & a \\ & a & h & h & h & h & a \\ & & h & h & h & h & h \\ v2 & h & h & h & h & h & h \\ v2 & v3 & h & h & h & h & h \\ v2 & v3 & v4 & h & h & h & h \\ & & & & & & a \end{bmatrix}$$

where a denotes an element of the original matrix $\text{sub}(A)$, h denotes a modified element of the upper Hessenberg matrix H , and v_i denotes an element of the vector defining $H(ja+ilo+i-2)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ormhr

Multiplies a general matrix by the orthogonal transformation matrix from a reduction to Hessenberg form determined by p?gehrd.

Syntax

```
void psormhr (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *ilo ,
MKL_INT *ihi , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau ,
float *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork ,
MKL_INT *info );
```

```
void pdormhr (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *ilo ,
MKL_INT *ihi , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau ,
double *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?ormhr function overwrites the general real distributed m -by- n matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|-----------------------|-----------------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q * \text{sub}(C)$ | $\text{sub}(C) * Q$ |
| $trans = 'T':$ | $Q^T * \text{sub}(C)$ | $\text{sub}(C) * Q^T$ |

where Q is a real orthogonal distributed matrix of order nq , with $nq = m$ if $side = 'L'$ and $nq = n$ if $side = 'R'$.

Q is defined as the product of $ihi-ilo$ elementary reflectors, as returned by p?gehrd.

$Q = H(ilo) H(ilo+1) \dots H(ihi-1)$.

Input Parameters

| | |
|------------|--|
| $side$ | (global) = 'L': Q or Q^T is applied from the left. = 'R': Q or Q^T is applied from the right. |
| $trans$ | (global) = 'N', no transpose, Q is applied. = 'T', transpose, Q^T is applied. |
| m | (global) The number of rows in the distributed matrix sub (C) ($m \geq 0$). |
| n | (global) The number of columns in he distributed matrix sub (C) ($n \geq 0$). |
| ilo, ihi | (global) ilo and ihi must have the same values as in the previous call of p?gehrd. Q is equal to the unit matrix except for the distributed submatrix $Q(ia+ilo:ia+ihi-1, ja+ilo:ja+ihi-1)$. |

If $side = 'L'$, $1 \leq ilo \leq ihi \leq \max(1, m)$;

If $side = 'R'$, $1 \leq ilo \leq ihi \leq \max(1, n)$;

ilo and ihi are relative indexes.

a

(local)

Pointer into the local memory to an array of size $lld_a * LOCC(ja+m-1)$ if $side = 'L'$, and $lld_a * LOCC(ja+n-1)$ if $side = 'R'$.

Contains the vectors which define the elementary reflectors, as returned by `p?gehrd`.

ia, ja

(global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively.

$desca$

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix A .

tau

(local)

Array of size $LOCC(ja+m-2)$ if $side = 'L'$, and $LOCC(ja+n-2)$ if $side = 'R'$.

$tau[j]$ contains the scalar factor of the elementary reflector $H(j+1)$ as returned by `p?gehrd` ($0 \leq j < \text{size}(tau)$). tau is tied to the distributed matrix A .

c

(local)

Pointer into the local memory to an array of size $lld_c * LOCC(jc+n-1)$.

Contains the local pieces of the distributed matrix $\text{sub}(C)$.

ic, jc

(global) The row and column indices in the global matrix C indicating the first row and the first column of the submatrix C , respectively.

$descc$

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix C .

$work$

(local)

Workspace array with size $lwork$.

$lwork$

(local or global)

The size of the array $work$.

$lwork$ must be at least $iaa = ia + ilo$; $jaa = ja + ilo - 1$;

If $side = 'L'$,

$mi = ihi - ilo$; $ni = n$; $icc = ic + ilo$; $jcc = jc$;

$lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + mpc0) * nb_a) + nb_a * nb_a$

else if $side = 'R'$,

$mi = m$; $ni = ihi - ilo$; $icc = ic$; $jcc = jc + ilo$;

$lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + \max(npa0 + \text{numroc}(\text{numroc}(ni + icoffc, nb_a, 0, 0, NPCOL), nb_a, 0, 0, lcmq), mpc0)) * nb_a) + nb_a * nb_a$

end if

```

where  $lcmq = lcm/NPCOL$  with  $lcm = ilcm(NPROW, NPCOL)$ ,
 $iroffa = \text{mod}(iaa-1, mb\_a)$ ,
 $icoffa = \text{mod}(jaa-1, nb\_a)$ ,
 $iarow = \text{indxg2p}(iaa, mb\_a, MYROW, rsrc\_a, NPROW)$ ,
 $npa0 = \text{numroc}(ni+iroffa, mb\_a, MYROW, iarow, NPROW)$ ,
 $iroffc = \text{mod}(icc-1, mb\_c)$ ,  $icoffc = \text{mod}(jcc-1, nb\_c)$ ,
 $icrow = \text{indxg2p}(icc, mb\_c, MYROW, rsrc\_c, NPROW)$ ,
 $iccol = \text{indxg2p}(jcc, nb\_c, MYCOL, csrc\_c, NPCOL)$ ,
 $mpc0 = \text{numroc}(mi+iroffc, mb\_c, MYROW, icrow, NPROW)$ ,
 $nqc0 = \text{numroc}(ni+icoffc, nb\_c, MYCOL, iccol, NPCOL)$ ,

```

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

$ilcm$, $indxg2p$ and $numroc$ are ScaLAPACK tool functions; $MYROW$, $MYCOL$, $NPROW$ and $NPCOL$ can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `p?xerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | <code>sub(C)</code> is overwritten by $Q*\text{sub}(C)$, or $Q'*\text{sub}(C)$, or $\text{sub}(C)*Q'$, or $\text{sub}(C)*Q$. |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of $lwork$ required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unmhr

Multiplies a general matrix by the unitary transformation matrix from a reduction to Hessenberg form determined by p?gehrd.

Syntax

```
void pcunmhr (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *ilo ,
MKL_INT *ihi , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
MKL_Complex8 *tau , MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc ,
MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzunmhr (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *ilo ,
MKL_INT *ihi , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
MKL_Complex16 *tau , MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

This function overwrites the general complex distributed m -by- n matrix $\text{sub}(C) = C(ic:ic+m-1,jc:jc+n-1)$ with

| | | |
|----------------|---------------------|---------------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q*\text{sub}(C)$ | $\text{sub}(C)*Q$ |
| $trans = 'H':$ | $Q^H*\text{sub}(C)$ | $\text{sub}(C)*Q^H$ |

where Q is a complex unitary distributed matrix of order nq , with $nq = m$ if $side = 'L'$ and $nq = n$ if $side = 'R'$.

Q is defined as the product of $ihi-ilo$ elementary reflectors, as returned by [p?gehrd](#).

$Q = H(ilo) H(ilo+1) \dots H(ihi-1)$.

Input Parameters

| | |
|------------|--|
| $side$ | (global) $= 'L'$: Q or Q^H is applied from the left. $= 'R'$: Q or Q^H is applied from the right. |
| $trans$ | (global) $= 'N'$, no transpose, Q is applied. $= 'C'$, conjugate transpose, Q^H is applied. |
| m | (global) The number of rows in the distributed matrix $\text{sub}(C)$ ($m \geq 0$). |
| n | (global) The number of columns in the distributed matrix $\text{sub}(C)$ ($n \geq 0$). |
| ilo, ihi | (global) These must be the same parameters ilo and ihi , respectively, as supplied to p?gehrd . Q is equal to the unit matrix except in the distributed submatrix $Q(ia+ilo:ia+ihi-1,ja+ilo:ja+ihi-1)$. If $side = 'L'$, then $1 \leq ilo \leq ihi \leq \max(1, m)$. If $side = 'R'$, then $1 \leq ilo \leq ihi \leq \max(1, n)$ ilo and ihi are relative indexes. |

| | |
|---------------|---|
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a*LOCc(ja+m-1)$ if $side = 'L'$, and $lld_a*LOCc(ja+n-1)$ if $side = 'R'$. Contains the vectors which define the elementary reflectors, as returned by <code>p?gehrd</code> . |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array of size $LOCc(ja+m-2)$, if $side = 'L'$, and $LOCc(ja+n-2)$ if $side = 'R'$. <i>tau</i> [<i>j</i>] contains the scalar factor of the elementary reflector $H(j+1)$ as returned by <code>p?gehrd</code> ($0 \leq j < \text{size}(\text{tau})$). <i>tau</i> is tied to the distributed matrix <i>A</i> . |
| <i>c</i> | (local) Pointer into the local memory to an array of size $lld_c*LOCc(jc+n-1)$. Contains the local pieces of the distributed matrix sub(<i>C</i>). |
| <i>ic, jc</i> | (global) The row and column indices in the global matrix <i>C</i> indicating the first row and the first column of the submatrix <i>C</i> , respectively. |
| <i>desc</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>C</i> . |
| <i>work</i> | (local) Workspace array with size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> . <i>lwork</i> must be at least $iaa = ia + ilo; jaa = ja + ilo - 1;$ If $side = 'L'$, $mi = ihi - ilo; ni = n; icc = ic + ilo; jcc = jc;$ $lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 + mpc0) * nb_a) + nb_a * nb_a$ else if $side = 'R'$, $mi = m; ni = ihi - ilo; icc = ic; jcc = jc + ilo;$ $lwork \geq \max((nb_a * (nb_a - 1)) / 2, (nqc0 +$ $\max(npa0 + \text{numroc}(\text{numroc}(ni + icoffc, nb_a, 0, 0, NPCOL), nb_a,$ $0, 0, lcmq), mpc0)) * nb_a) + nb_a * nb_a$ end if where $lcmq = lcm / NPCOL$ with $lcm = ilcm(NPROW, NPCOL),$ $iroffa = \text{mod}(iaa - 1, mb_a),$ $icoffa = \text{mod}(jaa - 1, nb_a),$ $iarow = \text{indxg2p}(iaa, mb_a, MYROW, rsrc_a, NPROW),$ |

```

npa0 = numroc(ni+iroffa, mb_a, MYROW, iarow, NPROW),
iroffc = mod(icc-1, mb_c),
icoffc = mod(jcc-1, nb_c),
icrow = indxg2p(icc, mb_c, MYROW, rsrc_c, NPROW),
iccol = indxg2p(jcc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0 = numroc(mi+iroffc, mb_c, MYROW, icrow, NPROW),
nqc0 = numroc(ni+icoffc, nb_c, MYCOL, iccol, NPCOL),

```

NOTE

`mod(x,y)` is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|---|
| <code>c</code> | C is overwritten by $Q^* \text{sub}(C)$ or $Q'^* \text{sub}(C)$ or $\text{sub}(C) * Q'$ or $\text{sub}(C) * Q$. |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <code>info = -(i*100+j)</code> ; if the <i>i</i> -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lahqr

Computes the Schur decomposition and/or eigenvalues of a matrix already in Hessenberg form.

Syntax

```

void pslahqr (MKL_INT *wantt , MKL_INT *wantz , MKL_INT *n , MKL_INT *ilo , MKL_INT
*ihi , float *a , MKL_INT *desca , float *wr , float *wi , MKL_INT *iloz , MKL_INT
*ihiz , float *z , MKL_INT *descz , float *work , MKL_INT *lwork , MKL_INT *iwork ,
MKL_INT *ilwork , MKL_INT *info );

```

```
void pdlahqr (MKL_INT *wantt , MKL_INT *wantz , MKL_INT *n , MKL_INT *ilo , MKL_INT
*ihi , double *a , MKL_INT *desca , double *wr , double *wi , MKL_INT *iloz , MKL_INT
*ihiz , double *z , MKL_INT *descz , double *work , MKL_INT *lwork , MKL_INT *iwork ,
MKL_INT *ilwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

This is an auxiliary function used to find the Schur decomposition and/or eigenvalues of a matrix already in Hessenberg form from columns *ilo* and *ihi*.

Input Parameters

| | |
|-------------------|---|
| <i>wantt</i> | (global) If <i>wantt</i> ≠ 0, the full Schur form <i>T</i> is required; If <i>wantt</i> = 0, only eigenvalues are required. |
| <i>wantz</i> | (global) If <i>wantz</i> ≠ 0, the matrix of Schur vectors <i>Z</i> is required; If <i>wantz</i> = 0, Schur vectors are not required. |
| <i>n</i> | (global) The order of the Hessenberg matrix <i>A</i> (and <i>z</i> if <i>wantz</i> is non-zero). $n \geq 0$. |
| <i>ilo, ihi</i> | (global) It is assumed that <i>A</i> is already upper quasi-triangular in rows and columns <i>ihi</i> +1: <i>n</i> , and that $A(ilo, ilo-1) = 0$ (unless $ilo = 1$). <i>pdlahqr</i> works primarily with the Hessenberg submatrix in rows and columns <i>ilo</i> to <i>ihi</i> , but applies transformations to all of <i>H</i> if <i>wantt</i> is non-zero. $1 \leq ilo \leq \max(1, ihi)$; $ihiz \leq n$. |
| <i>a</i> | (global) Array, of size $lld_a * LOCC(n)$. On entry, the upper Hessenberg matrix <i>A</i> . |
| <i>desca</i> | (global and local) array of size <i>dlen</i> _· . The array descriptor for the distributed matrix <i>A</i> . |
| <i>iloz, ihiz</i> | (global) Specify the rows of the matrix <i>Z</i> to which transformations must be applied if <i>wantz</i> is non-zero. $1 \leq iloz \leq ilo$; $ihiz \leq ihiz \leq n$. |
| <i>z</i> | (global) Array. If <i>wantz</i> is non-zero, on entry <i>z</i> must contain the current matrix <i>Z</i> of transformations accumulated by <i>pdhseqr</i> . If <i>wantz</i> is zero, <i>z</i> is not referenced. |
| <i>descz</i> | (global and local) array of size <i>dlen</i> _· . The array descriptor for the distributed matrix <i>Z</i> . |
| <i>work</i> | (local) Workspace array with size <i>lwork</i> . |

lwork (local) The size of *work*. *lwork* is assumed big enough so that $lwork \geq 3*n + \max(2*\max(lld_z, lld_a) + 2*LOCq(n), 7*\text{ceil}(n/hbl) / lcm(NPROW, NPCOL))$.

If *lwork* = -1, then *work*[0] gets set to the above number and the code returns immediately.

iwork (global and local) array of size *ilwork*.

ilwork (local) This holds some of the *iblk* integer arrays.

Output Parameters

a On exit, if *wantt* is non-zero, *A* is upper quasi-triangular in rows and columns *ilo:ihi*, with any 2-by-2 or larger diagonal blocks not yet in standard form. If *wantt* is zero, the contents of *A* are unspecified on exit.

work[0] On exit *work*[0] contains the minimum value of *lwork* required for optimum performance.

wr, wi (global replicated output)

Arrays of size *n* each. The real and imaginary parts, respectively, of the computed eigenvalues *ilo* to *ihi* are stored in the corresponding elements of *wr* and *wi*. If two eigenvalues are computed as a complex conjugate pair, they are stored in consecutive elements of *wr* and *wi*, say the *i*-th and (*i* +1)-th, with $wi[i-1] > 0$ and $wi[i] < 0$. If *wantt* is zero, the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in *A*. *A* may be returned with larger diagonal blocks until the next release.

z On exit *z* has been updated; transformations are applied only to the submatrix *Z(ilo:ihiz, ilo:ihi)*.

info (global)

= 0: the execution is successful.

< 0: the parameter number - *info* is incorrect or inconsistent

> 0: p?lahqr failed to compute all the eigenvalues *ilo* to *ihi* in a total of $30*(ihi-ilo+1)$ iterations; if *info* = *i*, elements *i*+1: *ihi* of *wr* and *wi* contain the eigenvalues that have been successfully computed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trevc

Computes right and/or left eigenvectors of a complex upper triangular matrix in parallel.

Syntax

```
void pctrevc (const char* side, const char* howmny, const MKL_INT* select, const
MKL_INT* n, MKL_Complex8* t, const MKL_INT* descvt, MKL_Complex8* vl, const MKL_INT*
descv1, MKL_Complex8* vr, const MKL_INT* descvr, const MKL_INT* mm, MKL_INT* m,
MKL_Complex8* work, float* rwork, MKL_INT* info);
```

```
void pztrevc (const char* side, const char* howmny, const MKL_INT* select, const
MKL_INT* n, MKL_Complex16* t, const MKL_INT* descvt, MKL_Complex16* vl, const MKL_INT*
descv1, MKL_Complex16* vr, const MKL_INT* descvr, const MKL_INT* mm, MKL_INT* m,
MKL_Complex16* work, double* rwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

`pztrevc` computes some or all of the right and/or left eigenvectors of a complex upper triangular matrix T in parallel.

The right eigenvector x and the left eigenvector y of T corresponding to an eigenvalue w are defined by:

$$T^*x = w^*x,$$

$$y^*T = w^*y'$$

where y' denotes the conjugate transpose of the vector y .

If all eigenvectors are requested, the routine may either return the matrices X and/or Y of right or left eigenvectors of T , or the products Q^*X and/or Q^*Y , where Q is an input unitary matrix. If T was obtained from the Schur factorization of an original matrix $A = Q^*T^*Q'$, then Q^*X and Q^*Y are the matrices of right or left eigenvectors of A .

Input Parameters

| | |
|---------------|---|
| <i>side</i> | (global) = 'R': compute right eigenvectors only; = 'L': compute left eigenvectors only; = 'B': compute both right and left eigenvectors. |
| <i>howmny</i> | (global) = 'A': compute all right and/or left eigenvectors; = 'B': compute all right and/or left eigenvectors, and backtransform them using the input matrices supplied in <i>vr</i> and/or <i>vl</i> ; = 'S': compute selected right and/or left eigenvectors, specified by the logical array <i>select</i> . |
| <i>select</i> | (global) Array, size (<i>n</i>) If <i>howmny</i> = 'S', <i>select</i> specifies the eigenvectors to be computed. If <i>howmny</i> = 'A' or 'B', <i>select</i> is not referenced. To select the eigenvector corresponding to the j -th eigenvalue, <i>select</i> [$j - 1$] must be set to non-zero. |
| <i>n</i> | (global) The order of the matrix T . $n \geq 0$. |
| <i>t</i> | (local) Array, size $lld_t * LOCC(n)$. |

| | |
|---------------|---|
| | The upper triangular matrix T . T is modified, but restored on exit. |
| <i>desct</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix T . |
| <i>v1</i> | (local) Array, size (<i>descv1</i> (<i>lld_</i>), <i>mm</i>) On entry, if <i>side</i> = 'L' or 'B' and <i>howmny</i> = 'B', <i>v1</i> must contain an n -by- n matrix Q (usually the unitary matrix Q of Schur vectors returned by ? hseqr). |
| <i>descv1</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix VL . |
| <i>vr</i> | (local) Array, size <i>descvr</i> (<i>lld_</i>)* <i>mm</i> . On entry, if <i>side</i> = 'R' or 'B' and <i>howmny</i> = 'B', <i>vr</i> must contain an n -by- n matrix Q (usually the unitary matrix Q of Schur vectors returned by ? hseqr). |
| <i>descvr</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix VR . |
| <i>mm</i> | (global) The number of columns in the arrays <i>v1</i> and/or <i>vr</i> . <i>mm</i> $\geq m$. |
| <i>work</i> | (local) Array, size (2* <i>desct</i> (<i>lld_</i>)) Additional workspace may be required if <i>p?lattr</i> s is updated to use <i>work</i> . |
| <i>rwork</i> | Array, size (<i>desct</i> (<i>lld_</i>)) |

Output Parameters

| | |
|-----------|---|
| <i>t</i> | The upper triangular matrix T . T is modified, but restored on exit. |
| <i>v1</i> | On exit, if <i>side</i> = 'L' or 'B', <i>v1</i> contains: if <i>howmny</i> = 'A', the matrix Y of left eigenvectors of T ; if <i>howmny</i> = 'B', the matrix $Q*Y$; if <i>howmny</i> = 'S', the left eigenvectors of T specified by <i>select</i> , stored consecutively in the columns of <i>v1</i> , in the same order as their eigenvalues. If <i>side</i> = 'R', <i>v1</i> is not referenced. |
| <i>vr</i> | On exit, if <i>side</i> = 'R' or 'B', <i>vr</i> contains: |

if *howmny* = 'A', the matrix *X* of right eigenvectors of *T*;
 if *howmny* = 'B', the matrix *Q***X*;
 if *howmny* = 'S', the right eigenvectors of *T* specified by *select*, stored consecutively in the columns of *vr*, in the same order as their eigenvalues. If *side* = 'L', *vr* is not referenced.

m (global)
 The number of columns in the arrays *vl* and/or *vr* actually used to store the eigenvectors. If *howmny* = 'A' or 'B', *m* is set to *n*. Each selected eigenvector occupies one column.

info (global)
 = 0: successful exit
 < 0: if *info* = -*i*, the *i*-th argument had an illegal value

Application Notes

The algorithm used in this program is basically backward (forward) substitution. Scaling should be used to make the code robust against possible overflow. But scaling has not yet been implemented in `p?lattr`s which is called by this routine to solve the triangular systems. `p?lattr`s just calls `p?trsv`.

Each eigenvector is normalized so that the element of largest magnitude has magnitude 1; here the magnitude of a complex number (*x*,*y*) is taken to be $|x| + |y|$.

Singular Value Decomposition: ScaLAPACK Driver Routines

This section describes ScaLAPACK routines for computing the singular value decomposition (SVD) of a general *m*-by-*n* matrix *A* (see "Singular Value Decomposition" in LAPACK chapter).

To find the SVD of a general matrix *A*, this matrix is first reduced to a bidiagonal matrix *B* by a unitary (orthogonal) transformation, and then SVD of the bidiagonal matrix is computed. Note that the SVD of *B* is computed using the LAPACK routine `?bdsqr`.

Table "Computational Routines for Singular Value Decomposition (SVD)" lists ScaLAPACK computational routines for performing this decomposition.

Computational Routines for Singular Value Decomposition (SVD)

| Operation | General matrix | Orthogonal/unitary matrix |
|--|----------------------|------------------------------|
| Reduce <i>A</i> to a bidiagonal matrix | <code>p?gebrd</code> | |
| Multiply matrix after reduction | | <code>p?ormbr/p?unmbr</code> |

`p?gebrd`

Reduces a general matrix to bidiagonal form.

Syntax

```
void psgebrd (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *d , float *e , float *tauq , float *taup , float *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pdgebrd (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *d , double *e , double *tauq , double *taup , double *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pcgebrd (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *d , float *e , MKL_Complex8 *tauq , MKL_Complex8 *taup ,
MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );

void pzgebrd (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *d , double *e , MKL_Complex16 *tauq , MKL_Complex16 *taup ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?gebrd` function reduces a real/complex general m -by- n distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$ to upper or lower bidiagonal form B by an orthogonal/unitary transformation:

$$Q^* \text{sub}(A) P = B.$$

If $m \geq n$, B is upper bidiagonal; if $m < n$, B is lower bidiagonal.

Input Parameters

| | |
|---------------------|---|
| <code>m</code> | (global) The number of rows in the distributed matrix $\text{sub}(A)$ ($m \geq 0$). |
| <code>n</code> | (global) The number of columns in the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <code>a</code> | (local) Real pointer into the local memory to an array of size $\text{lld}_a * \text{LOCc}(\text{ja}+n-1)$. On entry, this array contains the distributed matrix $\text{sub}(A)$. |
| <code>ia, ja</code> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <code>desca</code> | (global and local) array of size $\text{dlen}_$. The array descriptor for the distributed matrix A . |
| <code>work</code> | (local) Workspace array of size lwork . |
| <code>lwork</code> | (local or global) size of work , must be at least: $\text{lwork} \geq \text{nb} * (\text{mpa0} + \text{nqa0} + 1) + \text{nqa0}$ where $\text{nb} = \text{mb}_a = \text{nb}_a$, $\text{iroffa} = \text{mod}(\text{ia}-1, \text{nb})$, $\text{icoffa} = \text{mod}(\text{ja}-1, \text{nb})$, $\text{iarow} = \text{indxg2p}(\text{ia}, \text{nb}, \text{MYROW}, \text{rsrc}_a, \text{NPROW})$, $\text{iacol} = \text{indxg2p}(\text{ja}, \text{nb}, \text{MYCOL}, \text{csrc}_a, \text{NPCOL})$, $\text{mpa0} = \text{numroc}(m + \text{iroffa}, \text{nb}, \text{MYROW}, \text{iarow}, \text{NPROW})$, $\text{nqa0} = \text{numroc}(n + \text{icoffa}, \text{nb}, \text{MYCOL}, \text{iacol}, \text{NPCOL})$, |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|-------------------------|--|
| <code>a</code> | On exit, if $m \geq n$, the diagonal and the first superdiagonal of $\text{sub}(A)$ are overwritten with the upper bidiagonal matrix B ; the elements below the diagonal, with the array <code>tauq</code> , represent the orthogonal/unitary matrix Q as a product of elementary reflectors, and the elements above the first superdiagonal, with the array <code>taup</code> , represent the orthogonal matrix P as a product of elementary reflectors. If $m < n$, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix B ; the elements below the first subdiagonal, with the array <code>tauq</code> , represent the orthogonal/unitary matrix Q as a product of elementary reflectors, and the elements above the diagonal, with the array <code>taup</code> , represent the orthogonal matrix P as a product of elementary reflectors. See <i>Application Notes</i> below. |
| <code>d</code> | (local) Array of size <code>LOCc(ja+min(m,n)-1)</code> if $m \geq n$ and <code>LOCr(ia+min(m,n)-1)</code> otherwise. The distributed diagonal elements of the bidiagonal matrix B : $d[i] = A(i+1,i+1)$, $0 \leq i < \text{size}(d)$. <code>d</code> is tied to the distributed matrix A . |
| <code>e</code> | (local) Array of size <code>LOCr(ia+min(m,n)-1)</code> if $m \geq n$; <code>LOCc(ja+min(m,n)-2)</code> otherwise. The distributed off-diagonal elements of the bidiagonal distributed matrix B : If $m \geq n$, $e[i] = A(i+1,i+2)$ for $i = 0, 1, \dots, n-2$; if $m < n$, $e[i] = A(i+2,i+1)$ for $i = 0, 1, \dots, m-2$. <code>e</code> is tied to the distributed matrix A . |
| <code>tauq, taup</code> | (local) Arrays of size <code>LOCc(ja+min(m,n)-1)</code> for <code>tauq</code> and <code>LOCr(ia+min(m,n)-1)</code> for <code>taup</code> . Contain the scalar factors of the elementary reflectors that represent the orthogonal/unitary matrices Q and P , respectively. <code>tauq</code> and <code>taup</code> are tied to the distributed matrix A . See <i>Application Notes</i> below. |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then <code>info = -(i*100+j)</code> ; if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

Application Notes

The matrices Q and P are represented as products of elementary reflectors:

If $m \geq n$,

$$Q = H(1)*H(2)*...*H(n), \text{ and } P = G(1)*G(2)*...*G(n-1).$$

Each $H(i)$ and $G(i)$ has the form:

$$H(i) = I - \tau u q^* v^* v' \text{ and } G(i) = I - \tau u p^* u^* u'$$

where $\tau u q$ and $\tau u p$ are real/complex scalars, and v and u are real/complex vectors;

$v(1:i-1) = 0$, $v(i) = 1$, and $v(i+1:m)$ is stored on exit in $A(ia+i:ia+m-1, ja+i-1)$;

$u(1:i) = 0$, $u(i+1) = 1$, and $u(i+2:n)$ is stored on exit in $A(ia+i-1, ja+i+1:ja+n-1)$;

$\tau u q$ is stored in $\tau u q[ja+i-2]$ and $\tau u p$ in $\tau u p[ia+i-2]$.

If $m < n$,

$$Q = H(1)*H(2)*...*H(m-1), \text{ and } P = G(1)*G(2)*...*G(m)$$

Each $H(i)$ and $G(i)$ has the form:

$$H(i) = I - \tau u q^* v^* v' \text{ and } G(i) = I - \tau u p^* u^* u'$$

here $\tau u q$ and $\tau u p$ are real/complex scalars, and v and u are real/complex vectors;

$v(1:i) = 0$, $v(i+1) = 1$, and $v(i+2:m)$ is stored on exit in $A(ia+i:ia+m-1, ja+i-1)$; $u(1:i-1) = 0$, $u(i) = 1$, and $u(i+1:n)$ is stored on exit in $A(ia+i-1, ja+i+1:ja+n-1)$;

$\tau u q$ is stored in $\tau u q[ja+i-2]$ and $\tau u p$ in $\tau u p[ia+i-2]$.

The contents of sub(A) on exit are illustrated by the following examples:

$m = 6$ and $n = 5$ ($m > n$):

$$\begin{bmatrix} d & e & u1 & u1 & u1 \\ v1 & d & e & u2 & u2 \\ v1 & v2 & d & e & u3 \\ v1 & v2 & v3 & d & e \\ v1 & v2 & v3 & v4 & d \\ v1 & v2 & v3 & v4 & v5 \end{bmatrix}$$

$m = 5$ and $n = 6$ ($m < n$):

$$\begin{bmatrix} d & u1 & u1 & u1 & u1 & u1 \\ e & d & u2 & u2 & u2 & u2 \\ v1 & e & d & u3 & u3 & u3 \\ v1 & v2 & e & d & u4 & u4 \\ v1 & v2 & v3 & e & d & u5 \end{bmatrix}$$

where d and e denote diagonal and off-diagonal elements of B , vi denotes an element of the vector defining $H(i)$, and ui an element of the vector defining $G(i)$.

See Also

Overview of [ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ormbr

Multiplies a general matrix by one of the orthogonal matrices from a reduction to bidiagonal form determined by p?gebrd.

Syntax

```
void psormbr (char *vect , char *side , char *trans , MKL_INT *m , MKL_INT *n ,
MKL_INT *k , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau ,
float *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork ,
MKL_INT *info );
```

```
void pdormbr (char *vect , char *side , char *trans , MKL_INT *m , MKL_INT *n ,
MKL_INT *k , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau ,
double *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

If $vect = 'Q'$, the `p?ormbr` function overwrites the general real distributed m -by- n matrix $sub(C) = C(ic:ic+m-1, jc:jc+n-1)$ with

| | | |
|----------------|--------------|--------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q sub(C)$ | $sub(C) Q$ |
| $trans = 'T':$ | $Q^T sub(C)$ | $sub(C) Q^T$ |

If $vect = 'P'$, the function overwrites $sub(C)$ with

| | | |
|----------------|--------------|--------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $P sub(C)$ | $sub(C) P$ |
| $trans = 'T':$ | $P^T sub(C)$ | $sub(C) P^T$ |

Here Q and P^T are the orthogonal distributed matrices determined by `p?gebrd` when reducing a real distributed matrix $A(ia:*, ja:*)$ to bidiagonal form: $A(ia:*, ja:*) = Q*B*P^T$. Q and P^T are defined as products of elementary reflectors $H(i)$ and $G(i)$ respectively.

Let $nq = m$ if $side = 'L'$ and $nq = n$ if $side = 'R'$. Therefore nq is the order of the orthogonal matrix Q or P^T that is applied.

If $vect = 'Q'$, $A(ia:*, ja:*)$ is assumed to have been an nq -by- k matrix:

If $nq \geq k$, $Q = H(1) H(2) \dots H(k)$;

If $nq < k$, $Q = H(1) H(2) \dots H(nq-1)$.

If $vect = 'P'$, $A(ia:*, ja:*)$ is assumed to have been a k -by- nq matrix:

If $k < nq$, $P = G(1) G(2) \dots G(k)$;

If $k \geq nq$, $P = G(1) G(2) \dots G(nq-1)$.

Input Parameters

| | |
|---------------|---|
| <i>vect</i> | (global) If <i>vect</i> = 'Q', then Q or Q^T is applied. If <i>vect</i> = 'P', then P or P^T is applied. |
| <i>side</i> | (global) If <i>side</i> = 'L', then Q or Q^T , P or P^T is applied from the left. If <i>side</i> = 'R', then Q or Q^T , P or P^T is applied from the right. |
| <i>trans</i> | (global) If <i>trans</i> = 'N', no transpose, Q or P is applied. If <i>trans</i> = 'T', then Q^T or P^T is applied. |
| <i>m</i> | (global) The number of rows in the distributed matrix sub (C). |
| <i>n</i> | (global) The number of columns in the distributed matrix sub (C). |
| <i>k</i> | (global) If <i>vect</i> = 'Q', the number of columns in the original distributed matrix reduced by <code>p?gebrd</code> ; If <i>vect</i> = 'P', the number of rows in the original distributed matrix reduced by <code>p?gebrd</code> . Constraints: $k \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja + \min(nq, k) - 1)$ if <i>vect</i> = 'Q', and $lld_a * LOCC(ja + nq - 1)$ if <i>vect</i> = 'P'. $nq = m$ if <i>side</i> = 'L', and $nq = n$ otherwise. The vectors that define the elementary reflectors $H(i)$ and $G(i)$, whose products determine the matrices Q and P , as returned by <code>p?gebrd</code> . If <i>vect</i> = 'Q', $lld_a \geq \max(1, LOCr(ia + nq - 1))$; If <i>vect</i> = 'P', $lld_a \geq \max(1, LOCr(ia + \min(nq, k) - 1))$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size $LOCC(ja + \min(nq, k) - 1)$, if <i>vect</i> = 'Q', and $LOCr(ia + \min(nq, k) - 1)$, if <i>vect</i> = 'P'. <i>tau</i> [<i>i</i>] must contain the scalar factor of the elementary reflector $H(i+1)$ or $G(i+1)$ |

which determines Q or P , as returned by `pdgebrd` in its array argument `tauq` or `taup`. `tau` is tied to the distributed matrix A .

| | |
|---------------------|---|
| <code>c</code> | (local) Pointer into the local memory to an array of size <code>lld_c*LOCc(jc+n-1)</code> . Contains the local pieces of the distributed matrix sub (C). |
| <code>ic, jc</code> | (global) The row and column indices in the global matrix C indicating the first row and the first column of the submatrix C , respectively. |
| <code>desc</code> | (global and local) array of size <code>dlen_</code> . The array descriptor for the distributed matrix C . |
| <code>work</code> | (local) Workspace array of size <code>lwork</code> . |
| <code>lwork</code> | (local or global) size of <code>work</code> , must be at least: <pre> If side = 'L' nq = m; if ((vect = 'Q' and nq ≥ k) or (vect is not equal to 'Q' and nq > k)), iaa=ia; jaa=ja; mi=m; ni=n; icc=ic; jcc=jc; else iaa= ia+1; jaa=ja; mi=m-1; ni=n; icc=ic+1; jcc= jc; end if else If side = 'R', nq = n; if((vect = 'Q' and nq ≥ k) or (vect is not equal to 'Q' and nq > k)), iaa=ia; jaa=ja; mi=m; ni=n; icc=ic; jcc=jc; else iaa= ia; jaa= ja+1; mi= m; ni= n-1; icc= ic; jcc= jc+1; end if end if If vect = 'Q', If side = 'L', lwork ≥ max((nb_a*(nb_a-1))/2, (nqc0 + mpc0)*nb_a) + nb_a * nb_a else if side = 'R', lwork ≥ max((nb_a*(nb_a-1))/2, (nqc0 + max(npa0 + numroc(numroc(ni+icoffc, nb_a, 0, 0, NPCOL), nb_a, 0, 0, lcmq), mpc0))*nb_a) + nb_a*nb_a end if else if vect is not equal to 'Q', if side = 'L', </pre> |

```

lwork ≥ max((mb_a*(mb_a-1))/2, (mpc0 + max(mqa0 +
numroc(numroc(mi+iroffc, mb_a, 0, 0, NPROW), mb_a, 0, 0,
lcmp), nqc0))*mb_a) + mb_a*mb_a
else if side = 'R',
lwork ≥ max((mb_a*(mb_a-1))/2, (mpc0 + nqc0)*mb_a) + mb_a*mb_a
end if
end if
where lcmp = lcm/NPROW, lcmq = lcm/NPCOL, with lcm =
ilcm(NPROW, NPCOL),
iroffa = mod(iaa-1, mb_a),
icoffa = mod(jaa-1, nb_a),
iarow = indxg2p(iaa, mb_a, MYROW, rsrc_a, NPROW),
iacol = indxg2p(jaa, nb_a, MYCOL, csrc_a, NPCOL),
mqa0 = numroc(mi+icoffa, nb_a, MYCOL, iacol, NPCOL),
npa0 = numroc(ni+iroffa, mb_a, MYROW, iarow, NPROW),
iroffc = mod(icc-1, mb_c),
icoffc = mod(jcc-1, nb_c),
icrow = indxg2p(icc, mb_c, MYROW, rsrc_c, NPROW),
iccol = indxg2p(jcc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0 = numroc(mi+iroffc, mb_c, MYROW, icrow, NPROW),
nqc0 = numroc(ni+icoffc, nb_c, MYCOL, iccol, NPCOL),

```

NOTE

`mod(x, y)` is the integer remainder of x/y .

`indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW` and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | On exit, if <code>vect='Q'</code> , <code>sub(C)</code> is overwritten by $Q*\text{sub}(C)$, or $Q'*\text{sub}(C)$, or $\text{sub}(C)*Q'$, or $\text{sub}(C)*Q$; if <code>vect='P'</code> , <code>sub(C)</code> is overwritten by $P*\text{sub}(C)$, or $P'*\text{sub}(C)$, or $\text{sub}(C)*P'$, or $\text{sub}(C)*P$. |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. |

< 0 : if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?unmbr

Multiplies a general matrix by one of the unitary transformation matrices from a reduction to bidiagonal form determined by p?gebrd.

Syntax

```
void pcunmbr (char *vect , char *side , char *trans , MKL_INT *m , MKL_INT *n ,
MKL_INT *k , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
MKL_Complex8 *tau , MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc ,
MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzunmbr (char *vect , char *side , char *trans , MKL_INT *m , MKL_INT *n ,
MKL_INT *k , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca ,
MKL_Complex16 *tau , MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

If $vect = 'Q'$, the `p?unmbr` function overwrites the general complex distributed m -by- n matrix $sub(C) = C(ic:ic+m-1,jc:jc+n-1)$ with

| | | |
|----------------|--------------|--------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $Q*sub(C)$ | $sub(C)*Q$ |
| $trans = 'C':$ | $Q^H*sub(C)$ | $sub(C)*Q^H$ |

If $vect = 'P'$, the function overwrites $sub(C)$ with

| | | |
|----------------|--------------|--------------|
| | $side = 'L'$ | $side = 'R'$ |
| $trans = 'N':$ | $P*sub(C)$ | $sub(C)*P$ |
| $trans = 'C':$ | $P^H*sub(C)$ | $sub(C)*P^H$ |

Here Q and P^H are the unitary distributed matrices determined by `p?gebrd` when reducing a complex distributed matrix $A(ia:*, ja:*)$ to bidiagonal form: $A(ia:*, ja:*) = Q*B*P^H$.

Q and P^H are defined as products of elementary reflectors $H(i)$ and $G(i)$ respectively.

Let $nq = m$ if $side = 'L'$ and $nq = n$ if $side = 'R'$. Therefore nq is the order of the unitary matrix Q or P^H that is applied.

If $vect = 'Q'$, $A(ia:*, ja:*)$ is assumed to have been an nq -by- k matrix:

If $nq \geq k$, $Q = H(1) H(2) \dots H(k)$;

If $nq < k$, $Q = H(1) H(2) \dots H(nq-1)$.

If $vect = 'P'$, $A(ia:*, ja:*)$ is assumed to have been a k -by- nq matrix:

If $k < nq$, $P = G(1) G(2) \dots G(k)$;

If $k \geq nq$, $P = G(1) G(2) \dots G(nq-1)$.

Input Parameters

| | |
|---------------|---|
| <i>vect</i> | (global) If <i>vect</i> = 'Q', then Q or Q^H is applied. If <i>vect</i> = 'P', then P or P^H is applied. |
| <i>side</i> | (global) If <i>side</i> = 'L', then Q or Q^H , P or P^H is applied from the left. If <i>side</i> = 'R', then Q or Q^H , P or P^H is applied from the right. |
| <i>trans</i> | (global) If <i>trans</i> = 'N', no transpose, Q or P is applied. If <i>trans</i> = 'C', conjugate transpose, Q^H or P^H is applied. |
| <i>m</i> | (global) The number of rows in the distributed matrix sub (C) $m \geq 0$. |
| <i>n</i> | (global) The number of columns in the distributed matrix sub (C) $n \geq 0$. |
| <i>k</i> | (global) If <i>vect</i> = 'Q', the number of columns in the original distributed matrix reduced by <code>p?gebrd</code> ; If <i>vect</i> = 'P', the number of rows in the original distributed matrix reduced by <code>p?gebrd</code> . Constraints: $k \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCc(ja + \min(nq, k) - 1)$ if <i>vect</i> = 'Q', and $lld_a * LOCc(ja + nq - 1)$ if <i>vect</i> = 'P'. $nq = m$ if <i>side</i> = 'L', and $nq = n$ otherwise. The vectors that define the elementary reflectors $H(i)$ and $G(i)$, whose products determine the matrices Q and P , as returned by <code>p?gebrd</code> . If <i>vect</i> = 'Q', $lld_a \geq \max(1, LOCr(ia + nq - 1))$; If <i>vect</i> = 'P', $lld_a \geq \max(1, LOCr(ia + \min(nq, k) - 1))$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size $LOCc(ja + \min(nq, k) - 1)$, if <i>vect</i> = 'Q', and $LOCr(ia + \min(nq, k) - 1)$, if <i>vect</i> = 'P'. <i>tau</i> [<i>i</i>] must contain the scalar factor of the elementary reflector $H(i+1)$ or $G(i+1)$, which determines Q or P , as returned by <code>p?gebrd</code> in its array argument <i>tauq</i> or <i>taup</i> . <i>tau</i> is tied to the distributed matrix A . |

| | |
|---------------|--|
| <i>c</i> | (local) Pointer into the local memory to an array of size $lld_c * LOCC(jc+n-1)$. Contains the local pieces of the distributed matrix sub (C). |
| <i>ic, jc</i> | (global) The row and column indices in the global matrix <i>C</i> indicating the first row and the first column of the submatrix <i>C</i> , respectively. |
| <i>desc</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>C</i> . |
| <i>work</i> | (local) Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) size of <i>work</i> , must be at least: <pre> If <i>side</i> = 'L' <i>nq</i> = <i>m</i>; if ((<i>vect</i> = 'Q' and <i>nq</i> ≥ <i>k</i>) or (<i>vect</i> is not equal to 'Q' and <i>nq</i> > <i>k</i>)), <i>iaa</i> = <i>ia</i>; <i>jaa</i> = <i>ja</i>; <i>mi</i> = <i>m</i>; <i>ni</i> = <i>n</i>; <i>icc</i> = <i>ic</i>; <i>jcc</i> = <i>jc</i>; else <i>iaa</i> = <i>ia</i>+1; <i>jaa</i> = <i>ja</i>; <i>mi</i> = <i>m</i>-1; <i>ni</i> = <i>n</i>; <i>icc</i> = <i>ic</i>+1; <i>jcc</i> = <i>jc</i>; end if else If <i>side</i> = 'R', <i>nq</i> = <i>n</i>; if ((<i>vect</i> = 'Q' and <i>nq</i> ≥ <i>k</i>) or (<i>vect</i> is not equal to 'Q' and <i>nq</i> > <i>k</i>)), <i>iaa</i> = <i>ia</i>; <i>jaa</i> = <i>ja</i>; <i>mi</i> = <i>m</i>; <i>ni</i> = <i>n</i>; <i>icc</i> = <i>ic</i>; <i>jcc</i> = <i>jc</i>; else <i>iaa</i> = <i>ia</i>; <i>jaa</i> = <i>ja</i>+1; <i>mi</i> = <i>m</i>; <i>ni</i> = <i>n</i>-1; <i>icc</i> = <i>ic</i>; <i>jcc</i> = <i>jc</i>+1; end if end if If <i>vect</i> = 'Q', If <i>side</i> = 'L', <i>lwork</i> ≥ max((<i>nb_a</i>*(<i>nb_a</i>-1))/2, (<i>nqc</i>0+<i>mpc</i>0)*<i>nb_a</i>) + <i>nb_a</i>*<i>nb_a</i> else if <i>side</i> = 'R', <i>lwork</i> ≥ max((<i>nb_a</i>*(<i>nb_a</i>-1))/2, (<i>nqc</i>0 + max(<i>npa</i>0+numroc(numroc(<i>ni</i>+<i>icoffc</i>, <i>nb_a</i>, 0, 0, NPCOL), <i>nb_a</i>, 0, 0, <i>lcmq</i>), <i>mpc</i>0))*<i>nb_a</i>) + <i>nb_a</i>*<i>nb_a</i> end if else if <i>vect</i> is not equal to 'Q', if <i>side</i> = 'L', <i>lwork</i> ≥ max((<i>mb_a</i>*(<i>mb_a</i>-1))/2, (<i>mpc</i>0 + max(<i>mqa</i>0+numroc(numroc(<i>mi</i>+<i>iroffc</i>, <i>mb_a</i>, 0, 0, NPROW), <i>mb_a</i>, 0, 0, <i>lcmp</i>), <i>nqc</i>0))*<i>mb_a</i>) + <i>mb_a</i>*<i>mb_a</i> </pre> |

```

else if side = 'R',
  lwork ≥ max((mb_a*(mb_a-1))/2, (mpc0 + nqc0)*mb_a) + mb_a*mb_a
end if
end if
where lcmp = lcm/NPROW, lcmq = lcm/NPCOL, with lcm =
  ilcm(NPROW, NPCOL),
irowfa = mod(iaa-1, mb_a),
icoffa = mod(jaa-1, nb_a),
iarow = indxg2p(iaa, mb_a, MYROW, rsrc_a, NPROW),
iacol = indxg2p(jaa, nb_a, MYCOL, csrc_a, NPCOL),
mqa0 = numroc(mi+icoffa, nb_a, MYCOL, iacol, NPCOL),
npa0 = numroc(ni+irowfa, mb_a, MYROW, iarow, NPROW),
irowfc = mod(icc-1, mb_c),
icoffc = mod(jcc-1, nb_c),
icrow = indxg2p(icc, mb_c, MYROW, rsrc_c, NPROW),
icol = indxg2p(jcc, nb_c, MYCOL, csrc_c, NPCOL),
mpc0 = numroc(mi+irowfc, mb_c, MYROW, icrow, NPROW),
nqc0 = numroc(ni+icoffc, nb_c, MYCOL, icol, NPCOL),

```

NOTE

mod(*x*, *y*) is the integer remainder of *x*/*y*.

`indxg2p` and `numroc` are ScaLAPACK tool functions; MYROW, MYCOL, NPROW and NPCOL can be determined by calling the function `blacs_gridinfo`.

If `lwork` = -1, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>c</code> | On exit, if <code>vect='Q'</code> , <code>sub(C)</code> is overwritten by <code>Q*sub(C)</code> , or <code>Q'*sub(C)</code> , or <code>sub(C)*Q'</code> , or <code>sub(C)*Q</code> ; if <code>vect='P'</code> , <code>sub(C)</code> is overwritten by <code>P*sub(C)</code> , or <code>P'*sub(C)</code> , or <code>sub(C)*P</code> , or <code>sub(C)*P'</code> . |
| <code>work[0]</code> | On exit <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> - 1, had an illegal value, then <code>info</code> = -(<i>i</i> *100+ <i>j</i>); if the <i>i</i> -th argument is a scalar and had an illegal value, then <code>info</code> = - <i>i</i> . |

See Also

Overview of ScaLAPACK Routines for details of ScaLAPACK array descriptor structures and related notations.

Generalized Symmetric-Definite Eigenvalue Problems: ScaLAPACK Computational Routines

This section describes ScaLAPACK routines that allow you to reduce the *generalized symmetric-definite eigenvalue problems* (see [Generalized Symmetric-Definite Eigenvalue Problems](#) in LAPACK chapters) to standard symmetric eigenvalue problem $Cy = \lambda y$, which you can solve by calling ScaLAPACK routines described earlier in this chapter (see [Symmetric Eigenproblems](#)).

Table "Computational Routines for Reducing Generalized Eigenproblems to Standard Problems" lists these routines.

Computational Routines for Reducing Generalized Eigenproblems to Standard Problems

| Operation | Real symmetric matrices | Complex Hermitian matrices |
|-----------------------------|-------------------------|----------------------------|
| Reduce to standard problems | p?sygst | p?hegst |

p?sygst

Reduces a real symmetric-definite generalized eigenvalue problem to the standard form.

Syntax

```
void pssygst (MKL_INT *ibtype , char *uplo , MKL_INT *n , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
float *scale , MKL_INT *info );

void pdsygst (MKL_INT *ibtype , char *uplo , MKL_INT *n , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*scale , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?sygst function reduces real symmetric-definite generalized eigenproblems to the standard form.

In the following sub(A) denotes $A(ia:ia+n-1, ja:ja+n-1)$ and sub(B) denotes $B(ib:ib+n-1, jb:jb+n-1)$.

If $ibtype = 1$, the problem is

$$\text{sub}(A)*x = \lambda*\text{sub}(B)*x,$$

and sub(A) is overwritten by $\text{inv}(U^T)*\text{sub}(A)*\text{inv}(U)$, or $\text{inv}(L)*\text{sub}(A)*\text{inv}(L^T)$.

If $ibtype = 2$ or 3 , the problem is

$$\text{sub}(A)*\text{sub}(B)*x = \lambda*x, \text{ or } \text{sub}(B)*\text{sub}(A)*x = \lambda*x,$$

and sub(A) is overwritten by $U*\text{sub}(A)*U^T$, or $L^T*\text{sub}(A)*L$.

sub(B) must have been previously factorized as U^T*U or $L*L^T$ by p?potrf.

Input Parameters

ibtype (global) Must be 1 or 2 or 3.

If $itype = 1$, compute $\text{inv}(U^T)*\text{sub}(A)*\text{inv}(U)$, or $\text{inv}(L)*\text{sub}(A)*\text{inv}(L^T)$;

| | |
|---------------|---|
| | If $itype = 2$ or 3 , compute $U*\text{sub}(A)*U^T$, or $L^T*\text{sub}(A)*L$. |
| <i>uplo</i> | (global) Must be 'U' or 'L'. If $uplo = 'U'$, the upper triangle of $\text{sub}(A)$ is stored and $\text{sub}(B)$ is factored as U^T*U . If $uplo = 'L'$, the lower triangle of $\text{sub}(A)$ is stored and $\text{sub}(B)$ is factored as $L*L^T$. |
| <i>n</i> | (global) The order of the matrices $\text{sub}(A)$ and $\text{sub}(B)$ ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a*LOCc(ja+n-1)$. On entry, the array contains the local pieces of the n -by- n symmetric distributed matrix $\text{sub}(A)$. If $uplo = 'U'$, the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If $uplo = 'L'$, the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>b</i> | (local) Pointer into the local memory to an array of size $lld_b*LOCc(jb+n-1)$. On entry, the array contains the local pieces of the triangular factor from the Cholesky factorization of $\text{sub}(B)$ as returned by <code>p?potrf</code> . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of the submatrix B , respectively. |
| <i>descb</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |

Output Parameters

| | |
|--------------|---|
| <i>a</i> | On exit, if $info = 0$, the transformed matrix, stored in the same format as $\text{sub}(A)$. |
| <i>scale</i> | (global) Amount by which the eigenvalues should be scaled to compensate for the scaling performed in this function. At present, <i>scale</i> is always returned as 1.0, it is returned here to allow for future enhancement. |
| <i>info</i> | (global) |

If $info = 0$, the execution is successful. If $info < 0$, if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?hegst

Reduces a Hermitian positive-definite generalized eigenvalue problem to the standard form.

Syntax

```
void pchegst (MKL_INT *ibtype , char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , float *scale , MKL_INT *info );
```

```
void pzhegst (MKL_INT *ibtype , char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , double *scale , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?hegst function reduces complex Hermitian positive-definite generalized eigenproblems to the standard form.

In the following $sub(A)$ denotes $A(ia:ia+n-1, ja:ja+n-1)$ and $sub(B)$ denotes $B(ib:ib+n-1, jb:jb+n-1)$.

If $ibtype = 1$, the problem is

$$sub(A)*x = \lambda*sub(B)*x,$$

and $sub(A)$ is overwritten by $inv(U^H)*sub(A)*inv(U)$, or $inv(L)*sub(A)*inv(L^H)$.

If $ibtype = 2$ or 3 , the problem is

$$sub(A)*sub(B)*x = \lambda*x, \text{ or } sub(B)*sub(A)*x = \lambda*x,$$

and $sub(A)$ is overwritten by $U*sub(A)*U^H$, or $L^H*sub(A)*L$.

$sub(B)$ must have been previously factorized as U^H*U or $L*L^H$ by p?potrf.

Input Parameters

ibtype (global) Must be 1 or 2 or 3.
 If $ibtype = 1$, compute $inv(U^H)*sub(A)*inv(U)$, or $inv(L)*sub(A)*inv(L^H)$;
 If $ibtype = 2$ or 3 , compute $U*sub(A)*U^H$, or $L^H*sub(A)*L$.

uplo (global) Must be 'U' or 'L'.
 If $uplo = 'U'$, the upper triangle of $sub(A)$ is stored and $sub(B)$ is factored as U^H*U .

If $uplo = 'L'$, the lower triangle of $sub(A)$ is stored and $sub(B)$ is factored as $L*L^H$.

| | |
|----------|--|
| n | (global) The order of the matrices $sub(A)$ and $sub(B)$ ($n \geq 0$). |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, the array contains the local pieces of the n -by- n Hermitian distributed matrix $sub(A)$. If $uplo = 'U'$, the leading n -by- n upper triangular part of $sub(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If $uplo = 'L'$, the leading n -by- n lower triangular part of $sub(A)$ contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| b | (local) Pointer into the local memory to an array of size $lld_b * LOCC(jb+n-1)$. On entry, the array contains the local pieces of the triangular factor from the Cholesky factorization of $sub(B)$ as returned by <code>p?potrf</code> . |
| ib, jb | (global) The row and column indices in the global matrix B indicating the first row and the first column of the submatrix B , respectively. |
| $descb$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |

Output Parameters

| | |
|---------|---|
| a | On exit, if $info = 0$, the transformed matrix, stored in the same format as $sub(A)$. |
| $scale$ | (global) Amount by which the eigenvalues should be scaled to compensate for the scaling performed in this function. At present, $scale$ is always returned as 1.0, it is returned here to allow for future enhancement. |
| $info$ | (global) If $info = 0$, the execution is successful. If $info < 0$, if the i -th argument is an array and the j -th entry, indexed $j - 1$, had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

ScaLAPACK Driver Routines

Table "ScaLAPACK Driver Routines" lists ScaLAPACK driver routines available for solving systems of linear equations, linear least-squares problems, standard eigenvalue and singular value problems, and generalized symmetric definite eigenproblems.

ScaLAPACK Driver Routines

| Type of Problem | Matrix type, storage scheme | Driver |
|---|--|--|
| Linear equations | general (partial pivoting) | p?gesv (simple driver) / p?gesvx (expert driver) |
| | general band (partial pivoting) | p?gbsv (simple driver) |
| | general band (no pivoting) | p?dbsv (simple driver) |
| | general tridiagonal (no pivoting) | p?dtsv (simple driver) |
| | symmetric/Hermitian positive-definite | p?posv (simple driver) / p?posvx (expert driver) |
| | symmetric/Hermitian positive-definite, band | p?pbsv (simple driver) |
| | symmetric/Hermitian positive-definite, tridiagonal | p?ptsv (simple driver) |
| Linear least squares problem | general m -by- n | p?gels |
| Symmetric eigenvalue problem | symmetric/Hermitian | p?syev / p?heev (simple driver); p?syevd / p?heevd (simple driver with a divide and conquer algorithm); p?syevx / p?heevx (expert driver); p?syevr / p?heevr (simple driver with MRRR algorithm) |
| Singular value decomposition | general m -by- n | p?gesvd |
| Generalized symmetric definite eigenvalue problem | symmetric/Hermitian, one matrix also positive-definite | p?sygvx / p?hegvx (expert driver) |

[p?gesv](#)

Computes the solution to the system of linear equations with a square distributed matrix and multiple right-hand sides.

Syntax

```
void psgesv (MKL_INT *n , MKL_INT *nrhs , float *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );
```

```
void pdgesv (MKL_INT *n , MKL_INT *nrhs , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );
```

```
void pcgesv (MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );
```

```
void pzgesv (MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?gesv` function computes the solution to a real or complex system of linear equations $\text{sub}(A) * X = \text{sub}(B)$, where $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ is an n -by- n distributed matrix and X and $\text{sub}(B) = B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+nrhs-1)$ are n -by- $nrhs$ distributed matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor $\text{sub}(A)$ as $\text{sub}(A) = P * L * U$, where P is a permutation matrix, L is unit lower triangular, and U is upper triangular. L and U are stored in $\text{sub}(A)$. The factored form of $\text{sub}(A)$ is then used to solve the system of equations $\text{sub}(A) * X = \text{sub}(B)$.

Input Parameters

| | |
|---------------|---|
| <i>n</i> | (global) The number of rows and columns to be operated on, that is, the order of the distributed submatrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right hand sides, that is, the number of columns of the distributed submatrices B and X ($nrhs \geq 0$). |
| <i>a, b</i> | (local) Pointers into the local memory to arrays of local size <i>a</i> : $lld_a * LOCC(\text{ja} + n - 1)$ and <i>b</i> : $lld_b * LOCC(\text{jb} + nrhs - 1)$, respectively. On entry, the array <i>a</i> contains the local pieces of the n -by- n distributed matrix $\text{sub}(A)$ to be factored. On entry, the array <i>b</i> contains the right hand side distributed matrix $\text{sub}(B)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of $\text{sub}(B)$, respectively. |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix B . |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | Overwritten by the factors L and U from the factorization $\text{sub}(A) = P * L * U$; the unit diagonal elements of L are not stored . |
| <i>b</i> | Overwritten by the solution distributed matrix X . |
| <i>ipiv</i> | (local) Array of size $LOCr(m_a) + mb_a$. This array contains the pivoting information. The (local) row <i>i</i> of the matrix was interchanged with the (global) row $ipiv[i - 1]$. This array is tied to the distributed matrix A . |
| <i>info</i> | (global) If $info=0$, the execution is successful. $info < 0$: |

If the i -th argument is an array and the j -th entry had an illegal value, then $info = -(i*100+j)$; if the i -th argument is a scalar and had an illegal value, then $info = -i$.

$info > 0$:

If $info = k$, $U(ia+k-1, ja+k-1)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution could not be computed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gesvx

Uses the LU factorization to compute the solution to the system of linear equations with a square matrix A and multiple right-hand sides, and provides error bounds on the solution.

Syntax

```
void psgesvx (char *fact , char *trans , MKL_INT *n , MKL_INT *nrhs , float *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *af , MKL_INT *iaf , MKL_INT *jaf ,
MKL_INT *descaf , MKL_INT *ipiv , char *equed , float *r , float *c , float *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , float *x , MKL_INT *ix , MKL_INT *jx ,
MKL_INT *descx , float *rcond , float *ferr , float *berr , float *work , MKL_INT
*lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pdgesvx (char *fact , char *trans , MKL_INT *n , MKL_INT *nrhs , double *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *af , MKL_INT *iaf , MKL_INT
*jaf , MKL_INT *descaf , MKL_INT *ipiv , char *equed , double *r , double *c , double
*b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , double *x , MKL_INT *ix , MKL_INT
*jx , MKL_INT *descx , double *rcond , double *ferr , double *berr , double *work ,
MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pcgesvx (char *fact , char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *af , MKL_INT *iaf , MKL_INT
*jaf , MKL_INT *descaf , MKL_INT *ipiv , char *equed , float *r , float *c ,
MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_Complex8 *x ,
MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , float *rcond , float *ferr , float
*berr , MKL_Complex8 *work , MKL_INT *lwork , float *rwork , MKL_INT *lrwork , MKL_INT
*info );
```

```
void pzgesvx (char *fact , char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *af , MKL_INT *iaf ,
MKL_INT *jaf , MKL_INT *descaf , MKL_INT *ipiv , char *equed , double *r , double *c ,
MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_Complex16 *x ,
MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , double *rcond , double *ferr , double
*berr , MKL_Complex16 *work , MKL_INT *lwork , double *rwork , MKL_INT *lrwork ,
MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?gesvx` function uses the *LU* factorization to compute the solution to a real or complex system of linear equations $AX = B$, where A denotes the n -by- n submatrix $A(ia:ia+n-1, ja:ja+n-1)$, B denotes the n -by- $nrhs$ submatrix $B(ib:ib+n-1, jb:jb+nrhs-1)$ and X denotes the n -by- $nrhs$ submatrix $X(ix:ix+n-1, jx:jx+nrhs-1)$.

Error bounds on the solution and a condition estimate are also provided.

In the following description, af stands for the subarray of af from row iaf and column jaf to row $iaf+n-1$ and column $jaf+n-1$.

The function `p?gesvx` performs the following steps:

1. If $fact = 'E'$, real scaling factors R and C are computed to equilibrate the system:

$trans = 'N'$: $diag(R) * A * diag(C) * diag(C)^{-1} * X = diag(R) * B$

$trans = 'T'$: $(diag(R) * A * diag(C))^T * diag(R)^{-1} * X = diag(C) * B$

$trans = 'C'$: $(diag(R) * A * diag(C))^H * diag(R)^{-1} * X = diag(C) * B$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $diag(R) * A * diag(C)$ and B by $diag(R) * B$ (if $trans = 'N'$) or $diag(C) * B$ (if $trans = 'T'$ or $'C'$).

2. If $fact = 'N'$ or $'E'$, the *LU* decomposition is used to factor the matrix A (after equilibration if $fact = 'E'$) as $A = PLU$, where P is a permutation matrix, L is a unit lower triangular matrix, and U is upper triangular.
3. The factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than relative machine precision, steps 4 - 6 are skipped.
4. The system of equations is solved for X using the factored form of A .
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $diag(C)$ (if $trans = 'N'$) or $diag(R)$ (if $trans = 'T'$ or $'C'$) so that it solves the original system before equilibration.

Input Parameters

| | |
|--------------------|--|
| <code>fact</code> | <p>(global) Must be $'F'$, $'N'$, or $'E'$.</p> <p>Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.</p> <p>If $fact = 'F'$ then, on entry, af and $ipiv$ contain the factored form of A. If $equed$ is not $'N'$, the matrix A has been equilibrated with scaling factors given by r and c. Arrays a, af, and $ipiv$ are not modified.</p> <p>If $fact = 'N'$, the matrix A is copied to af and factored.</p> <p>If $fact = 'E'$, the matrix A is equilibrated if necessary, then copied to af and factored.</p> |
| <code>trans</code> | <p>(global) Must be $'N'$, $'T'$, or $'C'$.</p> <p>Specifies the form of the system of equations:</p> <p>If $trans = 'N'$, the system has the form $A * X = B$ (No transpose);</p> <p>If $trans = 'T'$, the system has the form $A^T * X = B$ (Transpose);</p> <p>If $trans = 'C'$, the system has the form $A^H * X = B$ (Conjugate transpose);</p> |

| | |
|-----------------------|--|
| <i>n</i> | (global) The number of linear equations; the order of the submatrix A ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right hand sides; the number of columns of the distributed submatrices B and X ($nrhs \geq 0$). |
| <i>a, af, b, work</i> | (local) Pointers into the local memory to arrays of local size <i>a</i> : $lld_a * LOCC(ja + n - 1)$, <i>af</i> : $lld_af * LOCC(ja + n - 1)$, <i>b</i> : $lld_b * LOCC(jb + nrhs - 1)$, <i>work</i> : $lwork$. The array <i>a</i> contains the matrix A . If <i>fact</i> = 'F' and <i>equed</i> is not 'N', then A must have been equilibrated by the scaling factors in <i>r</i> and/or <i>c</i> . The array <i>af</i> is an input argument if <i>fact</i> = 'F'. In this case it contains on entry the factored form of the matrix A , that is, the factors L and U from the factorization $A = P * L * U$ as computed by <code>p?getrf</code> . If <i>equed</i> is not 'N', then <i>af</i> is the factored form of the equilibrated matrix A . The array <i>b</i> contains on entry the matrix B whose columns are the right-hand sides for the systems of equations. <i>work</i> is a workspace array. The size of <i>work</i> is (<i>lwork</i>). |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix $A(ia:ia+n-1, ja:ja+n-1)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>iaf, jaf</i> | (global) The row and column indices in the global matrix AF indicating the first row and the first column of the subarray <i>af</i> , respectively. |
| <i>descaf</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix AF . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of the submatrix $B(ib:ib+n-1, jb:jb+nrhs-1)$, respectively. |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix B . |
| <i>ipiv</i> | (local) Array of size $LOCr(m_a) + mb_a$. The array <i>ipiv</i> is an input argument if <i>fact</i> = 'F'. On entry, it contains the pivot indices from the factorization $A = P * L * U$ as computed by <code>p?getrf</code> ; (local) row <i>i</i> of the matrix was interchanged with the (global) row <i>ipiv</i> [<i>i</i> - 1]. This array must be aligned with $A(ia:ia+n-1, *)$. |
| <i>equed</i> | (global) Must be 'N', 'R', 'C', or 'B'. <i>equed</i> is an input argument if <i>fact</i> = 'F'. It specifies the form of equilibration that was done: If <i>equed</i> = 'N', no equilibration was done (always true if <i>fact</i> = 'N'); |

If *equed* = 'R', row equilibration was done, that is, *A* has been premultiplied by *diag(r)*;
 If *equed* = 'C', column equilibration was done, that is, *A* has been postmultiplied by *diag(c)*;
 If *equed* = 'B', both row and column equilibration was done; *A* has been replaced by *diag(r) * A * diag(c)*.

r, c

(local)

Arrays of size *LOCr(m_a)* and *LOCc(n_a)*, respectively.

The array *r* contains the row scale factors for *A*, and the array *c* contains the column scale factors for *A*. These arrays are input arguments if *fact* = 'F' only; otherwise they are output arguments. If *equed* = 'R' or 'B', *A* is multiplied on the left by *diag(r)*; if *equed* = 'N' or 'C', *r* is not accessed.

If *fact* = 'F' and *equed* = 'R' or 'B', each element of *r* must be positive.

If *equed* = 'C' or 'B', *A* is multiplied on the right by *diag(c)*; if *equed* = 'N' or 'R', *c* is not accessed.

If *fact* = 'F' and *equed* = 'C' or 'B', each element of *c* must be positive. Array *r* is replicated in every process column, and is aligned with the distributed matrix *A*. Array *c* is replicated in every process row, and is aligned with the distributed matrix *A*.

ix, jx

(global) The row and column indices in the global matrix *X* indicating the first row and the first column of the submatrix *X(ix:ix+n-1, jx:jx+nrhs-1)*, respectively.

descx

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix *X*.

lwork

(local or global) The size of the array *work*; must be at least $\max(p?gecon(lwork), p?gerfs(lwork)) + LOCr(n_a)$.

iwork

(local, *psgesvx*/*pdgesvx* only). Workspace array. The size of *iwork* is (*liwork*).

liwork

(local, *psgesvx*/*pdgesvx* only). The size of the array *iwork*, must be at least *LOCr(n_a)*.

rwork

(local)

Workspace array, used in complex flavors only.

The size of *rwork* is (*lrwork*).

lrwork

(local or global, *pcgesvx*/*pzgesvx* only). The size of the array *rwork*; must be at least $2 * LOCc(n_a)$.

Output Parameters

x

(local)

Pointer into the local memory to an array of local size $l1d_x * LOCC(jx + nrhs - 1)$.

If $info = 0$, the array x contains the solution matrix X to the *original* system of equations. Note that A and B are modified on exit if $equed \neq 'N'$, and the solution to the *equilibrated* system is:

$diag(C)^{-1} * X$, if $trans = 'N'$ and $equed = 'C'$ or $'B'$; and
 $diag(R)^{-1} * X$, if $trans = 'T'$ or $'C'$ and $equed = 'R'$ or $'B'$.

a

Array a is not modified on exit if $fact = 'F'$ or $'N'$, or if $fact = 'E'$ and $equed = 'N'$.

If $equed \neq 'N'$, A is scaled on exit as follows:

$equed = 'R'$: $A = diag(R) * A$

$equed = 'C'$: $A = A * diag(c)$

$equed = 'B'$: $A = diag(R) * A * diag(c)$

af

If $fact = 'N'$ or $'E'$, then af is an output argument and on exit returns the factors L and U from the factorization $A = P * L * U$ of the original matrix A (if $fact = 'N'$) or of the equilibrated matrix A (if $fact = 'E'$). See the description of a for the form of the equilibrated matrix.

b

Overwritten by $diag(R) * B$ if $trans = 'N'$ and $equed = 'R'$ or $'B'$;

overwritten by $diag(c) * B$ if $trans = 'T'$ and $equed = 'C'$ or $'B'$; not changed if $equed = 'N'$.

r, c

These arrays are output arguments if $fact \neq 'F'$.

See the description of r, c in *Input Arguments* section.

$rcond$

(global).

An estimate of the reciprocal condition number of the matrix A after equilibration (if done). The function sets $rcond = 0$ if the estimate underflows; in this case the matrix is singular (to working precision). However, anytime $rcond$ is small compared to 1.0, for the working precision, the matrix may be poorly conditioned or even singular.

$ferr, berr$

(local)

Arrays of size $LOCC(n_b)$ each. Contain the component-wise forward and relative backward errors, respectively, for each solution vector.

Arrays $ferr$ and $berr$ are both replicated in every process row, and are aligned with the matrices B and X .

$ipiv$

If $fact = 'N'$ or $'E'$, then $ipiv$ is an output argument and on exit contains the pivot indices from the factorization $A = P * L * U$ of the original matrix A (if $fact = 'N'$) or of the equilibrated matrix A (if $fact = 'E'$).

$equed$

If $fact \neq 'F'$, then $equed$ is an output argument. It specifies the form of equilibration that was done (see the description of $equed$ in *Input Arguments* section).

| | |
|-----------------------|--|
| <code>work[0]</code> | If <code>info=0</code> , on exit <code>work[0]</code> returns the minimum value of <code>lwork</code> required for optimum performance. |
| <code>iwork[0]</code> | If <code>info=0</code> , on exit <code>iwork[0]</code> returns the minimum value of <code>liwork</code> required for optimum performance. |
| <code>rwork[0]</code> | If <code>info=0</code> , on exit <code>rwork[0]</code> returns the minimum value of <code>lrwork</code> required for optimum performance. |
| <code>info</code> | If <code>info=0</code> , the execution is successful. <i>info</i> < 0: if the <i>i</i> th argument is an array and the <i>j</i> th entry had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>); if the <i>i</i> th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . If <i>info</i> = <i>i</i> , and $i \leq n$, then $U(i,i)$ is exactly zero. The factorization has been completed, but the factor U is exactly singular, so the solution and error bounds could not be computed. If <i>info</i> = <i>i</i> , and $i = n + 1$, then U is nonsingular, but <i>rcond</i> is less than machine precision. The factorization has been completed, but the matrix is singular to working precision and the solution and error bounds have not been computed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gbsv

Computes the solution to the system of linear equations with a general banded distributed matrix and multiple right-hand sides.

Syntax

```
void psgbsv (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs , float *a ,
MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , float *b , MKL_INT *ib , MKL_INT
*descb , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdgbsv (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs , double *a ,
MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , double *b , MKL_INT *ib , MKL_INT
*descb , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcgbsv (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs , MKL_Complex8
*a , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex8 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzgbsv (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs , MKL_Complex16
*a , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_Complex16 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?gbsv` function computes the solution to a real or complex system of linear equations

$$\text{sub}(A) * X = \text{sub}(B),$$

where $\text{sub}(A) = A(1:n, ja:ja+n-1)$ is an n -by- n real/complex general banded distributed matrix with bwl subdiagonals and bwu superdiagonals, and X and $\text{sub}(B) = B(ib:ib+n-1, 1:rhs)$ are n -by- $nrhs$ distributed matrices.

The LU decomposition with partial pivoting and row interchanges is used to factor $\text{sub}(A)$ as $\text{sub}(A) = P^*L^*U^*Q$, where P and Q are permutation matrices, and L and U are banded lower and upper triangular matrices, respectively. The matrix Q represents reordering of columns for the sake of parallelism, while P represents reordering of rows for numerical stability using classic partial pivoting.

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Notice revision #20110804

Input Parameters

| | |
|---------|---|
| n | (global) The number of rows and columns to be operated on, that is, the order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| bwl | (global) The number of subdiagonals within the band of A ($0 \leq bwl \leq n-1$). |
| bwu | (global) The number of superdiagonals within the band of A ($0 \leq bwu \leq n-1$). |
| $nrhs$ | (global) The number of right hand sides; the number of columns of the distributed matrix $\text{sub}(B)$ ($nrhs \geq 0$). |
| a, b | (local) Pointers into the local memory to arrays of local size a : $lld_a * LOCC(ja + n - 1)$ and b : $lld_b * LOCC(nrhs)$. On entry, the array a contains the local pieces of the global array A . On entry, the array b contains the right hand side distributed matrix $\text{sub}(B)$. |
| ja | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . If $desca[dtype_ - 1] = 501$, then $dlen_ \geq 7$; else if $desca[dtype_ - 1] = 1$, then $dlen_ \geq 9$. |
| ib | (global) The row index in the global matrix B indicating the first row of the matrix to be operated on (which may be either all of B or a submatrix of B). |
| $descb$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . If $descb[dtype_ - 1] = 502$, then $dlen_ \geq 7$; |

else if $descb[dtype_ -1] = 1$, then $dlen \geq 9$.

work

(local)

Workspace array of size *lwork*.

lwork

(local or global) The size of the array *work*, must be at least $lwork \geq (NB + bwu) * (bwl + bwu) + 6 * (bwl + bwu) * (bwl + 2 * bwu) + \max(nrhs * (NB + 2 * bwl + 4 * bwu), 1)$.

Output Parameters

a

On exit, contains details of the factorization. Note that the resulting factorization is not the same factorization as returned from LAPACK. Additional permutations are performed on the matrix for the sake of parallelism.

b

On exit, this array contains the local pieces of the solution distributed matrix *X*.

ipiv

(local) array.

The size of *ipiv* must be at least $desca[NB - 1]$. This array contains pivot indices for local factorizations. You should not alter the contents between factorization and solve.

work[0]

On exit, *work*[0] contains the minimum value of *lwork* required for optimum performance.

info

If *info*=0, the execution is successful. *info* < 0:

If the *i*th argument is an array and the *j*-th entry had an illegal value, then $info = -(i * 100 + j)$; if the *i*th argument is a scalar and had an illegal value, then $info = -i$.

info > 0:

If $info = k \leq NPROCS$, the submatrix stored on processor *info* and factored locally was not nonsingular, and the factorization was not completed. If $info = k > NPROCS$, the submatrix stored on processor *info*-NPROCS representing interactions with other processors was not nonsingular, and the factorization was not completed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?dbsv

Solves a general band system of linear equations.

Syntax

```
void psdbsv (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs , float *a ,
MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb , float *work ,
MKL_INT *lwork , MKL_INT *info );
```

```

void pddbsv (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs , double *a ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb , double
*work , MKL_INT *lwork , MKL_INT *info );

void pcdbsv (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs , MKL_Complex8
*a , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *descb ,
MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );

void pzdbsv (MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu , MKL_INT *nrhs , MKL_Complex16
*a , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *descb ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );

```

Include Files

- mkl_scalapack.h

Description

The `p?dbsv` function solves the following system of linear equations:

$$A(1:n, ja:ja+n-1) * X = B(ib:ib+n-1, 1:nrhs),$$

where $A(1:n, ja:ja+n-1)$ is an n -by- n real/complex banded diagonally dominant-like distributed matrix with bandwidth bwl , bwu .

Gaussian elimination without pivoting is used to factor a reordering of the matrix into LU .

Optimization Notice

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Notice revision #20110804

Input Parameters

| | |
|--------|--|
| n | (global) The order of the distributed submatrix A , ($n \geq 0$). |
| bwl | (global) Number of subdiagonals. $0 \leq bwl \leq n-1$. |
| bwu | (global) Number of subdiagonals. $0 \leq bwu \leq n-1$. |
| $nrhs$ | (global) The number of right-hand sides; the number of columns of the distributed submatrix B , ($nrhs \geq 0$). |
| a | (local). Pointer into the local memory to an array with leading size $lld_a \geq (bwl + bwu + 1)$ (stored in $desca$). On entry, this array contains the local pieces of the distributed matrix. |
| ja | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |

| | |
|--------------|---|
| <i>desca</i> | (global and local) array of size <i>dlen</i> . If 1d type (<i>dtype_a</i> =501 or 502), <i>dlen</i> ≥ 7; If 2d type (<i>dtype_a</i> =1), <i>dlen</i> ≥ 9. The array descriptor for the distributed matrix <i>A</i> . Contains information of mapping of <i>A</i> to memory. |
| <i>b</i> | (local) Pointer into the local memory to an array of local lead size <i>lld_b</i> ≥ <i>nb</i> . On entry, this array contains the local pieces of the right hand sides $B(ib:ib+n-1, 1:nrhs)$. |
| <i>ib</i> | (global) The row index in the global matrix <i>B</i> indicating the first row of the matrix to be operated on (which may be either all of <i>b</i> or a submatrix of <i>B</i>). |
| <i>descb</i> | (global and local) array of size <i>dlen</i> . If 1d type (<i>dtype_b</i> =502), <i>dlen</i> ≥ 7; If 2d type (<i>dtype_b</i> =1), <i>dlen</i> ≥ 9. The array descriptor for the distributed matrix <i>B</i> . Contains information of mapping of <i>B</i> to memory. |
| <i>work</i> | (local). Temporary workspace. This space may be overwritten in between calls to functions. <i>work</i> must be the size given in <i>lwork</i> . |
| <i>lwork</i> | (local or global) Size of user-input workspace <i>work</i> . If <i>lwork</i> is too small, the minimal acceptable size will be returned in <i>work</i> [0] and an error code is returned. $lwork \geq nb(bw1+bwu) + 6\max(bw1, bwu) * \max(bw1, bwu) + \max(\max(bw1, bwu) nrhs, \max(bw1, bwu) * \max(bw1, bwu))$ |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | On exit, this array contains information containing details of the factorization. Note that permutations are performed on the matrix, so that the factors returned are different from those returned by LAPACK. |
| <i>b</i> | On exit, this contains the local piece of the solutions distributed matrix <i>X</i> . |
| <i>work</i> | On exit, <i>work</i> [0] contains the minimal <i>lwork</i> . |
| <i>info</i> | (local) If <i>info</i> =0, the execution is successful. < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -entry had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . > 0: If <i>info</i> = <i>k</i> < NPROCS, the submatrix stored on processor <i>info</i> and factored locally was not positive definite, and the factorization was not completed. |

If $info = k > NPROCS$, the submatrix stored on processor $info - NPROCS$ representing interactions with other processors was not positive definite, and the factorization was not completed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?dtsv

Solves a general tridiagonal system of linear equations.

Syntax

```
void psdtsv (MKL_INT *n , MKL_INT *nrhs , float *dl , float *d , float *du , MKL_INT
*ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb , float *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pddtsv (MKL_INT *n , MKL_INT *nrhs , double *dl , double *d , double *du ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb , double
*work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcdtsv (MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *dl , MKL_Complex8 *d ,
MKL_Complex8 *du , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzdtsv (MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *dl , MKL_Complex16 *d ,
MKL_Complex16 *du , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The function solves a system of linear equations

$$A(1:n, ja:ja+n-1) * X = B(ib:ib+n-1, 1:nrhs),$$

where $A(1:n, ja:ja+n-1)$ is an n -by- n complex tridiagonal diagonally dominant-like distributed matrix.

Gaussian elimination without pivoting is used to factor a reordering of the matrix into LU .

Optimization Notice

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Notice revision #20110804

Input Parameters

| | |
|--------------|--|
| <i>n</i> | (global) The order of the distributed submatrix A ($n \geq 0$). |
| <i>nrhs</i> | The number of right hand sides; the number of columns of the distributed matrix B ($nrhs \geq 0$). |
| <i>d1</i> | (local). Pointer to local part of global vector storing the lower diagonal of the matrix. Globally, $d1[0]$ is not referenced, and $d1$ must be aligned with d . Must be of size $> desca[nb_ - 1]$. |
| <i>d</i> | (local). Pointer to local part of global vector storing the main diagonal of the matrix. |
| <i>du</i> | (local). Pointer to local part of global vector storing the upper diagonal of the matrix. Globally, $du[n - 1]$ is not referenced, and du must be aligned with d . |
| <i>ja</i> | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| <i>desca</i> | (global and local) array of size $dlen$. If 1d type ($dtype_a=501$ or 502), $dlen \geq 7$; If 2d type ($dtype_a=1$), $dlen \geq 9$. The array descriptor for the distributed matrix A . Contains information of mapping of A to memory. |
| <i>b</i> | (local) Pointer into the local memory to an array of local lead size $lld_b > nb$. On entry, this array contains the local pieces of the right hand sides $B(ib:ib+n-1, 1:nrhs)$. |
| <i>ib</i> | (global) The row index in the global matrix B indicating the first row of the matrix to be operated on (which may be either all of b or a submatrix of B). |
| <i>descb</i> | (global and local) array of size $dlen$. If 1d type ($dtype_b=502$), $dlen \geq 7$; If 2d type ($dtype_b=1$), $dlen \geq 9$. The array descriptor for the distributed matrix B . Contains information of mapping of B to memory. |
| <i>work</i> | (local). |
| <i>lwork</i> | (local or global) Size of user-input workspace $work$. If $lwork$ is too small, the minimal acceptable size will be returned in $work[0]$ and an error code is returned. $lwork > (12*NPCOL+3*nb)+\max((10+2*\min(100, nrhs))*NPCOL+4*nrhs, 8*NPCOL)$ |

Output Parameters

| | |
|-------------|---|
| <i>dl</i> | On exit, this array contains information containing the * factors of the matrix. |
| <i>d</i> | On exit, this array contains information containing the * factors of the matrix. Must be of size $> \text{desca}[\text{nb}_- - 1]$. |
| <i>du</i> | On exit, this array contains information containing the * factors of the matrix. Must be of size $> \text{desca}[\text{nb}_- - 1]$. |
| <i>b</i> | On exit, this contains the local piece of the solutions distributed matrix <i>X</i> . |
| <i>work</i> | On exit, <i>work</i> [0] contains the minimal <i>lwork</i> . |
| <i>info</i> | <p>(local) If <i>info</i>=0, the execution is successful.</p> <p>< 0: If the <i>i</i>-th argument is an array and the <i>j</i>-entry had an illegal value, then $\text{info} = -(i*100+j)$, if the <i>i</i>-th argument is a scalar and had an illegal value, then $\text{info} = -i$.</p> <p>> 0: If $\text{info} = k < \text{NPROCS}$, the submatrix stored on processor <i>info</i> and factored locally was not positive definite, and the factorization was not completed.</p> <p>If $\text{info} = k > \text{NPROCS}$, the submatrix stored on processor $\text{info} - \text{NPROCS}$ representing interactions with other processors was not positive definite, and the factorization was not completed.</p> |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?posv

Solves a symmetric positive definite system of linear equations.

Syntax

```
void psposv (char *uplo , MKL_INT *n , MKL_INT *nrhs , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
MKL_INT *info );

void pdposv (char *uplo , MKL_INT *n , MKL_INT *nrhs , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );

void pcposv (char *uplo , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );

void pzposv (char *uplo , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?posv` function computes the solution to a real/complex system of linear equations

$$\text{sub}(A) * X = \text{sub}(B),$$

where $\text{sub}(A)$ denotes $A(ia:ia+n-1, ja:ja+n-1)$ and is an n -by- n symmetric/Hermitian distributed positive definite matrix and X and $\text{sub}(B)$ denoting $B(ib:ib+n-1, jb:jb+nrhs-1)$ are n -by- $nrhs$ distributed matrices. The Cholesky decomposition is used to factor $\text{sub}(A)$ as

$$\text{sub}(A) = U^T * U, \text{ if } \text{uplo} = 'U', \text{ or}$$

$$\text{sub}(A) = L * L^T, \text{ if } \text{uplo} = 'L',$$

where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of $\text{sub}(A)$ is then used to solve the system of equations.

Input Parameters

| | |
|---------------|--|
| <i>uplo</i> | (global) Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of $\text{sub}(A)$ is stored. |
| <i>n</i> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>nrhs</i> | The number of right-hand sides; the number of columns of the distributed matrix $\text{sub}(B)$ ($nrhs \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the n -by- n symmetric distributed matrix $\text{sub}(A)$ to be factored. If $\text{uplo} = 'U'$, the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If $\text{uplo} = 'L'$, the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the distributed matrix, and its strictly upper triangular part is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>b</i> | (local) Pointer into the local memory to an array of size $lld_b * LOCC(jb+nrhs-1)$. On entry, the local pieces of the right hand sides distributed matrix $\text{sub}(B)$. |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of the submatrix B , respectively. |
| <i>descb</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | On exit, if <i>info</i> = 0, this array contains the local pieces of the factor <i>U</i> or <i>L</i> from the Cholesky factorization $\text{sub}(A) = U^H * U$, or $L * L^H$. |
| <i>b</i> | On exit, if <i>info</i> = 0, $\text{sub}(B)$ is overwritten by the solution distributed matrix <i>X</i> . |
| <i>info</i> | (global) If <i>info</i> = 0, the execution is successful. If <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . If <i>info</i> > 0: If <i>info</i> = <i>k</i> , the leading minor of order <i>k</i> , $A(\text{ia}:\text{ia}+\text{k}-1, \text{ja}:\text{ja}+\text{k}-1)$ is not positive definite, and the factorization could not be completed, and the solution has not been computed. |

See Also

Overview of [ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?posvx

Solves a symmetric or Hermitian positive definite system of linear equations.

Syntax

```
void psposvx (char *fact , char *uplo , MKL_INT *n , MKL_INT *nrhs , float *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *af , MKL_INT *iaf , MKL_INT *jaf ,
MKL_INT *descaf , char *equed , float *sr , float *sc , float *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , float *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx ,
float *rcond , float *ferr , float *berr , float *work , MKL_INT *lwork , MKL_INT
*iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pdposvx (char *fact , char *uplo , MKL_INT *n , MKL_INT *nrhs , double *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *af , MKL_INT *iaf , MKL_INT
*jaf , MKL_INT *descaf , char *equed , double *sr , double *sc , double *b , MKL_INT
*ib , MKL_INT *jb , MKL_INT *descb , double *x , MKL_INT *ix , MKL_INT *jx , MKL_INT
*descx , double *rcond , double *ferr , double *berr , double *work , MKL_INT *lwork ,
MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pcposvx (char *fact , char *uplo , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *af , MKL_INT *iaf , MKL_INT
*jaf , MKL_INT *descaf , char *equed , float *sr , float *sc , MKL_Complex8 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_Complex8 *x , MKL_INT *ix , MKL_INT
*jx , MKL_INT *descx , float *rcond , float *ferr , float *berr , MKL_Complex8 *work ,
MKL_INT *lwork , float *rwork , MKL_INT *lrwork , MKL_INT *info );
```

```
void pzposvx (char *fact , char *uplo , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *af , MKL_INT *iaf ,
MKL_INT *jaf , MKL_INT *descaf , char *equed , double *sr , double *sc , MKL_Complex16
*b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_Complex16 *x , MKL_INT *ix ,
```

```
MKL_INT *jx , MKL_INT *descx , double *rcond , double *ferr , double *berr ,
MKL_Complex16 *work , MKL_INT *lwork , double *rwork , MKL_INT *lrwork , MKL_INT
*info );
```

Include Files

- mkl_scalapack.h

Description

The `p?posvx` function uses the Cholesky factorization $A=U^T*U$ or $A=L*L^T$ to compute the solution to a real or complex system of linear equations

$$A(ia:ia+n-1, ja:ja+n-1)*X = B(ib:ib+n-1, jb:jb+nrhs-1),$$

where $A(ia:ia+n-1, ja:ja+n-1)$ is a n -by- n matrix and X and $B(ib:ib+n-1, jb:jb+nrhs-1)$ are n -by- $nrhs$ matrices.

Error bounds on the solution and a condition estimate are also provided.

In the following comments y denotes $Y(iy:iy+m-1, jy:jy+k-1)$, an m -by- k matrix where y can be a , af , b and x .

The function `p?posvx` performs the following steps:

1. If `fact = 'E'`, real scaling factors s are computed to equilibrate the system:

$$\text{diag}(sr)*A*\text{diag}(sc)*\text{inv}(\text{diag}(sc))*X = \text{diag}(sr)*B$$

Whether or not the system will be equilibrated depends on the scaling of the matrix A , but if equilibration is used, A is overwritten by $\text{diag}(sr)*A*\text{diag}(sc)$ and B by $\text{diag}(sr)*B$.

2. If `fact = 'N'` or `'E'`, the Cholesky decomposition is used to factor the matrix A (after equilibration if `fact = 'E'`) as

$$A = U^T*U, \text{ if } \text{uplo} = 'U', \text{ or}$$

$$A = L*L^T, \text{ if } \text{uplo} = 'L',$$

where U is an upper triangular matrix and L is a lower triangular matrix.

3. The factored form of A is used to estimate the condition number of the matrix A . If the reciprocal of the condition number is less than machine precision, steps 4-6 are skipped
4. The system of equations is solved for X using the factored form of A .
5. Iterative refinement is applied to improve the computed solution matrix and calculate error bounds and backward error estimates for it.
6. If equilibration was used, the matrix X is premultiplied by $\text{diag}(sr)$ so that it solves the original system before equilibration.

Input Parameters

`fact` (global) Must be 'F', 'N', or 'E'.

Specifies whether or not the factored form of the matrix A is supplied on entry, and if not, whether the matrix A should be equilibrated before it is factored.

If `fact = 'F'`: on entry, `af` contains the factored form of A . If `equed = 'Y'`, the matrix A has been equilibrated with scaling factors given by s . a and `af` will not be modified.

If `fact = 'N'`, the matrix A will be copied to `af` and factored.

If `fact = 'E'`, the matrix A will be equilibrated if necessary, then copied to `af` and factored.

| | |
|-----------------|--|
| <i>uplo</i> | (global) Must be 'U' or 'L'. Indicates whether the upper or lower triangular part of A is stored. |
| <i>n</i> | (global) The order of the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right-hand sides; the number of columns of the distributed submatrices B and X . ($nrhs \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of local size $lld_a * LOCC(ja + n - 1)$. On entry, the symmetric/Hermitian matrix A , except if $fact = 'F'$ and $equed = 'Y'$, then A must contain the equilibrated matrix $\text{diag}(sr) * A * \text{diag}(sc)$. If $uplo = 'U'$, the leading n -by- n upper triangular part of A contains the upper triangular part of the matrix A , and the strictly lower triangular part of A is not referenced. If $uplo = 'L'$, the leading n -by- n lower triangular part of A contains the lower triangular part of the matrix A , and the strictly upper triangular part of A is not referenced. A is not modified if $fact = 'F'$ or 'N', or if $fact = 'E'$ and $equed = 'N'$ on exit. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>af</i> | (local) Pointer into the local memory to an array of local size $lld_af * LOCC(ja + n - 1)$. If $fact = 'F'$, then af is an input argument and on entry contains the triangular factor U or L from the Cholesky factorization $A = U^T * U$ or $A = L * L^T$, in the same storage format as A . If $equed \neq 'N'$, then af is the factored form of the equilibrated matrix $\text{diag}(sr) * A * \text{diag}(sc)$. |
| <i>iaf, jaf</i> | (global) The row and column indices in the global matrix AF indicating the first row and the first column of the submatrix AF , respectively. |
| <i>descaf</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix AF . |
| <i>equed</i> | (global) Must be 'N' or 'Y'. $equed$ is an input argument if $fact = 'F'$. It specifies the form of equilibration that was done: If $equed = 'N'$, no equilibration was done (always true if $fact = 'N'$); If $equed = 'Y'$, equilibration was done and A has been replaced by $\text{diag}(sr) * A * \text{diag}(sc)$. |
| <i>sr</i> | (local) Array of size lld_a . |

The array *s* contains the scale factors for *A*. This array is an input argument if *fact* = 'F' only; otherwise it is an output argument.

If *equed* = 'N', *s* is not accessed.

If *fact* = 'F' and *equed* = 'Y', each element of *s* must be positive.

| | |
|---------------|---|
| <i>b</i> | (local) Pointer into the local memory to an array of local size $lld_b * LOCC(jb + nrhs - 1)$. On entry, the <i>n</i> -by- <i>nrhs</i> right-hand side matrix <i>B</i> . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix <i>B</i> indicating the first row and the first column of the submatrix <i>B</i> , respectively. |
| <i>descb</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>B</i> . |
| <i>x</i> | (local) Pointer into the local memory to an array of local size $lld_x * LOCC(jx + nrhs - 1)$. |
| <i>ix, jx</i> | (global) The row and column indices in the global matrix <i>X</i> indicating the first row and the first column of the submatrix <i>X</i> , respectively. |
| <i>descx</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>X</i> . |
| <i>work</i> | (local) Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> . <i>lwork</i> is local input and must be at least $lwork = \max(p?pocon(lwork), p?porfs(lwork)) + LOCr(n_a)$. $lwork = 3 * desca[lld_ - 1]$. If <i>lwork</i> = -1, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <i>pxerbla</i> . |
| <i>iwork</i> | (local) Workspace array of size <i>liwork</i> . |
| <i>liwork</i> | (local or global) The size of the array <i>iwork</i> . <i>liwork</i> is local input and must be at least $liwork = desca[lld_ - 1]$ $liwork = LOCr(n_a)$. If <i>liwork</i> = -1, then <i>liwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <i>pxerbla</i> . |

Output Parameters

| | |
|--------------|---|
| <i>a</i> | On exit, if <i>fact</i> = 'E' and <i>equed</i> = 'Y', <i>a</i> is overwritten by $\text{diag}(sr) * a * \text{diag}(sc)$. |
| <i>af</i> | If <i>fact</i> = 'N', then <i>af</i> is an output argument and on exit returns the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization $A = U^T * U$ or $A = L * L^T$ of the original matrix <i>A</i> . If <i>fact</i> = 'E', then <i>af</i> is an output argument and on exit returns the triangular factor <i>U</i> or <i>L</i> from the Cholesky factorization $A = U^T * U$ or $A = L * L^T$ of the equilibrated matrix <i>A</i> (see the description of <i>A</i> for the form of the equilibrated matrix). |
| <i>equed</i> | If <i>fact</i> ≠ 'F', then <i>equed</i> is an output argument. It specifies the form of equilibration that was done (see the description of <i>equed</i> in <i>Input Arguments</i> section). |
| <i>sr</i> | This array is an output argument if <i>fact</i> ≠ 'F'. See the description of <i>sr</i> in <i>Input Arguments</i> section. |
| <i>sc</i> | This array is an output argument if <i>fact</i> ≠ 'F'. See the description of <i>sc</i> in <i>Input Arguments</i> section. |
| <i>b</i> | On exit, if <i>equed</i> = 'N', <i>b</i> is not modified; if <i>trans</i> = 'N' and <i>equed</i> = 'R' or 'B', <i>b</i> is overwritten by $\text{diag}(r) * b$; if <i>trans</i> = 'T' or 'C' and <i>equed</i> = 'C' or 'B', <i>b</i> is overwritten by $\text{diag}(c) * b$. |
| <i>x</i> | (local) If <i>info</i> = 0 the <i>n</i> -by- <i>nrhs</i> solution matrix <i>X</i> to the original system of equations. Note that <i>A</i> and <i>B</i> are modified on exit if <i>equed</i> ≠ 'N', and the solution to the equilibrated system is $\text{inv}(\text{diag}(sc)) * X$ if <i>trans</i> = 'N' and <i>equed</i> = 'C' or 'B', or $\text{inv}(\text{diag}(sr)) * X$ if <i>trans</i> = 'T' or 'C' and <i>equed</i> = 'R' or 'B'. |
| <i>rcond</i> | (global) An estimate of the reciprocal condition number of the matrix <i>A</i> after equilibration (if done). If <i>rcond</i> is less than the machine precision (in particular, if <i>rcond</i> =0), the matrix is singular to working precision. This condition is indicated by a return code of <i>info</i> > 0. |
| <i>ferr</i> | Arrays of size at least $\max(LOC, n_b)$. The estimated forward error bounds for each solution vector <i>X</i> (<i>j</i>) (the <i>j</i> -th column of the solution matrix <i>X</i>). If <i>xtrue</i> is the true solution, <i>ferr</i> [<i>j</i> - 1] bounds the magnitude of the largest entry in $(X(j) - xtrue)$ divided by the magnitude of the largest entry in <i>X</i> (<i>j</i>). The quality of the error bound depends on the quality of the estimate of $\text{norm}(\text{inv}(A))$ computed in the code; if the estimate of $\text{norm}(\text{inv}(A))$ is accurate, the error bound is guaranteed. |
| <i>berr</i> | (local) |

Arrays of size at least $\max(LOC, n_b)$. The componentwise relative backward error of each solution vector $X(j)$ (the smallest relative change in any entry of A or B that makes $X(j)$ an exact solution).

`work[0]`

(local) On exit, `work[0]` returns the minimal and optimal `liwork`.

`info`

(global)

If `info=0`, the execution is successful.

< 0: if `info = -i`, the i -th argument had an illegal value

> 0: if `info = i`, and $i \leq n$: if `info = i`, the leading minor of order i of a is not positive definite, so the factorization could not be completed, and the solution and error bounds could not be computed.

= $n+1$: `rcond` is less than machine precision. The factorization has been completed, but the matrix is singular to working precision, and the solution and error bounds have not been computed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?pbsv

Solves a symmetric/Hermitian positive definite banded system of linear equations.

Syntax

```
void pspbsv (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs , float *a ,
MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb , float *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pdpbsv (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs , double *a ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb , double
*work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcpbsv (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs , MKL_Complex8 *a ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *descb ,
MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzpbsv (char *uplo , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs , MKL_Complex16 *a ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *descb ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?pbsv` function solves a system of linear equations

$$A(1:n, ja:ja+n-1) * X = B(ib:ib+n-1, 1:nrhs),$$

where $A(1:n, ja:ja+n-1)$ is an n -by- n real/complex banded symmetric positive definite distributed matrix with bandwidth `bw`.

Cholesky factorization is used to factor a reordering of the matrix into $L * L'$.

Optimization Notice

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Notice revision #20110804

Input Parameters

| | |
|--------------|--|
| <i>uplo</i> | (global) Must be 'U' or 'L'. Indicates whether the upper or lower triangular of <i>A</i> is stored. If <i>uplo</i> = 'U', the upper triangular <i>A</i> is stored If <i>uplo</i> = 'L', the lower triangular of <i>A</i> is stored. |
| <i>n</i> | (global) The order of the distributed matrix <i>A</i> ($n \geq 0$). |
| <i>bw</i> | (global) The number of subdiagonals in <i>L</i> or <i>U</i> . $0 \leq bw \leq n-1$. |
| <i>nrhs</i> | (global) The number of right-hand sides; the number of columns in <i>B</i> ($nrhs \geq 0$). |
| <i>a</i> | (local). Pointer into the local memory to an array with leading size $lld_a \geq (bw+1)$ (stored in <i>desca</i>). On entry, this array contains the local pieces of the distributed matrix $sub(A)$ to be factored. |
| <i>ja</i> | (global) The index in the global matrix <i>A</i> indicating the start of the matrix to be operated on (which may be either all of <i>A</i> or a submatrix of <i>A</i>). |
| <i>desca</i> | (global and local) array of size <i>dlen</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>b</i> | (local) Pointer into the local memory to an array of local lead size $lld_b \geq nb$. On entry, this array contains the local pieces of the right hand sides $B(ib:ib+n-1, 1:nrhs)$. |
| <i>ib</i> | (global) The row index in the global matrix <i>B</i> indicating the first row of the matrix to be operated on (which may be either all of <i>b</i> or a submatrix of <i>B</i>). |
| <i>descb</i> | (global and local) array of size <i>dlen</i> . If 1D type ($dtype_b = 502$), $dlen \geq 7$; If 2D type ($dtype_b = 1$), $dlen \geq 9$. The array descriptor for the distributed matrix <i>B</i> . Contains information of mapping of <i>B</i> to memory. |

| | |
|--------------|--|
| <i>work</i> | (local). Temporary workspace. This space may be overwritten in between calls to functions. <i>work</i> must be the size given in <i>lwork</i> . |
| <i>lwork</i> | (local or global) Size of user-input workspace <i>work</i> . If <i>lwork</i> is too small, the minimal acceptable size will be returned in <i>work</i> [0] and an error code is returned. $lwork \geq (nb+2*bw)*bw + \max((bw*nrhs), bw*bw)$ |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | On exit, this array contains information containing details of the factorization. Note that permutations are performed on the matrix, so that the factors returned are different from those returned by LAPACK. |
| <i>b</i> | On exit, contains the local piece of the solutions distributed matrix X. |
| <i>work</i> | On exit, <i>work</i> [0] contains the minimal <i>lwork</i> . |
| <i>info</i> | (global) If <i>info</i> =0, the execution is successful. < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -entry had an illegal value, then <i>info</i> = $-(i*100+j)$, if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = $-i$. > 0: If <i>info</i> = $k \leq NPROCS$, the submatrix stored on processor <i>info</i> and factored locally was not positive definite, and the factorization was not completed. If <i>info</i> = $k > NPROCS$, the submatrix stored on processor <i>info</i> -NPROCS representing interactions with other processors was not positive definite, and the factorization was not completed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?ptsv

Syntax

Solves a symmetric or Hermitian positive definite tridiagonal system of linear equations.

```
void psptsv (MKL_INT *n , MKL_INT *nrhs , float *d , float *e , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdptsv (MKL_INT *n , MKL_INT *nrhs , double *d , double *e , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcptsv (char *uplo , MKL_INT *n , MKL_INT *nrhs , float *d , MKL_Complex8 *e , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *descb , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzptsv (char *uplo , MKL_INT *n , MKL_INT *nrhs , double *d , MKL_Complex16 *e , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *descb , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?ptsv` function solves a system of linear equations

$$A(1:n, ja:ja+n-1)*X = B(ib:ib+n-1, 1:nrhs),$$

where $A(1:n, ja:ja+n-1)$ is an n -by- n real tridiagonal symmetric positive definite distributed matrix.

Cholesky factorization is used to factor a reordering of the matrix into $L*L'$.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|---------|--|
| n | (global) The order of matrix A ($n \geq 0$). |
| $nrhs$ | (global) The number of right-hand sides; the number of columns of the distributed submatrix B ($nrhs \geq 0$). |
| d | (local) Pointer to local part of global vector storing the main diagonal of the matrix. |
| e | (local) Pointer to local part of global vector storing the upper diagonal of the matrix. Globally, $du(n)$ is not referenced, and du must be aligned with d . |
| ja | (global) The index in the global matrix A indicating the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| $desca$ | (global and local) array of size $dlen$. If 1d type ($dtype_a=501$ or 502), $dlen \geq 7$; If 2d type ($dtype_a=1$), $dlen \geq 9$. The array descriptor for the distributed matrix A . Contains information of mapping of A to memory. |
| b | (local) Pointer into the local memory to an array of local lead size $lld_b \geq nb$. On entry, this array contains the local pieces of the right hand sides $B(ib:ib+n-1, 1:nrhs)$. |

| | |
|--------------|--|
| <i>ib</i> | (global) The row index in the global matrix <i>B</i> indicating the first row of the matrix to be operated on (which may be either all of <i>b</i> or a submatrix of <i>B</i>). |
| <i>descb</i> | (global and local) array of size <i>dlen</i> . If 1d type (<i>dtype_b</i> = 502), <i>dlen</i> ≥ 7; If 2d type (<i>dtype_b</i> = 1), <i>dlen</i> ≥ 9. The array descriptor for the distributed matrix <i>B</i> . Contains information of mapping of <i>B</i> to memory. |
| <i>work</i> | (local). Temporary workspace. This space may be overwritten in between calls to functions. <i>work</i> must be the size given in <i>lwork</i> . |
| <i>lwork</i> | (local or global) Size of user-input workspace <i>work</i> . If <i>lwork</i> is too small, the minimal acceptable size will be returned in <i>work</i> [0] and an error code is returned. $lwork > (12 * NPCOL + 3 * nb) + \max((10 + 2 * \min(100, nrhs)) * NPCOL + 4 * nrhs, 8 * NPCOL)$. |

Output Parameters

| | |
|-------------|--|
| <i>d</i> | On exit, this array contains information containing the factors of the matrix. Must be of size greater than or equal to <i>desca</i> [<i>nb</i> - 1]. |
| <i>e</i> | On exit, this array contains information containing the factors of the matrix. Must be of size greater than or equal to <i>desca</i> [<i>nb</i> - 1]. |
| <i>b</i> | On exit, this contains the local piece of the solutions distributed matrix <i>X</i> . |
| <i>work</i> | On exit, <i>work</i> [0] contains the minimal <i>lwork</i> . |
| <i>info</i> | (local) If <i>info</i> =0, the execution is successful. < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -entry had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . > 0: If <i>info</i> = <i>k</i> ≤ NPROCS, the submatrix stored on processor <i>info</i> and factored locally was not positive definite, and the factorization was not completed. If <i>info</i> = <i>k</i> > NPROCS, the submatrix stored on processor <i>info</i> -NPROCS representing interactions with other processors was not positive definite, and the factorization was not completed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gels

Solves overdetermined or underdetermined linear systems involving a matrix of full rank.

Syntax

```
void psgels (char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *nrhs , float *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , float *work , MKL_INT *lwork , MKL_INT *info );

void pdgels (char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *nrhs , double *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , double *work , MKL_INT *lwork , MKL_INT *info );

void pcgels (char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT
*jb , MKL_INT *descb , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );

void pzgels (char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT
*jb , MKL_INT *descb , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?gels` function solves overdetermined or underdetermined real/ complex linear systems involving an m -by- n matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$, or its transpose/ conjugate-transpose, using a QTQ or LQ factorization of $\text{sub}(A)$. It is assumed that $\text{sub}(A)$ has full rank.

The following options are provided:

1. If $\text{trans} = 'N'$ and $m \geq n$: find the least squares solution of an overdetermined system, that is, solve the least squares problem

$$\text{minimize } ||\text{sub}(B) - \text{sub}(A) * X||$$

2. If $\text{trans} = 'N'$ and $m < n$: find the minimum norm solution of an underdetermined system $\text{sub}(A) * X = \text{sub}(B)$.
3. If $\text{trans} = 'T'$ and $m \geq n$: find the minimum norm solution of an undetermined system $\text{sub}(A)^T * X = \text{sub}(B)$.
4. If $\text{trans} = 'T'$ and $m < n$: find the least squares solution of an overdetermined system, that is, solve the least squares problem

$$\text{minimize } ||\text{sub}(B) - \text{sub}(A)^T * X||,$$

where $\text{sub}(B)$ denotes $B(\text{ib}:\text{ib}+m-1, \text{jb}:\text{jb}+nrhs-1)$ when $\text{trans} = 'N'$ and $B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+nrhs-1)$ otherwise. Several right hand side vectors b and solution vectors x can be handled in a single call; when $\text{trans} = 'N'$, the solution vectors are stored as the columns of the n -by- $nrhs$ right hand side matrix $\text{sub}(B)$ and the m -by- $nrhs$ right hand side matrix $\text{sub}(B)$ otherwise.

Input Parameters

| | |
|--------------|---|
| <i>trans</i> | (global) Must be 'N', or 'T'. If $\text{trans} = 'N'$, the linear system involves matrix $\text{sub}(A)$; If $\text{trans} = 'T'$, the linear system involves the transposed matrix A^T (for real flavors only). |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$ ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$ ($n \geq 0$). |

| | |
|---------------|--|
| <i>nrhs</i> | (global) The number of right-hand sides; the number of columns in the distributed submatrices $\text{sub}(B)$ and X . ($nrhs \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, contains the m -by- n matrix A . |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>b</i> | (local) Pointer into the local memory to an array of local size $lld_b * LOCC(jb + nrhs - 1)$. On entry, this array contains the local pieces of the distributed matrix B of right-hand side vectors, stored columnwise; $\text{sub}(B)$ is m -by- $nrhs$ if $trans = 'N'$, and n -by- $nrhs$ otherwise. |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of the submatrix B , respectively. |
| <i>descb</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |
| <i>work</i> | (local) Workspace array with size $lwork$. |
| <i>lwork</i> | (local or global) . The size of the array $work/lwork$ is local input and must be at least $lwork \geq ltau + \max(lwf, lws)$, where if $m > n$, then $ltau = \text{numroc}(ja + \min(m, n) - 1, nb_a, MYCOL, csrc_a, NPCOL),$ $lwf = nb_a * (mpa0 + nqa0 + nb_a)$ $lws = \max((nb_a * (nb_a - 1)) / 2, (nrhsqb0 + mpb0) * nb_a) + nb_a * nb_a$ else $ltau = \text{numroc}(ia + \min(m, n) - 1, mb_a, MYROW, rsrc_a, NPROW),$ $lwf = mb_a * (mpa0 + nqa0 + mb_a)$ $lws = \max((mb_a * (mb_a - 1)) / 2, (npb0 + \max(nqa0 + \text{numroc}(\text{numroc}(n + iroffb, mb_a, 0, 0, NPROW), mb_a, 0, 0, lcmp), nrhsqb0)) * mb_a) + mb_a * mb_a$ end if, where $lcmp = lcm / NPROW$ with $lcm = \text{ilcm}(NPROW, NPCOL)$, $iroffa = \text{mod}(ia - 1, mb_a),$ $icoffa = \text{mod}(ja - 1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, MYROW, rsrc_a, NPROW),$ $iacol = \text{indxg2p}(ja, nb_a, MYROW, rsrc_a, NPROW)$ |

```

mpa0 = numroc(m+iroffa, mb_a, MYROW, iarow, NPROW),
nqa0 = numroc(n+icoffa, nb_a, MYCOL, iacol, NPCOL),
iroffb = mod(ib-1, mb_b),
icoffb = mod(jb-1, nb_b),
ibrow = indxg2p(ib, mb_b, MYROW, rsrc_b, NPROW),
ibcol = indxg2p(jb, nb_b, MYCOL, csrc_b, NPCOL),
mpb0 = numroc(m+iroffb, mb_b, MYROW, icrow, NPROW),
nqb0 = numroc(n+icoffb, nb_b, MYCOL, ibcol, NPCOL),

```

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

`ilcm`, `indxg2p` and `numroc` are ScaLAPACK tool functions; `MYROW`, `MYCOL`, `NPROW`, and `NPCOL` can be determined by calling the function `blacs_gridinfo`.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|----------------------|---|
| <code>a</code> | On exit, If $m \geq n$, <code>sub(A)</code> is overwritten by the details of its <i>QR</i> factorization as returned by <code>p?geqrf</code> ; if $m < n$, <code>sub(A)</code> is overwritten by details of its <i>LQ</i> factorization as returned by <code>p?gelqf</code> . |
| <code>b</code> | On exit, <code>sub(B)</code> is overwritten by the solution vectors, stored columnwise: if <code>trans = 'N'</code> and $m \geq n$, rows 1 to n of <code>sub(B)</code> contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of elements $n+1$ to m in that column; If <code>trans = 'N'</code> and $m < n$, rows 1 to n of <code>sub(B)</code> contain the minimum norm solution vectors; If <code>trans = 'T'</code> and $m \geq n$, rows 1 to m of <code>sub(B)</code> contain the minimum norm solution vectors; if <code>trans = 'T'</code> and $m < n$, rows 1 to m of <code>sub(B)</code> contain the least squares solution vectors; the residual sum of squares for the solution in each column is given by the sum of squares of elements $m+1$ to n in that column. |
| <code>work[0]</code> | On exit, <code>work[0]</code> contains the minimum value of <code>lwork</code> required for optimum performance. |
| <code>info</code> | (global) = 0: the execution is successful. < 0: if the i -th argument is an array and the j -entry had an illegal value, then <code>info = - (i* 100+j)</code> , if the i -th argument is a scalar and had an illegal value, then <code>info = -i</code> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?syev

Computes selected eigenvalues and eigenvectors of a symmetric matrix.

Syntax

```
void pssyev (char *jobz , char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *w , float *z , MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdsyev (char *jobz , char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *w , double *z , MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , double *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?syev` function computes all eigenvalues and, optionally, eigenvectors of a real symmetric matrix A by calling the recommended sequence of ScaLAPACK functions.

In its present form, the function assumes a homogeneous system and makes no checks for consistency of the eigenvalues or eigenvectors across the different processes. Because of this, it is possible that a heterogeneous system may return incorrect results without any error messages.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

| | |
|-------------|--|
| <i>jobz</i> | (global) Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | (global) Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the symmetric matrix A is stored: If <i>uplo</i> = 'U', a stores the upper triangular part of A . If <i>uplo</i> = 'L', a stores the lower triangular part of A . |
| <i>n</i> | (global) The number of rows and columns of the matrix A ($n \geq 0$). |
| <i>a</i> | (local) Block cyclic array of global size $n*n$ and local size $lld_a*LOCc(ja+n-1)$. On entry, the symmetric matrix A . If <i>uplo</i> = 'U', only the upper triangular part of A is used to define the elements of the symmetric matrix. |

If *uplo* = 'L', only the lower triangular part of *A* is used to define the elements of the symmetric matrix.

ia, ja

(global) The row and column indices in the global matrix *A* indicating the first row and the first column of the submatrix *A*, respectively.

desca

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix *A*.

iz, jz

(global) The row and column indices in the global matrix *Z* indicating the first row and the first column of the submatrix *Z*, respectively.

descz

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix *Z*.

work

(local)

Array of size *lwork*.

lwork

(local) See below for definitions of variables used to define *lwork*.

If no eigenvectors are requested (*jobz* = 'N'), then $lwork \geq 5*n + sizesytrd + 1$,

where *sizesytrd* is the workspace for *p?sytrd* and is $\max(NB*(np + 1), 3*NB)$.

If eigenvectors are requested (*jobz* = 'V') then the amount of workspace required to guarantee that all eigenvectors are computed is:

$$qrmem = 2*n-2$$

$$lwmin = 5*n + n*ldc + \max(sizemqrleft, qrmem) + 1$$

Variable definitions:

$$nb = desca[mb_ - 1] = desca[nb_ - 1] = descz[mb_ - 1] = descz[nb_ - 1];$$

$$nn = \max(n, nb, 2);$$

$$desca[rsrc_ - 1] = desca[rsrc_ - 1] = descz[rsrc_ - 1] = descz[csrc_ - 1] = 0$$

$$np = \text{numroc}(nn, nb, 0, 0, \text{NPROW})$$

$$nq = \text{numroc}(\max(n, nb, 2), nb, 0, 0, \text{NPCOL})$$

$$nrc = \text{numroc}(n, nb, \text{myprowc}, 0, \text{NPROCS})$$

$$ldc = \max(1, nrc)$$

sizemqrleft is the workspace for *p?ormtr* when its *side* argument is 'L'.

myprowc is defined when a new context is created as follows:

```
call blacs_get(desca[ctxt_ - 1], 0, contextc)
```

```
call blacs_gridinit(contextc, 'R', NPROCS, 1)
```

```
call blacs_gridinfo(contextc, nprowc, npcoldc, myprowc, mypcoldc)
```

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `p?xerbla`.

Output Parameters

| | |
|----------------------|--|
| <code>a</code> | On exit, the lower triangle (if $uplo='L'$) or the upper triangle (if $uplo='U'$) of A , including the diagonal, is destroyed. |
| <code>w</code> | (global). Array of size n . On normal exit, the first m entries contain the selected eigenvalues in ascending order. |
| <code>z</code> | (local). Array, global size $n*n$, local size $lld_z*LOCc(jz+n-1)$. If $jobz = 'V'$, then on normal exit the first m columns of z contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If $jobz = 'N'$, then z is not referenced. |
| <code>work[0]</code> | On output, <code>work[0]</code> returns the workspace needed to guarantee completion. If the input parameters are incorrect, <code>work[0]</code> may also be incorrect. If $jobz = 'N'$ <code>work[0]</code> = minimal (optimal) amount of workspace If $jobz = 'V'$ <code>work[0]</code> = minimal workspace required to generate all the eigenvectors. |
| <code>info</code> | (global) If $info = 0$, the execution is successful. If $info < 0$: If the i -th argument is an array and the j -entry had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$. If $info > 0$: If $info=1$ through n , the i -th eigenvalue did not converge in <code>?steqr2</code> after a total of $30n$ iterations. If $info= n+1$, then <code>p?syev</code> has detected heterogeneity by finding that eigenvalues were not identical across the process grid. In this case, the accuracy of the results from <code>p?syev</code> cannot be guaranteed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?syevd

Computes all eigenvalues and eigenvectors of a real symmetric matrix by using a divide and conquer algorithm.

Syntax

```
void pssyevd (char *jobz , char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , float *w , float *z , MKL_INT *iz , MKL_INT *jz , MKL_INT
*descz , float *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT
*info );
```

```
void pdsyevd (char *jobz , char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , double *w , double *z , MKL_INT *iz , MKL_INT *jz , MKL_INT
*descz , double *work , MKL_INT *lwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT
*info );
```

Include Files

- mkl_scalapack.h

Description

The `p?syevd` function computes all eigenvalues and eigenvectors of a real symmetric matrix A by using a divide and conquer algorithm.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

| | |
|---------------|--|
| <i>jobz</i> | (global) Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | (global) Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored: If <i>uplo</i> = 'U', a stores the upper triangular part of A . If <i>uplo</i> = 'L', a stores the lower triangular part of A . |
| <i>n</i> | (global) The number of rows and columns of the matrix A ($n \geq 0$). |
| <i>a</i> | (local). Block cyclic array of global size $n*n$ and local size $lld_a*LOCc(ja+n-1)$. On entry, the symmetric matrix A . If <i>uplo</i> = 'U', only the upper triangular part of A is used to define the elements of the symmetric matrix. If <i>uplo</i> = 'L', only the lower triangular part of A is used to define the elements of the symmetric matrix. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . If <i>desca</i> [<i>ctxt_</i> - 1] is incorrect, <code>p?syevd</code> cannot guarantee correct error reporting. |

| | |
|---------------|---|
| <i>iz, jz</i> | (global) The row and column indices in the global matrix <i>Z</i> indicating the first row and the first column of the submatrix <i>Z</i> , respectively. |
| <i>descz</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>Z</i> . <i>descz[ctxt_ - 1]</i> must equal <i>desca[ctxt_ - 1]</i> . |
| <i>work</i> | (local). Array of size <i>lwork</i> . |
| <i>lwork</i> | (local) The size of the array <i>work</i> . If eigenvalues are requested: $lwork \geq \max(1+6*n + 2*np*nq, trilwmin) + 2*n$ with $trilwmin = 3*n + \max(nb*(np + 1), 3*nb)$ $np = \text{numroc}(n, nb, myrow, iarow, NPROW)$ $nq = \text{numroc}(n, nb, mycol, iacol, NPCOL)$ If <i>lwork</i> = -1, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. The required workspace is returned as the first element of the corresponding work arrays, and no error message is issued by <i>pxerbla</i> . |
| <i>iwork</i> | (local) Workspace array of size <i>liwork</i> . |
| <i>liwork</i> | (local) , size of <i>iwork</i> . $liwork = 7*n + 8*npcol + 2.$ |

Output Parameters

| | |
|------------------|---|
| <i>a</i> | On exit, the lower triangle (if <i>uplo</i> = 'L'), or the upper triangle (if <i>uplo</i> = 'U') of <i>A</i> , including the diagonal, is overwritten. |
| <i>w</i> | (global). Array of size <i>n</i> . If <i>info</i> = 0, <i>w</i> contains the eigenvalues in the ascending order. |
| <i>z</i> | (local). Array, global size (<i>n, n</i>), local size $lld_z*LOCc(jz+n-1)$. The <i>z</i> parameter contains the orthonormal eigenvectors of the matrix <i>A</i> . |
| <i>work</i> [0] | On exit, returns adequate workspace to allow optimal performance. |
| <i>iwork</i> [0] | (local). On exit, if <i>liwork</i> > 0, <i>iwork</i> [0] returns the optimal <i>liwork</i> . |
| <i>info</i> | (global) If <i>info</i> = 0, the execution is successful. If <i>info</i> < 0: |

If the i -th argument is an array and the j -entry had an illegal value, then $info = -(i*100+j)$. If the i -th argument is a scalar and had an illegal value, then $info = -i$.

If $info > 0$:

The algorithm failed to compute the $info/(n+1)$ -th eigenvalue while working on the submatrix lying in global rows and columns $\text{mod}(info, n+1)$.

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?syevr

Computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix using Relatively Robust Representation.

Syntax

```
void pssyevr(char* jobz, char* range, char* uplo, MKL_INT* n, float* a, MKL_INT* ia,
MKL_INT* ja, MKL_INT* desca, float* vl, float* vu, MKL_INT* il, MKL_INT* iu, MKL_INT*
m, MKL_INT* nz, float* w, float* z, MKL_INT* iz, MKL_INT* jz, MKL_INT* descz, float*
work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT* info);
```

```
void pdsyevr(char* jobz, char* range, char* uplo, MKL_INT* n, double* a, MKL_INT* ia,
MKL_INT* ja, MKL_INT* desca, double* vl, double* vu, MKL_INT* il, MKL_INT* iu,
MKL_INT* m, MKL_INT* nz, double* w, double* z, MKL_INT* iz, MKL_INT* jz, MKL_INT*
descz, double* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

`p?syevr` computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A distributed in 2D blockcyclic format by calling the recommended sequence of ScaLAPACK functions.

First, the matrix A is reduced to real symmetric tridiagonal form. Then, the eigenproblem is solved using the parallel MRRR algorithm. Last, if eigenvectors have been computed, a backtransformation is done.

Upon successful completion, each processor stores a copy of all computed eigenvalues in w . The eigenvector matrix z is stored in 2D block-cyclic format distributed over all processors.

Note that subsets of eigenvalues/vectors can be selected by specifying a range of values or a range of indices for the desired eigenvalues.

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Optimization Notice

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Input Parameters

| | |
|--------------|---|
| <i>jobz</i> | (global) Specifies whether or not to compute the eigenvectors: = 'N': Compute eigenvalues only. = 'V': Compute eigenvalues and eigenvectors. |
| <i>range</i> | (global) = 'A': all eigenvalues will be found. = 'V': all eigenvalues in the interval $[vl, vu]$ will be found. = 'I': the il -th through iu -th eigenvalues will be found. |
| <i>uplo</i> | (global) Specifies whether the upper or lower triangular part of the symmetric matrix A is stored: = 'U': Upper triangular = 'L': Lower triangular |
| <i>n</i> | (global) The number of rows and columns of the matrix a . $n \geq 0$ |
| <i>a</i> | Block cyclic array of global size $n * n$, local size $lld_a * LOC_c(ja+n-1)$. This array contains the local pieces of the symmetric distributed matrix A . If $uplo = 'U'$, only the upper triangular part of a is used to define the elements of the symmetric matrix. If $uplo = 'L'$, only the lower triangular part of a is used to define the elements of the symmetric matrix. On exit, the lower triangle (if $uplo='L'$) or the upper triangle (if $uplo='U'$) of a , including the diagonal, is destroyed. |
| <i>ia</i> | (global) Global row index in the global matrix A that points to the beginning of the submatrix which is to be operated on. It should be set to 1 when operating on a full matrix. |
| <i>ja</i> | (global) Global column index in the global matrix A that points to the beginning of the submatrix which is to be operated on. It should be set to 1 when operating on a full matrix. |
| <i>desca</i> | (global and local) array of size $dlen_ = 9$. |

| | |
|---------|---|
| | The array descriptor for the distributed matrix a . |
| vl | (global) If $range='V'$, the lower bound of the interval to be searched for eigenvalues. Not referenced if $range = 'A'$ or $'I'$. |
| vu | (global) If $range='V'$, the upper bound of the interval to be searched for eigenvalues. Not referenced if $range = 'A'$ or $'I'$. |
| il | (global) If $range='I'$, the index (from smallest to largest) of the smallest eigenvalue to be returned. $il \geq 1$. Not referenced if $range = 'A'$. |
| iu | (global) If $range='I'$, the index (from smallest to largest) of the largest eigenvalue to be returned. $\min(il, n) \leq iu \leq n$. Not referenced if $range = 'A'$. |
| iz | (global) Global row index in the global matrix Z that points to the beginning of the submatrix which is to be operated on. It should be set to 1 when operating on a full matrix. |
| jz | (global) Global column index in the global matrix Z that points to the beginning of the submatrix which is to be operated on. It should be set to 1 when operating on a full matrix. |
| $descz$ | array of size $dlen_$. The array descriptor for the distributed matrix z . The context $descz[ctxt_ - 1]$ must equal $desca[ctxt_ - 1]$. Also note the array alignment requirements specified below. |
| $work$ | (local workspace) array of size $lwork$ |
| $lwork$ | (local) Size of $work$, must be at least 3. See below for definitions of variables used to define $lwork$. If no eigenvectors are requested ($jobz = 'N'$) then $lwork \geq 2 + 5*n + \max(12 * nn, neig * (np0 + 1))$ If eigenvectors are requested ($jobz = 'V'$) then the amount of workspace required is: $lwork \geq 2 + 5*n + \max(18*nn, np0 * mq0 + 2 * neig * neig) + (2 + \text{iceil}(neig, nprow*npcol)) * nn$ Variable definitions: |

$neig$ = number of eigenvectors requested

$nb = desca[mb_ - 1] = desca(nb_) = descz[mb_ - 1] = descz(nb_)$

$nn = \max(n, neig, 2)$

$desca[rsrc_ - 1] = desca[csrc_nb_ - 1] = descz[rsrc_ - 1] = descz[csrc_ - 1] = 0$

$np0 = \text{numroc}(nn, neig, 0, 0, nprow)$

$mq0 = \text{numroc}(\max(neig, neig, 2), neig, 0, 0, npcol)$

$\text{ceil}(x, y)$ is a ScaLAPACK function returning $\text{ceil}(x/y)$, and $nprow$ and $npcol$ can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by `pxerbla`.

$liwork$

(local)

size of $iwork$

Let $nnp = \max(n, nprow*npcol + 1, 4)$. Then:

$liwork \geq 12*nnp + 2*n$ when the eigenvectors are desired

$liwork \geq 10*nnp + 2*n$ when only the eigenvalues have to be computed

If $liwork = -1$, then $liwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

OUTPUT Parameters

m

(global)

Total number of eigenvalues found. $0 \leq m \leq n$.

nz

(global)

Total number of eigenvectors computed. $0 \leq nz \leq m$.

The number of columns of z that are filled.

If $jobz \neq 'V'$, nz is not referenced.

If $jobz = 'V'$, $nz = m$

w

(global) array of size n

Upon successful exit, the first m entries contain the selected eigenvalues in ascending order.

z

Block-cyclic array, global size $n*n$, local size $lld_z*LOC_c(jz+n-1)$.

On exit, contains local pieces of distributed matrix Z .

| | |
|--------------|--|
| <i>work</i> | On return, <i>work</i> [0] contains the optimal amount of workspace required for efficient execution. If <i>jobz</i> ='N' <i>work</i> [0] = optimal amount of workspace required to compute the eigenvalues. If <i>jobz</i> ='V' <i>work</i> [0] = optimal amount of workspace required to compute eigenvalues and eigenvectors. |
| <i>iwork</i> | (local workspace) array On return, <i>iwork</i> [0] contains the amount of integer workspace required. |
| <i>info</i> | (global) = 0: successful exit < 0: If the <i>i</i> -th argument is an array and the <i>j</i> th-entry had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

Application Notes

The distributed submatrices $a(ia:*, ja:*)$ and $z(iz:iz+m-1, jz:jz+n-1)$ must satisfy the following alignment properties:

1. Identical (quadratic) dimension: $desca[m_ - 1] = descz[m_ - 1] = desca[n_ - 1] = descz[n_ - 1]$
2. Quadratic conformal blocking: $desca[mb_ - 1] = desca[nb_ - 1] = descz[mb_ - 1] = descz[nb_ - 1]$,
 $desca[rsrc_ - 1] = descz[rsrc_ - 1]$
3. $\text{mod}(ia-1, mb_a) = \text{mod}(iz-1, mb_z) = 0$

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?syevx

Computes selected eigenvalues and, optionally, eigenvectors of a symmetric matrix.

Syntax

```
void pssyevx (char *jobz , char *range , char *uplo , MKL_INT *n , float *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , float *vl , float *vu , MKL_INT *il , MKL_INT
*iu , float *abstol , MKL_INT *m , MKL_INT *nz , float *w , float *orfac , float *z ,
MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , float *work , MKL_INT *lwork , MKL_INT
*iwork , MKL_INT *liwork , MKL_INT *ifail , MKL_INT *iclustr , float *gap , MKL_INT
*info );
```

```
void pdsyevx (char *jobz , char *range , char *uplo , MKL_INT *n , double *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , double *vl , double *vu , MKL_INT *il , MKL_INT
*iu , double *abstol , MKL_INT *m , MKL_INT *nz , double *w , double *orfac , double
*z , MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , double *work , MKL_INT *lwork ,
MKL_INT *iwork , MKL_INT *liwork , MKL_INT *ifail , MKL_INT *iclustr , double *gap ,
MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?syevx` function computes selected eigenvalues and, optionally, eigenvectors of a real symmetric matrix A by calling the recommended sequence of ScaLAPACK functions. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

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Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

| | |
|---------|---|
| $jobz$ | (global) Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If $jobz = 'N'$, then only eigenvalues are computed. If $jobz = 'V'$, then eigenvalues and eigenvectors are computed. |
| $range$ | (global) Must be 'A', 'V', or 'I'. If $range = 'A'$, all eigenvalues will be found. If $range = 'V'$, all eigenvalues in the half-open interval $[vl, vu]$ will be found. If $range = 'I'$, the eigenvalues with indices il through iu will be found. |
| $uplo$ | (global) Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the symmetric matrix A is stored: If $uplo = 'U'$, a stores the upper triangular part of A . If $uplo = 'L'$, a stores the lower triangular part of A . |
| n | (global) The number of rows and columns of the matrix A ($n \geq 0$). |
| a | (local). Block cyclic array of global size $n*n$ and local size $lld_a*LOCc(ja+n-1)$. On entry, the symmetric matrix A . If $uplo = 'U'$, only the upper triangular part of A is used to define the elements of the symmetric matrix. |

| | |
|---------------|--|
| | If <i>uplo</i> = 'L', only the lower triangular part of <i>A</i> is used to define the elements of the symmetric matrix. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>vl, vu</i> | (global) If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues; $vl \leq vu$. Not referenced if <i>range</i> = 'A' or 'I'. |
| <i>il, iu</i> | (global) If <i>range</i> = 'I', the indices of the smallest and largest eigenvalues to be returned. Constraints: $il \geq 1$ $\min(il, n) \leq iu \leq n$ Not referenced if <i>range</i> = 'A' or 'V'. |
| <i>abstol</i> | (global). If <i>jobz</i> ='V', setting <i>abstol</i> to <code>p?lamch(context, 'U')</code> yields the most orthogonal eigenvectors. The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to $abstol + eps * \max(a , b)$, where <i>eps</i> is the machine precision. If <i>abstol</i> is less than or equal to zero, then $eps * \text{norm}(T)$ will be used in its place, where $\text{norm}(T)$ is the 1-norm of the tridiagonal matrix obtained by reducing <i>A</i> to tridiagonal form. Eigenvalues will be computed most accurately when <i>abstol</i> is set to twice the underflow threshold $2 * p?lamch('S')$ not zero. If this function returns with $(\text{mod}(\text{info}, 2) \neq 0)$ or $(\text{mod}(\text{info}/8, 2) \neq 0)$, indicating that some eigenvalues or eigenvectors did not converge, try setting <i>abstol</i> to $2 * p?lamch('S')$. |
| <i>orfac</i> | (global). Specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within $tol = orfac * \text{norm}(A)$ of each other are to be reorthogonalized. However, if the workspace is insufficient (see <i>lwork</i>), <i>tol</i> may be decreased until all eigenvectors to be reorthogonalized can be stored in one process. No reorthogonalization will be done if <i>orfac</i> equals zero. A default value of 1.0e-3 is used if <i>orfac</i> is negative. <i>orfac</i> should be identical on all processes. |
| <i>iz, jz</i> | (global) The row and column indices in the global matrix <i>Z</i> indicating the first row and the first column of the submatrix <i>Z</i> , respectively. |
| <i>descz</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>Z</i> . <code>descz[ctxt_ - 1]</code> must equal <code>desca[ctxt_ - 1]</code> . |

work (local)
 Array of size *lwork*.

lwork (local) The size of the array *work*.
 See below for definitions of variables used to define *lwork*.
 If no eigenvectors are requested (*jobz* = 'N'), then $lwork \geq 5*n + \max(5*nn, NB*(np0 + 1))$.
 If eigenvectors are requested (*jobz* = 'V'), then the amount of workspace required to guarantee that all eigenvectors are computed is:
 $lwork \geq 5*n + \max(5*nn, np0*mq0 + 2*NB*NB) + \text{iceil}(neig, NPROW*NPCOL)*nn$
 The computed eigenvectors may not be orthogonal if the minimal workspace is supplied and *orfac* is too small. If you want to guarantee orthogonality (at the cost of potentially poor performance) you should add the following to *lwork*:
 $(clustersize-1)*n$,
 where *clustersize* is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:
 $\{w[k - 1], \dots, w[k+clustersize-2] | w[j] \leq w[j-1]\} + orfac*2*norm(A)$,
 where
neig = number of eigenvectors requested
 $nb = desca[mb_ - 1] = desca[nb_ - 1] = descz[mb_ - 1] = descz[nb_ - 1]$;
 $nn = \max(n, nb, 2)$;
 $desca[rsrc_ - 1] = desca[nb_ - 1] = descz[rsrc_ - 1] = descz[csrc_ - 1] = 0$;
 $np0 = \text{numroc}(nn, nb, 0, 0, NPROW)$;
 $mq0 = \text{numroc}(\max(neig, nb, 2), nb, 0, 0, NPCOL)$
iceil(*x*, *y*) is a ScaLAPACK function returning ceiling(*x*/*y*)
 If *lwork* is too small to guarantee orthogonality, *p?syevx* attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues.
 If *lwork* is too small to compute all the eigenvectors requested, no computation is performed and *info*= -23 is returned.
 Note that when *range*='V', number of requested eigenvectors are not known until the eigenvalues are computed. In this case and if *lwork* is large enough to compute the eigenvalues, *p?sygvx* computes the eigenvalues and as many eigenvectors as possible.
Relationship between workspace, orthogonality & performance:
 Greater performance can be achieved if adequate workspace is provided. In some situations, performance can decrease as the provided workspace increases above the workspace amount shown below:
 $lwork \geq \max(lwork, 5*n + nsytrd_lwopt)$,

where *lwork*, as defined previously, depends upon the number of eigenvectors requested, and

```
nsytrd_lwopt = n + 2*(anb+1)*(4*nps+2) + (nps + 3)*nps;
```

```
anb = pjlaenv(desca[ctxt_ - 1], 3, 'p?sytrd', 'L', 0, 0, 0, 0);
```

```
sqnpc = int(sqrt(dble(NPROW * NPCOL)));
```

```
nps = max(numroc(n, 1, 0, 0, sqnpc), 2*anb);
```

numroc is a ScaLAPACK tool functions;

pjlaenv is a ScaLAPACK environmental inquiry function

MYROW, MYCOL, NPROW and NPCOL can be determined by calling the function blacs_gridinfo.

For large *n*, no extra workspace is needed, however the biggest boost in performance comes for small *n*, so it is wise to provide the extra workspace (typically less than a megabyte per process).

If *clustersize* > $n/\sqrt{\text{NPROW}*\text{NPCOL}}$, then providing enough space to compute all the eigenvectors orthogonally will cause serious degradation in performance. At the limit (that is, *clustersize* = *n*-1) *p?stein* will perform no better than *?stein* on single processor.

For *clustersize* = $n/\sqrt{\text{NPROW}*\text{NPCOL}}$ reorthogonalizing all eigenvectors will increase the total execution time by a factor of 2 or more.

For *clustersize* > $n/\sqrt{\text{NPROW}*\text{NPCOL}}$ execution time will grow as the square of the cluster size, all other factors remaining equal and assuming enough workspace. Less workspace means less reorthogonalization but faster execution.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by *pxerbla*.

iwork

(local) Workspace array.

liwork

(local) , size of *iwork*. $liwork \geq 6*nnp$

Where: $nnp = \max(n, \text{NPROW}*\text{NPCOL} + 1, 4)$

If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by *pxerbla*.

Output Parameters

a

On exit, the lower triangle (if *uplo* = 'L') or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.

m

(global) The total number of eigenvalues found; $0 \leq m \leq n$.

nz

(global) Total number of eigenvectors computed. $0 \leq nz \leq m$.

The number of columns of z that are filled.

If $jobz \neq 'V'$, nz is not referenced.

If $jobz = 'V'$, $nz = m$ unless the user supplies insufficient space and $p?syevx$ is not able to detect this before beginning computation. To get all the eigenvectors requested, the user must supply both sufficient space to hold the eigenvectors in z ($m \leq descz[n_ - 1]$) and sufficient workspace to compute them. (See $lwork$). $p?syevx$ is always able to detect insufficient space without computation unless $range = 'V'$.

w

(global).

Array of size n . The first m elements contain the selected eigenvalues in ascending order.

z

(local).

Array, global size $n*n$, local size $lld_z*LOCc(jz+n-1)$.

If $jobz = 'V'$, then on normal exit the first m columns of z contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in $ifail$.

If $jobz = 'N'$, then z is not referenced.

$work[0]$

On exit, returns workspace adequate workspace to allow optimal performance.

$iwork[0]$

On return, $iwork[0]$ contains the amount of integer workspace required.

$ifail$

(global).

Array of size n .

If $jobz = 'V'$, then on normal exit, the first m elements of $ifail$ are zero. If $(\text{mod}(\text{info}, 2) \neq 0)$ on exit, then $ifail$ contains the indices of the eigenvectors that failed to converge.

If $jobz = 'N'$, then $ifail$ is not referenced.

$iclustr$

(global) Array of size $(2*NPROW*NPCOL)$

This array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace (see $lwork$, $orfac$ and $info$). Eigenvectors corresponding to clusters of eigenvalues indexed $iclustr(2*i-1)$ to $iclustr(2*i)$, could not be reorthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal. $iclustr$ is a zero terminated array. $iclustr[2*k - 1] \neq 0$ and $iclustr[2*k] = 0$ if and only if k is the number of clusters.

$iclustr$ is not referenced if $jobz = 'N'$.

gap

(global)

Array of size $NPROW*NPCOL$

This array contains the gap between eigenvalues whose eigenvectors could not be reorthogonalized. The output values in this array correspond to the clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the *i*th cluster may be as high as $(C*n) / \text{gap}[i - 1]$ where *C* is a small constant.

info

(global)

If *info* = 0, the execution is successful.If *info* < 0:If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = $-(i*100+j)$, if the *i*-th argument is a scalar and had an illegal value, then *info* = $-i$.If *info* > 0: if $(\text{mod}(\text{info}, 2) \neq 0)$, then one or more eigenvectors failed to converge. Their indices are stored in *ifail*. Ensure $\text{abstol} = 2.0 * p * \text{lamch}('U')$.If $(\text{mod}(\text{info}/2, 2) \neq 0)$, then eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in the array *iclustr*.If $(\text{mod}(\text{info}/4, 2) \neq 0)$, then space limit prevented *p?syevxf* from computing all of the eigenvectors between *vl* and *vu*. The number of eigenvectors computed is returned in *nz*.If $(\text{mod}(\text{info}/8, 2) \neq 0)$, then *p?stebz* failed to compute eigenvalues. Ensure $\text{abstol} = 2.0 * p * \text{lamch}('U')$.**NOTE** $\text{mod}(x, y)$ is the integer remainder of x/y .**See Also**

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?heev

Computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix.

Syntax

```
void pcheev (char *jobz , char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *w , MKL_Complex8 *z , MKL_INT *iz , MKL_INT
*jz , MKL_INT *descz , MKL_Complex8 *work , MKL_INT *lwork , float *rwork , MKL_INT
*lrwork , MKL_INT *info );
```

```
void pzheev (char *jobz , char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *w , MKL_Complex16 *z , MKL_INT *iz , MKL_INT
*jz , MKL_INT *descz , MKL_Complex16 *work , MKL_INT *lwork , double *rwork , MKL_INT
*lrwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?heev` function computes all eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A by calling the recommended sequence of ScaLAPACK functions. The function assumes a homogeneous system and makes spot checks of the consistency of the eigenvalues across the different processes. A heterogeneous system may return incorrect results without any error messages.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

| | |
|----------|--|
| $jobz$ | (global) Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If $jobz = 'N'$, then only eigenvalues are computed. If $jobz = 'V'$, then eigenvalues and eigenvectors are computed. |
| $uplo$ | (global) Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the Hermitian matrix A is stored: If $uplo = 'U'$, a stores the upper triangular part of A . If $uplo = 'L'$, a stores the lower triangular part of A . |
| n | (global) The number of rows and columns of the matrix A ($n \geq 0$). |
| a | (local). Block cyclic array of global size $n*n$ and local size $lld_a*LOCc(ja+n-1)$. On entry, the Hermitian matrix A . If $uplo = 'U'$, only the upper triangular part of A is used to define the elements of the Hermitian matrix. If $uplo = 'L'$, only the lower triangular part of A is used to define the elements of the Hermitian matrix. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . If $desca[ctxt_ - 1]$ is incorrect, <code>p?heev</code> cannot guarantee correct error reporting. |
| iz, jz | (global) The row and column indices in the global matrix Z indicating the first row and the first column of the submatrix Z , respectively. |
| $descz$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix Z . $descz[ctxt_ - 1]$ must equal $desca[ctxt_ - 1]$. |
| $work$ | (local). Array of size $lwork$. |
| $lwork$ | (local) The size of the array $work$. If only eigenvalues are requested ($jobz = 'N'$): |

$$lwork \geq \max(nb*(np0 + 1), 3) + 3*n$$

If eigenvectors are requested ($jobz = 'V'$), then the amount of workspace required:

$$lwork \geq (np0+nq0+nb)*nb + 3*n + n^2$$

with $nb = desca[mb_ - 1] = desca[nb_ - 1] = nb = descz[mb_ - 1] = descz[nb_ - 1]$

$$np0 = \text{numroc}(nn, nb, 0, 0, \text{NPROW}).$$

$$nq0 = \text{numroc}(\max(n, nb, 2), nb, 0, 0, \text{NPCOL}).$$

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. The required workspace is returned as the first element of the corresponding work arrays, and no error message is issued by `pxerbla`.

rwork

(local).

Workspace array of size *lrwork*.*lrwork*(local) The size of the array *rwork*.See below for definitions of variables used to define *lrwork*.If no eigenvectors are requested ($jobz = 'N'$), then $lrwork \geq 2*n$.If eigenvectors are requested ($jobz = 'V'$), then $lrwork \geq 2*n + 2*n-2$.

If $lrwork = -1$, then $lrwork$ is global input and a workspace query is assumed; the function only calculates the minimum size required for the *rwork* array. The required workspace is returned as the first element of *rwork*, and no error message is issued by `pxerbla`.

Output Parameters

a

On exit, the lower triangle (if $uplo = 'L'$), or the upper triangle (if $uplo = 'U'$) of *A*, including the diagonal, is overwritten.

w

(global).

Array of size n . The first m elements contain the selected eigenvalues in ascending order.

z

(local).

Array, global size $n*n$, local size $lld_z*LOCc(jz+n-1)$.

If $jobz = 'V'$, then on normal exit the first m columns of *z* contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of *z* contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

If $jobz = 'N'$, then *z* is not referenced.

work[0]

On exit, returns adequate workspace to allow optimal performance.

If $jobz = 'N'$, then *work[0]* = minimal workspace only for eigenvalues.

If `jobz = 'V'`, then `work[0]` = minimal workspace required to generate all the eigenvectors.

`rwork[0]`

(local)

On output, `rwork[0]` returns workspace required to guarantee completion.

`info`

(global)

If `info = 0`, the execution is successful.

If `info < 0`:

If the i -th argument is an array and the j -entry had an illegal value, then `info = -(i*100+j)`. If the i -th argument is a scalar and had an illegal value, then `info = -i`.

If `info > 0`:

If `info = 1` through n , the i -th eigenvalue did not converge in `?stegr2` after a total of $30*n$ iterations.

If `info = n+1`, then `p?heev` detected heterogeneity, and the accuracy of the results cannot be guaranteed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?heevd

Computes all eigenvalues and eigenvectors of a complex Hermitian matrix by using a divide and conquer algorithm.

Syntax

```
void pcheevd (char *jobz , char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *w , MKL_Complex8 *z , MKL_INT *iz , MKL_INT
*jz , MKL_INT *descz , MKL_Complex8 *work , MKL_INT *lwork , float *rwork , MKL_INT
*lrwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

```
void pzheevd (char *jobz , char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *w , MKL_Complex16 *z , MKL_INT *iz , MKL_INT
*jz , MKL_INT *descz , MKL_Complex16 *work , MKL_INT *lwork , double *rwork , MKL_INT
*lrwork , MKL_INT *iwork , MKL_INT *liwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?heevd` function computes all eigenvalues and eigenvectors of a complex Hermitian matrix A by using a divide and conquer algorithm.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

| | |
|---------------|---|
| <i>jobz</i> | (global) Must be 'N' or 'V'. Specifies if it is necessary to compute the eigenvectors: If <i>jobz</i> = 'N', then only eigenvalues are computed. If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>uplo</i> | (global) Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the Hermitian matrix <i>A</i> is stored: If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular part of <i>A</i> . If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular part of <i>A</i> . |
| <i>n</i> | (global) The number of rows and columns of the matrix <i>A</i> ($n \geq 0$). |
| <i>a</i> | (local). Block cyclic array of global size $n*n$ and local size $lld_a*LOCc(ja+n-1)$. On entry, the Hermitian matrix <i>A</i> . If <i>uplo</i> = 'U', only the upper triangular part of <i>A</i> is used to define the elements of the Hermitian matrix. If <i>uplo</i> = 'L', only the lower triangular part of <i>A</i> is used to define the elements of the Hermitian matrix. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . If <i>desca</i> [<i>ctxt_</i> - 1] is incorrect, p?heevd cannot guarantee correct error reporting. |
| <i>iz, jz</i> | (global) The row and column indices in the global matrix <i>Z</i> indicating the first row and the first column of the submatrix <i>Z</i> , respectively. |
| <i>descz</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>Z</i> . <i>descz</i> [<i>ctxt_</i> - 1] must equal <i>desca</i> [<i>ctxt_</i> - 1]. |
| <i>work</i> | (local). Array of size <i>lwork</i> . |
| <i>lwork</i> | (local) The size of the array <i>work</i> . If eigenvalues are requested: $lwork = n + (nb0 + mq0 + nb)*nb$ with $np0 = \text{numroc}(\max(n, nb, 2), nb, 0, 0, \text{NPROW})$ $mq0 = \text{numroc}(\max(n, nb, 2), nb, 0, 0, \text{NPCOL})$ If <i>lwork</i> = -1, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. The required workspace is returned as the first element of the corresponding work arrays, and no error message is issued by p?xerbla. |
| <i>rwork</i> | (local). |

lwork Workspace array of size *lwork*.
 (local) The size of the array *rwork*.
 $lwork \geq 1 + 9*n + 3*np*nq$,
 with $np = \text{numroc}(n, nb, myrow, iarow, NPROW)$
 $nq = \text{numroc}(n, nb, mycol, iacol, NPCOL)$

iwork (local) Workspace array of size *liwork*.

liwork (local) , size of *iwork*.
 $liwork = 7*n + 8*npcol + 2$.

Output Parameters

a On exit, the lower triangle (if *uplo* = 'L'), or the upper triangle (if *uplo* = 'U') of *A*, including the diagonal, is overwritten.

w (global).
 Array of size *n*. If *info* = 0, *w* contains the eigenvalues in the ascending order.

z (local).
 Array, global size $n*n$, local size $lld_z*LOCc(jz+n-1)$.
 The *z* parameter contains the orthonormal eigenvectors of the matrix *A*.

work[0] On exit, returns adequate workspace to allow optimal performance.

rwork[0] (local)
 On output, *rwork*[0] returns workspace required to guarantee completion.

iwork[0] (local).
 On return, *iwork*[0] contains the amount of integer workspace required.

info (global)
 If *info* = 0, the execution is successful.
 If *info* < 0:
 If the *i*-th argument is an array and the *j*-entry had an illegal value, then $info = -(i*100+j)$. If the *i*-th argument is a scalar and had an illegal value, then $info = -i$.
 If *info* > 0:
 If *info* = 1 through *n*, the *i*-th eigenvalue did not converge.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?heevr

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix using Relatively Robust Representation.

Syntax

```
void pcheevr(char* jobz, char* range, char* uplo, MKL_INT* n, MKL_Complex8* a,
MKL_INT* ia, MKL_INT* ja, MKL_INT* desca, float* vl, float* vu, MKL_INT* il, MKL_INT*
iu, MKL_INT* m, MKL_INT* nz, float* w, MKL_Complex8* z, MKL_INT* iz, MKL_INT* jz,
MKL_INT* descz, MKL_Complex8* work, MKL_INT* lwork, float* rwork, MKL_INT* lrwork,
MKL_INT* iwork, MKL_INT* liwork, MKL_INT* info);
```

```
void pzheevr(char* jobz, char* range, char* uplo, MKL_INT* n, MKL_Complex16* a,
MKL_INT* ia, MKL_INT* ja, MKL_INT* desca, double* vl, double* vu, MKL_INT* il,
MKL_INT* iu, MKL_INT* m, MKL_INT* nz, double* w, MKL_Complex16* z, MKL_INT* iz,
MKL_INT* jz, MKL_INT* descz, MKL_Complex16* work, MKL_INT* lwork, double* rwork,
MKL_INT* lrwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

p?heevr computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A distributed in 2D blockcyclic format by calling the recommended sequence of ScaLAPACK functions.

First, the matrix A is reduced to complex Hermitian tridiagonal form. Then, the eigenproblem is solved using the parallel MRRR algorithm. Last, if eigenvectors have been computed, a backtransformation is done.

Upon successful completion, each processor stores a copy of all computed eigenvalues in w . The eigenvector matrix Z is stored in 2D block-cyclic format distributed over all processors.

Optimization Notice

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Notice revision #20110804

Input Parameters

jobz (global)
Specifies whether or not to compute the eigenvectors:
= 'N': Compute eigenvalues only.
= 'V': Compute eigenvalues and eigenvectors.

range (global)
= 'A': all eigenvalues will be found.

| | |
|--------------|---|
| | <p>= 'V': all eigenvalues in the interval $[v_l, v_u]$ will be found.</p> <p>= 'I': the i_l-th through i_u-th eigenvalues will be found.</p> |
| <i>uplo</i> | <p>(global)</p> <p>Specifies whether the upper or lower triangular part of the Hermitian matrix <i>A</i> is stored:</p> <p>= 'U': Upper triangular</p> <p>= 'L': Lower triangular</p> |
| <i>n</i> | <p>(global)</p> <p>The number of rows and columns of the matrix <i>A</i>. $n \geq 0$</p> |
| <i>a</i> | <p>Block-cyclic array, global size $n * n$), local size $lld_a * LOC_c(ja+n-1)$</p> <p>Contains the local pieces of the Hermitian distributed matrix <i>A</i>. If <i>uplo</i> = 'U', only the upper triangular part of <i>a</i> is used to define the elements of the Hermitian matrix. If <i>uplo</i> = 'L', only the lower triangular part of <i>a</i> is used to define the elements of the Hermitian matrix.</p> |
| <i>ia</i> | <p>(global)</p> <p>Global row index in the global matrix <i>A</i> that points to the beginning of the submatrix which is to be operated on. It should be set to 1 when operating on a full matrix.</p> |
| <i>ja</i> | <p>(global)</p> <p>Global column index in the global matrix <i>A</i> that points to the beginning of the submatrix which is to be operated on. It should be set to 1 when operating on a full matrix.</p> |
| <i>desca</i> | <p>(global and local) array of size <i>dlen_</i>. (The ScaLAPACK descriptor length is <i>dlen_</i> = 9.)</p> <p>The array descriptor for the distributed matrix <i>a</i>. The descriptor stores details about the 2D block-cyclic storage, see the notes below. If <i>desca</i> is incorrect, <i>p?heevr</i> cannot work correctly.</p> <p>Also note the array alignment requirements specified below</p> |
| <i>v_l</i> | <p>(global)</p> <p>If <i>range</i>='V', the lower bound of the interval to be searched for eigenvalues. Not referenced if <i>range</i> = 'A' or 'I'.</p> |
| <i>v_u</i> | <p>(global)</p> <p>If <i>range</i>='V', the upper bound of the interval to be searched for eigenvalues. Not referenced if <i>range</i> = 'A' or 'I'.</p> |
| <i>i_l</i> | <p>(global)</p> <p>If <i>range</i>='I', the index (from smallest to largest) of the smallest eigenvalue to be returned. $i_l \geq 1$.</p> <p>Not referenced if <i>range</i> = 'A'.</p> |
| <i>i_u</i> | <p>(global)</p> |

If *range*='I', the index (from smallest to largest) of the largest eigenvalue to be returned. $\min(il,n) \leq iu \leq n$.

Not referenced if *range* = 'A'.

iz

(global)

Global row index in the global matrix *Z* that points to the beginning of the submatrix which is to be operated on. It should be set to 1 when operating on a full matrix.

jz

(global)

Global column index in the global matrix *Z* that points to the beginning of the submatrix which is to be operated on. It should be set to 1 when operating on a full matrix.

descz

(global and local) array of size *dlen_*.

The array descriptor for the distributed matrix *z*. *descz*[*ctxt_* - 1] must equal *desca*[*ctxt_* - 1]

work

(local workspace) array of size *lwork*

lwork

(local)

Size of *work* array, must be at least 3.

If only eigenvalues are requested:

$$lwork \geq n + \max(nb * (np00 + 1), nb * 3)$$

If eigenvectors are requested:

$$lwork \geq n + (np00 + mq00 + nb) * nb$$

For definitions of *np00* and *mq00*, see *lrwork*.

For optimal performance, greater workspace is needed, i.e.

$$lwork \geq \max(lwork, nhetr_d_lwork)$$

Where *lwork* is as defined above, and

$$nhetr_d_lwork = n + 2*(an_b+1)*(4*nps+2) + (nps + 1) * nps$$

$$ictxt = desca[ctxt_ - 1]$$

$$an_b = p_jlaenv(ictxt, 3, 'PCHETTRD', 'L', 0, 0, 0, 0)$$

$$sqnpc = \text{sqrt}(\text{real}(nprow * npc col))$$

$$nps = \max(\text{numroc}(n, 1, 0, 0, sqnpc), 2*an_b)$$

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by [pxerbla](#).

rwork

(local workspace) array of size *lrwork*

lrwork

(local)

Size of *rwork*, must be at least 3.

See below for definitions of variables used to define *lrwork*.

If no eigenvectors are requested (*jobz* = 'N') then

$$lwork \geq 2 + 5 * n + \max(12 * n, nb * (np00 + 1))$$

If eigenvectors are requested (*jobz* = 'V') then the amount of workspace required is:

$$lwork \geq 2 + 5 * n + \max(18 * n, np00 * mq00 + 2 * nb * nb) + (2 + \text{iceil}(neig, nprow * npc0l)) * n$$

NOTE

iceil(*x*, *y*) is the ceiling of *x/y*.

Variable definitions:

neig = number of eigenvectors requested

nb = *desca*[*mb_* - 1] = *desca*[*nb_* - 1] = *descz*[*mb_* - 1] = *descz*[*nb_* - 1]

nn = $\max(n, nb, 2)$

desca[*rsrc_* - 1] = *desca*[*csrc_* - 1] = *descz*[*rsrc_* - 1] = *descz*[*csrc_* - 1] = 0

np00 = *numroc*(*nn*, *nb*, 0, 0, *nprow*)

mq00 = *numroc*($\max(neig, nb, 2)$, *nb*, 0, 0, *npcol*)

iceil(*x*, *y*) is a ScaLAPACK function returning *ceiling(x/y)*, and *nprow* and *npcol* can be determined by calling the function *blacs_gridinfo*.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by *pxerbla*

iwork (local workspace) array of size *liwork*

liwork (local)

size of *iwork*

Let *nnp* = $\max(n, nprow * npc0l + 1, 4)$. Then:

liwork $\geq 12 * nnp + 2 * n$ when the eigenvectors are desired

liwork $\geq 10 * nnp + 2 * n$ when only the eigenvalues have to be computed

If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by *pxerbla*

OUTPUT Parameters

a The lower triangle (if *uplo*='L') or the upper triangle (if *uplo*='U') of *a*, including the diagonal, is destroyed.

m (global)

| | |
|--------------|---|
| | Total number of eigenvalues found. $0 \leq m \leq n$. |
| <i>nz</i> | (global) Total number of eigenvectors computed. $0 \leq nz \leq m$. The number of columns of <i>z</i> that are filled. If <i>jobz</i> ≠ 'V', <i>nz</i> is not referenced. If <i>jobz</i> = 'V', <i>nz</i> = <i>m</i> |
| <i>w</i> | (global) array of size <i>n</i> On normal exit, the first <i>m</i> entries contain the selected eigenvalues in ascending order. |
| <i>z</i> | (local) array, global size $n * n$, local size $lld_z * LOC_c(jz+n-1)$ If <i>jobz</i> = 'V', then on normal exit the first <i>m</i> columns of <i>z</i> contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If <i>jobz</i> = 'N', then <i>z</i> is not referenced. |
| <i>work</i> | <i>work</i> [0] returns workspace adequate workspace to allow optimal performance. |
| <i>rwork</i> | On return, <i>rwork</i> [0] contains the optimal amount of workspace required for efficient execution. if <i>jobz</i> ='N' <i>rwork</i> [0] = optimal amount of workspace required to compute the eigenvalues. if <i>jobz</i> ='V' <i>rwork</i> [0] = optimal amount of workspace required to compute eigenvalues and eigenvectors. |
| <i>iwork</i> | On return, <i>iwork</i> [0] contains the amount of integer workspace required. |
| <i>info</i> | (global) = 0: successful exit < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry had an illegal value, then <i>info</i> = $-(i*100+j)$, if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = $-i$. |

Application Notes

The distributed submatrices $a(ia:*, ja:*)$ and $z(iz:iz+m-1, jz:jz+n-1)$ must satisfy the following alignment properties:

1. Identical (quadratic) dimension: $desca[m_ - 1] = descz[m_ - 1] = desca[n_ - 1] = descz[n_ - 1]$
2. Quadratic conformal blocking: $desca[mb_ - 1] = desca[nb_ - 1] = descz[mb_ - 1] = descz[nb_ - 1]$,
 $desca[rsrc_ - 1] = descz[rsrc_ - 1]$
3. $\text{mod}(ia-1, mb_a) = \text{mod}(iz-1, mb_z) = 0$

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

See Also

Overview of ScaLAPACK Routines for details of ScaLAPACK array descriptor structures and related notations.

p?heevx

Computes selected eigenvalues and, optionally, eigenvectors of a Hermitian matrix.

Syntax

```
void pcheevx (char *jobz , char *range , char *uplo , MKL_INT *n , MKL_Complex8 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *vl , float *vu , MKL_INT *il ,
MKL_INT *iu , float *abstol , MKL_INT *m , MKL_INT *nz , float *w , float *orfac ,
MKL_Complex8 *z , MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , MKL_Complex8 *work ,
MKL_INT *lwork , float *rwork , MKL_INT *lrwork , MKL_INT *iwork , MKL_INT *liwork ,
MKL_INT *ifail , MKL_INT *iclustr , float *gap , MKL_INT *info );
```

```
void pzheevx (char *jobz , char *range , char *uplo , MKL_INT *n , MKL_Complex16 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *vl , double *vu , MKL_INT *il ,
MKL_INT *iu , double *abstol , MKL_INT *m , MKL_INT *nz , double *w , double *orfac ,
MKL_Complex16 *z , MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , MKL_Complex16 *work ,
MKL_INT *lwork , double *rwork , MKL_INT *lrwork , MKL_INT *iwork , MKL_INT *liwork ,
MKL_INT *ifail , MKL_INT *iclustr , double *gap , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?heevx function computes selected eigenvalues and, optionally, eigenvectors of a complex Hermitian matrix A by calling the recommended sequence of ScaLAPACK functions. Eigenvalues and eigenvectors can be selected by specifying either a range of values or a range of indices for the desired eigenvalues.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

$jobz$ (global) Must be 'N' or 'V'.

Specifies if it is necessary to compute the eigenvectors:

If $jobz = 'N'$, then only eigenvalues are computed.

| | |
|---------------|---|
| | If <i>jobz</i> = 'V', then eigenvalues and eigenvectors are computed. |
| <i>range</i> | (global) Must be 'A', 'V', or 'I'. If <i>range</i> = 'A', all eigenvalues will be found. If <i>range</i> = 'V', all eigenvalues in the half-open interval [<i>vl</i> , <i>vu</i>] will be found. If <i>range</i> = 'I', the eigenvalues with indices <i>il</i> through <i>iu</i> will be found. |
| <i>uplo</i> | (global) Must be 'U' or 'L'. Specifies whether the upper or lower triangular part of the Hermitian matrix <i>A</i> is stored: If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular part of <i>A</i> . If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular part of <i>A</i> . |
| <i>n</i> | (global) The number of rows and columns of the matrix <i>A</i> ($n \geq 0$). |
| <i>a</i> | (local). Block cyclic array of global size $n*n$ and local size $lld_a*LOCc(ja+n-1)$. On entry, the Hermitian matrix <i>A</i> . If <i>uplo</i> = 'U', only the upper triangular part of <i>A</i> is used to define the elements of the Hermitian matrix. If <i>uplo</i> = 'L', only the lower triangular part of <i>A</i> is used to define the elements of the Hermitian matrix. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . If <i>desca</i> [<i>ctxt_</i> - 1] is incorrect, p?heevx cannot guarantee correct error reporting. |
| <i>vl, vu</i> | (global) If <i>range</i> = 'V', the lower and upper bounds of the interval to be searched for eigenvalues; not referenced if <i>range</i> = 'A' or 'I'. |
| <i>il, iu</i> | (global) If <i>range</i> = 'I', the indices of the smallest and largest eigenvalues to be returned. Constraints: $il \geq 1; \min(il, n) \leq iu \leq n$. Not referenced if <i>range</i> = 'A' or 'V'. |
| <i>abstol</i> | (global). If <i>jobz</i> ='V', setting <i>abstol</i> to <code>p?lamch(context, 'U')</code> yields the most orthogonal eigenvectors. |

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to $abstol + eps * \max(|a|, |b|)$, where eps is the machine precision. If $abstol$ is less than or equal to zero, then $eps * \text{norm}(T)$ will be used in its place, where $\text{norm}(T)$ is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues are computed most accurately when $abstol$ is set to twice the underflow threshold $2 * \text{p?lamch}('S')$, not zero. If this function returns with $((\text{mod}(\text{info}, 2) \neq 0) . \text{or.} (\text{mod}(\text{info}/8, 2) \neq 0))$, indicating that some eigenvalues or eigenvectors did not converge, try setting $abstol$ to $2 * \text{p?lamch}('S')$.

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

orfac

(global).

Specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within $tol = orfac * \text{norm}(A)$ of each other are to be reorthogonalized. However, if the workspace is insufficient (see *lwork*), *tol* may be decreased until all eigenvectors to be reorthogonalized can be stored in one process. No reorthogonalization will be done if *orfac* equals zero. A default value of 1.0e-3 is used if *orfac* is negative.

orfac should be identical on all processes.

iz, jz

(global) The row and column indices in the global matrix Z indicating the first row and the first column of the submatrix Z , respectively.

descz

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix Z . $\text{descz}[\text{ctxt}_ - 1]$ must equal $\text{desca}[\text{ctxt}_ - 1]$.

work

(local).

Array of size *lwork*.

lwork

(local) The size of the array *work*.

If only eigenvalues are requested:

$$lwork \geq n + \max(nb * (np0 + 1), 3)$$

If eigenvectors are requested:

$$lwork \geq n + (np0 + mq0 + nb) * nb$$

with $nq0 = \text{numroc}(nn, nb, 0, 0, \text{NPCOL})$.

$$lwork \geq 5 * n + \max(5 * nn, np0 * mq0 + 2 * nb * nb) + \text{iceil}(neig, \text{NPROW} * \text{NPCOL}) * nn$$

For optimal performance, greater workspace is needed, that is

$$lwork \geq \max(lwork, \text{nhetrd_lwork})$$

where *lwork* is as defined above, and $\text{nhetrd_lwork} = n + 2 * (anb + 1) * (4 * nps + 2) + (nps + 1) * nps$

$$ictxt = \text{desca}[\text{ctxt}_ - 1]$$

```

anb = pjlavenv(ictxt, 3, 'pchettrd', 'L', 0, 0, 0, 0)
sqnpc = sqrt(dble(NPROW * NPCOL))
nps = max(numroc(n, 1, 0, 0, sqnpc), 2*anb)

```

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by `pxerbla`.

rwork

(local)

Workspace array of size $lwork$.*lwork*(local) The size of the array *work*.See below for definitions of variables used to define $lwork$.If no eigenvectors are requested ($jobz = 'N'$), then $lwork \geq 5*nn+4*n$.If eigenvectors are requested ($jobz = 'V'$), then the amount of workspace required to guarantee that all eigenvectors are computed is:
$$lwork \geq 4*n + \max(5*nn, np0*mq0+2*nb*nb) + \text{iceil}(neig, \text{NPROW}*NPCOL)*nn$$

The computed eigenvectors may not be orthogonal if the minimal workspace is supplied and $orfac$ is too small. If you want to guarantee orthogonality (at the cost of potentially poor performance) you should add the following values to $lwork$:

$$(clustersize-1)*n,$$

where $clustersize$ is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:

$$\{w[k-1], \dots, w[k+clustersize-2] | w[j] \leq w[j-1] + orfac*2*norm(A)\}.$$

Variable definitions:

 $neig$ = number of eigenvectors requested;
$$nb = desca[mb_ - 1] = desca[nb_ - 1] = descz[mb_ - 1] = descz[nb_ - 1];$$

$$nn = \max(n, NB, 2);$$

$$desca[rsrc_ - 1] = desca[nb_ - 1] = descz[rsrc_ - 1] = descz[csrc_ - 1] = 0;$$

$$np0 = \text{numroc}(nn, nb, 0, 0, \text{NPROW});$$

$$mq0 = \text{numroc}(\max(neig, nb, 2), nb, 0, 0, \text{NPCOL});$$

$$\text{iceil}(x, y)$$
 is a ScaLAPACK function returning $\text{ceiling}(x/y)$
When $lwork$ is too small:

If $lwork$ is too small to guarantee orthogonality, `p?heevx` attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues. If $lwork$ is too small to compute all the eigenvectors requested, no computation is performed and $info = -23$ is returned. Note that when $range = 'V'$, `p?heevx` does not know how many eigenvectors are requested

until the eigenvalues are computed. Therefore, when `range='V'` and as long as `lwork` is large enough to allow `p?heevx` to compute the eigenvalues, `p?heevx` will compute the eigenvalues and as many eigenvectors as it can.

Relationship between workspace, orthogonality and performance:

If $clustersize \geq n/\sqrt{NPROW * NPCOL}$, then providing enough space to compute all the eigenvectors orthogonally will cause serious degradation in performance. In the limit (that is, $clustersize = n-1$) `p?stein` will perform no better than `?stein` on 1 processor.

For $clustersize = n/\sqrt{NPROW * NPCOL}$ reorthogonalizing all eigenvectors will increase the total execution time by a factor of 2 or more.

For $clustersize > n/\sqrt{NPROW * NPCOL}$ execution time will grow as the square of the cluster size, all other factors remaining equal and assuming enough workspace. Less workspace means less reorthogonalization but faster execution.

If `lwork = -1`, then `lwork` is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by `p?xerbla`.

`iwork` (local) Workspace array.

`liwork` (local), size of `iwork`.

$$liwork \geq 6 * nnp$$

Where: $nnp = \max(n, NPROW * NPCOL + 1, 4)$

If `liwork = -1`, then `liwork` is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `p?xerbla`.

Output Parameters

`a` On exit, the lower triangle (if `uplo = 'L'`), or the upper triangle (if `uplo = 'U'`) of A , including the diagonal, is overwritten.

`m` (global) The total number of eigenvalues found; $0 \leq m \leq n$.

`nz` (global) Total number of eigenvectors computed. $0 \leq nz \leq m$.

The number of columns of z that are filled.

If `jobz ≠ 'V'`, `nz` is not referenced.

If `jobz = 'V'`, `nz = m` unless the user supplies insufficient space and `p?heevx` is not able to detect this before beginning computation. To get all the eigenvectors requested, the user must supply both sufficient space to hold the eigenvectors in z ($m \leq descz[n_ - 1]$) and sufficient workspace to compute them. (See `lwork`). `p?heevx` is always able to detect insufficient space without computation unless `range='V'`.

`w` (global).

| | |
|------------|--|
| | <p>Array of size n. The first m elements contain the selected eigenvalues in ascending order.</p> |
| z | <p>(local).</p> <p>Array, global size $n*n$, local size $lld_z*LOCc(jz+n-1)$.</p> <p>If $jobz = 'V'$, then on normal exit the first m columns of z contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in <i>ifail</i>.</p> <p>If $jobz = 'N'$, then z is not referenced.</p> |
| $work[0]$ | <p>On exit, returns adequate workspace to allow optimal performance.</p> |
| $rwork$ | <p>(local).</p> <p>Array of size $lwork$. On return, $rwork[0]$ contains the optimal amount of workspace required for efficient execution.</p> <p>If $jobz='N'$ $rwork[0]$ = optimal amount of workspace required to compute eigenvalues efficiently.</p> <p>If $jobz='V'$ $rwork[0]$ = optimal amount of workspace required to compute eigenvalues and eigenvectors efficiently with no guarantee on orthogonality.</p> <p>If $range='V'$, it is assumed that all eigenvectors may be required.</p> |
| $iwork[0]$ | <p>(local)</p> <p>On return, $iwork[0]$ contains the amount of integer workspace required.</p> |
| $ifail$ | <p>(global)</p> <p>Array of size n.</p> <p>If $jobz = 'V'$, then on normal exit, the first m elements of $ifail$ are zero. If $(\text{mod}(\text{info}, 2) \neq 0)$ on exit, then $ifail$ contains the indices of the eigenvectors that failed to converge.</p> <p>If $jobz = 'N'$, then $ifail$ is not referenced.</p> |
| $iclustr$ | <p>(global)</p> <p>Array of size $2*NPROW*NPCOL$.</p> <p>This array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace (see <i>lwork</i>, <i>orfac</i> and <i>info</i>). Eigenvectors corresponding to clusters of eigenvalues indexed $iclustr[2*i - 2]$ to $iclustr[2*i - 1]$, could not be reorthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal. $iclustr$ is a zero terminated array. ($iclustr[2*k - 1] \neq 0$ and $iclustr[2*k] = 0$) if and only if k is the number of clusters. $iclustr$ is not referenced if $jobz = 'N'$.</p> |
| gap | <p>(global)</p> <p>Array of size $(NPROW*NPCOL)$</p> |

This array contains the gap between eigenvalues whose eigenvectors could not be reorthogonalized. The output values in this array correspond to the clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the *i*-th cluster may be as high as $(C*n)/gap(i)$ where *C* is a small constant.

info

(global)

If *info* = 0, the execution is successful.

If *info* < 0:

If the *i*-th argument is an array and the *j*-entry had an illegal value, then *info* = $-(i*100+j)$. If the *i*-th argument is a scalar and had an illegal value, then *info* = $-i$.

If *info* > 0:

If $(\text{mod}(\text{info}, 2) \neq 0)$, then one or more eigenvectors failed to converge. Their indices are stored in *ifail*. Ensure $\text{abstol} = 2.0 * p?lamch('U')$

If $(\text{mod}(\text{info}/2, 2) \neq 0)$, then eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in the array *iclustr*.

If $(\text{mod}(\text{info}/4, 2) \neq 0)$, then space limit prevented *p?syevx* from computing all of the eigenvectors between *vl* and *vu*. The number of eigenvectors computed is returned in *nz*.

If $(\text{mod}(\text{info}/8, 2) \neq 0)$, then *p?stebz* failed to compute eigenvalues. Ensure $\text{abstol} = 2.0 * p?lamch('U')$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gesvd

Computes the singular value decomposition of a general matrix, optionally computing the left and/or right singular vectors.

Syntax

```
void psgesvd (char *jobu , char *jobvt , MKL_INT *m , MKL_INT *n , float *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , float *s , float *u , MKL_INT *iu , MKL_INT *ju ,
MKL_INT *descu , float *vt , MKL_INT *ivt , MKL_INT *jvt , MKL_INT *descvt , float
*work , MKL_INT *lwork , float *rwork , MKL_INT *info );
```

```
void pdgesvd (char *jobu , char *jobvt , MKL_INT *m , MKL_INT *n , double *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , double *s , double *u , MKL_INT *iu , MKL_INT
*ju , MKL_INT *descu , double *vt , MKL_INT *ivt , MKL_INT *jvt , MKL_INT *descvt ,
double *work , MKL_INT *lwork , double *rwork , MKL_INT *info );
```

```
void pcgesvd (char *jobu , char *jobvt , MKL_INT *m , MKL_INT *n , MKL_Complex8 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *s , MKL_Complex8 *u , MKL_INT
*iu , MKL_INT *ju , MKL_INT *descu , MKL_Complex8 *vt , MKL_INT *ivt , MKL_INT *jvt ,
MKL_INT *descvt , MKL_Complex8 *work , MKL_INT *lwork , float *rwork , MKL_INT *info );
```

```
void pzgesvd (char *jobu , char *jobvt , MKL_INT *m , MKL_INT *n , MKL_Complex16 *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *s , MKL_Complex16 *u , MKL_INT
*iu , MKL_INT *ju , MKL_INT *descu , MKL_Complex16 *vt , MKL_INT *ivt , MKL_INT *jvt ,
MKL_INT *descvt , MKL_Complex16 *work , MKL_INT *lwork , double *rwork , MKL_INT
*info );
```

Include Files

- mkl_scalapack.h

Description

The `p?gesvd` function computes the singular value decomposition (SVD) of an m -by- n matrix A , optionally computing the left and/or right singular vectors. The SVD is written

$$A = U \Sigma V^T,$$

where Σ is an m -by- n matrix that is zero except for its $\min(m, n)$ diagonal elements, U is an m -by- m orthogonal matrix, and V is an n -by- n orthogonal matrix. The diagonal elements of Σ are the singular values of A and the columns of U and V are the corresponding right and left singular vectors, respectively. The singular values are returned in array s in decreasing order and only the first $\min(m, n)$ columns of U and rows of $vt = V^T$ are computed.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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Input Parameters

mp = number of local rows in A and U

nq = number of local columns in A and VT

$size$ = $\min(m, n)$

$sizeq$ = number of local columns in U

$sizep$ = number of local rows in VT

$jobu$ (global) Specifies options for computing all or part of the matrix U .
 If $jobu = 'V'$, the first $size$ columns of U (the left singular vectors) are returned in the array u ;
 If $jobu = 'N'$, no columns of U (no left singular vectors) are computed.

$jobvt$ (global)
 Specifies options for computing all or part of the matrix V^T .
 If $jobvt = 'V'$, the first $size$ rows of V^T (the right singular vectors) are returned in the array vt ;
 If $jobvt = 'N'$, no rows of V^T (no right singular vectors) are computed.

| | |
|-----------------|---|
| <i>m</i> | (global) The number of rows of the matrix A ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in A ($n \geq 0$). |
| <i>a</i> | (local). Block cyclic array, global size (m, n) , local size (mp, nq) . |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>iu, ju</i> | (global) The row and column indices in the global matrix U indicating the first row and the first column of the submatrix U , respectively. |
| <i>descu</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix U . |
| <i>ivt, jvt</i> | (global) The row and column indices in the global matrix VT indicating the first row and the first column of the submatrix VT , respectively. |
| <i>descvt</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix VT . |
| <i>work</i> | (local). Workspace array of size <i>lwork</i> |
| <i>lwork</i> | (local) The size of the array <i>work</i> ; $lwork > 2 + 6 * sizeb + \max(watobd, wbdtosvd)$, where $sizeb = \max(m, n)$, and <i>watobd</i> and <i>wbdtosvd</i> refer, respectively, to the workspace required to bidiagonalize the matrix A and to go from the bidiagonal matrix to the singular value decomposition $USVT$. For <i>watobd</i> , the following holds: $watobd = \max(\max(wp?lange, wp?gebrd), \max(wp?lared2d, wp?lared1d))$, where <i>wp?lange</i> , <i>wp?lared1d</i> , <i>wp?lared2d</i> , <i>wp?gebrd</i> are the workspaces required respectively for the subprograms <i>p?lange</i> , <i>p?lared1d</i> , <i>p?lared2d</i> , <i>p?gebrd</i> . Using the standard notation $mp = \text{numroc}(m, mb, \text{MYROW}, \text{desca}[\text{ctxt}_ - 1], \text{NPROW})$, $nq = \text{numroc}(n, nb, \text{MYCOL}, \text{desca}[\text{lld}_ - 1], \text{NPCOL})$, the workspaces required for the above subprograms are $wp?lange = mp$, $wp?lared1d = nq0$, $wp?lared2d = mp0$, $wp?gebrd = nb * (mp + nq + 1) + nq$, where <i>nq0</i> and <i>mp0</i> refer, respectively, to the values obtained at <code>MYCOL = 0</code> and <code>MYROW = 0</code> . In general, the upper limit for the workspace is given by a workspace required on processor (0,0): |

$$w_{atobd} \leq nb * (mp0 + nq0 + 1) + nq0.$$

In case of a homogeneous process grid this upper limit can be used as an estimate of the minimum workspace for every processor.

For *wbdtosvd*, the following holds:

$$wbdtosvd = size * (wantu * nru + wantvt * ncv) + \max(w?bdsqr, \max(wantu * wp?ormbrqln, wantvt * wp?ormbrprt)),$$

where

$wantu(wantvt) = 1$, if left/right singular vectors are wanted, and $wantu(wantvt) = 0$, otherwise. $w?bdsqr$, $wp?ormbrqln$, and $wp?ormbrprt$ refer respectively to the workspace required for the subprograms $?bdsqr$, $p?ormbr(qln)$, and $p?ormbr(prt)$, where qln and prt are the values of the arguments *vect*, *side*, and *trans* in the call to $p?ormbr$. nru is equal to the local number of rows of the matrix U when distributed 1-dimensional "column" of processes. Analogously, ncv is equal to the local number of columns of the matrix VT when distributed across 1-dimensional "row" of processes. Calling the LAPACK procedure $?bdsqr$ requires

$$w?bdsqr = \max(1, 2 * size + (2 * size - 4) * \max(wantu, wantvt))$$

on every processor. Finally,

$$wp?ormbrqln = \max((nb * (nb - 1)) / 2, (sizeq + mp) * nb) + nb * nb,$$

$$wp?ormbrprt = \max((mb * (mb - 1)) / 2, (sizep + nq) * mb) + mb * mb,$$

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum size for the work array. The required workspace is returned as the first element of *work* and no error message is issued by `pxerbla`.

rwork

Workspace array of size $1 + 4 * sizeb$. Not used for `psgesvd` and `pdgesvd`.

Output Parameters

a

On exit, the contents of *a* are destroyed.

s

(global).

Array of size *size*.

Contains the singular values of A sorted so that $s(i) \geq s(i+1)$.

u

(local).

local size $mp * sizeq$, global size $m * size$)

If $jobu = 'V'$, *u* contains the first $\min(m, n)$ columns of U .

If $jobu = 'N'$ or $'O'$, *u* is not referenced.

vt

(local).

local size (*sizep*, *nq*), global size (*size*, *n*)

If $jobvt = 'V'$, *vt* contains the first *size* rows of V^T if $jobu = 'N'$, *vt* is not referenced.

| | |
|--------------|--|
| <i>work</i> | On exit, if <i>info</i> = 0, then <i>work</i> [0] returns the required minimal size of <i>lwork</i> . |
| <i>rwork</i> | On exit, if <i>info</i> = 0, then <i>rwork</i> [0] returns the required size of <i>rwork</i> . |
| <i>info</i> | (global) If <i>info</i> = 0, the execution is successful. If <i>info</i> < 0, If <i>info</i> = - <i>i</i> , the <i>i</i> th parameter had an illegal value. If <i>info</i> > 0 <i>i</i> , then if ?bdsqr did not converge, If <i>info</i> = min(<i>m</i> , <i>n</i>) + 1, then p?gesvd has detected heterogeneity by finding that eigenvalues were not identical across the process grid. In this case, the accuracy of the results from p?gesvd cannot be guaranteed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?sygvx

Computes selected eigenvalues and, optionally, eigenvectors of a real generalized symmetric definite eigenproblem.

Syntax

```
void pssygvx (MKL_INT *ibtype , char *jobz , char *range , char *uplo , MKL_INT *n ,
float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , float *vl , float *vu , MKL_INT *il , MKL_INT *iu ,
float *abstol , MKL_INT *m , MKL_INT *nz , float *w , float *orfac , float *z ,
MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , float *work , MKL_INT *lwork , MKL_INT
*iwork , MKL_INT *liwork , MKL_INT *ifail , MKL_INT *iclustr , float *gap , MKL_INT
*info );
```

```
void pdsygvx (MKL_INT *ibtype , char *jobz , char *range , char *uplo , MKL_INT *n ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , double *vl , double *vu , MKL_INT *il , MKL_INT *iu ,
double *abstol , MKL_INT *m , MKL_INT *nz , double *w , double *orfac , double *z ,
MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , double *work , MKL_INT *lwork , MKL_INT
*iwork , MKL_INT *liwork , MKL_INT *ifail , MKL_INT *iclustr , double *gap , MKL_INT
*info );
```

Include Files

- mkl_scalapack.h

Description

The p?sygvx function computes all the eigenvalues, and optionally, the eigenvectors of a real generalized symmetric-definite eigenproblem, of the form

$$\text{sub}(A) * x = \lambda * \text{sub}(B) * x, \text{sub}(A) \text{ sub}(B) * x = \lambda * x, \text{ or } \text{sub}(B) * \text{sub}(A) * x = \lambda * x.$$

Here *x* denotes eigen vectors, λ (*lambda*) denotes eigenvalues, *sub*(*A*) denoting *A*(*ia:ia+n-1, ja:ja+n-1*) is assumed to symmetric, and *sub*(*B*) denoting *B*(*ib:ib+n-1, jb:jb+n-1*) is also positive definite.

Optimization Notice

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Input Parameters

| | |
|---------------|---|
| <i>ibtype</i> | (global) Must be 1 or 2 or 3. Specifies the problem type to be solved: If <i>ibtype</i> = 1, the problem type is $\text{sub}(A) * x = \text{lambda} * \text{sub}(B) * x$; If <i>ibtype</i> = 2, the problem type is $\text{sub}(A) * \text{sub}(B) * x = \text{lambda} * x$; If <i>ibtype</i> = 3, the problem type is $\text{sub}(B) * \text{sub}(A) * x = \text{lambda} * x$. |
| <i>jobz</i> | (global) Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>range</i> | (global) Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the function computes all eigenvalues. If <i>range</i> = 'V', the function computes eigenvalues in the interval: [<i>vl</i> , <i>vu</i>] If <i>range</i> = 'I', the function computes eigenvalues with indices <i>il</i> through <i>iu</i> . |
| <i>uplo</i> | (global) Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>a</i> and <i>b</i> store the upper triangles of $\text{sub}(A)$ and $\text{sub}(B)$; If <i>uplo</i> = 'L', arrays <i>a</i> and <i>b</i> store the lower triangles of $\text{sub}(A)$ and $\text{sub}(B)$. |
| <i>n</i> | (global) The order of the matrices $\text{sub}(A)$ and $\text{sub}(B)$, $n \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $\text{lld_a} * \text{LOCc}(j_{a+n-1})$. On entry, this array contains the local pieces of the <i>n</i> -by- <i>n</i> symmetric distributed matrix $\text{sub}(A)$. If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix. If <i>uplo</i> = 'L', the leading <i>n</i> -by- <i>n</i> lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix. |

| | |
|---------------|--|
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . If <i>desca[ctxt_ - 1]</i> is incorrect, <i>p?sygvx</i> cannot guarantee correct error reporting. |
| <i>b</i> | (local). Pointer into the local memory to an array of size <i>lld_b*LOCc(jb+n-1)</i> . On entry, this array contains the local pieces of the <i>n</i> -by- <i>n</i> symmetric distributed matrix sub(<i>B</i>). If <i>uplo = 'U'</i> , the leading <i>n</i> -by- <i>n</i> upper triangular part of sub(<i>B</i>) contains the upper triangular part of the matrix. If <i>uplo = 'L'</i> , the leading <i>n</i> -by- <i>n</i> lower triangular part of sub(<i>A</i>) contains the lower triangular part of the matrix. |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix <i>B</i> indicating the first row and the first column of the submatrix <i>B</i> , respectively. |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>B</i> . <i>descb[ctxt_ - 1]</i> must be equal to <i>desca[ctxt_ - 1]</i> . |
| <i>vl, vu</i> | (global) If <i>range = 'V'</i> , the lower and upper bounds of the interval to be searched for eigenvalues. If <i>range = 'A'</i> or <i>'I'</i> , <i>vl</i> and <i>vu</i> are not referenced. |
| <i>il, iu</i> | (global) If <i>range = 'I'</i> , the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $il \geq 1, \min(il, n) \leq iu \leq n$ If <i>range = 'A'</i> or <i>'V'</i> , <i>il</i> and <i>iu</i> are not referenced. |
| <i>abstol</i> | (global) If <i>jobz='V'</i> , setting <i>abstol</i> to <i>p?lamch(context, 'U')</i> yields the most orthogonal eigenvectors. The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to $abstol + eps * \max(a , b),$ where <i>eps</i> is the machine precision. If <i>abstol</i> is less than or equal to zero, then <i>eps*norm(T)</i> will be used in its place, where <i>norm(T)</i> is the 1-norm of the tridiagonal matrix obtained by reducing <i>A</i> to tridiagonal form. Eigenvalues will be computed most accurately when <i>abstol</i> is set to twice the underflow threshold <i>2*p?lamch('S')</i> not zero. If this function returns with $((\text{mod}(\text{info}, 2) \neq 0) \text{ or } (\text{mod}(\text{info}/8, 2) \neq 0))$, indicating that some eigenvalues or eigenvectors did not converge, try setting <i>abstol</i> to <i>2*p?lamch('S')</i> . |

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

| | |
|---------------|---|
| <i>orfac</i> | (global). Specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within $\text{tol} = \text{orfac} * \text{norm}(A)$ of each other are to be reorthogonalized. However, if the workspace is insufficient (see <i>lwork</i>), <i>tol</i> may be decreased until all eigenvectors to be reorthogonalized can be stored in one process. No reorthogonalization will be done if <i>orfac</i> equals zero. A default value of $1.0\text{e-}3$ is used if <i>orfac</i> is negative. <i>orfac</i> should be identical on all processes. |
| <i>iz, jz</i> | (global) The row and column indices in the global matrix <i>Z</i> indicating the first row and the first column of the submatrix <i>Z</i> , respectively. |
| <i>descz</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>Z</i> . <i>descz</i> [<i>ctxt_</i> - 1] must equal <i>desca</i> [<i>ctxt_</i> - 1]. |
| <i>work</i> | (local) Workspace array of size <i>lwork</i> |
| <i>lwork</i> | (local) Size of the array <i>work</i> . See below for definitions of variables used to define <i>lwork</i> . If no eigenvectors are requested (<i>jobz</i> = 'N'), then $\text{lwork} \geq 5 * n + \max(5 * nn, \text{NB} * (\text{np0} + 1))$. If eigenvectors are requested (<i>jobz</i> = 'V'), then the amount of workspace required to guarantee that all eigenvectors are computed is: $\text{lwork} \geq 5 * n + \max(5 * nn, \text{np0} * \text{mq0} + 2 * \text{nb} * \text{nb}) + \text{iceil}(\text{neig}, \text{NPROW} * \text{NPCOL}) * nn$ The computed eigenvectors may not be orthogonal if the minimal workspace is supplied and <i>orfac</i> is too small. If you want to guarantee orthogonality at the cost of potentially poor performance you should add the following to <i>lwork</i> : $(\text{clustersize} - 1) * n,$ where <i>clustersize</i> is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues: $\{w[k - 1], \dots, w[k + \text{clustersize} - 2] \mid w[j] \leq w[j - 1] + \text{orfac} * 2 * \text{norm}(A)\}$ Variable definitions: <i>neig</i> = number of eigenvectors requested, $\text{nb} = \text{desca}[\text{mb}_ - 1] = \text{desca}[\text{nb}_ - 1] = \text{descz}[\text{mb}_ - 1] = \text{descz}[\text{nb}_ - 1],$ $\text{nn} = \max(n, \text{nb}, 2),$ $\text{desca}[\text{rsrc}_ - 1] = \text{desca}[\text{nb}_ - 1] = \text{descz}[\text{rsrc}_ - 1] = \text{descz}[\text{csrc}_ - 1] = 0,$ |

$np0 = \text{numroc}(nn, nb, 0, 0, \text{NPROW}),$

$mq0 = \text{numroc}(\max(neig, nb, 2), nb, 0, 0, \text{NPCOL})$

$\text{iceil}(x, y)$ is a ScaLAPACK function returning $\text{ceiling}(x/y)$

If $lwork$ is too small to guarantee orthogonality, $p?syevx$ attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues.

If $lwork$ is too small to compute all the eigenvectors requested, no computation is performed and $info = -23$ is returned.

Note that when $range = 'V'$, number of requested eigenvectors are not known until the eigenvalues are computed. In this case and if $lwork$ is large enough to compute the eigenvalues, $p?sygvx$ computes the eigenvalues and as many eigenvectors as possible.

Greater performance can be achieved if adequate workspace is provided. In some situations, performance can decrease as the provided workspace increases above the workspace amount shown below:

$lwork \geq \max(lwork, 5*n + nsytrd_lwopt, nsygst_lwopt),$ where

$lwork$, as defined previously, depends upon the number of eigenvectors requested, and

$nsytrd_lwopt = n + 2*(anb+1)*(4*nps+2) + (nps+3)*nps$

$nsygst_lwopt = 2*np0*nb + nq0*nb + nb*nb$

$anb = \text{pjlaenv}(\text{desca}[\text{ctxt}_ - 1], 3, p?\text{sytrd} \text{ ', 'L', 0, 0, 0, 0})$

$sqnpc = \text{int}(\text{sqrt}(\text{dble}(\text{NPROW} * \text{NPCOL})))$

$nps = \max(\text{numroc}(n, 1, 0, 0, sqnpc), 2*anb)$

$\text{NB} = \text{desca}[\text{mb}_ - 1]$

$np0 = \text{numroc}(n, nb, 0, 0, \text{NPROW})$

$nq0 = \text{numroc}(n, nb, 0, 0, \text{NPCOL})$

numroc is a ScaLAPACK tool functions;

pjlaenv is a ScaLAPACK environmental inquiry function

$\text{MYROW}, \text{MYCOL}, \text{NPROW}$ and NPCOL can be determined by calling the function blacs_gridinfo .

For large n , no extra workspace is needed, however the biggest boost in performance comes for small n , so it is wise to provide the extra workspace (typically less than a Megabyte per process).

If $clustersize \geq n/\text{sqrt}(\text{NPROW}*\text{NPCOL})$, then providing enough space to compute all the eigenvectors orthogonally will cause serious degradation in performance. At the limit (that is, $clustersize = n-1$) $p?\text{stein}$ will perform no better than $?\text{stein}$ on a single processor.

For $clustersize = n/\text{sqrt}(\text{NPROW}*\text{NPCOL})$ reorthogonalizing all eigenvectors will increase the total execution time by a factor of 2 or more.

For $clustersize > n/\sqrt{NPROW * NPCOL}$ execution time will grow as the square of the cluster size, all other factors remaining equal and assuming enough workspace. Less workspace means less reorthogonalization but faster execution.

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by `pxerbla`.

iwork (local) Workspace array.

liwork (local) , size of *iwork*.

$liwork \geq 6 * nnp$

Where:

$nnp = \max(n, NPROW * NPCOL + 1, 4)$

If $liwork = -1$, then $liwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

a On exit,
If $jobz = 'V'$, and if $info = 0$, sub(A) contains the distributed matrix Z of eigenvectors. The eigenvectors are normalized as follows:

for $ibtype = 1$ or 2 , $Z^T * \text{sub}(B) * Z = I$;

for $ibtype = 3$, $Z^T * \text{inv}(\text{sub}(B)) * Z = I$.

If $jobz = 'N'$, then on exit the upper triangle (if $uplo = 'U'$) or the lower triangle (if $uplo = 'L'$) of sub(A), including the diagonal, is destroyed.

b On exit, if $info \leq n$, the part of sub(B) containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization $\text{sub}(B) = U^T * U$ or $\text{sub}(B) = L * L^T$.

m (global) The total number of eigenvalues found, $0 \leq m \leq n$.

nz (global)

Total number of eigenvectors computed. $0 \leq nz \leq m$. The number of columns of z that are filled.

If $jobz \neq 'V'$, *nz* is not referenced.

If $jobz = 'V'$, $nz = m$ unless the user supplies insufficient space and `p?sygvx` is not able to detect this before beginning computation. To get all the eigenvectors requested, the user must supply both sufficient space to hold the eigenvectors in z ($m \leq \text{descz}(n_)$) and sufficient workspace to compute them. (See *lwork* below.) `p?sygvx` is always able to detect insufficient space without computation unless $range = 'V'$.

w (global)

Array of size n . On normal exit, the first m entries contain the selected eigenvalues in ascending order.

z

(local).

global size $n*n$, local size $lld_z*LOCc(jz+n-1)$.

If $jobz = 'V'$, then on normal exit the first m columns of z contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in *ifail*.

If $jobz = 'N'$, then z is not referenced.

work

If $jobz='N'$ $work[0]$ = optimal amount of workspace required to compute eigenvalues efficiently

If $jobz = 'V'$ $work[0]$ = optimal amount of workspace required to compute eigenvalues and eigenvectors efficiently with no guarantee on orthogonality.

If $range='V'$, it is assumed that all eigenvectors may be required.

ifail

(global)

Array of size n .

ifail provides additional information when $info \neq 0$

If $(mod(info/16,2) \neq 0)$ then $ifail[0]$ indicates the order of the smallest minor which is not positive definite. If $(mod(info,2) \neq 0)$ on exit, then *ifail* contains the indices of the eigenvectors that failed to converge.

If neither of the above error conditions hold and $jobz = 'V'$, then the first m elements of *ifail* are set to zero.

iclustr

(global)

Array of size $(2*NPROW*NPCOL)$. This array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace (see *lwork*, *orfac* and *info*). Eigenvectors corresponding to clusters of eigenvalues indexed $iclustr[2*i - 2]$ to $iclustr[2*i - 1]$, could not be reorthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal. *iclustr* is a zero terminated array.

$(iclustr[2*k - 1] \neq 0$.and. $iclustr[2*k]=0)$ if and only if k is the number of clusters *iclustr* is not referenced if $jobz = 'N'$.

gap

(global)

Array of size $NPROW*NPCOL$. This array contains the gap between eigenvalues whose eigenvectors could not be reorthogonalized. The output values in this array correspond to the clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the i -th cluster may be as high as $(C*n)/gap[i - 1]$, where C is a small constant.

info

(global)

If $info = 0$, the execution is successful.

If $info < 0$: the i -th argument is an array and the j -entry had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

If $info > 0$:

If $(\text{mod}(info, 2) \neq 0)$, then one or more eigenvectors failed to converge. Their indices are stored in *ifail*.

If $(\text{mod}(info, 2, 2) \neq 0)$, then eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in the array *iclustr*.

If $(\text{mod}(info/4, 2) \neq 0)$, then space limit prevented $p?sygvx$ from computing all of the eigenvectors between vl and vu . The number of eigenvectors computed is returned in *nz*.

If $(\text{mod}(info/8, 2) \neq 0)$, then $p?stebz$ failed to compute eigenvalues.

If $(\text{mod}(info/16, 2) \neq 0)$, then B was not positive definite. *ifail(1)* indicates the order of the smallest minor which is not positive definite.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?hegvx

Computes selected eigenvalues and, optionally, eigenvectors of a complex generalized Hermitian positive-definite eigenproblem.

Syntax

```
void pchegvx (MKL_INT *ibtype , char *jobz , char *range , char *uplo , MKL_INT *n ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , float *vl , float *vu , MKL_INT *il ,
MKL_INT *iu , float *abstol , MKL_INT *m , MKL_INT *nz , float *w , float *orfac ,
MKL_Complex8 *z , MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , MKL_Complex8 *work ,
MKL_INT *lwork , float *rwork , MKL_INT *lrwork , MKL_INT *iwork , MKL_INT *liwork ,
MKL_INT *ifail , MKL_INT *iclustr , float *gap , MKL_INT *info );
```

```
void pzhegvx (MKL_INT *ibtype , char *jobz , char *range , char *uplo , MKL_INT *n ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , double *vl , double *vu , MKL_INT *il ,
MKL_INT *iu , double *abstol , MKL_INT *m , MKL_INT *nz , double *w , double *orfac ,
MKL_Complex16 *z , MKL_INT *iz , MKL_INT *jz , MKL_INT *descz , MKL_Complex16 *work ,
MKL_INT *lwork , double *rwork , MKL_INT *lrwork , MKL_INT *iwork , MKL_INT *liwork ,
MKL_INT *ifail , MKL_INT *iclustr , double *gap , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The $p?hegvx$ function computes all the eigenvalues, and optionally, the eigenvectors of a complex generalized Hermitian positive-definite eigenproblem, of the form

$$\text{sub}(A) * x = \lambda * \text{sub}(B) * x, \quad \text{sub}(A) * \text{sub}(B) * x = \lambda * x, \quad \text{or} \quad \text{sub}(B) * \text{sub}(A) * x = \lambda * x.$$

Here $\text{sub}(A)$ denoting $A(ia:ia+n-1, ja:ja+n-1)$ and $\text{sub}(B)$ are assumed to be Hermitian and $\text{sub}(B)$ denoting $B(ib:ib+n-1, jb:jb+n-1)$ is also positive definite.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|---------------|--|
| <i>ibtype</i> | (global) Must be 1 or 2 or 3. Specifies the problem type to be solved: If <i>ibtype</i> = 1, the problem type is $\text{sub}(A) * x = \text{lambda} * \text{sub}(B) * x;$ If <i>ibtype</i> = 2, the problem type is $\text{sub}(A) * \text{sub}(B) * x = \text{lambda} * x;$ If <i>ibtype</i> = 3, the problem type is $\text{sub}(B) * \text{sub}(A) * x = \text{lambda} * x.$ |
| <i>jobz</i> | (global) Must be 'N' or 'V'. If <i>jobz</i> = 'N', then compute eigenvalues only. If <i>jobz</i> = 'V', then compute eigenvalues and eigenvectors. |
| <i>range</i> | (global) Must be 'A' or 'V' or 'I'. If <i>range</i> = 'A', the function computes all eigenvalues. If <i>range</i> = 'V', the function computes eigenvalues in the interval: [<i>v1</i> , <i>vu</i>] If <i>range</i> = 'I', the function computes eigenvalues with indices <i>il</i> through <i>iu</i> . |
| <i>uplo</i> | (global) Must be 'U' or 'L'. If <i>uplo</i> = 'U', arrays <i>a</i> and <i>b</i> store the upper triangles of $\text{sub}(A)$ and $\text{sub}(B)$; If <i>uplo</i> = 'L', arrays <i>a</i> and <i>b</i> store the lower triangles of $\text{sub}(A)$ and $\text{sub}(B)$. |
| <i>n</i> | (global) The order of the matrices $\text{sub}(A)$ and $\text{sub}(B)$ ($n \geq 0$). |
| <i>a</i> | (local) |

Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the n -by- n Hermitian distributed matrix $sub(A)$. If $uplo = 'U'$, the leading n -by- n upper triangular part of $sub(A)$ contains the upper triangular part of the matrix. If $uplo = 'L'$, the leading n -by- n lower triangular part of $sub(A)$ contains the lower triangular part of the matrix.

ia, ja

(global)

The row and column indices in the global matrix A indicating the first row and the first column of the submatrix A , respectively.

desca(global and local) array of size $dlen_$.

The array descriptor for the distributed matrix A . If $desca[ctxt_ - 1]$ is incorrect, `p?hegvx` cannot guarantee correct error reporting.

b

(local).

Pointer into the local memory to an array of size $lld_b * LOCC(jb+n-1)$. On entry, this array contains the local pieces of the n -by- n Hermitian distributed matrix $sub(B)$.

If $uplo = 'U'$, the leading n -by- n upper triangular part of $sub(B)$ contains the upper triangular part of the matrix.

If $uplo = 'L'$, the leading n -by- n lower triangular part of $sub(B)$ contains the lower triangular part of the matrix.

ib, jb

(global)

The row and column indices in the global matrix B indicating the first row and the first column of the submatrix B , respectively.

descb(global and local) array of size $dlen_$.

The array descriptor for the distributed matrix B . $descb[ctxt_ - 1]$ must be equal to $desca[ctxt_ - 1]$.

vl, vu

(global)

If $range = 'V'$, the lower and upper bounds of the interval to be searched for eigenvalues.

If $range = 'A'$ or $'I'$, vl and vu are not referenced.

il, iu

(global)

If $range = 'I'$, the indices in ascending order of the smallest and largest eigenvalues to be returned. Constraint: $il \geq 1$, $\min(il, n) \leq iu \leq n$

If $range = 'A'$ or $'V'$, il and iu are not referenced.

abstol

(global)

If $jobz = 'V'$, setting $abstol$ to `p?lamch(context, 'U')` yields the most orthogonal eigenvectors.

The absolute error tolerance for the eigenvalues. An approximate eigenvalue is accepted as converged when it is determined to lie in an interval $[a, b]$ of width less than or equal to

$abstol + eps * \max(|a|, |b|)$,

where eps is the machine precision. If $abstol$ is less than or equal to zero, then $eps * \text{norm}(T)$ will be used in its place, where $\text{norm}(T)$ is the 1-norm of the tridiagonal matrix obtained by reducing A to tridiagonal form.

Eigenvalues will be computed most accurately when $abstol$ is set to twice the underflow threshold $2 * p?lamch('S')$ not zero. If this function returns with $((\text{mod}(\text{info}, 2) \neq 0) .or. * (\text{mod}(\text{info}/8, 2) \neq 0))$, indicating that some eigenvalues or eigenvectors did not converge, try setting $abstol$ to $2 * p?lamch('S')$.

NOTE

$\text{mod}(x, y)$ is the integer remainder of x/y .

orfac

(global).

Specifies which eigenvectors should be reorthogonalized. Eigenvectors that correspond to eigenvalues which are within $tol = orfac * \text{norm}(A)$ of each other are to be reorthogonalized. However, if the workspace is insufficient (see *lwork*), tol may be decreased until all eigenvectors to be reorthogonalized can be stored in one process. No reorthogonalization will be done if *orfac* equals zero. A default value of 1.0E-3 is used if *orfac* is negative. *orfac* should be identical on all processes.

iz, jz

(global) The row and column indices in the global matrix Z indicating the first row and the first column of the submatrix Z , respectively.

descz

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix Z . $descz[ctxt_ - 1]$ must equal $desca[ctxt_ - 1]$.

work

(local)

Workspace array of size *lwork*

lwork

(local).

The size of the array *work*.

If only eigenvalues are requested:

$$lwork \geq n + \max(NB * (np0 + 1), 3)$$

If eigenvectors are requested:

$$lwork \geq n + (np0 + mq0 + NB) * NB$$

with $nq0 = \text{numroc}(nn, NB, 0, 0, NPCOL)$.

For optimal performance, greater workspace is needed, that is

$$lwork \geq \max(lwork, n, nhetrd_lwopt, nhegst_lwopt)$$

where *lwork* is as defined above, and

$$nhetrd_lwork = 2 * (anb + 1) * (4 * nps + 2) + (nps + 1) * nps;$$

$$nhegst_lwopt = 2 * np0 * nb + nq0 * nb + nb * nb$$

$$nb = desca[mb_ - 1]$$

$$np0 = \text{numroc}(n, nb, 0, 0, NPROW)$$

```

ng0 = numroc(n, nb, 0, 0, NPCOL)
ictxt = desca[ctxt_ - 1]
anb = pjlenv(ictxt, 3, 'p?hettrd', 'L', 0, 0, 0, 0)
sqnpc = sqrt(dble(NPROW * NPCOL))
nps = max(numroc(n, 1, 0, 0, sqnpc), 2*anb)

```

numroc is a ScaLAPACK tool functions;

pjlenv is a ScaLAPACK environmental inquiry function MYROW, MYCOL, NPROW and NPCOL can be determined by calling the function blacs_gridinfo.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by p_xerbla.

rwork

(local)

Workspace array of size *lwork*.

lwork

(local) The size of the array *rwork*.

See below for definitions of variables used to define *lwork*.

If no eigenvectors are requested (*jobz* = 'N'), then $lwork \geq 5*nn+4*n$

If eigenvectors are requested (*jobz* = 'V'), then the amount of workspace required to guarantee that all eigenvectors are computed is:

$$lwork \geq 4*n + \max(5*nn, np0*mq0) + \text{iceil}(neig, NPROW*NPCOL)*nn$$

The computed eigenvectors may not be orthogonal if the minimal workspace is supplied and *orfac* is too small. If you want to guarantee orthogonality (at the cost of potentially poor performance) you should add the following value to *lwork*:

$$(clustersize-1)*n,$$

where *clustersize* is the number of eigenvalues in the largest cluster, where a cluster is defined as a set of close eigenvalues:

$$\{w[k-1], \dots, w[k+clustersize-2] \mid |w[j]-w[j-1]| \leq orfac*2*norm(A)\}$$

Variable definitions:

neig = number of eigenvectors requested;

$$nb = desca[mb_ - 1] = desca[nb_ - 1] = descz[mb_ - 1] = descz[nb_ - 1];$$

$$nn = \max(n, nb, 2);$$

$$desca[rsrc_ - 1] = desca[nb_ - 1] = descz[rsrc_ - 1] = descz[csrc_ - 1] = 0;$$

$$np0 = \text{numroc}(nn, nb, 0, 0, NPROW);$$

$$mq0 = \text{numroc}(\max(neig, nb, 2), nb, 0, 0, NPCOL);$$

iceil(*x*, *y*) is a ScaLAPACK function returning ceiling(*x*/*y*).

When *lwork* is too small:

If *lwork* is too small to guarantee orthogonality, `p?hegvx` attempts to maintain orthogonality in the clusters with the smallest spacing between the eigenvalues.

If *lwork* is too small to compute all the eigenvectors requested, no computation is performed and *info*= -25 is returned. Note that when *range*='V', `p?hegvx` does not know how many eigenvectors are requested until the eigenvalues are computed. Therefore, when *range*='V' and as long as *lwork* is large enough to allow `p?hegvx` to compute the eigenvalues, `p?hegvx` will compute the eigenvalues and as many eigenvectors as it can.

Relationship between workspace, orthogonality & performance:

If $clustersize > n/\sqrt{NPROW*NPCOL}$, then providing enough space to compute all the eigenvectors orthogonally will cause serious degradation in performance. In the limit (that is, $clustersize = n-1$) `p?stein` will perform no better than `?stein` on 1 processor.

For $clustersize = n/\sqrt{NPROW*NPCOL}$ reorthogonalizing all eigenvectors will increase the total execution time by a factor of 2 or more.

For $clustersize > n/\sqrt{NPROW*NPCOL}$ execution time will grow as the square of the cluster size, all other factors remaining equal and assuming enough workspace. Less workspace means less reorthogonalization but faster execution.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the size required for optimal performance for all work arrays. Each of these values is returned in the first entry of the corresponding work arrays, and no error message is issued by `pxerbla`.

iwork (local) Workspace array.

liwork (local) , size of *iwork*.

$liwork \geq 6 * nnp$

Where: $nnp = \max(n, NPROW*NPCOL + 1, 4)$

If *liwork* = -1, then *liwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

a On exit, if *jobz* = 'V', then if *info* = 0, sub(A) contains the distributed matrix *Z* of eigenvectors.

The eigenvectors are normalized as follows:

If *ibtype* = 1 or 2, then $Z^H * \text{sub}(B) * Z = i$;

If *ibtype* = 3, then $Z^H * \text{inv}(\text{sub}(B)) * Z = i$.

If *jobz* = 'N', then on exit the upper triangle (if *uplo*='U') or the lower triangle (if *uplo*='L') of sub(A), including the diagonal, is destroyed.

| | |
|--------------|--|
| <i>b</i> | On exit, if $info \leq n$, the part of $sub(B)$ containing the matrix is overwritten by the triangular factor U or L from the Cholesky factorization $sub(B) = U^H * U$, or $sub(B) = L * L^H$. |
| <i>m</i> | (global) The total number of eigenvalues found, $0 \leq m \leq n$. |
| <i>nz</i> | (global) Total number of eigenvectors computed. $0 < nz < m$. The number of columns of z that are filled. If $jobz \neq 'V'$, nz is not referenced. If $jobz = 'V'$, $nz = m$ unless the user supplies insufficient space and <code>p?hegvx</code> is not able to detect this before beginning computation. To get all the eigenvectors requested, the user must supply both sufficient space to hold the eigenvectors in z ($m \leq descz[n_ - 1]$) and sufficient workspace to compute them. (See <i>lwork</i> below.) The function <code>p?hegvx</code> is always able to detect insufficient space without computation unless $range = 'V'$. |
| <i>w</i> | (global) Array of size n . On normal exit, the first m entries contain the selected eigenvalues in ascending order. |
| <i>z</i> | (local). global size $n * n$, local size $lld_z * LOcc(jz+n-1)$. If $jobz = 'V'$, then on normal exit the first m columns of z contain the orthonormal eigenvectors of the matrix corresponding to the selected eigenvalues. If an eigenvector fails to converge, then that column of z contains the latest approximation to the eigenvector, and the index of the eigenvector is returned in <i>ifail</i> . If $jobz = 'N'$, then z is not referenced. |
| <i>work</i> | On exit, $work[0]$ returns the optimal amount of workspace. |
| <i>rwork</i> | On exit, $rwork[0]$ contains the amount of workspace required for optimal efficiency If $jobz = 'N'$ $rwork[0] =$ optimal amount of workspace required to compute eigenvalues efficiently If $jobz = 'V'$ $rwork[0] =$ optimal amount of workspace required to compute eigenvalues and eigenvectors efficiently with no guarantee on orthogonality. If $range = 'V'$, it is assumed that all eigenvectors may be required when computing optimal workspace. |
| <i>ifail</i> | (global) Array of size n . <i>ifail</i> provides additional information when $info \neq 0$ If $(mod(info/16,2) \neq 0)$, then $ifail[0]$ indicates the order of the smallest minor which is not positive definite. If $(mod(info,2) \neq 0)$ on exit, then $ifail[0]$ contains the indices of the eigenvectors that failed to converge. |

If neither of the above error conditions are held, and $jobz = 'V'$, then the first m elements of $ifail$ are set to zero.

iclustr

(global)

Array of size $(2*NPROW*NPCOL)$. This array contains indices of eigenvectors corresponding to a cluster of eigenvalues that could not be reorthogonalized due to insufficient workspace (see *lwork*, *orfac* and *info*). Eigenvectors corresponding to clusters of eigenvalues indexed $iclustr(2*i-1)$ to $iclustr(2*i)$, could not be reorthogonalized due to lack of workspace. Hence the eigenvectors corresponding to these clusters may not be orthogonal.

iclustr() is a zero terminated array. ($iclustr(2*k) \neq 0$ and $iclustr(2*k+1)=0$) if and only if k is the number of clusters.

iclustr is not referenced if $jobz = 'N'$.

gap

(global)

Array of size $NPROW*NPCOL$.

This array contains the gap between eigenvalues whose eigenvectors could not be reorthogonalized. The output values in this array correspond to the clusters indicated by the array *iclustr*. As a result, the dot product between eigenvectors corresponding to the i -th cluster may be as high as $(C*n)/gap(i)$, where C is a small constant.

info

(global)

If $info = 0$, the execution is successful.

If $info < 0$: the i -th argument is an array and the j -entry had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

If $info > 0$:

If $(\text{mod}(info, 2) \neq 0)$, then one or more eigenvectors failed to converge. Their indices are stored in *ifail*.

If $(\text{mod}(info, 2, 2) \neq 0)$, then eigenvectors corresponding to one or more clusters of eigenvalues could not be reorthogonalized because of insufficient workspace. The indices of the clusters are stored in the array *iclustr*.

If $(\text{mod}(info/4, 2) \neq 0)$, then space limit prevented `p?sygvx` from computing all of the eigenvectors between vl and vu . The number of eigenvectors computed is returned in *nz*.

If $(\text{mod}(info/8, 2) \neq 0)$, then `p?stebz` failed to compute eigenvalues.

If $(\text{mod}(info/16, 2) \neq 0)$, then B was not positive definite. *ifail(1)* indicates the order of the smallest minor which is not positive definite.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

ScaLAPACK Auxiliary Routines

ScaLAPACK Auxiliary Routines

| Routine Name | Data Types | Description |
|-------------------------|-------------------------|---|
| <code>b?laapp</code> | <code>s, d</code> | Multiplies a matrix with an orthogonal matrix. |
| <code>b?laexc</code> | <code>s, d</code> | Swaps adjacent diagonal blocks of a real upper quasi-triangular matrix in Schur canonical form, by an orthogonal similarity transformation. |
| <code>b?trexc</code> | <code>s, d</code> | Reorders the Schur factorization of a general matrix. |
| <code>p?lacgv</code> | <code>c, z</code> | Conjugates a complex vector. |
| <code>p?max1</code> | <code>c, z</code> | Finds the index of the element whose real part has maximum absolute value (similar to the Level 1 PBLAS <code>p?amax</code> , but using the absolute value to the real part). |
| <code>pmpcol</code> | <code>s, d</code> | <i>Finds the collaborators of a process.</i> |
| <code>pmpim2</code> | <code>s, d</code> | Computes the eigenpair range assignments for all processes. |
| <code>?combamax1</code> | <code>c, z</code> | Finds the element with maximum real part absolute value and its corresponding global index. |
| <code>p?sum1</code> | <code>sc, dz</code> | Forms the 1-norm of a complex vector similar to Level 1 PBLAS <code>p?asum</code> , but using the true absolute value. |
| <code>p?dbtrsv</code> | <code>s, d, c, z</code> | Computes an <i>LU</i> factorization of a general tridiagonal matrix with no pivoting. The routine is called by <code>p?dbtrs</code> . |
| <code>p?dttrsv</code> | <code>s, d, c, z</code> | Computes an <i>LU</i> factorization of a general band matrix, using partial pivoting with row interchanges. The routine is called by <code>p?dttrs</code> . |
| <code>p?gebal</code> | <code>s, d</code> | Balances a general real matrix. |
| <code>p?gebd2</code> | <code>s, d, c, z</code> | Reduces a general rectangular matrix to real bidiagonal form by an orthogonal/unitary transformation (unblocked algorithm). |
| <code>p?gehd2</code> | <code>s, d, c, z</code> | Reduces a general matrix to upper Hessenberg form by an orthogonal/unitary similarity transformation (unblocked algorithm). |
| <code>p?gelq2</code> | <code>s, d, c, z</code> | Computes an <i>LQ</i> factorization of a general rectangular matrix (unblocked algorithm). |
| <code>p?geql2</code> | <code>s, d, c, z</code> | Computes a <i>QL</i> factorization of a general rectangular matrix (unblocked algorithm). |
| <code>p?geqr2</code> | <code>s, d, c, z</code> | Computes a <i>QR</i> factorization of a general rectangular matrix (unblocked algorithm). |
| <code>p?gerq2</code> | <code>s, d, c, z</code> | Computes an <i>RQ</i> factorization of a general rectangular matrix (unblocked algorithm). |
| <code>p?getf2</code> | <code>s, d, c, z</code> | Computes an <i>LU</i> factorization of a general matrix, using partial pivoting with row interchanges (local blocked algorithm). |
| <code>p?labrd</code> | <code>s, d, c, z</code> | Reduces the first <i>nb</i> rows and columns of a general rectangular matrix <i>A</i> to real bidiagonal form by an orthogonal/unitary transformation, and returns auxiliary matrices that are needed to apply the transformation to the unreduced part of <i>A</i> . |

| Routine Name | Data Types | Description |
|-------------------------------|------------------------------|--|
| <code>p?lacon</code> | <code>s, d, c, z</code> | Estimates the 1-norm of a square matrix, using the reverse communication for evaluating matrix-vector products. |
| <code>p?laconsb</code> | <code>s, d</code> | Looks for two consecutive small subdiagonal elements. |
| <code>p?lacr2</code> | <code>s, d, c, z</code> | Copies all or part of a distributed matrix to another distributed matrix. |
| <code>p?lacr3</code> | <code>s, d</code> | Copies from a global parallel array into a local replicated array or vice versa. |
| <code>p?lacpy</code> | <code>s, d, c, z</code> | Copies all or part of one two-dimensional array to another. |
| <code>p?laevswp</code> | <code>s, d, c, z</code> | Moves the eigenvectors from where they are computed to ScaLAPACK standard block cyclic array. |
| <code>p?lahrd</code> | <code>s, d, c, z</code> | Reduces the first nb columns of a general rectangular matrix A so that elements below the k^{th} subdiagonal are zero, by an orthogonal/unitary transformation, and returns auxiliary matrices that are needed to apply the transformation to the unreduced part of A. |
| <code>p?laiect</code> | <code>s, d, c, z</code> | Exploits IEEE arithmetic to accelerate the computations of eigenvalues. |
| <code>p?lamve</code> | <code>s, d</code> | Copies all or part of one two-dimensional distributed array to another. |
| <code>p?lange</code> | <code>s, d, c, z</code> | Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a general rectangular matrix. |
| <code>p?lanhs</code> | <code>s, d, c, z</code> | Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of an upper Hessenberg matrix. |
| <code>p?lansy, p?lanhe</code> | <code>s, d, c, z/c, z</code> | Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element of a real symmetric or complex Hermitian matrix. |
| <code>p?lantr</code> | <code>s, d, c, z</code> | Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a triangular matrix. |
| <code>p?lapiv</code> | <code>s, d, c, z</code> | Applies a permutation matrix to a general distributed matrix, resulting in row or column pivoting. |
| <code>p?laqge</code> | <code>s, d, c, z</code> | Scales a general rectangular matrix, using row and column scaling factors computed by <code>p?geequ</code> . |
| <code>p?laqr0</code> | <code>s, d</code> | Computes the eigenvalues of a Hessenberg matrix and optionally returns the matrices from the Schur decomposition. |
| <code>p?laqr1</code> | <code>s, d</code> | Sets a scalar multiple of the first column of the product of a 2-by-2 or 3-by-3 matrix and specified shifts. |
| <code>p?laqr2</code> | <code>s, d</code> | Performs the orthogonal/unitary similarity transformation of a Hessenberg matrix to detect and deflate fully converged eigenvalues from a trailing principal submatrix (aggressive early deflation). |

| Routine Name | Data Types | Description |
|------------------------|-------------------------|--|
| <code>p?laqr3</code> | <code>s, d</code> | Performs the orthogonal/unitary similarity transformation of a Hessenberg matrix to detect and deflate fully converged eigenvalues from a trailing principal submatrix (aggressive early deflation). |
| <code>p?laqr4</code> | <code>s, d</code> | Computes the eigenvalues of a Hessenberg matrix, and optionally computes the matrices from the Schur decomposition. |
| <code>p?laqr5</code> | <code>s, d</code> | Performs a single small-bulge multi-shift QR sweep. |
| <code>p?laqsy</code> | <code>s, d, c, z</code> | Scales a symmetric/Hermitian matrix, using scaling factors computed by <code>p?poequ</code> . |
| <code>p?lared1d</code> | <code>s, d</code> | Redistributes an array assuming that the input array <i>bycol</i> is distributed across rows and that all process columns contain the same copy of <i>bycol</i> . |
| <code>p?lared2d</code> | <code>s, d</code> | Redistributes an array assuming that the input array <i>byrow</i> is distributed across columns and that all process rows contain the same copy of <i>byrow</i> . |
| <code>p?larf</code> | <code>s, d, c, z</code> | Applies an elementary reflector to a general rectangular matrix. |
| <code>p?larfb</code> | <code>s, d, c, z</code> | Applies a block reflector or its transpose/conjugate-transpose to a general rectangular matrix. |
| <code>p?larfc</code> | <code>c, z</code> | Applies the conjugate transpose of an elementary reflector to a general matrix. |
| <code>p?larfg</code> | <code>s, d, c, z</code> | Generates an elementary reflector (Householder matrix). |
| <code>p?larft</code> | <code>s, d, c, z</code> | Forms the triangular vector T of a block reflector $H=I-VTV^H$ |
| <code>p?larz</code> | <code>s, d, c, z</code> | Applies an elementary reflector as returned by <code>p?tzzrf</code> to a general matrix. |
| <code>p?larzb</code> | <code>s, d, c, z</code> | Applies a block reflector or its transpose/conjugate-transpose as returned by <code>p?tzzrf</code> to a general matrix. |
| <code>p?larzc</code> | <code>c, z</code> | Applies (multiplies by) the conjugate transpose of an elementary reflector as returned by <code>p?tzzrf</code> to a general matrix. |
| <code>p?larzt</code> | <code>s, d, c, z</code> | Forms the triangular factor T of a block reflector $H=I-VTV^H$ as returned by <code>p?tzzrf</code> . |
| <code>p?lascl</code> | <code>s, d, c, z</code> | Multiplies a general rectangular matrix by a real scalar defined as C_{to}/C_{from} . |
| <code>p?laset</code> | <code>s, d, c, z</code> | Initializes the off-diagonal elements of a matrix to α and the diagonal elements to β . |
| <code>p?lasmsub</code> | <code>s, d</code> | Looks for a small subdiagonal element from the bottom of the matrix that it can safely set to zero. |
| <code>p?lassq</code> | <code>s, d, c, z</code> | Updates a sum of squares represented in scaled form. |
| <code>p?laswp</code> | <code>s, d, c, z</code> | Performs a series of row interchanges on a general rectangular matrix. |
| <code>p?latra</code> | <code>s, d, c, z</code> | Computes the trace of a general square distributed matrix. |

| Routine Name | Data Types | Description |
|---------------------------------|---------------------|--|
| p?latrd | <i>s, d, c, z</i> | Reduces the first <i>nb</i> rows and columns of a symmetric/Hermitian matrix <i>A</i> to real tridiagonal form by an orthogonal/unitary similarity transformation. |
| p?latrz | <i>s, d, c, z</i> | Reduces an upper trapezoidal matrix to upper triangular form by means of orthogonal/unitary transformations. |
| p?lauu2 | <i>s, d, c, z</i> | Computes the product UU^H or $L^H L$, where <i>U</i> and <i>L</i> are upper or lower triangular matrices (local unblocked algorithm). |
| p?lauum | <i>s, d, c, z</i> | Computes the product UU^H or $L^H L$, where <i>U</i> and <i>L</i> are upper or lower triangular matrices. |
| p?lawil | <i>s, d</i> | Forms the Wilkinson transform. |
| p?org2l/p?ung2l | <i>s, d, c, z</i> | Generates all or part of the orthogonal/unitary matrix <i>Q</i> from a <i>QL</i> factorization determined by p?geqlf (unblocked algorithm). |
| p?org2r/p?ung2r | <i>s, d, c, z</i> | Generates all or part of the orthogonal/unitary matrix <i>Q</i> from a <i>QR</i> factorization determined by p?geqrf (unblocked algorithm). |
| p?orgl2/p?ungl2 | <i>s, d, c, z</i> | Generates all or part of the orthogonal/unitary matrix <i>Q</i> from an <i>LQ</i> factorization determined by p?gelqf (unblocked algorithm). |
| p?orgr2/p?ungr2 | <i>s, d, c, z</i> | Generates all or part of the orthogonal/unitary matrix <i>Q</i> from an <i>RQ</i> factorization determined by p?gerqf (unblocked algorithm). |
| p?orm2l/p?unm2l | <i>s, d, c, z</i> | Multiplies a general matrix by the orthogonal/unitary matrix from a <i>QL</i> factorization determined by p?geqlf (unblocked algorithm). |
| p?orm2r/p?unm2r | <i>s, d, c, z</i> | Multiplies a general matrix by the orthogonal/unitary matrix from a <i>QR</i> factorization determined by p?geqrf (unblocked algorithm). |
| p?orml2/p?unml2 | <i>s, d, c, z</i> | Multiplies a general matrix by the orthogonal/unitary matrix from an <i>LQ</i> factorization determined by p?gelqf (unblocked algorithm). |
| p?ormr2/p?unmr2 | <i>s, d, c, z</i> | Multiplies a general matrix by the orthogonal/unitary matrix from an <i>RQ</i> factorization determined by p?gerqf (unblocked algorithm). |
| p?pbtrsv | <i>s, d, c, z</i> | Solves a single triangular linear system via <code>frontsolve</code> or <code>backsolve</code> where the triangular matrix is a factor of a banded matrix computed by p?pbtrf . |
| p?pttrsv | <i>s, d, c, z</i> | Solves a single triangular linear system via <code>frontsolve</code> or <code>backsolve</code> where the triangular matrix is a factor of a tridiagonal matrix computed by p?pttrf . |
| p?potf2 | <i>s, d, c, z</i> | Computes the Cholesky factorization of a symmetric/Hermitian positive definite matrix (local unblocked algorithm). |
| p?rot | <i>s, d</i> | Applies a planar rotation to two distributed vectors. |
| p?rscl | <i>s, d, cs, zd</i> | Multiplies a vector by the reciprocal of a real scalar. |
| p?sygs2/p?hegs2 | <i>s, d, c, z</i> | Reduces a symmetric/Hermitian positive-definite generalized eigenproblem to standard form, using the factorization results obtained from p?potrf (local unblocked algorithm). |
| p?syt2/p?hetd2 | <i>s, d, c, z</i> | Reduces a symmetric/Hermitian matrix to real symmetric tridiagonal form by an orthogonal/unitary similarity transformation (local unblocked algorithm). |

| Routine Name | Data Types | Description |
|-----------------------|-------------------------|--|
| <code>p?trord</code> | <code>s, d</code> | Reorders the Schur factorization of a general matrix. |
| <code>p?trsen</code> | <code>s, d</code> | Reorders the Schur factorization of a matrix and (optionally) computes the reciprocal condition numbers and invariant subspace for the selected cluster of eigenvalues. |
| <code>p?trti2</code> | <code>s, d, c, z</code> | Computes the inverse of a triangular matrix (local unblocked algorithm). |
| <code>?lamsh</code> | <code>s, d</code> | Sends multiple shifts through a small (single node) matrix to maximize the number of bulges that can be sent through. |
| <code>?laqr6</code> | <code>s, d</code> | Performs a single small-bulge multi-shift QR sweep collecting the transformations. |
| <code>?lar1va</code> | <code>s, d</code> | <i>Computes scaled eigenvector corresponding to given eigenvalue.</i> |
| <code>?laref</code> | <code>s, d</code> | Applies Householder reflectors to matrices on either their rows or columns. |
| <code>?larrb2</code> | <code>s, d</code> | Provides limited bisection to locate eigenvalues for more accuracy. |
| <code>?larrd2</code> | <code>s, d</code> | Computes the eigenvalues of a symmetric tridiagonal matrix to suitable accuracy. |
| <code>?larre2</code> | <code>s, d</code> | Given a tridiagonal matrix, sets small off-diagonal elements to zero and for each unreduced block, finds base representations and eigenvalues. |
| <code>?larre2a</code> | <code>s, d</code> | <i>Given a tridiagonal matrix, sets small off-diagonal elements to zero and for each unreduced block, finds base representations and eigenvalues.</i> |
| <code>?larrf2</code> | <code>s, d</code> | Finds a new relatively robust representation such that at least one of the eigenvalues is relatively isolated. |
| <code>?larrv2</code> | <code>s, d</code> | Computes the eigenvectors of the tridiagonal matrix $T = L*D*L^T$ given L, D and the eigenvalues of $L*D*L^T$. |
| <code>?lasorte</code> | <code>s, d</code> | Sorts eigenpairs by real and complex data types. |
| <code>?lasrt2</code> | <code>s, d</code> | Sorts numbers in increasing or decreasing order. |
| <code>?stegr2</code> | <code>s, d</code> | Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix. |
| <code>?stegr2a</code> | <code>s, d</code> | Computes selected eigenvalues and initial representations needed for eigenvector computations. |
| <code>?stegr2b</code> | <code>s, d</code> | From eigenvalues and initial representations computes the selected eigenvalues and eigenvectors of the real symmetric tridiagonal matrix in parallel on multiple processors. |
| <code>?stein2</code> | <code>s, d</code> | Computes the eigenvectors corresponding to specified eigenvalues of a real symmetric tridiagonal matrix, using inverse iteration. |
| <code>?dbtff2</code> | <code>s, d, c, z</code> | Computes an LU factorization of a general band matrix with no pivoting (local unblocked algorithm). |
| <code>?dbtrf</code> | <code>s, d, c, z</code> | Computes an LU factorization of a general band matrix with no pivoting (local blocked algorithm). |

| Routine Name | Data Types | Description |
|-----------------------|-------------------------|---|
| <code>?dttrf</code> | <code>s, d, c, z</code> | Computes an <i>LU</i> factorization of a general tridiagonal matrix with no pivoting (local blocked algorithm). |
| <code>?dttrsv</code> | <code>s, d, c, z</code> | Solves a general tridiagonal system of linear equations using the <i>LU</i> factorization computed by <code>?dttrf</code> . |
| <code>?pttrsv</code> | <code>s, d, c, z</code> | Solves a symmetric (Hermitian) positive-definite tridiagonal system of linear equations, using the <i>LDL^H</i> factorization computed by <code>?pttrf</code> . |
| <code>?steqr2</code> | <code>s, d</code> | Computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit <i>QL</i> or <i>QR</i> method. |
| <code>?trmvt</code> | <code>s, d, c, z</code> | Performs matrix-vector operations. |
| <code>pilaenv</code> | NA | Returns the positive integer value of the logical blocking size. |
| <code>pilaenvx</code> | NA | Called from the ScaLAPACK routines to choose problem-dependent parameters for the local environment. |
| <code>pjlaenv</code> | NA | Called from the ScaLAPACK symmetric and Hermitian tailored eigen-routines to choose problem-dependent parameters for the local environment. |

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Notice revision #20110804

p?lacgv

Conjugates a complex vector.

Syntax

```
void pclacgv (MKL_INT *n , MKL_Complex8 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *incx );
```

```
void pzlacgv (MKL_INT *n , MKL_Complex16 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *incx );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lacgv` function conjugates a complex vector $\text{sub}(X)$ of length n , where $\text{sub}(X)$ denotes $X(ix, jx:jx+n-1)$ if `incx = m_x`, and $X(ix:ix+n-1, jx)$ if `incx = 1`.

Input Parameters

| | |
|--------------|---|
| <i>n</i> | (global) The length of the distributed vector $\text{sub}(X)$. |
| <i>x</i> | (local). Pointer into the local memory to an array of size $\text{lld}_x * \text{LOCc}(n_x)$. On entry the vector to be conjugated $x[i] = X(ix+(j_x-1)*m_x+i*incx)$, $0 \leq i < n$. |
| <i>ix</i> | (global) The row index in the global matrix X indicating the first row of $\text{sub}(X)$. |
| <i>jx</i> | (global) The column index in the global matrix X indicating the first column of $\text{sub}(X)$. |
| <i>descx</i> | (global and local) Array of size $\text{dlen}_x=9$. The array descriptor for the distributed matrix X . |
| <i>incx</i> | (global) The global increment for the elements of X . Only two values of <i>incx</i> are supported in this version, namely 1 and m_x . <i>incx</i> must not be zero. |

Output Parameters

| | |
|----------|--|
| <i>x</i> | (local). On exit, the local pieces of conjugated distributed vector $\text{sub}(X)$. |
|----------|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?max1

Finds the index of the element whose real part has maximum absolute value (similar to the Level 1 PBLAS $p?amax$, but using the absolute value to the real part).

Syntax

```
void pcmax1 (MKL_INT *n , MKL_Complex8 *amax , MKL_INT *indx , MKL_Complex8 *x ,
MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *incx );

void pzmax1 (MKL_INT *n , MKL_Complex16 *amax , MKL_INT *indx , MKL_Complex16 *x ,
MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *incx );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?max1` function computes the global index of the maximum element in absolute value of a distributed vector $\text{sub}(X)$. The global index is returned in *indx* and the value is returned in *amax*, where $\text{sub}(X)$ denotes $X(ix:ix+n-1, jx)$ if $incx = 1$, $X(ix, jx:jx+n-1)$ if $incx = m_x$.

Input Parameters

| | |
|--------------|--|
| <i>n</i> | (global). The number of components of the distributed vector $\text{sub}(X)$. $n \geq 0$. |
| <i>x</i> | (local) Pointer into the local memory to an array of size $\text{lld}_x * \text{LOCc}(j_x+n-1)$. On entry this array contains the local pieces of the distributed vector $\text{sub}(X)$. |
| <i>ix</i> | (global) The row index in the global matrix X indicating the first row of $\text{sub}(X)$. |
| <i>jx</i> | (global) The column index in the global matrix X indicating the first column of $\text{sub}(X)$. |
| <i>descx</i> | (global and local) Array of size $dlen_$. The array descriptor for the distributed matrix X . |
| <i>incx</i> | (global).The global increment for the elements of X . Only two values of <i>incx</i> are supported in this version, namely 1 and m_x . <i>incx</i> must not be zero. |

Output Parameters

| | |
|-------------|--|
| <i>amax</i> | (global output).The absolute value of the largest entry of the distributed vector $\text{sub}(X)$ only in the scope of $\text{sub}(X)$. |
| <i>indx</i> | (global output).The global index of the element of the distributed vector $\text{sub}(X)$ whose real part has maximum absolute value. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

pilaver

Returns the ScaLAPACK version.

Syntax

```
void pilaver (MKL_INT* vers_major, MKL_INT* vers_minor, MKL_INT* vers_patch);
```

Include Files

- `mkl_scalapack.h`

Description

This function returns the ScaLAPACK version.

Output Parameters

| | |
|-------------------|--|
| <i>vers_major</i> | Return the ScaLAPACK major version. |
| <i>vers_minor</i> | Return the ScaLAPACK minor version from the major version. |
| <i>vers_patch</i> | Return the ScaLAPACK patch version from the minor version. |

pmpcol

Finds the collaborators of a process.

Syntax

```
void pmpcol(MKL_INT* myproc, MKL_INT* nprocs, MKL_INT* iil, MKL_INT* needil, MKL_INT*
neediu, MKL_INT* pmyils, MKL_INT* pmyius, MKL_INT* colbrt, MKL_INT* frstcl, MKL_INT*
lastcl);
```

Include Files

- mkl_scalapack.h

Description

Using the output from [pmpim2](#) and given the information on eigenvalue clusters, `pmpcol` finds the collaborators of `myproc`.

Optimization Notice

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Input Parameters

| | |
|---------------------|--|
| <code>myproc</code> | The processor number, $0 \leq myproc < nprocs$. |
| <code>nprocs</code> | The total number of processors available. |
| <code>iil</code> | The index of the leftmost eigenvalue in the eigenvalue cluster. |
| <code>needil</code> | The leftmost position in the eigenvalue cluster needed by <code>myproc</code> . |
| <code>neediu</code> | The rightmost position in the eigenvalue cluster needed by <code>myproc</code> . |
| <code>pmyils</code> | array For each processor p , $0 < p \leq nprocs$, <code>pmyils[p-1]</code> is the index of the first eigenvalue in the eigenvalue cluster to be computed. <code>pmyils[p-1]</code> equals zero if p stays idle. |
| <code>pmyius</code> | array For each processor p , <code>pmyius[p-1]</code> is the index of the last eigenvalue in the eigenvalue cluster to be computed. <code>pmyius[p-1]</code> equals zero if p stays idle. |

OUTPUT Parameters

| | |
|-----------------------|---|
| <i>colbrt</i> | Non-zero if <i>myproc</i> collaborates. |
| <i>frstcl, lastcl</i> | First and last collaborator of <i>myproc</i> . <i>myproc</i> collaborates with: <i>frstcl, ..., myproc-1, myproc+1, ...,lastcl</i> If <i>myproc</i> = <i>frstcl</i> , there are no collaborators on the left. If <i>myproc</i> = <i>lastcl</i> , there are no collaborators on the right. If <i>frstcl</i> = 0 and <i>lastcl</i> = <i>nprocs-1</i> , then <i>myproc</i> collaborates with everybody |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

pmpim2

Computes the eigenpair range assignments for all processes.

Syntax

```
void pmpim2(MKL_INT* il, MKL_INT* iu, MKL_INT* nprocs, MKL_INT* pmyils, MKL_INT* pmyius);
```

Include Files

- `mkl_scalapack.h`

Description

`pmpim2` is the scheduling function. It computes for all processors the eigenpair range assignments.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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Input Parameters

| | |
|---------------|---|
| <i>il, iu</i> | The range of eigenpairs to be computed. |
| <i>nprocs</i> | The total number of processors available. |

Output Parameters

| | |
|---------------|-------|
| <i>pmyils</i> | array |
|---------------|-------|

For each processor p , $pmyils[p-1]$ is the index of the first eigenvalue in a cluster to be computed.

$pmyils[p-1]$ equals zero if p stays idle.

$pmyius$

array

For each processor p , $pmyius[p-1]$ is the index of the last eigenvalue in a cluster to be computed.

$pmyius[p-1]$ equals zero if p stays idle.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?combamax1

Finds the element with maximum real part absolute value and its corresponding global index.

Syntax

```
void ccombamax1 (MKL_Complex8 *v1 , MKL_Complex8 *v2 );
void zcombamax1 (MKL_Complex16 *v1 , MKL_Complex16 *v2 );
```

Include Files

- `mk1_scalapack.h`

Description

The `?combamax1` function finds the element having maximum real part absolute value as well as its corresponding global index.

Input Parameters

| | |
|------|---|
| $v1$ | (local) Array of size 2. The first maximum absolute value element and its global index. $v1[0]=amax$, $v1[1]=indx$. |
| $v2$ | (local) Array of size 2. The second maximum absolute value element and its global index. $v2[0]=amax$, $v2[1]=indx$. |

Output Parameters

| | |
|------|---|
| $v1$ | (local). The first maximum absolute value element and its global index. $v1[0]=amax$, $v1[1]=indx$. |
|------|---|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?sum1

Forms the 1-norm of a complex vector similar to Level 1 PBLAS `p?asum`, but using the true absolute value.

Syntax

```
void pscsum1 (MKL_INT *n , float *asum , MKL_Complex8 *x , MKL_INT *ix , MKL_INT *jx ,
MKL_INT *descx , MKL_INT *incx );
```

```
void pdzsum1 (MKL_INT *n , double *asum , MKL_Complex16 *x , MKL_INT *ix , MKL_INT
*jx , MKL_INT *descx , MKL_INT *incx );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?sum1` function returns the sum of absolute values of a complex distributed vector `sub(x)` in `asum`, where `sub(x)` denotes $X(ix:ix+n-1, jx:jx)$, if `incx = 1`, $X(ix:ix, jx:jx+n-1)$, if `incx = m_x`.

Based on `p?asum` from the Level 1 PBLAS. The change is to use the 'genuine' absolute value.

Input Parameters

| | |
|--------------------|--|
| <code>n</code> | (global). The number of components of the distributed vector <code>sub(x)</code> . $n \geq 0$. |
| <code>x</code> | (local) Pointer into the local memory to an array of size <code>lld_x * LOCc(jx+n-1)</code> . This array contains the local pieces of the distributed vector <code>sub(x)</code> . |
| <code>ix</code> | (global) The row index in the global matrix X indicating the first row of <code>sub(X)</code> . |
| <code>jx</code> | (global) The column index in the global matrix X indicating the first column of <code>sub(X)</code> . |
| <code>descx</code> | (local) Array of size <code>dlen_ = 9</code> . The array descriptor for the distributed matrix X . |
| <code>incx</code> | (global) The global increment for the elements of X . Only two values of <code>incx</code> are supported in this version, namely 1 and <code>m_x</code> . |

Output Parameters

| | |
|-------------------|--|
| <code>asum</code> | (local) The sum of absolute values of the distributed vector <code>sub(X)</code> only in its scope. |
|-------------------|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?dbtrsv

Computes an LU factorization of a general triangular matrix with no pivoting. The function is called by p?dbtrs.

Syntax

```
void psdbtrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu ,
MKL_INT *nrhs , float *a , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib ,
MKL_INT *descb , float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pddbtrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu ,
MKL_INT *nrhs , double *a , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib ,
MKL_INT *descb , double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pcdbtrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu ,
MKL_INT *nrhs , MKL_Complex8 *a , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b ,
MKL_INT *ib , MKL_INT *descb , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pzdbtrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *bwl , MKL_INT *bwu ,
MKL_INT *nrhs , MKL_Complex16 *a , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b ,
MKL_INT *ib , MKL_INT *descb , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16
*work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?dbtrsv function solves a banded triangular system of linear equations

$A(1:n, ja:ja+n-1) * X = B(ib:ib+n-1, 1:nrhs)$ or

$A(1:n, ja:ja+n-1)^T * X = B(ib:ib+n-1, 1:nrhs)$ (for real flavors); $A(1:n, ja:ja+n-1)^H * X = B(ib:ib+n-1, 1:nrhs)$ (for complex flavors),

where $A(1:n, ja:ja+n-1)$ is a banded triangular matrix factor produced by the Gaussian elimination code of p?dbtrf and is stored in $A(1:n, ja:ja+n-1)$ and af . The matrix stored in $A(1:n, ja:ja+n-1)$ is either upper or lower triangular according to $uplo$, and the choice of solving $A(1:n, ja:ja+n-1)$ or $A(1:n, ja:ja+n-1)^T$ is dictated by the user by the parameter $trans$.

The function p?dbtrf must be called first.

Input Parameters

| | |
|---------|--|
| $uplo$ | (global) If $uplo = 'U'$, the upper triangle of $A(1:n, ja:ja+n-1)$ is stored, if $uplo = 'L'$, the lower triangle of $A(1:n, ja:ja+n-1)$ is stored. |
| $trans$ | (global) If $trans = 'N'$, solve with $A(1:n, ja:ja+n-1)$, if $trans = 'C'$, solve with conjugate transpose $A(1:n, ja:ja+n-1)$. |

| | |
|--------------|---|
| <i>n</i> | (global) The order of the distributed submatrix <i>A</i> ; ($n \geq 0$). |
| <i>bwl</i> | (global) Number of subdiagonals. $0 \leq bwl \leq n-1$. |
| <i>bwu</i> | (global) Number of subdiagonals. $0 \leq bwu \leq n-1$. |
| <i>nrhs</i> | (global) The number of right-hand sides; the number of columns of the distributed submatrix <i>B</i> ($nrhs \geq 0$). |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$, where $lld_a \geq (bwl+bwu+1)$. On entry, this array contains the local pieces of the <i>n</i> -by- <i>n</i> unsymmetric banded distributed Cholesky factor <i>L</i> or L^T , represented in global <i>A</i> as $A(1:n, ja:ja+n-1)$. This local portion is stored in the packed banded format used in LAPACK. See the <i>Application Notes</i> below and the ScaLAPACK manual for more detail on the format of distributed matrices. |
| <i>ja</i> | (global) The index in the global matrix <i>A</i> that points to the start of the matrix to be operated on (which may be either all of <i>A</i> or a submatrix of <i>A</i>). |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . if 1d type ($dtype_a = 501$ or 502), $dlen \geq 7$; if 2d type ($dtype_a = 1$), $dlen \geq 9$. The array descriptor for the distributed matrix <i>A</i> . Contains information of mapping of <i>A</i> to memory. |
| <i>b</i> | (local) Pointer into the local memory to an array of local lead dimension $lld_b \geq nb$. On entry, this array contains the local pieces of the right-hand sides $B(ib:ib+n-1, 1:nrhs)$. |
| <i>ib</i> | (global) The row index in the global matrix <i>B</i> that points to the first row of the matrix to be operated on (which may be either all of <i>B</i> or a submatrix of <i>B</i>). |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . if 1d type ($dtype_b = 502$), $dlen \geq 7$; if 2d type ($dtype_b = 1$), $dlen \geq 9$. The array descriptor for the distributed matrix <i>B</i> . Contains information of mapping <i>B</i> to memory. |
| <i>laf</i> | (local) Size of user-input auxiliary fill-in space <i>af</i> . $laf \geq nb * (bwl+bwu) + 6 * \max(bwl, bwu) * \max(bwl, bwu)$. If <i>laf</i> is not large enough, an error code is returned and the minimum acceptable size will be returned in $af[0]$. |
| <i>work</i> | (local). Temporary workspace. This space may be overwritten in between function calls. <i>work</i> must be the size given in <i>lwork</i> . |

lwork (local or global)
 Size of user-input workspace *work*. If *lwork* is too small, the minimal acceptable size will be returned in *work[0]* and an error code is returned.
 $lwork \geq \max(bwl, bwu) * nrhs.$

Output Parameters

a (local).
 This local portion is stored in the packed banded format used in LAPACK. Please see the ScaLAPACK manual for more detail on the format of distributed matrices.

b On exit, this contains the local piece of the solutions distributed matrix *X*.

af (local).
 auxiliary fill-in space. The fill-in space is created in a call to the factorization function *p?dbtrf* and is stored in *af*. If a linear system is to be solved using *p?dbtrf* after the factorization function, *af* must not be altered after the factorization.

work On exit, *work[0]* contains the minimal *lwork*.

info (local).
 If *info* = 0, the execution is successful.
 < 0: If the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?dttrsv

Computes an LU factorization of a general band matrix, using partial pivoting with row interchanges. The function is called by p?dttrs.

Syntax

```
void psdttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , float *dl ,
float *d , float *du , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT
*descb , float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );

void pddttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , double *dl ,
double *d , double *du , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib ,
MKL_INT *descb , double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT
*info );

void pcdttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8
*d , MKL_Complex8 *d , MKL_Complex8 *du , MKL_INT *ja , MKL_INT *desca , MKL_Complex8
*b , MKL_INT *ib , MKL_INT *descb , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8
*work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzdttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16
*d1 , MKL_Complex16 *d , MKL_Complex16 *du , MKL_INT *ja , MKL_INT *desca ,
MKL_Complex16 *b , MKL_INT *ib , MKL_INT *descb , MKL_Complex16 *af , MKL_INT *laf ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The pzdttrsv function solves a tridiagonal triangular system of linear equations

$$A(1:n, ja:ja+n-1) * X = B(ib:ib+n-1, 1:nrhs) \text{ or}$$

$$A(1:n, ja:ja+n-1)^T * X = B(ib:ib+n-1, 1:nrhs) \text{ for real flavors; } A(1:n, ja:ja+n-1)^H * X = B(ib:ib+n-1, 1:nrhs) \text{ for complex flavors,}$$

where $A(1:n, ja:ja+n-1)$ is a tridiagonal matrix factor produced by the Gaussian elimination code of pzdttrf and is stored in $A(1:n, ja:ja+n-1)$ and af .

The matrix stored in $A(1:n, ja:ja+n-1)$ is either upper or lower triangular according to *uplo*, and the choice of solving $A(1:n, ja:ja+n-1)$ or $A(1:n, ja:ja+n-1)^T$ is dictated by the user by the parameter *trans*.

The function pzdttrf must be called first.

Input Parameters

| | |
|--------------|--|
| <i>uplo</i> | (global) If <i>uplo</i> ='U', the upper triangle of $A(1:n, ja:ja+n-1)$ is stored, if <i>uplo</i> = 'L', the lower triangle of $A(1:n, ja:ja+n-1)$ is stored. |
| <i>trans</i> | (global) If <i>trans</i> = 'N', solve with $A(1:n, ja:ja+n-1)$, if <i>trans</i> = 'C', solve with conjugate transpose $A(1:n, ja:ja+n-1)$. |
| <i>n</i> | (global) The order of the distributed submatrix A; ($n \geq 0$). |
| <i>nrhs</i> | (global) The number of right-hand sides; the number of columns of the distributed submatrix $B(ib:ib+n-1, 1:nrhs)$. ($nrhs \geq 0$). |
| <i>d1</i> | (local). Pointer to local part of global vector storing the lower diagonal of the matrix. Globally, $d1[0]$ is not referenced, and <i>d1</i> must be aligned with <i>d</i> . Must be of size $\geq nb_a$. |
| <i>d</i> | (local). Pointer to local part of global vector storing the main diagonal of the matrix. |
| <i>du</i> | (local). Pointer to local part of global vector storing the upper diagonal of the matrix. |

| | |
|--------------|---|
| | Globally, $du[n-1]$ is not referenced, and du must be aligned with d . |
| <i>ja</i> | (global) The index in the global matrix A that points to the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| <i>desca</i> | (global and local) array of size $dlen_$. if 1d type ($dtype_a = 501$ or 502), $dlen \geq 7$; if 2d type ($dtype_a = 1$), $dlen \geq 9$. The array descriptor for the distributed matrix A . Contains information of mapping of A to memory. |
| <i>b</i> | (local) Pointer into the local memory to an array of local lead dimension $lld_b \geq nb$. On entry, this array contains the local pieces of the right-hand sides $B(ib:ib+n-1, 1:nrhs)$. |
| <i>ib</i> | (global) The row index in the global matrix B that points to the first row of the matrix to be operated on (which may be either all of B or a submatrix of B). |
| <i>descb</i> | (global and local) array of size $dlen_$. if 1d type ($dtype_b = 502$), $dlen \geq 7$; if 2d type ($dtype_b = 1$), $dlen \geq 9$. The array descriptor for the distributed matrix B . Contains information of mapping B to memory. |
| <i>laf</i> | (local). Size of user-input auxiliary fill-in space af . $laf \geq 2*(nb+2)$. If laf is not large enough, an error code is returned and the minimum acceptable size will be returned in $af[0]$. |
| <i>work</i> | (local). Temporary workspace. This space may be overwritten in between function calls. $work$ must be the size given in $lwork$. |
| <i>lwork</i> | (local or global) Size of user-input workspace $work$. If $lwork$ is too small, the minimal acceptable size will be returned in $work[0]$ and an error code is returned. $lwork \geq 10*npcol+4*nrhs$. |

Output Parameters

| | |
|-----------|---|
| <i>d1</i> | (local). On exit, this array contains information containing the factors of the matrix. |
| <i>d</i> | On exit, this array contains information containing the factors of the matrix. Must be of size $\geq nb_a$. |

| | |
|-------------|---|
| <i>b</i> | On exit, this contains the local piece of the solutions distributed matrix X. |
| <i>af</i> | (local). Auxiliary fill-in space. The fill-in space is created in a call to the factorization function <code>p?dttrf</code> and is stored in <i>af</i> . If a linear system is to be solved using <code>p?dttrs</code> after the factorization function, <i>af</i> must not be altered after the factorization. |
| <i>work</i> | On exit, <code>work[0]</code> contains the minimal <i>lwork</i> . |
| <i>info</i> | (local). If <i>info</i> =0, the execution is successful. if <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then <i>info</i> = - (<i>i</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gebal

Balances a general real matrix.

Syntax

```
void psgebal(char* job, MKL_INT* n, float* a, MKL_INT* desca, MKL_INT* ilo, MKL_INT*
ihi, float* scale, MKL_INT* info);
```

```
void pdgebal(char* job, MKL_INT* n, double* a, MKL_INT* desca, MKL_INT* ilo, MKL_INT*
ihi, double* scale, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

`p?gebal` balances a general real matrix *A*. This involves, first, permuting *A* by a similarity transformation to isolate eigenvalues in the first 1 to *ilo*-1 and last *ihi*+1 to *n* elements on the diagonal; and second, applying a diagonal similarity transformation to rows and columns *ilo* to *ihi* to make the rows and columns as close in norm as possible. Both steps are optional.

Balancing may reduce the 1-norm of the matrix, and improve the accuracy of the computed eigenvalues and/or eigenvectors.

Input Parameters

| | |
|------------|---|
| <i>job</i> | (global) Specifies the operations to be performed on <i>a</i> : = 'N': none: simply set <i>ilo</i> = 1, <i>ihi</i> = <i>n</i> , <i>scale</i> [<i>i</i>] = 1.0 for <i>i</i> = 0,..., <i>n</i> -1; = 'P': permute only; = 'S': scale only; = 'B': both permute and scale. |
|------------|---|

| | |
|---------|---|
| n | (global) The order of the matrix A ($n \geq 0$). |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOC_c(n)$ This array contains the local pieces of global input matrix A . |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |

OUTPUT Parameters

| | |
|------------|---|
| a | On exit, a is overwritten by the balanced matrix A . If $job = 'N'$, a is not referenced. See Notes for further details. |
| ilo, ihi | (global) ilo and ihi are set to integers such that on exit matrix elements $A(i,j)$ are zero if $i > j$ and $j = 1, \dots, ilo-1$ or $i = ihi+1, \dots, n$. If $job = 'N'$ or $'S'$, $ilo = 1$ and $ihi = n$. |
| $scale$ | (global) array of size n . Details of the permutations and scaling factors applied to a . If pj is the index of the row and column interchanged with row and column j and dj is the scaling factor applied to row and column j , then $scale[j-1] = pj$ for $j = 1, \dots, ilo-1$ $scale[j-1] = dj$ for $j = ilo, \dots, ihi$ The order in which the interchanges are made is n to $ihi+1$, then 1 to $ilo-1$. |
| $info$ | (global) = 0: successful exit. < 0: if $info = -i$, the i -th argument had an illegal value. |

Application Notes

The permutations consist of row and column interchanges which put the matrix in the form

$$PAP = \begin{pmatrix} T_1 & X & Y \\ \mathbf{0} & B & Z \\ \mathbf{0} & \mathbf{0} & T_2 \end{pmatrix}$$

where T_1 and T_2 are upper triangular matrices whose eigenvalues lie along the diagonal. The column indices ilo and ihi mark the starting and ending columns of the submatrix B . Balancing consists of applying a diagonal similarity transformation $D^{-1}BD$ to make the 1-norms of each row of B and its corresponding column nearly equal. The output matrix is

$$\begin{pmatrix} T_1 & XD & Y \\ 0 & D^{-1}BD & D^{-1}Z \\ 0 & 0 & T_2 \end{pmatrix}$$

Information about the permutations P and the diagonal matrix D is returned in the vector *scale*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gebd2

Reduces a general rectangular matrix to real bidiagonal form by an orthogonal/unitary transformation (unblocked algorithm).

Syntax

```
void psgebd2 (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *d , float *e , float *tauq , float *taup , float *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pdgebd2 (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *d , double *e , double *tauq , double *taup , double *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pcgebd2 (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *d , float *e , MKL_Complex8 *tauq , MKL_Complex8 *taup ,
MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzgebd2 (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *d , double *e , MKL_Complex16 *tauq , MKL_Complex16 *taup ,
MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?gebd2 function reduces a real/complex general m -by- n distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$ to upper or lower bidiagonal form B by an orthogonal/unitary transformation:

$$Q^* \text{sub}(A) P = B.$$

If $m \geq n$, B is the upper bidiagonal; if $m < n$, B is the lower bidiagonal.

Input Parameters

- | | |
|-----|--|
| m | (global) The number of rows of the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| n | (global) |

| | |
|---------------|---|
| | The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOC_c(ja+n-1)$. On entry, this array contains the local pieces of the general distributed matrix $\text{sub}(A)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>work</i> | (local). This is a workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> . <i>lwork</i> is local input and must be at least $lwork \geq \max(mpa0, nqa0)$, where $nb = mb_a = nb_a$, $irowfa = \text{mod}(ia-1, nb)$, $iarow = \text{indxg2p}(ia, nb, myrow, rsrc_a, nprow)$, $iacol = \text{indxg2p}(ja, nb, mycol, csrc_a, npcot)$, $mpa0 = \text{numroc}(m+irowfa, nb, myrow, iarow, nprow)$, $nqa0 = \text{numroc}(n+icoffa, nb, mycol, iacol, npcot)$. indxg2p and numroc are ScaLAPACK tool functions; <i>myrow</i> , <i>mycol</i> , <i>nprow</i> , and <i>npcol</i> can be determined by calling the function <code>blacs_gridinfo</code> . If $lwork = -1$, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <code>pxerbla</code> . |

Output Parameters

| | |
|----------|---|
| <i>a</i> | (local). On exit, if $m \geq n$, the diagonal and the first superdiagonal of $\text{sub}(A)$ are overwritten with the upper bidiagonal matrix B ; the elements below the diagonal, with the array <i>tauq</i> , represent the orthogonal/unitary matrix Q as a product of elementary reflectors, and the elements above the first superdiagonal, with the array <i>taup</i> , represent the orthogonal matrix P as a product of elementary reflectors. If $m < n$, the diagonal and the first subdiagonal are overwritten with the lower bidiagonal matrix B ; the elements below the first subdiagonal, with the array <i>tauq</i> , represent the orthogonal/unitary matrix Q as a product of elementary reflectors, and the elements above the diagonal, with the array <i>taup</i> , represent the orthogonal matrix P as a product of elementary reflectors. See <i>Applications Notes</i> below. |
| <i>d</i> | (local) |

Array of size $LOCc(ja+\min(m,n)-1)$ if $m \geq n$; $LOCr(ia+\min(m,n)-1)$ otherwise. The distributed diagonal elements of the bidiagonal matrix B : $d[i] = A(i+1,i+1)$, $i=0, 1, \dots, \text{size}(d) - 1$. d is tied to the distributed matrix A .

e

(local)

Array of size $LOCc(ja+\min(m,n)-1)$ if $m \geq n$; $LOCr(ia+\min(m,n)-2)$ otherwise. The distributed diagonal elements of the bidiagonal matrix B :
if $m \geq n$, $e[i] = A(i+1,i+2)$ for $i = 0, 1, \dots, n-2$;
if $m < n$, $e[i] = A(i+2,i+1)$ for $i = 0, 1, \dots, m-2$. e is tied to the distributed matrix A .

tauq

(local).

Array of size $LOCc(ja+\min(m,n)-1)$. The scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix Q . *tauq* is tied to the distributed matrix A .

taup

(local).

Array of size $LOCr(ia+\min(m,n)-1)$. The scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix P . *taup* is tied to the distributed matrix A .

work

On exit, *work*[0] returns the minimal and optimal *lwork*.

info

(local)

If *info* = 0, the execution is successful.

if *info* < 0: If the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value, then *info* = - (*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

Application Notes

The matrices Q and P are represented as products of elementary reflectors:

If $m \geq n$,

$$Q = H(1)*H(2)*\dots*H(n), \text{ and } P = G(1)*G(2)*\dots*G(n-1)$$

Each $H(i)$ and $G(i)$ has the form:

$$H(i) = I - \tau_{uq} * v * v', \text{ and } G(i) = I - \tau_{up} * u * u',$$

where τ_{uq} and τ_{up} are real/complex scalars, and v and u are real/complex vectors. $v(1:i-1) = 0$, $v(i) = 1$, and $v(i+1:m)$ is stored on exit in

$$A(ia+i-ia+m-1, ja+i-1);$$

$$u(1:i) = 0, u(i+1) = 1, \text{ and } u(i+2:n) \text{ is stored on exit in } A(ia+i-1, ja+i+1:ja+n-1);$$

τ_{uq} is stored in $\tau_{uq}[ja+i-2]$ and τ_{up} in $\tau_{up}[ia+i-2]$.

If $m < n$,

$$v(1:i) = 0, v(i+1) = 1, \text{ and } v(i+2:m) \text{ is stored on exit in } A(ia+i+1: ia+m-1, ja+i-1);$$

$$u(1:i-1) = 0, u(i) = 1, \text{ and } u(i+1:n) \text{ is stored on exit in } A(ia+i-1, ja+i:ja+n-1);$$

τ_{uq} is stored in $\tau_{uq}[ja+i-2]$ and τ_{up} in $\tau_{up}[ia+i-2]$.

The contents of $\text{sub}(A)$ on exit are illustrated by the following examples:

$$\begin{array}{l}
 m = 6 \text{ and } n = 5 (m > n) : \\
 \begin{bmatrix} d & e & u1 & u1 & u1 \\ v1 & d & e & u2 & u2 \\ v1 & v2 & d & e & u3 \\ v1 & v2 & v3 & d & e \\ v1 & v2 & v3 & v4 & d \\ v1 & v2 & v3 & v4 & v5 \end{bmatrix}
 \end{array}
 \qquad
 \begin{array}{l}
 m = 5 \text{ and } n = 6 (m < n) : \\
 \begin{bmatrix} d & u1 & u1 & u1 & u1 & u1 \\ e & d & u2 & u2 & u2 & u2 \\ v1 & e & d & u3 & u3 & u3 \\ v1 & v2 & e & d & u4 & u4 \\ v1 & v2 & v3 & e & d & u5 \end{bmatrix}
 \end{array}$$

where d and e denote diagonal and off-diagonal elements of B , v_i denotes an element of the vector defining $H(i)$, and u_i an element of the vector defining $G(i)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gehd2

Reduces a general matrix to upper Hessenberg form by an orthogonal/unitary similarity transformation (unblocked algorithm).

Syntax

```
void psgehd2 (MKL_INT *n , MKL_INT *ilo , MKL_INT *ihi , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pdgehd2 (MKL_INT *n , MKL_INT *ilo , MKL_INT *ihi , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pcgehd2 (MKL_INT *n , MKL_INT *ilo , MKL_INT *ihi , MKL_Complex8 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pzgehd2 (MKL_INT *n , MKL_INT *ilo , MKL_INT *ihi , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?gehd2` function reduces a real/complex general distributed matrix $\text{sub}(A)$ to upper Hessenberg form H by an orthogonal/unitary similarity transformation: $Q^* \text{sub}(A) Q = H$, where $\text{sub}(A) = A(\text{ia}+\text{n}-1 : \text{ia}+\text{n}-1, \text{ja}+\text{n}-1 : \text{ja}+\text{n}-1)$.

Input Parameters

`n` (global) The order of the distributed submatrix A . ($n \geq 0$).

`ilo, ihi` (global) It is assumed that the matrix $\text{sub}(A)$ is already upper triangular in rows $\text{ia}:\text{ia}+\text{ilo}-2$ and $\text{ia}+\text{ihi}:\text{ia}+\text{n}-1$ and columns $\text{ja}:\text{ja}+\text{jlo}-2$ and $\text{ja}+\text{jhi}:\text{ja}+\text{n}-1$. See *Application Notes* for further information.

If $n \geq 0$, $1 \leq ilo \leq ihi \leq n$; otherwise set $ilo = 1$, $ihi = n$.

a (local).
Pointer into the local memory to an array of size $lld_a * LOC_c(ja+n-1)$.

On entry, this array contains the local pieces of the n -by- n general distributed matrix $sub(A)$ to be reduced.

ia, ja (global) The row and column indices in the global matrix A indicating the first row and the first column of $sub(A)$, respectively.

desca (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A .

work (local).
This is a workspace array of size *lwork*.

lwork (local or global)
The size of the array *work*.
lwork is local input and must be at least $lwork \geq nb + \max(npa0, nb)$, where $nb = mb_a = nb_a$, $irow = \text{mod}(ia-1, nb)$, $iarow = \text{indxg2p}(ia, nb, myrow, rsrc_a, nprow)$, $npa0 = \text{numroc}(ihi + irow, nb, myrow, iarow, nprow)$.

indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *ncol* can be determined by calling the function `blacs_gridinfo`.

If $lwork = -1$, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

a (local). On exit, the upper triangle and the first subdiagonal of $sub(A)$ are overwritten with the upper Hessenberg matrix H , and the elements below the first subdiagonal, with the array *tau*, represent the orthogonal/unitary matrix Q as a product of elementary reflectors. (see *Application Notes* below).

tau (local).
Array of size $LOC_c(ja+n-2)$ The scalar factors of the elementary reflectors (see *Application Notes* below). Elements $ja:ja+ilo-2$ and $ja+ihi:ja+n-2$ of the global vector *tau* are set to zero. *tau* is tied to the distributed matrix A .

work On exit, *work*[0] returns the minimal and optimal *lwork*.

info (local)
If $info = 0$, the execution is successful.
if $info < 0$: If the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

Application Notes

The matrix Q is represented as a product of (*ihi-ilo*) elementary reflectors

$$Q = H(ilo) * H(ilo+1) * \dots * H(ihi-1).$$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v',$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1:i) = 0$, $v(i+1) = 1$ and $v(ihi+1:n) = 0$; $v(i+2:ihi)$ is stored on exit in $A(ia+ilo+i:i+ihi-1, ia+ilo+i-2)$, and τ in $\tau[ja+ilo+i-3]$.

The contents of $A(ia:ia+n-1, ja:ja+n-1)$ are illustrated by the following example, with $n = 7$, $ilo = 2$ and $ihi = 6$:

| on entry | on exit |
|---|---|
| $\begin{bmatrix} a & a & a & a & a & a & a \\ a & a & a & a & a & a & a \\ a & a & a & a & a & a & a \\ a & a & a & a & a & a & a \\ a & a & a & a & a & a & a \\ a & a & a & a & a & a & a \\ & & & & & & a \end{bmatrix}$ | $\begin{bmatrix} a & a & h & h & h & h & a \\ a & h & h & h & h & h & a \\ h & h & h & h & h & h & h \\ v2 & h & h & h & h & h & h \\ v2 & v3 & h & h & h & h & h \\ v2 & v3 & v4 & h & h & h & h \\ & & & & & & a \end{bmatrix}$ |

where a denotes an element of the original matrix $\text{sub}(A)$, h denotes a modified element of the upper Hessenberg matrix H , and v_i denotes an element of the vector defining $H(ja+ilo+i-2)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gelq2

Computes an LQ factorization of a general rectangular matrix (unblocked algorithm).

Syntax

```
void psgelq2 (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdgelq2 (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcgelq2 (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzgelq2 (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `pxerbla` function computes an LQ factorization of a real/complex distributed m -by- n matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1) = L*Q$.

Input Parameters

| | |
|---------------------|--|
| <code>m</code> | (global) The number of rows of the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <code>n</code> | (global) The number of columns of the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <code>a</code> | (local). Pointer into the local memory to an array of size $\text{lld}_a * \text{LOCc}(\text{ja}+n-1)$. On entry, this array contains the local pieces of the m -by- n distributed matrix $\text{sub}(A)$ which is to be factored. |
| <code>ia, ja</code> | (global) The row and column indices in the global matrix A indicating the first row and the first column of $\text{sub}(A)$, respectively. |
| <code>desca</code> | (global and local) array of size $\text{dlen}_$. The array descriptor for the distributed matrix A . |
| <code>work</code> | (local). This is a workspace array of size lwork . |
| <code>lwork</code> | (local or global) The size of the array work . lwork is local input and must be at least $\text{lwork} \geq nq0 + \max(1, mp0)$, where $\text{iroff} = \text{mod}(\text{ia}-1, \text{mb}_a)$, $\text{icoff} = \text{mod}(\text{ja}-1, \text{nb}_a)$, $\text{iarow} = \text{indxg2p}(\text{ia}, \text{mb}_a, \text{myrow}, \text{rsrc}_a, \text{nprow})$, $\text{iacol} = \text{indxg2p}(\text{ja}, \text{nb}_a, \text{mycol}, \text{csrc}_a, \text{npcol})$, $\text{mp0} = \text{numroc}(m+\text{iroff}, \text{mb}_a, \text{myrow}, \text{iarow}, \text{nprow})$, $\text{nq0} = \text{numroc}(n+\text{icoff}, \text{nb}_a, \text{mycol}, \text{iacol}, \text{npcol})$, indxg2p and numroc are ScaLAPACK tool functions; myrow , mycol , nprow , and npcol can be determined by calling the function <code>blacs_gridinfo</code> . If $\text{lwork} = -1$, then lwork is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <code>pxerbla</code> . |

Output Parameters

| | |
|----------------|----------|
| <code>a</code> | (local). |
|----------------|----------|

On exit, the elements on and below the diagonal of sub(A) contain the m by $\min(m,n)$ lower trapezoidal matrix L (L is lower triangular if $m \leq n$); the elements above the diagonal, with the array τ , represent the orthogonal/unitary matrix Q as a product of elementary reflectors (see *Application Notes* below).

τ (local).
 Array of size $LOCr(ia+\min(m, n)-1)$. This array contains the scalar factors of the elementary reflectors. τ is tied to the distributed matrix A .

$work$ On exit, $work[0]$ returns the minimal and optimal $lwork$.

$info$ (local) If $info = 0$, the execution is successful. if $info < 0$: If the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

Application Notes

The matrix Q is represented as a product of elementary reflectors

$Q = H(ia+k-1) * H(ia+k-2) * \dots * H(ia)$ for real flavors, $Q = (H(ia+k-1))^{H*} (H(ia+k-2))^{H} \dots (H(ia))^{H}$ for complex flavors,

where $k = \min(m,n)$.

Each $H(i)$ has the form

$H(i) = I - \tau * v * v'$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1: i-1) = 0$ and $v(i) = 1$; $v(i+1: n)$ (for real flavors) or $\text{conjg}(v(i+1: n))$ (for complex flavors) is stored on exit in $A(ia+i-1, ja+i: ja+n-1)$, and τ in $\tau[ia+i-2]$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?geql2

Computes a QL factorization of a general rectangular matrix (unblocked algorithm).

Syntax

```
void psgeql2 (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdgeql2 (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcgeql2 (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzgeql2 (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?geql2` function computes a QL factorization of a real/complex distributed m -by- n matrix $\text{sub}(A) = A(ia:ia+m-1, ja:ja+n-1) = Q * L$.

Input Parameters

| | |
|---------------------|---|
| <code>m</code> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <code>n</code> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <code>a</code> | (local). Pointer into the local memory to an array of size $lld_a * LOC_c(ja+n-1)$. On entry, this array contains the local pieces of the m -by- n distributed matrix $\text{sub}(A)$ which is to be factored. |
| <code>ia, ja</code> | (global) The row and column indices in the global matrix A indicating the first row and the first column of $\text{sub}(A)$, respectively. |
| <code>desca</code> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <code>work</code> | (local). This is a workspace array of size $lwork$. |
| <code>lwork</code> | (local or global) The size of the array $work$. $lwork$ is local input and must be at least $lwork \geq mp0 + \max(1, nq0)$, where $iroff = \text{mod}(ia-1, mb_a)$, $icoff = \text{mod}(ja-1, nb_a)$, $iarow = \text{indxg2p}(ia, mb_a, myrow, rsrc_a, nprow)$, $iacol = \text{indxg2p}(ja, nb_a, mycol, csrc_a, npcot)$, $mp0 = \text{numroc}(m+iroff, mb_a, myrow, iarow, nprow)$, $nq0 = \text{numroc}(n+icoff, nb_a, mycol, iacol, npcot)$, indxg2p and numroc are ScaLAPACK tool functions; $myrow$, $mycol$, $nprow$, and $npcot$ can be determined by calling the function <code>blacs_gridinfo</code> . If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <code>pxerbla</code> . |

Output Parameters

| | |
|----------------|----------|
| <code>a</code> | (local). |
|----------------|----------|

On exit,

if $m \geq n$, the lower triangle of the distributed submatrix $A(ia+m-n:ia+m-1, ja:ja+n-1)$ contains the n -by- n lower triangular matrix L ;

if $m \leq n$, the elements on and below the $(n-m)$ -th superdiagonal contain the m -by- n lower trapezoidal matrix L ; the remaining elements, with the array τ , represent the orthogonal/ unitary matrix Q as a product of elementary reflectors (see *Application Notes* below).

τ

(local).

Array of size $LOCc(ja+n-1)$. This array contains the scalar factors of the elementary reflectors. τ is tied to the distributed matrix A .

$work$

On exit, $work[0]$ returns the minimal and optimal $lwork$.

$info$

(local).

If $info = 0$, the execution is successful. if $info < 0$: If the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

Application Notes

The matrix Q is represented as a product of elementary reflectors

$$Q = H(ja+k-1) * \dots * H(ja+1) * H(ja), \text{ where } k = \min(m, n).$$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v'$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(m-k+i+1:m) = 0$ and $v(m-k+i) = 1$; $v(1:m-k+i-1)$ is stored on exit in $A(ia:ia+m-k+i-2, ja+n-k+i-1)$, and τ in $\tau[ja+n-k+i-2]$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?geqr2

Computes a QR factorization of a general rectangular matrix (unblocked algorithm).

Syntax

```
void psgeqr2 (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdgeqr2 (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcgeqr2 (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzgeqr2 (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?geqr2` function computes a QR factorization of a real/complex distributed m -by- n matrix $\text{sub}(A) = A(ia:ia+m-1, ja:ja+n-1) = Q^*R$.

Input Parameters

| | |
|---------------------|---|
| <code>m</code> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <code>n</code> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <code>a</code> | (local). Pointer into the local memory to an array of size $lld_a * LOC_c(ja+n-1)$. On entry, this array contains the local pieces of the m -by- n distributed matrix $\text{sub}(A)$ which is to be factored. |
| <code>ia, ja</code> | (global) The row and column indices in the global matrix A indicating the first row and the first column of $\text{sub}(A)$, respectively. |
| <code>desca</code> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <code>work</code> | (local). This is a workspace array of size $lwork$. |
| <code>lwork</code> | (local or global) The size of the array $work$. $lwork$ is local input and must be at least $lwork \geq mp0 + \max(1, nq0)$, where $iroff = \text{mod}(ia-1, mb_a)$, $icoff = \text{mod}(ja-1, nb_a)$, $iarow = \text{indxg2p}(ia, mb_a, myrow, rsrc_a, nprow)$, $iacol = \text{indxg2p}(ja, nb_a, mycol, csrc_a, npcot)$, $mp0 = \text{numroc}(m+iroff, mb_a, myrow, iarow, nprow)$, $nq0 = \text{numroc}(n+icoff, nb_a, mycol, iacol, npcot)$. indxg2p and numroc are ScaLAPACK tool functions; $myrow$, $mycol$, $nprow$, and $npcol$ can be determined by calling the function <code>blacs_gridinfo</code> . If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <code>pxerbla</code> . |

Output Parameters

| | |
|----------------|----------|
| <code>a</code> | (local). |
|----------------|----------|

On exit, the elements on and above the diagonal of sub(A) contain the $\min(m,n)$ by n upper trapezoidal matrix R (R is upper triangular if $m \geq n$); the elements below the diagonal, with the array τ , represent the orthogonal/unitary matrix Q as a product of elementary reflectors (see *Application Notes* below).

τ (local).
 Array of size $LOCc(ja+\min(m,n)-1)$. This array contains the scalar factors of the elementary reflectors. τ is tied to the distributed matrix A .

$work$ On exit, $work[0]$ returns the minimal and optimal $lwork$.

$info$ (local)
 If $info = 0$, the execution is successful. if $info < 0$:
 If the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value, then $info = -(i*100+j)$,
 if the i -th argument is a scalar and had an illegal value, then $info = -i$.

Application Notes

The matrix Q is represented as a product of elementary reflectors

$$Q = H(ja) * H(ja+1) * \dots * H(ja+k-1), \text{ where } k = \min(m,n).$$

Each $H(i)$ has the form

$$H(j) = I - \tau * v * v',$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1:i-1) = 0$ and $v(i) = 1$; $v(i+1:m)$ is stored on exit in $A(ia+i:ia+m-1, ja+i-1)$, and τ in $\tau[ja+i-2]$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?gerq2

Computes an RQ factorization of a general rectangular matrix (unblocked algorithm).

Syntax

```
void psgqrq2 (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdgqrq2 (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcgqrq2 (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT
*info );
```

```
void pzgqrq2 (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT
*info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?gerq2` function computes an RQ factorization of a real/complex distributed m -by- n matrix $\text{sub}(A) = A(ia:ia+m-1, ja:ja+n-1) = R*Q$.

Input Parameters

| | |
|---------------|---|
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOC_c(ja+n-1)$. On entry, this array contains the local pieces of the m -by- n distributed matrix $\text{sub}(A)$ which is to be factored. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>work</i> | (local). This is a workspace array of size $lwork$. |
| <i>lwork</i> | (local or global) The size of the array $work$. $lwork$ is local input and must be at least $lwork \geq nq0 + \max(1, mp0)$, where $iroff = \text{mod}(ia-1, mb_a), \quad icoff = \text{mod}(ja-1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, myrow, rsrc_a, nprow),$ $iacol = \text{indxg2p}(ja, nb_a, mycol, csrc_a, npcot), \quad mp0 = \text{numroc}(m+iroff, mb_a, myrow, iarow, nprow),$ $nq0 = \text{numroc}(n+icoff, nb_a, mycol, iacol, npcot),$ indxg2p and numroc are ScaLAPACK tool functions; $myrow$, $mycol$, $nprow$, and $npcol$ can be determined by calling the function <code>blacs_gridinfo</code> . If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by <code>pxerbla</code> . |

Output Parameters

| | |
|----------|----------------------|
| <i>a</i> | (local). On exit, |
|----------|----------------------|

if $m \leq n$, the upper triangle of $A(ia+m-n:ia+m-1, ja:ja+n-1)$ contains the m -by- m upper triangular matrix R ;

if $m \geq n$, the elements on and above the $(m-n)$ -th subdiagonal contain the m -by- n upper trapezoidal matrix R ; the remaining elements, with the array τ , represent the orthogonal/ unitary matrix Q as a product of elementary reflectors (see *Application Notes* below).

τ (local).

Array of size $LOCr(ia+m-1)$. This array contains the scalar factors of the elementary reflectors. τ is tied to the distributed matrix A .

$work$ On exit, $work[0]$ returns the minimal and optimal $lwork$.

$info$ (local)

If $info = 0$, the execution is successful.

if $info < 0$: If the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

Application Notes

The matrix Q is represented as a product of elementary reflectors

$Q = H(ia) * H(ia+1) * \dots * H(ia+k-1)$ for real flavors,

$Q = (H(ia))^H * (H(ia+1))^H * \dots * (H(ia+k-1))^H$ for complex flavors,

where $k = \min(m, n)$.

Each $H(i)$ has the form

$H(i) = I - \tau * v * v'$,

where τ is a real/complex scalar, and v is a real/complex vector with $v(n-k+i+1:n) = 0$ and $v(n-k+i) = 1$; $v(1:n-k+i-1)$ for real flavors or $\text{conjg}(v(1:n-k+i-1))$ for complex flavors is stored on exit in $A(ia+m-k+i-1, ja:ja+n-k+i-2)$, and τ in $\tau[ia+m-k+i-2]$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?getf2

Computes an LU factorization of a general matrix, using partial pivoting with row interchanges (local blocked algorithm).

Syntax

```
void psgetf2 (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , MKL_INT *ipiv , MKL_INT *info );
```

```
void pdgetf2 (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , MKL_INT *info );
```

```
void pcgetf2 (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , MKL_INT *info );
```

```
void pzgetf2 (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *ipiv , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `pzgetf2` function computes an LU factorization of a general m -by- n distributed matrix $\text{sub}(A) = A(ia:ia+m-1, ja:ja+n-1)$ using partial pivoting with row interchanges.

The factorization has the form $\text{sub}(A) = P * L * U$, where P is a permutation matrix, L is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$), and U is upper triangular (upper trapezoidal if $m < n$). This is the right-looking Parallel Level 2 BLAS version of the algorithm.

Optimization Notice

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Notice revision #20110804

Input Parameters

| | |
|----------|---|
| m | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| n | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($nb_a - \text{mod}(ja-1, nb_a) \geq n \geq 0$). |
| a | (local). Pointer into the local memory to an array of size $lld_a * LOC_c(ja+n-1)$. On entry, this array contains the local pieces of the m -by- n distributed matrix $\text{sub}(A)$. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |

Output Parameters

| | |
|--------|--|
| $ipiv$ | (local) Array of size $(LOC_r(m_a) + mb_a)$. This array contains the pivoting information. $ipiv[i] \rightarrow$ The global row that local row $(i + 1)$ was swapped with, $i = 0, 1, \dots, LOC_r(m_a) + mb_a - 1$. This array is tied to the distributed matrix A . |
|--------|--|

info (local).

If *info* = 0: successful exit.

If *info* < 0:

- if the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value, then *info* = $-(i*100+j)$,
- if the *i*-th argument is a scalar and had an illegal value, then *info* = $-i$.

If *info* > 0: If *info* = *k*, the matrix element $U(ia+k-1, ja+k-1)$ is exactly zero. The factorization has been completed, but the factor *U* is exactly singular, and division by zero will occur if it is used to solve a system of equations.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?labrd

Reduces the first nb rows and columns of a general rectangular matrix A to real bidiagonal form by an orthogonal/unitary transformation, and returns auxiliary matrices that are needed to apply the transformation to the unreduced part of A.

Syntax

```
void pslabrd (MKL_INT *m , MKL_INT *n , MKL_INT *nb , float *a , MKL_INT *ia , MKL_INT
 *ja , MKL_INT *desca , float *d , float *e , float *tauq , float *taup , float *x ,
 MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , float *y , MKL_INT *iy , MKL_INT *jy ,
 MKL_INT *descy , float *work );
```

```
void pdlabrd (MKL_INT *m , MKL_INT *n , MKL_INT *nb , double *a , MKL_INT *ia ,
 MKL_INT *ja , MKL_INT *desca , double *d , double *e , double *tauq , double *taup ,
 double *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , double *y , MKL_INT *iy ,
 MKL_INT *jy , MKL_INT *descy , double *work );
```

```
void pclabrd (MKL_INT *m , MKL_INT *n , MKL_INT *nb , MKL_Complex8 *a , MKL_INT *ia ,
 MKL_INT *ja , MKL_INT *desca , float *d , float *e , MKL_Complex8 *tauq , MKL_Complex8
 *taup , MKL_Complex8 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_Complex8
 *y , MKL_INT *iy , MKL_INT *jy , MKL_INT *descy , MKL_Complex8 *work );
```

```
void pzlabrd (MKL_INT *m , MKL_INT *n , MKL_INT *nb , MKL_Complex16 *a , MKL_INT *ia ,
 MKL_INT *ja , MKL_INT *desca , double *d , double *e , MKL_Complex16 *tauq ,
 MKL_Complex16 *taup , MKL_Complex16 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx ,
 MKL_Complex16 *y , MKL_INT *iy , MKL_INT *jy , MKL_INT *descy , MKL_Complex16 *work );
```

Include Files

- mkl_scalapack.h

Description

The p?labrd function reduces the first *nb* rows and columns of a real/complex general *m*-by-*n* distributed matrix $\text{sub}(A) = A(ia:ia+m-1, ja:ja+n-1)$ to upper or lower bidiagonal form by an orthogonal/unitary transformation $Q^* A P$, and returns the matrices *X* and *Y* necessary to apply the transformation to the unreduced part of $\text{sub}(A)$.

If $m \geq n$, $\text{sub}(A)$ is reduced to upper bidiagonal form; if $m < n$, $\text{sub}(A)$ is reduced to lower bidiagonal form.

This is an auxiliary function called by `p?gebrd`.

Input Parameters

| | |
|---------------|---|
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <i>nb</i> | (global) The number of leading rows and columns of $\text{sub}(A)$ to be reduced. |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOC_c(ja+n-1)$. On entry, this array contains the local pieces of the general distributed matrix $\text{sub}(A)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>ix, jx</i> | (global) The row and column indices in the global matrix X indicating the first row and the first column of the matrix $\text{sub}(X)$, respectively. |
| <i>descx</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix X . |
| <i>iy, jy</i> | (global) The row and column indices in the global matrix Y indicating the first row and the first column of the matrix $\text{sub}(Y)$, respectively. |
| <i>descy</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix Y . |
| <i>work</i> | (local). Workspace array of size <i>lwork</i> . $lwork \geq nb_a + nq$, with $nq = \text{numroc}(n + \text{mod}(ia-1, nb_y), nb_y, mycol, iacol, npc1)$ $iacol = \text{indxg2p}(ja, nb_a, mycol, csrc_a, npc1)$ <i>indxg2p</i> and <i>numroc</i> are ScaLAPACK tool functions; <i>myrow</i> , <i>mycol</i> , <i>nrow</i> , and <i>npc1</i> can be determined by calling the function <code>blacs_gridinfo</code> . |

Output Parameters

| | |
|----------|--|
| <i>a</i> | (local) On exit, the first <i>nb</i> rows and columns of the matrix are overwritten; the rest of the distributed matrix $\text{sub}(A)$ is unchanged. |
|----------|--|

If $m \geq n$, elements on and below the diagonal in the first nb columns, with the array $tauq$, represent the orthogonal/unitary matrix Q as a product of elementary reflectors; and elements above the diagonal in the first nb rows, with the array $taup$, represent the orthogonal/unitary matrix P as a product of elementary reflectors.

If $m < n$, elements below the diagonal in the first nb columns, with the array $tauq$, represent the orthogonal/unitary matrix Q as a product of elementary reflectors, and elements on and above the diagonal in the first nb rows, with the array $taup$, represent the orthogonal/unitary matrix P as a product of elementary reflectors. See *Application Notes* below.

 d

(local).

Array of size $LOCr(ia+\min(m,n)-1)$ if $m \geq n$; $LOCc(ja+\min(m,n)-1)$ otherwise. The distributed diagonal elements of the bidiagonal distributed matrix B :

$$d[i] = A(ia+i, ja+i), i = 0, 1, \dots, \text{size}(d)-1$$

d is tied to the distributed matrix A .

 e

(local).

Array of size $LOCr(ia+\min(m,n)-1)$ if $m \geq n$; $LOCc(ja+\min(m,n)-2)$ otherwise. The distributed off-diagonal elements of the bidiagonal distributed matrix B :

$$\text{if } m \geq n, e[i] = A(ia+i, ja+i+1) \text{ for } i = 0, 1, \dots, n-2;$$

$$\text{if } m < n, e[i] = A(ia+i+1, ja+i) \text{ for } i = 0, 1, \dots, m-2.$$

e is tied to the distributed matrix A .

 $tauq, taup$

(local).

Array size $LOCc(ja+\min(m,n)-1)$ for $tauq$, size $LOCr(ia+\min(m,n)-1)$ for $taup$. The scalar factors of the elementary reflectors which represent the orthogonal/unitary matrix Q for $tauq$, P for $taup$. $tauq$ and $taup$ are tied to the distributed matrix A . See *Application Notes* below.

 x

(local)

Pointer into the local memory to an array of size $lld_x * nb$. On exit, the local pieces of the distributed m -by- nb matrix $X(ix:ix+m-1, jx:jx+nb-1)$ required to update the unreduced part of $\text{sub}(A)$.

 y

(local).

Pointer into the local memory to an array of size $lld_y * nb$. On exit, the local pieces of the distributed n -by- nb matrix $Y(iy:iy+n-1, jy:jy+nb-1)$ required to update the unreduced part of $\text{sub}(A)$.

Application Notes

The matrices Q and P are represented as products of elementary reflectors:

$$Q = H(1) * H(2) * \dots * H(nb), \text{ and } P = G(1) * G(2) * \dots * G(nb)$$

Each $H(i)$ and $G(i)$ has the form:

$$H(i) = I - tauq * v * v', \text{ and } G(i) = I - taup * u * u',$$

where τ_{uq} and τ_{up} are real/complex scalars, and v and u are real/complex vectors.

If $m \geq n$, $v(1:i-1) = 0$, $v(i) = 1$, and $v(i:m)$ is stored on exit in

$A(ia+i-1:ia+m-1, ja+i-1)$; $u(1:i) = 0$, $u(i+1:n) = 1$, and $u(i+1:n)$ is stored on exit in $A(ia+i-1, ja+i:ja+n-1)$; τ_{uq} is stored in $\tau_{uq}[ja+i-2]$ and τ_{up} in $\tau_{up}[ia+i-2]$.

If $m < n$, $v(1:i) = 0$, $v(i+1:n) = 1$, and $v(i+1:m)$ is stored on exit in

$A(ia+i+1:ia+m-1, ja+i-1)$; $u(1:i-1) = 0$, $u(i) = 1$, and $u(i:n)$ is stored on exit in $A(ia+i-1, ja+i:ja+n-1)$; τ_{uq} is stored in $\tau_{uq}[ja+i-2]$ and τ_{up} in $\tau_{up}[ia+i-2]$. The elements of the vectors v and u together form the m -by- nb matrix V and the nb -by- n matrix U' which are necessary, with X and Y , to apply the transformation to the unreduced part of the matrix, using a block update of the form: $\text{sub}(A) := \text{sub}(A) - V*Y' - X*U'$. The contents of $\text{sub}(A)$ on exit are illustrated by the following examples with $nb = 2$:

$$\begin{array}{l}
 m = 6 \text{ and } n = 5 (m > n): \\
 \begin{bmatrix} 1 & 1 & u1 & u1 & u1 \\ v1 & 1 & 1 & u2 & u2 \\ v1 & v2 & a & a & a \\ v1 & v2 & a & a & a \\ v1 & v2 & a & a & a \\ v1 & v2 & a & a & a \end{bmatrix} \\
 \\
 m = 5 \text{ and } n = 6 (m < n): \\
 \begin{bmatrix} 1 & u1 & u1 & u1 & u1 & u1 \\ 1 & 1 & u2 & u2 & u2 & u2 \\ v1 & 1 & a & a & a & a \\ v1 & v2 & a & a & a & a \\ v1 & v2 & a & a & a & a \end{bmatrix}
 \end{array}$$

where a denotes an element of the original matrix which is unchanged, v_i denotes an element of the vector defining $H(i)$, and u_i an element of the vector defining $G(i)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lacon

Estimates the 1-norm of a square matrix, using the reverse communication for evaluating matrix-vector products.

Syntax

```
void pslacon (MKL_INT *n , float *v , MKL_INT *iv , MKL_INT *jv , MKL_INT *descv ,
float *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *isgn , float *est ,
MKL_INT *kase );
```

```
void pdlacon (MKL_INT *n , double *v , MKL_INT *iv , MKL_INT *jv , MKL_INT *descv ,
double *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *isgn , double *est ,
MKL_INT *kase );
```

```
void pclacon (MKL_INT *n , MKL_Complex8 *v , MKL_INT *iv , MKL_INT *jv , MKL_INT
*descv , MKL_Complex8 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , float *est ,
MKL_INT *kase );
```

```
void pzlacon (MKL_INT *n , MKL_Complex16 *v , MKL_INT *iv , MKL_INT *jv , MKL_INT
*descv , MKL_Complex16 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , double *est ,
MKL_INT *kase );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lacon` function estimates the 1-norm of a square, real/unitary distributed matrix A . Reverse communication is used for evaluating matrix-vector products. x and v are aligned with the distributed matrix A , this information is implicitly contained within iv , ix , $descv$, and $descx$.

Input Parameters

| | |
|----------|---|
| n | (global) The length of the distributed vectors v and x . $n \geq 0$. |
| v | (local). Pointer into the local memory to an array of size $LOCr(n+\text{mod}(iv-1, mb_v))$. On the final return, $v = a*w$, where $est = \text{norm}(v)/\text{norm}(w)$ (w is not returned). |
| iv, jv | (global) The row and column indices in the global matrix V indicating the first row and the first column of the submatrix V , respectively. |
| $descv$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix V . |
| x | (local). Pointer into the local memory to an array of size $LOCr(n+\text{mod}(ix-1, mb_x))$. |
| ix, jx | (global) The row and column indices in the global matrix X indicating the first row and the first column of the submatrix X , respectively. |
| $descx$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix X . |
| $isgn$ | (local). Array of size $LOCr(n+\text{mod}(ix-1, mb_x))$. $isgn$ is aligned with x and v . |
| $kase$ | (local). On the initial call to <code>p?lacon</code> , $kase$ should be 0. |

Output Parameters

| | |
|--------|---|
| x | (local). On an intermediate return, X should be overwritten by $A*X$, if $kase=1$, $A'*X$, if $kase=2$, <code>p?lacon</code> must be re-called with all the other parameters unchanged. |
| est | (global). |
| $kase$ | (local) On an intermediate return, $kase$ is 1 or 2, indicating whether X should be overwritten by $A*X$, or $A'*X$. On the final return from <code>p?lacon</code> , $kase$ is again 0. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laconsb

Looks for two consecutive small subdiagonal elements.

Syntax

```
void pslaconsb (const float *a, const MKL_INT *desca, const MKL_INT *i, const MKL_INT *l, MKL_INT *m, const float *h44, const float *h33, const float *h43h34, float *buf, const MKL_INT *lwork );
```

```
void pdlaconsb (const double *a, const MKL_INT *desca, const MKL_INT *i, const MKL_INT *l, MKL_INT *m, const double *h44, const double *h33, const double *h43h34, double *buf, const MKL_INT *lwork );
```

```
void pclaconsb (const MKL_Complex8 *a , const MKL_INT *desca , const MKL_INT *i , const MKL_INT *l , MKL_INT *m , const MKL_Complex8 *h44 , const MKL_Complex8 *h33 , const MKL_Complex8 *h43h34 , MKL_Complex8 *buf , const MKL_INT *lwork );
```

```
void pzlaconsb (const MKL_Complex16 *a , const MKL_INT *desca , const MKL_INT *i , const MKL_INT *l , MKL_INT *m , const MKL_Complex16 *h44 , const MKL_Complex16 *h33 , const MKL_Complex16 *h43h34 , MKL_Complex16 *buf , const MKL_INT *lwork );
```

Include Files

- mkl_scalapack.h

Description

The p?laconsb function looks for two consecutive small subdiagonal elements by analyzing the effect of starting a double shift *QR* iteration given by *h44*, *h33*, and *h43h34* to see if this process makes a subdiagonal negligible.

Input Parameters

| | |
|---|--|
| <i>a</i> | (local) Array of size $lld_a * LOCc(n_a)$. On entry, the Hessenberg matrix whose tridiagonal part is being scanned. Unchanged on exit. |
| <i>desca</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>i</i> | (global) The global location of the bottom of the unreduced submatrix of <i>A</i> . Unchanged on exit. |
| <i>l</i> | (global) The global location of the top of the unreduced submatrix of <i>A</i> . Unchanged on exit. |
| <i>h44</i> , <i>h33</i> , <i>h43h34</i> | (global). These three values are for the double shift <i>QR</i> iteration. |
| <i>lwork</i> | (local) |

This must be at least $7 * \text{ceil}(\text{ceil}((i-1)/mb_a) / \text{lcm}(nprow, npc1))$. Here lcm is the least common multiple and $nprow * npc1$ is the logical grid size.

Output Parameters

m (global). On exit, this yields the starting location of the QR double shift. This will satisfy:
 $i \leq m \leq i-2$.

buf (local).
 Array of size *lwork*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lacp2

Copies all or part of a distributed matrix to another distributed matrix.

Syntax

```
void pslacp2 (char *uplo , MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb );
void pdlacp2 (char *uplo , MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb );
void pclacp2 (char *uplo , MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb );
void pzlacp2 (char *uplo , MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb );
```

Include Files

- mkl_scalapack.h

Description

The p?lacp2 function copies all or part of a distributed matrix *A* to another distributed matrix *B*. No communication is performed, p?lacp2 performs a local copy $\text{sub}(A) := \text{sub}(B)$, where $\text{sub}(A)$ denotes $A(ia:ia+m-1, a:ja+n-1)$ and $\text{sub}(B)$ denotes $B(ib:ib+m-1, jb:jb+n-1)$.

p?lacp2 requires that only dimension of the matrix operands is distributed.

Input Parameters

uplo (global) Specifies the part of the distributed matrix $\text{sub}(A)$ to be copied:
 = 'U': Upper triangular part is copied; the strictly lower triangular part of $\text{sub}(A)$ is not referenced;

= 'L': Lower triangular part is copied; the strictly upper triangular part of $\text{sub}(A)$ is not referenced.

Otherwise: all of the matrix $\text{sub}(A)$ is copied.

| | |
|---------------|--|
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the m -by- n distributed matrix $\text{sub}(A)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of $\text{sub}(B)$, respectively. |
| <i>descb</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |

Output Parameters

| | |
|----------|--|
| <i>b</i> | (local). Pointer into the local memory to an array of size $lld_b * LOCC(jb+n-1)$. This array contains on exit the local pieces of the distributed matrix $\text{sub}(B)$ set as follows: if $uplo = 'U'$, $B(ib+i-1, jb+j-1) = A(ia+i-1, ja+j-1)$, $1 \leq i \leq m$, $1 \leq j \leq n$; if $uplo = 'L'$, $B(ib+i-1, jb+j-1) = A(ia+i-1, ja+j-1)$, $j \leq i \leq m$, $1 \leq j \leq n$; otherwise, $B(ib+i-1, jb+j-1) = A(ia+i-1, ja+j-1)$, $1 \leq i \leq m$, $1 \leq j \leq n$. |
|----------|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lcp3

Copies from a global parallel array into a local replicated array or vice versa.

Syntax

```
void pslcp3 (const MKL_INT *m, const MKL_INT *i, float *a, const MKL_INT *desca, float *b, const MKL_INT *ldb, const MKL_INT *ii, const MKL_INT *jj, const MKL_INT *rev );
```

```

void pdlacp3 (const MKL_INT *m, const MKL_INT *i, double *a, const MKL_INT *desca,
double *b, const MKL_INT *ldb, const MKL_INT *ii, const MKL_INT *jj, const MKL_INT
*rev );

void pclacp3 (const MKL_INT *m, const MKL_INT *i, MKL_Complex8 *a, const MKL_INT
*desca, MKL_Complex8 *b, const MKL_INT *ldb, const MKL_INT *ii, const MKL_INT *jj,
const MKL_INT *rev);

void pzlacp3 (const MKL_INT *m, const MKL_INT *i, MKL_Complex16 *a, const MKL_INT
*desca, MKL_Complex16 *b, const MKL_INT *ldb, const MKL_INT *ii, const MKL_INT *jj,
const MKL_INT *rev);

```

Include Files

- mkl_scalapack.h

Description

This is an auxiliary function that copies from a global parallel array into a local replicated array or vice versa. Note that the entire submatrix that is copied gets placed on one node or more. The receiving node can be specified precisely, or all nodes can receive, or just one row or column of nodes.

Optimization Notice

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Input Parameters

| | |
|--------------|--|
| <i>m</i> | (global) <i>m</i> is the order of the square submatrix that is copied. <i>m</i> ≥ 0. Unchanged on exit. |
| <i>i</i> | (global) The matrix element $A(i, i)$ is the global location that the copying starts from. Unchanged on exit. |
| <i>a</i> | (local) Array of size $lld_a * LOCc(n_a)$. On entry, the parallel matrix to be copied into or from. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>b</i> | (local) Array of size $ldb * LOCc(m)$. If <i>rev</i> = 0, this is the global portion of the matrix $A(i:i+m-1, i:i+m-1)$. If <i>rev</i> = 1, this is unchanged on exit. |
| <i>ldb</i> | (local) |

The leading dimension of B .

ii, jj

(global) By using rev 0 and 1, data can be sent out and returned again. If $rev = 0$, then ii is destination row index and jj is destination column index for the node(s) receiving the replicated matrix B . If $ii \geq 0, jj \geq 0$, then node (ii, jj) receives the data. If $ii = -1, jj \geq 0$, then all rows in column jj receive the data. If $ii \geq 0, jj = -1$, then all cols in row ii receive the data. If $ii = -1, jj = -1$, then all nodes receive the data. If $rev \neq 0$, then ii is the source row index for the node(s) sending the replicated B .

rev

(global) Use $rev = 0$ to send global matrix A into locally replicated matrix B (on node (ii, jj)). Use $rev \neq 0$ to send locally replicated B from node (ii, jj) to its owner (which changes depending on its location in A) into the global A .

Output Parameters

a

On exit, if $rev = 1$, the copied data. Unchanged on exit if $rev = 0$.

b

If $rev = 1$, this is unchanged on exit.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lacpy

Copies all or part of one two-dimensional array to another.

Syntax

```
void pslacpy (char *uplo , MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb );
```

```
void pdlacpy (char *uplo , MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb );
```

```
void pclacpy (char *uplo , MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb );
```

```
void pzlacpy (char *uplo , MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lacpy` function copies all or part of a distributed matrix A to another distributed matrix B . No communication is performed, `p?lacpy` performs a local copy $\text{sub}(B) := \text{sub}(A)$, where $\text{sub}(A)$ denotes $A(ia:ia+m-1, ja:ja+n-1)$ and $\text{sub}(B)$ denotes $B(ib:ib+m-1, jb:jb+n-1)$.

Input Parameters

| | |
|---------------|--|
| <i>uplo</i> | (global) Specifies the part of the distributed matrix $\text{sub}(A)$ to be copied: = 'U': Upper triangular part; the strictly lower triangular part of $\text{sub}(A)$ is not referenced; = 'L': Lower triangular part; the strictly upper triangular part of $\text{sub}(A)$ is not referenced. Otherwise: all of the matrix $\text{sub}(A)$ is copied. |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOCc(ja+n-1)$. On entry, this array contains the local pieces of the distributed matrix $\text{sub}(A)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>ib, jb</i> | (global) The row and column indices in the global matrix B indicating the first row and the first column of $\text{sub}(B)$ respectively. |
| <i>descb</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . |

Output Parameters

| | |
|----------|--|
| <i>b</i> | (local). Pointer into the local memory to an array of size $lld_b * LOCc(jb+n-1)$. This array contains on exit the local pieces of the distributed matrix $\text{sub}(B)$ set as follows: if $uplo = 'U'$, $B(ib+i-1, jb+j-1) = A(ia+i-1, ja+j-1)$, $1 \leq j \leq n$; if $uplo = 'L'$, $B(ib+i-1, jb+j-1) = A(ia+i-1, ja+j-1)$, $j \leq m$, $1 \leq j \leq n$; otherwise, $B(ib+i-1, jb+j-1) = A(ia+i-1, ja+j-1)$, $1 \leq j \leq m$, $1 \leq j \leq n$. |
|----------|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laevswp

Moves the eigenvectors from where they are computed to ScaLAPACK standard block cyclic array.

Syntax

```
void pslaevswp (MKL_INT *n , float *zin , MKL_INT *ldzi , float *z , MKL_INT *iz ,
MKL_INT *jz , MKL_INT *descz , MKL_INT *nvs , MKL_INT *key , float *work , MKL_INT
*lwork );
```

```
void pdlaevswp (MKL_INT *n , double *zin , MKL_INT *ldzi , double *z , MKL_INT *iz ,
MKL_INT *jz , MKL_INT *descz , MKL_INT *nvs , MKL_INT *key , double *work , MKL_INT
*lwork );
```

```
void pclaevswp (MKL_INT *n , float *zin , MKL_INT *ldzi , MKL_Complex8 *z , MKL_INT
*iz , MKL_INT *jz , MKL_INT *descz , MKL_INT *nvs , MKL_INT *key , float *rwork ,
MKL_INT *lrwork );
```

```
void pzlaevswp (MKL_INT *n , double *zin , MKL_INT *ldzi , MKL_Complex16 *z , MKL_INT
*iz , MKL_INT *jz , MKL_INT *descz , MKL_INT *nvs , MKL_INT *key , double *rwork ,
MKL_INT *lrwork );
```

Include Files

- mkl_scalapack.h

Description

The `p?laevswp` function moves the eigenvectors (potentially unsorted) from where they are computed, to a ScaLAPACK standard block cyclic array, sorted so that the corresponding eigenvalues are sorted.

Input Parameters

np = the number of rows local to a given process.

nq = the number of columns local to a given process.

| | |
|---------------|---|
| <i>n</i> | (global) The order of the matrix <i>A</i> . $n \geq 0$. |
| <i>zin</i> | (local). Array of size $ldzi * nvs[iam+1]$. The eigenvectors on input. <i>iam</i> is a process rank from $[0, nprocs)$ interval. Each eigenvector resides entirely in one process. Each process holds a contiguous set of $nvs[iam+1]$ eigenvectors. The global number of the first eigenvector that the process holds is: ((sum for $i=[0, iam]$ of $nvs[i]$)+1). |
| <i>ldzi</i> | (local) The leading dimension of the <i>zin</i> array. |
| <i>iz, jz</i> | (global) The row and column indices in the global matrix <i>Z</i> indicating the first row and the first column of the submatrix <i>Z</i> , respectively. |
| <i>descz</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>Z</i> . |
| <i>nvs</i> | (global) Array of size $nprocs+1$ $nvs[i]$ = number of eigenvectors held by processes $[0, i)$ $nvs[0]$ = number of eigenvectors held by processes $[0, 0) = 0$ |

| | |
|---------------|--|
| | $nvs[nprocs]$ = number of eigenvectors held by $[0, nprocs)$ = total number of eigenvectors. |
| <i>key</i> | (global) Array of size n . Indicates the actual index (after sorting) for each of the eigenvectors. |
| <i>rwork</i> | (local). Array of size $lrwork$. |
| <i>lrwork</i> | (local) Size of <i>work</i> . |

Output Parameters

| | |
|----------|---|
| <i>z</i> | (local). Array of global size $n * n$ and of local size $lld_z * nq$. The eigenvectors on output. The eigenvectors are distributed in a block cyclic manner in both dimensions, with a block size of nb . |
|----------|---|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lahrd

Reduces the first nb columns of a general rectangular matrix A so that elements below the k -th subdiagonal are zero, by an orthogonal/unitary transformation, and returns auxiliary matrices that are needed to apply the transformation to the unreduced part of A .

Syntax

```
void pslahrd (MKL_INT *n , MKL_INT *k , MKL_INT *nb , float *a , MKL_INT *ia , MKL_INT
 *ja , MKL_INT *desca , float *tau , float *t , float *y , MKL_INT *iy , MKL_INT *jy ,
MKL_INT *descy , float *work );
```

```
void pdlahrd (MKL_INT *n , MKL_INT *k , MKL_INT *nb , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *tau , double *t , double *y , MKL_INT *iy ,
MKL_INT *jy , MKL_INT *descy , double *work );
```

```
void pclahrd (MKL_INT *n , MKL_INT *k , MKL_INT *nb , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *t , MKL_Complex8 *y ,
MKL_INT *iy , MKL_INT *jy , MKL_INT *descy , MKL_Complex8 *work );
```

```
void pzlahrd (MKL_INT *n , MKL_INT *k , MKL_INT *nb , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *t , MKL_Complex16
*y , MKL_INT *iy , MKL_INT *jy , MKL_INT *descy , MKL_Complex16 *work );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lahrd` function reduces the first nb columns of a real general n -by- $(n-k+1)$ distributed matrix $A(ia:ia+n-1, ja:ja+n-k)$ so that elements below the k -th subdiagonal are zero. The reduction is performed by an orthogonal/unitary similarity transformation Q^*A*Q . The function returns the matrices V and T which determine Q as a block reflector $I-V*T*V'$, and also the matrix $Y = A*V*T$.

This is an auxiliary function called by `p?gehrd`. In the following comments $sub(A)$ denotes $A(ia:ia+n-1, ja:ja+n-1)$.

Input Parameters

| | |
|----------|---|
| n | (global) The order of the distributed matrix $sub(A)$. $n \geq 0$. |
| k | (global) The offset for the reduction. Elements below the k -th subdiagonal in the first nb columns are reduced to zero. |
| nb | (global) The number of columns to be reduced. |
| a | (local). Pointer into the local memory to an array of size $lld_a * LOCc(ja+n-k)$. On entry, this array contains the local pieces of the n -by- $(n-k+1)$ general distributed matrix $A(ia:ia+n-1, ja:ja+n-k)$. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $sub(A)$, respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| iy, jy | (global) The row and column indices in the global matrix Y indicating the first row and the first column of the matrix $sub(Y)$, respectively. |
| $descy$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix Y . |
| $work$ | (local). Array of size nb . |

Output Parameters

| | |
|-------|---|
| a | (local). On exit, the elements on and above the k -th subdiagonal in the first nb columns are overwritten with the corresponding elements of the reduced distributed matrix; the elements below the k -th subdiagonal, with the array tau , represent the matrix Q as a product of elementary reflectors. The other columns of the matrix $A(ia:ia+n-1, ja:ja+n-k)$ are unchanged. (See <i>Application Notes</i> below.) |
| tau | (local) |

Array of size $LOCc(j_{a+n-2})$. The scalar factors of the elementary reflectors (see *Application Notes* below). τ is tied to the distributed matrix A .

t (local)
Array of size $nb_a * nb_a$. The upper triangular matrix T .

y (local).
Pointer into the local memory to an array of size $lld_y * nb_a$. On exit, this array contains the local pieces of the n -by- nb distributed matrix Y . $lld_y \geq LOCr(i_{a+n-1})$.

Application Notes

The matrix Q is represented as a product of nb elementary reflectors

$$Q = H(1) * H(2) * \dots * H(nb).$$

Each $H(i)$ has the form

$$H(i) = I - \tau * v * v',$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1:i+k-1) = 0$, $v(i+k) = 1$; $v(i+k+1:n)$ is stored on exit in $A(i_{a+i+k}:i_{a+n-1}, j_{a+i-1})$, and τ in $\tau[j_{a+i-2}]$.

The elements of the vectors v together form the $(n-k+1)$ -by- nb matrix V which is needed, with T and Y , to apply the transformation to the unreduced part of the matrix, using an update of the form: $A(i_{a:i_{a+n-1}}, j_{a:j_{a+n-k}}) := (I - V * T * V') * (A(i_{a:i_{a+n-1}}, j_{a:j_{a+n-k}}) - Y * V')$. The contents of $A(i_{a:i_{a+n-1}}, j_{a:j_{a+n-k}})$ on exit are illustrated by the following example with $n = 7$, $k = 3$, and $nb = 2$:

$$\begin{bmatrix} a & h & a & a & a \\ a & h & a & a & a \\ a & h & a & a & a \\ h & h & a & a & a \\ v1 & h & a & a & a \\ v1 & v2 & a & a & a \\ v1 & v2 & a & a & a \end{bmatrix}$$

where a denotes an element of the original matrix $A(i_{a:i_{a+n-1}}, j_{a:j_{a+n-k}})$, h denotes a modified element of the upper Hessenberg matrix H , and v_i denotes an element of the vector defining $H(i)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laiect

Exploits IEEE arithmetic to accelerate the computations of eigenvalues.

Syntax

```
void pslaiect (float *sigma , MKL_INT *n , float *d , MKL_INT *count );
void pdlaiectb (float *sigma , MKL_INT *n , float *d , MKL_INT *count );
void pdlaiectl (float *sigma , MKL_INT *n , float *d , MKL_INT *count );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?laiect` function computes the number of negative eigenvalues of $(A - \sigma I)$. This implementation of the Sturm Sequence loop exploits IEEE arithmetic and has no conditionals in the innermost loop. The signbit for real function `pslaiect` is assumed to be bit 32. Double-precision functions `pdlaiectb` and `pdlaiectl` differ in the order of the double precision word storage and, consequently, in the signbit location. For `pdlaiectb`, the double precision word is stored in the big-endian word order and the signbit is assumed to be bit 32. For `pdlaiectl`, the double precision word is stored in the little-endian word order and the signbit is assumed to be bit 64.

This is a ScaLAPACK internal function and arguments are not checked for unreasonable values.

Input Parameters

| | |
|--------------|--|
| <i>sigma</i> | The shift. <code>p?laiect</code> finds the number of eigenvalues less than equal to <i>sigma</i> . |
| <i>n</i> | The order of the tridiagonal matrix <i>T</i> . $n \geq 1$. |
| <i>d</i> | Array of size $2n-1$. On entry, this array contains the diagonals and the squares of the off-diagonal elements of the tridiagonal matrix <i>T</i> . These elements are assumed to be interleaved in memory for better cache performance. The diagonal entries of <i>T</i> are in the entries $d[0], d[2], \dots, d[2n-2]$, while the squares of the off-diagonal entries are $d[1], d[3], \dots, d[2n-3]$. To avoid overflow, the matrix must be scaled so that its largest entry is no greater than $overflow^{(1/2)} * underflow^{(1/4)}$ in absolute value, and for greatest accuracy, it should not be much smaller than that. |

Output Parameters

| | |
|----------|---|
| <i>n</i> | The count of the number of eigenvalues of <i>T</i> less than or equal to <i>sigma</i> . |
|----------|---|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lamve

Copies all or part of one two-dimensional distributed array to another.

Syntax

```
void pslamve(char* uplo, MKL_INT* m, MKL_INT* n, float* a, MKL_INT* ia, MKL_INT* ja,
MKL_INT* desca, float* b, MKL_INT* ib, MKL_INT* jb, MKL_INT* descb, float* dwork);
void pdlamve(char* uplo, MKL_INT* m, MKL_INT* n, double* a, MKL_INT* ia, MKL_INT* ja,
MKL_INT* desca, double* b, MKL_INT* ib, MKL_INT* jb, MKL_INT* descb, double* dwork);
```

Include Files

- `mkl_scalapack.h`

Description

`p?lamve` copies all or part of a distributed matrix A to another distributed matrix B . There is no alignment assumptions at all except that A and B are of the same size.

Optimization Notice

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Input Parameters

| | |
|--------------|--|
| <i>uplo</i> | (global) Specifies the part of the distributed matrix $\text{sub}(A)$ to be copied: = 'U': Upper triangular part is copied; the strictly lower triangular part of $\text{sub}(A)$ is not referenced; = 'L': Lower triangular part is copied; the strictly upper triangular part of $\text{sub}(A)$ is not referenced; Otherwise: All of the matrix $\text{sub}(A)$ is copied. |
| <i>m</i> | (global) The number of rows to be operated on, which is the number of rows of the distributed matrix $\text{sub}(A)$. $m \geq 0$. |
| <i>n</i> | (global) The number of columns to be operated on, which is the number of columns of the distributed matrix $\text{sub}(A)$. $n \geq 0$. |
| <i>a</i> | (local) pointer into the local memory to an array of size $lld_a * LOC_c(ja + n - 1)$. This array contains the local pieces of the distributed matrix $\text{sub}(A)$ to be copied from. |
| <i>ia</i> | (global) The row index in the global matrix A indicating the first row of $\text{sub}(A)$. |
| <i>ja</i> | (global) The column index in the global matrix A indicating the first column of $\text{sub}(A)$. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>ib</i> | (global) The row index in the global matrix B indicating the first row of $\text{sub}(B)$. |

| | |
|--------------|--|
| <i>jb</i> | (global) The column index in the global matrix <i>B</i> indicating the first column of sub(<i>B</i>). |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>B</i> . |
| <i>dwork</i> | (local workspace) array If <i>uplo</i> = 'U' or <i>uplo</i> = 'L' and number of processors > 1, the length of <i>dwork</i> is at least as large as the length of <i>b</i> . Otherwise, <i>dwork</i> is not referenced. |

OUTPUT Parameters

| | |
|----------|--|
| <i>b</i> | (local) pointer into the local memory to an array of size <i>lld_b</i> * <i>LOC_c(jb + n - 1)</i> . This array contains on exit the local pieces of the distributed matrix sub(<i>B</i>). |
|----------|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lange

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a general rectangular matrix.

Syntax

```
float pslange (char *norm , MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *work );
double pdlange (char *norm , MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *work );
float pclange (char *norm , MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *work );
double pzlange (char *norm , MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *work );
```

Include Files

- mkl_scalapack.h

Description

The p?lange function returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$.

Input Parameters

| | |
|-------------|---|
| <i>norm</i> | (global) Specifies what value is returned by the function: = 'M' or 'm': $\text{val} = \max(\text{abs}(A_{ij}))$, largest absolute value of the matrix <i>A</i> , it s not a matrix norm. |
|-------------|---|

| | |
|---------------|--|
| | = '1' or 'O' or 'o': $val = \text{norm1}(A)$, 1-norm of the matrix A (maximum column sum), |
| | = 'I' or 'i': $val = \text{normI}(A)$, infinity norm of the matrix A (maximum row sum), |
| | = 'F', 'f', 'E' or 'e': $val = \text{normF}(A)$, Frobenius norm of the matrix A (square root of sum of squares). |
| <i>m</i> | (global) The number of rows in the distributed matrix sub(A). When $m = 0$, <i>p?</i> <i>lange</i> is set to zero. $m \geq 0$. |
| <i>n</i> | (global) The number of columns in the distributed matrix sub(A). When $n = 0$, <i>p?</i> <i>lange</i> is set to zero. $n \geq 0$. |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOCc(ja+n-1)$ containing the local pieces of the distributed matrix sub(A). |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix sub(A), respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>work</i> | (local). Array size <i>lwork</i> . $lwork \geq 0$ if <i>norm</i> = 'M' or 'm' (not referenced), $nq0$ if <i>norm</i> = '1', 'O' or 'o', $mp0$ if <i>norm</i> = 'I' or 'i', 0 if <i>norm</i> = 'F', 'f', 'E' or 'e' (not referenced), where $irow = \text{mod}(ia-1, mb_a)$, $icol = \text{mod}(ja-1, nb_a)$, $iarow = \text{indxg2p}(ia, mb_a, myrow, rsrc_a, nprow)$, $iacol = \text{indxg2p}(ja, nb_a, mycol, csrc_a, npcot)$, $mp0 = \text{numroc}(m+irow, mb_a, myrow, iarow, nprow)$, $nq0 = \text{numroc}(n+icol, nb_a, mycol, icol, npcot)$, <i>indxg2p</i> and <i>numroc</i> are ScaLAPACK tool functions; <i>myrow</i> , <i>mycol</i> , <i>nprow</i> , and <i>npcol</i> can be determined by calling the function <i>blacs_gridinfo</i> . |

Output Parameters

val The value returned by the function.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lanhs

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of an upper Hessenberg matrix.

Syntax

```
float pslanhs (char *norm , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *work );
```

```
double pdlanhs (char *norm , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *work );
```

```
float pclanhs (char *norm , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *work );
```

```
double pzlanhs (char *norm , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , double *work );
```

Include Files

- mkl_scalapack.h

Description

The p?lanhs function returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of an upper Hessenberg distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$.

Input Parameters

| | |
|---------------|--|
| <i>norm</i> | Specifies the value to be returned by the function: = 'M' or 'm': $val = \max(\text{abs}(A_{ij}))$, largest absolute value of the matrix <i>A</i> . = '1' or 'O' or 'o': $val = \text{norm1}(A)$, 1-norm of the matrix <i>A</i> (maximum column sum), = 'I' or 'i': $val = \text{normI}(A)$, infinity norm of the matrix <i>A</i> (maximum row sum), = 'F', 'f', 'E' or 'e': $val = \text{normF}(A)$, Frobenius norm of the matrix <i>A</i> (square root of sum of squares). |
| <i>n</i> | (global) The number of columns in the distributed matrix sub(<i>A</i>). When $n = 0$, p?lanhs is set to zero. $n \geq 0$. |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOCC(\text{ja}+n-1)$ containing the local pieces of the distributed matrix sub(<i>A</i>). |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix sub(<i>A</i>), respectively. |

desca (global and local) array of size *dlen_*. The array descriptor for the distributed matrix A.

work (local).
 Array of size *lwork*.
lwork \geq 0 if *norm* = 'M' or 'm' (not referenced),
*nq*0 if *norm* = '1', 'O' or 'o',
*mp*0 if *norm* = 'I' or 'i',
 0 if *norm* = 'F', 'f', 'E' or 'e' (not referenced),
 where
iroffa = mod(*ia*-1, *mb_a*), *icoffa* = mod(*ja*-1, *nb_a*),
iarow = indxg2p(*ia*, *mb_a*, *myrow*, *rsrc_a*, *nrow*),
iacol = indxg2p(*ja*, *nb_a*, *mycol*, *csrc_a*, *ncol*),
mp0 = numroc(*m*+*iroffa*, *mb_a*, *myrow*, *iarow*, *nrow*),
nq0 = numroc(*n*+*icoffa*, *nb_a*, *mycol*, *iacol*, *ncol*),
 indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *imycol*, *nrow*,
 and *ncol* can be determined by calling the function `blacs_gridinfo`.

Output Parameters

val The value returned by the function.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lansy, p?lanhe

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a real symmetric or a complex Hermitian matrix.

Syntax

```
float pslansy (char *norm , char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT
 *ja , MKL_INT *desca , float *work );

double pdlansy (char *norm , char *uplo , MKL_INT *n , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *work );

float pclansy (char *norm , char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *work );

double pzlansy (char *norm , char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *work );

float pplanhe (char *norm , char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *work );

double pzlanhe (char *norm , char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *work );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lansy` and `p?lanhe` functions return the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$.

Input Parameters

| | |
|---------------|---|
| <i>norm</i> | <p>(global) Specifies what value is returned by the function:</p> <ul style="list-style-type: none"> = 'M' or 'm': $\text{val} = \max(\text{abs}(A_{ij}))$, largest absolute value of the matrix <i>A</i>, it s not a matrix norm. = '1' or 'O' or 'o': $\text{val} = \text{norm1}(A)$, 1-norm of the matrix <i>A</i> (maximum column sum), = 'I' or 'i': $\text{val} = \text{normI}(A)$, infinity norm of the matrix <i>A</i> (maximum row sum), = 'F', 'f', 'E' or 'e': $\text{val} = \text{normF}(A)$, Frobenius norm of the matrix <i>A</i> (square root of sum of squares). |
| <i>uplo</i> | <p>(global) Specifies whether the upper or lower triangular part of the symmetric matrix $\text{sub}(A)$ is to be referenced.</p> <ul style="list-style-type: none"> = 'U': Upper triangular part of $\text{sub}(A)$ is referenced, = 'L': Lower triangular part of $\text{sub}(A)$ is referenced. |
| <i>n</i> | <p>(global)</p> <p>The number of columns in the distributed matrix $\text{sub}(A)$. When $n = 0$, <code>p?lansy</code> is set to zero. $n \geq 0$.</p> |
| <i>a</i> | <p>(local).</p> <p>Pointer into the local memory to an array of size $\text{lld}_a * \text{LOCc}(\text{ja}+n-1)$ containing the local pieces of the distributed matrix $\text{sub}(A)$.</p> <p>If <i>uplo</i> = 'U', the leading <i>n</i>-by-<i>n</i> upper triangular part of $\text{sub}(A)$ contains the upper triangular matrix whose norm is to be computed, and the strictly lower triangular part of this matrix is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i>-by-<i>n</i> lower triangular part of $\text{sub}(A)$ contains the lower triangular matrix whose norm is to be computed, and the strictly upper triangular part of $\text{sub}(A)$ is not referenced.</p> |
| <i>ia, ja</i> | <p>(global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively.</p> |
| <i>desca</i> | <p>(global and local) array of size <i>dlen_</i>. The array descriptor for the distributed matrix <i>A</i>.</p> |
| <i>work</i> | <p>(local).</p> <p>Array of size <i>lwork</i>.</p> <p>$\text{lwork} \geq 0$ if <i>norm</i> = 'M' or 'm' (not referenced),</p> <p>$2 * nq0 + mp0 + ldw$ if <i>norm</i> = '1', 'O' or 'o', 'I' or 'i',</p> |

where ldw is given by:

```
if( nprow≠npcol ) then
  ldw = mb_a*iceil(iceil(np0,mb_a), (lcm/nprow))
else
  ldw = 0
end if
```

0 if $norm = 'F', 'f', 'E' \text{ or } 'e'$ (not referenced),

where lcm is the least common multiple of $nprow$ and $npcol$, $lcm = ilcm(nprow, npcol)$ and $iceil(x,y)$ is a ScaLAPACK function that returns ceiling (x/y) .

```
iroffa = mod(ia-1, mb_a), icoffa = mod(ja-1, nb_a),
```

```
iarow = indxg2p(ia, mb_a, myrow, rsrc_a, nprow),
```

```
iacol = indxg2p(ja, nb_a, mycol, csrc_a, npcol),
```

```
mp0 = numroc(m+iroffa, mb_a, myrow, iarow, nprow),
```

```
nq0 = numroc(n+icoffa, nb_a, mycol, iacol, npcol),
```

$ilcm$, $iceil$, $indxg2p$, and $numroc$ are ScaLAPACK tool functions; $myrow$, $mycol$, $nprow$, and $npcol$ can be determined by calling the function `blacs_gridinfo`.

Output Parameters

val The value returned by the function.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lantr

Returns the value of the 1-norm, Frobenius norm, infinity-norm, or the largest absolute value of any element, of a triangular matrix.

Syntax

```
float pslantr (char *norm , char *uplo , char *diag , MKL_INT *m , MKL_INT *n , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *work );
```

```
double pdlantr (char *norm , char *uplo , char *diag , MKL_INT *m , MKL_INT *n ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *work );
```

```
float pclantr (char *norm , char *uplo , char *diag , MKL_INT *m , MKL_INT *n ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *work );
```

```
double pzlantr (char *norm , char *uplo , char *diag , MKL_INT *m , MKL_INT *n ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *work );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lantr` function returns the value of the 1-norm, or the Frobenius norm, or the infinity norm, or the element of largest absolute value of a trapezoidal or triangular distributed matrix $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$.

Input Parameters

| | |
|---------------|--|
| <i>norm</i> | (global) Specifies what value is returned by the function: = 'M' or 'm': $\text{val} = \max(\text{abs}(A_{ij}))$, largest absolute value of the matrix A , it is not a matrix norm. = '1' or 'O' or 'o': $\text{val} = \text{norm1}(A)$, 1-norm of the matrix A (maximum column sum), = 'I' or 'i': $\text{val} = \text{normI}(A)$, infinity norm of the matrix A (maximum row sum), = 'F', 'f', 'E' or 'e': $\text{val} = \text{normF}(A)$, Frobenius norm of the matrix A (square root of sum of squares). |
| <i>uplo</i> | (global) Specifies whether the upper or lower triangular part of the symmetric matrix $\text{sub}(A)$ is to be referenced. = 'U': Upper trapezoidal, = 'L': Lower trapezoidal. Note that $\text{sub}(A)$ is triangular instead of trapezoidal if $m = n$. |
| <i>diag</i> | (global) Specifies whether the distributed matrix $\text{sub}(A)$ has unit diagonal. = 'N': Non-unit diagonal. = 'U': Unit diagonal. |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. When $m = 0$, <code>p?lantr</code> is set to zero. $m \geq 0$. |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. When $n = 0$, <code>p?lantr</code> is set to zero. $n \geq 0$. |
| <i>a</i> | (local). Pointer into the local memory to an array of size $\text{lld_a} * \text{LOCc}(\text{ja}+n-1)$ containing the local pieces of the distributed matrix $\text{sub}(A)$. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size dlen_ . The array descriptor for the distributed matrix A . |

work (local).
 Array size *lwork*.
 $lwork \geq 0$ if *norm* = 'M' or 'm' (not referenced),
 $nq0$ if *norm* = '1', 'O' or 'o',
 $mp0$ if *norm* = 'I' or 'i',
 0 if *norm* = 'F', 'f', 'E' or 'e' (not referenced),
 $irowfa = \text{mod}(ia-1, mb_a)$, $icoffa = \text{mod}(ja-1, nb_a)$,
 $iarow = \text{indxg2p}(ia, mb_a, myrow, rsrc_a, nprow)$,
 $iacol = \text{indxg2p}(ja, nb_a, mycol, csrc_a, npcot)$,
 $mp0 = \text{numroc}(m+irowfa, mb_a, myrow, iarow, nprow)$,
 $nq0 = \text{numroc}(n+icoffa, nb_a, mycol, iacol, npcot)$,
 indxg2p and numroc are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*,
 and *npcot* can be determined by calling the function `blacs_gridinfo`.

Output Parameters

val The value returned by the function.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lapiv

Applies a permutation matrix to a general distributed matrix, resulting in row or column pivoting.

Syntax

```
void pslapiv (char *direc , char *rowcol , char *pivroc , MKL_INT *m , MKL_INT *n ,
float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_INT *ip ,
MKL_INT *jp , MKL_INT *descip , MKL_INT *iwork );

void pdlapiv (char *direc , char *rowcol , char *pivroc , MKL_INT *m , MKL_INT *n ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_INT *ip ,
MKL_INT *jp , MKL_INT *descip , MKL_INT *iwork );

void pclapiv (char *direc , char *rowcol , char *pivroc , MKL_INT *m , MKL_INT *n ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv , MKL_INT
*ip , MKL_INT *jp , MKL_INT *descip , MKL_INT *iwork );

void pzlapiiv (char *direc , char *rowcol , char *pivroc , MKL_INT *m , MKL_INT *n ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *ipiv ,
MKL_INT *ip , MKL_INT *jp , MKL_INT *descip , MKL_INT *iwork );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lapiv` function applies either P (permutation matrix indicated by $ipiv$) or $inv(P)$ to a general m -by- n distributed matrix $sub(A) = A(ia:ia+m-1, ja:ja+n-1)$, resulting in row or column pivoting. The pivot vector may be distributed across a process row or a column. The pivot vector should be aligned with the distributed matrix A . This function will transpose the pivot vector, if necessary.

For example, if the row pivots should be applied to the columns of $sub(A)$, pass `rowcol='C'` and `pivroc='C'`.

Input Parameters

| | |
|---------------------|--|
| <code>direc</code> | (global) Specifies in which order the permutation is applied: = 'F' (Forward): Applies pivots forward from top of matrix. Computes $P * sub(A)$. = 'B' (Backward): Applies pivots backward from bottom of matrix. Computes $inv(P) * sub(A)$. |
| <code>rowcol</code> | (global) Specifies if the rows or columns are to be permuted: = 'R': Rows will be permuted, = 'C': Columns will be permuted. |
| <code>pivroc</code> | (global) Specifies whether $ipiv$ is distributed over a process row or column: = 'R': $ipiv$ is distributed over a process row, = 'C': $ipiv$ is distributed over a process column. |
| <code>m</code> | (global) The number of rows in the distributed matrix $sub(A)$. When $m = 0$, <code>p?lapiv</code> is set to zero. $m \geq 0$. |
| <code>n</code> | (global) The number of columns in the distributed matrix $sub(A)$. When $n = 0$, <code>p?lapiv</code> is set to zero. $n \geq 0$. |
| <code>a</code> | (local). Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$ containing the local pieces of the distributed matrix $sub(A)$. |
| <code>ia, ja</code> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $sub(A)$, respectively. |
| <code>desca</code> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <code>ipiv</code> | (local) Array of size $lipiv$; |

```

when rowcol='R' or 'r':
  lipiv≥LOCr(ia+m-1) + mb_a if pivroc='C' or 'c',
  lipiv≥LOCc(m + mod(jp-1, nb_p)) if pivroc='R' or 'r', and,
when rowcol='C' or 'c':
  lipiv≥LOCr(n + mod(ip-1, mb_p)) if pivroc='C' or 'c',
  lipiv≥LOCc(ja+n-1) + nb_a if pivroc='R' or 'r'.

```

This array contains the pivoting information. $ipiv(i)$ is the global row (column), local row (column) i was swapped with. When $rowcol='R'$ or $'r'$ and $pivroc='C'$ or $'c'$, or $rowcol='C'$ or $'c'$ and $pivroc='R'$ or $'r'$, the last piece of this array of size mb_a (resp. nb_a) is used as workspace. In those cases, this array is tied to the distributed matrix A .

ip, jp (global) The row and column indices in the global matrix P indicating the first row and the first column of the matrix sub(P), respectively.

$descip$ (global and local) array of size $dlen_$. The array descriptor for the distributed vector $ipiv$.

$iwork$ (local).

Array of size ldw , where ldw is equal to the workspace necessary for transposition, and the storage of the transposed $ipiv$:

Let lcm be the least common multiple of $npro$ and $npcol$.

```

if( *rowcol == 'r' && *pivroc == 'r') {
  if( npro == npc ) {
    ldw = LOCr( n_p + (*jp-1)%nb_p ) + nb_p;
  } else {
    ldw = LOCr( n_p + (*jp-1)%nb_p ) +
    nb_p * ceil( ceil(LOCc(n_p)/nb_p) / (lcm/npc) );
  }
} else if( *rowcol == 'c' && *pivroc == 'c') {
  if( npro == npc ) {
    ldw = LOCc( m_p + (*ip-1)%mb_p ) + mb_p;
  } else {
    ldw = LOCc( m_p + (*ip-1)%mb_p ) +
    mb_p *ceil(ceil(LOCr(m_p)/mb_p) / (lcm/npro) );
  }
} else {
  // iwork is not referenced.

```

Output Parameters

a (local).
On exit, the local pieces of the permuted distributed submatrix.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lapv2

Applies a permutation to an m -by- n distributed matrix.

Syntax

```
void pslapv2 (const char* direc, const char* rowcol, const MKL_INT* m, const MKL_INT*
n, float* a, const MKL_INT* ia, const MKL_INT* ja, const MKL_INT* desca, const
MKL_INT* ipiv, const MKL_INT* ip, const MKL_INT* jp, const MKL_INT* descip);
```

```
void pdlapv2 (const char* direc, const char* rowcol, const MKL_INT* m, const MKL_INT*
n, double* a, const MKL_INT* ia, const MKL_INT* ja, const MKL_INT* desca, const
MKL_INT* ipiv, const MKL_INT* ip, const MKL_INT* jp, const MKL_INT* descip);
```

```
void pclapv2 (const char* direc, const char* rowcol, const MKL_INT* m, const MKL_INT*
n, MKL_Complex8* a, const MKL_INT* ia, const MKL_INT* ja, const MKL_INT* desca, const
MKL_INT* ipiv, const MKL_INT* ip, const MKL_INT* jp, const MKL_INT* descip);
```

```
void pzlapv2 (const char* direc, const char* rowcol, const MKL_INT* m, const MKL_INT*
n, MKL_Complex16* a, const MKL_INT* ia, const MKL_INT* ja, const MKL_INT* desca, const
MKL_INT* ipiv, const MKL_INT* ip, const MKL_INT* jp, const MKL_INT* descip);
```

Include Files

- mkl_scalapack.h

Description

`p?lapv2` applies either P (permutation matrix indicated by `ipiv`) or $\text{inv}(P)$ to an m -by- n distributed matrix $\text{sub}(A)$ denoting $A(ia:ia+m-1, ja:ja+n-1)$, resulting in row or column pivoting. The pivot vector should be aligned with the distributed matrix A . For pivoting the rows of $\text{sub}(A)$, `ipiv` should be distributed along a process column and replicated over all process rows. Similarly, `ipiv` should be distributed along a process row and replicated over all process columns for column pivoting.

Input Parameters

| | |
|---------------------|---|
| <code>direc</code> | (global) Specifies in which order the permutation is applied: = 'F' (Forward) Applies pivots Forward from top of matrix. Computes $P * \text{sub}(A)$; = 'B' (Backward) Applies pivots Backward from bottom of matrix. Computes $\text{inv}(P) * \text{sub}(A)$. |
| <code>rowcol</code> | (global) Specifies if the rows or columns are to be permuted: = 'R' Rows will be permuted, = 'C' Columns will be permuted. |
| <code>m</code> | (global) The number of rows to be operated on, i.e. the number of rows of the distributed submatrix $\text{sub}(A)$. $m \geq 0$. |
| <code>n</code> | (global) The number of columns to be operated on, i.e. the number of columns of the distributed submatrix $\text{sub}(A)$. $n \geq 0$. |
| <code>a</code> | Pointer into local memory to an array of size $lld_a * LOCC(ja+n-1)$. |

On entry, this local array contains the local pieces of the distributed matrix sub(*A*) to which the row or columns interchanges will be applied.

| | |
|---------------|--|
| <i>ia</i> | (global) The row index in the global array <i>a</i> indicating the first row of sub(<i>A</i>). |
| <i>ja</i> | (global) The column index in the global array <i>a</i> indicating the first column of sub(<i>A</i>). |
| <i>desca</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>ipiv</i> | Array, size $\geq \text{LOCr}(m_a) + mb_a$ if <i>rowcol</i> = 'R', $\text{LOCc}(n_a) + nb_a$ otherwise. It contains the pivoting information. <i>ipiv</i> [<i>i</i> - 1] is the global row (column), local row (column) <i>i</i> was swapped with. The last piece of the array of size <i>mb_a</i> or <i>nb_a</i> is used as workspace. <i>ipiv</i> is tied to the distributed matrix <i>A</i> . |
| <i>ip</i> | (global) The global row index of <i>ipiv</i> , which points to the beginning of the submatrix on which to operate. |
| <i>jp</i> | (global) The global column index of <i>ipiv</i> , which points to the beginning of the submatrix on which to operate. |
| <i>descip</i> | (global and local) Array of size 8. The array descriptor for the distributed matrix <i>ipiv</i> . |

Output Parameters

| | |
|----------|---|
| <i>a</i> | On exit, this array contains the local pieces of the permuted distributed matrix. |
|----------|---|

p?laqge

Scales a general rectangular matrix, using row and column scaling factors computed by p?geequ.

Syntax

```
void pslaqge (MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *r , float *c , float *rowcnd , float *colcnd , float *amax , char
*equed );
```

```
void pdlaqge (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *r , double *c , double *rowcnd , double *colcnd , double
*amax , char *equed );
```

```
void pclaqge (MKL_INT *m , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *r , float *c , float *rowcnd , float *colcnd , float *amax ,
char *equed );
```

```
void pzlaqge (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *r , double *c , double *rowcnd , double *colcnd , double
*amax , char *equed );
```

Include Files

- mkl_scalapack.h

Description

The `p?laqge` function equilibrates a general m -by- n distributed matrix $\text{sub}(A) = A(ia:ia+m-1, ja:ja+n-1)$ using the row and scaling factors in the vectors r and c computed by `p?geequ`.

Input Parameters

| | |
|----------|---|
| m | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| n | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| a | (local). Pointer into the local memory to an array of size $lld_a * LOCc(ja+n-1)$. On entry, this array contains the distributed matrix $\text{sub}(A)$. |
| ia, ja | (global) The row and column indices in the global matrix A indicating the first row and the first column of the matrix $\text{sub}(A)$, respectively. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| r | (local). Array of size $LOCr(m_a)$. The row scale factors for $\text{sub}(A)$. r is aligned with the distributed matrix A , and replicated across every process column. r is tied to the distributed matrix A . |
| c | (local). Array of size $LOCc(n_a)$. The row scale factors for $\text{sub}(A)$. c is aligned with the distributed matrix A , and replicated across every process column. c is tied to the distributed matrix A . |
| $rowcnd$ | (local). The global ratio of the smallest $r[i]$ to the largest $r[i]$, $ia-1 \leq i \leq ia+m-2$. |
| $colcnd$ | (local). The global ratio of the smallest $c[i]$ to the largest $c[i]$, $ia-1 \leq i \leq ia+n-2$. |
| $amax$ | (global). Absolute value of largest distributed submatrix entry. |

Output Parameters

| | |
|--------------|--|
| <i>a</i> | (local). On exit, the equilibrated distributed matrix. See <i>equed</i> for the form of the equilibrated distributed submatrix. |
| <i>equed</i> | (global) Specifies the form of equilibration that was done. = 'N': No equilibration = 'R': Row equilibration, that is, sub(<i>A</i>) has been pre-multiplied by $\text{diag}(r[ia-1:ia+m-2])$, = 'C': column equilibration, that is, sub(<i>A</i>) has been post-multiplied by $\text{diag}(c[ja-1:ja+n-2])$, = 'B': Both row and column equilibration, that is, sub(<i>A</i>) has been replaced by $\text{diag}(r[ia-1:ia+m-2]) * \text{sub}(A) * \text{diag}(c[ja-1:ja+n-2])$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laqr0

Computes the eigenvalues of a Hessenberg matrix and optionally returns the matrices from the Schur decomposition.

Syntax

```
void pslaqr0(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* n, MKL_INT* ilo, MKL_INT* ihi,
float* h, MKL_INT* desch, float* wr, float* wi, MKL_INT* iloz, MKL_INT* ihiz, float*
z, MKL_INT* descz, float* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork,
MKL_INT* info, MKL_INT* recllevel);
```

```
void pdlaqr0(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* n, MKL_INT* ilo, MKL_INT* ihi,
double* h, MKL_INT* desch, double* wr, double* wi, MKL_INT* iloz, MKL_INT* ihiz,
double* z, MKL_INT* descz, double* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT*
liwork, MKL_INT* info, MKL_INT* recllevel);
```

Include Files

- mkl_scalapack.h

Description

p?laqr0 computes the eigenvalues of a Hessenberg matrix *H* and, optionally, the matrices *T* and *Z* from the Schur decomposition $H = Z * T * Z^T$, where *T* is an upper quasi-triangular matrix (the Schur form), and *Z* is the orthogonal matrix of Schur vectors.

Optionally *Z* may be postmultiplied into an input orthogonal matrix *Q* so that this function can give the Schur factorization of a matrix *A* which has been reduced to the Hessenberg form *H* by the orthogonal matrix *Q*: $A = Q * H * Q^T = (QZ) * T * (QZ)^T$.

Input Parameters

| | |
|-------------------|---|
| <i>wantt</i> | (global) Non-zero : the full Schur form T is required; Zero : only eigenvalues are required. |
| <i>wantz</i> | (global) Non-zero : the matrix of Schur vectors Z is required; Zero: Schur vectors are not required. |
| <i>n</i> | (global) The order of the Hessenberg matrix H (and Z if <i>wantz</i> is non-zero). $n \geq 0$. |
| <i>ilo, ihi</i> | (global) It is assumed that the matrix H is already upper triangular in rows and columns $1:ilo-1$ and $ihi+1:n$. <i>ilo</i> and <i>ihi</i> are normally set by a previous call to <code>p?gebal</code> , and then passed to <code>p?gehrd</code> when the matrix output by <i>ihi</i> is reduced to Hessenberg form. Otherwise <i>ilo</i> and <i>ihi</i> should be set to 1 and n , respectively. If $n > 0$, then $1 \leq ilo \leq ihi \leq n$. If $n = 0$, then $ilo = 1$ and $ihi = 0$. |
| <i>h</i> | (global) array of size $lld_h * LOC_c(n)$ The upper Hessenberg matrix H . |
| <i>desch</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix H . |
| <i>iloz, ihiz</i> | Specify the rows of the matrix Z to which transformations must be applied if <i>wantz</i> is non-zero, $1 \leq iloz \leq ilo; ihi \leq ihiz \leq n$. |
| <i>z</i> | Array of size $lld_z * LOC_c(n)$. If <i>wantz</i> is non-zero, contains the matrix Z . If <i>wantz</i> equals zero, <i>z</i> is not referenced. |
| <i>descz</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix Z . |
| <i>work</i> | (local workspace) array of size <i>lwork</i> |
| <i>lwork</i> | (local) The length of the workspace array <i>work</i> . |
| <i>iwork</i> | (local workspace) array of size <i>liwork</i> |
| <i>liwork</i> | (local) The length of the workspace array <i>iwork</i> . |
| <i>recllevel</i> | (local) |

Level of recursion. *recllevel* = 0 must hold on entry.

OUTPUT Parameters

| | |
|------------------|--|
| <i>h</i> | On exit, if <i>wantt</i> is non-zero, the matrix <i>H</i> is upper quasi-triangular in rows and columns <i>ilo:ihi</i> , with 1-by-1 and 2-by-2 blocks on the main diagonal. The 2-by-2 diagonal blocks (corresponding to complex conjugate pairs of eigenvalues) are returned in standard form, with $H(i,i) = H(i+1,i+1)$ and $H(i+1,i)*H(i,i+1) < 0$. If <i>info</i> = 0 and <i>wantt</i> equals zero, the contents of <i>h</i> are unspecified on exit. |
| <i>wr, wi</i> | The real and imaginary parts, respectively, of the computed eigenvalues <i>ilo</i> to <i>ihi</i> are stored in the corresponding elements of <i>wr</i> and <i>wi</i> . If two eigenvalues are computed as a complex conjugate pair, they are stored in consecutive elements of <i>wr</i> and <i>wi</i> , say the <i>i</i> -th and (<i>i</i> +1)th, with $wi[i-1] > 0$ and $wi[i] < 0$. If <i>wantt</i> is non-zero, the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in <i>h</i> . |
| <i>z</i> | Updated matrix with transformations applied only to the submatrix $Z(ilo:ihi,ilo:ihi)$. If COMPZ = 'I', on exit, if <i>info</i> = 0, <i>z</i> contains the orthogonal matrix <i>Z</i> of the Schur vectors of <i>H</i> . If <i>wantz</i> is non-zero, then $Z(ilo:ihi,iloz:ihiz)$ is replaced by $Z(ilo:ihi,iloz:ihiz)*U$, when <i>U</i> is the orthogonal/unitary Schur factor of $H(ilo:ihi,ilo:ihi)$. If <i>wantz</i> equals zero, then <i>z</i> is not defined. |
| <i>work</i> [0] | On exit, if <i>info</i> = 0, <i>work</i> [0] returns the optimal <i>lwork</i> . |
| <i>iwork</i> [0] | On exit, if <i>info</i> = 0, <i>iwork</i> [0] returns the optimal <i>liwork</i> . |
| <i>info</i> | > 0: if <i>info</i> = <i>i</i> , then the function failed to compute all the eigenvalues. Elements 0: <i>ilo</i> -2 and <i>i</i> : <i>n</i> -1 of <i>wr</i> and <i>wi</i> contain those eigenvalues which have been successfully computed. > 0: if <i>wantt</i> equals zero, then the remaining unconverged eigenvalues are the eigenvalues of the upper Hessenberg matrix rows and columns <i>ilo</i> through <i>ihi</i> of the final output value of <i>H</i> . > 0: if <i>wantt</i> is non-zero, then (initial value of <i>H</i>)* <i>U</i> = <i>U</i> *(final value of <i>H</i>), where <i>U</i> is an orthogonal/unitary matrix. The final value of <i>H</i> is upper Hessenberg and quasi-triangular/triangular in rows and columns <i>info</i> +1 through <i>ihi</i> . > 0: if <i>wantz</i> is non-zero, then (final value of $Z(ilo:ihi,iloz:ihiz)$)=(initial value of $Z(ilo:ihi,iloz:ihiz)$)* <i>U</i> , where <i>U</i> is the orthogonal/unitary matrix in the previous expression (regardless of the value of <i>wantt</i>). > 0: if <i>wantz</i> equals zero, then <i>z</i> is not accessed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laqr1

Sets a scalar multiple of the first column of the product of a 2-by-2 or 3-by-3 matrix and specified shifts.

Syntax

```
void pslaqr1(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* n, MKL_INT* ilo, MKL_INT* ihi,
float* a, MKL_INT* desca, float* wr, float* wi, MKL_INT* iloz, MKL_INT* ihiz, float*
z, MKL_INT* descz, float* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* ilwork,
MKL_INT* info);
```

```
void pdlaqr1(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* n, MKL_INT* ilo, MKL_INT* ihi,
double* a, MKL_INT* desca, double* wr, double* wi, MKL_INT* iloz, MKL_INT* ihiz,
double* z, MKL_INT* descz, double* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT*
ilwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

p?laqr1 is an auxiliary function used to find the Schur decomposition and/or eigenvalues of a matrix already in Hessenberg form from columns *ilo* to *ihi*.

This is a modified version of p?lahqr from ScaLAPACK version 1.7.3. The following modifications were made:

- Workspace query functionality was added.
- Aggressive early deflation is implemented.
- Aggressive deflation (looking for two consecutive small subdiagonal elements by PSLACONS) is abandoned.
- The returned Schur form is now in canonical form, i.e., the returned 2-by-2 blocks really correspond to complex conjugate pairs of eigenvalues.
- For some reason, the original version of p?lahqr sometimes did not read out the converged eigenvalues correctly. This is now fixed.

Optimization Notice

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Notice revision #20110804

Input Parameters

wantt (global)
 Non-zero : the full Schur form *T* is required;
 Zero: only eigenvalues are required.

| | |
|-------------------|---|
| <i>wantz</i> | Non-zero : the matrix of Schur vectors Z is required; Zero: Schur vectors are not required. |
| <i>n</i> | (global) The order of the Hessenberg matrix A (and Z if <i>wantz</i> is non-zero). $n \geq 0$. |
| <i>ilo, ihi</i> | (global) It is assumed that the matrix A is already upper quasi-triangular in rows and columns $ihi+1:n$, and that $A(ilo,ilo-1) = 0$ (unless $ilo = 1$). <i>p?laqr1</i> works primarily with the Hessenberg submatrix in rows and columns ilo to ihi , but applies transformations to all of H if <i>wantt</i> is non-zero. $1 \leq ilo \leq \max(1, ihi)$; $ihi \leq n$. |
| <i>a</i> | (global) array of size $lld_a * LOC_c(n)$ On entry, the upper Hessenberg matrix A . |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>iloz, ihiz</i> | (global) Specify the rows of the matrix Z to which transformations must be applied if <i>wantz</i> is non-zero. $1 \leq iloz \leq ilo$; $ihi \leq ihiz \leq n$. |
| <i>z</i> | (global) array of size $lld_z * LOC_c(n)$. If <i>wantz</i> is non-zero, on entry z must contain the current matrix Z of transformations accumulated by <i>p?hseqr</i> . If <i>wantz</i> is zero, z is not referenced. |
| <i>descz</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix Z . |
| <i>work</i> | (local output) array of size $lwork$ |
| <i>lwork</i> | (local) The size of the work array ($lwork \geq 1$). If $lwork = -1$, then a workspace query is assumed. |
| <i>iwork</i> | (global and local) array of size $ilwork$ This holds the some of the IBLK integer arrays. |
| <i>ilwork</i> | (local) The size of the <i>iwork</i> array ($ilwork \geq 3$). |

OUTPUT Parameters

| | |
|-----------------|---|
| <i>a</i> | If <i>wantt</i> is non-zero, the matrix <i>A</i> is upper quasi-triangular in rows and columns <i>ilo:ihi</i> , with any 2-by-2 or larger diagonal blocks not yet in standard form. If <i>wantt</i> equals zero, the contents of <i>a</i> are unspecified on exit. |
| <i>wr, wi</i> | (global replicated) array of size <i>n</i> The real and imaginary parts, respectively, of the computed eigenvalues <i>ilo</i> to <i>ihi</i> are stored in the corresponding elements of <i>wr</i> and <i>wi</i> . If two eigenvalues are computed as a complex conjugate pair, they are stored in consecutive elements of <i>wr</i> and <i>wi</i> , say the <i>i</i> -th and (<i>i</i> +1)th, with <i>wi</i> [<i>i</i> -1] > 0 and <i>wi</i> [<i>i</i>] < 0. If <i>wantt</i> is non-zero, the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in <i>a</i> . <i>a</i> may be returned with larger diagonal blocks until the next release. |
| <i>z</i> | On exit <i>z</i> is updated; transformations are applied only to the submatrix <i>Z</i> (<i>iloz:ihiz,ilo:ihi</i>). If <i>wantz</i> equals zero, <i>z</i> is not referenced. |
| <i>work</i> [0] | On exit, if <i>info</i> = 0, <i>work</i> [0] returns the optimal <i>lwork</i> . |
| <i>info</i> | (global) < 0: parameter number <i>-info</i> incorrect or inconsistent = 0: successful exit > 0: <i>p?laqr1</i> failed to compute all the eigenvalues <i>ilo</i> to <i>ihi</i> in a total of 30*(<i>ihi-ilo</i> +1) iterations; if <i>info</i> = <i>i</i> , elements <i>i:ihi-1</i> of <i>wr</i> and <i>wi</i> contain those eigenvalues which have been successfully computed. |

Application Notes

This algorithm is very similar to *p?ahqr*. Unlike *p?lahqr*, instead of sending one double shift through the largest unreduced submatrix, this algorithm sends multiple double shifts and spaces them apart so that there can be parallelism across several processor row/columns. Another critical difference is that this algorithm aggregates multiple transforms together in order to apply them in a block fashion.

Current Notes and/or Restrictions:

- This code requires the distributed block size to be square and at least six (6); unlike simpler codes like LU, this algorithm is extremely sensitive to block size. Unwise choices of too small a block size can lead to bad performance.
- This code requires *a* and *z* to be distributed identically and have identical contexts.
- This release currently does not have a function for resolving the Schur blocks into regular 2x2 form after this code is completed. Because of this, a significant performance impact is required while the deflation is done by sometimes a single column of processors.
- This code does not currently block the initial transforms so that none of the rows or columns for any bulge are completed until all are started. To offset pipeline start-up it is recommended that at least 2*LCM(NPROW,NPCOL) bulges are used (if possible)
- The maximum number of bulges currently supported is fixed at 32. In future versions this will be limited only by the incoming *work* array.
- The matrix *A* must be in upper Hessenberg form. If elements below the subdiagonal are nonzero, the resulting transforms may be nonsimilar. This is also true with the LAPACK function.
- For this release, it is assumed *rsrc_*=*csrc_*=0

- Currently, all the eigenvalues are distributed to all the nodes. Future releases will probably distribute the eigenvalues by the column partitioning.
- The internals of this function are subject to change.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laqr2

Performs the orthogonal/unitary similarity transformation of a Hessenberg matrix to detect and deflate fully converged eigenvalues from a trailing principal submatrix (aggressive early deflation).

Syntax

```
void pslaqr2(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* n, MKL_INT* ktop, MKL_INT* kbot,
MKL_INT* nw, float* a, MKL_INT* desca, MKL_INT* iloz, MKL_INT* ihiz, float* z,
MKL_INT* descz, MKL_INT* ns, MKL_INT* nd, float* sr, float* si, float* t, MKL_INT*
ldt, float* v, MKL_INT* ldv, float* wr, float* wi, float* work, MKL_INT* lwork);
```

```
void pdlaqr2(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* n, MKL_INT* ktop, MKL_INT* kbot,
MKL_INT* nw, double* a, MKL_INT* desca, MKL_INT* iloz, MKL_INT* ihiz, double* z,
MKL_INT* descz, MKL_INT* ns, MKL_INT* nd, double* sr, double* si, double* t, MKL_INT*
ldt, double* v, MKL_INT* ldv, double* wr, double* wi, double* work, MKL_INT* lwork);
```

Include Files

- mkl_scalapack.h

Description

p?laqr2 accepts as input an upper Hessenberg matrix A and performs an orthogonal similarity transformation designed to detect and deflate fully converged eigenvalues from a trailing principal submatrix. On output A is overwritten by a new Hessenberg matrix that is a perturbation of an orthogonal similarity transformation of A . It is to be hoped that the final version of A has many zero subdiagonal entries.

This function handles small deflation windows which is affordable by one processor. Normally, it is called by [p?laqr1](#). All the inputs are assumed to be valid without checking.

Optimization Notice

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Input Parameters

`wantt` (global)

| | |
|-------------------|--|
| | <p>If <i>wantt</i> is non-zero, then the Hessenberg matrix <i>A</i> is fully updated so that the quasi-triangular Schur factor may be computed (in cooperation with the calling function).</p> <p>If <i>wantt</i> equals zero, then only enough of <i>A</i> is updated to preserve the eigenvalues.</p> |
| <i>wantz</i> | <p>(global)</p> <p>If <i>wantz</i> is non-zero, then the orthogonal matrix <i>Z</i> is updated so that the orthogonal Schur factor may be computed (in cooperation with the calling function).</p> <p>If <i>wantz</i> equals zero, then <i>z</i> is not referenced.</p> |
| <i>n</i> | <p>(global)</p> <p>The order of the matrix <i>A</i> and (if <i>wantz</i> is non-zero) the order of the orthogonal matrix <i>Z</i>.</p> |
| <i>ktop, kbot</i> | <p>(global)</p> <p>It is assumed without a check that either $kbot = n$ or $A(kbot+1, kbot)=0$. <i>kbot</i> and <i>ktop</i> together determine an isolated block along the diagonal of the Hessenberg matrix. However, $A(ktop, ktop-1)=0$ is not essentially necessary if <i>wantt</i> is non-zero .</p> |
| <i>nw</i> | <p>(global)</p> <p>Deflation window size. $1 \leq nw \leq (kbot-ktop+1)$. Normally $nw \geq 3$ if <i>p?laqr2</i> is called by <i>p?laqr1</i>.</p> |
| <i>a</i> | <p>(local) array of size $lld_a * LOC_c(n)$</p> <p>The initial <i>n</i>-by-<i>n</i> section of <i>a</i> stores the Hessenberg matrix undergoing aggressive early deflation.</p> |
| <i>desca</i> | <p>(global and local) array of size <i>dlen_</i>.</p> <p>The array descriptor for the distributed matrix <i>A</i>.</p> |
| <i>iloz, ihiz</i> | <p>(global)</p> <p>Specify the rows of the matrix <i>Z</i> to which transformations must be applied if <i>wantz</i> is non-zero. $1 \leq iloz \leq ihiz \leq n$.</p> |
| <i>z</i> | <p>Array of size $lld_z * LOC_c(n)$</p> <p>If <i>wantz</i> is non-zero, then on output, the orthogonal similarity transformation mentioned above has been accumulated into the matrix $Z(iloz:ihiz, kbot:ktop)$, stored in <i>z</i>, from the right.</p> <p>If <i>wantz</i> is zero, then <i>z</i> is unreferenced.</p> |
| <i>descz</i> | <p>(global and local) array of size <i>dlen_</i>.</p> <p>The array descriptor for the distributed matrix <i>Z</i>.</p> |
| <i>t</i> | <p>(local workspace) array of size $ldt * nw$.</p> |

| | |
|---------------|---|
| <i>ldt</i> | (local) The leading dimension of the array <i>t</i> . $ldt \geq nw$. |
| <i>v</i> | (local workspace) array of size $ldv * nw$. |
| <i>ldv</i> | (local) The leading dimension of the array <i>v</i> . $ldv \geq nw$. |
| <i>wr, wi</i> | (local workspace) array of size <i>kbot</i> . |
| <i>work</i> | (local workspace) array of size <i>lwork</i> . |
| <i>lwork</i> | (local) <i>work(lwork)</i> is a local array and <i>lwork</i> is assumed big enough so that $lwork \geq nw * nw$. |

OUTPUT Parameters

| | |
|---------------|---|
| <i>a</i> | On output <i>a</i> has been transformed by an orthogonal similarity transformation, perturbed, and returned to Hessenberg form that (it is to be hoped) has some zero subdiagonal entries. |
| <i>z</i> | |
| <i>ns</i> | (global) The number of unconverged (that is, approximate) eigenvalues returned in <i>sr</i> and <i>si</i> that may be used as shifts by the calling function. |
| <i>nd</i> | (global) The number of converged eigenvalues uncovered by this function. |
| <i>sr, si</i> | (global) array of size <i>kbot</i> On output, the real and imaginary parts of approximate eigenvalues that may be used for shifts are stored in <i>sr</i> [<i>kbot-nd-ns</i>] through <i>sr</i> [<i>kbot-nd-1</i>] and <i>si</i> [<i>kbot-nd-ns</i>] through <i>si</i> [<i>kbot-nd-1</i>], respectively. On processor #0, the real and imaginary parts of converged eigenvalues are stored in <i>sr</i> [<i>kbot-nd</i>] through <i>sr</i> [<i>kbot-1</i>] and <i>si</i> [<i>kbot-nd</i>] through <i>si</i> [<i>kbot-1</i>], respectively. On other processors, these entries are set to zero. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laqr3

Performs the orthogonal/unitary similarity transformation of a Hessenberg matrix to detect and deflate fully converged eigenvalues from a trailing principal submatrix (aggressive early deflation).

Syntax

```
void pslaqr3(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* n, MKL_INT* ktop, MKL_INT* kbot,
MKL_INT* nw, float* h, MKL_INT* desch, MKL_INT* iloz, MKL_INT* ihiz, float* z,
MKL_INT* descz, MKL_INT* ns, MKL_INT* nd, float* sr, float* si, float* v, MKL_INT*
descv, MKL_INT* nh, float* t, MKL_INT* desct, MKL_INT* nv, float* wv, MKL_INT* descw,
float* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT* recllevel);
```

```
void pdlaqr3(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* n, MKL_INT* ktop, MKL_INT* kbot,
MKL_INT* nw, double* h, MKL_INT* desch, MKL_INT* iloz, MKL_INT* ihiz, double* z,
MKL_INT* descz, MKL_INT* ns, MKL_INT* nd, double* sr, double* si, double* v, MKL_INT*
descv, MKL_INT* nh, double* t, MKL_INT* desct, MKL_INT* nv, double* wv, MKL_INT*
descw, double* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT*
recllevel);
```

Include Files

- mkl_scalapack.h

Description

This function accepts as input an upper Hessenberg matrix H and performs an orthogonal similarity transformation designed to detect and deflate fully converged eigenvalues from a trailing principal submatrix. On output H is overwritten by a new Hessenberg matrix that is a perturbation of an orthogonal similarity transformation of H . It is to be hoped that the final version of H has many zero subdiagonal entries.

Input Parameters

| | | |
|--------------|-----------|--|
| <i>wantt</i> | (global) | If <i>wantt</i> is non-zero, then the Hessenberg matrix H is fully updated so that the quasi-triangular Schur factor may be computed (in cooperation with the calling function). If <i>wantt</i> equals zero, then only enough of H is updated to preserve the eigenvalues. |
| <i>wantz</i> | (global) | If <i>wantz</i> is non-zero, then the orthogonal matrix Z is updated so that the orthogonal Schur factor may be computed (in cooperation with the calling function). If <i>wantz</i> equals zero, then z is not referenced. |
| <i>n</i> | (global) | The order of the matrix H and (if <i>wantz</i> is non-zero), the order of the orthogonal matrix Z . |
| <i>ktop</i> | (global) | It is assumed that either $ktop = 1$ or $H(ktop,ktop-1)=0$. <i>kbot</i> and <i>ktop</i> together determine an isolated block along the diagonal of the Hessenberg matrix. |
| <i>kbot</i> | (global) | It is assumed without a check that either $kbot = n$ or $H(kbot+1,kbot)=0$. <i>kbot</i> and <i>ktop</i> together determine an isolated block along the diagonal of the Hessenberg matrix. |

| | |
|-------------------|---|
| <i>nw</i> | (global) Deflation window size. $1 \leq nw \leq (kbot-ktop+1)$. |
| <i>h</i> | (local) array of size $lld_h * LOC_c(n)$ The initial n -by- n section of H stores the Hessenberg matrix undergoing aggressive early deflation. |
| <i>desch</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix H . |
| <i>iloz, ihiz</i> | (global) Specify the rows of the matrix Z to which transformations must be applied if <i>wantz</i> is non-zero. $1 \leq iloz \leq ihiz \leq n$. |
| <i>z</i> | Array of size $lld_z * LOC_c(n)$ If <i>wantz</i> is non-zero, then on output, the orthogonal similarity transformation mentioned above has been accumulated into the matrix $Z(iloz:ihiz,kbot:ktop)$ from the right. If <i>wantz</i> is zero, then <i>z</i> is unreferenced. |
| <i>descz</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix Z . |
| <i>v</i> | (global workspace) array of size $lld_v * LOC_c(nw)$ An nw -by- nw distributed work array. |
| <i>descv</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix V . |
| <i>nh</i> | The number of columns of t . $nh \geq nw$. |
| <i>t</i> | (global workspace) array of size $lld_t * LOC_c(nh)$ |
| <i>desct</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix T . |
| <i>nv</i> | (global) The number of rows of work array wv available for workspace. $nv \geq nw$. |
| <i>wv</i> | (global workspace) array of size $lld_w * LOC_c(nw)$ |
| <i>descw</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix wv . |
| <i>work</i> | (local workspace) array of size $lwork$. |
| <i>lwork</i> | (local) The size of the work array <i>work</i> ($lwork \geq 1$). $lwork = 2*nw$ suffices, but greater efficiency may result from larger values of <i>lwork</i> . |

If $lwork = -1$, then a workspace query is assumed; `p?laqr3` only estimates the optimal workspace size for the given values of n , nw , $ktop$ and $kbot$. The estimate is returned in $work[0]$. No error message related to $lwork$ is issued by `xerbla`. Neither h nor z are accessed.

$iwork$ (local workspace) array of size $liwork$

$liwork$ (local)
 The length of the workspace array $iwork$ ($liwork \geq 1$).
 If $liwork = -1$, then a workspace query is assumed.

OUTPUT Parameters

h On output h has been transformed by an orthogonal similarity transformation, perturbed, and the returned to Hessenberg form that (it is to be hoped) has some zero subdiagonal entries.

z IF $wantz$ is non-zero, then on output, the orthogonal similarity transformation mentioned above has been accumulated into the matrix $Z(i_{loz}:i_{hiz}, kbot:ktop)$ from the right.
 If $wantz$ is zero, then z is unreferenced.

ns (global)
 The number of unconverged (that is, approximate) eigenvalues returned in sr and si that may be used as shifts by the calling function.

nd (global)
 The number of converged eigenvalues uncovered by this function.

sr, si (global) array of size $kbot$. The real and imaginary parts of approximate eigenvalues that may be used for shifts are stored in $sr[kbot-nd-ns]$ through $sr[kbot-nd-1]$ and $si[kbot-nd-ns]$ through $si[kbot-nd-1]$, respectively. The real and imaginary parts of converged eigenvalues are stored in $sr[kbot-nd]$ through $sr[kbot-1]$ and $si[kbot-nd]$ through $si[kbot-1]$, respectively.

$work[0]$ On exit, if $info = 0$, $work[0]$ returns the optimal $lwork$

$iwork[0]$ On exit, if $info = 0$, $iwork[0]$ returns the optimal $liwork$

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laqr5

Performs a single small-bulge multi-shift QR sweep.

Syntax

```
void pslaqr5(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* kacc22, MKL_INT* n, MKL_INT*
ktop, MKL_INT* kbot, MKL_INT* nshfts, float* sr, float* si, float* h, MKL_INT* desch,
MKL_INT* iloz, MKL_INT* ihiz, float* z, MKL_INT* descz, float* work, MKL_INT* lwork,
MKL_INT* iwork, MKL_INT* liwork);
```

```
void pdlaqr5(MKL_INT* wantt, MKL_INT* wantz, MKL_INT* kacc22, MKL_INT* n, MKL_INT*
ktop, MKL_INT* kbot, MKL_INT* nshfts, double* sr, double* si, double* h, MKL_INT*
desch, MKL_INT* iloz, MKL_INT* ihiz, double* z, MKL_INT* descz, double* work, MKL_INT*
lwork, MKL_INT* iwork, MKL_INT* liwork);
```

Include Files

- mkl_scalapack.h

Description

This auxiliary function called by [p?laqr0](#) performs a single small-bulge multi-shift QR sweep by chasing separated groups of bulges along the main block diagonal of a Hessenberg matrix H .

Input Parameters

| | |
|-------------------|--|
| <i>wantt</i> | (global) scalar <i>wantt</i> is non-zero if the quasi-triangular Schur factor is being computed. <i>wantt</i> is set to zero otherwise. |
| <i>wantz</i> | (global) scalar <i>wantz</i> is non-zero if the orthogonal Schur factor is being computed. <i>wantz</i> is set to zero otherwise. |
| <i>kacc22</i> | (global) Value 0, 1, or 2. Specifies the computation mode of far-from-diagonal orthogonal updates. = 0: p?laqr5 does not accumulate reflections and does not use matrix-matrix multiply to update far-from-diagonal matrix entries. = 1: p?laqr5 accumulates reflections and uses matrix-matrix multiply to update the far-from-diagonal matrix entries. = 2: p?laqr5 accumulates reflections, uses matrix-matrix multiply to update the far-from-diagonal matrix entries, and takes advantage of 2-by-2 block structure during matrix multiplies. |
| <i>n</i> | (global) scalar The order of the Hessenberg matrix H and, if <i>wantz</i> is non-zero, the order of the orthogonal matrix Z . |
| <i>ktop, kbot</i> | (global) scalar These are the first and last rows and columns of an isolated diagonal block upon which the QR sweep is to be applied. It is assumed without a check that either $ktop = 1$ or $H(ktop,ktop-1) = 0$ and either $kbot = n$ or $H(kbot+1,kbot) = 0$. |
| <i>nshfts</i> | (global) scalar |

| | |
|-------------------|--|
| | <i>nshfts</i> gives the number of simultaneous shifts. <i>nshfts</i> must be positive and even. |
| <i>sr, si</i> | (global) Array of size <i>nshfts</i> <i>sr</i> contains the real parts and <i>si</i> contains the imaginary parts of the <i>nshfts</i> shifts of origin that define the multi-shift QR sweep. |
| <i>h</i> | (local) Array of size <i>lld_h</i> * <i>LOC_c(n)</i> On input <i>h</i> contains a Hessenberg matrix <i>H</i> . |
| <i>desch</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>H</i> . |
| <i>iloz, ihiz</i> | (global) Specify the rows of the matrix <i>Z</i> to which transformations must be applied if <i>wantzis</i> non-zero. $1 \leq iloz \leq ihiz \leq n$ |
| <i>z</i> | (local) array of size <i>lld_z</i> * <i>LOC_c(n)</i> If <i>wantzis</i> non-zero, then the QR Sweep orthogonal similarity transformation is accumulated into the matrix <i>Z</i> (<i>iloz:ihiz,kbot:ktop</i>) from the right. If <i>wantzequals</i> zero, then <i>z</i> is unreferenced. |
| <i>descz</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>Z</i> . |
| <i>work</i> | (local workspace) array of size <i>lwork</i> |
| <i>lwork</i> | (local) The size of the <i>work</i> array (<i>lwork</i> ≥ 1). If <i>lwork</i> = -1, then a workspace query is assumed. |
| <i>iwork</i> | (local workspace) array of size <i>liwork</i> |
| <i>liwork</i> | (local) The size of the <i>iwork</i> array (<i>liwork</i> ≥ 1). If <i>liwork</i> = -1, then a workspace query is assumed. |

Output Parameters

| | |
|------------------|---|
| <i>h</i> | A multi-shift QR sweep with shifts $sr(j) + i * si(j)$ is applied to the isolated diagonal block in rows and columns <i>ktop</i> through <i>kbot</i> of the matrix <i>H</i> . |
| <i>z</i> | If <i>wantzis</i> non-zero, <i>z</i> is updated with transformations applied only to the submatrix <i>Z</i> (<i>iloz:ihiz,kbot:ktop</i>). |
| <i>work</i> [0] | On exit, if <i>info</i> = 0, <i>work</i> [0] returns the optimal <i>lwork</i> . |
| <i>iwork</i> [0] | On exit, if <i>info</i> = 0, <i>iwork</i> [0] returns the optimal <i>liwork</i> . |

See Also

Overview of ScaLAPACK Routines for details of ScaLAPACK array descriptor structures and related notations.

p?laqsy

Scales a symmetric/Hermitian matrix, using scaling factors computed by p?poequ.

Syntax

```
void pslaqsy (char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *sr , float *sc , float *scond , float *amax , char *equed );
```

```
void pdlaqsy (char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *sr , double *sc , double *scond , double *amax , char
*equed );
```

```
void pclaqsy (char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , float *sr , float *sc , float *scond , float *amax , char *equed );
```

```
void pzlaqsy (char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *sr , double *sc , double *scond , double *amax , char
*equed );
```

Include Files

- mkl_scalapack.h

Description

The p?laqsy function equilibrates a symmetric distributed matrix $\text{sub}(A) = A(ia:ia+n-1, ja:ja+n-1)$ using the scaling factors in the vectors *sr* and *sc*. The scaling factors are computed by p?poequ.

Input Parameters

| | |
|-------------|--|
| <i>uplo</i> | (global) Specifies the upper or lower triangular part of the symmetric distributed matrix $\text{sub}(A)$ is to be referenced: = 'U': Upper triangular part; = 'L': Lower triangular part. |
| <i>n</i> | (global) The order of the distributed matrix $\text{sub}(A)$. $n \geq 0$. |
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the distributed matrix $\text{sub}(A)$. On entry, the local pieces of the distributed symmetric matrix $\text{sub}(A)$. If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and the strictly lower triangular part of $\text{sub}(A)$ is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i> -by- <i>n</i> lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix, and the strictly upper triangular part of $\text{sub}(A)$ is not referenced. |

| | |
|---------------|--|
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the matrix sub(<i>A</i>), respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>sr</i> | (local) Array of size <i>LOCr(m_a)</i> . The scale factors for the matrix <i>A(ia:ia+m-1, ja:ja+n-1)</i> . <i>sr</i> is aligned with the distributed matrix <i>A</i> , and replicated across every process column. <i>sr</i> is tied to the distributed matrix <i>A</i> . |
| <i>sc</i> | (local) Array of size <i>LOCc(m_a)</i> . The scale factors for the matrix <i>A(ia:ia+m-1, ja:ja+n-1)</i> . <i>sc</i> is aligned with the distributed matrix <i>A</i> , and replicated across every process column. <i>sc</i> is tied to the distributed matrix <i>A</i> . |
| <i>scond</i> | (global). Ratio of the smallest <i>sr[i]</i> (respectively <i>sc[j]</i>) to the largest <i>sr[i]</i> (respectively <i>sc[j]</i>), with $ia - 1 \leq i < ia + n - 1$ and $ja - 1 \leq j < ja + n - 1$. |
| <i>amax</i> | (global). Absolute value of largest distributed submatrix entry. |

Output Parameters

| | |
|--------------|---|
| <i>a</i> | On exit, if <i>equed</i> = 'Y', the equilibrated matrix: $\text{diag}(sr_{ia}, \dots, sr_{ia+n-1}) * \text{sub}(A) * \text{diag}(sc_{ja}, \dots, sc_{ja+n-1})$. |
| <i>equed</i> | (global). Specifies whether or not equilibration was done. = 'N': No equilibration. = 'Y': Equilibration was done, that is, sub(<i>A</i>) has been replaced by: $\text{diag}(sr_{ia}, \dots, sr_{ia+n-1}) * \text{sub}(A) * \text{diag}(sc_{ja}, \dots, sc_{ja+n-1})$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lared1d

Redistributes an array assuming that the input array, bycol, is distributed across rows and that all process columns contain the same copy of bycol.

Syntax

```
void pslared1d (MKL_INT *n , MKL_INT *ia , MKL_INT *ja , MKL_INT *desc , float *bycol ,
float *byall , float *work , MKL_INT *lwork );
```

```
void pdlared1d (MKL_INT *n , MKL_INT *ia , MKL_INT *ja , MKL_INT *desc , double
*bycol , double *byall , double *work , MKL_INT *lwork );
```

Include Files

- mkl_scalapack.h

Description

The `pdlared1d` function redistributes a 1D array. It assumes that the input array `bycol` is distributed across rows and that all process column contain the same copy of `bycol`. The output array `byall` is identical on all processes and contains the entire array.

Input Parameters

`np` = Number of local rows in `bycol()`

| | |
|---------------------|--|
| <code>n</code> | (global) The number of elements to be redistributed. $n \geq 0$. |
| <code>ia, ja</code> | (global) <code>ia, ja</code> must be equal to 1. |
| <code>desc</code> | (local) array of size 9. A 2D array descriptor, which describes <code>bycol</code> . |
| <code>bycol</code> | (local). Distributed block cyclic array of global size <code>n</code> and of local size <code>np</code> . <code>bycol</code> is distributed across the process rows. All process columns are assumed to contain the same value. |
| <code>work</code> | (local). size <code>lwork</code> . Used to hold the buffers sent from one process to another. |
| <code>lwork</code> | (local) The size of the <code>work</code> array. $lwork \geq \text{numroc}(n, \text{desc}[\text{nb}_], 0, 0, \text{npcol})$. |

Output Parameters

| | |
|--------------------|---|
| <code>byall</code> | (global). Global size <code>n</code> , local size <code>n</code> . <code>byall</code> is exactly duplicated on all processes. It contains the same values as <code>bycol</code> , but it is replicated across all processes rather than being distributed. |
|--------------------|---|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lared2d

Redistributes an array assuming that the input array `byrow` is distributed across columns and that all process rows contain the same copy of `byrow`.

Syntax

```
void pslared2d (MKL_INT *n , MKL_INT *ia , MKL_INT *ja , MKL_INT *desc , float *byrow ,
float *byall , float *work , MKL_INT *lwork );
```

```
void pdlared2d (MKL_INT *n , MKL_INT *ia , MKL_INT *ja , MKL_INT *desc , double
*byrow , double *byall , double *work , MKL_INT *lwork );
```

Include Files

- mkl_scalapack.h

Description

The `pdlared2d` function redistributes a 1D array. It assumes that the input array `byrow` is distributed across columns and that all process rows contain the same copy of `byrow`. The output array `byall` will be identical on all processes and will contain the entire array.

Input Parameters

`np` = Number of local rows in `byrow()`

| | |
|---------------------|--|
| <code>n</code> | (global) The number of elements to be redistributed. $n \geq 0$. |
| <code>ia, ja</code> | (global) <code>ia, ja</code> must be equal to 1. |
| <code>desc</code> | (local) array of size <code>dlen_</code> . A 2D array descriptor, which describes <code>byrow</code> . |
| <code>byrow</code> | (local). Distributed block cyclic array of global size <code>n</code> and of local size <code>np</code> . <code>byrow</code> is distributed across the process columns. All process rows are assumed to contain the same value. |
| <code>work</code> | (local). size <code>lwork</code> . Used to hold the buffers sent from one process to another. |
| <code>lwork</code> | (local) The size of the <code>work</code> array. $lwork \geq \text{numroc}(n, \text{desc}[nb_], 0, 0, \text{npcol})$. |

Output Parameters

| | |
|--------------------|---|
| <code>byall</code> | (global). Global size <code>n</code> , local size <code>n</code> . <code>byall</code> is exactly duplicated on all processes. It contains the same values as <code>byrow</code> , but it is replicated across all processes rather than being distributed. |
|--------------------|---|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larf

Applies an elementary reflector to a general rectangular matrix.

Syntax

```
void pslarf (char *side , MKL_INT *m , MKL_INT *n , float *v , MKL_INT *iv , MKL_INT
*jv , MKL_INT *descv , MKL_INT *incv , float *tau , float *c , MKL_INT *ic , MKL_INT
*jc , MKL_INT *descc , float *work );
```

```

void pdlarf (char *side , MKL_INT *m , MKL_INT *n , double *v , MKL_INT *iv , MKL_INT
*jv , MKL_INT *descv , MKL_INT *incv , double *tau , double *c , MKL_INT *ic , MKL_INT
*jc , MKL_INT *desc , double *work );

void pclarf (char *side , MKL_INT *m , MKL_INT *n , MKL_Complex8 *v , MKL_INT *iv ,
MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , MKL_Complex8 *tau , MKL_Complex8 *c ,
MKL_INT *ic , MKL_INT *jc , MKL_INT *desc , MKL_Complex8 *work );

void pzlarf (char *side , MKL_INT *m , MKL_INT *n , MKL_Complex16 *v , MKL_INT *iv ,
MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , MKL_Complex16 *tau , MKL_Complex16 *c ,
MKL_INT *ic , MKL_INT *jc , MKL_INT *desc , MKL_Complex16 *work );

```

Include Files

- mkl_scalapack.h

Description

The `p?larf` function applies a real/complex elementary reflector Q (or Q^T) to a real/complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$, from either the left or the right. Q is represented in the form

$$Q = I - \tau * v * v',$$

where τ is a real/complex scalar and v is a real/complex vector.

If $\tau = 0$, then Q is taken to be the unit matrix.

Input Parameters

| | |
|---------------|--|
| <i>side</i> | (global). = 'L': form $Q * \text{sub}(C)$, = 'R': form $\text{sub}(C) * Q$, $Q = Q^T$. |
| <i>m</i> | (global) The number of rows in the distributed submatrix $\text{sub}(A)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed submatrix $\text{sub}(A)$. ($n \geq 0$). |
| <i>v</i> | (local). Pointer into the local memory to an array of size $lld_v * LOCC(n_v)$, containing the local pieces of the global distributed matrix V representing the Householder transformation Q , $V(iv:iv+m-1, jv)$ if $side = 'L'$ and $incv = 1$, $V(iv, jv:jv+m-1)$ if $side = 'L'$ and $incv = m_v$, $V(iv:iv+n-1, jv)$ if $side = 'R'$ and $incv = 1$, $V(iv, jv:jv+n-1)$ if $side = 'R'$ and $incv = m_v$. The array v is the representation of Q . v is not used if $\tau = 0$. |
| <i>iv, jv</i> | (global) The row and column indices in the global matrix V indicating the first row and the first column of the matrix $\text{sub}(V)$, respectively. |

| | |
|---------------|--|
| <i>descv</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>V</i> . |
| <i>incv</i> | (global) The global increment for the elements of <i>V</i> . Only two values of <i>incv</i> are supported in this version, namely 1 and <i>m_v</i> . <i>incv</i> must not be zero. |
| <i>tau</i> | (local). Array of size <i>LOCc(jv)</i> if <i>incv</i> = 1, and <i>LOCr(iv)</i> otherwise. This array contains the Householder scalars related to the Householder vectors. <i>tau</i> is tied to the distributed matrix <i>V</i> . |
| <i>c</i> | (local). Pointer into the local memory to an array of size <i>lld_c</i> * <i>LOCc(jc+n-1)</i> , containing the local pieces of sub(<i>C</i>). |
| <i>ic, jc</i> | (global) The row and column indices in the global matrix <i>C</i> indicating the first row and the first column of the matrix sub(<i>C</i>), respectively. |
| <i>descC</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>C</i> . |
| <i>work</i> | (local). Array of size <i>lwork</i> . If <i>incv</i> = 1, if <i>side</i> = 'L', if <i>ivcol</i> = <i>iccol</i> , <i>lwork</i> ≥ <i>nqc0</i> else <i>lwork</i> ≥ <i>mpc0</i> + max(1, <i>nqc0</i>) end if else if <i>side</i> = 'R', <i>lwork</i> ≥ <i>nqc0</i> + max(max(1, <i>mpc0</i>), numroc(numroc(<i>n</i> + <i>icoffc</i> , <i>nb_v</i> , 0, 0, <i>npcol</i>), <i>nb_v</i> , 0, 0, <i>lcmq</i>)) end if else if <i>incv</i> = <i>m_v</i> , if <i>side</i> = 'L', <i>lwork</i> ≥ <i>mpc0</i> + max(max(1, <i>nqc0</i>), numroc(numroc(<i>m</i> + <i>iroffc</i> , <i>mb_v</i> , 0, 0, <i>nprow</i>), <i>mb_v</i> , 0, 0, <i>lcmp</i>)) else if <i>side</i> = 'R', if <i>ivrow</i> = <i>icrow</i> , <i>lwork</i> ≥ <i>mpc0</i> |

```

        else
             $lwork \geq nqc0 + \max(1, mpc0)$ 
        end if
    end if
end if,

```

where lcm is the least common multiple of $nrow$ and $ncol$ and $lcm = ilcm(nrow, ncol)$, $lcmp = lcm/nrow$, $lcmq = lcm/ncol$,
 $iroffc = \text{mod}(ic-1, mb_c)$, $icoffc = \text{mod}(jc-1, nb_c)$,
 $icrow = \text{indxg2p}(ic, mb_c, myrow, rsrc_c, nrow)$,
 $iccol = \text{indxg2p}(jc, nb_c, mycol, csrc_c, ncol)$,
 $mpc0 = \text{numroc}(m+iroffc, mb_c, myrow, icrow, nrow)$,
 $nqc0 = \text{numroc}(n+icoffc, nb_c, mycol, iccol, ncol)$,
 $ilcm$, indxg2p , and numroc are ScaLAPACK tool functions; $myrow$, $mycol$, $nrow$, and $ncol$ can be determined by calling the function `blacs_gridinfo`.

Output Parameters

`c` (local).
 On exit, `sub(C)` is overwritten by the $Q \cdot \text{sub}(C)$ if `side = 'L'`,
 or `sub(C) * Q` if `side = 'R'`.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larfb

Applies a block reflector or its transpose/conjugate-transpose to a general rectangular matrix.

Syntax

```

void pslarfb (char *side , char *trans , char *direct , char *storev , MKL_INT *m ,
MKL_INT *n , MKL_INT *k , float *v , MKL_INT *iv , MKL_INT *jv , MKL_INT *descv ,
float *t , float *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , float *work );

void pdlarfb (char *side , char *trans , char *direct , char *storev , MKL_INT *m ,
MKL_INT *n , MKL_INT *k , double *v , MKL_INT *iv , MKL_INT *jv , MKL_INT *descv ,
double *t , double *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , double *work );

void pclarfb (char *side , char *trans , char *direct , char *storev , MKL_INT *m ,
MKL_INT *n , MKL_INT *k , MKL_Complex8 *v , MKL_INT *iv , MKL_INT *jv , MKL_INT
*descv , MKL_Complex8 *t , MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT
*descc , MKL_Complex8 *work );

void pzlarfb (char *side , char *trans , char *direct , char *storev , MKL_INT *m ,
MKL_INT *n , MKL_INT *k , MKL_Complex16 *v , MKL_INT *iv , MKL_INT *jv , MKL_INT
*descv , MKL_Complex16 *t , MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT
*descc , MKL_Complex16 *work );

```

Include Files

- `mkl_scalapack.h`

Description

The `p?larfb` function applies a real/complex block reflector Q or its transpose Q^T /conjugate transpose Q^H to a real/complex distributed m -by- n matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ from the left or the right.

Input Parameters

| | |
|---------------------|---|
| <code>side</code> | (global) if <code>side = 'L'</code> : apply Q or Q^T for real flavors (Q^H for complex flavors) from the Left; if <code>side = 'R'</code> : apply Q or Q^T for real flavors (Q^H for complex flavors) from the Right. |
| <code>trans</code> | (global) if <code>trans = 'N'</code> : no transpose, apply Q ; for real flavors, if <code>trans='T'</code> : transpose, apply Q^T for complex flavors, if <code>trans = 'C'</code> : conjugate transpose, apply Q^H ; |
| <code>direct</code> | (global) Indicates how Q is formed from a product of elementary reflectors. if <code>direct = 'F'</code> : $Q = H(1)*H(2)*\dots*H(k)$ (Forward) if <code>direct = 'B'</code> : $Q = H(k)*\dots*H(2)*H(1)$ (Backward) |
| <code>storev</code> | (global) Indicates how the vectors that define the elementary reflectors are stored: if <code>storev = 'C'</code> : Columnwise if <code>storev = 'R'</code> : Rowwise. |
| <code>m</code> | (global) The number of rows in the distributed matrix $\text{sub}(C)$. ($m \geq 0$). |
| <code>n</code> | (global) The number of columns in the distributed matrix $\text{sub}(C)$. ($n \geq 0$). |
| <code>k</code> | (global) The order of the matrix T . |
| <code>v</code> | (local). Pointer into the local memory to an array of size <code>lld_v * LOCc(jv+k-1)</code> if <code>storev = 'C'</code> , <code>lld_v * LOCc(jv+m-1)</code> if <code>storev = 'R'</code> and <code>side = 'L'</code> , <code>lld_v * LOCc(jv+n-1)</code> if <code>storev = 'R'</code> and <code>side = 'R'</code> . It contains the local pieces of the distributed vectors V representing the Householder transformation. if <code>storev = 'C'</code> and <code>side = 'L'</code> , <code>lld_v</code> \geq <code>max(1, LOCr(iv+m-1))</code> ; |

```

if storev = 'C' and side = 'R', lld_v ≥ max(1, LOCr(iv+n-1));
if storev = 'R', lld_v ≥ LOCr(jv+k-1).

```

iv, jv (global)
The row and column indices in the global matrix V indicating the first row and the first column of the matrix sub(V), respectively.

descv (global and local) array of size $dlen_$. The array descriptor for the distributed matrix V .

c (local).
Pointer into the local memory to an array of size $lld_c * LOCc(jc+n-1)$, containing the local pieces of sub(C).

ic, jc (global) The row and column indices in the global matrix C indicating the first row and the first column of the matrix sub(C), respectively.

desc (global and local) array of size $dlen_$. The array descriptor for the distributed matrix C .

work (local).
Workspace array of size $lwork$.
If $storev = 'C'$,
if $side = 'L'$,
 $lwork \geq (nqc0 + mpc0) * k$
else if $side = 'R'$,
 $lwork \geq (nqc0 + \max(npv0 + \text{numroc}(\text{numroc}(n + icoffc, nb_v, 0, 0, npc0), nb_v, 0, 0, lcmq), mpc0)) * k$
end if
else if $storev = 'R'$,
if $side = 'L'$,
 $lwork \geq (mpc0 + \max(mqv0 + \text{numroc}(\text{numroc}(m + iroffv, mb_v, 0, 0, nprow), mb_v, 0, 0, lcmp), nqc0)) * k$
else if $side = 'R'$,
 $lwork \geq (mpc0 + nqc0) * k$
end if
end if,
where
 $lcmq = lcm / npc0$ with $lcm = iclm(nprow, npc0)$,
 $iroffv = \text{mod}(iv-1, mb_v)$, $icoffv = \text{mod}(jv-1, nb_v)$,
 $ivrow = \text{indxg2p}(iv, mb_v, myrow, rsrc_v, nprow)$,

```

ivcol = indxg2p( jv, nb_v, mycol, csrc_v, npcov ),
MqV0 = numroc( m+icoffv, nb_v, mycol, ivcol, npcov ),
NpV0 = numroc( n+iroffv, mb_v, myrow, ivrow, nprow ),
iroffc = mod( ic-1, mb_c ), icoffc = mod( jc-1, nb_c ),
icrow = indxg2p( ic, mb_c, myrow, rsrc_c, nprow ),
iccol = indxg2p( jc, nb_c, mycol, csrc_c, npcov ),
MpC0 = numroc( m+iroffc, mb_c, myrow, icrow, nprow ),
NpC0 = numroc( n+icoffc, mb_c, myrow, icrow, nprow ),
NqC0 = numroc( n+icoffc, nb_c, mycol, iccol, npcov ),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions; myrow, mycol,
nprow, and npcov can be determined by calling the function
blacs_gridinfo.

```

Output Parameters

t (local).
 Array of size $mb_v * mb_vif$ *storev* = 'R', and $nb_v * nb_vif$ *storev* = 'C'. The triangular matrix *t* is the representation of the block reflector.

c (local).
 On exit, sub(C) is overwritten by the $Q * sub(C)$, or $Q' * sub(C)$, or $sub(C) * Q$, or $sub(C) * Q'$. *Q'* is transpose (conjugate transpose) of *Q*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larfc

Applies the conjugate transpose of an elementary reflector to a general matrix.

Syntax

```

void pclarfc (char *side , MKL_INT *m , MKL_INT *n , MKL_Complex8 *v , MKL_INT *iv ,
MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , MKL_Complex8 *tau , MKL_Complex8 *c ,
MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work );

void pzlarfc (char *side , MKL_INT *m , MKL_INT *n , MKL_Complex16 *v , MKL_INT *iv ,
MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , MKL_Complex16 *tau , MKL_Complex16 *c ,
MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work );

```

Include Files

- mkl_scalapack.h

Description

The p?larfc function applies a complex elementary reflector Q^H to a complex *m*-by-*n* distributed matrix $sub(C) = C(ic:ic+m-1, jc:jc+n-1)$, from either the left or the right. *Q* is represented in the form

$$Q = I - \tau * v * v',$$

where τ is a complex scalar and v is a complex vector.

If $\tau = 0$, then Q is taken to be the unit matrix.

Input Parameters

| | |
|---------------|--|
| <i>side</i> | (global) if <i>side</i> = 'L': form $Q^H * \text{sub}(C)$; if <i>side</i> = 'R': form $\text{sub}(C) * Q^H$. |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(C)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(C)$. ($n \geq 0$). |
| <i>v</i> | (local). Pointer into the local memory to an array of size $lld_v * LOCC(n_v)$, containing the local pieces of the global distributed matrix V representing the Householder transformation Q , $V(iv:iv+m-1, jv)$ if <i>side</i> = 'L' and <i>incv</i> = 1, $V(iv, jv:jv+m-1)$ if <i>side</i> = 'L' and <i>incv</i> = m_v , $V(iv:iv+n-1, jv)$ if <i>side</i> = 'R' and <i>incv</i> = 1, $V(iv, jv:jv+n-1)$ if <i>side</i> = 'R' and <i>incv</i> = m_v . The array v is the representation of Q . v is not used if $\tau = 0$. |
| <i>iv, jv</i> | (global) The row and column indices in the global matrix V indicating the first row and the first column of the matrix $\text{sub}(V)$, respectively. |
| <i>descv</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix V . |
| <i>incv</i> | (global) The global increment for the elements of v . Only two values of <i>incv</i> are supported in this version, namely 1 and m_v . <i>incv</i> must not be zero. |
| <i>tau</i> | (local) Array of size $LOCc(jv)$ if <i>incv</i> = 1, and $LOCr(iv)$ otherwise. This array contains the Householder scalars related to the Householder vectors. τ is tied to the distributed matrix V . |
| <i>c</i> | (local). Pointer into the local memory to an array of size $lld_c * LOCC(jc+n-1)$, containing the local pieces of $\text{sub}(C)$. |
| <i>ic, jc</i> | (global) |

The row and column indices in the global matrix *C* indicating the first row and the first column of the matrix sub(*C*), respectively.

desc

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix *C*.

work

(local).

Workspace array of size *lwork*.

If *incv* = 1,

if *side* = 'L' ,

if *ivcol* = *iccol*,

$lwork \geq nqc0$

else

$lwork \geq mpc0 + \max(1, nqc0)$

end if

else if *side* = 'R' ,

$lwork \geq nqc0 + \max(\max(1, mpc0), \text{numroc}(\text{numroc}(n+icoffc, nb_v, 0, 0, npc0), nb_v, 0, 0, lcmq))$

end if

else if *incv* = *m_v*,

if *side* = 'L' ,

$lwork \geq mpc0 + \max(\max(1, nqc0), \text{numroc}(\text{numroc}(m+iroffc, mb_v, 0, 0, nprow), mb_v, 0, 0, lcmp))$

else if *side* = 'R' ,

if *ivrow* = *icrow*,

$lwork \geq mpc0$

else

$lwork \geq nqc0 + \max(1, mpc0)$

end if

end if

end if,

where *lcm* is the least common multiple of *nprow* and *npcol* and $lcm = \text{ilcm}(nprow, npc0)$,

$lcmp = lcm/nprow, lcmq = lcm/ncol,$

$iroffc = \text{mod}(ic-1, mb_c), icoffc = \text{mod}(jc-1, nb_c),$

$icrow = \text{indxg2p}(ic, mb_c, myrow, rsrc_c, nprow),$

$iccol = \text{indxg2p}(jc, nb_c, mycol, csrc_c, npc0),$

$mpc0 = \text{numroc}(m+iroffc, mb_c, myrow, icrow, nprow),$

$nqc0 = \text{numroc}(n+icoffc, nb_c, mycol, iccol, npc0),$

`ilcm`, `indxg2p`, and `numroc` are ScaLAPACK tool functions; `myrow`, `mycol`, `nrow`, and `ncol` can be determined by calling the function `blacs_gridinfo`.

Output Parameters

`c`

(local).

On exit, `sub(C)` is overwritten by the $Q^H \text{sub}(C)$ if `side = 'L'`, or `sub(C) * Q^H` if `side = 'R'`.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larfg

Generates an elementary reflector (Householder matrix).

Syntax

```
void pslarfg (MKL_INT *n , float *alpha , MKL_INT *iax , MKL_INT *jax , float *x ,
MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *incx , float *tau );
```

```
void pdlarfg (MKL_INT *n , double *alpha , MKL_INT *iax , MKL_INT *jax , double *x ,
MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *incx , double *tau );
```

```
void pclarfg (MKL_INT *n , MKL_Complex8 *alpha , MKL_INT *iax , MKL_INT *jax ,
MKL_Complex8 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *incx ,
MKL_Complex8 *tau );
```

```
void pzlarfg (MKL_INT *n , MKL_Complex16 *alpha , MKL_INT *iax , MKL_INT *jax ,
MKL_Complex16 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , MKL_INT *incx ,
MKL_Complex16 *tau );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?larfg` function generates a real/complex elementary reflector H of order n , such that

$$H * \text{sub}(X) = H * \begin{pmatrix} x(iax, jax) \\ x \end{pmatrix} = \begin{pmatrix} alpha \\ 0 \end{pmatrix}, \quad H^H * H = I,$$

where $alpha$ is a scalar (a real scalar - for complex flavors), and `sub(X)` is an $(n-1)$ -element real/complex distributed vector $X(ix:ix+n-2, jx)$ if `incx = 1` and $X(ix, jx:jx+n-2)$ if `incx = m_x`. H is represented in the form

$$H = I - tau * \begin{pmatrix} 1 \\ v \end{pmatrix} * (1 \ v')$$

where τ is a real/complex scalar and v is a real/complex $(n-1)$ -element vector. Note that H is not Hermitian.

If the elements of $\text{sub}(X)$ are all zero (and $X(iax, jax)$ is real for complex flavors), then $\tau = 0$ and H is taken to be the unit matrix.

Otherwise $1 \leq \text{real}(\tau) \leq 2$ and $\text{abs}(\tau-1) \leq 1$.

Input Parameters

| | | |
|------------|--------------------|--|
| n | (global) | The global order of the elementary reflector. $n \geq 0$. |
| iax, jax | (global) | The global row and column indices of $X(iax, jax)$ in the global matrix X . |
| x | (local). | Pointer into the local memory to an array of size $lld_x * LOCc(n_x)$. This array contains the local pieces of the distributed vector $\text{sub}(X)$. Before entry, the incremented array $\text{sub}(X)$ must contain vector x . |
| ix, jx | (global) | The row and column indices in the global matrix X indicating the first row and the first column of $\text{sub}(X)$, respectively. |
| $descx$ | (global and local) | Array of size $dlen_$. The array descriptor for the distributed matrix X . |
| $incx$ | (global) | The global increment for the elements of x . Only two values of $incx$ are supported in this version, namely 1 and m_x . $incx$ must not be zero. |

Output Parameters

| | | |
|----------|----------|--|
| α | (local) | On exit, α is computed in the process scope having the vector $\text{sub}(X)$. |
| x | (local). | On exit, it is overwritten with the vector v . |
| τ | (local). | Array of size $LOCc(jx)$ if $incx = 1$, and $LOCr(ix)$ otherwise. This array contains the Householder scalars related to the Householder vectors. τ is tied to the distributed matrix X . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larft

Forms the triangular vector T of a block reflector $H=I-V^*T^*V^H$.

Syntax

```
void pslarft (char *direct , char *storev , MKL_INT *n , MKL_INT *k , float *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , float *tau , float *t , float *work );
void pdlarft (char *direct , char *storev , MKL_INT *n , MKL_INT *k , double *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , double *tau , double *t , double *work );
void pclarft (char *direct , char *storev , MKL_INT *n , MKL_INT *k , MKL_Complex8 *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , MKL_Complex8 *tau , MKL_Complex8 *t ,
MKL_Complex8 *work );
void pzlarft (char *direct , char *storev , MKL_INT *n , MKL_INT *k , MKL_Complex16
*v , MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , MKL_Complex16 *tau , MKL_Complex16
*t , MKL_Complex16 *work );
```

Include Files

- mkl_scalapack.h

Description

The p?larft function forms the triangular factor T of a real/complex block reflector H of order n , which is defined as a product of k elementary reflectors.

If $direct = 'F'$, $H = H(1)*H(2) \dots *H(k)$, and T is upper triangular;

If $direct = 'B'$, $H = H(k)* \dots *H(2)*H(1)$, and T is lower triangular.

If $storev = 'C'$, the vector which defines the elementary reflector $H(i)$ is stored in the i -th column of the distributed matrix V , and

$$H = I - V^* T^* V'$$

If $storev = 'R'$, the vector which defines the elementary reflector $H(i)$ is stored in the i -th row of the distributed matrix V , and

$$H = I - V' * T^* V.$$

Input Parameters

| | |
|---------------|--|
| <i>direct</i> | (global) |
| | Specifies the order in which the elementary reflectors are multiplied to form the block reflector: |
| | if $direct = 'F'$: $H = H(1)*H(2)* \dots *H(k)$ (forward) |
| | if $direct = 'B'$: $H = H(k)* \dots *H(2)*H(1)$ (backward). |
| <i>storev</i> | (global) |
| | Specifies how the vectors that define the elementary reflectors are stored (See <i>Application Notes</i> below): |
| | if $storev = 'C'$: columnwise; |
| | if $storev = 'R'$: rowwise. |

| | |
|----------|---|
| n | (global) The order of the block reflector H . $n \geq 0$. |
| k | (global) The order of the triangular factor T , is equal to the number of elementary reflectors. $1 \leq k \leq mb_v$ ($= nb_v$). |
| v | Pointer into the local memory to an array of local size $LOCr(iv+n-1) * LOCc(jv+k-1)$ if $storev = 'C'$, and $LOCr(iv+k-1) * LOCc(jv+n-1)$ if $storev = 'R'$. The distributed matrix V contains the Householder vectors. (See <i>Application Notes</i> below). |
| iv, jv | (global) The row and column indices in the global matrix V indicating the first row and the first column of the matrix $sub(V)$, respectively. |
| $descv$ | (local) array of size $dlen_$. The array descriptor for the distributed matrix V . |
| tau | (local) Array of size $LOCr(iv+k-1)$ if $incv = m_v$, and $LOCc(jv+k-1)$ otherwise. This array contains the Householder scalars related to the Householder vectors. tau is tied to the distributed matrix V . |
| $work$ | (local). Workspace array of size $k*(k-1)/2$. |

Output Parameters

| | |
|-----|---|
| v | |
| t | (local) Array of size $nb_v * nb_v$ if $storev = 'C'$, and $mb_v * mb_v$ otherwise. It contains the k -by- k triangular factor of the block reflector associated with v . If $direct = 'F'$, t is upper triangular; if $direct = 'B'$, t is lower triangular. |

Application Notes

The shape of the matrix V and the storage of the vectors that define the $H(i)$ is best illustrated by the following example with $n = 5$ and $k = 3$. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

direct = 'F' and *storev* = 'C' :

$$V(iv : iv + n - 1, \\ jv : jv + k - 1) = \begin{bmatrix} 1 & & & & \\ v1 & 1 & & & \\ v1 & v2 & 1 & & \\ v1 & v2 & v3 & & \\ v1 & v2 & v3 & & \end{bmatrix}$$

direct = 'F' and *storev* = 'R' :

$$V(iv : iv + k - 1, \\ jv : jv + n - 1) = \begin{bmatrix} 1 & v1 & v1 & v1 & v1 \\ & 1 & v2 & v2 & v2 \\ & & 1 & v3 & v3 \end{bmatrix}$$

direct = 'B' and *storev* = 'C' :

$$V(iv : iv + n - 1, \\ jv : jv + k - 1) = \begin{bmatrix} v1 & v2 & v3 \\ v1 & v2 & v3 \\ 1 & v2 & v3 \\ & 1 & v3 \\ & & 1 \end{bmatrix}$$

direct = 'B' and *storev* = 'R' :

$$V(iv : iv + k - 1, \\ jv : jv + n - 1) = \begin{bmatrix} v1 & v1 & 1 \\ v2 & v2 & v2 & 1 \\ v3 & v3 & v3 & v3 & 1 \end{bmatrix}$$

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larz

Applies an elementary reflector as returned by `p?tzrzf` to a general matrix.

Syntax

```
void pslarz (char *side , MKL_INT *m , MKL_INT *n , MKL_INT *l , float *v , MKL_INT
*iv , MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work );
```

```
void pdlarz (char *side , MKL_INT *m , MKL_INT *n , MKL_INT *l , double *v , MKL_INT
*iv , MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work );
```

```
void pclarz (char *side , MKL_INT *m , MKL_INT *n , MKL_INT *l , MKL_Complex8 *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work );
```

```
void pzlarz (char *side , MKL_INT *m , MKL_INT *n , MKL_INT *l , MKL_Complex16 *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?larz` function applies a real/complex elementary reflector Q (or Q^T) to a real/complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$, from either the left or the right. Q is represented in the form

$$Q = I - \tau * v * v',$$

where τ is a real/complex scalar and v is a real/complex vector.

If $\tau = 0$, then Q is taken to be the unit matrix.

Q is a product of k elementary reflectors as returned by `p?tzzrf`.

Input Parameters

| | |
|---------------|--|
| <i>side</i> | (global) if <i>side</i> = 'L': form $Q \cdot \text{sub}(C)$, if <i>side</i> = 'R': form $\text{sub}(C) \cdot Q$, $Q = Q^T$ (for real flavors). |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(C)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(C)$. ($n \geq 0$). |
| <i>l</i> | (global) The columns of the distributed matrix $\text{sub}(A)$ containing the meaningful part of the Householder reflectors. If <i>side</i> = 'L', $m \geq l \geq 0$, if <i>side</i> = 'R', $n \geq l \geq 0$. |
| <i>v</i> | (local). Pointer into the local memory to an array of size $ld_v * LOcc(n_v)$ containing the local pieces of the global distributed matrix V representing the Householder transformation Q , $V(iv:iv+1-1, jv)$ if <i>side</i> = 'L' and <i>incv</i> = 1, $V(iv, jv:jv+1-1)$ if <i>side</i> = 'L' and <i>incv</i> = m_v , $V(iv:iv+1-1, jv)$ if <i>side</i> = 'R' and <i>incv</i> = 1, $V(iv, jv:jv+1-1)$ if <i>side</i> = 'R' and <i>incv</i> = m_v . The vector v in the representation of Q . v is not used if $\tau = 0$. |
| <i>iv, jv</i> | (global) The row and column indices in the global distributed matrix V indicating the first row and the first column of the matrix $\text{sub}(V)$, respectively. |
| <i>descv</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix V . |
| <i>incv</i> | (global) The global increment for the elements of V . Only two values of <i>incv</i> are supported in this version, namely 1 and m_v . <i>incv</i> must not be zero. |
| <i>tau</i> | (local) Array of size $LOcc(jv)$ if <i>incv</i> = 1, and $LOCr(iv)$ otherwise. This array contains the Householder scalars related to the Householder vectors. <i>tau</i> is tied to the distributed matrix V . |
| <i>c</i> | (local). |

Pointer into the local memory to an array of size $lld_c * LOCC(jc+n-1)$, containing the local pieces of sub(C).

ic, jc

(global)

The row and column indices in the global matrix C indicating the first row and the first column of the matrix sub(C), respectively.

desc

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix C.

work

(local).

Array of size *lwork*

If *incv* = 1,

if *side* = 'L' ,

if *ivcol* = *iccol*,

$lwork \geq NqC0$

else

$lwork \geq MpC0 + \max(1, NqC0)$

end if

else if *side* = 'R' ,

$lwork \geq NqC0 + \max(\max(1, MpC0), \text{numroc}(\text{numroc}(n+icoffc,nb_v, 0,0,npcol),nb_v,0,0,lcmq))$

end if

else if *incv* = *m_v*,

if *side* = 'L' ,

$lwork \geq MpC0 + \max(\max(1, NqC0), \text{numroc}(\text{numroc}(m+iroffc,mb_v, 0,0,nprow),mb_v,0,0,lcmp))$

else if *side* = 'R' ,

if *ivrow* = *icrow*,

$lwork \geq MpC0$

else

$lwork \geq NqC0 + \max(1, MpC0)$

end if

end if

end if.

Here *lcm* is the least common multiple of *nprow* and *npcol* and

$lcm = \text{ilcm}(nprow, npcol)$, $lcmp = lcm / nprow$,

$lcmq = lcm / npcpl$,

$iroffc = \text{mod}(ic-1, mb_c)$, $icoffc = \text{mod}(jc-1, nb_c)$,

$icrow = \text{indxg2p}(ic, mb_c, myrow, rsrc_c, nprow)$,

$iccol = \text{indxg2p}(jc, nb_c, mycol, csrc_c, npcpl)$,

```

mpc0 = numroc( m+iroffc, mb_c, myrow, icrow, nprow ),
nqc0 = numroc( n+icoffc, nb_c, mycol, iccol, npc0l ),
ilcm, indxcg2p, and numroc are ScaLAPACK tool functions; myrow, mycol,
nprow, and npc0l can be determined by calling the function
blacs_gridinfo.

```

Output Parameters

c (local).
On exit, sub(C) is overwritten by the $Q^* \text{sub}(C)$ if *side* = 'L', or $\text{sub}(C) * Q$ if *side* = 'R'.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larzb

Applies a block reflector or its transpose/conjugate-transpose as returned by p?tzzrf to a general matrix.

Syntax

```

void pslarzb (char *side , char *trans , char *direct , char *storev , MKL_INT *m ,
MKL_INT *n , MKL_INT *k , MKL_INT *l , float *v , MKL_INT *iv , MKL_INT *jv , MKL_INT
*descv , float *t , float *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , float
*work );

void pdlarzb (char *side , char *trans , char *direct , char *storev , MKL_INT *m ,
MKL_INT *n , MKL_INT *k , MKL_INT *l , double *v , MKL_INT *iv , MKL_INT *jv , MKL_INT
*descv , double *t , double *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , double
*work );

void pclarzb (char *side , char *trans , char *direct , char *storev , MKL_INT *m ,
MKL_INT *n , MKL_INT *k , MKL_INT *l , MKL_Complex8 *v , MKL_INT *iv , MKL_INT *jv ,
MKL_INT *descv , MKL_Complex8 *t , MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc ,
MKL_INT *descc , MKL_Complex8 *work );

void pzlarzb (char *side , char *trans , char *direct , char *storev , MKL_INT *m ,
MKL_INT *n , MKL_INT *k , MKL_INT *l , MKL_Complex16 *v , MKL_INT *iv , MKL_INT *jv ,
MKL_INT *descv , MKL_Complex16 *t , MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc ,
MKL_INT *descc , MKL_Complex16 *work );

```

Include Files

- mkl_scalapack.h

Description

The p?larzb function applies a real/complex block reflector Q or its transpose Q^T (conjugate transpose Q^H for complex flavors) to a real/complex distributed m -by- n matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$ from the left or the right.

Q is a product of k elementary reflectors as returned by p?tzzrf.

Currently, only *storev* = 'R' and *direct* = 'B' are supported.

Input Parameters

| | |
|---------------|--|
| <i>side</i> | (global) if <i>side</i> = 'L': apply Q or Q^T (Q^H for complex flavors) from the Left; if <i>side</i> = 'R': apply Q or Q^T (Q^H for complex flavors) from the Right. |
| <i>trans</i> | (global) if <i>trans</i> = 'N': No transpose, apply Q ; If <i>trans</i> ='T': Transpose, apply Q^T (real flavors); If <i>trans</i> ='C': Conjugate transpose, apply Q^H (complex flavors). |
| <i>direct</i> | (global) Indicates how H is formed from a product of elementary reflectors. if <i>direct</i> = 'F': $H = H(1)*H(2)*\dots*H(k)$ - forward (not supported) ; if <i>direct</i> = 'B': $H = H(k)*\dots*H(2)*H(1)$ - backward. |
| <i>storev</i> | (global) Indicates how the vectors that define the elementary reflectors are stored: if <i>storev</i> = 'C': columnwise (not supported). if <i>storev</i> = 'R': rowwise. |
| <i>m</i> | (global) The number of rows in the distributed submatrix sub(C). ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed submatrix sub(C). ($n \geq 0$). |
| <i>k</i> | (global) The order of the matrix T . (= the number of elementary reflectors whose product defines the block reflector). |
| <i>l</i> | (global) The columns of the distributed submatrix sub(A) containing the meaningful part of the Householder reflectors. If <i>side</i> = 'L', $m \geq l \geq 0$, if <i>side</i> = 'R', $n \geq l \geq 0$. |
| <i>v</i> | (local). Pointer into the local memory to an array of size $lld_v * LOCC(jv+m-1)$ if <i>side</i> = 'L', $lld_v * LOCC(jv+m-1)$ if <i>side</i> = 'R'. It contains the local pieces of the distributed vectors V representing the Householder transformation as returned by <code>p?tzrzf</code> . $lld_v \geq LOCr(iv+k-1)$. |
| <i>iv, jv</i> | (global) |

The row and column indices in the global matrix V indicating the first row and the first column of the submatrix $\text{sub}(V)$, respectively.

descv (global and local) array of size *dlen_*. The array descriptor for the distributed matrix V .

t (local)
Array of size $mb_v * mb_v$.
The lower triangular matrix T in the representation of the block reflector.

c (local).
Pointer into the local memory to an array of size $lld_c * LOCc(jc+n-1)$.
On entry, the m -by- n distributed matrix $\text{sub}(C)$.

ic, jc (global)
The row and column indices in the global matrix C indicating the first row and the first column of the submatrix $\text{sub}(C)$, respectively.

desc (global and local) array of size *dlen_*. The array descriptor for the distributed matrix C .

work (local).
Array of size *lwork*.

```

If storev = 'C' ,
    if side = 'L' ,
        lwork ≥ (nqc0 + mpc0) * k
    else if side = 'R' ,
        lwork ≥ (nqc0 + max(npv0 + numroc(numroc(n+icoffc, nb_v, 0, 0,
            npcol),
                nb_v, 0, 0, lcmq), mpc0)) * k
    end if
else if storev = 'R' ,
    if side = 'L' ,
        lwork ≥ (mpc0 + max(mqv0 + numroc(numroc(m+iroffc, mb_v, 0, 0,
            nprw),
                mb_v, 0, 0, lcmp), nqc0)) * k
    else if side = 'R' ,
        lwork ≥ (mpc0 + nqc0) * k
    end if
end if.

```

Here $lcmq = lcm/npcol$ with $lcm = iclm(nprw, npcol)$,
 $iroffv = \text{mod}(iv-1, mb_v)$, $icoffv = \text{mod}(jv-1, nb_v)$,
 $ivrow = \text{indxg2p}(iv, mb_v, myrow, rsrc_v, nprw)$,

```

ivcol = indxg2p(jv, nb_v, mycol, csrc_v, npc0l),
mqv0 = numroc(m+icoffv, nb_v, mycol, ivcol, npc0l),
npv0 = numroc(n+iroffv, mb_v, myrow, ivrow, nprow),
iroffc = mod(ic-1, mb_c), icoffc= mod(jc-1, nb_c),
icrow= indxg2p(ic, mb_c, myrow, rsrc_c, nprow),
iccol= indxg2p(jc, nb_c, mycol, csrc_c, npc0l),
mpc0 = numroc(m+iroffc, mb_c, myrow, icrow, nprow),
npc0 = numroc(n+icoffc, mb_c, myrow, icrow, nprow),
nqc0 = numroc(n+icoffc, nb_c, mycol, iccol, npc0l),
ilcm, indxg2p, and numroc are ScaLAPACK tool functions; myrow, mycol,
nprow, and npc0l can be determined by calling the function
blacs_gridinfo.

```

Output Parameters

c (local).
 On exit, $\text{sub}(C)$ is overwritten by the $Q \cdot \text{sub}(C)$, or $Q' \cdot \text{sub}(C)$, or $\text{sub}(C) \cdot Q$, or $\text{sub}(C) \cdot Q'$, where Q' is the transpose (conjugate transpose) of Q .

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larzc

Applies (multiplies by) the conjugate transpose of an elementary reflector as returned by p?tzzrf to a general matrix.

Syntax

```

void pclarzc (char *side , MKL_INT *m , MKL_INT *n , MKL_INT *l , MKL_Complex8 *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work );
void pzlarzc (char *side , MKL_INT *m , MKL_INT *n , MKL_INT *l , MKL_Complex16 *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , MKL_INT *incv , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work );

```

Include Files

- mkl_scalapack.h

Description

The `p?larzc` function applies a complex elementary reflector Q^H to a complex m -by- n distributed matrix $\text{sub}(C) = C(ic:ic+m-1, jc:jc+n-1)$, from either the left or the right. Q is represented in the form

$$Q = I - \tau \cdot v \cdot v',$$

where τ is a complex scalar and v is a complex vector.

If $\tau = 0$, then Q is taken to be the unit matrix.

Q is a product of k elementary reflectors as returned by `p?tzrzf`.

Input Parameters

| | |
|---------------|--|
| <i>side</i> | (global) if <i>side</i> = 'L': form $Q^H * \text{sub}(C)$; if <i>side</i> = 'R': form $\text{sub}(C) * Q^H$. |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(C)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(C)$. ($n \geq 0$). |
| <i>l</i> | (global) The columns of the distributed matrix $\text{sub}(A)$ containing the meaningful part of the Householder reflectors. If <i>side</i> = 'L', $m \geq l \geq 0$, if <i>side</i> = 'R', $n \geq l \geq 0$. |
| <i>v</i> | (local). Pointer into the local memory to an array of size $lld_v * LOCC(n_v)$ containing the local pieces of the global distributed matrix V representing the Householder transformation Q , $V(iv:iv+1-1, jv)$ if <i>side</i> = 'L' and <i>incv</i> = 1, $V(iv, jv:jv+1-1)$ if <i>side</i> = 'L' and <i>incv</i> = m_v , $V(iv:iv+1-1, jv)$ if <i>side</i> = 'R' and <i>incv</i> = 1, $V(iv, jv:jv+1-1)$ if <i>side</i> = 'R' and <i>incv</i> = m_v . The vector v in the representation of Q . v is not used if <i>tau</i> = 0. |
| <i>iv, jv</i> | (global) The row and column indices in the global matrix V indicating the first row and the first column of the matrix $\text{sub}(V)$, respectively. |
| <i>descv</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix V . |
| <i>incv</i> | (global) The global increment for the elements of V . Only two values of <i>incv</i> are supported in this version, namely 1 and m_v . <i>incv</i> must not be zero. |
| <i>tau</i> | (local) Array of size $LOCC(jv)$ if <i>incv</i> = 1, and $LOCr(iv)$ otherwise. This array contains the Householder scalars related to the Householder vectors. <i>tau</i> is tied to the distributed matrix V . |
| <i>c</i> | (local). |

Pointer into the local memory to an array of size $lld_c * LOCC(jc+n-1)$, containing the local pieces of sub(C).

ic, jc

(global)

The row and column indices in the global matrix C indicating the first row and the first column of the matrix sub(C), respectively.

desc

(global and local) array of size $dlen_$. The array descriptor for the distributed matrix C .

work

(local).

```

If incv = 1,
  if side = 'L' ,
    if ivcol = iccol,
      lwork ≥ nqc0
    else
      lwork ≥ mpc0 + max(1, nqc0)
    end if
  else if side = 'R' ,
    lwork ≥ nqc0 + max(max(1, mpc0), numroc(numroc(n+icoffc, nb_v, 0,
0, npcol),
      nb_v, 0, 0, lcmq)) end if
else if incv = m_v,
  if side = 'L' ,
    lwork ≥ mpc0 + max(max(1, nqc0), numroc(numroc(m+iroffc, mb_v, 0,
0, nprow),
      mb_v, 0, 0, lcmp))
  else if side = 'R',
    if ivrow = icrow,
      lwork ≥ mpc0
    else
      lwork ≥ nqc0 + max(1, mpc0)
    end if
  end if
end if
end if

```

Here lcm is the least common multiple of $nprow$ and $npcol$;

$lcm = ilcm(nprow, npcol)$, $lcmp = lcm/nprow$, $lcmq = lcm/npcol$,

$iroffc = mod(ic-1, mb_c)$, $icoffc = mod(jc-1, nb_c)$,

$icrow = indxg2p(ic, mb_c, myrow, rsrc_c, nprow)$,

$iccol = indxg2p(jc, nb_c, mycol, csrc_c, npcol)$,

$mpc0 = numroc(m+iroffc, mb_c, myrow, icrow, nprow)$,

$nqc0 = numroc(n+icoffc, nb_c, mycol, iccol, npcol)$,

$ilcm$, $indxg2p$, and $numroc$ are ScaLAPACK tool functions;

$myrow$, $mycol$, $nprow$, and $npcol$ can be determined by calling the function `blacs_gridinfo`.

Output Parameters

c

(local).

On exit, $\text{sub}(C)$ is overwritten by the $Q^H * \text{sub}(C)$ if $\text{side} = 'L'$, or $\text{sub}(C) * Q^H$ if $\text{side} = 'R'$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?larzt

Forms the triangular factor T of a block reflector $H = I - V * T * V^H$ as returned by [p?tzrzf](#).

Syntax

```
void pslarzt (char *direct , char *storev , MKL_INT *n , MKL_INT *k , float *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , float *tau , float *t , float *work );
void pdlarzt (char *direct , char *storev , MKL_INT *n , MKL_INT *k , double *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , double *tau , double *t , double *work );
void pclarzt (char *direct , char *storev , MKL_INT *n , MKL_INT *k , MKL_Complex8 *v ,
MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , MKL_Complex8 *tau , MKL_Complex8 *t ,
MKL_Complex8 *work );
void pzlarzt (char *direct , char *storev , MKL_INT *n , MKL_INT *k , MKL_Complex16
*v , MKL_INT *iv , MKL_INT *jv , MKL_INT *descv , MKL_Complex16 *tau , MKL_Complex16
*t , MKL_Complex16 *work );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?larzt` function forms the triangular factor T of a real/complex block reflector H of order greater than n , which is defined as a product of k elementary reflectors as returned by [p?tzrzf](#).

If $\text{direct} = 'F'$, $H = H(1) * H(2) * \dots * H(k)$, and T is upper triangular;

If $\text{direct} = 'B'$, $H = H(k) * \dots * H(2) * H(1)$, and T is lower triangular.

If $\text{storev} = 'C'$, the vector which defines the elementary reflector $H(i)$, is stored in the i -th column of the array v , and

$$H = I - v * t * v^H.$$

If $\text{storev} = 'R'$, the vector, which defines the elementary reflector $H(i)$, is stored in the i -th row of the array v , and

$$H = I - v^H * t * v.$$

Currently, only $\text{storev} = 'R'$ and $\text{direct} = 'B'$ are supported.

Input Parameters

| | |
|---------------------|--|
| <code>direct</code> | (global) Specifies the order in which the elementary reflectors are multiplied to form the block reflector: if $\text{direct} = 'F'$: $H = H(1) * H(2) * \dots * H(k)$ (Forward, not supported) |
|---------------------|--|

| | |
|---------------|--|
| | if <i>direct</i> = 'B': $H = H(k) * \dots * H(2) * H(1)$ (Backward). |
| <i>storev</i> | (global) Specifies how the vectors which defines the elementary reflectors are stored: if <i>storev</i> = 'C': columnwise (not supported); if <i>storev</i> = 'R': rowwise. |
| <i>n</i> | (global) The order of the block reflector H . $n \geq 0$. |
| <i>k</i> | (global) The order of the triangular factor T (= the number of elementary reflectors). $1 \leq k \leq mb_v (= nb_v)$. |
| <i>v</i> | Pointer into the local memory to an array of local size $LOCr(iv+k-1) * LOCc(jv+n-1)$. The distributed matrix V contains the Householder vectors. See <i>Application Notes</i> below. |
| <i>iv, jv</i> | (global) The row and column indices in the global matrix V indicating the first row and the first column of the matrix $sub(V)$, respectively. |
| <i>descv</i> | (local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix V . |
| <i>tau</i> | (local) Array of size $LOCr(iv+k-1)$ if <i>incv</i> = <i>m_v</i> , and $LOCc(jv+k-1)$ otherwise. This array contains the Householder scalars related to the Householder vectors. <i>tau</i> is tied to the distributed matrix V . |
| <i>work</i> | (local). Workspace array of size $(k * (k-1) / 2)$. |

Output Parameters

| | |
|----------|---|
| <i>v</i> | |
| <i>t</i> | (local) Array of size $mb_v * mb_v$. It contains the k -by- k triangular factor of the block reflector associated with v . t is lower triangular. |

Application Notes

The shape of the matrix V and the storage of the vectors which define the $H(i)$ is best illustrated by the following example with $n = 5$ and $k = 3$. The elements equal to 1 are not stored; the corresponding array elements are modified but restored on exit. The rest of the array is not used.

direct='F' and storev='C'

$$\begin{matrix}
 \begin{bmatrix} v1 & v2 & v3 \\ v1 & v2 & v3 \end{bmatrix} \\
 v = \begin{matrix} . & . & . \\ . & . & . \\ 1 & . & . \\ & 1 & . \\ & & 1 \end{matrix}
 \end{matrix}$$

direct='F' and storev='R':

$$\begin{bmatrix} \overbrace{v1 \ v1 \ v1 \ v1 \ v1}^{\times} \ . \ . \ . \ . \ . \ 1 \\ v2 \ v2 \ v2 \ v2 \ v2 \ . \ . \ . \ 1 \\ v3 \ v3 \ v3 \ v3 \ v3 \ . \ . \ 1 \end{bmatrix}$$

direct='B' and storev='C':

$$\begin{matrix}
 1 \\
 . \ 1 \\
 . \ . \ 1 \\
 . \ . \ . \\
 v = \begin{matrix} . \ . \ . \\ v1 \ v2 \ v3 \end{matrix}
 \end{matrix}$$

direct='B' and storev='R':

$$\begin{bmatrix} 1 \ . \ . \ . \ . \ \overbrace{v1 \ v1 \ v1 \ v1 \ v1}^{\times} \\ . \ 1 \ . \ . \ . \ v2 \ v2 \ v2 \ v2 \ v2 \\ . \ . \ 1 \ . \ . \ v3 \ v3 \ v3 \ v3 \ v3 \end{bmatrix}$$

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lascl

Multiplies a general rectangular matrix by a real scalar defined as C_{to}/C_{from} .

Syntax

```
void pslascl (char *type , float *cfrom , float *cto , MKL_INT *m , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
```

```

void pdlascl (char *type , double *cfrom , double *cto , MKL_INT *m , MKL_INT *n ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );

void pclascl (char *type , float *cfrom , float *cto , MKL_INT *m , MKL_INT *n ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );

void pzlascl (char *type , double *cfrom , double *cto , MKL_INT *m , MKL_INT *n ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );

```

Include Files

- mkl_scalapack.h

Description

The `p?lascl` function multiplies the m -by- n real/complex distributed matrix $\text{sub}(A)$ denoting $A(ia:ia+m-1, ja:ja+n-1)$ by the real/complex scalar $cto/cfrom$. This is done without over/underflow as long as the final result $cto*A(i,j)/cfrom$ does not over/underflow. `type` specifies that $\text{sub}(A)$ may be full, upper triangular, lower triangular or upper Hessenberg.

Input Parameters

| | |
|-------------------------|---|
| <code>type</code> | (global) <code>type</code> indicates the storage type of the input distributed matrix. if <code>type = 'G'</code> : $\text{sub}(A)$ is a full matrix, if <code>type = 'L'</code> : $\text{sub}(A)$ is a lower triangular matrix, if <code>type = 'U'</code> : $\text{sub}(A)$ is an upper triangular matrix, if <code>type = 'H'</code> : $\text{sub}(A)$ is an upper Hessenberg matrix. |
| <code>cfrom, cto</code> | (global) The distributed matrix $\text{sub}(A)$ is multiplied by $cto/cfrom$. $A(i,j)$ is computed without over/underflow if the final result $cto*A(i,j)/cfrom$ can be represented without over/underflow. <code>cfrom</code> must be nonzero. |
| <code>m</code> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <code>n</code> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <code>a</code> | (local input/local output) Pointer into the local memory to an array of size $lld_a * LOCc(ja+n-1)$. This array contains the local pieces of the distributed matrix $\text{sub}(A)$. |
| <code>ia, ja</code> | (global) The column and row indices in the global matrix A indicating the first row and column of the matrix $\text{sub}(A)$, respectively. |
| <code>desca</code> | (global and local) Array of size $dlen_$. The array descriptor for the distributed matrix A . |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | (local). On exit, this array contains the local pieces of the distributed matrix multiplied by <i>cto/cfrom</i> . |
| <i>info</i> | (local) if <i>info</i> = 0: the execution is successful. if <i>info</i> < 0: If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lase2

Initializes an m-by-n distributed matrix.

Syntax

```
void pslase2 (const char* uplo, const MKL_INT* m, const MKL_INT* n, const float* alpha,
const float* beta, float* a, const MKL_INT* ia, const MKL_INT* ja, const MKL_INT*
desca);
```

```
void pdlase2 (const char* uplo, const MKL_INT* m, const MKL_INT* n, const double*
alpha, const double* beta, double* a, const MKL_INT* ia, const MKL_INT* ja, const
MKL_INT* desca);
```

```
void pclase2 (const char* uplo, const MKL_INT* m, const MKL_INT* n, const MKL_Complex8*
alpha, const MKL_Complex8* beta, MKL_Complex8* a, const MKL_INT* ia, const MKL_INT*
ja, const MKL_INT* desca);
```

```
void pzlase2 (const char* uplo, const MKL_INT* m, const MKL_INT* n, const
MKL_Complex16* alpha, const MKL_Complex16* beta, MKL_Complex16* a, const MKL_INT* ia,
const MKL_INT* ja, const MKL_INT* desca);
```

Include Files

- mkl_scalapack.h

Description

p?lase2 initializes an *m*-by-*n* distributed matrix sub(*A*) denoting $A(ia:ia+m-1,ja:ja+n-1)$ to *beta* on the diagonal and *alpha* on the off-diagonals. p?lase2 requires that only the dimension of the matrix operand is distributed.

Input Parameters

| | |
|-------------|---|
| <i>uplo</i> | (global) Specifies the part of the distributed matrix sub(<i>A</i>) to be set: = 'U': Upper triangular part is set; the strictly lower triangular part of sub(<i>A</i>) is not changed; |
|-------------|---|

= 'L': Lower triangular part is set; the strictly upper triangular part of $\text{sub}(A)$ is not changed;

Otherwise: All of the matrix $\text{sub}(A)$ is set.

| | |
|--------------|---|
| <i>m</i> | (global) The number of rows to be operated on i.e the number of rows of the distributed submatrix $\text{sub}(A)$. $m \geq 0$. |
| <i>n</i> | (global) The number of columns to be operated on i.e the number of columns of the distributed submatrix $\text{sub}(A)$. $n \geq 0$. |
| <i>alpha</i> | (global) The constant to which the off-diagonal elements are to be set. |
| <i>beta</i> | (global) The constant to which the diagonal elements are to be set. |
| <i>ia</i> | (global) The row index in the global array <i>a</i> indicating the first row of $\text{sub}(A)$. |
| <i>ja</i> | (global) The column index in the global array <i>a</i> indicating the first column of $\text{sub}(A)$. |
| <i>desca</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |

Output Parameters

| | |
|----------|--|
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. This array contains the local pieces of the distributed matrix $\text{sub}(A)$ to be set. On exit, the leading <i>m</i> -by- <i>n</i> submatrix $\text{sub}(A)$ is set as follows: if <i>uplo</i> = 'U', $A(ia+i-1, ja+j-1) = alpha, 1 \leq i \leq j-1, 1 \leq j \leq n$, if <i>uplo</i> = 'L', $A(ia+i-1, ja+j-1) = alpha, j+1 \leq i \leq m, 1 \leq j \leq n$, otherwise, $A(ia+i-1, ja+j-1) = alpha, 1 \leq i \leq m, 1 \leq j \leq n, ia+i \neq ja+j$, and, for all <i>uplo</i> , $A(ia+i-1, ja+i-1) = beta, 1 \leq i \leq \min(m, n)$. |
|----------|--|

p?laset

Initializes the offdiagonal elements of a matrix to *alpha* and the diagonal elements to *beta*.

Syntax

```
void pslaset (char *uplo , MKL_INT *m , MKL_INT *n , float *alpha , float *beta ,
float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );

void pdlaset (char *uplo , MKL_INT *m , MKL_INT *n , double *alpha , double *beta ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );

void pclaset (char *uplo , MKL_INT *m , MKL_INT *n , MKL_Complex8 *alpha , MKL_Complex8
*beta , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );

void pzaset (char *uplo , MKL_INT *m , MKL_INT *n , MKL_Complex16 *alpha ,
MKL_Complex16 *beta , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

Include Files

- mkl_scalapack.h

Description

The `p?laset` function initializes an m -by- n distributed matrix $\text{sub}(A)$ denoting $A(ia:ia+m-1, ja:ja+n-1)$ to β on the diagonal and α on the offdiagonals.

Input Parameters

| | |
|--------------|---|
| <i>uplo</i> | (global) Specifies the part of the distributed matrix $\text{sub}(A)$ to be set: if <i>uplo</i> = 'U': upper triangular part; the strictly lower triangular part of $\text{sub}(A)$ is not changed; if <i>uplo</i> = 'L': lower triangular part; the strictly upper triangular part of $\text{sub}(A)$ is not changed. Otherwise: all of the matrix $\text{sub}(A)$ is set. |
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(A)$. ($m \geq 0$). |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(A)$. ($n \geq 0$). |
| <i>alpha</i> | (global). The constant to which the offdiagonal elements are to be set. |
| <i>beta</i> | (global). The constant to which the diagonal elements are to be set. |

Output Parameters

| | |
|----------|--|
| <i>a</i> | (local). Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. This array contains the local pieces of the distributed matrix $\text{sub}(A)$ to be set. On exit, the leading m -by- n matrix $\text{sub}(A)$ is set as follows: if <i>uplo</i> = 'U', $A(ia+i-1, ja+j-1) = \alpha, 1 \leq i \leq m-1, 1 \leq j \leq n$, if <i>uplo</i> = 'L', $A(ia+i-1, ja+j-1) = \alpha, j+1 \leq i \leq m, 1 \leq j \leq n$, |
|----------|--|

otherwise, $A(i_a+i-1, j_a+j-1) = \alpha$, $1 \leq i \leq m$, $1 \leq j \leq n$, $i_a+i \neq j_a+j$, and, for all $uplo$, $A(i_a+i-1, j_a+i-1) = \beta$, $1 \leq j \leq \min(m, n)$.

ia, ja

(global)

The column and row indices in the distributed matrix A indicating the first row and column of the matrix sub(A), respectively.

desca

(global and local)

Array of size *dlen_*. The array descriptor for the distributed matrix A .

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lasmsub

Looks for a small subdiagonal element from the bottom of the matrix that it can safely set to zero.

Syntax

```
void pslasmsub (const float *a, const MKL_INT *desca, const MKL_INT *i, const MKL_INT *l, MKL_INT *k, const float *smlnum, float *buf, const MKL_INT *lwork );
```

```
void pdlasmsub (const double *a, const MKL_INT *desca, const MKL_INT *i, const MKL_INT *l, MKL_INT *k, const double *smlnum, double *buf, const MKL_INT *lwork );
```

```
void pclasmsub (const MKL_Complex8 *a , const MKL_INT *desca , const MKL_INT *i , const MKL_INT *l , MKL_INT *k , const float *smlnum , MKL_Complex8 *buf , const MKL_INT *lwork );
```

```
void pzlasmsub (const MKL_Complex16 *a , const MKL_INT *desca , const MKL_INT *i , const MKL_INT *l , MKL_INT *k , const double *smlnum , MKL_Complex16 *buf , const MKL_INT *lwork );
```

Include Files

- mkl_scalapack.h

Description

The p?lasmsubfunction looks for a small subdiagonal element from the bottom of the matrix that it can safely set to zero. This function performs a global maximum and must be called by all processes.

Input Parameters

a

(local)

Array of size $lld_a * LOCC(n_a)$.

On entry, the Hessenberg matrix whose tridiagonal part is being scanned. Unchanged on exit.

desca

(global and local)

Array of size *dlen_*. The array descriptor for the distributed matrix A .

i

(global)

| | |
|----------|--|
| | The global location of the bottom of the unreduced submatrix of A . Unchanged on exit. |
| l | (global) The global location of the top of the unreduced submatrix of A . Unchanged on exit. |
| $smlnum$ | (global) On entry, a "small number" for the given matrix. Unchanged on exit. The machine-dependent constants for the stopping criterion. |
| $lwork$ | (local) This must be at least $2 * \text{ceil}(\text{ceil}((i-1)/mb_a) / \text{lcm}(nprow, npcol))$. Here lcm is least common multiple and $nprow \times npcold$ is the logical grid size. |

Output Parameters

| | |
|-------|--|
| k | (global) On exit, this yields the bottom portion of the unreduced submatrix. This will satisfy: $l \leq k \leq i-1$. |
| buf | (local). Array of size $lwork$. |

Application Notes

This routine parallelizes the code from `?lahqr` that looks for a single small subdiagonal element.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lasrt

Sorts the numbers in an array and the corresponding vectors in increasing order.

Syntax

```
void pslasrt (const char* id, const MKL_INT* n, float* d, const float* q, const
MKL_INT* iq, const MKL_INT* jq, const MKL_INT* descq, float* work, const MKL_INT*
lwork, MKL_INT* iwork, const MKL_INT* liwork, MKL_INT* info);
```

```
void pdlasrt (const char* id, const MKL_INT* n, double* d, const double* q, const
MKL_INT* iq, const MKL_INT* jq, const MKL_INT* descq, double* work, const MKL_INT*
lwork, MKL_INT* iwork, const MKL_INT* liwork, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

`p?lasrt` sorts the numbers in d and the corresponding vectors in q in increasing order.

Input Parameters

| | |
|---------------|---|
| <i>id</i> | (global) = 'I': sort <i>d</i> in increasing order; = 'D': sort <i>d</i> in decreasing order. (NOT IMPLEMENTED YET) |
| <i>n</i> | (global) The number of columns to be operated on i.e the number of columns of the distributed submatrix sub(<i>Q</i>). $n \geq 0$. |
| <i>d</i> | (global) Array, size (<i>n</i>) |
| <i>q</i> | (local) Pointer into the local memory to an array of size $lld_q * LOCc(jq+n-1)$. This array contains the local pieces of the distributed matrix sub(<i>A</i>) to be copied from. |
| <i>iq</i> | (global) The row index in the global array <i>A</i> indicating the first row of sub(<i>Q</i>). |
| <i>jq</i> | (global) The column index in the global array <i>A</i> indicating the first column of sub(<i>Q</i>). |
| <i>descq</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Array, size (<i>lwork</i>) |
| <i>lwork</i> | (local) The size of the array <i>work</i> . $lwork = \text{MAX}(n, NP * (NB + NQ))$, where $NP = \text{numroc}(n, NB, MYROW, IAROW, NPROW)$, $NQ = \text{numroc}(n, NB, MYCOL, DESCQ(csrc_), NPCOL)$. <i>numroc</i> is a ScaLAPACK tool function. |
| <i>iwork</i> | (local) Array, size (<i>liwork</i>) |
| <i>liwork</i> | (local) The size of the array <i>iwork</i> . $liwork = n + 2 * NB + 2 * NPCOL$ |

Output Parameters

| | |
|----------|--|
| <i>d</i> | On exit, the numbers in <i>d</i> are sorted in increasing order. |
|----------|--|

info (global)
 = 0: successful exit
 < 0: If the *i*-th argument is an array and the *j*-th entry had an illegal value, then *info* = -(*i**100+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.

p?lassq

Updates a sum of squares represented in scaled form.

Syntax

```
void plassq (MKL_INT *n , float *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx ,
MKL_INT *incx , float *scale , float *sumsq );
```

```
void pdlassq (MKL_INT *n , double *x , MKL_INT *ix , MKL_INT *jx , MKL_INT *descx ,
MKL_INT *incx , double *scale , double *sumsq );
```

```
void pclassq (MKL_INT *n , MKL_Complex8 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT
*descx , MKL_INT *incx , float *scale , float *sumsq );
```

```
void pzlassq (MKL_INT *n , MKL_Complex16 *x , MKL_INT *ix , MKL_INT *jx , MKL_INT
*descx , MKL_INT *incx , double *scale , double *sumsq );
```

Include Files

- mkl_scalapack.h

Description

The p?lassq function returns the values *scl* and *sumsq* such that

$$scl^2 * sumsq = x_1^2 + \dots + x_n^2 + scale^2 * sumsq,$$

where

$x_i = \text{sub}(X) = X(ix + (jx-1)*m_x + (i-1)*incx)$ for plassq/pdlassq ,

$x_i = \text{sub}(X) = \text{abs}(X(ix + (jx-1)*m_x + (i-1)*incx))$ for pclassq/pzlassq.

For real functions plassq/pdlassq the value of *sumsq* is assumed to be non-negative and *scl* returns the value

$$scl = \max(scale, \text{abs}(x_i)).$$

For complex functions pclassq/pzlassq the value of *sumsq* is assumed to be at least unity and the value of *ssq* will then satisfy

$$1.0 \leq ssq \leq sumsq + 2n$$

Value *scale* is assumed to be non-negative and *scl* returns the value

$$scl = \max_i \left(scale, \text{abs}(\text{real}(x_i)), \text{abs}(\text{aimag}(x_i)) \right)$$

For all functions p?lassq values *scale* and *sumsq* must be supplied in *scale* and *sumsq* respectively, and *scale* and *sumsq* are overwritten by *scl* and *ssq* respectively.

All functions p?lassq make only one pass through the vector sub(*X*).

Input Parameters

| | |
|--------------|--|
| <i>n</i> | (global) The length of the distributed vector $\text{sub}(x)$. |
| <i>x</i> | The array that stores the vector for which a scaled sum of squares is computed: $x[ix + (jx-1)*m_x + i*incx], 0 \leq i < n.$ |
| <i>ix</i> | (global) The row index in the global matrix X indicating the first row of $\text{sub}(X)$. |
| <i>jx</i> | (global) The column index in the global matrix X indicating the first column of $\text{sub}(X)$. |
| <i>descx</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix X . |
| <i>incx</i> | (global) The global increment for the elements of X . Only two values of <i>incx</i> are supported in this version, namely 1 and <i>m_x</i> . The argument <i>incx</i> must not equal zero. |
| <i>scale</i> | (local). On entry, the value <i>scale</i> in the equation above. |
| <i>sumsq</i> | (local) On entry, the value <i>sumsq</i> in the equation above. |

Output Parameters

| | |
|--------------|--|
| <i>scale</i> | (local). On exit, <i>scale</i> is overwritten with <i>scl</i> , the scaling factor for the sum of squares. |
| <i>sumsq</i> | (local). On exit, <i>sumsq</i> is overwritten with the value <i>sumsq</i> , the basic sum of squares from which <i>scl</i> has been factored out. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?laswp

Performs a series of row interchanges on a general rectangular matrix.

Syntax

```
void pslaswp (char *direc , char *rowcol , MKL_INT *n , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_INT *k1 , MKL_INT *k2 , MKL_INT *ipiv );
```

```
void pdlaswp (char *direc , char *rowcol , MKL_INT *n , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_INT *k1 , MKL_INT *k2 , MKL_INT *ipiv );

void pclaswp (char *direc , char *rowcol , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_INT *k1 , MKL_INT *k2 , MKL_INT *ipiv );

void pzlaswp (char *direc , char *rowcol , MKL_INT *n , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *k1 , MKL_INT *k2 , MKL_INT *ipiv );
```

Include Files

- mkl_scalapack.h

Description

The `p?laswp` function performs a series of row or column interchanges on the distributed matrix $\text{sub}(A)=A(ia:ia+n-1, ja:ja+n-1)$. One interchange is initiated for each of rows or columns $k1$ through $k2$ of $\text{sub}(A)$. This function assumes that the pivoting information has already been broadcast along the process row or column. Also note that this function will only work for $k1-k2$ being in the same mb (or nb) block. If you want to pivot a full matrix, use `p?lapiv`.

Input Parameters

| | |
|---------------|---|
| <i>direc</i> | (global) Specifies in which order the permutation is applied: = 'F' - forward, = 'B' - backward. |
| <i>rowcol</i> | (global) Specifies if the rows or columns are permuted: = 'R' - rows, = 'C' - columns. |
| <i>n</i> | (global) If <i>rowcol</i> ='R', the length of the rows of the distributed matrix $A(*, ja:ja+n-1)$ to be permuted; If <i>rowcol</i> ='C', the length of the columns of the distributed matrix $A(ia:ia+n-1, *)$ to be permuted; |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCc(n_a)$. On entry, this array contains the local pieces of the distributed matrix to which the row/columns interchanges will be applied. |
| <i>ia</i> | (global) The row index in the global matrix A indicating the first row of $\text{sub}(A)$. |
| <i>ja</i> | (global) The column index in the global matrix A indicating the first column of $\text{sub}(A)$. |
| <i>desca</i> | (global and local) array of size $dlen_$. |

| | |
|--------|--|
| | The array descriptor for the distributed matrix A . |
| $k1$ | (global) The first element of $ipiv$ for which a row or column interchange will be done. |
| $k2$ | (global) The last element of $ipiv$ for which a row or column interchange will be done. |
| $ipiv$ | (local) Array of size $LOCr(m_a)+mb_a$ for row pivoting and $LOCr(n_a)+nb_a$ for column pivoting. This array is tied to the matrix A , $ipiv[k]=l$ implies rows (or columns) $k+1$ and l are to be interchanged, $k = 0, 1, \dots, \text{size}(ipiv) - 1$. |

Output Parameters

| | |
|-----|--|
| A | (local) On exit, the permuted distributed matrix. |
|-----|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?latra

Computes the trace of a general square distributed matrix.

Syntax

```
float pslatra (MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
double pdlatra (MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
void pclatra (MKL_Complex8 * , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
void pzlatra (MKL_Complex16 * , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

Include Files

- `mkl_scalapack.h`

Description

This function computes the trace of an n -by- n distributed matrix $\text{sub}(A)$ denoting $A(ia:ia+n-1, ja:ja+n-1)$. The result is left on every process of the grid.

Input Parameters

| | |
|-----|---|
| n | (global) The number of rows and columns to be operated on, that is, the order of the distributed matrix $\text{sub}(A)$. $n \geq 0$. |
| a | (local). |

Pointer into the local memory to an array of size $lld_a * LOCc(ja+n-1)$ containing the local pieces of the distributed matrix, the trace of which is to be computed.

ia, ja (global) The row and column indices respectively in the global matrix A indicating the first row and the first column of the matrix $sub(A)$, respectively.

desca (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A .

Output Parameters

val The value returned by the function.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?latrd

Reduces the first nb rows and columns of a symmetric/Hermitian matrix A to real tridiagonal form by an orthogonal/unitary similarity transformation.

Syntax

```
void pslatrd (char *uplo , MKL_INT *n , MKL_INT *nb , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *d , float *e , float *tau , float *w , MKL_INT *iw , MKL_INT *jw , MKL_INT *descw , float *work );
```

```
void pdlatrd (char *uplo , MKL_INT *n , MKL_INT *nb , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *d , double *e , double *tau , double *w , MKL_INT *iw , MKL_INT *jw , MKL_INT *descw , double *work );
```

```
void pclatrd (char *uplo , MKL_INT *n , MKL_INT *nb , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *d , float *e , MKL_Complex8 *tau , MKL_Complex8 *w , MKL_INT *iw , MKL_INT *jw , MKL_INT *descw , MKL_Complex8 *work );
```

```
void pzlatrd (char *uplo , MKL_INT *n , MKL_INT *nb , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *d , double *e , MKL_Complex16 *tau , MKL_Complex16 *w , MKL_INT *iw , MKL_INT *jw , MKL_INT *descw , MKL_Complex16 *work );
```

Include Files

- mkl_scalapack.h

Description

The `p?latrd` function reduces nb rows and columns of a real symmetric or complex Hermitian matrix $sub(A) = A(ia:ia+n-1, ja:ja+n-1)$ to symmetric/complex tridiagonal form by an orthogonal/unitary similarity transformation $Q'^*sub(A)Q$, and returns the matrices V and W , which are needed to apply the transformation to the unreduced part of $sub(A)$.

If $uplo = U$, `p?latrd` reduces the last nb rows and columns of a matrix, of which the upper triangle is supplied;

if $uplo = L$, `p?latrd` reduces the first nb rows and columns of a matrix, of which the lower triangle is supplied.

This is an auxiliary function called by `p?sytrd/p?hetrd`.

Input Parameters

| | | |
|--------------|--------------------|--|
| <i>uplo</i> | (global) | Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix $\text{sub}(A)$ is stored: = 'U': Upper triangular = L: Lower triangular. |
| <i>n</i> | (global) | The number of rows and columns to be operated on, that is, the order of the distributed matrix $\text{sub}(A)$. $n \geq 0$. |
| <i>nb</i> | (global) | The number of rows and columns to be reduced. |
| <i>a</i> | | Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the symmetric/Hermitian distributed matrix $\text{sub}(A)$. If $uplo = U$, the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and its strictly lower triangular part is not referenced. If $uplo = L$, the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix, and its strictly upper triangular part is not referenced. |
| <i>ia</i> | (global) | The row index in the global matrix A indicating the first row of $\text{sub}(A)$. |
| <i>ja</i> | (global) | The column index in the global matrix A indicating the first column of $\text{sub}(A)$. |
| <i>desca</i> | (global and local) | array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>iw</i> | (global) | The row index in the global matrix W indicating the first row of $\text{sub}(W)$. |
| <i>jwt</i> | (global) | The column index in the global matrix W indicating the first column of $\text{sub}(W)$. |
| <i>descw</i> | (global and local) | array of size $dlen_$. The array descriptor for the distributed matrix W . |
| <i>work</i> | (local) | Workspace array of size nb_a . |

Output Parameters

| | |
|------------|---|
| <i>a</i> | (local) On exit, if <i>uplo</i> = 'U', the last <i>nb</i> columns have been reduced to tridiagonal form, with the diagonal elements overwriting the diagonal elements of sub(<i>A</i>); the elements above the diagonal with the array <i>tau</i> represent the orthogonal/unitary matrix <i>Q</i> as a product of elementary reflectors; if <i>uplo</i> = 'L', the first <i>nb</i> columns have been reduced to tridiagonal form, with the diagonal elements overwriting the diagonal elements of sub(<i>A</i>); the elements below the diagonal with the array <i>tau</i> represent the orthogonal/unitary matrix <i>Q</i> as a product of elementary reflectors. |
| <i>d</i> | (local) Array of size $LOCc(ja+n-1)$. The diagonal elements of the tridiagonal matrix <i>T</i> : $d[i] = A(i+1,i+1)$, $i = 0, 1, \dots, LOCc(ja+n-1)-1$. <i>d</i> is tied to the distributed matrix <i>A</i> . |
| <i>e</i> | (local) Array of size $LOCc(ja+n-1)$ if <i>uplo</i> = 'U', $LOCc(ja+n-2)$ otherwise. The off-diagonal elements of the tridiagonal matrix <i>T</i> : $e[i] = A(i+1, i+2)$ if <i>uplo</i> = 'U', $e[i] = A(i+2, i+1)$ if <i>uplo</i> = 'L', $i = 0, 1, \dots, LOCc(ja+n-1)-1$. <i>e</i> is tied to the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array of size $LOCc(ja+n-1)$. This array contains the scalar factors of the elementary reflectors. <i>tau</i> is tied to the distributed matrix <i>A</i> . |
| <i>w</i> | (local) Pointer into the local memory to an array of size $lld_w * nb_w$. This array contains the local pieces of the <i>n</i> -by- <i>nb_w</i> matrix <i>w</i> required to update the unreduced part of sub(<i>A</i>). |

Application Notes

If *uplo* = 'U', the matrix *Q* is represented as a product of elementary reflectors

$$Q = H(n) * H(n-1) * \dots * H(n-nb+1)$$

Each *H(i)* has the form

$$H(i) = I - tau * v * v^T,$$

where *tau* is a real/complex scalar, and *v* is a real/complex vector with $v(i:n) = 0$ and $v(i-1) = 1$; $v(1:i-1)$ is stored on exit in $A(ia:ia+i-1, ja+i)$, and *tau* in $tau[ja+i-2]$.

If *uplo* = 'L', the matrix *Q* is represented as a product of elementary reflectors

$$Q = H(1) * H(2) * \dots * H(nb)$$

Each $H(i)$ has the form

$$H(i) = I - \tau v v^T,$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1:i) = 0$ and $v(i+1) = 1$; $v(i+2:n)$ is stored on exit in $A(ia+i+1:ia+n-1, ja+i-1)$, and τ in $\tau[ja+i-2]$.

The elements of the vectors v together form the n -by- nb matrix V which is needed, with W , to apply the transformation to the unreduced part of the matrix, using a symmetric/Hermitian rank- $2k$ update of the form:

$$\text{sub}(A) := \text{sub}(A) - v w^T - w v^T.$$

The contents of a on exit are illustrated by the following examples with

$n = 5$ and $nb = 2$:

| | |
|---|---|
| if $uplo='U'$: | if $uplo='L'$: |
| $\begin{bmatrix} a & a & a & v_4 & v_5 \\ & a & a & v_4 & v_5 \\ & & a & 1 & v_5 \\ & & & d & 1 \\ & & & & d \end{bmatrix}$ | $\begin{bmatrix} d & & & & \\ 1 & d & & & \\ v_1 & 1 & a & & \\ v_1 & v_2 & a & a & \\ v_1 & v_2 & a & a & a \end{bmatrix}$ |

where d denotes a diagonal element of the reduced matrix, a denotes an element of the original matrix that is unchanged, and v_i denotes an element of the vector defining $H(i)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?latrs

Solves a triangular system of equations with the scale factor set to prevent overflow.

Syntax

```
void pslatrs (char *uplo , char *trans , char *diag , char *normin , MKL_INT *n ,
float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *x , MKL_INT *ix ,
MKL_INT *jx , MKL_INT *descx , float *scale , float *cnorm , float *work );
```

```
void pdlatrs (char *uplo , char *trans , char *diag , char *normin , MKL_INT *n ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *x , MKL_INT *ix ,
MKL_INT *jx , MKL_INT *descx , double *scale , double *cnorm , double *work );
```

```
void pclatrs (char *uplo , char *trans , char *diag , char *normin , MKL_INT *n ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *x ,
MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , float *scale , float *cnorm ,
MKL_Complex8 *work );
```

```
void pzlatrs (char *uplo , char *trans , char *diag , char *normin , MKL_INT *n ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *x ,
MKL_INT *ix , MKL_INT *jx , MKL_INT *descx , double *scale , double *cnorm ,
MKL_Complex16 *work );
```

Include Files

- mkl_scalapack.h

Description

The `p?latrs` function solves a triangular system of equations $Ax = sb$, $A^T x = sb$ or $A^H x = sb$, where s is a scale factor set to prevent overflow. The description of the function will be extended in the future releases.

Input Parameters

| | |
|---------------|--|
| <i>uplo</i> | Specifies whether the matrix A is upper or lower triangular. = 'U': Upper triangular = 'L': Lower triangular |
| <i>trans</i> | Specifies the operation applied to Ax . = 'N': Solve $Ax = s*b$ (no transpose) = 'T': Solve $A^T x = s*b$ (transpose) = 'C': Solve $A^H x = s*b$ (conjugate transpose), where s - is a scale factor |
| <i>diag</i> | Specifies whether or not the matrix A is unit triangular. = 'N': Non-unit triangular = 'U': Unit triangular |
| <i>normin</i> | Specifies whether <i>cnorm</i> has been set or not. = 'Y': <i>cnorm</i> contains the column norms on entry; = 'N': <i>cnorm</i> is not set on entry. On exit, the norms will be computed and stored in <i>cnorm</i> . |
| <i>n</i> | The order of the matrix A . $n \geq 0$ |
| <i>a</i> | Array of size $lda * n$. Contains the triangular matrix A . If <i>uplo</i> = U, the leading n -by- n upper triangular part of the array <i>a</i> contains the upper triangular matrix, and the strictly lower triangular part of <i>a</i> is not referenced. If <i>uplo</i> = 'L', the leading n -by- n lower triangular part of the array <i>a</i> contains the lower triangular matrix, and the strictly upper triangular part of <i>a</i> is not referenced. If <i>diag</i> = 'U', the diagonal elements of <i>a</i> are also not referenced and are assumed to be 1. |

| | |
|---------------|--|
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the submatrix <i>A</i> , respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>x</i> | Array of size <i>n</i> . On entry, the right hand side <i>b</i> of the triangular system. |
| <i>ix</i> | (global).The row index in the global matrix <i>X</i> indicating the first row of sub(<i>x</i>). |
| <i>jx</i> | (global) The column index in the global matrix <i>X</i> indicating the first column of sub(<i>X</i>). |
| <i>descx</i> | (global and local) Array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>X</i> . |
| <i>cnorm</i> | Array of size <i>n</i> . If <i>normin</i> = 'Y', <i>cnorm</i> is an input argument and <i>cnorm</i> [<i>j</i>] contains the norm of the off-diagonal part of the (<i>j</i> +1)-th column of the matrix <i>A</i> , <i>j</i> =0, 1, ..., <i>n</i> -1. If <i>trans</i> = 'N', <i>cnorm</i> [<i>j</i>] must be greater than or equal to the infinity-norm, and if <i>trans</i> = 'T' or 'C', <i>cnorm</i> [<i>j</i>] must be greater than or equal to the 1-norm. |
| <i>work</i> | (local). Temporary workspace. |

Output Parameters

| | |
|--------------|--|
| <i>X</i> | On exit, <i>x</i> is overwritten by the solution vector <i>x</i> . |
| <i>scale</i> | Array of size <i>lda</i> * <i>n</i> . The scaling factor <i>s</i> for the triangular system as described above. If <i>scale</i> = 0, the matrix <i>A</i> is singular or badly scaled, and the vector <i>x</i> is an exact or approximate solution to $Ax = 0$. |
| <i>cnorm</i> | If <i>normin</i> = 'N', <i>cnorm</i> is an output argument and <i>cnorm</i> [<i>j</i>] returns the 1-norm of the off-diagonal part of the (<i>j</i> +1)-th column of <i>A</i> , <i>j</i> =0, 1, ..., <i>n</i> -1. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?latrz

Reduces an upper trapezoidal matrix to upper triangular form by means of orthogonal/unitary transformations.

Syntax

```
void pslatz (MKL_INT *m , MKL_INT *n , MKL_INT *l , float *a , MKL_INT *ia , MKL_INT
 *ja , MKL_INT *desca , float *tau , float *work );

void pdlatrz (MKL_INT *m , MKL_INT *n , MKL_INT *l , double *a , MKL_INT *ia , MKL_INT
 *ja , MKL_INT *desca , double *tau , double *work );
```

```
void pclatrz (MKL_INT *m , MKL_INT *n , MKL_INT *l , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work );

void pzlatrz (MKL_INT *m , MKL_INT *n , MKL_INT *l , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work );
```

Include Files

- mkl_scalapack.h

Description

The `p?latrz` function reduces the m -by- n ($m \leq n$) real/complex upper trapezoidal matrix $\text{sub}(A) = [A(ia:ia+m-1, ja:ja+m-1)A(ia:ia+m-1, ja+n-l:ja+n-1)]$ to upper triangular form by means of orthogonal/unitary transformations.

The upper trapezoidal matrix $\text{sub}(A)$ is factored as

$$\text{sub}(A) = (R \ 0) * Z,$$

where Z is an n -by- n orthogonal/unitary matrix and R is an m -by- m upper triangular matrix.

Input Parameters

| | |
|---------|---|
| m | (global) The number of rows in the distributed matrix $\text{sub}(A)$. $m \geq 0$. |
| n | (global) The number of columns in the distributed matrix $\text{sub}(A)$. $n \geq 0$. |
| l | (global) The number of columns of the distributed matrix $\text{sub}(A)$ containing the meaningful part of the Householder reflectors. $l > 0$. |
| a | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, the local pieces of the m -by- n distributed matrix $\text{sub}(A)$, which is to be factored. |
| ia | (global) The row index in the global matrix A indicating the first row of $\text{sub}(A)$. |
| ja | (global) The column index in the global matrix A indicating the first column of $\text{sub}(A)$. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| $work$ | (local) Workspace array of size $lwork$. $lwork \geq nq0 + \max(1, mp0)$, where $iroff = \text{mod}(ia-1, mb_a)$, $icoff = \text{mod}(ja-1, nb_a)$, |

```

iarow = indxg2p(ia, mb_a, myrow, rsrc_a, nrow),
iacol = indxg2p(ja, nb_a, mycol, csrc_a, ncol),
mp0 = numroc(m+iroff, mb_a, myrow, iarow, nrow),
nq0 = numroc(n+icoff, nb_a, mycol, iacol, ncol),
numroc, indxg2p, and numroc are ScaLAPACK tool functions; myrow,
mycol, nrow, and ncol can be determined by calling the function
blacs_gridinfo.

```

Output Parameters

| | |
|------------|---|
| <i>a</i> | On exit, the leading m -by- m upper triangular part of $\text{sub}(A)$ contains the upper triangular matrix R , and elements $n-l+1$ to n of the first m rows of $\text{sub}(A)$, with the array tau , represent the orthogonal/unitary matrix Z as a product of m elementary reflectors. |
| <i>tau</i> | (local) Array of size $\text{LOCr}(ja+m-1)$. This array contains the scalar factors of the elementary reflectors. tau is tied to the distributed matrix A . |

Application Notes

The factorization is obtained by Householder's method. The k -th transformation matrix, $Z(k)$, which is used (or, in case of complex functions, whose conjugate transpose is used) to introduce zeros into the $(m - k + 1)$ -th row of $\text{sub}(A)$, is given in the form

$$Z(k) = \begin{bmatrix} I & 0 \\ 0 & T(k) \end{bmatrix},$$

where

$$T(k) = I - \mathit{tau} * u(k) * u(k)', \quad u(k) = \begin{bmatrix} I \\ 0 \\ z(k) \end{bmatrix}$$

tau is a scalar and $z(k)$ is an $(n-m)$ -element vector. tau and $z(k)$ are chosen to annihilate the elements of the k -th row of $\text{sub}(A)$. The scalar tau is returned in the k -th element of tau , indexed $k-1$, and the vector $u(k)$ in the k -th row of $\text{sub}(A)$, such that the elements of $z(k)$ are in $A(k, m+1), \dots, A(k, n)$. The elements of R are returned in the upper triangular part of $\text{sub}(A)$.

Z is given by

$$Z = Z(1) Z(2) \dots Z(m).$$

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lauu2

Computes the product $U*U'$ or $L'*L$, where U and L are upper or lower triangular matrices (local unblocked algorithm).

Syntax

```
void pslauu2 (char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

```
void pdlauu2 (char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

```
void pclauu2 (char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

```
void pzlauu2 (char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

Include Files

- mkl_scalapack.h

Description

The p?lauu2 function computes the product $U*U'$ or $L'*L$, where the triangular factor U or L is stored in the upper or lower triangular part of the distributed matrix

$sub(A) = A(ia:ia+n-1, ja:ja+n-1)$.

If *uplo* = 'U' or 'u', then the upper triangle of the result is stored, overwriting the factor U in $sub(A)$.

If *uplo* = 'L' or 'l', then the lower triangle of the result is stored, overwriting the factor L in $sub(A)$.

This is the unblocked form of the algorithm, calling [BLAS Level 2 Routines](#). No communication is performed by this function, the matrix to operate on should be strictly local to one process.

Input Parameters

| | |
|-------------|---|
| <i>uplo</i> | (global) Specifies whether the triangular factor stored in the <i>matrix</i> $sub(A)$ is upper or lower triangular: = U: upper triangular = L: lower triangular. |
| <i>n</i> | (global) The number of rows and columns to be operated on, that is, the order of the triangular factor U or L . $n \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, the local pieces of the triangular factor U or L . |
| <i>ia</i> | (global) The row index in the global matrix A indicating the first row of $sub(A)$. |
| <i>ja</i> | (global) |

The column index in the global matrix A indicating the first column of $\text{sub}(A)$.

desca (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A .

Output Parameters

a (local)
On exit, if $uplo = 'U'$, the upper triangle of the distributed matrix $\text{sub}(A)$ is overwritten with the upper triangle of the product $U*U'$; if $uplo = 'L'$, the lower triangle of $\text{sub}(A)$ is overwritten with the lower triangle of the product $L'*L$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lauum

*Computes the product $U*U'$ or $L'*L$, where U and L are upper or lower triangular matrices.*

Syntax

```
void pslauum (char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

```
void pdlauum (char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

```
void pclauum (char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

```
void pzlauum (char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lauum` function computes the product $U*U'$ or $L'*L$, where the triangular factor U or L is stored in the upper or lower triangular part of the matrix $\text{sub}(A) = A(ia:ia+n-1, ja:ja+n-1)$.

If $uplo = 'U'$ or $'u'$, then the upper triangle of the result is stored, overwriting the factor U in $\text{sub}(A)$. If $uplo = 'L'$ or $'l'$, then the lower triangle of the result is stored, overwriting the factor L in $\text{sub}(A)$.

This is the blocked form of the algorithm, calling Level 3 PBLAS.

Input Parameters

uplo (global)
Specifies whether the triangular factor stored in the matrix $\text{sub}(A)$ is upper or lower triangular:
= $'U'$: upper triangular

= 'L': lower triangular.

| | | |
|--------------|---|--|
| <i>n</i> | (global) | The number of rows and columns to be operated on, that is, the order of the triangular factor <i>U</i> or <i>L</i> . $n \geq 0$. |
| <i>a</i> | (local) | Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, the local pieces of the triangular factor <i>U</i> or <i>L</i> . |
| <i>ia</i> | (global) | The row index in the global matrix <i>A</i> indicating the first row of sub(<i>A</i>). |
| <i>ja</i> | (global) | The column index in the global matrix <i>A</i> indicating the first column of sub(<i>A</i>). |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . | The array descriptor for the distributed matrix <i>A</i> . |

Output Parameters

| | | |
|----------|---------|---|
| <i>a</i> | (local) | On exit, if <i>uplo</i> = 'U', the upper triangle of the distributed matrix sub(<i>A</i>) is overwritten with the upper triangle of the product $U*U'$; if <i>uplo</i> = 'L', the lower triangle of sub(<i>A</i>) is overwritten with the lower triangle of the product $L'*L$. |
|----------|---------|---|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lawil

Forms the Wilkinson transform.

Syntax

```
void pslawil (const MKL_INT *ii, const MKL_INT *jj, const MKL_INT *m, const float *a,
const MKL_INT *desca, const float *h44, const float *h33, const float *h43h34, float
*v );
```

```
void pdlawil (const MKL_INT *ii, const MKL_INT *jj, const MKL_INT *m, const double *a,
const MKL_INT *desca, const double *h44, const double *h33, const double *h43h34,
double *v );
```

```
void pclawil (const MKL_INT *ii , const MKL_INT *jj , const MKL_INT *m , const
MKL_Complex8 *a , const MKL_INT *desca , const MKL_Complex8 *h44 , const MKL_Complex8
*h33 , const MKL_Complex8 *h43h34 , MKL_Complex8 *v );
```

```
void pzlawil (const MKL_INT *ii , const MKL_INT *jj , const MKL_INT *m , const
MKL_Complex16 *a , const MKL_INT *desca , const MKL_Complex16 *h44 , const
MKL_Complex16 *h33 , const MKL_Complex16 *h43h34 , MKL_Complex16 *v );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lawil` function gets the transform given by `h44`, `h33`, and `h43h34` into `v` starting at row `m`.

Input Parameters

| | |
|---------------------|--|
| <code>ii</code> | (global) Number of the process row which owns the matrix element $A(m+2, m+2)$. |
| <code>jj</code> | (global) Number of the process column which owns the matrix element $A(m+2, m+2)$. |
| <code>m</code> | (global) On entry, the location from where the transform starts (row <code>m</code>). Unchanged on exit. |
| <code>a</code> | (local) Array of size <code>lld_a*LOCc(n_a)</code> . On entry, the Hessenberg matrix. Unchanged on exit. |
| <code>desca</code> | (global and local) Array of size <code>dlen_</code> . The array descriptor for the distributed matrix <code>A</code> . Unchanged on exit. |
| <code>h43h34</code> | (global) These three values are for the double shift <code>QR</code> iteration. Unchanged on exit. |

Output Parameters

| | |
|----------------|--|
| <code>v</code> | (global) Array of size 3 that contains the transform on output. |
|----------------|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

[p?org2l/p?ung2l](#)

Generates all or part of the orthogonal/unitary matrix `Q` from a `QL` factorization determined by `p?geqlf` (unblocked algorithm).

Syntax

```
void psorg2l (MKL_INT *m , MKL_INT *n , MKL_INT *k , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
void pdorg2l (MKL_INT *m , MKL_INT *n , MKL_INT *k , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcung21 (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );

void pzung21 (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?org21/p?ung21` function generates an m -by- n real/complex distributed matrix Q denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal columns, which is defined as the last n columns of a product of k elementary reflectors of order m :

$Q = H(k)*...*H(2)*H(1)$ as returned by `p?geqlf`.

Input Parameters

| | |
|---------|---|
| m | (global) The number of rows in the distributed submatrix Q . $m \geq 0$. |
| n | (global) The number of columns in the distributed submatrix Q . $m \geq n \geq 0$. |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . $n \geq k \geq 0$. |
| a | Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, the j -th column of the matrix stored in a must contain the vector that defines the elementary reflector $H(j)$, $ja+n-k \leq j \leq ja+n-k$, as returned by <code>p?geqlf</code> in the k columns of its <i>distributed matrix</i> argument $A(ia:*, ja+n-k:ja+n-1)$. |
| ia | (global) The row index in the global matrix A indicating the first row of sub(A). |
| ja | (global) The column index in the global matrix A indicating the first column of sub(A). |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| tau | (local) Array of size $LOCC(ja+n-1)$. $tau[j]$ contains the scalar factor of the elementary reflector $H(j+1)$, $j = 0, 1, \dots, LOCC(ja+n-1)-1$, as returned by <code>p?geqlf</code> . |
| $work$ | (local) |

Workspace array of size *lwork*.

lwork (local or global)
The size of the array *work*.
lwork is local input and must be at least $lwork \geq mpa0 + \max(1, nqa0)$, where

```

iroffa = mod(ia-1, mb_a),
icoffa = mod(ja-1, nb_a),
iarow = indxg2p(ia, mb_a, myrow, rsrc_a, nprow),
iacol = indxg2p(ja, nb_a, mycol, csrc_a, npcot),
mpa0 = numroc(m+iroffa, mb_a, myrow, iarow, nprow),
nqa0 = numroc(n+icoffa, nb_a, mycol, iacol, npcot).

```

`indxg2p` and `numroc` are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcol* can be determined by calling the function `blacs_gridinfo`.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

a On exit, this array contains the local pieces of the *m*-by-*n* distributed matrix *Q*.

work On exit, *work*[0] returns the minimal and optimal *lwork*.

info (local).
= 0: successful exit
< 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value,
then *info* = - (*i**100 + *j*),
if the *i*-th argument is a scalar and had an illegal value,
then *info* = -*i*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?org2r/p?ung2r

Generates all or part of the orthogonal/unitary matrix Q from a QR factorization determined by p?geqrf (unblocked algorithm).

Syntax

```
void psorg2r (MKL_INT *m , MKL_INT *n , MKL_INT *k , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdorg2r (MKL_INT *m , MKL_INT *n , MKL_INT *k , double *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
void pcung2r (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
void pzung2r (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?org2r/p?ung2r` function generates an m -by- n real/complex matrix Q denoting $A(ia:ia+m-1, ja:ja+n-1)$ with orthonormal columns, which is defined as the first n columns of a product of k elementary reflectors of order m :

$$Q = H(1)*H(2)*...*H(k)$$

as returned by `p?geqrf`.

Input Parameters

| | |
|---------|--|
| m | (global) The number of rows in the distributed submatrix Q . $m \geq 0$. |
| n | (global) The number of columns in the distributed submatrix Q . $m \geq n \geq 0$. |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . $n \geq k \geq 0$. |
| a | Pointer into the local memory to an array of size $ld_a * LOCC(ja+n-1)$ On entry, the j -th column of the matrix stored in a must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by <code>p?geqrf</code> in the k columns of its <i>distributed matrix</i> argument $A(ia:*, ja:ja+k-1)$. |
| ia | (global) The row index in the global matrix A indicating the first row of sub(A). |
| ja | (global) The column index in the global matrix A indicating the first column of sub(A). |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| tau | (local) Array of size $LOCC(ja+k-1)$. |

$\tau[j]$ contains the scalar factor of the elementary reflector $H(j+1)$, $j = 0, 1, \dots, LOCC(ja+k-1)-1$, as returned by [p?geqrf](#). This array is tied to the distributed matrix A .

work (local)
Workspace array of size *lwork*.

lwork (local or global)
The size of the array *work*.
lwork is local input and must be at least $lwork \geq mpa0 + \max(1, nqa0)$, where

```
iroffa = mod(ia-1, mb_a), icoffa = mod(ja-1, nb_a),
iarow = indxg2p(ia, mb_a, myrow, rsrc_a, nprow),
iacol = indxg2p(ja, nb_a, mycol, csrc_a, npcot),
mpa0 = numroc(m+iroffa, mb_a, myrow, iarow, nprow),
nqa0 = numroc(n+icoffa, nb_a, mycol, iacol, npcot).
```

`indxg2p` and `numroc` are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npcot* can be determined by calling the function `blacs_gridinfo`.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by [pxerbla](#).

Output Parameters

a On exit, this array contains the local pieces of the m -by- n distributed matrix Q .

work On exit, *work*[0] returns the minimal and optimal *lwork*.

info (local).
= 0: successful exit
< 0: if the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value,
then $info = -(i*100 + j)$,
if the i -th argument is a scalar and had an illegal value,
then $info = -i$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

[p?orgl2/p?ungl2](#)

Generates all or part of the orthogonal/unitary matrix Q from an LQ factorization determined by [p?gelqf](#) (unblocked algorithm).

Syntax

```
void psorgl2 (MKL_INT *m , MKL_INT *n , MKL_INT *k , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdorgl2 (MKL_INT *m , MKL_INT *n , MKL_INT *k , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcungl2 (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pzungl2 (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?orgl2/p?ungl2` function generates a m -by- n real/complex matrix Q denoting A ($ia:ia+m-1$, $ja:ja+n-1$) with orthonormal rows, which is defined as the first m rows of a product of k elementary reflectors of order n

$Q = H(k) * \dots * H(2) * H(1)$ (for real flavors),

$Q = (H(k))^H * \dots * (H(2))^H * (H(1))^H$ (for complex flavors) as returned by `p?gelqf`.

Input Parameters

| | |
|------|--|
| m | (global) The number of rows in the distributed submatrix Q . $m \geq 0$. |
| n | (global) The number of columns in the distributed submatrix Q . $n \geq m \geq 0$. |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . $m \geq k \geq 0$. |
| a | Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, the i -th row of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by <code>p?gelqf</code> in the k rows of its <i>distributed matrix</i> argument A ($ia:ia+k-1$, $ja:*$). |
| ia | (global) The row index in the global matrix A indicating the first row of $sub(A)$. |
| ja | (global) The column index in the global matrix A indicating the first column of $sub(A)$. |

| | |
|--------------|---|
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array of size $LOCr(ja+k-1)$. $tau[j]$ contains the scalar factor of the elementary reflectors $H(j+1)$, $j = 0, 1, \dots, LOCr(ja+k-1) - 1$, as returned by p?gelqf . This array is tied to the distributed matrix <i>A</i> . |
| <i>WORK</i> | (local) Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> . <i>lwork</i> is local input and must be at least $lwork \geq nqa0 + \max(1, mpa0)$, where $iroffa = \text{mod}(ia-1, mb_a),$ $icoffa = \text{mod}(ja-1, nb_a),$ $iarow = \text{indxg2p}(ia, mb_a, myrow, rsrc_a, nprow),$ $iacol = \text{indxg2p}(ja, nb_a, mycol, csrc_a, npcot),$ $mpa0 = \text{numroc}(m+iroffa, mb_a, myrow, iarow, nprow),$ $nqa0 = \text{numroc}(n+icoffa, nb_a, mycol, iacol, npcot).$ <i>indxg2p</i> and <i>numroc</i> are ScaLAPACK tool functions; <i>myrow</i> , <i>mycol</i> , <i>nprow</i> , and <i>npcol</i> can be determined by calling the function <code>blacs_gridinfo</code> . If <i>lwork</i> = -1, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla . |

Output Parameters

| | |
|-------------|---|
| <i>a</i> | On exit, this array contains the local pieces of the <i>m</i> -by- <i>n</i> distributed matrix <i>Q</i> . |
| <i>work</i> | On exit, <i>work</i> [0] returns the minimal and optimal <i>lwork</i> . |
| <i>info</i> | (local) = 0: successful exit < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then $info = -(i*100 + j)$, if the <i>i</i> -th argument is a scalar and had an illegal value, then $info = -i$. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?orgr2/p?ungr2

Generates all or part of the orthogonal/unitary matrix Q from an RQ factorization determined by [p?gerqf](#) (unblocked algorithm).

Syntax

```
void psorgr2 (MKL_INT *m , MKL_INT *n , MKL_INT *k , float *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , float *tau , float *work , MKL_INT *lwork , MKL_INT *info );
void pdorgr2 (MKL_INT *m , MKL_INT *n , MKL_INT *k , double *a , MKL_INT *ia , MKL_INT
*ja , MKL_INT *desca , double *tau , double *work , MKL_INT *lwork , MKL_INT *info );
void pcungr2 (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex8 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
void pzungr2 (MKL_INT *m , MKL_INT *n , MKL_INT *k , MKL_Complex16 *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The [p?orgr2/p?ungr2](#) function generates an m -by- n real/complex matrix Q denoting A ($ia:ia+m-1$, $ja:ja+n-1$) with orthonormal rows, which is defined as the last m rows of a product of k elementary reflectors of order n

$Q = H(1)*H(2)*...*H(k)$ (for real flavors);

$Q = (H(1))^H*(H(2))^H*...*(H(k))^H$ (for complex flavors) as returned by [p?gerqf](#).

Input Parameters

| | |
|------|---|
| m | (global) The number of rows in the distributed submatrix Q . $m \geq 0$. |
| n | (global) The number of columns in the distributed submatrix Q . $n \geq m \geq 0$. |
| k | (global) The number of elementary reflectors whose product defines the matrix Q . $m \geq k \geq 0$. |
| a | Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, the i -th row of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia+m-k \leq i \leq ia+m-1$, as returned by p?gerqf in the k rows of its <i>distributed matrix</i> argument A ($ia+m-k:ia+m-1$, $ja:*$). |
| ia | (global) The row index in the global matrix A indicating the first row of sub(A). |

| | |
|--------------|---|
| <i>ja</i> | (global) The column index in the global matrix <i>A</i> indicating the first column of <i>sub(A)</i> . |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |
| <i>tau</i> | (local) Array of size $LOCr(ja+m-1)$. <i>tau[j]</i> contains the scalar factor of the elementary reflectors $H(j+1)$, $j = 0, 1, \dots, LOCr(ja+m-1)-1$, as returned by p?gerqf . This array is tied to the distributed matrix <i>A</i> . |
| <i>work</i> | (local) Workspace array of size <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the array <i>work</i> . <i>lwork</i> is local input and must be at least $lwork \geq nqa0 + \max(1, mpa0)$, where $irowfa = \text{mod}(ia-1, mb_a)$, $icoffa = \text{mod}(ja-1, nb_a)$, $iarow = \text{indxg2p}(ia, mb_a, myrow, rsrc_a, nprow)$, $iacol = \text{indxg2p}(ja, nb_a, mycol, csrc_a, npcot)$, $mpa0 = \text{numroc}(m+irowfa, mb_a, myrow, iarow, nprow)$, $nqa0 = \text{numroc}(n+icoffa, nb_a, mycol, iacol, npcot)$. <i>indxg2p</i> and <i>numroc</i> are ScaLAPACK tool functions; <i>myrow</i> , <i>mycol</i> , <i>nprow</i> , and <i>npcol</i> can be determined by calling the function <i>blacs_gridinfo</i> . If <i>lwork</i> = -1, then <i>lwork</i> is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla . |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | On exit, this array contains the local pieces of the <i>m</i> -by- <i>n</i> distributed matrix <i>Q</i> . |
| <i>work</i> | On exit, <i>work</i> [0] returns the minimal and optimal <i>lwork</i> . |
| <i>info</i> | (local) = 0: successful exit < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then <i>info</i> = - (<i>i</i> *100 + <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?orm2l/p?unm2l

Multiplies a general matrix by the orthogonal/unitary matrix from a QL factorization determined by p?geqlf (unblocked algorithm).

Syntax

```
void psorm2l (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );

void pdorm2l (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );

void pcunm2l (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );

void pzunm2l (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?orm2l/p?unm2l function overwrites the general real/complex m -by- n distributed matrix sub (C)= $C(ic:ic+m-1,jc:jc+n-1)$ with

$Q*\text{sub}(C)$ if $side = 'L'$ and $trans = 'N'$, or

$Q^T*\text{sub}(C) / Q^H*\text{sub}(C)$ if $side = 'L'$ and $trans = 'T'$ (for real flavors) or $trans = 'C'$ (for complex flavors), or

$\text{sub}(C)*Q$ if $side = 'R'$ and $trans = 'N'$, or

$\text{sub}(C)*Q^T / \text{sub}(C)*Q^H$ if $side = 'R'$ and $trans = 'T'$ (for real flavors) or $trans = 'C'$ (for complex flavors).

where Q is a real orthogonal or complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(k)*\dots*H(2)*H(1)$ as returned by p?geqlf . Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|--------------|---|
| <i>side</i> | (global) |
| | = 'L': apply Q or Q^T for real flavors (Q^H for complex flavors) from the left, |
| | = 'R': apply Q or Q^T for real flavors (Q^H for complex flavors) from the right. |
| <i>trans</i> | (global) |

| | |
|--------------|---|
| | <p>= 'N': apply Q (no transpose)</p> <p>= 'T': apply Q^T (transpose, for real flavors)</p> <p>= 'C': apply Q^H (conjugate transpose, for complex flavors)</p> |
| <i>m</i> | <p>(global)</p> <p>The number of rows in the distributed matrix $\text{sub}(C)$. $m \geq 0$.</p> |
| <i>n</i> | <p>(global)</p> <p>The number of columns in the distributed matrix $\text{sub}(C)$. $n \geq 0$.</p> |
| <i>k</i> | <p>(global)</p> <p>The number of elementary reflectors whose product defines the matrix Q.</p> <p>If $\text{side} = \text{'L'}$, $m \geq k \geq 0$;</p> <p>if $\text{side} = \text{'R'}$, $n \geq k \geq 0$.</p> |
| <i>a</i> | <p>(local)</p> <p>Pointer into the local memory to an array of size $\text{lld}_a * \text{LOCc}(ja+k-1)$.</p> <p>On entry, the j-th row of the matrix stored in <i>a</i> must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by p?geqlf in the k columns of its distributed matrix argument $A(\text{ia} : *, ja : ja+k-1)$. The argument $A(\text{ia} : *, ja : ja+k-1)$ is modified by the function but restored on exit.</p> <p>If $\text{side} = \text{'L'}$, $\text{lld}_a \geq \max(1, \text{LOCr}(\text{ia}+m-1))$,</p> <p>if $\text{side} = \text{'R'}$, $\text{lld}_a \geq \max(1, \text{LOCr}(\text{ia}+n-1))$.</p> |
| <i>ia</i> | <p>(global)</p> <p>The row index in the global matrix A indicating the first row of $\text{sub}(A)$.</p> |
| <i>ja</i> | <p>(global)</p> <p>The column index in the global matrix A indicating the first column of $\text{sub}(A)$.</p> |
| <i>desca</i> | <p>(global and local) array of size $dlen_$. The array descriptor for the distributed matrix A.</p> |
| <i>tau</i> | <p>(local)</p> <p>Array of size $\text{LOCc}(ja+n-1)$. $\text{tau}[j]$ contains the scalar factor of the elementary reflector $H(j+1)$, $j = 0, 1, \dots, \text{LOCc}(ja+n-1)-1$, as returned by p?geqlf. This array is tied to the distributed matrix A.</p> |
| <i>c</i> | <p>(local)</p> <p>Pointer into the local memory to an array of size $\text{lld}_c * \text{LOCc}(jc+n-1)$. On entry, the local pieces of the distributed matrix $\text{sub}(C)$.</p> |
| <i>ic</i> | <p>(global)</p> <p>The row index in the global matrix C indicating the first row of $\text{sub}(C)$.</p> |
| <i>jc</i> | <p>(global)</p> |

The column index in the global matrix C indicating the first column of $\text{sub}(C)$.

desc (global and local) array of size *dlen_*. The array descriptor for the distributed matrix C .

work (local)
Workspace array of size *lwork*.
On exit, *work*(1) returns the minimal and optimal *lwork*.

lwork (local or global)
The size of the array *work*.
lwork is local input and must be at least
if *side* = 'L', $lwork \geq mpc0 + \max(1, nqc0)$,
if *side* = 'R', $lwork \geq nqc0 + \max(\max(1, mpc0), \text{numroc}(\text{numroc}(n + icoffc, nb_a, 0, 0, npc0), nb_a, 0, 0, lcmq))$,

where

$$lcmq = lcm / npc0,$$

$$lcm = iclm(nprow, npc0),$$

$$iroffc = \text{mod}(ic-1, mb_c),$$

$$icoffc = \text{mod}(jc-1, nb_c),$$

$$icrow = \text{indxg2p}(ic, mb_c, myrow, rsrc_c, nprow),$$

$$iccol = \text{indxg2p}(jc, nb_c, mycol, csrc_c, npc0),$$

$$Mqc0 = \text{numroc}(m + icoffc, nb_c, mycol, icrow, nprow),$$

$$Npc0 = \text{numroc}(n + iroffc, mb_c, myrow, iccol, npc0),$$

iclm, *indxg2p*, and *numroc* are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npc0* can be determined by calling the function *blacs_gridinfo*.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by *pxerbla*.

Output Parameters

c On exit, *c* is overwritten by $Q * \text{sub}(C)$, or $Q^T * \text{sub}(C) / Q^H * \text{sub}(C)$, or $\text{sub}(C) * Q$, or $\text{sub}(C) * Q^T / \text{sub}(C) * Q^H$

work On exit, *work*[0] returns the minimal and optimal *lwork*.

info (local)
= 0: successful exit
< 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value,
then *info* = - (*i**100 + *j*),

if the i -th argument is a scalar and had an illegal value,
then $info = -i$.

NOTE

The distributed submatrices $A(ia:*, ja:*)$ and $C(ic:ic+m-1, jc:jc+n-1)$ must verify some alignment properties, namely the following expressions should be true:

If $side = 'L'$, ($mb_a == mb_c$ && $iroffa == iroffc$ && $iarow == icrow$)

If $side = 'R'$, ($mb_a == nb_c$ && $iroffa == iroffc$).

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?orm2r/p?unm2r

Multiplies a general matrix by the orthogonal/unitary matrix from a QR factorization determined by p?geqrf (unblocked algorithm).

Syntax

```
void psorm2r (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );
void pdorm2r (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );
void pcunm2r (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
void pzunm2r (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?orm2r/p?unm2r` function overwrites the general real/complex m -by- n distributed matrix sub (C)= $C(ic:ic+m-1, jc:jc+n-1)$ with

$Q*\text{sub}(C)$ if $side = 'L'$ and $trans = 'N'$, or

$Q^T*\text{sub}(C)$ / $Q^H*\text{sub}(C)$ if $side = 'L'$ and $trans = 'T'$ (for real flavors) or $trans = 'C'$ (for complex flavors), or

$\text{sub}(C)*Q$ if $side = 'R'$ and $trans = 'N'$, or

$\text{sub}(C)*Q^T$ / $\text{sub}(C)*Q^H$ if $side = 'R'$ and $trans = 'T'$ (for real flavors) or $trans = 'C'$ (for complex flavors).

where Q is a real orthogonal or complex unitary matrix defined as the product of k elementary reflectors

$Q = H(k) * \dots * H(2) * H(1)$ as returned by `p?geqrf`. Q is of order m if `side = 'L'` and of order n if `side = 'R'`.

Input Parameters

| | |
|--------------------|---|
| <code>side</code> | (global) = 'L': apply Q or Q^T for real flavors (Q^H for complex flavors) from the left, = 'R': apply Q or Q^T for real flavors (Q^H for complex flavors) from the right. |
| <code>trans</code> | (global) = 'N': apply Q (no transpose) = 'T': apply Q^T (transpose, for real flavors) = 'C': apply Q^H (conjugate transpose, for complex flavors) |
| <code>m</code> | (global) The number of rows in the distributed matrix <code>sub(C)</code> . $m \geq 0$. |
| <code>n</code> | (global) The number of columns in the distributed matrix <code>sub(C)</code> . $n \geq 0$. |
| <code>k</code> | (global) The number of elementary reflectors whose product defines the matrix Q . If <code>side = 'L'</code> , $m \geq k \geq 0$; if <code>side = 'R'</code> , $n \geq k \geq 0$. |
| <code>a</code> | (local) Pointer into the local memory to an array of size <code>lld_a * LOCC(ja+k-1)</code> . On entry, the j -th column of the matrix stored in <code>a</code> must contain the vector that defines the elementary reflector $H(j)$, $ja \leq j \leq ja+k-1$, as returned by <code>p?geqrf</code> in the k columns of its distributed matrix argument $A(ia:*, ja:ja+k-1)$. The argument $A(ia:*, ja:ja+k-1)$ is modified by the function but restored on exit. If <code>side = 'L'</code> , <code>lld_a</code> \geq <code>max(1, LOCr(ia+m-1))</code> , if <code>side = 'R'</code> , <code>lld_a</code> \geq <code>max(1, LOCr(ia+n-1))</code> . |
| <code>ia</code> | (global) The row index in the global matrix A indicating the first row of <code>sub(A)</code> . |
| <code>ja</code> | (global) The column index in the global matrix A indicating the first column of <code>sub(A)</code> . |
| <code>desca</code> | (global and local) array of size <code>dlen_</code> . The array descriptor for the distributed matrix A . |
| <code>tau</code> | (local) |

Array of size $LOCc(ja+k-1) \cdot \tau[j]$ contains the scalar factor of the elementary reflector $H(j+1)$, $j = 0, 1, \dots, LOCc(ja+k-1)-1$, as returned by [p?geqrf](#). This array is tied to the distributed matrix A .

| | |
|--------------|--|
| <i>c</i> | (local) Pointer into the local memory to an array of size $lld_c * LOCc(jc+n-1)$. On entry, the local pieces of the distributed matrix sub (C). |
| <i>ic</i> | (global) The row index in the global matrix C indicating the first row of sub(C). |
| <i>jc</i> | (global) The column index in the global matrix C indicating the first column of sub(C). |
| <i>desc</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix C . |
| <i>work</i> | (local) Workspace array of size $lwork$. |
| <i>lwork</i> | (local or global) The size of the array $work$. <i>lwork</i> is local input and must be at least if $side = 'L'$, $lwork \geq mpc0 + \max(1, nqc0)$, if $side = 'R'$, $lwork \geq nqc0 + \max(\max(1, mpc0), \text{numroc}(\text{numroc}(n + icoffc, nb_a, 0, 0, npc0), nb_a, 0, 0, lcmq))$, where $lcmq = lcm/npcol$, $lcm = iclm(nprow, npc0)$, $iroffc = \text{mod}(ic-1, mb_c)$, $icoffc = \text{mod}(jc-1, nb_c)$, $icrow = \text{indxg2p}(ic, mb_c, myrow, rsrc_c, nprow)$, $iccol = \text{indxg2p}(jc, nb_c, mycol, csrc_c, npc0)$, $Mqc0 = \text{numroc}(m+icoffc, nb_c, mycol, icrow, nprow)$, $Npc0 = \text{numroc}(n+iroffc, mb_c, myrow, iccol, npc0)$, $iclm$, $indxg2p$ and numroc are ScaLAPACK tool functions; $myrow$, $mycol$, $nprow$, and $npc0$ can be determined by calling the function <code>blacs_gridinfo</code> . If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla . |

Output Parameters

| | |
|-------------|--|
| <i>c</i> | On exit, <i>c</i> is overwritten by $Q \cdot \text{sub}(C)$, or $Q^T \cdot \text{sub}(C) / Q^H \cdot \text{sub}(C)$, or $\text{sub}(C) \cdot Q$, or $\text{sub}(C) \cdot Q^T / \text{sub}(C) \cdot Q^H$ |
| <i>work</i> | On exit, <i>work</i> [0] returns the minimal and optimal <i>lwork</i> . |
| <i>info</i> | (local) = 0: successful exit < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then <i>info</i> = - (<i>i</i> *100 + <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

NOTE

The distributed submatrices $A(ia:*, ja:*)$ and $C(ic:ic+m-1, jc:jc+n-1)$ must verify some alignment properties, namely the following expressions should be true:

If *side* = 'L', (*mb_a* == *mb_c*) && (*irowfa* == *irowfc*) && (*iarow* == *icrow*).

If *side* = 'R', (*mb_a* == *nb_c*) && (*irowfa* == *irowfc*).

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?orml2/p?unml2

Multiplies a general matrix by the orthogonal/unitary matrix from an LQ factorization determined by p?gelqf (unblocked algorithm).

Syntax

```
void psorml2 (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdorml2 (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcunml2 (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );
```

```
void pzunml2 (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?orml2/p?unml2` function overwrites the general real/complex m -by- n distributed matrix sub(C)= $C(ic:ic+m-1, jc:jc+n-1)$ with

$Q*\text{sub}(C)$ if $side = 'L'$ and $trans = 'N'$, or

$Q^T*\text{sub}(C) / Q^H*\text{sub}(C)$ if $side = 'L'$ and $trans = 'T'$ (for real flavors) or $trans = 'C'$ (for complex flavors), or

$\text{sub}(C)*Q$ if $side = 'R'$ and $trans = 'N'$, or

$\text{sub}(C)*Q^T / \text{sub}(C)*Q^H$ if $side = 'R'$ and $trans = 'T'$ (for real flavors) or $trans = 'C'$ (for complex flavors).

where Q is a real orthogonal or complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(k) * \dots * H(2) * H(1)$ (for real flavors)

$Q = (H(k))^H * \dots * (H(2))^H * (H(1))^H$ (for complex flavors)

as returned by `p?gelqf`. Q is of order m if $side = 'L'$ and of order n if $side = 'R'$.

Input Parameters

| | |
|--------------|---|
| <i>side</i> | (global) = 'L': apply Q or Q^T for real flavors (Q^H for complex flavors) from the left, = 'R': apply Q or Q^T for real flavors (Q^H for complex flavors) from the right. |
| <i>trans</i> | (global) = 'N': apply Q (no transpose) = 'T': apply Q^T (transpose, for real flavors) = 'C': apply Q^H (conjugate transpose, for complex flavors) |
| <i>m</i> | (global) The number of rows in the distributed matrix sub(C). $m \geq 0$. |
| <i>n</i> | (global) The number of columns in the distributed matrix sub(C). $n \geq 0$. |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q . If $side = 'L'$, $m \geq k \geq 0$; if $side = 'R'$, $n \geq k \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+m-1)$ if $side='L'$, $lld_a * LOCC(ja+n-1)$ if $side='R'$, |

where $lld_a \geq \max(1, LOCr(ia+k-1))$.

On entry, the i -th row of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by `p?gelqf` in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. The argument $A(ia:ia+k-1, ja:*)$ is modified by the function but restored on exit.

| | |
|--------------------|---|
| <code>ia</code> | (global) The row index in the global matrix A indicating the first row of $sub(A)$. |
| <code>ja</code> | (global) The column index in the global matrix A indicating the first column of $sub(A)$. |
| <code>desca</code> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <code>tau</code> | (local) Array of size $LOCc(ia+k-1)$. $tau[i]$ contains the scalar factor of the elementary reflector $H(i+1)$, $i = 0, 1, \dots, LOCc(ja+k-1)-1$, as returned by <code>p?gelqf</code> . This array is tied to the distributed matrix A . |
| <code>c</code> | (local) Pointer into the local memory to an array of size $lld_c * LOCc(jc+n-1)$. On entry, the local pieces of the distributed matrix $sub(C)$. |
| <code>ic</code> | (global) The row index in the global matrix C indicating the first row of $sub(C)$. |
| <code>jc</code> | (global) The column index in the global matrix C indicating the first column of $sub(C)$. |
| <code>descc</code> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix C . |
| <code>work</code> | (local) Workspace array of size $lwork$. |
| <code>lwork</code> | (local or global) The size of the array $work$. |

$lwork$ is local input and must be at least

if $side = 'L'$, $lwork \geq mqc0 + \max(\max(1, npc0), \text{numroc}(\text{numroc}(m + icoffc, mb_a, 0, 0, nprow), mb_a, 0, 0, lcm)),$

if $side = 'R'$, $lwork \geq npc0 + \max(1, mqc0),$

where

$lcmp = lcm / nprow,$

$lcm = iclm(nprow, npc0),$

```

iroffc = mod(ic-1, mb_c),
icoffc = mod(jc-1, nb_c),
icrow = indxg2p(ic, mb_c, myrow, rsrc_c, nprow),
iccol = indxg2p(jc, nb_c, mycol, csrc_c, npc1),
Mpc0 = numroc(m+icoffc, mb_c, mycol, icrow, nprow),
Nqc0 = numroc(n+iroffc, nb_c, myrow, iccol, npc1),
ilcm, indxg2p and numroc are ScaLAPACK tool functions; myrow, mycol,
nprow, and npc1 can be determined by calling the function
blacs_gridinfo.

```

If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by `pxerbla`.

Output Parameters

| | |
|--------|---|
| c | On exit, c is overwritten by $Q*\text{sub}(C)$, or $Q^T*\text{sub}(C)$ / $Q^H*\text{sub}(C)$, or $\text{sub}(C)*Q$, or $\text{sub}(C)*Q^T$ / $\text{sub}(C)*Q^H$ |
| $work$ | On exit, $work[0]$ returns the minimal and optimal $lwork$. |
| $info$ | (local) = 0: successful exit < 0: if the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value, then $info = -(i*100 + j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

NOTE

The distributed submatrices $A(ia:*, ja:*)$ and $C(ic:ic+m-1, jc:jc+n-1)$ must verify some alignment properties, namely the following expressions should be true:

If $side = 'L'$, $(nb_a == mb_c \ \&\& \ icoffa == iroffc)$

If $side = 'R'$, $(nb_a == nb_c \ \&\& \ icoffa == icoffc \ \&\& \ iacol == iccol)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

[p?ormr2/p?unmr2](#)

Multiplies a general matrix by the orthogonal/unitary matrix from an RQ factorization determined by $p?$ `gerqf` (unblocked algorithm).

Syntax

```
void psormr2 (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , float
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , float *tau , float *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , float *work , MKL_INT *lwork , MKL_INT *info );

void pdormr2 (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k , double
*a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *tau , double *c , MKL_INT
*ic , MKL_INT *jc , MKL_INT *descc , double *work , MKL_INT *lwork , MKL_INT *info );

void pcunmr2 (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *tau ,
MKL_Complex8 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex8 *work ,
MKL_INT *lwork , MKL_INT *info );

void pzunmr2 (char *side , char *trans , MKL_INT *m , MKL_INT *n , MKL_INT *k ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *tau ,
MKL_Complex16 *c , MKL_INT *ic , MKL_INT *jc , MKL_INT *descc , MKL_Complex16 *work ,
MKL_INT *lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?ormr2/p?unmr2` function overwrites the general real/complex m -by- n distributed matrix sub $(C)=C(ic:ic+m-1, jc:jc+n-1)$ with

Q *sub(C) if `side = 'L'` and `trans = 'N'`, or

Q^T *sub(C) / Q^H *sub(C) if `side = 'L'` and `trans = 'T'` (for real flavors) or `trans = 'C'` (for complex flavors), or

sub(C)* Q if `side = 'R'` and `trans = 'N'`, or

sub(C)* Q^T / sub(C)* Q^H if `side = 'R'` and `trans = 'T'` (for real flavors) or `trans = 'C'` (for complex flavors).

where Q is a real orthogonal or complex unitary distributed matrix defined as the product of k elementary reflectors

$Q = H(1)*H(2)*...*H(k)$ (for real flavors)

$Q = (H(1))^H*(H(2))^H*...*(H(k))^H$ (for complex flavors)

as returned by `p?gerqf`. Q is of order m if `side = 'L'` and of order n if `side = 'R'`.

Input Parameters

| | |
|--------------------|---|
| <code>side</code> | (global) |
| | = 'L': apply Q or Q^T for real flavors (Q^H for complex flavors) from the left, |
| | = 'R': apply Q or Q^T for real flavors (Q^H for complex flavors) from the right. |
| <code>trans</code> | (global) |
| | = 'N': apply Q (no transpose) |
| | = 'T': apply Q^T (transpose, for real flavors) |
| | = 'C': apply Q^H (conjugate transpose, for complex flavors) |

| | |
|--------------|--|
| <i>m</i> | (global) The number of rows in the distributed matrix $\text{sub}(C)$. $m \geq 0$. |
| <i>n</i> | (global) The number of columns in the distributed matrix $\text{sub}(C)$. $n \geq 0$. |
| <i>k</i> | (global) The number of elementary reflectors whose product defines the matrix Q . If $\text{side} = 'L', m \geq k \geq 0$; if $\text{side} = 'R', n \geq k \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $\text{lld}_a * \text{LOCc}(ja+m-1)$ if $\text{side}='L'$, $\text{lld}_a * \text{LOCc}(ja+n-1)$ if $\text{side}='R'$, where $\text{lld}_a \geq \max(1, \text{LOCr}(ia+k-1))$. On entry, the i -th row of the matrix stored in a must contain the vector that defines the elementary reflector $H(i)$, $ia \leq i \leq ia+k-1$, as returned by p?gerqf in the k rows of its distributed matrix argument $A(ia:ia+k-1, ja:*)$. The argument $A(ia:ia+k-1, ja:*)$ is modified by the function but restored on exit. |
| <i>ia</i> | (global) The row index in the global matrix A indicating the first row of $\text{sub}(A)$. |
| <i>ja</i> | (global) The column index in the global matrix A indicating the first column of $\text{sub}(A)$. |
| <i>desca</i> | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . |
| <i>tau</i> | (local) Array of size $\text{LOCc}(ia+k-1)$. $\text{tau}[j]$ contains the scalar factor of the elementary reflector $H(j+1)$, $j = 0, 1, \dots, \text{LOCc}(ja+k-1)-1$, as returned by p?gerqf . This array is tied to the distributed matrix A . |
| <i>c</i> | (local) Pointer into the local memory to an array of size $\text{lld}_c * \text{LOCc}(jc+n-1)$. On entry, the local pieces of the distributed matrix $\text{sub}(C)$. |
| <i>ic</i> | (global) The row index in the global matrix C indicating the first row of $\text{sub}(C)$. |
| <i>jc</i> | (global) |

The column index in the global matrix C indicating the first column of $\text{sub}(C)$.

desc

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix C .

work

(local)

Workspace array of size *lwork*.

lwork

(local or global)

The size of the array *work*.

lwork is local input and must be at least

if *side* = 'L', $lwork \geq mpc0 + \max(\max(1, nqc0), \text{numroc}(\text{numroc}(m + iroffc, mb_a, 0, 0, nprow), mb_a, 0, 0, lcm)),$

if *side* = 'R', $lwork \geq nqc0 + \max(1, mpc0),$

where $lcmp = lcm/nprow,$

$lcm = iclm(nprow, npc0),$

$iroffc = \text{mod}(ic-1, mb_c),$

$icoffc = \text{mod}(jc-1, nb_c),$

$icrow = \text{indxg2p}(ic, mb_c, myrow, rsrc_c, nprow),$

$iccol = \text{indxg2p}(jc, nb_c, mycol, csrc_c, npc0),$

$Mpc0 = \text{numroc}(m + iroffc, mb_c, myrow, icrow, nprow),$

$Nqc0 = \text{numroc}(n + icoffc, nb_c, mycol, iccol, npc0),$

iclm, *indxg2p* and *numroc* are ScaLAPACK tool functions; *myrow*, *mycol*, *nprow*, and *npc0* can be determined by calling the function *blacs_gridinfo*.

If *lwork* = -1, then *lwork* is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by *pxerbla*.

Output Parameters

c

On exit, *c* is overwritten by $Q * \text{sub}(C)$, or $Q^T * \text{sub}(C) / Q^H * \text{sub}(C)$, or $\text{sub}(C) * Q$, or $\text{sub}(C) * Q^T / \text{sub}(C) * Q^H$

work

On exit, *work*[0] returns the minimal and optimal *lwork*.

info

(local)

= 0: successful exit

< 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value,

then $info = -(i * 100 + j),$

if the *i*-th argument is a scalar and had an illegal value,

then $info = -i.$

NOTE

The distributed submatrices $A(ia:*, ja:*)$ and $C(ic:ic+m-1, jc:jc+n-1)$ must verify some alignment properties, namely the following expressions should be true:

If $side = 'L'$, $(nb_a == mb_c) \ \&\& \ (icoffa == iroffc)$.

If $side = 'R'$, $(nb_a == nb_c) \ \&\& \ (icoffa == icoffc) \ \&\& \ (iacol == iccol)$.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?pbtrsv

Solves a single triangular linear system via frontsolve or backsolve where the triangular matrix is a factor of a banded matrix computed by p?pbtrf.

Syntax

```
void pspbtrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs ,
float *a , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb ,
float *af , MKL_INT *laf , float *work , MKL_INT *lwork , MKL_INT *info );

void pdpbtrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs ,
double *a , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb ,
double *af , MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );

void pcpbtrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs ,
MKL_Complex8 *a , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );

void pzpbttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *bw , MKL_INT *nrhs ,
MKL_Complex16 *a , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?pbtrsv function solves a banded triangular system of linear equations

$$A(1:n, ja:ja+n-1) * X = B(jb:jb+n-1, 1:nrhs)$$

or

$$A(1:n, ja:ja+n-1)^T * X = B(jb:jb+n-1, 1:nrhs) \text{ for real flavors,}$$

$$A(1:n, ja:ja+n-1)^H * X = B(jb:jb+n-1, 1:nrhs) \text{ for complex flavors,}$$

where $A(1:n, ja:ja+n-1)$ is a banded triangular matrix factor produced by the Cholesky factorization code p?pbtrf and is stored in $A(1:n, ja:ja+n-1)$ and af . The matrix stored in $A(1:n, ja:ja+n-1)$ is either upper or lower triangular according to $uplo$.

The function p?pbtrf must be called first.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|--------------|---|
| <i>uplo</i> | (global) Must be 'U' or 'L'. If <i>uplo</i> = 'U', upper triangle of $A(1:n, ja:ja+n-1)$ is stored; If <i>uplo</i> = 'L', lower triangle of $A(1:n, ja:ja+n-1)$ is stored. |
| <i>trans</i> | (global) Must be 'N' or 'T' or 'C'. If <i>trans</i> = 'N', solve with $A(1:n, ja:ja+n-1)$; If <i>trans</i> = 'T' or 'C' for real flavors, solve with $A(1:n, ja:ja+n-1)^T$. If <i>trans</i> = 'C' for complex flavors, solve with conjugate transpose $(A(1:n, ja:ja+n-1))^H$. |
| <i>n</i> | (global) The number of rows and columns to be operated on, that is, the order of the distributed submatrix $A(1:n, ja:ja+n-1)$. $n \geq 0$. |
| <i>bw</i> | (global) The number of subdiagonals in 'L' or 'U', $0 \leq bw \leq n-1$. |
| <i>nrhs</i> | (global) The number of right hand sides; the number of columns of the distributed submatrix $B(jb:jb+n-1, 1:nrhs)$; $nrhs \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array with the first size $lld_a \geq (bw + 1)$, stored in <i>desca</i> . On entry, this array contains the local pieces of the n -by- n symmetric banded distributed Cholesky factor L or $L^T * A(1:n, ja:ja+n-1)$. This local portion is stored in the packed banded format used in LAPACK. See the <i>Application Notes</i> below and the ScaLAPACK manual for more detail on the format of distributed matrices. |
| <i>ja</i> | (global) The index in the global in the global matrix A that points to the start of the matrix to be operated on (which may be either all of A or a submatrix of A). |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |

If 1D type ($dtype_a = 501$), then $dlen \geq 7$;

If 2D type ($dtype_a = 1$), then $dlen \geq 9$.

Contains information on mapping of A to memory. (See ScaLAPACK manual for full description and options.)

b (local)

Pointer into the local memory to an array of local lead size $lld_b \geq nb$.

On entry, this array contains the local pieces of the right hand sides $B(jb:jb+n-1, 1:nrhs)$.

ib (global) The row index in the global matrix B that points to the first row of the matrix to be operated on (which may be either all of B or a submatrix of B).

descb (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B .

If 1D type ($dtype_b = 502$), then $dlen \geq 7$;

If 2D type ($dtype_b = 1$), then $dlen \geq 9$.

Contains information on mapping of B to memory. Please, see ScaLAPACK manual for full description and options.

laf (local)

The size of user-input auxiliary fill-in space af . Must be $laf \geq (nb + 2*bw)*bw$. If laf is not large enough, an error code will be returned and the minimum acceptable size will be returned in $af[0]$.

work (local)

The array $work$ is a temporary workspace array of size $lwork$. This space may be overwritten in between function calls.

lwork (local or global) The size of the user-input workspace $work$, must be at least $lwork \geq bw*nrhs$. If $lwork$ is too small, the minimal acceptable size will be returned in $work[0]$ and an error code is returned.

Output Parameters

af (local)

The array af is of size laf . It contains auxiliary fill-in space. The fill-in space is created in a call to the factorization function [p?pbtrf](#) and is stored in af . If a linear system is to be solved using [p?pbtrs](#) after the factorization function, af must not be altered after the factorization.

b On exit, this array contains the local piece of the solutions distributed matrix X .

work[0] On exit, $work[0]$ contains the minimum value of $lwork$.

info (local)

= 0: successful exit

< 0: if the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value,
 then $info = - (i*100 + j)$,
 if the i -th argument is a scalar and had an illegal value,
 then $info = -i$.

Application Notes

If the factorization function and the solve function are to be called separately to solve various sets of right-hand sides using the same coefficient matrix, the auxiliary space af must not be altered between calls to the factorization function and the solve function.

The best algorithm for solving banded and tridiagonal linear systems depends on a variety of parameters, especially the bandwidth. Currently, only algorithms designed for the case $N/P \gg bw$ are implemented. These algorithms go by many names, including Divide and Conquer, Partitioning, domain decomposition-type, etc.

The Divide and Conquer algorithm assumes the matrix is narrowly banded compared with the number of equations. In this situation, it is best to distribute the input matrix A one-dimensionally, with columns atomic and rows divided amongst the processes. The basic algorithm divides the banded matrix up into P pieces with one stored on each processor, and then proceeds in 2 phases for the factorization or 3 for the solution of a linear system.

- 1. Local Phase :** The individual pieces are factored independently and in parallel. These factors are applied to the matrix creating fill-in, which is stored in a non-inspectable way in auxiliary space af . Mathematically, this is equivalent to reordering the matrix A as PAP^T and then factoring the principal leading submatrix of size equal to the sum of the sizes of the matrices factored on each processor. The factors of these submatrices overwrite the corresponding parts of A in memory.
- 2. Reduced System Phase :** A small ($bw*(P-1)$) system is formed representing interaction of the larger blocks and is stored (as are its factors) in the space af . A parallel Block Cyclic Reduction algorithm is used. For a linear system, a parallel front solve followed by an analogous backsolve, both using the structure of the factored matrix, are performed.
- 3. Back Substitution Phase:** For a linear system, a local backsubstitution is performed on each processor in parallel.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?pttrsv

Solves a single triangular linear system via `frontsolve` or `backsolve` where the triangular matrix is a factor of a tridiagonal matrix computed by `p?pttrf`.

Syntax

```
void pspttrsv (char *uplo , MKL_INT *n , MKL_INT *nrhs , float *d , float *e , MKL_INT
*ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *descb , float *af , MKL_INT
*laf , float *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pdpttrsv (char *uplo , MKL_INT *n , MKL_INT *nrhs , double *d , double *e ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *descb , double *af ,
MKL_INT *laf , double *work , MKL_INT *lwork , MKL_INT *info );
```

```
void pcpttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , float *d ,
MKL_Complex8 *e , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex8 *af , MKL_INT *laf , MKL_Complex8 *work , MKL_INT
*lwork , MKL_INT *info );
```

```
void pzpttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , double *d ,
MKL_Complex16 *e , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib ,
MKL_INT *descb , MKL_Complex16 *af , MKL_INT *laf , MKL_Complex16 *work , MKL_INT
*lwork , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `p?pttrsv` function solves a tridiagonal triangular system of linear equations

$$A(1:n, ja:ja+n-1) * X = B(jb:jb+n-1, 1:nrhs)$$

or

$$A(1:n, ja:ja+n-1)^T * X = B(jb:jb+n-1, 1:nrhs) \text{ for real flavors,}$$

$$A(1:n, ja:ja+n-1)^H * X = B(jb:jb+n-1, 1:nrhs) \text{ for complex flavors,}$$

where $A(1:n, ja:ja+n-1)$ is a tridiagonal triangular matrix factor produced by the Cholesky factorization code `p?pttrf` and is stored in $A(1:n, ja:ja+n-1)$ and af . The matrix stored in $A(1:n, ja:ja+n-1)$ is either upper or lower triangular according to `uplo`.

The function `p?pttrf` must be called first.

Input Parameters

| | |
|--------------------|---|
| <code>uplo</code> | (global) Must be 'U' or 'L'. If <code>uplo = 'U'</code> , upper triangle of $A(1:n, ja:ja+n-1)$ is stored; If <code>uplo = 'L'</code> , lower triangle of $A(1:n, ja:ja+n-1)$ is stored. |
| <code>trans</code> | (global) Must be 'N' or 'C'. If <code>trans = 'N'</code> , solve with $A(1:n, ja:ja+n-1)$; If <code>trans = 'C'</code> (for complex flavors), solve with conjugate transpose $(A(1:n, ja:ja+n-1))^H$. |
| <code>n</code> | (global) The number of rows and columns to be operated on, that is, the order of the distributed submatrix $A(1:n, ja:ja+n-1)$. $n \geq 0$. |
| <code>nrhs</code> | (global) The number of right hand sides; the number of columns of the distributed submatrix $B(jb:jb+n-1, 1:nrhs)$; $nrhs \geq 0$. |
| <code>d</code> | (local) Pointer to the local part of the global vector storing the main diagonal of the matrix; must be of size $\geq nb_a$. |
| <code>e</code> | (local) |

| | |
|--------------|---|
| | <p>Pointer to the local part of the global vector du storing the upper diagonal of the matrix; must be of size $\geq nb_a$. Globally, $du(n)$ is not referenced, and du must be aligned with d.</p> |
| <i>ja</i> | <p>(global) The index in the global matrix A that points to the start of the matrix to be operated on (which may be either all of A or a submatrix of A).</p> |
| <i>desca</i> | <p>(global and local) array of size $dlen_$. The array descriptor for the distributed matrix A.</p> <p>If 1D type ($dtype_a = 501$ or 502), then $dlen \geq 7$;</p> <p>If 2D type ($dtype_a = 1$), then $dlen \geq 9$.</p> <p>Contains information on mapping of A to memory. See ScaLAPACK manual for full description and options.</p> |
| <i>b</i> | <p>(local)</p> <p>Pointer into the local memory to an array of local lead size $lld_b \geq nb$.</p> <p>On entry, this array contains the local pieces of the right hand sides $B(jb:jb+n-1, 1:nrhs)$.</p> |
| <i>ib</i> | <p>(global) The row index in the global matrix B that points to the first row of the matrix to be operated on (which may be either all of B or a submatrix of B).</p> |
| <i>descb</i> | <p>(global and local) array of size $dlen_$. The array descriptor for the distributed matrix B.</p> <p>If 1D type ($dtype_b = 502$), then $dlen \geq 7$;</p> <p>If 2D type ($dtype_b = 1$), then $dlen \geq 9$.</p> <p>Contains information on mapping of B to memory. See ScaLAPACK manual for full description and options.</p> |
| <i>laf</i> | <p>(local)</p> <p>The size of user-input auxiliary fill-in space af. Must be $laf \geq (nb + 2 * bw) * bw$.</p> <p>If laf is not large enough, an error code will be returned and the minimum acceptable size will be returned in $af[0]$.</p> |
| <i>work</i> | <p>(local)</p> <p>The array $work$ is a temporary workspace array of size $lwork$. This space may be overwritten in between function calls.</p> |
| <i>lwork</i> | <p>(local or global) The size of the user-input workspace $work$, must be at least $lwork \geq (10 + 2 * \min(100, nrhs)) * npc01 + 4 * nrhs$. If $lwork$ is too small, the minimal acceptable size will be returned in $work[0]$ and an error code is returned.</p> |

Output Parameters

| | |
|-------------|--|
| <i>d, e</i> | <p>(local).</p> <p>On exit, these arrays contain information on the factors of the matrix.</p> |
|-------------|--|

| | |
|-----------------|--|
| <i>af</i> | (local) The array <i>af</i> is of size <i>laf</i> . It contains auxiliary fill-in space. The fill-in space is created in a call to the factorization function <code>p?pbturf</code> and is stored in <i>af</i> . If a linear system is to be solved using <code>p?pttrs</code> after the factorization function, <i>af</i> must not be altered after the factorization. |
| <i>b</i> | On exit, this array contains the local piece of the solutions distributed matrix <i>X</i> . |
| <i>work</i> [0] | On exit, <i>work</i> [0] contains the minimum value of <i>lwork</i> . |
| <i>info</i> | (local) = 0: successful exit < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then <i>info</i> = - (<i>i</i> *100 + <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?potf2

Computes the Cholesky factorization of a symmetric/Hermitian positive definite matrix (local unblocked algorithm).

Syntax

```
void pspotf2 (char *uplo , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , MKL_INT *info );

void pdpotf2 (char *uplo , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *info );

void pcpotf2 (char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *info );

void pzpotf2 (char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?potf2` function computes the Cholesky factorization of a real symmetric or complex Hermitian positive definite distributed matrix sub (*A*)=*A*(*ia*:*ia*+*n*-1, *ja*:*ja*+*n*-1).

The factorization has the form

sub(*A*) = *U***U*, if *uplo* = 'U', or sub(*A*) = *L***L'*, if *uplo* = 'L',

where *U* is an upper triangular matrix, *L* is lower triangular. *X'* denotes transpose (conjugate transpose) of *X*.

Input Parameters

| | |
|---------------|---|
| <i>uplo</i> | (global) Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix <i>A</i> is stored. = 'U': upper triangle of sub (<i>A</i>) is stored; = 'L': lower triangle of sub (<i>A</i>) is stored. |
| <i>n</i> | (global) The number of rows and columns to be operated on, that is, the order of the distributed matrix sub (<i>A</i>). $n \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$ containing the local pieces of the <i>n</i> -by- <i>n</i> symmetric distributed matrix sub(<i>A</i>) to be factored. If <i>uplo</i> = 'U', the leading <i>n</i> -by- <i>n</i> upper triangular part of sub(<i>A</i>) contains the upper triangular matrix and the strictly lower triangular part of this matrix is not referenced. If <i>uplo</i> = 'L', the leading <i>n</i> -by- <i>n</i> lower triangular part of sub(<i>A</i>) contains the lower triangular matrix and the strictly upper triangular part of sub(<i>A</i>) is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix <i>A</i> indicating the first row and the first column of the sub(<i>A</i>), respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . |

Output Parameters

| | |
|-------------|--|
| <i>a</i> | (local) On exit, if <i>uplo</i> = 'U', the upper triangular part of the distributed matrix contains the Cholesky factor <i>U</i> ; if <i>uplo</i> = 'L', the lower triangular part of the distributed matrix contains the Cholesky factor <i>L</i> . |
| <i>info</i> | (local) = 0: successful exit < 0: if the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then <i>info</i> = - (<i>i</i> *100 + <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

> 0 : if $info = k$, the leading minor of order k is not positive definite, and the factorization could not be completed.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?rot

Applies a planar rotation to two distributed vectors.

Syntax

```
void psrot(MKL_INT* n, float* x, MKL_INT* ix, MKL_INT* jx, MKL_INT* descx, MKL_INT*
incx, float* y, MKL_INT* iy, MKL_INT* jy, MKL_INT* descy, MKL_INT* incy, float* cs,
float* sn, float* work, MKL_INT* lwork, MKL_INT* info);
```

```
void pdrot(MKL_INT* n, double* x, MKL_INT* ix, MKL_INT* jx, MKL_INT* descx, MKL_INT*
incx, double* y, MKL_INT* iy, MKL_INT* jy, MKL_INT* descy, MKL_INT* incy, double* cs,
double* sn, double* work, MKL_INT* lwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

p?rot applies a planar rotation defined by cs and sn to the two distributed vectors $sub(x)$ and $sub(y)$.

Input Parameters

| | |
|---------|--|
| n | (global) The number of elements to operate on when applying the planar rotation to x and y ($n \geq 0$). |
| x | (local) array of size $((j_x-1)*m_x + ix + (n-1)*abs(incx))$ This array contains the entries of the distributed vector $sub(x)$. |
| ix | (global) The global row index of the submatrix of the distributed matrix x to operate on. If $incx = 1$, then it is required that $ix = iy$. $1 \leq ix \leq m_x$. |
| jx | (global) The global column index of the submatrix of the distributed matrix x to operate on. If $incx = m_x$, then it is required that $jx = jy$. $1 \leq ix \leq n_x$. |
| $descx$ | (global and local) array of size 9 The array descriptor of the distributed matrix x . |
| $incx$ | (global) The global increment for the elements of x . Only two values of $incx$ are supported in this version, namely 1 and m_x . Moreover, it must hold that $incx = m_x$ if $incy = m_y$ and that $incx = 1$ if $incy = 1$. |

| | |
|---------------|--|
| <i>y</i> | (local) array of size $((jy-1)*m_y + iy + (n-1)*abs(incy))$ This array contains the entries of the distributed vector <code>sub(y)</code> . |
| <i>iy</i> | (global) The global row index of the submatrix of the distributed matrix <i>y</i> to operate on. If <i>incy</i> = 1, then it is required that <i>iy</i> = <i>ix</i> . $1 \leq iy \leq m_y$. |
| <i>jy</i> | (global) The global column index of the submatrix of the distributed matrix <i>y</i> to operate on. If <i>incy</i> = <i>m_x</i> , then it is required that <i>jy</i> = <i>jx</i> . $1 \leq jy \leq m_y$. |
| <i>descy</i> | (global and local) array of size 9 The array descriptor of the distributed matrix <i>y</i> . |
| <i>incy</i> | (global) The global increment for the elements of <i>y</i> . Only two values of <i>incy</i> are supported in this version, namely 1 and <i>m_y</i> . Moreover, it must hold that <i>incy</i> = <i>m_y</i> if <i>incx</i> = <i>m_x</i> and that <i>incy</i> = 1 if <i>incx</i> = 1. |
| <i>cs, sn</i> | (global) The parameters defining the properties of the planar rotation. It must hold that $0 \leq cs, sn \leq 1$ and that $sn^2 + cs^2 = 1$. The latter is hardly checked in finite precision arithmetics. |
| <i>work</i> | (local workspace) array of size <i>lwork</i> |
| <i>lwork</i> | (local) The length of the workspace array <i>work</i> . If <i>incx</i> = 1 and <i>incy</i> = 1, then <i>lwork</i> = $2*m_x$ If <i>lwork</i> = -1, then a workspace query is assumed; the function only calculates the optimal size of the <i>work</i> array, returns this value as the first entry of the IWORK array, and no error message related to LIWORK is issued by pxerbla . |

OUTPUT Parameters

| | |
|-----------------|--|
| <i>x</i> | |
| <i>y</i> | |
| <i>work</i> [0] | On exit, if <i>info</i> = 0, <i>work</i> [0] returns the optimal <i>lwork</i> |
| <i>info</i> | (global) = 0: successful exit < 0: if <i>info</i> = - <i>i</i> , the <i>i</i> -th argument had an illegal value. If the <i>i</i> -th argument is an array and the <i>j</i> -th entry, indexed <i>j</i> -1, had an illegal value, then <i>info</i> = -(<i>i</i> *100+ <i>j</i>), if the <i>i</i> -th argument is a scalar and had an illegal value, then <i>info</i> = - <i>i</i> . |

See Also

Overview of ScaLAPACK Routines for details of ScaLAPACK array descriptor structures and related notations.

p?rscl

Multiplies a vector by the reciprocal of a real scalar.

Syntax

```
void p?rscl (MKL_INT *n , float *sa , float *sx , MKL_INT *ix , MKL_INT *jx , MKL_INT
*descx , MKL_INT *incx );

void pdrsc1 (MKL_INT *n , double *sa , double *sx , MKL_INT *ix , MKL_INT *jx ,
MKL_INT *descx , MKL_INT *incx );

void pc?rscl (MKL_INT *n , float *sa , MKL_Complex8 *sx , MKL_INT *ix , MKL_INT *jx ,
MKL_INT *descx , MKL_INT *incx );

void pzdrsc1 (MKL_INT *n , double *sa , MKL_Complex16 *sx , MKL_INT *ix , MKL_INT *jx ,
MKL_INT *descx , MKL_INT *incx );
```

Include Files

- mkl_scalapack.h

Description

The p?rscl function multiplies an n -element real/complex vector $\text{sub}(X)$ by the real scalar $1/a$. This is done without overflow or underflow as long as the final result $\text{sub}(X)/a$ does not overflow or underflow.

$\text{sub}(X)$ denotes $X(ix:ix+n-1, jx:jx)$, if $incx = 1$,

and $X(ix:ix, jx:jx+n-1)$, if $incx = m_x$.

Input Parameters

| | |
|---------|---|
| n | (global) The number of components of the distributed vector $\text{sub}(X)$. $n \geq 0$. |
| sa | The scalar a that is used to divide each component of the vector $\text{sub}(X)$. This parameter must be ≥ 0 . |
| sx | Array containing the local pieces of a distributed matrix of size of at least $((jx-1)*m_x + ix + (n-1)*abs(incx))$. This array contains the entries of the distributed vector $\text{sub}(X)$. |
| ix | (global) The row index of the submatrix of the distributed matrix X to operate on. |
| jx | (global) The column index of the submatrix of the distributed matrix X to operate on. |
| $descx$ | (global and local) Array of size 9. The array descriptor for the distributed matrix X . |
| $incx$ | (global) |

The increment for the elements of X . This version supports only two values of $incx$, namely 1 and m_x .

Output Parameters

sx On exit, the result x/a .

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?sygs2/p?hegs2

Reduces a symmetric/Hermitian positive-definite generalized eigenproblem to standard form, using the factorization results obtained from p?potrf (local unblocked algorithm).

Syntax

```
void pssygs2 (MKL_INT *ibtype , char *uplo , MKL_INT *n , float *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
MKL_INT *info );
```

```
void pdsygs2 (MKL_INT *ibtype , char *uplo , MKL_INT *n , double *a , MKL_INT *ia ,
MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *info );
```

```
void pchegs2 (MKL_INT *ibtype , char *uplo , MKL_INT *n , MKL_Complex8 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_INT *info );
```

```
void pzhegs2 (MKL_INT *ibtype , char *uplo , MKL_INT *n , MKL_Complex16 *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?sygs2/p?hegs2 function reduces a real symmetric-definite or a complex Hermitian positive-definite generalized eigenproblem to standard form.

Here $\text{sub}(A)$ denotes $A(ia:ia+n-1, ja:ja+n-1)$, and $\text{sub}(B)$ denotes $B(ib:ib+n-1, jb:jb+n-1)$.

If $ibtype = 1$, the problem is

$$\text{sub}(A) * x = \lambda * \text{sub}(B) * x$$

and $\text{sub}(A)$ is overwritten by

$\text{inv}(U^T) * \text{sub}(A) * \text{inv}(U)$ or $\text{inv}(L) * \text{sub}(A) * \text{inv}(L^T)$ - for real flavors, and

$\text{inv}(U^H) * \text{sub}(A) * \text{inv}(U)$ or $\text{inv}(L) * \text{sub}(A) * \text{inv}(L^H)$ - for complex flavors.

If $ibtype = 2$ or 3 , the problem is

$$\text{sub}(A) * \text{sub}(B) x = \lambda * x \text{ or } \text{sub}(B) * \text{sub}(A) x = \lambda * x$$

and $\text{sub}(A)$ is overwritten by

$U^* \text{sub}(A) * U^T$ or $L^* * T^* \text{sub}(A) * L^-$ for real flavors and

$U^* \text{sub}(A) * U^H$ or $L^* * H^* \text{sub}(A) * L^-$ for complex flavors.

The matrix $\text{sub}(B)$ must have been previously factorized as $U^T * U$ or $L^* L^T$ (for real flavors), or as $U^H * U$ or $L^* L^H$ (for complex flavors) by [p?potrf](#).

Input Parameters

| | |
|---------------|---|
| <i>ibtype</i> | (global) = 1: compute $\text{inv}(U^T) * \text{sub}(A) * \text{inv}(U)$, or $\text{inv}(L) * \text{sub}(A) * \text{inv}(L^T)$ for real functions, and $\text{inv}(U^H) * \text{sub}(A) * \text{inv}(U)$, or $\text{inv}(L) * \text{sub}(A) * \text{inv}(L^H)$ for complex functions; = 2 or 3: compute $U^* \text{sub}(A) * U^T$, or $L^T * \text{sub}(A) * L$ for real functions, and $U^* \text{sub}(A) * U^H$ or $L^H * \text{sub}(A) * L$ for complex functions. |
| <i>uplo</i> | (global) Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix $\text{sub}(A)$ is stored, and how $\text{sub}(B)$ is factorized. = 'U': Upper triangular of $\text{sub}(A)$ is stored and $\text{sub}(B)$ is factorized as $U^T * U$ (for real functions) or as $U^H * U$ (for complex functions). = 'L': Lower triangular of $\text{sub}(A)$ is stored and $\text{sub}(B)$ is factorized as $L^* L^T$ (for real functions) or as $L^* L^H$ (for complex functions) |
| <i>n</i> | (global) The order of the matrices $\text{sub}(A)$ and $\text{sub}(B)$. $n \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array of size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the n -by- n symmetric/Hermitian distributed matrix $\text{sub}(A)$. If <i>uplo</i> = 'U', the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and the strictly lower triangular part of $\text{sub}(A)$ is not referenced. If <i>uplo</i> = 'L', the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix, and the strictly upper triangular part of $\text{sub}(A)$ is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the $\text{sub}(A)$, respectively. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix A . |
| <i>B</i> | (local) Pointer into the local memory to an array of size $lld_b * LOCC(jb+n-1)$. |

On entry, this array contains the local pieces of the triangular factor from the Cholesky factorization of $\text{sub}(B)$ as returned by `p?potrf`.

ib, jb

(global)

The row and column indices in the global matrix B indicating the first row and the first column of the $\text{sub}(B)$, respectively.

descb

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix B .

Output Parameters

a

(local)

On exit, if *info* = 0, the transformed matrix is stored in the same format as $\text{sub}(A)$.

info

= 0: successful exit.

< 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value,

then *info* = - (*i**100+ *j*),

if the *i*-th argument is a scalar and had an illegal value,

then *info* = -*i*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?sytd2/p?hetd2

Reduces a symmetric/Hermitian matrix to real symmetric tridiagonal form by an orthogonal/unitary similarity transformation (local unblocked algorithm).

Syntax

```
void pssytd2 (char *uplo, MKL_INT *n, float *a, MKL_INT *ia, MKL_INT *ja, MKL_INT *desca, float *d, float *e, float *tau, float *work, MKL_INT *lwork, MKL_INT *info);
```

```
void pdsytd2 (char *uplo, MKL_INT *n, double *a, MKL_INT *ia, MKL_INT *ja, MKL_INT *desca, double *d, double *e, double *tau, double *work, MKL_INT *lwork, MKL_INT *info);
```

```
void pchetd2 (char *uplo, MKL_INT *n, MKL_Complex8 *a, MKL_INT *ia, MKL_INT *ja, MKL_INT *desca, float *d, float *e, MKL_Complex8 *tau, MKL_Complex8 *work, MKL_INT *lwork, MKL_INT *info);
```

```
void pzhetd2 (char *uplo, MKL_INT *n, MKL_Complex16 *a, MKL_INT *ia, MKL_INT *ja, MKL_INT *desca, double *d, double *e, MKL_Complex16 *tau, MKL_Complex16 *work, MKL_INT *lwork, MKL_INT *info);
```

Include Files

- `mkl_scalapack.h`

Description

The `p?sytd2/p?hetd2` function reduces a real symmetric/complex Hermitian matrix $\text{sub}(A)$ to symmetric/Hermitian tridiagonal form T by an orthogonal/unitary similarity transformation:

$$Q^* \text{sub}(A) Q = T, \text{ where } \text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1).$$

Input Parameters

| | |
|---------------------|---|
| <code>uplo</code> | (global) Specifies whether the upper or lower triangular part of the symmetric/Hermitian matrix $\text{sub}(A)$ is stored: = 'U': upper triangular = 'L': lower triangular |
| <code>n</code> | (global) The number of rows and columns to be operated on, that is, the order of the distributed matrix $\text{sub}(A)$. $n \geq 0$. |
| <code>a</code> | (local) Pointer into the local memory to an array of size $lld_a * LOC_c(\text{ja}+n-1)$. On entry, this array contains the local pieces of the n -by- n symmetric/Hermitian distributed matrix $\text{sub}(A)$. If <code>uplo = 'U'</code> , the leading n -by- n upper triangular part of $\text{sub}(A)$ contains the upper triangular part of the matrix, and the strictly lower triangular part of $\text{sub}(A)$ is not referenced. If <code>uplo = 'L'</code> , the leading n -by- n lower triangular part of $\text{sub}(A)$ contains the lower triangular part of the matrix, and the strictly upper triangular part of $\text{sub}(A)$ is not referenced. |
| <code>ia, ja</code> | (global) The row and column indices in the global matrix A indicating the first row and the first column of the $\text{sub}(A)$, respectively. |
| <code>desca</code> | (global and local) array of size <code>dlen_</code> . The array descriptor for the distributed matrix A . |
| <code>work</code> | (local) The array <code>work</code> is a temporary workspace array of size <code>lwork</code> . |

Output Parameters

| | |
|----------------|---|
| <code>a</code> | On exit, if <code>uplo = 'U'</code> , the diagonal and first superdiagonal of $\text{sub}(A)$ are overwritten by the corresponding elements of the tridiagonal matrix T , and the elements above the first superdiagonal, with the array <code>tau</code> , represent the orthogonal/unitary matrix Q as a product of elementary reflectors; if <code>uplo = 'L'</code> , the diagonal and first subdiagonal of A are overwritten by the corresponding elements of the tridiagonal matrix T , and the elements below the first subdiagonal, with the array <code>tau</code> , represent the orthogonal/unitary matrix Q as a product of elementary reflectors. See the <i>Application Notes</i> below. |
|----------------|---|

| | |
|----------------|---|
| <i>d</i> | (local) Array of size $LOCc(ja+n-1)$. The diagonal elements of the tridiagonal matrix T : $d[i] = A(i+1,i+1)$, where $i=0,1, \dots, LOCc(ja+n-1) - 1$; d is tied to the distributed matrix A . |
| <i>e</i> | (local) Array of size $LOCc(ja+n-1)$, if $uplo = 'U'$, $LOCc(ja+n-2)$ otherwise. The off-diagonal elements of the tridiagonal matrix T : $e[i] = A(i+1,i+2)$ if $uplo = 'U'$, $e[i] = A(i+2,i+1)$ if $uplo = 'L'$, where $i=0,1, \dots, LOCc(ja+n-1) - 1$. e is tied to the distributed matrix A . |
| <i>tau</i> | (local) Array of size $LOCc(ja+n-1)$. The scalar factors of the elementary reflectors. tau is tied to the distributed matrix A . |
| <i>work[0]</i> | On exit, <i>work[0]</i> returns the minimal and optimal value of <i>lwork</i> . |
| <i>lwork</i> | (local or global) The size of the workspace array <i>work</i> . $lwork$ is local input and must be at least $lwork \geq 3n$. If $lwork = -1$, then $lwork$ is global input and a workspace query is assumed; the function only calculates the minimum and optimal size for all work arrays. Each of these values is returned in the first entry of the corresponding work array, and no error message is issued by pxerbla . |
| <i>info</i> | (local) = 0: successful exit < 0: if the i -th argument, indexed $i-1$, is an array and the j -th entry had an illegal value, then $info = -(i*100+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$. |

Application Notes

If $uplo = 'U'$, the matrix Q is represented as a product of elementary reflectors

$$Q = H(n-1) * \dots * H(2) * H(1)$$

Each $H(i)$ has the form

$$H(i) = I - tau * v * v'$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(i+1:n) = 0$ and $v(i) = 1$; $v(1:i-1)$ is stored on exit in $A(ia:ia+i-2, ja+i)$, and τ in $\tau[ja+i-2]$.

If $uplo = 'L'$, the matrix Q is represented as a product of elementary reflectors

$$Q = H(1)*H(2)*\dots*H(n-1).$$

Each $H(i)$ has the form

$$H(i) = I - \tau*v*v',$$

where τ is a real/complex scalar, and v is a real/complex vector with $v(1:i) = 0$ and $v(i+1) = 1$; $v(i+2:n)$ is stored on exit in $A(ia+i+1:ia+n-1, ja+i-1)$, and τ in $\tau[ja+i-2]$.

The contents of sub (A) on exit are illustrated by the following examples with $n = 5$:

$$\begin{array}{l} \text{if } uplo='U': \\ \left[\begin{array}{ccccc} d & e & v_2 & v_3 & v_4 \\ & d & e & v_3 & v_4 \\ & & d & e & v_4 \\ & & & d & e \\ & & & & d \end{array} \right] \end{array} \qquad \begin{array}{l} \text{if } uplo='L': \\ \left[\begin{array}{ccccc} d & & & & \\ e & d & & & \\ v_1 & e & d & & \\ v_1 & v_2 & e & d & \\ v_1 & v_2 & v_3 & e & d \end{array} \right] \end{array}$$

where d and e denotes diagonal and off-diagonal elements of T , and v_i denotes an element of the vector defining $H(i)$.

NOTE

The distributed matrix sub(A) must verify some alignment properties, namely the following expression should be true:

$$(mb_a == nb_a \ \&\& \ iroffa == icoffa) \text{ where } iroffa = \text{mod}(ia - 1, mb_a) \text{ and } icoffa = \text{mod}(ja - 1, nb_a).$$

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trord

Reorders the Schur factorization of a general matrix.

Syntax

```
void pstrord( char* compq, MKL_INT* select, MKL_INT* para, MKL_INT* n, float* t,
MKL_INT* it, MKL_INT* jt, MKL_INT* desct, float* q, MKL_INT* iq, MKL_INT* jq, MKL_INT*
descq, float* wr, float* wi, MKL_INT* m, float* work, MKL_INT* lwork, MKL_INT* iwork,
MKL_INT* liwork, MKL_INT* info);
```

```
void pdtrord(char* compq, MKL_INT* select, MKL_INT* para, MKL_INT* n, double* t,
MKL_INT* it, MKL_INT* jt, MKL_INT* desct, double* q, MKL_INT* iq, MKL_INT* jq,
MKL_INT* descq, double* wr, double* wi, MKL_INT* m, double* work, MKL_INT* lwork,
MKL_INT* iwork, MKL_INT* liwork, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

`p?trord` reorders the real Schur factorization of a real matrix $A = Q^*T^*Q^T$, so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix T , and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace.

T must be in Schur form (as returned by `p?lahqr`), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks.

This function uses a delay and accumulate procedure for performing the off-diagonal updates.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|----------------------|---|
| <code>compq</code> | (global) = 'V': update the matrix q of Schur vectors; = 'N': do not update q . |
| <code>select</code> | (global) array of size n <i>select</i> specifies the eigenvalues in the selected cluster. To select a real eigenvalue $w(j)$, <i>select</i> [$j-1$] must be set to 1. To select a complex conjugate pair of eigenvalues $w(j)$ and $w(j+1)$, corresponding to a 2-by-2 diagonal block, either <i>select</i> [$j-1$] or <i>select</i> [j] or both must be set to 1; a complex conjugate pair of eigenvalues must be either both included in the cluster or both excluded. |
| <code>para</code> | (global) Block parameters: |
| <code>para[0]</code> | maximum number of concurrent computational windows allowed in the algorithm; $0 < para[0] \leq \min(nprow, npcot)$ must hold; |
| <code>para[1]</code> | number of eigenvalues in each window; $0 < para[1] < para[2]$ must hold; |
| <code>para[2]</code> | window size; $para[1] < para[2] < mb_t$ must hold; |

| | |
|-----------------|---|
| <i>para</i> [3] | minimal percentage of FLOPS required for performing matrix-matrix multiplications instead of pipelined orthogonal transformations; $0 \leq \textit{para}[3] \leq 100$ must hold; |
| <i>para</i> [4] | width of block column slabs for row-wise application of pipelined orthogonal transformations in their factorized form; $0 < \textit{para}[4] \leq \textit{mb_t}$ must hold. |
| <i>para</i> [5] | the maximum number of eigenvalues moved together over a process border; in practice, this will be approximately half of the cross border window size; $0 < \textit{para}[5] \leq \textit{para}[1]$ must hold. |
| <i>n</i> | (global) The order of the globally distributed matrix <i>t</i> . $n \geq 0$. |
| <i>t</i> | (local) array of size $\textit{lld_t} * \textit{LOC}_c(n)$. The local pieces of the global distributed upper quasi-triangular matrix <i>T</i> , in Schur form. |
| <i>it, jt</i> | (global) The row and column index in the global matrix <i>T</i> indicating the first column of <i>T</i> . $it = jt = 1$ must hold (see Application Notes). |
| <i>desct</i> | (global and local) array of size $\textit{dlen_}$. The array descriptor for the global distributed matrix <i>T</i> . |
| <i>q</i> | (local) array of size $\textit{lld_q} * \textit{LOC}_c(n)$. On entry, if $\textit{compq} = 'V'$, the local pieces of the global distributed matrix <i>Q</i> of Schur vectors. If $\textit{compq} = 'N'$, <i>q</i> is not referenced. |
| <i>iq, jq</i> | (global) The column index in the global matrix <i>Q</i> indicating the first column of <i>Q</i> . $iq = jq = 1$ must hold (see Application Notes). |
| <i>descq</i> | (global and local) array of size $\textit{dlen_}$. The array descriptor for the global distributed matrix <i>Q</i> . |
| <i>work</i> | (local workspace) array of size \textit{lwork} |
| <i>lwork</i> | (local) The size of the array <i>work</i> . If $\textit{lwork} = -1$, then a workspace query is assumed; the function only calculates the optimal size of the <i>work</i> array, returns this value as the first entry of the <i>work</i> array, and no error message related to <i>lwork</i> is issued by pxerbla . |

iwork (local workspace) array of size *liwork*

liwork (local)
 The size of the array *iwork*.
 If *liwork* = -1, then a workspace query is assumed; the function only calculates the optimal size of the *iwork* array, returns this value as the first entry of the *iwork* array, and no error message related to *liwork* is issued by `pxerbla`

OUTPUT Parameters

select (global) array of size *n*
 The (partial) reordering is displayed.

t On exit, *t* is overwritten by the local pieces of the reordered matrix *T*, again in Schur form, with the selected eigenvalues in the globally leading diagonal blocks.

q On exit, if *compq* = 'V', *q* has been postmultiplied by the global orthogonal transformation matrix which reorders *t*; the leading *m* columns of *q* form an orthonormal basis for the specified invariant subspace.
 If *compq* = 'N', *q* is not referenced.

wr, wi (global) array of size *n*
 The real and imaginary parts, respectively, of the reordered eigenvalues of the matrix *T*. The eigenvalues are in principle stored in the same order as on the diagonal of *T*, with $wr[i] = T(i+1,i+1)$ and, if $T(i:i+1,i:i+1)$ is a 2-by-2 diagonal block, $wi[i-1] > 0$ and $wi[i] = -wi[i-1]$.
 Note also that if a complex eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering.

m (global)
 The size of the specified invariant subspace.
 $0 \leq m \leq n$.

work[0] On exit, if *info* = 0, *work*[0] returns the optimal *lwork*.

iwork[0] On exit, if *info* = 0, *iwork*[0] returns the optimal *liwork*.

info (global)
 = 0: successful exit
 < 0: if *info* = -*i*, the *i*-th argument had an illegal value. If the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value, then *info* = -(*i**1000+*j*), if the *i*-th argument is a scalar and had an illegal value, then *info* = -*i*.
 > 0: here we have several possibilities

- Reordering of *t* failed because some eigenvalues are too close to separate (the problem is very ill-conditioned);

t may have been partially reordered, and wr and wi contain the eigenvalues in the same order as in t .

On exit, $info = \{\text{the index of } t \text{ where the swap failed (indexing starts at 1)}\}$.

- A 2-by-2 block to be reordered split into two 1-by-1 blocks and the second block failed to swap with an adjacent block.

On exit, $info = \{\text{the index of } t \text{ where the swap failed}\}$.

- If $info = n+1$, there is no valid BLACS context (see the BLACS documentation for details).

Application Notes

The following alignment requirements must hold:

- $mb_t = nb_t = mb_q = nb_q$
- $rsrc_t = rsrc_q$
- $csrc_t = csrc_q$

All matrices must be blocked by a block factor larger than or equal to two (3). This is to simplify reordering across processor borders in the presence of 2-by-2 blocks.

This algorithm cannot work on submatrices of t and q , i.e., $it = jt = iq = jq = 1$ must hold. This is however no limitation since [p?lahqr](#) does not compute Schur forms of submatrices anyway.

Parallel execution recommendations:

- Use a square grid, if possible, for maximum performance. The block parameters in $para$ should be kept well below the data distribution block size.
- In general, the parallel algorithm strives to perform as much work as possible without crossing the block borders on the main block diagonal.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trsen

Reorders the Schur factorization of a matrix and (optionally) computes the reciprocal condition numbers and invariant subspace for the selected cluster of eigenvalues.

Syntax

```
void pstrsen(char* job, char* compq, MKL_INT* select, MKL_INT* para, MKL_INT* n,
float* t, MKL_INT* it, MKL_INT* jt, MKL_INT* desct, float* q, MKL_INT* iq, MKL_INT*
jq, MKL_INT* descq, float* wr, float* wi, MKL_INT* m, float* s, float* sep, float*
work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT* info);
```

```
void pdtrsen(char* job, char* compq, MKL_INT* select, MKL_INT* para, MKL_INT* n,
double* t, MKL_INT* it, MKL_INT* jt, MKL_INT* desct, double* q, MKL_INT* iq, MKL_INT*
jq, MKL_INT* descq, double* wr, double* wi, MKL_INT* m, double* s, double* sep,
double* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

`p?trsen` reorders the real Schur factorization of a real matrix $A = Q^*T^*Q^T$, so that a selected cluster of eigenvalues appears in the leading diagonal blocks of the upper quasi-triangular matrix T , and the leading columns of Q form an orthonormal basis of the corresponding right invariant subspace. The reordering is performed by `p?trord`.

Optionally the function computes the reciprocal condition numbers of the cluster of eigenvalues and/or the invariant subspace.

T must be in Schur form (as returned by `p?lahqr`), that is, block upper triangular with 1-by-1 and 2-by-2 diagonal blocks.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|---------------|--|
| <i>job</i> | (global) Specifies whether condition numbers are required for the cluster of eigenvalues (<i>s</i>) or the invariant subspace (<i>sep</i>): = 'N': no condition numbers are required; = 'E': only the condition number for the cluster of eigenvalues is computed (<i>s</i>); = 'V': only the condition number for the invariant subspace is computed (<i>sep</i>); = 'B': condition numbers for both the cluster and the invariant subspace are computed (<i>s</i> and <i>sep</i>). |
| <i>compq</i> | (global) = 'V': update the matrix <i>q</i> of Schur vectors; = 'N': do not update <i>q</i> . |
| <i>select</i> | (global) array of size <i>n</i> <i>select</i> specifies the eigenvalues in the selected cluster. To select a real eigenvalue $w(j)$, <i>select</i> [<i>j</i> -1] must be set to a non-zero number. To select a complex conjugate pair of eigenvalues $w(j)$ and $w(j+1)$, corresponding to a 2-by-2 diagonal block, either <i>select</i> [<i>j</i> -1] or <i>select</i> [<i>j</i>] or both must be set to a non-zero number; a complex conjugate pair of eigenvalues must be either both included in the cluster or both excluded. |
| <i>para</i> | (global) Block parameters: |

| | |
|-----------------|---|
| <i>para</i> [0] | maximum number of concurrent computational windows allowed in the algorithm; $0 < para[0] \leq \min(NPROW, NPCOL)$ must hold; |
| <i>para</i> [1] | number of eigenvalues in each window; $0 < para[1] < para[2]$ must hold; |
| <i>para</i> [2] | window size; $para[1] < para[2] < mb_t$ must hold; |
| <i>para</i> [3] | minimal percentage of flops required for performing matrix-matrix multiplications instead of pipelined orthogonal transformations; $0 \leq para[3] \leq 100$ must hold; |
| <i>para</i> [4] | width of block column slabs for row-wise application of pipelined orthogonal transformations in their factorized form; $0 < para[4] \leq mb_t$ must hold. |
| <i>para</i> [5] | the maximum number of eigenvalues moved together over a process border; in practice, this will be approximately half of the cross border window size $0 < para[5] \leq para[1]$ must hold; |
| <i>n</i> | (global) The order of the globally distributed matrix <i>t</i> . $n \geq 0$. |
| <i>t</i> | (local) array of size $lld_t * LOC_c(n)$. The local pieces of the global distributed upper quasi-triangular matrix <i>T</i> , in Schur form. |
| <i>it, jt</i> | (global) The row and column index in the global matrix <i>T</i> indicating the first column of <i>T</i> . $it = jt = 1$ must hold (see Application Notes). |
| <i>desct</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the global distributed matrix <i>T</i> . |
| <i>q</i> | (local) array of size $lld_q * LOC_c(n)$. On entry, if <i>compq</i> = 'V', the local pieces of the global distributed matrix <i>Q</i> of Schur vectors. If <i>compq</i> = 'N', <i>q</i> is not referenced. |
| <i>iq, jq</i> | (global) The column index in the global matrix <i>Q</i> indicating the first column of <i>Q</i> . $iq = jq = 1$ must hold (see Application Notes). |
| <i>descq</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the global distributed matrix <i>Q</i> . |

| | |
|---------------|--|
| <i>work</i> | (local workspace) array of size <i>lwork</i> |
| <i>lwork</i> | (local) The size of the array <i>work</i> . If <i>lwork</i> = -1, then a workspace query is assumed; the function only calculates the optimal size of the <i>work</i> array, returns this value as the first entry of the <i>work</i> array, and no error message related to <i>lwork</i> is issued by pxerbla . |
| <i>iwork</i> | (local workspace) array of size <i>liwork</i> |
| <i>liwork</i> | (local) The size of the array <i>iwork</i> . If <i>liwork</i> = -1, then a workspace query is assumed; the function only calculates the optimal size of the <i>iwork</i> array, returns this value as the first entry of the <i>iwork</i> array, and no error message related to <i>liwork</i> is issued by pxerbla . |

OUTPUT Parameters

| | |
|---------------|---|
| <i>t</i> | <i>t</i> is overwritten by the local pieces of the reordered matrix <i>T</i> , again in Schur form, with the selected eigenvalues in the globally leading diagonal blocks. |
| <i>q</i> | On exit, if <i>compq</i> = 'V', <i>q</i> has been postmultiplied by the global orthogonal transformation matrix which reorders <i>t</i> ; the leading <i>m</i> columns of <i>q</i> form an orthonormal basis for the specified invariant subspace. If <i>compq</i> = 'N', <i>q</i> is not referenced. |
| <i>wr, wi</i> | (global) array of size <i>n</i> The real and imaginary parts, respectively, of the reordered eigenvalues of the matrix <i>T</i> . The eigenvalues are in principle stored in the same order as on the diagonal of <i>T</i> , with <i>wr</i> [<i>j</i>] = <i>T</i> (<i>i</i> +1, <i>i</i> +1) and, if <i>T</i> (<i>i</i> : <i>i</i> +1, <i>i</i> : <i>i</i> +1) is a 2-by-2 diagonal block, <i>wi</i> [<i>j</i> -1] > 0 and <i>wi</i> [<i>j</i>] = - <i>wi</i> [<i>j</i> -1]. Note also that if a complex eigenvalue is sufficiently ill-conditioned, then its value may differ significantly from its value before reordering. |
| <i>m</i> | (global) The size of the specified invariant subspace. $0 \leq m \leq n$. |
| <i>s</i> | (global) If <i>job</i> = 'E' or 'B', <i>s</i> is a lower bound on the reciprocal condition number for the selected cluster of eigenvalues. <i>s</i> cannot underestimate the true reciprocal condition number by more than a factor of \sqrt{n} . If <i>m</i> = 0 or <i>n</i> , <i>s</i> = 1. If <i>job</i> = 'N' or 'V', <i>s</i> is not referenced. |
| <i>sep</i> | (global) |

If $job = 'V'$ or $'B'$, sep is the estimated reciprocal condition number of the specified invariant subspace. If

$m = 0$ or n , $sep = \text{norm}(t)$.

If $job = 'N'$ or $'E'$, sep is not referenced.

$work[0]$ On exit, if $info = 0$, $work[0]$ returns the optimal $lwork$.

$iwork[0]$ On exit, if $info = 0$, $iwork[0]$ returns the optimal $liwork$.

$info$ (global)

= 0: successful exit

< 0: if $info = -i$, the i -th argument had an illegal value.

If the i -th argument is an array and the j -th entry, indexed $j-1$, had an illegal value, then $info = -(i*1000+j)$, if the i -th argument is a scalar and had an illegal value, then $info = -i$.

> 0: here we have several possibilities

- Reordering of t failed because some eigenvalues are too close to separate (the problem is very ill-conditioned); t may have been partially reordered, and wr and wi contain the eigenvalues in the same order as in t .

On exit, $info = \{\text{the index of } t \text{ where the swap failed (indexing starts at 1)}\}$.

- A 2-by-2 block to be reordered split into two 1-by-1 blocks and the second block failed to swap with an adjacent block.

On exit, $info = \{\text{the index of } t \text{ where the swap failed}\}$.

- If $info = n+1$, there is no valid BLACS context (see the BLACS documentation for details).

Application Notes

The following alignment requirements must hold:

- $mb_t = nb_t = mb_q = nb_q$
- $rsrc_t = rsrc_q$
- $csrc_t = csrc_q$

All matrices must be blocked by a block factor larger than or equal to two (3). This to simplify reordering across processor borders in the presence of 2-by-2 blocks.

This algorithm cannot work on submatrices of t and q , i.e., $it = jt = iq = jq = 1$ must hold. This is however no limitation since [p?lahqr](#) does not compute Schur forms of submatrices anyway.

For parallel execution, use a square grid, if possible, for maximum performance. The block parameters in $para$ should be kept well below the data distribution block size.

In general, the parallel algorithm strives to perform as much work as possible without crossing the block borders on the main block diagonal.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trti2

Computes the inverse of a triangular matrix (local unblocked algorithm).

Syntax

```
void pstrti2 (char *uplo , char *diag , MKL_INT *n , float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
void pdtrti2 (char *uplo , char *diag , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
void pctrti2 (char *uplo , char *diag , MKL_INT *n , MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
void pztrti2 (char *uplo , char *diag , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The p?trti2 function computes the inverse of a real/complex upper or lower triangular block matrix sub (A) = $A(ia:ia+n-1, ja:ja+n-1)$.

This matrix should be contained in one and only one process memory space (local operation).

Input Parameters

| | |
|-------------|--|
| <i>uplo</i> | (global) Specifies whether the matrix sub (A) is upper or lower triangular. = 'U': sub (A) is upper triangular = 'L': sub (A) is lower triangular. |
| <i>diag</i> | (global) Specifies whether or not the matrix A is unit triangular. = 'N': sub (A) is non-unit triangular = 'U': sub (A) is unit triangular. |
| <i>n</i> | (global) The number of rows and columns to be operated on, i.e., the order of the distributed submatrix sub (A). $n \geq 0$. |
| <i>a</i> | (local) Pointer into the local memory to an array, size $lld_a * LOCC(ja+n-1)$. On entry, this array contains the local pieces of the triangular matrix sub(A). If <i>uplo</i> = 'U', the leading n -by- n upper triangular part of the matrix sub(A) contains the upper triangular part of the matrix, and the strictly lower triangular part of sub(A) is not referenced. |

If *uplo* = 'L', the leading *n*-by-*n* lower triangular part of the matrix sub(*A*) contains the lower triangular part of the matrix, and the strictly upper triangular part of sub(*A*) is not referenced. If *diag* = 'U', the diagonal elements of sub(*A*) are not referenced either and are assumed to be 1.

ia, ja

(global)

The row and column indices in the global matrix *A* indicating the first row and the first column of the sub(*A*), respectively.

desca

(global and local) array of size *dlen_*. The array descriptor for the distributed matrix *A*.

Output Parameters

a

On exit, the (triangular) inverse of the original matrix, in the same storage format.

info

= 0: successful exit

< 0: if the *i*-th argument is an array and the *j*-th entry, indexed *j*-1, had an illegal value,

then *info* = - (*i**100+*j*),

if the *i*-th argument is a scalar and had an illegal value,

then *info* = -*i*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?lahqr2

Updates the eigenvalues and Schur decomposition.

Syntax

```
void clahqr2 (const MKL_INT* wantt, const MKL_INT* wantz, const MKL_INT* n, const
MKL_INT* ilo, const MKL_INT* ihi, MKL_Complex8* h, const MKL_INT* ldh, MKL_Complex8*
w, const MKL_INT* iloz, const MKL_INT* ihiz, MKL_Complex8* z, const MKL_INT* ldz,
MKL_INT* info);
```

```
void zlahqr2 (const MKL_INT* wantt, const MKL_INT* wantz, const MKL_INT* n, const
MKL_INT* ilo, const MKL_INT* ihi, MKL_Complex16* h, const MKL_INT* ldh, MKL_Complex16*
w, const MKL_INT* iloz, const MKL_INT* ihiz, MKL_Complex16* z, const MKL_INT* ldz,
MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

?lahqr2 is an auxiliary routine called by ?hseqr to update the eigenvalues and Schur decomposition already computed by ?hseqr, by dealing with the Hessenberg submatrix in rows and columns *ilo* to *ihi*. This version of ?lahqr (not the standard LAPACK version) uses a double-shift algorithm (like LAPACK's ?lahqr). Unlike the standard LAPACK convention, this does not assume the subdiagonal is real, nor does it work to preserve this quality if given.

Input Parameters

| | |
|-------------------|--|
| <i>wantt</i> | <p>≠ 0: the full Schur form <i>T</i> is required; = 0: only eigenvalues are required.</p> |
| <i>wantz</i> | <p>≠ 0: the matrix of Schur vectors <i>Z</i> is required; = 0: Schur vectors are not required.</p> |
| <i>n</i> | The order of the matrix <i>H</i> . $n \geq 0$. |
| <i>ilo, ihi</i> | <p>It is assumed that the matrix <i>H</i> is upper triangular in rows and columns <i>ihi</i> + 1 : <i>n</i>, and that matrix element $H(i_{lo}, i_{lo}-1) = 0$ (unless $i_{lo} = 1$). ?lahqr works primarily with the Hessenberg submatrix in rows and columns <i>ilo</i> to <i>ihi</i>, but applies transformations to all of <i>h</i> if <i>wantt</i> is nonzero. $1 \leq i_{lo} \leq \max(1, i_{hi}); i_{hi} \leq n$.</p> |
| <i>h</i> | <p>Array, size $ldh * n$. On entry, the upper Hessenberg matrix <i>H</i>.</p> |
| <i>ldh</i> | The leading dimension of the array <i>h</i> . $ldh \geq \max(1, n)$. |
| <i>iloz, ihiz</i> | <p>Specify the rows of <i>Z</i> to which transformations must be applied if <i>wantz</i> ≠ 0. $1 \leq i_{loz} \leq i_{lo}; i_{hi} \leq i_{hiz} \leq n$.</p> |
| <i>z</i> | <p>Array, size $ldz * n$. If <i>wantz</i> ≠ 0, on entry <i>z</i> must contain the current matrix <i>Z</i> of transformations. If <i>wantz</i> = 0, <i>z</i> is not referenced.</p> |
| <i>ldz</i> | The leading dimension of the array <i>z</i> . $ldz \geq \max(1, n)$. |

Output Parameters

| | |
|----------|---|
| <i>h</i> | On exit, if <i>wantt</i> ≠ 0, <i>h</i> is upper triangular in rows and columns <i>ilo</i> : <i>ihi</i> . If <i>wantt</i> = 0, the contents of <i>h</i> are unspecified on exit. |
| <i>w</i> | <p>Array, size (<i>n</i>) The computed eigenvalues <i>ilo</i> to <i>ihi</i> are stored in the corresponding elements of <i>w</i>. If <i>wantt</i> ≠ 0, the eigenvalues are stored in the same order as on the diagonal of the Schur form returned in <i>h</i>, with $w[i] = H(i, i)$.</p> |

z If *wantz* ≠ 0, on exit *z* has been updated; transformations are applied only to the submatrix $Z(i_{loz}:i_{hiz}, i_{lo}:i_{hi})$. If *wantz* = 0, *z* is not referenced.

info = 0: successful exit
 > 0: if *info* = *i*, ?lahqr failed to compute all the eigenvalues *ilo* to *ihi* in a total of $30*(i_{hi}-i_{lo}+1)$ iterations; elements $w[i:i_{hi} - 1]$ contain those eigenvalues which have been successfully computed.

?lamsh

Sends multiple shifts through a small (single node) matrix to maximize the number of bulges that can be sent through.

Syntax

```
void slamsh (float *s, const MKL_INT *lds, MKL_INT *nbulge, const MKL_INT *jblk, float
*h, const MKL_INT *ldh, const MKL_INT *n, const float *ulp );

void dlamsh (double *s, const MKL_INT *lds, MKL_INT *nbulge, const MKL_INT *jblk,
double *h, const MKL_INT *ldh, const MKL_INT *n, const double *ulp );

void clamsh (MKL_Complex8 *s , const MKL_INT *lds , MKL_INT *nbulge , const MKL_INT
*jblk , MKL_Complex8 *h , const MKL_INT *ldh , const MKL_INT *n , const float *ulp );

void zlamsh (MKL_Complex16 *s , const MKL_INT *lds , MKL_INT *nbulge , const MKL_INT
*jblk , MKL_Complex16 *h , const MKL_INT *ldh , const MKL_INT *n , const double *ulp );
```

Include Files

- mkl_scalapack.h

Description

The ?lamsh function sends multiple shifts through a small (single node) matrix to see how small consecutive subdiagonal elements are modified by subsequent shifts in an effort to maximize the number of bulges that can be sent through. The function should only be called when there are multiple shifts/bulges (*nbulge* > 1) and the first shift is starting in the middle of an unreduced Hessenberg matrix because of two or more small consecutive subdiagonal elements.

Input Parameters

s (local)
 Array of size $lds*2*jblk$.
 On entry, the matrix of shifts. Only the 2x2 diagonal of *s* is referenced. It is assumed that *s* has *jblk* double shifts (size 2).

lds (local)
 On entry, the leading dimension of *S*; unchanged on exit. $1 < nbulge \leq jblk \leq lds/2$.

nbulge (local)
 On entry, the number of bulges to send through *h* (>1). *nbulge* should be less than the maximum determined (*jblk*). $1 < nbulge \leq jblk \leq lds/2$.

| | |
|-------------|---|
| <i>jblk</i> | (local) On entry, the number of double shifts determined for <i>S</i> ; unchanged on exit. |
| <i>h</i> | (local) Array of size $ldh*n$. On entry, the local matrix to apply the shifts on. <i>h</i> should be aligned so that the starting row is 2. |
| <i>ldh</i> | (local) On entry, the leading dimension of <i>H</i> ; unchanged on exit. |
| <i>n</i> | (local) On entry, the size of <i>H</i> . If all the bulges are expected to go through, <i>n</i> should be at least $4nbulge+2$. Otherwise, <i>nbulge</i> may be reduced by this function. |
| <i>ulp</i> | (local) On entry, machine precision. Unchanged on exit. |

Output Parameters

| | |
|---------------|---|
| <i>s</i> | On exit, the data is rearranged in the best order for applying. |
| <i>nbulge</i> | On exit, the maximum number of bulges that can be sent through. |
| <i>h</i> | On exit, the data is destroyed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?lapst

Sorts the numbers in increasing or decreasing order.

Syntax

```
void slapst (const char* id, const MKL_INT* n, const float* d, MKL_INT* indx, MKL_INT* info);
```

```
void dlapst (const char* id, const MKL_INT* n, const double* d, MKL_INT* indx, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

?lapst is a modified version of the LAPACK routine ?lasrt.

Define a permutation *indx* that sorts the numbers in *d* in increasing order (if *id* = 'I') or in decreasing order (if *id* = 'D').

Use Quick Sort, reverting to Insertion sort on arrays of size ≤ 20 . Dimension of STACK limits *n* to about 2^{32} .

Input Parameters

| | |
|-----------|--|
| <i>id</i> | = 'I': sort <i>d</i> in increasing order; = 'D': sort <i>d</i> in decreasing order. |
| <i>n</i> | The length of the array <i>d</i> . |
| <i>d</i> | Array, size (<i>n</i>) The array to be sorted. |

Output Parameters

| | |
|-------------|---|
| <i>indx</i> | Array, size (<i>n</i>). The permutation which sorts the array <i>d</i> . |
| <i>info</i> | = 0: successful exit < 0: if <i>info</i> = - <i>i</i> , the <i>i</i> -th argument had an illegal value |

?laqr6

Performs a single small-bulge multi-shift QR sweep collecting the transformations.

Syntax

```
void slaqr6(char* job, MKL_INT* wantt, MKL_INT* wantz, MKL_INT* kacc22, MKL_INT* n,
MKL_INT* ktop, MKL_INT* kbot, MKL_INT* nshfts, float* sr, float* si, float* h,
MKL_INT* ldh, MKL_INT* iloz, MKL_INT* ihiz, float* z, MKL_INT* ldz, float* v, MKL_INT*
ldv, float* u, MKL_INT* ldu, MKL_INT* nv, float* wv, MKL_INT* ldwv, MKL_INT* nh,
float* wh, MKL_INT* ldwh);
```

```
void dlaqr6(char* job, MKL_INT* wantt, MKL_INT* wantz, MKL_INT* kacc22, MKL_INT* n,
MKL_INT* ktop, MKL_INT* kbot, MKL_INT* nshfts, double* sr, double* si, double* h,
MKL_INT* ldh, MKL_INT* iloz, MKL_INT* ihiz, double* z, MKL_INT* ldz, double* v,
MKL_INT* ldv, double* u, MKL_INT* ldu, MKL_INT* nv, double* wv, MKL_INT* ldwv,
MKL_INT* nh, double* wh, MKL_INT* ldwh);
```

Include Files

- mkl_scalapack.h

Description

This auxiliary function performs a single small-bulge multi-shift QR sweep, moving the chain of bulges from top to bottom in the submatrix $H(k_{top}:k_{bot}, k_{top}:k_{bot})$, collecting the transformations in the matrix *V* or accumulating the transformations in the matrix *Z* (see below).

This is a modified version of ?laqr5 from LAPACK 3.1.

Input Parameters

job Set the kind of *job* to do in ?laqr6, as follows:

| | |
|-------------------|--|
| | <p><i>job</i> = 'I': Introduce and chase bulges in submatrix</p> <p><i>job</i> = 'C': Chase bulges from top to bottom of submatrix</p> <p><i>job</i> = 'O': Chase bulges off submatrix</p> |
| <i>wantt</i> | <i>wantt</i> is non-zero if the quasi-triangular Schur factor is being computed. <i>wantt</i> is set to zero otherwise. |
| <i>wantz</i> | <i>wantz</i> is non-zero if the orthogonal Schur factor is being computed. <i>wantz</i> is set to zero otherwise. |
| <i>kacc22</i> | Specifies the computation mode of far-from-diagonal orthogonal updates. <ul style="list-style-type: none"> = 0: ?laqr6 does not accumulate reflections and does not use matrix-matrix multiply to update far-from-diagonal matrix entries. = 1: ?laqr6 accumulates reflections and uses matrix-matrix multiply to update the far-from-diagonal matrix entries. = 2: ?laqr6 accumulates reflections, uses matrix-matrix multiply to update the far-from-diagonal matrix entries, and takes advantage of 2-by-2 block structure during matrix multiplies. |
| <i>n</i> | <i>n</i> is the order of the Hessenberg matrix <i>H</i> upon which this function operates. |
| <i>ktop, kbot</i> | These are the first and last rows and columns of an isolated diagonal block upon which the QR sweep is to be applied. It is assumed without a check that either $ktop = 1$ or $H(ktop,ktop-1) = 0$ and either $kbot = n$ or $H(kbot+1,kbot) = 0$. |
| <i>nshfts</i> | <i>nshfts</i> gives the number of simultaneous shifts. <i>nshfts</i> must be positive and even. |
| <i>sr, si</i> | Array of size <i>nshfts</i> <i>sr</i> contains the real parts and <i>si</i> contains the imaginary parts of the <i>nshfts</i> shifts of origin that define the multi-shift QR sweep. |
| <i>h</i> | Array of size $ldh * n$ On input <i>h</i> contains a Hessenberg matrix <i>H</i> . |
| <i>ldh</i> | <i>ldh</i> is the leading dimension of <i>H</i> just as declared in the calling function. $ldh \geq \max(1,n)$. |
| <i>iloz, ihiz</i> | Specify the rows of the matrix <i>Z</i> to which transformations must be applied if <i>wantz</i> is non-zero. $1 \leq iloz \leq ihiz \leq n$ |
| <i>z</i> | Array of size $ldz * ktop$ If <i>wantz</i> is non-zero, then the QR sweep orthogonal similarity transformation is accumulated into the matrix $Z(iloz:ihiz,kbot:ktop)$, stored in the array <i>z</i> , from the right. If <i>wantz</i> equals zero, then <i>z</i> is unreferenced. |

| | |
|-------------|---|
| <i>ldz</i> | <i>ldz</i> is the leading dimension of <i>z</i> just as declared in the calling function. <i>ldz</i> ≥ <i>n</i> . |
| <i>v</i> | (workspace) array of size <i>ldv</i> * <i>nshfts</i> /2 |
| <i>ldv</i> | <i>ldv</i> is the leading dimension of <i>v</i> as declared in the calling function. <i>ldv</i> ≥ 3. |
| <i>u</i> | (workspace) array of size <i>ldu</i> * (3* <i>nshfts</i> -3) |
| <i>ldu</i> | <i>ldu</i> is the leading dimension of <i>u</i> just as declared in the calling function. <i>ldu</i> ≥ 3* <i>nshfts</i> -3. |
| <i>nh</i> | <i>nh</i> is the number of columns in array <i>wh</i> available for workspace. <i>nh</i> ≥ 1 is required for usage of this workspace, otherwise the updates of the far-from-diagonal elements will be updated without level 3 BLAS. |
| <i>wh</i> | (workspace) array of size <i>ldwh</i> * <i>nh</i> |
| <i>ldwh</i> | Leading dimension of <i>wh</i> just as declared in the calling function. <i>ldwh</i> ≥ 3* <i>nshfts</i> -3. |
| <i>nv</i> | <i>nv</i> is the number of rows in <i>wv</i> available for workspace. <i>nv</i> ≥ 1 is required for usage of this workspace, otherwise the updates of the far-from-diagonal elements will be updated without level 3 BLAS. |
| <i>wv</i> | (workspace) array of size <i>ldwv</i> * 3* <i>nshfts</i> |
| <i>ldwv</i> | scalar <i>ldwv</i> is the leading dimension of <i>wv</i> as declared in the in the calling function. <i>ldwv</i> ≥ <i>nv</i> . |

OUTPUT Parameters

| | |
|----------|--|
| <i>h</i> | A multi-shift QR sweep with shifts $sr[j]+i*si[j]$ is applied to the isolated diagonal block in matrix rows and columns <i>ktop</i> through <i>kbot</i> . |
| <i>z</i> | If <i>wantzis</i> non-zero, then the QR sweep orthogonal/unitary similarity transformation is accumulated into the matrix $Z(i_{loz}:i_{hiz},k_{bot}:k_{top})$ from the right. If <i>wantze</i> equals zero, then <i>z</i> is unreferenced. |

Application Notes

Notes

Based on contributions by Karen Braman and Ralph Byers, Department of Mathematics, University of Kansas, USA Robert Granat, Department of Computing Science and HPC2N, Umea University, Sweden

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?slar1va

Computes scaled eigenvector corresponding to given eigenvalue.

Syntax

```
void slar1va(MKL_INT* n, MKL_INT* b1, MKL_INT* bn, float* lambda, float* d, float* l,
float* ld, float* lld, float* pivmin, float* gaptol, float* z, MKL_INT* wantnc,
MKL_INT* negcnt, float* ztz, float* mingma, MKL_INT* r, MKL_INT* isuppz, float*
nrminv, float* resid, float* rqcrr, float* work);
```

```
void dlar1va(MKL_INT* n, MKL_INT* b1, MKL_INT* bn, double* lambda, double* d, double*
l, double* ld, double* lld, double* pivmin, double* gaptol, double* z, MKL_INT*
wantnc, MKL_INT* negcnt, double* ztz, double* mingma, MKL_INT* r, MKL_INT* isuppz,
double* nrminv, double* resid, double* rqcrr, double* work);
```

Include Files

- mkl_scalapack.h

Description

?slar1va computes the (scaled) r -th column of the inverse of the submatrix in rows $b1$ through bn of the tridiagonal matrix $LDL^T - \lambda I$. When λ is close to an eigenvalue, the computed vector is an accurate eigenvector. Usually, r corresponds to the index where the eigenvector is largest in magnitude. The following steps accomplish this computation :

1. Stationary qd transform, $LDL^T - \lambda I = L_+ D_+ L_+^T$,
2. Progressive qd transform, $LDL^T - \lambda I = U \cdot D \cdot U^T$,
3. Computation of the diagonal elements of the inverse of $LDL^T - \lambda I$ by combining the above transforms, and choosing r as the index where the diagonal of the inverse is (one of the) largest in magnitude.
4. Computation of the (scaled) r -th column of the inverse using the twisted factorization obtained by combining the top part of the stationary and the bottom part of the progressive transform.

Input Parameters

| | |
|----------|--|
| n | The order of the matrix LDL^T . |
| $b1$ | First index of the submatrix of LDL^T . |
| bn | Last index of the submatrix of LDL^T . |
| $lambda$ | The shift λ . In order to compute an accurate eigenvector, $lambda$ should be a good approximation to an eigenvalue of LDL^T . |
| l | Array of size $n-1$ The $(n-1)$ subdiagonal elements of the unit bidiagonal matrix L , in elements 0 to $n-2$. |
| d | Array of size n The n diagonal elements of the diagonal matrix D . |
| ld | Array of size $n-1$ The $n-1$ elements $l[i]*d[i]$, $i=0, \dots, n-2$. |

| | |
|---------------|---|
| <i>lld</i> | Array of size $n-1$ The $n-1$ elements $l[i]*l[i]*d[i]$, $i=0,\dots,n-2$. |
| <i>pivmin</i> | The minimum pivot in the Sturm sequence. |
| <i>gaptol</i> | Tolerance that indicates when eigenvector entries are negligible with respect to their contribution to the residual. |
| <i>z</i> | Array of size n On input, all entries of z must be set to 0. |
| <i>wantnc</i> | Specifies whether <i>negcnt</i> has to be computed. |
| <i>r</i> | The twist index for the twisted factorization used to compute z . On input, $0 \leq r \leq n$. If r is input as 0, r is set to the index where $(LDL^T - \sigma I)^{-1}$ is largest in magnitude. If $1 \leq r \leq n$, r is unchanged. Ideally, r designates the position of the maximum entry in the eigenvector. |
| <i>work</i> | (Workspace) array of size $4*n$ |

OUTPUT Parameters

| | |
|---------------|---|
| <i>z</i> | On output, z contains the (scaled) r -th column of the inverse. The scaling is such that $z[r-1]$ equals 1. |
| <i>negcnt</i> | If <i>wantnc</i> is non-zero then <i>negcnt</i> = the number of pivots $< pivmin$ in the matrix factorization LDL^T , and <i>negcnt</i> = -1 otherwise. |
| <i>ztz</i> | The square of the 2-norm of z . |
| <i>mingma</i> | The reciprocal of the largest (in magnitude) diagonal element of the inverse of $LDL^T - \sigma I$. |
| <i>r</i> | On output, r contains the twist index used to compute z . |
| <i>isuppz</i> | array of size 2 The support of the vector in z , i.e., the vector z is non-zero only in elements <i>isuppz</i> [0] and <i>isuppz</i> [1]. |
| <i>nrminv</i> | $nrminv = 1/\text{SQRT}(ztz)$ |
| <i>resid</i> | The residual of the FP vector. $resid = \text{ABS}(mingma)/\text{SQRT}(ztz)$ |
| <i>rqcorr</i> | The Rayleigh Quotient correction to <i>lambda</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?laref

Applies Householder reflectors to matrices on their rows or columns.

Syntax

```
void slaref (const char* type, float* a, const MKL_INT* lda, const MKL_INT* wantz,
float* z, const MKL_INT* ldz, const MKL_INT* block, MKL_INT* irow1, MKL_INT* icoll,
const MKL_INT* istart, const MKL_INT* istop, const MKL_INT* itmp1, const MKL_INT*
itmp2, const MKL_INT* liloz, const MKL_INT* lihiz, const float* vecs, float* v2,
float* v3, float* t1, float* t2, float* t3);
```

```
void dlaref (const char* type, double* a, const MKL_INT* lda, const MKL_INT* wantz,
double* z, const MKL_INT* ldz, const MKL_INT* block, MKL_INT* irow1, MKL_INT* icoll,
const MKL_INT* istart, const MKL_INT* istop, const MKL_INT* itmp1, const MKL_INT*
itmp2, const MKL_INT* liloz, const MKL_INT* lihiz, const double* vecs, double* v2,
double* v3, double* t1, double* t2, double* t3);
```

```
void claref (const char* type, MKL_Complex8* a, const MKL_INT* lda, const MKL_INT*
wantz, MKL_Complex8* z, const MKL_INT* ldz, const MKL_INT* block, MKL_INT* irow1,
MKL_INT* icoll, const MKL_INT* istart, const MKL_INT* istop, const MKL_INT* itmp1,
const MKL_INT* itmp2, const MKL_INT* liloz, const MKL_INT* lihiz, const MKL_Complex8*
vecs, MKL_Complex8* v2, MKL_Complex8* v3, MKL_Complex8* t1, MKL_Complex8* t2,
MKL_Complex8* t3);
```

```
void zlaref (const char* type, MKL_Complex16* a, const MKL_INT* lda, const MKL_INT*
wantz, MKL_Complex16* z, const MKL_INT* ldz, const MKL_INT* block, MKL_INT* irow1,
MKL_INT* icoll, const MKL_INT* istart, const MKL_INT* istop, const MKL_INT* itmp1,
const MKL_INT* itmp2, const MKL_INT* liloz, const MKL_INT* lihiz, const MKL_Complex16*
vecs, MKL_Complex16* v2, MKL_Complex16* v3, MKL_Complex16* t1, MKL_Complex16* t2,
MKL_Complex16* t3);
```

Include Files

- mkl_scalapack.h

Description

?laref applies one or several Householder reflectors of size 3 to one or two matrices (if column is specified) on either their rows or columns.

Input Parameters

| | |
|-------------|---|
| <i>type</i> | (local) If 'R': Apply reflectors to the rows of the matrix (apply from left) Otherwise: Apply reflectors to the columns of the matrix Unchanged on exit. |
| <i>a</i> | (local) Array, $lld_a * LOCc(ja+n-1)$ On entry, the matrix to receive the reflections. |
| <i>lda</i> | (local) |

| | |
|---------------|---|
| | On entry, the leading dimension of a . Unchanged on exit. |
| <i>wantz</i> | (local) If $wantz \neq 0$, then apply any column reflections to z as well. If $wantz = 0$, then do no additional work on z . |
| z | (local) Array, $ldz * ncols$, where the value $ncols$ depends on other arguments. If $wantz \neq 0$ and $type \neq 'R'$ then $ncols = icoll + 3 * (lihiz - liloz + 1)$. Otherwise, $ncols$ is unused. On entry, the second matrix to receive column reflections. This is changed only if $wantz$ is set. |
| <i>ldz</i> | (local) On entry, the leading dimension of z . Unchanged on exit. |
| <i>block</i> | (local) If nonzero, then apply several reflectors at once and read their data from the <i>vecs</i> array. If zero, apply the single reflector given by $v2, v3, t1, t2$, and $t3$. |
| <i>irowl</i> | (local) On entry, the local row element of a . |
| <i>icoll</i> | (local) On entry, the local column element of a . |
| <i>istart</i> | (local) Specifies the "number" of the first reflector. This is used as an index into <i>vecs</i> if <i>block</i> is set. <i>istart</i> is ignored if <i>block</i> is zero. |
| <i>istop</i> | (local) Specifies the "number" of the last reflector. This is used as an index into <i>vecs</i> if <i>block</i> is set. <i>istop</i> is ignored if <i>block</i> is zero. |
| <i>itmpl</i> | (local) Starting range into a . For rows, this is the local first column. For columns, this is the local first row. |
| <i>itmp2</i> | (local) Ending range into a . For rows, this is the local last column. For columns, this is the local last row. |

| | | |
|---------------------------|---------|---|
| <i>liloz, lihiz</i> | (local) | These serve the same purpose as <i>itmp1, itmp2</i> but for <i>z</i> when <i>wantz</i> is set. |
| <i>vecs</i> | (local) | Array of size 3*N (matrix size) This holds the size 3 reflectors one after another and this is only accessed when <i>block</i> is nonzero |
| <i>v2, v3, t1, t2, t3</i> | (local) | This holds information on a single size 3 Householder reflector and is read when <i>block</i> is zero, and overwritten when <i>block</i> is nonzero |

Output Parameters

| | |
|---------------------------|--|
| <i>a</i> | The updated matrix on exit. |
| <i>z</i> | This is changed only if <i>wantz</i> is set. |
| <i>irow1</i> | Undefined on output. |
| <i>icol1</i> | Undefined on output. |
| <i>v2, v3, t1, t2, t3</i> | Overwritten when <i>block</i> is nonzero. |

?larrb2

Provides limited bisection to locate eigenvalues for more accuracy.

Syntax

```
void slarrb2(MKL_INT* n, float* d, float* lld, MKL_INT* ifirst, MKL_INT* ilast, float*
rtol1, float* rtol2, MKL_INT* offset, float* w, float* wgap, float* werr, float* work,
MKL_INT* iwork, float* pivmin, float* lgpvmn, float* lgspdm, MKL_INT* twist, MKL_INT*
info);
```

```
void dlarrb2(MKL_INT* n, double* d, double* lld, MKL_INT* ifirst, MKL_INT* ilast,
double* rtol1, double* rtol2, MKL_INT* offset, double* w, double* wgap, double* werr,
double* work, MKL_INT* iwork, double* pivmin, double* lgpvmn, double* lgspdm, MKL_INT*
twist, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

Given the relatively robust representation (RRR) LDL^T , ?larrb2 does "limited" bisection to refine the eigenvalues of LDL^T with indices in a given range to more accuracy. Initial guesses for these eigenvalues are input in *w*, the corresponding estimate of the error in these guesses and their gaps are input in *werr* and *wgap*, respectively. During bisection, intervals [*left*, *right*] are maintained by storing their mid-points and semi-widths in the arrays *w* and *werr* respectively. The range of indices is specified by the *ifirst*, *ilast*, and *offset* parameters, as explained in [Input Parameters](#).

NOTE

There are very few minor differences between `larrb` from LAPACK and this current function `?larrb2`. The most important reason for creating this nearly identical copy is profiling: in the ScaLAPACK MRRR algorithm, eigenvalue computation using `?larrb2` is used for refinement in the construction of the representation tree, as opposed to the initial computation of the eigenvalues for the root RRR which uses `?larrb`. When profiling, this allows an easy quantification of refinement work vs. computing eigenvalues of the root.

Input Parameters

| | |
|---------------------|--|
| <i>n</i> | The order of the matrix. |
| <i>d</i> | Array of size <i>n</i> . The <i>n</i> diagonal elements of the diagonal matrix <i>D</i> . |
| <i>lld</i> | Array of size <i>n</i> -1. The (<i>n</i> -1) elements $l_{i+1} * l_{i+1} * d[i]$, $i=0, \dots, n-2$. |
| <i>ifirst</i> | The index of the first eigenvalue to be computed. |
| <i>ilast</i> | The index of the last eigenvalue to be computed. |
| <i>rtol1, rtol2</i> | Tolerance for the convergence of the bisection intervals. An interval [<i>left</i> , <i>right</i>] has converged if $right - left < \max(rtol1 * gap, rtol2 * \max(left , right))$ where <i>gap</i> is the (estimated) distance to the nearest eigenvalue. |
| <i>offset</i> | Offset for the arrays <i>w</i> , <i>wgap</i> and <i>werr</i> , i.e., the elements indexed <i>ifirst</i> - <i>offset</i> - 1 through <i>ilast</i> - <i>offset</i> - 1 of these arrays are to be used. |
| <i>w</i> | Array of size <i>n</i> On input, $w[ifirst - offset - 1]$ through $w[ilast - offset - 1]$ are estimates of the eigenvalues of LDL^T indexed <i>ifirst</i> through <i>ilast</i> . |
| <i>wgap</i> | Array of size <i>n</i> -1. On input, the (estimated) gaps between consecutive eigenvalues of LDL^T , i.e., $wgap[I - offset - 1]$ is the gap between eigenvalues <i>I</i> and <i>I</i> + 1. Note that if <i>ifirst</i> = <i>ilast</i> then $wgap[ifirst - offset - 1]$ must be set to zero. |
| <i>werr</i> | Array of size <i>n</i> . On input, $werr[ifirst - offset - 1]$ through $werr[ilast - offset - 1]$ are the errors in the estimates of the corresponding elements in <i>w</i> . |
| <i>work</i> | (workspace) array of size $4*n$. Workspace. |
| <i>iwork</i> | (workspace) array of size $2*n$. Workspace. |

| | |
|---------------|--|
| <i>pivmin</i> | The minimum pivot in the Sturm sequence. |
| <i>lgpvmn</i> | Logarithm of <i>pivmin</i> , precomputed. |
| <i>lgspdm</i> | Logarithm of the spectral diameter, precomputed. |
| <i>twist</i> | The twist index for the twisted factorization that is used for the <i>negcount</i> . <i>twist</i> = <i>n</i> : Compute <i>negcount</i> from $LDL^T - \lambda I = L_+ D_+ L_+^T$ <i>twist</i> = 1: Compute <i>negcount</i> from $LDL^T - \lambda I = U_+ D_+ U_+^T$ <i>twist</i> = <i>r</i> , $1 < r < n$: Compute <i>negcount</i> from $LDL^T - \lambda I = N_r A_r N_r^T$ |

OUTPUT Parameters

| | |
|-------------|--|
| <i>w</i> | On output, the eigenvalue estimates in <i>w</i> are refined. |
| <i>wgap</i> | On output, the eigenvalue gaps in <i>wgap</i> are refined. |
| <i>werr</i> | On output, the errors in <i>werr</i> are refined. |
| <i>info</i> | Error flag. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?larrd2

Computes the eigenvalues of a symmetric tridiagonal matrix to suitable accuracy.

Syntax

```
void slarrd2(char* range, char* order, MKL_INT* n, float* vl, float* vu, MKL_INT* il,
MKL_INT* iu, float* gers, float* reltol, float* d, float* e, float* e2, float* pivmin,
MKL_INT* nsplit, MKL_INT* isplit, MKL_INT* m, float* w, float* werr, float* wl, float*
wu, MKL_INT* iblock, MKL_INT* indexw, float* work, MKL_INT* iwork, MKL_INT* dol,
MKL_INT* dou, MKL_INT* info);
```

```
void dlarrd2(char* range, char* order, MKL_INT* n, double* vl, double* vu, MKL_INT*
il, MKL_INT* iu, double* gers, double* reltol, double* d, double* e, double* e2,
double* pivmin, MKL_INT* nsplit, MKL_INT* isplit, MKL_INT* m, double* w, double* werr,
double* wl, double* wu, MKL_INT* iblock, MKL_INT* indexw, double* work, MKL_INT*
iwork, MKL_INT* dol, MKL_INT* dou, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

?larrd2 computes the eigenvalues of a symmetric tridiagonal matrix *T* to limited initial accuracy. This is an auxiliary code to be called from [larre2a](#).

`?larrd2` has been created using the LAPACK code `larrd` which itself stems from `stebz`. The motivation for creating `?larrd2` is efficiency: When computing eigenvalues in parallel and the input tridiagonal matrix splits into blocks, `?larrd2` can skip over blocks which contain none of the eigenvalues from DOL to DOU for which the processor responsible. In extreme cases (such as large matrices consisting of many blocks of small size like 2x2), the gain can be substantial.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|-----------------|---|
| <i>range</i> | <p>= 'A': ("All") all eigenvalues will be found.</p> <p>= 'V': ("Value") all eigenvalues in the half-open interval $(v_l, v_u]$ will be found.</p> <p>= 'I': ("Index") eigenvalues of the entire matrix with the indices in a given range will be found.</p> |
| <i>order</i> | <p>= 'B': ("By Block") the eigenvalues will be grouped by split-off block (see <i>iblock</i>, <i>isplit</i>) and ordered from smallest to largest within the block.</p> <p>= 'E': ("Entire matrix") the eigenvalues for the entire matrix will be ordered from smallest to largest.</p> |
| <i>n</i> | The order of the tridiagonal matrix T . $n \geq 0$. |
| <i>v_l, v_u</i> | <p>If <i>range</i>='V', the lower and upper bounds of the interval to be searched for eigenvalues. Eigenvalues less than or equal to v_l, or greater than v_u, will not be returned. $v_l < v_u$.</p> <p>Not referenced if <i>range</i> = 'A' or 'I'.</p> |
| <i>il, iu</i> | <p>If <i>range</i>='I', the indices (in ascending order) of the smallest eigenvalue, to be returned in $w[il-1]$, and largest eigenvalue, to be returned in $w[iu-1]$.</p> <p>$1 \leq il \leq iu \leq n$, if $n > 0$; $il = 1$ and $iu = 0$ if $n = 0$.</p> <p>Not referenced if <i>range</i> = 'A' or 'V'.</p> |
| <i>gers</i> | <p>Array of size $2*n$</p> <p>The n Gerschgorin intervals (the i-th Gerschgorin interval is $(gers[2*i-2], gers[2*i-1])$).</p> |
| <i>reltol</i> | The minimum relative width of an interval. When an interval is narrower than <i>reltol</i> times the larger (in magnitude) endpoint, then it is considered to be sufficiently small, i.e., converged. Note: this should always be at least $\text{radix} * \text{machine epsilon}$. |

| | |
|-----------------|--|
| <i>d</i> | Array of size n The n diagonal elements of the tridiagonal matrix T . |
| <i>e</i> | Array of size $n-1$ The $(n-1)$ off-diagonal elements of the tridiagonal matrix T . |
| <i>e2</i> | Array of size $n-1$ The $(n-1)$ squared off-diagonal elements of the tridiagonal matrix T . |
| <i>pivmin</i> | The minimum pivot allowed in the sturm sequence for T . |
| <i>nsplit</i> | The number of diagonal blocks in the matrix T . $1 \leq nsplit \leq n$. |
| <i>isplit</i> | Array of size n The splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to <i>isplit</i> [0], the second of rows/columns <i>isplit</i> [0]+1 through <i>isplit</i> [1], etc., and the <i>nsplit</i> -th submatrix consists of rows/columns <i>isplit</i> [<i>nsplit</i> -2]+1 through <i>isplit</i> [<i>nsplit</i> -1]= n . (Only the first <i>nsplit</i> elements will actually be used, but since the user cannot know a priori what value <i>nsplit</i> will have, n words must be reserved for <i>isplit</i> .) |
| <i>work</i> | (workspace) Array of size $4*n$ |
| <i>iwork</i> | (workspace) Array of size $3*n$ |
| <i>dol, dou</i> | Specifying an index range <i>dol</i> : <i>dou</i> allows the user to work on only a selected part of the representation tree. Otherwise, the setting <i>dol</i> =1, <i>dou</i> = n should be applied. Note that <i>dol</i> and <i>dou</i> refer to the order in which the eigenvalues are stored in W . |

OUTPUT Parameters

| | |
|-------------|---|
| <i>m</i> | The actual number of eigenvalues found. $0 \leq m \leq n$. (See also the description of <i>info</i> =2,3.) |
| <i>w</i> | Array of size n On exit, the first m elements of <i>w</i> will contain the eigenvalue approximations. ?larrd2 computes an interval $I_j = (a_j, b_j]$ that includes eigenvalue j . The eigenvalue approximation is given as the interval midpoint $w[j-1] = (a_j + b_j)/2$. The corresponding error is bounded by $werr[j-1] = \text{abs}(a_j - b_j)/2$. |
| <i>werr</i> | Array of size n The error bound on the corresponding eigenvalue approximation in <i>w</i> . |

| | |
|---------------|---|
| <i>wl, wu</i> | <p>The interval $(wl, wu]$ contains all the wanted eigenvalues.</p> <p>If <i>range</i>='V', then $wl=v_l$ and $wu=v_u$.</p> <p>If <i>range</i>='A', then <i>wl</i> and <i>wu</i> are the global Gerschgorin bounds on the spectrum.</p> <p>If <i>range</i>='I', then <i>wl</i> and <i>wu</i> are computed by SLAEBZ from the index range specified.</p> |
| <i>iblock</i> | <p>Array of size <i>n</i></p> <p>At each row/column <i>j</i> where $e[j-1]$ is zero or small, the matrix <i>T</i> is considered to split into a block diagonal matrix. On exit, if <i>info</i> = 0, <i>iblock</i>[<i>i</i>] specifies to which block (from 0 to the number of blocks minus one) the eigenvalue $w[i]$ belongs. (<i>?larre2</i> may use the remaining <i>n-m</i> elements as workspace.)</p> |
| <i>indexw</i> | <p>Array of size <i>n</i></p> <p>The indices of the eigenvalues within each block (submatrix); for example, <i>indexw</i>[<i>i</i>] = <i>j</i> and <i>iblock</i>[<i>i</i>] = <i>k</i> imply that the (<i>i</i>+1)-th eigenvalue $w[i]$ is the <i>j</i>-th eigenvalue in block <i>k</i>.</p> |
| <i>info</i> | <p>= 0: successful exit</p> <p>< 0: if <i>info</i> = -<i>i</i>, the <i>i</i>-th argument had an illegal value</p> <p>> 0: some or all of the eigenvalues failed to converge or were not computed:</p> <ul style="list-style-type: none"> • =1 or 3: Bisection failed to converge for some eigenvalues; these eigenvalues are flagged by a negative block number. The effect is that the eigenvalues may not be as accurate as the absolute and relative tolerances. • =2 or 3: <i>range</i>='I' only: Not all of the eigenvalues <i>il:iu</i> were found. • = 4: <i>range</i>='I', and the Gershgorin interval initially used was too small. No eigenvalues were computed. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?larre2

Given a tridiagonal matrix, sets small off-diagonal elements to zero and for each unreduced block, finds base representations and eigenvalues.

Syntax

```
void slarre2(char* range, MKL_INT* n, float* vl, float* vu, MKL_INT* il, MKL_INT* iu,
float* d, float* e, float* e2, float* rtol1, float* rtol2, float* spltol, MKL_INT*
nsplit, MKL_INT* isplit, MKL_INT* m, MKL_INT* dol, MKL_INT* dou, float* w, float*
werr, float* wgap, MKL_INT* iblock, MKL_INT* indexw, float* gers, float* pivmin,
float* work, MKL_INT* iwork, MKL_INT* info);
```

```
void dlarre2(char* range, MKL_INT* n, double* vl, double* vu, MKL_INT* il, MKL_INT*
iu, double* d, double* e, double* e2, double* rtol1, double* rtol2, double* spltol,
MKL_INT* nsplit, MKL_INT* isplit, MKL_INT* m, MKL_INT* dol, MKL_INT* dou, double* w,
double* werr, double* wgap, MKL_INT* iblock, MKL_INT* indexw, double* gers, double*
pivmin, double* work, MKL_INT* iwork, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

To find the desired eigenvalues of a given real symmetric tridiagonal matrix T , `?larre2` sets, via `?larra`, "small" off-diagonal elements to zero. For each block T_i , it finds

- a suitable shift at one end of the block's spectrum,
- the root RRR, $T_i - \sigma_i I = L_i D_i L_i^T$, and
- eigenvalues of each $L_i D_i L_i^T$.

The representations and eigenvalues found are then returned to `?stegr2` to compute the eigenvectors T .

`?larre2` is more suitable for parallel computation than the original LAPACK code for computing the root RRR and its eigenvalues. When computing eigenvalues in parallel and the input tridiagonal matrix splits into blocks, `?larre2` can skip over blocks which contain none of the eigenvalues from `dol` to `dou` for which the processor is responsible. In extreme cases (such as large matrices consisting of many blocks of small size, e.g. 2x2), the gain can be substantial.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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Input Parameters

| | |
|---------------------|---|
| <code>range</code> | <p>= 'A': ("All") all eigenvalues will be found.</p> <p>= 'V': ("Value") all eigenvalues in the half-open interval $(vl, vu]$ will be found.</p> <p>= 'I': ("Index") eigenvalues of the entire matrix with the indices in a given range will be found.</p> |
| <code>n</code> | The order of the matrix. $n > 0$. |
| <code>vl, vu</code> | <p>If <code>range='V'</code>, the lower and upper bounds for the eigenvalues.</p> <p>Eigenvalues less than or equal to <code>vl</code>, or greater than <code>vu</code>, will not be returned. $vl < vu$.</p> |
| <code>il, iu</code> | If <code>range='I'</code> , the indices (in ascending order) of the smallest eigenvalue, to be returned in <code>w[il-1]</code> , and largest eigenvalue, to be returned in <code>w[iu-1]</code> . |

| | |
|---------------------|---|
| | $1 \leq i_l \leq i_u \leq n.$ |
| <i>d</i> | Array of size <i>n</i> The <i>n</i> diagonal elements of the tridiagonal matrix <i>T</i> . |
| <i>e</i> | Array of size <i>n</i> The first (<i>n</i> -1) entries contain the subdiagonal elements of the tridiagonal matrix <i>T</i> ; <i>e</i> [<i>n</i> -1] need not be set. |
| <i>e2</i> | Array of size <i>n</i> The first (<i>n</i> -1) entries contain the squares of the subdiagonal elements of the tridiagonal matrix <i>T</i> ; <i>e2</i> [<i>n</i> -1] need not be set. |
| <i>rtol1, rtol2</i> | Parameters for bisection. An interval [<i>left, right</i>] has converged if $right-left < \max(rtol1*gap, rtol2*\max(left , right))$ |
| <i>spltol</i> | The threshold for splitting. |
| <i>dol, dou</i> | Specifying an index range <i>dol:dou</i> allows the user to work on only a selected part of the representation tree. Otherwise, the setting <i>dol</i> =1, <i>dou</i> = <i>n</i> should be applied. Note that <i>dol</i> and <i>dou</i> refer to the order in which the eigenvalues are stored in <i>w</i> . |
| <i>work</i> | Workspace array of size $6*n$ |
| <i>iwork</i> | Workspace array of size $5*n$ |

OUTPUT Parameters

| | |
|---------------|---|
| <i>vl, vu</i> | If <i>range</i> ='I' or 'A', <i>vl</i> and <i>vu</i> contain bounds on the desired part of the spectrum. |
| <i>d</i> | The <i>n</i> diagonal elements of the diagonal matrices <i>D_i</i> . |
| <i>e</i> | <i>e</i> contains the subdiagonal elements of the unit bidiagonal matrices <i>L_i</i> . The entries <i>e</i> [<i>isplit</i> [<i>i</i>]], $0 \leq i < nsplit$, contain the base points σ_{i+1} on output. |
| <i>e2</i> | The entries <i>e2</i> [<i>isplit</i> [<i>i</i>]], $0 \leq i < nsplit$, are set to zero. |
| <i>nsplit</i> | The number of blocks <i>T</i> splits into. $1 \leq nsplit \leq n$. |
| <i>isplit</i> | Array of size <i>n</i> The splitting points, at which <i>T</i> breaks up into blocks. The first block consists of rows/columns 1 to <i>isplit</i> [0], the second of rows/columns <i>isplit</i> [0]+1 through <i>isplit</i> [1], etc., and the <i>nsplit</i> -th block consists of rows/columns <i>isplit</i> [<i>nsplit</i> -2]+1 through <i>isplit</i> [<i>nsplit</i> -1]= <i>n</i> . |
| <i>m</i> | The total number of eigenvalues (of all $L_i D_i L_i^T$) found. |

| | |
|---------------|---|
| <i>w</i> | <p>Array of size n</p> <p>The first m elements contain the eigenvalues. The eigenvalues of each of the blocks, $L_i D_i L_i^T$, are sorted in ascending order (<code>?larre2</code> may use the remaining $n-m$ elements as workspace).</p> <p>Note that immediately after exiting this function, only the eigenvalues in w with indices in range $dol-1:dou-1$ might rely on this processor when the eigenvalue computation is done in parallel.</p> |
| <i>werr</i> | <p>Array of size n</p> <p>The error bound on the corresponding eigenvalue in w.</p> <p>Note that immediately after exiting this function, only the uncertainties in $werr$ with indices in range $dol-1:dou-1$ might rely on this processor when the eigenvalue computation is done in parallel.</p> |
| <i>wgap</i> | <p>Array of size n</p> <p>The separation from the right neighbor eigenvalue in w.</p> <p>The gap is only with respect to the eigenvalues of the same block as each block has its own representation tree.</p> <p>Exception: at the right end of a block we store the left gap</p> <p>Note that immediately after exiting this function, only the gaps in $wgap$ with indices in range $dol-1:dou-1$ might rely on this processor when the eigenvalue computation is done in parallel.</p> |
| <i>iblock</i> | <p>Array of size n</p> <p>The indices of the blocks (submatrices) associated with the corresponding eigenvalues in w; $iblock[i]=1$ if eigenvalue $w[i]$ belongs to the first block from the top, $iblock[i]=2$ if $w[i]$ belongs to the second block, and so on.</p> |
| <i>indexw</i> | <p>Array of size n</p> <p>The indices of the eigenvalues within each block (submatrix); for example, $indexw[i]=10$ and $iblock[i]=2$ imply that the $(i+1)$-th eigenvalue $w[i]$ is the 10th eigenvalue in block 2.</p> |
| <i>gers</i> | <p>Array of size $2*n$</p> <p>The n Gerschgorin intervals (the i-th Gerschgorin interval is $(gers[2*i-2], gers[2*i-1])$).</p> |
| <i>pivmin</i> | <p>The minimum pivot in the sturm sequence for T.</p> |
| <i>info</i> | <p>= 0: successful exit</p> <p>> 0: A problem occurred in <code>?larre2</code>.</p> <p>< 0: One of the called functions signaled an internal problem.</p> <p>Needs inspection of the corresponding parameter <i>info</i> for further information.</p> <p>=-1: Problem in <code>?larrd</code>.</p> <p>=-2: Not enough internal iterations to find the base representation.</p> |

=-3: Problem in ?larrb when computing the refined root representation for ?lasq2.

=-4: Problem in ?larrb when performing bisection on the desired part of the spectrum.

=-5: Problem in ?lasq2

=-6: Problem in ?lasq2

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?larre2a

Given a tridiagonal matrix, sets small off-diagonal elements to zero and for each unreduced block, finds base representations and eigenvalues.

Syntax

```
void slarre2a(char* range, MKL_INT* n, float* vl, float* vu, MKL_INT* il, MKL_INT* iu,
float* d, float* e, float* e2, float* rtol1, float* rtol2, float* spltol, MKL_INT*
nsplit, MKL_INT* isplit, MKL_INT* m, MKL_INT* dol, MKL_INT* dou, MKL_INT* needil,
MKL_INT* neediu, float* w, float* werr, float* wgap, MKL_INT* iblock, MKL_INT* indexw,
float* gers, float* sdiam, float* pivmin, float* work, MKL_INT* iwork, float* minrgp,
MKL_INT* info);
```

```
void dlarre2a(char* range, MKL_INT* n, double* vl, double* vu, MKL_INT* il, MKL_INT*
iu, double* d, double* e, double* e2, double* rtol1, double* rtol2, double* spltol,
MKL_INT* nsplit, MKL_INT* isplit, MKL_INT* m, MKL_INT* dol, MKL_INT* dou, MKL_INT*
needil, MKL_INT* neediu, double* w, double* werr, double* wgap, MKL_INT* iblock,
MKL_INT* indexw, double* gers, double* sdiam, double* pivmin, double* work, MKL_INT*
iwork, double* minrgp, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

To find the desired eigenvalues of a given real symmetric tridiagonal matrix T , ?larre2a sets any "small" off-diagonal elements to zero, and for each unreduced block T_i , it finds

- a suitable shift at one end of the block's spectrum,
- the base representation, $T_i - \sigma_i I = L_i D_i L_i^T$, and
- eigenvalues of each $L_i D_i L_i^T$.

NOTE

The algorithm obtains a crude picture of all the wanted eigenvalues (as selected by *range*). However, to reduce work and improve scalability, only the eigenvalues *dol* to *dou* are refined. Furthermore, if the matrix splits into blocks, RRRs for blocks that do not contain eigenvalues from *dol* to *dou* are skipped. The DQDS algorithm (function ?lasq2) is not used, unlike in the sequential case. Instead, eigenvalues are computed in parallel to some figures using bisection.

Optimization Notice

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Input Parameters

| | |
|---------------------|--|
| <i>range</i> | <p>= 'A': ("All") all eigenvalues will be found.</p> <p>= 'V': ("Value") all eigenvalues in the half-open interval $(v_l, v_u]$ will be found.</p> <p>= 'I': ("Index") eigenvalues of the entire matrix with the indices in a given range will be found.</p> |
| <i>n</i> | The order of the matrix. $n > 0$. |
| <i>v_l, v_u</i> | <p>If <i>range</i>='V', the lower and upper bounds for the eigenvalues. Eigenvalues less than or equal to v_l, or greater than v_u, will not be returned. $v_l < v_u$.</p> <p>If <i>range</i>='I' or 'A', <code>?larre2a</code> computes bounds on the desired part of the spectrum.</p> |
| <i>il, iu</i> | <p>If <i>range</i>='I', the indices (in ascending order) of the smallest eigenvalue, to be returned in $w[il-1]$, and largest eigenvalue, to be returned in $w[iu-1]$.</p> <p>$1 \leq il \leq iu \leq n$.</p> |
| <i>d</i> | <p>Array of size n</p> <p>On entry, the n diagonal elements of the tridiagonal matrix T.</p> |
| <i>e</i> | <p>Array of size n</p> <p>The first $(n-1)$ entries contain the subdiagonal elements of the tridiagonal matrix T; $e[n-1]$ need not be set.</p> |
| <i>e2</i> | <p>Array of size n</p> <p>The first $(n-1)$ entries contain the squares of the subdiagonal elements of the tridiagonal matrix T; $e2[n-1]$ need not be set.</p> |
| <i>rtol1, rtol2</i> | <p>Parameters for bisection.</p> <p>An interval $[left, right]$ has converged if $right - left < \max(rtol1*gap, rtol2*\max(left , right))$</p> |
| <i>spltol</i> | The threshold for splitting. |
| <i>dol, dou</i> | <p>If the user wants to work on only a selected part of the representation tree, he can specify an index range $dol:dou$.</p> <p>Otherwise, the setting $dol=1, dou=n$ should be applied.</p> |

Note that *dol* and *dou* refer to the order in which the eigenvalues are stored in *w*.

| | |
|---------------|--|
| <i>work</i> | Workspace array of size $6*n$ |
| <i>iwork</i> | Workspace array of size $5*n$ |
| <i>minrgp</i> | The minimum relative gap threshold to decide whether an eigenvalue or a cluster boundary is reached. |

OUTPUT Parameters

| | |
|-----------------------|--|
| <i>vl, vu</i> | <p>If <i>range</i>='V', the lower and upper bounds for the eigenvalues. Eigenvalues less than or equal to <i>vl</i>, or greater than <i>vu</i>, are not returned. $vl < vu$.</p> <p>If <i>range</i>='I' or <i>range</i>='A', <i>?larre2a</i> computes bounds on the desired part of the spectrum.</p> |
| <i>d</i> | The n diagonal elements of the diagonal matrices D_j . |
| <i>e</i> | <i>e</i> contains the subdiagonal elements of the unit bidiagonal matrices L_j . The entries $e[isplit[i]]$, $0 \leq i < nsplit$, contain the base points σ_{i+1} on output. |
| <i>e2</i> | The entries $e2[isplit[i]]$, $0 \leq i < nsplit$ have been set to zero. |
| <i>nsplit</i> | The number of blocks <i>T</i> splits into. $1 \leq nsplit \leq n$. |
| <i>isplit</i> | <p>Array of size n</p> <p>The splitting points, at which <i>T</i> breaks up into blocks.</p> <p>The first block consists of rows/columns 1 to $isplit[0]$, the second of rows/columns $isplit[0]+1$ through $isplit[1]$, etc., and the <i>nsplit</i>-th block consists of rows/columns $isplit[nsplit-2]+1$ through $isplit[nsplit-1]=n$.</p> |
| <i>m</i> | The total number of eigenvalues (of all $L_j D_j L_j^T$) found. |
| <i>needil, neediu</i> | The indices of the leftmost and rightmost eigenvalues of the root node RRR which are needed to accurately compute the relevant part of the representation tree. |
| <i>w</i> | <p>Array of size n</p> <p>The first m elements contain the eigenvalues. The eigenvalues of each of the blocks, $L_j D_j L_j^T$, are sorted in ascending order (<i>?larre2a</i> may use the remaining $n-m$ elements as workspace).</p> <p>Note that immediately after exiting this function, only the eigenvalues in <i>w</i> with indices in range $dol-1:dou-1$ rely on this processor because the eigenvalue computation is done in parallel.</p> |
| <i>werr</i> | <p>Array of size n</p> <p>The error bound on the corresponding eigenvalue in <i>w</i>.</p> |

Note that immediately after exiting this function, only the uncertainties in w_{err} with indices in range $dol-1:dou-1$ are reliable on this processor because the eigenvalue computation is done in parallel.

wgap

Array of size n

The separation from the right neighbor eigenvalue in w . The gap is only with respect to the eigenvalues of the same block as each block has its own representation tree.

Exception: at the right end of a block we store the left gap

Note that immediately after exiting this function, only the gaps in $wgap$ with indices in range $dol-1:dou-1$ are reliable on this processor because the eigenvalue computation is done in parallel.

iblock

Array of size n

The indices of the blocks (submatrices) associated with the corresponding eigenvalues in w ; $iblock[i]=1$ if eigenvalue $w[i]$ belongs to the first block from the top, $iblock[i]=2$ if $w[i]$ belongs to the second block, and so on.

indexw

Array of size n

The indices of the eigenvalues within each block (submatrix); for example, $indexw[i]=10$ and $iblock[i]=2$ imply that the $(i+1)$ -th eigenvalue $w[i]$ is the 10th eigenvalue in block 2.

gers

Array of size $2*n$

The n Gerschgorin intervals (the i -th Gerschgorin interval is $(gers[2*i-2], gers[2*i-1])$).

pivmin

The minimum pivot in the sturm sequence for T .

info

= 0: successful exit

> 0: A problem occurred in `?larre2a`.

< 0: One of the called functions signaled an internal problem. Needs inspection of the corresponding parameter *info* for further information.

=-1: Problem in `?larrd2`.

=-2: Not enough internal iterations to find base representation.

=-3: Problem in `?larrb2` when computing the refined root representation.

=-4: Problem in `?larrb2` when performing bisection on the desired part of the spectrum.

= -9 Problem: $m < dou-dol+1$, that is the code found fewer eigenvalues than it was supposed to.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?larrf2

Finds a new relatively robust representation such that at least one of the eigenvalues is relatively isolated.

Syntax

```
void slarrf2(MKL_INT* n, float* d, float* l, float* ld, MKL_INT* clstrt, MKL_INT*
clend, MKL_INT* clmid1, MKL_INT* clmid2, float* w, float* wgap, float* werr, MKL_INT*
trymid, float* spdiam, float* clgapl, float* clgapr, float* pivmin, float* sigma,
float* dplus, float* lplus, float* work, MKL_INT* info);
```

```
void dlarrf2(MKL_INT* n, double* d, double* l, double* ld, MKL_INT* clstrt, MKL_INT*
clend, MKL_INT* clmid1, MKL_INT* clmid2, double* w, double* wgap, double* werr,
MKL_INT* trymid, double* spdiam, double* clgapl, double* clgapr, double* pivmin,
double* sigma, double* dplus, double* lplus, double* work, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

Given the initial representation LDL^T and its cluster of close eigenvalues (in a relative measure), defined by the indices of the first and last eigenvalues in the cluster, `?larrf2` finds a new relatively robust representation $LDL^T - \sigma I = L_+ D_+ L_+^T$ such that at least one of the eigenvalues of $L_+ D_+ L_+^T$ is relatively isolated.

This is an enhanced version of `?larrf` that also tries shifts in the middle of the cluster, should there be a large gap, in order to break large clusters into at least two pieces.

Input Parameters

| | |
|-----------------------|--|
| <i>n</i> | The order of the matrix (subblock, if the matrix was split). |
| <i>d</i> | Array of size <i>n</i> The <i>n</i> diagonal elements of the diagonal matrix <i>D</i> . |
| <i>l</i> | Array of size <i>n</i> -1 The (<i>n</i> -1) subdiagonal elements of the unit bidiagonal matrix <i>L</i> . |
| <i>ld</i> | Array of size <i>n</i> -1 The (<i>n</i> -1) elements $l[i]*d[i]$. |
| <i>clstrt</i> | The index of the first eigenvalue in the cluster. |
| <i>clend</i> | The index of the last eigenvalue in the cluster. |
| <i>clmid1, clmid2</i> | The index of a middle eigenvalue pair with large gap. |
| <i>w</i> | Array of size $\geq (clend-clstrt+1)$ The eigenvalue approximations of LDL^T in ascending order. $w[clstrt - 1]$ through $w[clend - 1]$ form the cluster of relatively close eigenvalues. |
| <i>wgap</i> | Array of size $\geq (clend-clstrt+1)$ The separation from the right neighbor eigenvalue in <i>w</i> . |
| <i>werr</i> | Array of size $\geq (clend-clstrt+1)$ <i>werr</i> contains the semiwidth of the uncertainty interval of the corresponding eigenvalue approximation in <i>w</i> . |

| | |
|-----------------------|---|
| <i>spdiam</i> | Estimate of the spectral diameter obtained from the Gerschgorin intervals |
| <i>clgapl, clgapr</i> | Absolute gap on each end of the cluster. Set by the calling function to protect against shifts too close to eigenvalues outside the cluster. |
| <i>pivmin</i> | The minimum pivot allowed in the Sturm sequence. |
| <i>work</i> | Workspace array of size $2*n$ |

OUTPUT Parameters

| | |
|--------------|--|
| <i>wgap</i> | Contains refined values of its input approximations. Very small gaps are unchanged. |
| <i>sigma</i> | The shift (σ) used to form $L_+D_+L_+^T$. |
| <i>dplus</i> | Array of size n The n diagonal elements of the diagonal matrix D_+ . |
| <i>lplus</i> | Array of size $n-1$ The first $(n-1)$ elements of <i>lplus</i> contain the subdiagonal elements of the unit bidiagonal matrix L_+ . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?larrv2

*Computes the eigenvectors of the tridiagonal matrix $T = L*D*L^T$ given L, D and the eigenvalues of $L*D*L^T$.*

Syntax

```
void slarrv2(MKL_INT* n, float* vl, float* vu, float* d, float* l, float* pivmin,
MKL_INT* isplit, MKL_INT* m, MKL_INT* dol, MKL_INT* dou, MKL_INT* needil, MKL_INT*
neediu, float* minrgp, float* rtol1, float* rtol2, float* w, float* werr, float* wgap,
MKL_INT* iblock, MKL_INT* indexw, float* gers, float* sdiam, float* z, MKL_INT* ldz,
MKL_INT* isuppz, float* work, MKL_INT* iwork, MKL_INT* vstart, MKL_INT* finish,
MKL_INT* maxcls, MKL_INT* ndepth, MKL_INT* parity, MKL_INT* zoffset, MKL_INT* info);

void dlarrv2(MKL_INT* n, double* vl, double* vu, double* d, double* l, double* pivmin,
MKL_INT* isplit, MKL_INT* m, MKL_INT* dol, MKL_INT* dou, MKL_INT* needil, MKL_INT*
neediu, double* minrgp, double* rtol1, double* rtol2, double* w, double* werr, double*
wgap, MKL_INT* iblock, MKL_INT* indexw, double* gers, double* sdiam, double* z,
MKL_INT* ldz, MKL_INT* isuppz, double* work, MKL_INT* iwork, MKL_INT* vstart, MKL_INT*
finish, MKL_INT* maxcls, MKL_INT* ndepth, MKL_INT* parity, MKL_INT* zoffset, MKL_INT*
info);
```

Include Files

- `mkl_scalapack.h`

Description

?larrv2 computes the eigenvectors of the tridiagonal matrix $T = LDL^T$ given L , D and approximations to the eigenvalues of LDL^T . The input eigenvalues should have been computed by [larre2a](#) or by previous calls to ?larrv2.

The major difference between the parallel and the sequential construction of the representation tree is that in the parallel case, not all eigenvalues of a given cluster might be computed locally. Other processors might "own" and refine part of an eigenvalue cluster. This is crucial for scalability. Thus there might be communication necessary before the current level of the representation tree can be parsed.

Please note:

- The calling sequence has two additional integer parameters, dol and dou , that should satisfy $m \geq dou \geq dol \geq 1$. These parameters are only relevant when both eigenvalues and eigenvectors are computed (`stegr2b` parameter `jobz = 'V'`). ?larrv2 only computes the eigenvectors corresponding to eigenvalues dol through dou in w . (That is, instead of computing the eigenvectors belonging to $w[0]$ through $w[m-1]$, only the eigenvectors belonging to eigenvalues $w[dol - 1]$ through $w[dou - 1]$ are computed. In this case, only the eigenvalues $dol:dou$ are guaranteed to be accurately refined to all figures by Rayleigh-Quotient iteration.
- The additional arguments `vstart`, `finish`, `ndepth`, `parity`, `zoffset` are included as a thread-safe implementation equivalent to save variables. These variables store details about the local representation tree which is computed layerwise. For scalability reasons, eigenvalues belonging to the locally relevant representation tree might be computed on other processors. These need to be communicated before the inspection of the RRRs can proceed on any given layer. Note that only when the variable `finish` is non-zero, the computation has ended. All eigenpairs between dol and dou have been computed. m is set to $dou - dol + 1$.
- ?larrv2 needs more workspace in z than the sequential `slarrv`. It is used to store the conformal embedding of the local representation tree.

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Input Parameters

| | |
|----------|--|
| n | The order of the matrix. $n \geq 0$. |
| vl, vu | Lower and upper bounds of the interval that contains the desired eigenvalues. $vl < vu$. Needed to compute gaps on the left or right end of the extremal eigenvalues in the desired range. vu is currently not used but kept as parameter in case needed. |
| d | Array of size n The n diagonal elements of the diagonal matrix d . On exit, d is overwritten. |
| l | Array of size n |

The $(n-1)$ subdiagonal elements of the unit bidiagonal matrix L are in elements 0 to $n-2$ of l (if the matrix is not split.) At the end of each block is stored the corresponding shift as given by `?larre`. On exit, l is overwritten.

| | |
|-----------------------------|---|
| <code>pivmin</code> | The minimum pivot allowed in the sturm sequence. |
| <code>isplit</code> | <p>Array of size n</p> <p>The splitting points, at which the matrix T breaks up into blocks. The first block consists of rows/columns 1 to <code>isplit[0]</code>, the second of rows/columns <code>isplit[0] + 1</code> through <code>isplit[1]</code>, etc.</p> |
| <code>m</code> | The total number of input eigenvalues. $0 \leq m \leq n$. |
| <code>dol, dou</code> | <p>If you want to compute only selected eigenvectors from all the eigenvalues supplied, you can specify an index range <code>dol:dou</code>. Or else the setting <code>dol=1, dou=m</code> should be applied. Note that <code>dol</code> and <code>dou</code> refer to the order in which the eigenvalues are stored in w. If you want to compute only selected eigenpairs, the columns <code>dol-1</code> to <code>dou+1</code> of the eigenvector space Z contain the computed eigenvectors. All other columns of Z are set to zero.</p> <p>If <code>dol > 1</code>, then <code>Z(:,dol-1-zoffset)</code> is used.</p> <p>If <code>dou < m</code>, then <code>Z(:,dou+1-zoffset)</code> is used.</p> |
| <code>needil, neediu</code> | <p>Describe which are the left and right outermost eigenvalues that still need to be included in the computation. These indices indicate whether eigenvalues from other processors are needed to correctly compute the conformally embedded representation tree.</p> <p>When <code>dol ≤ needil ≤ neediu ≤ dou</code>, all required eigenvalues are local to the processor and no communication is required to compute its part of the representation tree.</p> |
| <code>minrgp</code> | The minimum relative gap threshold to decide whether an eigenvalue or a cluster boundary is reached. |
| <code>rtol1, rtol2</code> | Parameters for bisection. An interval <code>[left,right]</code> has converged if <code>right-left < max(rtol1*gap, rtol2*max(left , right))</code> |
| <code>w</code> | <p>Array of size n</p> <p>The first m elements of w contain the approximate eigenvalues for which eigenvectors are to be computed. The eigenvalues should be grouped by split-off block and ordered from smallest to largest within the block. (The output array w from <code>?stegr2a</code> is expected here.) Furthermore, they are with respect to the shift of the corresponding root representation for their block.</p> |
| <code>werr</code> | <p>Array of size n</p> <p>The first m elements contain the semiwidth of the uncertainty interval of the corresponding eigenvalue in w.</p> |
| <code>wgap</code> | Array of size n |

| | |
|----------------|---|
| | The separation from the right neighbor eigenvalue in w . |
| <i>iblock</i> | Array of size n The indices of the blocks (submatrices) associated with the corresponding eigenvalues in w ; <i>iblock</i> [i]=1 if eigenvalue w [i] belongs to the first block from the top, <i>iblock</i> [i]=2 if w [i] belongs to the second block, and so on. |
| <i>indexw</i> | Array of size n The indices of the eigenvalues within each block (submatrix). For example: <i>indexw</i> [i]= 10 and <i>iblock</i> [i]=2 imply that the ($i+1$)-th eigenvalue w [i] is the 10th eigenvalue in block 2. |
| <i>gers</i> | Array of size $2*n$ The n Gerschgorin intervals (the i -th Gerschgorin interval is (<i>gers</i> [$2*i-2$], <i>gers</i> [$2*i-1$])). The Gerschgorin intervals should be computed from the original unshifted matrix. Not used but kept as parameter for possible future use. |
| <i>sdiam</i> | Array of size n The spectral diameters for all unreduced blocks. |
| <i>ldz</i> | The leading dimension of the array z . $ldz \geq 1$, and if <i>stegr2b</i> parameter <i>jobz</i> = 'V', $ldz \geq \max(1, n)$. |
| <i>work</i> | (workspace) array of size $12*n$ |
| <i>iwork</i> | (workspace) Array of size $7*n$ |
| <i>vstart</i> | Non-zero on initialization, set to zero afterwards. |
| <i>finish</i> | A flag that indicates whether all eigenpairs have been computed. |
| <i>maxcls</i> | The largest cluster worked on by this processor in the representation tree. |
| <i>ndepth</i> | The current depth of the representation tree. Set to zero on initial pass, changed when the deeper levels of the representation tree are generated. |
| <i>parity</i> | An internal parameter needed for the storage of the clusters on the current level of the representation tree. |
| <i>zoffset</i> | Offset for storing the eigenpairs when z is distributed in 1D-cyclic fashion. |

OUTPUT Parameters

| | |
|-----------------------|---|
| <i>needil, neediu</i> | |
| w | Unshifted eigenvalues for which eigenvectors have already been computed. |
| <i>werr</i> | Contains refined values of its input approximations. |
| <i>wgap</i> | Contains refined values of its input approximations. Very small gaps are changed. |
| z | Array of size $ldz * \max(1, m)$ |

If *info* = 0, the first *m* columns of the matrix *Z*, stored in the array *z*, contain the orthonormal eigenvectors of the matrix *T* corresponding to the input eigenvalues, with the *i*-th column of *Z* holding the eigenvector associated with *w*[*i* - 1].

In the distributed version, only a subset of columns is accessed, see *dol*, *dou*, and *zoffset*.

isuppz

Array of size $2 * \max(1, m)$

The support of the eigenvectors in *z*, i.e., the indices indicating the non-zero elements in *z*. The *i*-th eigenvector is non-zero only in elements *isuppz*[$2 * i - 2$] through *isuppz*[$2 * i - 1$].

vstart

Non-zero on initialization, set to zero afterwards.

finish

A flag that indicates whether all eigenpairs have been computed.

maxcls

The largest cluster worked on by this processor in the representation tree.

ndepth

The current depth of the representation tree. Set to zero on initial pass, changed when the deeper levels of the representation tree are generated.

parity

An internal parameter needed for the storage of the clusters on the current level of the representation tree.

info

= 0: successful exit

> 0: A problem occurred in ?*larrv2*.

< 0: One of the called functions signaled an internal problem.

Needs inspection of the corresponding parameter *info* for further information.

== -1: Problem in ?*larrb2* when refining a child's eigenvalues.

== -2: Problem in ?*larrf2* when computing the RRR of a child. When a child is inside a tight cluster, it can be difficult to find an RRR. A partial remedy from the user's point of view is to make the parameter *minrgp* smaller and recompile. However, as the orthogonality of the computed vectors is proportional to $1 / \text{minrgp}$, be aware that decreasing *minrgp* might be reduce precision.

== -3: Problem in ?*larrb2* when refining a single eigenvalue after the Rayleigh correction was rejected.

= 5: The Rayleigh Quotient Iteration failed to converge to full accuracy.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?*lasorte*

Sorts eigenpairs by real and complex data types.

Syntax

```
void slasorte (float *s , MKL_INT *lds , MKL_INT *j , float *out , MKL_INT *info );
```

```
void dlasorte (double *s , MKL_INT *lds , MKL_INT *j , double *out , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `?lasorte` function sorts eigenpairs so that real eigenpairs are together and complex eigenpairs are together. This helps to employ 2x2 shifts easily since every second subdiagonal is guaranteed to be zero. This function does no parallel work and makes no calls.

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Input Parameters

| | |
|-------------|---|
| <i>s</i> | (local) Array of size <i>lds</i> . On entry, a matrix already in Schur form. |
| <i>lds</i> | (local) On entry, the leading dimension of the array <i>s</i> ; unchanged on exit. |
| <i>j</i> | (local) On entry, the order of the matrix <i>S</i> ; unchanged on exit. |
| <i>out</i> | (local) Array of size $2*j$. The work buffer required by the function. |
| <i>info</i> | (local) Set, if the input matrix had an odd number of real eigenvalues and things could not be paired or if the input matrix <i>S</i> was not originally in Schur form. 0 indicates successful completion. |

Output Parameters

| | |
|------------|---|
| <i>s</i> | On exit, the diagonal blocks of <i>S</i> have been rewritten to pair the eigenvalues. The resulting matrix is no longer similar to the input. |
| <i>out</i> | Work buffer. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?lasrt2

Sorts numbers in increasing or decreasing order.

Syntax

```
void slasrt2 (char *id , MKL_INT *n , float *d , MKL_INT *key , MKL_INT *info );
void dlasrt2 (char *id , MKL_INT *n , double *d , MKL_INT *key , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The ?lasrt2 function is modified LAPACK function ?lasrt, which sorts the numbers in *d* in increasing order (if *id* = 'I') or in decreasing order (if *id* = 'D'). It uses Quick Sort, reverting to Insertion Sort on arrays of size ≤ 20 . The size of *STACK* limits *n* to about 2^{32} .

Input Parameters

| | |
|------------|---|
| <i>id</i> | = 'I': sort <i>d</i> in increasing order; = 'D': sort <i>d</i> in decreasing order. |
| <i>n</i> | The length of the array <i>d</i> . |
| <i>d</i> | Array of size <i>n</i> . On entry, the array to be sorted. |
| <i>key</i> | Array of size <i>n</i> . On entry, <i>key</i> contains a key to each of the entries in <i>d</i> . Typically, $key[i] = i+1$ for all $i = 0, \dots, n-1$. |

Output Parameters

| | |
|-------------|--|
| <i>d</i> | On exit, <i>d</i> has been sorted into increasing order ($d[0] \leq \dots \leq d[n - 1]$) or into decreasing order ($d[0] \geq \dots \geq d[n - 1]$), depending on <i>id</i> . |
| <i>info</i> | = 0: successful exit < 0: if <i>info</i> = - <i>i</i> , the <i>i</i> -th argument had an illegal value. |
| <i>key</i> | On exit, <i>key</i> is permuted in exactly the same manner as <i>d</i> was permuted from input to output. Therefore, if $key[i] = i+1$ for all $i = 0, \dots, n-1$ on input, $d[i]$ on output equals $d[key[i]-1]$ on input. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?stegr2

Computes selected eigenvalues and eigenvectors of a real symmetric tridiagonal matrix.

Syntax

```
void sstegr2(char* jobz, char* range, MKL_INT* n, float* d, float* e, float* vl,
float* vu, MKL_INT* il, MKL_INT* iu, MKL_INT* m, float* w, float* z, MKL_INT* ldz,
MKL_INT* nzc, MKL_INT* isuppz, float* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT*
liwork, MKL_INT* dol, MKL_INT* dou, MKL_INT* zoffset, MKL_INT* info);
```

```
void dstegr2(char* jobz, char* range, MKL_INT* n, double* d, double* e, double* vl,
double* vu, MKL_INT* il, MKL_INT* iu, MKL_INT* m, double* w, double* z, MKL_INT* ldz,
MKL_INT* nzc, MKL_INT* isuppz, double* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT*
liwork, MKL_INT* dol, MKL_INT* dou, MKL_INT* zoffset, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

?stegr2 computes selected eigenvalues and, optionally, eigenvectors of a real symmetric tridiagonal matrix T . It is invoked in the ScaLAPACK MRRR driver `p?syevr` and the corresponding Hermitian version either when only eigenvalues are to be computed, or when only a single processor is used (the sequential-like case).

?stegr2 has been adapted from LAPACK's ?stegr. Please note the following crucial changes.

1. The calling sequence has two additional integer parameters, dol and dou , that should satisfy $m \geq dou \geq dol \geq 1$. ?stegr2 only computes the eigenpairs corresponding to eigenvalues dol through dou in w , indexed $dol-1$ through $dou-1$. (That is, instead of computing the eigenpairs belonging to $w[0]$ through $w[m-1]$, only the eigenvectors belonging to eigenvalues $w[dol-1]$ through $w[dou-1]$ are computed. In this case, only the eigenvalues dol through dou are guaranteed to be fully accurate.
2. m is *not* the number of eigenvalues specified by $range$, but is $m = dou - dol + 1$. This concerns the case where only eigenvalues are computed, but on more than one processor. Thus, in this case m refers to the number of eigenvalues computed on this processor.
3. The arrays w and z might not contain all the wanted eigenpairs locally, instead this information is distributed over other processors.

Optimization Notice

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Input Parameters

- $jobz$ = 'N': Compute eigenvalues only;
= 'V': Compute eigenvalues and eigenvectors.

| | |
|---------------|--|
| <i>range</i> | <p>= 'A': all eigenvalues will be found.</p> <p>= 'V': all eigenvalues in the half-open interval $(vl, vu]$ will be found.</p> <p>= 'I': eigenvalues of the entire matrix with the indices in a given range will be found.</p> |
| <i>n</i> | The order of the matrix. $n \geq 0$. |
| <i>d</i> | <p>Array of size n</p> <p>On entry, the n diagonal elements of the tridiagonal matrix T. Overwritten on exit.</p> |
| <i>e</i> | <p>Array of size n</p> <p>On entry, the $(n-1)$ subdiagonal elements of the tridiagonal matrix T in elements 0 to $n-2$ of e. $e[n-1]$ need not be set on input, but is used internally as workspace. Overwritten on exit.</p> |
| <i>vl</i> | |
| <i>vu</i> | <p>If $range='V'$, the lower and upper bounds of the interval to be searched for eigenvalues. $vl < vu$.</p> <p>Not referenced if $range = 'A'$ or 'I'.</p> |
| <i>il, iu</i> | <p>If $range='I'$, the indices (in ascending order) of the smallest eigenvalue, to be returned in $w[il-1]$, and largest eigenvalue, to be returned in $w[iu-1]$.</p> <p>$1 \leq il \leq iu \leq n$, if $n > 0$.</p> <p>Not referenced if $range = 'A'$ or 'V'.</p> |
| <i>ldz</i> | The leading dimension of the array z . $ldz \geq 1$, and if $jobz = 'V'$, then $ldz \geq \max(1, n)$. |
| <i>nzc</i> | <p>The number of eigenvectors to be held in the array z, storing the matrix Z.</p> <p>If $range = 'A'$, then $nzc \geq \max(1, n)$.</p> <p>If $range = 'V'$, then $nzc \geq$ the number of eigenvalues in $(vl, vu]$.</p> <p>If $range = 'I'$, then $nzc \geq iu - il + 1$.</p> <p>If $nzc = -1$, then a workspace query is assumed; the function calculates the number of columns of the matrix Z that are needed to hold the eigenvectors. This value is returned as the first entry of the z array, and no error message related to nzc is issued.</p> |
| <i>lwork</i> | <p>The size of the array $work$. $lwork \geq \max(1, 18*n)$</p> <p>if $jobz = 'V'$, and $lwork \geq \max(1, 12*n)$ if $jobz = 'N'$. If $lwork = -1$, then a workspace query is assumed; the function only calculates the optimal size of the $work$ array, returns this value as the first entry of the $work$ array, and no error message related to $lwork$ is issued.</p> |
| <i>liwork</i> | The size of the array $iwork$. $liwork \geq \max(1, 10*n)$ if the eigenvectors are desired, and $liwork \geq \max(1, 8*n)$ if only the eigenvalues are to be computed. |

If $liwork = -1$, then a workspace query is assumed; the function only calculates the optimal size of the $iwork$ array, returns this value as the first entry of the $iwork$ array, and no error message related to $liwork$ is issued.

dol, dou

From the eigenvalues $w[0]$ through $w[m-1]$, only eigenvectors $Z(:,dol)$ to $Z(:,dou)$ are computed.

If $dol > 1$, then $Z(:,dol-1-zoffset)$ is used and overwritten.

If $dou < m$, then $Z(:,dou+1-zoffset)$ is used and overwritten.

$zoffset$

Offset for storing the eigenpairs when z is distributed in 1D-cyclic fashion

OUTPUT Parameters

m

Globally summed over all processors, m equals the total number of eigenvalues found. $0 \leq m \leq n$. If $range = 'A'$, $m = n$, and if $range = 'I'$, $m = iu - il + 1$. The local output equals $m = dou - dol + 1$.

w

Array of size n

The first m elements contain the selected eigenvalues in ascending order. Note that immediately after exiting this function, only the eigenvalues indexed $dol-1$ through $dou-1$ are reliable on this processor because the eigenvalue computation is done in parallel. Other processors will hold reliable information on other parts of the w array. This information is communicated in the ScaLAPACK driver.

z

Array of size $ldz * \max(1,m)$.

If $jobz = 'V'$, and if $info = 0$, then the first m columns of the matrix Z stored in z contain some of the orthonormal eigenvectors of the matrix T corresponding to the selected eigenvalues, with the i -th column of Z holding the eigenvector associated with $w[i-1]$.

If $jobz = 'N'$, then z is not referenced.

Note: the user must ensure that at least $\max(1,m)$ columns of the matrix are supplied in the array z ; if $range = 'V'$, the exact value of m is not known in advance and can be computed with a workspace query by setting $nzc = -1$, see below.

$isuppz$

array of size $2 * \max(1,m)$

The support of the eigenvectors in z , i.e., the indices indicating the nonzero elements in z . The i -th computed eigenvector is nonzero only in elements $isuppz[2*i-2]$ through $isuppz[2*i-1]$. This is relevant in the case when the matrix is split. $isuppz$ is only set if $n > 2$.

$work$

On exit, if $info = 0$, $work[0]$ returns the optimal (and minimal) $lwork$.

$iwork$

On exit, if $info = 0$, $iwork[0]$ returns the optimal $liwork$.

$info$

On exit, $info$

= 0: successful exit

other: if $info = -i$, the i -th argument had an illegal value

if *info* = 10X, internal error in ?larre2,

if *info* = 20X, internal error in ?larrv.

Here, the digit X = ABS(*iinfo*) < 10, where *iinfo* is the nonzero error code returned by ?larre2 or ?larrv, respectively.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?stegr2a

Computes selected eigenvalues and initial representations needed for eigenvector computations.

Syntax

```
void sstegr2a(char* jobz, char* range, MKL_INT* n, float* d, float* e, float* vl,
float* vu, MKL_INT* il, MKL_INT* iu, MKL_INT* m, float* w, float* z, MKL_INT* ldz,
MKL_INT* nzc, float* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT*
dol, MKL_INT* dou, MKL_INT* needil, MKL_INT* neediu, MKL_INT* inderr, MKL_INT* nsplit,
float* pivmin, float* scale, float* wl, float* wu, MKL_INT* info);
```

```
void dstegr2a(char* jobz, char* range, MKL_INT* n, double* d, double* e, double* vl,
double* vu, MKL_INT* il, MKL_INT* iu, MKL_INT* m, double* w, double* z, MKL_INT* ldz,
MKL_INT* nzc, double* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT*
dol, MKL_INT* dou, MKL_INT* needil, MKL_INT* neediu, MKL_INT* inderr, MKL_INT* nsplit,
double* pivmin, double* scale, double* wl, double* wu, MKL_INT* info);
```

Include Files

- mkl_scalapack.h

Description

?stegr2a computes selected eigenvalues and initial representations needed for eigenvector computations in ?stegr2b. It is invoked in the ScaLAPACK MRRR driver p?syevr and the corresponding Hermitian version when both eigenvalues and eigenvectors are computed in parallel on multiple processors. For this case, ?stegr2a implements the first part of the MRRR algorithm, parallel eigenvalue computation and finding the root RRR. At the end of ?stegr2a, other processors might have a part of the spectrum that is needed to continue the computation locally. Once this eigenvalue information has been received by the processor, the computation can then proceed by calling the second part of the parallel MRRR algorithm, ?stegr2b.

Please note:

- The calling sequence has two additional integer parameters, (compared to LAPACK's [stegr](#)), these are *dol* and *dou* and should satisfy $m \geq dou \geq dol \geq 1$. These parameters are only relevant for the case *jobz* = 'V'.

Globally invoked over all processors, ?stegr2a computes all the eigenvalues specified by *range*.

?stegr2a locally only computes the eigenvalues corresponding to eigenvalues *dol* through *dou* in *w*, indexed *dol*-1 through *dou*-1. (That is, instead of computing the eigenvectors belonging to $w[[0]$ through $w[[m-1]$, only the eigenvectors belonging to eigenvalues $w[[dol-1]$ through $w[[dou-1]$ are computed. In this case, only the eigenvalues *dol* through *dou* are guaranteed to be fully accurate.

- *m* is not the number of eigenvalues specified by *range*, but it is $m = dou - dol + 1$. Instead, *m* refers to the number of eigenvalues computed on this processor.
- While no eigenvectors are computed in ?stegr2a itself (this is done later in ?stegr2b), the interface

If $jobz = 'V'$ then, depending on $range$ and $dol, dou, ?stegr2a$ might need more workspace in z than the original $?stegr$. In particular, the arrays w and z might not contain all the wanted eigenpairs locally, instead this information is distributed over other processors.

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Input Parameters

| | |
|----------|---|
| $jobz$ | <p>= 'N': Compute eigenvalues only;</p> <p>= 'V': Compute eigenvalues and eigenvectors.</p> |
| $range$ | <p>= 'A': all eigenvalues will be found.</p> <p>= 'V': all eigenvalues in the half-open interval $(vl, vu]$ will be found.</p> <p>= 'I': eigenvalues of the entire matrix with the indices in a given range will be found.</p> |
| n | The order of the matrix. $n \geq 0$. |
| d | <p>Array of size n</p> <p>The n diagonal elements of the tridiagonal matrix T. Overwritten on exit.</p> |
| e | <p>Array of size n</p> <p>On entry, the $(n-1)$ subdiagonal elements of the tridiagonal matrix T in elements 0 to $n-2$ of e. $e[n-1]$ need not be set on input, but is used internally as workspace. Overwritten on exit.</p> |
| vl, vu | <p>If $range='V'$, the lower and upper bounds of the interval to be searched for eigenvalues. $vl < vu$.</p> <p>Not referenced if $range = 'A'$ or 'I'.</p> |
| il, iu | <p>If $range='I'$, the indices (in ascending order) of the smallest eigenvalue, to be returned in $w[il-1]$, and largest eigenvalue, to be returned in $w[iu-1]$. $1 \leq il \leq iu \leq n$, if $n > 0$.</p> <p>Not referenced if $range = 'A'$ or 'V'.</p> |
| ldz | The leading dimension of the array z . $ldz \geq 1$, and if $jobz = 'V'$, then $ldz \geq \max(1, n)$. |
| nzc | <p>The number of eigenvectors to be held in the array z.</p> <p>If $range = 'A'$, then $nzc \geq \max(1, n)$.</p> <p>If $range = 'V'$, then $nzc \geq$ the number of eigenvalues in $(vl, vu]$.</p> |

If $range = 'I'$, then $nzc \geq iu - il + 1$.

If $nzc = -1$, then a workspace query is assumed; the function calculates the number of columns of the matrix stored in array z that are needed to hold the eigenvectors. This value is returned as the first entry of the z array, and no error message related to nzc is issued.

$lwork$

The size of the array $work$. $lwork \geq \max(1, 18 * n)$ if $jobz = 'V'$, and $lwork \geq \max(1, 12 * n)$ if $jobz = 'N'$.

If $lwork = -1$, then a workspace query is assumed; the function only calculates the optimal size of the $work$ array, returns this value as the first entry of the $work$ array, and no error message related to $lwork$ is issued.

$liwork$

The size of the array $iwork$. $liwork \geq \max(1, 10 * n)$ if the eigenvectors are desired, and $liwork \geq \max(1, 8 * n)$ if only the eigenvalues are to be computed.

If $liwork = -1$, then a workspace query is assumed; the function only calculates the optimal size of the $iwork$ array, returns this value as the first entry of the $iwork$ array, and no error message related to $liwork$ is issued.

dol, dou

From all the eigenvalues $w[0]$ through $w[m-1]$, only eigenvalues $w[dol-1]$ through $w[dou-1]$ are computed.

OUTPUT Parameters

m

Globally summed over all processors, m equals the total number of eigenvalues found. $0 \leq m \leq n$.

If $range = 'A'$, $m = n$, and if $range = 'I'$, $m = iu - il + 1$.

The local output equals $m = dou - dol + 1$.

w

Array of size n

The first m elements contain approximations to the selected eigenvalues in ascending order. Note that immediately after exiting this function, only the eigenvalues indexed $dol-1$ through $dou-1$ are reliable on this processor because the eigenvalue computation is done in parallel. The other entries are very crude preliminary approximations. Other processors hold reliable information on these other parts of the w array.

This information is communicated in the ScaLAPACK driver.

z

Array of size $ldz * \max(1, m)$.

?stegr2a does not compute eigenvectors, this is done in ?stegr2b. The argument z as well as all related other arguments only appear to keep the interface consistent and to signal to the user that this function is meant to be used when eigenvectors are computed.

$work$

On exit, if $info = 0$, $work[0]$ returns the optimal (and minimal) $lwork$.

$iwork$

On exit, if $info = 0$, $iwork[0]$ returns the optimal $liwork$.

| | |
|-----------------------|---|
| <i>needil, neediu</i> | The indices of the leftmost and rightmost eigenvalues needed to accurately compute the relevant part of the representation tree. This information can be used to find out which processors have the relevant eigenvalue information needed so that it can be communicated. |
| <i>inderr</i> | <i>inderr</i> points to the place in the work space where the eigenvalue uncertainties (errors) are stored. |
| <i>nsplit</i> | The number of blocks into which T splits. $1 \leq nsplit \leq n$. |
| <i>pivmin</i> | The minimum pivot in the sturm sequence for T . |
| <i>scale</i> | The scaling factor for the tridiagonal T . |
| <i>wl, wu</i> | The interval $(wl, wu]$ contains all the wanted eigenvalues. It is either given by the user or computed in ?larre2a . |
| <i>info</i> | On exit, <i>info</i> = 0: successful exit other: if <i>info</i> = - <i>i</i> , the <i>i</i> -th argument had an illegal value if <i>info</i> = 10 <i>x</i> , internal error in ?larre2a , Here, the digit $x = \text{abs}(iinfo) < 10$, where <i>iinfo</i> is the nonzero error code returned by ?larre2a . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?stegr2b

From eigenvalues and initial representations computes the selected eigenvalues and eigenvectors of the real symmetric tridiagonal matrix in parallel on multiple processors.

Syntax

```
void sstegr2b(char* jobz, MKL_INT* n, float* d, float* e, MKL_INT* m, float* w, float* z, MKL_INT* ldz, MKL_INT* nzc, MKL_INT* isuppz, float* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT* dol, MKL_INT* dou, MKL_INT* needil, MKL_INT* neediu, MKL_INT* indwlc, float* pivmin, float* scale, float* wl, float* wu, MKL_INT* vstart, MKL_INT* finish, MKL_INT* maxcls, MKL_INT* ndepth, MKL_INT* parity, MKL_INT* zoffset, MKL_INT* info);
```

```
void dstegr2b(char* jobz, MKL_INT* n, double* d, double* e, MKL_INT* m, double* w, double* z, MKL_INT* ldz, MKL_INT* nzc, MKL_INT* isuppz, double* work, MKL_INT* lwork, MKL_INT* iwork, MKL_INT* liwork, MKL_INT* dol, MKL_INT* dou, MKL_INT* needil, MKL_INT* neediu, MKL_INT* indwlc, double* pivmin, double* scale, double* wl, double* wu, MKL_INT* vstart, MKL_INT* finish, MKL_INT* maxcls, MKL_INT* ndepth, MKL_INT* parity, MKL_INT* zoffset, MKL_INT* info);
```

Include Files

- `mkl_scalapack.h`

Description

?stegr2b should only be called after a call to ?stegr2a. From eigenvalues and initial representations computed by ?stegr2a, ?stegr2b computes the selected eigenvalues and eigenvectors of the real symmetric tridiagonal matrix in parallel on multiple processors. It is potentially invoked multiple times on a given processor because the locally relevant representation tree might depend on spectral information that is "owned" by other processors and might need to be communicated.

Please note:

- The calling sequence has two additional integer parameters, *dol* and *dou*, that should satisfy $m \geq dou \geq dol \geq 1$. These parameters are only relevant for the case *jobz* = 'V'. ?stegr2b only computes the eigenvectors corresponding to eigenvalues *dol* through *dou* in *w*, indexed *dol*-1 through *dou*-1. (That is, instead of computing the eigenvectors belonging to $w[0]$ through $w[m-1]$, only the eigenvectors belonging to eigenvalues $w[dol-1]$ through $w[dou-1]$ are computed. In this case, only the eigenvalues *dol* through *dou* are guaranteed to be accurately refined to all figures by Rayleigh-Quotient iteration.
- The additional arguments *vstart*, *finish*, *ndepth*, *parity*, *zoffset* are included as a thread-safe implementation equivalent to save variables. These variables store details about the local representation tree which is computed layerwise. For scalability reasons, eigenvalues belonging to the locally relevant representation tree might be computed on other processors. These need to be communicated before the inspection of the RRRs can proceed on any given layer. Note that only when the variable *finish* is non-zero, the computation has ended. All eigenpairs between *dol* and *dou* have been computed. *m* is set to $dou - dol + 1$.
- ?stegr2b needs more workspace in *z* than the sequential ?stegr. It is used to store the conformal embedding of the local representation tree.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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Input Parameters

| | |
|-------------|---|
| <i>jobz</i> | = 'N': Compute eigenvalues only; = 'V': Compute eigenvalues and eigenvectors. |
| <i>n</i> | The order of the matrix. $n \geq 0$. |
| <i>d</i> | Array of size <i>n</i> The <i>n</i> diagonal elements of the tridiagonal matrix T. Overwritten on exit. |
| <i>e</i> | Array of size <i>n</i> The (<i>n</i> -1) subdiagonal elements of the tridiagonal matrix T in elements 0 to <i>n</i> -2 of <i>e</i> . $e[n-1]$ need not be set on input, but is used internally as workspace. Overwritten on exit. |
| <i>m</i> | The total number of eigenvalues found in ?stegr2a. $0 \leq m \leq n$. |

| | |
|-----------------------|--|
| <i>w</i> | <p>Array of size n</p> <p>The first m elements contain approximations to the selected eigenvalues in ascending order. Note that only the eigenvalues from the locally relevant part of the representation tree, that is all the clusters that include eigenvalues from dol through dou, are reliable on this processor. (It does not need to know about any others anyway.)</p> |
| <i>ldz</i> | <p>The leading dimension of the array z. $ldz \geq 1$, and if $jobz = 'V'$, then $ldz \geq \max(1, n)$.</p> |
| <i>nzc</i> | <p>The number of eigenvectors to be held in the array z, storing the matrix Z.</p> |
| <i>lwork</i> | <p>The size of the array $work$. $lwork \geq \max(1, 18 * n)$ if $jobz = 'V'$, and $lwork \geq \max(1, 12 * n)$ if $jobz = 'N'$.</p> <p>If $lwork = -1$, then a workspace query is assumed; the function only calculates the optimal size of the $work$ array, returns this value as the first entry of the $work$ array, and no error message related to $lwork$ is issued.</p> |
| <i>liwork</i> | <p>The size of the array $iwork$. $liwork \geq \max(1, 10 * n)$ if the eigenvectors are desired, and $liwork \geq \max(1, 8 * n)$ if only the eigenvalues are to be computed.</p> <p>If $liwork = -1$, then a workspace query is assumed; the function only calculates the optimal size of the $iwork$ array, returns this value as the first entry of the $iwork$ array, and no error message related to $liwork$ is issued.</p> |
| <i>dol, dou</i> | <p>From the eigenvalues $w[0]$ through $w[m-1]$, only eigenvectors $Z(:, dol)$ to $Z(:, dou)$ are computed.</p> <p>If $dol > 1$, then $Z(:, dol-1-zoffset)$ is used and overwritten.</p> <p>If $dou < m$, then $Z(:, dou+1-zoffset)$ is used and overwritten.</p> |
| <i>needil, neediu</i> | <p>Describes which are the left and right outermost eigenvalues still to be computed. Initially computed by ?larre2a, modified in the course of the algorithm.</p> |
| <i>pivmin</i> | <p>The minimum pivot in the sturm sequence for T.</p> |
| <i>scale</i> | <p>The scaling factor for T. Used for unscaling the eigenvalues at the very end of the algorithm.</p> |
| <i>wl, wu</i> | <p>The interval $(wl, wu]$ contains all the wanted eigenvalues.</p> |
| <i>vstart</i> | <p>Non-zero on initialization, set to zero afterwards.</p> |
| <i>finish</i> | <p>Indicates whether all eigenpairs have been computed.</p> |
| <i>maxcls</i> | <p>The largest cluster worked on by this processor in the representation tree.</p> |
| <i>ndepth</i> | <p>The current depth of the representation tree. Set to zero on initial pass, changed when the deeper levels of the representation tree are generated.</p> |
| <i>parity</i> | <p>An internal parameter needed for the storage of the clusters on the current level of the representation tree.</p> |

zoffset Offset for storing the eigenpairs when *z* is distributed in 1D-cyclic fashion.

OUTPUT Parameters

z Array of size $ldz * \max(1,m)$

If *jobz* = 'V', and if *info* = 0, then a subset of the first *m* columns of the matrix *Z*, stored in *z*, contain the orthonormal eigenvectors of the matrix *T* corresponding to the selected eigenvalues, with the *i*-th column of *Z* holding the eigenvector associated with *w*[*i*-1].

See *dol*, *dou* for more information.

isuppz array of size $2 * \max(1,m)$.

The support of the eigenvectors in *z*, i.e., the indices indicating the nonzero elements in *z*. The *i*-th computed eigenvector is nonzero only in elements *isuppz*[$2*i-2$] through *isuppz*[$2*i-1$]. This is relevant in the case when the matrix is split. *isuppz* is only set if *n*>2.

work On exit, if *info* = 0, *work*[0] returns the optimal (and minimal) *lwork*.

iwork On exit, if *info* = 0, *iwork*[0] returns the optimal *liwork*.

needil, *neediu* Modified in the course of the algorithm.

indwlc Pointer into the workspace location where the local eigenvalue representations are stored. ("Local eigenvalues" are those relative to the individual shifts of the RRRs.)

vstart Non-zero on initialization, set to zero afterwards.

finish Indicates whether all eigenpairs have been computed

maxcls The largest cluster worked on by this processor in the representation tree.

ndepth The current depth of the representation tree. Set to zero on initial pass, changed when the deeper levels of the representation tree are generated.

parity An internal parameter needed for the storage of the clusters on the current level of the representation tree.

info On exit, *info*

= 0: successful exit

other: if *info* = -*i*, the *i*-th argument had an illegal value

if *info* = 20*x*, internal error in ?*larrv2*.

Here, the digit *x* = $\text{abs}(iinfo) < 10$, where *iinfo* is the nonzero error code returned by ?*larrv2*

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?stein2

Computes the eigenvectors corresponding to specified eigenvalues of a real symmetric tridiagonal matrix, using inverse iteration.

Syntax

```
void sstein2 (MKL_INT *n , float *d , float *e , MKL_INT *m , float *w , MKL_INT
*iblock , MKL_INT *isplit , float *orfac , float *z , MKL_INT *ldz , float *work ,
MKL_INT *iwork , MKL_INT *ifail , MKL_INT *info );

void dstein2 (MKL_INT *n , double *d , double *e , MKL_INT *m , double *w , MKL_INT
*iblock , MKL_INT *isplit , double *orfac , double *z , MKL_INT *ldz , double *work ,
MKL_INT *iwork , MKL_INT *ifail , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The ?stein2 function is a modified LAPACK function ?stein. It computes the eigenvectors of a real symmetric tridiagonal matrix T corresponding to specified eigenvalues, using inverse iteration.

The maximum number of iterations allowed for each eigenvector is specified by an internal parameter *maxits* (currently set to 5).

Input Parameters

| | |
|-----------|--|
| n | The order of the matrix T ($n \geq 0$). |
| m | The number of eigenvectors to be found ($0 \leq m \leq n$). |
| d, e, w | <p>Arrays:</p> <p>d, of size n. The n diagonal elements of the tridiagonal matrix T.</p> <p>e, of size n.</p> <p>The $(n-1)$ subdiagonal elements of the tridiagonal matrix T, in elements 1 to $n-1$. $e[n-1]$ need not be set.</p> <p>w, of size n.</p> <p>The first m elements of w contain the eigenvalues for which eigenvectors are to be computed. The eigenvalues should be grouped by split-off block and ordered from smallest to largest within the block. (The output array w from ?stebz with ORDER = 'B' is expected here).</p> <p>The size of w must be at least $\max(1, n)$.</p> |
| $iblock$ | <p>Array of size n.</p> <p>The submatrix indices associated with the corresponding eigenvalues in w;</p> <p>$iblock[j] = 1$, if eigenvalue $w[j]$ belongs to the first submatrix from the top,</p> <p>$iblock[j] = 2$, if eigenvalue $w[j]$ belongs to the second submatrix, etc. (The output array $iblock$ from ?stebz is expected here).</p> |

| | |
|---------------|--|
| <i>isplit</i> | Array of size n . The splitting points, at which T breaks up into submatrices. The first submatrix consists of rows/columns 1 to <i>isplit</i> [0], the second submatrix consists of rows/columns <i>isplit</i> [0]+1 through <i>isplit</i> [1], etc. (The output array <i>isplit</i> from <i>?stebz</i> is expected here). |
| <i>orfac</i> | <i>orfac</i> specifies which eigenvectors should be orthogonalized. Eigenvectors that correspond to eigenvalues which are within $orfac * T $ of each other are to be orthogonalized. |
| <i>ldz</i> | The leading dimension of the output array z ; $ldz \geq \max(1, n)$. |
| <i>work</i> | Workspace array of size $5n$. |
| <i>iwork</i> | Workspace array of size n . |

Output Parameters

| | |
|--------------|--|
| <i>z</i> | Array of size $ldz * m$. The computed eigenvectors. The eigenvector associated with the eigenvalue $w[i]$ is stored in the $(i+1)$ -th column of the matrix Z represented by z , $i=0, \dots, m-1$. Any vector that fails to converge is set to its current iterate after <i>maxits</i> iterations. |
| <i>ifail</i> | Array of size m . On normal exit, all elements of <i>ifail</i> are zero. If one or more eigenvectors fail to converge after <i>maxits</i> iterations, then their indices are stored in the array <i>ifail</i> . |
| <i>info</i> | <i>info</i> = 0, the exit is successful. <i>info</i> < 0: if <i>info</i> = $-i$, the i -th had an illegal value. <i>info</i> > 0: if <i>info</i> = i , then i eigenvectors failed to converge in <i>maxits</i> iterations. Their indices are stored in the array <i>ifail</i> . |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?dbtf2

Computes an LU factorization of a general band matrix with no pivoting (local unblocked algorithm).

Syntax

```
void sdbtf2 (MKL_INT *m , MKL_INT *n , MKL_INT *kl , MKL_INT *ku , float *ab , MKL_INT *ldab , MKL_INT *info );
void ddbtf2 (MKL_INT *m , MKL_INT *n , MKL_INT *kl , MKL_INT *ku , double *ab , MKL_INT *ldab , MKL_INT *info );
void cdbtf2 (MKL_INT *m , MKL_INT *n , MKL_INT *kl , MKL_INT *ku , MKL_Complex8 *ab , MKL_INT *ldab , MKL_INT *info );
```

```
void zdbtf2 (MKL_INT *m , MKL_INT *n , MKL_INT *kl , MKL_INT *ku , MKL_Complex16 *ab ,
MKL_INT *ldab , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `zdbtf2` function computes an LU factorization of a general real/complex m -by- n band matrix A without using partial pivoting with row interchanges.

This is the unblocked version of the algorithm, calling [BLAS Routines and Functions](#).

Input Parameters

| | |
|--------|--|
| m | The number of rows of the matrix A ($m \geq 0$). |
| n | The number of columns in A ($n \geq 0$). |
| kl | The number of sub-diagonals within the band of A ($kl \geq 0$). |
| ku | The number of super-diagonals within the band of A ($ku \geq 0$). |
| ab | Array of size $ldab * n$. The matrix A in band storage, in rows $kl+1$ to $2kl+ku+1$; rows 1 to kl of the matrix need not be set. The j -th column of A is stored in the array ab as follows: $ab[kl+ku+i-j+(j-1)*ldab] = A(i,j)$ for $\max(1, j-ku) \leq j \leq \min(m, j+kl)$. |
| $ldab$ | The leading dimension of the array ab . ($ldab \geq 2kl + ku + 1$) |

Output Parameters

| | |
|--------|--|
| ab | On exit, details of the factorization: U is stored as an upper triangular band matrix with $kl+ku$ superdiagonals in rows 1 to $kl+ku+1$, and the multipliers used during the factorization are stored in rows $kl+ku+2$ to $2*kl+ku+1$. See the <i>Application Notes</i> below for further details. |
| $info$ | = 0: successful exit < 0: if $info = -i$, the i -th argument had an illegal value, > 0: if $info = +i$, the matrix element $U(i,i)$ is 0. The factorization has been completed, but the factor U is exactly singular. Division by 0 will occur if you use the factor U for solving a system of linear equations. |

Application Notes

The band storage scheme is illustrated by the following example, when $m = n = 6$, $kl = 2$, $ku = 1$:

| on entry | on exit |
|--|--|
| $\begin{bmatrix} * & a_{12} & a_{23} & a_{34} & a_{45} & a_{56} \\ a_{11} & a_{22} & a_{33} & a_{44} & a_{55} & a_{66} \\ a_{21} & a_{32} & a_{43} & a_{54} & a_{65} & * \\ a_{31} & a_{42} & a_{53} & a_{64} & * & * \end{bmatrix}$ | $\begin{bmatrix} * & u_{12} & u_{23} & u_{34} & u_{45} & u_{56} \\ u_{11} & u_{22} & u_{33} & u_{44} & u_{55} & u_{66} \\ m_{21} & m_{32} & m_{43} & m_{54} & m_{65} & * \\ m_{31} & m_{42} & m_{53} & m_{64} & * & * \end{bmatrix}$ |

The function does not use array elements marked *; elements marked + need not be set on entry, but the function requires them to store elements of U , because of fill-in resulting from the row interchanges.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?dbtrf

Computes an LU factorization of a general band matrix with no pivoting (local blocked algorithm).

Syntax

```
void sdbtrf (MKL_INT *m , MKL_INT *n , MKL_INT *kl , MKL_INT *ku , float *ab , MKL_INT *ldab , MKL_INT *info );
```

```
void ddbtrf (MKL_INT *m , MKL_INT *n , MKL_INT *kl , MKL_INT *ku , double *ab , MKL_INT *ldab , MKL_INT *info );
```

```
void cdbtrf (MKL_INT *m , MKL_INT *n , MKL_INT *kl , MKL_INT *ku , MKL_Complex8 *ab , MKL_INT *ldab , MKL_INT *info );
```

```
void zdbtrf (MKL_INT *m , MKL_INT *n , MKL_INT *kl , MKL_INT *ku , MKL_Complex16 *ab , MKL_INT *ldab , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

This function computes an LU factorization of a real m -by- n band matrix A without using partial pivoting or row interchanges.

This is the blocked version of the algorithm, calling [BLAS Routines and Functions](#).

Input Parameters

| | |
|------|---|
| m | The number of rows of the matrix A ($m \geq 0$). |
| n | The number of columns in A ($n \geq 0$). |
| kl | The number of sub-diagonals within the band of A ($kl \geq 0$). |
| ku | The number of super-diagonals within the band of A ($ku \geq 0$). |
| ab | Array of size $ldab * n$. |

The matrix A in band storage, in rows $kl+1$ to $2kl+ku+1$; rows 1 to kl need not be set. The j -th column of A is stored in the array ab as follows: $ab[kl+ku+i-j+(j-1)*ldab] = A(i, j)$ for $\max(1, j-ku) \leq i \leq \min(m, j+kl)$.

$ldab$

The leading dimension of the array ab .

($ldab \geq 2kl + ku + 1$)

Output Parameters

ab

On exit, details of the factorization: U is stored as an upper triangular band matrix with $kl+ku$ superdiagonals in rows 1 to $kl+ku+1$, and the multipliers used during the factorization are stored in rows $kl+ku+2$ to $2*kl+ku+1$. See the *Application Notes* below for further details.

$info$

= 0: successful exit

< 0: if $info = -i$, the i -th argument had an illegal value,

> 0: if $info = +i$, the matrix element $U(i,i)$ is 0. The factorization has been completed, but the factor U is exactly singular. Division by 0 will occur if you use the factor U for solving a system of linear equations.

Application Notes

The band storage scheme is illustrated by the following example, when $m = n = 6$, $kl = 2$, $ku = 1$:

| on entry | on exit |
|--|--|
| $\begin{bmatrix} * & a_{12} & a_{23} & a_{34} & a_{45} & a_{56} \\ a_{11} & a_{22} & a_{33} & a_{44} & a_{55} & a_{66} \\ a_{21} & a_{32} & a_{43} & a_{54} & a_{65} & * \\ a_{31} & a_{42} & a_{53} & a_{64} & * & * \end{bmatrix}$ | $\begin{bmatrix} * & u_{12} & u_{23} & u_{34} & u_{45} & u_{56} \\ u_{11} & u_{22} & u_{33} & u_{44} & u_{55} & u_{66} \\ m_{21} & m_{32} & m_{43} & m_{54} & m_{65} & * \\ m_{31} & m_{42} & m_{53} & m_{64} & * & * \end{bmatrix}$ |

The function does not use array elements marked *.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?dttf

Computes an LU factorization of a general tridiagonal matrix with no pivoting (local blocked algorithm).

Syntax

```
void sdttrf (MKL_INT *n , float *dl , float *d , float *du , MKL_INT *info );
```

```
void ddttrf (MKL_INT *n , double *dl , double *d , double *du , MKL_INT *info );
```

```
void cdttrf (MKL_INT *n , MKL_Complex8 *dl , MKL_Complex8 *d , MKL_Complex8 *du , MKL_INT *info );
```

```
void zdttrf (MKL_INT *n , MKL_Complex16 *dl , MKL_Complex16 *d , MKL_Complex16 *du , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `?dtttrf` function computes an LU factorization of a real or complex tridiagonal matrix A using elimination without partial pivoting.

The factorization has the form $A = L*U$, where L is a product of unit lower bidiagonal matrices and U is upper triangular with nonzeros only in the main diagonal and first superdiagonal.

Input Parameters

| | |
|-------------|--|
| n | The order of the matrix A ($n \geq 0$). |
| dl, d, du | Arrays containing elements of A . The array dl of size $(n-1)$ contains the sub-diagonal elements of A . The array d of size n contains the diagonal elements of A . The array du of size $(n-1)$ contains the super-diagonal elements of A . |

Output Parameters

| | |
|--------|---|
| dl | Overwritten by the $(n-1)$ multipliers that define the matrix L from the LU factorization of A . |
| d | Overwritten by the n diagonal elements of the upper triangular matrix U from the LU factorization of A . |
| du | Overwritten by the $(n-1)$ elements of the first super-diagonal of U . |
| $info$ | = 0: successful exit < 0: if $info = -i$, the i -th argument had an illegal value, > 0: if $info = i$, the matrix element $U(i,i)$ is exactly 0. The factorization has been completed, but the factor U is exactly singular. Division by 0 will occur if you use the factor U for solving a system of linear equations. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?dtttrsv

Solves a general tridiagonal system of linear equations using the LU factorization computed by `?dtttrf`.

Syntax

```
void sdttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , float *dl ,
float *d , float *du , float *b , MKL_INT *ldb , MKL_INT *info );
```

```
void ddttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , double *dl ,
double *d , double *du , double *b , MKL_INT *ldb , MKL_INT *info );
```

```

void cdtttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex8
*d1 , MKL_Complex8 *d , MKL_Complex8 *du , MKL_Complex8 *b , MKL_INT *ldb , MKL_INT
*info );

void zdtttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , MKL_Complex16
*d1 , MKL_Complex16 *d , MKL_Complex16 *du , MKL_Complex16 *b , MKL_INT *ldb , MKL_INT
*info );

```

Include Files

- mkl_scalapack.h

Description

The `cdtttrsv` function solves one of the following systems of linear equations:

$$L * X = B, L^T * X = B, \text{ or } L^H * X = B,$$

$$U * X = B, U^T * X = B, \text{ or } U^H * X = B$$

with factors of the tridiagonal matrix A from the LU factorization computed by `cdttrf`.

Input Parameters

| | |
|------------------|---|
| <i>uplo</i> | Specifies whether to solve with L or U . |
| <i>trans</i> | Must be 'N' or 'T' or 'C'. Indicates the form of the equations: If <i>trans</i> = 'N', then $A * X = B$ is solved for X (no transpose). If <i>trans</i> = 'T', then $A^T * X = B$ is solved for X (transpose). If <i>trans</i> = 'C', then $A^H * X = B$ is solved for X (conjugate transpose). |
| <i>n</i> | The order of the matrix A ($n \geq 0$). |
| <i>nrhs</i> | The number of right-hand sides, that is, the number of columns in the matrix B ($nrhs \geq 0$). |
| <i>d1,d,du,b</i> | The array <i>d1</i> of size $(n - 1)$ contains the $(n - 1)$ multipliers that define the matrix L from the LU factorization of A . The array <i>d</i> of size n contains n diagonal elements of the upper triangular matrix U from the LU factorization of A . The array <i>du</i> of size $(n - 1)$ contains the $(n - 1)$ elements of the first super-diagonal of U . On entry, the array <i>b</i> of size $ldb * nrhs$ contains the right-hand side of matrix B . |
| <i>ldb</i> | The leading dimension of the array <i>b</i> ; $ldb \geq \max(1, n)$. |

Output Parameters

| | |
|-------------|--|
| <i>b</i> | Overwritten by the solution matrix X . |
| <i>info</i> | If <i>info</i> =0, the execution is successful. If <i>info</i> = $-i$, the i -th parameter had an illegal value. |

See Also

Overview of ScaLAPACK Routines for details of ScaLAPACK array descriptor structures and related notations.

?pttrsv

Solves a symmetric (Hermitian) positive-definite tridiagonal system of linear equations, using the L^*D*L^H factorization computed by `?pttrf`.

Syntax

```
void spttrsv (char *trans , MKL_INT *n , MKL_INT *nrhs , float *d , float *e , float *b , MKL_INT *ldb , MKL_INT *info );
```

```
void dpttrsv (char *trans , MKL_INT *n , MKL_INT *nrhs , double *d , double *e , double *b , MKL_INT *ldb , MKL_INT *info );
```

```
void cpttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , float *d , MKL_Complex8 *e , MKL_Complex8 *b , MKL_INT *ldb , MKL_INT *info );
```

```
void zpttrsv (char *uplo , char *trans , MKL_INT *n , MKL_INT *nrhs , double *d , MKL_Complex16 *e , MKL_Complex16 *b , MKL_INT *ldb , MKL_INT *info );
```

Include Files

- `mkl_scalapack.h`

Description

The `?pttrsv` function solves one of the triangular systems:

$L^T * X = B$, or $L * X = B$ for real flavors,

or

$L * X = B$, or $L^H * X = B$,

$U * X = B$, or $U^H * X = B$ for complex flavors,

where L (or U for complex flavors) is the Cholesky factor of a Hermitian positive-definite tridiagonal matrix A such that

$A = L^*D*L^H$ (computed by `spttrf/dpttrf`)

or

$A = U^H * D * U$ or $A = L^*D*L^H$ (computed by `cpttrf/zpttrf`).

Input Parameters

| | |
|--------------------|--|
| <code>uplo</code> | Must be 'U' or 'L'. Specifies whether the superdiagonal or the subdiagonal of the tridiagonal matrix A is stored and the form of the factorization: If <code>uplo = 'U'</code> , e is the superdiagonal of U , and $A = U^H * D * U$ or $A = L^*D*L^H$; if <code>uplo = 'L'</code> , e is the subdiagonal of L , and $A = L^*D*L^H$. The two forms are equivalent, if A is real. |
| <code>trans</code> | Specifies the form of the system of equations: |

| | |
|--------|---|
| | for real flavors: if $trans = 'N'$: $L^*X = B$ (no transpose) if $trans = 'T'$: $L^T * X = B$ (transpose) for complex flavors: if $trans = 'N'$: $U^*X = B$ or $L^*X = B$ (no transpose) if $trans = 'C'$: $U^H * X = B$ or $L^H * X = B$ (conjugate transpose). |
| n | The order of the tridiagonal matrix A . $n \geq 0$. |
| $nrhs$ | The number of right hand sides, that is, the number of columns of the matrix B . $nrhs \geq 0$. |
| d | array of size n . The n diagonal elements of the diagonal matrix D from the factorization computed by ?pttrf . |
| e | array of size $(n-1)$. The $(n-1)$ off-diagonal elements of the unit bidiagonal factor U or L from the factorization computed by ?pttrf . See <i>uplo</i> . |
| b | array of size $ldb * nrhs$. On entry, the right hand side matrix B . |
| ldb | The leading dimension of the array b . $ldb \geq \max(1, n)$. |

Output Parameters

| | |
|--------|--|
| b | On exit, the solution matrix X . |
| $info$ | = 0: successful exit < 0: if $info = -i$, the i -th argument had an illegal value. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?steqr2

Computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method.

Syntax

```
void ssteqr2 (char *compz , MKL_INT *n , float *d , float *e , float *z , MKL_INT
*ldz , MKL_INT *nr , float *work , MKL_INT *info );
void dsteqr2 (char *compz , MKL_INT *n , double *d , double *e , double *z , MKL_INT
*ldz , MKL_INT *nr , double *work , MKL_INT *info );
```

Include Files

- mkl_scalapack.h

Description

The `?steqr2` function is a modified version of LAPACK function `?steqr`. The `?steqr2` function computes all eigenvalues and, optionally, eigenvectors of a symmetric tridiagonal matrix using the implicit QL or QR method. `?steqr2` is modified from `?steqr` to allow each ScaLAPACK process running `?steqr2` to perform updates on a distributed matrix Q . Proper usage of `?steqr2` can be gleaned from examination of ScaLAPACK function `p?syeval`.

Input Parameters

| | |
|-------------------------|--|
| <code>compz</code> | <p>Must be 'N' or 'I'.</p> <p>If <code>compz = 'N'</code>, the function computes eigenvalues only. If <code>compz = 'I'</code>, the function computes the eigenvalues and eigenvectors of the tridiagonal matrix T.</p> <p>z must be initialized to the identity matrix by <code>p?laset</code> or <code>?laset</code> prior to entering this function.</p> |
| <code>n</code> | The order of the matrix T ($n \geq 0$). |
| <code>d, e, work</code> | <p>Arrays:</p> <p>d contains the diagonal elements of T. The size of d must be at least $\max(1, n)$.</p> <p>e contains the $(n-1)$ subdiagonal elements of T. The size of e must be at least $\max(1, n-1)$.</p> <p>$work$ is a workspace array. The size of $work$ is $\max(1, 2*n-2)$. If <code>compz = 'N'</code>, then $work$ is not referenced.</p> |
| <code>z</code> | <p>(local)</p> <p>Array of global size $n * n$ and of local size $ldz * nr$.</p> <p>If <code>compz = 'V'</code>, then z contains the orthogonal matrix used in the reduction to tridiagonal form.</p> |
| <code>ldz</code> | <p>The leading dimension of the array z. Constraints:</p> <p>$ldz \geq 1$,</p> <p>$ldz \geq \max(1, n)$, if eigenvectors are desired.</p> |
| <code>nr</code> | <p>$nr = \max(1, \text{numroc}(n, nb, myprow, 0, nprocs))$.</p> <p>If <code>compz = 'N'</code>, then nr is not referenced.</p> |

Output Parameters

| | |
|----------------|--|
| <code>d</code> | <p>On exit, the eigenvalues in ascending order, if <code>info = 0</code>.</p> <p>See also <code>info</code>.</p> |
| <code>e</code> | On exit, e has been destroyed. |
| <code>z</code> | On exit, if <code>info = 0</code> , then, |

if `compz = 'V'`, `z` contains the orthonormal eigenvectors of the original symmetric matrix, and if `compz = 'I'`, `z` contains the orthonormal eigenvectors of the symmetric tridiagonal matrix. If `compz = 'N'`, then `z` is not referenced.

`info`

`info = 0`, the exit is successful.

`info < 0`: if `info = -i`, the `i`-th had an illegal value.

`info > 0`: the algorithm has failed to find all the eigenvalues in a total of $30n$ iterations;

if `info = i`, then `i` elements of `e` have not converged to zero; on exit, `d` and `e` contain the elements of a symmetric tridiagonal matrix, which is orthogonally similar to the original matrix.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

?trmvt

Performs matrix-vector operations.

Syntax

```
void strmvt (const char* uplo, const MKL_INT* n, const float* t, const MKL_INT* ldt,
float* x, const MKL_INT* incx, const float* y, const MKL_INT* incy, float* w, const
MKL_INT* incw, const float* z, const MKL_INT* incz);
```

```
void dtrmvt (const char* uplo, const MKL_INT* n, const double* t, const MKL_INT* ldt,
double* x, const MKL_INT* incx, const double* y, const MKL_INT* incy, double* w, const
MKL_INT* incw, const double* z, const MKL_INT* incz);
```

```
void ctrmvt (const char* uplo, const MKL_INT* n, const MKL_Complex8* t, const MKL_INT*
ldt, MKL_Complex8* x, const MKL_INT* incx, const MKL_Complex8* y, const MKL_INT* incy,
MKL_Complex8* w, const MKL_INT* incw, const MKL_Complex8* z, const MKL_INT* incz);
```

```
void ztrmvt (const char* uplo, const MKL_INT* n, const MKL_Complex16* t, const MKL_INT*
ldt, MKL_Complex16* x, const MKL_INT* incx, const MKL_Complex16* y, const MKL_INT*
incy, MKL_Complex16* w, const MKL_INT* incw, const MKL_Complex16* z, const MKL_INT*
incz);
```

Include Files

- `mkl_scalapack.h`

Description

?trmvt performs the matrix-vector operations as follows:

strmvt and dtrmvt: $x := T^* y$, and $w := T^* z$

ctrmvt and ztrmvt: $x := \text{conjg}(T^*) y$, and $w := T^* z$,

where `x` is an `n` element vector and `T` is an `n`-by-`n` upper or lower triangular matrix.

Input Parameters

| | |
|-------------|--|
| <i>uplo</i> | <p>On entry, <i>uplo</i> specifies whether the matrix is an upper or lower triangular matrix as follows:</p> <p><i>uplo</i> = 'U' or 'u' <i>A</i> is an upper triangular matrix.</p> <p><i>uplo</i> = 'L' or 'l' <i>A</i> is a lower triangular matrix.</p> <p>Unchanged on exit.</p> |
| <i>n</i> | <p>On entry, <i>n</i> specifies the order of the matrix <i>A</i>. <i>n</i> must be at least zero.</p> <p>Unchanged on exit.</p> |
| <i>t</i> | <p>Array of size (<i>ldt</i>, <i>n</i>).</p> <p>Before entry with <i>uplo</i> = 'U' or 'u', the leading <i>n</i>-by-<i>n</i> upper triangular part of the array <i>t</i> must contain the upper triangular matrix and the strictly lower triangular part of <i>t</i> is not referenced.</p> <p>Before entry with <i>uplo</i> = 'L' or 'l', the leading <i>n</i>-by-<i>n</i> lower triangular part of the array <i>t</i> must contain the lower triangular matrix and the strictly upper triangular part of <i>t</i> is not referenced.</p> |
| <i>ldt</i> | <p>On entry, <i>lda</i> specifies the first dimension of <i>A</i> as declared in the calling (sub) program. <i>lda</i> must be at least max(1, <i>n</i>).</p> <p>Unchanged on exit.</p> |
| <i>incx</i> | <p>On entry, <i>incx</i> specifies the increment for the elements of <i>x</i>. <i>incx</i> must not be zero.</p> <p>Unchanged on exit.</p> |
| <i>y</i> | <p>Array of size at least (1 + (<i>n</i> - 1) * abs(<i>incy</i>)).</p> <p>Before entry, the incremented array <i>y</i> must contain the <i>n</i> element vector <i>y</i>.</p> <p>Unchanged on exit.</p> |
| <i>incy</i> | <p>On entry, <i>incy</i> specifies the increment for the elements of <i>y</i>. <i>incy</i> must not be zero.</p> <p>Unchanged on exit.</p> |
| <i>incw</i> | <p>On entry, <i>incw</i> specifies the increment for the elements of <i>w</i>. <i>incw</i> must not be zero.</p> <p>Unchanged on exit.</p> |
| <i>z</i> | <p>Array of size at least (1 + (<i>n</i> - 1) * abs(<i>incz</i>)).</p> <p>Before entry, the incremented array <i>z</i> must contain the <i>n</i> element vector <i>z</i>.</p> <p>Unchanged on exit.</p> |
| <i>incz</i> | <p>On entry, <i>incz</i> specifies the increment for the elements of <i>z</i>. <i>incz</i> must not be zero.</p> |

Unchanged on exit.

Output Parameters

| | |
|----------|---|
| <i>t</i> | <p>Before entry with <i>uplo</i> = 'U' or 'u', the leading <i>n</i>-by-<i>n</i> upper triangular part of the array <i>t</i> must contain the upper triangular matrix and the strictly lower triangular part of <i>t</i> is not referenced.</p> <p>Before entry with <i>uplo</i> = 'L' or 'l', the leading <i>n</i>-by-<i>n</i> lower triangular part of the array <i>t</i> must contain the lower triangular matrix and the strictly upper triangular part of <i>t</i> is not referenced.</p> |
| <i>x</i> | <p>Array of size at least $(1 + (n - 1) * \text{abs}(incx))$.</p> <p>On exit, $x = T * y$.</p> |
| <i>w</i> | <p>Array of size at least $(1 + (n - 1) * \text{abs}(incw))$.</p> <p>On exit, $w = T * z$.</p> |

pilaenv

Returns the positive integer value of the logical blocking size.

Syntax

```
MKL_INT pilaenv (const MKL_INT *ictxt , const char *prec);
```

Include Files

- mkl_pblas.h

Description

`pilaenv` returns the positive integer value of the logical blocking size. This value is machine and precision specific. This version provides a logical blocking size which should give good though not optimal performance on many of the currently available distributed-memory concurrent computers. You are encouraged to modify this subroutine to set this tuning parameter for your particular machine.

Input Parameters

| | |
|--------------|---|
| <i>ictxt</i> | <p>On entry, <i>ictxt</i> specifies the BLACS context handle, indicating the global context of the operation. The context itself is global, but the value of <i>ictxt</i> is local.</p> |
| <i>prec</i> | <p>On input, <i>prec</i> specifies the precision for which the logical block size should be returned as follows:</p> <p><i>prec</i> = 'S' or 's' single precision real, <i>prec</i> = 'D' or 'd' double precision real, <i>prec</i> = 'C' or 'c' single precision complex, <i>prec</i> = 'Z' or 'z' double precision complex, <i>prec</i> = 'I' or 'i' integer.</p> |

Application Notes

Before modifying this routine to tune the library performance on your system, be aware of the following:

1. The value this function returns must be strictly larger than zero,
2. If you are planning to link your program with different instances of the library (for example, on a heterogeneous machine), you *must* compile each instance of the library with exactly the same version of this routine for obvious interoperability reasons.

pilaenvx

Called from the ScaLAPACK routines to choose problem-dependent parameters for the local environment.

Syntax

```
MKL_INT pilaenvx (const MKL_INT* ictxt, const MKL_INT* ispec, const char* name, const
char* opts, const MKL_INT* n1, const MKL_INT* n2, const MKL_INT* n3, const MKL_INT*
```

Include Files

- mkl.h

Description

`pilaenvx` is called from the ScaLAPACK routines to choose problem-dependent parameters for the local environment. See `ispec` for a description of the parameters. This version provides a set of parameters which should give good, though not optimal, performance on many of the currently available computers. You are encouraged to modify this subroutine to set the tuning parameters for your particular machine using the option and problem size information in the arguments.

Input Parameters

| | |
|--------------------|---|
| <code>ictxt</code> | (local input) On entry, <code>ictxt</code> specifies the BLACS context handle, indicating the global context of the operation. The context itself is global, but the value of <code>ictxt</code> is local. |
| <code>ispec</code> | (global input) Specifies the parameter to be returned as the value of <code>pilaenvx</code> . = 1: the optimal blocksize; if this value is 1, an unblocked algorithm will give the best performance (unlikely). = 2: the minimum block size for which the block routine should be used; if the usable block size is less than this value, an unblocked routine should be used. = 3: the crossover point (in a block routine, for N less than this value, an unblocked routine should be used). = 4: the number of shifts, used in the nonsymmetric eigenvalue routines (DEPRECATED). = 5: the minimum column dimension for blocking to be used; rectangular blocks must have dimension at least k by m , where k is given by <code>pilaenvx(2, ...)</code> and m by <code>pilaenvx(5, ...)</code> . = 6: the crossover point for the SVD (when reducing an m by n matrix to bidiagonal form, if $\max(m,n)/\min(m,n)$ exceeds this value, a QR factorization is used first to reduce the matrix to a triangular form). |

- = 7: the number of processors.
- = 8: the crossover point for the multishift QR method for nonsymmetric eigenvalue problems (DEPRECATED).
- = 9: maximum size of the subproblems at the bottom of the computation tree in the divide-and-conquer algorithm (used by `?gelsd` and `?gesdd`).
- =10: IEEE NaN arithmetic can be trusted not to trap.
- =11: infinity arithmetic can be trusted not to trap.

12 <= *ispec* <= 16:

`p?hseqr` or one of its subroutines, see `piparmq` for detailed explanation.

17 <= *ispec* <= 22:

Parameters for `pb?trord/p?hseqr` (not all), as follows:

- =17: maximum number of concurrent computational windows;
- =18: number of eigenvalues/bulges in each window;
- =19: computational window size;
- =20: minimal percentage of FLOPS required for performing matrix-matrix multiplications instead of pipelined orthogonal transformations;
- =21: width of block column slabs for row-wise application of pipelined orthogonal transformations in their factorized form;
- =22: the maximum number of eigenvalues moved together over a process border;
- =23: the number of processors involved in Aggressive Early Deflation (AED);
- =99: Maximum iteration chunksize in OpenMP parallelization.

name

(global input)

The name of the calling subroutine, in either upper case or lower case.

opts

(global input) The character options to the subroutine name, concatenated into a single character string. For example, *uplo* = 'U', *trans* = 'T', and *diag* = 'N' for a triangular routine would be specified as *opts* = 'UTN'.

n1, *n2*, *n3*, and *n4*

(global input) Problem dimensions for the subroutine name; these may not all be required.

Output Parameters

result

(global output)

>= 0: the value of the parameter specified by *ispec*.

< 0: if *pilaenvx* = -*k*, the *k*-th argument had an illegal value.

Application Notes

The following conventions have been used when calling `ilaenv` from the LAPACK routines:

1. *opts* is a concatenation of all of the character options to subroutine name, in the same order that they appear in the argument list for name, even if they are not used in determining the value of the parameter specified by *ispec*.
2. The problem dimensions *n1*, *n2*, *n3*, and *n4* are specified in the order that they appear in the argument list for *name*. *n1* is used first, *n2* second, and so on, and unused problem dimensions are passed a value of -1.
3. The parameter value returned by `ilaenv` is checked for validity in the calling subroutine. For example, `ilaenv` is used to retrieve the optimal block size for `strtri` as follows:

```
NB = ilaenv( 1, 'STRTRI', UPLO // DIAG, N, -1, -1, -1 );
if( NB<=1 ) {
    NB = MAX( 1, N );
}
```

The same conventions hold for this ScaLAPACK-style variant.

pjlaenv

Called from the ScaLAPACK symmetric and Hermitian tailored eigen-routines to choose problem-dependent parameters for the local environment.

Syntax

```
MKL_INT pjlaenv (const MKL_INT* ictxt, const MKL_INT* ispec, const char* name, const
char* opts, const MKL_INT* n1, const MKL_INT* n2, const MKL_INT* n3, const MKL_INT*
n4);
```

Include Files

- `mkl.h`

Description

`pjlaenv` is called from the ScaLAPACK symmetric and Hermitian tailored eigen-routines to choose problem-dependent parameters for the local environment. See *ispec* for a description of the parameters. This version provides a set of parameters which should give good, though not optimal, performance on many of the currently available computers. You are encouraged to modify this subroutine to set the tuning parameters for your particular machine using the option and problem size information in the arguments.

Input Parameters

ispec (global input) Specifies the parameter to be returned as the value of `pjlaenv`.

= 1: the data layout blocksize;

= 2: the panel blocking factor;

= 3: the algorithmic blocking factor;

= 4: execution path control;

= 5: maximum size for direct call to the LAPACK routine.

name (global input) The name of the calling subroutine, in either upper case or lower case.

| | |
|---|---|
| <i>opts</i> | (global input) The character options to the subroutine name, concatenated into a single character string. For example, <i>uplo</i> = 'U', <i>trans</i> = 'T', and <i>diag</i> = 'N' for a triangular routine would be specified as <i>opts</i> = 'UTN'. |
| <i>n1</i> , <i>n2</i> , <i>n3</i> , and <i>n4</i> | (global input) Problem dimensions for the subroutine name; these may not all be required. At present, only <i>n1</i> is used, and it (<i>n1</i>) is used only for 'TTRD'. |

Output Parameters

| | |
|---------------|--|
| <i>result</i> | (global or local output) ≥ 0 : the value of the parameter specified by <i>ispec</i> . < 0 : if <i>pjlaenv</i> = $-k$, the k -th argument had an illegal value. Most parameters set via a call to <i>pjlaenv</i> must be identical on all processors and hence <i>pjlaenv</i> will return the same value to all procesors (i.e. global output). However some, in particular, the panel blocking factor can be different on each processor and hence <i>pjlaenv</i> can return different values on different processors (i.e. local output). |
|---------------|--|

Application Notes

The following conventions have been used when calling *pjlaenv* from the ScaLAPACK routines:

- opts* is a concatenation of all of the character options to subroutine name, in the same order that they appear in the argument list for name, even if they are not used in determining the value of the parameter specified by *ispec*.
- The problem dimensions *n1*, *n2*, *n3*, and *n4* are specified in the order that they appear in the argument list for name. *n1* is used first, *n2* second, and so on, and unused problem dimensions are passed a value of -1.
 - The parameter value returned by *pjlaenv* is checked for validity in the calling subroutine. For example, *pjlaenv* is used to retrieve the optimal blocksize for *STRTRI* as follows:

```
NB = pjlaenv( 1, 'STRTRI', UPLO // DIAG, N, -1, -1, -1 );
IF( NB>=1 ) {
    NB = MAX( 1, N );
}
```

pjlaenv is patterned after *ilaenv* and keeps the same interface in anticipation of future needs, even though *pjlaenv* is only sparsely used at present in ScaLAPACK. Most ScaLAPACK codes use the input data layout blocking factor as the algorithmic blocking factor - hence there is no need or opportunity to set the algorithmic or data decomposition blocking factor. *pXYytevx.f* and *pXYytgvx.f* and *pXYyttrd.f* are the only codes which call *pjlaenv*. *pXYytevx.f* and *pXYytgvx.f* redistribute the data to the best data layout for each transformation. *pXYyttrd.f* uses a data layout blocking factor of 1.

Additional ScaLAPACK Routines

```
void pchettrd (const char *uplo , const MKL_INT *n , MKL_Complex8 *a , const MKL_INT
*ia , const MKL_INT *ja , const MKL_INT *desca , float *d , float *e , MKL_Complex8
*tau , MKL_Complex8 *work , const MKL_INT *lwork , MKL_INT *info );
```

```
void pzhettrd (const char *uplo , const MKL_INT *n , MKL_Complex16 *a , const MKL_INT
*ia , const MKL_INT *ja , const MKL_INT *desca , double *d , double *e , MKL_Complex16
*tau , MKL_Complex16 *work , const MKL_INT *lwork , MKL_INT *info );
```

```

void pslaed0 (const MKL_INT *n , float *d , float *e , float *q , const MKL_INT *iq ,
const MKL_INT *jq , const MKL_INT *descq , float *work , MKL_INT *iwork , MKL_INT
*info );

void pdlaed0 (const MKL_INT *n , double *d , double *e , double *q , const MKL_INT
*iq , const MKL_INT *jq , const MKL_INT *descq , double *work , MKL_INT *iwork ,
MKL_INT *info );

void pslaed1 (const MKL_INT *n , const MKL_INT *n1 , float *d , const MKL_INT *id ,
float *q , const MKL_INT *iq , const MKL_INT *jq , const MKL_INT *descq , const float
*rho , float *work , MKL_INT *iwork , MKL_INT *info );

void pdlaed1 (const MKL_INT *n , const MKL_INT *n1 , double *d , const MKL_INT *id ,
double *q , const MKL_INT *iq , const MKL_INT *jq , const MKL_INT *descq , const double
*rho , double *work , MKL_INT *iwork , MKL_INT *info );

void pslaed2 (const MKL_INT *ictxt , MKL_INT *k , const MKL_INT *n , const MKL_INT
*n1 , const MKL_INT *nb , float *d , const MKL_INT *drow , const MKL_INT *dcol , float
*q , const MKL_INT *ldq , float *rho , const float *z , float *w , float *dlamda ,
float *q2 , const MKL_INT *ldq2 , float *qbuf , MKL_INT *ctot , MKL_INT *psm , const
MKL_INT *npcol , MKL_INT *indx , MKL_INT *indxc , MKL_INT *indxp , MKL_INT *indcol ,
MKL_INT *coltyp , MKL_INT *nn , MKL_INT *nn1 , MKL_INT *nn2 , MKL_INT *ib1 , MKL_INT
*ib2 );

void pdlaed2 (const MKL_INT *ictxt , MKL_INT *k , const MKL_INT *n , const MKL_INT
*n1 , const MKL_INT *nb , double *d , const MKL_INT *drow , const MKL_INT *dcol ,
double *q , const MKL_INT *ldq , double *rho , const double *z , double *w , double
*dlamda , double *q2 , const MKL_INT *ldq2 , double *qbuf , MKL_INT *ctot , MKL_INT
*psm , const MKL_INT *npcol , MKL_INT *indx , MKL_INT *indxc , MKL_INT *indxp ,
MKL_INT *indcol , MKL_INT *coltyp , MKL_INT *nn , MKL_INT *nn1 , MKL_INT *nn2 ,
MKL_INT *ib1 , MKL_INT *ib2 );

void pslaed3 (const MKL_INT *ictxt , MKL_INT *k , const MKL_INT *n , const MKL_INT
*nb , float *d , const MKL_INT *drow , const MKL_INT *dcol , float *rho , float
*dlamda , float *w , const float *z , float *u , const MKL_INT *ldu , float *buf ,
MKL_INT *indx , MKL_INT *indcol , MKL_INT *indrow , MKL_INT *indxr , MKL_INT *indxc ,
MKL_INT *ctot , const MKL_INT *npcol , MKL_INT *info );

void pdlaed3 (const MKL_INT *ictxt , MKL_INT *k , const MKL_INT *n , const MKL_INT
*nb , double *d , const MKL_INT *drow , const MKL_INT *dcol , double *rho , double
*dlamda , double *w , const double *z , double *u , const MKL_INT *ldu , double *buf ,
MKL_INT *indx , MKL_INT *indcol , MKL_INT *indrow , MKL_INT *indxr , MKL_INT *indxc ,
MKL_INT *ctot , const MKL_INT *npcol , MKL_INT *info );

void pslaedz (const MKL_INT *n , const MKL_INT *n1 , const MKL_INT *id , const float
*q , const MKL_INT *iq , const MKL_INT *jq , const MKL_INT *ldq , const MKL_INT
*descq , float *z , float *work );

void pdlaedz (const MKL_INT *n , const MKL_INT *n1 , const MKL_INT *id , const double
*q , const MKL_INT *iq , const MKL_INT *jq , const MKL_INT *ldq , const MKL_INT
*descq , double *z , double *work );

void pdlaiectb (const double *sigma , const MKL_INT *n , const double *d , MKL_INT
*count );

void pdlaiectl (const double *sigma , const MKL_INT *n , const double *d , MKL_INT
*count );

void slamov (const char *UPLO , const MKL_INT *M , const MKL_INT *N , const float *A ,
const MKL_INT *LDA , float *B , const MKL_INT *LDB );

void dlamov (const char *UPLO , const MKL_INT *M , const MKL_INT *N , const double *A ,
const MKL_INT *LDA , double *B , const MKL_INT *LDB );

```

```

void clamov (const char *UPLO , const MKL_INT *M , const MKL_INT *N , const
MKL_Complex8 *A , const MKL_INT *LDA , MKL_Complex8 *B , const MKL_INT *LDB );

void zlamov (const char *UPLO , const MKL_INT *M , const MKL_INT *N , const
MKL_Complex16 *A , const MKL_INT *LDA , MKL_Complex16 *B , const MKL_INT *LDB );

void pslamr1d (const MKL_INT *n , float *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , float *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT
*descb );

void pdlamr1d (const MKL_INT *n , double *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , double *b , const MKL_INT *ib , const MKL_INT *jb , const
MKL_INT *descb );

void pclamr1d (const MKL_INT *n , MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT
*ja , const MKL_INT *desca , MKL_Complex8 *b , const MKL_INT *ib , const MKL_INT *jb ,
const MKL_INT *descb );

void pzlamr1d (const MKL_INT *n , MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT
*ja , const MKL_INT *desca , MKL_Complex16 *b , const MKL_INT *ib , const MKL_INT
*jb , const MKL_INT *descb );

void clanv2 (MKL_Complex8 *a , MKL_Complex8 *b , MKL_Complex8 *c , MKL_Complex8 *d ,
MKL_Complex8 *rt1 , MKL_Complex8 *rt2 , float *cs , MKL_Complex8 *sn );

void zlanv2 (MKL_Complex16 *a , MKL_Complex16 *b , MKL_Complex16 *c , MKL_Complex16
*d , MKL_Complex16 *rt1 , MKL_Complex16 *rt2 , double *cs , MKL_Complex16 *sn );

void pclatrs (const char *uplo , const char *trans , const char *diag , const char
*normin , const MKL_INT *n , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT
*ja , const MKL_INT *desca , MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx ,
const MKL_INT *descx , float *scale , float *cnorm , MKL_INT *info );

void pzlatrs (const char *uplo , const char *trans , const char *diag , const char
*normin , const MKL_INT *n , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT
*ja , const MKL_INT *desca , MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT
*jx , const MKL_INT *descx , double *scale , double *cnorm , MKL_INT *info );

void pssytrd (const char *uplo , const MKL_INT *n , float *a , const MKL_INT *ia ,
const MKL_INT *ja , const MKL_INT *desca , float *d , float *e , float *tau , float
*work , const MKL_INT *lwork , MKL_INT *info );

void pdsytrd (const char *uplo , const MKL_INT *n , double *a , const MKL_INT *ia ,
const MKL_INT *ja , const MKL_INT *desca , double *d , double *e , double *tau ,
double *work , const MKL_INT *lwork , MKL_INT *info );

MKL_INT piparmq (const MKL_INT *ictxt , const MKL_INT *ispec , const char *name , const
char *opts , const MKL_INT *n , const MKL_INT *ilo , const MKL_INT *ihi , const MKL_INT
*lworknb );

```

For descriptions of these functions, please see <http://www.netlib.org/scalapack/explore-html/files.html>.

ScaLAPACK Utility Functions and Routines

This section describes ScaLAPACK utility functions and routines. Summary information about these routines is given in the following table:

ScaLAPACK Utility Functions and Routines

| Routine Name | Data Types | Description |
|--------------------------|------------|---|
| <code>p?labad</code> | s,d | Returns the square root of the underflow and overflow thresholds if the exponent-range is very large. |
| <code>p?lachkieee</code> | s,d | Performs a simple check for the features of the IEEE standard. |

| Routine Name | Data Types | Description |
|-----------------------|-------------------|---|
| <code>p?lamch</code> | <code>s, d</code> | Determines machine parameters for floating-point arithmetic. |
| <code>p?lasnbt</code> | <code>s, d</code> | Computes the position of the sign bit of a floating-point number. |

See Also

`pxerbla` Error handling routine called by ScaLAPACK routines.

p?labad

Returns the square root of the underflow and overflow thresholds if the exponent-range is very large.

Syntax

```
void pslabad (MKL_INT *ictxt , float *small , float *large );
void pdlabad (MKL_INT *ictxt , double *small , double *large );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?labad` function takes as input the values computed by `p?lamch` for underflow and overflow, and returns the square root of each of these values if the log of `large` is sufficiently large. This function is intended to identify machines with a large exponent range, such as the Crays, and redefine the underflow and overflow limits to be the square roots of the values computed by `p?lamch`. This function is needed because `p?lamch` does not compensate for poor arithmetic in the upper half of the exponent range, as is found on a Cray.

In addition, this function performs a global minimization and maximization on these values, to support heterogeneous computing networks.

Input Parameters

| | |
|--------------------|---|
| <code>ictxt</code> | (global) The BLACS context handle in which the computation takes place. |
| <code>small</code> | (local). On entry, the underflow threshold as computed by <code>p?lamch</code> . |
| <code>large</code> | (local). On entry, the overflow threshold as computed by <code>p?lamch</code> . |

Output Parameters

| | |
|--------------------|---|
| <code>small</code> | (local). On exit, if $\log_{10}(\text{large})$ is sufficiently large, the square root of <code>small</code> , otherwise unchanged. |
| <code>large</code> | (local). On exit, if $\log_{10}(\text{large})$ is sufficiently large, the square root of <code>large</code> , otherwise unchanged. |

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lachskeee

Performs a simple check for the features of the IEEE standard.

Syntax

```
void pslachskeee (MKL_INT *isieee , float *rmax , float *rmin );  
void pdlachskeee (MKL_INT *isieee , float *rmax , float *rmin );
```

Include Files

- mkl_scalapack.h

Description

The p?lachskeee function performs a simple check to make sure that the features of the IEEE standard are implemented. In some implementations, p?lachskeee may not return.

This is a ScaLAPACK internal function and arguments are not checked for unreasonable values.

Input Parameters

| | |
|-------------|---|
| <i>rmax</i> | (local). The overflow threshold (= ?lamch ('O')). |
| <i>rmin</i> | (local). The underflow threshold (= ?lamch ('U')). |

Output Parameters

| | |
|---------------|--|
| <i>isieee</i> | (local). On exit, <i>isieee</i> = 1 implies that all the features of the IEEE standard that we rely on are implemented. On exit, <i>isieee</i> = 0 implies that some the features of the IEEE standard that we rely on are missing. |
|---------------|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lamch

Determines machine parameters for floating-point arithmetic.

Syntax

```
float pslamch (MKL_INT *ictxt , char *cmach );  
double pdlamch (MKL_INT *ictxt , char *cmach );
```

Include Files

- `mkl_scalapack.h`

Description

The `p?lamch` function determines single precision machine parameters.

Input Parameters

`ictxt` (global). The BLACS context handle in which the computation takes place.

`cmach` (global)
Specifies the value to be returned by `p?lamch`:

- = 'E' or 'e', `p?lamch := eps`
- = 'S' or 's', `p?lamch := sfmin`
- = 'B' or 'b', `p?lamch := base`
- = 'P' or 'p', `p?lamch := eps*base`
- = 'N' or 'n', `p?lamch := t`
- = 'R' or 'r', `p?lamch := rnd`
- = 'M' or 'm', `p?lamch := emin`
- = 'U' or 'u', `p?lamch := rmin`
- = 'L' or 'l', `p?lamch := emax`
- = 'O' or 'o', `p?lamch := rmax,`

where

- `eps` = relative machine precision
- `sfmin` = safe minimum, such that $1/sfmin$ does not overflow
- `base` = base of the machine
- `prec` = `eps*base`
- `t` = number of (base) digits in the mantissa
- `rnd` = 1.0 when rounding occurs in addition, 0.0 otherwise
- `emin` = minimum exponent before (gradual) underflow
- `rmin` = underflow threshold - $base^{(emin-1)}$
- `emax` = largest exponent before overflow
- `rmax` = overflow threshold - $(base^{emax}) * (1-eps)$

Output Parameters

`val` Value returned by the function.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?lasnbt

Computes the position of the sign bit of a floating-point number.

Syntax

```
void pslasnbt (MKL_INT *ieflag );
void pdlasnbt (MKL_INT *ieflag );
```

Include Files

- mkl_scalapack.h

Description

The p?lasnbt function finds the position of the signbit of a single/double precision floating point number. This function assumes IEEE arithmetic, and hence, tests only the 32-nd bit (for single precision) or 32-nd and 64-th bits (for double precision) as a possibility for the signbit. `sizeof(int)` is assumed equal to 4 bytes.

If a compile time flag (`NO_IEEE`) indicates that the machine does not have IEEE arithmetic, `ieflag = 0` is returned.

Output Parameters

ieflag This flag indicates the position of the signbit of any single/double precision floating point number.

ieflag = 0, if the compile time flag `NO_IEEE` indicates that the machine does not have IEEE arithmetic, or if `sizeof(int)` is different from 4 bytes.

ieflag = 1 indicates that the signbit is the 32-nd bit for a single precision function.

In the case of a double precision function:

ieflag = 1 indicates that the signbit is the 32-nd bit (Big Endian).

ieflag = 2 indicates that the signbit is the 64-th bit (Little Endian).

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

ScaLAPACK Redistribution/Copy Routines

This section describes ScaLAPACK redistribution/copy routines. Summary information about these routines is given in the following table:

ScaLAPACK Redistribution/Copy Routines

| Routine Name | Data Types | Description |
|--------------|---------------|--|
| p?gemr2d | s, d, c, z, i | Copies a submatrix from one general rectangular matrix to another. |
| p?trmr2d | s, d, c, z, i | Copies a submatrix from one trapezoidal matrix to another. |

See Also

[pxerbla](#) Error handling routine called by ScaLAPACK routines.

p?gemr2d

Copies a submatrix from one general rectangular matrix to another.

Syntax

```
void psgemr2d (MKL_INT *m, MKL_INT *n, float *a , MKL_INT *ia , MKL_INT *ja , MKL_INT
*desca , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_INT *ictxt );

void pdgemr2d (MKL_INT *m , MKL_INT *n , double *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , double *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_INT
*ictxt );

void pcgemr2d (MKL_INT *m , MKL_INT *n MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_Complex8 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
MKL_INT *ictxt );

void pzgemr2d (MKL_INT *m , MKL_INT *n , MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_Complex16 *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb ,
MKL_INT *ictxt );

void pigemr2d (MKL_INT *m , MKL_INT *n , MKL_INT *a , MKL_INT *ia , MKL_INT *ja ,
MKL_INT *desca , MKL_INT *b , MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_INT
*ictxt );
```

Include Files

- mkl_scalapack.h

Description

The p?gemr2d function copies the indicated matrix or submatrix of A to the indicated matrix or submatrix of B . It provides a truly general copy from any block cyclicly-distributed matrix or submatrix to any other block cyclicly-distributed matrix or submatrix. With p?trmr2d, these functions are the only ones in the ScaLAPACK library which provide inter-context operations: they can take a matrix or submatrix A in context A (distributed over process grid A) and copy it to a matrix or submatrix B in context B (distributed over process grid B).

There does not need to be a relationship between the two operand matrices or submatrices other than their global size and the fact that they are both legal block cyclicly-distributed matrices or submatrices. This means that they can, for example, be distributed across different process grids, have varying block sizes and differing matrix starting points, or be contained in different sized distributed matrices.

Take care when context A is disjoint from context B . The general rules for which parameters need to be set are:

- All calling processes must have the correct m and n .
- Processes in context A must correctly define all parameters describing A .
- Processes in context B must correctly define all parameters describing B .
- Processes which are not members of context A must pass $ctxt_a = -1$ and need not set other parameters describing A .
- Processes which are not members of context B must pass $ctxt_b = -1$ and need not set other parameters describing B .

Because of its generality, p?gemr2d can be used for many operations not usually associated with copy functions. For instance, it can be used to take a matrix on one process and distribute it across a process grid, or the reverse. If a supercomputer is grouped into a virtual parallel machine with a workstation, for instance, this function can be used to move the matrix from the workstation to the supercomputer and back.

In ScaLAPACK, it is called to copy matrices from a two-dimensional process grid to a one-dimensional process grid. It can be used to redistribute matrices so that distributions providing maximal performance can be used by various component libraries, as well.

Note that this function requires an array descriptor with $dtype_ = 1$.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Input Parameters

| | |
|----------|--|
| m | (global) The number of rows of matrix A to be copied ($m \geq 0$). |
| n | (global) The number of columns of matrix A to be copied ($n \geq 0$). |
| a | (local) Pointer into the local memory to array of size $lld_a * LOCC(ja+n-1)$ containing the source matrix A . |
| ia, ja | (global) The row and column indices in the array A indicating the first row and the first column, respectively, of the submatrix of A to copy. $1 \leq ia \leq total_rows_in_a - m + 1$, $1 \leq ja \leq total_columns_in_a - n + 1$. |
| $desca$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix A . Only $dtype_a = 1$ is supported, so $dlen_ = 9$. If the calling process is not part of the context of A , $ctxt_a$ must be equal to -1. |
| ib, jb | (global) The row and column indices in the array B indicating the first row and the first column, respectively, of the submatrix B to which to copy the matrix. $1 \leq ib \leq total_rows_in_b - m + 1$, $1 \leq jb \leq total_columns_in_b - n + 1$. |
| $descb$ | (global and local) array of size $dlen_$. The array descriptor for the distributed matrix B . Only $dtype_b = 1$ is supported, so $dlen_ = 9$. If the calling process is not part of the context of B , $ctxt_b$ must be equal to -1. |
| $ictxt$ | (global). The context encompassing at least the union of all processes in context A and context B . All processes in the context $ictxt$ must call this function, even if they do not own a piece of either matrix. |

Output Parameters

b Pointer into the local memory to array of size $lld_b * LOCC(jb+n-1)$.
Overwritten by the submatrix from *A*.

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

p?trmr2d

Copies a submatrix from one trapezoidal matrix to another.

Syntax

```
void pstrmr2d (char *uplo , char *diag , MKL_INT *m , MKL_INT *n , float *a , MKL_INT
*ia , MKL_INT *ja , MKL_INT *desca , float *b , MKL_INT *ib , MKL_INT *jb , MKL_INT
*descb , MKL_INT *ictxt );
```

```
void pdtrmr2d (char *uplo , char *diag , MKL_INT *m , MKL_INT *n , MKL_INT *nrhs ,
double *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , double *b , MKL_INT *ib ,
MKL_INT *jb , MKL_INT *descb , MKL_INT *ictxt );
```

```
void pctrmr2d (char *uplo , char *diag , MKL_INT *m , MKL_INT *n , MKL_INT *nrhs ,
MKL_Complex8 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex8 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_INT *ictxt );
```

```
void pztrmr2d (char *uplo , char *diag , MKL_INT *m , MKL_INT *n , MKL_INT *nrhs ,
MKL_Complex16 *a , MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_Complex16 *b ,
MKL_INT *ib , MKL_INT *jb , MKL_INT *descb , MKL_INT *ictxt );
```

```
void pitrmr2d (char *uplo , char *diag , MKL_INT *m , MKL_INT *n , MKL_INT *a ,
MKL_INT *ia , MKL_INT *ja , MKL_INT *desca , MKL_INT *b , MKL_INT *ib , MKL_INT *jb ,
MKL_INT *descb , MKL_INT *ictxt );
```

Include Files

- mkl_scalapack.h

Description

The `p?trmr2d` function copies the indicated matrix or submatrix of *A* to the indicated matrix or submatrix of *B*. It provides a truly general copy from any block cyclicly-distributed matrix or submatrix to any other block cyclicly-distributed matrix or submatrix. With `p?gemr2d`, these functions are the only ones in the ScaLAPACK library which provide inter-context operations: they can take a matrix or submatrix *A* in context *A* (distributed over process grid *A*) and copy it to a matrix or submatrix *B* in context *B* (distributed over process grid *B*).

The `p?trmr2d` function assumes the matrix or submatrix to be trapezoidal. Only the upper or lower part is copied, and the other part is unchanged.

There does not need to be a relationship between the two operand matrices or submatrices other than their global size and the fact that they are both legal block cyclicly-distributed matrices or submatrices. This means that they can, for example, be distributed across different process grids, have varying block sizes and differing matrix starting points, or be contained in different sized distributed matrices.

Take care when context *A* is disjoint from context *B*. The general rules for which parameters need to be set are:

- All calling processes must have the correct m and n .
- Processes in context A must correctly define all parameters describing A .
- Processes in context B must correctly define all parameters describing B .
- Processes which are not members of context A must pass $ctxt_a = -1$ and need not set other parameters describing A .
- Processes which are not members of context B must pass $ctxt_b = -1$ and need not set other parameters describing B .

Because of its generality, `p?trmr2d` can be used for many operations not usually associated with copy functions. For instance, it can be used to take a matrix on one process and distribute it across a process grid, or the reverse. If a supercomputer is grouped into a virtual parallel machine with a workstation, for instance, this function can be used to move the matrix from the workstation to the supercomputer and back. In ScaLAPACK, it is called to copy matrices from a two-dimensional process grid to a one-dimensional process grid. It can be used to redistribute matrices so that distributions providing maximal performance can be used by various component libraries, as well.

Note that this function requires an array descriptor with $dtype_ = 1$.

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Input Parameters

| | |
|-------------|--|
| <i>uplo</i> | (global) Specifies whether to copy the upper or lower part of the matrix or submatrix. |
| | <i>uplo</i> = 'U' Copy the upper triangular part. |
| | <i>uplo</i> = 'L' Copy the lower triangular part. |
| <i>diag</i> | (global) Specifies whether to copy the diagonal of the matrix or submatrix. |
| | <i>diag</i> = 'U' Do not copy the diagonal. |
| | <i>diag</i> = 'N' Copy the diagonal. |
| <i>m</i> | (global) The number of rows of matrix A to be copied ($m \geq 0$). |
| <i>n</i> | (global) The number of columns of matrix A to be copied ($n \geq 0$). |
| <i>a</i> | (local) Pointer into the local memory to array of size $lld_a * LOCC(ja+n-1)$ containing the source matrix A . |

| | |
|---------------|--|
| <i>ia, ja</i> | (global) The row and column indices in the array <i>A</i> indicating the first row and the first column, respectively, of the submatrix of <i>A</i> to copy. $1 \leq ia \leq total_rows_in_a - m + 1, 1 \leq ja \leq total_columns_in_a - n + 1$. |
| <i>desca</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>A</i> . Only <i>dtype_a</i> = 1 is supported, so <i>dlen_</i> = 9. If the calling process is not part of the context of <i>A</i> , <i>ctxt_a</i> must be equal to -1. |
| <i>ib, jb</i> | (global) The row and column indices in the array <i>B</i> indicating the first row and the first column, respectively, of the submatrix <i>B</i> to which to copy the matrix. $1 \leq ib \leq total_rows_in_b - m + 1, 1 \leq jb \leq total_columns_in_b - n + 1$. |
| <i>descb</i> | (global and local) array of size <i>dlen_</i> . The array descriptor for the distributed matrix <i>B</i> . Only <i>dtype_b</i> = 1 is supported, so <i>dlen_</i> = 9. If the calling process is not part of the context of <i>B</i> , <i>ctxt_b</i> must be equal to -1. |
| <i>ictxt</i> | (global). The context encompassing at least the union of all processes in context <i>A</i> and context <i>B</i> . All processes in the context <i>ictxt</i> must call this function, even if they do not own a piece of either matrix. |

Output Parameters

| | |
|----------|--|
| <i>b</i> | Pointer into the local memory to array of size $lld_b * LOCC(jb+n-1)$. Overwritten by the submatrix from <i>A</i> . |
|----------|--|

See Also

[Overview of ScaLAPACK Routines](#) for details of ScaLAPACK array descriptor structures and related notations.

Sparse Solver Routines

Intel® Math Kernel Library (Intel® MKL) provides user-callable sparse solver software to solve real or complex, symmetric, structurally symmetric or nonsymmetric, positive definite, indefinite or Hermitian square sparse linear system of algebraic equations.

The terms and concepts required to understand the use of the Intel MKL sparse solver routines are discussed in the [Appendix A "Linear Solvers Basics"](#). If you are familiar with linear sparse solvers and sparse matrix storage schemes, you can skip these sections and go directly to the interface descriptions.

This chapter describes

- the direct sparse solver based on PARDISO* which is referred to here as [Intel MKL PARDISO](#);
- the alternative interface for the direct sparse solver which is referred to here as the [DSS interface](#);
- [iterative sparse solvers \(ISS\)](#) based on the reverse communication interface (RCI);
- and [two preconditioners](#) based on the incomplete LU factorization technique.

Intel MKL PARDISO - Parallel Direct Sparse Solver Interface

This section describes the interface to the shared-memory multiprocessing parallel direct sparse solver known as the Intel MKL PARDISO solver.

The Intel MKL PARDISO package is a high-performance, robust, memory efficient, and easy to use software package for solving large sparse linear systems of equations on shared memory multiprocessors. The solver uses a combination of left- and right-looking Level-3 BLAS supernode techniques [[Schenk00-2](#)]. To improve sequential and parallel sparse numerical factorization performance, the algorithms are based on a Level-3 BLAS update and pipelining parallelism is used with a combination of left- and right-looking supernode techniques [[Schenk00](#), [Schenk01](#), [Schenk02](#), [Schenk03](#)]. The parallel pivoting methods allow complete supernode pivoting to compromise numerical stability and scalability during the factorization process. For sufficiently large problem sizes, numerical experiments demonstrate that the scalability of the parallel algorithm is nearly independent of the shared-memory multiprocessing architecture.

The following table lists the names of the Intel MKL PARDISO routines and describes their general use.

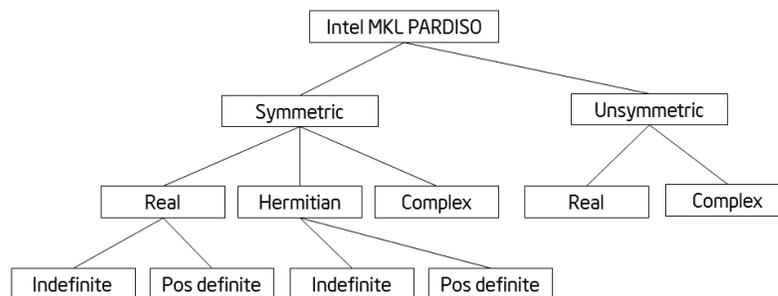
Intel MKL PARDISO Routines

| Routine | Description |
|--------------------------------|---|
| <code>pardisoinit</code> | Initializes Intel MKL PARDISO with default parameters depending on the matrix type. |
| <code>pardiso</code> | Calculates the solution of a set of sparse linear equations with single or multiple right-hand sides. |
| <code>pardiso_64</code> | Calculates the solution of a set of sparse linear equations with single or multiple right-hand sides, 64-bit integer version. |
| <code>pardiso_getenv</code> | Retrieves additional values from the Intel MKL PARDISO handle. |
| <code>pardiso_setenv</code> | Sets additional values in the Intel MKL PARDISO handle. |
| <code>mkl_pardiso_pivot</code> | Replaces routine which handles Intel MKL PARDISO pivots with user-defined routine. |
| <code>pardiso_getdiag</code> | Returns diagonal elements of initial and factorized matrix. |

| Routine | Description |
|--|--|
| <code>pardiso_handle_store</code> | Store internal structures from <code>pardiso</code> to a file. |
| <code>pardiso_handle_restore</code> | Restore <code>pardiso</code> internal structures from a file. |
| <code>pardiso_handle_delete</code> | Delete files with <code>pardiso</code> internal structure data. |
| <code>pardiso_handle_store_64</code> | Store internal structures from <code>pardiso_64</code> to a file. |
| <code>pardiso_handle_restore_64</code> | Restore <code>pardiso_64</code> internal structures from a file. |
| <code>pardiso_handle_delete_64</code> | Delete files with <code>pardiso_64</code> internal structure data. |

The Intel MKL PARDISO solver supports a wide range of real and complex sparse matrix types (see [the figure below](#)).

Sparse Matrices That Can Be Solved with the Intel MKL PARDISO Solver



The Intel MKL PARDISO solver performs four tasks:

- analysis and symbolic factorization
- numerical factorization
- forward and backward substitution including iterative refinement
- termination to release all internal solver memory.

You can find code examples that use Intel MKL PARDISO routines to solve systems of linear equations in the `examples` folder of the Intel MKL installation directory:

- `examples/solverc/source`

Supported Matrix Types

The analysis steps performed by Intel MKL PARDISO depend on the structure of the input matrix A .

Symmetric Matrices

The solver first computes a symmetric fill-in reducing permutation P based on either the minimum degree algorithm [Liu85] or the nested dissection algorithm from the METIS package [Karypis98] (both included with Intel MKL), followed by the parallel left-right looking numerical Cholesky factorization [Schenk00-2] of $PAP^T = LL^T$ for symmetric positive-definite matrices, or $PAP^T = LDL^T$ for symmetric indefinite matrices. The solver uses diagonal pivoting, or 1x1 and 2x2 Bunch-Kaufman pivoting for symmetric indefinite matrices. An approximation of X is found by forward and backward substitution and optional iterative refinement.

Whenever numerically acceptable 1x1 and 2x2 pivots cannot be found within the diagonal supernode block, the coefficient matrix is perturbed. One or two passes of iterative refinement may be required to correct the effect of the perturbations. This restricting notion of pivoting with iterative refinement is effective for highly indefinite symmetric systems. Furthermore, for a large set of matrices from different applications areas, this method is as accurate as a direct factorization method that uses complete sparse pivoting techniques [Schenk04].

Another method of improving the pivoting accuracy is to use symmetric weighted matching algorithms. These algorithms identify large entries in the coefficient matrix A that, if permuted close to the diagonal, permit the factorization process to identify more acceptable pivots and proceed with fewer pivot perturbations. These algorithms are based on maximum weighted matchings and improve the quality of the factor in a complementary way to the alternative of using more complete pivoting techniques.

The inertia is also computed for real symmetric indefinite matrices.

Structurally Symmetric Matrices

The solver first computes a symmetric fill-in reducing permutation P followed by the parallel numerical factorization of $PAP^T = QLU^T$. The solver uses partial pivoting in the supernodes and an approximation of X is found by forward and backward substitution and optional iterative refinement.

Nonsymmetric Matrices

The solver first computes a nonsymmetric permutation P_{MPS} and scaling matrices D_r and D_c with the aim of placing large entries on the diagonal to enhance reliability of the numerical factorization process [Duff99]. In the next step the solver computes a fill-in reducing permutation P based on the matrix $P_{MPS}A + (P_{MPS}A)^T$ followed by the parallel numerical factorization

$$QLUR = PP_{MPS}D_rAD_cP$$

with supernode pivoting matrices Q and R . When the factorization algorithm reaches a point where it cannot factor the supernodes with this pivoting strategy, it uses a pivoting perturbation strategy similar to [Li99]. The magnitude of the potential pivot is tested against a constant threshold of

$$\alpha = \text{eps} * ||A2||_{\text{inf}}$$

where eps is the machine precision, $A2 = P * P_{MPS} * D_r * A * D_c * P$, and $||A2||_{\text{inf}}$ is the infinity norm of A . Any tiny pivots encountered during elimination are set to the sign $(1_{IT}) * \text{eps} * ||A2||_{\text{inf}}$, which trades off some numerical stability for the ability to keep pivots from getting too small. Although many failures could render the factorization well-defined but essentially useless, in practice the diagonal elements are rarely modified for a large class of matrices. The result of this pivoting approach is that the factorization is, in general, not exact and iterative refinement may be needed.

Sparse Data Storage

Intel MKL PARDISO stores sparse data in several formats:

- CSR3: The 3-array variation of the compressed sparse row format described in [Three Array Variation of CSR Format](#).
- BSR3: The three-array variation of the block compressed sparse row format described in [Three Array Variation of BSR Format](#). Use `iparm[36]` to specify the block size.
- VBSR: Variable BSR format. Intel MKL PARDISO analyzes the matrix provided in CSR3 format and converts it into an internal structure which can improve performance for matrices with a block structure. Use `iparm[36] = -t` ($0 < t \leq 100$) to specify use of internal VBSR format and to set the degree of similarity required to combine elements of the matrix. For example, if you set `iparm[36] = -80`, two rows of the input matrix are combined when their non-zero patterns are 80% or more similar.

NOTE

Intel MKL only supports VBSR format for real and symmetric positive definite or indefinite matrices ($mtype = 2$ or $mtype = -2$).

For all storage formats, the Intel MKL PARDISO parameter ja is used for the *columns* array, ia is used for *rowIndex*, and a is used for *values*. The algorithms in Intel MKL PARDISO require column indices ja to be in increasing order per row and that the diagonal element in each row be present for any structurally symmetric matrix. For symmetric or nonsymmetric matrices the diagonal elements which are equal to zero are not necessary.

CAUTION

Intel MKL PARDISO column indices ja must be in increasing order per row. You can validate the sparse matrix structure with the matrix checker ([iparm\[26\]](#))

NOTE

While the presence of zero diagonal elements for symmetric matrices is not required, you should explicitly set zero diagonal elements for symmetric matrices. Otherwise, Intel MKL PARDISO creates internal copies of arrays ia , ja , and a full of diagonal elements, which require additional memory and computational time. However, the memory and time required the diagonal elements in internal arrays is usually not significant compared to the memory and the time required to factor and solve the matrix.

Storage of Matrices

By default, Intel MKL PARDISO stores matrices in RAM. However, you can specify that Intel MKL PARDISO store matrices on disk by setting [iparm\[59\]](#). This is referred to as in-core (IC) and out-of-core (OOC), respectively.

OOC parameters can be set in a configuration file. You can set the path to this file and its name using environmental variables `MKL_PARDISO_OOC_CFG_PATH` and `MKL_PARDISO_OOC_CFG_FILE_NAME`.

These variables specify the path and filename as follows:

`<MKL_PARDISO_OOC_CFG_PATH>/<MKL_PARDISO_OOC_CFG_FILE_NAME>` for Linux* OS and macOS*, and
`<MKL_PARDISO_OOC_CFG_PATH>\<MKL_PARDISO_OOC_CFG_FILE_NAME>` for Windows* OS.

By default, the name of the file is `pardiso_ooc.cfg` and it is placed in the current directory.

All temporary data files can be deleted or stored when the calculations are completed in accordance with the value of the environmental variable `MKL_PARDISO_OOC_KEEP_FILE`. If it is not set or if it is set to 1, then all files are deleted. If it is set to 0, then all files are stored.

By default, the OOC version of Intel MKL PARDISO uses the current directory for storing data, and all work arrays associated with the matrix factors are stored in files named `ooc_temp` with different extensions. These default values can be changed by using the environmental variable `MKL_PARDISO_OOC_PATH`.

To set the environmental variables `MKL_PARDISO_OOC_MAX_CORE_SIZE`, `MKL_PARDISO_OOC_MAX_SWAP_SIZE`, `MKL_PARDISO_OOC_KEEP_FILE`, and `MKL_PARDISO_OOC_PATH`, create the configuration file with the following lines:

```
MKL_PARDISO_OOC_PATH = <path>\ooc_file
MKL_PARDISO_OOC_MAX_CORE_SIZE = N
MKL_PARDISO_OOC_MAX_SWAP_SIZE = K
MKL_PARDISO_OOC_KEEP_FILE = 0 (or 1)
```

where $\langle path \rangle$ is the directory for storing data, ooc_file is the file name without any extension, N is the maximum size of RAM in megabytes available for Intel MKL PARDISO (default value is 2000 MB), K is the maximum swap size in megabytes available for Intel MKL PARDISO (default value is 0 MB). Do not set $N+K$ greater than the size of the RAM plus the size of the swap. Be sure to allow enough free memory for the operating system and any other processes which are necessary.

CAUTION

The maximum length of the path lines in the configuration files is 1000 characters.

Alternatively the environment variables can be set via command line.

For Linux* OS and macOS*:

```
export MKL_PARDISO_OOC_PATH = <path>/ooc_file
export MKL_PARDISO_OOC_MAX_CORE_SIZE = N
export MKL_PARDISO_OOC_MAX_SWAP_SIZE = K
export MKL_PARDISO_OOC_KEEP_FILE = 0 (or 1)
```

For Windows* OS:

```
set MKL_PARDISO_OOC_PATH = <path>\ooc_file
set MKL_PARDISO_OOC_MAX_CORE_SIZE = N
set MKL_PARDISO_OOC_MAX_SWAP_SIZE = K
set MKL_PARDISO_OOC_KEEP_FILE = 0 (or 1)
```

NOTE

The values specified in a command line have higher priorities: if a variable is changed in the configuration file and in the command line, OOC version of Intel MKL PARDISO uses only the value defined in the command line.

Direct-Iterative Preconditioning for Nonsymmetric Linear Systems

The solver uses a combination of direct and iterative methods [Sonn89] to accelerate the linear solution process for transient simulation. Most applications of sparse solvers require solutions of systems with gradually changing values of the nonzero coefficient matrix, but with an identical sparsity pattern. In these applications, the analysis phase of the solvers has to be performed only once and the numerical factorizations are the important time-consuming steps during the simulation. Intel MKL PARDISO uses a numerical factorization and applies the factors in a preconditioned Krylow-Subspace iteration. If the iteration does not converge, the solver automatically switches back to the numerical factorization. This method can be applied to nonsymmetric matrices in Intel MKL PARDISO. You can select the method using the $iparm[3]$ input parameter. The $iparm[19]$ parameter returns the error status after running Intel MKL PARDISO.

Single and Double Precision Computations

Intel MKL PARDISO solves tasks using single or double precision. Each precision has its benefits and drawbacks. Double precision variables have more digits to store value, so the solver uses more memory for keeping data. But this mode solves matrices with better accuracy, which is especially important for input matrices with large condition numbers.

Single precision variables have fewer digits to store values, so the solver uses less memory than in the double precision mode. Additionally this mode usually takes less time. But as computations are made less precisely, only some systems of equations can be solved accurately enough using single precision.

Separate Forward and Backward Substitution

The solver execution step (see $parameter_{phase} = 33$ below) can be divided into two or three separate substitutions: forward, backward, and possible diagonal. This separation can be explained by the examples of solving systems with different matrix types.

A real symmetric positive definite matrix A ($mtype = 2$) is factored by Intel MKL PARDISO as $A = L^*L^T$. In this case the solution of the system $A*x=b$ can be found as sequence of substitutions: $L*y=b$ (forward substitution, $phase = 331$) and $L^T*x=y$ (backward substitution, $phase = 333$).

A real nonsymmetric matrix A ($mtype = 11$) is factored by Intel MKL PARDISO as $A = L*U$. In this case the solution of the system $A*x=b$ can be found by the following sequence: $L*y=b$ (forward substitution, $phase = 331$) and $U*x=y$ (backward substitution, $phase = 333$).

Solving a system with a real symmetric indefinite matrix A ($mtype = -2$) is slightly different from the cases above. Intel MKL PARDISO factors this matrix as $A=LDL^T$, and the solution of the system $A*x=b$ can be calculated as the following sequence of substitutions: $L*y=b$ (forward substitution, $phase = 331$), $D*v=y$ (diagonal substitution, $phase = 332$), and finally $L^T*x=v$ (backward substitution, $phase = 333$). Diagonal substitution makes sense only for symmetric indefinite matrices ($mtype = -2, -4, 6$). For matrices of other types a solution can be found as described in the first two examples.

CAUTION

The number of refinement steps (`iparm[7]`) must be set to zero if a solution is calculated with separate substitutions ($phase = 331, 332, 333$), otherwise Intel MKL PARDISO produces the wrong result.

NOTE

Different pivoting (`iparm[20]`) produces different LDL^T factorization. Therefore results of forward, diagonal and backward substitutions with diagonal pivoting can differ from results of the same steps with Bunch-Kaufman pivoting. Of course, the final results of sequential execution of forward, diagonal and backward substitution are equal to the results of the full solving step ($phase=33$) regardless of the pivoting used.

Callback Function for Pivoting Control

In-core Intel MKL PARDISO allows you to control pivoting with a callback routine, `mkl_pardiso_pivot`. You can then use the `pardiso_getdiag` routine to access the diagonal elements. Set `iparm[55]` to 1 in order to use the callback functionality.

pardiso

Calculates the solution of a set of sparse linear equations with single or multiple right-hand sides.

Syntax

```
void pardiso (_MKL_DSS_HANDLE_t pt, const MKL_INT *maxfct, const MKL_INT *mnum, const
MKL_INT *mtype, const MKL_INT *phase, const MKL_INT *n, const void *a, const MKL_INT
*ia, const MKL_INT *ja, MKL_INT *perm, const MKL_INT *nrhs, MKL_INT *iparm, const
MKL_INT *msglvl, void *b, void *x, MKL_INT *error);
```

Include Files

- `mkl.h`

Description

The routine `pardiso` calculates the solution of a set of sparse linear equations

$$A*X = B$$

with single or multiple right-hand sides, using a parallel LU , LDL , or LL^T factorization, where A is an n -by- n matrix, and X and B are n -by- $nrhs$ vectors or matrices.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

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Input Parameters

pt Array with size of 64.
Handle to internal data structure. The entries must be set to zero prior to the first call to `pardiso`. Unique for factorization.

CAUTION

After the first call to `pardiso` do not directly modify *pt*, as that could cause a serious memory leak.

Use the `pardiso_handle_store` or `pardiso_handle_store_64` routine to store the content of *pt* to a file. Restore the contents of *pt* from the file using `pardiso_handle_restore` or `pardiso_handle_restore_64`. Use `pardiso_handle_store` and `pardiso_handle_restore` with `pardiso`, and `pardiso_handle_store_64` and `pardiso_handle_restore_64` with `pardiso_64`.

maxfct Maximum number of factors with identical sparsity structure that must be kept in memory at the same time. In most applications this value is equal to 1. It is possible to store several different factorizations with the same nonzero structure at the same time in the internal data structure management of the solver.

`pardiso` can process several matrices with an identical matrix sparsity pattern and it can store the factors of these matrices at the same time. Matrices with a different sparsity structure can be kept in memory with different memory address pointers *pt*.

mnum Indicates the actual matrix for the solution phase. With this scalar you can define which matrix to factorize. The value must be: $1 \leq mnum \leq maxfct$.
In most applications this value is 1.

mtype Defines the matrix type, which influences the pivoting method. The Intel MKL PARDISO solver supports the following matrices:

| | |
|----|---|
| 1 | real and structurally symmetric |
| 2 | real and symmetric positive definite |
| -2 | real and symmetric indefinite |
| 3 | complex and structurally symmetric |
| 4 | complex and Hermitian positive definite |
| -4 | complex and Hermitian indefinite |
| 6 | complex and symmetric |
| 11 | real and nonsymmetric |
| 13 | complex and nonsymmetric |

phase

Controls the execution of the solver. Usually it is a two- or three-digit integer. The first digit indicates the starting phase of execution and the second digit indicates the ending phase. Intel MKL PARDISO has the following phases of execution:

- Phase 1: Fill-reduction analysis and symbolic factorization
- Phase 2: Numerical factorization
- Phase 3: Forward and Backward solve including optional iterative refinement

This phase can be divided into two or three separate substitutions: forward, backward, and diagonal (see [Separate Forward and Backward Substitution](#)).

- Memory release phase (*phase*= 0 or *phase*= -1)

If a previous call to the routine has computed information from previous phases, execution may start at any phase. The *phase* parameter can have the following values:

| <i>phase</i> | Solver Execution Steps |
|--------------|--|
| 11 | Analysis |
| 12 | Analysis, numerical factorization |
| 13 | Analysis, numerical factorization, solve, iterative refinement |
| 22 | Numerical factorization |
| 23 | Numerical factorization, solve, iterative refinement |
| 33 | Solve, iterative refinement |
| 331 | like <i>phase</i> =33, but only forward substitution |
| 332 | like <i>phase</i> =33, but only diagonal substitution (if available) |
| 333 | like <i>phase</i> =33, but only backward substitution |

| <i>phase</i> | Solver Execution Steps |
|--------------|---|
| 0 | Release internal memory for L and U matrix number <i>mnum</i> |
| -1 | Release all internal memory for all matrices |

If $iparm[35] = 0$, phases 331, 332, and 333 perform this decomposition:

$$A = \begin{bmatrix} L_{11} & 0 \\ L_{12} & L_{22} \end{bmatrix} \begin{bmatrix} D_{11} & 0 \\ 0 & D_{22} \end{bmatrix} \begin{bmatrix} U_{11} & U_{21} \\ 0 & U_{22} \end{bmatrix}$$

If $iparm[35] = 2$, phases 331, 332, and 333 perform a different decomposition:

$$A = \begin{bmatrix} L_{11} & 0 \\ L_{12} & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} U_{11} & U_{21} \\ 0 & I \end{bmatrix}$$

You can supply a custom implementation for phase 332 instead of calling `pardiso`. For example, it can be implemented with dense LAPACK functionality. Custom implementation also allows you to substitute the matrix S with your own.

NOTE

For very large Schur complement matrices use LAPACK functionality to compute the Schur complement vector instead of the Intel MKL PARDISO phase 332 implementation.

n Number of equations in the sparse linear systems of equations $A^*X = B$.
Constraint: $n > 0$.

a Array. Contains the non-zero elements of the coefficient matrix A corresponding to the indices in ja . The coefficient matrix can be either real or complex. The matrix must be stored in the three-array variant of the compressed sparse row (CSR3) or in the three-array variant of the block compressed sparse row (BSR3) format, and the matrix must be stored with increasing values of ja for each row.

For CSR3 format, the size of a is the same as that of ja . Refer to the *values* array description in [Three Array Variation of CSR Format](#) for more details.

For BSR3 format the size of a is the size of ja multiplied by the square of the block size. Refer to the *values* array description in [Three Array Variation of BSR Format](#) for more details.

NOTE

If you set $iparm[36]$ to a negative value, Intel MKL PARDISO converts the data from CSR3 format to an internal variable BSR (VBSR) format. See [Sparse Data Storage](#).

ia Array, size $(n+1)$.

For CSR3 format, $ia[i]$ ($i < n$) points to the first column index of row i in the array ja . That is, $ia[i]$ gives the index of the element in array a that contains the first non-zero element from row i of A . The last element $ia[n]$ is taken to be equal to the number of non-zero elements in A , plus one. Refer to *rowIndex* array description in [Three Array Variation of CSR Format](#) for more details.

For BSR3 format, $ia[i]$ ($i < n$) points to the first column index of row i in the array ja . That is, $ia[i]$ gives the index of the element in array a that contains the first non-zero block from row i of A . The last element $ia[n]$ is taken to be equal to the number of non-zero blocks in A , plus one. Refer to *rowIndex* array description in [Three Array Variation of BSR Format](#) for more details.

The array ia is accessed in all phases of the solution process.

Indexing of ia is one-based by default, but it can be changed to zero-based by setting the appropriate value to the parameter $iparm[34]$.

ja

For CSR3 format, array ja contains column indices of the sparse matrix A . It is important that the indices are in increasing order per row. For structurally symmetric matrices it is assumed that all diagonal elements are stored (even if they are zeros) in the list of non-zero elements in a and ja . For symmetric matrices, the solver needs only the upper triangular part of the system as is shown for *columns* array in [Three Array Variation of CSR Format](#).

For BSR3 format, array ja contains column indices of the sparse matrix A . It is important that the indices are in increasing order per row. For structurally symmetric matrices it is assumed that all diagonal blocks are stored (even if they are zeros) in the list of non-zero blocks in a and ja . For symmetric matrices, the solver needs only the upper triangular part of the system as is shown for *columns* array in [Three Array Variation of BSR Format](#).

The array ja is accessed in all phases of the solution process.

Indexing of ja is one-based by default, but it can be changed to zero-based by setting the appropriate value to the parameter $iparm[34]$.

$perm$

Array, size (n). Depending on the value of $iparm[4]$ and $iparm[30]$, either holds the permutation vector of size n or specifies elements used for computing a partial solution.

- If $iparm[4] = 1$, $iparm[30] = 0$, and $iparm[35] = 0$, $perm$ specifies the fill-in reducing ordering to the solver. Let A be the original matrix and $C = P^*A^*P^T$ be the permuted matrix. Row (column) i of C is the $perm[i]$ row (column) of A . The array $perm$ is also used to return the permutation vector calculated during fill-in reducing ordering stage.

NOTE

Be aware that setting $iparm[4] = 1$ prevents use of a parallel algorithm for the solve step.

- If $iparm[4] = 2$, $iparm[30] = 0$, and $iparm[35] = 0$, the permutation vector computed in phase 11 is returned in the $perm$ array.

- If $iparm[4] = 0$, $iparm[30] > 0$, and $iparm[35] = 0$, $perm$ specifies elements of the right-hand side to use or of the solution to compute for a partial solution.
- If $iparm[4] = 0$, $iparm[30] = 0$, and $iparm[35] > 0$, $perm$ specifies elements for a Schur complement.

See [iparm\[4\]](#) and [iparm\[30\]](#) for more details.

Indexing of $perm$ is one-based by default, but it can be changed to zero-based by setting the appropriate value to the parameter [iparm\[34\]](#).

| | |
|---------------|--|
| <i>nrhs</i> | Number of right-hand sides that need to be solved for. |
| <i>iparm</i> | Array, size (64). This array is used to pass various parameters to Intel MKL PARDISO and to return some useful information after execution of the solver. See pardiso iparm Parameter for more details about the <i>iparm</i> parameters. |
| <i>msglvl</i> | Message level information. If $msglvl = 0$ then <code>pardiso</code> generates no output, if $msglvl = 1$ the solver prints statistical information to the screen. |
| <i>b</i> | Array, size ($n*nrhs$). On entry, contains the right-hand side vector/matrix B , which is placed in memory contiguously. The $b[+k*nrhs]$ element must hold the i -th component of k -th right-hand side vector. Note that b is only accessed in the solution phase. |

Output Parameters

(See also [Intel MKL PARDISO Parameters in Tabular Form.](#))

| | |
|--------------|---|
| <i>pt</i> | Handle to internal data structure. |
| <i>perm</i> | See the Input Parameter description of the <i>perm</i> array. |
| <i>iparm</i> | On output, some <i>iparm</i> values report information such as the numbers of non-zero elements in the factors. See pardiso iparm Parameter for more details about the <i>iparm</i> parameters. |
| <i>b</i> | On output, the array is replaced with the solution if $iparm[5] = 1$. |
| <i>x</i> | Array, size ($n*nrhs$). If $iparm[5] = 0$ it contains solution vector/matrix X , which is placed contiguously in memory. The $x[i + k*n]$ element must hold the i -th component of the k -th solution vector. Note that x is only accessed in the solution phase. |
| <i>error</i> | The error indicator according to the below table: |

| error | Information |
|--------------|--------------------|
| 0 | no error |
| -1 | input inconsistent |
| -2 | not enough memory |

| error | Information |
|--------------|---|
| -3 | reordering problem |
| -4 | zero pivot, numerical factorization or iterative refinement problem |
| -5 | unclassified (internal) error |
| -6 | reordering failed (matrix types 11 and 13 only) |
| -7 | diagonal matrix is singular |
| -8 | 32-bit integer overflow problem |
| -9 | not enough memory for OOC |
| -10 | error opening OOC files |
| -11 | read/write error with OOC files |
| -12 | (pardiso_64 only) pardiso_64 called from 32-bit library |

pardisoinit

Initialize Intel MKL PARDISO with default parameters in accordance with the matrix type.

Syntax

```
void pardisoinit (_MKL_DSS_HANDLE_t pt, const MKL_INT *mtype, MKL_INT *iparm );
```

Include Files

- mkl.h

Description

This function initializes Intel MKL PARDISO internal address pointer *pt* with zero values (as needed for the very first call of *pardiso*) and sets default *iparm* values in accordance with the matrix type. Intel MKL supplies the *pardisoinit* routine to be compatible with PARDISO 3.2 or lower.

NOTE

An alternative way to set default *iparm* values is to call *pardiso* in the analysis phase with *iparm(1)=0*. In this case you must initialize the internal address pointer *pt* with zero values manually.

NOTE

The *pardisoinit* routine initializes only the in-core version of Intel MKL PARDISO. Switching on the out-of core version of Intel MKL PARDISO as well as changing default *iparm* values can be done after the call to *pardisoinit* but before the first call to *pardiso*.

Input Parameters

| | |
|--------------|--|
| <i>perm</i> | Ignored. |
| <i>mtype</i> | This scalar value defines the matrix type. Based on this value <code>pardisoinit</code> sets default values for the <i>iparm</i> array. Refer to the section Intel MKL PARDISO Parameters in Tabular Form for more details about the default values of Intel MKL PARDISO |

Output Parameters

| | |
|-----------|--|
| <i>pt</i> | Array with a dimension of 64. Solver internal data address pointer. These addresses are passed to the solver, and all related internal memory management is organized through this array. The <code>pardisoinit</code> routine nullifies the array <i>pt</i> . |
|-----------|--|

NOTE

It is very important that the pointer *pt* is initialized with zero before the first call of Intel MKL PARDISO. After that first call you should never modify the pointer, because it could cause a serious memory leak or a crash.

| | |
|--------------|---|
| <i>iparm</i> | Array with a dimension of 64. This array is used to pass various parameters to Intel MKL PARDISO and to return some useful information after execution of the solver. The <code>pardisoinit</code> routine fills-in the <i>iparm</i> array with the default values. Refer to the section Intel MKL PARDISO Parameters in Tabular Form for more details about the default values of Intel MKL PARDISO. |
|--------------|---|

pardiso_64

Calculates the solution of a set of sparse linear equations with single or multiple right-hand sides, 64-bit integer version.

Syntax

```
void pardiso_64 (_MKL_DSS_HANDLE_t pt, const long long int *maxfct, const long long int *mnum, const long long int *mtype, const long long int *phase, const long long int *n, const void *a, const long long int *ia, const long long int *ja, long long int *perm, const long long int *nrhs, long long int *iparm, const long long int *msglvl, void *b, void *x, long long int *error);
```

Include Files

- `mkl.h`

Description

`pardiso_64` is an alternative ILP64 (64-bit integer) version of the `pardiso` routine (see [Description](#) section for more details). The interface of `pardiso_64` is the same as the interface of `pardiso`, but it accepts and returns all integer data as `long long int`.

Use `pardiso_64` when `pardiso` for solving large matrices (with the number of non-zero elements on the order of 500 million or more). You can use it together with the usual LP64 interfaces for the rest of Intel MKL functionality. In other words, if you use 64-bit integer version (`pardiso_64`), you do not need to re-link your applications with ILP64 libraries. Take into account that `pardiso_64` may perform slower than regular `pardiso` on the reordering and symbolic factorization phase.

NOTE

`pardiso_64` is supported only in the 64-bit libraries. If `pardiso_64` is called from the 32-bit libraries, it returns `error = -12`.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

The input parameters of `pardiso_64` are the same as the input parameters of `pardiso`, but `pardiso_64` accepts all integer data as `long long int`.

Output Parameters

The output parameters of `pardiso_64` are the same as the [output parameters of `pardiso`](#), but `pardiso_64` returns all integer data as `long long int`.

`pardiso_getenv`, `pardiso_setenv`

Retrieves additional values from or sets additional values in the Intel MKL PARDISO handle.

Syntax

```
MKL_INT pardiso_getenv (const _MKL_DSS_HANDLE_t handle, const enum PARDISO_ENV_PARAM *param, char *value);
```

```
MKL_INT pardiso_setenv (_MKL_DSS_HANDLE_t handle , const enum PARDISO_ENV_PARAM *param, const char *value);
```

Include Files

- `mkl.h`

Description

These functions operate with the Intel MKL PARDISO handle. The `pardiso_getenv` routine retrieves additional values from the Intel MKL PARDISO handle, and `pardiso_setenv` sets specified values in the Intel MKL PARDISO handle.

These functions enable retrieving and setting the name of the Intel MKL PARDISO OOC file.

To retrieve the Intel MKL PARDISO OOC file name, you can apply this function to any properly-created handle.

To set the Intel MKL PARDISO OOC file name in the handle you must call the function before the reordering stage. This is because the OOC file name is stored in the handle after the reordering stage and it is not changed during further computations.

NOTE

A 1024-byte internal buffer is used inside Intel MKL PARDISO for storing the OOC file name. Allocate a 1024-byte buffer for passing to the `pardiso_getenv` function as the `value` parameter.

Input Parameters

| | |
|---------------------|---|
| <code>handle</code> | Intel MKL PARDISO handle for which to set and from which to retrieve information. (See DSS Interface Description for structure description) |
| <code>param</code> | Specifies the required parameter. The only value is <code>PARDISO_OCC_FILE_NAME</code> , defined in the corresponding include file. |
| <code>value</code> | Input parameter for <code>pardiso_setenv</code> . Contains the name of the OOC file that must be used in the handle. |

Output Parameters

| | |
|---------------------|--|
| <code>handle</code> | Output parameter for <code>pardiso_setenv</code> . Data object of the <code>MKL_DSS_HANDLE</code> type (see DSS Interface Description). |
| <code>value</code> | Output parameter for <code>pardiso_getenv</code> . Contains the name of the OOC file which must be used in the handle. |

mkl_pardiso_pivot

Replaces routine which handles Intel MKL PARDISO pivots with user-defined routine.

Syntax

```
void mkl_pardiso_pivot (const void *ai, void *bi, const void *eps);
```

Include Files

- `mkl.h`

Description

The `mkl_pardiso_pivot` routine allows you to handle diagonal elements which arise during numerical factorization that are zero or near zero. By default, Intel MKL PARDISO determines that a diagonal element `bi` is a pivot if `bi < eps`, and if so, replaces it with `eps`. But you can provide your own routine to modify the resulting factorized matrix in case there are small elements on the diagonal during the factorization step.

NOTE

To use this routine, you must set `iparm[55]` to 1 before the main `pardiso` loop.

Input Parameters

| | |
|-----------------|--|
| <code>ai</code> | Diagonal element of initial matrix corresponding to pivot element. |
| <code>bi</code> | Diagonal element of factorized matrix that could be chosen as a pivot element. |

eps Scalar to compare with diagonal of factorized matrix. On input equal to parameter described by *iparm[9]*.

Output Parameters

bi In case element is chosen as a pivot, value with which to replace the pivot.

pardiso_getdiag

Returns diagonal elements of initial and factorized matrix.

Syntax

```
void pardiso_getdiag (const _MKL_DSS_HANDLE_t pt, void *df, void *da, const MKL_INT *mnum, MKL_INT *error);
```

Include Files

- mkl.h

Description

This routine returns the diagonal elements of the initial and factorized matrix for a real or Hermitian matrix.

NOTE

In order to use this routine, you must set *iparm[55]* to 1 before the main *pardiso* loop.

Input Parameters

pt Array with a size of 64. Handle to internal data structure for the Intel MKL PARDISO solver. The entries must be set to zero prior to the first call to *pardiso*. Unique for factorization.

mnum Indicates the actual matrix for the solution phase of the Intel MKL PARDISO solver. With this scalar you can define the diagonal elements of the factorized matrix that you want to obtain. The value must be: 1 $\leq mnum \leq maxfct$. In most applications this value is 1.

Output Parameters

df Array with a dimension of *n*. Contains diagonal elements of the factorized matrix after factorization.

NOTE

Elements of *df* correspond to diagonal elements of matrix *L* computed during phase 22. Because during phase 22 Intel MKL PARDISO makes additional permutations to improve stability, it is possible that array *df* is not in line with the *perm* array computed during phase 11.

da Array with a dimension of *n*. Contains diagonal elements of the initial matrix.

NOTE

Elements of *da* correspond to diagonal elements of matrix *L* computed during phase 22. Because during phase 22 Intel MKL PARDISO makes additional permutations to improve stability, it is possible that array *da* is not in line with the *perm* array computed during phase 11.

error The error indicator.

| error | Information |
|--------------|--|
| 0 | no error |
| -1 | Diagonal information not turned on before <code>pardiso</code> main loop (<code>iparm[55]=0</code>). |

pardiso_handle_store

Store internal structures from `pardiso` to a file.

Syntax

```
void pardiso_handle_store (_MKL_DSS_HANDLE_t pt, const char *dirname, MKL_INT *error);
```

Include Files

- `mkl.h`

Description

This function stores Intel MKL PARDISO structures to a file, allowing you to store Intel MKL PARDISO internal structures between the stages of the `pardiso` routine. The `pardiso_handle_restore` routine can restore the Intel MKL PARDISO internal structures from the file.

Input Parameters

| | |
|----------------|--|
| <i>pt</i> | Array with a size of 64. Handle to internal data structure. |
| <i>dirname</i> | String containing the name of the directory to which to write the files with the content of the internal structures. Use an empty string ("") to specify the current directory. The routine creates a file named <code>handle.pds</code> in the directory. |

Output Parameters

| <i>pt</i> | Handle to internal data structure. |
|--------------|------------------------------------|
| <i>error</i> | The error indicator. |
| error | Information |
| 0 | No error. |
| -2 | Not enough memory. |
| -10 | Cannot open file for writing. |

| error | Information |
|--------------|------------------------------|
| -11 | Error while writing to file. |
| -13 | Wrong file format. |

pardiso_handle_restore

Restore pardiso internal structures from a file.

Syntax

```
void pardiso_handle_restore (_MKL_DSS_HANDLE_t pt, const char *dirname, MKL_INT *error);
```

Include Files

- mkl.h

Description

This function restores Intel MKL PARDISO structures from a file. This allows you to restore Intel MKL PARDISO internal structures stored by `pardiso_handle_store` after a phase of the `pardiso` routine and continue execution of the next phase.

Input Parameters

dirname String containing the name of the directory in which the file with the content of the internal structures are located. Use an empty string ("") to specify the current directory.

Output Parameters

pt Array with a dimension of 64. Handle to internal data structure.

error The error indicator.

| error | Information |
|--------------|--------------------------------|
| 0 | No error. |
| -2 | Not enough memory. |
| -10 | Cannot open file for reading. |
| -11 | Error while reading from file. |
| -13 | Wrong file format. |

pardiso_handle_delete

Delete files with pardiso internal structure data.

Syntax

```
void pardiso_handle_delete (const char *dirname, MKL_INT *error);
```

Include Files

- `mkl.h`

Description

This function deletes files generated with `pardiso_handle_store` that contain Intel MKL PARDISO internal structures.

Input Parameters

dirname String containing the name of the directory in which the file with the content of the internal structures are located. Use an empty string ("") to specify the current directory.

Output Parameters

error The error indicator.

| error | Information |
|--------------|----------------------|
| 0 | No error. |
| -10 | Cannot delete files. |

`pardiso_handle_store_64`

Store internal structures from `pardiso_64` to a file.

Syntax

```
void pardiso_handle_store_64 (_MKL_DSS_HANDLE_t pt, const char *dirname, MKL_INT *error);
```

Include Files

- `mkl.h`

Description

This function stores Intel MKL PARDISO structures to a file, allowing you to store Intel MKL PARDISO internal structures between the stages of the `pardiso_64` routine. The `pardiso_handle_restore_64` routine can restore the Intel MKL PARDISO internal structures from the file.

Input Parameters

pt Array with a dimension of 64. Handle to internal data structure.

dirname String containing the name of the directory to which to write the files with the content of the internal structures. Use an empty string ("") to specify the current directory. The routine creates a file named `handle.pds` in the directory.

Output Parameters

pt Handle to internal data structure.

error The error indicator.

| error | Information |
|--------------|--|
| 0 | No error. |
| -2 | Not enough memory. |
| -10 | Cannot open file for writing. |
| -11 | Error while writing to file. |
| -12 | Not supported in 32-bit library - routine is only supported in 64-bit libraries. |
| -13 | Wrong file format. |

pardiso_handle_restore_64

Restore pardiso_64 internal structures from a file.

Syntax

```
void pardiso_handle_restore_64 (_MKL_DSS_HANDLE_t pt, const char *dirname, MKL_INT *error);
```

Include Files

- mkl.h

Description

This function restores Intel MKL PARDISO structures from a file. This allows you to restore Intel MKL PARDISO internal structures stored by `pardiso_handle_store_64` after a phase of the `pardiso_64` routine and continue execution of the next phase.

Input Parameters

dirname String containing the name of the directory in which the file with the content of the internal structures are located. Use an empty string ("") to specify the current directory.

Input Parameters

pt Array with a dimension of 64. Handle to internal data structure.

error The error indicator.

| error | Information |
|--------------|--------------------------------|
| 0 | No error. |
| -2 | Not enough memory. |
| -10 | Cannot open file for reading. |
| -11 | Error while reading from file. |
| -13 | Wrong file format. |

pardiso_handle_delete_64

Syntax

Delete files with `pardiso_64` internal structure data.

```
void pardiso_handle_delete_64 (const char *dirname, MKL_INT *error);
```

Include Files

- `mkl.h`

Description

This function deletes files generated with `pardiso_handle_store_64` that contain Intel MKL PARDISO internal structures.

Input Parameters

dirname String containing the name of the directory in which the file with the content of the internal structures are located. Use an empty string ("") to specify the current directory.

Output Parameters

error The error indicator.

| error | Information |
|--------------|--|
| 0 | No error. |
| -10 | Cannot delete files. |
| -12 | Not supported in 32-bit library - routine is only supported in 64-bit libraries. |

Intel MKL PARDISO Parameters in Tabular Form

The following table lists all parameters of Intel MKL PARDISO and gives their brief descriptions.

| Parameter | Type | Description | Values | Comments | In/Out |
|---------------|----------|--|---------------------|---|--------|
| <i>pt</i> | void* | Solver internal data address pointer | 0 | Must be initialized with zeros and never be modified later | in/out |
| <i>maxfct</i> | MKL_INT* | Maximal number of factors in memory | >0 | Generally used value is 1 | in |
| <i>mnum</i> | MKL_INT* | The number of matrix (from 1 to <i>maxfct</i>) to solve | [1: <i>maxfct</i>] | Generally used value is 1 | in |
| <i>mtype</i> | MKL_INT* | Matrix type | 1 2 | Real and structurally symmetric Real and symmetric positive definite | in |

| Parameter | Type | Description | Values | Comments | In/Out |
|--------------|----------|---|--------|---|--------|
| | | | -2 | Real and symmetric indefinite | |
| | | | 3 | Complex and structurally symmetric | |
| | | | 4 | Complex and Hermitian positive definite | |
| | | | -4 | Complex and Hermitian indefinite | |
| | | | 6 | Complex and symmetric matrix | |
| | | | 11 | Real and nonsymmetric matrix | |
| | | | 13 | Complex and nonsymmetric matrix | |
| <i>phase</i> | MKL_INT* | Controls the execution of the solver For <i>iparm</i> [35] > 0, phases 331, 332, and 333 perform a different decomposition. See the <i>phase</i> parameter of pardiso for details. | 11 | Analysis | in |
| | | | 12 | Analysis, numerical factorization | |
| | | | 13 | Analysis, numerical factorization, solve | |
| | | | 22 | Numerical factorization | |
| | | | 23 | Numerical factorization, solve | |
| | | | 33 | Solve, iterative refinement | |
| | | | 331 | <i>phase</i> =33, but only forward substitution | |
| | | | 332 | <i>phase</i> =33, but only diagonal substitution | |
| | | | 333 | <i>phase</i> =33, but only backward substitution | |
| | | | 0 | Release internal memory for L and U of the matrix number <i>mnum</i> | |
| | | | -1 | Release all internal memory for all matrices | |
| <i>n</i> | MKL_INT* | Number of equations in the sparse linear system $A*X = B$ | >0 | | in |
| <i>a</i> | void* | Contains the non-zero elements of the coefficient matrix <i>A</i> | * | The size of <i>a</i> is the same as that of <i>ja</i> , and the coefficient matrix can be either real or complex. The matrix must be stored in the 3-array variation of | in |

| Parameter | Type | Description | Values | Comments | In/Out |
|--------------------------|----------|---|--------|---|--------|
| <i>ia</i> [<i>n</i>] | MKL_INT* | <i>rowIndex</i> array in CSR3 format | >=0 | <p>compressed sparse row (CSR3) format with increasing values of <i>ja</i> for each row</p> <p><i>ia</i>[<i>i</i>] gives the index of the element in array <i>a</i> that contains the first non-zero element from row <i>i</i> of <i>A</i>. The last element <i>ia</i>(<i>n</i>) is taken to be equal to the number of non-zero elements in <i>A</i>.</p> <p>Note: <i>iparm</i>[34] indicates whether row/column indexing starts from 1 or 0.</p> | in |
| <i>ja</i> | MKL_INT* | <i>columns</i> array in CSR3 format | >=0 | <p>The column indices for each row of <i>A</i> must be sorted in increasing order. For structurally symmetric matrices zero diagonal elements must be stored in <i>a</i> and <i>ja</i>. Zero diagonal elements should be stored for symmetric matrices, although they are not required. For symmetric matrices, the solver needs only the upper triangular part of the system.</p> <p>Note: <i>iparm</i>[34] indicates whether row/column indexing starts from 1 or 0.</p> | in |
| <i>perm</i> [<i>n</i>] | MKL_INT* | Holds the permutation vector of size <i>n</i> or specifies elements used for computing a partial solution | >=0 | <p>You can apply your own fill-in reducing ordering (<i>iparm</i>[4] = 1) or return the permutation from the solver (<i>iparm</i>[4] = 2).</p> <p>Let $C = P^* A^* P^T$ be the permuted matrix. Row (column) <i>i</i> of <i>C</i> is the <i>perm</i>(<i>i</i>) row (column) of <i>A</i>. The numbering of the array must describe a permutation.</p> <p>To specify elements for a partial solution, set <i>iparm</i>[4] = 0, <i>iparm</i>[30] > 0, and <i>iparm</i>[35] = 0.</p> | in/out |

| Parameter | Type | Description | Values | Comments | In/Out |
|------------------|----------|---|----------|---|--------|
| | | | | <p>To specify elements for a Schur complement, set $iparm[4] = 0$, $iparm[30] = 0$, and $iparm[35] > 0$.</p> <p>Note: $iparm[34]$ indicates whether row/column indexing starts from 1 or 0.</p> | |
| <i>nrhs</i> | MKL_INT* | Number of right-hand sides that need to be solved for | ≥ 0 | <p>Generally used value is 1</p> <p>To obtain better Intel MKL PARDISO performance, during the numerical factorization phase you can provide the maximum number of right-hand sides, which can be used further during the solving phase.</p> | in |
| <i>iparm[64]</i> | MKL_INT* | This array is used to pass various parameters to Intel MKL PARDISO and to return some useful information after execution of the solver (see pardiso iparm Parameter for more details) | * | If $iparm[0]=0$, Intel MKL PARDISO fills $iparm[1]$ through $iparm[63]$ with default values and uses them. | in/out |
| <i>msglvl</i> | MKL_INT* | Message level information | 0 1 | <p>Intel MKL PARDISO generates no output</p> <p>Intel MKL PARDISO prints statistical information</p> | in |
| <i>b[n*nrhs]</i> | void* | Right-hand side vectors | * | <p>On entry, contains the right-hand side vector/matrix B, which is placed contiguously in memory. The $b[i+k*n]$ element must hold the i-th component of k-th right-hand side vector. Note that b is only accessed in the solution phase.</p> <p>On output, the array is replaced with the solution if $iparm[5]=1$.</p> | in/out |
| <i>x[n*nrhs]</i> | void* | Solution vectors | * | <p>On output, if $iparm[5]=0$, contains solution vector/matrix X which is placed contiguously in memory. The $x[i+k*n]$ element</p> | out |

| Parameter | Type | Description | Values | Comments | In/Out |
|--------------|----------|-----------------|--------|---|--------|
| | | | | must hold the i -th component of k -th solution vector. Note that x is only accessed in the solution phase. | |
| <i>error</i> | MKL_INT* | Error indicator | 0 | No error | out |
| | | | -1 | Input inconsistent | |
| | | | -2 | Not enough memory | |
| | | | -3 | Reordering problem | |
| | | | -4 | Zero pivot, numerical factorization or iterative refinement problem | |
| | | | -5 | Unclassified (internal) error | |
| | | | -6 | Reordering failed (matrix types 11 and 13 only) | |
| | | | -7 | Diagonal matrix is singular | |
| | | | -8 | 32-bit integer overflow problem | |
| | | | -9 | Not enough memory for OOC | |
| | | | -10 | Problems with opening OOC temporary files | |
| | | | -11 | Read/write problems with the OOC data file | |

¹⁾ See description of PARDISO_DATA_TYPE in [PARDISO_DATA_TYPE](#).

pardiso iparm Parameter

The following table describes all individual components of the Intel MKL PARDISO *iparm* parameter. Components which are not used must be initialized with 0. Default values are denoted with an asterisk (*).

| Component | Description |
|------------------|---|
| <i>iparm</i> [0] | Use default values. |
| input | 0 <i>iparm</i> [1] - <i>iparm</i> [63] are filled with default values. #0 You must supply all values in components <i>iparm</i> [1] - <i>iparm</i> [63]. |
| <i>iparm</i> [1] | Fill-in reducing ordering for the input matrix. |
| input | <p>CAUTION</p> <p>You can control the parallel execution of the solver by explicitly setting the MKL_NUM_THREADS environment variable. If fewer OpenMP threads are available than specified, the execution may slow down instead of speeding up. If MKL_NUM_THREADS is not defined, then the solver uses all available processors.</p> |

| Component | Description |
|-----------|---|
| 0 | The minimum degree algorithm [Li99]. |
| 2* | The nested dissection algorithm from the METIS package [Karypis98]. |
| 3 | The parallel (OpenMP) version of the nested dissection algorithm. It can decrease the time of computations on multi-core computers, especially when Intel MKL PARDISO Phase 1 takes significant time. |

NOTE

Setting $iparm[1] = 3$ prevents the use of CNR mode ($iparm[33] > 0$) because Intel MKL PARDISO uses dynamic parallelism.

| | |
|---------------------|---|
| $iparm[2]$ | Reserved. Set to zero. |
| $iparm[3]$ input | Preconditioned CGS/CG. This parameter controls preconditioned CGS [Sonn89] for nonsymmetric or structurally symmetric matrices and Conjugate-Gradients for symmetric matrices. $iparm[3]$ has the form $iparm[3] = 10 * L + K$. |
| K=0 | The factorization is always computed as required by <i>phase</i> . |
| K=1 | CGS iteration replaces the computation of <i>LU</i> . The preconditioner is <i>LU</i> that was computed at a previous step (the first step or last step with a failure) in a sequence of solutions needed for identical sparsity patterns. |
| K=2 | CGS iteration for symmetric positive definite matrices replaces the computation of LL^T . The preconditioner is LL^T that was computed at a previous step (the first step or last step with a failure) in a sequence of solutions needed for identical sparsity patterns. |

The value *L* controls the stopping criterion of the Krylow-Subspace iteration:

$eps_{CGS} = 10^{-L}$ is used in the stopping criterion

$$||dx_i|| / ||dx_0|| < eps_{CGS}$$

where $||dx_i|| = ||inv(L*U)*r_i||$ for $K = 1$ or $||dx_i|| = ||inv(L*L^T)*r_i||$ for $K = 2$ and r_i is the residue at iteration *i* of the preconditioned Krylow-Subspace iteration.

A maximum number of 150 iterations is fixed with the assumption that the iteration will converge before consuming half the factorization time. Intermediate convergence rates and residue excursions are checked and can terminate the iteration process. If *phase* =23, then the factorization for a given *A* is automatically recomputed in cases where the Krylow-Subspace iteration failed, and the corresponding direct solution is returned. Otherwise the solution from the preconditioned Krylow-Subspace iteration is returned. Using *phase* =33 results in an error message (*error*=-4) if the stopping criteria for the Krylow-Subspace iteration can not be reached. More information on the failure can be obtained from $iparm[19]$.

The default is $iparm[3]=0$, and other values are only recommended for an advanced user. $iparm[3]$ must be greater than or equal to zero.

Examples:

| $iparm[3]$ | Description |
|------------|---|
| 31 | <i>LU</i> -preconditioned CGS iteration with a stopping criterion of 1.0E-3 for nonsymmetric matrices |
| 61 | <i>LU</i> -preconditioned CGS iteration with a stopping criterion of 1.0E-6 for nonsymmetric matrices |
| 62 | LL^T -preconditioned CGS iteration with a stopping criterion of 1.0E-6 for symmetric positive definite matrices |

| | |
|------------|-------------------|
| $iparm[4]$ | User permutation. |
|------------|-------------------|

| Component | Description |
|----------------------------|--|
| input | <p>This parameter controls whether user supplied fill-in reducing permutation is used instead of the integrated multiple-minimum degree or nested dissection algorithms. Another use of this parameter is to control obtaining the fill-in reducing permutation vector calculated during the reordering stage of Intel MKL PARDISO.</p> <p>This option is useful for testing reordering algorithms, adapting the code to special applications problems (for instance, to move zero diagonal elements to the end of $P^*A^*P^T$), or for using the permutation vector more than once for matrices with identical sparsity structures. For definition of the permutation, see the description of the <i>perm</i> parameter.</p> <hr/> <p>CAUTION You can only set one of <i>iparm</i>[4], <i>iparm</i>[30], and <i>iparm</i>[35], so be sure that the <i>iparm</i>[30] (partial solution) and the <i>iparm</i>[35] (Schur complement) parameters are 0 if you set <i>iparm</i>[4].</p> <hr/> <p>0* User permutation in the <i>perm</i> array is ignored.</p> <hr/> <p>1 Intel MKL PARDISO uses the user supplied fill-in reducing permutation from the <i>perm</i> array. <i>iparm</i>[1] is ignored.</p> <hr/> <p>NOTE Setting <i>iparm</i>[4] = 1 prevents use of a parallel algorithm for the solve step.</p> <hr/> <p>2 Intel MKL PARDISO returns the permutation vector computed at phase 1 in the <i>perm</i> array.</p> |
| <i>iparm</i> [5] input | <p>Write solution on <i>x</i>.</p> <hr/> <p>NOTE The array <i>x</i> is always used.</p> <hr/> <p>0* The array <i>x</i> contains the solution; right-hand side vector <i>b</i> is kept unchanged.</p> <hr/> <p>1 The solver stores the solution on the right-hand side <i>b</i>.</p> |
| <i>iparm</i> [6] output | <p>Number of iterative refinement steps performed.</p> <p>Reports the number of iterative refinement steps that were actually performed during the solve step.</p> |
| <i>iparm</i> [7] input | <p>Iterative refinement step.</p> <p>On entry to the solve and iterative refinement step, <i>iparm</i>[7] must be set to the maximum number of iterative refinement steps that the solver performs.</p> <hr/> <p>0* The solver automatically performs two steps of iterative refinement when perturbed pivots are obtained during the numerical factorization.</p> <hr/> <p>>0 Maximum number of iterative refinement steps that the solver performs. The solver performs not more than the absolute value of <i>iparm</i>[7] steps of iterative refinement. The solver might stop the process before the maximum number of steps if</p> <ul style="list-style-type: none"> • a satisfactory level of accuracy of the solution in terms of backward error is achieved, |

| Component | Description |
|---------------------------|--|
| | <ul style="list-style-type: none"> or if it determines that the required accuracy cannot be reached. In this case Intel MKL PARDISO returns -4 in the <i>error</i> parameter. <p>The number of executed iterations is reported in <i>iparm[6]</i>.</p> |
| <0 | <p>Same as above, but the accumulation of the residuum uses extended precision real and complex data types.</p> <p>Perturbed pivots result in iterative refinement (independent of <i>iparm[7]=0</i>) and the number of executed iterations is reported in <i>iparm[6]</i>.</p> |
| <i>iparm[8]</i> | Reserved. Set to zero. |
| <i>iparm[9]</i> input | <p>Pivoting perturbation.</p> <p>This parameter instructs Intel MKL PARDISO how to handle small pivots or zero pivots for nonsymmetric matrices (<i>mtype</i> =11 or <i>mtype</i> =13) and symmetric matrices (<i>mtype</i> =-2, <i>mtype</i> =-4, or <i>mtype</i> =6). For these matrices the solver uses a complete supernode pivoting approach. When the factorization algorithm reaches a point where it cannot factor the supernodes with this pivoting strategy, it uses a pivoting perturbation strategy similar to [Li99], [Schenk04].</p> <p>Small pivots are perturbed with $\text{eps} = 10^{-\text{iparm}[9]}$.</p> <p>The magnitude of the potential pivot is tested against a constant threshold of $\alpha = \text{eps} * \ A2\ _{\text{inf}}$, where $\text{eps} = 10^{(-\text{iparm}[9])}$, $A2 = P * P_{\text{MPS}} * D_r * A * D_c * P$, and $\ A2\ _{\text{inf}}$ is the infinity norm of the scaled and permuted matrix <i>A</i>. Any tiny pivots encountered during elimination are set to the sign (<i>1</i> or <i>-1</i>) * $\text{eps} * \ A2\ _{\text{inf}}$, which trades off some numerical stability for the ability to keep pivots from getting too small. Small pivots are therefore perturbed with $\text{eps} = 10^{(-\text{iparm}[9])}$.</p> |
| 13* | The default value for nonsymmetric matrices(<i>mtype</i> =11, <i>mtype</i> =13), $\text{eps} = 10^{-13}$. |
| 8* | The default value for symmetric indefinite matrices (<i>mtype</i> =-2, <i>mtype</i> =-4, <i>mtype</i> =6), $\text{eps} = 10^{-8}$. |
| <i>iparm[10]</i> input | <p>Scaling vectors.</p> <p>Intel MKL PARDISO uses a maximum weight matching algorithm to permute large elements on the diagonal and to scale so that the diagonal elements are equal to 1 and the absolute values of the off-diagonal entries are less than or equal to 1. This scaling method is applied only to nonsymmetric matrices (<i>mtype</i> = 11 or <i>mtype</i> = 13). The scaling can also be used for symmetric indefinite matrices (<i>mtype</i> = -2, <i>mtype</i> =-4, or <i>mtype</i> = 6) when the symmetric weighted matchings are applied (<i>iparm[12]</i> = 1).</p> <p>Use <i>iparm[10]</i> = 1 (scaling) and <i>iparm[12]</i> = 1 (matching) for highly indefinite symmetric matrices, for example, from interior point optimizations or saddle point problems. Note that in the analysis phase (<i>phase</i>=11) you must provide the numerical values of the matrix <i>A</i> in array <i>a</i> in case of scaling and symmetric weighted matching.</p> |
| 0* | Disable scaling. Default for symmetric indefinite matrices. |
| 1* | <p>Enable scaling. Default for nonsymmetric matrices.</p> <p>Scale the matrix so that the diagonal elements are equal to 1 and the absolute values of the off-diagonal entries are less than or equal to 1. This scaling method is applied to nonsymmetric matrices (<i>mtype</i> = 11, <i>mtype</i> = 13). The scaling can also be used for symmetric indefinite matrices (<i>mtype</i> = -2, <i>mtype</i> = -4, <i>mtype</i> = 6) when the symmetric weighted matchings are applied (<i>iparm[12]</i> = 1).</p> |

| Component | Description |
|-----------------------------|---|
| | Note that in the analysis phase (<i>phase</i> =11) you must provide the numerical values of the matrix <i>A</i> in case of scaling. |
| <i>iparm</i> [11] input | Solve with transposed or conjugate transposed matrix <i>A</i> . |
| | NOTE For real matrices the terms <i>transposed</i> and <i>conjugate transposed</i> are equivalent. |
| | 0* Solve a linear system $AX = B$. |
| | 1 Solve a conjugate transposed system $A^HX = B$ based on the factorization of the matrix <i>A</i> . |
| | 2 Solve a transposed system $A^TX = B$ based on the factorization of the matrix <i>A</i> . |
| <i>iparm</i> [12] input | Improved accuracy using (non-) symmetric weighted matching. Intel MKL PARDISO can use a maximum weighted matching algorithm to permute large elements close the diagonal. This strategy adds an additional level of reliability to the factorization methods and complements the alternative of using more complete pivoting techniques during the numerical factorization. |
| | 0* Disable matching. Default for symmetric indefinite matrices. |
| | 1* Enable matching. Default for nonsymmetric matrices. Maximum weighted matching algorithm to permute large elements close to the diagonal. It is recommended to use <i>iparm</i> [10] = 1 (scaling) and <i>iparm</i> [12] = 1 (matching) for highly indefinite symmetric matrices, for example from interior point optimizations or saddle point problems. Note that in the analysis phase (<i>phase</i> =11) you must provide the numerical values of the matrix <i>A</i> in case of symmetric weighted matching. |
| <i>iparm</i> [13] output | Number of perturbed pivots. After factorization, contains the number of perturbed pivots for the matrix types: 11, 13, -2, -4 and -6. |
| <i>iparm</i> [14] output | Peak memory on symbolic factorization. The total peak memory in kilobytes that the solver needs during the analysis and symbolic factorization phase. This value is only computed in phase 1. |
| <i>iparm</i> [15] output | Permanent memory on symbolic factorization. Permanent memory from the analysis and symbolic factorization phase in kilobytes that the solver needs in the factorization and solve phases. This value is only computed in phase 1. |
| <i>iparm</i> [16] output | Size of factors/Peak memory on numerical factorization and solution. This parameter provides the size in kilobytes of the total memory consumed by in-core Intel MKL PARDISO for internal floating point arrays. This parameter is computed in phase 1. See <i>iparm</i> [62] for the OOC mode. The total peak memory consumed by Intel MKL PARDISO is $\max(iparm[14], iparm[15]+iparm[16])$ |

| Component | Description |
|-----------------------------------|--|
| <i>iparm</i> [17] input/output | Report the number of non-zero elements in the factors. <hr/> <0 Enable reporting if <i>iparm</i> [17] < 0 on entry. The default value is -1. <hr/> >=0 Disable reporting. |
| <i>iparm</i> [18] input/output | Report number of floating point operations (in 10 ⁶ floating point operations) that are necessary to factor the matrix <i>A</i> . <hr/> <0 Enable report if <i>iparm</i> [18] < 0 on entry. This increases the reordering time. <hr/> >=0 Disable report. * |
| <i>iparm</i> [19] output | Report CG/CGS diagnostics. <hr/> >0 CGS succeeded, reports the number of completed iterations. <hr/> <0 CG/CGS failed (<i>error</i> =-4 after the solution phase). If <i>phase</i> = 23, then the factors <i>L</i> and <i>U</i> are recomputed for the matrix <i>A</i> and the error flag <i>error</i> =0 in case of a successful factorization. If <i>phase</i> = 33, then <i>error</i> = -4 signals failure. $iparm[19] = - it_cgs * 10 - cgs_error.$ Possible values of <i>cgs_error</i> : 1 - fluctuations of the residuum are too large 2 - $ dx_{max_it_cgs}/2 $ is too large (slow convergence) 3 - stopping criterion is not reached at <i>max_it_cgs</i> 4 - perturbed pivots caused iterative refinement 5 - factorization is too fast for this matrix. It is better to use the factorization method with <i>iparm</i> [3] = 0 |
| <i>iparm</i> [20] input | Pivoting for symmetric indefinite matrices. <hr/> NOTE Use <i>iparm</i> [10] = 1 (scaling) and <i>iparm</i> [12] = 1 (matchings) for highly indefinite symmetric matrices, for example from interior point optimizations or saddle point problems. <hr/> 0 Apply 1x1 diagonal pivoting during the factorization process. <hr/> 1* Apply 1x1 and 2x2 Bunch-Kaufman pivoting during the factorization process. Bunch-Kaufman pivoting is available for matrices of <i>mtype</i> =-2, <i>mtype</i> =-4, or <i>mtype</i> =6. <hr/> 2 Apply 1x1 diagonal pivoting during the factorization process. Using this value is the same as using <i>iparm</i> [20] = 0 except that the solve step does not automatically make iterative refinements when perturbed pivots are obtained during numerical factorization. The number of iterations is limited to the number of iterative refinements specified by <i>iparm</i> [7] (0 by default). <hr/> 3 Apply 1x1 and 2x2 Bunch-Kaufman pivoting during the factorization process. Bunch-Kaufman pivoting is available for matrices of <i>mtype</i> =-2, <i>mtype</i> =-4, or <i>mtype</i> =6. Using this value is the same as using <i>iparm</i> [20] = 1 except that the |

| Component | Description |
|-----------------------------|--|
| | solve step does not automatically make iterative refinements when perturbed pivots are obtained during numerical factorization. The number of iterations is limited to the number of iterative refinements specified by <i>iparm</i> [7] (0 by default). |
| <i>iparm</i> [21] output | Inertia: number of positive eigenvalues. Intel MKL PARDISO reports the number of positive eigenvalues for symmetric indefinite matrices. |
| <i>iparm</i> [22] output | Inertia: number of negative eigenvalues. Intel MKL PARDISO reports the number of negative eigenvalues for symmetric indefinite matrices. |
| <i>iparm</i> [23] input | Parallel factorization control. |
| | NOTE The two-level factorization algorithm does not improve performance in OOC mode. |
| | 0* Intel MKL PARDISO uses the classic algorithm for factorization. |
| | 1 Intel MKL PARDISO uses a two-level factorization algorithm. This algorithm generally improves scalability in case of parallel factorization on many OpenMP threads (more than eight). |
| <i>iparm</i> [24] input | Parallel forward/backward solve control. |
| | 0* Intel MKL PARDISO uses a parallel algorithm for the solve step. |
| | 1 Intel MKL PARDISO uses the sequential forward and backward solve. This feature is available only for in-core Intel MKL PARDISO (see <i>iparm</i> [59]). |
| <i>iparm</i> [25] | Reserved. Set to zero. |
| <i>iparm</i> [26] input | Matrix checker. |
| | 0* Intel MKL PARDISO does not check the sparse matrix representation for errors. |
| | 1 Intel MKL PARDISO checks integer arrays <i>ia</i> and <i>ja</i> . In particular, Intel MKL PARDISO checks whether column indices are sorted in increasing order within each row. |
| <i>iparm</i> [27] input | Single or double precision Intel MKL PARDISO. See <i>iparm</i> [7] for information on controlling the precision of the refinement steps. |
| | Important The <i>iparm</i> [27] value is stored in the Intel MKL PARDISO handle between Intel MKL PARDISO calls, so the precision mode can be changed only during phase 1. |
| | 0* Input arrays (<i>a</i> , <i>x</i> and <i>b</i>) and all internal arrays must be presented in double precision. |
| | 1 Input arrays (<i>a</i> , <i>x</i> and <i>b</i>) must be presented in single precision. In this case all internal computations are performed in single precision. |
| <i>iparm</i> [28] | Reserved. Set to zero. |
| <i>iparm</i> [29] | Number of zero or negative pivots. |

| Component | Description |
|---|--|
| output | If Intel MKL PARDISO detects zero or negative pivot for $mtype=2$ or $mtype=4$ matrix types, the factorization is stopped, Intel MKL PARDISO returns immediately with an $error = -4$, and $iparm[29]$ reports the number of the equation where the first zero or negative pivot is detected. |
| $iparm[30]$ | Partial solve and computing selected components of the solution vectors. |
| input | <p>This parameter controls the solve step of Intel MKL PARDISO. It can be used if only a few components of the solution vectors are needed or if you want to reduce the computation cost at the solve step by utilizing the sparsity of the right-hand sides. To use this option the input permutation vector define $perm$ so that when $perm(i) = 1$ it means that either the i-th component in the right-hand sides is nonzero, or the i-th component in the solution vectors is computed, or both, depending on the value of $iparm[30]$.</p> <p>The permutation vector $perm$ must be present in all phases of Intel MKL PARDISO software. At the reordering step, the software overwrites the input vector $perm$ by a permutation vector used by the software at the factorization and solver step. If m is the number of components such that $perm(i) = 1$, then the last m components of the output vector $perm$ are a set of the indices i satisfying the condition $perm(i) = 1$ on input.</p> |
| <p>NOTE</p> <p>Turning on this option often increases the time used by Intel MKL PARDISO for factorization and reordering steps, but it can reduce the time required for the solver step.</p> | |
| <p>Important</p> <p>This feature is only available for in-core Intel MKL PARDISO, so to use it you must set $iparm[59] = 0$. Set the parameters $iparm[7]$ (iterative refinement steps), $iparm[3]$ (preconditioned CGS), $iparm[4]$ (user permutation), and $iparm[35]$ (Schur complement) to 0 as well.</p> | |
| 0* | Disables this option. |
| 1 | it is assumed that the right-hand sides have only a few non-zero components* and the input permutation vector $perm$ is defined so that $perm(i) = 1$ means that the (i)-th component in the right-hand sides is nonzero. In this case Intel MKL PARDISO only uses the non-zero components of the right-hand side vectors and computes only corresponding components in the solution vectors. That means the i -th component in the solution vectors is only computed if $perm(i) = 1$. |
| 2 | <p>It is assumed that the right-hand sides have only a few non-zero components* and the input permutation vector $perm$ is defined so that $perm(i) = 1$ means that the i-th component in the right-hand sides is nonzero.</p> <p>Unlike for $iparm[30]=1$, all components of the solution vector are computed for this setting and all components of the right-hand sides are used. Because all components are used, for $iparm[30]=2$ you must set the i-th component of the right-hand sides to zero explicitly if $perm(i)$ is not equal to 1.</p> |
| 3 | Selected components of the solution vectors are computed. The $perm$ array is not related to the right-hand sides and it only indicates which components of the solution vectors should be computed. In this case $perm(i) = 1$ means that the i -th component in the solution vectors is computed. |
| $iparm[31]$ | Reserved. Set to zero. |
| $iparm[32]$ | |

| Component | Description |
|----------------------------|---|
| <i>iparm</i> [33] input | <p>Optimal number of OpenMP threads for conditional numerical reproducibility (CNR) mode. Intel MKL PARDISO reads the value of <i>iparm</i>[33] during the analysis phase (phase 1), so you cannot change it later.</p> <p>Because Intel MKL PARDISO uses C random number generator facilities during the analysis phase (phase 1) you must take these precautions to get numerically reproducible results:</p> <ul style="list-style-type: none"> • Do not alter the states of the random number generators. • Do not run multiple instances of Intel MKL PARDISO in parallel in the analysis phase (phase 1). <hr/> <p>NOTE</p> <p>CNR is only available for the in-core version of Intel MKL PARDISO and the non-parallel version of the nested dissection algorithm. You must also:</p> <ul style="list-style-type: none"> • set <i>iparm</i>[59] to 0 in order to use the in-core version, • not set <i>iparm</i>[1] to 3 in order to not use the parallel version of the nested dissection algorithm. <p>Otherwise Intel MKL PARDISO does not produce numerically repeatable results even if CNR is enabled for Intel MKL using the functionality described in Support Functions for CNR.</p> <hr/> <p>0*</p> <p>CNR mode for Intel MKL PARDISO is enabled only if it is enabled for Intel MKL using the functionality described in Support Functions for CNR and the in-core version is used. Intel MKL PARDISO determines the optimal number of OpenMP threads automatically, and produces numerically reproducible results regardless of the number of threads.</p> <hr/> <p>>0</p> <p>CNR mode is enabled for Intel MKL PARDISO if in-core version is used and the optimal number of OpenMP threads for Intel MKL PARDISO to rely on is defined by the value of <i>iparm</i>[33]. You can use <i>iparm</i>[33] to enable CNR mode independent from other Intel MKL domains. To get the best performance, set <i>iparm</i>[33] to the actual number of hardware threads dedicated for Intel MKL PARDISO. Setting <i>iparm</i>[33] to fewer OpenMP threads than the maximum number of them in use reduces the scalability of the problem being solved. Setting <i>iparm</i>[33] to more threads than are available can reduce the performance of Intel MKL PARDISO.</p> |
| <i>iparm</i> [34] input | <p>One- or zero-based indexing of columns and rows.</p> <hr/> <p>0*</p> <p>One-based indexing: columns and rows indexing in arrays <i>ia</i>, <i>ja</i>, and <i>perm</i> starts from 1 (Fortran-style indexing).</p> <hr/> <p>1</p> <p>Zero-based indexing: columns and rows indexing in arrays <i>ia</i>, <i>ja</i>, and <i>perm</i> starts from 0 (C-style indexing).</p> |
| <i>iparm</i> [35] input | <p>Schur complement matrix computation control. To calculate this matrix, you must set the input permutation vector <i>perm</i> to a set of indexes such that when $perm(i) = 1$, the <i>i</i>-th element of the initial matrix is an element of the Schur matrix.</p> <hr/> <p>CAUTION</p> <p>You can only set one of <i>iparm</i>[4], <i>iparm</i>[30], and <i>iparm</i>[35], so be sure that the <i>iparm</i>[4] (user permutation) and the <i>iparm</i>[30] (partial solution) parameters are 0 if you set <i>iparm</i>[35].</p> <hr/> <p>0*</p> <p>Do not compute Schur complement.</p> |

| Component | Description |
|-------------------|---|
| | <p>1 Compute Schur complement matrix as part of Intel MKL PARDISO factorization step and return it in the solution vector.</p> <hr/> <p>NOTE This option only computes the Schur complement matrix, and does not calculate factorization arrays.</p> |
| | <p>2 Compute Schur complement matrix as part of Intel MKL PARDISO factorization step and return it in the solution vector. Since this option calculates factorization arrays you can use it to launch partial or full solution of the entire problem after the factorization step.</p> |
| <i>iparm</i> [36] | Format for matrix storage. |
| input | <p>0* Use CSR format (see Three Array Variation of CSR Format) for matrix storage.</p> <hr/> <p>> 0 Use BSR format (see Three Array Variation of BSR Format) for matrix storage with blocks of size <i>iparm</i>[36].</p> <hr/> <p>NOTE Intel MKL does not support BSR format in these cases:</p> <ul style="list-style-type: none"> • <i>iparm</i>[10] > 0: Scaling vectors • <i>iparm</i>[12] > 0: Weighted matching • <i>iparm</i>[30] > 0: Partial solution • <i>iparm</i>[35] > 0: Schur complement • <i>iparm</i>[55] > 0: Pivoting control • <i>iparm</i>[59] > 0: OOC Intel MKL PARDISO |
| | <p>< 0 Convert supplied matrix to variable BSR (VBSR) format (see Sparse Data Storage) for matrix storage. Intel MKL PARDISO analyzes the matrix provided in CSR3 format and converts it to an internal VBSR format. Set <i>iparm</i>[36] = -<i>t</i>, 0 < <i>t</i> ≤ 100.</p> <hr/> <p>NOTE Intel MKL only supports VBSR format for real and symmetric positive definite or indefinite matrices (<i>mtype</i> = 2 or <i>mtype</i> = -2). Intel MKL does not support VBSR format in these cases:</p> <ul style="list-style-type: none"> • <i>iparm</i>[10] > 0: Scaling vectors • <i>iparm</i>[12] > 0: Weighted matching • <i>iparm</i>[30] > 0: Partial solution • <i>iparm</i>[35] > 0: Schur complement • <i>iparm</i>[55] > 0: Pivoting control • <i>iparm</i>[59] > 0: OOC Intel MKL PARDISO |
| <i>iparm</i> [37] | Reserved. Set to zero. |
| <i>iparm</i> [54] | |
| <i>iparm</i> [55] | Diagonal and pivoting control. |
| | <p>0* Internal function used to work with pivot and calculation of diagonal arrays turned off.</p> |

| Component | Description |
|---------------------------------|--|
| | <p>1 You can use the mkl_pardiso_pivot callback routine to control pivot elements which appear during numerical factorization. Additionally, you can obtain the elements of initial matrix and factorized matrices after the <code>pardiso</code> factorization step diagonal using the pardiso_getdiag routine. This parameter can be turned on only in the in-core version of Intel MKL PARDISO.</p> |
| <code>iparm[56]</code> - | Reserved. Set to zero. |
| <code>iparm[58]</code> | |
| <code>iparm[59]</code> input | <p>Intel MKL PARDISO mode.</p> <p><code>iparm[59]</code> switches between in-core (IC) and out-of-core (OOC) Intel MKL PARDISO. OOC can solve very large problems by holding the matrix factors in files on the disk, which requires a reduced amount of main memory compared to IC.</p> <p>Unless you are operating in sequential mode, you can switch between IC and OOC modes after the reordering phase. However, you can get better Intel MKL PARDISO performance by setting <code>iparm[59]</code> before the reordering phase.</p> <hr/> <p>NOTE The amount of memory used in OOC mode depends on the number of OpenMP threads.</p> <hr/> <p>WARNING Do not increase the number of OpenMP threads used for Intel MKL PARDISO between the first call to <code>pardiso</code> and the factorization or solution phase. Because the minimum amount of memory required for out-of-core execution depends on the number of OpenMP threads, increasing it after the initial call can cause incorrect results.</p> <hr/> |
| 0* | IC mode. |
| 1 | <p>IC mode is used if the total amount of RAM (in megabytes) needed for storing the matrix factors is less than sum of two values of the environment variables: <code>MKL_PARDISO_OOC_MAX_CORE_SIZE</code> (default value 2000 MB) and <code>MKL_PARDISO_OOC_MAX_SWAP_SIZE</code> (default value 0 MB); otherwise OOC mode is used. In this case amount of RAM used by OOC mode cannot exceed the value of <code>MKL_PARDISO_OOC_MAX_CORE_SIZE</code>.</p> <p>If the total peak memory needed for storing the local arrays is more than <code>MKL_PARDISO_OOC_MAX_CORE_SIZE</code>, increase <code>MKL_PARDISO_OOC_MAX_CORE_SIZE</code> if possible.</p> <hr/> <p>NOTE Conditional numerical reproducibility (CNR) is not supported for this mode.</p> <hr/> |
| 2 | <p>OOC mode.</p> <p>The OOC mode can solve very large problems by holding the matrix factors in files on the disk. Hence the amount of RAM required by OOC mode is significantly reduced compared to IC mode.</p> <p>If the total peak memory needed for storing the local arrays is more than <code>MKL_PARDISO_OOC_MAX_CORE_SIZE</code>, increase <code>MKL_PARDISO_OOC_MAX_CORE_SIZE</code> if possible.</p> |

| Component | Description |
|-------------------|---|
| | To obtain better Intel MKL PARDISO performance, during the numerical factorization phase you can provide the maximum number of right-hand sides, which can be used further during the solving phase. |
| <i>iparm</i> [60] | Reserved. Set to zero. |
| <i>iparm</i> [61] | |
| <i>iparm</i> [62] | Size of the minimum OOC memory for numerical factorization and solution. |
| output | This parameter provides the size in kilobytes of the minimum memory required by OOC Intel MKL PARDISO for internal floating point arrays. This parameter is computed in phase 1. Total peak memory consumption of OOC Intel MKL PARDISO can be estimated as $\max(iparm[14], iparm[15] + iparm[62])$. |
| <i>iparm</i> [63] | Reserved. Set to zero. |

NOTE

Generally in sparse matrices, components which are equal to zero can be considered non-zero if necessary. For example, in order to make a matrix structurally symmetric, elements which are zero can be considered non-zero. See [Sparse Matrix Storage Formats](#) for an example.

Optimization Notice

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Notice revision #20110804

PARDISO_DATA_TYPE

The following table lists the values of `PARDISO_DATA_TYPE` depending on the matrix types and values of the parameter `iparm[27]`.

| Data type value | Matrix type <i>mtype</i> | <i>iparm</i> [27] | comments |
|-----------------|--------------------------|-------------------|------------------------------------|
| double | 1, 2, -2, 11 | 0 | Real matrices, double precision |
| float | | 1 | Real matrices, single precision |
| MKL_Complex16 | 3, 6, 13, 4, -4 | 0 | Complex matrices, double precision |
| MKL_Complex8 | | 1 | Complex matrices, single precision |

Parallel Direct Sparse Solver for Clusters Interface

The Parallel Direct Sparse Solver for Clusters Interface solves large linear systems of equations with sparse matrices on clusters. It is

- high performing
- robust
- memory efficient
- easy to use

A hybrid implementation combines Message Passing Interface (MPI) technology for data exchange between parallel tasks (processes) running on different nodes, and OpenMP* technology for parallelism inside each node of the cluster. This approach effectively uses modern hardware resources such as clusters consisting of nodes with multi-core processors. The solver code is optimized for the latest Intel processors, but also performs well on clusters consisting of non-Intel processors.

Code examples are available in the Intel MKL installation `examples` directory.

Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

Parallel Direct Sparse Solver for Clusters Interface Algorithm

Parallel Direct Sparse Solver for Clusters Interface solves a set of sparse linear equations

$$A * X = B$$

with multiple right-hand sides using a distributed LU , LL^T , LDL^T or LDL^* factorization, where A is an n -by- n matrix, and X and B are n -by- $nrhs$ matrices.

The solution comprises four tasks:

- analysis and symbolic factorization;
- numerical factorization;
- forward and backward substitution including iterative refinement;
- termination to release all internal solver memory.

The solver first computes a symmetric fill-in reducing permutation P based on the nested dissection algorithm from the METIS package [Karypis98] (included with Intel MKL), followed by the Cholesky or other type of factorization (depending on matrix type) [Schenk00-2] of PAP^T . The solver uses either diagonal pivoting, or 1x1 and 2x2 Bunch and Kaufman pivoting for symmetric indefinite or Hermitian matrices before finding an approximation of X by forward and backward substitution and iterative refinement.

The initial matrix A is perturbed whenever numerically acceptable 1x1 and 2x2 pivots cannot be found within the diagonal blocks. One or two passes of iterative refinement may be required to correct the effect of the perturbations. This restricted notion of pivoting with iterative refinement is effective for highly indefinite symmetric systems. For a large set of matrices from different application areas, the accuracy of this method is comparable to a direct factorization method that uses complete sparse pivoting techniques [Schenk04].

Parallel Direct Sparse Solver for Clusters additionally improves the pivoting accuracy by applying symmetric weighted matching algorithms. These methods identify large entries in the coefficient matrix A that, if permuted close to the diagonal, enable the factorization process to identify more acceptable pivots and

proceed with fewer pivot perturbations. The methods are based on maximum weighted matching and improve the quality of the factor in a complementary way to the alternative idea of using more complete pivoting techniques.

Parallel Direct Sparse Solver for Clusters Interface Matrix Storage

The sparse data storage in the Parallel Direct Sparse Solver for Clusters Interface follows the scheme described in the [Sparse Matrix Storage Formats](#) section using the variable *ja* for *columns*, *ia* for *rowIndex*, and *a* for *values*. Column indices *ja* must be in increasing order per row.

When an input data structure is not accessed in a call, a NULL pointer or any valid address can be passed as a placeholder for that argument.

Algorithm Parallelization and Data Distribution

Intel® MKL Parallel Direct Sparse Solver for Clusters enables parallel execution of the solution algorithm with efficient data distribution.

The master MPI process performs the symbolic factorization phase to represent matrix *A* as computational tree. Then matrix *A* is divided among all MPI processes in a one-dimensional manner. The same distribution is used for *L-factor* (the lower triangular matrix in Cholesky decomposition). Matrix *A* and all required internal data are broadcast to slave MPI processes. Each MPI process fills in its own parts of *L-factor* with initial values of the matrix *A*.

Parallel Direct Sparse Solver for Clusters Interface computes all independent parts of *L-factor* completely in parallel. When a block of the factor must be updated by other blocks, these updates are independently passed to a temporary array on each updating MPI process. It further gathers the result into an updated block using the `MPI_Reduce()` routine. The computations within an MPI process are dynamically divided among OpenMP threads using pipelining parallelism with a combination of left- and right-looking techniques similar to those of the PARDISO* software. Level 3 BLAS operations from Intel MKL ensure highly efficient performance of block-to-block update operations.

During forward/backward substitutions, respective Right Hand Side (RHS) parts are distributed among all MPI processes. All these processes participate in the computation of the solution. Finally, the solution is gathered on the master MPI process.

This approach demonstrates good scalability on clusters with Infiniband* technology. Another advantage of the approach is the effective distribution of *L-factor* among cluster nodes. This enables the solution of tasks with a much higher number of non-zero elements than it is possible with any Symmetric Multiprocessing (SMP) in-core direct solver.

The algorithm ensures that the memory required to keep internal data on each MPI process is decreased when the number of MPI processes in a run increases. However, the solver requires that matrix *A* and some other internal arrays completely fit into the memory of each MPI process.

To get the best performance, run one MPI process per physical node and set the number of OpenMP* threads per node equal to the number of physical cores on the node.

NOTE

Instead of calling `MPI_Init()`, initialize MPI with `MPI_Init_thread()` and set the MPI threading level to `MPI_THREAD_FUNNELED` or higher. For details, see the code examples in `<install_dir>/examples`.

cluster_sparse_solver

Calculates the solution of a set of sparse linear equations with single or multiple right-hand sides.

Syntax

```
void cluster_sparse_solver (_MKL_DSS_HANDLE_t pt, const MKL_INT *maxfct, const MKL_INT *mnum, const MKL_INT *mtype, const MKL_INT *phase, const MKL_INT *n, const void *a, const MKL_INT *ia, const MKL_INT *ja, MKL_INT *perm, const MKL_INT *nrhs, MKL_INT *iparm, const MKL_INT *msglvl, void *b, void *x, const int *comm, MKL_INT *error);
```

Include Files

- `mkl_cluster_sparse_solver.h`

Description

The routine `cluster_sparse_solver` calculates the solution of a set of sparse linear equations

$$A * X = B$$

with single or multiple right-hand sides, using a parallel *LU*, *LDL*, or *LL^T* factorization, where *A* is an *n*-by-*n* matrix, and *X* and *B* are *n*-by-*nrhs* vectors or matrices.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

NOTE

Most of the input parameters (except for the *pt*, *phase*, and *comm* parameters and, for the distributed format, the *a*, *ia*, and *ja* arrays) must be set on the master MPI process only, and ignored on other processes. Other MPI processes get all required data from the master MPI process using the MPI communicator, *comm*.

| | |
|-----------|---|
| <i>pt</i> | Array of size 64. Handle to internal data structure. The entries must be set to zero before the first call to <code>cluster_sparse_solver</code> . |
|-----------|---|

CAUTION

After the first call to `cluster_sparse_solver` do not modify *pt*, as that could cause a serious memory leak.

| | | | | | | | | | |
|---------------|---|---|---------------------------------|---|--------------------------------------|----|-------------------------------|---|------------------------------------|
| <i>maxfct</i> | Ignored; assumed equal to 1. | | | | | | | | |
| <i>mnum</i> | Ignored; assumed equal to 1. | | | | | | | | |
| <i>mtype</i> | Defines the matrix type, which influences the pivoting method. The Parallel Direct Sparse Solver for Clusters solver supports the following matrices: <table style="margin-left: 2em;"> <tr> <td>1</td> <td>real and structurally symmetric</td> </tr> <tr> <td>2</td> <td>real and symmetric positive definite</td> </tr> <tr> <td>-2</td> <td>real and symmetric indefinite</td> </tr> <tr> <td>3</td> <td>complex and structurally symmetric</td> </tr> </table> | 1 | real and structurally symmetric | 2 | real and symmetric positive definite | -2 | real and symmetric indefinite | 3 | complex and structurally symmetric |
| 1 | real and structurally symmetric | | | | | | | | |
| 2 | real and symmetric positive definite | | | | | | | | |
| -2 | real and symmetric indefinite | | | | | | | | |
| 3 | complex and structurally symmetric | | | | | | | | |

| | |
|----|---|
| 4 | complex and Hermitian positive definite |
| -4 | complex and Hermitian indefinite |
| 6 | complex and symmetric |
| 11 | real and nonsymmetric |
| 13 | complex and nonsymmetric |

phase

Controls the execution of the solver. Usually it is a two- or three-digit integer. The first digit indicates the starting phase of execution and the second digit indicates the ending phase. Parallel Direct Sparse Solver for Clusters has the following phases of execution:

- Phase 1: Fill-reduction analysis and symbolic factorization
- Phase 2: Numerical factorization
- Phase 3: Forward and Backward solve including optional iterative refinement
- Memory release (*phase*= -1)

If a previous call to the routine has computed information from previous phases, execution may start at any phase. The *phase* parameter can have the following values:

| <i>phase</i> | Solver Execution Steps |
|--------------|--|
| 11 | Analysis |
| 12 | Analysis, numerical factorization |
| 13 | Analysis, numerical factorization, solve, iterative refinement |
| 22 | Numerical factorization |
| 23 | Numerical factorization, solve, iterative refinement |
| 33 | Solve, iterative refinement |
| -1 | Release all internal memory for all matrices |

n

Number of equations in the sparse linear systems of equations $A^*X = B$.
Constraint: $n > 0$.

a

Array. Contains the non-zero elements of the coefficient matrix *A* corresponding to the indices in *ja*. The coefficient matrix can be either real or complex. The matrix must be stored in the three-array variant of the compressed sparse row (CSR3) or in the three-array variant of the block compressed sparse row (BSR3) format, and the matrix must be stored with increasing values of *ja* for each row.

For CSR3 format, the size of *a* is the same as that of *ja*. Refer to the *values* array description in [Three Array Variation of CSR Format](#) for more details.

For BSR3 format the size of a is the size of ja multiplied by the square of the block size. Refer to the *values* array description in [Three Array Variation of BSR Format](#) for more details.

NOTE

For centralized input ($iparm[39]=0$), provide the a array for the master MPI process only. For distributed assembled input ($iparm[39]=1$ or $iparm[39]=2$), provide it for all MPI processes.

Important

The column indices of non-zero elements of each row of the matrix A must be stored in increasing order.

ia

For CSR3 format, $ia[i]$ ($i < n$) points to the first column index of row i in the array ja . That is, $ia[i]$ gives the index of the element in array a that contains the first non-zero element from row i of A . The last element $ia[n]$ is taken to be equal to the number of non-zero elements in A , plus one. Refer to *rowIndex* array description in [Three Array Variation of CSR Format](#) for more details.

For BSR3 format, $ia[i]$ ($i < n$) points to the first column index of row i in the array ja . That is, $ia[i]$ gives the index of the element in array a that contains the first non-zero block from row i of A . The last element $ia[n]$ is taken to be equal to the number of non-zero blocks in A , plus one. Refer to *rowIndex* array description in [Three Array Variation of BSR Format](#) for more details.

The array ia is accessed in all phases of the solution process.

Indexing of ia is one-based by default, but it can be changed to zero-based by setting the appropriate value to the parameter $iparm[34]$. For zero-based indexing, the last element $ia[n]$ is assumed to be equal to the number of non-zero elements in matrix A .

NOTE

For centralized input ($iparm[39]=0$), provide the ia array at the master MPI process only. For distributed assembled input ($iparm[39]=1$ or $iparm[39]=2$), provide it at all MPI processes.

ja

For CSR3 format, array ja contains column indices of the sparse matrix A . It is important that the indices are in increasing order per row. For symmetric matrices, the solver needs only the upper triangular part of the system as is shown for *columns* array in [Three Array Variation of CSR Format](#).

For BSR3 format, array ja contains column indices of the sparse matrix A . It is important that the indices are in increasing order per row. For symmetric matrices, the solver needs only the upper triangular part of the system as is shown for *columns* array in [Three Array Variation of BSR Format](#).

The array ja is accessed in all phases of the solution process.

Indexing of *ja* is one-based by default, but it can be changed to zero-based by setting the appropriate value to the parameter *iparm(35)*.

NOTE

For centralized input (*iparm(40)=0*), provide the *ja* array at the master MPI process only. For distributed assembled input (*iparm(40)=1* or *iparm(40)=2*), provide it at all MPI processes.

| | |
|---------------|--|
| <i>perm</i> | Ignored. |
| <i>nrhs</i> | Number of right-hand sides that need to be solved for. |
| <i>iparm</i> | <p>Array, size 64. This array is used to pass various parameters to Parallel Direct Sparse Solver for Clusters Interface and to return some useful information after execution of the solver.</p> <p>See cluster_sparse_solver iparm Parameter for more details about the <i>iparm</i> parameters.</p> |
| <i>msglvl</i> | <p>Message level information. If <i>msglvl = 0</i> then <i>cluster_sparse_solver</i> generates no output, if <i>msglvl = 1</i> the solver prints statistical information to the screen.</p> <p>Statistics include information such as the number of non-zero elements in <i>L-factor</i> and the timing for each phase.</p> <p>Set <i>msglvl = 1</i> if you report a problem with the solver, since the additional information provided can facilitate a solution.</p> |
| <i>b</i> | <p>Array, size $n * nrhs$. On entry, contains the right-hand side vector/matrix <i>B</i>, which is placed in memory contiguously. The $b[i+k*n]$ must hold the <i>i</i>-th component of <i>k</i>-th right-hand side vector. Note that <i>b</i> is only accessed in the solution phase.</p> |
| <i>comm</i> | <p>MPI communicator. The solver uses the Fortran MPI communicator internally. Convert the MPI communicator to Fortran using the <code>MPI_Comm_c2f()</code> function. See the examples in the <code><install_dir>/examples</code> directory.</p> |

Output Parameters

| | |
|--------------|---|
| <i>pt</i> | Handle to internal data structure. |
| <i>perm</i> | Ignored. |
| <i>iparm</i> | <p>On output, some <i>iparm</i> values report information such as the numbers of non-zero elements in the factors.</p> <p>See cluster_sparse_solver iparm Parameter for more details about the <i>iparm</i> parameters.</p> |
| <i>b</i> | On output, the array is replaced with the solution if <i>iparm[5] = 1</i> . |

x Array, size ($n * nrhs$). If *iparm*[5]=0 it contains solution vector/matrix *X*, which is placed contiguously in memory. The $x[i+k*n]$ element must hold the *i*-th component of the *k*-th solution vector. Note that *x* is only accessed in the solution phase.

error The error indicator according to the below table:

| error | Information |
|--------------|-------------------------------|
| 0 | no error |
| -1 | input inconsistent |
| -2 | not enough memory |
| -3 | reordering problem |
| -5 | unclassified (internal) error |

cluster_sparse_solver_64

Calculates the solution of a set of sparse linear equations with single or multiple right-hand sides.

Syntax

```
void cluster_sparse_solver_64 (void *pt, const long long int *maxfct, const long long int *mnum, const long long int *mtype, const long long int *phase, const long long int *n, const void *a, const long long int *ia, const long long int *ja, long long int *perm, const long long int *nrhs, long long int *iparm, const long long int *msglvl, void *b, void *x, const int *comm, long long int *error);
```

Include Files

- mkl_cluster_sparse_solver.h

Description

The routine `cluster_sparse_solver_64` is an alternative ILP64 (64-bit integer) version of the `cluster_sparse_solver` routine (see the Description section for more details). The interface of `cluster_sparse_solver_64` is the same as the interface of `cluster_sparse_solver`, but it accepts and returns all integer data as `long long int`.

Use `cluster_sparse_solver_64` when `cluster_sparse_solver` for solving large matrices (with the number of non-zero elements on the order of 500 million or more). You can use it together with the usual LP64 interfaces for the rest of Intel MKL functionality. In other words, if you use 64-bit integer version (`cluster_sparse_solver_64`), you do not need to re-link your applications with ILP64 libraries. Take into account that `cluster_sparse_solver_64` may perform slower than regular `cluster_sparse_solver` on the reordering and symbolic factorization phase.

NOTE

`cluster_sparse_solver_64` is supported only in the 64-bit libraries. If `cluster_sparse_solver_64` is called from the 32-bit libraries, it returns `error == -12`.

NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

Input Parameters

The input parameters of `cluster_sparse_solver_64` are the same as the input parameters of `cluster_sparse_solver`, but `cluster_sparse_solver_64` accepts all integer data as long long int.

Output Parameters

The output parameters of `cluster_sparse_solver_64` are the same as the output parameters of `cluster_sparse_solver`, but `cluster_sparse_solver_64` returns all integer data as long long int.

cluster_sparse_solver iparm Parameter

The following table describes all individual components of the Parallel Direct Sparse Solver for Clusters Interface `iparm` parameter. Components which are not used must be initialized with 0. Default values are denoted with an asterisk (*).

| Component | Description |
|---|---|
| <code>iparm[0]</code> | Use default values. |
| input | 0 <code>iparm[1] - iparm(64)</code> are filled with default values. !=0 You must supply all values in components <code>iparm[1] - iparm(64)</code> . |
| <code>iparm[1]</code> | Fill-in reducing ordering for the input matrix. |
| input | 2* The nested dissection algorithm from the METIS package [Karypis98] . 3 The parallel version of the nested dissection algorithm. It can decrease the time of computations on multi-core computers, especially when Phase 1 takes significant time. 10 The MPI version of the nested dissection and symbolic factorization algorithms. The input matrix for the reordering must be distributed among different MPI processes without any intersection. Use <code>iparm[40]</code> and <code>iparm[41]</code> to set the bounds of the domain. During all of Phase 1, the entire matrix is not gathered on any one process, which can decrease computation time (especially when Phase 1 takes significant time) and decrease memory usage for each MPI process on the cluster. |
| <p>NOTE If you set <code>iparm[1] = 10</code>, <code>comm = -1</code> (MPI communicator), and if there is one MPI process, optimization and full parallelization with the OpenMP version of the nested dissection and symbolic factorization algorithms proceeds. This can decrease computation time on multi-core computers. In this case, set <code>iparm[40] = 1</code> and <code>iparm[41] = n</code> for one-based indexing, or to 0 and <code>n - 1</code>, respectively, for zero-based indexing.</p> | |
| <code>iparm[2]</code> | Reserved. Set to zero. |
| <code>iparm[5]</code> | Write solution on <code>x</code> . |
| input | <p>NOTE The array <code>x</code> is always used.</p> <p>0* The array <code>x</code> contains the solution; right-hand side vector <code>b</code> is kept unchanged. 1 The solver stores the solution on the right-hand side <code>b</code>.</p> |

| Component | Description |
|----------------------------|---|
| <i>iparm</i> [6] output | Number of iterative refinement steps performed. Reports the number of iterative refinement steps that were actually performed during the solve step. |
| <i>iparm</i> [7] input | Iterative refinement step. On entry to the solve and iterative refinement step, <i>iparm</i> [7] must be set to the maximum number of iterative refinement steps that the solver performs. |
| | 0* The solver automatically performs two steps of iterative refinement when perturbed pivots are obtained during the numerical factorization. |
| | >0 Maximum number of iterative refinement steps that the solver performs. The solver performs not more than the absolute value of <i>iparm</i> [7] steps of iterative refinement. The solver might stop the process before the maximum number of steps if <ul style="list-style-type: none"> a satisfactory level of accuracy of the solution in terms of backward error is achieved, or if it determines that the required accuracy cannot be reached. In this case Parallel Direct Sparse Solver for Clusters Interface returns -4 in the <i>error</i> parameter. The number of executed iterations is reported in <i>iparm</i> [6]. |
| | <0 Same as above, but the accumulation of the residuum uses extended precision real and complex data types. Perturbed pivots result in iterative refinement (independent of <i>iparm</i> [7]=0) and the number of executed iterations is reported in <i>iparm</i> [6]. |
| <i>iparm</i> [8] | Reserved. Set to zero. |
| <i>iparm</i> [9] input | Pivoting perturbation. This parameter instructs Parallel Direct Sparse Solver for Clusters Interface how to handle small pivots or zero pivots for nonsymmetric matrices (<i>mtype</i> =11 or <i>mtype</i> =13) and symmetric matrices (<i>mtype</i> =-2, <i>mtype</i> =-4, or <i>mtype</i> =6). For these matrices the solver uses a complete supernode pivoting approach. When the factorization algorithm reaches a point where it cannot factor the supernodes with this pivoting strategy, it uses a pivoting perturbation strategy similar to [Li99], [Schenk04]. Small pivots are perturbed with $\text{eps} = 10^{-iparm[9]}$. The magnitude of the potential pivot is tested against a constant threshold of $\alpha = \text{eps} * A2 _{inf}$, where $\text{eps} = 10^{(-iparm[9])}$, $A2 = P * P_{MPS} * D_r * A * D_c * P$, and $ A2 _{inf}$ is the infinity norm of the scaled and permuted matrix <i>A</i> . Any tiny pivots encountered during elimination are set to the sign (<i>111</i>)* $\text{eps} * A2 _{inf}$, which trades off some numerical stability for the ability to keep pivots from getting too small. Small pivots are therefore perturbed with $\text{eps} = 10^{(-iparm[9])}$. |
| | 13* The default value for nonsymmetric matrices(<i>mtype</i> =11, <i>mtype</i> =13), $\text{eps} = 10^{-13}$. |
| | 8* The default value for symmetric indefinite matrices (<i>mtype</i> =-2, <i>mtype</i> =-4, <i>mtype</i> =6), $\text{eps} = 10^{-8}$. |
| <i>iparm</i> [10] input | Scaling vectors. Parallel Direct Sparse Solver for Clusters Interface uses a maximum weight matching algorithm to permute large elements on the diagonal and to scale. |

| Component | Description |
|---|---|
| | Use <code>iparm[10] = 1</code> (scaling) and <code>iparm[12] = 1</code> (matching) for highly indefinite symmetric matrices, for example, from interior point optimizations or saddle point problems. Note that in the analysis phase (<code>phase=11</code>) you must provide the numerical values of the matrix <i>A</i> in array <i>a</i> in case of scaling and symmetric weighted matching. |
| | 0* Disable scaling. Default for symmetric indefinite matrices. |
| | 1* Enable scaling. Default for nonsymmetric matrices. Scale the matrix so that the diagonal elements are equal to 1 and the absolute values of the off-diagonal entries are less or equal to 1. This scaling method is applied to nonsymmetric matrices (<code>mtype = 11</code> , <code>mtype = 13</code>). The scaling can also be used for symmetric indefinite matrices (<code>mtype = -2</code> , <code>mtype = -4</code> , <code>mtype = 6</code>) when the symmetric weighted matchings are applied (<code>iparm[12] = 1</code>). Note that in the analysis phase (<code>phase=11</code>) you must provide the numerical values of the matrix <i>A</i> in case of scaling. |
| <code>iparm[11]</code> | Reserved. Set to zero. |
| <code>iparm[12]</code> input | Improved accuracy using (non-) symmetric weighted matching. Parallel Direct Sparse Solver for Clusters Interface can use a maximum weighted matching algorithm to permute large elements close the diagonal. This strategy adds an additional level of reliability to the factorization methods and complements the alternative of using more complete pivoting techniques during the numerical factorization. |
| | 0* Disable matching. Default for symmetric indefinite matrices. |
| | 1* Enable matching. Default for nonsymmetric matrices. Maximum weighted matching algorithm to permute large elements close to the diagonal. It is recommended to use <code>iparm[10] = 1</code> (scaling) and <code>iparm[12] = 1</code> (matching) for highly indefinite symmetric matrices, for example from interior point optimizations or saddle point problems. Note that in the analysis phase (<code>phase=11</code>) you must provide the numerical values of the matrix <i>A</i> in case of symmetric weighted matching. |
| <code>iparm[13]</code> - | Reserved. Set to zero. |
| <code>iparm[19]</code> <code>iparm[20]</code> input | Pivoting for symmetric indefinite matrices. |
| | 0 Apply 1x1 diagonal pivoting during the factorization process. |
| | 1* Apply 1x1 and 2x2 Bunch-Kaufman pivoting during the factorization process. Bunch-Kaufman pivoting is available for matrices of <code>mtype=-2</code> , <code>mtype=-4</code> , or <code>mtype=6</code> . |
| <code>iparm[21]</code> - | Reserved. Set to zero. |
| <code>iparm[25]</code> <code>iparm[26]</code> input | Matrix checker. |
| | 0* Do not check the sparse matrix representation for errors. |
| | 1 Check integer arrays <i>ia</i> and <i>ja</i> . In particular, check whether the column indices are sorted in increasing order within each row. |

| Component | Description |
|----------------------------|--|
| <i>iparm</i> [27] input | Single or double precision Parallel Direct Sparse Solver for Clusters Interface. See <i>iparm</i> [7] for information on controlling the precision of the refinement steps. |
| | 0* Input arrays (<i>a</i> , <i>x</i> and <i>b</i>) and all internal arrays must be presented in double precision. |
| | 1 Input arrays (<i>a</i> , <i>x</i> and <i>b</i>) must be presented in single precision. In this case all internal computations are performed in single precision. |
| <i>iparm</i> [28] - | Reserved. Set to zero. |
| <i>iparm</i> [33] | |
| <i>iparm</i> [34] input | One- or zero-based indexing of columns and rows. |
| | 0* One-based indexing: columns and rows indexing in arrays <i>ia</i> , <i>ja</i> , and <i>perm</i> starts from 1 (Fortran-style indexing). |
| | 1 Zero-based indexing: columns and rows indexing in arrays <i>ia</i> , <i>ja</i> , and <i>perm</i> starts from 0 (C-style indexing). |
| <i>iparm</i> [36] input | Format for matrix storage. |
| | 0* Use CSR format (see Three Array Variation of BSR Format) for matrix storage. |
| | > 0 Use BSR format (see Three Array Variation of BSR Format) for matrix storage with blocks of size <i>iparm</i> [36]. |
| | NOTE Intel MKL does not support BSR format in these cases: |
| | <i>iparm</i> [10] > 0 Scaling vectors |
| | <i>iparm</i> [12] > 0 Weighted matching |
| | <i>iparm</i> [30] > 0 Partial solution |
| | <i>iparm</i> [35] > 0 Schur complement |
| | <i>iparm</i> [55] > 0 Pivoting control |
| | <i>iparm</i> [59] > 0 OOC Intel MKL PARDISO |
| <i>iparm</i> [37] - | Reserved. Set to zero. |
| <i>iparm</i> [38] | |
| <i>iparm</i> [39] input | Matrix input format. |
| | NOTE Performance of the reordering step of the Parallel Direct Sparse Solver for Clusters Interface is slightly better for assembled format (CSR, <i>iparm</i> [39] = 0) than for distributed format (DCSR, <i>iparm</i> [39] > 0) for the same matrices, so if the matrix is assembled on one node do not distribute it before calling <code>cluster_sparse_solver</code> . |
| | 0* Provide the matrix in usual centralized input format: the master MPI process stores all data from matrix <i>A</i> , with rank=0. |

| Component | Description |
|--|---|
| 1 | Provide the matrix in distributed assembled matrix input format. In this case, each MPI process stores only a part (or domain) of the matrix <i>A</i> data. Set the bounds of the domain using <i>iparm</i> [40] and <i>iparm</i> [41]. The solution vector is placed on the master process. |
| 2 | Provide the matrix in distributed assembled matrix input format. In this case, each MPI process stores only a part (or domain) of the matrix <i>A</i> data. Set the bounds of the domain using <i>iparm</i> [40] and <i>iparm</i> [41]. The solution vector, <i>A</i> , and RHS elements are distributed between processes in same manner. |
| 3 | Provide the matrix in distributed assembled matrix input format. In this case, each MPI process stores only a part (or domain) of the matrix <i>A</i> data. Set the bounds of the domain using <i>iparm</i> [40] and <i>iparm</i> [41]. The <i>A</i> and RHS elements are distributed between processes in same manner and the solution vector is the same on each process |
| <i>iparm</i> [40] input | <p>Beginning of input domain.</p> <p>The number of the matrix <i>A</i> row, RHS element, and, for <i>iparm</i>[39]=2, solution vector that begins the input domain belonging to this MPI process.</p> <p>Only applicable to the distributed assembled matrix input format (<i>iparm</i>[39] > 0).</p> <p>See Sparse Matrix Storage Formats for more details.</p> |
| <i>iparm</i> [41] input | <p>End of input domain.</p> <p>The number of the matrix <i>A</i> row, RHS element, and, for <i>iparm</i>[39]=2, solution vector that ends the input domain belonging to this MPI process.</p> <p>Only applicable to the distributed assembled matrix input format (<i>iparm</i>[39] > 0).</p> <p>See Sparse Matrix Storage Formats for more details.</p> |
| <i>iparm</i> [42] - <i>iparm</i> [63] input | <p>Reserved. Set to zero.</p> |

NOTE

Generally in sparse matrices, components which are equal to zero can be considered non-zero if necessary. For example, in order to make a matrix structurally symmetric, elements which are zero can be considered non-zero. See [Sparse Matrix Storage Formats](#) for an example.

Optimization Notice

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Direct Sparse Solver (DSS) Interface Routines

Intel MKL supports the DSS interface, an alternative to the Intel MKL PARDISO interface for the direct sparse solver. The DSS interface implements a group of user-callable routines that are used in the step-by-step solving process and utilizes the general scheme described in [Appendix A Linear Solvers Basics](#) for solving sparse systems of linear equations. This interface also includes one routine for gathering statistics related to the solving process.

The DSS interface also supports the out-of-core (OOC) mode.

Table "DSS Interface Routines" lists the names of the routines and describes their general use.

DSS Interface Routines

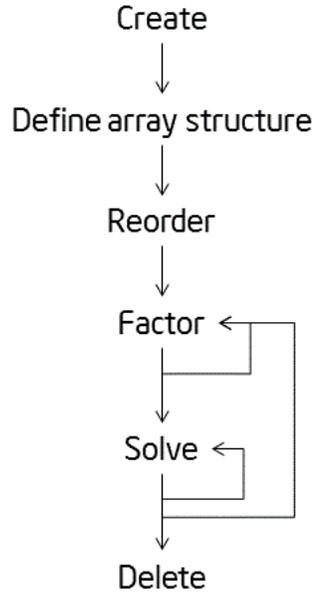
| Routine | Description |
|--|--|
| <code>dss_create</code> | Initializes the solver and creates the basic data structures necessary for the solver. This routine must be called before any other DSS routine. |
| <code>dss_define_structure</code> | Informs the solver of the locations of the non-zero elements of the matrix. |
| <code>dss_reorder</code> | Based on the non-zero structure of the matrix, computes a permutation vector to reduce fill-in during the factoring process. |
| <code>dss_factor_real</code> , <code>dss_factor_complex</code> | Computes the LU , LDL^T or LL^T factorization of a real or complex matrix. |
| <code>dss_solve_real</code> , <code>dss_solve_complex</code> | Computes the solution vector for a system of equations based on the factorization computed in the previous phase. |
| <code>dss_delete</code> | Deletes all data structures created during the solving process. |
| <code>dss_statistics</code> | Returns statistics about various phases of the solving process. |

To find a single solution vector for a single system of equations with a single right-hand side, invoke the Intel MKL DSS interface routines in this order:

1. `dss_create`
2. `dss_define_structure`
3. `dss_reorder`
4. `dss_factor_real`, `dss_factor_complex`
5. `dss_solve_real`, `dss_solve_complex`
6. `dss_delete`

However, in certain applications it is necessary to produce solution vectors for multiple right-hand sides for a given factorization and/or factor several matrices with the same non-zero structure. Consequently, it is sometimes necessary to invoke the Intel MKL sparse routines in an order other than that listed, which is possible using the DSS interface. The solving process is conceptually divided into six phases. [Figure "Typical order for invoking DSS interface routines"](#) indicates the typical order in which the DSS interface routines can be invoked.

Typical order for invoking DSS interface routines



See the code examples that use the DSS interface routines to solve systems of linear equations in the Intel MKL installation directory (`dss_*.c`).

- `examples/solverc/source`

DSS Interface Description

Each DSS routine reads from or writes to a data object called a *handle*. Refer to [Memory Allocation and Handles](#) to determine the correct method for declaring a handle argument for each language. For simplicity, the descriptions in DSS routines refer to the data type as `MKL_DSS_HANDLE`.

Routine Options

The DSS routines have an integer argument (referred below to as *opt*) for passing various options to the routines. The permissible values for *opt* should be specified using only the symbol constants defined in the language-specific header files (see [Implementation Details](#)). The routines accept options for setting the message and termination levels as described in [Table "Symbolic Names for the Message and Termination Levels Options"](#). Additionally, each routine accepts the option `MKL_DSS_DEFAULTS` that sets the default values (as documented) for *opt* to the routine.

Symbolic Names for the Message and Termination Levels Options

| Message Level | Termination Level |
|--------------------------------------|---------------------------------------|
| <code>MKL_DSS_MSG_LVL_SUCCESS</code> | <code>MKL_DSS_TERM_LVL_SUCCESS</code> |
| <code>MKL_DSS_MSG_LVL_INFO</code> | <code>MKL_DSS_TERM_LVL_INFO</code> |
| <code>MKL_DSS_MSG_LVL_WARNING</code> | <code>MKL_DSS_TERM_LVL_WARNING</code> |
| <code>MKL_DSS_MSG_LVL_ERROR</code> | <code>MKL_DSS_TERM_LVL_ERROR</code> |
| <code>MKL_DSS_MSG_LVL_FATAL</code> | <code>MKL_DSS_TERM_LVL_FATAL</code> |

The settings for message and termination levels can be set on any call to a DSS routine. However, once set to a particular level, they remain at that level until they are changed in another call to a DSS routine.

You can specify both message and termination level for a DSS routine by adding the options together. For example, to set the message level to `debug` and the termination level to `error` for all the DSS routines, use the following call:

```
dss_create( handle, MKL_DSS_MSG_LVL_INFO + MKL_DSS_TERM_LVL_ERROR)
```

User Data Arrays

Many of the DSS routines take arrays of user data as input. For example, pointers to user arrays are passed to the routine `dss_define_structure` to describe the location of the non-zero entries in the matrix.

CAUTION

Do not modify the contents of these arrays after they are passed to one of the solver routines.

DSS Implementation Details

To promote portability across platforms and ease of use across different languages, use the appropriate Intel MKL DSS header file:

- `mkl_dss.h`

The header file defines symbolic constants for returned error values, function options, certain defined data types, and function prototypes.

NOTE

Constants for options, returned error values, and message severities must be referred only by the symbolic names that are defined in these header files. Use of the Intel MKL DSS software without including one of the above header files is not supported.

Memory Allocation and Handles

You do not need to allocate any temporary working storage in order to use the Intel MKL DSS routines, because the solver itself allocates any required storage. To enable multiple users to access the solver simultaneously, the solver keeps track of the storage allocated for a particular application by using a *handle* data object.

Each of the Intel MKL DSS routines creates, uses, or deletes a handle. Consequently, any program calling an Intel MKL DSS routine must be able to allocate storage for a handle. The exact syntax for allocating storage for a handle varies from language to language. To standardize the handle declarations, the language-specific header files declare constants and defined data types that must be used when declaring a handle object in your code.

- ```
#include "mkl_dss.h"
_MKL_DSS_HANDLE_t handle;
```

In addition to the definition for the correct declaration of a handle, the include file also defines the following:

- function prototypes for languages that support prototypes
- symbolic constants that are used for the returned error values
- user options for the solver routines
- constants indicating message severity.

## DSS Routines

## dss\_create

Initializes the solver.

---

### Syntax

```
MKL_INT dss_create(_MKL_DSS_HANDLE_t *handle, MKL_INT const *opt)
```

### Include Files

- mkl.h

### Description

The `dss_create` routine initializes the solver. After the call to `dss_create`, all subsequent invocations of the Intel MKL DSS routines must use the value of the handle returned by `dss_create`.

---

**WARNING**

Do not write the value of handle directly.

---

The default value of the parameter `opt` is

```
MKL_DSS_MSG_LVL_WARNING + MKL_DSS_TERM_LVL_ERROR.
```

By default, the DSS routines use double precision for solving systems of linear equations. The precision used by the DSS routines can be set to single mode by adding the following value to the `opt` parameter:

```
MKL_DSS_SINGLE_PRECISION.
```

Input data and internal arrays are required to have single precision.

By default, the DSS routines use Fortran style (one-based) indexing for input arrays of integer types (the first value is referenced as array element 1). To set indexing to C style (the first value is referenced as array element 0), add the following value to the `opt` parameter:

```
MKL_DSS_ZERO_BASED_INDEXING.
```

The `opt` parameter can also control number of refinement steps used on the solution stage by specifying the two following values:

```
MKL_DSS_REFINEMENT_OFF - maximum number of refinement steps is set to zero;
```

```
MKL_DSS_REFINEMENT_ON (default value) - maximum number of refinement steps is set to 2.
```

By default, DSS uses in-core computations. To launch the out-of-core version of DSS (OOC DSS) you can add to this parameter one of two possible values: `MKL_DSS_OOC_STRONG` and `MKL_DSS_OOC_VARIABLE`.

```
MKL_DSS_OOC_STRONG - OOC DSS is used.
```

```
MKL_DSS_OOC_VARIABLE - if the memory needed for the matrix factors is less than the value of the environment variable MKL_PARDISO_OOC_MAX_CORE_SIZE, then the OOC DSS uses the in-core kernels of Intel MKL PARDISO, otherwise it uses the OOC computations.
```

The variable `MKL_PARDISO_OOC_MAX_CORE_SIZE` defines the maximum size of RAM allowed for storing work arrays associated with the matrix factors. It is ignored if `MKL_DSS_OOC_STRONG` is set. The default value of `MKL_PARDISO_OOC_MAX_CORE_SIZE` is 2000 MB. This value and default path and file name for storing temporary data can be changed using the configuration file `pardiso_ooc.cfg` or command line (See more details in the description of the [pardiso](#) routine).

---

**WARNING**

Other than message and termination level options, do not change the OOC DSS settings after they are specified in the routine `dss_create`.

---

## Input Parameters

*opt* Parameter to pass the DSS options. The default value is MKL\_DSS\_MSG\_LVL\_WARNING + MKL\_DSS\_TERM\_LVL\_ERROR.

## Output Parameters

*handle* Pointer to the data structure storing internal DSS results (MKL\_DSS\_HANDLE).

## Return Values

MKL\_DSS\_SUCCESS

MKL\_DSS\_INVALID\_OPTION

MKL\_DSS\_OUT\_OF\_MEMORY

MKL\_DSS\_MSG\_LVL\_ERR

MKL\_DSS\_TERM\_LVL\_ERR

## dss\_define\_structure

*Communicates locations of non-zero elements in the matrix to the solver.*

---

### Syntax

```
MKL_INT dss_define_structure(_MKL_DSS_HANDLE_t *handle, MKL_INT const *opt, MKL_INT const *rowIndex, MKL_INT const *nRows, MKL_INT const *nCols, MKL_INT const *columns, MKL_INT const *nNonZeros);
```

### Include Files

- mkl.h

### Description

The routine `dss_define_structure` communicates the locations of the `nNonZeros` number of non-zero elements in a matrix of `nRows * nCols` size to the solver.

---

#### NOTE

The Intel MKL DSS software operates only on square matrices, so `nRows` must be equal to `nCols`.

---

To communicate the locations of non-zero elements in the matrix, do the following:

1. Define the general non-zero structure of the matrix by specifying the value for the options argument `opt`. You can set the following values for real matrices:

- MKL\_DSS\_SYMMETRIC\_STRUCTURE
- MKL\_DSS\_SYMMETRIC
- MKL\_DSS\_NON\_SYMMETRIC

and for complex matrices:

- MKL\_DSS\_SYMMETRIC\_STRUCTURE\_COMPLEX
- MKL\_DSS\_SYMMETRIC\_COMPLEX

- MKL\_DSS\_NON\_SYMMETRIC\_COMPLEX

The information about the matrix type must be defined in `dss_define_structure`.

2. Provide the actual locations of the non-zeros by means of the arrays `rowIndex` and `columns` (see [Sparse Matrix Storage Format](#)).

## Input Parameters

|                        |                                                                                                                   |
|------------------------|-------------------------------------------------------------------------------------------------------------------|
| <code>opt</code>       | Parameter to pass the DSS options. The default value for the matrix structure is <code>MKL_DSS_SYMMETRIC</code> . |
| <code>rowIndex</code>  | Array of size <code>nRows+1</code> . Defines the location of non-zero entries in the matrix.                      |
| <code>nRows</code>     | Number of rows in the matrix.                                                                                     |
| <code>nCols</code>     | Number of columns in the matrix; must be equal to <code>nRows</code> .                                            |
| <code>columns</code>   | Array of size <code>nNonZeros</code> . Defines the column location of non-zero entries in the matrix.             |
| <code>nNonZeros</code> | Number of non-zero elements in the matrix.                                                                        |

## Output Parameters

|                     |                                                                                             |
|---------------------|---------------------------------------------------------------------------------------------|
| <code>handle</code> | Pointer to the data structure storing internal DSS results ( <code>MKL_DSS_HANDLE</code> ). |
|---------------------|---------------------------------------------------------------------------------------------|

## Return Values

`MKL_DSS_SUCCESS`  
`MKL_DSS_STATE_ERR`  
`MKL_DSS_INVALID_OPTION`  
`MKL_DSS_STRUCTURE_ERR`  
`MKL_DSS_ROW_ERR`  
`MKL_DSS_COL_ERR`  
`MKL_DSS_NOT_SQUARE`  
`MKL_DSS_TOO_FEW_VALUES`  
`MKL_DSS_TOO_MANY_VALUES`  
`MKL_DSS_OUT_OF_MEMORY`  
`MKL_DSS_MSG_LVL_ERR`  
`MKL_DSS_TERM_LVL_ERR`

## **dss\_reorder**

*Computes or sets a permutation vector that minimizes the fill-in during the factorization phase.*

---

## Syntax

```
MKL_INT dss_reorder(_MKL_DSS_HANDLE_t *handle, MKL_INT const *opt, MKL_INT const *perm)
```

## Include Files

- `mk1.h`

## Description

If `opt` contains the option `MKL_DSS_AUTO_ORDER`, then the routine `dss_reorder` computes a permutation vector that minimizes the fill-in during the factorization phase. For this option, the routine ignores the contents of the `perm` array.

If `opt` contains the option `MKL_DSS_METIS_OPENMP_ORDER`, then the routine `dss_reorder` computes permutation vector using the parallel nested dissections algorithm to minimize the fill-in during the factorization phase. This option can be used to decrease the time of `dss_reorder` call on multi-core computers. For this option, the routine ignores the contents of the `perm` array.

If `opt` contains the option `MKL_DSS_MY_ORDER`, then you must supply a permutation vector in the array `perm`. In this case, the array `perm` is of length `nRows`, where `nRows` is the number of rows in the matrix as defined by the previous call to `dss_define_structure`.

If `opt` contains the option `MKL_DSS_GET_ORDER`, then the permutation vector computed during the `dss_reorder` call is copied to the array `perm`. In this case you must allocate the array `perm` beforehand. The permutation vector is computed in the same way as if the option `MKL_DSS_AUTO_ORDER` is set.

### Optimization Notice

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## Input Parameters

|                   |                                                                                                                                                                                                |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>opt</code>  | Parameter to pass the DSS options. The default value for the permutation type is <code>MKL_DSS_AUTO_ORDER</code> .                                                                             |
| <code>perm</code> | Array of length <code>nRows</code> . Contains a user-defined permutation vector (accessed only if <code>opt</code> contains <code>MKL_DSS_MY_ORDER</code> or <code>MKL_DSS_GET_ORDER</code> ). |

## Output Parameters

|                     |                                                                                             |
|---------------------|---------------------------------------------------------------------------------------------|
| <code>handle</code> | Pointer to the data structure storing internal DSS results ( <code>MKL_DSS_HANDLE</code> ). |
|---------------------|---------------------------------------------------------------------------------------------|

## Return Values

`MKL_DSS_SUCCESS`  
`MKL_DSS_STATE_ERR`  
`MKL_DSS_INVALID_OPTION`  
`MKL_DSS_REORDER_ERR`  
`MKL_DSS_REORDER1_ERR`

MKL\_DSS\_I32BIT\_ERR  
MKL\_DSS\_FAILURE  
MKL\_DSS\_OUT\_OF\_MEMORY  
MKL\_DSS\_MSG\_LVL\_ERR  
MKL\_DSS\_TERM\_LVL\_ERR

## dss\_factor\_real, dss\_factor\_complex

Compute factorization of the matrix with previously specified location of non-zero elements.

---

### Syntax

```
MKL_INT dss_factor_real(_MKL_DSS_HANDLE_t *handle, MKL_INT const *opt, void const *rValues)
```

```
MKL_INT dss_factor_complex(_MKL_DSS_HANDLE_t *handle, MKL_INT const *opt, void const *cValues)
```

### Include Files

- mkl.h

### Description

These routines compute factorization of the matrix whose non-zero locations were previously specified by a call to [dss\\_define\\_structure](#) and whose non-zero values are given in the array *rValues*, *cValues* or *Values*. Data type These arrays must be of length *nNonZeros* as defined in a previous call to [dss\\_define\\_structure](#).

---

#### NOTE

The data type (single or double precision) of *rValues*, *cValues*, *Values* must be in correspondence with precision specified by the parameter *opt* in the routine [dss\\_create](#).

---

The *opt* argument can contain one of the following options:

- MKL\_DSS\_POSITIVE\_DEFINITE
- MKL\_DSS\_INDEFINITE
- MKL\_DSS\_HERMITIAN\_POSITIVE\_DEFINITE
- MKL\_DSS\_HERMITIAN\_INDEFINITE

depending on your matrix's type.

---

#### NOTE

This routine supports the Progress Routine feature. See [Progress Function](#) section for details.

---

### Input Parameters

|               |                                                                                    |
|---------------|------------------------------------------------------------------------------------|
| <i>handle</i> | Pointer to the data structure storing internal DSS results (MKL_DSS_HANDLE).       |
| <i>opt</i>    | Parameter to pass the DSS options. The default value is MKL_DSS_POSITIVE_DEFINITE. |

|                |                                                                                                                                                                             |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>rValues</i> | Array of elements of the matrix <i>A</i> . Real data, single or double precision as it is specified by the parameter <i>opt</i> in the routine <code>dss_create</code> .    |
| <i>cValues</i> | Array of elements of the matrix <i>A</i> . Complex data, single or double precision as it is specified by the parameter <i>opt</i> in the routine <code>dss_create</code> . |

## Return Values

MKL\_DSS\_SUCCESS  
 MKL\_DSS\_STATE\_ERR  
 MKL\_DSS\_INVALID\_OPTION  
 MKL\_DSS\_OPTION\_CONFLICT  
 MKL\_DSS\_VALUES\_ERR  
 MKL\_DSS\_OUT\_OF\_MEMORY  
 MKL\_DSS\_ZERO\_PIVOT  
 MKL\_DSS\_FAILURE  
 MKL\_DSS\_MSG\_LVL\_ERR  
 MKL\_DSS\_TERM\_LVL\_ERR  
 MKL\_DSS\_OOC\_MEM\_ERR  
 MKL\_DSS\_OOC\_OC\_ERR  
 MKL\_DSS\_OOC\_RW\_ERR

## `dss_solve_real`, `dss_solve_complex`

Compute the corresponding solution vector and place it in the output array.

### Syntax

```

MKL_INT dss_solve_real(_MKL_DSS_HANDLE_t *handle, MKL_INT const *opt, void const
*rRhsValues, MKL_INT const *nRhs, void *rSolValues)

MKL_INT dss_solve_complex(_MKL_DSS_HANDLE_t *handle, MKL_INT const *opt, void const
*cRhsValues, MKL_INT const *nRhs, void *cSolValues)

```

### Include Files

- `mkl.h`

### Description

For each right-hand side column vector defined in the arrays *rRhsValues*, *cRhsValues*, or *RhsValues*, these routines compute the corresponding solution vector and place it in the arrays *rSolValues*, *cSolValues*, or *SolValues* respectively.

---

#### NOTE

The data type (single or double precision) of all arrays must be in correspondence with precision specified by the parameter *opt* in the routine `dss_create`.

---

The lengths of the right-hand side and solution vectors, *nRows* and *nCols* respectively, must be defined in a previous call to `dss_define_structure`.

By default, both routines perform the full solution step (it corresponds to *phase* = 33 in Intel MKL PARDISO). The parameter *opt* enables you to calculate the final solution step-by-step, calling forward and backward substitutions.

If it is set to `MKL_DSS_FORWARD_SOLVE`, the forward substitution (corresponding to *phase* = 331 in Intel MKL PARDISO) is performed;

if it is set to `MKL_DSS_DIAGONAL_SOLVE`, the diagonal substitution (corresponding to *phase* = 332 in Intel MKL PARDISO) is performed, if possible;

if it is set to `MKL_DSS_BACKWARD_SOLVE`, the backward substitution (corresponding to *phase* = 333 in Intel MKL PARDISO) is performed.

For more details about using these substitutions for different types of matrices, see [Separate Forward and Backward Substitution](#) in the Intel MKL PARDISO solver description.

This parameter also can control the number of refinement steps that is used on the solution stage: if it is set to `MKL_DSS_REFINEMENT_OFF`, the maximum number of refinement steps equal to zero, and if it is set to `MKL_DSS_REFINEMENT_ON` (default value), the maximum number of refinement steps is equal to 2.

`MKL_DSS_CONJUGATE_SOLVE` option added to the parameter *opt* enables solving a conjugate transposed system  $A^HX = B$  based on the factorization of the matrix *A*. This option is equivalent to the parameter *iparm*[11]= 1 in Intel MKL PARDISO.

`MKL_DSS_TRANSPOSE_SOLVE` option added to the parameter *opt* enables solving a transposed system  $A^TX = B$  based on the factorization of the matrix *A*. This option is equivalent to the parameter *iparm*[11]= 2 in Intel MKL PARDISO.

## Input Parameters

|                   |                                                                                                                                                                                                                       |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>handle</i>     | Pointer to the data structure storing internal DSS results ( <code>MKL_DSS_HANDLE</code> ).                                                                                                                           |
| <i>opt</i>        | Parameter to pass the DSS options.                                                                                                                                                                                    |
| <i>nRhs</i>       | Number of the right-hand sides in the system of linear equations.                                                                                                                                                     |
| <i>rRhsValues</i> | Array of size <i>nRows</i> * <i>nRhs</i> . Contains real right-hand side vectors. Real data, single or double precision as it is specified by the parameter <i>opt</i> in the routine <code>dss_create</code> .       |
| <i>cRhsValues</i> | Array of size <i>nRows</i> * <i>nRhs</i> . Contains complex right-hand side vectors. Complex data, single or double precision as it is specified by the parameter <i>opt</i> in the routine <code>dss_create</code> . |
| <i>RhsValues</i>  | Array of size <i>nRows</i> * <i>nRhs</i> . Contains right-hand side vectors. Real or complex data, single or double precision as it is specified by the parameter <i>opt</i> in the routine <code>dss_create</code> . |

## Output Parameters

|                   |                                                                                                                                                                                                                |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>rSolValues</i> | Array of size <i>nCols</i> * <i>nRhs</i> . Contains real solution vectors. Real data, single or double precision as it is specified by the parameter <i>opt</i> in the routine <code>dss_create</code> .       |
| <i>cSolValues</i> | Array of size <i>nCols</i> * <i>nRhs</i> . Contains complex solution vectors. Complex data, single or double precision as it is specified by the parameter <i>opt</i> in the routine <code>dss_create</code> . |



## dss\_statistics

Returns statistics about various phases of the solving process.

---

### Syntax

```
MKL_INT dss_statistics(_MKL_DSS_HANDLE_t *handle, MKL_INT const *opt, _CHARACTER_STR_t const *statArr, _DOUBLE_PRECISION_t *retValues)
```

### Include Files

- mkl.h

### Description

The `dss_statistics` routine returns statistics about various phases of the solving process. This routine gathers the following statistics:

- time taken to do reordering,
- time taken to do factorization,
- duration of problem solving,
- determinant of the symmetric indefinite input matrix,
- inertia of the symmetric indefinite input matrix,
- number of floating point operations taken during factorization,
- total peak memory needed during the analysis and symbolic factorization,
- permanent memory needed from the analysis and symbolic factorization,
- memory consumption for the factorization and solve phases.

Statistics are returned in accordance with the input string specified by the parameter `statArr`. The value of the statistics is returned in double precision in a return array, which you must allocate beforehand.

For multiple statistics, multiple string constants separated by commas can be used as input. Return values are put into the return array in the same order as specified in the input string.

Statistics can only be requested at the appropriate stages of the solving process. For example, requesting `FactorTime` before a matrix is factored leads to an error.

The following table shows the point at which each individual statistics item can be requested:

### Statistics Calling Sequences

| Type of Statistics | When to Call                                                                                                    |
|--------------------|-----------------------------------------------------------------------------------------------------------------|
| ReorderTime        | After <code>dss_reorder</code> is completed successfully.                                                       |
| FactorTime         | After <code>dss_factor_real</code> or <code>dss_factor_complex</code> is completed successfully.                |
| SolveTime          | After <code>dss_solve_real</code> or <code>dss_solve_complex</code> is completed successfully.                  |
| Determinant        | After <code>dss_factor_real</code> or <code>dss_factor_complex</code> is completed successfully.                |
| Inertia            | After <code>dss_factor_real</code> is completed successfully and the matrix is real, symmetric, and indefinite. |
| Flops              | After <code>dss_factor_real</code> or <code>dss_factor_complex</code> is completed successfully.                |
| Peakmem            | After <code>dss_reorder</code> is completed successfully.                                                       |
| Factormem          | After <code>dss_reorder</code> is completed successfully.                                                       |
| Solvemem           | After <code>dss_factor_real</code> or <code>dss_factor_complex</code> is completed successfully.                |

## Input Parameters

|                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|--------------------------------------------|------------|-----------------------------------------------|-----------|----------------------------------------------------------------|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-------------------------------------------------------------------------|---------|------------------------------------------------------------------------------------------------------------|
| <i>handle</i>  | Pointer to the data structure storing internal DSS results (MKL_DSS_HANDLE).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
| <i>opt</i>     | Parameter to pass the DSS options.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
| <i>statArr</i> | Input string that defines the type of the returned statistics. The parameter can include one or more of the following string constants (case of the input string has no effect): <table> <tr> <td>ReorderTime</td> <td>Amount of time taken to do the reordering.</td> </tr> <tr> <td>FactorTime</td> <td>Amount of time taken to do the factorization.</td> </tr> <tr> <td>SolveTime</td> <td>Amount of time taken to solve the problem after factorization.</td> </tr> <tr> <td>Determinant</td> <td>Determinant of the matrix <math>A</math>.<br/><br/>For real matrices: the determinant is returned as <math>det\_pow</math>, <math>det\_base</math> in two consecutive return array locations, where <math>1.0 \leq abs(det\_base) &lt; 10.0</math> and <math>determinant = det\_base * 10^{(det\_pow)}</math>.<br/><br/>For complex matrices: the determinant is returned as <math>det\_pow</math>, <math>det\_re</math>, <math>det\_im</math> in three consecutive return array locations, where <math>1.0 \leq abs(det\_re) + abs(det\_im) &lt; 10.0</math> and <math>determinant = (det\_re, det\_im) * 10^{(det\_pow)}</math>.</td> </tr> <tr> <td>Inertia</td> <td>Inertia of a real symmetric matrix is defined as a triplet of nonnegative integers <math>(p, n, z)</math>, where <math>p</math> is the number of positive eigenvalues, <math>n</math> is the number of negative eigenvalues, and <math>z</math> is the number of zero eigenvalues.<br/><br/>Inertia is returned as three consecutive return array locations <math>p, n, z</math>.<br/><br/>Computing inertia can lead to incorrect results for matrixes with a cluster of eigenvalues which are near 0.<br/><br/>Inertia of a <math>k</math>-by-<math>k</math> real symmetric positive definite matrix is always <math>(k, 0, 0)</math>. Therefore Inertia is returned only in cases of real symmetric indefinite matrices. For all other matrix types, an error message is returned.</td> </tr> <tr> <td>Flops</td> <td>Number of floating point operations performed during the factorization.</td> </tr> <tr> <td>Peakmem</td> <td>Total peak memory in kilobytes that the solver needs during the analysis and symbolic factorization phase.</td> </tr> </table> | ReorderTime | Amount of time taken to do the reordering. | FactorTime | Amount of time taken to do the factorization. | SolveTime | Amount of time taken to solve the problem after factorization. | Determinant | Determinant of the matrix $A$ .<br><br>For real matrices: the determinant is returned as $det\_pow$ , $det\_base$ in two consecutive return array locations, where $1.0 \leq abs(det\_base) < 10.0$ and $determinant = det\_base * 10^{(det\_pow)}$ .<br><br>For complex matrices: the determinant is returned as $det\_pow$ , $det\_re$ , $det\_im$ in three consecutive return array locations, where $1.0 \leq abs(det\_re) + abs(det\_im) < 10.0$ and $determinant = (det\_re, det\_im) * 10^{(det\_pow)}$ . | Inertia | Inertia of a real symmetric matrix is defined as a triplet of nonnegative integers $(p, n, z)$ , where $p$ is the number of positive eigenvalues, $n$ is the number of negative eigenvalues, and $z$ is the number of zero eigenvalues.<br><br>Inertia is returned as three consecutive return array locations $p, n, z$ .<br><br>Computing inertia can lead to incorrect results for matrixes with a cluster of eigenvalues which are near 0.<br><br>Inertia of a $k$ -by- $k$ real symmetric positive definite matrix is always $(k, 0, 0)$ . Therefore Inertia is returned only in cases of real symmetric indefinite matrices. For all other matrix types, an error message is returned. | Flops | Number of floating point operations performed during the factorization. | Peakmem | Total peak memory in kilobytes that the solver needs during the analysis and symbolic factorization phase. |
| ReorderTime    | Amount of time taken to do the reordering.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
| FactorTime     | Amount of time taken to do the factorization.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
| SolveTime      | Amount of time taken to solve the problem after factorization.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
| Determinant    | Determinant of the matrix $A$ .<br><br>For real matrices: the determinant is returned as $det\_pow$ , $det\_base$ in two consecutive return array locations, where $1.0 \leq abs(det\_base) < 10.0$ and $determinant = det\_base * 10^{(det\_pow)}$ .<br><br>For complex matrices: the determinant is returned as $det\_pow$ , $det\_re$ , $det\_im$ in three consecutive return array locations, where $1.0 \leq abs(det\_re) + abs(det\_im) < 10.0$ and $determinant = (det\_re, det\_im) * 10^{(det\_pow)}$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
| Inertia        | Inertia of a real symmetric matrix is defined as a triplet of nonnegative integers $(p, n, z)$ , where $p$ is the number of positive eigenvalues, $n$ is the number of negative eigenvalues, and $z$ is the number of zero eigenvalues.<br><br>Inertia is returned as three consecutive return array locations $p, n, z$ .<br><br>Computing inertia can lead to incorrect results for matrixes with a cluster of eigenvalues which are near 0.<br><br>Inertia of a $k$ -by- $k$ real symmetric positive definite matrix is always $(k, 0, 0)$ . Therefore Inertia is returned only in cases of real symmetric indefinite matrices. For all other matrix types, an error message is returned.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
| Flops          | Number of floating point operations performed during the factorization.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |
| Peakmem        | Total peak memory in kilobytes that the solver needs during the analysis and symbolic factorization phase.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |             |                                            |            |                                               |           |                                                                |             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |                                                                         |         |                                                                                                            |

|           |                                                                                                                                               |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| Factormem | Permanent memory in kilobytes that the solver needs from the analysis and symbolic factorization phase in the factorization and solve phases. |
| Solvemem  | Total double precision memory consumption (kilobytes) of the solver for the factorization and solve phases.                                   |

## Output Parameters

*retValues* Value of the statistics returned.

## Finding 'time used to reorder' and 'inertia' of a matrix

The example below illustrates the use of the `dss_statistics` routine.

To find the above values, call `dss_statistics(handle, opt, statArr, retValue)`, where *statArr* is "ReorderTime,Inertia"

In this example, *retValue* has the following values:

|                     |                       |
|---------------------|-----------------------|
| <i>retValue</i> [0] | Time to reorder.      |
| <i>retValue</i> [1] | Positive Eigenvalues. |
| <i>retValue</i> [2] | Negative Eigenvalues. |
| <i>retValue</i> [3] | Zero Eigenvalues.     |

## Return Values

MKL\_DSS\_SUCCESS  
MKL\_DSS\_INVALID\_OPTION  
MKL\_DSS\_STATISTICS\_INVALID\_MATRIX  
MKL\_DSS\_STATISTICS\_INVALID\_STATE  
MKL\_DSS\_STATISTICS\_INVALID\_STRING  
MKL\_DSS\_MSG\_LVL\_ERR  
MKL\_DSS\_TERM\_LVL\_ERR

## Iterative Sparse Solvers based on Reverse Communication Interface (RCI ISS)

---

Intel MKL supports iterative sparse solvers (ISS) based on the reverse communication interface (RCI), referred to here as the RCI ISS interface. The RCI ISS interface implements a group of user-callable routines that are used in the step-by-step solving process of a symmetric positive definite system (RCI conjugate gradient solver, or RCI CG), and of a non-symmetric indefinite (non-degenerate) system (RCI flexible generalized minimal residual solver, or RCI FGMRES) of linear algebraic equations. This interface uses the general RCI scheme described in [Dong95].

See the [Appendix A Linear Solvers Basics](#) for discussion of terms and concepts related to the ISS routines.

The term *RCI* indicates that when the solver needs the results of certain operations (for example, matrix-vector multiplications), the user performs them and passes the result to the solver. This makes the solver more universal as it is independent of the specific implementation of the operations like the matrix-vector multiplication. To perform such operations, the user can use the built-in sparse matrix-vector multiplications and triangular solvers routines described in [Sparse BLAS Level 2 and Level 3 Routines](#).

**NOTE**

The RCI CG solver is implemented in two versions: for system of equations with a single right-hand side, and for systems of equations with multiple right-hand sides.

The CG method may fail to compute the solution or compute the wrong solution if the matrix of the system is not symmetric and not positive definite.

The FGMRES method may fail if the matrix is degenerate.

Table "RCI CG Interface Routines" lists the names of the routines, and describes their general use.

**RCI ISS Interface Routines**

| Routine                                                                                   | Description                                                      |
|-------------------------------------------------------------------------------------------|------------------------------------------------------------------|
| <a href="#">dcg_init</a> , <a href="#">dcgmrhs_init</a> , <a href="#">dfgmres_init</a>    | Initializes the solver.                                          |
| <a href="#">dcg_check</a> , <a href="#">dcgmrhs_check</a> , <a href="#">dfgmres_check</a> | Checks the consistency and correctness of the user defined data. |
| <a href="#">dcg</a> , <a href="#">dcgmrhs</a> , <a href="#">dfgmres</a>                   | Computes the approximate solution vector.                        |
| <a href="#">dcg_get</a> , <a href="#">dcgmrhs_get</a> , <a href="#">dfgmres_get</a>       | Retrieves the number of the current iteration.                   |

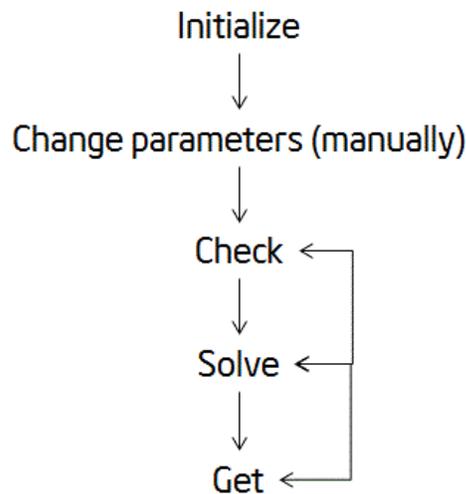
The Intel MKL RCI ISS interface routines are normally invoked in this order:

1. `<system_type>_init`
2. `<system_type>_check`
3. `<system_type>`
4. `<system_type>_get`

Advanced users can change that order if they need it. Others should follow the above order of calls.

The following diagram indicates the typical order in which the RCI ISS interface routines are invoked.

**Typical Order for Invoking RCI ISS interface Routines**



See the code examples that use the RCI ISS interface routines to solve systems of linear equations in the Intel MKL installation directory.

- `examples/solverc/source`

## CG Interface Description

Each routine for the RCI CG solver is implemented in two versions: for a system of equations with a single right-hand side (SRHS), and for a system of equations with multiple right-hand sides (MRHS). The names of routines for a system with MRHS contain the suffix `mrhs`.

### Routine Options

All of the RCI CG routines have common parameters for passing various options to the routines (see [CG Common Parameters](#)). The values for these parameters can be changed during computations.

### User Data Arrays

Many of the RCI CG routines take arrays of user data as input. For example, user arrays are passed to the routine `dcg` to compute the solution of a system of linear algebraic equations. The Intel MKL RCI CG routines do not make copies of the user input arrays to minimize storage requirements and improve overall run-time efficiency.

### CG Common Parameters

---

#### NOTE

The default and initial values listed below are assigned to the parameters by calling the `dcg_init/dcgmrhs_init` routine.

---

|                    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>           | MKL_INT, this parameter sets the size of the problem in the <code>dcg_init/dcgmrhs_init</code> routine. All the other routines use the <code>ipar[0]</code> parameter instead. Note that the coefficient matrix <i>A</i> is a square matrix of size $n*n$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>x</i>           | double array of size <i>n</i> for SRHS, or matrix of size $(n*nrhs)$ for MRHS. This parameter contains the current approximation to the solution. Before the first call to the <code>dcg/dcgmrhs</code> routine, it contains the initial approximation to the solution.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <i>nrhs</i>        | MKL_INT, this parameter sets the number of right-hand sides for MRHS routines.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <i>b</i>           | double array containing a single right-hand side vector, or matrix of size $n*nrhs$ containing right-hand side vectors.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <i>RCI_request</i> | <p>MKL_INT, this parameter gives information about the result of work of the RCI CG routines. Negative values of the parameter indicate that the routine completed with errors or warnings. The 0 value indicates successful completion of the task. Positive values mean that you must perform specific actions:</p> <p><i>RCI_request</i>= 1      multiply the matrix by <code>tmp [0:n - 1)</code>, put the result in <code>tmp[n:2*n - 1)</code>, and return the control to the <code>dcg/dcgmrhs</code> routine;</p> <p><i>RCI_request</i>= 2      to perform the stopping tests. If they fail, return the control to the <code>dcg/dcgmrhs</code> routine. If the stopping tests succeed, it indicates that the solution is found and stored in the <i>x</i> array;</p> |

*RCI\_request*= 3      for SRHS: apply the preconditioner to *tmp*[2\**n*:3\**n* - 1], put the result in *tmp*[3\**n*:4\**n* - 1], and return the control to the *dcg* routine;

                          for MRHS: apply the preconditioner to *tmp*[2+*ipar*[2]\**n*:(3 + *ipar*[2])\**n* - 1], put the result in *tmp*[3\**n*:4\**n* - 1], and return the control to the *dcgmrhs* routine.

Note that the *dcg\_get/dcgmrhs\_get* routine does not change the parameter *RCI\_request*. This enables use of this routine inside the *reverse communication* computations.

*ipar*

MKL\_INT array, of size 128 for SRHS, and of size (128+2\**nrhs*) for MRHS. This parameter specifies the integer set of data for the RCI CG computations:

*ipar*[0]                specifies the size of the problem. The *dcg\_init/dcgmrhs\_init* routine assigns *ipar*[0]=*n*. All the other routines use this parameter instead of *n*. There is no default value for this parameter.

*ipar*[1]                specifies the type of output for error and warning messages generated by the RCI CG routines. The default value 6 means that all messages are displayed on the screen. Otherwise, the error and warning messages are written to the newly created files *dcg\_errors.txt* and *dcg\_check\_warnings.txt*, respectively. Note that if *ipar*[5] and *ipar*[6] parameters are set to 0, error and warning messages are not generated at all.

*ipar*[2]                for SRHS: contains the current stage of the RCI CG computations. The initial value is 1;  
                           for MRHS: contains the number of the right-hand side for which the calculations are currently performed.

---

**WARNING**

Avoid altering this variable during computations.

---

*ipar*[3]                contains the current iteration number. The initial value is 0.

*ipar*[4]                specifies the maximum number of iterations. The default value is  $\min(150, n)$ .

*ipar*[5]                if the value is not equal to 0, the routines output error messages in accordance with the parameter *ipar*[1]. Otherwise, the routines do not output error messages at all, but return a negative value of the parameter *RCI\_request*. The default value is 1.

*ipar*[6] if the value is not equal to 0, the routines output warning messages in accordance with the parameter *ipar*[1]. Otherwise, the routines do not output warning messages at all, but they return a negative value of the parameter *RCI\_request*. The default value is 1.

*ipar*[7] if the value is not equal to 0, the *dcg/dcgmrhs* routine performs the stopping test for the maximum number of iterations:  $ipar[3] \leq ipar[4]$ . Otherwise, the method is stopped and the corresponding value is assigned to the *RCI\_request*. If the value is 0, the routine does not perform this stopping test. The default value is 1.

*ipar*[8] if the value is not equal to 0, the *dcg/dcgmrhs* routine performs the residual stopping test:  $dpar(5) \leq dpar(4) = dpar(1) * dpar(3) + dpar(2)$ . Otherwise, the method is stopped and corresponding value is assigned to the *RCI\_request*. If the value is 0, the routine does not perform this stopping test. The default value is 0.

*ipar*[9] if the value is not equal to 0, the *dcg/dcgmrhs* routine requests a user-defined stopping test by setting the output parameter *RCI\_request*=2. If the value is 0, the routine does not perform the user defined stopping test. The default value is 1.

---

**NOTE**

At least one of the parameters *ipar*[7]-*ipar*[9] must be set to 1.

---

*ipar*[10] if the value is equal to 0, the *dcg/dcgmrhs* routine runs the non-preconditioned version of the corresponding CG method. Otherwise, the routine runs the preconditioned version of the CG method, and by setting the output parameter *RCI\_request*=3, indicates that you must perform the preconditioning step. The default value is 0.

*ipar*[11:127] are reserved and not used in the current RCI CG SRHS and MRHS routines.

---

**NOTE**

For future compatibility, you must declare the array *ipar* with length 128 for a single right-hand side.

---

*ipar*[11:127 + 2\**nrhs*] are reserved for internal use in the current RCI CG SRHS and MRHS routines.

**NOTE**

For future compatibility, you must declare the array *ipar* with length  $128+2*nrhs$  for multiple right-hand sides.

*dpar*

double array, for SRHS of size 128, for MRHS of size  $(128+2*nrhs)$ ; this parameter is used to specify the double precision set of data for the RCI CG computations, specifically:

|                     |                                                                                                                                      |
|---------------------|--------------------------------------------------------------------------------------------------------------------------------------|
| <i>dpar</i> [0]     | specifies the relative tolerance. The default value is $1.0 \times 10^{-6}$ .                                                        |
| <i>dpar</i> [1]     | specifies the absolute tolerance. The default value is 0.0.                                                                          |
| <i>dpar</i> [2]     | specifies the square norm of the initial residual (if it is computed in the <i>dcg/dcgmrhs</i> routine). The initial value is 0.0.   |
| <i>dpar</i> [3]     | service variable equal to $dpar[0]*dpar[2]+dpar[1]$ (if it is computed in the <i>dcg/dcgmrhs</i> routine). The initial value is 0.0. |
| <i>dpar</i> [4]     | specifies the square norm of the current residual. The initial value is 0.0.                                                         |
| <i>dpar</i> [5]     | specifies the square norm of residual from the previous iteration step (if available). The initial value is 0.0.                     |
| <i>dpar</i> [6]     | contains the <i>alpha</i> parameter of the CG method. The initial value is 0.0.                                                      |
| <i>dpar</i> [7]     | contains the <i>beta</i> parameter of the CG method, it is equal to $dpar[4]/dpar[5]$ The initial value is 0.0.                      |
| <i>dpar</i> [8:127] | are reserved and not used in the current RCI CG SRHS and MRHS routines.                                                              |

**NOTE**

For future compatibility, you must declare the array *dpar* with length 128 for a single right-hand side.

*dpar*(9:128+2\*nrhs) are reserved for internal use in the current RCI CG  
 ) [8:127 + 2\*nrhs] SRHS and MRHS routines.

---

**NOTE**

For future compatibility, you must declare the array *dpar* with length  $128+2*nrhs$  for multiple right-hand sides.

---

*tmp*

double array of size  $(n*4)$  for SRHS, and  $(n*(3+nrhs))$  for MRHS. This parameter is used to supply the double precision temporary space for the RCI CG computations, specifically:

- tmp*[0:*n* - 1] specifies the current search direction. The initial value is 0.0.
- tmp*[*n*:2\**n* - 1] contains the matrix multiplied by the current search direction. The initial value is 0.0.
- tmp*[2\**n*:3\**n* - 1] contains the current residual. The initial value is 0.0.
- tmp*[3\**n*:4\**n* - 1] contains the inverse of the preconditioner applied to the current residual for the SRHS version of CG. There is no initial value for this parameter.
- tmp*[4\**n*: (4 + *nrhs*) \* *n* - 1] contains the inverse of the preconditioner applied to the current residual for the MRHS version of CG. There is no initial value for this parameter.

---

**NOTE**

You can define this array in the code using RCI CG SRHS as `double tmp[3*n]` if you run only non-preconditioned CG iterations.

---

## FGMRES Interface Description

### Routine Options

All of the RCI FGMRES routines have common parameters for passing various options to the routines (see [FGMRES Common Parameters](#)). The values for these parameters can be changed during computations.

### User Data Arrays

Many of the RCI FGMRES routines take arrays of user data as input. For example, user arrays are passed to the routine `dfgmres` to compute the solution of a system of linear algebraic equations. To minimize storage requirements and improve overall run-time efficiency, the Intel MKL RCI FGMRES routines do not make copies of the user input arrays.

### FGMRES Common Parameters

---

**NOTE**

The default and initial values listed below are assigned to the parameters by calling the `dfgmres_init` routine.

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|                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
|------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>               | MKL_INT, this parameter sets the size of the problem in the <code>dfgmres_init</code> routine. All the other routines use the <code>ipar[0]</code> parameter instead. Note that the coefficient matrix <i>A</i> is a square matrix of size $n*n$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>x</i>               | double array, this parameter contains the current approximation to the solution vector. Before the first call to the <code>dfgmres</code> routine, it contains the initial approximation to the solution vector.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>b</i>               | double array, this parameter contains the right-hand side vector. Depending on user requests (see the parameter <code>ipar[12]</code> ), it might contain the approximate solution after execution.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>RCI_request</i>     | MKL_INT, this parameter gives information about the result of work of the RCI FGMRES routines. Negative values of the parameter indicate that the routine completed with errors or warnings. The 0 value indicates successful completion of the task. Positive values mean that you must perform specific actions: <table border="0" style="margin-left: 2em;"> <tr> <td style="vertical-align: top;"><i>RCI_request</i>= 1</td> <td>multiply the matrix by <code>tmp[ipar[21] - 1:ipar[21] + n - 2]</code>, put the result in <code>tmp[ipar[22] - 1:ipar[22] + n - 2]</code>, and return the control to the <code>dfgmres</code> routine;</td> </tr> <tr> <td style="vertical-align: top;"><i>RCI_request</i>= 2</td> <td>perform the stopping tests. If they fail, return the control to the <code>dfgres</code> routine. Otherwise, the solution can be updated by a subsequent call to <code>dfgmres_get</code> routine;</td> </tr> <tr> <td style="vertical-align: top;"><i>RCI_request</i>= 3</td> <td>apply the preconditioner to <code>tmp[ipar[21] - 1:ipar[21] + n - 2]</code>, put the result in <code>tmp[ipar[22] - 1:ipar[22] + n - 2]</code>, and return the control to the <code>dfgmres</code> routine.</td> </tr> <tr> <td style="vertical-align: top;"><i>RCI_request</i>= 4</td> <td>check if the norm of the current orthogonal vector is zero, within the rounding or computational errors. Return the control to the <code>dfgmres</code> routine if it is not zero, otherwise complete the solution process by calling <code>dfgmres_get</code> routine.</td> </tr> </table> | <i>RCI_request</i> = 1 | multiply the matrix by <code>tmp[ipar[21] - 1:ipar[21] + n - 2]</code> , put the result in <code>tmp[ipar[22] - 1:ipar[22] + n - 2]</code> , and return the control to the <code>dfgmres</code> routine;                 | <i>RCI_request</i> = 2 | perform the stopping tests. If they fail, return the control to the <code>dfgres</code> routine. Otherwise, the solution can be updated by a subsequent call to <code>dfgmres_get</code> routine;                                                                                                             | <i>RCI_request</i> = 3 | apply the preconditioner to <code>tmp[ipar[21] - 1:ipar[21] + n - 2]</code> , put the result in <code>tmp[ipar[22] - 1:ipar[22] + n - 2]</code> , and return the control to the <code>dfgmres</code> routine. | <i>RCI_request</i> = 4 | check if the norm of the current orthogonal vector is zero, within the rounding or computational errors. Return the control to the <code>dfgmres</code> routine if it is not zero, otherwise complete the solution process by calling <code>dfgmres_get</code> routine. |
| <i>RCI_request</i> = 1 | multiply the matrix by <code>tmp[ipar[21] - 1:ipar[21] + n - 2]</code> , put the result in <code>tmp[ipar[22] - 1:ipar[22] + n - 2]</code> , and return the control to the <code>dfgmres</code> routine;                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>RCI_request</i> = 2 | perform the stopping tests. If they fail, return the control to the <code>dfgres</code> routine. Otherwise, the solution can be updated by a subsequent call to <code>dfgmres_get</code> routine;                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>RCI_request</i> = 3 | apply the preconditioner to <code>tmp[ipar[21] - 1:ipar[21] + n - 2]</code> , put the result in <code>tmp[ipar[22] - 1:ipar[22] + n - 2]</code> , and return the control to the <code>dfgmres</code> routine.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>RCI_request</i> = 4 | check if the norm of the current orthogonal vector is zero, within the rounding or computational errors. Return the control to the <code>dfgmres</code> routine if it is not zero, otherwise complete the solution process by calling <code>dfgmres_get</code> routine.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>ipar[128]</i>       | MKL_INT array, this parameter specifies the integer set of data for the RCI FGMRES computations: <table border="0" style="margin-left: 2em;"> <tr> <td style="vertical-align: top;"><i>ipar</i>[0]</td> <td>specifies the size of the problem. The <code>dfgmres_init</code> routine assigns <code>ipar[0]=n</code>. All the other routines uses this parameter instead of <i>n</i>. There is no default value for this parameter.</td> </tr> <tr> <td style="vertical-align: top;"><i>ipar</i>[1]</td> <td>specifies the type of output for error and warning messages that are generated by the RCI FGMRES routines. The default value 6 means that all messages are displayed on the screen. Otherwise the error and warning messages are written to the newly created file <code>MKL_RCI_FGMRES_Log.txt</code>. Note</td> </tr> </table>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | <i>ipar</i> [0]        | specifies the size of the problem. The <code>dfgmres_init</code> routine assigns <code>ipar[0]=n</code> . All the other routines uses this parameter instead of <i>n</i> . There is no default value for this parameter. | <i>ipar</i> [1]        | specifies the type of output for error and warning messages that are generated by the RCI FGMRES routines. The default value 6 means that all messages are displayed on the screen. Otherwise the error and warning messages are written to the newly created file <code>MKL_RCI_FGMRES_Log.txt</code> . Note |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>ipar</i> [0]        | specifies the size of the problem. The <code>dfgmres_init</code> routine assigns <code>ipar[0]=n</code> . All the other routines uses this parameter instead of <i>n</i> . There is no default value for this parameter.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |
| <i>ipar</i> [1]        | specifies the type of output for error and warning messages that are generated by the RCI FGMRES routines. The default value 6 means that all messages are displayed on the screen. Otherwise the error and warning messages are written to the newly created file <code>MKL_RCI_FGMRES_Log.txt</code> . Note                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                        |                                                                                                                                                                                                                          |                        |                                                                                                                                                                                                                                                                                                               |                        |                                                                                                                                                                                                               |                        |                                                                                                                                                                                                                                                                         |

that if *ipar*[5] and *ipar*[6] parameters are set to 0, error and warning messages are not generated at all.

*ipar*[2] contains the current stage of the RCI FGMRES computations. The initial value is 1.

---

**WARNING**

Avoid altering this variable during computations.

---

*ipar*[3] contains the current iteration number. The initial value is 0.

*ipar*[4] specifies the maximum number of iterations. The default value is *min* (150,*n*).

*ipar*[5] if the value is not 0, the routines output error messages in accordance with the parameter *ipar*[1]. If it is 0, the routines do not output error messages at all, but return a negative value of the parameter *RCI\_request*. The default value is 1.

*ipar*[6] if the value is not 0, the routines output warning messages in accordance with the parameter *ipar*[1]. Otherwise, the routines do not output warning messages at all, but they return a negative value of the parameter *RCI\_request*. The default value is 1.

*ipar*[7] if the value is not equal to 0, the *dfgmres* routine performs the stopping test for the maximum number of iterations: *ipar*[3] ≤ *ipar*[4]. If the value is 0, the *dfgmres* routine does not perform this stopping test. The default value is 1.

*ipar*[8] if the value is not 0, the *dfgmres* routine performs the residual stopping test: *dpar*[4] ≤ *dpar*[3]. If the value is 0, the *dfgmres* routine does not perform this stopping test. The default value is 0.

*ipar*[9] if the value is not 0, the *dfgmres* routine indicates that the user-defined stopping test should be performed by setting *RCI\_request*=2. If the value is 0, the *dfgmres* routine does not perform the user-defined stopping test. The default value is 1.

---

**NOTE**

At least one of the parameters *ipar*[7]-*ipar*[9] must be set to 1.

---

---

|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> [10] | if the value is 0, the <code>dfgmres</code> routine runs the non-preconditioned version of the FGMRES method. Otherwise, the routine runs the preconditioned version of the FGMRES method, and requests that you perform the preconditioning step by setting the output parameter <code>RCI_request=3</code> . The default value is 0.                                                                                          |
| <i>ipar</i> [11] | if the value is not equal to 0, the <code>dfgmres</code> routine performs the automatic test for zero norm of the currently generated vector: $dpar[6] \leq dpar[7]$ , where <code>dpar[7]</code> contains the tolerance value. Otherwise, the routine indicates that you must perform this check by setting the output parameter <code>RCI_request=4</code> . The default value is 0.                                          |
| <i>ipar</i> [12] | if the value is equal to 0, the <code>dfgmres_get</code> routine updates the solution to the vector $x$ according to the computations done by the <code>dfgmres</code> routine. If the value is positive, the routine writes the solution to the right-hand side vector $b$ . If the value is negative, the routine returns only the number of the current iteration, and does not update the solution. The default value is 0. |

**NOTE**


---

It is possible to call the `dfgmres_get` routine at any place in the code, but you must pay special attention to the parameter `ipar[12]`. The RCI FGMRES iterations can be continued after the call to `dfgmres_get` routine only if the parameter `ipar[12]` is not equal to zero. If `ipar[12]` is positive, then the updated solution overwrites the right-hand side in the vector  $b$ . If you want to run the restarted version of FGMRES with the same right-hand side, then it must be saved in a different memory location before the first call to the `dfgmres_get` routine with positive `ipar[12]`.

---

|                  |                                                                                                                                      |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> [13] | contains the internal iteration counter that counts the number of iterations before the restart takes place. The initial value is 0. |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------|

**WARNING**


---

Do not alter this variable during computations.

---

|                  |                                                                                                                                                    |
|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> [14] | specifies the number of the non-restarted FGMRES iterations. To run the restarted version of the FGMRES method, assign the number of iterations to |
|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------|

|  |                      |                                                                                                                                                                                                                                                                      |
|--|----------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <i>ipar</i> [14]     | before the restart. The default value is $\min(150, n)$ , which means that by default the non-restarted version of FGMRES method is used.                                                                                                                            |
|  | <i>ipar</i> [15]     | service variable specifying the location of the rotated Hessenberg matrix from which the matrix stored in the packed format (see <a href="#">Matrix Arguments</a> in the Appendix B for details) is started in the <i>tmp</i> array.                                 |
|  | <i>ipar</i> [16]     | service variable specifying the location of the rotation cosines from which the vector of cosines is started in the <i>tmp</i> array.                                                                                                                                |
|  | <i>ipar</i> [17]     | service variable specifying the location of the rotation sines from which the vector of sines is started in the <i>tmp</i> array.                                                                                                                                    |
|  | <i>ipar</i> [18]     | service variable specifying the location of the rotated residual vector from which the vector is started in the <i>tmp</i> array.                                                                                                                                    |
|  | <i>ipar</i> [19]     | service variable, specifies the location of the least squares solution vector from which the vector is started in the <i>tmp</i> array.                                                                                                                              |
|  | <i>ipar</i> [20]     | service variable specifying the location of the set of preconditioned vectors from which the set is started in the <i>tmp</i> array. The memory locations in the <i>tmp</i> array starting from <i>ipar</i> [20] are used only for the preconditioned FGMRES method. |
|  | <i>ipar</i> [21]     | specifies the memory location from which the first vector (source) used in operations requested via <i>RCI_request</i> is started in the <i>tmp</i> array.                                                                                                           |
|  | <i>ipar</i> [22]     | specifies the memory location from which the second vector (output) used in operations requested via <i>RCI_request</i> is started in the <i>tmp</i> array.                                                                                                          |
|  | <i>ipar</i> [23:127] | are reserved and not used in the current RCI FGMRES routines.                                                                                                                                                                                                        |

**NOTE**

You must declare the array *ipar* with length 128. While defining the array in the code as *ipar*[23] works, there is no guarantee of future compatibility with Intel MKL.

*dpar*(128)

double array, this parameter specifies the double precision set of data for the RCI CG computations, specifically:

|                      |                                                                                                                                            |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| <code>dpar[0]</code> | specifies the relative tolerance. The default value is 1.0e-6.                                                                             |
| <code>dpar[1]</code> | specifies the absolute tolerance. The default value is 0.0e-0.                                                                             |
| <code>dpar[2]</code> | specifies the Euclidean norm of the initial residual (if it is computed in the <code>dfgmres</code> routine). The initial value is 0.0.    |
| <code>dpar[3]</code> | service variable equal to $dpar[0] * dpar[2] + dpar[1]$ (if it is computed in the <code>dfgmres</code> routine). The initial value is 0.0. |
| <code>dpar[4]</code> | specifies the Euclidean norm of the current residual. The initial value is 0.0.                                                            |
| <code>dpar[5]</code> | specifies the Euclidean norm of residual from the previous iteration step (if available). The initial value is 0.0.                        |
| <code>dpar[6]</code> | contains the norm of the generated vector. The initial value is 0.0.                                                                       |

**NOTE**

In terms of [Saad03] this parameter is the coefficient  $h_{k+1,k}$  of the Hessenberg matrix.

|                          |                                                                                                           |
|--------------------------|-----------------------------------------------------------------------------------------------------------|
| <code>dpar[7]</code>     | contains the tolerance for the zero norm of the currently generated vector. The default value is 1.0e-12. |
| <code>dpar[8:127]</code> | are reserved and not used in the current RCI FGMRES routines.                                             |

**NOTE**

You must declare the array `dpar` with length 128. While defining the array in the code as `double dpar[8]` works, there is no guarantee of future compatibility with Intel MKL.

`tmp`

double array of size  $((2 * ipar[14] + 1) * n + ipar[14] * (ipar[14] + 9) / 2 + 1)$  used to supply the double precision temporary space for the RCI FGMRES computations, specifically:

|                                             |                                                                                                                                                                                      |
|---------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>tmp[0:ipar[15] - 2]</code>            | contains the sequence of vectors generated by the FGMRES method. The initial value is 0.0.                                                                                           |
| <code>tmp[ipar[15] - 1:ipar[16] - 2]</code> | contains the rotated Hessenberg matrix generated by the FGMRES method; the matrix is stored in the packed format. There is no initial value for this part of <code>tmp</code> array. |

|                                             |                                                                                                                                                                                                                                                                          |
|---------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>tmp[ipar[16] - 1:ipar[17] - 2]</code> | contains the rotation cosines vector generated by the FGMRES method. There is no initial value for this part of <code>tmp</code> array.                                                                                                                                  |
| <code>tmp[ipar[17] - 1:ipar[18] - 2]</code> | contains the rotation sines vector generated by the FGMRES method. There is no initial value for this part of <code>tmp</code> array.                                                                                                                                    |
| <code>tmp[ipar[18] - 1:ipar[19] - 2]</code> | contains the rotated residual vector generated by the FGMRES method. There is no initial value for this part of <code>tmp</code> array.                                                                                                                                  |
| <code>tmp[ipar[19] - 1:ipar[20] - 2]</code> | contains the solution vector to the least squares problem generated by the FGMRES method. There is no initial value for this part of <code>tmp</code> array.                                                                                                             |
| <code>tmp[ipar[20] - 1:*]</code>            | contains the set of preconditioned vectors generated for the FGMRES method by the user. This part of <code>tmp</code> array is not used if the non-preconditioned version of FGMRES method is called. There is no initial value for this part of <code>tmp</code> array. |

---

#### NOTE

You can define this array in the code as `double tmp[(2*ipar[14] + 1)*n + ipar[14]*(ipar[14] + 9)/2 + 1]` if you run only non-preconditioned FGMRES iterations.

---

## RCI ISS Routines

### dcg\_init

*Initializes the solver.*

---

#### Syntax

```
void dcg_init (const MKL_INT *n , const double *x , const double *b , MKL_INT
*RCI_request , MKL_INT *ipar , double *dpar , double *tmp);
```

#### Include Files

- `mkl.h`

#### Description

The routine `dcg_init` initializes the solver. After initialization, all subsequent invocations of the Intel MKL RCI CG routines use the values of all parameters returned by the routine `dcg_init`. Advanced users can skip this step and set the values in the `ipar` and `dpar` arrays directly.

**CAUTION**

You can modify the contents of these arrays after they are passed to the solver routine only if you are sure that the values are correct and consistent. You can perform a basic check for correctness and consistency by calling the `dcg_check` routine, but it does not guarantee that the method will work correctly.

**Input Parameters**

|          |                                                                                                                                |
|----------|--------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i> | Sets the size of the problem.                                                                                                  |
| <i>x</i> | Array of size <i>n</i> . Contains the initial approximation to the solution vector. Normally it is equal to 0 or to <i>b</i> . |
| <i>b</i> | Array of size <i>n</i> . Contains the right-hand side vector.                                                                  |

**Output Parameters**

|                    |                                                                             |
|--------------------|-----------------------------------------------------------------------------|
| <i>RCI_request</i> | Gives information about the result of the routine.                          |
| <i>ipar</i>        | Array of size 128. Refer to the <a href="#">CG Common Parameters</a> .      |
| <i>dpar</i>        | Array of size 128. Refer to the <a href="#">CG Common Parameters</a> .      |
| <i>tmp</i>         | Array of size $(n*4)$ . Refer to the <a href="#">CG Common Parameters</a> . |

**Return Values**

|                             |                                             |
|-----------------------------|---------------------------------------------|
| <i>RCI_request</i> = 0      | Indicates that the task completed normally. |
| <i>RCI_request</i> = -10000 | Indicates failure to complete the task.     |

**dcg\_check**

*Checks consistency and correctness of the user defined data.*

**Syntax**

```
void dcg_check (const MKL_INT *n , const double *x , const double *b , MKL_INT
*RCI_request , MKL_INT *ipar , double *dpar , double *tmp);
```

**Include Files**

- `mkl.h`

**Description**

The routine `dcg_check` checks consistency and correctness of the parameters to be passed to the solver routine `dcg`. However this operation does not guarantee that the solver returns the correct result. It only reduces the chance of making a mistake in the parameters of the method. Skip this operation only if you are sure that the correct data is specified in the solver parameters.

The lengths of all vectors must be defined in a previous call to the `dcg_init` routine.

## Input Parameters

|     |                                                                                                                      |
|-----|----------------------------------------------------------------------------------------------------------------------|
| $n$ | Sets the size of the problem.                                                                                        |
| $x$ | Array of size $n$ . Contains the initial approximation to the solution vector. Normally it is equal to 0 or to $b$ . |
| $b$ | Array of size $n$ . Contains the right-hand side vector.                                                             |

## Output Parameters

|                |                                                                             |
|----------------|-----------------------------------------------------------------------------|
| $RCI\_request$ | Gives information about result of the routine.                              |
| $ipar$         | Array of size 128. Refer to the <a href="#">CG Common Parameters</a> .      |
| $dpar$         | Array of size 128. Refer to the <a href="#">CG Common Parameters</a> .      |
| $tmp$          | Array of size $(n*4)$ . Refer to the <a href="#">CG Common Parameters</a> . |

## Return Values

|                        |                                                                                              |
|------------------------|----------------------------------------------------------------------------------------------|
| $RCI\_request = 0$     | Indicates that the task completed normally.                                                  |
| $RCI\_request = -1100$ | Indicates that the task is interrupted and the errors occur.                                 |
| $RCI\_request = -1001$ | Indicates that there are some warning messages.                                              |
| $RCI\_request = -1010$ | Indicates that the routine changed some parameters to make them consistent or correct.       |
| $RCI\_request = -1011$ | Indicates that there are some warning messages and that the routine changed some parameters. |

## dcg

*Computes the approximate solution vector.*

---

### Syntax

```
void dcg (const MKL_INT *n , double *x , const double *b , MKL_INT *RCI_request ,
MKL_INT *ipar , double *dpar , double *tmp);
```

### Include Files

- mkl.h

### Description

The `dcg` routine computes the approximate solution vector using the CG method [Young71]. The routine `dcg` uses the vector in the array  $x$  before the first call as an initial approximation to the solution. The parameter  $RCI\_request$  gives you information about the task completion and requests results of certain operations that are required by the solver.

Note that lengths of all vectors must be defined in a previous call to the `dcg_init` routine.

## Input Parameters

|       |                                                                                |
|-------|--------------------------------------------------------------------------------|
| $n$   | Sets the size of the problem.                                                  |
| $x$   | Array of size $n$ . Contains the initial approximation to the solution vector. |
| $b$   | Array of size $n$ . Contains the right-hand side vector.                       |
| $tmp$ | Array of size $(n*4)$ . Refer to the <a href="#">CG Common Parameters</a> .    |

## Output Parameters

|                |                                                                                |
|----------------|--------------------------------------------------------------------------------|
| $RCI\_request$ | Gives information about result of work of the routine.                         |
| $x$            | Array of size $n$ . Contains the updated approximation to the solution vector. |
| $ipar$         | Array of size 128. Refer to the <a href="#">CG Common Parameters</a> .         |
| $dpar$         | Array of size 128. Refer to the <a href="#">CG Common Parameters</a> .         |
| $tmp$          | Array of size $(n*4)$ . Refer to the <a href="#">CG Common Parameters</a> .    |

## Return Values

|                     |                                                                                                                                                                                                                                                     |
|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $RCI\_request=0$    | Indicates that the task completed normally and the solution is found and stored in the vector $x$ . This occurs only if the stopping tests are fully automatic. For the user defined stopping tests, see the description of the $RCI\_request= 2$ . |
| $RCI\_request=-1$   | Indicates that the routine was interrupted because the maximum number of iterations was reached, but the relative stopping criterion was not met. This situation occurs only if you request both tests.                                             |
| $RCI\_request=-2$   | Indicates that the routine was interrupted because of an attempt to divide by zero. This situation happens if the matrix is non-positive definite or almost non-positive definite.                                                                  |
| $RCI\_request=- 10$ | Indicates that the routine was interrupted because the residual norm is invalid. This usually happens because the value $dpar[5]$ was altered outside of the routine, or the $dcg\_check$ routine was not called.                                   |
| $RCI\_request=-11$  | Indicates that the routine was interrupted because it enters the infinite cycle. This usually happens because the values $ipar[7]$ , $ipar[8]$ , $ipar[9]$ were altered outside of the routine, or the $dcg\_check$ routine was not called.         |
| $RCI\_request= 1$   | Indicates that you must multiply the matrix by $tmp[0:n - 1]$ , put the result in the $tmp[n:2*n - 1]$ , and return control back to the routine $dcg$ .                                                                                             |

`RCI_request= 2`

Indicates that you must perform the stopping tests. If they fail, return control back to the `dcg` routine. Otherwise, the solution is found and stored in the vector  $x$ .

`RCI_request= 3`

Indicates that you must apply the preconditioner to  $[2*n:3*n - 1]$ , put the result in the  $[3*n:4*n - 1]$ , and return control back to the routine `dcg`.

## dcg\_get

*Retrieves the number of the current iteration.*

---

### Syntax

```
void dcg_get (const MKL_INT *n , const double *x , const double *b , const MKL_INT *RCI_request , const MKL_INT *ipar , const double *dpar , const double *tmp , MKL_INT *itercount);
```

### Include Files

- `mkl.h`

### Description

The routine `dcg_get` retrieves the current iteration number of the solutions process.

### Input Parameters

|                          |                                                                             |
|--------------------------|-----------------------------------------------------------------------------|
| <code>n</code>           | Sets the size of the problem.                                               |
| <code>x</code>           | Array of size $n$ . Contains the approximation vector to the solution.      |
| <code>b</code>           | Array of size $n$ . Contains the right-hand side vector.                    |
| <code>RCI_request</code> | This parameter is not used.                                                 |
| <code>ipar</code>        | Array of size 128. Refer to the <a href="#">CG Common Parameters</a> .      |
| <code>dpar</code>        | Array of size 128. Refer to the <a href="#">CG Common Parameters</a> .      |
| <code>tmp</code>         | Array of size $(n*4)$ . Refer to the <a href="#">CG Common Parameters</a> . |

### Output Parameters

|                        |                                       |
|------------------------|---------------------------------------|
| <code>itercount</code> | Returns the current iteration number. |
|------------------------|---------------------------------------|

### Return Values

The routine `dcg_get` has no return values.

## dcgmrhs\_init

*Initializes the RCI CG solver with MHRS.*

---

### Syntax

```
void dcgmrhs_init (const MKL_INT *n , const double *x , const MKL_INT *nrhs , const double *b , const MKL_INT *method , MKL_INT *RCI_request , MKL_INT *ipar , double *dpar , double *tmp);
```

## Include Files

- `mkl.h`

## Description

The routine `dcgmrhs_init` initializes the solver. After initialization all subsequent invocations of the Intel MKL RCI CG with multiple right-hand sides (MRHS) routines use the values of all parameters that are returned by `dcgmrhs_init`. Advanced users may skip this step and set the values to these parameters directly in the appropriate routines.

---

### WARNING

You can modify the contents of these arrays after they are passed to the solver routine only if you are sure that the values are correct and consistent. You can perform a basic check for correctness and consistency by calling the `dcgmrhs_check` routine, but it does not guarantee that the method will work correctly.

---

## Input Parameters

|                     |                                                                                                                            |
|---------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>      | Sets the size of the problem.                                                                                              |
| <code>x</code>      | Array of size $n*nrhs$ . Contains the initial approximation to the solution vectors. Normally it is equal to 0 or to $b$ . |
| <code>nrhs</code>   | Sets the number of right-hand sides.                                                                                       |
| <code>b</code>      | Array of size $n*nrhs$ . Contains the right-hand side vectors.                                                             |
| <code>method</code> | Specifies the method of solution:<br>A value of 1 indicates CG with multiple right-hand sides (default value)              |

## Output Parameters

|                          |                                                                                    |
|--------------------------|------------------------------------------------------------------------------------|
| <code>RCI_request</code> | Gives information about the result of the routine.                                 |
| <code>ipar</code>        | Array of size $(128+2*nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> . |
| <code>dpar</code>        | Array of size $(128+2*nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> . |
| <code>tmp</code>         | Array of size $(n*(3+nrhs))$ . Refer to the <a href="#">CG Common Parameters</a> . |

## Return Values

|                                  |                                             |
|----------------------------------|---------------------------------------------|
| <code>RCI_request= 0</code>      | Indicates that the task completed normally. |
| <code>RCI_request= -10000</code> | Indicates failure to complete the task.     |

## `dcgmrhs_check`

*Checks consistency and correctness of the user defined data.*

---

## Syntax

```
void dcgmrhs_check (const MKL_INT *n , const double *x , const MKL_INT *nrhs , const
double *b , MKL_INT *RCI_request , MKL_INT *ipar , double *dpar , double *tmp);
```

## Include Files

- `mkl.h`

## Description

The routine `dcgmrhs_check` checks the consistency and correctness of the parameters to be passed to the solver routine `dcgmrhs`. While this operation reduces the chance of making a mistake in the parameters, it does not guarantee that the solver returns the correct result.

If you are sure that the correct data is *specified* in the solver parameters, you can skip this operation.

The lengths of all vectors must be defined in a previous call to the `dcgmrhs_init` routine.

## Input Parameters

|                   |                                                                                                                                       |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | Sets the size of the problem.                                                                                                         |
| <code>x</code>    | Array of size $n*nrhs$ . Contains the initial approximation to the solution vectors. Normally it is equal to 0 or to <code>b</code> . |
| <code>nrhs</code> | This parameter sets the number of right-hand sides.                                                                                   |
| <code>b</code>    | Array of size $n*nrhs$ . Contains the right-hand side vectors.                                                                        |

## Output Parameters

|                          |                                                                                    |
|--------------------------|------------------------------------------------------------------------------------|
| <code>RCI_request</code> | Returns information about the results of the routine.                              |
| <code>ipar</code>        | Array of size $(128+2*nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> . |
| <code>dpar</code>        | Array of size $(128+2*nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> . |
| <code>tmp</code>         | Array of size $(n*(3+nrhs))$ . Refer to the <a href="#">CG Common Parameters</a> . |

## Return Values

|                                 |                                                                                              |
|---------------------------------|----------------------------------------------------------------------------------------------|
| <code>RCI_request= 0</code>     | Indicates that the task completed normally.                                                  |
| <code>RCI_request= -1100</code> | Indicates that the task is interrupted and the errors occur.                                 |
| <code>RCI_request= -1001</code> | Indicates that there are some warning messages.                                              |
| <code>RCI_request= -1010</code> | Indicates that the routine changed some parameters to make them consistent or correct.       |
| <code>RCI_request= -1011</code> | Indicates that there are some warning messages and that the routine changed some parameters. |

## dcgmrhs

*Computes the approximate solution vectors.*

---

## Syntax

```
void dcgmrhs (const MKL_INT *n , double *x , const MKL_INT *nrhs , const double *b ,
MKL_INT *RCI_request , MKL_INT *ipar , double *dpar , double *tmp);
```

## Include Files

- `mkl.h`

## Description

The routine `dcgmrhs` computes approximate solution vectors using the CG with multiple right-hand sides (MRHS) method [Young71]. The routine `dcgmrhs` uses the value that was in the  $x$  before the first call as an initial approximation to the solution. The parameter `RCI_request` gives information about task completion status and requests results of certain operations that are required by the solver.

Note that lengths of all vectors are assumed to have been defined in a previous call to the `dcgmrhs_init` routine.

## Input Parameters

|        |                                                                                      |
|--------|--------------------------------------------------------------------------------------|
| $n$    | Sets the size of the problem, and the sizes of arrays $x$ and $b$ .                  |
| $x$    | Array of size $n*nrhs$ . Contains the initial approximation to the solution vectors. |
| $nrhs$ | Sets the number of right-hand sides.                                                 |
| $b$    | Array of size $n*nrhs$ . Contains the right-hand side vectors.                       |
| $tmp$  | Array of size $(n, 3+nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> .    |

## Output Parameters

|                          |                                                                                        |
|--------------------------|----------------------------------------------------------------------------------------|
| <code>RCI_request</code> | Gives information about result of work of the routine.                                 |
| $x$                      | Array of size $(n*nrhs)$ . Contains the updated approximation to the solution vectors. |
| $ipar$                   | Array of size $(128+2*nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> .     |
| $dpar$                   | Array of size $(128+2*nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> .     |
| $tmp$                    | Array of size $(n*(3+nrhs))$ . Refer to the <a href="#">CG Common Parameters</a> .     |

## Return Values

|                             |                                                                                                                                                                                                                                                               |
|-----------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>RCI_request=0</code>  | Indicates that the task completed normally and the solution is found and stored in the vector $x$ . This occurs only if the stopping tests are fully automatic. For the user defined stopping tests, see the description of the <code>RCI_request= 2</code> . |
| <code>RCI_request=-1</code> | Indicates that the routine was interrupted because the maximum number of iterations was reached, but the relative stopping criterion was not met. This situation occurs only if both tests are requested by the user.                                         |
| <code>RCI_request=-2</code> | The routine was interrupted because of an attempt to divide by zero. This situation happens if the matrix is non-positive definite or almost non-positive definite.                                                                                           |

|                               |                                                                                                                                                                                                                                                                                        |
|-------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>RCI_request=- 10</code> | Indicates that the routine was interrupted because the residual norm is invalid. This usually happens because the value <code>dpar[5]</code> was altered outside of the routine, or the <code>dcg_check</code> routine was not called.                                                 |
| <code>RCI_request=-11</code>  | Indicates that the routine was interrupted because it enters the infinite cycle. This usually happens because the values <code>ipar[7]</code> , <code>ipar[8]</code> , <code>ipar[9]</code> were altered outside of the routine, or the <code>dcg_check</code> routine was not called. |
| <code>RCI_request= 1</code>   | Indicates that you must multiply the matrix by <code>tmp[0:n - 1]</code> , put the result in the <code>tmp[n:2*n - 1]</code> , and return control back to the routine <code>dcg</code> .                                                                                               |
| <code>RCI_request= 2</code>   | Indicates that you must perform the stopping tests. If they fail, return control back to the <code>dcg</code> routine. Otherwise, the solution is found and stored in the vector <code>x</code> .                                                                                      |
| <code>RCI_request= 3</code>   | Indicates that you must apply the preconditioner to <code>tmp[2*n:3*n - 1]</code> , put the result in the <code>tmp[3*n:4*n - 1]</code> , and return control back to the routine <code>dcg</code> .                                                                                    |

## dcgmrhs\_get

Retrieves the number of the current iteration.

### Syntax

```
void dcgmrhs_get (const MKL_INT *n , const double *x , const MKL_INT *nrhs , const double *b , const MKL_INT *RCI_request , const MKL_INT *ipar , const double *dpar , const double *tmp , MKL_INT *itercount);
```

### Include Files

- `mkl.h`

### Description

The routine `dcgmrhs_get` retrieves the current iteration number of the solving process.

### Input Parameters

|                          |                                                                                        |
|--------------------------|----------------------------------------------------------------------------------------|
| <code>n</code>           | Sets the size of the problem.                                                          |
| <code>x</code>           | Array of size $n * nrhs$ . Contains the initial approximation to the solution vectors. |
| <code>nrhs</code>        | Sets the number of right-hand sides.                                                   |
| <code>b</code>           | Array of size $n * nrhs$ . Contains the right-hand side .                              |
| <code>RCI_request</code> | This parameter is not used.                                                            |
| <code>ipar</code>        | Array of size $(128 + 2 * nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> . |
| <code>dpar</code>        | Array of size $(128 + 2 * nrhs)$ . Refer to the <a href="#">CG Common Parameters</a> . |

*tmp* Array of size  $(n * (3 + nrhs))$ . Refer to the [CG Common Parameters](#).

## Output Parameters

*itercount* Array of size *nrhs*. Returns the current iteration number for each right-hand side.

## Return Values

The routine `dcgmrhs_get` has no return values.

## dfgmres\_init

*Initializes the solver.*

---

### Syntax

```
void dfgmres_init (const MKL_INT *n , const double *x , const double *b , MKL_INT
*RCI_request , MKL_INT *ipar , double *dpar , double *tmp);
```

### Include Files

- `mkl.h`

### Description

The routine `dfgmres_init` initializes the solver. After initialization all subsequent invocations of Intel MKL RCI FGMRES routines use the values of all parameters that are returned by `dfgmres_init`. Advanced users can skip this step and set the values in the *ipar* and *dpar* arrays directly.

---

#### WARNING

You can modify the contents of these arrays after they are passed to the solver routine only if you are sure that the values are correct and consistent. You can perform a basic check for correctness and consistency by calling the `dfgmres_check` routine, but it does not guarantee that the method will work correctly.

---

## Input Parameters

*n* Sets the size of the problem.

*x* Array of size *n*. Contains the initial approximation to the solution vector. Normally it is equal to 0 or to *b*.

*b* Array of size *n*. Contains the right-hand side vector.

## Output Parameters

*RCI\_request* Gives information about the result of the routine.

*ipar* Array of size 128. Refer to the [FGMRES Common Parameters](#).

*dpar* Array of size 128. Refer to the [FGMRES Common Parameters](#).

*tmp* Array of size  $((2 * ipar[14] + 1) * n + ipar[14] * (ipar[14] + 9) / 2 + 1)$ . Refer to the [FGMRES Common Parameters](#).

## Return Values

|                                  |                                             |
|----------------------------------|---------------------------------------------|
| <code>RCI_request= 0</code>      | Indicates that the task completed normally. |
| <code>RCI_request= -10000</code> | Indicates failure to complete the task.     |

## dfgmres\_check

Checks consistency and correctness of the user defined data.

---

## Syntax

```
void dfgmres_check (const MKL_INT *n, const double *x, const double *b, MKL_INT *RCI_request, MKL_INT *ipar, double *dpar, double *tmp);
```

## Include Files

- mkl.h

## Description

The routine `dfgmres_check` checks consistency and correctness of the parameters to be passed to the solver routine `dfgmres`. However, this operation does not guarantee that the method gives the correct result. It only reduces the chance of making a mistake in the parameters of the routine. Skip this operation only if you are sure that the correct data is specified in the solver parameters.

The lengths of all vectors are assumed to have been defined in a previous call to the `dfgmres_init` routine.

## Input Parameters

|                |                                                                                                                                            |
|----------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code> | Sets the size of the problem.                                                                                                              |
| <code>x</code> | Array of size <code>n</code> . Contains the initial approximation to the solution vector. Normally it is equal to 0 or to <code>b</code> . |
| <code>b</code> | Array of size <code>n</code> . Contains the right-hand side vector.                                                                        |

## Output Parameters

|                          |                                                                                                                                        |
|--------------------------|----------------------------------------------------------------------------------------------------------------------------------------|
| <code>RCI_request</code> | Gives information about result of the routine.                                                                                         |
| <code>ipar</code>        | Array of size 128. Refer to the <a href="#">FGMRES Common Parameters</a> .                                                             |
| <code>dpar</code>        | Array of size 128. Refer to the <a href="#">FGMRES Common Parameters</a> .                                                             |
| <code>tmp</code>         | Array of size $((2 * ipar[14] + 1) * n + ipar[14] * (ipar[14] + 9) / 2 + 1)$ . Refer to the <a href="#">FGMRES Common Parameters</a> . |

## Return Values

|                                 |                                                              |
|---------------------------------|--------------------------------------------------------------|
| <code>RCI_request= 0</code>     | Indicates that the task completed normally.                  |
| <code>RCI_request= -1100</code> | Indicates that the task is interrupted and the errors occur. |
| <code>RCI_request= -1001</code> | Indicates that there are some warning messages.              |

|                                 |                                                                                              |
|---------------------------------|----------------------------------------------------------------------------------------------|
| <code>RCI_request= -1010</code> | Indicates that the routine changed some parameters to make them consistent or correct.       |
| <code>RCI_request= -1011</code> | Indicates that there are some warning messages and that the routine changed some parameters. |

## dfgmres

*Makes the FGMRES iterations.*

---

### Syntax

```
void dfgmres (const MKL_INT *n, double *x, double *b, MKL_INT *RCI_request, MKL_INT
*ipar, double *dpar, double *tmp);
```

### Include Files

- `mkl.h`

### Description

The routine `dfgmres` performs the FGMRES iterations [Saad03], using the value that was in the array  $x$  before the first call as an initial approximation of the solution vector. To update the current approximation to the solution, the `dfgmres_get` routine must be called. The RCI FGMRES iterations can be continued after the call to the `dfgmres_get` routine only if the value of the parameter `ipar[12]` is not equal to 0 (default value). Note that the updated solution overwrites the right-hand side in the vector  $b$  if the parameter `ipar[12]` is positive, and the restarted version of the FGMRES method can not be run. If you want to keep the right-hand side, you must save it in a different memory location before the first call to the `dfgmres_get` routine with a positive `ipar[12]`.

The parameter `RCI_request` gives information about the task completion and requests results of certain operations that the solver requires.

The lengths of all the vectors must be defined in a previous call to the `dfgmres_init` routine.

### Input Parameters

|                  |                                                                                |
|------------------|--------------------------------------------------------------------------------|
| <code>n</code>   | Sets the size of the problem.                                                  |
| <code>x</code>   | Array of size $n$ . Contains the initial approximation to the solution vector. |
| <code>b</code>   | Array of size $n$ . Contains the right-hand side vector.                       |
| <code>tmp</code> | Array of size [12]. Refer to the <a href="#">FGMRES Common Parameters</a> .    |

### Output Parameters

|                          |                                                                                                                                      |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------|
| <code>RCI_request</code> | Informs about result of work of the routine.                                                                                         |
| <code>ipar</code>        | Array of size 128. Refer to the <a href="#">FGMRES Common Parameters</a> .                                                           |
| <code>dpar</code>        | Array of size 128. Refer to the <a href="#">FGMRES Common Parameters</a> .                                                           |
| <code>tmp</code>         | Array of size $((2 * ipar(15) + 1) * n + ipar(15) * ipar(15) + 9) / 2 + 1$ . Refer to the <a href="#">FGMRES Common Parameters</a> . |

## Return Values

|                               |                                                                                                                                                                                                                                                                                                                                             |
|-------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>RCI_request=0</code>    | Indicates that the task completed normally and the solution is found and stored in the vector $x$ . This occurs only if the stopping tests are fully automatic. For the user defined stopping tests, see the description of the <code>RCI_request= 2</code> or <code>4</code> .                                                             |
| <code>RCI_request=-1</code>   | Indicates that the routine was interrupted because the maximum number of iterations was reached, but the relative stopping criterion was not met. This situation occurs only if you request both tests.                                                                                                                                     |
| <code>RCI_request= -10</code> | Indicates that the routine was interrupted because of an attempt to divide by zero. Usually this happens if the matrix is degenerate or almost degenerate. However, it may happen if the parameter <code>dpar</code> is altered, or if the method is not stopped when the solution is found.                                                |
| <code>RCI_request= -11</code> | Indicates that the routine was interrupted because it entered an infinite cycle. Usually this happens because the values <code>ipar[7]</code> , <code>ipar[8]</code> , <code>ipar[9]</code> were altered outside of the routine, or the <code>dfgmres_check</code> routine was not called.                                                  |
| <code>RCI_request= -12</code> | Indicates that the routine was interrupted because errors were found in the method parameters. Usually this happens if the parameters <code>ipar</code> and <code>dpar</code> were altered by mistake outside the routine.                                                                                                                  |
| <code>RCI_request= 1</code>   | Indicates that you must multiply the matrix by <code>tmp[ipar[21] - 1:ipar[21] + n - 2]</code> , put the result in the <code>tmp[ipar[22] - 1:ipar[22] + n - 2]</code> , and return control back to the routine <code>dfgmres</code> .                                                                                                      |
| <code>RCI_request= 2</code>   | Indicates that you must perform the stopping tests. If they fail, return control to the <code>dfgmres</code> routine. Otherwise, the FGMRES solution is found, and you can run the <code>fgmres_get</code> routine to update the computed solution in the vector $x$ .                                                                      |
| <code>RCI_request= 3</code>   | Indicates that you must apply the inverse preconditioner to <code>tmp[ipar[21] - 1:ipar[21] + n - 2]</code> , put the result in the <code>tmp[ipar[22] - 1:ipar[22] + n - 2]</code> , and return control back to the routine <code>dfgmres</code> .                                                                                         |
| <code>RCI_request= 4</code>   | Indicates that you must check the norm of the currently generated vector. If it is not zero within the computational/rounding errors, return control to the <code>dfgmres</code> routine. Otherwise, the FGMRES solution is found, and you can run the <code>dfgmres_get</code> routine to update the computed solution in the vector $x$ . |

## dfgmres\_get

Retrieves the number of the current iteration and updates the solution.

---

### Syntax

```
void dfgmres_get (const MKL_INT *n, double *x, double *b, MKL_INT *RCI_request, const
MKL_INT *ipar, const double *dpar, double *tmp, MKL_INT *itercount);
```

### Include Files

- mkl.h

### Description

The routine `dfgmres_get` retrieves the current iteration number of the solution process and updates the solution according to the computations performed by the `dfgmres` routine. To retrieve the current iteration number only, set the parameter `ipar[12] = -1` beforehand. Normally, you should do this before proceeding further with the computations. If the intermediate solution is needed, the method parameters must be set properly. For details see [FGMRES Common Parameters](#) and the Iterative Sparse Solver code examples in the Intel MKL installation directory:

- `examples/solverc/source`

### Input Parameters

|             |                                                                                                                                      |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | Sets the size of the problem.                                                                                                        |
| <i>ipar</i> | Array of size 128. Refer to the <a href="#">FGMRES Common Parameters</a> .                                                           |
| <i>dpar</i> | Array of size 128. Refer to the <a href="#">FGMRES Common Parameters</a> .                                                           |
| <i>tmp</i>  | Array of size $((2 * ipar[14] + 1) * n + ipar[14] * ipar[14] + 9) / 2 + 1$ . Refer to the <a href="#">FGMRES Common Parameters</a> . |

### Output Parameters

|                    |                                                                                                                                                                                                                    |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>x</i>           | Array of size <i>n</i> . If <code>ipar[12] = 0</code> , it contains the updated approximation to the solution according to the computations done in <code>dfgmres</code> routine. Otherwise, it is not changed.    |
| <i>b</i>           | Array of size <i>n</i> . If <code>ipar(13) &gt; 0</code> , it contains the updated approximation to the solution according to the computations done in <code>dfgmres</code> routine. Otherwise, it is not changed. |
| <i>RCI_request</i> | Gives information about result of the routine.                                                                                                                                                                     |
| <i>itercount</i>   | Contains the value of the current iteration number.                                                                                                                                                                |

### Return Values

`RCI_request = 0` Indicates that the task completed normally.

`RCI_request= -12`

Indicates that the routine was interrupted because errors were found in the routine parameters. Usually this happens if the parameters `ipar` and `dpar` were altered by mistake outside of the routine.

`RCI_request= -10000`

Indicates that the routine failed to complete the task.

## RCI ISS Implementation Details

Several aspects of the Intel MKL RCI ISS interface are platform-specific and language-specific. To promote portability across platforms and ease of use across different languages, include one of the Intel MKL RCI ISS language-specific header files.

---

### NOTE

Intel MKL does not support the RCI ISS interface unless you include the language-specific header file.

---

## Preconditioners based on Incomplete LU Factorization Technique

---

Preconditioners, or accelerators are used to accelerate an iterative solution process. In some cases, their use can reduce the number of iterations dramatically and thus lead to better solver performance. Although the terms *preconditioner* and *accelerator* are synonyms, hereafter only *preconditioner* is used.

Intel MKL provides two preconditioners, ILU0 and ILUT, for sparse matrices presented in the format accepted in the Intel MKL direct sparse solvers (three-array variation of the CSR storage format described in [Sparse Matrix Storage Format](#) ). The algorithms used are described in [[Saad03](#)].

The ILU0 preconditioner is based on a well-known factorization of the original matrix into a product of two triangular matrices: lower and upper triangular matrices. Usually, such decomposition leads to some fill-in in the resulting matrix structure in comparison with the original matrix. The distinctive feature of the ILU0 preconditioner is that it preserves the structure of the original matrix in the result.

Unlike the ILU0 preconditioner, the ILUT preconditioner preserves some resulting fill-in in the preconditioner matrix structure. The distinctive feature of the ILUT algorithm is that it calculates each element of the preconditioner and saves each one if it satisfies two conditions simultaneously: its value is greater than the product of the given tolerance and matrix row norm, and its value is in the given bandwidth of the resulting preconditioner matrix.

Both ILU0 and ILUT preconditioners can apply to any non-degenerate matrix. They can be used alone or together with the Intel MKL RCI FGMRES solver (see [Sparse Solver Routines](#)). Avoid using these preconditioners with MKL RCI CG solver because in general, they produce a non-symmetric resulting matrix even if the original matrix is symmetric. Usually, an inverse of the preconditioner is required in this case. To do this the Intel MKL triangular solver routine `mkl_dcsrtrsv` must be applied twice: for the lower triangular part of the preconditioner, and then for its upper triangular part.

---

### NOTE

Although ILU0 and ILUT preconditioners apply to any non-degenerate matrix, in some cases the algorithm may fail to ensure successful termination and the required result. Whether or not the preconditioner produces an acceptable result can only be determined in practice.

A preconditioner may increase the number of iterations for an arbitrary case of the system and the initial solution, and even ruin the convergence. It is your responsibility as a user to choose a suitable preconditioner.

---

## General Scheme of Using ILUT and RCI FGMRES Routines

The general scheme for use is the same for both preconditioners. Some differences exist in the calling parameters of the preconditioners and in the subsequent call of two triangular solvers. You can see all these differences in the preconditioner code examples (`dcsrilu*.*`) in the `examples` folder of the Intel MKL installation directory:

- `examples/solverc/source`

## ILU0 and ILUT Preconditioners Interface Description

The concepts required to understand the use of the Intel MKL preconditioner routines are discussed in the [Appendix A Linear Solvers Basics](#).

### User Data Arrays

The preconditioner routines take arrays of user data as input. To minimize storage requirements and improve overall run-time efficiency, the Intel MKL preconditioner routines do not make copies of the user input arrays.

### Common Parameters

Some parameters of the preconditioners are common with the [FGMRES Common Parameters](#). The routine `dfgmres_init` specifies their default and initial values. However, some parameters can be redefined with other values. These parameters are listed below.

#### For the ILU0 preconditioner:

`ipar[1]` - specifies the destination of error messages generated by the ILU0 routine. The default value 6 means that all error messages are displayed on the screen. Otherwise routine creates a log file called `MKL_PREC_log.txt` and writes error messages to it. Note if the parameter `ipar[5]` is set to 0, then error messages are not generated at all.

`ipar[5]` - specifies whether error messages are generated. If its value is not equal to 0, the ILU0 routine returns error messages as specified by the parameter `ipar[1]`. Otherwise, the routine does not generate error messages at all, but returns a negative value for the parameter `ierr`. The default value is 1.

#### For the ILUT preconditioner:

`ipar[1]` - specifies the destination of error messages generated by the ILUT routine. The default value 6 means that all messages are displayed on the screen. Otherwise routine creates a log file called `MKL_PREC_log.txt` and writes error messages to it. Note if the parameter `ipar[5]` is set to 0, then error messages are not generated at all.

`ipar[5]` - specifies whether error messages are generated. If its value is not equal to 0, the ILUT routine returns error messages as specified by the parameter `ipar[1]`. Otherwise, the routine does not generate error messages at all, but returns a negative value for the parameter `ierr`. The default value is 1.

`ipar[6]` - if its value is greater than 0, the ILUT routine generates warning messages as specified by the parameter `ipar[1]` and continues calculations. If its value is equal to 0, the routine returns a positive value of the parameter `ierr`. If its value is less than 0, the routine generates a warning message as specified by the parameter `ipar[1]` and returns a positive value of the parameter `ierr`. The default value is 1.

## dcsrilu0

*ILU0 preconditioner based on incomplete LU factorization of a sparse matrix.*

---

### Syntax

```
void dcsrilu0 (const MKL_INT *n , const double *a , const MKL_INT *ia , const MKL_INT *ja , double *bilu0 , const MKL_INT *ipar , const double *dpar , MKL_INT *ierr);
```

## Include Files

- `mkl.h`

## Description

The routine `dcsrilu0` computes a preconditioner  $B$  [Saad03] of a given sparse matrix  $A$  stored in the format accepted in the direct sparse solvers:

$A \sim B = L * U$ , where  $L$  is a lower triangular matrix with a unit diagonal,  $U$  is an upper triangular matrix with a non-unit diagonal, and the portrait of the original matrix  $A$  is used to store the incomplete factors  $L$  and  $U$ .

---

### CAUTION

This routine supports only one-based indexing of the array parameters.

---

## Input Parameters

|           |                                                                                                                                                                                                                                                                                                                                                                                                                     |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>  | Size (number of rows or columns) of the original square $n$ -by- $n$ matrix $A$ .                                                                                                                                                                                                                                                                                                                                   |
| <i>a</i>  | Array containing the set of elements of the matrix $A$ . Its length is equal to the number of non-zero elements in the matrix $A$ . Refer to the <i>values</i> array description in the <a href="#">Sparse Matrix Storage Format</a> for more details.                                                                                                                                                              |
| <i>ia</i> | Array of size $(n+1)$ containing begin indices of rows of the matrix $A$ such that $ia[i]$ is the index in the array <i>a</i> of the first non-zero element from the row $i$ . The value of the last element $ia[n]$ is equal to the number of non-zero elements in the matrix $A$ , plus one. Refer to the <i>rowIndex</i> array description in the <a href="#">Sparse Matrix Storage Format</a> for more details. |
| <i>ja</i> | Array containing the column indices for each non-zero element of the matrix $A$ . It is important that the indices are in increasing order per row. The matrix size is equal to the size of the array <i>a</i> . Refer to the <i>columns</i> array description in the <a href="#">Sparse Matrix Storage Format</a> for more details.                                                                                |

---

### CAUTION

If column indices are not stored in ascending order for each row of matrix, the result of the routine might not be correct.

---

*ipar* Array of size 128. This parameter specifies the integer set of data for both the ILU0 and RCI FGMRES computations. Refer to the *ipar* array description in the [FGMRES Common Parameters](#) for more details on FGMRES parameter entries. The entries that are specific to ILU0 are listed below.

|                  |                                                                                                                                                                                                                                                                                                                                                                                      |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> [30] | specifies how the routine operates when a zero diagonal element occurs during calculation. If this parameter is set to 0 (the default value set by the routine <code>dfgmres_init</code> ), then the calculations are stopped and the routine returns a non-zero error value. Otherwise, the diagonal element is set to the value of <i>dpar</i> [31] and the calculations continue. |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

**NOTE**

You can declare the *ipar* array with a size of 32. However, for future compatibility you must declare the array *ipar* with length 128.

*dpar*

Array of size 128. This parameter specifies the double precision set of data for both the ILU0 and RCI FGMRES computations. Refer to the *dpar* array description in the [FGMRES Common Parameters](#) for more details on FGMRES parameter entries. The entries specific to ILU0 are listed below.

*dpar*[30] specifies a small value, which is compared with the computed diagonal elements. When *ipar*[30] is not 0, then diagonal elements less than *dpar*[30] are set to *dpar*[31]. The default value is 1.0e-16.

**NOTE**

This parameter can be set to the negative value, because the calculation uses its absolute value.

If this parameter is set to 0, the comparison with the diagonal element is not performed.

*dpar*[31] specifies the value that is assigned to the diagonal element if its value is less than *dpar*[30] (see above). The default value is 1.0e-10.

**NOTE**

You can declare the *dpar* array with a size of 32. However, for future compatibility you must declare the array *dpar* with length 128.

**Output Parameters***bilu0*

Array *B* containing non-zero elements of the resulting preconditioning matrix *B*, stored in the format accepted in direct sparse solvers. Its size is equal to the number of non-zero elements in the matrix *A*. Refer to the *values* array description in the [Sparse Matrix Storage Format](#) section for more details.

*ierr*

Error flag, gives information about the routine completion.

**NOTE**

To present the resulting preconditioning matrix in the CSR3 format the arrays *ia* (row indices) and *ja* (column indices) of the input matrix must be used.

**Return Values***ierr=0*

Indicates that the task completed normally.

|                        |                                                                                                                                                                                                  |
|------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ierr=-101</code> | Indicates that the routine was interrupted and that error occurred: at least one diagonal element is omitted from the matrix in CSR3 format (see <a href="#">Sparse Matrix Storage Format</a> ). |
| <code>ierr=-102</code> | Indicates that the routine was interrupted because the matrix contains a diagonal element with the value of zero.                                                                                |
| <code>ierr=-103</code> | Indicates that the routine was interrupted because the matrix contains a diagonal element which is so small that it could cause an overflow, or that it would cause a bad approximation to ILU0. |
| <code>ierr=-104</code> | Indicates that the routine was interrupted because the memory is insufficient for the internal work array.                                                                                       |
| <code>ierr=-105</code> | Indicates that the routine was interrupted because the input matrix size $n$ is less than or equal to 0.                                                                                         |
| <code>ierr=-106</code> | Indicates that the routine was interrupted because the column indices $ja$ are not in the ascending order.                                                                                       |

## dcsrilut

*ILUT preconditioner based on the incomplete LU factorization with a threshold of a sparse matrix.*

---

### Syntax

```
void dcsrilit (const MKL_INT *n, const double *a, const MKL_INT *ia, const MKL_INT *ja,
double *bilut, MKL_INT *ibilut, MKL_INT *jbilut, const double *tol, const MKL_INT
*maxfil, const MKL_INT *ipar, const double *dpar, MKL_INT *ierr);
```

### Include Files

- `mkl.h`

### Description

The routine `dcscrilit` computes a preconditioner  $B$  [Saad03] of a given sparse matrix  $A$  stored in the format accepted in the direct sparse solvers:

$A \sim B = L * U$ , where  $L$  is a lower triangular matrix with unit diagonal and  $U$  is an upper triangular matrix with non-unit diagonal.

The following threshold criteria are used to generate the incomplete factors  $L$  and  $U$ :

- 1) the resulting entry must be greater than the matrix current row norm multiplied by the parameter `tol`, and
- 2) the number of the non-zero elements in each row of the resulting  $L$  and  $U$  factors must not be greater than the value of the parameter `maxfil`.

---

### CAUTION

This routine supports only one-based indexing of the array parameters.

---

## Input Parameters

|           |                                                                                                                                                                                                                                                                                                                                                                                     |
|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>  | Size (number of rows or columns) of the original square $n$ -by- $n$ matrix $A$ .                                                                                                                                                                                                                                                                                                   |
| <i>a</i>  | Array containing all non-zero elements of the matrix $A$ . The length of the array is equal to their number. Refer to <i>values</i> array description in the <a href="#">Sparse Matrix Storage Format</a> section for more details.                                                                                                                                                 |
| <i>ia</i> | Array of size $(n+1)$ containing indices of non-zero elements in the array $a$ . $ia[i]$ is the index of the first non-zero element from the row $i$ . The value of the last element $ia[n]$ is equal to the number of non-zeros in the matrix $A$ , plus one. Refer to the <i>rowIndex</i> array description in the <a href="#">Sparse Matrix Storage Format</a> for more details. |
| <i>ja</i> | Array of size equal to the size of the array $a$ . This array contains the column numbers for each non-zero element of the matrix $A$ . It is important that the indices are in increasing order per row. Refer to the <i>columns</i> array description in the <a href="#">Sparse Matrix Storage Format</a> for more details.                                                       |

---

### CAUTION

If column indices are not stored in ascending order for each row of matrix, the result of the routine might not be correct.

---

|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>tol</i>       | Tolerance for threshold criterion for the resulting entries of the preconditioner.                                                                                                                                                                                                                                                                                                                                                               |
| <i>maxfil</i>    | Maximum fill-in, which is half of the preconditioner bandwidth. The number of non-zero elements in the rows of the preconditioner cannot exceed $(2*maxfil+1)$ .                                                                                                                                                                                                                                                                                 |
| <i>ipar</i>      | Array of size 128. This parameter is used to specify the integer set of data for both the ILUT and RCI FGMRES computations. Refer to the <i>ipar</i> array description in the <a href="#">FGMRES Common Parameters</a> for more details on FGMRES parameter entries. The entries specific to ILUT are listed below.                                                                                                                              |
| <i>ipar</i> [30] | specifies how the routine operates if the value of the computed diagonal element is less than the current matrix row norm multiplied by the value of the parameter <i>tol</i> . If <i>ipar</i> [30] = 0, then the calculation is stopped and the routine returns non-zero error value. Otherwise, the value of the diagonal element is set to a value determined by <i>dpar</i> [30] (see its description below), and the calculations continue. |

---

### NOTE

There is no default value for *ipar*[30] even if the preconditioner is used within the RCI ISS context. Always set the value of this entry.

---

**NOTE**

You must declare the array *ipar* with length 128. While defining the array in the code as *ipar*[30] works, there is no guarantee of future compatibility with Intel MKL.

---

*dpar*

Array of size 128. This parameter specifies the double precision set of data for both ILUT and RCI FGMRES computations. Refer to the *dpar* array description in the [FGMRES Common Parameters](#) for more details on FGMRES parameter entries. The entries that are specific to ILUT are listed below.

*dpar*[30]

used to adjust the value of small diagonal elements. Diagonal elements with a value less than the current matrix row norm multiplied by *tol* are replaced with the value of *dpar*[30] multiplied by the matrix row norm.

**NOTE**

There is no default value for *dpar*[30] entry even if the preconditioner is used within RCI ISS context. Always set the value of this entry.

---

**NOTE**

You must declare the array *dpar* with length 128. While defining the array in the code as *ipar*[30] works, there is no guarantee of future compatibility with Intel MKL.

---

## Output Parameters

*bilut*

Array containing non-zero elements of the resulting preconditioning matrix *B*, stored in the format accepted in the direct sparse solvers. Refer to the *values* array description in the [Sparse Matrix Storage Format](#) for more details. The size of the array is equal to  $(2 * \text{maxfil} + 1) * n - \text{maxfil} * (\text{maxfil} + 1) + 1$ .

**NOTE**

Provide enough memory for this array before calling the routine. Otherwise, the routine may fail to complete successfully with a correct result.

---

*ibilut*

Array of size  $(n+1)$  containing indices of non-zero elements in the array *bilut*. *ibilut*[*i*] is the index of the first non-zero element from the row *i*. The value of the last element *ibilut*[*n*] is equal to the number of non-zeros in the matrix *B*, plus one. Refer to the *rowIndex* array description in the [Sparse Matrix Storage Format](#) for more details.

|               |                                                                                                                                                                                                                                                                                |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>jbilut</i> | Array, its size is equal to the size of the array <i>bilut</i> . This array contains the column numbers for each non-zero element of the matrix <i>B</i> . Refer to the <i>columns</i> array description in the <a href="#">Sparse Matrix Storage Format</a> for more details. |
| <i>ierr</i>   | Error flag, gives information about the routine completion.                                                                                                                                                                                                                    |

## Return Values

|                   |                                                                                                                                                                                                                                 |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ierr</i> =0    | Indicates that the task completed normally.                                                                                                                                                                                     |
| <i>ierr</i> =-101 | Indicates that the routine was interrupted because of an error: the number of elements in some matrix row specified in the sparse format is equal to or less than 0.                                                            |
| <i>ierr</i> =-102 | Indicates that the routine was interrupted because the value of the computed diagonal element is less than the product of the given tolerance and the current matrix row norm, and it cannot be replaced as <i>ipar</i> [30]=0. |
| <i>ierr</i> =-103 | Indicates that the routine was interrupted because the element <i>ia</i> [ <i>i</i> ] is less than or equal to the element <i>ia</i> [ <i>i</i> - 1] (see <a href="#">Sparse Matrix Storage Format</a> ).                       |
| <i>ierr</i> =-104 | Indicates that the routine was interrupted because the memory is insufficient for the internal work arrays.                                                                                                                     |
| <i>ierr</i> =-105 | Indicates that the routine was interrupted because the input value of <i>maxfil</i> is less than 0.                                                                                                                             |
| <i>ierr</i> =-106 | Indicates that the routine was interrupted because the size <i>n</i> of the input matrix is less than 0.                                                                                                                        |
| <i>ierr</i> =-107 | Indicates that the routine was interrupted because an element of the array <i>ja</i> is less than 1, or greater than <i>n</i> (see <a href="#">Sparse Matrix Storage Format</a> ).                                              |
| <i>ierr</i> =101  | The value of <i>maxfil</i> is greater than or equal to <i>n</i> . The calculation is performed with the value of <i>maxfil</i> set to ( <i>n</i> -1).                                                                           |
| <i>ierr</i> =102  | The value of <i>tol</i> is less than 0. The calculation is performed with the value of the parameter set to (- <i>tol</i> ).                                                                                                    |
| <i>ierr</i> =103  | The absolute value of <i>tol</i> is greater than value of <i>dpar</i> [30]; it can result in instability of the calculation.                                                                                                    |
| <i>ierr</i> =104  | The value of <i>dpar</i> [30] is equal to 0. It can cause calculations to fail.                                                                                                                                                 |

## Sparse Matrix Checker Routines

Intel MKL provides a sparse matrix checker so that you can find errors in the storage of sparse matrices before calling Intel MKL PARDISO, DSS, or Sparse BLAS routines.

## sparse\_matrix\_checker

Checks correctness of sparse matrix.

### Syntax

```
MKL_INT sparse_matrix_checker (sparse_struct* handle);
```

### Include Files

- mkl.h

### Description

The `sparse_matrix_checker` routine checks a user-defined array used to store a sparse matrix in order to detect issues which could cause problems in routines that require sparse input matrices, such as Intel MKL PARDISO, DSS, or Sparse BLAS.

### Input Parameters

*handle* Pointer to the data structure describing the sparse array to check.

### Return Values

The routine returns a value *error*. Additionally, the *check\_result* parameter returns information about where the error occurred, which can be used when *message\_level* is MKL\_NO\_PRINT.

### Sparse Matrix Checker Error Values

| <i>error value</i>                | Meaning                                                                                                                                                              | Location                                                                                                                                                                                                                                  |
|-----------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| MKL_SPARSE_CHECKER_SUCCESS        | The input array successfully passed all checks.                                                                                                                      |                                                                                                                                                                                                                                           |
| MKL_SPARSE_CHECKER_NON_MONOTONIC  | The input array is not 0 or 1 based ( <i>ia</i> [0] is not 0 or 1) or elements of <i>ia</i> are not in non-decreasing order as required.                             | <b>C:</b><br><i>ia</i> [ <i>i</i> ] and <i>ia</i> [ <i>i</i> + 1] are incompatible.<br><i>check_result</i> [0] = <i>i</i><br><i>check_result</i> [1] = <i>ia</i> [ <i>i</i> ]<br><i>check_result</i> [2] = <i>ia</i> [ <i>i</i> + 1]      |
| MKL_SPARSE_CHECKER_OUT_OF_RANGE   | The value of the <i>ja</i> array is lower than the number of the first column or greater than the number of the last column.                                         | <b>C:</b><br><i>ia</i> [ <i>i</i> ] and <i>ia</i> [ <i>i</i> + 1] are incompatible.<br><i>check_result</i> [0] = <i>i</i><br><i>check_result</i> [1] = <i>ia</i> [ <i>i</i> ]<br><i>check_result</i> [2] = <i>ia</i> [ <i>i</i> + 1]      |
| MKL_SPARSE_CHECKER_NON_TRIANGULAR | The <i>matrix_structure</i> parameter is MKL_UPPER_TRIANGULAR and both <i>ia</i> and <i>ja</i> are not upper triangular, or the <i>matrix_structure</i> parameter is | <b>C:</b><br><i>ia</i> [ <i>i</i> ] and <i>ja</i> [ <i>j</i> ] are incompatible.<br><i>check_result</i> [0] = <i>i</i><br><i>check_result</i> [1] = <i>ia</i> [ <i>i</i> ] = <i>j</i><br><i>check_result</i> [2] = <i>ja</i> [ <i>j</i> ] |

| <i>error value</i>                | Meaning                                                                                                | Location                                                                                                                                                                                                                             |
|-----------------------------------|--------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                   | MKL_LOWER_TRIANGULAR<br>and both <i>ia</i> and <i>ja</i> are not<br>lower triangular                   |                                                                                                                                                                                                                                      |
| MKL_SPARSE_CHECKER_NON<br>ORDERED | The elements of the <i>ja</i><br>array are not in non-<br>decreasing order in each<br>row as required. | <b>C:</b><br><i>ia</i> [ <i>i</i> ] and <i>ia</i> [ <i>i</i> + 1] are incompatible.<br><i>check_result</i> [0] = <i>j</i><br><i>check_result</i> [1] = <i>ja</i> [ <i>j</i> ]<br><i>check_result</i> [2] = <i>ja</i> [ <i>j</i> + 1] |

**See Also**

[sparse\\_matrix\\_checker\\_init](#) Initializes handle for sparse matrix checker.  
[Intel MKL PARDISO - Parallel Direct Sparse Solver Interface](#)  
[Sparse BLAS Level 2 and Level 3 Routines](#)  
[Sparse Matrix Storage Formats](#)

**sparse\_matrix\_checker\_init**

*Initializes handle for sparse matrix checker.*

**Syntax**

```
void sparse_matrix_checker_init (sparse_struct* handle);
```

**Include Files**

- mkl.h

**Description**

The `sparse_matrix_checker_init` routine initializes the handle for the `sparse_matrix_checker` routine. The `handle` variable contains this data:

**Description of sparse\_matrix\_checker handle Data**

| Field                        | Type     | Possible Values                                                                                                                | Meaning                                                                                   |
|------------------------------|----------|--------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| <code>n</code>               | MKL_INT  |                                                                                                                                | Order of the matrix stored in sparse array.                                               |
| <code>csr_ia</code>          | MKL_INT* | Pointer to <i>ia</i> array for<br><code>matrix_format =</code><br>MKL_CSR                                                      |                                                                                           |
| <code>csr_ja</code>          | MKL_INT* | Pointer to <i>ja</i> array for<br><code>matrix_format =</code><br>MKL_CSR                                                      |                                                                                           |
| <code>check_result[3]</code> | MKL_INT  | See <a href="#">Sparse Matrix Checker Error Values</a> for a description of the values returned in <code>check_result</code> . | Indicates location of problem in array when <code>message_level =</code><br>MKL_NO_PRINT. |

| Field            | Type                         | Possible Values                                                                                   | Meaning                                                       |
|------------------|------------------------------|---------------------------------------------------------------------------------------------------|---------------------------------------------------------------|
| indexing         | sparse_matrix_indexing       | MKL_ZERO_BASED<br>MKL_ONE_BASED                                                                   | Indexing style used in array.                                 |
| matrix_structure | sparse_matrix_structures     | MKL_GENERAL_STRUCTURE<br>MKL_UPPER_TRIANGULAR<br>MKL_LOWER_TRIANGULAR<br>MKL_STRUCTURAL_SYMMETRIC | Type of sparse matrix stored in array.                        |
| matrix_format    | sparse_matrix_formats        | MKL_CSR                                                                                           | Format of array used for sparse matrix storage.               |
| message_level    | sparse_matrix_message_levels | MKL_NO_PRINT<br>MKL_PRINT                                                                         | Determines whether or not feedback is provided on the screen. |
| print_style      | sparse_matrix_print_styles   | MKL_C_STYLE<br>MKL_FORTRAN_STYLE                                                                  | Determines style of messages when message_level = MKL_PRINT.  |

## Input Parameters

*handle* Pointer to the data structure describing the sparse array to check.

## Output Parameters

*handle* Pointer to the initialized data structure.

## See Also

[sparse\\_matrix\\_checker](#) Checks correctness of sparse matrix.  
[Intel MKL PARDISO - Parallel Direct Sparse Solver Interface](#)  
[Sparse BLAS Level 2 and Level 3 Routines](#)  
[Sparse Matrix Storage Formats](#)

# Extended Eigensolver Routines

The Extended Eigensolver functionality in Intel Math Kernel Library (Intel MKL) is based on the FEAST Eigenvalue Solver 2.0 (<http://www.ecs.umass.edu/~polizzi/feast>) in compliance with a BSD license agreement.

Extended Eigensolver uses the same naming convention as the original FEAST package.

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## NOTE

Intel MKL only supports the shared memory programming (SMP) version of the eigenvalue solver.

---

- [The FEAST Algorithm](#) gives a brief description of the algorithm underlying the Extended Eigensolver.
- [Extended Eigensolver Functionality](#) describes the problems that can and cannot be solved with the Extended Eigensolver and how to get the best results from the routines.
- [Extended Eigensolver Interfaces](#) gives a reference for calling Extended Eigensolver routines.

## The FEAST Algorithm

---

The Extended Eigensolver functionality is a set of high-performance numerical routines for solving symmetric standard eigenvalue problems,  $Ax=\lambda x$ , or generalized symmetric-definite eigenvalue problems,  $Ax=\lambda Bx$ . It yields all the eigenvalues ( $\lambda$ ) and eigenvectors ( $x$ ) within a given search interval  $[\lambda_{\min}, \lambda_{\max}]$ . It is based on the FEAST algorithm, an innovative fast and stable numerical algorithm presented in [\[Polizzi09\]](#), which fundamentally differs from the traditional Krylov subspace iteration based techniques (Arnoldi and Lanczos algorithms [\[Bai00\]](#)) or other Davidson-Jacobi techniques [\[Sleijpen96\]](#). The FEAST algorithm is inspired by the density-matrix representation and contour integration techniques in quantum mechanics.

The FEAST numerical algorithm obtains eigenpair solutions using a numerically efficient contour integration technique. The main computational tasks in the FEAST algorithm consist of solving a few independent linear systems along the contour and solving a reduced eigenvalue problem. Consider a circle centered in the middle of the search interval  $[\lambda_{\min}, \lambda_{\max}]$ . The numerical integration over the circle in the current version of FEAST is performed using  $N_e$ -point Gauss-Legendre quadrature with  $x_e$  the  $e$ -th Gauss node associated with the weight  $\omega_e$ . For example, for the case  $N_e = 8$ :

$(x_1, \omega_1) = (0.183434642495649, 0.362683783378361),$   
 $(x_2, \omega_2) = (-0.183434642495649, 0.362683783378361),$   
 $(x_3, \omega_3) = (0.525532409916328, 0.313706645877887),$   
 $(x_4, \omega_4) = (-0.525532409916328, 0.313706645877887),$   
 $(x_5, \omega_5) = (0.796666477413626, 0.222381034453374),$   
 $(x_6, \omega_6) = (-0.796666477413626, 0.222381034453374),$   
 $(x_7, \omega_7) = (0.960289856497536, 0.101228536290376),$  and  
 $(x_8, \omega_8) = (-0.960289856497536, 0.101228536290376).$

The figure [FEAST Pseudocode](#) shows the basic pseudocode for the FEAST algorithm for the case of real symmetric (left pane) and complex Hermitian (right pane) generalized eigenvalue problems, using  $N$  for the size of the system and  $M$  for the number of eigenvalues in the search interval (see [\[Polizzi09\]](#)).

---

## NOTE

The pseudocode presents a simplified version of the actual algorithm. Refer to <http://arxiv.org/abs/1302.0432> for an in-depth presentation and mathematical proof of convergence of FEAST.

---

## FEAST Pseudocode

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><math>A</math>: real symmetric<br/> <math>B</math>: symmetric positive definite (SPD)<br/> <math>\Re\{x\}</math>: real part of <math>x</math></p> <ol style="list-style-type: none"> <li>1. Select <math>M_0 &gt; M</math> random vectors <math>Y_{N \times M_0} \in \mathbb{R}^{N \times M_0}</math>.</li> <li>2. Set <math>Q = 0</math> with <math>Q \in \mathbb{R}^{N \times M_0}</math>; <math>r = (\lambda_{\max} - \lambda_{\min}) / 2</math>;<br/>           For <math>e = 1, \dots, N_e</math><br/>           compute <math>\theta_e = -(\pi / 2)(x_e - 1)</math>,<br/>           compute <math>Z_e = (\lambda_{\max} + \lambda_{\min}) / 2 + r \exp(i\theta_e)</math>,<br/>           solve <math>(Z_e B - A)Q_e = Y</math> to obtain <math>Q_e \in \mathbb{C}^{N \times M_0}</math><br/>           compute <math>Q = Q - (\omega_e / 2)\Re\{r \exp(i\theta_e)Q_e\}</math><br/>           End</li> <li>3. Form <math>A_{Q_{M_0 \times M_0}} = Q^T A Q</math> and <math>B_{Q_{M_0 \times M_0}} = Q^T B Q</math><br/>           reduce value of <math>M_0</math> if <math>B_Q</math> is not symmetric positive definite.</li> <li>4. Solve <math>A_Q \Phi = \varepsilon B_Q \Phi</math> to obtain the <math>M_0</math> eigenvalue <math>\varepsilon_m</math>, and eigenvectors <math>\Phi_{M_0 \times M_0} \in \mathbb{R}^{M_0 \times M_0}</math>.</li> <li>5. Set <math>\lambda_m = \varepsilon_m</math> and compute <math>X_{N \times M_0} = Q_{N \times M_0} \Phi_{M_0 \times M_0}</math>.<br/>           If <math>\lambda_m \in [\lambda_{\min}, \lambda_{\max}]</math>, <math>\lambda_m</math> is an eigenvalue solution and its eigenvector is <math>X_m</math> (the <math>m</math>-th column of <math>X</math>).</li> <li>6. Check convergence for the trace of the eigenvalues <math>\lambda_m</math>. If iterative refinement is needed, compute <math>Y = BX</math> and go back to step 2.</li> </ol> | <p><math>A</math>: complex Hermitian<br/> <math>B</math>: Hermitian positive definite (HPD)</p> <ol style="list-style-type: none"> <li>1. Select <math>M_0 &gt; M</math> random vectors <math>Y_{N \times M_0} \in \mathbb{C}^{N \times M_0}</math>.</li> <li>2. Set <math>Q = 0</math> with <math>Q \in \mathbb{R}^{N \times M_0}</math>; <math>r = (\lambda_{\max} - \lambda_{\min}) / 2</math>;<br/>           For <math>e = 1, \dots, N_e</math><br/>           compute <math>\theta_e = -(\pi / 2)(x_e - 1)</math>,<br/>           compute <math>Z_e = (\lambda_{\max} + \lambda_{\min}) / 2 + r \exp(i\theta_e)</math>,<br/>           solve <math>(Z_e B - A)Q_e = Y</math> to obtain <math>Q_e \in \mathbb{C}^{N \times M_0}</math><br/>           solve <math>(Z_e B - A)^H \hat{Q}_e = Y</math> to obtain <math>\hat{Q}_e \in \mathbb{C}^{N \times M_0}</math><br/> <math>Q = Q - (\omega_e / 4)r(\exp(i\theta_e)Q_e + \exp(-i\theta_e)\hat{Q}_e)</math><br/>           End</li> <li>3. Form <math>A_{Q_{M_0 \times M_0}} = Q^H A Q</math> and <math>B_{Q_{M_0 \times M_0}} = Q^H B Q</math><br/>           reduce value of <math>M_0</math> if <math>B_Q</math> is not Hermitian positive definite.</li> <li>4. Solve <math>A_Q \Phi = \varepsilon B_Q \Phi</math> to obtain the <math>M_0</math> eigenvalue <math>\varepsilon_m</math>, and eigenvectors <math>\Phi_{M_0 \times M_0} \in \mathbb{C}^{M_0 \times M_0}</math>.</li> <li>5. Set <math>\lambda_m = \varepsilon_m</math> and compute <math>X_{N \times M_0} = Q_{N \times M_0} \Phi_{M_0 \times M_0}</math>.<br/>           If <math>\lambda_m \in [\lambda_{\min}, \lambda_{\max}]</math>, <math>\lambda_m</math> is an eigenvalue solution and its eigenvector is <math>X_m</math> (the <math>m</math>-th column of <math>X</math>).</li> <li>6. Check convergence for the trace of the eigenvalues <math>\lambda_m</math>. If iterative refinement is needed, compute <math>Y = BX</math> and go back to step 2.</li> </ol> |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## Extended Eigensolver Functionality

Use Extended Eigensolver to compute all the eigenvalues and eigenvectors within a given search interval.

The eigenvalue problems covered are as follows:

- standard,  $Ax = \lambda x$ 
  - $A$  complex Hermitian
  - $A$  real symmetric
- generalized,  $Ax = \lambda Bx$ 
  - $A$  complex Hermitian,  $B$  Hermitian positive definite (hpd)
  - $A$  real symmetric and  $B$  real symmetric positive definite (spd)

The Extended Eigensolver functionality offers:

- Real/Complex and Single/Double precisions: double precision is recommended to provide better accuracy of eigenpairs.
- Reverse communication interfaces (RCI) provide maximum flexibility for specific applications. RCI are independent of matrix format and inner system solvers, so you must provide your own linear system solvers (direct or iterative) and matrix-matrix multiply routines.
- Predefined driver interfaces for dense, LAPACK banded, and sparse (CSR) formats are less flexible but are optimized and easy to use:
  - The Extended Eigensolver interfaces for dense matrices are likely to be slower than the comparable LAPACK routines because the FEAST algorithm has a higher computational cost.

- The Extended Eigensolver interfaces for banded matrices support banded LAPACK-type storage.
- The Extended Eigensolver sparse interfaces support compressed sparse row format and use the Intel MKL PARDISO solver.

#### Optimization Notice

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Notice revision #20110804

## Parallelism in Extended Eigensolver Routines

How you achieve parallelism in Extended Eigensolver routines depends on which interface you use. Parallelism (via shared memory programming) is not *explicitly* implemented in Extended Eigensolver routines within one node: the inner linear systems are currently solved one after another.

- Using the Extended Eigensolver RCI interfaces, you can achieve parallelism by providing a threaded inner system solver and a matrix-matrix multiplication routine. When using the RCI interfaces, you are responsible for activating the threaded capabilities of your BLAS and LAPACK libraries most likely using the shell variable `OMP_NUM_THREADS`.
- Using the predefined Extended Eigensolver interfaces, parallelism can be implicitly obtained within the shared memory version of BLAS, LAPACK or Intel MKL PARDISO. The shell variable `MKL_NUM_THREADS` can be used for automatically setting the number of OpenMP threads (cores) for BLAS, LAPACK, and Intel MKL PARDISO.

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Notice revision #20110804

## Achieving Performance With Extended Eigensolver Routines

In order to use the Extended Eigensolver Routines, you need to provide

- the search interval and the size of the subspace  $M_0$  (overestimation of the number of eigenvalues  $M$  within a given search interval);
- the system matrix in dense, banded, or sparse CSR format if the Extended Eigensolver predefined interfaces are used, or a high-performance complex direct or iterative system solver and matrix-vector multiplication routine if RCI interfaces are used.

In return, you can expect

- fast convergence with very high accuracy when seeking up to 1000 eigenpairs (in two to four iterations using  $M_0 = 1.5M$ , and  $N_e = 8$  or at most using  $N_e = 16$  contour points);
- an extremely robust approach.

The performance of the basic FEAST algorithm depends on a trade-off between the choices of the number of Gauss quadrature points  $N_e$ , the size of the subspace  $M_0$ , and the number of outer refinement loops to reach the desired accuracy. In practice you should use  $M_0 > 1.5 M$ ,  $N_e = 8$ , and at most two refinement loops.

For better performance:

- $M_0$  should be much smaller than the size of the eigenvalue problem, so that the arithmetic complexity depends mainly on the inner system solver ( $O(NM)$  for narrow-banded or sparse systems).
- Parallel scalability performance depends on the shared memory capabilities of the of the inner system solver.
- For very large sparse and challenging systems, application users should make use of the Extended Eigensolver RCI interfaces with customized highly-efficient iterative systems solvers and preconditioners.
- For the Extended Eigensolver interfaces for banded matrices, the parallel performance scalability is limited.

### Optimization Notice

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## Extended Eigensolver Interfaces

### Extended Eigensolver Naming Conventions

There are two different types of interfaces available in the Extended Eigensolver routines:

1. The reverse communication interfaces (RCI):

```
?feast_<matrix type>_rci
```

These interfaces are matrix free format (the interfaces are independent of the matrix data formats). You must provide matrix-vector multiply and direct/iterative linear system solvers for your own explicit or implicit data format.

2. The predefined interfaces:

```
?feast_<matrix type><type of eigenvalue problem>
```

are predefined drivers for `?feast` reverse communication interface that act on commonly used matrix data storage (dense, banded and compressed sparse row representation), using internal matrix-vector routines and selected inner linear system solvers.

For these interfaces:

- ? indicates the data type of matrix  $A$  (and matrix  $B$  if any) defined as follows:

|   |               |
|---|---------------|
| s | float         |
| d | double        |
| c | MKL_Complex8  |
| z | MKL_Complex16 |

- `<matrix type>` defined as follows:

|      | <b>Value of <code>&lt;matrix type&gt;</code></b> | <b>Matrix format</b>              | <b>Inner linear system solver used by Extended Eigensolver</b> |
|------|--------------------------------------------------|-----------------------------------|----------------------------------------------------------------|
| sy   | (symmetric real)                                 |                                   |                                                                |
| he   | (Hermitian complex)                              | Dense                             | LAPACK dense solvers                                           |
| sb   | (symmetric banded real)                          |                                   |                                                                |
| hb   | (Hermitian banded complex)                       | Banded-LAPACK                     | Internal banded solver                                         |
| scsr | (symmetric real)                                 |                                   |                                                                |
| hcsr | (Hermitian complex)                              | Compressed sparse row             | PARDISO solver                                                 |
| s    | (symmetric real)                                 | Reverse communications interfaces | User defined                                                   |
| h    | (Hermitian complex)                              |                                   |                                                                |

- `<type of eigenvalue problem>` is:

|    |                                |
|----|--------------------------------|
| gv | generalized eigenvalue problem |
| ev | standard eigenvalue problem    |

For example, `sfeast_scsrev` is a single-precision routine with a symmetric real matrix stored in sparse compressed-row format for a standard eigenvalue problem, and `zfeast_hrci` is a complex double-precision routine with a Hermitian matrix using the reverse communication interface.

Note that:

- ? can be `s` or `d` if a matrix is real symmetric: `<matrix type>` is `sy`, `sb`, or `scsr`.
- ? can be `c` or `z` if a matrix is complex Hermitian: `<matrix type>` is `he`, `hb`, or `hcsr`.
- ? can be `c` or `z` if the Extended Eigensolver RCI interface is used for solving a complex Hermitian problem.
- ? can be `s` or `d` if the Extended Eigensolver RCI interface is used for solving a real symmetric problem.

## feastinit

Initialize Extended Eigensolver input parameters with default values.

### Syntax

```
feastinit (MKL_INT* fpm);
```

### Include Files

- `mkl.h`

### Description

This routine sets all Extended Eigensolver parameters to their default values.

## Output Parameters

*fpm* Array, size 128. This array is used to pass various parameters to Extended Eigensolver routines. See [Extended Eigensolver Input Parameters](#) for a complete description of the parameters and their default values.

## Extended Eigensolver Input Parameters

The input parameters for Extended Eigensolver routines are contained in an MKL\_INT array named *fpm*. To call the Extended Eigensolver interfaces, this array should be initialized using the routine `feastinit`.

| Param          | Default          | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|----------------|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>fpm</i> [0] | 0                | Specifies whether Extended Eigensolver routines print runtime status.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|                | <i>fpm</i> [0]=0 | Extended Eigensolver routines do not generate runtime messages at all.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|                | <i>fpm</i> [0]=1 | Extended Eigensolver routines print runtime status to the screen.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>fpm</i> [1] | 8                | The number of contour points $N_e = 8$ (see the description of FEAST algorithm). Must be one of {3,4,5,6,8,10,12,16,20,24,32,40,48}.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>fpm</i> [2] | 12               | Error trace double precision stopping criteria $\epsilon$ ( $\epsilon = 10^{-fpm[2]}$ ).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>fpm</i> [3] | 20               | Maximum number of Extended Eigensolver refinement loops allowed. If no convergence is reached within <i>fpm</i> [3] refinement loops, Extended Eigensolver routines return <code>info=2</code> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <i>fpm</i> [4] | 0                | User initial subspace. If <i>fpm</i> [4]=0 then Extended Eigensolver routines generate initial subspace, if <i>fpm</i> [4]=1 the user supplied initial subspace is used.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>fpm</i> [5] | 0                | Extended Eigensolver stopping test.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|                | <i>fpm</i> [5]=0 | Extended Eigensolvers are stopped if this residual stopping test is satisfied: <ul style="list-style-type: none"> <li>generalized eigenvalue problem: <math display="block">\max_{i=1:mode} \frac{\ Ax_i - \lambda_i Bx_i\ _1}{\max( E_{min} ,  E_{max} ) \ Bx_i\ _1} &lt; \epsilon</math> </li> <li>standard eigenvalue problem: <math display="block">\max_{i=1:mode} \frac{\ Ax_i - \lambda_i x_i\ _1}{\max( E_{min} ,  E_{max} ) \ x_i\ _1} &lt; \epsilon</math> </li> </ul> where <i>mode</i> is the total number of eigenvalues found in the search interval and $\epsilon = 10^{-fpm[6]}$ for real and complex or $\epsilon = 10^{-fpm[2]}$ for double precision and double complex. |
|                | <i>fpm</i> [5]=1 | Extended Eigensolvers are stopped if this trace stopping test is satisfied: $\frac{ trace_j - trace_{j-1} }{\max( E_{min} ,  E_{max} )} < \epsilon$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |

| Param                                                                                                          | Default     | Description                                                                                                                                                                                                                                     |
|----------------------------------------------------------------------------------------------------------------|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                                                                                |             | where $trace_j$ denotes the sum of all eigenvalues found in the search interval $[emin, emax]$ at the $j$ -th Extended Eigensolver iteration:                                                                                                   |
|                                                                                                                |             | $trace_j = \sum_{i=1}^M \lambda_i^{(j)}$                                                                                                                                                                                                        |
| $fpm[6]$                                                                                                       | 5           | Error trace single precision stopping criteria ( $10^{-fpm[6]}$ ).                                                                                                                                                                              |
| $fpm[13]$                                                                                                      | 0           | $fpm[13]=0$ Standard use for Extended Eigensolver routines.<br>$fpm[13]=1$ Non-standard use for Extended Eigensolver routines: return the computed eigenvectors subspace after one single contour integration.                                  |
| $fpm[26]$                                                                                                      | 0           | Specifies whether Extended Eigensolver routines check input matrices (applies to CSR format only).<br>$fpm[26]=0$ Extended Eigensolver routines do not check input matrices.<br>$fpm[26]=1$ Extended Eigensolver routines check input matrices. |
| $fpm[27]$                                                                                                      | 0           | Check if matrix $B$ is positive definite. Set $fpm[27] = 1$ to check if $B$ is positive definite.                                                                                                                                               |
| $fpm[29]$<br>to<br>$fpm[62]$                                                                                   | -           | Reserved for future use.                                                                                                                                                                                                                        |
| $fpm[63]$                                                                                                      | 0           | Use the Intel MKL PARDISO solver with the user-defined PARDISO $iparm$ array settings.                                                                                                                                                          |
| <b>NOTE</b><br>This option can only be used by Extended Eigensolver Predefined Interfaces for Sparse Matrices. |             |                                                                                                                                                                                                                                                 |
|                                                                                                                | $fpm[63]=0$ | Extended Eigensolver routines use the Intel MKL PARDISO default $iparm$ settings defined by calling the <code>pardisoinit</code> subroutine.                                                                                                    |
|                                                                                                                | $fpm[63]=1$ | The values from $fpm[64]$ to $fpm[127]$ correspond to $iparm[0]$ to $iparm[63]$ respectively according to the formula $fpm[64 + i] = iparm[i]$ for $i = 0, 1, \dots, 63$ .                                                                      |

**Extended Eigensolver Output Details**

Errors and warnings encountered during a run of the Extended Eigensolver routines are stored in an integer variable, *info*. If the value of the output *info* parameter is not 0, either an error or warning was encountered. The possible return values for the *info* parameter along with the error code descriptions are given in the following table.

**Return Codes for info Parameter**

| <i>info</i> | Classification | Description                                        |
|-------------|----------------|----------------------------------------------------|
| 202         | Error          | Problem with size of the system $n$ ( $n \leq 0$ ) |

| <i>info</i> | Classification  | Description                                                                                                                                                                                                                         |
|-------------|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 201         | Error           | Problem with size of initial subspace $m0$ ( $m0 \leq 0$ or $m0 > n$ )                                                                                                                                                              |
| 200         | Error           | Problem with $emin, emax$ ( $emin \geq emax$ )                                                                                                                                                                                      |
| (100+i)     | Error           | Problem with $i$ -th value of the input Extended Eigensolver parameter ( $fpm[i - 1]$ ). Only the parameters in use are checked.                                                                                                    |
| 4           | Warning         | Successful return of only the computed subspace after call with $fpm[13] = 1$                                                                                                                                                       |
| 3           | Warning         | Size of the subspace $m0$ is too small ( $m0 < m$ )                                                                                                                                                                                 |
| 2           | Warning         | No Convergence (number of iteration loops $> fpm[3]$ )                                                                                                                                                                              |
| 1           | Warning         | No eigenvalue found in the search interval. See remark below for further details.                                                                                                                                                   |
| 0           | Successful exit |                                                                                                                                                                                                                                     |
| -1          | Error           | Internal error for allocation memory.                                                                                                                                                                                               |
| -2          | Error           | Internal error of the inner system solver. Possible reasons: not enough memory for inner linear system solver or inconsistent input.                                                                                                |
| -3          | Error           | Internal error of the reduced eigenvalue solver<br>Possible cause: matrix $B$ may not be positive definite. It can be checked by setting $fpm[27] = 1$ before calling an Extended Eigensolver routine, or by using LAPACK routines. |
| -4          | Error           | Matrix $B$ is not positive definite.                                                                                                                                                                                                |
| -(100+i)    | Error           | Problem with the $i$ -th argument of the Extended Eigensolver interface.                                                                                                                                                            |

In some extreme cases the return value  $info=1$  may indicate that the Extended Eigensolver routine has failed to find the eigenvalues in the search interval. This situation could arise if a very large search interval is used to locate a small and isolated cluster of eigenvalues (i.e. the dimension of the search interval is many orders of magnitude larger than the number of contour points. It is then either recommended to increase the number of contour points  $fpm[1]$  or simply rescale more appropriately the search interval. Rescaling means the initial problem of finding all eigenvalues the search interval  $[\lambda_{min}, \lambda_{max}]$  for the standard eigenvalue problem  $Ax = \lambda x$  is replaced with the problem of finding all eigenvalues in the search interval  $[\lambda_{min}/t, \lambda_{max}/t]$  for the standard eigenvalue problem  $(A/t)x = (\lambda/t)x$  where  $t$  is a scaling factor.

## Extended Eigensolver RCI Routines

If you do not require specific linear system solvers or matrix storage schemes, you can skip this section and go directly to [Extended Eigensolver Predefined Interfaces](#).

## Extended Eigensolver RCI Interface Description

The Extended Eigensolver RCI interfaces can be used to solve standard or generalized eigenvalue problems, and are independent of the format of the matrices. As mentioned earlier, the Extended Eigensolver algorithm is based on the contour integration techniques of the matrix resolvent  $G(\sigma) = (\sigma B - A)^{-1}$  over a circle. For solving a generalized eigenvalue problem, Extended Eigensolver has to perform one or more of the following operations at each contour point denoted below by  $Z_e$  :

- Factorize the matrix  $(Z_e * B - A)$
- Solve the linear system  $(Z_e * B - A)X = Y$  or  $(Z_e * B - A)^H X = Y$  with multiple right hand sides, where H means transpose conjugate

- Matrix-matrix multiply  $BX = Y$  or  $AX = Y$

For solving a standard eigenvalue problem, replace the matrix  $B$  with the identity matrix  $I$ .

The primary aim of RCI interfaces is to isolate these operations: the linear system solver, factorization of the matrix resolvent at each contour point, and matrix-matrix multiplication. This gives universality to RCI interfaces as they are independent of data structures and the specific implementation of the operations like matrix-vector multiplication or inner system solvers. However, this approach requires some additional effort when calling the interface. In particular, operations listed above are performed by routines that you supply on data structures that you find most appropriate for the problem at hand.

To initialize an Extended Eigensolver RCI routine, set the job indicator ( $ijob$ ) parameter to the value -1. When the routine requires the results of an operation, it generates a special value of  $ijob$  to indicate the operation that needs to be performed. The routine also returns  $ze$ , the coordinate along the complex contour, the values of array  $work$  or  $workc$ , and the number of columns to be used. Your subroutine then must perform the operation at the given contour point  $ze$ , store the results in prescribed array, and return control to the Extended Eigensolver RCI routine.

The following pseudocode shows the general scheme for using the Extended Eigensolver RCI functionality for a real symmetric problem:

```

ijob=-1; // initialization
do while (ijob!=0) {
 ?feast_src1(&ijob, &N, &Ze, work, workc, Aq, Bq,
 fpm, &epsout, &loop, &Emin, &Emax, &M0, E, lambda, &q, res, &info);

 switch(ijob) {
 case 10: // Factorize the complex matrix (ZeB-A)
 break;

 case 11: // Solve the complex linear system (ZeB-A)x=workc
 // Put result in workc
 break;

 case 30: // Perform multiplication A by Qi..Qj columns of QNxM0
 // where i = fpm[23] and j = fpm[23]+fpm[24]-1
 // Qi..Qj located in q starting from q+N*(i-1)
 break;

 case 40: // Perform multiplication B by Qi..Qj columns of QNxM0
 // where i = fpm[23] and j = fpm[23]+fpm[24]-1
 // Qi..Qj located in q starting from q+N*(i-1)
 // Result is stored in work+N*(i-1)

 break;
 }
}

```

---

#### NOTE

The ? option in ?feast in the pseudocode given above should be replaced by either *s* or *d*, depending on the matrix data type of the eigenvalue system.

---

The next pseudocode shows the general scheme for using the Extended Eigensolver RCI functionality for a complex Hermitian problem:

```

ijob=-1; // initialization
while (ijob!=0) {
 ?feast_hrc1(&ijob, &N, &Ze, work, workc, Aq, Bq,
 fpm, &epsout, &loop, &Emin, &Emax, &M0, E, lambda, &q, res, &info);

 switch (ijob) {
 case 10: // Factorize the complex matrix (ZeB-A)

```

```

break;

case 11: // Solve the linear system (ZeB-A)y=workc
 // Put result in workc
break;

case 20: // Factorize (if needed by case 21) the complex matrix (ZeB-A)^H
 // ATTENTION: This option requires additional memory storage
 // (i.e . the resulting matrix from case 10 cannot be overwritten)
break;

case 21: // Solve the linear system (ZeB-A)^Hy=workc
 // Put result in workc
 // REMARK: case 20 becomes obsolete if this solve can be performed
 // using the factorization in case 10
break;

case 30: // Multiply A by Q_i..Q_j columns of Q_{NxM0},
 // where i = fpm[23] and j = fpm[23]+fpm[24]-1
 // Q_i..Q_j located in q starting from q+N*(i-1)
 // Result is stored in work+N*(i-1)
break;

case 40: // Perform multiplication B by Q_i..Q_j columns of Q_{NxM0}
 // where i = fpm[23] and j = fpm[23]+fpm[24]-1
 // Q_i..Q_j located in q starting from q+N*(i-1)
 // Result is stored in work+N*(i-1)
break;

}
}
end do

```

**NOTE**

The `?feast` option in the pseudocode given above should be replaced by either `c` or `z`, depending on the matrix data type of the eigenvalue system.

If `case 20` can be avoided, performance could be up to twice as fast, and Extended Eigensolver functionality would use half of the memory.

If an iterative solver is used along with a preconditioner, the factorization of the preconditioner could be performed with `ijob = 10` (and `ijob = 20` if applicable) for a given value of  $Z_e$ , and the associated iterative solve would then be performed with `ijob = 11` (and `ijob = 21` if applicable).

**Optimization Notice**

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Notice revision #20110804

## ?feast\_srcil/?feast\_hrcil

Extended Eigensolver RCI interface.

### Syntax

```
void sfeast_srcil (MKL_INT* ijob, const MKL_INT* n, MKL_Complex8* ze, float* work,
MKL_Complex8* workc, float* aq, float* sq, MKL_INT* fpm, float* epsout, MKL_INT* loop,
const float* emin, const float* emax, MKL_INT* m0, float* lambda, float* q, MKL_INT*
m, float* res, MKL_INT* info);
```

```
void dfeast_srcil (MKL_INT* ijob, const MKL_INT* n, MKL_Complex16* ze, double* work,
MKL_Complex16* workc, double* aq, double* sq, MKL_INT* fpm, double* epsout, MKL_INT*
loop, const double* emin, const double* emax, MKL_INT* m0, double* lambda, double* q,
MKL_INT* m, double* res, MKL_INT* info);
```

```
void cfeast_hrcil (MKL_INT* ijob, const MKL_INT* n, MKL_Complex8* ze, MKL_Complex8*
work, MKL_Complex8* workc, MKL_Complex8* aq, MKL_Complex8* sq, MKL_INT* fpm, float*
epsout, MKL_INT* loop, const float* emin, const float* emax, MKL_INT* m0, float*
lambda, MKL_Complex8* q, MKL_INT* m, float* res, MKL_INT* info);
```

```
void zfeast_hrcil (MKL_INT* ijob, const MKL_INT* n, MKL_Complex16* ze, MKL_Complex16*
work, MKL_Complex16* workc, MKL_Complex16* aq, MKL_Complex16* sq, MKL_INT* fpm,
double* epsout, MKL_INT* loop, const double* emin, const double* emax, MKL_INT* m0,
double* lambda, MKL_Complex16* q, MKL_INT* m, double* res, MKL_INT* info);
```

### Include Files

- mkl.h

### Description

Compute eigenvalues as described in [Extended Eigensolver RCI Interface Description](#).

### Input Parameters

|                   |                                                                                                                                                                                                                                                                                 |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ijob</i>       | Job indicator variable. On entry, a call to ?feast_srcil/?feast_hrcil with <i>ijob</i> =-1 initializes the eigensolver.                                                                                                                                                         |
| <i>n</i>          | Sets the size of the problem. $n > 0$ .                                                                                                                                                                                                                                         |
| <i>work</i>       | Workspace array of size $n$ by $m0$ .                                                                                                                                                                                                                                           |
| <i>workc</i>      | Workspace array of size $n$ by $m0$ .                                                                                                                                                                                                                                           |
| <i>aq, sq</i>     | Workspace arrays of size $m0$ by $m0$ .                                                                                                                                                                                                                                         |
| <i>fpm</i>        | Array, size of 128. This array is used to pass various parameters to Extended Eigensolver routines. See <a href="#">Extended Eigensolver Input Parameters</a> for a complete description of the parameters and their default values.                                            |
| <i>emin, emax</i> | The lower and upper bounds of the interval to be searched for eigenvalues; $emin \leq emax$ .                                                                                                                                                                                   |
| <i>m0</i>         | On entry, specifies the initial guess for subspace size to be used, $0 < m0 \leq n$ . Set $m0 \geq m$ where $m$ is the total number of eigenvalues located in the interval $[emin, emax]$ . If the initial guess is wrong, Extended Eigensolver routines return <i>info</i> =3. |

*q* On entry, if  $fpm[4]=1$ , the array *q* of size *n* by *m* contains a basis of guess subspace where *n* is the order of the input matrix.

## Output Parameters

*ijob* On exit, the parameter carries the status flag that indicates the condition of the return. The status information is divided into three categories:

1. A zero value indicates successful completion of the task.
2. A positive value indicates that the solver requires a matrix-vector multiplication or solving a specific system with a complex coefficient.
3. A negative value indicates successful initiation.

A non-zero value of *ijob* specifically means the following:

- *ijob* = 10 - factorize the complex matrix  $Z_e * B - A$  at a given contour point  $Z_e$  and return the control to the `?feast_src`/`?feast_hrc` routine where  $Z_e$  is a complex number meaning contour point and its value is defined internally in `?feast_src`/`?feast_hrc`.
- *ijob* = 11 - solve the complex linear system  $(Z_e * B - A) * y = workc$ , put the solution in *workc* and return the control to the `?feast_src`/`?feast_hrc` routine.
- *ijob* = 20 - factorize the complex matrix  $(Z_e * B - A)^H$  at a given contour point  $Z_e$  and return the control to the `?feast_src`/`?feast_hrc` routine where  $Z_e$  is a complex number meaning contour point and its value is defined internally in `?feast_src`/`?feast_hrc`.

The symbol  $X^H$  means transpose conjugate of matrix *X*.

- *ijob* = 21 - solve the complex linear system  $(Z_e * B - A)^H * y = workc$ , put the solution in *workc* and return the control to the `?feast_src`/`?feast_hrc` routine. The case *ijob*=20 becomes obsolete if the solve can be performed using the factorization computed for *ijob*=10.

The symbol  $X^H$  mean transpose conjugate of matrix *X*.

- *ijob* = 30 - multiply matrix *A* by  $Q_j \cdot Q_i$ , put the result in  $work + N * (i - 1)$ , and return the control to the `?feast_src`/`?feast_hrc` routine.  
*i* is  $fpm[24]$ , and *j* is  $fpm[23] + fpm[24] - 1$ .
- *ijob* = 40 - multiply matrix *B* by  $Q_j \cdot Q_i$ , put the result in  $work + N * (i - 1)$  and return the control to the `?feast_src`/`?feast_hrc` routine. If a standard eigenvalue problem is solved, just return  $work = q$ .  
*i* is  $fpm[24]$ , and *j* is  $fpm[23] + fpm[24] - 1$ .
- *ijob* = -2 - rerun the `?feast_src`/`?feast_hrc` task with the same parameters.

*ze* Defines the coordinate along the complex contour. All values of *ze* are generated by `?feast_src`/`?feast_hrc` internally.

*fpm* On output, contains coordinates of columns of work array needed for iterative refinement. (See [Extended Eigensolver RCI Interface Description](#).)

*epsout* On output, contains the relative error on the trace:  $|trace_i - trace_{i-1}| / \max(|emin|, |emax|)$

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>loop</i>   | On output, contains the number of refinement loop executed. Ignored on input.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>lambda</i> | Array of length <i>m0</i> . On output, the first <i>m</i> entries of <i>lambda</i> are eigenvalues found in the interval.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>q</i>      | On output, <i>q</i> contains all eigenvectors corresponding to <i>lambda</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>m</i>      | The total number of eigenvalues found in the interval [ <i>emin</i> , <i>emax</i> ]: $0 \leq m \leq m0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>res</i>    | <p>Array of length <i>m0</i>. On exit, the first <i>m</i> components contain the relative residual vector:</p> <ul style="list-style-type: none"> <li>generalized eigenvalue problem:                     <math display="block">\frac{\ Ax_i - \lambda_i Bx_i\ _1}{\max( E_{\min} ,  E_{\max} ) \ Bx_i\ _1}</math> </li> <li>standard eigenvalue problem:                     <math display="block">\frac{\ Ax_i - \lambda_i x_i\ _1}{\max( E_{\min} ,  E_{\max} ) \ x_i\ _1}</math> </li> </ul> <p>for <math>i=0, 1, \dots, m-1</math>, and where <i>m</i> is the total number of eigenvalues found in the search interval.</p> |
| <i>info</i>   | If <i>info</i> =0, the execution is successful. If <i>info</i> ≠ 0, see <a href="#">Output Eigensolver info Details</a> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

### Extended Eigensolver Predefined Interfaces

The predefined interfaces include routines for standard and generalized eigenvalue problems, and for dense, banded, and sparse matrices.

| Matrix Type | Standard Eigenvalue Problem    | Generalized Eigenvalue Problem |
|-------------|--------------------------------|--------------------------------|
| Dense       | ?feast_syev<br>?feast_heev     | ?feast_sygv<br>?feast_hegv     |
| Banded      | ?feast_sbev<br>?feast_hbev     | ?feast_sbgv<br>?feast_hbgv     |
| Sparse      | ?feast_scsrev<br>?feast_hcsrev | ?feast_scsrgv<br>?feast_hcsrgv |

### Matrix Storage

The symmetric and Hermitian matrices used in Extended Eigensolvers predefined interfaces can be stored in full, band, and sparse formats.

- In the full storage format (described in [Full Storage](#) in additional detail) you store all elements, all of the elements in the upper triangle of the matrix, or all of the elements in the lower triangle of the matrix.

- In the band storage format (described in [Band storage](#) in additional detail), you store only the elements along a diagonal band of the matrix.
- In the sparse format (described in [Storage Arrays for a Matrix in CSR Format \(3-Array Variation\)](#)), you store only the non-zero elements of the matrix.

In generalized eigenvalue systems you must use the same family of storage format for both matrices  $A$  and  $B$ . The bandwidth can be different for the banded format ( $k_{lb}$  can be different from  $k_{la}$ ), and the position of the non-zero elements can also be different for the sparse format (CSR coordinates  $ib$  and  $jb$  can be different from  $ia$  and  $ja$ ).

## ?feast\_syev/?feast\_heev

*Extended Eigensolver interface for standard eigenvalue problem with dense matrices.*

### Syntax

```
void sfeast_syev (const char * uplo, const MKL_INT * n, const float * a, const MKL_INT * lda, MKL_INT * fpm, float * epsout, MKL_INT * loop, const float * emin, const float * emax, MKL_INT * m0, float * e, float * x, MKL_INT * m, float * res, MKL_INT * info);

void dfeast_syev (const char * uplo, const MKL_INT * n, const double * a, const MKL_INT * lda, MKL_INT * fpm, double * epsout, MKL_INT * loop, const double * emin, const double * emax, MKL_INT * m0, double * e, double * x, MKL_INT * m, double * res, MKL_INT * info);

void cfeast_heev (const char * uplo, const MKL_INT * n, const MKL_Complex8 * a, const MKL_INT * lda, MKL_INT * fpm, float * epsout, MKL_INT * loop, const float * emin, const float * emax, MKL_INT * m0, float * e, MKL_Complex8 * x, MKL_INT * m, float * res, MKL_INT * info);

void zfeast_heev (const char * uplo, const MKL_INT * n, const MKL_Complex16 * a, const MKL_INT * lda, MKL_INT * fpm, double * epsout, MKL_INT * loop, const double * emin, const double * emax, MKL_INT * m0, double * e, MKL_Complex16 * x, MKL_INT * m, double * res, MKL_INT * info);
```

### Include Files

- mkl.h

### Description

The routines compute all the eigenvalues and eigenvectors for standard eigenvalue problems,  $Ax = \lambda x$ , within a given search interval.

### Input Parameters

|             |                                                                                                                                                                                                                                                      |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i> | Must be 'U' or 'L' or 'F' .<br>If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular parts of $A$ .<br>If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular parts of $A$ .<br>If <i>uplo</i> = 'F', <i>a</i> stores the full matrix $A$ . |
| <i>n</i>    | Sets the size of the problem. $n > 0$ .                                                                                                                                                                                                              |
| <i>a</i>    | Array of dimension <i>lda</i> by <i>n</i> , contains either full matrix $A$ or upper or lower triangular part of the matrix $A$ , as specified by <i>uplo</i>                                                                                        |
| <i>lda</i>  | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$ .                                                                                                                                                                        |

|                   |                                                                                                                                                                                                                                                                                      |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>fpm</i>        | Array, dimension of 128. This array is used to pass various parameters to Extended Eigensolver routines. See <a href="#">Extended Eigensolver Input Parameters</a> for a complete description of the parameters and their default values.                                            |
| <i>emin, emax</i> | The lower and upper bounds of the interval to be searched for eigenvalues; $emin \leq emax$ .                                                                                                                                                                                        |
| <i>m0</i>         | On entry, specifies the initial guess for subspace dimension to be used, $0 < m0 \leq n$ . Set $m0 \geq m$ where $m$ is the total number of eigenvalues located in the interval $[emin, emax]$ . If the initial guess is wrong, Extended Eigensolver routines return <i>info</i> =3. |
| <i>x</i>          | On entry, if <i>fpm</i> [4]=1, the array <i>x</i> of size $n$ by $m$ contains a basis of guess subspace where $n$ is the order of the input matrix.                                                                                                                                  |

## Output Parameters

|               |                                                                                                                                                                                                                                                                                    |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>epsout</i> | On output, contains the relative error on the trace: $ trace_i - trace_{i-1}  / \max( emin ,  emax )$                                                                                                                                                                              |
| <i>loop</i>   | On output, contains the number of refinement loop executed. Ignored on input.                                                                                                                                                                                                      |
| <i>e</i>      | Array of length $m0$ . On output, the first $m$ entries of <i>e</i> are eigenvalues found in the interval.                                                                                                                                                                         |
| <i>x</i>      | On output, the first $m$ columns of <i>x</i> contain the orthonormal eigenvectors corresponding to the computed eigenvalues <i>e</i> , with the $i$ -th column of <i>x</i> holding the eigenvector associated with $e[i]$ .                                                        |
| <i>m</i>      | The total number of eigenvalues found in the interval $[emin, emax]$ : $0 \leq m \leq m0$ .                                                                                                                                                                                        |
| <i>res</i>    | Array of length $m0$ . On exit, the first $m$ components contain the relative residual vector: $\frac{\ Ax_i - \lambda_i x_i\ _1}{\max( E_{\min} ,  E_{\max} ) \ x_i\ _1}$ for $i=1, 2, \dots, m$ , and where $m$ is the total number of eigenvalues found in the search interval. |
| <i>info</i>   | If <i>info</i> =0, the execution is successful. If <i>info</i> ≠0, see <a href="#">Output Eigensolver info Details</a> .                                                                                                                                                           |

## ?feast\_sylv/?feast\_hgv

Extended Eigensolver interface for generalized eigenvalue problem with dense matrices.

### Syntax

```
void sfeast_sylv (const char * uplo, const MKL_INT * n, const float * a, const MKL_INT * lda, const float * b, const MKL_INT * ldb, MKL_INT * fpm, float * epsout, MKL_INT * loop, const float * emin, const float * emax, MKL_INT * m0, float * e, float * x, MKL_INT * m, float * res, MKL_INT * info);
```

```
void dfeast_sygv (const char * uplo, const MKL_INT * n, const double * a, const MKL_INT * lda, const double * b, const MKL_INT * ldb, MKL_INT * fpm, double * epsout, MKL_INT * loop, const double * emin, const double * emax, MKL_INT * m0, double * e, double * x, MKL_INT * m, double * res, MKL_INT * info);
```

```
void cfeast_hegv (const char * uplo, const MKL_INT * n, const MKL_Complex8 * a, const MKL_INT * lda, const MKL_Complex8 * b, const MKL_INT * ldb, MKL_INT * fpm, float * epsout, MKL_INT * loop, const float * emin, const float * emax, MKL_INT * m0, float * e, MKL_Complex8 * x, MKL_INT * m, float * res, MKL_INT * info);
```

```
void zfeast_hegv (const char * uplo, const MKL_INT * n, const MKL_Complex16 * a, const MKL_INT * lda, const MKL_Complex16 * b, const MKL_INT * ldb, MKL_INT * fpm, double * epsout, MKL_INT * loop, const double * emin, const double * emax, MKL_INT * m0, double * e, MKL_Complex16 * x, MKL_INT * m, double * res, MKL_INT * info);
```

## Include Files

- mkl.h

## Description

The routines compute all the eigenvalues and eigenvectors for generalized eigenvalue problems,  $Ax = \lambda Bx$ , within a given search interval.

## Input Parameters

|                   |                                                                                                                                                                                                                                                                                                                                                                                                  |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>       | Must be 'U' or 'L' or 'F' .<br><br>If <i>UPLO</i> = 'U', <i>a</i> and <i>b</i> store the upper triangular parts of <i>A</i> and <i>B</i> respectively.<br><br>If <i>UPLO</i> = 'L', <i>a</i> and <i>b</i> store the lower triangular parts of <i>A</i> and <i>B</i> respectively.<br><br>If <i>UPLO</i> = 'F', <i>a</i> and <i>b</i> store the full matrices <i>A</i> and <i>B</i> respectively. |
| <i>n</i>          | Sets the size of the problem. $n > 0$ .                                                                                                                                                                                                                                                                                                                                                          |
| <i>a</i>          | Array of dimension <i>lda</i> by <i>n</i> , contains either full matrix <i>A</i> or upper or lower triangular part of the matrix <i>A</i> , as specified by <i>uplo</i>                                                                                                                                                                                                                          |
| <i>lda</i>        | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$ .                                                                                                                                                                                                                                                                                                                    |
| <i>b</i>          | Array of dimension <i>ldb</i> by <i>n</i> , contains either full matrix <i>B</i> or upper or lower triangular part of the matrix <i>B</i> , as specified by <i>uplo</i>                                                                                                                                                                                                                          |
| <i>ldb</i>        | The leading dimension of the array <i>B</i> . Must be at least $\max(1, n)$ .                                                                                                                                                                                                                                                                                                                    |
| <i>fpm</i>        | Array, dimension of 128. This array is used to pass various parameters to Extended Eigensolver routines. See <a href="#">Extended Eigensolver Input Parameters</a> for a complete description of the parameters and their default values.                                                                                                                                                        |
| <i>emin, emax</i> | The lower and upper bounds of the interval to be searched for eigenvalues; $emin \leq emax$ .                                                                                                                                                                                                                                                                                                    |
| <i>m0</i>         | On entry, specifies the initial guess for subspace dimension to be used, $0 < m0 \leq n$ . Set $m0 \geq m$ where <i>m</i> is the total number of eigenvalues located in the interval [ <i>emin</i> , <i>emax</i> ]. If the initial guess is wrong, Extended Eigensolver routines return <i>info</i> =3.                                                                                          |

*x* On entry, if  $f_{pm}[4]=1$ , the array *x* of size *n* by *m* contains a basis of guess subspace where *n* is the order of the input matrix.

## Output Parameters

*epsout* On output, contains the relative error on the trace:  $|trace_i - trace_{i-1}| / \max(|e_{min}|, |e_{max}|)$

*loop* On output, contains the number of refinement loop executed. Ignored on input.

*e* Array of length *m0*. On output, the first *m* entries of *e* are eigenvalues found in the interval.

*x* On output, the first *m* columns of *x* contain the orthonormal eigenvectors corresponding to the computed eigenvalues *e*, with the *i*-th column of *x* holding the eigenvector associated with *e*[*i*].

*m* The total number of eigenvalues found in the interval [*e*<sub>min</sub>, *e*<sub>max</sub>]:  $0 \leq m \leq m0$ .

*res* Array of length *m0*. On exit, the first *m* components contain the relative residual vector:

$$\frac{\|Ax_i - \lambda_i Bx_i\|_1}{\max(|E_{min}|, |E_{max}|) \|Bx_i\|_1}$$

for  $i=1, 2, \dots, m$ , and where *m* is the total number of eigenvalues found in the search interval.

*info* If *info*=0, the execution is successful. If *info*≠ 0, see [Output Eigensolver info Details](#).

## ?feast\_sbev/?feast\_hbev

*Extended Eigensolver interface for standard eigenvalue problem with banded matrices.*

### Syntax

```
void sfeast_sbev (const char * uplo, const MKL_INT * n, const MKL_INT * kla, const
float * a, const MKL_INT * lda, MKL_INT * fpm, float * epsout, MKL_INT * loop, const
float * emin, const float * emax, MKL_INT * m0, float * e, float * x, MKL_INT * m,
float * res, MKL_INT * info);
```

```
void dfeast_sbev (const char * uplo, const MKL_INT * n, const MKL_INT * kla, const
double * a, const MKL_INT * lda, MKL_INT * fpm, double * epsout, MKL_INT * loop, const
double * emin, const double * emax, MKL_INT * m0, double * e, double * x, MKL_INT * m,
double * res, MKL_INT * info);
```

```
void cfeast_hbev (const char * uplo, const MKL_INT * n, const MKL_INT * kla, const
MKL_Complex8 * a, const MKL_INT * lda, MKL_INT * fpm, float * epsout, MKL_INT * loop,
const float * emin, const float * emax, MKL_INT * m0, float * e, MKL_Complex8 * x,
MKL_INT * m, float * res, MKL_INT * info);
```

```
void zfeast_hbev (const char * uplo, const MKL_INT * n, const MKL_INT * kla, const
MKL_Complex16 * a, const MKL_INT * lda, MKL_INT * fpm, double * epsout, MKL_INT *
loop, const double * emin, const double * emax, MKL_INT * m0, double * e,
MKL_Complex16 * x, MKL_INT * m, double * res, MKL_INT * info);
```

## Include Files

- mkl.h

## Description

The routines compute all the eigenvalues and eigenvectors for standard eigenvalue problems,  $Ax = \lambda x$ , within a given search interval.

## Input Parameters

|                   |                                                                                                                                                                                                                                                                                                         |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>       | Must be 'U' or 'L' or 'F' .<br>If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular parts of <i>A</i> .<br>If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular parts of <i>A</i> .<br>If <i>uplo</i> = 'F' , <i>a</i> stores the full matrix <i>A</i> .                                    |
| <i>n</i>          | Sets the size of the problem. $n > 0$ .                                                                                                                                                                                                                                                                 |
| <i>kla</i>        | The number of super- or sub-diagonals within the band in <i>A</i> ( $kla \geq 0$ ).                                                                                                                                                                                                                     |
| <i>a</i>          | Array of dimension <i>lda</i> by <i>n</i> , contains either full matrix <i>A</i> or upper or lower triangular part of the matrix <i>A</i> , as specified by <i>uplo</i>                                                                                                                                 |
| <i>lda</i>        | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$ .                                                                                                                                                                                                                           |
| <i>fpm</i>        | Array, dimension of 128. This array is used to pass various parameters to Extended Eigensolver routines. See <a href="#">Extended Eigensolver Input Parameters</a> for a complete description of the parameters and their default values.                                                               |
| <i>emin, emax</i> | The lower and upper bounds of the interval to be searched for eigenvalues; $emin \leq emax$ .                                                                                                                                                                                                           |
| <i>m0</i>         | On entry, specifies the initial guess for subspace dimension to be used, $0 < m0 \leq n$ . Set $m0 \geq m$ where <i>m</i> is the total number of eigenvalues located in the interval [ <i>emin</i> , <i>emax</i> ]. If the initial guess is wrong, Extended Eigensolver routines return <i>info</i> =3. |
| <i>x</i>          | On entry, if <i>fpm</i> [4]=1, the array <i>x</i> of size <i>n</i> by <i>m</i> contains a basis of guess subspace where <i>n</i> is the order of the input matrix.                                                                                                                                      |

## Output Parameters

|               |                                                                                                                      |
|---------------|----------------------------------------------------------------------------------------------------------------------|
| <i>epsout</i> | On output, contains the relative error on the trace: $ trace_j - trace_{j-1}  / \max( emin ,  emax )$                |
| <i>loop</i>   | On output, contains the number of refinement loop executed. Ignored on input.                                        |
| <i>e</i>      | Array of length <i>m0</i> . On output, the first <i>m</i> entries of <i>e</i> are eigenvalues found in the interval. |

|             |                                                                                                                                                                                                                                                                                                              |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>x</i>    | On output, the first <i>m</i> columns of <i>x</i> contain the orthonormal eigenvectors corresponding to the computed eigenvalues <i>e</i> , with the <i>i</i> -th column of <i>x</i> holding the eigenvector associated with <i>e</i> [ <i>i</i> ].                                                          |
| <i>m</i>    | The total number of eigenvalues found in the interval [ <i>emin</i> , <i>emax</i> ]: $0 \leq m \leq m0$ .                                                                                                                                                                                                    |
| <i>res</i>  | Array of length <i>m0</i> . On exit, the first <i>m</i> components contain the relative residual vector: $\frac{\ Ax_i - \lambda_i x_i\ _1}{\max( E_{\min} ,  E_{\max} ) \ x_i\ _1}$ for <i>i</i> =1, 2, ..., <i>m</i> , and where <i>m</i> is the total number of eigenvalues found in the search interval. |
| <i>info</i> | If <i>info</i> =0, the execution is successful. If <i>info</i> ≠ 0, see <a href="#">Output Eigensolver info Details</a> .                                                                                                                                                                                    |

### ?feast\_sbgv/?feast\_hbgv

*Extended Eigensolver interface for generalized eigenvalue problem with banded matrices.*

#### Syntax

```
void sfeast_sbgv (const char * uplo, const MKL_INT * n, const MKL_INT * kla, const
float * a, const MKL_INT * lda, const MKL_INT * klb, const float * b, const MKL_INT *
ldb, MKL_INT * fpm, float * epsout, MKL_INT * loop, const float * emin, const float *
emax, MKL_INT * m0, float * e, float * x, MKL_INT * m, float * res, MKL_INT * info);

void dfeast_sbgv (const char * uplo, const MKL_INT * n, const MKL_INT * kla, const
double * a, const MKL_INT * lda, const MKL_INT * klb, const double * b, const MKL_INT *
ldb, MKL_INT * fpm, double * epsout, MKL_INT * loop, const double * emin, const double
* emax, MKL_INT * m0, double * e, double * x, MKL_INT * m, double * res, MKL_INT *
info);

void cfeast_hbgv (const char * uplo, const MKL_INT * n, const MKL_INT * kla, const
MKL_Complex8 * a, const MKL_INT * lda, const MKL_INT * klb, const MKL_Complex8 * b,
const MKL_INT * ldb, MKL_INT * fpm, float * epsout, MKL_INT * loop, const float *
emin, const float * emax, MKL_INT * m0, float * e, MKL_Complex8 * x, MKL_INT * m,
float * res, MKL_INT * info);

void zfeast_hbgv (const char * uplo, const MKL_INT * n, const MKL_INT * kla, const
MKL_Complex16 * a, const MKL_INT * lda, const MKL_INT * klb, const MKL_Complex16 * b,
const MKL_INT * ldb, MKL_INT * fpm, double * epsout, MKL_INT * loop, const double *
emin, const double * emax, MKL_INT * m0, double * e, MKL_Complex16 * x, MKL_INT * m,
double * res, MKL_INT * info);
```

#### Include Files

- mkl.h

#### Description

The routines compute all the eigenvalues and eigenvectors for generalized eigenvalue problems,  $Ax = \lambda Bx$ , within a given search interval.

**NOTE**

Both matrices *A* and *B* must use the same family of storage format. The bandwidth, however, can be different (*klb* can be different from *kla*).

**Input Parameters**

|                   |                                                                                                                                                                                                                                                                                                                                                                                                  |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>       | Must be 'U' or 'L' or 'F' .<br><br>If <i>UPLO</i> = 'U', <i>a</i> and <i>b</i> store the upper triangular parts of <i>A</i> and <i>B</i> respectively.<br><br>If <i>UPLO</i> = 'L', <i>a</i> and <i>b</i> store the lower triangular parts of <i>A</i> and <i>B</i> respectively.<br><br>If <i>UPLO</i> = 'F', <i>a</i> and <i>b</i> store the full matrices <i>A</i> and <i>B</i> respectively. |
| <i>n</i>          | Sets the size of the problem. $n > 0$ .                                                                                                                                                                                                                                                                                                                                                          |
| <i>kla</i>        | The number of super- or sub-diagonals within the band in <i>A</i> ( $kla \geq 0$ ).                                                                                                                                                                                                                                                                                                              |
| <i>a</i>          | Array of dimension <i>lda</i> by <i>n</i> , contains either full matrix <i>A</i> or upper or lower triangular part of the matrix <i>A</i> , as specified by <i>uplo</i>                                                                                                                                                                                                                          |
| <i>lda</i>        | The leading dimension of the array <i>a</i> . Must be at least $\max(1, n)$ .                                                                                                                                                                                                                                                                                                                    |
| <i>klb</i>        | The number of super- or sub-diagonals within the band in <i>B</i> ( $klb \geq 0$ ).                                                                                                                                                                                                                                                                                                              |
| <i>b</i>          | Array of dimension <i>ldb</i> by <i>n</i> , contains either full matrix <i>B</i> or upper or lower triangular part of the matrix <i>B</i> , as specified by <i>uplo</i>                                                                                                                                                                                                                          |
| <i>ldb</i>        | The leading dimension of the array <i>B</i> . Must be at least $\max(1, n)$ .                                                                                                                                                                                                                                                                                                                    |
| <i>fpm</i>        | Array, dimension of 128. This array is used to pass various parameters to Extended Eigensolver routines. See <a href="#">Extended Eigensolver Input Parameters</a> for a complete description of the parameters and their default values.                                                                                                                                                        |
| <i>emin, emax</i> | The lower and upper bounds of the interval to be searched for eigenvalues; $emin \leq emax$ .                                                                                                                                                                                                                                                                                                    |
| <i>m0</i>         | On entry, specifies the initial guess for subspace dimension to be used, $0 < m0 \leq n$ . Set $m0 \geq m$ where <i>m</i> is the total number of eigenvalues located in the interval [ <i>emin</i> , <i>emax</i> ]. If the initial guess is wrong, Extended Eigensolver routines return <i>info</i> =3.                                                                                          |
| <i>x</i>          | On entry, if <i>fpm</i> [4]=1, the array <i>x</i> of size <i>n</i> by <i>m</i> contains a basis of guess subspace where <i>n</i> is the order of the input matrix.                                                                                                                                                                                                                               |

**Output Parameters**

|               |                                                                                                       |
|---------------|-------------------------------------------------------------------------------------------------------|
| <i>epsout</i> | On output, contains the relative error on the trace: $ trace_i - trace_{i-1}  / \max( emin ,  emax )$ |
| <i>loop</i>   | On output, contains the number of refinement loop executed. Ignored on input.                         |

|             |                                                                                                                                                                                                                                                                                                                |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>e</i>    | Array of length <i>m0</i> . On output, the first <i>m</i> entries of <i>e</i> are eigenvalues found in the interval.                                                                                                                                                                                           |
| <i>x</i>    | On output, the first <i>m</i> columns of <i>x</i> contain the orthonormal eigenvectors corresponding to the computed eigenvalues <i>e</i> , with the <i>i</i> -th column of <i>x</i> holding the eigenvector associated with <i>e</i> [ <i>i</i> ].                                                            |
| <i>m</i>    | The total number of eigenvalues found in the interval [ <i>emin</i> , <i>emax</i> ]: $0 \leq m \leq m0$ .                                                                                                                                                                                                      |
| <i>res</i>  | Array of length <i>m0</i> . On exit, the first <i>m</i> components contain the relative residual vector: $\frac{\ Ax_i - \lambda_i Bx_i\ _1}{\max( E_{\min} ,  E_{\max} ) \ Bx_i\ _1}$ for <i>i</i> =1, 2, ..., <i>m</i> , and where <i>m</i> is the total number of eigenvalues found in the search interval. |
| <i>info</i> | If <i>info</i> =0, the execution is successful. If <i>info</i> ≠ 0, see <a href="#">Output Eigensolver info Details</a> .                                                                                                                                                                                      |

### [?feast\\_scsrev/?feast\\_hcsrev](#)

*Extended Eigensolver interface for standard eigenvalue problem with sparse matrices.*

#### Syntax

```
void sfeast_scsrev (const char * uplo, const MKL_INT * n, const float * a, const
MKL_INT * ia, const MKL_INT * ja, MKL_INT * fpm, float * epsout, MKL_INT * loop, const
float * emin, const float * emax, MKL_INT * m0, float * e, float * x, MKL_INT * m,
float * res, MKL_INT * info);
```

```
void dfeast_scsrev (const char * uplo, const MKL_INT * n, const double * a, const
MKL_INT * ia, const MKL_INT * ja, MKL_INT * fpm, double * epsout, MKL_INT * loop,
const double * emin, const double * emax, MKL_INT * m0, double * e, double * x,
MKL_INT * m, double * res, MKL_INT * info);
```

```
void cfeast_hcsrev (const char * uplo, const MKL_INT * n, const MKL_Complex8 * a, const
MKL_INT * ia, const MKL_INT * ja, MKL_INT * fpm, float * epsout, MKL_INT * loop, const
float * emin, const float * emax, MKL_INT * m0, float * e, MKL_Complex8 * x, MKL_INT *
m, float * res, MKL_INT * info);
```

```
void zfeast_hcsrev (const char * uplo, const MKL_INT * n, const MKL_Complex16 * a,
const MKL_INT * ia, const MKL_INT * ja, MKL_INT * fpm, double * epsout, MKL_INT *
loop, const double * emin, const double * emax, MKL_INT * m0, double * e,
MKL_Complex16 * x, MKL_INT * m, double * res, MKL_INT * info);
```

#### Include Files

- mkl.h

#### Description

The routines compute all the eigenvalues and eigenvectors for standard eigenvalue problems,  $Ax = \lambda x$ , within a given search interval.

## Input Parameters

|                   |                                                                                                                                                                                                                                                                                                               |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>       | Must be 'U' or 'L' or 'F' .<br>If <i>uplo</i> = 'U', <i>a</i> stores the upper triangular parts of <i>A</i> .<br>If <i>uplo</i> = 'L', <i>a</i> stores the lower triangular parts of <i>A</i> .<br>If <i>uplo</i> = 'F' , <i>a</i> stores the full matrix <i>A</i> .                                          |
| <i>n</i>          | Sets the size of the problem. $n > 0$ .                                                                                                                                                                                                                                                                       |
| <i>a</i>          | Array containing the nonzero elements of either the full matrix <i>A</i> or the upper or lower triangular part of the matrix <i>A</i> , as specified by <i>uplo</i> .                                                                                                                                         |
| <i>ia</i>         | Array of length $n + 1$ , containing indices of elements in the array <i>a</i> , such that <i>ia</i> [ <i>i</i> ] is the index in the array <i>a</i> of the first non-zero element from the row <i>i</i> . The value of the last element <i>ia</i> [ <i>n</i> ] is equal to the number of non-zeros plus one. |
| <i>ja</i>         | Array containing the column indices for each non-zero element of the matrix <i>A</i> being represented in the array <i>a</i> . Its length is equal to the length of the array <i>a</i> .                                                                                                                      |
| <i>fpm</i>        | Array, dimension of 128. This array is used to pass various parameters to Extended Eigensolver routines. See <a href="#">Extended Eigensolver Input Parameters</a> for a complete description of the parameters and their default values.                                                                     |
| <i>emin, emax</i> | The lower and upper bounds of the interval to be searched for eigenvalues; $emin \leq emax$ .                                                                                                                                                                                                                 |
| <i>m0</i>         | On entry, specifies the initial guess for subspace dimension to be used, $0 < m0 \leq n$ . Set $m0 \geq m$ where <i>m</i> is the total number of eigenvalues located in the interval [ <i>emin</i> , <i>emax</i> ]. If the initial guess is wrong, Extended Eigensolver routines return <i>info</i> =3.       |
| <i>x</i>          | On entry, if <i>fpm</i> [4]=1, the array <i>x</i> of size <i>n</i> by <i>m</i> contains a basis of guess subspace where <i>n</i> is the order of the input matrix.                                                                                                                                            |

## Output Parameters

|               |                                                                                                                                                                                                                                                     |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>fpm</i>    | On output, the last 64 values correspond to Intel MKL PARDISO <i>iparm</i> [0] to <i>iparm</i> [63] (regardless of the value of <i>fpm</i> [63] on input).                                                                                          |
| <i>epsout</i> | On output, contains the relative error on the trace: $ trace_i - trace_{i-1}  / \max( emin ,  emax )$                                                                                                                                               |
| <i>loop</i>   | On output, contains the number of refinement loop executed. Ignored on input.                                                                                                                                                                       |
| <i>e</i>      | Array of length <i>m0</i> . On output, the first <i>m</i> entries of <i>e</i> are eigenvalues found in the interval.                                                                                                                                |
| <i>x</i>      | On output, the first <i>m</i> columns of <i>x</i> contain the orthonormal eigenvectors corresponding to the computed eigenvalues <i>e</i> , with the <i>i</i> -th column of <i>x</i> holding the eigenvector associated with <i>e</i> [ <i>i</i> ]. |

|             |                                                                                                                                                                                                                                                                                                   |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>m</i>    | The total number of eigenvalues found in the interval [ <i>emin</i> , <i>emax</i> ]: $0 \leq m \leq m0$ .                                                                                                                                                                                         |
| <i>res</i>  | Array of length <i>m0</i> . On exit, the first <i>m</i> components contain the relative residual vector: $\frac{\ Ax_i - \lambda_i x_i\ _1}{\max( E_{\min} ,  E_{\max} ) \ x_i\ _1}$ for $i=1, 2, \dots, m$ , and where <i>m</i> is the total number of eigenvalues found in the search interval. |
| <i>info</i> | If <i>info</i> =0, the execution is successful. If <i>info</i> ≠ 0, see <a href="#">Output Eigensolver info Details</a> .                                                                                                                                                                         |

### [?feast\\_scsrgv/?feast\\_hcsrgv](#)

*Extended Eigensolver interface for generalized eigenvalue problem with sparse matrices.*

#### Syntax

```
void sfeast_scsrgv (const char * uplo, const MKL_INT * n, const float * a, const
MKL_INT * ia, const MKL_INT * ja, const float * b, const MKL_INT * ib, const MKL_INT *
jb, MKL_INT * fpm, float * epsout, MKL_INT * loop, const float * emin, const float *
emax, MKL_INT * m0, float * e, float * x, MKL_INT * m, float * res, MKL_INT * info);

void dfeast_scsrgv (const char * uplo, const MKL_INT * n, const double * a, const
MKL_INT * ia, const MKL_INT * ja, const double * b, const MKL_INT * ib, const MKL_INT *
jb, MKL_INT * fpm, double * epsout, MKL_INT * loop, const double * emin, const double
* emax, MKL_INT * m0, double * e, double * x, MKL_INT * m, double * res, MKL_INT *
info);

void cfeast_hcsrgv (const char * uplo, const MKL_INT * n, const MKL_Complex8 * a, const
MKL_INT * ia, const MKL_INT * ja, const MKL_Complex8 * b, const MKL_INT * ib, const
MKL_INT * jb, MKL_INT * fpm, float * epsout, MKL_INT * loop, const float * emin, const
float * emax, MKL_INT * m0, float * e, MKL_Complex8 * x, MKL_INT * m, float * res,
MKL_INT * info);

void zfeast_hcsrgv (const char * uplo, const MKL_INT * n, const MKL_Complex16 * a,
const MKL_INT * ia, const MKL_INT * ja, const MKL_Complex16 * b, const MKL_INT * ib,
const MKL_INT * jb, MKL_INT * fpm, double * epsout, MKL_INT * loop, const double *
emin, const double * emax, MKL_INT * m0, double * e, MKL_Complex16 * x, MKL_INT * m,
double * res, MKL_INT * info);
```

#### Include Files

- mkl.h

#### Description

The routines compute all the eigenvalues and eigenvectors for generalized eigenvalue problems,  $Ax = \lambda Bx$ , within a given search interval.

**NOTE**

Both matrices  $A$  and  $B$  must use the same family of storage format. The position of the non-zero elements can be different (CSR coordinates  $ib$  and  $jb$  can be different from  $ia$  and  $ja$ ).

**Input Parameters**

|                   |                                                                                                                                                                                                                                                                                                                          |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>       | Must be 'U' or 'L' or 'F' .<br><br>If $UPLO = 'U'$ , $a$ and $b$ store the upper triangular parts of $A$ and $B$ respectively.<br><br>If $UPLO = 'L'$ , $a$ and $b$ store the lower triangular parts of $A$ and $B$ respectively.<br><br>If $UPLO = 'F'$ , $a$ and $b$ store the full matrices $A$ and $B$ respectively. |
| <i>n</i>          | Sets the size of the problem. $n > 0$ .                                                                                                                                                                                                                                                                                  |
| <i>a</i>          | Array containing the nonzero elements of either the full matrix $A$ or the upper or lower triangular part of the matrix $A$ , as specified by <i>uplo</i> .                                                                                                                                                              |
| <i>ia</i>         | Array of length $n + 1$ , containing indices of elements in the array $a$ , such that $ia[i - 1]$ is the index in the array $a$ of the first non-zero element from the row $i$ . The value of the last element $ia[n]$ is equal to the number of non-zeros plus one.                                                     |
| <i>ja</i>         | Array containing the column indices for each non-zero element of the matrix $A$ being represented in the array $a$ . Its length is equal to the length of the array $a$ .                                                                                                                                                |
| <i>b</i>          | Array of dimension $ldb$ by $*$ , contains the nonzero elements of either the full matrix $B$ or the upper or lower triangular part of the matrix $B$ , as specified by <i>uplo</i> .                                                                                                                                    |
| <i>ib</i>         | Array of length $n + 1$ , containing indices of elements in the array $b$ , such that $ib[i - 1]$ is the index in the array $b$ of the first non-zero element from the row $i$ . The value of the last element $ib[n]$ is equal to the number of non-zeros plus one.                                                     |
| <i>jb</i>         | Array containing the column indices for each non-zero element of the matrix $B$ being represented in the array $b$ . Its length is equal to the length of the array $b$ .                                                                                                                                                |
| <i>fpm</i>        | Array, dimension of 128. This array is used to pass various parameters to Extended Eigensolver routines. See <a href="#">Extended Eigensolver Input Parameters</a> for a complete description of the parameters and their default values.                                                                                |
| <i>emin, emax</i> | The lower and upper bounds of the interval to be searched for eigenvalues; $emin \leq emax$ .                                                                                                                                                                                                                            |
| <i>m0</i>         | On entry, specifies the initial guess for subspace dimension to be used, $0 < m0 \leq n$ . Set $m0 \geq m$ where $m$ is the total number of eigenvalues located in the interval $[emin, emax]$ . If the initial guess is wrong, Extended Eigensolver routines return $info=3$ .                                          |

$x$  On entry, if  $fpm[4]=1$ , the array  $x$  of size  $n$  by  $m$  contains a basis of guess subspace where  $n$  is the order of the input matrix.

## Output Parameters

$fpm$  On output, the last 64 values correspond to Intel MKL PARDISO  $iparm[0]$  to  $iparm[63]$  (regardless of the value of  $fpm[63]$  on input).

$epsout$  On output, contains the relative error on the trace:  $|trace_i - trace_{i-1}| / \max(|emin|, |emax|)$

$loop$  On output, contains the number of refinement loop executed. Ignored on input.

$e$  Array of length  $m0$ . On output, the first  $m$  entries of  $e$  are eigenvalues found in the interval.

$x$  On output, the first  $m$  columns of  $x$  contain the orthonormal eigenvectors corresponding to the computed eigenvalues  $e$ , with the  $i$ -th column of  $x$  holding the eigenvector associated with  $e[i]$ .

$m$  The total number of eigenvalues found in the interval  $[emin, emax]$ :  $0 \leq m \leq m0$ .

$res$  Array of length  $m0$ . On exit, the first  $m$  components contain the relative residual vector:

$$\frac{\|Ax_i - \lambda_i Bx_i\|_1}{\max(|E_{\min}|, |E_{\max}|) \|Bx_i\|_1}$$

for  $i=1, 2, \dots, m$ , and where  $m$  is the total number of eigenvalues found in the search interval.

$info$  If  $info=0$ , the execution is successful. If  $info \neq 0$ , see [Output Eigensolver info Details](#).



# Vector Mathematical Functions

This chapter describes Intel® MKL Vector Mathematics functions (VM), which compute a mathematical function of each of the vector elements. VM includes a set of highly optimized functions (arithmetic, power, trigonometric, exponential, hyperbolic, special, and rounding) that operate on vectors of real and complex numbers.

Application programs that improve performance with VM include nonlinear programming software, computation of integrals, financial calculations, computer graphics, and many others.

VM functions fall into the following groups according to the operations they perform:

- [VM Mathematical Functions](#) compute values of mathematical functions, such as sine, cosine, exponential, or logarithm, on vectors stored contiguously in memory.
- [VM Pack/Unpack Functions](#) convert to and from vectors with positive increment indexing, vector indexing, and mask indexing (see [Appendix B](#) for details on vector indexing methods).
- [VM Service Functions](#) set/get the accuracy modes and the error codes, and free memory.

The VM mathematical functions take an input vector as an argument, compute values of the respective function element-wise, and return the results in an output vector. All the VM mathematical functions can perform in-place operations, where the input and output arrays are at the same memory locations.

The Intel MKL interfaces are given in `mkl_vml_functions.h`.

Examples that demonstrate how to use the VM functions are located in:

```
$(MKL)/examples/vmlc/source
```

See VM performance and accuracy data in the online VM Performance and Accuracy Data document available at <http://software.intel.com/en-us/articles/intel-math-kernel-library-documentation/>

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## VM Data Types, Accuracy Modes, and Performance Tips

VM includes mathematical and pack/unpack vector functions for single and double precision vector arguments of real and complex types. Intel MKL provides Fortran and C interfaces for all VM functions, including the associated service functions. The Function Naming Conventions section below shows how to call these functions.

Performance depends on a number of factors, including vectorization and threading overhead. The recommended usage is as follows:

- Use VM for vector lengths larger than 40 elements.
- Use the Intel® Compiler for vector lengths less than 40 elements.

All VM vector functions support the following accuracy modes:

- High Accuracy (HA), the default mode
- Low Accuracy (LA), which improves performance by reducing accuracy of the two least significant bits
- Enhanced Performance (EP), which provides better performance at the cost of significantly reduced accuracy. Approximately half of the bits in the mantissa are correct.

Note that using the EP mode does not guarantee accurate processing of corner cases and special values. Although the default accuracy is HA, LA is sufficient in most cases. For applications that require less accuracy (for example, media applications, some Monte Carlo simulations, etc.), the EP mode may be sufficient.

VM handles special values in accordance with the C99 standard [C99].

Intel MKL offers both functions and environment variables to switch between modes for VM. See the *Intel MKL Developer Guide* for details about the environment variables. Use the `vmlSetMode(mode)` function (see [Table "Values of the mode Parameter"](#)) to switch between the HA, LA, and EP modes. The `vmlGetMode()` function returns the current mode.

### Optimization Notice

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### See Also

[VM Naming Conventions](#)

## VM Naming Conventions

The VM function names are of mixed (lower and upper) case.

The VM mathematical and pack/unpack function names have the following structure:

`v[m]<?><name><mod>`

where

- `v` is a prefix indicating vector operations.
- `[m]` is an optional prefix for mathematical functions that indicates additional argument to specify a VM mode for a given function call (see [vmlSetMode](#) for possible values and their description).
- `<?>` is a precision prefix that indicates one of the following the data types:

|                |                |
|----------------|----------------|
| <code>s</code> | float.         |
| <code>d</code> | double.        |
| <code>c</code> | MKL_Complex8.  |
| <code>z</code> | MKL_Complex16. |

- `<name>` indicates the function short name, with some of its letters in uppercase. See examples in [Table "VM Mathematical Functions"](#).
- `<mod>` field (written in uppercase) is present only in the pack/unpack functions and indicates the indexing method used:

|                |                                    |
|----------------|------------------------------------|
| <code>i</code> | indexing with a positive increment |
| <code>v</code> | indexing with an index vector      |

*m* indexing with a mask vector.

The VM service function names have the following structure:

*vm*<name>

where

<name> indicates the function short name, with some of its letters in uppercase. See examples in [Table "VM Service Functions"](#).

To call VM functions from an application program, use conventional function calls. For example, call the vector single precision real exponential function as

```
vsExp (n, a, y);
```

## VM Function Interfaces

VM interfaces include the function names and argument lists. The following sections describe the interfaces for the VM functions. Note that some of the functions have multiple input and output arguments

Some VM functions may also take scalar arguments as input. See the function description for the naming conventions of such arguments.

### VM Mathematical Function Interfaces

```
v<?><name>(n, a, [scalar input arguments],y);
v<?><name>(n, a, b, [scalar input arguments],y);
v<?><name>(n, a, y, z);
vm<?><name>(n, a, [scalar input arguments],y, mode);
vm<?><name>(n, a, b, [scalar input arguments],y, mode);
vm<?><name>(n, a, y, z, mode);
```

### VM Pack Function Interfaces

```
v<?>PackI(n, a, inca, y);
v<?>PackV(n, a, ia, y);
v<?>PackM(n, a, ma, y);
```

### VM Unpack Function Interfaces

```
v<?>UnpackI(n, a, y, incy);
v<?>UnpackV(n, a, y, iy);
v<?>UnpackM(n, a, y, my);
```

### VM Service Function Interfaces

```
oldmode = vmlSetMode(mode);
mode = vmlGetMode(void);
olderr = vmlSetErrStatus (err);
err = vmlGetErrStatus(void);
olderr = vmlClearErrStatus(void);
oldcallback = vmlSetErrorCallBack(callback);
callback = vmlGetErrorCallBack(void);
oldcallback = vmlClearErrorCallBack(void);
```

Note that *oldmode*, *oldcerr*, and *oldcallback* refer to settings prior to the call.

## VM Input Function Interfaces

|                 |                                                 |
|-----------------|-------------------------------------------------|
| <i>n</i>        | number of elements to be calculated             |
| <i>a</i>        | first input vector                              |
| <i>b</i>        | second input vector                             |
| <i>inca</i>     | vector increment for the input vector <i>a</i>  |
| <i>ia</i>       | index vector for the input vector <i>a</i>      |
| <i>ma</i>       | mask vector for the input vector <i>a</i>       |
| <i>incy</i>     | vector increment for the output vector <i>y</i> |
| <i>iy</i>       | index vector for the output vector <i>y</i>     |
| <i>my</i>       | mask vector for the output vector <i>y</i>      |
| <i>err</i>      | error code                                      |
| <i>mode</i>     | VM mode                                         |
| <i>callback</i> | address of the callback function                |

## VM Output Function Interfaces

|                    |                                         |
|--------------------|-----------------------------------------|
| <i>y</i>           | first output vector                     |
| <i>z</i>           | second output vector                    |
| <i>err</i>         | error code                              |
| <i>mode</i>        | VM mode                                 |
| <i>olderr</i>      | former error code                       |
| <i>oldmode</i>     | former VM mode                          |
| <i>callback</i>    | address of the callback function        |
| <i>oldcallback</i> | address of the former callback function |

See the data types of the parameters used in each function in the respective function description section. All the Intel MKL VM mathematical functions can perform in-place operations.

## Vector Indexing Methods

---

VM mathematical functions work only with unit stride. To accommodate arrays with other increments, or more complicated indexing, you can gather the elements into a contiguous vector and then scatter them after the computation is complete.

VM uses the three following indexing methods to do this task:

- positive increment
- index vector
- mask vector

The indexing method used in a particular function is indicated by the indexing modifier (see the description of the `<mod>` field in [Function Naming Conventions](#)). For more information on the indexing methods, see [Vector Arguments in VM](#) in Appendix B.

## VM Error Diagnostics

The VM mathematical functions incorporate the error handling mechanism, which is controlled by the following service functions:

|                                                                                                        |                                                                                                                                                                                                                         |
|--------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>vmlGetErrStatus,</code><br><code>vmlSetErrStatus,</code><br><code>vmlClearErrStatus</code>       | These functions operate with a global variable called VM Error Status. The VM Error Status flags an error, a warning, or a successful execution of a VM function.                                                       |
| <code>vmlGetErrCallback,</code><br><code>vmlSetErrCallback,</code><br><code>vmlClearErrCallback</code> | These functions enable you to customize the error handling. For example, you can identify a particular argument in a vector where an error occurred or that caused a warning.                                           |
| <code>vmlSetMode, vmlGetMode</code>                                                                    | These functions get and set a VM mode. If you set a new VM mode using the <code>vmlSetMode</code> function, you can store the previous VM mode returned by the routine and restore it at any point of your application. |

If both an error and a warning situation occur during the function call, the VM Error Status variable keeps only the value of the error code. See [Table "Values of the VM Error Status"](#) for possible values. If a VM function does not encounter errors or warnings, it sets the VM Error Status to `VML_STATUS_OK`.

If you use incorrect input arguments to a VM function (`VML_STATUS_BADSIZE` and `VML_STATUS_BADMEM`), the function calls `xerbla` to report errors. See [Table "Values of the VM Error Status"](#) for details

You can use the `vmlSetMode` and `vmlGetMode` functions to modify error handling behavior. Depending on the VM mode, the error handling behavior includes the following operations:

- setting the VM Error Status to a value corresponding to the observed error or warning
- setting the `errno` variable to one of the values described in [Table "Set Values of the `errno` Variable"](#)
- writing error text information to the `stderr` stream
- raising the appropriate exception on an error, if necessary
- calling the additional error handler callback function that is set by `vmlSetErrorCallback`.

### Set Values of the `errno` Variable

| Value of <code>errno</code> | Description                                                               |
|-----------------------------|---------------------------------------------------------------------------|
| <code>0</code>              | No errors are detected.                                                   |
| <code>EINVAL</code>         | The array dimension is not positive.                                      |
| <code>EACCES</code>         | NULL pointer is passed.                                                   |
| <code>EDOM</code>           | At least one of array values is out of a range of definition.             |
| <code>ERANGE</code>         | At least one of array values caused a singularity, overflow or underflow. |

### See Also

[vmlGetErrStatus](#) Gets the VM Error Status.

[vmlSetErrStatus](#) Sets the new VM Error Status according to `err` and stores the previous VM Error Status to `olderr`.

[vmlClearErrStatus](#) Sets the VM Error Status to `VML_STATUS_OK` and stores the previous VM Error Status to `olderr`.

[vmlSetErrorCallback](#) Sets the additional error handler callback function and gets the old callback function.

[vmlGetErrorCallback](#) Gets the additional error handler callback function.

**vmClearErrorCallback** Deletes the additional error handler callback function and retrieves the former callback function.

**vmGetMode** Gets the VM mode.

**vmSetMode** Sets a new mode for VM functions according to the *mode* parameter and stores the previous VM mode to *oldmode*.

## VM Mathematical Functions

This section describes VM functions that compute values of mathematical functions on real and complex vector arguments with unit increment.

Each function is introduced by its short name, a brief description of its purpose, and the calling sequence for each type of data, as well as a description of the input/output arguments.

The input range of parameters is equal to the mathematical range of the input data type, unless the function description specifies input threshold values, which mark off the precision overflow, as follows:

- *FLT\_MAX* denotes the maximum number representable in single precision real data type
- *DBL\_MAX* denotes the maximum number representable in double precision real data type

Table "VM Mathematical Functions" lists available mathematical functions and associated data types.

### VM Mathematical Functions

| Function                                     | Data Types        | Description                                                                          |
|----------------------------------------------|-------------------|--------------------------------------------------------------------------------------|
| <b>Arithmetic Functions</b>                  |                   |                                                                                      |
| v?Add                                        | <i>s, d, c, z</i> | Addition of vector elements                                                          |
| v?Sub                                        | <i>s, d, c, z</i> | Subtraction of vector elements                                                       |
| v?Sqr                                        | <i>s, d</i>       | Squaring of vector elements                                                          |
| v?Mul                                        | <i>s, d, c, z</i> | Multiplication of vector elements                                                    |
| v?MulByConj                                  | <i>c, z</i>       | Multiplication of elements of one vector by conjugated elements of the second vector |
| v?Conj                                       | <i>c, z</i>       | Conjugation of vector elements                                                       |
| v?Abs                                        | <i>s, d, c, z</i> | Computation of the absolute value of vector elements                                 |
| v?Arg                                        | <i>c, z</i>       | Computation of the argument of vector elements                                       |
| v?LinearFrac                                 | <i>s, d</i>       | Linear fraction transformation of vectors                                            |
| <b>Power and Root Functions</b>              |                   |                                                                                      |
| v?Inv                                        | <i>s, d</i>       | Inversion of vector elements                                                         |
| v?Div                                        | <i>s, d, c, z</i> | Division of elements of one vector by elements of the second vector                  |
| v?Sqrt                                       | <i>s, d, c, z</i> | Computation of the square root of vector elements                                    |
| v?InvSqrt                                    | <i>s, d</i>       | Computation of the inverse square root of vector elements                            |
| v?Cbirt                                      | <i>s, d</i>       | Computation of the cube root of vector elements                                      |
| v?InvCbirt                                   | <i>s, d</i>       | Computation of the inverse cube root of vector elements                              |
| v?Pow2o3                                     | <i>s, d</i>       | Raising each vector element to the power of 2/3                                      |
| v?Pow3o2                                     | <i>s, d</i>       | Raising each vector element to the power of 3/2                                      |
| v?Pow                                        | <i>s, d, c, z</i> | Raising each vector element to the specified power                                   |
| v?Powx                                       | <i>s, d, c, z</i> | Raising each vector element to the constant power                                    |
| v?Hypot                                      | <i>s, d</i>       | Computation of the square root of sum of squares                                     |
| <b>Exponential and Logarithmic Functions</b> |                   |                                                                                      |
| v?Exp                                        | <i>s, d, c, z</i> | Computation of the exponential of vector elements                                    |
| v?Expml                                      | <i>s, d</i>       | Computation of the exponential of vector elements decreased by 1                     |
| v?Ln                                         | <i>s, d, c, z</i> | Computation of the natural logarithm of vector elements                              |
| v?Log10                                      | <i>s, d, c, z</i> | Computation of the denary logarithm of vector elements                               |
| v?Log1p                                      | <i>s, d</i>       | Computation of the natural logarithm of vector elements that are increased by 1      |
| <b>Trigonometric Functions</b>               |                   |                                                                                      |
| v?Cos                                        | <i>s, d, c, z</i> | Computation of the cosine of vector elements                                         |

| Function                    | Data Types   | Description                                                                                          |
|-----------------------------|--------------|------------------------------------------------------------------------------------------------------|
| <code>v?Sin</code>          | $s, d, c, z$ | Computation of the sine of vector elements                                                           |
| <code>v?SinCos</code>       | $s, d$       | Computation of the sine and cosine of vector elements                                                |
| <code>v?CIS</code>          | $c, z$       | Computation of the complex exponent of vector elements (cosine and sine combined to complex value)   |
| <code>v?Tan</code>          | $s, d, c, z$ | Computation of the tangent of vector elements                                                        |
| <code>v?Acos</code>         | $s, d, c, z$ | Computation of the inverse cosine of vector elements                                                 |
| <code>v?Asin</code>         | $s, d, c, z$ | Computation of the inverse sine of vector elements                                                   |
| <code>v?Atan</code>         | $s, d, c, z$ | Computation of the inverse tangent of vector elements                                                |
| <code>v?Atan2</code>        | $s, d$       | Computation of the four-quadrant inverse tangent of elements of two vectors                          |
| <b>Hyperbolic Functions</b> |              |                                                                                                      |
| <code>v?Cosh</code>         | $s, d, c, z$ | Computation of the hyperbolic cosine of vector elements                                              |
| <code>v?Sinh</code>         | $s, d, c, z$ | Computation of the hyperbolic sine of vector elements                                                |
| <code>v?Tanh</code>         | $s, d, c, z$ | Computation of the hyperbolic tangent of vector elements                                             |
| <code>v?Acosh</code>        | $s, d, c, z$ | Computation of the inverse hyperbolic cosine of vector elements                                      |
| <code>v?Asinh</code>        | $s, d, c, z$ | Computation of the inverse hyperbolic sine of vector elements                                        |
| <code>v?Atanh</code>        | $s, d, c, z$ | Computation of the inverse hyperbolic tangent of vector elements.                                    |
| <b>Special Functions</b>    |              |                                                                                                      |
| <code>v?Erf</code>          | $s, d$       | Computation of the error function value of vector elements                                           |
| <code>v?Erfc</code>         | $s, d$       | Computation of the complementary error function value of vector elements                             |
| <code>v?CdfNorm</code>      | $s, d$       | Computation of the cumulative normal distribution function value of vector elements                  |
| <code>v?ErfInv</code>       | $s, d$       | Computation of the inverse error function value of vector elements                                   |
| <code>v?ErfcInv</code>      | $s, d$       | Computation of the inverse complementary error function value of vector elements                     |
| <code>v?CdfNormInv</code>   | $s, d$       | Computation of the inverse cumulative normal distribution function value of vector elements          |
| <code>v?LGamma</code>       | $s, d$       | Computation of the natural logarithm for the absolute value of the gamma function of vector elements |
| <code>v?TGamma</code>       | $s, d$       | Computation of the gamma function of vector elements                                                 |
| <code>v?ExpInt1</code>      | $s, d$       | Computation of the exponential integral of vector elements                                           |
| <b>Rounding Functions</b>   |              |                                                                                                      |
| <code>v?Floor</code>        | $s, d$       | Rounding towards minus infinity                                                                      |
| <code>v?Ceil</code>         | $s, d$       | Rounding towards plus infinity                                                                       |
| <code>v?Trunc</code>        | $s, d$       | Rounding towards zero infinity                                                                       |
| <code>v?Round</code>        | $s, d$       | Rounding to nearest integer                                                                          |
| <code>v?NearbyInt</code>    | $s, d$       | Rounding according to current mode                                                                   |
| <code>v?Rint</code>         | $s, d$       | Rounding according to current mode and raising inexact result exception                              |
| <code>v?Modf</code>         | $s, d$       | Computation of the integer and fractional parts                                                      |
| <code>v?Frac</code>         | $s, d$       | Computation of the fractional part                                                                   |

## Special Value Notations

This section defines notations of special values for complex functions. The definitions are provided in text, tables, or formulas.

- $z, z1, z2$ , etc. denote complex numbers.
- $i, i^2=-1$  is the imaginary unit.
- $x, X, x1, x2$ , etc. denote real imaginary parts.
- $y, Y, y1, y2$ , etc. denote imaginary parts.
- $X$  and  $Y$  represent any finite positive IEEE-754 floating point values, if not stated otherwise.
- Quiet NaN and signaling NaN are denoted with `QNaN` and `SNAN`, respectively.

- The IEEE-754 positive infinities or floating-point numbers are denoted with a + sign before  $x$ ,  $y$ , etc.
- The IEEE-754 negative infinities or floating-point numbers are denoted with a – sign before  $x$ ,  $y$ , etc.

CONJ( $z$ ) and CIS( $z$ ) are defined as follows:

$$\text{CONJ}(x+i \cdot y)=x-i \cdot y$$

$$\text{CIS}(y)=\cos (y)+i \cdot \sin (y) .$$

The special value tables show the result of the function for the  $z$  argument at the intersection of the  $\text{RE}(z)$  column and the  $i \cdot \text{IM}(z)$  row. If the function raises an exception on the argument  $z$ , the lower part of this cell shows the raised exception and the VM Error Status. An empty cell indicates that this argument is normal and the result is defined mathematically.

## Arithmetic Functions

Arithmetic functions perform the basic mathematical operations like addition, subtraction, multiplication or computation of the absolute value of the vector elements.

### v?Add

*Performs element by element addition of vector  $a$  and vector  $b$ .*

#### Syntax

```
vsAdd(n, a, b, y);
vmsAdd(n, a, b, y, mode);
vdAdd(n, a, b, y);
vmdAdd(n, a, b, y, mode);
vcAdd(n, a, b, y);
vmcAdd(n, a, b, y, mode);
vzAdd(n, a, b, y);
vmzAdd(n, a, b, y, mode);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| Name   | Type                                                                                                                                                                                                                                                                                                                       | Description                                                     |
|--------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------|
| $n$    | <code>const MKL_INT</code>                                                                                                                                                                                                                                                                                                 | Specifies the number of elements to be calculated.              |
| $a, b$ | <code>const float*</code> for <code>vsAdd</code> , <code>vmsAdd</code><br><code>const double*</code> for <code>vdAdd</code> , <code>vmdAdd</code><br><code>const MKL_Complex8*</code> for <code>vcAdd</code> ,<br><code>vmcAdd</code><br><code>const MKL_Complex16*</code> for <code>vzAdd</code> ,<br><code>vmzAdd</code> | Pointers to arrays that contain the input vectors $a$ and $b$ . |

| Name        | Type            | Description                                                                                                                        |
|-------------|-----------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>mode</i> | const MKL_INT64 | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                         | Description                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsAdd, vmsAdd<br>double* for vdAdd, vmdAdd<br>MKL_Complex8* for vcAdd, vmcAdd<br>MKL_Complex16* for vzAdd, vmzAdd | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The *v?Add* function performs element by element addition of vector *a* and vector *b*.

### Special values for Real Function *v?Add(x)*

| Argument 1 | Argument 2 | Result | Exception |
|------------|------------|--------|-----------|
| +0         | +0         | +0     |           |
| +0         | -0         | +0     |           |
| -0         | +0         | +0     |           |
| -0         | -0         | -0     |           |
| +∞         | +∞         | +∞     |           |
| +∞         | -∞         | QNAN   | INVALID   |
| -∞         | +∞         | QNAN   | INVALID   |
| -∞         | -∞         | -∞     |           |
| SNAN       | any value  | QNAN   | INVALID   |
| any value  | SNAN       | QNAN   | INVALID   |
| QNAN       | non-SNAN   | QNAN   |           |
| non-SNAN   | QNAN       | QNAN   |           |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Add}(x1+i*y1, x2+i*y2) = (x1+x2) + i*(y1+y2)$$

Overflow in a complex function occurs (supported in the HA/LA accuracy modes only) when *x1*, *x2*, *y1*, *y2* are finite numbers, but the real or imaginary part of the computed result is so large that it does not fit the target precision. In this case, the function returns ∞ in that part of the result, raises the OVERFLOW exception, and sets the VM Error Status to VML\_STATUS\_OVERFLOW (overriding any possible VML\_STATUS\_ACCURACYWARNING status).

## *v?Sub*

*Performs element by element subtraction of vector b from vector a.*

### Syntax

```
vsSub(n, a, b, y);
```

```
vmsSub(n, a, b, y, mode);
```

```
vdSub(n, a, b, y);
```

```
vmdSub(n, a, b, y, mode);
```

```
vcSub(n, a, b, y);
vmcSub(n, a, b, y, mode);
vzSub(n, a, b, y);
vmzSub(n, a, b, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                                                                                 | Description                                                                                                                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                        | Specifies the number of elements to be calculated.                                                                                 |
| <i>a, b</i> | const float* for vsSub, vmsSub<br>const double* for vdSub, vmdSub<br>const MKL_Complex8* for vcSub, vmcSub<br>const MKL_Complex16* for vzSub, vmzSub | Pointers to arrays that contain the input vectors <i>a</i> and <i>b</i> .                                                          |
| <i>mode</i> | const MKL_INT64                                                                                                                                      | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                         | Description                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsSub, vmsSub<br>double* for vdSub, vmdSub<br>MKL_Complex8* for vcSub, vmcSub<br>MKL_Complex16* for vzSub, vmzSub | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?Sub` function performs element by element subtraction of vector *b* from vector *a*.

### Special values for Real Function v?Sub(x)

| Argument 1 | Argument 2 | Result | Exception |
|------------|------------|--------|-----------|
| +0         | +0         | +0     |           |
| +0         | -0         | +0     |           |
| -0         | +0         | -0     |           |
| -0         | -0         | +0     |           |
| +∞         | +∞         | QNAN   | INVALID   |
| +∞         | -∞         | +∞     |           |
| -∞         | +∞         | -∞     |           |
| -∞         | -∞         | QNAN   | INVALID   |
| SNAN       | any value  | QNAN   | INVALID   |
| any value  | SNAN       | QNAN   | INVALID   |

| Argument 1 | Argument 2 | Result | Exception |
|------------|------------|--------|-----------|
| QNAN       | non-SNAN   | QNAN   |           |
| non-SNAN   | QNAN       | QNAN   |           |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Sub}(x1+i*y1, x2+i*y2) = (x1-x2) + i*(y1-y2).$$

Overflow in a complex function occurs (supported in the HA/LA accuracy modes only) when  $x1$ ,  $x2$ ,  $y1$ ,  $y2$  are finite numbers, but the real or imaginary part of the computed result is so large that it does not fit the target precision. In this case, the function returns  $\infty$  in that part of the result, raises the `OVERFLOW` exception, and sets the VM Error Status to `VML_STATUS_OVERFLOW` (overriding any possible `VML_STATUS_ACCURACYWARNING` status).

## v?Sqr

*Performs element by element squaring of the vector.*

### Syntax

```
vsSqr(n, a, y);
vmsSqr(n, a, y, mode);
vdSqr(n, a, y);
vmdSqr(n, a, y, mode);
```

### Include Files

- `mk1.h`

### Input Parameters

| Name              | Type                                                                                                                                                        | Description                                                                                                                        |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                  | Specifies the number of elements to be calculated.                                                                                 |
| <code>a</code>    | <code>const float*</code> for <code>vsSqr</code> ,<br><code>vmsSqr</code><br><br><code>const double*</code> for <code>vdSqr</code> ,<br><code>vmdSqr</code> | Pointer to an array that contains the input vector <code>a</code> .                                                                |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Output Parameters

| Name           | Type                                                                                                                                      | Description                                                          |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsSqr</code> , <code>vmsSqr</code><br><br><code>double*</code> for <code>vdSqr</code> , <code>vmdSqr</code> | Pointer to an array that contains the output vector <code>y</code> . |

### Description

The `v?Sqr` function performs element by element squaring of the vector.

## Special Values for Real Function v?Sqr(x)

| Argument | Result | Exception |
|----------|--------|-----------|
| +0       | +0     |           |
| -0       | +0     |           |
| +∞       | +∞     |           |
| -∞       | +∞     |           |
| QNAN     | QNAN   |           |
| SNAN     | QNAN   | INVALID   |

### v?Mul

Performs element by element multiplication of vector *a* and vector *b*.

### Syntax

```
vsMul(n, a, b, y);
vmsMul(n, a, b, y, mode);
vdMul(n, a, b, y);
vmdMul(n, a, b, y, mode);
vcMul(n, a, b, y);
vmcMul(n, a, b, y, mode);
vzMul(n, a, b, y);
vmzMul(n, a, b, y, mode);
```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type                                                                                                                                                 | Description                                                                                                                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                        | Specifies the number of elements to be calculated.                                                                                 |
| <i>a, b</i> | const float* for vsMul, vmsMul<br>const double* for vdMul, vmdMul<br>const MKL_Complex8* for vcMul, vmcMul<br>const MKL_Complex16* for vzMul, vmzMul | Pointers to arrays that contain the input vectors <i>a</i> and <i>b</i> .                                                          |
| <i>mode</i> | const MKL_INT64                                                                                                                                      | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name | Type                                                                                                                         | Description                                               |
|------|------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|
| $y$  | float* for vsMul, vmsMul<br>double* for vdMul, vmdMul<br>MKL_Complex8* for vcMul, vmcMul<br>MKL_Complex16* for vzMul, vmzMul | Pointer to an array that contains the output vector $y$ . |

## Description

The  $v?Mul$  function performs element by element multiplication of vector  $a$  and vector  $b$ .

### Special values for Real Function $v?Mul(x)$

| Argument 1 | Argument 2 | Result    | Exception |
|------------|------------|-----------|-----------|
| +0         | +0         | +0        |           |
| +0         | -0         | -0        |           |
| -0         | +0         | -0        |           |
| -0         | -0         | +0        |           |
| +0         | $+\infty$  | QNAN      | INVALID   |
| +0         | $-\infty$  | QNAN      | INVALID   |
| -0         | $+\infty$  | QNAN      | INVALID   |
| -0         | $-\infty$  | QNAN      | INVALID   |
| $+\infty$  | +0         | QNAN      | INVALID   |
| $+\infty$  | -0         | QNAN      | INVALID   |
| $-\infty$  | +0         | QNAN      | INVALID   |
| $-\infty$  | -0         | QNAN      | INVALID   |
| $+\infty$  | $+\infty$  | $+\infty$ |           |
| $+\infty$  | $-\infty$  | $-\infty$ |           |
| $-\infty$  | $+\infty$  | $-\infty$ |           |
| $-\infty$  | $-\infty$  | $+\infty$ |           |
| SNAN       | any value  | QNAN      | INVALID   |
| any value  | SNAN       | QNAN      | INVALID   |
| QNAN       | non-SNAN   | QNAN      |           |
| non-SNAN   | QNAN       | QNAN      |           |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Mul}(x_1+i*y_1, x_2+i*y_2) = (x_1*x_2-y_1*y_2) + i*(x_1*y_2+y_1*x_2).$$

Overflow in a complex function occurs (supported in the HA/LA accuracy modes only) when  $x_1$ ,  $x_2$ ,  $y_1$ ,  $y_2$  are finite numbers, but the real or imaginary part of the computed result is so large that it does not fit the target precision. In this case, the function returns  $\infty$  in that part of the result, raises the `OVERFLOW` exception, and sets the VM Error Status to `VML_STATUS_OVERFLOW` (overriding any possible `VML_STATUS_ACCURACYWARNING` status).

## $v?MulByConj$

*Performs element by element multiplication of vector  $a$  element and conjugated vector  $b$  element.*

### Syntax

```
vcMulByConj(n, a, b, y);
```

```
vmcMulByConj(n, a, b, y, mode);
vzMulByConj(n, a, b, y);
vmzMulByConj(n, a, b, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                                              | Description                                                                                                                        |
|-------------|-------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                     | Specifies the number of elements to be calculated.                                                                                 |
| <i>a, b</i> | const MKL_Complex8* for<br>vcMulByConj, vmcMulByConj<br><br>const MKL_Complex16* for<br>vzMulByConj, vmzMulByConj | Pointers to arrays that contain the input vectors <i>a</i> and <i>b</i> .                                                          |
| <i>mode</i> | const MKL_INT64                                                                                                   | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                  | Description                                                    |
|----------|-------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | MKL_Complex8* for vcMulByConj,<br>vmcMulByConj<br><br>MKL_Complex16* for vzMulByConj,<br>vmzMulByConj | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?MulByConj` function performs element by element multiplication of vector *a* element and conjugated vector *b* element.

Specifications for special values of the functions are found according to the formula

$$\text{MulByConj}(x1+i*y1, x2+i*y2) = \text{Mul}(x1+i*y1, x2-i*y2).$$

Overflow in a complex function occurs (supported in the HA/LA accuracy modes only) when *x1*, *x2*, *y1*, *y2* are finite numbers, but the real or imaginary part of the computed result is so large that it does not fit the target precision. In this case, the function returns  $\infty$  in that part of the result, raises the `OVERFLOW` exception, and sets the VM Error Status to `VML_STATUS_OVERFLOW` (overriding any possible `VML_STATUS_ACCURACYWARNING` status).

## v?Conj

*Performs element by element conjugation of the vector.*

---

## Syntax

```
vcConj(n, a, y);
vmcConj(n, a, y, mode);
vzConj(n, a, y);
```

```
vmzConj(n, a, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                | Description                                                                                                                        |
|-------------|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                       | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const MKL_Complex8* for vcConj, vmcConj<br>const MKL_Complex16* for vzConj, vmzConj | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                     | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                    | Description                                                    |
|----------|-------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | MKL_Complex8* for vcConj, vmcConj<br>MKL_Complex16* for vzConj, vmzConj | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?Conj` function performs element by element conjugation of the vector.

No special values are specified. The function does not raise floating-point exceptions.

## v?Abs

*Computes absolute value of vector elements.*

### Syntax

```
vsAbs(n, a, y);
```

```
vmsAbs(n, a, y, mode);
```

```
vdAbs(n, a, y);
```

```
vmdAbs(n, a, y, mode);
```

```
vcAbs(n, a, y);
```

```
vmcAbs(n, a, y, mode);
```

```
vzAbs(n, a, y);
```

```
vmzAbs(n, a, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                                                                                 | Description                                                                                                                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                        | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsAbs, vmsAbs<br>const double* for vdAbs, vmdAbs<br>const MKL_Complex8* for vcAbs, vmcAbs<br>const MKL_Complex16* for vzAbs, vmzAbs | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                      | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                | Description                                                    |
|----------|-------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsAbs, vmsAbs, vcAbs, vmcAbs<br>double* for vdAbs, vmdAbs, vzAbs, vmzAbs | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The *v?Abs* function computes an absolute value of vector elements.

### Special Values for Real Function *v?Abs(x)*

| Argument | Result | Exception |
|----------|--------|-----------|
| +0       | +0     |           |
| -0       | +0     |           |
| +∞       | +∞     |           |
| -∞       | +∞     |           |
| QNAN     | QNAN   |           |
| SNAN     | QNAN   | INVALID   |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Abs}(z) = \text{Hypot}(\text{RE}(z), \text{IM}(z)).$$

### *v?Arg*

*Computes argument of vector elements.*

### Syntax

```
vcArg(n, a, y);
```

```
vmcArg(n, a, y, mode);
```

```
vzArg(n, a, y);
```

```
vmzArg(n, a, y, mode);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name              | Type                                                                                                                                                                      | Description                                                                                                                     |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                                | Specifies the number of elements to be calculated.                                                                              |
| <code>a</code>    | <code>const MKL_Complex8*</code> for <code>vcArg</code> ,<br><code>vmcArg</code><br><br><code>const MKL_Complex16*</code> for <code>vzArg</code> ,<br><code>vmcArg</code> | Pointer to an array that contains the input vector <code>a</code> .                                                             |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                              | Overrides global VM mode setting for this function call. See <code>vm1SetMode</code> for possible values and their description. |

## Output Parameters

| Name           | Type                                                                                                                                      | Description                                                          |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vcArg</code> , <code>vmcArg</code><br><br><code>double*</code> for <code>vzArg</code> , <code>vmcArg</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The `v?Arg` function computes argument of vector elements.

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function `v?Arg(z)`

| <b>RE(z)</b><br><b>i·IM(z)</b><br><b>)</b> | <b>-∞</b>           | <b>-X</b>         | <b>-0</b>         | <b>+0</b>         | <b>+X</b>         | <b>+∞</b>         | <b>NAN</b> |
|--------------------------------------------|---------------------|-------------------|-------------------|-------------------|-------------------|-------------------|------------|
| <code>+i·∞</code>                          | <code>+3·π/4</code> | <code>+π/2</code> | <code>+π/2</code> | <code>+π/2</code> | <code>+π/2</code> | <code>+π/4</code> | NAN        |
| <code>+i·Y</code>                          | <code>+π</code>     |                   | <code>+π/2</code> | <code>+π/2</code> |                   | <code>+0</code>   | NAN        |
| <code>+i·0</code>                          | <code>+π</code>     | <code>+π</code>   | <code>+π</code>   | <code>+0</code>   | <code>+0</code>   | <code>+0</code>   | NAN        |
| <code>-i·0</code>                          | <code>-π</code>     | <code>-π</code>   | <code>-π</code>   | <code>-0</code>   | <code>-0</code>   | <code>-0</code>   | NAN        |
| <code>-i·Y</code>                          | <code>-π</code>     |                   | <code>-π/2</code> | <code>-π/2</code> |                   | <code>-0</code>   | NAN        |
| <code>-i·∞</code>                          | <code>-3·π/4</code> | <code>-π/2</code> | <code>-π/2</code> | <code>-π/2</code> | <code>-π/2</code> | <code>-π/4</code> | NAN        |
| <code>+i·NAN</code>                        | NAN                 | NAN               | NAN               | NAN               | NAN               | NAN               | NAN        |

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- $\text{Arg}(z) = \text{Atan2}(\text{IM}(z), \text{RE}(z))$ .

## `v?LinearFrac`

Performs linear fraction transformation of vectors `a` and `b` with scalar parameters.

## Syntax

```
vsLinearFrac(n, a, b, scalea, shifta, scaleb, shiftb, y);
vmsLinearFrac(n, a, b, scalea, shifta, scaleb, shiftb, y, mode);
vdLinearFrac(n, a, b, scalea, shifta, scaleb, shiftb, y)
vmdLinearFrac(n, a, b, scalea, shifta, scaleb, shiftb, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name                  | Type                                                                                              | Description                                                                                                                        |
|-----------------------|---------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>              | const MKL_INT                                                                                     | Specifies the number of elements to be calculated.                                                                                 |
| <i>a, b</i>           | const float* for vsLinearFrac, vmsLinearFrac<br><br>const double* for vdLinearFrac, vmdLinearFrac | Pointers to arrays that contain the input vectors <i>a</i> and <i>b</i> .                                                          |
| <i>scalea, scaleb</i> | const float for vsLinearFrac, vmsLinearFrac<br><br>const double for vdLinearFrac, vmdLinearFrac   | Constant values for scaling multipliers of vectors <i>a</i> and <i>b</i> .                                                         |
| <i>shifta, shiftb</i> | const float for vsLinearFrac, vmsLinearFrac<br><br>const double for vdLinearFrac, vmdLinearFrac   | Constant values for shifting addends of vectors <i>a</i> and <i>b</i> .                                                            |
| <i>mode</i>           | const MKL_INT64                                                                                   | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                  | Description                                                    |
|----------|---------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsLinearFrac, vmsLinearFrac<br><br>double* for vdLinearFrac, vmdLinearFrac | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?LinearFrac` function performs a linear fraction transformation of vector *a* by vector *b* with scalar parameters: scaling multipliers *scalea*, *scaleb* and shifting addends *shifta*, *shiftb*:

$$y[i] = (scalea \cdot a[i] + shifta) / (scaleb \cdot b[i] + shiftb), i=1,2 \dots n$$

The `v?LinearFrac` function is implemented in the EP accuracy mode only, therefore no special values are defined for this function. If used in HA or LA mode, `v?LinearFrac` sets the VM Error Status to `VML_STATUS_ACCURACYWARNING` (see the [Values of the VM Status](#) table). Correctness is guaranteed within the threshold limitations defined for each input parameter (see the table below); otherwise, the behavior is unspecified.

---

#### Threshold Limitations on Input Parameters

---

$$2^{E_{\text{MIN}}/2} \leq |scalea| \leq 2^{(E_{\text{MAX}}-2)/2}$$

$$2^{E_{\text{MIN}}/2} \leq |scaleb| \leq 2^{(E_{\text{MAX}}-2)/2}$$

$$|shiffta| \leq 2^{E_{\text{MAX}}-2}$$

$$|shifftb| \leq 2^{E_{\text{MAX}}-2}$$

$$2^{E_{\text{MIN}}/2} \leq a[i] \leq 2^{(E_{\text{MAX}}-2)/2}$$

$$2^{E_{\text{MIN}}/2} \leq b[i] \leq 2^{(E_{\text{MAX}}-2)/2}$$

$$a[i] \neq - (shiffta/scalea) * (1-\delta_1), \quad |\delta_1| \leq 2^{1-(p-1)/2}$$

$$b[i] \neq - (shifftb/scaleb) * (1-\delta_2), \quad |\delta_2| \leq 2^{1-(p-1)/2}$$


---

$E_{\text{MIN}}$  and  $E_{\text{MAX}}$  are the minimum and maximum exponents and  $p$  is the number of significant bits (precision) for the corresponding data type according to the ANSI/IEEE Standard 754-2008 ([IEEE754]):

- for single precision  $E_{\text{MIN}} = -126$ ,  $E_{\text{MAX}} = 127$ ,  $p = 24$
- for double precision  $E_{\text{MIN}} = -1022$ ,  $E_{\text{MAX}} = 1023$ ,  $p = 53$

The thresholds become less strict for common cases with  $scalea=0$  and/or  $scaleb=0$ :

- if  $scalea=0$ , there are no limitations for the values of  $a[i]$  and  $shiffta$ .
- if  $scaleb=0$ , there are no limitations for the values of  $b[i]$  and  $shifftb$ .

### Example

To use the `v?LinearFrac` to shift vector  $a$  by a scalar value, set  $scaleb$  to 0. Note that even if  $scaleb$  is 0,  $b$  must be declared.

```
#include <stdio.h>
#include "mkl_vml.h"

int main()
{
 double a[10], *b;
 double r[10];
 double scalea = 1.0, scaleb = 0.0;
 double shiffta = -1.0, shifftb = 1.0;

 MKL_INT i=0,n=10;

 a[0]=-10000.0000;
 a[1]=-7777.7777;
 a[2]=-5555.5555;
 a[3]=-3333.3333;
 a[4]=-1111.1111;
 a[5]=1111.1111;
 a[6]=3333.3333;
 a[7]=5555.5555;
 a[8]=7777.7777;
 a[9]=10000.0000;
```

```

vdLinearFrac(n, a, b, scalea, shifta, scaleb, shiftb, r);

for(i=0;i<10;i++) {
 printf("%25.14f %25.14f\n",a[i],r[i]);
}

return 0;
}

```

To use the `v?LinearFrac` to compute  $shifta/(scaleb \cdot b[i] + shiftb)$ , set `scalea` to 0. Note that even if `scalea` is 0, `a` must be declared.

## Power and Root Functions

### v?Inv

*Performs element by element inversion of the vector.*

#### Syntax

```

vsInv(n, a, y);
vmsInv(n, a, y, mode);
vdInv(n, a, y);
vmdInv(n, a, y, mode);

```

#### Include Files

- `mkl.h`

#### Input Parameters

| Name              | Type                                                                                                                                                        | Description                                                                                                                        |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                  | Specifies the number of elements to be calculated.                                                                                 |
| <code>a</code>    | <code>const float*</code> for <code>vsInv</code> ,<br><code>vmsInv</code><br><br><code>const double*</code> for <code>vdInv</code> ,<br><code>vmdInv</code> | Pointer to an array that contains the input vector <code>a</code> .                                                                |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

#### Output Parameters

| Name           | Type                                                                                                                                      | Description                                                          |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsInv</code> , <code>vmsInv</code><br><br><code>double*</code> for <code>vdInv</code> , <code>vmdInv</code> | Pointer to an array that contains the output vector <code>y</code> . |

#### Description

The `v?Inv` function performs element by element inversion of the vector.

**Special Values for Real Function v?Inv(x)**

| Argument  | Result    | VM Error Status | Exception  |
|-----------|-----------|-----------------|------------|
| +0        | $+\infty$ | VML_STATUS_SING | ZERODIVIDE |
| -0        | $-\infty$ | VML_STATUS_SING | ZERODIVIDE |
| $+\infty$ | +0        |                 |            |
| $-\infty$ | -0        |                 |            |
| QNAN      | QNAN      |                 |            |
| SNAN      | QNAN      |                 | INVALID    |

**v?Div**

Performs element by element division of vector *a* by vector *b*

**Syntax**

```
vsDiv(n, a, b, y);
vmsDiv(n, a, b, y, mode);
vdDiv(n, a, b, y);
vmdDiv(n, a, b, y, mode);
vcDiv(n, a, b, y);
vmcDiv(n, a, b, y, mode);
vzDiv(n, a, b, y);
vmzDiv(n, a, b, y, mode);
```

**Include Files**

- mkl.h

**Input Parameters**

| Name        | Type                                                                                                                                                 | Description                                                                                                                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                        | Specifies the number of elements to be calculated.                                                                                 |
| <i>a, b</i> | const float* for vsDiv, vmsDiv<br>const double* for vdDiv, vmdDiv<br>const MKL_Complex8* for vcDiv, vmcDiv<br>const MKL_Complex16* for vzDiv, vmzDiv | Pointers to arrays that contain the input vectors <i>a</i> and <i>b</i> .                                                          |
| <i>mode</i> | const MKL_INT64                                                                                                                                      | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

**Precision Overflow Thresholds for Real v?Div Function**

| Data Type        | Threshold Limitations on Input Parameters               |
|------------------|---------------------------------------------------------|
| single precision | $\text{abs}(a[i]) < \text{abs}(b[i]) * \text{FLT\_MAX}$ |
| double precision | $\text{abs}(a[i]) < \text{abs}(b[i]) * \text{DBL\_MAX}$ |

Precision overflow thresholds for the complex `v?Div` function are beyond the scope of this document.

## Output Parameters

| Name           | Type                                                                                                                                                                                                                                                                                         | Description                                                          |
|----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsDiv</code> , <code>vmsDiv</code><br><code>double*</code> for <code>vdDiv</code> , <code>vmdDiv</code><br><code>MKL_Complex8*</code> for <code>vcDiv</code> , <code>vmcDiv</code><br><code>MKL_Complex16*</code> for <code>vzDiv</code> , <code>vmzDiv</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The `v?Div` function performs element by element division of vector `a` by vector `b`.

### Special values for Real Function `v?Div(x)`

| Argument 1             | Argument 2        | Result            | VM Error Status              | Exception               |
|------------------------|-------------------|-------------------|------------------------------|-------------------------|
| <code>X &gt; +0</code> | <code>+0</code>   | <code>+∞</code>   | <code>VML_STATUS_SING</code> | <code>ZERODIVIDE</code> |
| <code>X &gt; +0</code> | <code>-0</code>   | <code>-∞</code>   | <code>VML_STATUS_SING</code> | <code>ZERODIVIDE</code> |
| <code>X &lt; +0</code> | <code>+0</code>   | <code>-∞</code>   | <code>VML_STATUS_SING</code> | <code>ZERODIVIDE</code> |
| <code>X &lt; +0</code> | <code>-0</code>   | <code>+∞</code>   | <code>VML_STATUS_SING</code> | <code>ZERODIVIDE</code> |
| <code>+0</code>        | <code>+0</code>   | <code>QNAN</code> | <code>VML_STATUS_SING</code> |                         |
| <code>-0</code>        | <code>-0</code>   | <code>QNAN</code> | <code>VML_STATUS_SING</code> |                         |
| <code>X &gt; +0</code> | <code>+∞</code>   | <code>+0</code>   |                              |                         |
| <code>X &gt; +0</code> | <code>-∞</code>   | <code>-0</code>   |                              |                         |
| <code>+∞</code>        | <code>+∞</code>   | <code>QNAN</code> |                              |                         |
| <code>-∞</code>        | <code>-∞</code>   | <code>QNAN</code> |                              |                         |
| <code>QNAN</code>      | <code>QNAN</code> | <code>QNAN</code> |                              |                         |
| <code>SNAN</code>      | <code>SNAN</code> | <code>QNAN</code> |                              | <code>INVALID</code>    |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Div}(x_1+i*y_1, x_2+i*y_2) = (x_1+i*y_1) * (x_2-i*y_2) / (x_2*x_2+y_2*y_2).$$

Overflow in a complex function occurs when `x2+i*y2` is not zero, `x1`, `x2`, `y1`, `y2` are finite numbers, but the real or imaginary part of the exact result is so large that it does not fit the target precision. In that case, the function returns `∞` in that part of the result, raises the `OVERFLOW` exception, and sets the VM Error Status to `VML_STATUS_OVERFLOW`.

## `v?Sqrt`

*Computes a square root of vector elements.*

### Syntax

```
vsSqrt(n, a, y);
vmsSqrt(n, a, y, mode);
vdSqrt(n, a, y);
vmdSqrt(n, a, y, mode);
vcSqrt(n, a, y);
vmcSqrt(n, a, y, mode);
vzSqrt(n, a, y);
vmzSqrt(n, a, y, mode);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name   | Type                                                                                                                                                                                                                                                                                                                         | Description                                                                                                                     |
|--------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| $n$    | <code>const MKL_INT</code>                                                                                                                                                                                                                                                                                                   | Specifies the number of elements to be calculated.                                                                              |
| $a$    | <code>const float*</code> for <code>vsSqrt</code> , <code>vmsSqrt</code><br><code>const double*</code> for <code>vdSqrt</code> , <code>vmdSqrt</code><br><code>const MKL_Complex8*</code> for <code>vcSqrt</code> , <code>vmcSqrt</code><br><code>const MKL_Complex16*</code> for <code>vzSqrt</code> , <code>vmzSqrt</code> | Pointer to an array that contains the input vector $a$ .                                                                        |
| $mode$ | <code>const MKL_INT64</code>                                                                                                                                                                                                                                                                                                 | Overrides global VM mode setting for this function call. See <code>vm1SetMode</code> for possible values and their description. |

## Output Parameters

| Name | Type                                                                                                                                                                                                                                                                                                 | Description                                               |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|
| $y$  | <code>float*</code> for <code>vsSqrt</code> , <code>vmsSqrt</code><br><code>double*</code> for <code>vdSqrt</code> , <code>vmdSqrt</code><br><code>MKL_Complex8*</code> for <code>vcSqrt</code> , <code>vmcSqrt</code><br><code>MKL_Complex16*</code> for <code>vzSqrt</code> , <code>vmzSqrt</code> | Pointer to an array that contains the output vector $y$ . |

## Description

The `v?Sqrt` function computes a square root of vector elements.

### Special Values for Real Function `v?Sqrt(x)`

| Argument  | Result    | VM Error Status   | Exception |
|-----------|-----------|-------------------|-----------|
| $X < +0$  | QNAN      | VML_STATUS_ERRDOM | INVALID   |
| $+0$      | $+0$      |                   |           |
| $-0$      | $-0$      |                   |           |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID   |
| $+\infty$ | $+\infty$ |                   |           |
| QNAN      | QNAN      |                   |           |
| SNAN      | QNAN      |                   | INVALID   |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function `v?Sqrt(z)`

| $\text{RE}(z)$<br>$i \cdot \text{IM}(z)$ | $-\infty$                | $-X$                     | $-0$                     | $+0$                     | $+X$                     | $+\infty$                | NAN                      |
|------------------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $+i \cdot \infty$                        | $+\infty+i \cdot \infty$ | $+\infty+i \cdot \infty$ | $+\infty+i \cdot \infty$ | $+\infty+i \cdot \infty$ | $+\infty+i \cdot \infty$ | $+\infty+i \cdot \infty$ | $+\infty+i \cdot \infty$ |

| <b>RE(z)</b><br><b>i·IM(z)</b> | <b>-∞</b>   | <b>-X</b>   | <b>-0</b>   | <b>+0</b>   | <b>+X</b>   | <b>+∞</b> | <b>NAN</b>  |
|--------------------------------|-------------|-------------|-------------|-------------|-------------|-----------|-------------|
| +i·Y                           | +0+i·∞      |             |             |             |             | +∞+i·0    | QNAN+i·QNAN |
| +i·0                           | +0+i·∞      |             | +0+i·0      | +0+i·0      |             | +∞+i·0    | QNAN+i·QNAN |
| -i·0                           | +0-i·∞      |             | +0-i·0      | +0-i·0      |             | +∞-i·0    | QNAN+i·QNAN |
| -i·Y                           | +0-i·∞      |             |             |             |             | +∞-i·0    | QNAN+i·QNAN |
| -i·∞                           | +∞-i·∞      | +∞-i·∞      | +∞-i·∞      | +∞-i·∞      | +∞-i·∞      | +∞-i·∞    | +∞-i·∞      |
| +i·NAN                         | QNAN+i·QNAN | QNAN+i·QNAN | QNAN+i·QNAN | QNAN+i·QNAN | QNAN+i·QNAN | +∞+i·QNAN | QNAN+i·QNAN |

Notes:

- raises `INVALID` exception when the real or imaginary part of the argument is `SNAN`
- $\text{Sqrt}(\text{CONJ}(z)) = \text{CONJ}(\text{Sqrt}(z))$ .

## v?InvSqrt

*Computes an inverse square root of vector elements.*

### Syntax

```
vsInvSqrt(n, a, y);
vmsInvSqrt(n, a, y, mode);
vdInvSqrt(n, a, y);
vmdInvSqrt(n, a, y, mode);
```

### Include Files

- `mk1.h`

### Input Parameters

| <b>Name</b>       | <b>Type</b>                                                                                                                                                                 | <b>Description</b>                                                                                                                 |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                                  | Specifies the number of elements to be calculated.                                                                                 |
| <code>a</code>    | <code>const float*</code> for <code>vsInvSqrt</code> ,<br><code>vmsInvSqrt</code><br><br><code>const double*</code> for <code>vdInvSqrt</code> ,<br><code>vmdInvSqrt</code> | Pointer to an array that contains the input vector <code>a</code> .                                                                |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                                | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

### Output Parameters

| <b>Name</b>    | <b>Type</b>                                                                                                                                           | <b>Description</b>                                                   |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsInvSqrt</code> , <code>vmsInvSqrt</code><br><code>double*</code> for <code>vdInvSqrt</code> , <code>vmdInvSqrt</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The `v?InvSqrt` function computes an inverse square root of vector elements.

### Special Values for Real Function `v?InvSqrt(x)`

| Argument  | Result    | VM Error Status   | Exception  |
|-----------|-----------|-------------------|------------|
| $X < +0$  | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $+0$      | $+\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| $-0$      | $-\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $+\infty$ | $+0$      |                   |            |
| QNAN      | QNAN      |                   |            |
| SNAN      | QNAN      |                   | INVALID    |

## `v?Cbrt`

*Computes a cube root of vector elements.*

### Syntax

```
vsCbrt(n, a, y);
```

```
vmsCbrt(n, a, y, mode);
```

```
vdCbrt(n, a, y);
```

```
vmdCbrt(n, a, y, mode);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name              | Type                                                                                                                                                  | Description                                                                                                                        |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                            | Specifies the number of elements to be calculated.                                                                                 |
| <code>a</code>    | <code>const float*</code> for <code>vsCbrt</code> , <code>vmsCbrt</code><br><code>const double*</code> for <code>vdCbrt</code> , <code>vmdCbrt</code> | Pointer to an array that contains the input vector <code>a</code> .                                                                |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                          | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Output Parameters

| Name           | Type                                                                                                                                      | Description                                                          |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsCbrt</code> , <code>vmsCbrt</code><br><code>double*</code> for <code>vdCbrt</code> , <code>vmdCbrt</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The `v?Cbrt` function computes a cube root of vector elements.

## Special Values for Real Function $v?Cbrt(x)$

| Argument  | Result    | Exception |
|-----------|-----------|-----------|
| +0        | +0        |           |
| -0        | -0        |           |
| $+\infty$ | $+\infty$ |           |
| $-\infty$ | $-\infty$ |           |
| QNAN      | QNAN      |           |
| SNAN      | QNAN      | INVALID   |

## $v?InvCbrt$

Computes an inverse cube root of vector elements.

### Syntax

```
vsInvCbrt(n, a, y);
vmsInvCbrt(n, a, y, mode);
vdInvCbrt(n, a, y);
vmdInvCbrt(n, a, y, mode);
```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type                                                                              | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                     | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsInvCbrt, vmsInvCbrt<br>const double* for vdInvCbrt, vmdInvCbrt | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                   | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Output Parameters

| Name     | Type                                                                  | Description                                                    |
|----------|-----------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsInvCbrt, vmsInvCbrt<br>double* for vdInvCbrt, vmdInvCbrt | Pointer to an array that contains the output vector <i>y</i> . |

### Description

The  $v?InvCbrt$  function computes an inverse cube root of vector elements.

## Special Values for Real Function $v?InvCbrt(x)$

| Argument | Result    | VM Error Status | Exception  |
|----------|-----------|-----------------|------------|
| +0       | $+\infty$ | VML_STATUS_SING | ZERODIVIDE |

| Argument  | Result    | VM Error Status | Exception  |
|-----------|-----------|-----------------|------------|
| -0        | $-\infty$ | VML_STATUS_SING | ZERODIVIDE |
| $+\infty$ | +0        |                 |            |
| $-\infty$ | -0        |                 |            |
| QNAN      | QNAN      |                 |            |
| SNAN      | QNAN      |                 | INVALID    |

## v?Pow2o3

Raises each element of a vector to the constant power  $2/3$ .

### Syntax

```
vsPow2o3(n, a, y);
```

```
vmsPow2o3(n, a, y, mode);
```

```
vdPow2o3(n, a, y);
```

```
vmdPow2o3(n, a, y, mode);
```

### Include Files

- `mk1.h`

### Input Parameters

| Name        | Type                                                                                                                                                                    | Description                                                                                                                        |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | <code>const MKL_INT</code>                                                                                                                                              | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | <code>const float*</code> for <code>vsPow2o3</code> ,<br><code>vmsPow2o3</code><br><br><code>const double*</code> for <code>vdPow2o3</code> ,<br><code>vmdPow2o3</code> | Pointers to arrays that contain the input vector <i>a</i> .                                                                        |
| <i>mode</i> | <code>const MKL_INT64</code>                                                                                                                                            | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Output Parameters

| Name     | Type                                                                                                                                                  | Description                                                    |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | <code>float*</code> for <code>vsPow2o3</code> , <code>vmsPow2o3</code><br><br><code>double*</code> for <code>vdPow2o3</code> , <code>vmdPow2o3</code> | Pointer to an array that contains the output vector <i>y</i> . |

### Description

The `v?Pow2o3` function raises each element of a vector to the constant power  $2/3$ .

### Special Values for Real Function v?Pow2o3(x)

| Argument  | Result    | Exception |
|-----------|-----------|-----------|
| +0        | +0        |           |
| -0        | +0        |           |
| $+\infty$ | $+\infty$ |           |

| Argument  | Result    | Exception |
|-----------|-----------|-----------|
| $-\infty$ | $+\infty$ |           |
| QNAN      | QNAN      |           |
| SNAN      | QNAN      | INVALID   |

## v?Pow3o2

Raises each element of a vector to the constant power  $3/2$ .

### Syntax

```
vsPow3o2(n, a, y);
vmsPow3o2(n, a, y, mode);
vdPow3o2(n, a, y);
vmdPow3o2(n, a, y, mode);
```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type                                                                                    | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                           | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsPow3o2,<br>vmsPow3o2<br><br>const double* for vdPow3o2,<br>vmdPow3o2 | Pointers to arrays that contain the input vector <i>a</i> .                                                                        |
| <i>mode</i> | const MKL_INT64                                                                         | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

### Precision Overflow Thresholds for Pow3o2 Function

| Data Type        | Threshold Limitations on Input Parameters    |
|------------------|----------------------------------------------|
| single precision | $\text{abs}(a[i]) < (\text{FLT\_MAX})^{2/3}$ |
| double precision | $\text{abs}(a[i]) < (\text{DBL\_MAX})^{2/3}$ |

### Output Parameters

| Name     | Type                                                                  | Description                                                    |
|----------|-----------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsPow3o2, vmsPow3o2<br><br>double* for vdPow3o2, vmdPow3o2 | Pointer to an array that contains the output vector <i>y</i> . |

### Description

The v?Pow3o2 function raises each element of a vector to the constant power  $3/2$ .

**Special Values for Real Function v?Pow3o2(x)**

| Argument  | Result    | VM Error Status   | Exception |
|-----------|-----------|-------------------|-----------|
| $X < +0$  | QNAN      | VML_STATUS_ERRDOM | INVALID   |
| +0        | +0        |                   |           |
| -0        | -0        |                   |           |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID   |
| $+\infty$ | $+\infty$ |                   |           |
| QNAN      | QNAN      |                   |           |
| SNAN      | QNAN      |                   | INVALID   |

**v?Pow**

Computes  $a$  to the power  $b$  for elements of two vectors.

**Syntax**

```
vsPow(n, a, b, y);
vmsPow(n, a, b, y, mode);
vdPow(n, a, b, y);
vmdPow(n, a, b, y, mode);
vcPow(n, a, b, y);
vmcPow(n, a, b, y, mode);
vzPow(n, a, b, y);
vmzPow(n, a, b, y, mode);
```

**Include Files**

- `mk1.h`

**Input Parameters**

| Name        | Type                                                                                                                                                                                                                                                                                                                           | Description                                                                                                                     |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | <code>const MKL_INT</code>                                                                                                                                                                                                                                                                                                     | Specifies the number of elements to be calculated.                                                                              |
| <i>a, b</i> | <code>const float*</code> for <code>vsPow</code> , <code>vmsPow</code><br><code>const double*</code> for <code>vdPow</code> , <code>vmdPow</code><br><code>const MKL_Complex8*</code> for <code>vcPow</code> ,<br><code>vmcPow</code><br><br><code>const MKL_Complex16*</code> for <code>vzPow</code> ,<br><code>vmzPow</code> | Pointers to arrays that contain the input vectors <i>a</i> and <i>b</i> .                                                       |
| <i>mode</i> | <code>const MKL_INT64</code>                                                                                                                                                                                                                                                                                                   | Overrides global VM mode setting for this function call. See <code>vmlSetMode</code> for possible values and their description. |

**Precision Overflow Thresholds for Real v?Pow Function**

| Data Type        | Threshold Limitations on Input Parameters       |
|------------------|-------------------------------------------------|
| single precision | $\text{abs}(a[i]) < (\text{FLT\_MAX})^{1/b[i]}$ |

| Data Type        | Threshold Limitations on Input Parameters       |
|------------------|-------------------------------------------------|
| double precision | $\text{abs}(a[i]) < (\text{DBL\_MAX})^{1/b[i]}$ |

Precision overflow thresholds for the complex `v?Pow` function are beyond the scope of this document.

## Output Parameters

| Name           | Type                                                                                                                                                                                                                                     | Description                                                          |
|----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | float* for <code>vsPow</code> , <code>vmsPow</code><br>double* for <code>vdPow</code> , <code>vmdPow</code><br>MKL_Complex8* for <code>vcPow</code> , <code>vmcPow</code><br>MKL_Complex16* for <code>vzPow</code> , <code>vmzPow</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The `v?Pow` function computes  $a$  to the power  $b$  for elements of two vectors.

The real function `v(s/d)Pow` has certain limitations on the input range of  $a$  and  $b$  parameters. Specifically, if  $a[i]$  is positive, then  $b[i]$  may be arbitrary. For negative  $a[i]$ , the value of  $b[i]$  must be an integer (either positive or negative).

The complex function `v(c/z)Pow` has no input range limitations.

### Special values for Real Function `v?Pow(x)`

| Argument 1 | Argument 2        | Result    | VM Error Status   | Exception  |
|------------|-------------------|-----------|-------------------|------------|
| +0         | neg. odd integer  | $+\infty$ | VML_STATUS_ERRDOM | ZERODIVIDE |
| -0         | neg. odd integer  | $-\infty$ | VML_STATUS_ERRDOM | ZERODIVIDE |
| +0         | neg. even integer | $+\infty$ | VML_STATUS_ERRDOM | ZERODIVIDE |
| -0         | neg. even integer | $+\infty$ | VML_STATUS_ERRDOM | ZERODIVIDE |
| +0         | neg. non-integer  | $+\infty$ | VML_STATUS_ERRDOM | ZERODIVIDE |
| -0         | neg. non-integer  | $+\infty$ | VML_STATUS_ERRDOM | ZERODIVIDE |
| -0         | pos. odd integer  | +0        |                   |            |
| -0         | pos. odd integer  | -0        |                   |            |
| +0         | pos. even integer | +0        |                   |            |
| -0         | pos. even integer | +0        |                   |            |
| +0         | pos. non-integer  | +0        |                   |            |
| -0         | pos. non-integer  | +0        |                   |            |
| -1         | $+\infty$         | +1        |                   |            |
| -1         | $-\infty$         | +1        |                   |            |
| +1         | any value         | +1        |                   |            |
| +1         | +0                | +1        |                   |            |
| +1         | -0                | +1        |                   |            |
| +1         | $+\infty$         | +1        |                   |            |
| +1         | $-\infty$         | +1        |                   |            |
| +1         | QNaN              | +1        |                   |            |
| any value  | +0                | +1        |                   |            |
| +0         | +0                | +1        |                   |            |
| -0         | +0                | +1        |                   |            |
| $+\infty$  | +0                | +1        |                   |            |
| $-\infty$  | +0                | +1        |                   |            |
| QNaN       | +0                | +1        |                   |            |

| Argument 1 | Argument 2        | Result    | VM Error Status   | Exception  |
|------------|-------------------|-----------|-------------------|------------|
| any value  | -0                | +1        |                   |            |
| +0         | -0                | +1        |                   |            |
| -0         | -0                | +1        |                   |            |
| $+\infty$  | -0                | +1        |                   |            |
| $-\infty$  | -0                | +1        |                   |            |
| QNAN       | -0                | +1        |                   |            |
| $X < +0$   | non-integer       | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $ X  < 1$  | $-\infty$         | $+\infty$ |                   |            |
| +0         | $-\infty$         | $+\infty$ | VML_STATUS_ERRDOM | ZERODIVIDE |
| -0         | $-\infty$         | $+\infty$ | VML_STATUS_ERRDOM | ZERODIVIDE |
| $ X  > 1$  | $-\infty$         | +0        |                   |            |
| $+\infty$  | $-\infty$         | +0        |                   |            |
| $-\infty$  | $-\infty$         | +0        |                   |            |
| $ X  < 1$  | $+\infty$         | +0        |                   |            |
| +0         | $+\infty$         | +0        |                   |            |
| -0         | $+\infty$         | +0        |                   |            |
| $ X  > 1$  | $+\infty$         | $+\infty$ |                   |            |
| $+\infty$  | $+\infty$         | $+\infty$ |                   |            |
| $-\infty$  | $+\infty$         | $+\infty$ |                   |            |
| $-\infty$  | neg. odd integer  | -0        |                   |            |
| $-\infty$  | neg. even integer | +0        |                   |            |
| $-\infty$  | neg. non-integer  | +0        |                   |            |
| $-\infty$  | pos. odd integer  | $-\infty$ |                   |            |
| $-\infty$  | pos. even integer | $+\infty$ |                   |            |
| $-\infty$  | pos. non-integer  | $+\infty$ |                   |            |
| $+\infty$  | $X < +0$          | +0        |                   |            |
| $+\infty$  | $X > +0$          | $+\infty$ |                   |            |
| QNAN       | QNAN              | QNAN      |                   |            |
| QNAN       | SNAN              | QNAN      |                   | INVALID    |
| SNAN       | QNAN              | QNAN      |                   | INVALID    |
| SNAN       | SNAN              | QNAN      |                   | INVALID    |

The complex double precision versions of this function, `vzPow` and `vmzPow`, are implemented in the EP accuracy mode only. If used in HA or LA mode, `vzPow` and `vmzPow` set the VM Error Status to `VML_STATUS_ACCURACYWARNING` (see the [Values of the VM Status](#) table).

Overflow in a complex function occurs (supported in the HA/LA accuracy modes only) when  $x_1$ ,  $x_2$ ,  $y_1$ ,  $y_2$  are finite numbers, but the real or imaginary part of the computed result is so large that it does not fit the target precision. In this case, the function returns  $\infty$  in that part of the result, raises the `OVERFLOW` exception, and sets the VM Error Status to `VML_STATUS_OVERFLOW` (overriding any possible `VML_STATUS_ACCURACYWARNING` status).

### **v?Powx**

*Raises each element of a vector to the constant power.*

### **Syntax**

```
vsPowx(n, a, b, y);
```

```
vmsPowx(n, a, b, y, mode);
```

```
vdPowx(n, a, b, y);
vmdPowx(n, a, b, y, mode);
vcPowx(n, a, b, y);
vmcPowx(n, a, b, y, mode);
vzPowx(n, a, b, y);
vmzPowx(n, a, b, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                                                                                         | Description                                                                                                                        |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                                | Number of elements to be calculated.                                                                                               |
| <i>a</i>    | const float* for vsPowx, vmsPowx<br>const double* for vdPowx, vmdPowx<br>const MKL_Complex8* for vcPowx, vmcPowx<br>const MKL_Complex16* for vzPowx, vmzPowx | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>b</i>    | const float for vsPowx, vmsPowx<br>const double for vdPowx, vmdPowx<br>const MKL_Complex8 for vcPowx, vmcPowx<br>const MKL_Complex16 for vzPowx, vmzPowx     | Constant value for power <i>b</i> .                                                                                                |
| <i>mode</i> | const MKL_INT64                                                                                                                                              | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Precision Overflow Thresholds for Real v?Powx Function

| Data Type        | Threshold Limitations on Input Parameters    |
|------------------|----------------------------------------------|
| single precision | $\text{abs}(a[i]) < (\text{FLT\_MAX})^{1/b}$ |
| double precision | $\text{abs}(a[i]) < (\text{DBL\_MAX})^{1/b}$ |

Precision overflow thresholds for the complex v?Powx function are beyond the scope of this document.

## Output Parameters

| Name     | Type                                                                                           | Description                                                    |
|----------|------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsPowx, vmsPowx<br>double* for vdPowx, vmdPowx<br>MKL_Complex8* for vcPowx, vmcPowx | Pointer to an array that contains the output vector <i>y</i> . |

| Name | Type                                  | Description |
|------|---------------------------------------|-------------|
|      | MKL_Complex16* for vzPowx,<br>vmzPowx |             |

## Description

The `v?Powx` function raises each element of a vector to the constant power.

The real function `v(s/d)Powx` has certain limitations on the input range of  $a$  and  $b$  parameters. Specifically, if  $a[i]$  is positive, then  $b$  may be arbitrary. For negative  $a[i]$ , the value of  $b$  must be an integer (either positive or negative).

The complex function `v(c/z)Powx` has no input range limitations.

Special values and VM Error Status treatment are the same as for the `v?Pow` function.

## v?Hypot

Computes a square root of sum of two squared elements.

## Syntax

```
vsHypot(n, a, b, y);
vmsHypot(n, a, b, y, mode);
vdHypot(n, a, b, y);
vmdHypot(n, a, b, y, mode);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name   | Type                                                                                                                                                                | Description                                                                                                                        |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| $n$    | <code>const MKL_INT</code>                                                                                                                                          | Number of elements to be calculated.                                                                                               |
| $a, b$ | <code>const float*</code> for <code>vsHypot</code> ,<br><code>vmsHypot</code><br><br><code>const double*</code> for <code>vdHypot</code> ,<br><code>vmdHypot</code> | Pointers to arrays that contain the input vectors $a$ and $b$ .                                                                    |
| $mode$ | <code>const MKL_INT64</code>                                                                                                                                        | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Precision Overflow Thresholds for Hypot Function

| Data Type        | Threshold Limitations on Input Parameters                                                              |
|------------------|--------------------------------------------------------------------------------------------------------|
| single precision | $\text{abs}(a[i]) < \text{sqrt}(\text{FLT\_MAX})$<br>$\text{abs}(b[i]) < \text{sqrt}(\text{FLT\_MAX})$ |
| double precision | $\text{abs}(a[i]) < \text{sqrt}(\text{DBL\_MAX})$<br>$\text{abs}(b[i]) < \text{sqrt}(\text{DBL\_MAX})$ |

## Output Parameters

| Name | Type                                                          | Description                                               |
|------|---------------------------------------------------------------|-----------------------------------------------------------|
| $y$  | float* for vsHypot, vmsHypot<br>double* for vdHypot, vmdHypot | Pointer to an array that contains the output vector $y$ . |

## Description

The function  $v?Hypot$  computes a square root of sum of two squared elements.

### Special values for Real Function $v?Hypot(x)$

| Argument 1 | Argument 2 | Result    | Exception |
|------------|------------|-----------|-----------|
| +0         | +0         | +0        |           |
| -0         | -0         | +0        |           |
| $+\infty$  | any value  | $+\infty$ |           |
| any value  | $+\infty$  | $+\infty$ |           |
| SNAN       | any value  | QNAN      | INVALID   |
| any value  | SNAN       | QNAN      | INVALID   |
| QNAN       | any value  | QNAN      |           |
| any value  | QNAN       | QNAN      |           |

## Exponential and Logarithmic Functions

### $v?Exp$

*Computes an exponential of vector elements.*

### Syntax

```
vsExp(n, a, y);
vmsExp(n, a, y, mode);
vdExp(n, a, y);
vmdExp(n, a, y, mode);
vcExp(n, a, y);
vmcExp(n, a, y, mode);
vzExp(n, a, y);
vmzExp(n, a, y, mode);
```

### Include Files

- mkl.h

### Input Parameters

| Name | Type          | Description                                        |
|------|---------------|----------------------------------------------------|
| $n$  | const MKL_INT | Specifies the number of elements to be calculated. |

| Name        | Type                                                                                                                                                 | Description                                                                                                                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>    | const float* for vsExp, vmsExp<br>const double* for vdExp, vmdExp<br>const MKL_Complex8* for vcExp, vmcExp<br>const MKL_Complex16* for vzExp, vmzExp | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                      | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Precision Overflow Thresholds for Real v?Exp Function

| Data Type        | Threshold Limitations on Input Parameters |
|------------------|-------------------------------------------|
| single precision | $a[i] < \text{Ln}(\text{FLT\_MAX})$       |
| double precision | $a[i] < \text{Ln}(\text{DBL\_MAX})$       |

Precision overflow thresholds for the complex v?Exp function are beyond the scope of this document.

### Output Parameters

| Name     | Type                                                                                                                         | Description                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsExp, vmsExp<br>double* for vdExp, vmdExp<br>MKL_Complex8* for vcExp, vmcExp<br>MKL_Complex16* for vzExp, vmzExp | Pointer to an array that contains the output vector <i>y</i> . |

### Description

The v?Exp function computes an exponential of vector elements.

### Special Values for Real Function v?Exp(x)

| Argument      | Result | VM Error Status      | Exception |
|---------------|--------|----------------------|-----------|
| +0            | +1     |                      |           |
| -0            | +1     |                      |           |
| X > overflow  | +∞     | VML_STATUS_OVERFLOW  | OVERFLOW  |
| X < underflow | +0     | VML_STATUS_UNDERFLOW | UNDERFLOW |
| +∞            | +∞     |                      |           |
| -∞            | +0     |                      |           |
| QNAN          | QNAN   |                      |           |
| SNAN          | QNAN   |                      | INVALID   |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function v?Exp(z)

| RE(z)<br>i·IM(z) | -∞     | -X                     | -0                     | +0                     | +X                     | +∞                   | NAN                    |
|------------------|--------|------------------------|------------------------|------------------------|------------------------|----------------------|------------------------|
| +i∞              | +0+i·0 | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | +∞+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID |

| <b>RE(z)</b><br><b>i·IM(z)</b> | <b>-∞</b> | <b>-X</b>              | <b>-0</b>              | <b>+0</b>              | <b>+X</b>              | <b>+∞</b>            | <b>NAN</b>  |
|--------------------------------|-----------|------------------------|------------------------|------------------------|------------------------|----------------------|-------------|
| <b>+i·Y</b>                    | +0·CIS(Y) |                        |                        |                        |                        | +∞·CIS(Y)            | QNAN+i·QNAN |
| <b>+i·0</b>                    | +0·CIS(0) |                        | +1+i·0                 | +1+i·0                 |                        | +∞+i·0               | QNAN+i·0    |
| <b>-i·0</b>                    | +0·CIS(0) |                        | +1-i·0                 | +1-i·0                 |                        | +∞-i·0               | QNAN-i·0    |
| <b>-i·Y</b>                    | +0·CIS(Y) |                        |                        |                        |                        | +∞·CIS(Y)            | QNAN+i·QNAN |
| <b>-i·∞</b>                    | +0-i·0    | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | +∞+i·QNAN<br>INVALID | QNAN+i·QNAN |
| <b>+i·NAN</b>                  | +0+i·0    | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | +∞+i·QNAN            | QNAN+i·QNAN |

## Notes:

- raises the `INVALID` exception when real or imaginary part of the argument is `SNAN`
- raises the `INVALID` exception on argument  $z = -\infty + i \cdot \text{QNAN}$
- raises the `OVERFLOW` exception and sets the VM Error Status to `VML_STATUS_OVERFLOW` in the case of overflow, that is, when  $\text{RE}(z)$ ,  $\text{IM}(z)$  are finite non-zero numbers, but the real or imaginary part of the exact result is so large that it does not meet the target precision.

## v?Expml

Computes an exponential of vector elements decreased by 1.

### Syntax

```
vsExpml(n, a, y);
vmsExpml(n, a, y, mode);
vdExpml(n, a, y);
vmdExpml(n, a, y, mode);
```

### Include Files

- `mkl.h`

### Input Parameters

| <b>Name</b>       | <b>Type</b>                                                                                                                                                     | <b>Description</b>                                                                                                                 |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                      | Specifies the number of elements to be calculated.                                                                                 |
| <code>a</code>    | <code>const float*</code> for <code>vsExpml</code> ,<br><code>vmsExpml</code><br><code>const double*</code> for <code>vdExpml</code> ,<br><code>vmdExpml</code> | Pointer to an array that contains the input vector <code>a</code> .                                                                |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                    | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

**Precision Overflow Thresholds for Exp<sub>m</sub>1 Function**

| Data Type        | Threshold Limitations on Input Parameters |
|------------------|-------------------------------------------|
| single precision | $a[i] < \text{Ln}(\text{FLT\_MAX})$       |
| double precision | $a[i] < \text{Ln}(\text{DBL\_MAX})$       |

**Output Parameters**

| Name | Type                                                                                                              | Description                                               |
|------|-------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|
| $y$  | float* for vsExp <sub>m</sub> 1, vmsExp <sub>m</sub> 1<br>double* for vdExp <sub>m</sub> 1, vmdExp <sub>m</sub> 1 | Pointer to an array that contains the output vector $y$ . |

**Description**

The  $v?Exp_{m1}$  function computes an exponential of vector elements decreased by 1.

**Special Values for Real Function  $v?Exp_{m1}(x)$** 

| Argument              | Result    | VM Error Status     | Exception |
|-----------------------|-----------|---------------------|-----------|
| +0                    | +0        |                     |           |
| -0                    | +0        |                     |           |
| $X > \text{overflow}$ | $+\infty$ | VML_STATUS_OVERFLOW | OVERFLOW  |
| $+\infty$             | $+\infty$ |                     |           |
| $-\infty$             | -1        |                     |           |
| QNAN                  | QNAN      |                     |           |
| SNAN                  | QNAN      |                     | INVALID   |

 **$v?Ln$** 

*Computes natural logarithm of vector elements.*

**Syntax**

```
vsLn(n, a, y);
vmsLn(n, a, y, mode);
vdLn(n, a, y);
vmdLn(n, a, y, mode);
vcLn(n, a, y);
vmcLn(n, a, y, mode);
vzLn(n, a, y);
vmzLn(n, a, y, mode);
```

**Include Files**

- mkl.h

**Input Parameters**

| Name | Type          | Description                                        |
|------|---------------|----------------------------------------------------|
| $n$  | const MKL_INT | Specifies the number of elements to be calculated. |

| Name        | Type                                                                                                                                         | Description                                                                                                                        |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>    | const float* for vsLn, vmsLn<br>const double* for vdLn, vmdLn<br>const MKL_Complex8* for vcLn, vmcLn<br>const MKL_Complex16* for vzLn, vmzLn | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                              | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                 | Description                                                    |
|----------|----------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsLn, vmsLn<br>double* for vdLn, vmdLn<br>MKL_Complex8* for vcLn, vmcLn<br>MKL_Complex16* for vzLn, vmzLn | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The *v?Ln* function computes natural logarithm of vector elements.

### Special Values for Real Function *v?Ln(x)*

| Argument | Result | VM Error Status   | Exception  |
|----------|--------|-------------------|------------|
| +1       | +0     |                   |            |
| X < +0   | QNAN   | VML_STATUS_ERRDOM | INVALID    |
| +0       | -∞     | VML_STATUS_SING   | ZERODIVIDE |
| -0       | -∞     | VML_STATUS_SING   | ZERODIVIDE |
| -∞       | QNAN   | VML_STATUS_ERRDOM | INVALID    |
| +∞       | +∞     |                   |            |
| QNAN     | QNAN   |                   |            |
| SNAN     | QNAN   |                   | INVALID    |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function *v?Ln(z)*

| RE(z)<br>i·IM(z) | -∞                        | -X       | -0                       | +0                       | +X       | +∞       | NAN                    |
|------------------|---------------------------|----------|--------------------------|--------------------------|----------|----------|------------------------|
| +i·∞             | +∞ + i · $\frac{3\pi}{4}$ | +∞+i·π/2 | +∞+i·π/2                 | +∞+i·π/2                 | +∞+i·π/2 | +∞+i·π/4 | +∞+i·QNAN              |
| +i·Y             | +∞+i·π                    |          |                          |                          |          | +∞+i·0   | QNAN+i·QNAN<br>INVALID |
| +i·0             | +∞+i·π                    |          | -∞+i·π<br>ZERODIVID<br>E | -∞+i·0<br>ZERODIVID<br>E |          | +∞+i·0   | QNAN+i·QNAN<br>INVALID |

| <b>RE(z)<br/>i·IM(z)</b> | <b>-∞</b>                 | <b>-X</b>              | <b>-0</b>                | <b>+0</b>                | <b>+X</b>              | <b>+∞</b> | <b>NAN</b>             |
|--------------------------|---------------------------|------------------------|--------------------------|--------------------------|------------------------|-----------|------------------------|
| -i·0                     | +∞-i·π                    |                        | -∞-i·π<br>ZERODIVID<br>E | -∞-i·0<br>ZERODIVID<br>E |                        | +∞-i·0    | QNAN+i·QNAN<br>INVALID |
| -i·Y                     | +∞-i·π                    |                        |                          |                          |                        | +∞-i·0    | QNAN+i·QNAN<br>INVALID |
| -i·∞                     | +∞ - i · $\frac{3\pi}{4}$ | +∞-i·π/2               | +∞-i·π/2                 | +∞-i·π/2                 | +∞-i·π/2               | +∞-i·π/4  | +∞+i·QNAN              |
| +i·NAN                   | +∞+i·QNAN                 | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID   | QNAN+i·QNAN<br>INVALID   | QNAN+i·QNAN<br>INVALID | +∞+i·QNAN | QNAN+i·QNAN<br>INVALID |

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`

### v?Log10

*Computes denary logarithm of vector elements.*

#### Syntax

```
vsLog10(n, a, y);
vmsLog10(n, a, y, mode);
vdLog10(n, a, y);
vmdLog10(n, a, y, mode);
vcLog10(n, a, y);
vmcLog10(n, a, y, mode);
vzLog10(n, a, y);
vmzLog10(n, a, y, mode);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| <b>Name</b>    | <b>Type</b>                                                                                                                                                                                                                                                     | <b>Description</b>                                                  |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|
| <code>n</code> | <code>const MKL_INT</code>                                                                                                                                                                                                                                      | Specifies the number of elements to be calculated.                  |
| <code>a</code> | <code>const float*</code> for <code>vsLog10</code> ,<br><code>vmsLog10</code><br><br><code>const double*</code> for <code>vdLog10</code> ,<br><code>vmdLog10</code><br><br><code>const MKL_Complex8*</code> for <code>vcLog10</code> ,<br><code>vmcLog10</code> | Pointer to an array that contains the input vector <code>a</code> . |

| Name | Type                                          | Description                                                                                                                        |
|------|-----------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
|      | const MKL_Complex16* for vzLog10,<br>vmzLog10 |                                                                                                                                    |
| mode | const MKL_INT64                               | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name | Type                                                                                                                                                       | Description                                            |
|------|------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------|
| y    | float* for vsLog10, vmsLog10<br>double* for vdLog10, vmdLog10<br><br>MKL_Complex8* for vcLog10,<br>vmcLog10<br><br>MKL_Complex16* for vzLog10,<br>vmzLog10 | Pointer to an array that contains the output vector y. |

## Description

The v?Log10 function computes a denary logarithm of vector elements.

### Special Values for Real Function v?Log10(x)

| Argument | Result | VM Error Status   | Exception  |
|----------|--------|-------------------|------------|
| +1       | +0     |                   |            |
| X < +0   | QNAN   | VML_STATUS_ERRDOM | INVALID    |
| +0       | -∞     | VML_STATUS_SING   | ZERODIVIDE |
| -0       | -∞     | VML_STATUS_SING   | ZERODIVIDE |
| -∞       | QNAN   | VML_STATUS_ERRDOM | INVALID    |
| +∞       | +∞     |                   |            |
| QNAN     | QNAN   |                   |            |
| SNAN     | QNAN   |                   | INVALID    |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function v?Log10(z)

| RE(z)<br>i·IM(z) | -∞                                            | -X                                            | -0                                             | +0                                            | +X                                            | +∞                                            | NAN                    |
|------------------|-----------------------------------------------|-----------------------------------------------|------------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|------------------------|
| +i·∞             | $+\infty + i \frac{3}{4} \frac{\pi}{\ln(10)}$ | $+\infty + i \frac{\pi}{2} \frac{1}{\ln(10)}$ | $+\infty + i \frac{\pi}{2} \frac{1}{\ln(10)}$  | $+\infty + i \frac{\pi}{2} \frac{1}{\ln(10)}$ | $+\infty + i \frac{\pi}{2} \frac{1}{\ln(10)}$ | $+\infty + i \frac{\pi}{4} \frac{1}{\ln(10)}$ | +∞+i·QNAN<br>INVALID   |
| +i·Y             | $+\infty + i \frac{\pi}{\ln(10)}$             |                                               |                                                |                                               |                                               | +∞+i·0                                        | QNAN+i·QNAN<br>INVALID |
| +i·0             | $+\infty + i \frac{\pi}{\ln(10)}$             |                                               | $-\infty + i \frac{\pi}{\ln(10)}$<br>ZERODRIVE | $-\infty + i \cdot 0$<br>ZERODRIVE            |                                               | +∞+i·0                                        | QNAN+i·QNAN<br>INVALID |

| <b>RE(z)<br/>i·IM(z)</b> | <b>-∞</b>                                     | <b>-X</b>                                     | <b>-0</b>                                       | <b>+0</b>                                     | <b>+X</b>                                     | <b>+∞</b>                                     | <b>NAN</b>                      |
|--------------------------|-----------------------------------------------|-----------------------------------------------|-------------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|---------------------------------|
| <b>-i·0</b>              | $+\infty - i \frac{\pi}{\ln(10)}$             |                                               | $-\infty - i \frac{\pi}{\ln(10)}$<br>ZERODIVIDE | $-\infty - i \cdot 0$<br>ZERODIVIDE           |                                               | $+\infty - i \cdot 0$                         | QNaN-i·QNaN<br>INVALID          |
| <b>-i·Y</b>              | $+\infty - i \frac{\pi}{\ln(10)}$             |                                               |                                                 |                                               |                                               | $+\infty - i \cdot 0$                         | QNaN+i·QNaN<br>INVALID          |
| <b>-i·∞</b>              | $+\infty + i \frac{3}{4} \frac{\pi}{\ln(10)}$ | $+\infty - i \frac{\pi}{2} \frac{1}{\ln(10)}$ | $+\infty - i \frac{\pi}{2} \frac{1}{\ln(10)}$   | $+\infty - i \frac{\pi}{2} \frac{1}{\ln(10)}$ | $+\infty - i \frac{\pi}{2} \frac{1}{\ln(10)}$ | $+\infty - i \frac{\pi}{4} \frac{1}{\ln(10)}$ | $+\infty + i \cdot \text{QNaN}$ |
| <b>+i·NAN</b>            | $+\infty + i \cdot \text{QNaN}$               | QNaN+i·QNaN<br>INVALID                        | QNaN+i·QNaN<br>INVALID                          | QNaN+i·QNaN<br>INVALID                        | QNaN+i·QNaN<br>INVALID                        | $+\infty + i \cdot \text{QNaN}$               | QNaN+i·QNaN<br>INVALID          |

Notes:

- raises INVALID exception when real or imaginary part of the argument is SNAN

### v?Log1p

Computes a natural logarithm of vector elements that are increased by 1.

#### Syntax

```
vsLog1p(n, a, y);
vmsLog1p(n, a, y, mode);
vdLog1p(n, a, y);
vmdLog1p(n, a, y, mode);
```

#### Include Files

- mkl.h

#### Input Parameters

| <b>Name</b> | <b>Type</b>                                                                         | <b>Description</b>                                                                                                                 |
|-------------|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                       | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsLog1p,<br>vmsLog1p<br><br>const double* for vdLog1p,<br>vmdLog1p | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                     | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name | Type                                                          | Description                                               |
|------|---------------------------------------------------------------|-----------------------------------------------------------|
| $y$  | float* for vsLog1p, vmsLog1p<br>double* for vdLog1p, vmdLog1p | Pointer to an array that contains the output vector $y$ . |

## Description

The `v?Log1p` function computes a natural logarithm of vector elements that are increased by 1.

### Special Values for Real Function `v?Log1p(x)`

| Argument  | Result    | VM Error Status   | Exception  |
|-----------|-----------|-------------------|------------|
| -1        | $-\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| $X < -1$  | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| +0        | +0        |                   |            |
| -0        | -0        |                   |            |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $+\infty$ | $+\infty$ |                   |            |
| QNAN      | QNAN      |                   |            |
| SNAN      | QNAN      |                   | INVALID    |

## Trigonometric Functions

### `v?Cos`

*Computes cosine of vector elements.*

### Syntax

```
vsCos(n, a, y);
vmsCos(n, a, y, mode);
vdCos(n, a, y);
vmdCos(n, a, y, mode);
vcCos(n, a, y);
vmcCos(n, a, y, mode);
vzCos(n, a, y);
vmzCos(n, a, y, mode);
```

### Include Files

- `mk1.h`

### Input Parameters

| Name | Type          | Description                                        |
|------|---------------|----------------------------------------------------|
| $n$  | const MKL_INT | Specifies the number of elements to be calculated. |

| Name        | Type                                                                                                                                                 | Description                                                                                                                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>    | const float* for vsCos, vmsCos<br>const double* for vdCos, vmdCos<br>const MKL_Complex8* for vcCos, vmcCos<br>const MKL_Complex16* for vzCos, vmzCos | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                      | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                         | Description                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsCos, vmsCos<br>double* for vdCos, vmdCos<br>MKL_Complex8* for vcCos, vmcCos<br>MKL_Complex16* for vzCos, vmzCos | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The *v?Cos* function computes cosine of vector elements.

Note that arguments  $\text{abs}(a[i]) \leq 2^{13}$  and  $\text{abs}(a[i]) \leq 2^{16}$  for single and double precisions respectively are called fast computational path. These are trigonometric function arguments for which VM provides the best possible performance. Avoid arguments that do not belong to the fast computational path in the VM High Accuracy (HA) and Low Accuracy (LA) functions. Alternatively, you can use VM Enhanced Performance (EP) functions that are fast on the entire function domain. However, these functions provide less accuracy.

### Special Values for Real Function *v?Cos(x)*

| Argument  | Result | VM Error Status   | Exception |
|-----------|--------|-------------------|-----------|
| +0        | +1     |                   |           |
| -0        | +1     |                   |           |
| $+\infty$ | QNAN   | VML_STATUS_ERRDOM | INVALID   |
| $-\infty$ | QNAN   | VML_STATUS_ERRDOM | INVALID   |
| QNAN      | QNAN   |                   |           |
| SNAN      | QNAN   |                   | INVALID   |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Cos}(z) = \text{Cosh}(i*z).$$

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Notice revision #20110804

## v?Sin

*Computes sine of vector elements.*

### Syntax

```
vsSin(n, a, y);
vmsSin(n, a, y, mode);
vdSin(n, a, y);
vmdSin(n, a, y, mode);
vcSin(n, a, y);
vmcSin(n, a, y, mode);
vzSin(n, a, y);
vmzSin(n, a, y, mode);
```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type                                                                                                                                                 | Description                                                                                                                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                        | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsSin, vmsSin<br>const double* for vdSin, vmdSin<br>const MKL_Complex8* for vcSin, vmcSin<br>const MKL_Complex16* for vzSin, vmzSin | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                      | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Output Parameters

| Name     | Type                                                                                     | Description                                                    |
|----------|------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsSin, vmsSin<br>double* for vdSin, vmdSin<br>MKL_Complex8* for vcSin, vmcSin | Pointer to an array that contains the output vector <i>y</i> . |

| Name | Type | Description |
|------|------|-------------|
|------|------|-------------|

MKL\_Complex16\* for vzSin, vmzSin

## Description

The function computes sine of vector elements.

Note that arguments  $\text{abs}(a[i]) \leq 2^{13}$  and  $\text{abs}(a[i]) \leq 2^{16}$  for single and double precisions respectively are called fast computational path. These are trigonometric function arguments for which VM provides the best possible performance. Avoid arguments that do not belong to the fast computational path in the VM High Accuracy (HA) and Low Accuracy (LA) functions. Alternatively, you can use VM Enhanced Performance (EP) functions that are fast on the entire function domain. However, these functions provide less accuracy.

### Special Values for Real Function v?Sin(x)

| Argument  | Result | VM Error Status   | Exception |
|-----------|--------|-------------------|-----------|
| +0        | +0     |                   |           |
| -0        | -0     |                   |           |
| $+\infty$ | QNAN   | VML_STATUS_ERRDOM | INVALID   |
| $-\infty$ | QNAN   | VML_STATUS_ERRDOM | INVALID   |
| QNAN      | QNAN   |                   |           |
| SNAN      | QNAN   |                   | INVALID   |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Sin}(z) = -i * \text{Sinh}(i * z).$$

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Notice revision #20110804

## v?SinCos

*Computes sine and cosine of vector elements.*

### Syntax

```
vsSinCos(n, a, y, z);
```

```
vmsSinCos(n, a, y, z, mode);
```

```
vdSinCos(n, a, y, z);
```

```
vmdSinCos(n, a, y, z, mode);
```

### Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                    | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                           | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsSinCos,<br>vmsSinCos<br><br>const double* for vdSinCos,<br>vmdSinCos | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                         | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name        | Type                                                              | Description                                                                                                    |
|-------------|-------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| <i>y, z</i> | float* for vsSinCos, vmsSinCos<br>double* for vdSinCos, vmdSinCos | Pointers to arrays that contain the output vectors <i>y</i> (for sinevalues) and <i>z</i> (for cosine values). |

## Description

The function computes sine and cosine of vector elements.

Note that arguments  $\text{abs}(a[i]) \leq 2^{13}$  and  $\text{abs}(a[i]) \leq 2^{16}$  for single and double precisions respectively are called fast computational path. These are trigonometric function arguments for which VM provides the best possible performance. Avoid arguments that do not belong to the fast computational path in the VM High Accuracy (HA) and Low Accuracy (LA) functions. Alternatively, you can use VM Enhanced Performance (EP) functions that are fast on the entire function domain. However, these functions provide less accuracy.

### Special Values for Real Function $v?SinCos(x)$

| Argument  | Result 1 | Result 2 | VM Error Status   | Exception |
|-----------|----------|----------|-------------------|-----------|
| +0        | +0       | +1       |                   |           |
| -0        | -0       | +1       |                   |           |
| $+\infty$ | QNAN     | QNAN     | VML_STATUS_ERRDOM | INVALID   |
| $-\infty$ | QNAN     | QNAN     | VML_STATUS_ERRDOM | INVALID   |
| QNAN      | QNAN     | QNAN     |                   |           |
| SNAN      | QNAN     | QNAN     |                   | INVALID   |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Sin}(z) = -i * \text{Sinh}(i * z).$$

### Optimization Notice

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**Optimization Notice**

Notice revision #20110804

**v?CIS**

Computes complex exponent of real vector elements (cosine and sine of real vector elements combined to complex value).

**Syntax**

```
vcCIS(n, a, y);
vmcCIS(n, a, y, mode);
vzCIS(n, a, y);
vmzCIS(n, a, y, mode);
```

**Include Files**

- mkl.h

**Input Parameters**

| Name        | Type                                                              | Description                                                                                                                        |
|-------------|-------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                     | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vcCIS, vmcCIS<br>const double* for vzCIS, vmzCIS | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                   | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

**Output Parameters**

| Name     | Type                                                                | Description                                                    |
|----------|---------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | MKL_Complex8* for vcCIS, vmcCIS<br>MKL_Complex16* for vzCIS, vmzCIS | Pointer to an array that contains the output vector <i>y</i> . |

**Description**

The v?CIS function computes complex exponent of real vector elements (cosine and sine of real vector elements combined to complex value).

See the [Special Value Notations](#) section for the conventions used in the table below.

**Special Values for Complex Function v?CIS(x)**

| <b>x</b> | <b>CIS(x)</b>          |
|----------|------------------------|
| + ∞      | QNAN+i·QNAN<br>INVALID |
| + 0      | +1+i·0                 |

| <b>x</b> | <b>CIS(x)</b>          |
|----------|------------------------|
| - 0      | +1-i·0                 |
| - ∞      | QNAN+i·QNAN<br>INVALID |
| NAN      | QNAN+i·QNAN            |

Notes:

- raises `INVALID` exception when the argument is `SNAN`
- raises `INVALID` exception and sets the VM Error Status to `VML_STATUS_ERRDOM` for  $x=+\infty$ ,  $x=-\infty$

## v?Tan

*Computes tangent of vector elements.*

### Syntax

```
vsTan(n, a, y);
vmsTan(n, a, y, mode);
vdTan(n, a, y);
vmdTan(n, a, y, mode);
vcTan(n, a, y);
vmcTan(n, a, y, mode);
vzTan(n, a, y);
vmzTan(n, a, y, mode);
```

### Include Files

- `mkl.h`

### Input Parameters

| <b>Name</b>       | <b>Type</b>                                                                                                                                                                                                                                                                                                                | <b>Description</b>                                                                                                                 |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                                                                                                                                                                                 | Specifies the number of elements to be calculated.                                                                                 |
| <code>a</code>    | <code>const float*</code> for <code>vsTan</code> , <code>vmsTan</code><br><code>const double*</code> for <code>vdTan</code> , <code>vmdTan</code><br><code>const MKL_Complex8*</code> for <code>vcTan</code> ,<br><code>vmcTan</code><br><code>const MKL_Complex16*</code> for <code>vzTan</code> ,<br><code>vmzTan</code> | Pointer to an array that contains the input vector <code>a</code> .                                                                |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                                                                                                                                                                               | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name | Type                                                                                                                         | Description                                               |
|------|------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|
| $y$  | float* for vsTan, vmsTan<br>double* for vdTan, vmdTan<br>MKL_Complex8* for vcTan, vmcTan<br>MKL_Complex16* for vzTan, vmzTan | Pointer to an array that contains the output vector $y$ . |

## Description

The  $v?Tan$  function computes tangent of vector elements.

Note that arguments  $abs(a[i]) \leq 2^{13}$  and  $abs(a[i]) \leq 2^{16}$  for single and double precisions respectively are called fast computational path. These are trigonometric function arguments for which VM provides the best possible performance. Avoid arguments that do not belong to the fast computational path in the VM High Accuracy (HA) and Low Accuracy (LA) functions. Alternatively, you can use VM Enhanced Performance (EP) functions that are fast on the entire function domain. However, these functions provide less accuracy.

### Special Values for Real Function $v?Tan(x)$

| Argument  | Result | VM Error Status   | Exception |
|-----------|--------|-------------------|-----------|
| +0        | +0     |                   |           |
| -0        | -0     |                   |           |
| $+\infty$ | QNAN   | VML_STATUS_ERRDOM | INVALID   |
| $-\infty$ | QNAN   | VML_STATUS_ERRDOM | INVALID   |
| QNAN      | QNAN   |                   |           |
| SNAN      | QNAN   |                   | INVALID   |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Tan}(z) = -i * \text{Tanh}(i * z).$$

### Optimization Notice

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Notice revision #20110804

## $v?Acos$

*Computes inverse cosine of vector elements.*

### Syntax

```
vsAcos(n, a, y);
```

```
vmsAcos(n, a, y, mode);
```

```
vdAcos(n, a, y);
```

```
vmdAcos(n, a, y, mode);
```

```
vcAcos(n, a, y);
```

```
vmcAcos(n, a, y, mode);
vzAcos(n, a, y);
vmzAcos(n, a, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                                                                                         | Description                                                                                                                        |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const int                                                                                                                                                    | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsAcos, vmsAcos<br>const double* for vdAcos, vmdAcos<br>const MKL_Complex8* for vcAcos, vmcAcos<br>const MKL_Complex16* for vzAcos, vmzAcos | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                              | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                                 | Description                                                    |
|----------|--------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsAcos, vmsAcos<br>double* for vdAcos, vmdAcos<br>MKL_Complex8* for vcAcos, vmcAcos<br>MKL_Complex16* for vzAcos, vmzAcos | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The *v?Acos* function computes inverse cosine of vector elements.

### Special Values for Real Function *v?Acos(x)*

| Argument  | Result   | VM Error Status   | Exception |
|-----------|----------|-------------------|-----------|
| +0        | $+\pi/2$ |                   |           |
| -0        | $+\pi/2$ |                   |           |
| +1        | +0       |                   |           |
| -1        | $+\pi$   |                   |           |
| $ X  > 1$ | QNAN     | VML_STATUS_ERRDOM | INVALID   |
| $+\infty$ | QNAN     | VML_STATUS_ERRDOM | INVALID   |
| $-\infty$ | QNAN     | VML_STATUS_ERRDOM | INVALID   |
| QNAN      | QNAN     |                   |           |
| SNAN      | QNAN     |                   | INVALID   |

See the [Special Value Notations](#) section for the conventions used in the table below.

**Special Values for Complex Function v?Acos(z)**

| <b>RE(z)<br/>i·IM(z)</b> | <b>-∞</b>                          | <b>-X</b>                         | <b>-0</b>                              | <b>+0</b>                              | <b>+X</b>                         | <b>+∞</b>                         | <b>NAN</b>  |
|--------------------------|------------------------------------|-----------------------------------|----------------------------------------|----------------------------------------|-----------------------------------|-----------------------------------|-------------|
| <b>+i·∞</b>              | $+\frac{3\pi}{4} - i \cdot \infty$ | $+\frac{\pi}{2} - i \cdot \infty$ | $+\frac{\pi}{2} - i \cdot \infty$      | $+\frac{\pi}{2} - i \cdot \infty$      | $+\frac{\pi}{2} - i \cdot \infty$ | $+\frac{\pi}{4} - i \cdot \infty$ | QNAN-i·∞    |
| <b>+i·Y</b>              | $+\pi - i \cdot \infty$            |                                   |                                        |                                        |                                   | $+0 - i \cdot \infty$             | QNAN+i·QNAN |
| <b>+i·0</b>              | $+\pi - i \cdot \infty$            |                                   | $+\frac{\pi}{2} - i \cdot 0$           | $+\frac{\pi}{2} - i \cdot 0$           |                                   | $+0 - i \cdot \infty$             | QNAN+i·QNAN |
| <b>-i·0</b>              | $+\pi + i \cdot \infty$            |                                   | $+\frac{\pi}{2} + i \cdot \infty$      | $+\frac{\pi}{2} + i \cdot \infty$      |                                   | $+0 + i \cdot \infty$             | QNAN+i·QNAN |
| <b>-i·Y</b>              | $+\pi + i \cdot \infty$            |                                   |                                        |                                        |                                   | $+0 + i \cdot \infty$             | QNAN+i·QNAN |
| <b>-i·∞</b>              | $+\frac{3\pi}{4} + i \cdot \infty$ | $+\frac{\pi}{2} + i \cdot \infty$ | $+\frac{\pi}{2} + i \cdot \infty$      | $+\frac{\pi}{2} + i \cdot \infty$      | $+\frac{\pi}{2} + i \cdot \infty$ | $+\frac{\pi}{4} + i \cdot \infty$ | QNAN+i·∞    |
| <b>+i·NAN</b>            | QNAN+i·∞                           | QNAN+i·QNAN                       | $+\frac{\pi}{2} + i \cdot \text{QNAN}$ | $+\frac{\pi}{2} + i \cdot \text{QNAN}$ | QNAN+i·QNAN                       | QNAN+i·∞                          | QNAN+i·QNAN |

Notes:

- raises INVALID exception when real or imaginary part of the argument is SNAN
- $\text{Acos}(\text{CONJ}(z)) = \text{CONJ}(\text{Acos}(z))$ .

**v?Asin**

*Computes inverse sine of vector elements.*

**Syntax**

```
vsAsin(n, a, y);
vmsAsin(n, a, y, mode);
vdAsin(n, a, y);
vmdAsin(n, a, y, mode);
vcAsin(n, a, y);
vmcAsin(n, a, y, mode);
vzAsin(n, a, y);
vmzAsin(n, a, y, mode);
```

**Include Files**

- mkl.h

**Input Parameters**

| <b>Name</b> | <b>Type</b>                                                           | <b>Description</b>                                            |
|-------------|-----------------------------------------------------------------------|---------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                         | Specifies the number of elements to be calculated.            |
| <i>a</i>    | const float* for vsAsin, vmsAsin<br>const double* for vdAsin, vmdAsin | Pointer to an array that contains the input vector <i>a</i> . |

| Name | Type                                     | Description                                                                                                                        |
|------|------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
|      | const MKL_Complex8* for vcAsin, vmcAsin  |                                                                                                                                    |
|      | const MKL_Complex16* for vzAsin, vmzAsin |                                                                                                                                    |
| mode | const MKL_INT64                          | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name | Type                                                                                                                                 | Description                                                    |
|------|--------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| y    | float* for vsAsin, vmsAsin<br>double* for vdAsin, vmdAsin<br>MKL_Complex8* for vcAsin, vmcAsin<br>MKL_Complex16* for vzAsin, vmzAsin | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?Asin` function computes inverse sine of vector elements.

### Special Values for Real Function `v?Asin(x)`

| Argument  | Result   | VM Error Status   | Exception |
|-----------|----------|-------------------|-----------|
| +0        | +0       |                   |           |
| -0        | -0       |                   |           |
| +1        | $+\pi/2$ |                   |           |
| -1        | $-\pi/2$ |                   |           |
| $ X  > 1$ | QNAN     | VML_STATUS_ERRDOM | INVALID   |
| $+\infty$ | QNAN     | VML_STATUS_ERRDOM | INVALID   |
| $-\infty$ | QNAN     | VML_STATUS_ERRDOM | INVALID   |
| QNAN      | QNAN     |                   |           |
| SNAN      | QNAN     |                   | INVALID   |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Asin}(z) = -i \cdot \text{Asinh}(i \cdot z).$$

## `v?Atan`

*Computes inverse tangent of vector elements.*

### Syntax

```
vsAtan(n, a, y);
```

```
vmsAtan(n, a, y, mode);
```

```
vdAtan(n, a, y);
```

```
vmdAtan(n, a, y, mode);
```

```
vcAtan(n, a, y);
```

```
vmcAtan(n, a, y, mode);
```

```

vzAtan(n, a, y);
vmzAtan(n, a, y, mode);

```

## Include Files

- `mk1.h`

## Input Parameters

| Name              | Type                                                                                                                                                                                                                                                                                                                         | Description                                                                                                                        |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                                                                                                                                                                                   | Specifies the number of elements to be calculated.                                                                                 |
| <code>a</code>    | <code>const float*</code> for <code>vsAtan</code> , <code>vmsAtan</code><br><code>const double*</code> for <code>vdAsin</code> , <code>vmdAtan</code><br><code>const MKL_Complex8*</code> for <code>vcAtan</code> , <code>vmcAtan</code><br><code>const MKL_Complex16*</code> for <code>vzAsin</code> , <code>vmzAtan</code> | Pointer to an array that contains the input vector <code>a</code> .                                                                |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                                                                                                                                                                                 | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name           | Type                                                                                                                                                                                                                                                                                                 | Description                                                          |
|----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsAtan</code> , <code>vmsAtan</code><br><code>double*</code> for <code>vdAsin</code> , <code>vmdAtan</code><br><code>MKL_Complex8*</code> for <code>vcAtan</code> , <code>vmcAtan</code><br><code>MKL_Complex16*</code> for <code>vzAsin</code> , <code>vmzAtan</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The `v?Atan` function computes inverse tangent of vector elements.

### Special Values for Real Function `v?Atan(x)`

| Argument  | Result   | VM Error Status | Exception |
|-----------|----------|-----------------|-----------|
| +0        | +0       |                 |           |
| -0        | -0       |                 |           |
| $+\infty$ | $+\pi/2$ |                 |           |
| $-\infty$ | $-\pi/2$ |                 |           |
| QNAN      | QNAN     |                 |           |
| SNAN      | QNAN     |                 | INVALID   |

Specifications for special values of the complex functions are defined according to the following formula

$$\text{Atan}(z) = -i \cdot \text{Atanh}(i \cdot z).$$

## v?Atan2

Computes four-quadrant inverse tangent of elements of two vectors.

### Syntax

```
vsAtan2(n, a, b, y);
vmsAtan2(n, a, b, y, mode);
vdAtan2(n, a, b, y);
vmdAtan2(n, a, b, y, mode);
```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type                                                                                | Description                                                                                                                        |
|-------------|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                       | Specifies the number of elements to be calculated.                                                                                 |
| <i>a, b</i> | const float* for vsAtan2,<br>vmsAtan2<br><br>const double* for vdAtan2,<br>vmdAtan2 | Pointers to arrays that contain the input vectors <i>a</i> and <i>b</i> .                                                          |
| <i>mode</i> | const MKL_INT64                                                                     | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

### Output Parameters

| Name     | Type                                                          | Description                                                    |
|----------|---------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsAtan2, vmsAtan2<br>double* for vdAtan2, vmdAtan2 | Pointer to an array that contains the output vector <i>y</i> . |

### Description

The `v?Atan2` function computes four-quadrant inverse tangent of elements of two vectors.

The elements of the output vector *y* are computed as the four-quadrant arctangent of  $a[i] / b[i]$ .

### Special values for Real Function v?Atan2(x)

| Argument 1 | Argument 2 | Result     | Exception |
|------------|------------|------------|-----------|
| $-\infty$  | $-\infty$  | $-3*\pi/4$ |           |
| $-\infty$  | $X < +0$   | $-\pi/2$   |           |
| $-\infty$  | $-0$       | $-\pi/2$   |           |
| $-\infty$  | $+0$       | $-\pi/2$   |           |
| $-\infty$  | $X > +0$   | $-\pi/2$   |           |
| $-\infty$  | $+\infty$  | $-\pi/4$   |           |
| $X < +0$   | $-\infty$  | $-\pi$     |           |
| $X < +0$   | $-0$       | $-\pi/2$   |           |

| Argument 1 | Argument 2 | Result     | Exception |
|------------|------------|------------|-----------|
| X < +0     | +0         | $-\pi/2$   |           |
| X < +0     | $+\infty$  | -0         |           |
| -0         | $-\infty$  | $-\pi$     |           |
| -0         | X < +0     | $-\pi$     |           |
| -0         | -0         | $-\pi$     |           |
| -0         | +0         | -0         |           |
| -0         | X > +0     | -0         |           |
| -0         | $+\infty$  | -0         |           |
| +0         | $-\infty$  | $+\pi$     |           |
| +0         | X < +0     | $+\pi$     |           |
| +0         | -0         | $+\pi$     |           |
| +0         | +0         | +0         |           |
| +0         | X > +0     | +0         |           |
| +0         | $+\infty$  | +0         |           |
| X > +0     | $-\infty$  | $+\pi$     |           |
| X > +0     | -0         | $+\pi/2$   |           |
| X > +0     | +0         | $+\pi/2$   |           |
| X > +0     | $+\infty$  | +0         |           |
| $+\infty$  | $-\infty$  | $-3*\pi/4$ |           |
| $+\infty$  | X < +0     | $+\pi/2$   |           |
| $+\infty$  | -0         | $+\pi/2$   |           |
| $+\infty$  | +0         | $+\pi/2$   |           |
| $+\infty$  | X > +0     | $+\pi/2$   |           |
| $+\infty$  | $+\infty$  | $+\pi/4$   |           |
| X > +0     | QNAN       | QNAN       |           |
| X > +0     | SNAN       | QNAN       | INVALID   |
| QNAN       | X > +0     | QNAN       |           |
| SNAN       | X > +0     | QNAN       | INVALID   |
| QNAN       | QNAN       | QNAN       |           |
| QNAN       | SNAN       | QNAN       | INVALID   |
| SNAN       | QNAN       | QNAN       | INVALID   |
| SNAN       | SNAN       | QNAN       | INVALID   |

## Hyperbolic Functions

### v?Cosh

Computes hyperbolic cosine of vector elements.

#### Syntax

```
vsCosh(n, a, y);
```

```
vmsCosh(n, a, y, mode);
```

```
vdCosh(n, a, y);
```

```
vmdCosh(n, a, y, mode);
```

```
vcCosh(n, a, y);
```

```
vmcCosh(n, a, y, mode);
```

```
vzCosh(n, a, y);
```

```
vmzCosh(n, a, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                                                                                         | Description                                                                                                                        |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                                | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsCosh, vmsCosh<br>const double* for vdCosh, vmdCosh<br>const MKL_Complex8* for vcCosh, vmcCosh<br>const MKL_Complex16* for vzCosh, vmzCosh | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                              | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Precision Overflow Thresholds for Real v?Cosh Function

| Data Type        | Threshold Limitations on Input Parameters                             |
|------------------|-----------------------------------------------------------------------|
| single precision | $-\ln(\text{FLT\_MAX}) - \ln 2 < a[i] < \ln(\text{FLT\_MAX}) + \ln 2$ |
| double precision | $-\ln(\text{DBL\_MAX}) - \ln 2 < a[i] < \ln(\text{DBL\_MAX}) + \ln 2$ |

Precision overflow thresholds for the complex v?Cosh function are beyond the scope of this document.

## Output Parameters

| Name     | Type                                                                                                                                 | Description                                                    |
|----------|--------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsCosh, vmsCosh<br>double* for vdCosh, vmdCosh<br>MKL_Complex8* for vcCosh, vmcCosh<br>MKL_Complex16* for vzCosh, vmzCosh | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The v?Cosh function computes hyperbolic cosine of vector elements.

## Special Values for Real Function v?Cosh(x)

| Argument      | Result    | VM Error Status     | Exception |
|---------------|-----------|---------------------|-----------|
| +0            | +1        |                     |           |
| -0            | +1        |                     |           |
| X > overflow  | $+\infty$ | VML_STATUS_OVERFLOW | OVERFLOW  |
| X < -overflow | $+\infty$ | VML_STATUS_OVERFLOW | OVERFLOW  |
| $+\infty$     | $+\infty$ |                     |           |

| Argument  | Result    | VM Error Status | Exception |
|-----------|-----------|-----------------|-----------|
| $-\infty$ | $+\infty$ |                 |           |
| QNAN      | QNAN      |                 |           |
| SNAN      | QNAN      |                 | INVALID   |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function $v?Cosh(z)$

| RE(z)<br>i·IM(z)    | $-\infty$                                            | -X                     | -0                  | +0                  | +X                     | $+\infty$                        | NAN         |
|---------------------|------------------------------------------------------|------------------------|---------------------|---------------------|------------------------|----------------------------------|-------------|
| $+i\cdot\infty$     | $+\infty+i\cdot$ QNAN<br>INVALID                     | QNAN+i·QNAN<br>INVALID | QNAN-i·0<br>INVALID | QNAN+i·0<br>INVALID | QNAN+i·QNAN<br>INVALID | $+\infty+i\cdot$ QNAN<br>INVALID | QNAN+i·QNAN |
| $+i\cdot Y$         | $+\infty\cdot\cos(Y)-$<br>$i\cdot\infty\cdot\sin(Y)$ |                        |                     |                     |                        | $+\infty\cdot\text{CIS}(Y)$      | QNAN+i·QNAN |
| $+i\cdot 0$         | $+\infty-i\cdot 0$                                   |                        | $+1-i\cdot 0$       | $+1+i\cdot 0$       |                        | $+\infty+i\cdot 0$               | QNAN+i·0    |
| $-i\cdot 0$         | $+\infty+i\cdot 0$                                   |                        | $+1+i\cdot 0$       | $+1-i\cdot 0$       |                        | $+\infty-i\cdot 0$               | QNAN-i·0    |
| $-i\cdot Y$         | $+\infty\cdot\cos(Y)-$<br>$i\cdot\infty\cdot\sin(Y)$ |                        |                     |                     |                        | $+\infty\cdot\text{CIS}(Y)$      | QNAN+i·QNAN |
| $-i\cdot\infty$     | $+\infty+i\cdot$ QNAN<br>INVALID                     | QNAN+i·QNAN<br>INVALID | QNAN+i·0<br>INVALID | QNAN-i·0<br>INVALID | QNAN+i·QNAN<br>INVALID | $+\infty+i\cdot$ QNAN<br>INVALID | QNAN+i·QNAN |
| $+i\cdot\text{NAN}$ | $+\infty+i\cdot$ QNAN                                | QNAN+i·QNAN            | QNAN+i·QNAN         | QNAN-i·QNAN         | QNAN+i·QNAN            | $+\infty+i\cdot$ QNAN            | QNAN+i·QNAN |

Notes:

- raises the INVALID exception when the real or imaginary part of the argument is SNAN
- raises the OVERFLOW exception and sets the VM Error Status to VML\_STATUS\_OVERFLOW in the case of overflow, that is, when RE(z), IM(z) are finite non-zero numbers, but the real or imaginary part of the exact result is so large that it does not meet the target precision.
- $\text{Cosh}(\text{CONJ}(z)) = \text{CONJ}(\text{Cosh}(z))$
- $\text{Cosh}(-z) = \text{Cosh}(z)$ .

### $v?Sinh$

*Computes hyperbolic sine of vector elements.*

#### Syntax

```
vsSinh(n, a, y);
vmsSinh(n, a, y, mode);
vdSinh(n, a, y);
vmdSinh(n, a, y, mode);
vcSinh(n, a, y);
vmcSinh(n, a, y, mode);
vzSinh(n, a, y);
vmzSinh(n, a, y, mode);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name        | Type                                                                                                                                                                                                                                                                                                                         | Description                                                                                                                        |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | <code>const MKL_INT</code>                                                                                                                                                                                                                                                                                                   | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | <code>const float*</code> for <code>vsSinh</code> , <code>vmsSinh</code><br><code>const double*</code> for <code>vdSinh</code> , <code>vmdSinh</code><br><code>const MKL_Complex8*</code> for <code>vcSinh</code> , <code>vmcSinh</code><br><code>const MKL_Complex16*</code> for <code>vzSinh</code> , <code>vmzSinh</code> | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | <code>const MKL_INT64</code>                                                                                                                                                                                                                                                                                                 | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Precision Overflow Thresholds for Real $v?Sinh$ Function

| Data Type        | Threshold Limitations on Input Parameters                             |
|------------------|-----------------------------------------------------------------------|
| single precision | $-\ln(\text{FLT\_MAX}) - \ln 2 < a[i] < \ln(\text{FLT\_MAX}) + \ln 2$ |
| double precision | $-\ln(\text{DBL\_MAX}) - \ln 2 < a[i] < \ln(\text{DBL\_MAX}) + \ln 2$ |

Precision overflow thresholds for the complex  $v?Sinh$  function are beyond the scope of this document.

## Output Parameters

| Name     | Type                                                                                                                                                                                                                                                                                                 | Description                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | <code>float*</code> for <code>vsSinh</code> , <code>vmsSinh</code><br><code>double*</code> for <code>vdSinh</code> , <code>vmdSinh</code><br><code>MKL_Complex8*</code> for <code>vcSinh</code> , <code>vmcSinh</code><br><code>MKL_Complex16*</code> for <code>vzSinh</code> , <code>vmzSinh</code> | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The  $v?Sinh$  function computes hyperbolic sine of vector elements.

## Special Values for Real Function $v?Sinh(x)$

| Argument               | Result    | VM Error Status     | Exception |
|------------------------|-----------|---------------------|-----------|
| +0                     | +0        |                     |           |
| -0                     | -0        |                     |           |
| $X > \text{overflow}$  | $+\infty$ | VML_STATUS_OVERFLOW | OVERFLOW  |
| $X < \text{-overflow}$ | $-\infty$ | VML_STATUS_OVERFLOW | OVERFLOW  |
| $+\infty$              | $+\infty$ |                     |           |
| $-\infty$              | $-\infty$ |                     |           |
| QNAN                   | QNAN      |                     |           |
| SNAN                   | QNAN      |                     | INVALID   |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function $v?Sinh(z)$

| <b>RE(z)</b><br><b>i·IM(z)</b> | <b>-∞</b>                | <b>-X</b>              | <b>-0</b>            | <b>+0</b>            | <b>+X</b>              | <b>+∞</b>            | <b>NAN</b>  |
|--------------------------------|--------------------------|------------------------|----------------------|----------------------|------------------------|----------------------|-------------|
| <b>+i·∞</b>                    | -∞+i·QNAN<br>INVALID     | QNAN+i·QNAN<br>INVALID | -0+i·QNAN<br>INVALID | +0+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | +∞+i·QNAN<br>INVALID | QNAN+i·QNAN |
| <b>+i·Y</b>                    | -∞·Cos(Y)+<br>i·∞·Sin(Y) |                        |                      |                      |                        | +∞·CIS(Y)            | QNAN+i·QNAN |
| <b>+i·0</b>                    | -∞+i·0                   |                        | -0+i·0               | +0+i·0               |                        | +∞+i·0               | QNAN+i·0    |
| <b>-i·0</b>                    | -∞-i·0                   |                        | -0-i·0               | +0-i·0               |                        | +∞-i·0               | QNAN-i·0    |
| <b>-i·Y</b>                    | -∞·Cos(Y)+<br>i·∞·Sin(Y) |                        |                      |                      |                        | +∞·CIS(Y)            | QNAN+i·QNAN |
| <b>-i·∞</b>                    | -∞+i·QNAN<br>INVALID     | QNAN+i·QNAN<br>INVALID | -0+i·QNAN<br>INVALID | +0+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | +∞+i·QNAN<br>INVALID | QNAN+i·QNAN |
| <b>+i·NAN</b>                  | -∞+i·QNAN                | QNAN+i·QNAN            | -0+i·QNAN            | +0+i·QNAN            | QNAN+i·QNAN            | +∞+i·QNAN            | QNAN+i·QNAN |

**Notes:**

- raises the `INVALID` exception when the real or imaginary part of the argument is `SNAN`
- raises the `OVERFLOW` exception and sets the VM Error Status to `VML_STATUS_OVERFLOW` in the case of overflow, that is, when `RE(z)`, `IM(z)` are finite non-zero numbers, but the real or imaginary part of the exact result is so large that it does not meet the target precision.
- $Sinh(CONJ(z)) = CONJ(Sinh(z))$
- $Sinh(-z) = -Sinh(z)$ .

### $v?Tanh$

*Computes hyperbolic tangent of vector elements.*

#### Syntax

```
vsTanh(n, a, y);
vmsTanh(n, a, y, mode);
vdTanh(n, a, y);
vmdTanh(n, a, y, mode);
vcTanh(n, a, y);
vmcTanh(n, a, y, mode);
vzTanh(n, a, y);
vmzTanh(n, a, y, mode);
```

#### Include Files

- `mk1.h`

## Input Parameters

| Name        | Type                                                                                                                                                         | Description                                                                                                                        |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                                | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsTanh, vmsTanh<br>const double* for vdTanh, vmdTanh<br>const MKL_Complex8* for vcTanh, vmcTanh<br>const MKL_Complex16* for vzTanh, vmzTanh | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                              | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                                 | Description                                                    |
|----------|--------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsTanh, vmsTanh<br>double* for vdTanh, vmdTanh<br>MKL_Complex8* for vcTanh, vmcTanh<br>MKL_Complex16* for vzTanh, vmzTanh | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The  $v?Tanh$  function computes hyperbolic tangent of vector elements.

### Special Values for Real Function $v?Tanh(x)$

| Argument  | Result | Exception |
|-----------|--------|-----------|
| +0        | +0     |           |
| -0        | -0     |           |
| $+\infty$ | +1     |           |
| $-\infty$ | -1     |           |
| QNAN      | QNAN   |           |
| SNAN      | QNAN   | INVALID   |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function $v?Tanh(z)$

| RE(z)<br>i·IM(z) | $-\infty$     | -X                     | -0                     | +0                     | +X                     | $+\infty$     | NAN         |
|------------------|---------------|------------------------|------------------------|------------------------|------------------------|---------------|-------------|
| $+i\cdot\infty$  | -1+i·0        | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | +1+i·0        | QNAN+i·QNAN |
| $+i\cdot Y$      | -1+i·0·Tan(Y) |                        |                        |                        |                        | +1+i·0·Tan(Y) | QNAN+i·QNAN |
| $+i\cdot 0$      | -1+i·0        |                        | -0+i·0                 | +0+i·0                 |                        | +1+i·0        | QNAN+i·0    |

| <b>RE(z)</b><br><b>i·IM(z)</b> | <b>-∞</b>     | <b>-X</b>              | <b>-0</b>              | <b>+0</b>              | <b>+X</b>              | <b>+∞</b>     | <b>NAN</b>  |
|--------------------------------|---------------|------------------------|------------------------|------------------------|------------------------|---------------|-------------|
| <b>-i·0</b>                    | -1-i·0        |                        | -0-i·0                 | +0-i·0                 |                        | +1-i·0        | QNAN-i·0    |
| <b>-i·Y</b>                    | -1+i·0·Tan(Y) |                        |                        |                        |                        | +1+i·0·Tan(Y) | QNAN+i·QNAN |
| <b>-i·∞</b>                    | -1-i·0        | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | QNAN+i·QNAN<br>INVALID | +1-i·0        | QNAN+i·QNAN |
| <b>+i·NAN</b>                  | -1+i·0        | QNAN+i·QNAN            | QNAN+i·QNAN            | QNAN+i·QNAN            | QNAN+i·QNAN            | +1+i·0        | QNAN+i·QNAN |

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- $\text{Tanh}(\text{CONJ}(z)) = \text{CONJ}(\text{Tanh}(z))$
- $\text{Tanh}(-z) = -\text{Tanh}(z)$ .

### v?Acosh

Computes inverse hyperbolic cosine (nonnegative) of vector elements.

#### Syntax

```
vsAcosh(n, a, y);
vmsAcosh(n, a, y, mode);
vdAcosh(n, a, y);
vmdAcosh(n, a, y, mode);
vcAcosh(n, a, y);
vmcAcosh(n, a, y, mode);
vzAcosh(n, a, y);
vmzAcosh(n, a, y, mode);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| <b>Name</b>    | <b>Type</b>                                                                                                                                                                                                                                                                                                                                                  | <b>Description</b>                                                  |
|----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|
| <code>n</code> | <code>const MKL_INT</code>                                                                                                                                                                                                                                                                                                                                   | Specifies the number of elements to be calculated.                  |
| <code>a</code> | <code>const float*</code> for <code>vsAcosh</code> ,<br><code>vmsAcosh</code><br><br><code>const double*</code> for <code>vdAcosh</code> ,<br><code>vmdAcosh</code><br><br><code>const MKL_Complex8*</code> for <code>vcAcosh</code> ,<br><code>vmcAcosh</code><br><br><code>const MKL_Complex16*</code> for <code>vzAcosh</code> ,<br><code>vmzAcosh</code> | Pointer to an array that contains the input vector <code>a</code> . |

| Name              | Type                         | Description                                                                                                                        |
|-------------------|------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>mode</code> | <code>const MKL_INT64</code> | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name           | Type                                                                                                                                                                                                                                                                                                               | Description                                                          |
|----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsAcosh</code> , <code>vmsAcosh</code><br><code>double*</code> for <code>vdAcosh</code> , <code>vmdAcosh</code><br><code>MKL_Complex8*</code> for <code>vcAcosh</code> ,<br><code>vmcAcosh</code><br><code>MKL_Complex16*</code> for <code>vzAcosh</code> ,<br><code>vmzAcosh</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The `v?Acosh` function computes inverse hyperbolic cosine (nonnegative) of vector elements.

### Special Values for Real Function `v?Acosh(x)`

| Argument               | Result            | VM Error Status                | Exception            |
|------------------------|-------------------|--------------------------------|----------------------|
| <code>+1</code>        | <code>+0</code>   |                                |                      |
| <code>X &lt; +1</code> | <code>QNAN</code> | <code>VML_STATUS_ERRDOM</code> | <code>INVALID</code> |
| <code>-∞</code>        | <code>QNAN</code> | <code>VML_STATUS_ERRDOM</code> | <code>INVALID</code> |
| <code>+∞</code>        | <code>+∞</code>   |                                |                      |
| <code>QNAN</code>      | <code>QNAN</code> |                                |                      |
| <code>SNAN</code>      | <code>QNAN</code> |                                | <code>INVALID</code> |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function `v?Acosh(z)`

| RE(z)<br>i·IM(z)    | -∞                                                | -X                       | -0                       | +0                       | +X                       | +∞                     | NAN                      |
|---------------------|---------------------------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|------------------------|--------------------------|
| <code>+i·∞</code>   | <code>+∞ + i · <math>\frac{3\pi}{4}</math></code> | <code>+∞+i·π/2</code>    | <code>+∞+i·π/2</code>    | <code>+∞+i·π/2</code>    | <code>+∞+i·π/2</code>    | <code>+∞+i·π/4</code>  | <code>+∞+i·QNAN</code>   |
| <code>+i·Y</code>   | <code>+∞+i·π</code>                               |                          |                          |                          |                          | <code>+∞+i·0</code>    | <code>QNAN+i·QNAN</code> |
| <code>+i·0</code>   | <code>+∞+i·π</code>                               |                          | <code>+0+i·π/2</code>    | <code>+0+i·π/2</code>    |                          | <code>+∞+i·0</code>    | <code>QNAN+i·QNAN</code> |
| <code>-i·0</code>   | <code>+∞+i·π</code>                               |                          | <code>+0+i·π/2</code>    | <code>+0+i·π/2</code>    |                          | <code>+∞+i·0</code>    | <code>QNAN+i·QNAN</code> |
| <code>-i·Y</code>   | <code>+∞+i·π</code>                               |                          |                          |                          |                          | <code>+∞+i·0</code>    | <code>QNAN+i·QNAN</code> |
| <code>-i·∞</code>   | <code>+∞ - i · <math>\frac{3\pi}{4}</math></code> | <code>+∞-i·π/2</code>    | <code>+∞-i·π/2</code>    | <code>+∞-i·π/2</code>    | <code>+∞-i·π/2</code>    | <code>+∞-i·π/4</code>  | <code>+∞+i·QNAN</code>   |
| <code>+i·NAN</code> | <code>+∞+i·QNAN</code>                            | <code>QNAN+i·QNAN</code> | <code>QNAN+i·QNAN</code> | <code>QNAN+i·QNAN</code> | <code>QNAN+i·QNAN</code> | <code>+∞+i·QNAN</code> | <code>QNAN+i·QNAN</code> |

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- `Acosh (CONJ (z)) =CONJ (Acosh (z))`.

## v?Asinh

Computes inverse hyperbolic sine of vector elements.

### Syntax

```

vsAsinh(n, a, y);
vmsAsinh(n, a, y, mode);
vdAsinh(n, a, y);
vmdAsinh(n, a, y, mode);
vcAsinh(n, a, y);
vmcAsinh(n, a, y, mode);
vzAsinh(n, a, y);
vmzAsinh(n, a, y, mode);

```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type                                                                                                                                                                                         | Description                                                                                                                        |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                                                                | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsAsinh,<br>vmsAsinh<br><br>const double* for vdAsinh,<br>vmdAsinh<br><br>const MKL_Complex8* for vcAsinh,<br>vmcAsinh<br><br>const MKL_Complex16* for vzAsinh,<br>vmzAsinh | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                                                              | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Output Parameters

| Name     | Type                                                                                                                                                           | Description                                                    |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsAsinh, vmsAsinh<br><br>double* for vdAsinh, vmdAsinh<br><br>MKL_Complex8* for vcAsinh,<br>vmcAsinh<br><br>MKL_Complex16* for vzAsinh,<br>vmzAsinh | Pointer to an array that contains the output vector <i>y</i> . |

### Description

The v?Asinh function computes inverse hyperbolic sine of vector elements.

### Special Values for Real Function v?Asinh(x)

| Argument | Result | Exception |
|----------|--------|-----------|
| +0       | +0     |           |
| -0       | -0     |           |
| +∞       | +∞     |           |
| -∞       | -∞     |           |
| QNAN     | QNAN   |           |
| SNAN     | QNAN   | INVALID   |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function v?Asinh(z)

| RE(z)<br>i·IM(z) | -∞        | -X              | -0              | +0              | +X              | +∞        | NAN         |
|------------------|-----------|-----------------|-----------------|-----------------|-----------------|-----------|-------------|
| +i·∞             | -∞+i·π/4  | -∞+i·π/2        | +∞+i·π/2        | +∞+i·π/2        | +∞+i·π/2        | +∞+i·π/4  | +∞+i·QNAN   |
| +i·Y             | -∞+i·0    |                 |                 |                 |                 | +∞+i·0    | QNAN+i·QNAN |
| +i·0             | +∞+i·0    |                 | +0+i·0          | +0+i·0          |                 | +∞+i·0    | QNAN+i·QNAN |
| -i·0             | -∞-i·0    |                 | -0-i·0          | +0-i·0          |                 | +∞-i·0    | QNAN-i·QNAN |
| -i·Y             | -∞-i·0    |                 |                 |                 |                 | +∞-i·0    | QNAN+i·QNAN |
| -i·∞             | -∞-i·π/4  | -∞-i·π/2        | -∞-i·π/2        | +∞-i·π/2        | +∞-i·π/2        | +∞-i·π/4  | +∞+i·QNAN   |
| +i·NAN           | -∞+i·QNAN | QNAN<br>+i·QNAN | QNAN<br>+i·QNAN | QNAN<br>+i·QNAN | QNAN<br>+i·QNAN | +∞+i·QNAN | QNAN+i·QNAN |

Notes:

- raises `INVALID` exception when real or imaginary part of the argument is `SNAN`
- $\text{Asinh}(\text{CONJ}(z)) = \text{CONJ}(\text{Asinh}(z))$
- $\text{Asinh}(-z) = -\text{Asinh}(z)$ .

### v?Atanh

Computes inverse hyperbolic tangent of vector elements.

#### Syntax

```
vsAtanh(n, a, y);
vmsAtanh(n, a, y, mode);
vdAtanh(n, a, y);
vmdAtanh(n, a, y, mode);
vcAtanh(n, a, y);
vmcAtanh(n, a, y, mode);
vzAtanh(n, a, y);
vmzAtanh(n, a, y, mode);
```

#### Include Files

- `mk1.h`

## Input Parameters

| Name        | Type                                                                                                                                                                                         | Description                                                                                                                        |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                                                                                                                                | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsAtanh,<br>vmsAtanh<br><br>const double* for vdAtanh,<br>vmdAtanh<br><br>const MKL_Complex8* for vcAtanh,<br>vmcAtanh<br><br>const MKL_Complex16* for vzAtanh,<br>vmzAtanh | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                                                                                                              | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                                                       | Description                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsAtanh, vmsAtanh<br>double* for vdAtanh, vmdAtanh<br><br>MKL_Complex8* for vcAtanh,<br>vmcAtanh<br><br>MKL_Complex16* for vzAtanh,<br>vmzAtanh | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?Atanh` function computes inverse hyperbolic tangent of vector elements.

### Special Values for Real Function `v?Atanh(x)`

| Argument  | Result    | VM Error Status   | Exception  |
|-----------|-----------|-------------------|------------|
| +1        | $+\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| -1        | $-\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| $ X  > 1$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $+\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| QNAN      | QNAN      |                   |            |
| SNAN      | QNAN      |                   | INVALID    |

See the [Special Value Notations](#) section for the conventions used in the table below.

### Special Values for Complex Function `v?Atanh(z)`

| RE(z)<br>i·IM(z) | $-\infty$        | -X               | -0               | +0               | +X               | $+\infty$        | NAN              |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| $+i\cdot\infty$  | $-0+i\cdot\pi/2$ | $-0+i\cdot\pi/2$ | $-0+i\cdot\pi/2$ | $+0+i\cdot\pi/2$ | $+0+i\cdot\pi/2$ | $+0+i\cdot\pi/2$ | $+0+i\cdot\pi/2$ |

| <b>RE(z)<br/>i·IM(z)</b> | <b>-∞</b> | <b>-X</b>       | <b>-0</b> | <b>+0</b> | <b>+X</b>       | <b>+∞</b> | <b>NAN</b>  |
|--------------------------|-----------|-----------------|-----------|-----------|-----------------|-----------|-------------|
| +i·Y                     | -0+i·π/2  |                 |           |           |                 | +0+i·π/2  | QNAN+i·QNAN |
| +i·0                     | -0+i·π/2  |                 | -0+i·0    | +0+i·0    |                 | +0+i·π/2  | QNAN+i·QNAN |
| -i·0                     | -0-i·π/2  |                 | -0-i·0    | +0-i·0    |                 | +0-i·π/2  | QNAN-i·QNAN |
| -i·Y                     | -0-i·π/2  |                 |           |           |                 | +0-i·π/2  | QNAN+i·QNAN |
| -i·∞                     | -0-i·π/2  | -0-i·π/2        | -0-i·π/2  | +0-i·π/2  | +0-i·π/2        | +0-i·π/2  | +0-i·π/2    |
| +i·NAN                   | -0+i·QNAN | QNAN<br>+i·QNAN | -0+i·QNAN | +0+i·QNAN | QNAN<br>+i·QNAN | +0+i·QNAN | QNAN+i·QNAN |

Notes:

- $\text{Atanh} (+-1+-i*0) = +-∞+-i*0$ , and ZERODIVIDE exception is raised
- raises INVALID exception when real or imaginary part of the argument is SNAN
- $\text{Atanh}(\text{CONJ}(z)) = \text{CONJ}(\text{Atanh}(z))$
- $\text{Atanh}(-z) = -\text{Atanh}(z)$ .

## Special Functions

### v?Erf

*Computes the error function value of vector elements.*

#### Syntax

```
vsErf(n, a, y);
vmsErf(n, a, y, mode);
vdErf(n, a, y);
vmdErf(n, a, y, mode);
```

#### Include Files

- mkl.h

#### Input Parameters

| <b>Name</b> | <b>Type</b>                                                       | <b>Description</b>                                                                                                                 |
|-------------|-------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                     | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsErf, vmsErf<br>const double* for vdErf, vmdErf | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                   | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name | Type                                                  | Description                                               |
|------|-------------------------------------------------------|-----------------------------------------------------------|
| $y$  | float* for vsErf, vmsErf<br>double* for vdErf, vmdErf | Pointer to an array that contains the output vector $y$ . |

## Description

The `erf` function computes the error function values for elements of the input vector  $a$  and writes them to the output vector  $y$ .

The error function is defined as given by:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt .$$

Useful relations:

$$1. \quad \text{erfc}(x) = 1 - \text{erf}(x) ,$$

where `erfc` is the complementary error function.

$$2. \quad \Phi(x) = \frac{1}{2} \text{erf} (x/\sqrt{2}) ,$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp ( - t^2/2) dt$$

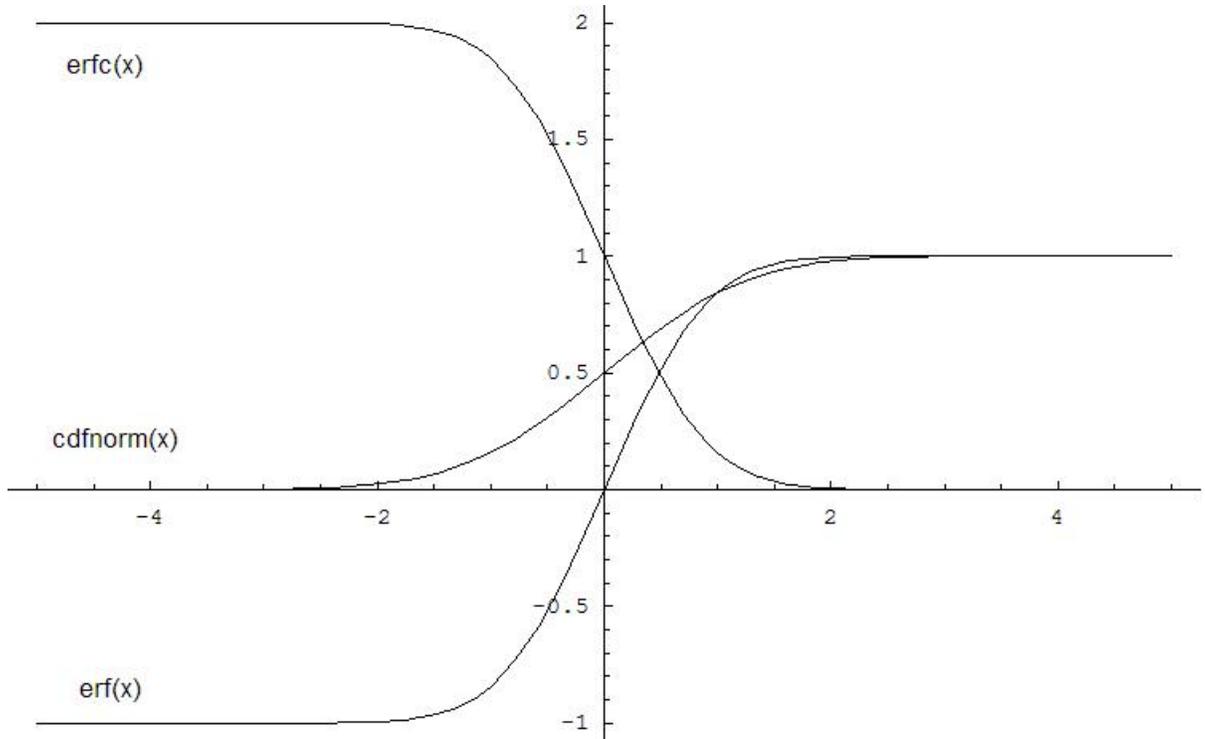
is the cumulative normal distribution function.

$$3. \quad \Phi^{-1}(x) = \sqrt{2} \text{erf}^{-1}(2x - 1) ,$$

where  $\Phi^{-1}(x)$  and  $\text{erf}^{-1}(x)$  are the inverses to  $\Phi(x)$  and  $\text{erf}(x)$  respectively.

The following figure illustrates the relationships among `Erf` family functions (`Erf`, `Erfc`, `CdfNorm`).

## Erf Family Functions Relationship



Useful relations for these functions:

$$\text{erf}(x) + \text{erfc}(x) = 1$$

$$\text{cdfnorm}(x) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right) = 1 - \frac{1}{2} \text{erfc} \left( \frac{x}{\sqrt{2}} \right)$$

### Special Values for Real Function v?Erf(x)

| Argument  | Result | Exception |
|-----------|--------|-----------|
| $+\infty$ | +1     |           |
| $-\infty$ | -1     |           |
| QNAN      | QNAN   |           |
| SNAN      | QNAN   | INVALID   |

### See Also

v?Erfc  
v?CdfNorm

### v?Erfc

Computes the complementary error function value of vector elements.

### Syntax

```
vsErfc(n, a, y);
vmsErfc(n, a, y, mode);
vdErfc(n, a, y);
vmdErfc(n, a, y, mode);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name   | Type                                                                                                                                                  | Description                                                                                                                        |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| $n$    | <code>const MKL_INT</code>                                                                                                                            | Specifies the number of elements to be calculated.                                                                                 |
| $a$    | <code>const float*</code> for <code>vsErfc</code> , <code>vmsErfc</code><br><code>const double*</code> for <code>vdErfc</code> , <code>vmdErfc</code> | Pointer to an array that contains the input vector $a$ .                                                                           |
| $mode$ | <code>const MKL_INT64</code>                                                                                                                          | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name | Type                                                                                                                                      | Description                                               |
|------|-------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------|
| $y$  | <code>float*</code> for <code>vsErfc</code> , <code>vmsErfc</code><br><code>double*</code> for <code>vdErfc</code> , <code>vmdErfc</code> | Pointer to an array that contains the output vector $y$ . |

## Description

The `Erfc` function computes the complementary error function values for elements of the input vector  $a$  and writes them to the output vector  $y$ .

The complementary error function is defined as follows:

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt .$$

Useful relations:

1.  $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$ .

2.  $\Phi(x) = \frac{1}{2} \operatorname{erf}(x/\sqrt{2})$ ,

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp(-t^2/2) dt$$

is the cumulative normal distribution function.

3.  $\Phi^{-1}(x) = \sqrt{2} \operatorname{erf}^{-1}(2x - 1)$ ,

where  $\Phi^{-1}(x)$  and  $\operatorname{erf}^{-1}(x)$  are the inverses to  $\Phi(x)$  and  $\operatorname{erf}(x)$  respectively.

See also [Figure "Erf Family Functions Relationship"](#) in `Erf` function description for `Erfc` function relationship with the other functions of `Erf` family.

## Special Values for Real Function `v?Erfc(x)`

| Argument        | Result | VM Error Status      | Exception |
|-----------------|--------|----------------------|-----------|
| $X >$ underflow | +0     | VML_STATUS_UNDERFLOW | UNDERFLOW |
| $+\infty$       | +0     |                      |           |
| $-\infty$       | +2     |                      |           |
| QNAN            | QNAN   |                      |           |
| SNAN            | QNAN   |                      | INVALID   |

## See Also

[v?Erf](#)

[v?CdfNorm](#)

## `v?CdfNorm`

*Computes the cumulative normal distribution function values of vector elements.*

## Syntax

```
vsCdfNorm(n, a, y);
```

```
vmsCdfNorm(n, a, y, mode);
```

```
vdCdfNorm(n, a, y);
```

```
vmdCdfNorm(n, a, y, mode);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name        | Type                                                                                                                                                                        | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | <code>const MKL_INT</code>                                                                                                                                                  | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | <code>const float*</code> for <code>vsCdfNorm</code> ,<br><code>vmsCdfNorm</code><br><br><code>const double*</code> for <code>vdCdfNorm</code> ,<br><code>vmdCdfNorm</code> | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | <code>const MKL_INT64</code>                                                                                                                                                | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                                                  | Description                                                    |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | <code>float*</code> for <code>vsCdfNorm</code> , <code>vmsCdfNorm</code><br><code>double*</code> for <code>vdCdfNorm</code> , <code>vmdCdfNorm</code> | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `CdfNorm` function computes the cumulative normal distribution function values for elements of the input vector  $a$  and writes them to the output vector  $y$ .

The cumulative normal distribution function is defined as given by:

$$\text{CdfNorm}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt .$$

Useful relations:

$$\text{cdfnorm}(x) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{x}{\sqrt{2}} \right) \right) = 1 - \frac{1}{2} \text{erfc} \left( \frac{x}{\sqrt{2}} \right)$$

where `Erf` and `Erfc` are the error and complementary error functions.

See also [Figure "Erf Family Functions Relationship"](#) in `Erf` function description for `CdfNorm` function relationship with the other functions of `Erf` family.

### Special Values for Real Function v?CdfNorm(x)

| Argument               | Result | VM Error Status      | Exception |
|------------------------|--------|----------------------|-----------|
| $X < \text{underflow}$ | +0     | VML_STATUS_UNDERFLOW | UNDERFLOW |
| $+\infty$              | +1     |                      |           |
| $-\infty$              | +0     |                      |           |
| QNAN                   | QNAN   |                      |           |
| SNAN                   | QNAN   |                      | INVALID   |

### See Also

v?Erf

v?Erfc

### v?ErfInv

*Computes inverse error function value of vector elements.*

### Syntax

```
vsErfInv(n, a, y);
```

```
vmsErfInv(n, a, y, mode);
```

```
vdErfInv(n, a, y);
```

```
vmdErfInv(n, a, y, mode);
```

### Include Files

- `mk1.h`

## Input Parameters

| Name        | Type                                                                                    | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                           | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsErfInv,<br>vmsErfInv<br><br>const double* for vdErfInv,<br>vmdErfInv | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                         | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                              | Description                                                    |
|----------|-------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsErfInv, vmsErfInv<br>double* for vdErfInv, vmdErfInv | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `ErfInv` function computes the inverse error function values for elements of the input vector *a* and writes them to the output vector *y*

$$y = \text{erf}^{-1}(a),$$

where `erf(x)` is the error function defined as given by:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt .$$

Useful relations:

- $\text{erf}^{-1}(x) = \text{erfc}^{-1}(1 - x),$

where `erfc` is the complementary error function.

- $\Phi(x) = \frac{1}{2} \text{erf}(x/\sqrt{2}),$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x \exp(-t^2/2) dt$$

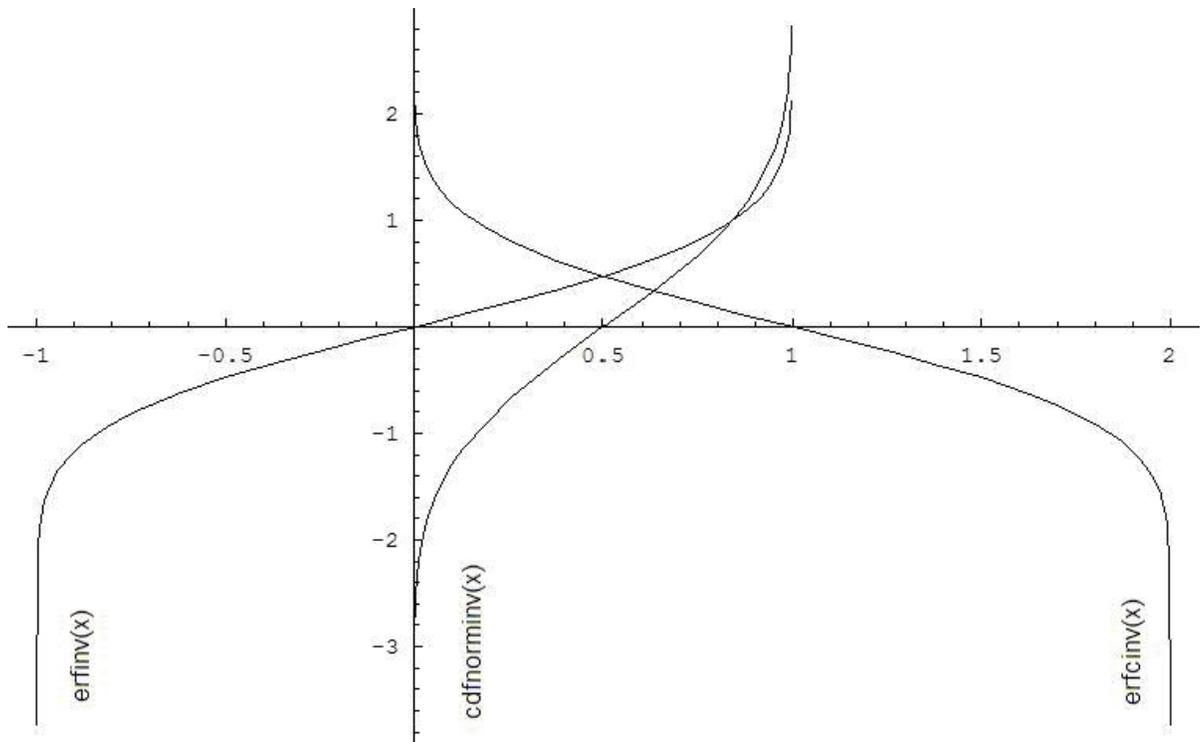
is the cumulative normal distribution function.

$$3. \Phi^{-1}(x) = \sqrt{2} \operatorname{erf}^{-1}(2x - 1),$$

where  $\Phi^{-1}(x)$  and  $\operatorname{erf}^{-1}(x)$  are the inverses to  $\Phi(x)$  and  $\operatorname{erf}(x)$  respectively.

Figure "ErfInv Family Functions Relationship" illustrates the relationships among ErfInv family functions (ErfInv, ErfcInv, CdfNormInv).

**ErfInv Family Functions Relationship**



Useful relations for these functions:

$$\operatorname{erfcinv}(x) = \operatorname{erfinv}(1 - x)$$

$$\operatorname{cdfnorminv}(x) = \sqrt{2}\operatorname{erfinv}(2x - 1) = \sqrt{2}\operatorname{erfcinv}(2 - 2x)$$

**Special Values for Real Function v?ErfInv(x)**

| Argument  | Result    | VM Error Status   | Exception  |
|-----------|-----------|-------------------|------------|
| +0        | +0        |                   |            |
| -0        | -0        |                   |            |
| +1        | $+\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| -1        | $-\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| $ X  > 1$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $+\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| QNAN      | QNAN      |                   |            |

| Argument | Result | VM Error Status | Exception |
|----------|--------|-----------------|-----------|
| SNAN     | QNAN   |                 | INVALID   |

## See Also

[v?ErfcInv](#)

[v?CdfNormInv](#)

## v?ErfcInv

Computes the inverse complementary error function value of vector elements.

## Syntax

```
vsErfcInv(n, a, y);
```

```
vmsErfcInv(n, a, y, mode);
```

```
vdErfcInv(n, a, y);
```

```
vmdErfcInv(n, a, y, mode);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name        | Type                                                                                                                                                                        | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | <code>const MKL_INT</code>                                                                                                                                                  | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | <code>const float*</code> for <code>vsErfcInv</code> ,<br><code>vmsErfcInv</code><br><br><code>const double*</code> for <code>vdErfcInv</code> ,<br><code>vmdErfcInv</code> | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | <code>const MKL_INT64</code>                                                                                                                                                | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                                                      | Description                                                    |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | <code>float*</code> for <code>vsErfcInv</code> , <code>vmsErfcInv</code><br><br><code>double*</code> for <code>vdErfcInv</code> , <code>vmdErfcInv</code> | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `ErfcInv` function computes the inverse complimentary error function values for elements of the input vector *a* and writes them to the output vector *y*.

The inverse complementary error function is defined as given by:

$$\text{erfcinv}(x) = \text{erfinv}(1 - x)$$

$$\operatorname{erfinv}(x) = \operatorname{erf}^{-1}(x)$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

where  $\operatorname{erf}(x)$  denotes the error function and  $\operatorname{erfinv}(x)$  denotes the inverse error function.

See also [Figure "ErfInv Family Functions Relationship"](#) in `ErfInv` function description for `ErfcInv` function relationship with the other functions of `ErfInv` family.

### Special Values for Real Function `v?ErfcInv(x)`

| Argument  | Result    | VM Error Status   | Exception  |
|-----------|-----------|-------------------|------------|
| +1        | +0        |                   |            |
| +2        | $-\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| -0        | $+\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| +0        | $+\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| $X < -0$  | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $X > +2$  | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $+\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| QNAN      | QNAN      |                   |            |
| SNAN      | QNAN      |                   | INVALID    |

### See Also

[v?ErfInv](#)

[v?CdfNormInv](#)

### `v?CdfNormInv`

*Computes the inverse cumulative normal distribution function values of vector elements.*

### Syntax

```
vsCdfNormInv(n, a, y);
```

```
vmsCdfNormInv(n, a, y, mode);
```

```
vdCdfNormInv(n, a, y);
```

```
vmdCdfNormInv(n, a, y, mode);
```

### Include Files

- `mk1.h`

### Input Parameters

| Name           | Type                       | Description                                        |
|----------------|----------------------------|----------------------------------------------------|
| <code>n</code> | <code>const MKL_INT</code> | Specifies the number of elements to be calculated. |

| Name        | Type                                                                                                    | Description                                                                                                                        |
|-------------|---------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>    | const float* for vsCdfNormInv,<br>vmsCdfNormInv<br><br>const double* for vdCdfNormInv,<br>vmdCdfNormInv | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                                         | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                        | Description                                                    |
|----------|---------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsCdfNormInv,<br>vmsCdfNormInv<br><br>double* for vdCdfNormInv,<br>vmdCdfNormInv | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `CdfNormInv` function computes the inverse cumulative normal distribution function values for elements of the input vector *a* and writes them to the output vector *y*.

The inverse cumulative normal distribution function is defined as given by:

$$\text{CdfNormInv}(x) = \text{CdfNorm}^{-1}(x) ,$$

where `CdfNorm(x)` denotes the cumulative normal distribution function.

Useful relations:

$$\text{cdfnorminv}(x) = \sqrt{2}\text{erfinv}(2x - 1) = \sqrt{2}\text{erfcinv}(2 - 2x)$$

where `erfinv(x)` denotes the inverse error function and `erfcinv(x)` denotes the inverse complementary error functions.

See also [Figure "ErfInv Family Functions Relationship"](#) in `ErfInv` function description for `CdfNormInv` function relationship with the other functions of `ErfInv` family.

## Special Values for Real Function v?CdfNormInv(x)

| Argument  | Result    | VM Error Status   | Exception  |
|-----------|-----------|-------------------|------------|
| +0.5      | +0        |                   |            |
| +1        | $+\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| -0        | $-\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| +0        | $-\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| $X < -0$  | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $X > +1$  | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $+\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |

| Argument | Result | VM Error Status | Exception |
|----------|--------|-----------------|-----------|
| QNaN     | QNaN   |                 |           |
| SNAN     | QNaN   |                 | INVALID   |

## See Also

[v?ErfInv](#)

[v?ErfcInv](#)

## v?LGamma

Computes the natural logarithm of the absolute value of gamma function for vector elements.

## Syntax

```
vsLGamma(n, a, y);
```

```
vmsLGamma(n, a, y, mode);
```

```
vdLGamma(n, a, y);
```

```
vmdLGamma(n, a, y, mode);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name        | Type                                                                                                                                                                    | Description                                                                                                                        |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | <code>const MKL_INT</code>                                                                                                                                              | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | <code>const float*</code> for <code>vsLGamma</code> ,<br><code>vmsLGamma</code><br><br><code>const double*</code> for <code>vdLGamma</code> ,<br><code>vmdLGamma</code> | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | <code>const MKL_INT64</code>                                                                                                                                            | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                                                  | Description                                                    |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | <code>float*</code> for <code>vsLGamma</code> , <code>vmsLGamma</code><br><br><code>double*</code> for <code>vdLGamma</code> , <code>vmdLGamma</code> | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?LGamma` function computes the natural logarithm of the absolute value of gamma function for elements of the input vector *a* and writes them to the output vector *y*. Precision overflow thresholds for the `v?LGamma` function are beyond the scope of this document. If the result does not meet the target precision, the function raises the `OVERFLOW` exception and sets the VM Error Status to `VML_STATUS_OVERFLOW`.

## Special Values for Real Function v?LGamma(x)

| Argument         | Result | VM Error Status     | Exception  |
|------------------|--------|---------------------|------------|
| +1               | +0     |                     |            |
| +2               | +0     |                     |            |
| +0               | +?     | VML_STATUS_SING     | ZERODIVIDE |
| -0               | +?     | VML_STATUS_SING     | ZERODIVIDE |
| negative integer | +?     | VML_STATUS_SING     | ZERODIVIDE |
| -?               | +?     |                     |            |
| +?               | +?     |                     |            |
| X > overflow     | +?     | VML_STATUS_OVERFLOW | OVERFLOW   |
| QNAN             | QNAN   |                     |            |
| SNAN             | QNAN   |                     | INVALID    |

## v?TGamma

*Computes the gamma function of vector elements.*

### Syntax

```
vsTGamma(n, a, y);
```

```
vmsTGamma(n, a, y, mode);
```

```
vdTGamma(n, a, y);
```

```
vmdTGamma(n, a, y, mode);
```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type                                                                                    | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                           | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsTGamma,<br>vmsTGamma<br><br>const double* for vdTGamma,<br>vmdTGamma | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                         | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Output Parameters

| Name     | Type                                                              | Description                                                    |
|----------|-------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsTGamma, vmsTGamma<br>double* for vdTGamma, vmdTGamma | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?TGamma` function computes the gamma function for elements of the input vector  $a$  and writes them to the output vector  $y$ . Precision overflow thresholds for the `v?TGamma` function are beyond the scope of this document. If the result does not meet the target precision, the function raises the `OVERFLOW` exception and sets the VM Error Status to `VML_STATUS_OVERFLOW`.

### Special Values for Real Function `v?TGamma(x)`

| Argument              | Result    | VM Error Status                  | Exception  |
|-----------------------|-----------|----------------------------------|------------|
| +0                    | $+\infty$ | <code>VML_STATUS_SING</code>     | ZERODIVIDE |
| -0                    | $-\infty$ | <code>VML_STATUS_SING</code>     | ZERODIVIDE |
| negative integer      | QNAN      | <code>VML_STATUS_ERRDOM</code>   | INVALID    |
| $-\infty$             | QNAN      | <code>VML_STATUS_ERRDOM</code>   | INVALID    |
| $+\infty$             | $+\infty$ |                                  |            |
| $X > \text{overflow}$ | $+\infty$ | <code>VML_STATUS_OVERFLOW</code> | OVERFLOW   |
| QNAN                  | QNAN      |                                  |            |
| SNAN                  | QNAN      |                                  | INVALID    |

## `v?ExpInt1`

*Computes the exponential integral of vector elements.*

### Syntax

```
vsExpInt1(n, a, y);
vmsExpInt1(n, a, y, mode);
vdExpInt1(n, a, y);
vmdExpInt1(n, a, y, mode);
```

### Include Files

- `mk1.h`

### Input Parameters

| Name   | Type                                                                                                                                                                        | Description                                                                                                                        |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| $n$    | <code>const MKL_INT</code>                                                                                                                                                  | Specifies the number of elements to be calculated.                                                                                 |
| $a$    | <code>const float*</code> for <code>vsExpInt1</code> ,<br><code>vmsExpInt1</code><br><br><code>const double*</code> for <code>vdExpInt1</code> ,<br><code>vmdExpInt1</code> | Pointer to an array that contains the input vector $a$ .                                                                           |
| $mode$ | <code>const MKL_INT64</code>                                                                                                                                                | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

### Output Parameters

| Name | Type                                                                     | Description                                               |
|------|--------------------------------------------------------------------------|-----------------------------------------------------------|
| $y$  | <code>float*</code> for <code>vsExpInt1</code> , <code>vmsExpInt1</code> | Pointer to an array that contains the output vector $y$ . |

| Name | Type | Description |
|------|------|-------------|
|------|------|-------------|

double\* for vdExpInt1, vmdExpInt1

## Description

The v?ExpInt1 function computes the exponential integral  $E_1$  of vector elements.

For positive real values  $x$ , this can be written as:

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt = \int_1^\infty \frac{e^{-xt}}{t} dt.$$

For negative real values  $x$ , the result is defined as NAN.

### Special Values for Real Function v?ExpInt1(x)

| Argument  | Result    | VM Error Status   | Exception  |
|-----------|-----------|-------------------|------------|
| $x < +0$  | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| +0        | $+\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| -0        | $+\infty$ | VML_STATUS_SING   | ZERODIVIDE |
| $+\infty$ | +0        |                   |            |
| $-\infty$ | QNAN      | VML_STATUS_ERRDOM | INVALID    |
| QNAN      | QNAN      |                   |            |
| SNAN      | QNAN      |                   | INVALID    |

## Rounding Functions

### v?Floor

Computes an integer value rounded towards minus infinity for each vector element.

### Syntax

```
vsFloor(n, a, y);
vmsFloor(n, a, y, mode);
vdFloor(n, a, y);
vmdFloor(n, a, y, mode);
```

### Include Files

- mkl.h

### Input Parameters

| Name | Type                                                                                | Description                                              |
|------|-------------------------------------------------------------------------------------|----------------------------------------------------------|
| $n$  | const MKL_INT                                                                       | Specifies the number of elements to be calculated.       |
| $a$  | const float* for vsFloor,<br>vmsFloor<br><br>const double* for vdFloor,<br>vmdFloor | Pointer to an array that contains the input vector $a$ . |

| Name              | Type                         | Description                                                                                                                        |
|-------------------|------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>mode</code> | <code>const MKL_INT64</code> | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name           | Type                                                                                                                                          | Description                                                          |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsFloor</code> , <code>vmsFloor</code><br><code>double*</code> for <code>vdFloor</code> , <code>vmdFloor</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The function computes an integer value rounded towards minus infinity for each vector element.

$$y_i = \lfloor a_i \rfloor$$

## Special Values for Real Function `v?Floor(x)`

| Argument  | Result    | Exception |
|-----------|-----------|-----------|
| +0        | +0        |           |
| -0        | -0        |           |
| $+\infty$ | $+\infty$ |           |
| $-\infty$ | $-\infty$ |           |
| SNAN      | QNAN      | INVALID   |
| QNAN      | QNAN      |           |

## `v?Ceil`

*Computes an integer value rounded towards plus infinity for each vector element.*

## Syntax

```
vsCeil(n, a, y);
```

```
vmsCeil(n, a, y, mode);
```

```
vdCeil(n, a, y);
```

```
vmdCeil(n, a, y, mode);
```

## Include Files

- `mkL.h`

## Input Parameters

| Name           | Type                       | Description                                        |
|----------------|----------------------------|----------------------------------------------------|
| <code>n</code> | <code>const MKL_INT</code> | Specifies the number of elements to be calculated. |

| Name        | Type                                                                  | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>    | const float* for vsCeil, vmsCeil<br>const double* for vdCeil, vmdCeil | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                       | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                      | Description                                                    |
|----------|-----------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsCeil, vmsCeil<br>double* for vdCeil, vmdCeil | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The function computes an integer value rounded towards plus infinity for each vector element.

$$y_i = \lceil a_i \rceil$$

## Special Values for Real Function v?Ceil(x)

| Argument  | Result    | Exception |
|-----------|-----------|-----------|
| +0        | +0        |           |
| -0        | -0        |           |
| $+\infty$ | $+\infty$ |           |
| $-\infty$ | $-\infty$ |           |
| SNAN      | QNAN      | INVALID   |
| QNAN      | QNAN      |           |

## v?Trunc

Computes an integer value rounded towards zero for each vector element.

## Syntax

```
vsTrunc(n, a, y);
```

```
vmsTrunc(n, a, y, mode);
```

```
vdTrunc(n, a, y);
```

```
vmdTrunc(n, a, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                                | Description                                                                                                                        |
|-------------|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                                       | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsTrunc,<br>vmsTrunc<br><br>const double* for vdTrunc,<br>vmdTrunc | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                                     | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                              | Description                                                    |
|----------|-------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsTrunc, vmsTrunc<br><br>double* for vdTrunc, vmdTrunc | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The function computes an integer value rounded towards zero for each vector element.

$$a_i \geq 0, y_i = \lfloor a_i \rfloor$$

$$a_i < 0, y_i = \lceil a_i \rceil$$

## Special Values for Real Function v?Trunc(x)

| Argument  | Result    | Exception |
|-----------|-----------|-----------|
| +0        | +0        |           |
| -0        | -0        |           |
| $+\infty$ | $+\infty$ |           |
| $-\infty$ | $-\infty$ |           |
| SNAN      | QNAN      | INVALID   |
| QNAN      | QNAN      |           |

## v?Round

Computes a value rounded to the nearest integer for each vector element.

## Syntax

```
vsRound(n, a, y);
```

```
vmsRound(n, a, y, mode);
```

```
vdRound(n, a, y);
```

```
vmdRound(n, a, y, mode);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name              | Type                                                                                                                                                                | Description                                                                                                                        |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                          | Specifies the number of elements to be calculated.                                                                                 |
| <code>a</code>    | <code>const float*</code> for <code>vsRound</code> ,<br><code>vmsRound</code><br><br><code>const double*</code> for <code>vdRound</code> ,<br><code>vmdRound</code> | Pointer to an array that contains the input vector <code>a</code> .                                                                |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                        | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name           | Type                                                                                                                                              | Description                                                          |
|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsRound</code> , <code>vmsRound</code><br><br><code>double*</code> for <code>vdRound</code> , <code>vmdRound</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The function computes a value rounded to the nearest integer for each vector element. Input elements that are halfway between two consecutive integers are always rounded away from zero regardless of the rounding mode.

### Special Values for Real Function `v?Round(x)`

| Argument | Result | Exception |
|----------|--------|-----------|
| +0       | +0     |           |
| -0       | -0     |           |
| +∞       | +∞     |           |
| -∞       | -∞     |           |
| SNAN     | QNAN   | INVALID   |
| QNAN     | QNAN   |           |

## `v?NearbyInt`

*Computes a rounded integer value in the current rounding mode for each vector element.*

## Syntax

```
vsNearbyInt(n, a, y);
```

```
vmsNearbyInt(n, a, y, mode);
```

```
vdNearbyInt(n, a, y);
```

```
vmdNearbyInt(n, a, y, mode);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name              | Type                                                                                                                                                                                | Description                                                                                                                     |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>const MKL_INT</code>                                                                                                                                                          | Specifies the number of elements to be calculated.                                                                              |
| <code>a</code>    | <code>const float*</code> for <code>vsNearbyInt</code> ,<br><code>vmsNearbyInt</code><br><br><code>const double*</code> for <code>vdNearbyInt</code> ,<br><code>vmdNearbyInt</code> | Pointer to an array that contains the input vector <code>a</code> .                                                             |
| <code>mode</code> | <code>const MKL_INT64</code>                                                                                                                                                        | Overrides global VM mode setting for this function call. See <code>vm1SetMode</code> for possible values and their description. |

## Output Parameters

| Name           | Type                                                                                                                                                                    | Description                                                          |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>y</code> | <code>float*</code> for <code>vsNearbyInt</code> ,<br><code>vmsNearbyInt</code><br><br><code>double*</code> for <code>vdNearbyInt</code> ,<br><code>vmdNearbyInt</code> | Pointer to an array that contains the output vector <code>y</code> . |

## Description

The `v?NearbyInt` function computes a rounded integer value in a current rounding mode for each vector element.

### Special Values for Real Function `v?NearbyInt(x)`

| Argument  | Argument  | Exception |
|-----------|-----------|-----------|
| +0        | +0        |           |
| -0        | -0        |           |
| $+\infty$ | $+\infty$ |           |
| $-\infty$ | $-\infty$ |           |
| SNAN      | QNAN      | INVALID   |
| QNAN      | QNAN      |           |

## `v?Rint`

*Computes a rounded integer value in the current rounding mode.*

### Syntax

```
vsRint(n, a, y);
```

```
vmsRint(n, a, y, mode);
```

```
vdRint(n, a, y);
```

```
vmdRint(n, a, y, mode);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name        | Type                                                                                                                                                  | Description                                                                                                                        |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | <code>const MKL_INT</code>                                                                                                                            | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | <code>const float*</code> for <code>vsRint</code> , <code>vmsRint</code><br><code>const double*</code> for <code>vdRint</code> , <code>vmdRint</code> | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | <code>const MKL_INT64</code>                                                                                                                          | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                                                                                                      | Description                                                    |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | <code>float*</code> for <code>vsRint</code> , <code>vmsRint</code><br><code>double*</code> for <code>vdRint</code> , <code>vmdRint</code> | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The `v?Rint` function computes a rounded floating-point integer value using the current rounding mode for each vector element.

The rounding mode affects the results computed for inputs that fall between consecutive integers. For example:

- $f(0.5) = 0$ , for rounding modes set to round to nearest round toward zero or to minus infinity.
- $f(0.5) = 1$ , for rounding modes set to plus infinity.
- $f(-1.5) = -2$ , for rounding modes set to round to nearest or to minus infinity.
- $f(-1.5) = -1$ , for rounding modes set to round toward zero or to plus infinity.

## Special Values for Real Function `v?Rint(x)`

| Argument  | Result    | Exception |
|-----------|-----------|-----------|
| +0        | +0        |           |
| -0        | -0        |           |
| $+\infty$ | $+\infty$ |           |
| $-\infty$ | $-\infty$ |           |
| SNAN      | QNAN      | INVALID   |
| QNAN      | QNAN      |           |

## `v?Modf`

*Computes a truncated integer value and the remaining fraction part for each vector element.*

## Syntax

```
vsModf(n, a, y, z);
```

```
vmsModf(n, a, y, z, mode);
```

```
vdModf(n, a, y, z);
```

```
vmdModf(n, a, y, z, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                  | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                         | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsModf, vmsModf<br>const double* for vdModf, vmdModf | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                       | Overrides global VM mode setting for this function call. See <a href="#">vm1SetMode</a> for possible values and their description. |

## Output Parameters

| Name        | Type                                                      | Description                                                                 |
|-------------|-----------------------------------------------------------|-----------------------------------------------------------------------------|
| <i>y, z</i> | float* for vsModf, vmsModf<br>double* for vdModf, vmdModf | Pointer to an array that contains the output vector <i>y</i> and <i>z</i> . |

## Description

The function computes a truncated integer value and the remaining fraction part for each vector element.

$$a_i \geq 0, \begin{cases} y_i = \lfloor a_i \rfloor \\ z_i = a_i - \lfloor a_i \rfloor \end{cases}$$

$$a_i < 0, \begin{cases} y_i = \lceil a_i \rceil \\ z_i = a_i - \lceil a_i \rceil \end{cases}$$

## Special Values for Real Function v?Modf(x)

| Argument  | Result: <i>y</i> ( <i>i</i> ) | Result: <i>z</i> ( <i>i</i> ) | Exception |
|-----------|-------------------------------|-------------------------------|-----------|
| +0        | +0                            | +0                            |           |
| -0        | -0                            | -0                            |           |
| $+\infty$ | $+\infty$                     | +0                            |           |
| $-\infty$ | $-\infty$                     | -0                            |           |
| SNAN      | QNAN                          | QNAN                          | INVALID   |
| QNAN      | QNAN                          | QNAN                          |           |

## v?Frac

Computes a signed fractional part for each vector element.

## Syntax

```
vsFrac(n, a, y);
vmsFrac(n, a, y, mode);
vdFrac(n, a, y);
vmdFrac(n, a, y, mode);
```

## Include Files

- mkl.h

## Input Parameters

| Name        | Type                                                                  | Description                                                                                                                        |
|-------------|-----------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | const MKL_INT                                                         | Specifies the number of elements to be calculated.                                                                                 |
| <i>a</i>    | const float* for vsFrac, vmsFrac<br>const double* for vdFrac, vmdFrac | Pointer to an array that contains the input vector <i>a</i> .                                                                      |
| <i>mode</i> | const MKL_INT64                                                       | Overrides global VM mode setting for this function call. See <a href="#">vmlSetMode</a> for possible values and their description. |

## Output Parameters

| Name     | Type                                                      | Description                                                    |
|----------|-----------------------------------------------------------|----------------------------------------------------------------|
| <i>y</i> | float* for vsFrac, vmsFrac<br>double* for vdFrac, vmdFrac | Pointer to an array that contains the output vector <i>y</i> . |

## Description

The function computes a signed fractional part for each vector element.

$$y_i = \begin{cases} a_i - \lfloor a_i \rfloor, & a_i \geq 0 \\ a_i - \lceil a_i \rceil, & a_i < 0 \end{cases}$$

## Special Values for Real Function v?Frac(x)

| Argument  | Result | Exception |
|-----------|--------|-----------|
| +0        | +0     |           |
| -0        | -0     |           |
| $+\infty$ | +0     |           |
| $-\infty$ | -0     |           |
| SNAN      | QNAN   | INVALID   |
| QNAN      | QNAN   |           |

## VM Pack/Unpack Functions

This section describes VM functions that convert vectors with unit increment to and from vectors with positive increment indexing, vector indexing, and mask indexing (see Appendix B for details on vector indexing methods).

The table below lists available VM Pack/Unpack functions, together with data types and indexing methods associated with them.

### VM Pack/Unpack Functions

| Function Short Name   | Data Types                  | Indexing Methods | Description                                                 |
|-----------------------|-----------------------------|------------------|-------------------------------------------------------------|
| <code>v?Pack</code>   | <i>s, d, c,</i><br><i>z</i> | <i>I, V, M</i>   | Gathers elements of arrays, indexed by different methods.   |
| <code>v?Unpack</code> | <i>s, d, c,</i><br><i>z</i> | <i>I, V, M</i>   | Scatters vector elements to arrays with different indexing. |

### See Also

Vector Arguments in VM

### v?Pack

*Copies elements of an array with specified indexing to a vector with unit increment.*

### Syntax

```
vsPackI(n, a, inca, y);
vsPackV(n, a, ia, y);
vsPackM(n, a, ma, y);
vdPackI(n, a, inca, y);
vdPackV(n, a, ia, y);
vdPackM(n, a, ma, y);
vcPackI(n, a, inca, y);
vcPackV(n, a, ia, y);
vcPackM(n, a, ma, y);
vzPackI(n, a, inca, y);
vzPackV(n, a, ia, y);
vzPackM(n, a, ma, y);
```

### Include Files

- `mk1.h`

### Input Parameters

| Name           | Type                                                                                                | Description                                                                                       |
|----------------|-----------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| <code>n</code> | <code>const MKL_INT</code>                                                                          | Specifies the number of elements to be calculated.                                                |
| <code>a</code> | <code>const float*</code> for <code>vsPackI</code> ,<br><code>vsPackV</code> , <code>vsPackM</code> | Specifies pointer to an array that contains the input vector <code>a</code> . The arrays must be: |

| Name        | Type                                                                                                                                                                    | Description                                                                                                                                   |
|-------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
|             | const double* for vdPackI,<br>vdPackV, vdPackM<br><br>const MKL_Complex8* for vcPackI,<br>vcPackV, vcPackM<br><br>const MKL_Complex16* for<br>vzPackI, vzPackV, vzPackM | for v?PackI, at least $(1 + (n-1) * inca)$<br>for v?PackV, at least $\max(n, \max(ia[j]))$ , $j=0, \dots, n-1$<br>for v?PackM, at least $n$ . |
| <i>inca</i> | const MKL_INT for vsPackI,<br>vdPackI, vcPackI, vzPackI                                                                                                                 | Specifies the increment for the elements of <i>a</i> .                                                                                        |
| <i>ia</i>   | const int* for vsPackV, vdPackV,<br>vcPackV, vzPackV                                                                                                                    | Specifies the pointer to an array of size at least <i>n</i> that contains the index vector for the elements of <i>a</i> .                     |
| <i>ma</i>   | const int* for vsPackM, vdPackM,<br>vcPackM, vzPackM                                                                                                                    | Specifies the pointer to an array of size at least <i>n</i> that contains the mask vector for the elements of <i>a</i> .                      |

## Output Parameters

| Name     | Type                                                                                                                                                                                                             | Description                                                                              |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| <i>y</i> | float* for vsPackI, vsPackV,<br>vsPackM<br><br>double* for vdPackI, vdPackV,<br>vdPackM<br><br>const MKL_Complex8* for vcPackI,<br>vcPackV, vcPackM<br><br>const MKL_Complex16* for vzPackI,<br>vzPackV, vzPackM | Pointer to an array of size at least <i>n</i> that contains the output vector <i>y</i> . |

## v?Unpack

*Copies elements of a vector with unit increment to an array with specified indexing.*

### Syntax

```

vsUnpackI(n, a, y, incy);
vsUnpackV(n, a, y, iy);
vsUnpackM(n, a, y, my);
vdUnpackI(n, a, y, incy);
vdUnpackV(n, a, y, iy);
vdUnpackM(n, a, y, my);
vcUnpackI(n, a, y, incy);
vcUnpackV(n, a, y, iy);
vcUnpackM(n, a, y, my);
vzUnpackI(n, a, y, incy);

```

```
vzUnpackV(n, a, y, iy);
```

```
vzUnpackM(n, a, y, my);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name        | Type                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Description                                                                                                               |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>    | <code>const MKL_INT</code>                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Specifies the number of elements to be calculated.                                                                        |
| <i>a</i>    | <code>const float*</code> for <code>vsUnpackI</code> ,<br><code>vsUnpackV</code> , <code>vsUnpackM</code><br><br><code>const double*</code> for <code>vdUnpackI</code> ,<br><code>vdUnpackV</code> , <code>vdUnpackM</code><br><br><code>const MKL_Complex8*</code> for<br><code>vcUnpackI</code> , <code>vcUnpackV</code> , <code>vcUnpackM</code><br><br><code>const MKL_Complex16*</code> for<br><code>vzUnpackI</code> , <code>vzUnpackV</code> , <code>vzUnpackM</code> | Specifies the pointer to an array of size at least <i>n</i> that contains the input vector <i>a</i> .                     |
| <i>incy</i> | <code>const MKL_INT</code> for <code>vsUnpackI</code> ,<br><code>vdUnpackI</code> , <code>vcUnpackI</code> , <code>vzUnpackI</code>                                                                                                                                                                                                                                                                                                                                          | Specifies the increment for the elements of <i>y</i> .                                                                    |
| <i>iy</i>   | <code>const int*</code> for <code>vsUnpackV</code> ,<br><code>vdUnpackV</code> , <code>vcUnpackV</code> , <code>vzUnpackV</code>                                                                                                                                                                                                                                                                                                                                             | Specifies the pointer to an array of size at least <i>n</i> that contains the index vector for the elements of <i>a</i> . |
| <i>my</i>   | <code>const int*</code> for <code>vsUnpackM</code> ,<br><code>vdUnpackM</code> , <code>vcUnpackM</code> , <code>vzUnpackM</code>                                                                                                                                                                                                                                                                                                                                             | Specifies the pointer to an array of size at least <i>n</i> that contains the mask vector for the elements of <i>a</i> .  |

## Output Parameters

| Name     | Type                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Description                                                                                                                                                                                                                                                                                                      |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>y</i> | <code>float*</code> for <code>vsUnpackI</code> , <code>vsUnpackV</code> ,<br><code>vsUnpackM</code><br><br><code>double*</code> for <code>vdUnpackI</code> ,<br><code>vdUnpackV</code> , <code>vdUnpackM</code><br><br><code>const MKL_Complex8*</code> for<br><code>vcUnpackI</code> , <code>vcUnpackV</code> , <code>vcUnpackM</code><br><br><code>const MKL_Complex16*</code> for<br><code>vzUnpackI</code> , <code>vzUnpackV</code> , <code>vzUnpackM</code> | Specifies the pointer to an array that contains the output vector <i>y</i> .<br><br>The array must be:<br><br>for <code>v?UnpackI</code> , at least $(1 + (n-1) * incy)$<br>for <code>v?UnpackV</code> , at least<br>$\max(n, \max(ia[j]), j=0, \dots, n-1,$<br>for <code>v?UnpackM</code> , at least <i>n</i> . |

## VM Service Functions

The VM Service functions enable you to set/get the accuracy mode and error code. These functions are available both in the Fortran and C interfaces. The table below lists available VM Service functions and their short description.

## VM Service Functions

| Function Short Name                | Description                                                                                      |
|------------------------------------|--------------------------------------------------------------------------------------------------|
| <code>vmlSetMode</code>            | Sets the VM mode                                                                                 |
| <code>vmlGetMode</code>            | Gets the VM mode                                                                                 |
| <code>MKLFreeTls</code>            | Frees allocated VM/VS thread local storage memory from within DllMain routine (Windows* OS only) |
| <code>vmlSetErrStatus</code>       | Sets the VM Error Status                                                                         |
| <code>vmlGetErrStatus</code>       | Gets the VM Error Status                                                                         |
| <code>vmlClearErrStatus</code>     | Clears the VM Error Status                                                                       |
| <code>vmlSetErrorCallBack</code>   | Sets the additional error handler callback function                                              |
| <code>vmlGetErrorCallBack</code>   | Gets the additional error handler callback function                                              |
| <code>vmlClearErrorCallBack</code> | Deletes the additional error handler callback function                                           |

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Notice revision #20110804

## vmlSetMode

Sets a new mode for VM functions according to the *mode* parameter and stores the previous VM mode to *oldmode*.

### Syntax

```
oldmode = vmlSetMode(mode);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name        | Type                        | Description                      |
|-------------|-----------------------------|----------------------------------|
| <i>mode</i> | <code>const MKL_UINT</code> | Specifies the VM mode to be set. |

### Output Parameters

| Name           | Type                      | Description                   |
|----------------|---------------------------|-------------------------------|
| <i>oldmode</i> | <code>unsigned int</code> | Specifies the former VM mode. |

### Description

The `vmlSetMode` function sets a new mode for VM functions according to the *mode* parameter and stores the previous VM mode to *oldmode*. The mode change has a global effect on all the VM functions within a thread.

**NOTE**

You can override the global mode setting and change the mode for a given VM function call by using a respective `vm[s, d]<Func>` variant of the function.

The *mode* parameter is designed to control accuracy, handling of denormalized numbers, and error handling. Table "Values of the *mode* Parameter" lists values of the *mode* parameter. You can obtain all other possible values of the *mode* parameter from the *mode* parameter values by using bitwise OR ( | ) operation to combine one value for accuracy, one value for handling of denormalized numbers, and one value for error control options. The default value of the *mode* parameter is `VML_HA | VML_FTZDAZ_OFF | VML_ERRMODE_DEFAULT`.

The `VML_FTZDAZ_ON` mode is specifically designed to improve the performance of computations that involve denormalized numbers at the cost of reasonable accuracy loss. This mode changes the numeric behavior of the functions: denormalized input values are treated as zeros (`DAZ = denormals-are-zero`) and denormalized results are flushed to zero (`FTZ = flush-to-zero`). Accuracy loss may occur if input and/or output values are close to denormal range.

**Values of the *mode* Parameter**

| Value of <i>mode</i>                         | Description                                                                                                             |
|----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| <b>Accuracy Control</b>                      |                                                                                                                         |
| <code>VML_HA</code>                          | high accuracy versions of VM functions                                                                                  |
| <code>VML_LA</code>                          | low accuracy versions of VM functions                                                                                   |
| <code>VML_EP</code>                          | enhanced performance accuracy versions of VM functions                                                                  |
| <b>Denormalized Numbers Handling Control</b> |                                                                                                                         |
| <code>VML_FTZDAZ_ON</code>                   | Faster processing of denormalized inputs is enabled.                                                                    |
| <code>VML_FTZDAZ_OFF</code>                  | Faster processing of denormalized inputs is disabled.                                                                   |
| <b>Error Mode Control</b>                    |                                                                                                                         |
| <code>VML_ERRMODE_IGNORE</code>              | No action is set for computation errors.                                                                                |
| <code>VML_ERRMODE_ERRNO</code>               | On error, the <i>errno</i> variable is set.                                                                             |
| <code>VML_ERRMODE_STDERR</code>              | On error, the error text information is written to <i>stderr</i> .                                                      |
| <code>VML_ERRMODE_EXCEPT</code>              | On error, an exception is raised.                                                                                       |
| <code>VML_ERRMODE_CALLBACK</code>            | On error, an additional error handler function is called.                                                               |
| <code>VML_ERRMODE_DEFAULT</code>             | On error, the <i>errno</i> variable is set, an exception is raised, and an additional error handler function is called. |

**Optimization Notice**

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**Examples**

The following example shows how to set low accuracy, fast processing for denormalized numbers and `stderr` error mode:

```
vmlSetMode(VML_LA);
vmlSetMode(VML_LA | VML_FTZDAZ_ON | VML_ERRMODE_STDERR);
```

## vmlGetMode

*Gets the VM mode.*

### Syntax

```
mod = vmlGetMode(void);
```

### Include Files

- mkl.h

### Output Parameters

| Name       | Type         | Description                                 |
|------------|--------------|---------------------------------------------|
| <i>mod</i> | unsigned int | Specifies the packed <i>mode</i> parameter. |

### Description

The function `vmlGetMode` returns the VM *mode* parameter that controls accuracy, handling of denormalized numbers, and error handling options. The *mod* variable value is a combination of the values listed in the table "Values of the *mode* Parameter". You can obtain these values using the respective mask from the table "Values of Mask for the *mode* Parameter".

#### Values of Mask for the *mode* Parameter

| Value of mask     | Description                                        |
|-------------------|----------------------------------------------------|
| VML_ACCURACY_MASK | Specifies mask for accuracy <i>mode</i> selection. |
| VML_FTZDAZ_MASK   | Specifies mask for FTZDAZ <i>mode</i> selection.   |
| VML_ERRMODE_MASK  | Specifies mask for error <i>mode</i> selection.    |

See example below:

### Examples

```
accm = vmlGetMode(void) & VML_ACCURACY_MASK;
denm = vmlGetMode(void) & VML_FTZDAZ_MASK;
errm = vmlGetMode(void) & VML_ERRMODE_MASK;
```

## MKLFreeTls

*Frees allocated VM/VS thread local storage memory from within DllMain routine. Use on Windows\* OS only.*

### Syntax

```
void MKLFreeTls(const MKL_UINT fdwReason);
```

### Include Files

- mkl\_vml\_functions\_win.h

## Description

The `MKLFreeTls` routine frees thread local storage (TLS) memory which has been allocated when using MKL static libraries to link into a DLL on the Windows\* OS. The routine should only be used within `DllMain`.

### NOTE

It is only necessary to use `MKLFreeTls` for TLS data in the VM and VS domains.

## Input Parameters

| Name                   | Type                        | Description                                     |
|------------------------|-----------------------------|-------------------------------------------------|
| <code>fdwReason</code> | <code>const MKL_UINT</code> | Reason code from the <code>DllMain</code> call. |

## Example

```
BOOL WINAPI DllMain(HINSTANCE hInst, DWORD fdwReason, LPVOID lpvReserved)
{
 /*customer code*/
 MKLFreeTls(fdwReason);
 /*customer code*/
}
```

## vmlSetErrStatus

Sets the new VM Error Status according to `err` and stores the previous VM Error Status to `olderr`.

## Syntax

```
olderr = vmlSetErrStatus(status);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name                | Type                       | Description                              |
|---------------------|----------------------------|------------------------------------------|
| <code>status</code> | <code>const MKL_INT</code> | Specifies the VM error status to be set. |

## Output Parameters

| Name                | Type             | Description                           |
|---------------------|------------------|---------------------------------------|
| <code>olderr</code> | <code>int</code> | Specifies the former VM error status. |

## Description

Table "Values of the VM Status" lists possible values of the `err` parameter.

## Values of the VM Status

| Status                      | Description                               |
|-----------------------------|-------------------------------------------|
| <b>Successful Execution</b> |                                           |
| <code>VML_STATUS_OK</code>  | The execution was completed successfully. |

| Status                     | Description                                                                                                    |
|----------------------------|----------------------------------------------------------------------------------------------------------------|
| <b>Warnings</b>            |                                                                                                                |
| VML_STATUS_ACCURACYWARNING | The execution was completed successfully in a different accuracy mode.                                         |
| <b>Errors</b>              |                                                                                                                |
| VML_STATUS_BADSIZE         | The function does not support the preset accuracy mode. The Low Accuracy mode is used instead.                 |
| VML_STATUS_BADMEM          | NULL pointer is passed.                                                                                        |
| VML_STATUS_ERRDOM          | At least one of array values is out of a range of definition.                                                  |
| VML_STATUS_SING            | At least one of the input array values causes a divide-by-zero exception or produces an invalid (QNaN) result. |
| VML_STATUS_OVERFLOW        | An overflow has happened during the calculation process.                                                       |
| VML_STATUS_UNDERFLOW       | An underflow has happened during the calculation process.                                                      |

## Examples

```
olderr = vmlSetErrStatus(VML_STATUS_OK);
```

```
olderr = vmlSetErrStatus(VML_STATUS_ERRDOM);
```

```
olderr = vmlSetErrStatus(VML_STATUS_UNDERFLOW);
```

## vmlGetErrStatus

*Gets the VM Error Status.*

### Syntax

```
err = vmlGetErrStatus(void);
```

### Include Files

- mkl.h

### Output Parameters

| Name | Type | Description                    |
|------|------|--------------------------------|
| err  | int  | Specifies the VM error status. |

## vmlClearErrStatus

*Sets the VM Error Status to VML\_STATUS\_OK and stores the previous VM Error Status to olderr.*

### Syntax

```
olderr = vmlClearErrStatus(void);
```

### Include Files

- mkl.h

## Output Parameters

| Name                | Type             | Description                           |
|---------------------|------------------|---------------------------------------|
| <code>olderr</code> | <code>int</code> | Specifies the former VM error status. |

## `vmlSetErrorCallback`

*Sets the additional error handler callback function and gets the old callback function.*

---

### Syntax

```
oldcallback = vmlSetErrorCallBack(callback);
```

### Include Files

- `mk1.h`

## Input Parameters

| Name                  | Description                       |
|-----------------------|-----------------------------------|
| <code>callback</code> | Pointer to the callback function. |

The callback function has the following format:

```
static int __stdcall
MyHandler(DefVmlErrorContext*
pContext)
{
 /* Handler body */
};
```

**Name****Description**

The passed error structure is defined as follows:

```
typedef struct _DefVmlErrorContext
{
 int iCode; /* Error status value */
 int iIndex; /* Index for bad array
 element, or bad array
 dimension, or bad
 array pointer */
 double dbA1; /* Error argument 1 */
 double dbA2; /* Error argument 2 */
 double dbR1; /* Error result 1 */
 double dbR2; /* Error result 2 */
 char cFuncName[64]; /* Function name */
 int iFuncNameLen; /* Length of
 functionname*/
 double dbA1Im; /* Error argument 1, imag
 part*/
 double dbA2Im; /* Error argument 2, imag
 part*/
 double dbR1Im; /* Error result 1, imag
 part*/
 double dbR2Im; /* Error result 2, imag
 part*/
} DefVmlErrorContext;
```

**Output Parameters****Name****Type****Description**

`oldcallback`    `int`

Pointer to the former callback function.

**Description**

The callback function is called on each VM mathematical function error if `VML_ERRMODE_CALLBACK` error mode is set (see "Values of the *mode* Parameter").

Use the `vmlSetErrorCallBack()` function if you need to define your own callback function instead of default empty callback function.

The input structure for a callback function contains the following information about the error encountered:

- the input value that caused an error
- location (array index) of this value
- the computed result value
- error code

- name of the function in which the error occurred.

You can insert your own error processing into the callback function. This may include correcting the passed result values in order to pass them back and resume computation. The standard error handler is called after the callback function only if it returns 0.

## **vmlGetErrorCallBack**

*Gets the additional error handler callback function.*

---

### **Syntax**

```
callback = vmlGetErrorCallBack(void);
```

### **Include Files**

- mkl.h

### **Output Parameters**

| <b>Name</b>     | <b>Description</b>               |
|-----------------|----------------------------------|
| <i>callback</i> | Pointer to the callback function |

## **vmlClearErrorCallBack**

*Deletes the additional error handler callback function and retrieves the former callback function.*

---

### **Syntax**

```
oldcallback = vmlClearErrorCallBack(void);
```

### **Include Files**

- mkl.h

### **Output Parameters**

| <b>Name</b>        | <b>Type</b> | <b>Description</b>                      |
|--------------------|-------------|-----------------------------------------|
| <i>oldcallback</i> | int         | Pointer to the former callback function |



# Statistical Functions

Statistical functions in Intel® MKL are known as the Vector Statistics (VS). They are designed for the purpose of

- generating vectors of pseudorandom, quasi-random, and non-deterministic random numbers
- performing mathematical operations of convolution and correlation
- computing basic statistical estimates for single and double precision multi-dimensional datasets

The corresponding functionality is described in the respective [Random Number Generators](#), [Convolution and Correlation](#), and [Summary Statistics](#) sections.

See VS performance data in the online VS Performance Data document available at <http://software.intel.com/en-us/articles/intel-math-kernel-library-documentation/>

The basic notion in VS is a task. The task object is a data structure or descriptor that holds the parameters related to a specific statistical operation: random number generation, convolution and correlation, or summary statistics estimation. Such parameters can be an identifier of a random number generator, its internal state and parameters, data arrays, their shape and dimensions, an identifier of the operation and so forth. You can modify the VS task parameters using the VS service functions.

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## Random Number Generators

Intel MKL VS provides a set of routines implementing commonly used pseudorandom, quasi-random, or non-deterministic random number generators with continuous and discrete distribution. To improve performance, all these routines were developed using the calls to the highly optimized *Basic Random Number Generators* (BRNGs) and vector mathematical functions (VM, see [Chapter 9, "Vector Mathematical Functions"](#)).

VS provides interfaces both for Fortran and C languages. For users of the C and C++ languages the `mk1_vs1.h` header file is provided. All header files are found in the following directory:

```
{MKL}/include
```

All VS routines can be classified into three major categories:

- Transformation routines for different types of statistical distributions, for example, uniform, normal (Gaussian), binomial, etc. These routines indirectly call basic random number generators, which are pseudorandom, quasi-random, or non-deterministic random number generators. Detailed description of the generators can be found in [Distribution Generators](#) section.
- Service routines to handle random number streams: create, initialize, delete, copy, save to a binary file, load from a binary file, get the index of a basic generator. The description of these routines can be found in [Service Routines](#) section.
- Registration routines for basic pseudorandom generators and routines that obtain properties of the registered generators (see [Advanced Service Routines](#) section).

The last two categories are referred to as service routines.

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**Random Number Generators Conventions**

This document makes no specific differentiation between random, pseudorandom, and quasi-random numbers, nor between random, pseudorandom, and quasi-random number generators unless the context requires otherwise. For details, refer to the 'Random Numbers' section in [VS Notes](#) document provided at the Intel® MKL web page.

All generators of nonuniform distributions, both discrete and continuous, are built on the basis of the uniform distribution generators, called Basic Random Number Generators (BRNGs). The pseudorandom numbers with nonuniform distribution are obtained through an appropriate transformation of the uniformly distributed pseudorandom numbers. Such transformations are referred to as *generation methods*. For a given distribution, several generation methods can be used. See [VS Notes](#) for the description of methods available for each generator.

An RNG task determines environment in which random number generation is performed, in particular parameters of the BRNG and its internal state. Output of VS generators is a stream of random numbers that are used in Monte Carlo simulations. A *random stream descriptor* and a *random stream* are used as synonyms of an *RNG task* in the document unless the context requires otherwise.

The *random stream descriptor* specifies which BRNG should be used in a given transformation method. See the *Random Streams and RNGs in Parallel Computation* section of [VS Notes](#).

The term *computational node* means a logical or physical unit that can process data in parallel.

**Random Number Generators Mathematical Notation**

The following notation is used throughout the text:

- $N$  The set of natural numbers  $N = \{1, 2, 3 \dots\}$ .
- $Z$  The set of integers  $Z = \{\dots -3, -2, -1, 0, 1, 2, 3 \dots\}$ .
- $R$  The set of real numbers.
- $\lfloor a \rfloor$  The floor of  $a$  (the largest integer less than or equal to  $a$ ).
- $\oplus$  or **xor** Bitwise exclusive OR.
- $C_{\alpha}^k$  or  $\binom{\alpha}{k}$  Binomial coefficient or combination ( $\alpha \in R, \alpha \geq 0; k \in \mathbb{N} \cup \{0\}$ ).
- $C_{\alpha}^0 = 1$
- For  $\alpha \geq k$  binomial coefficient is defined as

$$C_{\alpha}^{\kappa} = \frac{\alpha(\alpha - 1) \dots (\alpha - \kappa + 1)}{\kappa!}$$

If  $\alpha < \kappa$ , then

$$C_{\alpha}^{\kappa} = 0$$

$\Phi(x)$

Cumulative Gaussian distribution function

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy$$

defined over  $-\infty < x < +\infty$ .

$\Phi(-\infty) = 0$ ,  $\Phi(+\infty) = 1$ .

$\Gamma(\alpha)$

The complete gamma function

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt$$

where  $\alpha > 0$ .

$B(p, q)$

The complete beta function

$$B(p, q) = \int_0^1 t^{p-1} (1-t)^{q-1} dt$$

where  $p > 0$  and  $q > 0$ .

LCG( $a, c, m$ )

Linear Congruential Generator  $x_{n+1} = (ax_n + c) \bmod m$ , where  $a$  is called the *multiplier*,  $c$  is called the *increment*, and  $m$  is called the *modulus* of the generator.

MCG( $a, m$ )

Multiplicative Congruential Generator  $x_{n+1} = (ax_n) \bmod m$  is a special case of Linear Congruential Generator, where the increment  $c$  is taken to be 0.

GFSR( $p, q$ )

Generalized Feedback Shift Register Generator

$$x_n = x_{n-p} \oplus x_{n-q}$$

### Random Number Generators Naming Conventions

The names of the routines, types, and constants in VS random number generators are case-sensitive and can contain lowercase and uppercase characters (`viRngUniform`).

The names of generator routines have the following structure:

```
v<type of result>Rng<distribution>
```

where

- *v* is the prefix of a VS vector function.
- *<type of result>* is either *s*, *d*, or *i* and specifies one of the following types:

|          |        |
|----------|--------|
| <i>s</i> | float  |
| <i>d</i> | double |
| <i>i</i> | int    |

Prefixes *s* and *d* apply to continuous distributions only, prefix *i* applies only to discrete case.

- *rng* indicates that the routine is a random generator.
- *<distribution>* specifies the type of statistical distribution.

Names of service routines follow the template below:

```
vsl<name>
```

where

- *vsl* is the prefix of a VS service function.
- *<name>* contains a short function name.

For a more detailed description of service routines, refer to [Service Routines](#) and [Advanced Service Routines](#) sections.

The prototype of each generator routine corresponding to a given probability distribution fits the following structure:

```
status = <function name>(method, stream, n, r, [<distribution parameters>])
```

where

- *method* defines the method of generation. A detailed description of this parameter can be found in table "[Values of <method> in method parameter](#)". See the next page, where the structure of the *method* parameter name is explained.
- *stream* defines the descriptor of the random stream and must have a non-zero value. Random streams, descriptors, and their usage are discussed further in [Random Streams](#) and [Service Routines](#).
- *n* defines the number of random values to be generated. If *n* is less than or equal to zero, no values are generated. Furthermore, if *n* is negative, an error condition is set.
- *r* defines the destination array for the generated numbers. The dimension of the array must be large enough to store at least *n* random numbers.
- *status* defines the error status of a VS routine. See [Error Reporting](#) section for a detailed description of error status values.

Additional parameters included into *<distribution parameters>* field are individual for each generator routine and are described in detail in [Distribution Generators](#) section.

To invoke a distribution generator, use a call to the respective VS routine. For example, to obtain a vector *r*, composed of *n* independent and identically distributed random numbers with normal (Gaussian) distribution, that have the mean value *a* and standard deviation *sigma*, write the following:

```
status = vsRngGaussian(method, stream, n, r, a, sigma)
```

The name of a *method* parameter has the following structure:

```
VSL_RNG_METHOD_method<distribution>_<method>
```

```
VSL_RNG_METHOD_<distribution>_<method>_ACCURATE
```

where

- `<distribution>` is the probability distribution.
- `<method>` is the method name.

Type of the name structure for the `method` parameter corresponds to fast and accurate modes of random number generation (see "Distribution Generators" section and VS Notes for details).

Method names `VSL_RNG_METHOD_<distribution>_<method>`

and

`VSL_RNG_METHOD_<distribution>_<method>_ACCURATE`

should be used with

`v<precision>Rng<distribution>`

function only, where

- `<precision>` is
  - `s` for single precision continuous distribution
  - `d` for double precision continuous distribution
  - `i` for discrete distribution
- `<distribution>` is the probability distribution.

is the probability distribution. Table "Values of `<method>` in `method` parameter" provides specific predefined values of the `method` name. The third column contains names of the functions that use the given method.

**Values of `<method>` in `method` parameter**

| Method     | Short Description                                                                                                                                                                                                                                         | Functions                                                                           |
|------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| STD        | Standard method. Currently there is only one method for these functions.                                                                                                                                                                                  | Uniform (continuous), Uniform (discrete), UniformBits, UniformBits32, UniformBits64 |
| BOXMULLER  | BOXMULLER generates normally distributed random number $x$ thru the pair of uniformly distributed numbers $u_1$ and $u_2$ according to the formula:<br><br>$x = \sqrt{-2 \ln u_1} \sin 2\pi u_2$                                                          | Gaussian, GaussianMV                                                                |
| BOXMULLER2 | BOXMULLER2 generates normally distributed random numbers $x_1$ and $x_2$ thru the pair of uniformly distributed numbers $u_1$ and $u_2$ according to the formulas:<br><br>$x_1 = \sqrt{-2 \ln u_1} \sin 2\pi u_2$ $x_2 = \sqrt{-2 \ln u_1} \cos 2\pi u_2$ | Gaussian, GaussianMV, Lognormal                                                     |
| ICDF       | Inverse cumulative distribution function method.                                                                                                                                                                                                          | Exponential, Laplace, Weibull, Cauchy, Rayleigh,                                    |

| Method   | Short Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | Functions                                                                    |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------|
|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | Gumbel,<br>Bernoulli,<br>Geometric,<br>Gaussian,<br>GaussianMV,<br>Lognormal |
| GNORM    | For $\alpha > 1$ , a gamma distributed random number is generated as a cube of properly scaled normal random number; for $0.6 \leq \alpha < 1$ , a gamma distributed random number is generated using rejection from Weibull distribution; for $\alpha < 0.6$ , a gamma distributed random number is obtained using transformation of exponential power distribution; for $\alpha = 1$ , gamma distribution is reduced to exponential distribution.                                                                                                                                | Gamma                                                                        |
| CJA      | For $\min(p, q) > 1$ , Cheng method is used; for $\min(p, q) < 1$ , Johnk method is used, if $q + K \cdot p^2 + C \leq 0$ ( $K = 0.852\dots$ , $C = -0.956\dots$ ) otherwise, Atkinson switching algorithm is used; for $\max(p, q) < 1$ , method of Johnk is used; for $\min(p, q) < 1$ , $\max(p, q) > 1$ , Atkinson switching algorithm is used (CJA stands for the first letters of Cheng, Johnk, Atkinson); for $p = 1$ or $q = 1$ , inverse cumulative distribution function method is used; for $p = 1$ and $q = 1$ , beta distribution is reduced to uniform distribution. | Beta                                                                         |
| BTPE     | Acceptance/rejection method for $n \cdot \min(p, 1 - p) \geq 30$ with decomposition into 4 regions: <ul style="list-style-type: none"> <li>- 2 parallelograms</li> <li>- triangle</li> <li>- left exponential tail</li> <li>- right exponential tail</li> </ul>                                                                                                                                                                                                                                                                                                                    | Binomial                                                                     |
| H2PE     | Acceptance/rejection method for large mode of distribution with decomposition into 3 regions: <ul style="list-style-type: none"> <li>- rectangular</li> <li>- left exponential tail</li> <li>- right exponential tail</li> </ul>                                                                                                                                                                                                                                                                                                                                                   | Hypergeometric                                                               |
| PTPE     | Acceptance/rejection method for $\lambda \geq 27$ with decomposition into 4 regions: <ul style="list-style-type: none"> <li>- 2 parallelograms</li> <li>- triangle</li> <li>- left exponential tail</li> <li>- right exponential tail;</li> </ul> otherwise, table lookup method is used.                                                                                                                                                                                                                                                                                          | Poisson                                                                      |
| POISNORM | for $\lambda \geq 1$ , method based on Poisson inverse CDF approximation by Gaussian inverse CDF;<br>for $\lambda < 1$ , table lookup method is used.                                                                                                                                                                                                                                                                                                                                                                                                                              | Poisson,<br>PoissonV                                                         |
| NBAR     | Acceptance/rejection method for ,                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | NegBinomial                                                                  |

| Method | Short Description                                                                                                                                                                            | Functions |
|--------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|
|        | $\frac{(a - 1) \cdot (1 - p)}{p} \geq 100$                                                                                                                                                   |           |
|        | with decomposition into 5 regions: <ul style="list-style-type: none"> <li>- rectangular</li> <li>- 2 trapezoid</li> <li>- left exponential tail</li> <li>- right exponential tail</li> </ul> |           |

**NOTE**

In this document, routines are often referred to by their base name (`Gaussian`) when this does not lead to ambiguity. In the routine reference, the full name (`vsrnggaussian`, `vsRngGaussian`) is always used in prototypes and code examples.

## Basic Generators

VS provides pseudorandom, quasi-random, and non-deterministic random number generators. This includes the following BRNGs, which differ in speed and other properties:

- the 31-bit multiplicative congruential pseudorandom number generator `MCG(1132489760, 231 - 1)` [[L'Ecuyer99](#)]
- the 32-bit generalized feedback shift register pseudorandom number generator `GFSR(250, 103)` [[Kirkpatrick81](#)]
- the combined multiple recursive pseudorandom number generator `MRG32k3a` [[L'Ecuyer99a](#)]
- the 59-bit multiplicative congruential pseudorandom number generator `MCG(1313, 259)` from NAG Numerical Libraries [[NAG](#)]
- Wichmann-Hill pseudorandom number generator (a set of 273 basic generators) from NAG Numerical Libraries [[NAG](#)]
- Mersenne Twister pseudorandom number generator `MT19937` [[Matsumoto98](#)] with period length 2<sup>19937</sup>-1 of the produced sequence
- Set of 6024 Mersenne Twister pseudorandom number generators `MT2203` [[Matsumoto98](#)], [[Matsumoto00](#)]. Each of them generates a sequence of period length equal to 2<sup>2203</sup>-1. Parameters of the generators provide mutual independence of the corresponding sequences.
- SIMD-oriented Fast Mersenne Twister pseudorandom number generator `SFMT19937` [[Saito08](#)] with a period length equal to 2<sup>19937</sup>-1 of the produced sequence.
- Sobol quasi-random number generator [[Sobol76](#)], [[Bratley88](#)], which works in arbitrary dimension. For dimensions greater than 40 the user should supply initialization parameters (initial direction numbers and primitive polynomials or direction numbers) by using `vs1NewStreamEx` function. See additional details on interface for registration of the parameters in the library in [VS Notes](#).
- Niederreiter quasi-random number generator [[Bratley92](#)], which works in arbitrary dimension. For dimensions greater than 318 the user should supply initialization parameters (irreducible polynomials or direction numbers) by using `vs1NewStreamEx` function. See additional details on interface for registration of the parameters in the library in [VS Notes](#).
- Non-deterministic random number generator (RDRAND-based generators only) [[AVX](#)], [[IntelSWMan](#)].

**NOTE**

You can use a non-deterministic random number generator only if the underlying hardware supports it. For instructions on how to detect if an Intel CPU supports a non-deterministic random number generator see, for example, *Chapter 8: Post-32nm Processor Instructions* in [AVX] or *Chapter 4: RdRand Instruction Usage* in [BMT].

**NOTE**

The time required by some non-deterministic sources to generate a random number is not constant, so you might have to make multiple requests before the next random number is available. VS limits the number of retries for requests to the non-deterministic source to 10. You can redefine the maximum number of retries during the initialization of the non-deterministic random number generator with the `vslNewStreamEx` function.

For more details on the non-deterministic source implementation for Intel CPUs please refer to Section 7.3.17, Volume 1, *Random Number Generator Instruction* in [IntelSWMAn] and Section 4.2.2, *RdRand Retry Loop* in [BMT].

- Philox4x32-10 counter-based pseudorandom number generator with a period of  $2^{128}$  `PHILOX4X32X10` [Salmon11].
- ARS-5 counter-based pseudorandom number generator with a period of  $2^{128}$ , which uses instructions from the AES-NI set `ARS5` [Salmon11].

See some testing results for the generators in [VS Notes](#) and comparative performance data at <https://software.intel.com/en-us/articles/intel-math-kernel-library-documentation>.

VS provides means of registration of such user-designed generators through the steps described in [Advanced Service Routines](#) section.

For some basic generators, VS provides two methods of creating independent random streams in multiprocessor computations, which are the leapfrog method and the block-splitting method. These sequence splitting methods are also useful in sequential Monte Carlo.

In addition, MT2203 pseudorandom number generator is a set of 6024 generators designed to create up to 6024 independent random sequences, which might be used in parallel Monte Carlo simulations. Another generator that has the same feature is Wichmann-Hill. It allows creating up to 273 independent random streams. The properties of the generators designed for parallel computations are discussed in detail in [Coddington94].

You may want to design and use your own basic generators. VS provides means of registration of such user-designed generators through the steps described in [Advanced Service Routines](#) section.

There is also an option to utilize externally generated random numbers in VS distribution generator routines. For this purpose VS provides three additional basic random number generators:

- for external random data packed in 32-bit integer array
- for external random data stored in double precision floating-point array; data is supposed to be uniformly distributed over  $(a,b)$  interval
- for external random data stored in single precision floating-point array; data is supposed to be uniformly distributed over  $(a,b)$  interval.

Such basic generators are called the abstract basic random number generators.

See [VS Notes](#) for a more detailed description of the generator properties.

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**BRNG Parameter Definition**

Predefined values for the *brng* input parameter are as follows:

**Values of *brng* parameter**

| Value                  | Short Description                                                                                                                                       |
|------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_BRNG_MCG31         | A 31-bit multiplicative congruential generator.                                                                                                         |
| VSL_BRNG_R250          | A generalized feedback shift register generator.                                                                                                        |
| VSL_BRNG_MRG32K3A      | A combined multiple recursive generator with two components of order 3.                                                                                 |
| VSL_BRNG_MCG59         | A 59-bit multiplicative congruential generator.                                                                                                         |
| VSL_BRNG_WH            | A set of 273 Wichmann-Hill combined multiplicative congruential generators.                                                                             |
| VSL_BRNG_MT19937       | A Mersenne Twister pseudorandom number generator.                                                                                                       |
| VSL_BRNG_MT2203        | A set of 6024 Mersenne Twister pseudorandom number generators.                                                                                          |
| VSL_BRNG_SFMT19937     | A SIMD-oriented Fast Mersenne Twister pseudorandom number generator.                                                                                    |
| VSL_BRNG_SOBOL         | A 32-bit Gray code-based generator producing low-discrepancy sequences for dimensions $1 \leq s \leq 40$ ; user-defined dimensions are also available.  |
| VSL_BRNG_NIEDERR       | A 32-bit Gray code-based generator producing low-discrepancy sequences for dimensions $1 \leq s \leq 318$ ; user-defined dimensions are also available. |
| VSL_BRNG_IABSTRACT     | An abstract random number generator for integer arrays.                                                                                                 |
| VSL_BRNG_DABSTRACT     | An abstract random number generator for double precision floating-point arrays.                                                                         |
| VSL_BRNG_SABSTRACT     | An abstract random number generator for single precision floating-point arrays.                                                                         |
| VSL_BRNG_NONDETERM     | A non-deterministic random number generator.                                                                                                            |
| VSL_BRNG_PHILOX4X32X10 | A Philox4x32-10 counter-based pseudorandom number generator.                                                                                            |
| VSL_BRNG_ARS5          | An ARS-5 counter-based pseudorandom number generator that uses instructions from the AES-NI set.                                                        |

See [VS Notes](#) for detailed description.

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**Random Streams**

*Random stream* (or *stream*) is an abstract source of pseudo- and quasi-random sequences of uniform distribution. You can operate with stream state descriptors only. A stream state descriptor, which holds state descriptive information for a particular BRNG, is a necessary parameter in each routine of a distribution generator. Only the distribution generator routines operate with random streams directly. See [VS Notes](#) for details.

**NOTE**

Random streams associated with abstract basic random number generator are called the abstract random streams. See [VS Notes](#) for detailed description of abstract streams and their use.

You can create unlimited number of random streams by [VS Service Routines](#) like [NewStream](#) and utilize them in any distribution generator to get the sequence of numbers of given probability distribution. When they are no longer needed, the streams should be deleted calling service routine [DeleteStream](#).

VS provides service functions [SaveStreamF](#) and [LoadStreamF](#) to save random stream descriptive data to a binary file and to read this data from a binary file respectively. See [VS Notes](#) for detailed description.

**BRNG Data Types**

```
typedef(void*) VSLStreamStatePtr;
```

See [Advanced Service Routines](#) for the format of the stream state structure for user-designed generators.

**Error Reporting**

VS RNG routines return status codes of the performed operation to report errors to the calling program. The application should perform error-related actions and/or recover from the error. The status codes are of integer type and have the following format:

VSL\_ERROR\_<ERROR\_NAME> - indicates VS errors common for all VS domains.

VSL\_RNG\_ERROR\_<ERROR\_NAME> - indicates VS RNG errors.

VS RNG errors are of negative values while warnings are of positive values. The status code of zero value indicates successful completion of the operation: VSL\_ERROR\_OK (or synonymic VSL\_STATUS\_OK).

**Status Codes**

| Status Code                 | Description                        |
|-----------------------------|------------------------------------|
| <b>Common VSL</b>           |                                    |
| VSL_ERROR_OK, VSL_STATUS_OK | No error, execution is successful. |
| VSL_ERROR_BADARGS           | Input argument value is not valid. |

| <b>Status Code</b>                    | <b>Description</b>                                                                                                               |
|---------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_CPU_NOT_SUPPORTED           | CPU version is not supported.                                                                                                    |
| VSL_ERROR_FEATURE_NOT_IMPLEMENTED     | Feature invoked is not implemented.                                                                                              |
| VSL_ERROR_MEM_FAILURE                 | System cannot allocate memory.                                                                                                   |
| VSL_ERROR_NULL_PTR                    | Input pointer argument is NULL.                                                                                                  |
| VSL_ERROR_UNKNOWN                     | Unknown error.                                                                                                                   |
| <b>VS RNG Specific</b>                |                                                                                                                                  |
| VSL_RNG_ERROR_BAD_FILE_FORMAT         | File format is unknown.                                                                                                          |
| VSL_RNG_ERROR_BAD_MEM_FORMAT          | Descriptive random stream format is unknown.                                                                                     |
| VSL_RNG_ERROR_BAD_NBITS               | The value in <code>NBits</code> field is bad.                                                                                    |
| VSL_RNG_ERROR_BAD_NSEEDS              | The value in <code>NSeeds</code> field is bad.                                                                                   |
| VSL_RNG_ERROR_BAD_STREAM              | The random stream is invalid.                                                                                                    |
| VSL_RNG_ERROR_BAD_STREAM_STATE_SIZE   | The value in <code>StreamStateSize</code> field is bad.                                                                          |
| VSL_RNG_ERROR_BAD_UPDATE              | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, $< 0$ or $> n_{max}$ . |
| VSL_RNG_ERROR_BAD_WORD_SIZE           | The value in <code>WordSize</code> field is bad.                                                                                 |
| VSL_RNG_ERROR_BRNG_NOT_SUPPORTED      | BRNG is not supported by the function.                                                                                           |
| VSL_RNG_ERROR_BRNG_TABLE_FULL         | Registration cannot be completed due to lack of free entries in the table of registered BRNGs.                                   |
| VSL_RNG_ERROR_BRNGS_INCOMPATIBLE      | Two BRNGs are not compatible for the operation.                                                                                  |
| VSL_RNG_ERROR_FILE_CLOSE              | Error in closing the file.                                                                                                       |
| VSL_RNG_ERROR_FILE_OPEN               | Error in opening the file.                                                                                                       |
| VSL_RNG_ERROR_FILE_READ               | Error in reading the file.                                                                                                       |
| VSL_RNG_ERROR_FILE_WRITE              | Error in writing the file.                                                                                                       |
| VSL_RNG_ERROR_INVALID_ABSTRACT_STREAM | The abstract random stream is invalid.                                                                                           |
| VSL_RNG_ERROR_INVALID_BRNG_INDEX      | BRNG index is not valid.                                                                                                         |
| VSL_RNG_ERROR_LEAPFROG_UNSUPPORTED    | BRNG does not support Leapfrog method.                                                                                           |
| VSL_RNG_ERROR_NO_NUMBERS              | Callback function for an abstract BRNG returns zero as the number of updated entries in a buffer.                                |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED     | Period of the generator is exceeded.                                                                                             |
| VSL_RNG_ERROR_SKIPAHEAD_UNSUPPORTED   | BRNG does not support Skip-Ahead method.                                                                                         |

| Status Code                               | Description                                                                                                                                                                                    |
|-------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_RNG_ERROR_UNSUPPORTED_FILE_VER        | File format version is not supported.                                                                                                                                                          |
| VSL_RNG_ERROR_NONDETERM_NOT_SUPPORTED     | Non-deterministic random number generator is not supported on the CPU running the application.                                                                                                 |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number using non-deterministic random number generator exceeds threshold (see Section 7.2.1.12 <i>Non-deterministic</i> in [VS Notes] for more details) |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                                                                                             |

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## VS RNG Usage Model

A typical algorithm for VS random number generators is as follows:

1. Create and initialize stream/streams. Functions `vslNewStream`, `vslNewStreamEx`, `vslCopyStream`, `vslCopyStreamState`, `vslLeapfrogStream`, `vslSkipAheadStream`.
2. Call one or more RNGs.
3. Process the output.
4. Delete the stream/streams. Function `vslDeleteStream`.

### NOTE

You may reiterate steps 2-3. Random number streams may be generated for different threads.

The following example demonstrates generation of a random stream that is output of basic generator MT19937. The seed is equal to 777. The stream is used to generate 10,000 normally distributed random numbers in blocks of 1,000 random numbers with parameters  $a = 5$  and  $\sigma = 2$ . Delete the streams after completing the generation. The purpose of the example is to calculate the sample mean for normal distribution with the given parameters.

### Example of VS RNG Usage

```
#include <stdio.h>
#include "mkl_vsl.h"

int main()
{
 double r[1000]; /* buffer for random numbers */
 double s; /* average */
 VSLStreamStatePtr stream;
 int i, j;
```

```

/* Initializing */
s = 0.0;
vslNewStream(&stream, VSL_BRNG_MT19937, 777);

/* Generating */
for (i=0; i<10; i++) {
 vdRngGaussian(VSL_RNG_METHOD_GAUSSIAN_ICDF, stream, 1000, r, 5.0, 2.0);
 for (j=0; j<1000; j++) {
 s += r[j];
 }
}
s /= 10000.0;

/* Deleting the stream */
vslDeleteStream(&stream);

/* Printing results */
printf("Sample mean of normal distribution = %f\n", s);

return 0;
}

```

Additionally, examples that demonstrate usage of VS random number generators are available in:

`$(MKL)/examples/vslc/source`

## Service Routines

Stream handling comprises routines for creating, deleting, or copying the streams and getting the index of a basic generator. A random stream can also be saved to and then read from a binary file. [Table "Service Routines"](#) lists all available service routines

### Service Routines

| Routine                            | Short Description                                                                             |
|------------------------------------|-----------------------------------------------------------------------------------------------|
| <code>vslNewStream</code>          | Creates and initializes a random stream.                                                      |
| <code>vslNewStreamEx</code>        | Creates and initializes a random stream for the generators with multiple initial conditions.  |
| <code>vслиNewAbstractStream</code> | Creates and initializes an abstract random stream for integer arrays.                         |
| <code>vslдNewAbstractStream</code> | Creates and initializes an abstract random stream for double precision floating-point arrays. |
| <code>vslsNewAbstractStream</code> | Creates and initializes an abstract random stream for single precision floating-point arrays. |
| <code>vslDeleteStream</code>       | Deletes previously created stream.                                                            |
| <code>vslCopyStream</code>         | Copies a stream to another stream.                                                            |
| <code>vslCopyStreamState</code>    | Creates a copy of a random stream state.                                                      |
| <code>vslSaveStreamF</code>        | Writes a stream to a binary file.                                                             |
| <code>vslLoadStreamF</code>        | Reads a stream from a binary file.                                                            |
| <code>vslSaveStreamM</code>        | Writes a random stream descriptive data, including state, to a memory buffer.                 |

| Routine                               | Short Description                                                                                 |
|---------------------------------------|---------------------------------------------------------------------------------------------------|
| <a href="#">vslLoadStreamM</a>        | Creates a new stream and reads stream descriptive data, including state, from the memory buffer.  |
| <a href="#">vslGetStreamSize</a>      | Computes size of memory necessary to hold the random stream.                                      |
| <a href="#">vslLeapfrogStream</a>     | Initializes the stream by the leapfrog method to generate a subsequence of the original sequence. |
| <a href="#">vslSkipAheadStream</a>    | Initializes the stream by the skip-ahead method.                                                  |
| <a href="#">vslGetStreamStateBrng</a> | Obtains the index of the basic generator responsible for the generation of a given random stream. |
| <a href="#">vslGetNumRegBrngs</a>     | Obtains the number of currently registered basic generators.                                      |

Most of the generator-based work comprises three basic steps:

1. Creating and initializing a stream ([vslNewStream](#), [vslNewStreamEx](#), [vslCopyStream](#), [vslCopyStreamState](#), [vslLeapfrogStream](#), [vslSkipAheadStream](#)).
2. Generating random numbers with given distribution, see [Distribution Generators](#).
3. Deleting the stream ([vslDeleteStream](#)).

Note that you can concurrently create multiple streams and obtain random data from one or several generators by using the stream state. You must use the [vslDeleteStream](#) function to delete all the streams afterwards.

## vslNewStream

*Creates and initializes a random stream.*

### Syntax

```
status = vslNewStream(&stream, brng, seed);
```

### Include Files

- `mk1.h`

### Input Parameters

| Name              | Type                        | Description                                                                                                                                                                                                                                                                       |
|-------------------|-----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>brng</code> | <code>const MKL_INT</code>  | Index of the basic generator to initialize the stream. See <a href="#">Table Values of <code>brng</code> parameter</a> for specific value.                                                                                                                                        |
| <code>seed</code> | <code>const MKL_UINT</code> | Initial condition of the stream. In the case of a quasi-random number generator seed parameter is used to set the dimension. If the dimension is greater than the dimension that <code>brng</code> can support or is less than 1, then the dimension is assumed to be equal to 1. |

### Output Parameters

| Name                | Type                            | Description             |
|---------------------|---------------------------------|-------------------------|
| <code>stream</code> | <code>VSLStreamStatePtr*</code> | Stream state descriptor |

## Description

For a basic generator with number *brng*, this function creates a new stream and initializes it with a 32-bit seed. The seed is an initial value used to select a particular sequence generated by the basic generator *brng*. The function is also applicable for generators with multiple initial conditions. Use this function to create and initialize a new stream with a 32-bit seed only. If you need to provide multiple initial conditions such as several 32-bit or wider seeds, use the function [vslNewStreamEx](#). See [VS Notes](#) for a more detailed description of stream initialization for different basic generators.

### NOTE

This function is not applicable for abstract basic random number generators. Please use [vslNewAbstractStream](#), [vslsNewAbstractStream](#) or [vsldNewAbstractStream](#) to utilize integer, single-precision or double-precision external random data respectively.

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## Return Values

|                                              |                                                                                    |
|----------------------------------------------|------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK                  | Indicates no error, execution is successful.                                       |
| VSL_RNG_ERROR_INVALID_BRNG_INDEX             | BRNG index is invalid.                                                             |
| VSL_ERROR_MEM_FAILURE                        | System cannot allocate memory for <i>stream</i> .                                  |
| VSL_RNG_ERROR_NONDETERMINISTIC_NOT_SUPPORTED | Non-deterministic random number generator is not supported.                        |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED             | ARS-5 random number generator is not supported on the CPU running the application. |

## vslNewStreamEx

*Creates and initializes a random stream for generators with multiple initial conditions.*

### Syntax

```
status = vslNewStreamEx(&stream, brng, n, params);
```

### Include Files

- `mkl.h`

## Input Parameters

| Name          | Type               | Description                                                                                                                                                                                                                                                                                                                                                                       |
|---------------|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>brng</i>   | const MKL_INT      | Index of the basic generator to initialize the stream. See <a href="#">Table "Values of <i>brng</i> parameter"</a> for specific value.                                                                                                                                                                                                                                            |
| <i>n</i>      | const MKL_INT      | Number of initial conditions contained in <i>params</i>                                                                                                                                                                                                                                                                                                                           |
| <i>params</i> | const unsigned int | Array of initial conditions necessary for the basic generator <i>brng</i> to initialize the stream. In the case of a quasi-random number generator only the first element in <i>params</i> parameter is used to set the dimension. If the dimension is greater than the dimension that <i>brng</i> can support or is less than 1, then the dimension is assumed to be equal to 1. |

## Output Parameters

| Name          | Type               | Description             |
|---------------|--------------------|-------------------------|
| <i>stream</i> | VSLStreamStatePtr* | Stream state descriptor |

## Description

The `vslNewStreamEx` function provides an advanced tool to set the initial conditions for a basic generator if its input arguments imply several initialization parameters. Initial values are used to select a particular sequence generated by the basic generator *brng*. Whenever possible, use `vslNewStream`, which is analogous to `vslNewStreamEx` except that it takes only one 32-bit initial condition. In particular, `vslNewStreamEx` may be used to initialize the state table in Generalized Feedback Shift Register Generators (GFSRs). A more detailed description of this issue can be found in [VS Notes](#).

This function is also used to pass user-defined initialization parameters of quasi-random number generators into the library. See [VS Notes](#) for the format for their passing and registration in VS.

### NOTE

This function is not applicable for abstract basic random number generators. Please use `vslNewAbstractStream`, `vslsNewAbstractStream` or `vslldNewAbstractStream` to utilize integer, single-precision or double-precision external random data respectively.

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## Return Values

`VSL_ERROR_OK`, `VSL_STATUS_OK` Indicates no error, execution is successful.

|                                              |                                                                                    |
|----------------------------------------------|------------------------------------------------------------------------------------|
| VSL_RNG_ERROR_INVALID_BRNG_INDEX             | BRNG index is invalid.                                                             |
| VSL_ERROR_MEM_FAILURE                        | System cannot allocate memory for <i>stream</i> .                                  |
| VSL_RNG_ERROR_NONDETERMINISTIC_NOT_SUPPORTED | Non-deterministic random number generator is not supported.                        |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED             | ARS-5 random number generator is not supported on the CPU running the application. |

### vsliNewAbstractStream

Creates and initializes an abstract random stream for integer arrays.

#### Syntax

```
status = vsliNewAbstractStream(&stream, n, ibuf, icallback);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name             | Type               | Description                                                  |
|------------------|--------------------|--------------------------------------------------------------|
| <i>n</i>         | const MKL_INT      | Size of the array <i>ibuf</i>                                |
| <i>ibuf</i>      | const unsigned int | Array of <i>n</i> 32-bit integers                            |
| <i>icallback</i> |                    | Pointer to the callback function used for <i>ibuf</i> update |

#### NOTE

Format of the callback function:

```
int iUpdateFunc(VSLStreamStatePtrstream, int* n, unsigned int ibuf[], int* nmin, int* nmax, int* idx);
```

The callback function returns the number of elements in the array actually updated by the function. [Table \*icallback\* Callback Function Parameters](#) gives the description of the callback function parameters.

#### *icallback* Callback Function Parameters

| Parameters    | Short Description                                                        |
|---------------|--------------------------------------------------------------------------|
| <i>stream</i> | Abstract random stream descriptor                                        |
| <i>n</i>      | Size of <i>ibuf</i>                                                      |
| <i>ibuf</i>   | Array of random numbers associated with the stream <i>stream</i>         |
| <i>nmin</i>   | Minimal quantity of numbers to update                                    |
| <i>nmax</i>   | Maximal quantity of numbers that can be updated                          |
| <i>idx</i>    | Position in cyclic buffer <i>ibuf</i> to start update $0 \leq idx < n$ . |

## Output Parameters

| Name          | Type               | Description                              |
|---------------|--------------------|------------------------------------------|
| <i>stream</i> | VSLStreamStatePtr* | Descriptor of the stream state structure |

## Description

The `vslNewAbstractStream` function creates a new abstract stream and associates it with an integer array *ibuf* and your callback function *icallback* that is intended for updating of *ibuf* content.

## Return Values

|                             |                                                                 |
|-----------------------------|-----------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK | Indicates no error, execution is successful.                    |
| VSL_ERROR_BADARGS           | Parameter <i>n</i> is not positive.                             |
| VSL_ERROR_MEM_FAILURE       | System cannot allocate memory for <i>stream</i> .               |
| VSL_ERROR_NULL_PTR          | Either buffer or callback function parameter is a NULL pointer. |

## vslNewAbstractStream

*Creates and initializes an abstract random stream for double precision floating-point arrays.*

## Syntax

```
status = vslNewAbstractStream(&stream, n, dbuf, a, b, dcallback);
```

## Include Files

- mkl.h

## Input Parameters

| Name             | Type                  | Description                                                                                                    |
|------------------|-----------------------|----------------------------------------------------------------------------------------------------------------|
| <i>n</i>         | const MKL_INT         | Size of the array <i>dbuf</i>                                                                                  |
| <i>dbuf</i>      | const double          | Array of <i>n</i> double precision floating-point random numbers with uniform distribution over interval (a,b) |
| <i>a</i>         | const double          | Left boundary a                                                                                                |
| <i>b</i>         | const double          | Right boundary b                                                                                               |
| <i>dcallback</i> | See <i>Note</i> below | Pointer to the callback function used for update of the array <i>dbuf</i>                                      |

## Output Parameters

| Name          | Type               | Description                              |
|---------------|--------------------|------------------------------------------|
| <i>stream</i> | VSLStreamStatePtr* | Descriptor of the stream state structure |

**NOTE**

Format of the callback function:

```
int dUpdateFunc(VSLStreamStatePtr stream, int* n, double dbuf[], int* nmin, int* nmax, int*
idx);
```

The callback function returns the number of elements in the array actually updated by the function. [Table dcallback Callback Function Parameters](#) gives the description of the callback function parameters.

**dcallback Callback Function Parameters**

| Parameters    | Short Description                                                        |
|---------------|--------------------------------------------------------------------------|
| <i>stream</i> | Abstract random stream descriptor                                        |
| <i>n</i>      | Size of <i>dbuf</i>                                                      |
| <i>dbuf</i>   | Array of random numbers associated with the stream <i>stream</i>         |
| <i>nmin</i>   | Minimal quantity of numbers to update                                    |
| <i>nmax</i>   | Maximal quantity of numbers that can be updated                          |
| <i>idx</i>    | Position in cyclic buffer <i>dbuf</i> to start update $0 \leq idx < n$ . |

**Description**

The `vslNewAbstractStream` function creates a new abstract stream for double precision floating-point arrays with random numbers of the uniform distribution over interval  $(a,b)$ . The function associates the stream with a double precision array *dbuf* and your callback function *dcallback* that is intended for updating of *dbuf* content.

**Return Values**

|                             |                                                                 |
|-----------------------------|-----------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK | Indicates no error, execution is successful.                    |
| VSL_ERROR_BADARGS           | Parameter <i>n</i> is not positive.                             |
| VSL_ERROR_MEM_FAILURE       | System cannot allocate memory for <i>stream</i> .               |
| VSL_ERROR_NULL_PTR          | Either buffer or callback function parameter is a NULL pointer. |

**vslsNewAbstractStream**

*Creates and initializes an abstract random stream for single precision floating-point arrays.*

**Syntax**

```
status = vslsNewAbstractStream(&stream, n, sbuf, a, b, scallback);
```

**Include Files**

- `mkl.h`

## Input Parameters

| Name             | Type                  | Description                                                                                                    |
|------------------|-----------------------|----------------------------------------------------------------------------------------------------------------|
| <i>n</i>         | const MKL_INT         | Size of the array <i>sbuf</i>                                                                                  |
| <i>sbuf</i>      | const float           | Array of <i>n</i> single precision floating-point random numbers with uniform distribution over interval (a,b) |
| <i>a</i>         | const float           | Left boundary a                                                                                                |
| <i>b</i>         | const float           | Right boundary b                                                                                               |
| <i>scallback</i> | See <i>Note</i> below | Pointer to the callback function used for update of the array <i>sbuf</i>                                      |

## Output Parameters

| Name          | Type               | Description                              |
|---------------|--------------------|------------------------------------------|
| <i>stream</i> | VSLStreamStatePtr* | Descriptor of the stream state structure |

### NOTE

Format of the callback function in C:

```
int sUpdateFunc(VSLStreamStatePtr stream, int* n, float sbuf[], int* nmin, int* nmax, int* idx);
```

The callback function returns the number of elements in the array actually updated by the function. [Table \*scallback\* Callback Function Parameters](#) gives the description of the callback function parameters.

## scallback Callback Function Parameters

| Parameters    | Short Description                                                        |
|---------------|--------------------------------------------------------------------------|
| <i>stream</i> | Abstract random stream descriptor                                        |
| <i>n</i>      | Size of <i>sbuf</i>                                                      |
| <i>sbuf</i>   | Array of random numbers associated with the stream <i>stream</i>         |
| <i>nmin</i>   | Minimal quantity of numbers to update                                    |
| <i>nmax</i>   | Maximal quantity of numbers that can be updated                          |
| <i>idx</i>    | Position in cyclic buffer <i>sbuf</i> to start update $0 \leq idx < n$ . |

## Description

The `vslsNewAbstractStream` function creates a new abstract stream for single precision floating-point arrays with random numbers of the uniform distribution over interval (a,b). The function associates the stream with a single precision array *sbuf* and your callback function *scallback* that is intended for updating of *sbuf* content.

## Return Values

|                             |                                              |
|-----------------------------|----------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK | Indicates no error, execution is successful. |
| VSL_ERROR_BADARGS           | Parameter <i>n</i> is not positive.          |

|                       |                                                                 |
|-----------------------|-----------------------------------------------------------------|
| VSL_ERROR_MEM_FAILURE | System cannot allocate memory for <i>stream</i> .               |
| VSL_ERROR_NULL_PTR    | Either buffer or callback function parameter is a NULL pointer. |

## vslDeleteStream

*Deletes a random stream.*

---

### Syntax

```
status = vslDeleteStream(&stream);
```

### Include Files

- mkl.h

### Input/Output Parameters

| Name          | Type               | Description                                                                                                              |
|---------------|--------------------|--------------------------------------------------------------------------------------------------------------------------|
| <i>stream</i> | VSLStreamStatePtr* | Stream state descriptor. Must have non-zero value. After the stream is successfully deleted, the pointer is set to NULL. |

### Description

The function deletes the random stream created by one of the initialization functions.

### Return Values

|                             |                                              |
|-----------------------------|----------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK | Indicates no error, execution is successful. |
| VSL_ERROR_NULL_PTR          | <i>stream</i> parameter is a NULL pointer.   |
| VSL_RNG_ERROR_BAD_STREAM    | <i>stream</i> is not a valid random stream.  |

## vslCopyStream

*Creates a copy of a random stream.*

---

### Syntax

```
status = vslCopyStream(&newstream, srcstream);
```

### Include Files

- mkl.h

### Input Parameters

| Name             | Type                    | Description                                        |
|------------------|-------------------------|----------------------------------------------------|
| <i>srcstream</i> | const VSLStreamStatePtr | Pointer to the stream state structure to be copied |

## Output Parameters

| Name             | Type               | Description                     |
|------------------|--------------------|---------------------------------|
| <i>newstream</i> | VSLStreamStatePtr* | Copied random stream descriptor |

## Description

The function creates an exact copy of *srcstream* and stores its descriptor to *newstream*.

## Return Values

|                             |                                                      |
|-----------------------------|------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK | Indicates no error, execution is successful.         |
| VSL_ERROR_NULL_PTR          | <i>srcstream</i> parameter is a NULL pointer.        |
| VSL_RNG_ERROR_BAD_STREAM    | <i>srcstream</i> is not a valid random stream.       |
| VSL_ERROR_MEM_FAILURE       | System cannot allocate memory for <i>newstream</i> . |

## vslCopyStreamState

*Creates a copy of a random stream state.*

## Syntax

```
status = vslCopyStreamState(deststream, srcstream);
```

## Include Files

- mkl.h

## Input Parameters

| Name             | Type                    | Description                                                                     |
|------------------|-------------------------|---------------------------------------------------------------------------------|
| <i>srcstream</i> | const VSLStreamStatePtr | Pointer to the stream state structure, from which the state structure is copied |

## Output Parameters

| Name              | Type              | Description                                                            |
|-------------------|-------------------|------------------------------------------------------------------------|
| <i>deststream</i> | VSLStreamStatePtr | Pointer to the stream state structure where the stream state is copied |

## Description

The `vslCopyStreamState` function copies a stream state from *srcstream* to the existing *deststream* stream. Both the streams should be generated by the same basic generator. An error message is generated when the index of the BRNG that produced *deststream* stream differs from the index of the BRNG that generated *srcstream* stream.

Unlike `vslCopyStream` function, which creates a new stream and copies both the stream state and other data from *srcstream*, the function `vslCopyStreamState` copies only *srcstream* stream state data to the generated *deststream* stream.

## Return Values

|                                  |                                                                                                       |
|----------------------------------|-------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK      | Indicates no error, execution is successful.                                                          |
| VSL_ERROR_NULL_PTR               | Either <i>srcstream</i> or <i>deststream</i> is a NULL pointer.                                       |
| VSL_RNG_ERROR_BAD_STREAM         | Either <i>srcstream</i> or <i>deststream</i> is not a valid random stream.                            |
| VSL_RNG_ERROR_BRNGS_INCOMPATIBLE | BRNG associated with <i>srcstream</i> is not compatible with BRNG associated with <i>deststream</i> . |

## vslSaveStreamF

Writes random stream descriptive data, including stream state, to binary file.

### Syntax

```
errstatus = vslSaveStreamF(stream, fname);
```

### Include Files

- mkl.h

### Input Parameters

| Name          | Type                    | Description                                     |
|---------------|-------------------------|-------------------------------------------------|
| <i>stream</i> | const VSLStreamStatePtr | Random stream to be written to the file         |
| <i>fname</i>  | const char*             | File name specified as a null-terminated string |

### Output Parameters

| Name             | Type | Description                   |
|------------------|------|-------------------------------|
| <i>errstatus</i> | int  | Error status of the operation |

### Description

The `vslSaveStreamF` function writes the random stream descriptive data, including the stream state, to the binary file. Random stream descriptive data is saved to the binary file with the name *fname*. The random stream *stream* must be a valid stream created by `vslNewStream`-like or `vslCopyStream`-like service routines. If the stream cannot be saved to the file, *errstatus* has a non-zero value. The random stream can be read from the binary file using the `vslLoadStreamF` function.

## Return Values

|                             |                                                         |
|-----------------------------|---------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK | Indicates no error, execution is successful.            |
| VSL_ERROR_NULL_PTR          | Either <i>fname</i> or <i>stream</i> is a NULL pointer. |
| VSL_RNG_ERROR_BAD_STREAM    | <i>stream</i> is not a valid random stream.             |
| VSL_RNG_ERROR_FILE_OPEN     | Indicates an error in opening the file.                 |
| VSL_RNG_ERROR_FILE_WRITE    | Indicates an error in writing the file.                 |

|                          |                                                   |
|--------------------------|---------------------------------------------------|
| VSL_RNG_ERROR_FILE_CLOSE | Indicates an error in closing the file.           |
| VSL_ERROR_MEM_FAILURE    | System cannot allocate memory for internal needs. |

## vslLoadStreamF

Creates new stream and reads stream descriptive data, including stream state, from binary file.

### Syntax

```
errstatus = vslLoadStreamF(&stream, fname);
```

### Include Files

- mkl.h

### Input Parameters

| Name         | Type        | Description                                     |
|--------------|-------------|-------------------------------------------------|
| <i>fname</i> | const char* | File name specified as a null-terminated string |

### Output Parameters

| Name             | Type               | Description                    |
|------------------|--------------------|--------------------------------|
| <i>stream</i>    | VSLStreamStatePtr* | Pointer to a new random stream |
| <i>errstatus</i> | int                | Error status of the operation  |

### Description

The `vslLoadStreamF` function creates a new stream and reads stream descriptive data, including the stream state, from the binary file. A new random stream is created using the stream descriptive data from the binary file with the name *fname*. If the stream cannot be read (for example, an I/O error occurs or the file format is invalid), *errstatus* has a non-zero value. To save random stream to the file, use `vslSaveStreamF` function.

#### CAUTION

Calling `vslLoadStreamF` with a previously initialized *stream* pointer can have unintended consequences such as a memory leak. To initialize a stream which has been in use until calling `vslLoadStreamF`, you should call the `vslDeleteStream` function first to deallocate the resources.

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## Return Values

|                                              |                                                                                    |
|----------------------------------------------|------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK                  | Indicates no error, execution is successful.                                       |
| VSL_ERROR_NULL_PTR                           | <i>fname</i> is a NULL pointer.                                                    |
| VSL_RNG_ERROR_FILE_OPEN                      | Indicates an error in opening the file.                                            |
| VSL_RNG_ERROR_FILE_WRITE                     | Indicates an error in writing the file.                                            |
| VSL_RNG_ERROR_FILE_CLOSE                     | Indicates an error in closing the file.                                            |
| VSL_ERROR_MEM_FAILURE                        | System cannot allocate memory for internal needs.                                  |
| VSL_RNG_ERROR_BAD_FILE_FORMAT                | Unknown file format.                                                               |
| VSL_RNG_ERROR_UNSUPPORTED_FILE_VER           | File format version is unsupported.                                                |
| VSL_RNG_ERROR_NONDETERMINISTIC_NOT_SUPPORTED | Non-deterministic random number generator is not supported.                        |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED             | ARS-5 random number generator is not supported on the CPU running the application. |

## vslSaveStreamM

Writes random stream descriptive data, including stream state, to a memory buffer.

### Syntax

```
errstatus = vslSaveStreamM(stream, memptr);
```

### Include Files

- mkl.h

### Input Parameters

| Name          | Type                    | Description                                             |
|---------------|-------------------------|---------------------------------------------------------|
| <i>stream</i> | const VSLStreamStatePtr | Random stream to be written to the memory               |
| <i>memptr</i> | char*                   | Memory buffer to save random stream descriptive data to |

### Output Parameters

| Name             | Type | Description                   |
|------------------|------|-------------------------------|
| <i>errstatus</i> | int  | Error status of the operation |

### Description

The `vslSaveStreamM` function writes the random stream descriptive data, including the stream state, to the memory at `memptr`. Random stream `stream` must be a valid stream created by `vslNewStream`-like or `vslCopyStream`-like service routines. The `memptr` parameter must be a valid pointer to the memory of size sufficient to hold the random stream `stream`. Use the service routine `vslGetStreamSize` to determine this amount of memory.

If the stream cannot be saved to the memory, *errstatus* has a non-zero value. The random stream can be read from the memory pointed by *memptr* using the [vslLoadStreamM](#) function.

## Return Values

|                             |                                                          |
|-----------------------------|----------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK | Indicates no error, execution is successful.             |
| VSL_ERROR_NULL_PTR          | Either <i>memptr</i> or <i>stream</i> is a NULL pointer. |
| VSL_RNG_ERROR_BAD_STREAM    | <i>stream</i> is a NULL pointer.                         |

## vslLoadStreamM

*Creates a new stream and reads stream descriptive data, including stream state, from the memory buffer.*

## Syntax

```
errstatus = vslLoadStreamM(&stream, memptr);
```

## Include Files

- mkl.h

## Input Parameters

| Name          | Type        | Description                                               |
|---------------|-------------|-----------------------------------------------------------|
| <i>memptr</i> | const char* | Memory buffer to load random stream descriptive data from |

## Output Parameters

| Name             | Type               | Description                    |
|------------------|--------------------|--------------------------------|
| <i>stream</i>    | VSLStreamStatePtr* | Pointer to a new random stream |
| <i>errstatus</i> | int                | Error status of the operation  |

## Description

The [vslLoadStreamM](#) function creates a new stream and reads stream descriptive data, including the stream state, from the memory buffer. A new random stream is created using the stream descriptive data from the memory pointer by *memptr*. If the stream cannot be read (for example, *memptr* is invalid), *errstatus* has a non-zero value. To save random stream to the memory, use [vslSaveStreamM](#) function. Use the service routine [vslGetStreamSize](#) to determine the amount of memory sufficient to hold the random stream.

### CAUTION

Calling [LoadStreamM](#) with a previously initialized *stream* pointer can have unintended consequences such as a memory leak. To initialize a stream which has been in use until calling [vslLoadStreamM](#), you should call the [vslDeleteStream](#) function first to deallocate the resources.

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**Return Values**

|                                              |                                                                                    |
|----------------------------------------------|------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK                  | Indicates no error, execution is successful.                                       |
| VSL_ERROR_NULL_PTR                           | <i>memptr</i> is a NULL pointer.                                                   |
| VSL_ERROR_MEM_FAILURE                        | System cannot allocate memory for internal needs.                                  |
| VSL_RNG_ERROR_BAD_MEM_FORMAT                 | Descriptive random stream format is unknown.                                       |
| VSL_RNG_ERROR_NONDETERMINISTIC_NOT_SUPPORTED | Non-deterministic random number generator is not supported.                        |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED             | ARS-5 random number generator is not supported on the CPU running the application. |

**vslGetStreamSize**

Computes size of memory necessary to hold the random stream.

**Syntax**

```
memsize = vslGetStreamSize(stream);
```

**Include Files**

- mkl.h

**Input Parameters**

| Name          | Type                    | Description   |
|---------------|-------------------------|---------------|
| <i>stream</i> | const VSLStreamStatePtr | Random stream |

**Output Parameters**

| Name           | Type | Description                                                                                 |
|----------------|------|---------------------------------------------------------------------------------------------|
| <i>memsize</i> | int  | Amount of memory in bytes necessary to hold descriptive data of random stream <i>stream</i> |

**Description**

The `vslGetStreamSize` function returns the size of memory in bytes which is necessary to hold the given random stream. Use the output of the function to allocate the buffer to which you will save the random stream by means of the `vslSaveStreamM` function.

## Return Values

VSL\_ERROR\_OK, VSL\_STATUS\_OK Indicates no error, execution is successful.

VSL\_RNG\_ERROR\_BAD\_STREAM *stream* is a NULL pointer.

## vslLeapfrogStream

Initializes a stream using the leapfrog method.

### Syntax

```
status = vslLeapfrogStream(stream, k, nstreams);
```

### Include Files

- mkl.h

### Input Parameters

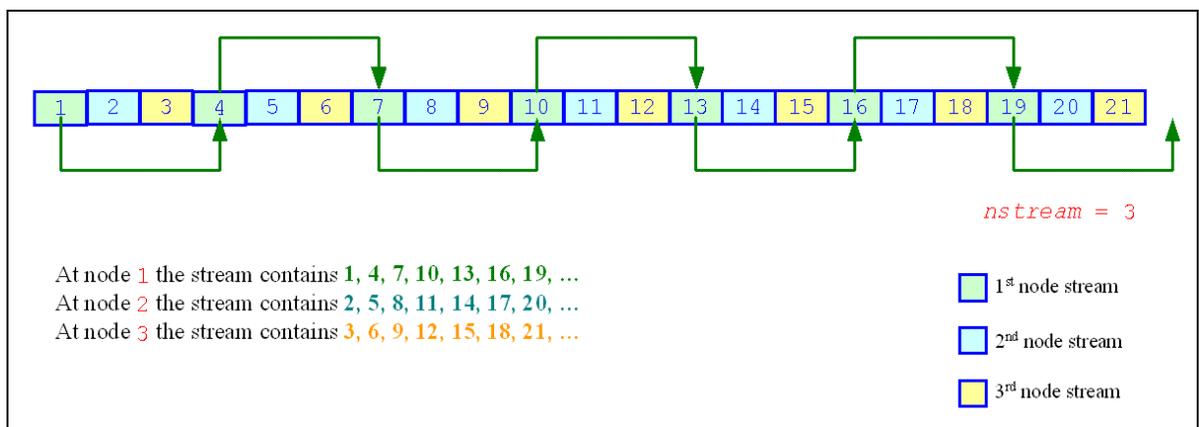
| Name            | Type              | Description                                                               |
|-----------------|-------------------|---------------------------------------------------------------------------|
| <i>stream</i>   | VSLStreamStatePtr | Pointer to the stream state structure to which leapfrog method is applied |
| <i>k</i>        | const MKL_INT     | Index of the computational node, or stream number                         |
| <i>nstreams</i> | const MKL_INT     | Largest number of computational nodes, or stride                          |

### Description

The `vslLeapfrogStream` function generates random numbers in a random stream with non-unit stride. This feature is particularly useful in distributing random numbers from the original stream across the `nstreams` buffers without generating the original random sequence with subsequent manual distribution.

One of the important applications of the leapfrog method is splitting the original sequence into non-overlapping subsequences across `nstreams` computational nodes. The function initializes the original random stream (see [Figure "Leapfrog Method"](#)) to generate random numbers for the computational node  $k$ ,  $0 \leq k < nstreams$ , where `nstreams` is the largest number of computational nodes used.

### Leapfrog Method



The leapfrog method is supported only for those basic generators that allow splitting elements by the leapfrog method, which is more efficient than simply generating them by a generator with subsequent manual distribution across computational nodes. See [VS Notes](#) for details.

For quasi-random basic generators, the leapfrog method allows generating individual components of quasi-random vectors instead of whole quasi-random vectors. In this case *nstreams* parameter should be equal to the dimension of the quasi-random vector while *k* parameter should be the index of a component to be generated ( $0 \leq k < nstreams$ ). Other parameters values are not allowed.

The following code illustrates the initialization of three independent streams using the leapfrog method:

### Code for Leapfrog Method

```
...
VSLStreamStatePtr stream1;
VSLStreamStatePtr stream2;
VSLStreamStatePtr stream3;

/* Creating 3 identical streams */
status = vslNewStream(&stream1, VSL_BRNG_MCG31, 174);
status = vslCopyStream(&stream2, stream1);
status = vslCopyStream(&stream3, stream1);

/* Leapfrogging the streams
*/
status = vslLeapfrogStream(stream1, 0, 3);
status = vslLeapfrogStream(stream2, 1, 3);
status = vslLeapfrogStream(stream3, 2, 3);

/* Generating random numbers
*/
...
/* Deleting the streams
*/
status = vslDeleteStream(&stream1);
status = vslDeleteStream(&stream2);
status = vslDeleteStream(&stream3);
...
```

### Return Values

|                                    |                                              |
|------------------------------------|----------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK        | Indicates no error, execution is successful. |
| VSL_ERROR_NULL_PTR                 | <i>stream</i> is a NULL pointer.             |
| VSL_RNG_ERROR_BAD_STREAM           | <i>stream</i> is not a valid random stream.  |
| VSL_RNG_ERROR_LEAPFROG_UNSUPPORTED | BRNG does not support Leapfrog method.       |

### vslSkipAheadStream

*Initializes a stream using the block-splitting method.*

#### Syntax

```
status = vslSkipAheadStream(stream, nskip);
```

## Include Files

- `mkl.h`

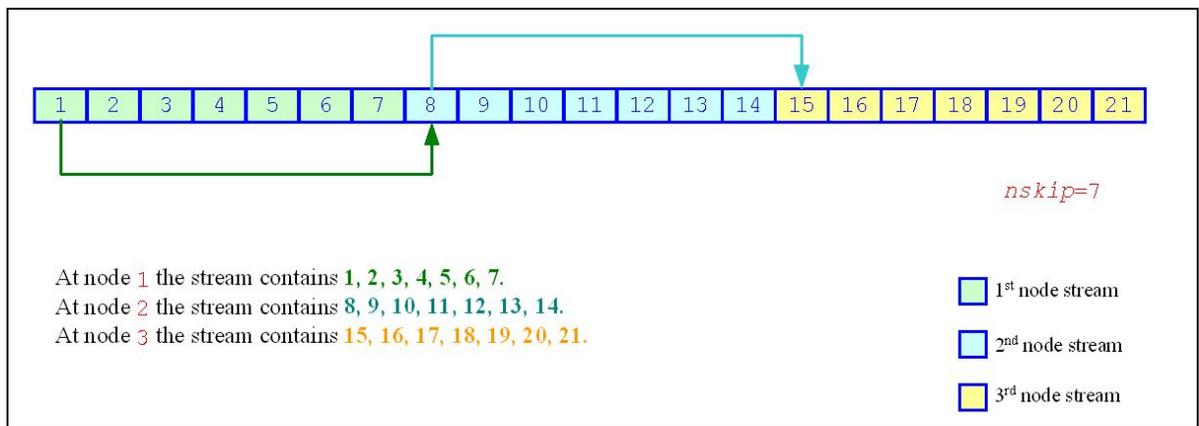
## Input Parameters

| Name                | Type                             | Description                                                                      |
|---------------------|----------------------------------|----------------------------------------------------------------------------------|
| <code>stream</code> | <code>VSLStreamStatePtr</code>   | Pointer to the stream state structure to which block-splitting method is applied |
| <code>nskip</code>  | <code>const long long int</code> | Number of skipped elements                                                       |

## Description

The `vslSkipAheadStream` function skips a given number of elements in a random stream. This feature is particularly useful in distributing random numbers from original random stream across different computational nodes. If the largest number of random numbers used by a computational node is `nskip`, then the original random sequence may be split by `vslSkipAheadStream` into non-overlapping blocks of `nskip` size so that each block corresponds to the respective computational node. The number of computational nodes is unlimited. This method is known as the block-splitting method or as the skip-ahead method. (see Figure "Block-Splitting Method").

## Block-Splitting Method



The skip-ahead method is supported only for those basic generators that allow skipping elements by the skip-ahead method, which is more efficient than simply generating them by generator with subsequent manual skipping. See [VS Notes](#) for details.

Please note that for quasi-random basic generators the skip-ahead method works with components of quasi-random vectors rather than with whole quasi-random vectors. Therefore, to skip `NS` quasi-random vectors, set the `nskip` parameter equal to the `NS*DIMEN`, where `DIMEN` is the dimension of the quasi-random vector. If this operation results in exceeding the period of the quasi-random number generator, which is  $2^{32}-1$ , the library returns the `VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED` error code.

The following code illustrates how to initialize three independent streams using the `vslSkipAheadStream` function:

## Code for Block-Splitting Method

```
VSLStreamStatePtr stream1;
VSLStreamStatePtr stream2;
VSLStreamStatePtr stream3;
```

```

/* Creating the 1st stream
*/
status = vslNewStream(&stream1, VSL_BRNG_MCG31, 174);

/* Skipping ahead by 7 elements the 2nd stream */
status = vslCopyStream(&stream2, stream1);
status = vslSkipAheadStream(stream2, 7);

/* Skipping ahead by 7 elements the 3rd stream */
status = vslCopyStream(&stream3, stream2);
status = vslSkipAheadStream(stream3, 7);

/* Generating random numbers
*/
...
/* Deleting the streams
*/
status = vslDeleteStream(&stream1);
status = vslDeleteStream(&stream2);
status = vslDeleteStream(&stream3);
...

```

## Return Values

|                                     |                                                          |
|-------------------------------------|----------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK         | Indicates no error, execution is successful.             |
| VSL_ERROR_NULL_PTR                  | <i>stream</i> is a NULL pointer.                         |
| VSL_RNG_ERROR_BAD_STREAM            | <i>stream</i> is not a valid random stream.              |
| VSL_RNG_ERROR_SKIPAHEAD_UNSUPPORTED | BRNG does not support the Skip-Ahead method.             |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED   | Period of the quasi-random number generator is exceeded. |

## vslGetStreamStateBrng

Returns index of a basic generator used for generation of a given random stream.

## Syntax

```
brng = vslGetStreamStateBrng(stream);
```

## Include Files

- mkl.h

## Input Parameters

| Name          | Type                    | Description                           |
|---------------|-------------------------|---------------------------------------|
| <i>stream</i> | const VSLStreamStatePtr | Pointer to the stream state structure |

## Output Parameters

| Name        | Type | Description                                                                                              |
|-------------|------|----------------------------------------------------------------------------------------------------------|
| <i>brng</i> | int  | Index of the basic generator assigned for the generation of <i>stream</i> ; negative in case of an error |

### Description

The `vslGetStreamStateBrng` function retrieves the index of a basic generator used for generation of a given random stream.

### Return Values

|                          |                                             |
|--------------------------|---------------------------------------------|
| VSL_ERROR_NULL_PTR       | <i>stream</i> is a NULL pointer.            |
| VSL_RNG_ERROR_BAD_STREAM | <i>stream</i> is not a valid random stream. |

## vslGetNumRegBrngs

Obtains the number of currently registered basic generators.

### Syntax

```
nregbrngs = vslGetNumRegBrngs(void);
```

### Include Files

- `mkl.h`

## Output Parameters

| Name             | Type | Description                                                              |
|------------------|------|--------------------------------------------------------------------------|
| <i>nregbrngs</i> | int  | Number of basic generators registered at the moment of the function call |

### Description

The `vslGetNumRegBrngs` function obtains the number of currently registered basic generators. Whenever user registers a user-designed basic generator, the number of registered basic generators is incremented. The maximum number of basic generators that can be registered is determined by the `VSL_MAX_REG_BRNGS` parameter.

## Distribution Generators

Intel MKL VS routines are used to generate random numbers with different types of distribution. Each function group is introduced below by the type of underlying distribution and contains a short description of its functionality, as well as specifications of the call sequence and the explanation of input and output parameters. [Table "Continuous Distribution Generators"](#) and [Table "Discrete Distribution Generators"](#) list the random number generator routines with data types and output distributions, and sets correspondence between data types of the generator routines and the basic random number generators.

### Continuous Distribution Generators

| Type of Distribution     | Data Types  | BRNG Data Type | Description                                             |
|--------------------------|-------------|----------------|---------------------------------------------------------|
| <code>vRngUniform</code> | <i>s, d</i> | <i>s, d</i>    | Uniform continuous distribution on the interval $[a,b)$ |

| Type of Distribution         | Data Types | BRNG Data Type | Description                                            |
|------------------------------|------------|----------------|--------------------------------------------------------|
| <code>vRngGaussian</code>    | s, d       | s, d           | Normal (Gaussian) distribution                         |
| <code>vRngGaussianMV</code>  | s, d       | s, d           | Multivariate normal (Gaussian) distribution            |
| <code>vRngExponential</code> | s, d       | s, d           | Exponential distribution                               |
| <code>vRngLaplace</code>     | s, d       | s, d           | Laplace distribution (double exponential distribution) |
| <code>vRngWeibull</code>     | s, d       | s, d           | Weibull distribution                                   |
| <code>vRngCauchy</code>      | s, d       | s, d           | Cauchy distribution                                    |
| <code>vRngRayleigh</code>    | s, d       | s, d           | Rayleigh distribution                                  |
| <code>vRngLognormal</code>   | s, d       | s, d           | Lognormal distribution                                 |
| <code>vRngGumbel</code>      | s, d       | s, d           | Gumbel (extreme value) distribution                    |
| <code>vRngGamma</code>       | s, d       | s, d           | Gamma distribution                                     |
| <code>vRngBeta</code>        | s, d       | s, d           | Beta distribution                                      |

### Discrete Distribution Generators

| Type of Distribution            | Data Types | BRNG Data Type                                                                                                                                                                         | Description                                            |
|---------------------------------|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------|
| <code>vRngUniform</code>        | i          | d                                                                                                                                                                                      | Uniform discrete distribution on the interval $[a,b]$  |
| <code>vRngUniformBits</code>    | i          | i                                                                                                                                                                                      | Underlying BRNG integer recurrence                     |
| <code>vRngUniformBits 32</code> | i          | i                                                                                                                                                                                      | Uniformly distributed bits in 32-bit chunks            |
| <code>vRngUniformBits 64</code> | i          | i                                                                                                                                                                                      | Uniformly distributed bits in 64-bit chunks            |
| <code>vRngBernoulli</code>      | i          | s                                                                                                                                                                                      | Bernoulli distribution                                 |
| <code>vRngGeometric</code>      | i          | s                                                                                                                                                                                      | Geometric distribution                                 |
| <code>vRngBinomial</code>       | i          | d                                                                                                                                                                                      | Binomial distribution                                  |
| <code>vRngHypergeometric</code> | i          | d                                                                                                                                                                                      | Hypergeometric distribution                            |
| <code>vRngPoisson</code>        | i          | s (for <code>VSL_RNG_METHOD_POISSON_POISNORM</code> )<br>s (for distribution parameter $\lambda \geq 27$ ) and d (for $\lambda < 27$ ) (for <code>VSL_RNG_METHOD_POISSON_PTPE</code> ) | Poisson distribution                                   |
| <code>vRngPoissonV</code>       | i          | s                                                                                                                                                                                      | Poisson distribution with varying mean                 |
| <code>vRngNegBinomial</code>    | i          | d                                                                                                                                                                                      | Negative binomial distribution, or Pascal distribution |

## Modes of random number generation

The library provides two modes of random number generation, accurate and fast. Accurate generation mode is intended for the applications that are highly demanding to accuracy of calculations. When used in this mode, the generators produce random numbers lying completely within definitional domain for all values of the distribution parameters. For example, random numbers obtained from the generator of continuous distribution that is uniform on interval  $[a,b]$  belong to this interval irrespective of what  $a$  and  $b$  values may be. Fast mode provides high performance of generation and also guarantees that generated random numbers belong to the definitional domain except for some specific values of distribution parameters. The generation mode is set by specifying relevant value of the method parameter in generator routines. List of distributions that support accurate mode of generation is given in the table below.

### Distribution Generators Supporting Accurate Mode

| Type of Distribution         | Data Types        |
|------------------------------|-------------------|
| <code>vRngUniform</code>     | <code>s, d</code> |
| <code>vRngExponential</code> | <code>s, d</code> |
| <code>vRngWeibull</code>     | <code>s, d</code> |
| <code>vRngRayleigh</code>    | <code>s, d</code> |
| <code>vRngLognormal</code>   | <code>s, d</code> |
| <code>vRngGamma</code>       | <code>s, d</code> |
| <code>vRngBeta</code>        | <code>s, d</code> |

See additional details about accurate and fast mode of random number generation in [VS Notes](#).

## New method names

The current version of Intel MKL has a modified structure of VS RNG method names. (See [RNG Naming Conventions](#) for details.) The old names are kept for backward compatibility. The set correspondence between the new and legacy method names for VS random number generators.

### Method Names for Continuous Distribution Generators

| RNG                              | Legacy Method Name                                                                                                                                                                                                | New Method Name                                                                                                      |
|----------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------|
| <code>vRngUniform</code>         | VSL_METHOD_SUNIFORM_STD,<br>VSL_METHOD_DUNIFORM_STD,<br>VSL_METHOD_SUNIFORM_STD_ACCURATE,<br>VSL_METHOD_DUNIFORM_STD_ACCURATE                                                                                     | VSL_RNG_METHOD_UNIFORM_STD,<br>VSL_RNG_METHOD_UNIFORM_STD_ACCURATE                                                   |
| <code>vRngGaussia<br/>n</code>   | VSL_METHOD_SGAUSSIAN_BOXMULLER,<br>VSL_METHOD_SGAUSSIAN_BOXMULLER2,<br>VSL_METHOD_SGAUSSIAN_ICDF,<br>VSL_METHOD_DGAUSSIAN_BOXMULLER,<br>VSL_METHOD_DGAUSSIAN_BOXMULLER2,<br>VSL_METHOD_DGAUSSIAN_ICDF             | VSL_RNG_METHOD_GAUSSIAN_BOXMULLER,<br>VSL_RNG_METHOD_GAUSSIAN_BOXMULLER2,<br>VSL_RNG_METHOD_GAUSSIAN_ICDF            |
| <code>vRngGaussia<br/>nMV</code> | VSL_METHOD_SGAUSSIANMV_BOXMULLER,<br>VSL_METHOD_SGAUSSIANMV_BOXMULLER2,<br>VSL_METHOD_SGAUSSIANMV_ICDF,<br>VSL_METHOD_DGAUSSIANMV_BOXMULLER,<br>VSL_METHOD_DGAUSSIANMV_BOXMULLER2,<br>VSL_METHOD_DGAUSSIANMV_ICDF | VSL_RNG_METHOD_GAUSSIANMV_BOXMULLER<br>,<br>VSL_RNG_METHOD_GAUSSIANMV_BOXMULLER<br>2, VSL_RNG_METHOD_GAUSSIANMV_ICDF |

| <b>RNG</b>                   | <b>Legacy Method Name</b>                                                                                                                                                                                                                                                                                                                                                                                                  | <b>New Method Name</b>                                                                                                                                                                                                   |
|------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>vRngExponential</code> | <code>VSL_METHOD_SEXPONENTIAL_ICDF,</code><br><code>VSL_METHOD_DEXPONENTIAL_ICDF,</code><br><code>VSL_METHOD_SEXPONENTIAL_ICDF_ACCURATE,</code><br><code>VSL_METHOD_DEXPONENTIAL_ICDF_ACCURATE</code>                                                                                                                                                                                                                      | <code>VSL_RNG_METHOD_EXPONENTIAL_ICDF,</code><br><code>VSL_RNG_METHOD_EXPONENTIAL_ICDF_ACCURATE</code>                                                                                                                   |
| <code>vRngLaplace</code>     | <code>VSL_METHOD_SLAPLACE_ICDF,</code><br><code>VSL_METHOD_DLAPLACEL_ICDF</code>                                                                                                                                                                                                                                                                                                                                           | <code>VSL_RNG_METHOD_LAPLACE_ICDF</code>                                                                                                                                                                                 |
| <code>vRngWeibull</code>     | <code>VSL_METHOD_SWEIBULL_ICDF,</code><br><code>VSL_METHOD_DWEIBULL_ICDF,</code><br><code>VSL_METHOD_SWEIBULL_ICDF_ACCURATE,</code><br><code>VSL_METHOD_DWEIBULL_ICDF_ACCURATE</code>                                                                                                                                                                                                                                      | <code>VSL_RNG_METHOD_WEIBULL_ICDF,</code><br><code>VSL_RNG_METHOD_WEIBULL_ICDF_ACCURATE</code>                                                                                                                           |
| <code>vRngCauchy</code>      | <code>VSL_METHOD_SCAUCHY_ICDF,</code><br><code>VSL_METHOD_DCAUCHY_ICDF</code>                                                                                                                                                                                                                                                                                                                                              | <code>VSL_RNG_METHOD_CAUCHY_ICDF</code>                                                                                                                                                                                  |
| <code>vRngRayleigh</code>    | <code>VSL_METHOD_SRAYLEIGH_ICDF,</code><br><code>VSL_METHOD_DRAYLEIGH_ICDF,</code><br><code>VSL_METHOD_SRAYLEIGH_ICDF_ACCURATE,</code><br><code>VSL_METHOD_DRAYLEIGH_ICDF_ACCURATE</code>                                                                                                                                                                                                                                  | <code>VSL_RNG_METHOD_RAYLEIGH_ICDF,</code><br><code>VSL_RNG_METHOD_RAYLEIGH_ICDF_ACCURATE</code>                                                                                                                         |
| <code>vRngLognormal</code>   | <code>VSL_METHOD_SLOGNORMAL_BOXMULLER2,</code><br><code>VSL_METHOD_DLOGNORMAL_BOXMULLER2,</code><br><code>VSL_METHOD_SLOGNORMAL_BOXMULLER2_ACCURATE,</code><br><code>VSL_METHOD_DLOGNORMAL_BOXMULLER2_ACCURATE</code><br><br><code>VSL_METHOD_SLOGNORMAL_ICDF,</code><br><code>VSL_METHOD_DLOGNORMAL_ICDF,</code><br><code>VSL_METHOD_SLOGNORMAL_ICDF_ACCURATE,</code><br><code>VSL_METHOD_DLOGNORMAL_ICDF_ACCURATE</code> | <code>VSL_RNG_METHOD_LOGNORMAL_BOXMULLER2,</code><br><code>VSL_RNG_METHOD_LOGNORMAL_BOXMULLER2_ACCURATE</code><br><br><code>VSL_RNG_METHOD_LOGNORMAL_ICDF,</code><br><code>VSL_RNG_METHOD_LOGNORMAL_ICDF_ACCURATE</code> |
| <code>vRngGumbel</code>      | <code>VSL_METHOD_SGUMBEL_ICDF,</code><br><code>VSL_METHOD_DGUMBEL_ICDF</code>                                                                                                                                                                                                                                                                                                                                              | <code>VSL_RNG_METHOD_GUMBEL_ICDF</code>                                                                                                                                                                                  |
| <code>vRngGamma</code>       | <code>VSL_METHOD_SGAMMA_GNORM,</code><br><code>VSL_METHOD_DGAMMA_GNORM,</code><br><code>VSL_METHOD_SGAMMA_GNORM_ACCURATE,</code><br><code>VSL_METHOD_DGAMMA_GNORM_ACCURATE</code>                                                                                                                                                                                                                                          | <code>VSL_RNG_METHOD_GAMMA_GNORM,</code><br><code>VSL_RNG_METHOD_GAMMA_GNORM_ACCURATE</code>                                                                                                                             |
| <code>vRngBeta</code>        | <code>VSL_METHOD_SBETA_CJA,</code><br><code>VSL_METHOD_DBETA_CJA,</code><br><code>VSL_METHOD_SBETA_CJA_ACCURATE,</code><br><code>VSL_METHOD_DBETA_CJA_ACCURATE</code>                                                                                                                                                                                                                                                      | <code>VSL_RNG_METHOD_BETA_CJA,</code><br><code>VSL_RNG_METHOD_BETA_CJA_ACCURATE</code>                                                                                                                                   |

**Method Names for Discrete Distribution Generators**

| <b>RNG</b>                   | <b>Legacy Method Name</b>                | <b>New Method Name</b>                      |
|------------------------------|------------------------------------------|---------------------------------------------|
| <code>vRngUniform</code>     | <code>VSL_METHOD_IUNIFORM_STD</code>     | <code>VSL_RNG_METHOD_UNIFORM_STD</code>     |
| <code>vRngUniformBits</code> | <code>VSL_METHOD_IUNIFORMBITS_STD</code> | <code>VSL_RNG_METHOD_UNIFORMBITS_STD</code> |

| RNG                             | Legacy Method Name                                                                  | New Method Name                                                                           |
|---------------------------------|-------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------|
| <code>vRngBernoulli</code>      | <code>VSL_METHOD_IBERNOULLI_ICDF</code>                                             | <code>VSL_RNG_METHOD_BERNOULLI_ICDF</code>                                                |
| <code>vRngGeometric</code>      | <code>VSL_METHOD_IGEOMETRIC_ICDF</code>                                             | <code>VSL_RNG_METHOD_GEOMETRIC_ICDF</code>                                                |
| <code>vRngBinomial</code>       | <code>VSL_METHOD_IBINOMIAL_BTPE</code>                                              | <code>VSL_RNG_METHOD_BINOMIAL_BTPE</code>                                                 |
| <code>vRngHypergeometric</code> | <code>VSL_METHOD_IHYPERGEOMETRIC_H2PE</code>                                        | <code>VSL_RNG_METHOD_HYPERGEOMETRIC_H2PE</code>                                           |
| <code>vRngPoisson</code>        | <code>VSL_METHOD_IPOISSON_PTPE,</code><br><code>VSL_METHOD_IPOISSON_POISNORM</code> | <code>VSL_RNG_METHOD_POISSON_PTPE,</code><br><code>VSL_RNG_METHOD_POISSON_POISNORM</code> |
| <code>vRngPoissonV</code>       | <code>VSL_METHOD_IPOISSONV_POISNORM</code>                                          | <code>VSL_RNG_METHOD_POISSONV_POISNORM</code>                                             |
| <code>vRngNegBinomial</code>    | <code>VSL_METHOD_INEGBINOMIAL_NBAR</code>                                           | <code>VSL_RNG_METHOD_NEGBINOMIAL_NBAR</code>                                              |

## Continuous Distributions

This section describes routines for generating random numbers with continuous distribution.

### vRngUniform Continuous Distribution Generators

Generates random numbers with uniform distribution.

#### Syntax

```
status = vsRngUniform(method, stream, n, r, a, b);
```

```
status = vdRngUniform(method, stream, n, r, a, b);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| Name                | Type                                                                                                              | Description                                                                                                                                                               |
|---------------------|-------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>method</code> | <code>const MKL_INT</code>                                                                                        | Generation method; the specific values are as follows:<br><code>VSL_RNG_METHOD_UNIFORM_STD</code><br><code>VSL_RNG_METHOD_UNIFORM_STD_ACCURATE</code><br>Standard method. |
| <code>stream</code> | <code>VSLStreamStatePtr</code>                                                                                    | Pointer to the stream state structure                                                                                                                                     |
| <code>n</code>      | <code>const MKL_INT</code>                                                                                        | Number of random values to be generated                                                                                                                                   |
| <code>a</code>      | <code>const float</code> for <code>vsRngUniform</code><br><code>const double</code> for <code>vdRngUniform</code> | Left bound a                                                                                                                                                              |
| <code>b</code>      | <code>const float</code> for <code>vsRngUniform</code>                                                            | Right bound b                                                                                                                                                             |

| Name | Type                             | Description |
|------|----------------------------------|-------------|
|      | const double for<br>vdRngUniform |             |

### Output Parameters

| Name     | Type                                                | Description                                                                        |
|----------|-----------------------------------------------------|------------------------------------------------------------------------------------|
| <i>r</i> | float* for vsRngUniform<br>double* for vdRngUniform | Vector of <i>n</i> random numbers uniformly distributed over the interval $[a, b)$ |

### Description

The `vdRngUniform` function generates random numbers uniformly distributed over the interval  $[a, b)$ , where  $a, b$  are the left and right bounds of the interval, respectively, and  $a, b \in \mathbb{R}; a < b$ .

The probability density function is given by:

$$f_{a,b}(x) = \begin{cases} \frac{1}{b-a}, & x \in [a, b) \\ 0, & x \notin [a, b) \end{cases}, \quad -\infty < x < +\infty$$

The cumulative distribution function is as follows:

$$F_{a,b}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x < b \\ 1, & x \geq b \end{cases}, \quad -\infty < x < +\infty.$$

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### Return Values

|                             |                                              |
|-----------------------------|----------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK | Indicates no error, execution is successful. |
| VSL_ERROR_NULL_PTR          | <i>stream</i> is a NULL pointer.             |

|                                           |                                                                                                                               |
|-------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                                   |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, $< 0$ or $> nmax$ . |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                                |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                                    |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.           |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                            |

## vRngGaussian

Generates normally distributed random numbers.

### Syntax

```
status = vsRngGaussian(method, stream, n, r, a, sigma);
```

```
status = vdRngGaussian(method, stream, n, r, a, sigma);
```

### Include Files

- mkl.h

### Input Parameters

| Name          | Type                                                            | Description                                                                                                                                                                                                                                                                                                       |
|---------------|-----------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT                                                   | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_GAUSSIAN_BOXMULLER<br>VSL_RNG_METHOD_GAUSSIAN_BOXMULLER2<br>VSL_RNG_METHOD_GAUSSIAN_ICDF<br>See brief description of the methods BOXMULLER, BOXMULLER2, and ICDF in <a href="#">Table "Values of &lt;method&gt; in method parameter"</a> |
| <i>stream</i> | VSLStreamStatePtr                                               | Pointer to the stream state structure                                                                                                                                                                                                                                                                             |
| <i>n</i>      | const MKL_INT                                                   | Number of random values to be generated                                                                                                                                                                                                                                                                           |
| <i>a</i>      | const float for vsRngGaussian<br>const double for vdRngGaussian | Mean value $\mu$ .                                                                                                                                                                                                                                                                                                |
| <i>sigma</i>  | const float for vsRngGaussian                                   | Standard deviation $\sigma$ .                                                                                                                                                                                                                                                                                     |

| Name | Type                              | Description |
|------|-----------------------------------|-------------|
|      | const double for<br>vdRngGaussian |             |

### Output Parameters

| Name     | Type                                                  | Description                                            |
|----------|-------------------------------------------------------|--------------------------------------------------------|
| <i>r</i> | float* for vsRngGaussian<br>double* for vdRngGaussian | Vector of <i>n</i> normally distributed random numbers |

### Description

The `vdRngGaussian` function generates random numbers with normal (Gaussian) distribution with mean value *a* and standard deviation  $\sigma$ , where

$$a, \sigma \in \mathbb{R} ; \sigma > 0.$$

The probability density function is given by:

$$f_{a, \sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - a)^2}{2\sigma^2}\right), \quad -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a, \sigma}(x) = \int_{-\infty}^x \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y - a)^2}{2\sigma^2}\right) dy, \quad -\infty < x < +\infty.$$

The cumulative distribution function  $F_{a, \sigma}(x)$  can be expressed in terms of standard normal distribution  $\Phi(x)$  as

$$F_{a, \sigma}(x) = \Phi((x - a)/\sigma)$$

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### Return Values

VSL\_ERROR\_OK, VSL\_STATUS\_OK Indicates no error, execution is successful.

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

### vRngGaussianMV

Generates random numbers from multivariate normal distribution.

### Syntax

```
status = vsRngGaussianMV(method, stream, n, r, dimen, mstorage, a, t);
status = vdRngGaussianMV(method, stream, n, r, dimen, mstorage, a, t);
```

### Include Files

- mkl.h

### Input Parameters

| Name            | Type              | Description                                                                                                                                                                                                                                                                                                             |
|-----------------|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i>   | const MKL_INT     | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_GAUSSIANMV_BOXMULLER<br>VSL_RNG_METHOD_GAUSSIANMV_BOXMULLER2<br>VSL_RNG_METHOD_GAUSSIANMV_ICDF<br>See brief description of the methods BOXMULLER, BOXMULLER2, and ICDF in <a href="#">Table "Values of &lt;method&gt; in method parameter"</a> |
| <i>stream</i>   | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                                                                                                                                                                                   |
| <i>n</i>        | const MKL_INT     | Number of <i>d</i> -dimensional vectors to be generated                                                                                                                                                                                                                                                                 |
| <i>dimen</i>    | const MKL_INT     | Dimension <i>d</i> ( $d \geq 1$ ) of output random vectors                                                                                                                                                                                                                                                              |
| <i>mstorage</i> | const MKL_INT     | Matrix storage scheme for lower triangular matrix <i>T</i> . The routine supports three matrix storage schemes:                                                                                                                                                                                                         |

| Name | Type                                                                      | Description                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|------|---------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|      |                                                                           | <ul style="list-style-type: none"> <li>VSL_MATRIX_STORAGE_FULL— all <math>d \times d</math> elements of the matrix <math>T</math> are passed, however, only the lower triangle part is actually used in the routine.</li> <li>VSL_MATRIX_STORAGE_PACKED— lower triangle elements of <math>T</math> are packed by rows into a one-dimensional array.</li> <li>VSL_MATRIX_STORAGE_DIAGONAL— only diagonal elements of <math>T</math> are passed.</li> </ul> |
| $a$  | const float* for vsRngGaussianMV<br><br>const double* for vdRngGaussianMV | Mean vector $a$ of dimension $d$                                                                                                                                                                                                                                                                                                                                                                                                                          |
| $t$  | const float* for vsRngGaussianMV<br><br>const double* for vdRngGaussianMV | Elements of the lower triangular matrix passed according to the matrix $T$ storage scheme $mstorage$ .                                                                                                                                                                                                                                                                                                                                                    |

### Output Parameters

| Name | Type                                                          | Description                                      |
|------|---------------------------------------------------------------|--------------------------------------------------|
| $r$  | float* for vsRngGaussianMV<br><br>double* for vdRngGaussianMV | Array of $n$ random vectors of dimension $dimen$ |

### Description

The `vsRngGaussianMV` function generates random numbers with  $d$ -variate normal (Gaussian) distribution with mean value  $a$  and variance-covariance matrix  $C$ , where  $a \in R^d$ ;  $C$  is a  $d \times d$  symmetric positive-definite matrix.

The probability density function is given by:

$$f_{a,C}(x) = \frac{1}{\sqrt{\det(2\pi C)}} \exp(-1/2(x - a)^T C^{-1}(x - a)),$$

where  $x \in R^d$ .

Matrix  $C$  can be represented as  $C = TT^T$ , where  $T$  is a lower triangular matrix - Cholesky factor of  $C$ .

Instead of variance-covariance matrix  $C$  the generation routines require Cholesky factor of  $C$  in input. To compute Cholesky factor of matrix  $C$ , the user may call Intel MKL LAPACK routines for matrix factorization: `?potrf` or `?pptrf` for `v?RngGaussianMV/v?rnggaussianmv` routines (? means either `s` or `d` for single and double precision respectively). See [Application Notes](#) for more details.

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**Application Notes**

Since matrices are stored in Fortran by columns, while in C they are stored by rows, the usage of Intel MKL factorization routines (assuming Fortran matrices storage) in combination with multivariate normal RNG (assuming C matrix storage) is slightly different in C and Fortran. The following tables help in using these routines in C and Fortran. For further information please refer to the appropriate VS example file.

**Using Cholesky Factorization Routines in C**

| <b>Matrix Storage Scheme</b> | <b>Variance-Covariance Matrix Argument</b>                       | <b>Factorization Routine</b>                                       | <b>UPLO Parameter in Factorization Routine</b> | <b>Result of Factorization as Input Argument for RNG</b>              |
|------------------------------|------------------------------------------------------------------|--------------------------------------------------------------------|------------------------------------------------|-----------------------------------------------------------------------|
| VSL_MATRIX_STORAGE_FULL      | C in C two-dimensional array                                     | spotrf for<br>vsRngGaussianMV<br><br>dpotrf for<br>vdRngGaussianMV | 'U'                                            | Lower triangle of $T$ . Upper triangle is not used.                   |
| VSL_MATRIX_STORAGE_PACKED    | Lower triangle of C packed by columns into one-dimensional array | spptrf for<br>vsRngGaussianMV<br><br>dpptrf for<br>vdRngGaussianMV | 'L'                                            | Lower triangle of $T$ packed by columns into a one-dimensional array. |

**Return Values**

|                                           |                                                                                                                               |
|-------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                                  |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                              |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                                   |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, $< 0$ or $> nmax$ . |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                                |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                                    |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.           |

VSL\_RNG\_ERROR\_ARS5\_NOT\_SUPPORTED

ARS-5 random number generator is not supported on the CPU running the application.

### vRngExponential

Generates exponentially distributed random numbers.

#### Syntax

```
status = vsRngExponential(method, stream, n, r, a, beta);
```

```
status = vdRngExponential(method, stream, n, r, a, beta);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name          | Type                                                                  | Description                                                                                                                                                                              |
|---------------|-----------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT                                                         | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_EXPONENTIAL_ICDF<br>VSL_RNG_METHOD_EXPONENTIAL_ICDF_ACCURATE<br>Inverse cumulative distribution function method |
| <i>stream</i> | VSLStreamStatePtr                                                     | Pointer to the stream state structure                                                                                                                                                    |
| <i>n</i>      | const MKL_INT                                                         | Number of random values to be generated                                                                                                                                                  |
| <i>a</i>      | const float for vsRngExponential<br>const double for vdRngExponential | Displacement <i>a</i>                                                                                                                                                                    |
| <i>beta</i>   | const float for vsRngExponential<br>const double for vdRngExponential | Scalefactor $\beta$ .                                                                                                                                                                    |

#### Output Parameters

| Name     | Type                                                        | Description                                                 |
|----------|-------------------------------------------------------------|-------------------------------------------------------------|
| <i>r</i> | float* for vsRngExponential<br>double* for vdRngExponential | Vector of <i>n</i> exponentially distributed random numbers |

#### Description

The `vsRngExponential` function generates random numbers with exponential distribution that has displacement *a* and scalefactor  $\beta$ , where  $a, \beta \in \mathbb{R}$ ;  $\beta > 0$ .

The probability density function is given by:

$$f_{a,\beta}(x) = \begin{cases} \frac{1}{\beta} \exp(-(x-a)/\beta), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a,\beta}(x) = \begin{cases} 1 - \exp(-(x-a)/\beta), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

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**Return Values**

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

**vRngLaplace**

Generates random numbers with Laplace distribution.

## Syntax

```
status = vsRngLaplace(method, stream, n, r, a, beta);
status = vdRngLaplace(method, stream, n, r, a, beta);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name          | Type                                                                                                              | Description                                                                                                                                           |
|---------------|-------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | <code>const MKL_INT</code>                                                                                        | Generation method. The specific values are as follows:<br><code>VSL_RNG_METHOD_LAPLACE_ICDF</code><br>Inverse cumulative distribution function method |
| <i>stream</i> | <code>VSLStreamStatePtr</code>                                                                                    | Pointer to the stream state structure                                                                                                                 |
| <i>n</i>      | <code>const MKL_INT</code>                                                                                        | Number of random values to be generated                                                                                                               |
| <i>a</i>      | <code>const float</code> for <code>vsRngLaplace</code><br><code>const double</code> for <code>vdRngLaplace</code> | Mean value <i>a</i>                                                                                                                                   |
| <i>beta</i>   | <code>const float</code> for <code>vsRngLaplace</code><br><code>const double</code> for <code>vdRngLaplace</code> | Scalefactor $\beta$ .                                                                                                                                 |

## Output Parameters

| Name     | Type                                                                                                    | Description                                           |
|----------|---------------------------------------------------------------------------------------------------------|-------------------------------------------------------|
| <i>r</i> | <code>float*</code> for <code>vsRngLaplace</code><br><code>double*</code> for <code>vdRngLaplace</code> | Vector of <i>n</i> Laplace distributed random numbers |

## Description

The `vsRngLaplace` function generates random numbers with Laplace distribution with mean value (or average) *a* and scalefactor  $\beta$ , where  $a, \beta \in \mathbb{R}$ ;  $\beta > 0$ . The scalefactor value determines the standard deviation as

$$\sigma = \beta\sqrt{2}$$

The probability density function is given by:

$$f_{a,\beta}(x) = \frac{1}{\sqrt{2}\beta} \exp\left(-\frac{|x-a|}{\beta}\right), -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$f_{\alpha,\beta}(x) = \begin{cases} \frac{1}{2} \exp\left(-\frac{|x-a|}{\beta}\right), & x \geq a \\ 1 - \frac{1}{2} \exp\left(-\frac{|x-a|}{\beta}\right), & x < a \end{cases}, -\infty < x < +\infty.$$

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**Return Values**

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

**vRngWeibull**

*Generates Weibull distributed random numbers.*

**Syntax**

```
status = vsRngWeibull(method, stream, n, r, alpha, a, beta);
```

```
status = vdRngWeibull(method, stream, n, r, alpha, a, beta);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name          | Type                                                                                                              | Description                                                                                                                                                                                                |
|---------------|-------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | <code>const MKL_INT</code>                                                                                        | Generation method. The specific values are as follows:<br><code>VSL_RNG_METHOD_WEIBULL_ICDF</code><br><code>VSL_RNG_METHOD_WEIBULL_ICDF_ACCURATE</code><br>Inverse cumulative distribution function method |
| <i>stream</i> | <code>VSLStreamStatePtr</code>                                                                                    | Pointer to the stream state structure                                                                                                                                                                      |
| <i>n</i>      | <code>const MKL_INT</code>                                                                                        | Number of random values to be generated                                                                                                                                                                    |
| <i>alpha</i>  | <code>const float</code> for <code>vsRngWeibull</code><br><code>const double</code> for <code>vdRngWeibull</code> | Shape $\alpha$ .                                                                                                                                                                                           |
| <i>a</i>      | <code>const float</code> for <code>vsRngWeibull</code><br><code>const double</code> for <code>vdRngWeibull</code> | Displacement $a$                                                                                                                                                                                           |
| <i>beta</i>   | <code>const float</code> for <code>vsRngWeibull</code><br><code>const double</code> for <code>vdRngWeibull</code> | Scalefactor $\beta$ .                                                                                                                                                                                      |

## Output Parameters

| Name     | Type                                                                                                    | Description                                      |
|----------|---------------------------------------------------------------------------------------------------------|--------------------------------------------------|
| <i>r</i> | <code>float*</code> for <code>vsRngWeibull</code><br><code>double*</code> for <code>vdRngWeibull</code> | Vector of $n$ Weibull distributed random numbers |

## Description

The `vdRngWeibull` function generates Weibull distributed random numbers with displacement  $a$ , scalefactor  $\beta$ , and shape  $\alpha$ , where  $\alpha, \beta, a \in \mathbb{R}; \alpha > 0, \beta > 0$ .

The probability density function is given by:

$$f_{a,\alpha,\beta}(x) = \begin{cases} \frac{\alpha}{\beta^\alpha} (x - a)^{\alpha-1} \exp\left(-\left(\frac{x - a}{\beta}\right)^\alpha\right), & x \geq a \\ 0, & x < a \end{cases}$$

The cumulative distribution function is as follows:

$$F_{a,\alpha,\beta}(x) = \begin{cases} 1 - \exp\left(-\left(\frac{x-a}{\beta}\right)^\alpha\right), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

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**Return Values**

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

**vRngCauchy**

*Generates Cauchy distributed random values.*

**Syntax**

```
status = vsRngCauchy(method, stream, n, r, a, beta);
```

```
status = vdRngCauchy(method, stream, n, r, a, beta);
```

**Include Files**

- mkl.h

## Input Parameters

| Name          | Type                                                        | Description                                                                                                                             |
|---------------|-------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT                                               | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_CAUCHY_ICDF<br>Inverse cumulative distribution function method |
| <i>stream</i> | VSLStreamStatePtr                                           | Pointer to the stream state structure                                                                                                   |
| <i>n</i>      | const MKL_INT                                               | Number of random values to be generated                                                                                                 |
| <i>a</i>      | const float for vsRngCauchy<br>const double for vdRngCauchy | Displacement $a$ .                                                                                                                      |
| <i>beta</i>   | const float for vsRngCauchy<br>const double for vdRngCauchy | Scale factor $\beta$ .                                                                                                                  |

## Output Parameters

| Name     | Type                                              | Description                                     |
|----------|---------------------------------------------------|-------------------------------------------------|
| <i>r</i> | float* for vsRngCauchy<br>double* for vdRngCauchy | Vector of $n$ Cauchy distributed random numbers |

## Description

The function generates Cauchy distributed random numbers with displacement  $a$  and scale factor  $\beta$ , where  $a, \beta \in \mathbb{R}$ ;  $\beta > 0$ .

The probability density function is given by:

$$f_{a,\beta}(x) = \frac{1}{\pi\beta \left(1 + \left(\frac{x-a}{\beta}\right)^2\right)}, \quad -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a,\beta}(x) = \frac{1}{2} + \frac{1}{\pi} \arctan\left(\frac{x-a}{\beta}\right), \quad -\infty < x < +\infty.$$

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**Return Values**

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

**vRngRayleigh**

*Generates Rayleigh distributed random values.*

**Syntax**

```
status = vsRngRayleigh(method, stream, n, r, a, beta);
status = vdRngRayleigh(method, stream, n, r, a, beta);
```

**Include Files**

- mkl.h

**Input Parameters**

| Name          | Type              | Description                                                                                                                                                                        |
|---------------|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_RAYLEIGH_ICDF<br>VSL_RNG_METHOD_RAYLEIGH_ICDF_ACCURATE<br>Inverse cumulative distribution function method |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                                              |

| Name   | Type                                                                      | Description                             |
|--------|---------------------------------------------------------------------------|-----------------------------------------|
| $n$    | const MKL_INT                                                             | Number of random values to be generated |
| $a$    | const float for<br>vsRngRayleigh<br><br>const double for<br>vdRngRayleigh | Displacement $a$                        |
| $beta$ | const float for<br>vsRngRayleigh<br><br>const double for<br>vdRngRayleigh | Scalefactor $\beta$ .                   |

### Output Parameters

| Name | Type                                                      | Description                                       |
|------|-----------------------------------------------------------|---------------------------------------------------|
| $r$  | float* for vsRngRayleigh<br><br>double* for vdRngRayleigh | Vector of $n$ Rayleigh distributed random numbers |

### Description

The `vsRngRayleigh` function generates Rayleigh distributed random numbers with displacement  $a$  and scalefactor  $\beta$ , where  $a, \beta \in \mathbb{R}$ ;  $\beta > 0$ .

The Rayleigh distribution is a special case of the `Weibull` distribution, where the shape parameter  $\alpha = 2$ .

The probability density function is given by:

$$f_{a,\beta}(x) = \begin{cases} \frac{2(x-a)}{\beta^2} \exp\left(-\frac{(x-a)^2}{\beta^2}\right), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a,\beta}(x) = \begin{cases} 1 - \exp\left(-\frac{(x-a)^2}{\beta^2}\right), & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty.$$

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**Return Values**

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

**vRngLognormal**

Generates lognormally distributed random numbers.

**Syntax**

```
status = vsRngLognormal(method, stream, n, r, a, sigma, b, beta);
status = vdRngLognormal(method, stream, n, r, a, sigma, b, beta);
```

**Include Files**

- mkl.h

**Input Parameters**

| Name          | Type          | Description                                                                                                                                                                                                                                           |
|---------------|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_LOGNORMAL_BOXMULLER2<br>VSL_RNG_METHOD_LOGNORMAL_BOXMULLER2_ACCURATE<br>Box Muller 2 based method<br>VSL_RNG_METHOD_LOGNORMAL_ICDF<br>VSL_RNG_METHOD_LOGNORMAL_ICDF_ACCURATE |

| Name          | Type                                                                        | Description                                                    |
|---------------|-----------------------------------------------------------------------------|----------------------------------------------------------------|
|               |                                                                             | Inverse cumulative distribution function based method          |
| <i>stream</i> | VSLStreamStatePtr                                                           | Pointer to the stream state structure                          |
| <i>n</i>      | const MKL_INT                                                               | Number of random values to be generated                        |
| <i>a</i>      | const float for<br>vsRngLognormal<br><br>const double for<br>vdRngLognormal | Average <i>a</i> of the subject normal distribution            |
| <i>sigma</i>  | const float for<br>vsRngLognormal<br><br>const double for<br>vdRngLognormal | Standard deviation $\sigma$ of the subject normal distribution |
| <i>b</i>      | const float for<br>vsRngLognormal<br><br>const double for<br>vdRngLognormal | Displacement <i>b</i>                                          |
| <i>beta</i>   | const float for<br>vsRngLognormal<br><br>const double for<br>vdRngLognormal | Scalefactor $\beta$ .                                          |

### Output Parameters

| Name     | Type                                                        | Description                                               |
|----------|-------------------------------------------------------------|-----------------------------------------------------------|
| <i>r</i> | float* for vsRngLognormal<br><br>double* for vdRngLognormal | Vector of <i>n</i> lognormally distributed random numbers |

### Description

The `vsRngLognormal` function generates lognormally distributed random numbers with average of distribution *a* and standard deviation  $\sigma$  of subject normal distribution, displacement *b*, and scalefactor  $\beta$ , where  $a, \sigma, b, \beta \in \mathbb{R}$ ;  $\sigma > 0, \beta > 0$ .

The probability density function is given by:

$$f_{a,\sigma,b,\beta}(x) = \begin{cases} \frac{1}{\sigma(x-b)\sqrt{2\pi}} \exp\left(-\frac{[\ln((x-b)/\beta) - a]^2}{2\sigma^2}\right), & x > b \\ 0, & x \leq b \end{cases}$$

The cumulative distribution function is as follows:

$$F_{a,\sigma,b,\beta}(x) = \begin{cases} \Phi((\ln((x-b)/\beta) - a)/\sigma), & x > b \\ 0, & x \leq b \end{cases}$$

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### Return Values

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

### vRngGumbel

*Generates Gumbel distributed random values.*

### Syntax

```
status = vsRngGumbel(method, stream, n, r, a, beta);
```

```
status = vdRngGumbel(method, stream, n, r, a, beta);
```

### Include Files

- mkl.h

## Input Parameters

| Name          | Type                                                        | Description                                                                                                                             |
|---------------|-------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT                                               | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_GUMBEL_ICDF<br>Inverse cumulative distribution function method |
| <i>stream</i> | VSLStreamStatePtr                                           | Pointer to the stream state structure                                                                                                   |
| <i>n</i>      | const MKL_INT                                               | Number of random values to be generated                                                                                                 |
| <i>a</i>      | const float for vsRngGumbel<br>const double for vdRngGumbel | Displacement $a$ .                                                                                                                      |
| <i>beta</i>   | const float for vsRngGumbel<br>const double for vdRngGumbel | Scalefactor $\beta$ .                                                                                                                   |

## Output Parameters

| Name     | Type                                              | Description                                           |
|----------|---------------------------------------------------|-------------------------------------------------------|
| <i>r</i> | float* for vsRngGumbel<br>double* for vdRngGumbel | Vector of $n$ random numbers with Gumbel distribution |

## Description

The `vsRngGumbel` function generates Gumbel distributed random numbers with displacement  $a$  and scalefactor  $\beta$ , where  $a, \beta \in R$ ;  $\beta > 0$ .

The probability density function is given by:

$$f_{a,\beta}(x) = \left\{ \frac{1}{\beta} \exp\left(\frac{x-a}{\beta}\right) \exp(-\exp((x-a)/\beta)) \right\}, -\infty < x < +\infty.$$

The cumulative distribution function is as follows:

$$F_{a,\beta}(x) = 1 - \exp(-\exp((x-a)/\beta)), -\infty < x < +\infty$$

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**Optimization Notice**

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**Return Values**

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

**vRngGamma**

*Generates gamma distributed random values.*

**Syntax**

```
status = vsRngGamma(method, stream, n, r, alpha, a, beta);
status = vdRngGamma(method, stream, n, r, alpha, a, beta);
```

**Include Files**

- mkl.h

**Input Parameters**

| <b>Name</b>   | <b>Type</b>       | <b>Description</b>                                                                                                                                                                                                                                                                                                            |
|---------------|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_GAMMA_GNORM<br>VSL_RNG_METHOD_GAMMA_GNORM_ACCURATE<br>Acceptance/rejection method using random numbers with Gaussian distribution. See brief description of the method GNORM in <a href="#">Table "Values of &lt;method&gt; in method parameter"</a> |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                                                                                                                                                                                         |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                                                                                                                                                                                                       |

| Name         | Type                                                      | Description           |
|--------------|-----------------------------------------------------------|-----------------------|
| <i>alpha</i> | const float for vsRngGamma<br>const double for vdRngGamma | Shape $\alpha$ .      |
| <i>a</i>     | const float for vsRngGamma<br>const double for vdRngGamma | Displacement $a$ .    |
| <i>beta</i>  | const float for vsRngGamma<br>const double for vdRngGamma | Scalefactor $\beta$ . |

### Output Parameters

| Name     | Type                                            | Description                                          |
|----------|-------------------------------------------------|------------------------------------------------------|
| <i>r</i> | float* for vsRngGamma<br>double* for vdRngGamma | Vector of $n$ random numbers with gamma distribution |

### Description

The `vrngGamma` function generates random numbers with gamma distribution that has shape parameter  $\alpha$ , displacement  $a$ , and scale parameter  $\beta$ , where  $\alpha, \beta$ , and  $a \in \mathbb{R}$ ;  $\alpha > 0, \beta > 0$ .

The probability density function is given by:

$$f_{\alpha,a,\beta}(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^\alpha} (x - a)^{\alpha-1} e^{-(x-a)/\beta}, & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty$$

where  $\Gamma(\alpha)$  is the complete gamma function.

The cumulative distribution function is as follows:

$$F_{\alpha,a,\beta}(x) = \begin{cases} \int_a^x \frac{1}{\Gamma(\alpha)\beta^\alpha} (y - a)^{\alpha-1} e^{-(y-a)/\beta} dy, & x \geq a \\ 0, & x < a \end{cases}, -\infty < x < +\infty$$

#### Optimization Notice

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## Return Values

|                                               |                                                                                                                          |
|-----------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK                   | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                            | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                      | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                      | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                      | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED             | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEED<br>ED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED              | ARS-5 random number generator is not supported on the CPU running the application.                                       |

## vRngBeta

*Generates beta distributed random values.*

## Syntax

```
status = vsRngBeta(method, stream, n, r, p, q, a, beta);
status = vdRngBeta(method, stream, n, r, p, q, a, beta);
```

## Include Files

- mkl.h

## Input Parameters

| Name          | Type                                                    | Description                                                                                                                                                                                                                              |
|---------------|---------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT                                           | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_BETA_CJA<br>VSL_RNG_METHOD_BETA_CJA_ACCURATE<br>See brief description of the method CJA in <a href="#">Table "Values of &lt;method&gt; in method parameter"</a> |
| <i>stream</i> | VSLStreamStatePtr                                       | Pointer to the stream state structure                                                                                                                                                                                                    |
| <i>n</i>      | const MKL_INT                                           | Number of random values to be generated                                                                                                                                                                                                  |
| <i>p</i>      | const float for vsRngBeta<br>const double for vdRngBeta | Shape <i>p</i>                                                                                                                                                                                                                           |
| <i>q</i>      | const float for vsRngBeta                               | Shape <i>q</i>                                                                                                                                                                                                                           |

| Name        | Type                                                    | Description            |
|-------------|---------------------------------------------------------|------------------------|
|             | const double for vdRngBeta                              |                        |
| <i>a</i>    | const float for vsRngBeta<br>const double for vdRngBeta | Displacement $a$ .     |
| <i>beta</i> | const float for vsRngBeta<br>const double for vdRngBeta | Scale factor $\beta$ . |

### Output Parameters

| Name     | Type                                          | Description                                         |
|----------|-----------------------------------------------|-----------------------------------------------------|
| <i>r</i> | float* for vsRngBeta<br>double* for vdRngBeta | Vector of $n$ random numbers with beta distribution |

### Description

The `vdRngBeta` function generates random numbers with beta distribution that has shape parameters  $p$  and  $q$ , displacement  $a$ , and scale parameter  $\beta$ , where  $p, q, a$ , and  $\beta \in \mathbb{R}$ ;  $p > 0$ ,  $q > 0$ ,  $\beta > 0$ .

The probability density function is given by:

$$f_{p,q,a,\beta}(x) = \begin{cases} \frac{1}{B(p,q)\beta^{p+q-1}} (x-a)^{p-1} (\beta+a-x)^{q-1}, & a \leq x < a + \beta \\ 0, & x < a, x \geq a + \beta \end{cases}, -\infty < x < \infty,$$

where  $B(p, q)$  is the complete beta function.

The cumulative distribution function is as follows:

$$F_{p,q,a,\beta}(x) = \begin{cases} 0, & x < a \\ \int_a^x \frac{1}{B(p,q)\beta^{p+q-1}} (y-a)^{p-1} (\beta+a-y)^{q-1} dy, & a \leq x < a + \beta \\ 1, & x \geq a + \beta \end{cases}, -\infty < x < \infty.$$

#### Optimization Notice

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## Return Values

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

## Discrete Distributions

This section describes routines for generating random numbers with discrete distribution.

### vRngUniform Discrete Distribution Generators

*Generates random numbers uniformly distributed over the interval [a, b).*

#### Syntax

```
status = viRngUniform(method, stream, n, r, a, b);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name          | Type              | Description                                                                                                                                                                        |
|---------------|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method; the specific value is as follows:<br><br>VSL_RNG_METHOD_UNIFORM_STD<br><br>Standard method. Currently there is only one method for this distribution generator. |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                                              |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                                                            |
| <i>a</i>      | const int         | Left interval bound a                                                                                                                                                              |
| <i>b</i>      | const int         | Right interval bound b                                                                                                                                                             |

## Output Parameters

| Name | Type              | Description                                                                   |
|------|-------------------|-------------------------------------------------------------------------------|
| $r$  | <code>int*</code> | Vector of $n$ random numbers uniformly distributed over the interval $[a, b)$ |

## Description

The `vRngUniform` function generates random numbers uniformly distributed over the interval  $[a, b)$ , where  $a, b$  are the left and right bounds of the interval respectively, and  $a, b \in \mathbb{Z}; a < b$ .

The probability distribution is given by:

$$P(X = k) = \frac{1}{b - a}, k \in \{a, a + 1, \dots, b - 1\}.$$

The cumulative distribution function is as follows:

$$F_{a,b}(x) = \begin{cases} 0, & x < a \\ \frac{\lfloor x - a + 1 \rfloor}{b - a}, & a \leq x < b, x \in \mathbb{R}. \\ 1, & x \geq b \end{cases}$$

### Optimization Notice

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## Return Values

|                                          |                                                                                                                               |
|------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| <code>VSL_ERROR_OK, VSL_STATUS_OK</code> | Indicates no error, execution is successful.                                                                                  |
| <code>VSL_ERROR_NULL_PTR</code>          | <i>stream</i> is a NULL pointer.                                                                                              |
| <code>VSL_RNG_ERROR_BAD_STREAM</code>    | <i>stream</i> is not a valid random stream.                                                                                   |
| <code>VSL_RNG_ERROR_BAD_UPDATE</code>    | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, $< 0$ or $> nmax$ . |

|                                           |                                                                                                                     |
|-------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                      |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                          |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold. |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                  |

## vRngUniformBits

Generates bits of underlying BRNG integer recurrence.

### Syntax

```
status = viRngUniformBits(method, stream, n, r);
```

### Include Files

- mkl.h

### Input Parameters

| Name          | Type              | Description                                                                          |
|---------------|-------------------|--------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method; the specific value is <code>VSL_RNG_METHOD_UNIFORMBITS_STD</code> |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                              |

### Output Parameters

| Name     | Type          | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|----------|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>r</i> | unsigned int* | Vector of <i>n</i> random integer numbers. If the <i>stream</i> was generated by a 64 or a 128-bit generator, each integer value is represented by two or four elements of <i>r</i> respectively. The number of bytes occupied by each integer is contained in the field <i>WordSize</i> of the structure <code>VSLBRngProperties</code> . The total number of bits that are actually used to store the value are contained in the field <i>NBits</i> of the same structure. See <a href="#">Advanced Service Routines</a> for a more detailed discussion of <code>VSLBRngProperties</code> . |

### Description

The `viRngUniformBits` function generates integer random values with uniform bit distribution. The generators of uniformly distributed numbers can be represented as recurrence relations over integer values in modular arithmetic. Apparently, each integer can be treated as a vector of several bits. In a truly random generator, these bits are random, while in pseudorandom generators this randomness can be violated. For example, a well known drawback of linear congruential generators is that lower bits are less random than

higher bits (for example, see [Knuth81]). For this reason, care should be taken when using this function. Typically, in a 32-bit LCG only 24 higher bits of an integer value can be considered random. See [VS Notes](#) for details.

### Optimization Notice

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### Return Values

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

### vRngUniformBits32

*Generates uniformly distributed bits in 32-bit chunks.*

### Syntax

```
status = viRngUniformBits32(method, stream, n, r);
```

### Include Files

- mkl.h

## Input Parameters

| Name          | Type              | Description                                                                  |
|---------------|-------------------|------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method; the specific value is<br>VSL_RNG_METHOD_UNIFORMBITS32_STD |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                        |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                      |

## Output Parameters

| Name     | Type          | Description                                                                     |
|----------|---------------|---------------------------------------------------------------------------------|
| <i>r</i> | unsigned int* | Vector of <i>n</i> 32-bit random integer numbers with uniform bit distribution. |

## Description

The `vRngUniformBits32` function generates uniformly distributed bits in 32-bit chunks. Unlike `vRngUniformBits`, which provides the output of underlying integer recurrence and does not guarantee uniform distribution across bits, `vRngUniformBits32` is designed to ensure each bit in the 32-bit chunk is uniformly distributed. See [VS Notes](#) for details.

### Optimization Notice

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## Return Values

|                                               |                                                                                                                     |
|-----------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK                   | Indicates no error, execution is successful.                                                                        |
| VSL_ERROR_NULL_PTR                            | <i>stream</i> is a NULL pointer.                                                                                    |
| VSL_RNG_ERROR_BAD_STREAM                      | <i>stream</i> is not a valid random stream.                                                                         |
| VSL_RNG_ERROR_BRNG_NOT_SUPPORTED              | BRNG is not supported by the function.                                                                              |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEED<br>ED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold. |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED              | ARS-5 random number generator is not supported on the CPU running the application.                                  |

**vRngUniformBits64**

*Generates uniformly distributed bits in 64-bit chunks.*

**Syntax**

```
status = viRngUniformBits64(method, stream, n, r);
```

**Include Files**

- mkl.h

**Input Parameters**

| Name          | Type              | Description                                                                               |
|---------------|-------------------|-------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method; the specific value is<br><code>VSL_RNG_METHOD_UNIFORMBITS64_STD</code> |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                     |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                   |

**Output Parameters**

| Name     | Type                | Description                                                                     |
|----------|---------------------|---------------------------------------------------------------------------------|
| <i>r</i> | unsigned MKL_INT64* | Vector of <i>n</i> 64-bit random integer numbers with uniform bit distribution. |

**Description**

The `vRngUniformBits64` function generates uniformly distributed bits in 64-bit chunks. Unlike `vRngUniformBits`, which provides the output of underlying integer recurrence and does not guarantee uniform distribution across bits, `vRngUniformBits64` is designed to ensure each bit in the 64-bit chunk is uniformly distributed. See [VS Notes](#) for details.

**Optimization Notice**

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**Return Values**

|                                  |                                              |
|----------------------------------|----------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK      | Indicates no error, execution is successful. |
| VSL_ERROR_NULL_PTR               | <i>stream</i> is a NULL pointer.             |
| VSL_RNG_ERROR_BAD_STREAM         | <i>stream</i> is not a valid random stream.  |
| VSL_RNG_ERROR_BRNG_NOT_SUPPORTED | BRNG is not supported by the function.       |

|                                               |                                                                                                                     |
|-----------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEED<br>ED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold. |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED              | ARS-5 random number generator is not supported on the CPU running the application.                                  |

## vRngBernoulli

*Generates Bernoulli distributed random values.*

### Syntax

```
status = viRngBernoulli(method, stream, n, r, p);
```

### Include Files

- mkl.h

### Input Parameters

| Name          | Type              | Description                                                                                                                                            |
|---------------|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific value is as follows:<br><code>VSL_RNG_METHOD_BERNOULLI_ICDF</code><br>Inverse cumulative distribution function method. |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                  |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                                |
| <i>p</i>      | const double      | Success probability $p$ of a trial                                                                                                                     |

### Output Parameters

| Name     | Type | Description                                       |
|----------|------|---------------------------------------------------|
| <i>r</i> | int* | Vector of $n$ Bernoulli distributed random values |

### Description

The `vRngBernoulli` function generates Bernoulli distributed random numbers with probability  $p$  of a single trial success, where

$$p \in \mathbb{R}; 0 \leq p \leq 1.$$

A variate is called Bernoulli distributed, if after a trial it is equal to 1 with probability of success  $p$ , and to 0 with probability  $1 - p$ .

The probability distribution is given by:

$$P(X = 1) = p$$

$$P(X = 0) = 1 - p$$

The cumulative distribution function is as follows:

$$F_p(x) = \begin{cases} 0, & x < 0 \\ 1 - p, & 0 \leq x < 1, x \in R. \\ 1, & x \geq 1 \end{cases}$$

### Optimization Notice

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### Return Values

|                                   |                                                                                                                          |
|-----------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK       | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM          | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE          | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS          | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED  | ARS-5 random number generator is not supported on the CPU running the application.                                       |

### vRngGeometric

Generates geometrically distributed random values.

### Syntax

```
status = viRngGeometric(method, stream, n, r, p);
```

### Include Files

- mkl.h

## Input Parameters

| Name          | Type              | Description                                                                                                                               |
|---------------|-------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific value is as follows:<br>VSL_RNG_METHOD_GEOMETRIC_ICDF<br>Inverse cumulative distribution function method. |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                     |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                   |
| <i>p</i>      | const double      | Success probability $p$ of a trial                                                                                                        |

## Output Parameters

| Name     | Type | Description                                           |
|----------|------|-------------------------------------------------------|
| <i>r</i> | int* | Vector of $n$ geometrically distributed random values |

## Description

The `vRngGeometric` function generates geometrically distributed random numbers with probability  $p$  of a single trial success, where  $p \in \mathbb{R}$ ;  $0 < p < 1$ .

A geometrically distributed variate represents the number of independent Bernoulli trials preceding the first success. The probability of a single Bernoulli trial success is  $p$ .

The probability distribution is given by:

$$P(X = k) = p \cdot (1 - p)^k, k \in \{0, 1, 2, \dots\}.$$

The cumulative distribution function is as follows:

$$F_p(x) = \begin{cases} 0, & x < 0 \\ 1 - (1 - p)^{\lfloor x+1 \rfloor}, & 0 \geq x \end{cases} \quad x \in \mathbb{R}.$$

### Optimization Notice

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## Return Values

VSL\_ERROR\_OK, VSL\_STATUS\_OK Indicates no error, execution is successful.

|                                               |                                                                                                                               |
|-----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_NULL_PTR                            | <i>stream</i> is a NULL pointer.                                                                                              |
| VSL_RNG_ERROR_BAD_STREAM                      | <i>stream</i> is not a valid random stream.                                                                                   |
| VSL_RNG_ERROR_BAD_UPDATE                      | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, $< 0$ or $> nmax$ . |
| VSL_RNG_ERROR_NO_NUMBERS                      | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                                |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED             | Period of the generator has been exceeded.                                                                                    |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEED<br>ED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.           |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED              | ARS-5 random number generator is not supported on the CPU running the application.                                            |

### vRngBinomial

*Generates binomially distributed random numbers.*

#### Syntax

```
status = viRngBinomial(method, stream, n, r, ntrial, p);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name          | Type              | Description                                                                                                                                                                                                        |
|---------------|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific value is as follows:<br><br>VSL_RNG_METHOD_BINOMIAL_BTPE<br><br>See brief description of the BTPE method in <a href="#">Table "Values of &lt;method&gt; in method parameter"</a> . |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                                                                              |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                                                                                            |
| <i>ntrial</i> | const int         | Number of independent trials $m$                                                                                                                                                                                   |
| <i>p</i>      | const double      | Success probability $p$ of a single trial                                                                                                                                                                          |

#### Output Parameters

| Name     | Type | Description                                        |
|----------|------|----------------------------------------------------|
| <i>r</i> | int* | Vector of $n$ binomially distributed random values |

#### Description

The `vRngBinomial` function generates binomially distributed random numbers with number of independent Bernoulli trials  $m$ , and with probability  $p$  of a single trial success, where  $p \in \mathbb{R}$ ;  $0 \leq p \leq 1$ ,  $m \in \mathbb{N}$ .

A binomially distributed variate represents the number of successes in  $m$  independent Bernoulli trials with probability of a single trial success  $p$ .

The probability distribution is given by:

$$P(X = k) = C_m^k p^k (1 - p)^{m-k}, k \in \{0, 1, \dots, m\}.$$

The cumulative distribution function is as follows:

$$F_{m,p}(x) = \begin{cases} 0, & x < 0 \\ \sum_{k=0}^{\lfloor x \rfloor} C_m^k p^k (1 - p)^{m-k}, & 0 \leq x < m, x \in R \\ 1, & x \geq m \end{cases}$$

### Optimization Notice

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### Return Values

|                                           |                                                                                                                               |
|-------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                                  |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                              |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                                   |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, $< 0$ or $> nmax$ . |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                                |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                                    |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.           |

VSL\_RNG\_ERROR\_ARS5\_NOT\_SUPPORTED

ARS-5 random number generator is not supported on the CPU running the application.

### vRngHypergeometric

Generates hypergeometrically distributed random values.

#### Syntax

```
status = viRngHypergeometric(method, stream, n, r, l, s, m);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name          | Type              | Description                                                                                                                                                                                                            |
|---------------|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific value is as follows:<br><br>VSL_RNG_METHOD_HYPERGEOMETRIC_H2PE<br><br>See brief description of the H2PE method in <a href="#">Table "Values of &lt;method&gt; in method parameter"</a> |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                                                                                  |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                                                                                                |
| <i>l</i>      | const int         | Lot size <i>l</i>                                                                                                                                                                                                      |
| <i>s</i>      | const int         | Size of sampling without replacement <i>s</i>                                                                                                                                                                          |
| <i>m</i>      | const int         | Number of marked elements <i>m</i>                                                                                                                                                                                     |

#### Output Parameters

| Name     | Type | Description                                                     |
|----------|------|-----------------------------------------------------------------|
| <i>r</i> | int* | Vector of <i>n</i> hypergeometrically distributed random values |

#### Description

The `viRngHypergeometric` function generates hypergeometrically distributed random values with lot size *l*, size of sampling *s*, and number of marked elements in the lot *m*, where  $l, m, s \in \mathbb{N} \cup \{0\}$ ;  $l \geq \max(s, m)$ .

Consider a lot of *l* elements comprising *m* "marked" and *l*-*m* "unmarked" elements. A trial sampling without replacement of exactly *s* elements from this lot helps to define the hypergeometric distribution, which is the probability that the group of *s* elements contains exactly *k* marked elements.

The probability distribution is given by:)

$$P(X = k) = \frac{C_m^k C_{l-m}^{s-k}}{C_l^s}$$

,  $k \in \{\max(0, s + m - l), \dots, \min(s, m)\}$

The cumulative distribution function is as follows:

$$F_{1,s,m}(x) = \begin{cases} 0, & x < \max(0, s + m - 1) \\ \sum_{k=\max(0, s+m-1)}^{\lfloor x \rfloor} \frac{C_m^k C_{1-m}^{s-k}}{C_1^s}, & \max(0, s + m - 1) \leq x \leq \min(s, m) \\ 1, & x > \min(s, m) \end{cases}$$

**Optimization Notice**

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**Return Values**

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

**vRngPoisson**

*Generates Poisson distributed random values.*

**Syntax**

```
status = viRngPoisson(method, stream, n, r, lambda);
```

**Include Files**

- mkl.h

## Input Parameters

| Name          | Type              | Description                                                                                                                                                                                                                                               |
|---------------|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific values are as follows:<br>VSL_RNG_METHOD_POISSON_PTPE<br>VSL_RNG_METHOD_POISSON_POISNORM<br>See brief description of the PTPE and POISNORM methods in Table "Values of <i>&lt;method&gt;</i> in <i>method</i> parameter". |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                                                                                                                     |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                                                                                                                                   |
| <i>lambda</i> | const double      | Distribution parameter $\lambda$ .                                                                                                                                                                                                                        |

## Output Parameters

| Name     | Type | Description                                     |
|----------|------|-------------------------------------------------|
| <i>r</i> | int* | Vector of $n$ Poisson distributed random values |

## Description

The `vRng"Poisson` function generates Poisson distributed random numbers with distribution parameter  $\lambda$ , where  $\lambda \in R$ ;  $\lambda > 0$ .

The probability distribution is given by:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!},$$

$k \in \{0, 1, 2, \dots\}$ .

The cumulative distribution function is as follows:

$$F_{\lambda}(x) = \begin{cases} \sum_{k=0}^{\lfloor x \rfloor} \frac{\lambda^k e^{-\lambda}}{k!}, & x \geq 0 \\ 0, & x < 0 \end{cases}, x \in R$$

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**Optimization Notice**

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**Return Values**

|                                           |                                                                                                                          |
|-------------------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK               | Indicates no error, execution is successful.                                                                             |
| VSL_ERROR_NULL_PTR                        | <i>stream</i> is a NULL pointer.                                                                                         |
| VSL_RNG_ERROR_BAD_STREAM                  | <i>stream</i> is not a valid random stream.                                                                              |
| VSL_RNG_ERROR_BAD_UPDATE                  | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or > nmax. |
| VSL_RNG_ERROR_NO_NUMBERS                  | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                           |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                               |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold.      |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                       |

**vRngPoissonV**

*Generates Poisson distributed random values with varying mean.*

**Syntax**

```
status = viRngPoissonV(method, stream, n, r, lambda);
```

**Include Files**

- mkl.h

**Input Parameters**

| <b>Name</b>   | <b>Type</b>       | <b>Description</b>                                                                                                                                                                                                       |
|---------------|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific value is as follows:<br><br>VSL_RNG_METHOD_POISSONV_POISNORM<br><br>See brief description of the POISNORM method in <a href="#">Table "Values of &lt;method&gt; in method parameter"</a> |
| <i>stream</i> | VSLStreamStatePtr | Pointer to the stream state structure                                                                                                                                                                                    |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                                                                                                  |
| <i>lambda</i> | const double*     | Array of <i>n</i> distribution parameters $\lambda_i$ .                                                                                                                                                                  |

## Output Parameters

| Name | Type              | Description                                     |
|------|-------------------|-------------------------------------------------|
| $r$  | <code>int*</code> | Vector of $n$ Poisson distributed random values |

## Description

The `vRngPoissonV` function generates  $n$  Poisson distributed random numbers  $x_i (i = 1, \dots, n)$  with distribution parameter  $\lambda_i$ , where  $\lambda_i \in \mathbb{R}; \lambda_i > 0$ .

The probability distribution is given by:

$$P(X_i = k) = \frac{\lambda_i^k \exp(-\lambda_i)}{k!}, k \in \{0, 1, 2, \dots\}.$$

The cumulative distribution function is as follows:

$$F_{\lambda_i}(x) = \begin{cases} \sum_{k=0}^{\lfloor x \rfloor} \frac{\lambda_i^k e^{-\lambda_i}}{k!}, & x \geq 0 \\ 0, & x < 0 \end{cases}, x \in \mathbb{R}$$

### Optimization Notice

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## Return Values

|                                          |                                                                                                                               |
|------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| <code>VSL_ERROR_OK, VSL_STATUS_OK</code> | Indicates no error, execution is successful.                                                                                  |
| <code>VSL_ERROR_NULL_PTR</code>          | <code>stream</code> is a NULL pointer.                                                                                        |
| <code>VSL_RNG_ERROR_BAD_STREAM</code>    | <code>stream</code> is not a valid random stream.                                                                             |
| <code>VSL_RNG_ERROR_BAD_UPDATE</code>    | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, $< 0$ or $> nmax$ . |
| <code>VSL_RNG_ERROR_NO_NUMBERS</code>    | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                                |

|                                           |                                                                                                                     |
|-------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED         | Period of the generator has been exceeded.                                                                          |
| VSL_RNG_ERROR_NONDETERM_NRETRIES_EXCEEDED | Number of retries to generate a random number by using non-deterministic random number generator exceeds threshold. |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED          | ARS-5 random number generator is not supported on the CPU running the application.                                  |

### vRngNegBinomial

Generates random numbers with negative binomial distribution.

#### Syntax

```
status = viRngNegbinomial(method, stream, n, r, a, p);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name          | Type              | Description                                                                                                                                                                                                   |
|---------------|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>method</i> | const MKL_INT     | Generation method. The specific value is:<br><code>VSL_RNG_METHOD_NEGBINOMIAL_NBAR</code><br>See brief description of the NBAR method in <a href="#">Table "Values of &lt;method&gt; in method parameter"</a> |
| <i>stream</i> | VSLStreamStatePtr | pointer to the stream state structure                                                                                                                                                                         |
| <i>n</i>      | const MKL_INT     | Number of random values to be generated                                                                                                                                                                       |
| <i>a</i>      | const double      | The first distribution parameter <i>a</i>                                                                                                                                                                     |
| <i>p</i>      | const double      | The second distribution parameter <i>p</i>                                                                                                                                                                    |

#### Output Parameters

| Name     | Type | Description                                                           |
|----------|------|-----------------------------------------------------------------------|
| <i>r</i> | int* | Vector of <i>n</i> random values with negative binomial distribution. |

#### Description

The `vRngNegBinomial` function generates random numbers with negative binomial distribution and distribution parameters *a* and *p*, where  $p, a \in \mathbb{R}; 0 < p < 1; a > 0$ .

If the first distribution parameter  $a \in \mathbb{N}$ , this distribution is the same as Pascal distribution. If  $a \in \mathbb{N}$ , the distribution can be interpreted as the expected time of *a*-th success in a sequence of Bernoulli trials, when the probability of success is *p*.

The probability distribution is given by:

$$P(X = k) = C_{a+k-1}^k p^a (1 - p)^k, k \in \{0, 1, 2, \dots\}.$$

The cumulative distribution function is as follows:

$$F_{a,p}(x) = \begin{cases} \sum_{k=0}^{\lfloor x \rfloor} C_{a+k-1}^k p^a (1-p)^k, & x \geq 0 \\ 0, & x < 0 \end{cases}, x \in \mathbb{R}$$

### Optimization Notice

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### Return Values

|                                   |                                                                                                                               |
|-----------------------------------|-------------------------------------------------------------------------------------------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK       | Indicates no error, execution is successful.                                                                                  |
| VSL_ERROR_NULL_PTR                | <i>stream</i> is a NULL pointer.                                                                                              |
| VSL_RNG_ERROR_BAD_STREAM          | <i>stream</i> is not a valid random stream.                                                                                   |
| VSL_RNG_ERROR_BAD_UPDATE          | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, $< 0$ or $> nmax$ . |
| VSL_RNG_ERROR_NO_NUMBERS          | Callback function for an abstract BRNG returns 0 as the number of updated entries in a buffer.                                |
| VSL_RNG_ERROR_QRNG_PERIOD_ELAPSED | Period of the generator has been exceeded.                                                                                    |
| VSL_RNG_ERROR_ARS5_NOT_SUPPORTED  | ARS-5 random number generator is not supported on the CPU running the application.                                            |

### Advanced Service Routines

This section describes service routines for registering a user-designed basic generator ([vslRegisterBrng](#)) and for obtaining properties of the previously registered basic generators ([vslGetBrngProperties](#)). See [VS Notes](#) ("Basic Generators" section of VS Structure chapter) for substantiation of the need for several basic generators including user-defined BRNGs.

### Advanced Service Routine Data Types

The Advanced Service routines refer to a structure defining the properties of the basic generator.

This structure is described as follows:

```
typedef struct _VSLBRngProperties {
 int StreamStateSize;
 int NSeeds;
 int IncludesZero;
 int WordSize;
 int NBits;
 InitStreamPtr InitStream;
 sBRngPtr sBRng;
 dBRngPtr dBRng;
 iBRngPtr iBRng;
} VSLBRngProperties;
```

The following table provides brief descriptions of the fields engaged in the above structure:

### Field Descriptions

| Field           | Short Description                                                                                                                                                                                                                                                                                                                                                                                        |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| StreamStateSize | The size, in bytes, of the stream state structure for a given basic generator.                                                                                                                                                                                                                                                                                                                           |
| NSeeds          | The number of 32-bit initial conditions (seeds) necessary to initialize the stream state structure for a given basic generator.                                                                                                                                                                                                                                                                          |
| IncludesZero    | Flag value indicating whether the generator can produce a random 0.                                                                                                                                                                                                                                                                                                                                      |
| WordSize        | Machine word size, in bytes, used in integer-value computations. Possible values: 4, 8, and 16 for 32, 64, and 128-bit generators, respectively.                                                                                                                                                                                                                                                         |
| NBits           | The number of bits required to represent a random value in integer arithmetic. Note that, for instance, 48-bit random values are stored to 64-bit (8 byte) memory locations. In this case, <code>wordsize/WordSize</code> is equal to 8 (number of bytes used to store the random value), while <code>nbits/NBits</code> contains the actual number of bits occupied by the value (in this example, 48). |
| InitStream      | Contains the pointer to the initialization routine of a given basic generator.                                                                                                                                                                                                                                                                                                                           |
| sBRng           | Contains the pointer to the basic generator of single precision real numbers uniformly distributed over the interval $(a,b)$ ( <code>float</code> ).                                                                                                                                                                                                                                                     |
| dBRng           | Contains the pointer to the basic generator of double precision real numbers uniformly distributed over the interval $(a,b)$ ( <code>double</code> ).                                                                                                                                                                                                                                                    |
| iBRng           | Contains the pointer to the basic generator of integer numbers with uniform bit distribution ( <code>unsigned int</code> ).                                                                                                                                                                                                                                                                              |

### vslRegisterBrng

*Registers user-defined basic generator.*

#### Syntax

```
brng = vslRegisterBrng(&properties);
```

<sup>1</sup> A specific generator that permits operations over single bits and bit groups of random numbers.

## Include Files

- `mkl.h`

## Input Parameters

| Name                    | Type                                  | Description                                                                            |
|-------------------------|---------------------------------------|----------------------------------------------------------------------------------------|
| <code>properties</code> | <code>const VSLBRngProperties*</code> | Pointer to the structure containing properties of the basic generator to be registered |

## Output Parameters

| Name              | Type             | Description                                                                                                                 |
|-------------------|------------------|-----------------------------------------------------------------------------------------------------------------------------|
| <code>brng</code> | <code>int</code> | Number (index) of the registered basic generator; used for identification. Negative values indicate the registration error. |

## Description

An example of a registration procedure can be found in the respective directory of the VS examples.

## Return Values

|                                                  |                                                                                                                                       |
|--------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| <code>VSL_ERROR_OK, VSL_STATUS_OK</code>         | Indicates no error, execution is successful.                                                                                          |
| <code>VSL_RNG_ERROR_BRNG_TABLE_FULL</code>       | Registration cannot be completed due to lack of free entries in the table of registered BRNGs.                                        |
| <code>VSL_RNG_ERROR_BAD_STREAM_STATE_SIZE</code> | Bad value in <code>StreamStateSize</code> field.                                                                                      |
| <code>VSL_RNG_ERROR_BAD_WORD_SIZE</code>         | Bad value in <code>WordSize</code> field.                                                                                             |
| <code>VSL_RNG_ERROR_BAD_NSEEDS</code>            | Bad value in <code>NSeeds</code> field.                                                                                               |
| <code>VSL_RNG_ERROR_BAD_NBITS</code>             | Bad value in <code>NBits</code> field.                                                                                                |
| <code>VSL_ERROR_NULL_PTR</code>                  | At least one of the fields <code>iBrng</code> , <code>dBrng</code> , <code>sBrng</code> or <code>InitStream</code> is a NULL pointer. |

## **vslGetBrngProperties**

Returns structure with properties of a given basic generator.

### Syntax

```
status = vslGetBrngProperties(brng, &properties);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name        | Type      | Description                                                                                                                                                                                                  |
|-------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>brng</i> | const int | Number (index) of the registered basic generator; used for identification. See specific values in <a href="#">Table "Values of <i>brng</i> parameter"</a> . Negative values indicate the registration error. |

## Output Parameters

| Name              | Type               | Description                                                                             |
|-------------------|--------------------|-----------------------------------------------------------------------------------------|
| <i>properties</i> | VSLBRngProperties* | Pointer to the structure containing properties of the generator with number <i>brng</i> |

## Description

The `vslGetBrngProperties` function returns a structure with properties of a given basic generator.

## Return Values

|                                  |                                              |
|----------------------------------|----------------------------------------------|
| VSL_ERROR_OK, VSL_STATUS_OK      | Indicates no error, execution is successful. |
| VSL_RNG_ERROR_INVALID_BRNG_INDEX | BRNG index is invalid.                       |

## Formats for User-Designed Generators

To register a user-designed basic generator using `vslRegisterBrng` function, you need to pass the pointer `iBrng` to the integer-value implementation of the generator; the pointers `sBrng` and `dBrng` to the generator implementations for single and double precision values, respectively; and pass the pointer `InitStream` to the stream initialization routine. See recommendations below on defining such functions with input and output arguments. An example of the registration procedure for a user-designed generator can be found in the respective directory of VS examples.

The respective pointers are defined as follows:

```
typedef int(*InitStreamPtr)(int method, VSLStreamStatePtr stream, int n, const unsigned int
params[]);
```

```
typedef int(*sBrngPtr)(VSLStreamStatePtr stream, int n, float r[], float a, float b);
```

```
typedef int(*dBrngPtr)(VSLStreamStatePtr stream, int n, double r[], double a, double b);
```

```
typedef int(*iBrngPtr)(VSLStreamStatePtr stream, int n, unsigned int r[]);
```

## InitStream

```
int MyBrngInitStream(int method, VSLStreamStatePtr stream, int n, const unsigned int params[])
{
 /* Initialize the stream */
 ...
} /* MyBrngInitStream */
```

## Description

The initialization routine of a user-designed generator must initialize `stream` according to the specified initialization `method`, initial conditions `params` and the argument `n`. The value of `method` determines the initialization method to be used.

- If `method` is equal to 1, the initialization is by the standard generation method, which must be supported by all basic generators. In this case the function assumes that the `stream` structure was not previously initialized. The value of `n` is used as the actual number of 32-bit values passed as initial conditions

through *params*. Note, that the situation when the actual number of initial conditions passed to the function is not sufficient to initialize the generator is not an error. Whenever it occurs, the basic generator must initialize the missing conditions using default settings.

- If *method* is equal to 2, the generation is by the leapfrog method, where *n* specifies the number of computational nodes (independent streams). Here the function assumes that the *stream* was previously initialized by the standard generation method. In this case *params* contains only one element, which identifies the computational node. If the generator does not support the leapfrog method, the function must return the error code `VSL_RNG_ERROR_LEAPFROG_UNSUPPORTED`.
- If *method* is equal to 3, the generation is by the block-splitting method. Same as above, the *stream* is assumed to be previously initialized by the standard generation method; *params* is not used, *n* identifies the number of skipped elements. If the generator does not support the block-splitting method, the function must return the error code `VSL_RNG_ERROR_SKIPAHEAD_UNSUPPORTED`.

For a more detailed description of the leapfrog and the block-splitting methods, refer to the description of [vslLeapfrogStream](#) and [vslSkipAheadStream](#), respectively.

Stream state structure is individual for every generator. However, each structure has a number of fields that are the same for all the generators:

```
typedef struct
{
 unsigned int Reserved1[2];
 unsigned int Reserved2[2];
 [fields specific for the given generator]
} MyStreamState;
```

The fields *Reserved1* and *Reserved2* are reserved for private needs only, and must not be modified by the user. When including specific fields into the structure, follow the rules below:

- The fields must fully describe the current state of the generator. For example, the state of a linear congruential generator can be identified by only one initial condition;
- If the generator can use both the leapfrog and the block-splitting methods, additional fields should be introduced to identify the independent streams. For example, in  $LCG(a, c, m)$ , apart from the initial conditions, two more fields should be specified: the value of the multiplier  $a^k$  and the value of the increment  $(a^k - 1)c / (a - 1)$ .

For a more detailed discussion, refer to [Knuth81], and [Gentle98]. An example of the registration procedure can be found in the respective directory of VS examples.

## iBRng

```
int iMyBrng(VSLStreamStatePtr stream, int n, unsigned int r[])
{
 int i; /* Loop variable */
 /* Generating integer random numbers */
 /* Pay attention to word size needed to
 store only random number */
 for(i = 0; i < n; i++)
 {
 r[i] = ...;
 }
 /* Update stream state */
 ...
 return errcode;
} /* iMyBrng */
```

**NOTE**

When using 64 and 128-bit generators, consider digit capacity to store the numbers to the random vector  $r$  correctly. For example, storing one 64-bit value requires two elements of  $r$ , the first to store the lower 32 bits and the second to store the higher 32 bits. Similarly, use 4 elements of  $r$  to store a 128-bit value.

**sBRng**

```
int sMyBrng(VSLStreamStatePtr stream, int n, float r[], float a, float b)
{
 int i; /* Loop variable */
 /* Generating float (a,b) random numbers */
 for (i = 0; i < n; i++)
 {
 r[i] = ...;
 }
 /* Update stream state */
 ...
 return errcode;
} /* sMyBrng */
```

**dBRng**

```
int dMyBrng(VSLStreamStatePtr stream, int n, double r[], double a, double b)
{
 int i; /* Loop variable */
 /* Generating double (a,b) random numbers */
 for (i = 0; i < n; i++)
 {
 r[i] = ...;
 }
 /* Update stream state */
 ...
 return errcode;
} /* dMyBrng */
```

## Convolution and Correlation

Intel MKL VS provides a set of routines intended to perform linear convolution and correlation transformations for single and double precision real and complex data.

For correct definition of implemented operations, see the [Mathematical Notation and Definitions](#) section.

The current implementation provides:

- Fourier algorithms for one-dimensional single and double precision real and complex data
- Fourier algorithms for multi-dimensional single and double precision real and complex data
- Direct algorithms for one-dimensional single and double precision real and complex data
- Direct algorithms for multi-dimensional single and double precision real and complex data

One-dimensional algorithms cover the following functions from the IBM\* ESSL library:

SCONF, SCORE

SCOND, SCORD

SDCON, SDCOR

DDCON, DDCOR

SDDCON, SDDCOR.

Special wrappers are designed to simulate these ESSL functions. The wrappers are provided as sample sources:

```
{MKL}/examples/vslc/essl/vsl_wrappers
```

Additionally, you can browse the examples demonstrating the calculation of the ESSL functions through the wrappers:

```
{MKL}/examples/vslc/essl
```

The convolution and correlation API provides interfaces for Fortran 90 and C/89 languages. You can use the C89 interface with later versions of the C/C++.

Intel MKL provides the `mkl_vsl.h` header file. All header files are in the directory

```
{MKL}/include
```

The convolution and correlation API is implemented through task objects, or tasks. Task object is a data structure, or descriptor, which holds parameters that determine the specific convolution or correlation operation. Such parameters may be precision, type, and number of dimensions of user data, an identifier of the computation algorithm to be used, shapes of data arrays, and so on.

All the Intel MKL VS convolution and correlation routines process task objects in one way or another: either create a new task descriptor, change the parameter settings, compute mathematical results of the convolution or correlation using the stored parameters, or perform other operations. Accordingly, all routines are split into the following groups:

**Task Constructors** - routines that create a new task object descriptor and set up most common parameters.

**Task Editors** - routines that can set or modify some parameter settings in the existing task descriptor.

**Task Execution Routines** - compute results of the convolution or correlation operation over the actual input data, using the operation parameters held in the task descriptor.

**Task Copy** - routines used to make several copies of the task descriptor.

**Task Destructors** - routines that delete task objects and free the memory.

When the task is executed or copied for the first time, a special process runs which is called task commitment. During this process, consistency of task parameters is checked and the required work data are prepared. If the parameters are consistent, the task is tagged as committed successfully. The task remains committed until you edit its parameters. Hence, the task can be executed multiple times after a single commitment process. Since the task commitment process may include costly intermediate calculations such as preparation of Fourier transform of input data, launching the process only once can help speed up overall performance.

## Convolution and Correlation Naming Conventions

The names of routines, types, and constants in the convolution and correlation API are case-sensitive and can contain both lowercase and uppercase characters (`vslsConvExec`).

The names of routines have the following structure:

```
vsl[datatype]{Conv|Corr}<base name>
```

where

- `vsl` is a prefix indicating that the routine belongs to Intel® MKL Vector Statistics.

- [*datatype*] is optional. If present, the symbol specifies the type of the input and output data and can be *s* (for single precision real type), *d* (for double precision real type), *c* (for single precision complex type), or *z* (for double precision complex type).
- `Conv` or `Corr` specifies whether the routine refers to convolution or correlation task, respectively.
- `<base name>` field specifies a particular functionality that the routine is designed for, for example, `NewTask`, `DeleteTask`.

## Convolution and Correlation Data Types

All convolution or correlation routines use the following types for specifying data objects:

| Type           | Data Object                                        |
|----------------|----------------------------------------------------|
| VSLConvTaskPtr | Pointer to a task descriptor for convolution       |
| VSLCorrTaskPtr | Pointer to a task descriptor for correlation       |
| float          | Input/output user real data in single precision    |
| double         | Input/output user real data in double precision    |
| MKL_Complex8   | Input/output user complex data in single precision |
| MKL_Complex16  | Input/output user complex data in double precision |
| int            | All other data                                     |

Generic integer type (without specifying the byte size) is used for all integer data.

---

### NOTE

The actual size of the generic integer type is platform-dependent. Before you compile your application, set an appropriate byte size for integers. See details in the 'Using the ILP64 Interface vs. LP64 Interface' section of the *Intel® MKL Developer Guide*.

---

## Convolution and Correlation Parameters

Basic parameters held by the task descriptor are assigned values when the task object is created, copied, or modified by task editors. Parameters of the correlation or convolution task are initially set up by task constructors when the task object is created. Parameter changes or additional settings are made by task editors. More parameters which define location of the data being convolved need to be specified when the task execution routine is invoked.

According to how the parameters are passed or assigned values, all of them can be categorized as either explicit (directly passed as routine parameters when a task object is created or executed) or optional (assigned some default or implicit values during task construction).

The following table lists all applicable parameters used in the Intel MKL convolution and correlation API.

### Convolution and Correlation Task Parameters

| Name        | Category | Type    | Default Value Label             | Description                                                                                        |
|-------------|----------|---------|---------------------------------|----------------------------------------------------------------------------------------------------|
| <i>job</i>  | explicit | integer | Implied by the constructor name | Specifies whether the task relates to convolution or correlation                                   |
| <i>type</i> | explicit | integer | Implied by the constructor name | Specifies the type (real or complex) of the input/output data. Set to real in the current version. |

| Name                             | Category | Type           | Default Value Label                        | Description                                                                                                                                                                                                                                           |
|----------------------------------|----------|----------------|--------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>precision</i>                 | explicit | integer        | Implied by the constructor name            | Specifies precision (single or double) of the input/output data to be provided in arrays <i>x,y,z</i> .                                                                                                                                               |
| <i>mode</i>                      | explicit | integer        | None                                       | Specifies whether the convolution/correlation computation should be done via Fourier transforms, or by a direct method, or by automatically choosing between the two. See <a href="#">SetMode</a> for the list of named constants for this parameter. |
| <i>method</i>                    | optional | integer        | "auto"                                     | Hints at a particular computation method if several methods are available for the given <i>mode</i> . Setting this parameter to "auto" means that software will choose the best available method.                                                     |
| <i>internal_precision</i>        | optional | integer        | Set equal to the value of <i>precision</i> | Specifies precision of internal calculations. Can enforce double precision calculations even when input/output data are single precision. See <a href="#">SetInternalPrecision</a> for the list of named constants for this parameter.                |
| <i>dims</i>                      | explicit | integer        | None                                       | Specifies the rank (number of dimensions) of the user data provided in arrays <i>x,y,z</i> . Can be in the range from 1 to 7.                                                                                                                         |
| <i>x,y</i>                       | explicit | real arrays    | None                                       | Specify input data arrays. See <a href="#">Data Allocation</a> for more information.                                                                                                                                                                  |
| <i>z</i>                         | explicit | real array     | None                                       | Specifies output data array. See <a href="#">Data Allocation</a> for more information.                                                                                                                                                                |
| <i>xshape, yshape, zshape</i>    | explicit | integer arrays | None                                       | Define shapes of the arrays <i>x, y, z</i> . See <a href="#">Data Allocation</a> for more information.                                                                                                                                                |
| <i>xstride, ystride, zstride</i> | explicit | integer arrays | None                                       | Define strides within arrays <i>x, y, z</i> , that is specify the physical location of the input and output data in these arrays. See <a href="#">Data Allocation</a> for more information.                                                           |
| <i>start</i>                     | optional | integer array  | Undefined                                  | Defines the first element of the mathematical result that will be stored to output array <i>z</i> . See <a href="#">SetStart</a> and <a href="#">Data Allocation</a> for more information.                                                            |
| <i>decimation</i>                | optional | integer array  | Undefined                                  | Defines how to thin out the mathematical result that will be stored to output array <i>z</i> . See <a href="#">SetDecimation</a> and <a href="#">Data Allocation</a> for more information.                                                            |

Users may pass the NULL pointer instead of either or all of the parameters *xstride, ystride, or zstride* for multi-dimensional calculations. In this case, the software assumes the dense data allocation for the arrays *x, y, or z* due to the Fortran-style "by columns" representation of multi-dimensional arrays.

## Convolution and Correlation Task Status and Error Reporting

The task status is an integer value, which is zero if no error has been detected while processing the task, or a specific non-zero error code otherwise. Negative status values indicate errors, and positive values indicate warnings.

An error can be caused by invalid parameter values, a system fault like a memory allocation failure, or can be an internal error self-detected by the software.

Each task descriptor contains the current status of the task. When creating a task object, the constructor assigns the `VSL_STATUS_OK` status to the task. When processing the task afterwards, other routines such as editors or executors can change the task status if an error occurs and write a corresponding error code into the task status field.

Note that at the stage of creating a task or editing its parameters, the set of parameters may be inconsistent. The parameter consistency check is only performed during the task commitment operation, which is implicitly invoked before task execution or task copying. If an error is detected at this stage, task execution or task copying is terminated and the task descriptor saves the corresponding error code. Once an error occurs, any further attempts to process that task descriptor is terminated and the task keeps the same error code.

Normally, every convolution or correlation function (except `DeleteTask`) returns the status assigned to the task while performing the function operation.

The header files define symbolic names for the status codes. These names are defined as macros via the `#define` statements.

If there is no error, the `VSL_STATUS_OK` status is returned, which is defined as zero:

```
#define VSL_STATUS_OK 0
```

In case of an error, a non-zero error code is returned, which indicates the origin of the failure. The following status codes for the convolution/correlation error codes are pre-defined in the header files.

### Convolution/Correlation Status Codes

| Status Code                                  | Description                                 |
|----------------------------------------------|---------------------------------------------|
| <code>VSL_CC_ERROR_NOT_IMPLEMENTED</code>    | Requested functionality is not implemented. |
| <code>VSL_CC_ERROR_ALLOCATION_FAILURE</code> | Memory allocation failure.                  |
| <code>VSL_CC_ERROR_BAD_DESCRIPTOR</code>     | Task descriptor is corrupted.               |
| <code>VSL_CC_ERROR_SERVICE_FAILURE</code>    | A service function has failed.              |
| <code>VSL_CC_ERROR_EDIT_FAILURE</code>       | Failure while editing the task.             |
| <code>VSL_CC_ERROR_EDIT_PROHIBITED</code>    | You cannot edit this parameter.             |
| <code>VSL_CC_ERROR_COMMIT_FAILURE</code>     | Task commitment has failed.                 |
| <code>VSL_CC_ERROR_COPY_FAILURE</code>       | Failure while copying the task.             |
| <code>VSL_CC_ERROR_DELETE_FAILURE</code>     | Failure while deleting the task.            |
| <code>VSL_CC_ERROR_BAD_ARGUMENT</code>       | Bad argument or task parameter.             |
| <code>VSL_CC_ERROR_JOB</code>                | Bad parameter: <i>job</i> .                 |
| <code>SL_CC_ERROR_KIND</code>                | Bad parameter: <i>kind</i> .                |
| <code>VSL_CC_ERROR_MODE</code>               | Bad parameter: <i>mode</i> .                |
| <code>VSL_CC_ERROR_METHOD</code>             | Bad parameter: <i>method</i> .              |
| <code>VSL_CC_ERROR_TYPE</code>               | Bad parameter: <i>type</i> .                |

| Status Code                     | Description                                                                                                             |
|---------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| VSL_CC_ERROR_EXTERNAL_PRECISION | Bad parameter: <i>external_precision</i> .                                                                              |
| VSL_CC_ERROR_INTERNAL_PRECISION | Bad parameter: <i>internal_precision</i> .                                                                              |
| VSL_CC_ERROR_PRECISION          | Incompatible external/internal precisions.                                                                              |
| VSL_CC_ERROR_DIMS               | Bad parameter: <i>dims</i> .                                                                                            |
| VSL_CC_ERROR_XSHAPE             | Bad parameter: <i>xshape</i> .                                                                                          |
| VSL_CC_ERROR_YSHAPE             | Bad parameter: <i>yshape</i> .                                                                                          |
|                                 | Callback function for an abstract BRNG returns an invalid number of updated entries in a buffer, that is, < 0 or >nmax. |
| VSL_CC_ERROR_ZSHAPE             | Bad parameter: <i>zshape</i> .                                                                                          |
| VSL_CC_ERROR_XSTRIDE            | Bad parameter: <i>xstride</i> .                                                                                         |
| VSL_CC_ERROR_YSTRIDE            | Bad parameter: <i>ystride</i> .                                                                                         |
| VSL_CC_ERROR_ZSTRIDE            | Bad parameter: <i>zstride</i> .                                                                                         |
| VSL_CC_ERROR_X                  | Bad parameter: <i>x</i> .                                                                                               |
| VSL_CC_ERROR_Y                  | Bad parameter: <i>y</i> .                                                                                               |
| VSL_CC_ERROR_Z                  | Bad parameter: <i>z</i> .                                                                                               |
| VSL_CC_ERROR_START              | Bad parameter: <i>start</i> .                                                                                           |
| VSL_CC_ERROR_DECIMATION         | Bad parameter: <i>decimation</i> .                                                                                      |
| VSL_CC_ERROR_OTHER              | Another error.                                                                                                          |

## Convolution and Correlation Task Constructors

Task constructors are routines intended for creating a new task descriptor and setting up basic parameters. No additional parameter adjustment is typically required and other routines can use the task object.

Intel® MKL implementation of the convolution and correlation API provides two different forms of constructors: a general form and an X-form. X-form constructors work in the same way as the general form constructors but also assign particular data to the first operand vector used in the convolution or correlation operation (stored in array *x*).

Using X-form constructors is recommended when you need to compute multiple convolutions or correlations with the same data vector held in array *x* against different vectors held in array *y*. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

Each constructor routine has an associated one-dimensional version that provides algorithmic and computational benefits.

---

### NOTE

If the constructor fails to create a task descriptor, it returns the `NULL` task pointer.

---

The [Table "Task Constructors"](#) lists available task constructors:

**Task Constructors**

| <b>Routine</b>                                        | <b>Description</b>                                                                                 |
|-------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| <code>vslConvNewTask/vslCorrNewTask</code>            | Creates a new convolution or correlation task descriptor for a multidimensional case.              |
| <code>vslConvNewTask1D/<br/>vslCorrNewTask1D</code>   | Creates a new convolution or correlation task descriptor for a one-dimensional case.               |
| <code>vslConvNewTaskX/vslCorrNewTaskX</code>          | Creates a new convolution or correlation task descriptor as an X-form for a multidimensional case. |
| <code>vslConvNewTaskX1D/<br/>vslCorrNewTaskX1D</code> | Creates a new convolution or correlation task descriptor as an X-form for a one-dimensional case.  |

**vslConvNewTask/vslCorrNewTask**

*Creates a new convolution or correlation task descriptor for multidimensional case.*

**Syntax**

```
status = vslsConvNewTask(task, mode, dims, xshape, yshape, zshape);
status = vsldConvNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslcConvNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslzConvNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslsCorrNewTask(task, mode, dims, xshape, yshape, zshape);
status = vsldCorrNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslcCorrNewTask(task, mode, dims, xshape, yshape, zshape);
status = vslzCorrNewTask(task, mode, dims, xshape, yshape, zshape);
```

**Include Files**

- mkl.h

**Input Parameters**

| <b>Name</b>         | <b>Type</b>                | <b>Description</b>                                                                                                                                                                                                                           |
|---------------------|----------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mode</code>   | <code>const MKL_INT</code> | Specifies whether convolution/correlation calculation must be performed by using a direct algorithm or through Fourier transform of the input data. See <a href="#">Table "Values of mode parameter"</a> for a list of possible values.      |
| <code>dims</code>   | <code>const MKL_INT</code> | Rank of user data. Specifies number of dimensions for the input and output arrays <i>x</i> , <i>y</i> , and <i>z</i> used during the execution stage. Must be in the range from 1 to 7. The value is explicitly assigned by the constructor. |
| <code>xshape</code> | <code>const int[]</code>   | Defines the shape of the input data for the source array <i>x</i> . See <a href="#">Data Allocation</a> for more information.                                                                                                                |
| <code>yshape</code> | <code>const int[]</code>   | Defines the shape of the input data for the source array <i>y</i> . See <a href="#">Data Allocation</a> for more information.                                                                                                                |

| Name                | Type                     | Description                                                                                                                           |
|---------------------|--------------------------|---------------------------------------------------------------------------------------------------------------------------------------|
| <code>zshape</code> | <code>const int[]</code> | Defines the shape of the output data to be stored in array <code>z</code> . See <a href="#">Data Allocation</a> for more information. |

## Output Parameters

| Name                | Type                                                                                                                                                                                                                                                                                                                 | Description                                                                                                    |
|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| <code>task</code>   | VSLConvTaskPtr* for<br><code>vslsConvNewTask,</code><br><code>vsldConvNewTask,</code><br><code>vslcConvNewTask,</code><br><code>vslzConvNewTask</code><br><br>VSLCorrTaskPtr* for<br><code>vslsCorrNewTask,</code><br><code>vsldCorrNewTask,</code><br><code>vslcConvNewTask,</code><br><code>vslzConvNewTask</code> | Pointer to the task descriptor if created successfully or NULL pointer otherwise.                              |
| <code>status</code> | <code>int</code>                                                                                                                                                                                                                                                                                                     | Set to <code>VSL_STATUS_OK</code> if the task is created successfully or set to non-zero error code otherwise. |

## Description

Each `vslConvNewTask/vslCorrNewTask` constructor creates a new convolution or correlation task descriptor with the user specified values for explicit parameters. The optional parameters are set to their default values (see [Table "Convolution and Correlation Task Parameters"](#)).

The parameters `xshape`, `yshape`, and `zshape` define the shapes of the input and output data provided by the arrays `x`, `y`, and `z`, respectively. Each shape parameter is an array of integers with its length equal to the value of `dims`. You explicitly assign the shape parameters when calling the constructor. If the value of the parameter `dims` is 1, then `xshape`, `yshape`, `zshape` are equal to the number of elements read from the arrays `x` and `y` or stored to the array `z`. Note that values of shape parameters may differ from physical shapes of arrays `x`, `y`, and `z` if non-trivial strides are assigned.

If the constructor fails to create a task descriptor, it returns a NULL task pointer.

## vslConvNewTask1D/vslCorrNewTask1D

*Creates a new convolution or correlation task descriptor for one-dimensional case.*

### Syntax

```
status = vslsConvNewTask1D(task, mode, xshape, yshape, zshape);
status = vsldConvNewTask1D(task, mode, xshape, yshape, zshape);
status = vslcConvNewTask1D(task, mode, xshape, yshape, zshape);
status = vslzConvNewTask1D(task, mode, xshape, yshape, zshape);
status = vslsCorrNewTask1D(task, mode, xshape, yshape, zshape);
status = vsldCorrNewTask1D(task, mode, xshape, yshape, zshape);
status = vslcCorrNewTask1D(task, mode, xshape, yshape, zshape);
status = vslzCorrNewTask1D(task, mode, xshape, yshape, zshape);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                | Type                       | Description                                                                                                                                                                                                                             |
|---------------------|----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mode</code>   | <code>const MKL_INT</code> | Specifies whether convolution/correlation calculation must be performed by using a direct algorithm or through Fourier transform of the input data. See <a href="#">Table "Values of mode parameter"</a> for a list of possible values. |
| <code>xshape</code> | <code>const MKL_INT</code> | Defines the length of the input data sequence for the source array <code>x</code> . See <a href="#">Data Allocation</a> for more information.                                                                                           |
| <code>yshape</code> | <code>const MKL_INT</code> | Defines the length of the input data sequence for the source array <code>y</code> . See <a href="#">Data Allocation</a> for more information.                                                                                           |
| <code>zshape</code> | <code>const MKL_INT</code> | Defines the length of the output data sequence to be stored in array <code>z</code> . See <a href="#">Data Allocation</a> for more information.                                                                                         |

## Output Parameters

| Name                | Type                                                                                                                                                                                                                                                                                                                                                                 | Description                                                                                                    |
|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| <code>task</code>   | <code>VSLConvTaskPtr*</code> for<br><code>vslsConvNewTask1D</code> ,<br><code>vsldConvNewTask1D</code> ,<br><code>vslcConvNewTask1D</code> ,<br><code>vslzConvNewTask1D</code><br><br><code>VSLCorrTaskPtr*</code> for<br><code>vslsCorrNewTask1D</code> ,<br><code>vsldCorrNewTask1D</code> ,<br><code>vslcCorrNewTask1D</code> ,<br><code>vslzCorrNewTask1D</code> | Pointer to the task descriptor if created successfully or <code>NULL</code> pointer otherwise.                 |
| <code>status</code> | <code>int</code>                                                                                                                                                                                                                                                                                                                                                     | Set to <code>VSL_STATUS_OK</code> if the task is created successfully or set to non-zero error code otherwise. |

## Description

Each `vslConvNewTask1D/vslCorrNewTask1D` constructor creates a new convolution or correlation task descriptor with the user specified values for explicit parameters. The optional parameters are set to their default values (see [Table "Convolution and Correlation Task Parameters"](#)). Unlike `vslConvNewTask/vslCorrNewTask`, these routines represent a special one-dimensional version of the constructor which assumes that the value of the parameter `dims` is 1. The parameters `xshape`, `yshape`, and `zshape` are equal to the number of elements read from the arrays `x` and `y` or stored to the array `z`. You explicitly assign the shape parameters when calling the constructor.

### **`vslConvNewTaskX/vslCorrNewTaskX`**

*Creates a new convolution or correlation task descriptor for multidimensional case and assigns source data to the first operand vector.*

### Syntax

```
status = vslsConvNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
```

```

status = vsldConvNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslcConvNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslzConvNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslsCorrNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vsldCorrNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslcCorrNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);
status = vslzCorrNewTaskX(task, mode, dims, xshape, yshape, zshape, x, xstride);

```

## Include Files

- mkl.h

## Input Parameters

| Name           | Type                                                                                                                                                                                                                                                              | Description                                                                                                                                                                                                                                  |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>mode</i>    | const MKL_INT                                                                                                                                                                                                                                                     | Specifies whether convolution/correlation calculation must be performed by using a direct algorithm or through Fourier transform of the input data. See <a href="#">Table "Values of mode parameter"</a> for a list of possible values.      |
| <i>dims</i>    | const MKL_INT                                                                                                                                                                                                                                                     | Rank of user data. Specifies number of dimensions for the input and output arrays <i>x</i> , <i>y</i> , and <i>z</i> used during the execution stage. Must be in the range from 1 to 7. The value is explicitly assigned by the constructor. |
| <i>xshape</i>  | const int[]                                                                                                                                                                                                                                                       | Defines the shape of the input data for the source array <i>x</i> . See <a href="#">Data Allocation</a> for more information.                                                                                                                |
| <i>yshape</i>  | const int[]                                                                                                                                                                                                                                                       | Defines the shape of the input data for the source array <i>y</i> . See <a href="#">Data Allocation</a> for more information.                                                                                                                |
| <i>zshape</i>  | const int[]                                                                                                                                                                                                                                                       | Defines the shape of the output data to be stored in array <i>z</i> . See <a href="#">Data Allocation</a> for more information.                                                                                                              |
| <i>x</i>       | const float[] for real data in single precision flavors,<br>const double[] for real data in double precision flavors,<br>const MKL_Complex8[] for complex data in single precision flavors,<br>const MKL_Complex16[] for complex data in double precision flavors | Pointer to the array containing input data for the first operand vector. See <a href="#">Data Allocation</a> for more information.                                                                                                           |
| <i>xstride</i> | const int[]                                                                                                                                                                                                                                                       | Strides for input data in the array <i>x</i> .                                                                                                                                                                                               |

## Output Parameters

| Name          | Type                                                                                                                                                                                                                 | Description                                                                                       |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
| <i>task</i>   | VSLConvTaskPtr* for<br>vslsConvNewTaskX,<br>vsldConvNewTaskX,<br>vslcConvNewTaskX,<br>vslzConvNewTaskX<br><br>VSLCorrTaskPtr* for<br>vslsCorrNewTaskX,<br>vsldCorrNewTaskX,<br>vslcCorrNewTaskX,<br>vslzCorrNewTaskX | Pointer to the task descriptor if created successfully or NULL pointer otherwise.                 |
| <i>status</i> | int                                                                                                                                                                                                                  | Set to VSL_STATUS_OK if the task is created successfully or set to non-zero error code otherwise. |

## Description

Each `vslConvNewTaskX/vslCorrNewTaskX` constructor creates a new convolution or correlation task descriptor with the user specified values for explicit parameters. The optional parameters are set to their default values (see [Table "Convolution and Correlation Task Parameters"](#)).

Unlike `vslConvNewTask/vslCorrNewTask`, these routines represent the so called X-form version of the constructor, which means that in addition to creating the task descriptor they assign particular data to the first operand vector in array *x* used in convolution or correlation operation. The task descriptor created by the `vslConvNewTaskX/vslCorrNewTaskX` constructor keeps the pointer to the array *x* all the time, that is, until the task object is deleted by one of the destructor routines (see `vslConvDeleteTask/vslCorrDeleteTask`).

Using this form of constructors is recommended when you need to compute multiple convolutions or correlations with the same data vector in array *x* against different vectors in array *y*. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

The parameters *xshape*, *yshape*, and *zshape* define the shapes of the input and output data provided by the arrays *x*, *y*, and *z*, respectively. Each shape parameter is an array of integers with its length equal to the value of *dims*. You explicitly assign the shape parameters when calling the constructor. If the value of the parameter *dims* is 1, then *xshape*, *yshape*, and *zshape* are equal to the number of elements read from the arrays *x* and *y* or stored to the array *z*. Note that values of shape parameters may differ from physical shapes of arrays *x*, *y*, and *z* if non-trivial strides are assigned.

The stride parameter *xstride* specifies the physical location of the input data in the array *x*. In a one-dimensional case, stride is an interval between locations of consecutive elements of the array. For example, if the value of the parameter *xstride* is *s*, then only every *s*<sup>th</sup> element of the array *x* will be used to form the input sequence. The stride value must be positive or negative but not zero.

### **vslConvNewTaskX1D/vslCorrNewTaskX1D**

*Creates a new convolution or correlation task descriptor for one-dimensional case and assigns source data to the first operand vector.*

### Syntax

```
status = vslsConvNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vsldConvNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslcConvNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
```

```

status = vslzConvNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslsCorrNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vsldCorrNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslcCorrNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);
status = vslzCorrNewTaskX1D(task, mode, xshape, yshape, zshape, x, xstride);

```

## Include Files

- mkl.h

## Input Parameters

| Name           | Type                                                                                                                                                                                                                                                              | Description                                                                                                                                                                                                                             |
|----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>mode</i>    | const MKL_INT                                                                                                                                                                                                                                                     | Specifies whether convolution/correlation calculation must be performed by using a direct algorithm or through Fourier transform of the input data. See <a href="#">Table "Values of mode parameter"</a> for a list of possible values. |
| <i>xshape</i>  | const MKL_INT                                                                                                                                                                                                                                                     | Defines the length of the input data sequence for the source array <i>x</i> . See <a href="#">Data Allocation</a> for more information.                                                                                                 |
| <i>yshape</i>  | const MKL_INT                                                                                                                                                                                                                                                     | Defines the length of the input data sequence for the source array <i>y</i> . See <a href="#">Data Allocation</a> for more information.                                                                                                 |
| <i>zshape</i>  | const MKL_INT                                                                                                                                                                                                                                                     | Defines the length of the output data sequence to be stored in array <i>z</i> . See <a href="#">Data Allocation</a> for more information.                                                                                               |
| <i>x</i>       | const float[] for real data in single precision flavors,<br>const double[] for real data in double precision flavors,<br>const MKL_Complex8[] for complex data in single precision flavors,<br>const MKL_Complex16[] for complex data in double precision flavors | Pointer to the array containing input data for the first operand vector. See <a href="#">Data Allocation</a> for more information.                                                                                                      |
| <i>xstride</i> | const MKL_INT                                                                                                                                                                                                                                                     | Stride for input data sequence in the array <i>x</i> .                                                                                                                                                                                  |

## Output Parameters

| Name        | Type                                                                                               | Description                                                                       |
|-------------|----------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
| <i>task</i> | VSLConvTaskPtr* for vslsConvNewTaskX1D, vsldConvNewTaskX1D, vslcConvNewTaskX1D, vslzConvNewTaskX1D | Pointer to the task descriptor if created successfully or NULL pointer otherwise. |

| Name          | Type                                                                                                           | Description                                                                                       |
|---------------|----------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|
|               | VSLCorrTaskPtr* for<br>vslsCorrNewTaskX1D,<br>vsldCorrNewTaskX1D,<br>vslcCorrNewTaskX1D,<br>vslzCorrNewTaskX1D |                                                                                                   |
| <i>status</i> | int                                                                                                            | Set to VSL_STATUS_OK if the task is created successfully or set to non-zero error code otherwise. |

## Description

Each `vslConvNewTaskX1D/vslCorrNewTaskX1D` constructor creates a new convolution or correlation task descriptor with the user specified values for explicit parameters. The optional parameters are set to their default values (see [Table "Convolution and Correlation Task Parameters"](#)).

These routines represent a special one-dimensional version of the so called X-form of the constructor. This assumes that the value of the parameter `dims` is 1 and that in addition to creating the task descriptor, constructor routines assign particular data to the first operand vector in array `x` used in convolution or correlation operation. The task descriptor created by the `vslConvNewTaskX1D/vslCorrNewTaskX1D` constructor keeps the pointer to the array `x` all the time, that is, until the task object is deleted by one of the destructor routines (see [vslConvDeleteTask/vslCorrDeleteTask](#)).

Using this form of constructors is recommended when you need to compute multiple convolutions or correlations with the same data vector in array `x` against different vectors in array `y`. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

The parameters `xshape`, `yshape`, and `zshape` are equal to the number of elements read from the arrays `x` and `y` or stored to the array `z`. You explicitly assign the shape parameters when calling the constructor.

The [stride parameters](#) `xstride` specifies the physical location of the input data in the array `x` and is an interval between locations of consecutive elements of the array. For example, if the value of the parameter `xstride` is `s`, then only every `s`<sup>th</sup> element of the array `x` will be used to form the input sequence. The stride value must be positive or negative but not zero.

## Convolution and Correlation Task Editors

Task editors in convolution and correlation API of Intel MKL are routines intended for setting up or changing the following task parameters (see [Table "Convolution and Correlation Task Parameters"](#)):

- `mode`
- `internal_precision`
- `start`
- `decimation`

For setting up or changing each of the above parameters, a separate routine exists.

---

### NOTE

Fields of the task descriptor structure are accessible only through the set of task editor routines provided with the software.

---

The work data computed during the last commitment process may become invalid with respect to new parameter settings. That is why after applying any of the editor routines to change the task descriptor settings, the task loses its commitment status and goes through the full commitment process again during the next execution or copy operation. For more information on task commitment, see the [Introduction to Convolution and Correlation](#).

Table "Task Editors" lists available task editors.

### Task Editors

| Routine                                                                   | Description                                                                                                   |
|---------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| <code>vslConvSetMode/vslCorrSetMode</code>                                | Changes the value of the parameter <i>mode</i> for the operation of convolution or correlation.               |
| <code>vslConvSetInternalPrecision/<br/>vslCorrSetInternalPrecision</code> | Changes the value of the parameter <i>internal_precision</i> for the operation of convolution or correlation. |
| <code>vslConvSetStart/vslCorrSetStart</code>                              | Sets the value of the parameter <i>start</i> for the operation of convolution or correlation.                 |
| <code>vslConvSetDecimation/<br/>vslCorrSetDecimation</code>               | Sets the value of the parameter <i>decimation</i> for the operation of convolution or correlation.            |

#### NOTE

You can use the `NULL` task pointer in calls to editor routines. In this case, the routine is terminated and no system crash occurs.

### `vslConvSetMode/vslCorrSetMode`

Changes the value of the parameter *mode* in the convolution or correlation task descriptor.

#### Syntax

```
status = vslConvSetMode(task, newmode);
```

```
status = vslCorrSetMode(task, newmode);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| Name           | Type                                                                                                                           | Description                              |
|----------------|--------------------------------------------------------------------------------------------------------------------------------|------------------------------------------|
| <i>task</i>    | <code>VSLConvTaskPtr</code> for <code>vslConvSetMode</code><br><br><code>VSLCorrTaskPtr</code> for <code>vslCorrSetMode</code> | Pointer to the task descriptor.          |
| <i>newmode</i> | <code>const MKL_INT</code>                                                                                                     | New value of the parameter <i>mode</i> . |

#### Output Parameters

| Name          | Type             | Description                 |
|---------------|------------------|-----------------------------|
| <i>status</i> | <code>int</code> | Current status of the task. |

#### Description

This function is declared in `mkl_vsl_functions.h`.

The function routine changes the value of the parameter `mode` for the operation of convolution or correlation. This parameter defines whether the computation should be done via Fourier transforms of the input/output data or using a direct algorithm. Initial value for `mode` is assigned by a task constructor.

Predefined values for the `mode` parameter are as follows:

### Values of `mode` parameter

| Value                             | Purpose                                                      |
|-----------------------------------|--------------------------------------------------------------|
| <code>VSL_CONV_MODE_FFT</code>    | Compute convolution by using fast Fourier transform.         |
| <code>VSL_CORR_MODE_FFT</code>    | Compute correlation by using fast Fourier transform.         |
| <code>VSL_CONV_MODE_DIRECT</code> | Compute convolution directly.                                |
| <code>VSL_CORR_MODE_DIRECT</code> | Compute correlation directly.                                |
| <code>VSL_CONV_MODE_AUTO</code>   | Automatically choose direct or Fourier mode for convolution. |
| <code>VSL_CORR_MODE_AUTO</code>   | Automatically choose direct or Fourier mode for correlation. |

### `vslConvSetInternalPrecision/vslCorrSetInternalPrecision`

Changes the value of the parameter `internal_precision` in the convolution or correlation task descriptor.

#### Syntax

```
status = vslConvSetInternalPrecision(task, precision);
```

```
status = vslCorrSetInternalPrecision(task, precision);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| Name                   | Type                                                                                                                                                             | Description                                                  |
|------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------|
| <code>task</code>      | <code>VSLConvTaskPtr</code> for<br><code>vslConvSetInternalPrecision</code><br>or<br><code>VSLCorrTaskPtr</code> for<br><code>vslCorrSetInternalPrecision</code> | Pointer to the task descriptor.                              |
| <code>precision</code> | <code>const MKL_INT</code>                                                                                                                                       | New value of the parameter <code>internal_precision</code> . |

#### Output Parameters

| Name                | Type             | Description                 |
|---------------------|------------------|-----------------------------|
| <code>status</code> | <code>int</code> | Current status of the task. |

## Description

The `vslConvSetInternalPrecision/vslCorrSetInternalPrecision` routine changes the value of the parameter `internal_precision` for the operation of convolution or correlation. This parameter defines whether the internal computations of the convolution or correlation result should be done in single or double precision. Initial value for `internal_precision` is assigned by a task constructor and set to either "single" or "double" according to the particular flavor of the constructor used.

Changing the `internal_precision` can be useful if the default setting of this parameter was "single" but you want to calculate the result with double precision even if input and output data are represented in single precision.

Predefined values for the `internal_precision` input parameter are as follows:

### Values of `internal_precision` Parameter

| Value                                  | Purpose                                    |
|----------------------------------------|--------------------------------------------|
| <code>VSL_CONV_PRECISION_SINGLE</code> | Compute convolution with single precision. |
| <code>VSL_CORR_PRECISION_SINGLE</code> | Compute correlation with single precision. |
| <code>VSL_CONV_PRECISION_DOUBLE</code> | Compute convolution with double precision. |
| <code>VSL_CORR_PRECISION_DOUBLE</code> | Compute correlation with double precision. |

### `vslConvSetStart/vslCorrSetStart`

*Changes the value of the parameter `start` in the convolution or correlation task descriptor.*

### Syntax

```
status = vslConvSetStart(task, start);
status = vslCorrSetStart(task, start);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name               | Type                                                                                                                             | Description                                     |
|--------------------|----------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------|
| <code>task</code>  | <code>VSLConvTaskPtr</code> for <code>vslConvSetStart</code><br><br><code>VSLCorrTaskPtr</code> for <code>vslCorrSetStart</code> | Pointer to the task descriptor.                 |
| <code>start</code> | <code>const int[]</code>                                                                                                         | New value of the parameter <code>start</code> . |

### Output Parameters

| Name                | Type             | Description                 |
|---------------------|------------------|-----------------------------|
| <code>status</code> | <code>int</code> | Current status of the task. |

## Description

The `vslConvSetStart/vslCorrSetStart` routine sets the value of the parameter `start` for the operation of convolution or correlation. In a one-dimensional case, this parameter points to the first element in the mathematical result that should be stored in the output array. In a multidimensional case, `start` is an array of indices and its length is equal to the number of dimensions specified by the parameter `dims`. For more information about the definition and effect of this parameter, see [Data Allocation](#).

During the initial task descriptor construction, the default value for `start` is undefined and this parameter is not used. Therefore the only way to set and use the `start` parameter is via assigning it some value by one of the `vslConvSetStart/vslCorrSetStart` routines.

## vslConvSetDecimation/vslCorrSetDecimation

*Changes the value of the parameter `decimation` in the convolution or correlation task descriptor.*

## Syntax

```
status = vslConvSetDecimation(task, decimation);
```

```
status = vslCorrSetDecimation(task, decimation);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                                      | Type                                                                                                                   | Description                                          |
|-------------------------------------------|------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|
| <code>task</code>                         | VSLConvTaskPtr for<br><code>vslConvSetDecimation</code><br><br>VSLCorrTaskPtr for<br><code>vslCorrSetDecimation</code> | Pointer to the task descriptor.                      |
| <code>decimation</code><br><code>n</code> | <code>const int[]</code>                                                                                               | New value of the parameter <code>decimation</code> . |

## Output Parameters

| Name                | Type             | Description                 |
|---------------------|------------------|-----------------------------|
| <code>status</code> | <code>int</code> | Current status of the task. |

## Description

The routine sets the value of the parameter `decimation` for the operation of convolution or correlation. This parameter determines how to thin out the mathematical result of convolution or correlation before writing it into the output data array. For example, in a one-dimensional case, if `decimation = d > 1`, only every `d`-th element of the mathematical result is written to the output array `z`. In a multidimensional case, `decimation` is an array of indices and its length is equal to the number of dimensions specified by the parameter `dims`. For more information about the definition and effect of this parameter, see [Data Allocation](#).

During the initial task descriptor construction, the default value for `decimation` is undefined and this parameter is not used. Therefore the only way to set and use the `decimation` parameter is via assigning it some value by one of the `vslSetDecimation` routines.

## Task Execution Routines

Task execution routines compute convolution or correlation results based on parameters held by the task descriptor and on the user data supplied for input vectors.

After you create and adjust a task, you can execute it multiple times by applying to different input/output data of the same type, precision, and shape.

Intel MKL provides the following forms of convolution/correlation execution routines:

- **General form** executors that use the task descriptor created by the general form constructor and expect to get two source data arrays *x* and *y* on input
- **X-form** executors that use the task descriptor created by the X-form constructor and expect to get only one source data array *y* on input because the first array *x* has been already specified on the construction stage

When the task is executed for the first time, the execution routine includes a task commitment operation, which involves two basic steps: parameters consistency check and preparation of auxiliary data (for example, this might be the calculation of Fourier transform for input data).

Each execution routine has an associated one-dimensional version that provides algorithmic and computational benefits.

---

### NOTE

You can use the `NULL` task pointer in calls to execution routines. In this case, the routine is terminated and no system crash occurs.

---

If the task is executed successfully, the execution routine returns the zero status code. If an error is detected, the execution routine returns an error code which signals that a specific error has occurred. In particular, an error status code is returned in the following cases:

- if the task pointer is `NULL`
- if the task descriptor is corrupted
- if calculation has failed for some other reason.

---

### NOTE

Intel® MKL does not control floating-point errors, like overflow or gradual underflow, or operations with NaNs, etc.

---

If an error occurs, the task descriptor stores the error code.

The table below lists all task execution routines.

### Task Execution Routines

| Routine                                    | Description                                                                |
|--------------------------------------------|----------------------------------------------------------------------------|
| <code>vslConvExec/vslCorrExec</code>       | Computes convolution or correlation for a multidimensional case.           |
| <code>vslConvExec1D/vslCorrExec1D</code>   | Computes convolution or correlation for a one-dimensional case.            |
| <code>vslConvExecX/vslCorrExecX</code>     | Computes convolution or correlation as X-form for a multidimensional case. |
| <code>vslConvExecX1D/vslCorrExecX1D</code> | Computes convolution or correlation as X-form for a one-dimensional case.  |

### `vslConvExec/vslCorrExec`

*Computes convolution or correlation for multidimensional case.*

---

## Syntax

```

status = vslsConvExec(task, x, xstride, y, ystride, z, zstride);
status = vsldConvExec(task, x, xstride, y, ystride, z, zstride);
status = vslcConvExec(task, x, xstride, y, ystride, z, zstride);
status = vslzConvExec(task, x, xstride, y, ystride, z, zstride);
status = vslsCorrExec(task, x, xstride, y, ystride, z, zstride);
status = vsldCorrExec(task, x, xstride, y, ystride, z, zstride);
status = vslcCorrExec(task, x, xstride, y, ystride, z, zstride);
status = vslzCorrExec(task, x, xstride, y, ystride, z, zstride);

```

## Include Files

- mkl.h

## Input Parameters

| Name                                                 | Type                                                                                                                                                                                                                                                            | Description                                                                                         |
|------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|
| <i>task</i>                                          | VSLConvTaskPtr for<br>vslsConvExec, vsldConvExec,<br>vslcConvExec, vslzConvExec<br><br>VSLCorrTaskPtr for<br>vslsCorrExec, vsldCorrExec,<br>vslcCorrExec, vslzCorrExec                                                                                          | Pointer to the task descriptor                                                                      |
| <i>x, y</i>                                          | const float[] for<br>vslsConvExec and<br>vslsCorrExec,<br><br>const double[] for<br>vsldConvExec and<br>vsldCorrExec,<br><br>const MKL_Complex8[] for<br>vslcConvExec and<br>vslcCorrExec,<br><br>const MKL_Complex16[] for<br>vslzConvExec and<br>vslzCorrExec | Pointers to arrays containing input data. See <a href="#">Data Allocation</a> for more information. |
| <i>xstride,</i><br><i>ystride,</i><br><i>zstride</i> | const int[]                                                                                                                                                                                                                                                     | Strides for input and output data. For more information, see <a href="#">stride parameters</a> .    |

## Output Parameters

| Name     | Type                                                   | Description                                                                                             |
|----------|--------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <i>z</i> | const float[] for<br>vslsConvExec and<br>vslsCorrExec, | Pointer to the array that stores output data. See <a href="#">Data Allocation</a> for more information. |

| Name          | Type                                                                                                                                                                                              | Description                                                                                        |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
|               | const double[] for<br>vsldConvExec and<br>vsldCorrExec,<br><br>const MKL_Complex8[] for<br>vslcConvExec and<br>vslcCorrExec,<br><br>const MKL_Complex16[] for<br>vslzConvExec and<br>vslzCorrExec |                                                                                                    |
| <i>status</i> | int                                                                                                                                                                                               | Set to VSL_STATUS_OK if the task is executed successfully or set to non-zero error code otherwise. |

## Description

Each of the `vslConvExec/vslCorrExec` routines computes convolution or correlation of the data provided by the arrays *x* and *y* and then stores the results in the array *z*. Parameters of the operation are read from the task descriptor created previously by a corresponding `vslConvNewTask/vslCorrNewTask` constructor and pointed to by *task*. If *task* is NULL, no operation is done.

The stride parameters *xstride*, *ystride*, and *zstride* specify the physical location of the input and output data in the arrays *x*, *y*, and *z*, respectively. In a one-dimensional case, stride is an interval between locations of consecutive elements of the array. For example, if the value of the parameter *zstride* is *s*, then only every *s*<sup>th</sup> element of the array *z* will be used to store the output data. The stride value must be positive or negative but not zero.

## vslConvExec1D/vslCorrExec1D

*Computes convolution or correlation for one-dimensional case.*

### Syntax

```

status = vslsConvExec1D(task, x, xstride, y, ystride, z, zstride);
status = vsldConvExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslcConvExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslzConvExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslsCorrExec1D(task, x, xstride, y, ystride, z, zstride);
status = vsldCorrExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslcCorrExec1D(task, x, xstride, y, ystride, z, zstride);
status = vslzCorrExec1D(task, x, xstride, y, ystride, z, zstride);

```

### Include Files

- `mk1.h`

## Input Parameters

| Name                                     | Type                                                                                                                                                                                                                                                                            | Description                                                                                         |
|------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------|
| <i>task</i>                              | VSLConvTaskPtr for<br>vs1sConvExec1D,<br>vs1dConvExec1D,<br>vs1cConvExec1D,<br>vs1zConvExec1D<br><br>VSLCorrTaskPtr for<br>vs1sCorrExec1D,<br>vs1dCorrExec1D,<br>vs1cCorrExec1D,<br>vs1zCorrExec1D                                                                              | Pointer to the task descriptor.                                                                     |
| <i>x, y</i>                              | const float[] for<br>vs1sConvExec1D and<br>vs1sCorrExec1D,<br><br>const double[] for<br>vs1dConvExec1D and<br>vs1dCorrExec1D,<br><br>const MKL_Complex8[] for<br>vs1cConvExec1D and<br>vs1cCorrExec1D,<br><br>const MKL_Complex16[] for<br>vs1zConvExec1D and<br>vs1zCorrExec1D | Pointers to arrays containing input data. See <a href="#">Data Allocation</a> for more information. |
| <i>xstride,<br/>ystride,<br/>zstride</i> | const MKL_INT                                                                                                                                                                                                                                                                   | Strides for input and output data. For more information, see <a href="#">stride parameters</a> .    |

## Output Parameters

| Name          | Type                                                                                                                                                                                                                                                                            | Description                                                                                             |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <i>z</i>      | const float[] for<br>vs1sConvExec1D and<br>vs1sCorrExec1D,<br><br>const double[] for<br>vs1dConvExec1D and<br>vs1dCorrExec1D,<br><br>const MKL_Complex8[] for<br>vs1cConvExec1D and<br>vs1cCorrExec1D,<br><br>const MKL_Complex16[] for<br>vs1zConvExec1D and<br>vs1zCorrExec1D | Pointer to the array that stores output data. See <a href="#">Data Allocation</a> for more information. |
| <i>status</i> | int                                                                                                                                                                                                                                                                             | Set to VSL_STATUS_OK if the task is executed successfully or set to non-zero error code otherwise.      |

## Description

Each of the `vslConvExec1D/vslCorrExec1D` routines computes convolution or correlation of the data provided by the arrays `x` and `y` and then stores the results in the array `z`. These routines represent a special one-dimensional version of the operation, assuming that the value of the parameter `dims` is 1. Using this version of execution routines can help speed up performance in case of one-dimensional data.

Parameters of the operation are read from the task descriptor created previously by a corresponding `vslConvNewTask1D/vslCorrNewTask1D` constructor and pointed to by `task`. If `task` is `NULL`, no operation is done.

## vslConvExecX/vslCorrExecX

*Computes convolution or correlation for multidimensional case with the fixed first operand vector.*

---

## Syntax

```
status = vslsConvExecX(task, y, ystride, z, zstride);
status = vsldConvExecX(task, y, ystride, z, zstride);
status = vslcConvExecX(task, y, ystride, z, zstride);
status = vslzConvExecX(task, y, ystride, z, zstride);
status = vslsCorrExecX(task, y, ystride, z, zstride);
status = vslcCorrExecX(task, y, ystride, z, zstride);
status = vslzCorrExecX(task, y, ystride, z, zstride);
status = vsldCorrExecX(task, y, ystride, z, zstride);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name              | Type                                                                                                                                                                                                                                                                                                     | Description                                                                                                                       |
|-------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| <code>task</code> | VSLConvTaskPtr for<br><code>vslsConvExecX</code> ,<br><code>vsldConvExecX</code> ,<br><code>vslcConvExecX</code> ,<br><code>vslzConvExecX</code><br><br>VSLCorrTaskPtr for<br><code>vslsCorrExecX</code> ,<br><code>vsldCorrExecX</code> ,<br><code>vslcCorrExecX</code> ,<br><code>vslzCorrExecX</code> | Pointer to the task descriptor.                                                                                                   |
| <code>x, y</code> | <code>const float[]</code> for<br><code>vslsConvExecX</code> and<br><code>vslsCorrExecX</code> ,<br><br><code>const double[]</code> for<br><code>vsldConvExecX</code> and<br><code>vsldCorrExecX</code> ,                                                                                                | Pointer to array containing input data (for the second operand vector). See <a href="#">Data Allocation</a> for more information. |

| Name                            | Type                                                                                                                                   | Description                                                                                      |
|---------------------------------|----------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|
|                                 | const MKL_Complex8[] for<br>vs1cConvExecX and<br>vs1cCorrExecX,<br><br>const MKL_Complex16[] for<br>vs1zConvExecX and<br>vs1zCorrExecX |                                                                                                  |
| <i>ystride</i> , <i>zstride</i> | const int[]                                                                                                                            | Strides for input and output data. For more information, see <a href="#">stride parameters</a> . |

## Output Parameters

| Name          | Type                                                                                                                                                                                                                                                                    | Description                                                                                                     |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|
| <i>z</i>      | const float[] for<br>vs1sConvExecX and<br>vs1sCorrExecX,<br><br>const double[] for<br>vs1dConvExecX and<br>vs1dCorrExecX,<br><br>const MKL_Complex8[] for<br>vs1cConvExecX and<br>vs1cCorrExecX,<br><br>const MKL_Complex16[] for<br>vs1zConvExecX and<br>vs1zCorrExecX | Pointer to the array that stores output data. See <a href="#">Data Allocation</a> for more information.         |
| <i>status</i> | int                                                                                                                                                                                                                                                                     | Set to <code>VSL_STATUS_OK</code> if the task is executed successfully or set to non-zero error code otherwise. |

## Description

Each of the `vs1ConvExecX/vs1CorrExecX` routines computes convolution or correlation of the data provided by the arrays *x* and *y* and then stores the results in the array *z*. These routines represent a special version of the operation, which assumes that the first operand vector was set on the task construction stage and the task object keeps the pointer to the array *x*.

Parameters of the operation are read from the task descriptor created previously by a corresponding `vs1ConvNewTaskX/vs1CorrNewTaskX` constructor and pointed to by *task*. If *task* is NULL, no operation is done.

Using this form of execution routines is recommended when you need to compute multiple convolutions or correlations with the same data vector in array *x* against different vectors in array *y*. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

### `vs1ConvExecX1D/vs1CorrExecX1D`

*Computes convolution or correlation for one-dimensional case with the fixed first operand vector.*

### Syntax

```
status = vs1sConvExecX1D(task, y, ystride, z, zstride);
```

```

status = vsldConvExecX1D(task, y, ystride, z, zstride);
status = vslcConvExecX1D(task, y, ystride, z, zstride);
status = vslzConvExecX1D(task, y, ystride, z, zstride);
status = vslsCorrExecX1D(task, y, ystride, z, zstride);
status = vslcCorrExecX1D(task, y, ystride, z, zstride);
status = vslzCorrExecX1D(task, y, ystride, z, zstride);
status = vsldCorrExecX1D(task, y, ystride, z, zstride);

```

## Include Files

- mkl.h

## Input Parameters

| Name                              | Type                                                                                                                                                                                                                                                                                    | Description                                                                                                                       |
|-----------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>                       | VSLConvTaskPtr for<br>vslsConvExecX1D,<br>vsldConvExecX1D,<br>vslcConvExecX1D,<br>vslzConvExecX1D<br><br>VSLCorrTaskPtr for<br>vslsCorrExecX1D,<br>vsldCorrExecX1D,<br>vslcCorrExecX1D,<br>vslzCorrExecX1D                                                                              | Pointer to the task descriptor.                                                                                                   |
| <i>x, y</i>                       | const float[] for<br>vslsConvExecX1D and<br>vslsCorrExecX1D,<br><br>const double[] for<br>vsldConvExecX1D and<br>vsldCorrExecX1D,<br><br>const MKL_Complex8[] for<br>vslcConvExecX1D and<br>vslcCorrExecX1D,<br><br>const MKL_Complex16[] for<br>vslzConvExecX1D and<br>vslzCorrExecX1D | Pointer to array containing input data (for the second operand vector). See <a href="#">Data Allocation</a> for more information. |
| <i>ystride,</i><br><i>zstride</i> | const MKL_INT                                                                                                                                                                                                                                                                           | Strides for input and output data. For more information, see <a href="#">stride parameters</a> .                                  |

## Output Parameters

| Name     | Type                                                         | Description                                                                                             |
|----------|--------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <i>z</i> | const float[] for<br>vslsConvExecX1D and<br>vslsCorrExecX1D, | Pointer to the array that stores output data. See <a href="#">Data Allocation</a> for more information. |

| Name          | Type                                                                                                                                                                                                                | Description                                                                                        |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
|               | const double[] for<br>vsldConvExecX1D and<br>vsldCorrExecX1D,<br><br>const MKL_Complex8[] for<br>vslcConvExecX1D and<br>vslcCorrExecX1D,<br><br>const MKL_Complex16[] for<br>vslzConvExecX1D and<br>vslzCorrExecX1D |                                                                                                    |
| <i>status</i> | int                                                                                                                                                                                                                 | Set to VSL_STATUS_OK if the task is executed successfully or set to non-zero error code otherwise. |

## Description

Each of the `vslConvExecX1D/vslCorrExecX1D` routines computes convolution or correlation of one-dimensional (assuming that `dims = 1`) data provided by the arrays `x` and `y` and then stores the results in the array `z`. These routines represent a special version of the operation, which expects that the first operand vector was set on the task construction stage.

Parameters of the operation are read from the task descriptor created previously by a corresponding `vslConvNewTaskX1D/vslCorrNewTaskX1D` constructor and pointed to by `task`. If `task` is NULL, no operation is done.

Using this form of execution routines is recommended when you need to compute multiple one-dimensional convolutions or correlations with the same data vector in array `x` against different vectors in array `y`. This helps improve performance by eliminating unnecessary overhead in repeated computation of intermediate data required for the operation.

## Convolution and Correlation Task Destructors

Task destructors are routines designed for deleting task objects and deallocating memory.

### `vslConvDeleteTask/vslCorrDeleteTask`

*Destroys the task object and frees the memory.*

#### Syntax

```
errcode = vslConvDeleteTask(task);
```

```
errcode = vslCorrDeleteTask(task);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| Name        | Type                                                  | Description                     |
|-------------|-------------------------------------------------------|---------------------------------|
| <i>task</i> | VSLConvTaskPtr* for<br><code>vslConvDeleteTask</code> | Pointer to the task descriptor. |

| Name | Type                                     | Description |
|------|------------------------------------------|-------------|
|      | VSLCorrTaskPtr* for<br>vslCorrDeleteTask |             |

### Output Parameters

| Name           | Type | Description                                                                                            |
|----------------|------|--------------------------------------------------------------------------------------------------------|
| <i>errcode</i> | int  | Contains 0 if the task object is deleted successfully.<br>Contains an error code if an error occurred. |

### Description

The `vslConvDeleteTask/vslCorrvDeleteTask` routine deletes the task descriptor object and frees any working memory and the memory allocated for the data structure. The task pointer is set to `NULL`.

Note that if the `vslConvDeleteTask/vslCorrvDeleteTask` routine does not delete the task successfully, the routine returns an error code. This error code has no relation to the task status code and does not change it.

---

#### NOTE

You can use the `NULL` task pointer in calls to destructor routines. In this case, the routine terminates with no system crash.

---

## Convolution and Correlation Task Copiers

The routines are designed for copying convolution and correlation task descriptors.

### `vslConvCopyTask/vslCorrCopyTask`

*Copies a descriptor for convolution or correlation task.*

#### Syntax

```
status = vslConvCopyTask(newtask, srctask);
```

```
status = vslCorrCopyTask(newtask, srctask);
```

#### Include Files

- `mkl.h`

### Input Parameters

| Name           | Type                                                                                                                     | Description                            |
|----------------|--------------------------------------------------------------------------------------------------------------------------|----------------------------------------|
| <i>srctask</i> | const VSLConvTaskPtr for<br><code>vslConvCopyTask</code><br><br>const VSLCorrTaskPtr for<br><code>vslCorrCopyTask</code> | Pointer to the source task descriptor. |

## Output Parameters

| Name           | Type                                                                                 | Description                         |
|----------------|--------------------------------------------------------------------------------------|-------------------------------------|
| <i>newtask</i> | VSLConvTaskPtr* for<br>vslConvCopyTask<br><br>VSLCorrTaskPtr* for<br>vslCorrCopyTask | Pointer to the new task descriptor. |
| <i>status</i>  | int                                                                                  | Current status of the source task.  |

## Description

If a task object *srctask* already exists, you can use an appropriate `vslConvCopyTask/vslCorrCopyTask` routine to make its copy in *newtask*. After the copy operation, both source and new task objects will become committed (see [Introduction to Convolution and Correlation](#) for information about task commitment). If the source task was not previously committed, the commitment operation for this task is implicitly invoked before copying starts. If an error occurs during source task commitment, the task stores the error code in the status field. If an error occurs during copy operation, the routine returns a `NULL` pointer instead of a reference to a new task object.

## Convolution and Correlation Usage Examples

This section demonstrates how you can use the Intel MKL routines to perform some common convolution and correlation operations both for single-threaded and multithreaded calculations. The following two sample functions `scond1` and `sconf1` simulate the convolution and correlation functions `SCOND` and `SCONF` found in IBM ESSL\* library. The functions assume single-threaded calculations and can be used with C or C++ compilers.

### Function `scond1` for Single-Threaded Calculations

```
#include "mkl_vsl.h"

int acond1(
 float h[], int inch,
 float x[], int incx,
 float y[], int incy,
 int nh, int nx, int iy0, int ny)
{
 int status;
 VSLConvTaskPtr task;
 vslsConvNewTask1D(&task, VSL_CONV_MODE_DIRECT, nh, nx, ny);
 vslConvSetStart(task, &iy0);
 status = vslsConvExec1D(task, h, inch, x, incx, y, incy);
 vslConvDeleteTask(&task);
 return status;
}
```

## Function `sconf1` for Single-Threaded Calculations

```
#include "mkl_vsl.h"

int sconf1(
 int init,
 float h[], int inclh,
 float x[], int inclx, int inc2x,
 float y[], int incly, int inc2y,
 int nh, int nx, int m, int iy0, int ny,
 void* aux1, int nau1, void* aux2, int nau2)
{
 int status;

 /* assume that aux1!=0 and nau1 is big enough */
 VSLConvTaskPtr* task = (VSLConvTaskPtr*)aux1;

 if (init != 0)
 /* initialization: */
 status = vslsConvNewTaskX1D(task, VSL_CONV_MODE_FFT,
 nh, nx, ny, h, inclh);

 if (init == 0) {
 /* calculations: */
 int i;
 vslConvSetStart(*task, &iy0);
 for (i=0; i<m; i++) {
 float* xi = &x[inc2x * i];
 float* yi = &y[inc2y * i];

 /* task is implicitly committed at i==0 */
 status = vslsConvExecX1D(*task, xi, inclx, yi, incly);
 };
 };

 vslConvDeleteTask(task);

 return status;
}
```

## Using Multiple Threads

For functions such as `sconf1` described in the previous example, parallel calculations may be more preferable instead of cycling. If  $m > 1$ , you can use multiple threads for invoking the task execution against different data sequences. For such cases, use task copy routines to create  $m$  copies of the task object before the calculations stage and then run these copies with different threads. Ensure that you make all necessary parameter adjustments for the task (using [Task Editors](#)) before copying it.

The sample code in this case may look as follows:

```
if (init == 0) {
 int i, status, ss[M];
 VSLConvTaskPtr tasks[M];
 /* assume that M is big enough */
 . . .
 vslConvSetStart(*task, &iy0);
 . . .
 for (i=0; i<m; i++)
 /* implicit commitment at i==0 */
 vslConvCopyTask(&tasks[i],*task);
 . . .
}
```

Then,  $m$  threads may be started to execute different copies of the task:

```
. . .
 float* xi = &x[inc2x * i];
 float* yi = &y[inc2y * i];
 ss[i]=vslsConvExecX1D(tasks[i], xi,inc1x, yi,inc1y);
 . . .
}
```

And finally, after all threads have finished the calculations, overall status should be collected from all task objects. The following code signals the first error found, if any:

```
. . .
for (i=0; i<m; i++) {
 status = ss[i];
 if (status != 0) /* 0 means "OK" */
 break;
};
return status;
}; /* end if init==0 */
```

Execution routines modify the task internal state (fields of the task structure). Such modifications may conflict with each other if different threads work with the same task object simultaneously. That is why different threads must use different copies of the task.

#### Optimization Notice

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|                            |
|----------------------------|
| <b>Optimization Notice</b> |
| Notice revision #20110804  |

### Convolution and Correlation Mathematical Notation and Definitions

The following notation is necessary to explain the underlying mathematical definitions used in the text:

|                                                            |                                                                                |
|------------------------------------------------------------|--------------------------------------------------------------------------------|
| $\mathbf{R} = (-\infty, +\infty)$                          | The set of real numbers.                                                       |
| $\mathbf{Z} = \{0, \pm 1, \pm 2, \dots\}$                  | The set of integer numbers.                                                    |
| $\mathbf{Z}^N = \mathbf{Z} \times \dots \times \mathbf{Z}$ | The set of N-dimensional series of integer numbers.                            |
| $p = (p_1, \dots, p_N) \in \mathbf{Z}^N$                   | N-dimensional series of integers.                                              |
| $u: \mathbf{Z}^N \rightarrow \mathbf{R}$                   | Function $u$ with arguments from $\mathbf{Z}^N$ and values from $\mathbf{R}$ . |
| $u(p) = u(p_1, \dots, p_N)$                                | The value of the function $u$ for the argument $(p_1, \dots, p_N)$ .           |
| $w = u * v$                                                | Function $w$ is the convolution of the functions $u, v$ .                      |
| $w = u \bullet v$                                          | Function $w$ is the correlation of the functions $u, v$ .                      |

Given series  $p, q \in \mathbf{Z}^N$ :

- series  $r = p + q$  is defined as  $r^n = p^n + q^n$  for every  $n=1, \dots, N$
- series  $r = p - q$  is defined as  $r^n = p^n - q^n$  for every  $n=1, \dots, N$
- series  $r = \sup\{p, q\}$  is defines as  $r^n = \max\{p^n, q^n\}$  for every  $n=1, \dots, N$
- series  $r = \inf\{p, q\}$  is defined as  $r^n = \min\{p^n, q^n\}$  for every  $n=1, \dots, N$
- inequality  $p \leq q$  means that  $p^n \leq q^n$  for every  $n=1, \dots, N$ .

A function  $u(p)$  is called a finite function if there exist series  $p^{\min}, p^{\max} \in \mathbf{Z}^N$  such that:

$$u(p) \neq 0$$

implies

$$p^{\min} \leq p \leq p^{\max}.$$

Operations of convolution and correlation are only defined for finite functions.

Consider functions  $u, v$  and series  $p^{\min}, p^{\max}, q^{\min}, q^{\max} \in \mathbf{Z}^N$  such that:

$$u(p) \neq 0 \text{ implies } p^{\min} \leq p \leq p^{\max}.$$

$$v(q) \neq 0 \text{ implies } q^{\min} \leq q \leq q^{\max}.$$

Definitions of linear correlation and linear convolution for functions  $u$  and  $v$  are given below.

#### Linear Convolution

If function  $w = u * v$  is the convolution of  $u$  and  $v$ , then:

$$w(r) \neq 0 \text{ implies } \mathbf{R}^{\min} \leq r \leq \mathbf{R}^{\max},$$

$$\text{where } \mathbf{R}^{\min} = p^{\min} + q^{\min} \text{ and } \mathbf{R}^{\max} = p^{\max} + q^{\max}.$$

If  $\mathbf{R}^{\min} \leq r \leq \mathbf{R}^{\max}$ , then:

$$w(r) = \sum u(t) \cdot v(r-t) \text{ is the sum for all } t \in \mathbf{Z}^N \text{ such that } \mathbf{T}^{\min} \leq t \leq \mathbf{T}^{\max},$$

$$\text{where } \mathbf{T}^{\min} = \sup\{p^{\min}, r - q^{\max}\} \text{ and } \mathbf{T}^{\max} = \inf\{p^{\max}, r - q^{\min}\}.$$

#### Linear Correlation

If function  $w = u \bullet v$  is the correlation of  $u$  and  $v$ , then:

$$w(r) \neq 0 \text{ implies } \mathbf{R}^{\min} \leq r \leq \mathbf{R}^{\max},$$

$$\text{where } \mathbf{R}^{\min} = q^{\min} - p^{\max} \text{ and } \mathbf{R}^{\max} = q^{\max} - p^{\min}.$$

If  $\mathbf{R}^{\min} \leq r \leq \mathbf{R}^{\max}$ , then:

$w(r) = \sum u(t) \cdot v(r+t)$  is the sum for all  $t \in \mathbf{Z}^N$  such that  $\mathbf{T}^{\min} \leq t \leq \mathbf{T}^{\max}$ , where  $\mathbf{T}^{\min} = \sup\{\mathbf{P}^{\min}, \mathbf{Q}^{\min}-r\}$  and  $\mathbf{T}^{\max} = \inf\{\mathbf{P}^{\max}, \mathbf{Q}^{\max}-r\}$ .

Representation of the functions  $u$ ,  $v$ ,  $w$  as the input/output data for the Intel MKL convolution and correlation functions is described in the [Data Allocation](#) section below.

## Convolution and Correlation Data Allocation

This section explains the relation between:

- mathematical finite functions  $u$ ,  $v$ ,  $w$  introduced in the section [Mathematical Notation and Definitions](#);
- multi-dimensional input and output data vectors representing the functions  $u$ ,  $v$ ,  $w$ ;
- arrays  $u$ ,  $v$ ,  $w$  used to store the input and output data vectors in computer memory

The convolution and correlation routine parameters that determine the allocation of input and output data are the following:

- Data arrays  $x$ ,  $y$ ,  $z$
- Shape arrays  $xshape$ ,  $yshape$ ,  $zshape$
- Strides within arrays  $xstride$ ,  $ystride$ ,  $zstride$
- Parameters  $start$ ,  $decimation$

### Finite Functions and Data Vectors

The finite functions  $u(p)$ ,  $v(q)$ , and  $w(r)$  introduced above are represented as multi-dimensional vectors of input and output data:

`inputu(i1, ..., idims)` for  $u(p_1, \dots, p_N)$

`inputv(j1, ..., jdims)` for  $v(q_1, \dots, q_N)$

`output(k1, ..., kdims)` for  $w(r_1, \dots, r_N)$ .

Parameter *dims* represents the number of dimensions and is equal to  $N$ .

The parameters *xshape*, *yshape*, and *zshape* define the shapes of input/output vectors:

`inputu(i1, ..., idims)` is defined if  $1 \leq i_n \leq xshape(n)$  for every  $n=1, \dots, dims$

`inputv(j1, ..., jdims)` is defined if  $1 \leq j_n \leq yshape(n)$  for every  $n=1, \dots, dims$

`output(k1, ..., kdims)` is defined if  $1 \leq k_n \leq zshape(n)$  for every  $n=1, \dots, dims$ .

Relation between the input vectors and the functions  $u$  and  $v$  is defined by the following formulas:

`inputu(i1, ..., idims)` =  $u(p_1, \dots, p_N)$ , where  $p_n = P_n^{\min} + (i_n - 1)$  for every  $n$

`inputv(j1, ..., jdims)` =  $v(q_1, \dots, q_N)$ , where  $q_n = Q_n^{\min} + (j_n - 1)$  for every  $n$ .

The relation between the output vector and the function  $w(r)$  is similar (but only in the case when parameters *start* and *decimation* are not defined):

`output(k1, ..., kdims)` =  $w(r_1, \dots, r_N)$ , where  $r_n = R_n^{\min} + (k_n - 1)$  for every  $n$ .

If the parameter *start* is defined, it must belong to the interval  $R_n^{\min} \leq start(n) \leq R_n^{\max}$ . If defined, the *start* parameter replaces  $R_n^{\min}$  in the formula:

`output(k1, ..., kdims)` =  $w(r_1, \dots, r_N)$ , where  $r_n = start(n) + (k_n - 1)$

If the parameter *decimation* is defined, it changes the relation according to the following formula:

`output(k1, ..., kdims)` =  $w(r_1, \dots, r_N)$ , where  $r_n = R_n^{\min} + (k_n - 1) * decimation(n)$

If both parameters *start* and *decimation* are defined, the formula is as follows:

$\text{output}(k_1, \dots, k_{\text{dims}}) = w(r_1, \dots, r_N)$ , where  $r_n = \text{start}(n) + (k_n - 1) \cdot \text{decimation}(n)$

The convolution and correlation software checks the values of *zshape*, *start*, and *decimation* during task commitment. If  $r_n$  exceeds  $R_n^{\text{max}}$  for some  $k_n, n=1, \dots, \text{dims}$ , an error is raised.

### Allocation of Data Vectors

Both parameter arrays *x* and *y* contain input data vectors in memory, while array *z* is intended for storing output data vector. To access the memory, the convolution and correlation software uses only pointers to these arrays and ignores the array shapes.

For parameters *x*, *y*, and *z*, you can provide one-dimensional arrays with the requirement that actual length of these arrays be sufficient to store the data vectors.

The allocation of the input and output data inside the arrays *x*, *y*, and *z* is described below assuming that the arrays are one-dimensional. Given multi-dimensional indices  $i, j, k \in \mathbf{Z}^N$ , one-dimensional indices  $e, f, g \in \mathbf{Z}$  are defined such that:

$\text{inputu}(i_1, \dots, i_{\text{dims}})$  is allocated at  $x(e)$   
 $\text{inputv}(j_1, \dots, j_{\text{dims}})$  is allocated at  $y(f)$   
 $\text{output}(k_1, \dots, k_{\text{dims}})$  is allocated at  $z(g)$ .

The indices *e*, *f*, and *g* are defined as follows:

$e = 1 + \sum x\text{stride}(n) \cdot dx(n)$  (the sum is for all  $n=1, \dots, \text{dims}$ )  
 $f = 1 + \sum y\text{stride}(n) \cdot dy(n)$  (the sum is for all  $n=1, \dots, \text{dims}$ )  
 $g = 1 + \sum z\text{stride}(n) \cdot dz(n)$  (the sum is for all  $n=1, \dots, \text{dims}$ )

The distances  $dx(n)$ ,  $dy(n)$ , and  $dz(n)$  depend on the signum of the stride:

$dx(n) = i_n - 1$  if  $x\text{stride}(n) > 0$ , or  $dx(n) = i_n - x\text{shape}(n)$  if  $x\text{stride}(n) < 0$   
 $dy(n) = j_n - 1$  if  $y\text{stride}(n) > 0$ , or  $dy(n) = j_n - y\text{shape}(n)$  if  $y\text{stride}(n) < 0$   
 $dz(n) = k_n - 1$  if  $z\text{stride}(n) > 0$ , or  $dz(n) = k_n - z\text{shape}(n)$  if  $z\text{stride}(n) < 0$

The definitions of indices *e*, *f*, and *g* assume that indexes for arrays *x*, *y*, and *z* are started from unity:

$x(e)$  is defined for  $e=1, \dots, \text{length}(x)$   
 $y(f)$  is defined for  $f=1, \dots, \text{length}(y)$   
 $z(g)$  is defined for  $g=1, \dots, \text{length}(z)$

Below is a detailed explanation about how elements of the multi-dimensional output vector are stored in the array *z* for one-dimensional and two-dimensional cases.

**One-dimensional case.** If  $\text{dims}=1$ , then *zshape* is the number of the output values to be stored in the array *z*. The actual length of array *z* may be greater than *zshape* elements.

If  $z\text{stride} > 1$ , output values are stored with the stride:  $\text{output}(1)$  is stored to  $z(1)$ ,  $\text{output}(2)$  is stored to  $z(1+z\text{stride})$ , and so on. Hence, the actual length of *z* must be at least  $1+z\text{stride} \cdot (z\text{shape}-1)$  elements or more.

If  $z\text{stride} < 0$ , it still defines the stride between elements of array *z*. However, the order of the used elements is the opposite. For the *k*-th output value,  $\text{output}(k)$  is stored in  $z(1+|z\text{stride}| \cdot (z\text{shape}-k))$ , where  $|z\text{stride}|$  is the absolute value of *zstride*. The actual length of the array *z* must be at least  $1+|z\text{stride}| \cdot (z\text{shape} - 1)$  elements.

**Two-dimensional case.** If  $\text{dims}=2$ , the output data is a two-dimensional matrix. The value  $z\text{stride}(1)$  defines the stride inside matrix columns, that is, the stride between the  $\text{output}(k_1, k_2)$  and  $\text{output}(k_1+1, k_2)$  for every pair of indices  $k_1, k_2$ . On the other hand,  $z\text{stride}(2)$  defines the stride between columns, that is, the stride between  $\text{output}(k_1, k_2)$  and  $\text{output}(k_1, k_2+1)$ .

If `zstride(2)` is greater than `zshape(1)`, this causes sparse allocation of columns. If the value of `zstride(2)` is smaller than `zshape(1)`, this may result in the transposition of the output matrix. For example, if `zshape = (2,3)`, you can define `zstride = (3,1)` to allocate output values like transposed matrix of the shape `3x2`.

Whether `zstride` assumes this kind of transformations or not, you need to ensure that different elements output ( $k_1, \dots, k_{\text{dims}}$ ) will be stored in different locations  $z(g)$ .

## Summary Statistics

The Summary Statistics domain provides routines that compute basic statistical estimates for single and double precision multi-dimensional datasets.

The Summary Statistics routines calculate:

- raw and central moments up to the fourth order
- skewness and excess kurtosis (further referred to as *kurtosis* for brevity)
- variation coefficient
- quantiles and order statistics
- minimum and maximum
- variance-covariance/correlation matrix
- pooled/group variance-covariance matrix and mean
- partial variance-covariance/correlation matrix
- robust estimators for variance-covariance matrix and mean in presence of outliers
- raw/central partial sums up to the fourth order (for brevity referred to as *raw/central sums*)
- matrix of cross-products and sums of squares (for brevity referred to as *cross-product matrix*)
- median absolute deviation, mean absolute deviation

The library also contains functions to perform the following tasks:

- Detect outliers in datasets
- Support missing values in datasets
- Parameterize correlation matrices
- Compute quantiles for streaming data

The [Mathematical Notation and Definitions](#) section defines the supported operations in the Summary Statistics routines.

You can access the Summary Statistics routines through the Fortran 90 and C89 language interfaces. You can use the C89 interface with later versions of the C/C++.

The `mk1_vs1.h` header file is in the `/${MKL}/include` directory.

You can find examples that demonstrate calculation of the Summary Statistics estimates in the `/${MKL}/examples/vslc` example directory.

The Summary Statistics API is implemented through task objects, or tasks. A task object is a data structure, or a descriptor, holding parameters that determine a specific Summary Statistics operation. For example, such parameters may be precision, dimensions of user data, the matrix of the observations, or shapes of data arrays.

All the Summary Statistics routines process a task object as follows:

1. Create a task.
2. Modify settings of the task parameters.
3. Compute statistical estimates.
4. Destroy the task.

The Summary Statistics functions fall into the following categories:

**Task Constructors** - routines that create a new task object descriptor and set up most common parameters (dimension, number of observations, and matrix of the observations).

**Task Editors** - routines that can set or modify some parameter settings in the existing task descriptor.

**Task Computation Routine** - a routine that computes specified statistical estimates.

**Task Destructor** - a routine that deletes the task object and frees the memory.

A Summary Statistics task object contains a series of pointers to the input and output data arrays. You can read and modify the datasets and estimates at any time but you should allocate and release memory for such data.

See detailed information on the algorithms, API, and their usage in the *Intel® MKL Summary Statistics Application Notes [SS Notes]*.

## Summary Statistics Naming Conventions

The names of Summary Statistics routines, types, and constants are case-sensitive and can contain lowercase and uppercase characters (`vslsSSEditQuantiles`).

The names of routines have the following structure:

```
vsl[datatype]SS<base name>
```

where

- `vsl` is a prefix indicating that the routine belongs to Intel MKL Vector Statistics.
- `[datatype]` specifies the type of the input and/or output data and can be `s` (single precision real type), `d` (double precision real type), or `i` (integer type).
- `SS/ss` indicates that the routine is intended for calculations of the Summary Statistics estimates.
- `<base name>` specifies a particular functionality that the routine is designed for, for example, `NewTask`, `Compute`, `DeleteTask`.

---

### NOTE

The Summary Statistics routine `vslDeleteTask` for deletion of the task is independent of the data type and its name omits the `[datatype]` field.

---

## Summary Statistics Data Types

The Summary Statistics routines use the following data types for calculations:

| Type                                           | Data Object                                |
|------------------------------------------------|--------------------------------------------|
| <code>VSLSSSTaskPtr</code>                     | Pointer to a Summary Statistics task       |
| <code>float</code>                             | Input/output user data in single precision |
| <code>double</code>                            | Input/output user data in double precision |
| <code>MKL_INT</code> or <code>long long</code> | Other data                                 |

---

### NOTE

The actual size of the generic integer type is platform-specific and can be 32 or 64 bits in length. Before you compile your application, set an appropriate size for integers. See details in the 'Using the ILP64 Interface vs. LP64 Interface' section of the *Intel® MKL Developer Guide*.

---

## Summary Statistics Parameters

The basic parameters in the task descriptor (addresses of dimensions, number of observations, and datasets) are assigned values when the task editors create or modify the task object. Other parameters are determined by the specific task and changed by the task editors.

## Summary Statistics Task Status and Error Reporting

The task status is an integer value, which is zero if no error is detected, or a specific non-zero error code otherwise. Negative status values indicate errors, and positive values indicate warnings. An error can be caused by invalid parameter values or a memory allocation failure.

The header files define symbolic names for the status codes. These names are defined as macros via `#define` statements.

The header files define the following status codes for the Summary Statistics error codes:

### Summary Statistics Status Codes

| Status Code                        | Description                                                     |
|------------------------------------|-----------------------------------------------------------------|
| VSL_STATUS_OK                      | Operation is successfully completed.                            |
| VSL_SS_ERROR_ALLOCATION_FAILURE    | Memory allocation has failed.                                   |
| VSL_SS_ERROR_BAD_DIMEN             | Dimension value is invalid.                                     |
| VSL_SS_ERROR_BAD_OBSERV_N          | Invalid number (zero or negative) of observations was obtained. |
| VSL_SS_ERROR_STORAGE_NOT_SUPPORTED | Storage format is not supported.                                |
| VSL_SS_ERROR_BAD_INDC_ADDR         | Array of indices is not defined.                                |
| VSL_SS_ERROR_BAD_WEIGHTS           | Array of weights contains negative values.                      |
| VSL_SS_ERROR_BAD_MEAN_ADDR         | Array of means is not defined.                                  |
| VSL_SS_ERROR_BAD_2R_MOM_ADDR       | Array of the second order raw moments is not defined.           |
| VSL_SS_ERROR_BAD_3R_MOM_ADDR       | Array of the third order raw moments is not defined.            |
| VSL_SS_ERROR_BAD_4R_MOM_ADDR       | Array of the fourth order raw moments is not defined.           |
| VSL_SS_ERROR_BAD_2C_MOM_ADDR       | Array of the second order central moments is not defined.       |
| VSL_SS_ERROR_BAD_3C_MOM_ADDR       | Array of the third order central moments is not defined.        |
| VSL_SS_ERROR_BAD_4C_MOM_ADDR       | Array of the fourth order central moments is not defined.       |
| VSL_SS_ERROR_BAD_KURTOSIS_ADDR     | Array of kurtosis values is not defined.                        |
| VSL_SS_ERROR_BAD_SKEWNESS_ADDR     | Array of skewness values is not defined.                        |
| VSL_SS_ERROR_BAD_MIN_ADDR          | Array of minimum values is not defined.                         |
| VSL_SS_ERROR_BAD_MAX_ADDR          | Array of maximum values is not defined.                         |

| Status Code                               | Description                                                                                |
|-------------------------------------------|--------------------------------------------------------------------------------------------|
| VSL_SS_ERROR_BAD_VARIATION_ADDR           | Array of variation coefficients is not defined.                                            |
| VSL_SS_ERROR_BAD_COV_ADDR                 | Covariance matrix is not defined.                                                          |
| VSL_SS_ERROR_BAD_COR_ADDR                 | Correlation matrix is not defined.                                                         |
| VSL_SS_ERROR_BAD_QUANT_ORDER_ADDR         | Array of quantile orders is not defined.                                                   |
| VSL_SS_ERROR_BAD_QUANT_ORDER              | Quantile order value is invalid.                                                           |
| VSL_SS_ERROR_BAD_QUANT_ADDR               | Array of quantiles is not defined.                                                         |
| VSL_SS_ERROR_BAD_ORDER_STATS_ADDR         | Array of order statistics is not defined.                                                  |
| VSL_SS_ERROR_MOMORDER_NOT_SUPPORTED       | Moment of requested order is not supported.                                                |
| VSL_SS_NOT_FULL_RANK_MATRIX               | Correlation matrix is not of full rank.                                                    |
| VSL_SS_ERROR_ALL_OBSERVS_OUTLIERS         | All observations are outliers. (At least one observation must not be an outlier.)          |
| VSL_SS_ERROR_BAD_ROBUST_COV_ADDR          | Robust covariance matrix is not defined.                                                   |
| VSL_SS_ERROR_BAD_ROBUST_MEAN_ADDR         | Array of robust means is not defined.                                                      |
| VSL_SS_ERROR_METHOD_NOT_SUPPORTED         | Requested method is not supported.                                                         |
| VSL_SS_ERROR_NULL_TASK_DESCRIPTOR         | Task descriptor is null.                                                                   |
| VSL_SS_ERROR_BAD_OBSERV_ADDR              | Dataset matrix is not defined.                                                             |
| VSL_SS_ERROR_BAD_ACCUM_WEIGHT_ADDR        | Pointer to the variable that holds the value of accumulated weight is not defined.         |
| VSL_SS_ERROR_SINGULAR_COV                 | Covariance matrix is singular.                                                             |
| VSL_SS_ERROR_BAD_POOLED_COV_ADDR          | Pooled covariance matrix is not defined.                                                   |
| VSL_SS_ERROR_BAD_POOLED_MEAN_ADDR         | Array of pooled means is not defined.                                                      |
| VSL_SS_ERROR_BAD_GROUP_COV_ADDR           | Group covariance matrix is not defined.                                                    |
| VSL_SS_ERROR_BAD_GROUP_MEAN_ADDR          | Array of group means is not defined.                                                       |
| VSL_SS_ERROR_BAD_GROUP_INDC_ADDR          | Array of group indices is not defined.                                                     |
| VSL_SS_ERROR_BAD_GROUP_INDC               | Group indices have improper values.                                                        |
| VSL_SS_ERROR_BAD_OUTLIERS_PARAMS_ADDR     | Array of parameters for the outlier detection algorithm is not defined.                    |
| VSL_SS_ERROR_BAD_OUTLIERS_PARAMS_N_ADDR   | Pointer to size of the parameter array for the outlier detection algorithm is not defined. |
| VSL_SS_ERROR_BAD_OUTLIERS_WEIGHTS_ADDR    | Output of the outlier detection algorithm is not defined.                                  |
| VSL_SS_ERROR_BAD_ROBUST_COV_PARAMS_ADDR   | Array of parameters of the robust covariance estimation algorithm is not defined.          |
| VSL_SS_ERROR_BAD_ROBUST_COV_PARAMS_N_ADDR | Pointer to the number of parameters of the algorithm for robust covariance is not defined. |

| Status Code                               | Description                                                                                                    |
|-------------------------------------------|----------------------------------------------------------------------------------------------------------------|
| VSL_SS_ERROR_BAD_STORAGE_ADDR             | Pointer to the variable that holds the storage format is not defined.                                          |
| VSL_SS_ERROR_BAD_PARTIAL_COV_IDX_ADDR     | Array that encodes sub-components of a random vector for the partial covariance algorithm is not defined.      |
| VSL_SS_ERROR_BAD_PARTIAL_COV_IDX          | Array that encodes sub-components of a random vector for partial covariance has improper values.               |
| VSL_SS_ERROR_BAD_PARTIAL_COV_ADDR         | Partial covariance matrix is not defined.                                                                      |
| VSL_SS_ERROR_BAD_PARTIAL_COR_ADDR         | Partial correlation matrix is not defined.                                                                     |
| VSL_SS_ERROR_BAD_MI_PARAMS_ADDR           | Array of parameters for the Multiple Imputation method is not defined.                                         |
| VSL_SS_ERROR_BAD_MI_PARAMS_N_ADDR         | Pointer to number of parameters for the Multiple Imputation method is not defined.                             |
| VSL_SS_ERROR_BAD_MI_BAD_PARAMS_N          | Size of the parameter array of the Multiple Imputation method is invalid.                                      |
| VSL_SS_ERROR_BAD_MI_PARAMS                | Parameters of the Multiple Imputation method are invalid.                                                      |
| VSL_SS_ERROR_BAD_MI_INIT_ESTIMATES_N_ADDR | Pointer to the number of initial estimates in the Multiple Imputation method is not defined.                   |
| VSL_SS_ERROR_BAD_MI_INIT_ESTIMATES_ADDR   | Array of initial estimates for the Multiple Imputation method is not defined.                                  |
| VSL_SS_ERROR_BAD_MI_SIMUL_VALS_ADDR       | Array of simulated missing values in the Multiple Imputation method is not defined.                            |
| VSL_SS_ERROR_BAD_MI_SIMUL_VALS_N_ADDR     | Pointer to the size of the array of simulated missing values in the Multiple Imputation method is not defined. |
| VSL_SS_ERROR_BAD_MI_ESTIMATES_N_ADDR      | Pointer to the number of parameter estimates in the Multiple Imputation method is not defined.                 |
| VSL_SS_ERROR_BAD_MI_ESTIMATES_ADDR        | Array of parameter estimates in the Multiple Imputation method is not defined.                                 |
| VSL_SS_ERROR_BAD_MI_SIMUL_VALS_N          | Invalid size of the array of simulated values in the Multiple Imputation method.                               |
| VSL_SS_ERROR_BAD_MI_ESTIMATES_N           | Invalid size of an array to hold parameter estimates obtained using the Multiple Imputation method.            |
| VSL_SS_ERROR_BAD_MI_OUTPUT_PARAMS         | Array of output parameters in the Multiple Imputation method is not defined.                                   |
| VSL_SS_ERROR_BAD_MI_PRIOR_N_ADDR          | Pointer to the number of prior parameters is not defined.                                                      |
| VSL_SS_ERROR_BAD_MI_PRIOR_ADDR            | Array of prior parameters is not defined.                                                                      |
| VSL_SS_ERROR_BAD_MI_MISSING_VALS_N        | Invalid number of missing values was obtained.                                                                 |

| <b>Status Code</b>                          | <b>Description</b>                                                                                            |
|---------------------------------------------|---------------------------------------------------------------------------------------------------------------|
| VSL_SS_SEMIDEFINITE_COR                     | Correlation matrix passed into the parameterization function is semi-definite.                                |
| VSL_SS_ERROR_BAD_PARAMTR_COR_ADDR           | Correlation matrix to be parameterized is not defined.                                                        |
| VSL_SS_ERROR_BAD_COR                        | All eigenvalues of the correlation matrix to be parameterized are non-positive.                               |
| VSL_SS_ERROR_BAD_STREAM_QUANT_PARAMS_N_ADDR | Pointer to the number of parameters for the quantile computation algorithm for streaming data is not defined. |
| VSL_SS_ERROR_BAD_STREAM_QUANT_PARAMS_ADDR   | Array of parameters of the quantile computation algorithm for streaming data is not defined.                  |
| VSL_SS_ERROR_BAD_STREAM_QUANT_PARAMS_N      | Invalid number of parameters of the quantile computation algorithm for streaming data has been obtained.      |
| VSL_SS_ERROR_BAD_STREAM_QUANT_PARAMS        | Invalid parameters of the quantile computation algorithm for streaming data have been passed.                 |
| VSL_SS_ERROR_BAD_STREAM_QUANT_ORDER_ADDR    | Array of the quantile orders for streaming data is not defined.                                               |
| VSL_SS_ERROR_BAD_STREAM_QUANT_ORDER         | Invalid quantile order for streaming data is defined.                                                         |
| VSL_SS_ERROR_BAD_STREAM_QUANT_ADDR          | Array of quantiles for streaming data is not defined.                                                         |
| VSL_SS_ERROR_BAD_SUM_ADDR                   | Array of sums is not defined.                                                                                 |
| VSL_SS_ERROR_BAD_2R_SUM_ADDR                | Array of raw sums of 2nd order is not defined.                                                                |
| VSL_SS_ERROR_BAD_3R_SUM_ADDR                | Array of raw sums of 3rd order is not defined.                                                                |
| VSL_SS_ERROR_BAD_4R_SUM_ADDR                | Array of raw sums of 4th order is not defined.                                                                |
| VSL_SS_ERROR_BAD_2C_SUM_ADDR                | Array of central sums of 2nd order is not defined.                                                            |
| VSL_SS_ERROR_BAD_3C_SUM_ADDR                | Array of central sums of 3rd order is not defined.                                                            |
| VSL_SS_ERROR_BAD_4C_SUM_ADDR                | Array of central sums of 4th order is not defined.                                                            |
| VSL_SS_ERROR_BAD_CP_SUM_ADDR                | Cross-product matrix is not defined.                                                                          |
| VSL_SS_ERROR_BAD_MDAD_ADDR                  | Array of median absolute deviations is not defined.                                                           |
| VSL_SS_ERROR_BAD_MNAD_ADDR                  | Array of mean absolute deviations is not defined.                                                             |
| VSL_SS_ERROR_BAD_SORTED_OBSERV_ADDR         | Array for storing observation sorting results is not defined.                                                 |
| VSL_SS_ERROR_ERROR_INDICES_NOT_SUPPORTED    | Array of indices is not supported.                                                                            |

Routines for robust covariance estimation, outlier detection, partial covariance estimation, multiple imputation, and parameterization of a correlation matrix can return internal error codes that are related to a specific implementation. Such error codes indicate invalid input data or other bugs in the Intel MKL routines other than the Summary Statistics routines.

## Summary Statistics Task Constructors

Task constructors are routines intended for creating a new task descriptor and setting up basic parameters.

---

### NOTE

If the constructor fails to create a task descriptor, it returns the `NULL` task pointer.

---

### vs1sSSNewTask

*Creates and initializes a new summary statistics task descriptor.*

---

#### Syntax

```
status = vs1sSSNewTask(&task, p, n, xstorage, x, w, indices);
```

```
status = vs1dSSNewTask(&task, p, n, xstorage, x, w, indices);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| Name                  | Type                                                                                                                  | Description                                                                                                                                                                          |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>p</code>        | <code>const MKL_INT*</code>                                                                                           | Dimension of the task, number of variables                                                                                                                                           |
| <code>n</code>        | <code>const MKL_INT*</code>                                                                                           | Number of observations                                                                                                                                                               |
| <code>xstorage</code> | <code>const MKL_INT*</code>                                                                                           | Storage format of matrix of observations                                                                                                                                             |
| <code>x</code>        | <code>const float*</code> for <code>vs1sSSNewTask</code><br><code>const double*</code> for <code>vs1dSSNewTask</code> | Matrix of observations                                                                                                                                                               |
| <code>w</code>        | <code>const float*</code> for <code>vs1sSSNewTask</code><br><code>const double*</code> for <code>vs1dSSNewTask</code> | Array of weights of size <code>n</code> . Elements of the arrays are non-negative numbers. If a <code>NULL</code> pointer is passed, each observation is assigned weight equal to 1. |
| <code>indices</code>  | <code>const MKL_INT*</code>                                                                                           | Array of vector components that will be processed. Size of array is <code>p</code> . If a <code>NULL</code> pointer is passed, all components of random vector are processed.        |

#### Output Parameters

| Name              | Type                       | Description            |
|-------------------|----------------------------|------------------------|
| <code>task</code> | <code>VSLSSTaskPtr*</code> | Descriptor of the task |

| Name                | Type             | Description                                                                                                         |
|---------------------|------------------|---------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code> | Set to <code>VSL_STATUS_OK</code> if the task is created successfully, otherwise a non-zero error code is returned. |

## Description

Each `vslSSNewTask` constructor routine creates a new summary statistics task descriptor with the user-specified value for a required parameter, dimension of the task. The optional parameters (matrix of observations, its storage format, number of observations, weights of observations, and indices of the random vector components) are set to their default values.

The observations of random  $p$ -dimensional vector  $\xi = (\xi_1, \dots, \xi_j, \dots, \xi_p)$ , which are  $n$  vectors of dimension  $p$ , are passed as a one-dimensional array  $x$ . The parameter `xstorage` defines the storage format of the observations and takes one of the possible values listed in [Table "Storage format of matrix of observations and order statistics"](#).

### Storage format of matrix of observations, order statistics, and matrix of sorted observations

| Parameter                               | Description                                                                                                                                                                                                              |
|-----------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>VSL_SS_MATRIX_STORAGE_ROWS</code> | The observations of random vector $\xi$ are packed by rows: $n$ data points for the vector component $\xi_1$ come first, $n$ data points for the vector component $\xi_2$ come second, and so forth.                     |
| <code>VSL_SS_MATRIX_STORAGE_COLS</code> | The observations of random vector $\xi$ are packed by columns: the first $p$ -dimensional observation of the vector $\xi$ comes first, the second $p$ -dimensional observation of the vector comes second, and so forth. |

A one-dimensional array  $w$  of size  $n$  contains non-negative weights assigned to the observations. You can pass a `NULL` array into the constructor. In this case, each observation is assigned the default value of the weight.

You can choose vector components for which you wish to compute statistical estimates. If an element of the vector indices of size  $p$  contains 0, the observations that correspond to this component are excluded from the calculations. If you pass the `NULL` value of the parameter into the constructor, statistical estimates for all random variables are computed.

If the constructor fails to create a task descriptor, it returns the `NULL` task pointer.

## Summary Statistics Task Editors

Task editors are intended to set up or change the task parameters listed in [Table "Parameters of Summary Statistics Task to Be Initialized or Modified"](#). As an example, to compute the sample mean for a one-dimensional dataset, initialize a variable for the mean value, and pass its address into the task as shown in the example below:

```
#define DIM 1
#define N 1000

int main()
{
 VSLSSTaskPtr task;
 double x[N];
 double mean;
 MKL_INT p, n, xstorage;
 int status;
```

```

/* initialize variables used in the computations of sample mean */
p = DIM;
n = N;
xstorage = VSL_SS_MATRIX_STORAGE_ROWS;
mean = 0.0;

/* create task */
status = vsldSSNewTask(&task, &p, &n, &xstorage, x, 0, 0);

/* initialize task parameters */
status = vsldSSEditTask(task, VSL_SS_ED_MEAN, &mean);

/* compute mean using SS fast method */
status = vsldSSCompute(task, VSL_SS_MEAN, VSL_SS_METHOD_FAST);

/* deallocate task resources */
status = vsldSSDeleteTask(&task);

return 0;
}

```

Use the single (`vsldssedittask`) or double (`vsldssedittask`) version of an editor, to initialize single or double precision version task parameters, respectively. Use an integer version of an editor (`vsldssedittask`) to initialize parameters of the integer type.

Table "Summary Statistics Task Editors" lists the task editors for Summary Statistics. Each of them initializes and/or modifies a respective group of related parameters.

### Summary Statistics Task Editors

| Editor                                   | Description                                                                                      |
|------------------------------------------|--------------------------------------------------------------------------------------------------|
| <code>vsldSSEditTask</code>              | Changes a pointer in the task descriptor.                                                        |
| <code>vsldSSEditMoments</code>           | Changes pointers to arrays associated with raw and central moments.                              |
| <code>vsldSSEditSums</code>              | Modifies the pointers to arrays that hold sum estimates.                                         |
| <code>vsldSSEditCovCor</code>            | Changes pointers to arrays associated with covariance and/or correlation matrices.               |
| <code>vsldSSEditCP</code>                | Modifies the pointers to cross-product matrix parameters.                                        |
| <code>vsldSSEditPartialCovCor</code>     | Changes pointers to arrays associated with partial covariance and/or correlation matrices.       |
| <code>vsldSSEditQuantiles</code>         | Changes pointers to arrays associated with quantile/order statistics calculations.               |
| <code>vsldSSEditStreamQuantiles</code>   | Changes pointers to arrays for quantile related calculations for streaming data.                 |
| <code>vsldSSEditPooledCovariance</code>  | Changes pointers to arrays associated with algorithms related to a pooled covariance matrix.     |
| <code>vsldSSEditRobustCovariance</code>  | Changes pointers to arrays for robust estimation of a covariance matrix and mean.                |
| <code>vsldSSEditOutliersDetection</code> | Changes pointers to arrays for detection of outliers.                                            |
| <code>vsldSSEditMissingValues</code>     | Changes pointers to arrays associated with the method of supporting missing values in a dataset. |

| Editor                                       | Description                                                                                            |
|----------------------------------------------|--------------------------------------------------------------------------------------------------------|
| <a href="#">vslSSEditCorParameterization</a> | Changes pointers to arrays associated with the algorithm for parameterization of a correlation matrix. |

**NOTE**

You can use the `NULL` task pointer in calls to editor routines. In this case, the routine is terminated and no system crash occurs.

**vslSSEditTask**

Modifies address of an input/output parameter in the task descriptor.

**Syntax**

```
status = vslsSSEditTask(task, parameter, par_addr);
```

```
status = vsldSSEditTask(task, parameter, par_addr);
```

```
status = vsliSSEditTask(task, parameter, par_addr);
```

**Include Files**

- `mkl.h`

**Input Parameters**

| Name                   | Type                                                                                                                                                                                   | Description                  |
|------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------|
| <code>task</code>      | <code>VSLSSTaskPtr</code>                                                                                                                                                              | Descriptor of the task       |
| <code>parameter</code> | <code>const MKL_INT</code>                                                                                                                                                             | Parameter to change          |
| <code>par_addr</code>  | <code>const float*</code> for <code>vslsSSEditTask</code><br><code>const double*</code> for <code>vsldSSEditTask</code><br><code>const MKL_INT*</code> for <code>vsliSSEditTask</code> | Address of the new parameter |

**Output Parameters**

| Name                | Type             | Description                |
|---------------------|------------------|----------------------------|
| <code>status</code> | <code>int</code> | Current status of the task |

**Description**

The `vslSSEditTask` routine replaces the pointer to the parameter stored in the Summary Statistics task descriptor with the `par_addr` pointer. If you pass the `NULL` pointer to the editor, no changes take place in the task and a corresponding error code is returned. See [Table "Parameters of Summary Statistics Task to Be Initialized or Modified"](#) for the predefined values of the parameter.

Use the single (`vslsssedittask`) or double (`vsldsssedittask`) version of the editor, to initialize single or double precision version task parameters, respectively. Use an integer version of the editor (`vsliSSEditTask`) to initialize parameters of the integer type.

**Parameters of Summary Statistics Task to Be Initialized or Modified**

| <b>Parameter Value</b>   | <b>Type</b> | <b>Purpose</b>                                                                 | <b>Initialization</b>                                                                                                                                                                                                                                                                          |
|--------------------------|-------------|--------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_ED_DIMEN          | i           | Address of a variable that holds the task dimension                            | Required. Positive integer value.                                                                                                                                                                                                                                                              |
| VSL_SS_ED_OBSERV_N       | i           | Address of a variable that holds the number of observations                    | Required. Positive integer value.                                                                                                                                                                                                                                                              |
| VSL_SS_ED_OBSERV         | d, s        | Address of the observation matrix                                              | Required. Provide the matrix containing your observations.                                                                                                                                                                                                                                     |
| VSL_SS_ED_OBSERV_STORAGE | i           | Address of a variable that holds the storage format for the observation matrix | Required. Provide a storage format supported by the library whenever you pass a matrix of observations. <sup>1</sup>                                                                                                                                                                           |
| VSL_SS_ED_INDC           | i           | Address of the array of indices                                                | Optional. Provide this array if you need to process individual components of the random vector. Set entry <i>i</i> of the array to one to include the <i>i</i> th coordinate in the analysis. Set entry <i>i</i> of the array to zero to exclude the <i>i</i> th coordinate from the analysis. |
| VSL_SS_ED_WEIGHTS        | d, s        | Address of the array of observation weights                                    | Optional. If the observations have weights different from the default weight (one), set entries of the array to non-negative floating point values.                                                                                                                                            |
| VSL_SS_ED_MEAN           | d, s        | Address of the array of means                                                  | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.                                                                                                                              |
| VSL_SS_ED_2R_MOM         | d, s        | Address of an array of raw moments of the second order                         | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.                                                                                                                              |
| VSL_SS_ED_3R_MOM         | d, s        | Address of an array of raw moments of the third order                          | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.                                                                                                                              |
| VSL_SS_ED_4R_MOM         | d, s        | Address of an array of raw moments of the fourth order                         | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.                                                                                                                              |

| <b>Parameter Value</b> | <b>Type</b> | <b>Purpose</b>                                             | <b>Initialization</b>                                                                                                                                                                                                                                              |
|------------------------|-------------|------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_ED_2C_MOM       | d, s        | Address of an array of central moments of the second order | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first and second order.                 |
| VSL_SS_ED_3C_MOM       | d, s        | Address of an array of central moments of the third order  | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first, second, and third order.         |
| VSL_SS_ED_4C_MOM       | d, s        | Address of an array of central moments of the fourth order | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first, second, third, and fourth order. |
| VSL_SS_ED_KURTOSIS     | d, s        | Address of the array of kurtosis estimates                 | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first, second, third, and fourth order. |
| VSL_SS_ED_SKEWNESS     | d, s        | Address of the array of skewness estimates                 | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first, second, and third order.         |
| VSL_SS_ED_MIN          | d, s        | Address of the array of minimum estimates                  | Optional. Set entries of array to meaningful values, such as the values of the first observation.                                                                                                                                                                  |
| VSL_SS_ED_MAX          | d, s        | Address of the array of maximum estimates                  | Optional. Set entries of array to meaningful values, such as the values of the first observation.                                                                                                                                                                  |

| Parameter Value         | Type | Purpose                                                                                                                                                       | Initialization                                                                                                                                                                                                                                                             |
|-------------------------|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_ED_VARIATION     | d, s | Address of the array of variation coefficients                                                                                                                | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. Make sure you also provide arrays for raw moments of the first and second order.                         |
| VSL_SS_ED_COV           | d, s | Address of a covariance matrix                                                                                                                                | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Make sure you also provide an array for the mean.                                                                                                |
| VSL_SS_ED_COV_STORAGE   | i    | Address of the variable that holds the storage format for a covariance matrix                                                                                 | Required. Provide a storage format supported by the library whenever you intend to compute the covariance matrix. <sup>2</sup>                                                                                                                                             |
| VSL_SS_ED_COR           | d, s | Address of a correlation matrix                                                                                                                               | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. If you initialize the matrix in non-trivial way, make sure that the main diagonal contains variance values. Also, provide an array for the mean. |
| VSL_SS_ED_COR_STORAGE   | i    | Address of the variable that holds the correlation storage format for a correlation matrix                                                                    | Required. Provide a storage format supported by the library whenever you intend to compute the correlation matrix. <sup>2</sup>                                                                                                                                            |
| VSL_SS_ED_ACCUM_WEIGHT  | d, s | Address of the array of size 2 that holds the accumulated weight (sum of weights) in the first position and the sum of weights squared in the second position | Optional. Set the entries of the matrix to meaningful values (typically zero) if you intend to do progressive processing of the dataset or need the sum of weights and sum of squared weights assigned to observations.                                                    |
| VSL_SS_ED_QUANT_ORDER_N | i    | Address of the variable that holds the number of quantile orders                                                                                              | Required. Positive integer value. Provide the number of quantile orders whenever you compute quantiles.                                                                                                                                                                    |
| VSL_SS_ED_QUANT_ORDER   | d, s | Address of the array of quantile orders                                                                                                                       | Required. Set entries of array to values from the interval (0,1). Provide this parameter whenever you compute quantiles.                                                                                                                                                   |

| Parameter Value                   | Type | Purpose                                                                                                              | Initialization                                                                                                                                                                |
|-----------------------------------|------|----------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_ED_QUANT_QUANTILE<br>S     | d, s | Address of the array of quantiles                                                                                    | None.                                                                                                                                                                         |
| VSL_SS_ED_ORDER_STATS             | d, s | Address of the array of order statistics                                                                             | None.                                                                                                                                                                         |
| VSL_SS_ED_GROUP_INDC              | i    | Address of the array of group indices used in computation of a pooled covariance matrix                              | Required. Set entry $i$ to integer value $k$ if the observation belongs to group $k$ . Values of $k$ take values in the range $[0, g-1]$ , where $g$ is the number of groups. |
| VSL_SS_ED_POOLED_COV_STO<br>RAGE  | i    | Address of a variable that holds the storage format for a pooled covariance matrix                                   | Required. Provide a storage format supported by the library whenever you intend to compute pooled covariance. <sup>2</sup>                                                    |
| VSL_SS_ED_POOLED_MEAN             | d, s | Address of an array of pooled means                                                                                  | None.                                                                                                                                                                         |
| VSL_SS_ED_POOLED_COV              | d, s | Address of pooled covariance matrices                                                                                | None.                                                                                                                                                                         |
| VSL_SS_ED_GROUP_COV_INDC          | i    | Address of an array of indices for which covariance/means should be computed                                         | Optional. Set the $k$ th entry of the array to 1 if you need group covariance and mean for group $k$ ; otherwise set it to zero.                                              |
| VSL_SS_ED_REQ_GROUP_INDC          | i    | Address of an array of indices for which group estimates such as covariance or means are requested                   | Optional. Set the $k$ th entry of the array to 1 if you need an estimate for group $k$ ; otherwise set it to zero.                                                            |
| VSL_SS_ED_GROUP_MEANS             | i    | Address of an array of group means                                                                                   | None.                                                                                                                                                                         |
| VSL_SS_ED_GROUP_COV_STOR<br>AGE   | d, s | Address of a variable that holds the storage format for a group covariance matrix                                    | Required. Provide a storage format supported by the library whenever you intend to get group covariance. <sup>2</sup>                                                         |
| VSL_SS_ED_GROUP_COV               | d, s | Address of group covariance matrices                                                                                 | None.                                                                                                                                                                         |
| VSL_SS_ED_ROBUST_COV_STO<br>RAGE  | d, s | Address of a variable that holds the storage format for a robust covariance matrix                                   | Required. Provide a storage format supported by the library whenever you compute robust covariance <sup>2</sup> .                                                             |
| VSL_SS_ED_ROBUST_COV_PAR<br>AMS_N | i    | Address of a variable that holds the number of algorithmic parameters of the method for robust covariance estimation | Required. Set to the number of TBS parameters,<br><i>VSL_SS_TBS_PARAMS_N</i> .                                                                                                |

| Parameter Value               | Type | Purpose                                                                                                  | Initialization                                                                                                              |
|-------------------------------|------|----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_ED_ROBUST_COV_PARAMS   | d, s | Address of an array of parameters of the method for robust estimation of a covariance                    | Required. Set the entries of the array according to the description in <a href="#">vslSSEditRobustCovariance</a> .          |
| VSL_SS_ED_ROBUST_MEAN         | i    | Address of an array of robust means                                                                      | None.                                                                                                                       |
| VSL_SS_ED_ROBUST_COV          | d, s | Address of a robust covariance matrix                                                                    | None.                                                                                                                       |
| VSL_SS_ED_OUTLIERS_PARAMS_N   | d, s | Address of a variable that holds the number of parameters of the outlier detection method                | Required. Set to the number of outlier detection parameters, <i>VSL_SS_BACON_PARAMS_N</i> .                                 |
| VSL_SS_ED_OUTLIERS_PARAMS     | i    | Address of an array of algorithmic parameters for the outlier detection method                           | Required. Set the entries of the array according to the description in <a href="#">vslSSEditOutliersDetection</a> .         |
| VSL_SS_ED_OUTLIERS_WEIGHT     | d, s | Address of an array of weights assigned to observations by the outlier detection method                  | None.                                                                                                                       |
| VSL_SS_ED_ORDER_STATS_STORAGE | d, s | Address of a variable that holds the storage format of an order statistics matrix                        | Required. Provide a storage format supported by the library whenever you compute a matrix of order statistics. <sup>1</sup> |
| VSL_SS_ED_PARTIAL_COV_ID_X    | i    | Address of an array that encodes subcomponents of a random vector                                        | Required. Set the entries of the array according to the description in <a href="#">vslSSEditPartialCovCor</a> .             |
| VSL_SS_ED_PARTIAL_COV         | d, s | Address of a partial covariance matrix                                                                   | None.                                                                                                                       |
| VSL_SS_ED_PARTIAL_COV_STORAGE | i    | Address of a variable that holds the storage format of a partial covariance matrix                       | Required. Provide a storage format supported by the library whenever you compute the partial covariance. <sup>2</sup>       |
| VSL_SS_ED_PARTIAL_COR         | d, s | Address of a partial correlation matrix                                                                  | None.                                                                                                                       |
| VSL_SS_ED_PARTIAL_COR_STORAGE | i    | Address of a variable that holds the storage format for a partial correlation matrix                     | Required. Provide a storage format supported by the library whenever you compute the partial correlation. <sup>2</sup>      |
| VSL_SS_ED_MI_PARAMS_N         | i    | Address of a variable that holds the number of algorithmic parameters for the Multiple Imputation method | Required. Set to the number of MI parameters, <i>VSL_SS_MI_PARAMS_SIZE</i> .                                                |

| Parameter Value               | Type | Purpose                                                                                                         | Initialization                                                                                                                                                                                                                                              |
|-------------------------------|------|-----------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_ED_MI_PARAMS           | d, s | Address of an array of algorithmic parameters for the Multiple Imputation method                                | Required. Set entries of the array according to the description in <a href="#">vslSSEditMissingValues</a> .                                                                                                                                                 |
| VSL_SS_ED_MI_INIT_ESTIMATES_N | i    | Address of a variable that holds the number of initial estimates for the Multiple Imputation method             | Optional. Set to $p+p*(p+1)/2$ , where $p$ is the task dimension.                                                                                                                                                                                           |
| VSL_SS_ED_MI_INIT_ESTIMATES   | d, s | Address of an array of initial estimates for the Multiple Imputation method                                     | Optional. Set the values of the array according to the description in "Basic Components of the Multiple Imputation Function in Summary Statistics" in the <i>Intel® MKL Summary Statistics Application Notes</i> document [ <a href="#">SS Notes</a> ].     |
| VSL_SS_ED_MI_SIMUL_VALS_N     | i    | Address of a variable that holds the number of simulated values in the Multiple Imputation method               | Optional. Positive integer indicating the number of missing points in the observation matrix.                                                                                                                                                               |
| VSL_SS_ED_MI_SIMUL_VALS       | d, s | Address of an array of simulated values in the Multiple Imputation method                                       | None.                                                                                                                                                                                                                                                       |
| VSL_SS_ED_MI_ESTIMATES_N      | i    | Address of a variable that holds the number of estimates obtained as a result of the Multiple Imputation method | Optional. Positive integer number defined according to the description in "Basic Components of the Multiple Imputation Function in Summary Statistics" in the <i>Intel® MKL Summary Statistics Application Notes</i> document [ <a href="#">SS Notes</a> ]. |
| VSL_SS_ED_MI_ESTIMATES        | d, s | Address of an array of estimates obtained as a result of the Multiple Imputation method                         | None.                                                                                                                                                                                                                                                       |
| VSL_SS_ED_MI_PRIOR_N          | i    | Address of a variable that holds the number of prior parameters for the Multiple Imputation method              | Optional. If you pass a user-defined array of prior parameters, set this parameter to $(p^2+3*p+4)/2$ , where $p$ is the task dimension.                                                                                                                    |
| VSL_SS_ED_MI_PRIOR            | d, s | Address of an array of prior parameters for the Multiple Imputation method                                      | Optional. Set entries of the array of prior parameters according to the description in "Basic Components of the Multiple Imputation Function in Summary                                                                                                     |

| Parameter Value                  | Type | Purpose                                                                                                       | Initialization                                                                                                                                                                                                       |
|----------------------------------|------|---------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                  |      |                                                                                                               | Statistics" in the <i>Intel® MKL Summary Statistics Application Notes</i> document [ <a href="#">SS Notes</a> ].                                                                                                     |
| VSL_SS_ED_PARAMTR_COR            | d, s | Address of a parameterized correlation matrix                                                                 | None.                                                                                                                                                                                                                |
| VSL_SS_ED_PARAMTR_COR_STORAGE    | i    | Address of a variable that holds the storage format of a parameterized correlation matrix                     | Required. Provide a storage format supported by the library whenever you compute the parameterized correlation matrix. <sup>2</sup>                                                                                  |
| VSL_SS_ED_STREAM_QUANT_PARAMS_N  | i    | Address of a variable that holds the number of parameters of a quantile computation method for streaming data | Required. Set to the number of quantile computation parameters, <i>VSL_SS_SQUANTS_ZW_PARAMS_N</i> .                                                                                                                  |
| VSL_SS_ED_STREAM_QUANT_PARAMS    | d, s | Address of an array of parameters of a quantile computation method for streaming data                         | Required. Set the entries of the array according to the description in "Computing Quantiles for Streaming Data" in the <i>Intel® MKL Summary Statistics Application Notes</i> document [ <a href="#">SS Notes</a> ]. |
| VSL_SS_ED_STREAM_QUANT_ORDER_N   | i    | Address of a variable that holds the number of quantile orders for streaming data                             | Required. Positive integer value.                                                                                                                                                                                    |
| VSL_SS_ED_STREAM_QUANT_ORDER     | d, s | Address of an array of quantile orders for streaming data                                                     | Required. Set entries of the array to values from the interval (0,1). Provide this parameter whenever you compute quantiles.                                                                                         |
| VSL_SS_ED_STREAM_QUANT_QUANTILES | d, s | Address of an array of quantiles for streaming data                                                           | None.                                                                                                                                                                                                                |
| VSL_SS_ED_SUM                    | d, s | Address of array of sums                                                                                      | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.                                                    |
| VSL_SS_ED_2R_SUM                 | d, s | Address of array of raw sums of 2nd order                                                                     | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.                                                    |
| VSL_SS_ED_3R_SUM                 | d, s | Address of array of raw sums of 3rd order                                                                     | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array.                                                    |

| Parameter Value                 | Type | Purpose                                                                 | Initialization                                                                                                                                                    |
|---------------------------------|------|-------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_ED_4R_SUM                | d, s | Address of array of raw sums of 4th order                               | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. |
| VSL_SS_ED_2C_SUM                | d, s | Address of array of central sums of 2nd order                           | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. |
| VSL_SS_ED_3C_SUM                | d, s | Address of array of central sums of 3rd order                           | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. |
| VSL_SS_ED_4C_SUM                | d, s | Address of array of central sums of 4th order                           | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. |
| VSL_SS_ED_CP                    | d, s | Address of cross-product matrix                                         | Optional. Set entries of the array to meaningful values (typically zero) if you intend to compute a progressive estimate. Otherwise, do not initialize the array. |
| VSL_SS_ED_MDAD                  | d, s | Address of array of median absolute deviations                          | None.                                                                                                                                                             |
| VSL_SS_ED_MNAD                  | d, s | Address of array of mean absolute deviations                            | None.                                                                                                                                                             |
| VSL_SS_ED_SORTED_OBSERV         | d, s | Address of the array that stores sorted results                         | None.                                                                                                                                                             |
| VSL_SS_ED_SORTED_OBSERV_STORAGE | i    | Address of a variable that holds the storage format of an output matrix | Required. Provide a supported storage format whenever you specify sorting of the observation matrix.                                                              |

1. See [Table: "Storage format of matrix of observations and order statistics"](#) for storage formats.
2. See [Table: "Storage formats of a variance-covariance/correlation matrix"](#) for storage formats.

### vsLSSEditMoments

*Modifies the pointers to arrays that hold moment estimates.*

#### Syntax

```
status = vsLSSEditMoments(task, mean, r2m, r3m, r4m, c2m, c3m, c4m);
status = vsLdSSEditMoments(task, mean, r2m, r3m, r4m, c2m, c3m, c4m);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name              | Type                                                                                                              | Description                                                          |
|-------------------|-------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <code>task</code> | <code>VSLSTaskPtr</code>                                                                                          | Descriptor of the task                                               |
| <code>mean</code> | <code>float*</code> for <code>vslsSSEditMoments</code><br><code>double*</code> for <code>vsldSSEditMoments</code> | Pointer to the array of means                                        |
| <code>r2m</code>  | <code>float*</code> for <code>vslsSSEditMoments</code><br><code>double*</code> for <code>vsldSSEditMoments</code> | Pointer to the array of raw moments of the 2 <sup>nd</sup> order     |
| <code>r3m</code>  | <code>float*</code> for <code>vslsSSEditMoments</code><br><code>double*</code> for <code>vsldSSEditMoments</code> | Pointer to the array of raw moments of the 3 <sup>rd</sup> order     |
| <code>r4m</code>  | <code>float*</code> for <code>vslsSSEditMoments</code><br><code>double*</code> for <code>vsldSSEditMoments</code> | Pointer to the array of raw moments of the 4 <sup>th</sup> order     |
| <code>c2m</code>  | <code>float*</code> for <code>vslsSSEditMoments</code><br><code>double*</code> for <code>vsldSSEditMoments</code> | Pointer to the array of central moments of the 2 <sup>nd</sup> order |
| <code>c3m</code>  | <code>float*</code> for <code>vslsSSEditMoments</code><br><code>double*</code> for <code>vsldSSEditMoments</code> | Pointer to the array of central moments of the 3 <sup>rd</sup> order |
| <code>c4m</code>  | <code>float*</code> for <code>vslsSSEditMoments</code><br><code>double*</code> for <code>vsldSSEditMoments</code> | Pointer to the array of central moments of the 4 <sup>th</sup> order |

## Output Parameters

| Name                | Type             | Description                |
|---------------------|------------------|----------------------------|
| <code>status</code> | <code>int</code> | Current status of the task |

## Description

The `vslsSSEditMoments` routine replaces pointers to the arrays that hold estimates of raw and central moments with values passed as corresponding parameters of the routine. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

## vslSSEditSums

*Modifies the pointers to arrays that hold sum estimates.*

---

### Syntax

```
status = vslsSSEditSums(task, sum, r2s, r3s, r4s, c2s, c3s, c4s);
```

```
status = vsldSSEditSums(task, sum, r2s, r3s, r4s, c2s, c3s, c4s);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name              | Type                                                                                                        | Description                                              |
|-------------------|-------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|
| <code>task</code> | <code>VSLSTaskPtr</code>                                                                                    | Descriptor of the task                                   |
| <code>sum</code>  | <code>float*</code> for <code>vslsSSEditSums</code><br><code>double*</code> for <code>vsldSSEditSums</code> | Pointer to the array of sums                             |
| <code>r2s</code>  | <code>float*</code> for <code>vslsSSEditSums</code><br><code>double*</code> for <code>vsldSSEditSums</code> | Pointer to the array of raw sums of the second order     |
| <code>r3s</code>  | <code>float*</code> for <code>vslsSSEditSums</code><br><code>double*</code> for <code>vsldSSEditSums</code> | Pointer to the array of raw sums of the third order      |
| <code>r4s</code>  | <code>float*</code> for <code>vslsSSEditSums</code><br><code>double*</code> for <code>vsldSSEditSums</code> | Pointer to the array of raw sums of the fourth order     |
| <code>c2s</code>  | <code>float*</code> for <code>vslsSSEditSums</code><br><code>double*</code> for <code>vsldSSEditSums</code> | Pointer to the array of central sums of the second order |
| <code>c3s</code>  | <code>float*</code> for <code>vslsSSEditSums</code><br><code>double*</code> for <code>vsldSSEditSums</code> | Pointer to the array of central sums of the third order  |
| <code>c4s</code>  | <code>float*</code> for <code>vslsSSEditSums</code><br><code>double*</code> for <code>vsldSSEditSums</code> | Pointer to the array of central sums of the fourth order |

## Output Parameters

| Name                | Type             | Description                |
|---------------------|------------------|----------------------------|
| <code>status</code> | <code>int</code> | Current status of the task |

## Description

The `vslsSSEditSums` routine replaces pointers to the arrays that hold estimates of raw and central sums with values passed as corresponding parameters of the routine. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

## `vslsSSEditCovCor`

*Modifies the pointers to covariance/correlation/cross-product parameters.*

## Syntax

```
status = vslsSSEditCovCor(task, mean, cov, cov_storage, cor, cor_storage);
status = vsldSSEditCovCor(task, mean, cov, cov_storage, cor, cor_storage);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name               | Type                                                        | Description                                             |
|--------------------|-------------------------------------------------------------|---------------------------------------------------------|
| <i>task</i>        | VSLSTaskPtr                                                 | Descriptor of the task                                  |
| <i>mean</i>        | float* for vslsSSEditCovCor<br>double* for vsldSSEditCovCor | Pointer to the array of means                           |
| <i>cov</i>         | float* for vslsSSEditCovCor<br>double* for vsldSSEditCovCor | Pointer to a covariance matrix                          |
| <i>cov_storage</i> | const MKL_INT*                                              | Pointer to the storage format of the covariance matrix  |
| <i>cor</i>         | float* for vslsSSEditCovCor<br>double* for vsldSSEditCovCor | Pointer to a correlation matrix                         |
| <i>cor_storage</i> | const MKL_INT*                                              | Pointer to the storage format of the correlation matrix |

## Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

## Description

The `vslsSSEditCovCor` routine replaces pointers to the array of means, covariance/correlation arrays, and their storage format with values passed as corresponding parameters of the routine. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

The storage parameters, `cov_storage` and `cor_storage`, describe the storage format used for the  $p$ -by- $p$  symmetric variance-covariance/correlation/cross-product matrix  $C$ . The matrix  $C$  can be described as

$$C = \begin{pmatrix} c_{1,1} & c_{1,2} & \cdots & \cdots & \cdots & c_{1,p} \\ c_{2,1} & c_{2,2} & \cdots & \cdots & \cdots & c_{2,p} \\ \vdots & \vdots & \ddots & & & \vdots \\ \vdots & \vdots & & c_{i,j} & & \vdots \\ \vdots & \vdots & & & \ddots & \vdots \\ c_{p,1} & c_{p,2} & \cdots & \cdots & \cdots & c_{p,p} \end{pmatrix}$$

Table "Storage formats of a variance-covariance/correlation/cross-product matrix" shows how the matrix is stored in a one-dimensional array  $cp$  for different values of the storage parameters.

## Storage formats of variance-covariance/correlation/cross-product matrices

| Parameter                  | Description                                                                                                                                                                           |
|----------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_MATRIX_STORAGE_FULL | The array $cp$ contains all elements of the matrix stored sequentially, row-by-row:<br><br>$cp[0]$ contains $c_{1,1}$<br>$cp[1]$ contains $c_{1,2}$<br>$cp[p - 1]$ contains $c_{1,p}$ |

| Parameter                      | Description                                                                                                                                                                                                                                                                                                                                                              |
|--------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                | <p><math>cp[p]</math> contains <math>c_{2,1}</math><br/> <math>cp[p*p - 1]</math> contains <math>c_{p,p}</math></p> <p>The size of array <math>cp</math> is <math>p*p</math>.</p>                                                                                                                                                                                        |
| VSL_SS_MATRIX_STORAGE_L_PACKED | <p>The array <math>cp</math> contains the lower triangular part of the symmetric matrix stored sequentially, row-by-row:</p> <p><math>cp[0]</math> contains <math>c_{1,1}</math><br/> <math>cp[1]</math> contains <math>c_{2,1}</math><br/> <math>cp[2]</math> contains <math>c_{2,2}</math><br/> and so on.</p> <p>The size of the array is <math>p*(p+1)/2</math>.</p> |
| VSL_SS_MATRIX_STORAGE_U_PACKED | <p>The array <math>cp</math> contains the upper triangular part of the symmetric matrix stored sequentially, row-by-row:</p> <p><math>cp[0]</math> contains <math>c_{1,1}</math><br/> <math>cp[1]</math> contains <math>c_{1,2}</math><br/> <math>cp[3]</math> contains <math>c_{1,3}</math><br/> and so on.</p> <p>The size of the array is <math>p*(p+1)/2</math>.</p> |

## vslSSEditCP

Modifies the pointers to cross-product matrix parameters.

### Syntax

```
status = vslsSSEditCP(task, mean, sum, cp, cp_storage);
status = vsldSSEditCP(task, mean, sum, cp, cp_storage);
```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type                                                | Description                       |
|-------------|-----------------------------------------------------|-----------------------------------|
| <i>task</i> | VSLSSTaskPtr                                        | Descriptor of the task            |
| <i>mean</i> | float* for vslsSSEditCP<br>double* for vsldSSEditCP | Pointer to array of means         |
| <i>sum</i>  | float* for vslsSSEditCP<br>double* for vsldSSEditCP | Pointer to array of sums          |
| <i>cp</i>   | float* for vslsSSEditCP<br>double* for vsldSSEditCP | Pointer to a cross-product matrix |

| Name              | Type           | Description                                               |
|-------------------|----------------|-----------------------------------------------------------|
| <i>cp_storage</i> | const MKL_INT* | Pointer to the storage format of the cross-product matrix |

### Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

### Description

The `vslSSEditCP` routine replaces pointers to the array of means, array of sums, cross-product matrix, and its storage format with values passed as corresponding parameters of the routine. See [Table: "Storage formats of a variance-covariance/correlation/cross-product matrix"](#) for possible values of the *cp\_storage* parameter. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

### Storage formats of variance-covariance/correlation/cross-product matrices

| Parameter                                   | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|---------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>VSL_SS_MATRIX_STORAGE_FULL</code>     | <p>The array <i>cp</i> contains all elements of the matrix stored sequentially, row-by-row:</p> <ul style="list-style-type: none"> <li><i>cp</i>[0] contains <math>c_{1,1}</math></li> <li><i>cp</i>[1] contains <math>c_{1,2}</math></li> <li><i>cp</i>[<i>p</i> - 1] contains <math>c_{1,p}</math></li> <li><i>cp</i>[<i>p</i>] contains <math>c_{2,1}</math></li> <li><i>cp</i>[<i>p</i>*<i>p</i> - 1] contains <math>c_{p,p}</math></li> </ul> <p>The size of array <i>cp</i> is <math>p*p</math>.</p> |
| <code>VSL_SS_MATRIX_STORAGE_L_PACKED</code> | <p>The array <i>cp</i> contains the lower triangular part of the symmetric matrix stored sequentially, row-by-row:</p> <ul style="list-style-type: none"> <li><i>cp</i>[0] contains <math>c_{1,1}</math></li> <li><i>cp</i>[1] contains <math>c_{2,1}</math></li> <li><i>cp</i>[2] contains <math>c_{2,2}</math></li> <li>and so on.</li> </ul> <p>The size of the array is <math>p*(p+1)/2</math>.</p>                                                                                                    |
| <code>VSL_SS_MATRIX_STORAGE_U_PACKED</code> | <p>The array <i>cp</i> contains the upper triangular part of the symmetric matrix stored sequentially, row-by-row:</p> <ul style="list-style-type: none"> <li><i>cp</i>[0] contains <math>c_{1,1}</math></li> <li><i>cp</i>[1] contains <math>c_{1,2}</math></li> <li><i>cp</i>[3] contains <math>c_{1,3}</math></li> <li>and so on.</li> </ul> <p>The size of the array is <math>p*(p+1)/2</math>.</p>                                                                                                    |

## vslsSSEditPartialCovCor

Modifies the pointers to partial covariance/correlation parameters.

### Syntax

```
status = vslsSSEditPartialCovCor(task, p_idx_array, cov, cov_storage, cor,
cor_storage, p_cov, p_cov_storage, p_cor, p_cor_storage);

status = vsldSSEditPartialCovCor(task, p_idx_array, cov, cov_storage, cor,
cor_storage, p_cov, p_cov_storage, p_cor, p_cor_storage);
```

### Include Files

- mkl.h

### Input Parameters

| Name                 | Type                                                                                            | Description                                                                                                                                                                                                                                                                                                                                             |
|----------------------|-------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>          | VSLSSTaskPtr                                                                                    | Descriptor of the task                                                                                                                                                                                                                                                                                                                                  |
| <i>p_idx_array</i>   | const MKL_INT*                                                                                  | Pointer to the array that encodes indices of subcomponents $Z$ and $Y$ of the random vector as described in section <a href="#">Mathematical Notation and Definitions</a> .<br><br>$p\_idx\_array[i]$ equals to<br>-1 if the $i$ -th component of the random vector belongs to $Z$<br>1, if the $i$ -th component of the random vector belongs to $Y$ . |
| <i>cov</i>           | const float* for<br>vslsSSEditPartialCovCor<br><br>const double* for<br>vsldSSEditPartialCovCor | Pointer to a covariance matrix                                                                                                                                                                                                                                                                                                                          |
| <i>cov_storage</i>   | const MKL_INT*                                                                                  | Pointer to the storage format of the covariance matrix                                                                                                                                                                                                                                                                                                  |
| <i>cor</i>           | const float* for<br>vslsSSEditPartialCovCor<br><br>const double* for<br>vsldSSEditPartialCovCor | Pointer to a correlation matrix                                                                                                                                                                                                                                                                                                                         |
| <i>cor_storage</i>   | const MKL_INT*                                                                                  | Pointer to the storage format of the correlation matrix                                                                                                                                                                                                                                                                                                 |
| <i>p_cov</i>         | float* for vslsSSEditPartialCovCor<br>double* for vsldSSEditPartialCovCor                       | Pointer to a partial covariance matrix                                                                                                                                                                                                                                                                                                                  |
| <i>p_cov_storage</i> | const MKL_INT*                                                                                  | Pointer to the storage format of the partial covariance matrix                                                                                                                                                                                                                                                                                          |
| <i>p_cor</i>         | float* for vslsSSEditPartialCovCor                                                              | Pointer to a partial correlation matrix                                                                                                                                                                                                                                                                                                                 |

| Name                 | Type                                | Description                                                     |
|----------------------|-------------------------------------|-----------------------------------------------------------------|
|                      | double* for vsldSSEditPartialCovCor |                                                                 |
| <i>p_cor_storage</i> | const MKL_INT*                      | Pointer to the storage format of the partial correlation matrix |

### Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

### Description

The `vs1sSEditPartialCovCor` routine replaces pointers to covariance/correlation arrays, partial covariance/correlation arrays, and their storage format with values passed as corresponding parameters of the routine. See [Table "Storage formats of a variance-covariance/correlation matrix"](#) for possible values of the `cov_storage`, `cor_storage`, `p_cov_storage`, and `p_cor_storage` parameters. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

### vs1sSEditQuantiles

*Modifies the pointers to parameters related to quantile computations.*

### Syntax

```
status = vs1sSEditQuantiles(task, quant_order_n, quant_order, quants, order_stats, order_stats_storage);
```

```
status = vsldSSEditQuantiles(task, quant_order_n, quant_order, quants, order_stats, order_stats_storage);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                 | Type                                                                         | Description                              |
|----------------------|------------------------------------------------------------------------------|------------------------------------------|
| <i>task</i>          | VSLSTaskPtr                                                                  | Descriptor of the task                   |
| <i>quant_order_n</i> | const MKL_INT*                                                               | Pointer to the number of quantile orders |
| <i>quant_order</i>   | const float* for vs1sSEditQuantiles<br>const double* for vsldSSEditQuantiles | Pointer to the array of quantile orders  |
| <i>quants</i>        | float* for vs1sSEditQuantiles<br>double* for vsldSSEditQuantiles             | Pointer to the array of quantiles        |
| <i>order_stats</i>   | float* for vs1sSEditQuantiles<br>double* for vsldSSEditQuantiles             | Pointer to the array of order statistics |

| Name                             | Type                        | Description                                                 |
|----------------------------------|-----------------------------|-------------------------------------------------------------|
| <code>order_stats_storage</code> | <code>const MKL_INT*</code> | Pointer to the storage format of the order statistics array |

## Output Parameters

| Name                | Type             | Description                |
|---------------------|------------------|----------------------------|
| <code>status</code> | <code>int</code> | Current status of the task |

## Description

The `vslsSSEditQuantiles` routine replaces pointers to the number of quantile orders, the array of quantile orders, the array of quantiles, the array that holds order statistics, and the storage format for the order statistics with values passed into the routine. See [Table "Storage format of matrix of observations and order statistics"](#) for possible values of the `order_statistics_storage` parameter. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

## vslsSSEditStreamQuantiles

*Modifies the pointers to parameters related to quantile computations for streaming data.*

## Syntax

```
status = vslsSSEditStreamQuantiles(task, quant_order_n, quant_order, quants, nparams,
params);
```

```
status = vsldSSEditStreamQuantiles(task, quant_order_n, quant_order, quants, nparams,
params);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                       | Type                                                                                                                                                | Description                                       |
|----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------|
| <code>task</code>          | <code>VSLSSTaskPtr</code>                                                                                                                           | Descriptor of the task                            |
| <code>quant_order_n</code> | <code>const MKL_INT*</code>                                                                                                                         | Pointer to the number of quantile orders          |
| <code>quant_order</code>   | <code>const float*</code> for<br><code>vslsSSEditStreamQuantiles</code><br><code>const double*</code> for<br><code>vsldSSEditStreamQuantiles</code> | Pointer to the array of quantile orders           |
| <code>quants</code>        | <code>float*</code> for<br><code>vslsSSEditStreamQuantiles</code><br><code>double*</code> for<br><code>vsldSSEditStreamQuantiles</code>             | Pointer to the array of quantiles                 |
| <code>nparams</code>       | <code>const MKL_INT*</code>                                                                                                                         | Pointer to the number of the algorithm parameters |

| Name          | Type                                                                                            | Description                                      |
|---------------|-------------------------------------------------------------------------------------------------|--------------------------------------------------|
| <i>params</i> | const float*<br>for vslsSSEditStreamQuantiles<br>const double*<br>for vsldSSEditStreamQuantiles | Pointer to the array of the algorithm parameters |

### Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

### Description

The `vslsSSEditStreamQuantiles` routine replaces pointers to the number of quantile orders, the array of quantile orders, the array of quantiles, the number of the algorithm parameters, and the array of the algorithm parameters with values passed into the routine. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

### vslsSSEditPooledCovariance

*Modifies pooled/group covariance matrix array pointers.*

### Syntax

```
status = vslsSSEditPooledCovariance(task, grp_indices, pld_mean, pld_cov,
req_grp_indices, grp_means, grp_cov);
status = vsldSSEditPooledCovariance(task, grp_indices, pld_mean, pld_cov,
req_grp_indices, grp_means, grp_cov);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name               | Type                                                                                  | Description                                                                                                                               |
|--------------------|---------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>        | VSLSTaskPtr                                                                           | Descriptor of the task                                                                                                                    |
| <i>grp_indices</i> | const MKL_INT*                                                                        | Pointer to an array of size <i>n</i> . The <i>i</i> -th element of the array contains the number of the group the observation belongs to. |
| <i>pld_mean</i>    | float* for<br>vslsSSEditPooledCovariance<br>double* for<br>vsldSSEditPooledCovariance | Pointer to the array of pooled means                                                                                                      |
| <i>pld_cov</i>     | float* for<br>vslsSSEditPooledCovariance<br>double* for<br>vsldSSEditPooledCovariance | Pointer to the array that holds a pooled covariance matrix                                                                                |

| Name                         | Type                                                                                                                  | Description                                                                                                      |
|------------------------------|-----------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| <code>req_grp_indices</code> | <code>const MKL_INT*</code>                                                                                           | Pointer to the array that contains indices of groups for which estimates to return (such as covariance and mean) |
| <code>grp_means</code>       | <code>float* for<br/>vslsSSEditPooledCovariance</code><br><br><code>double* for<br/>vsldSSEditPooledCovariance</code> | Pointer to the array of group means                                                                              |
| <code>grp_cov</code>         | <code>float* for<br/>vslsSSEditPooledCovariance</code><br><br><code>double* for<br/>vsldSSEditPooledCovariance</code> | Pointer to the array that holds group covariance matrices                                                        |

## Output Parameters

| Name                | Type             | Description                |
|---------------------|------------------|----------------------------|
| <code>status</code> | <code>int</code> | Current status of the task |

## Description

The `vslsSSEditPooledCovariance` routine replaces pointers to the array of group indices, the array of pooled means, the array for a pooled covariance matrix, and pointers to the array of indices of group matrices, the array of group means, and the array for group covariance matrices with values passed in the editors. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.. Use the `vslsSSEditTask` routine to replace the storage format for pooled and group covariance matrices.

## vslsSSEditRobustCovariance

*Modifies pointers to arrays related to a robust covariance matrix.*

## Syntax

```
status = vslsSSEditRobustCovariance(task, rcov_storage, nparams, params, rmean, rcov);
status = vsldSSEditRobustCovariance(task, rcov_storage, nparams, params, rmean, rcov);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                      | Type                        | Description                                                 |
|---------------------------|-----------------------------|-------------------------------------------------------------|
| <code>task</code>         | <code>VSLSTaskPtr</code>    | Descriptor of the task                                      |
| <code>rcov_storage</code> | <code>const MKL_INT*</code> | Pointer to the storage format of a robust covariance matrix |
| <code>nparams</code>      | <code>const MKL_INT*</code> | Pointer to the number of method parameters                  |

| Name          | Type                                                                                                  | Description                               |
|---------------|-------------------------------------------------------------------------------------------------------|-------------------------------------------|
| <i>params</i> | const float* for<br>vs1sSSEditRobustCovariance<br><br>const double* for<br>vs1dSSEditRobustCovariance | Pointer to the array of method parameters |
| <i>rmean</i>  | float* for<br>vs1sSSEditRobustCovariance<br><br>double* for<br>vs1dSSEditRobustCovariance             | Pointer to the array of robust means      |
| <i>rcov</i>   | float* for<br>vs1sSSEditRobustCovariance<br><br>double* for<br>vs1dSSEditRobustCovariance             | Pointer to a robust covariance matrix     |

### Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

### Description

The `vs1sSSEditRobustCovariance` routine uses values passed as parameters of the routine to replace:

- pointers to covariance matrix storage
- pointers to the number of method parameters and to the array of the method parameters of size *nparams*
- pointers to the arrays that hold robust means and covariance

See [Table "Storage formats of a variance-covariance/correlation matrix"](#) for possible values of the *rcov\_storage* parameter. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

Intel MKL provides a Translated Biweight S-estimator (TBS) for robust estimation of a variance-covariance matrix and mean [Rocke96]. Use one iteration of the Maronna algorithm with the reweighting step [Maronna02] to compute the initial point of the algorithm. Pack the parameters of the TBS algorithm into the *params* array and pass them into the editor. [Table "Structure of the Array of TBS Parameters"](#) describes the *params* structure.

### Structure of the Array of TBS Parameters

| Array Position | Algorithm Parameter | Description                                                                                                                 |
|----------------|---------------------|-----------------------------------------------------------------------------------------------------------------------------|
| 0              | $\epsilon$          | Breakdown point, the number of outliers the algorithm can hold. By default, the value is $(n-p) / (2n)$ .                   |
| 1              | $\alpha$            | Asymptotic rejection probability, see details in [Rocke96]. By default, the value is 0.001.                                 |
| 2              | $\delta$            | Stopping criterion: the algorithm is terminated if weights are changed less than $\delta$ . By default, the value is 0.001. |
| 3              | <i>max_iter</i>     | Maximum number of iterations. The algorithm terminates after <i>max_iter</i> iterations. By default, the value is 10.       |

| Array Position | Algorithm Parameter | Description                                                                                                                                                     |
|----------------|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                |                     | If you set this parameter to zero, the function returns a robust estimate of the variance-covariance matrix computed using the Maronna method [Maronna02] only. |

The robust estimator of variance-covariance implementation in Intel MKL requires that the number of observations  $n$  be greater than twice the number of variables:  $n > 2p$ .

See additional details of the algorithm usage model in the *Intel® MKL Summary Statistics Application Notes* document [SS Notes].

### vsLSSEditOutliersDetection

*Modifies array pointers related to multivariate outliers detection.*

#### Syntax

```
status = vsLSSEditOutliersDetection(task, nparams, params, w);
```

```
status = vsLDSEditOutliersDetection(task, nparams, params, w);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name           | Type                                                                                                  | Description                                                                                             |
|----------------|-------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------|
| <i>task</i>    | VSLSTaskPtr                                                                                           | Descriptor of the task                                                                                  |
| <i>nparams</i> | const MKL_INT*                                                                                        | Pointer to the number of method parameters                                                              |
| <i>params</i>  | const float* for<br>vsLSSEditOutliersDetection<br><br>const double* for<br>vsLDSEditOutliersDetection | Pointer to the array of method parameters                                                               |
| <i>w</i>       | float* for<br>vsLSSEditOutliersDetection<br><br>double* for<br>vsLDSEditOutliersDetection             | Pointer to an array of size $n$ . The array holds the weights of observations to be marked as outliers. |

#### Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

#### Description

The `vsLSSEditOutliersDetection` routine uses the parameters passed to replace

- the pointers to the number of method parameters and to the array of the method parameters of size *nparams*

- the pointer to the array that holds the calculated weights of the observations

If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

Intel MKL provides the BACON algorithm ([Billor00]) for the detection of multivariate outliers. Pack the parameters of the BACON algorithm into the `params` array and pass them into the editor. Table "Structure of the Array of BACON Parameters" describes the `params` structure.

### Structure of the Array of BACON Parameters

| Array Position | Algorithm Parameter           | Description                                                                                                                                                                                                                                                                                                                                |
|----------------|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0              | Method to start the algorithm | The parameter takes one of the following possible values:<br><br><code>VSL_SS_METHOD_BACON_MEDIAN_INIT</code> , if the algorithm is started using the median estimate. This is the default value of the parameter.<br><br><code>VSL_SS_METHOD_BACON_MAHALANOBIS_INIT</code> , if the algorithm is started using the Mahalanobis distances. |
| 1              | $\alpha$                      | One-tailed probability that defines the $(1 - \alpha)$ quantile of $\chi^2$ distribution with $p$ degrees of freedom. The recommended value is $\alpha/n$ , where $n$ is the number of observations. By default, the value is 0.05.                                                                                                        |
| 2              | $\delta$                      | Stopping criterion; the algorithm is terminated if the size of the basic subset is changed less than $\delta$ . By default, the value is 0.005.                                                                                                                                                                                            |

Output of the algorithm is the vector of weights, `BaconWeights`, such that `BaconWeights(i) = 0` if  $i$ -th observation is detected as an outlier. Otherwise `BaconWeights(i) = w(i)`, where  $w$  is the vector of input weights. If you do not provide the vector of input weights, `BaconWeights(i)` is set to 1 if the  $i$ -th observation is not detected as an outlier.

See additional details about usage model of the algorithm in the *Intel(R) MKL Summary Statistics Application Notes* document [SS Notes].

### vslsSSEditMissingValues

*Modifies pointers to arrays associated with the method of supporting missing values in a dataset.*

#### Syntax

```
status = vslsSSEditMissingValues(task, nparams, params, init_estimates_n,
init_estimates, prior_n, prior, simul_missing_vals_n, simul_missing_vals, estimates_n,
estimates);
```

```
status = vsldSSEditMissingValues(task, nparams, params, init_estimates_n,
init_estimates, prior_n, prior, simul_missing_vals_n, simul_missing_vals, estimates_n,
estimates);
```

#### Include Files

- `mkl.h`

## Input Parameters

| Name                        | Type                                                                                            | Description                                                                                                                                                                                                               |
|-----------------------------|-------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>                 | VSLSSTaskPtr                                                                                    | Descriptor of the task                                                                                                                                                                                                    |
| <i>nparams</i>              | const MKL_INT*                                                                                  | Pointer to the number of method parameters                                                                                                                                                                                |
| <i>params</i>               | const float* for<br>vslsSSEditMissingValues<br><br>const double* for<br>vsldSSEditMissingValues | Pointer to the array of method parameters                                                                                                                                                                                 |
| <i>init_estimates_n</i>     | const MKL_INT*                                                                                  | Pointer to the number of initial estimates for mean and a variance-covariance matrix                                                                                                                                      |
| <i>init_estimates</i>       | const float* for<br>vslsSSEditMissingValues<br><br>const double* for<br>vsldSSEditMissingValues | Pointer to the array that holds initial estimates for mean and a variance-covariance matrix                                                                                                                               |
| <i>prior_n</i>              | const MKL_INT*                                                                                  | Pointer to the number of prior parameters                                                                                                                                                                                 |
| <i>prior</i>                | const float* for<br>vslsSSEditMissingValues<br><br>const double* for<br>vsldSSEditMissingValues | Pointer to the array of prior parameters                                                                                                                                                                                  |
| <i>simul_missing_vals_n</i> | const MKL_INT*                                                                                  | Pointer to the size of the array that holds output of the Multiple Imputation method                                                                                                                                      |
| <i>simul_missing_vals</i>   | float* for vslsSSEditMissingValues<br>double* for vsldSSEditMissingValues                       | Pointer to the array of size $k*m$ , where $k$ is the total number of missing values, and $m$ is number of copies of missing values. The array holds $m$ sets of simulated missing values for the matrix of observations. |
| <i>estimates_n</i>          | const MKL_INT*                                                                                  | Pointer to the number of estimates to be returned by the routine                                                                                                                                                          |
| <i>estimates</i>            | float* for vslsSSEditMissingValues<br>double* for vsldSSEditMissingValues                       | Pointer to the array that holds estimates of the mean and a variance-covariance matrix.                                                                                                                                   |

## Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

## Description

The `vslSSEditMissingValues` routine uses values passed as parameters of the routine to replace pointers to the number and the array of the method parameters, pointers to the number and the array of initial mean/variance-covariance estimates, the pointer to the number and the array of prior parameters, pointers to the number and the array of simulated missing values, and pointers to the number and the array of the intermediate mean/covariance estimates. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

Before you call the Summary Statistics routines to process missing values, preprocess the dataset and denote missing observations with one of the following predefined constants:

- `VSL_SS_SNAN`, if the dataset is stored in single precision floating-point arithmetic
- `VSL_SS_DNAN`, if the dataset is stored in double precision floating-point arithmetic

Intel MKL provides the `VSL_SS_METHOD_MI` method to support missing values in the dataset based on the Multiple Imputation (MI) approach described in [Schafer97]. The following components support Multiple Imputation:

- Expectation Maximization (EM) algorithm to compute the start point for the Data Augmentation (DA) procedure
- DA function

---

### NOTE

The DA component of the MI procedure is simulation-based and uses the `VSL_BRNG_MCG59` basic random number generator with predefined  $seed = 2^{50}$  and the Gaussian distribution generator (`ICDF` method) available in Intel MKL [Gaussian].

---

Pack the parameters of the MI algorithm into the `params` array. Table "Structure of the Array of MI Parameters" describes the `params` structure.

### Structure of the Array of MI Parameters

| Array Position | Algorithm Parameter                | Description                                                                                                                                                                                                                         |
|----------------|------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0              | <code>em_iter_num</code>           | Maximal number of iterations for the EM algorithm. By default, this value is 50.                                                                                                                                                    |
| 1              | <code>da_iter_num</code>           | Maximal number of iterations for the DA algorithm. By default, this value is 30.                                                                                                                                                    |
| 2              | <code><math>\epsilon</math></code> | Stopping criterion for the EM algorithm. The algorithm terminates if the maximal module of the element-wise difference between the previous and current parameter values is less than $\epsilon$ . By default, this value is 0.001. |
| 3              | <code>m</code>                     | Number of sets to impute                                                                                                                                                                                                            |
| 4              | <code>missing_vals_num</code>      | Total number of missing values in the datasets                                                                                                                                                                                      |

You can also pass initial estimates into the EM algorithm by packing both the vector of means and the variance-covariance matrix as a one-dimensional array `init_estimates`. The size of the array should be at least  $p + p(p + 1)/2$ . For  $i=0, \dots, p-1$ , the `init_estimates[i]` array contains the initial estimate of means. The remaining positions of the array are occupied by the upper triangular part of the variance-covariance matrix.

If you provide no initial estimates for the EM algorithm, the editor uses the default values, that is, the vector of zero means and the unitary matrix as a variance-covariance matrix. You can also pass *prior* parameters for  $\mu$  and  $\Sigma$  into the library:  $\mu_0$ ,  $\tau$ ,  $m$ , and  $\Lambda^{-1}$ . Pack these parameters as a one-dimensional array *prior* with a size of at least

$$(p^2 + 3p + 4)/2.$$

The storage format is as follows:

- *prior*[0], ..., *prior*[*p*-1] contain the elements of the vector  $\mu_0$ .
- *prior*[*p*] contains the parameter  $\tau$ .
- *prior*[*p*+1] contains the parameter  $m$ .
- The remaining positions are occupied by the upper-triangular part of the inverted matrix  $\Lambda^{-1}$ .

If you provide no *prior* parameters, the editor uses their default values:

- The array of  $p$  zeros is used as  $\mu_0$ .
- $\tau$  is set to 0.
- $m$  is set to  $p$ .
- The zero matrix is used as an initial approximate of  $\Lambda^{-1}$ .

The `EditMissingValues` editor returns  $m$  sets of imputed values and/or a sequence of parameter estimates drawn during the DA procedure.

The editor returns the imputed values as the *simul\_missing\_vals* array. The size of the array should be sufficient to hold  $m$  sets each of the *missing\_vals\_num* size, that is, at least  $m \cdot \text{missing\_vals\_num}$  in total. The editor packs the imputed values one by one in the order of their appearance in the matrix of observations.

For example, consider a task of dimension 4. The total number of observations  $n$  is 10. The second observation vector misses variables 1 and 2, and the seventh observation vector lacks variable 1. The number of sets to impute is  $m=2$ . Then, *simul\_missing\_vals*[0] and *simul\_missing\_vals*[1] contains the first and the second points for the second observation vector, and *simul\_missing\_vals*[2] holds the first point for the seventh observation. Positions 3, 4, and 5 are formed similarly.

To estimate convergence of the DA algorithm and choose a proper value of the number of DA iterations, request the sequence of parameter estimates that are produced during the DA procedure. The editor returns the sequence of parameters as a single array. The size of the array is

$$m \cdot \text{da\_iter\_num} \cdot (p + (p^2 + p) / 2)$$

where

- $m$  is the number of sets of values to impute.
- *da\_iter\_num* is the number of DA iterations.
- The value  $p + (p^2 + p) / 2$  determines the size of the memory to hold one set of the parameter estimates.

In each set of the parameters, the vector of means occupies the first  $p$  positions and the remaining  $(p^2 + p) / 2$  positions are intended for the upper triangular part of the variance-covariance matrix.

Upon successful generation of  $m$  sets of imputed values, you can place them in cells of the data matrix with missing values and use the Summary Statistics routines to analyze and get estimates for each of the  $m$  complete datasets.

---

**NOTE**

Intel MKL implementation of the MI algorithm rewrites cells of the dataset that contain the `VSL_SS_SNaN/VSL_SS_DNaN` values. If you want to use the Summary Statistics routines to process the data with missing values again, mask the positions of the empty cells.

---

See additional details of the algorithm usage model in the *Intel® MKL Summary Statistics Application Notes* document [[SS Notes](#)].

## vsSSEditCorParameterization

Modifies pointers to arrays related to the algorithm of correlation matrix parameterization.

### Syntax

```
status = vsSSEditCorParameterization(task, cor, cor_storage, pcor, pcor_storage);
```

```
status = vsdSSEditCorParameterization(task, cor, cor_storage, pcor, pcor_storage);
```

### Include Files

- mkl.h

### Input Parameters

| Name               | Type                                                                                                 | Description                                                           |
|--------------------|------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------|
| <i>task</i>        | VSLSTaskPtr                                                                                          | Descriptor of the task                                                |
| <i>cor</i>         | const float* for<br>vsSSEditCorParameterization<br>const double* for<br>vsdSSEditCorParameterization | Pointer to the correlation matrix                                     |
| <i>cor_storage</i> | const MKL_INT*                                                                                       | Pointer to the storage format of the correlation matrix               |
| <i>pcor</i>        | float* for<br>vsSSEditCorParameterization<br>double* for<br>vsdSSEditCorParameterization             | Pointer to the parameterized correlation matrix                       |
| <i>por_storage</i> | const MKL_INT*                                                                                       | Pointer to the storage format of the parameterized correlation matrix |

### Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

### Description

The `vsSSEditCorParameterization` routine uses values passed as parameters of the routine to replace pointers to the correlation matrix, pointers to the correlation matrix storage format, a pointer to the parameterized correlation matrix, and a pointer to the parameterized correlation matrix storage format. See [Table "Storage formats of a variance-covariance/correlation matrix"](#) for possible values of the `cor_storage` and `pcor_storage` parameters. If you pass a value of `NULL` for a specific input parameter, the value of that parameter in the task descriptor is unchanged.

## Summary Statistics Task Computation Routines

Task computation routines calculate statistical estimates on the data provided and parameters held in the task descriptor. After you create the task and initialize its parameters, you can call the computation routines as many times as necessary. [Table "Summary Statistics Estimates Obtained with vsSCompute Routine"](#) lists the statistical estimates that you can obtain using the `vsSCompute` routine.

**NOTE**

The Summary Statistics computation routines do not signal floating-point errors, such as overflow or gradual underflow, or operations with NaNs (except for the missing values in the observations).

---

**Summary Statistics Estimates Obtained with vsISSCompute Routine**


---

| <b>Estimate</b>  | <b>Support of Observations Available in Blocks</b> | <b>Description</b>                                                                                                          |
|------------------|----------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_MEAN      | Yes                                                | Computes the array of means.                                                                                                |
| VSL_SS_SUM       | Yes                                                | Computes the array of sums.                                                                                                 |
| VSL_SS_2R_MOM    | Yes                                                | Computes the array of the 2 <sup>nd</sup> order raw moments.                                                                |
| VSL_SS_2R_SUM    | Yes                                                | Computes the array of raw sums of the 2 <sup>nd</sup> order.                                                                |
| VSL_SS_3R_MOM    | Yes                                                | Computes the array of the 3 <sup>rd</sup> order raw moments.                                                                |
| VSL_SS_3R_SUM    | Yes                                                | Computes the array of raw sums of the 3 <sup>rd</sup> order.                                                                |
| VSL_SS_4R_MOM    | Yes                                                | Computes the array of the 4 <sup>th</sup> order raw moments.                                                                |
| VSL_SS_4R_SUM    | Yes                                                | Computes the array of raw sums of the 4 <sup>th</sup> order.                                                                |
| VSL_SS_2C_MOM    | Yes                                                | Computes the array of the 2 <sup>nd</sup> order central moments.                                                            |
| VSL_SS_2C_SUM    | Yes                                                | Computes the array of central sums of the 2 <sup>nd</sup> order.                                                            |
| VSL_SS_3C_MOM    | Yes                                                | Computes the array of the 3 <sup>rd</sup> order central moments.                                                            |
| VSL_SS_3C_SUM    | Yes                                                | Computes the array of central sums of the 3 <sup>rd</sup> order.                                                            |
| VSL_SS_4C_MOM    | Yes                                                | Computes the array of the 4 <sup>th</sup> order central moments.                                                            |
| VSL_SS_4C_SUM    | Yes                                                | Computes the array of central sums of the 4 <sup>th</sup> order.                                                            |
| VSL_SS_KURTOSIS  | Yes                                                | Computes the array of kurtosis values.                                                                                      |
| VSL_SS_SKEWNESS  | Yes                                                | Computes the array of skewness values.                                                                                      |
| VSL_SS_MIN       | Yes                                                | Computes the array of minimum values.                                                                                       |
| VSL_SS_MAX       | Yes                                                | Computes the array of maximum values.                                                                                       |
| VSL_SS_VARIATION | Yes                                                | Computes the array of variation coefficients.                                                                               |
| VSL_SS_COV       | Yes                                                | Computes a covariance matrix.                                                                                               |
| VSL_SS_COR       | Yes                                                | Computes a correlation matrix. The main diagonal of the correlation matrix holds variances of the random vector components. |

| Estimate             | Support of Observations Available in Blocks | Description                                                      |
|----------------------|---------------------------------------------|------------------------------------------------------------------|
| VSL_SS_CP            | Yes                                         | Computes a cross-product matrix.                                 |
| VSL_SS_POOLED_COV    | No                                          | Computes a pooled covariance matrix.                             |
| VSL_SS_POOLED_MEAN   | No                                          | Computes an array of pooled means.                               |
| VSL_SS_GROUP_COV     | No                                          | Computes group covariance matrices.                              |
| VSL_SS_GROUP_MEAN    | No                                          | Computes group means.                                            |
| VSL_SS_QUANTS        | No                                          | Computes quantiles.                                              |
| VSL_SS_ORDER_STATS   | No                                          | Computes order statistics.                                       |
| VSL_SS_ROBUST_COV    | No                                          | Computes a robust covariance matrix.                             |
| VSL_SS_OUTLIERS      | No                                          | Detects outliers in the dataset.                                 |
| VSL_SS_PARTIAL_COV   | No                                          | Computes a partial covariance matrix.                            |
| VSL_SS_PARTIAL_COR   | No                                          | Computes a partial correlation matrix.                           |
| VSL_SS_MISSING_VALS  | No                                          | Supports missing values in datasets.                             |
| VSL_SS_PARAMTR_COR   | No                                          | Computes a parameterized correlation matrix.                     |
| VSL_SS_STREAM_QUANTS | Yes                                         | Computes quantiles for streaming data.                           |
| VSL_SS_MDAD          | No                                          | Computes median absolute deviation.                              |
| VSL_SS_MNAD          | No                                          | Computes mean absolute deviation.                                |
| VSL_SS_SORTED_OBSERV | No                                          | Sorts the dataset by the components of the random vector $\xi$ . |

Table "Summary Statistics Computation Method" lists estimate calculation methods supported by Intel MKL. See the *Intel(R) MKL Summary Statistics Application Notes* document [SS Notes] for a detailed description of the methods.

### Summary Statistics Computation Method

| Method                       | Description                                                                                                                                                                                                                                                                                                                           |
|------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| VSL_SS_METHOD_FAST           | Fast method for calculation of the estimates: <ul style="list-style-type: none"> <li>raw/central moments/sums, skewness, kurtosis, variation, variance-covariance/correlation/cross-product matrix</li> <li>min/max/quantile/order statistics</li> <li>partial variance-covariance</li> <li>median/mean absolute deviation</li> </ul> |
| VSL_SS_METHOD_FAST_USER_MEAN | Fast method for calculation of the estimates given user-defined mean: <ul style="list-style-type: none"> <li>central moments/sums of 2-4 order, skewness, kurtosis, variation, variance-covariance/correlation/cross-product matrix, mean absolute deviation</li> </ul>                                                               |
| VSL_SS_METHOD_1PASS          | One-pass method for calculation of estimates:                                                                                                                                                                                                                                                                                         |

| Method                        | Description                                                                                                                                                                                             |
|-------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                               | <ul style="list-style-type: none"> <li>raw/central moments/sums, skewness, kurtosis, variation, variance-covariance/correlation/cross-product matrix</li> <li>pooled/group covariance matrix</li> </ul> |
| VSL_SS_METHOD_TBS             | TBS method for robust estimation of covariance and mean                                                                                                                                                 |
| VSL_SS_METHOD_BACON           | BACON method for detection of multivariate outliers                                                                                                                                                     |
| VSL_SS_METHOD_MI              | Multiple imputation method for support of missing values                                                                                                                                                |
| VSL_SS_METHOD_SD              | Spectral decomposition method for parameterization of a correlation matrix                                                                                                                              |
| VSL_SS_METHOD_SQUANTS_ZW      | Zhang-Wang (ZW) method for quantile estimation for streaming data                                                                                                                                       |
| VSL_SS_METHOD_SQUANTS_ZW_FAST | Fast ZW method for quantile estimation for streaming data                                                                                                                                               |
| VSL_SS_METHOD_RADIX           | Radix method for dataset sorting                                                                                                                                                                        |

You can calculate all requested estimates in one call of the routine. For example, to compute a kurtosis and covariance matrix using a fast method, pass a combination of the pre-defined parameters into the `Compute` routine as shown in the example below:

```
...
method = VSL_SS_METHOD_FAST;
task_params = VSL_SS_KURTOSIS|VSL_SS_COV;
...
status = vsldSSCompute(task, task_params, method);
```

To compute statistical estimates for the next block of observations, you can do one of the following:

- copy the observations to memory, starting with the address available to the task
- use one of the appropriate [Editors](#) to modify the pointer to the new dataset in the task.

The library does not detect your changes of the dataset and computed statistical estimates. To obtain statistical estimates for a new matrix, change the observations and initialize relevant arrays. You can follow this procedure to compute statistical estimates for observations that come in portions. See [Table "Summary Statistics Estimates Obtained with vsldSSCompute Routine"](#) for information on such observations supported by the Intel MKL Summary Statistics estimators.

To modify parameters of the task using the Task Editors, set the address of the targeted matrix of the observations or change the respective vector component indices. After you complete editing the task parameters, you can compute statistical estimates in the modified environment.

If the task completes successfully, the computation routine returns the zero status code. If an error is detected, the computation routine returns an error code. In particular, an error status code is returned in the following cases:

- the task pointer is `NULL`
- memory allocation has failed
- the calculation has failed for some other reason

#### NOTE

You can use the `NULL` task pointer in calls to editor routines. In this case, the routine is terminated and no system crash occurs.

## vslSSCompute

Computes Summary Statistics estimates.

---

### Syntax

```
status = vslsSSCompute(task, estimates, method);
```

```
status = vsldSSCompute(task, estimates, method);
```

### Include Files

- mkl.h

### Input Parameters

| Name             | Type            | Description                              |
|------------------|-----------------|------------------------------------------|
| <i>task</i>      | VSLSSTaskPtr    | Descriptor of the task                   |
| <i>estimates</i> | const MKL_INT64 | List of statistical estimates to compute |
| <i>method</i>    | const MKL_INT   | Method to be used in calculations        |

### Output Parameters

| Name          | Type | Description                |
|---------------|------|----------------------------|
| <i>status</i> | int  | Current status of the task |

### Description

The `vslSSCompute` routine calculates statistical estimates passed as the *estimates* parameter using the algorithms passed as the *method* parameter of the routine. The computations are done in the context of the task descriptor that contains pointers to all required and optional, if necessary, properly initialized arrays. In one call of the function, you can compute several estimates using proper methods for their calculation. See [Table "Summary Statistics Estimates Obtained with Compute Routine"](#) for the list of the estimates that you can calculate with the `vslSSCompute` routine. See [Table "Summary Statistics Computation Methods"](#) for the list of possible values of the *method* parameter.

To initialize single or double precision version task parameters, use the single (`vslssscompute`) or double (`vsldsscompute`) version of the editor, respectively. To initialize parameters of the integer type, use an integer version of the editor (`vslisscompute`).

---

**NOTE**

Requesting a combination of the `VSL_SS_MISSING_VALS` value and any other estimate parameter in the `Compute` function results in processing only the missing values.

---

### Application Notes

Be aware that when computing a correlation matrix, the `vslSSCompute` routine allocates an additional array for each thread which is running the task. If you are running on a large number of threads `vslSSCompute` might consume large amounts of memory.

When calculating covariance, correlation, or cross product, the number of bytes of memory required is at least  $(P*P*T + P*T)*b$ , where  $P$  is the dimension of the task or number of variables,  $T$  is the number of threads, and  $b$  is the number of bytes required for each unit of data. If observation is weighted and the method is `VSL_SS_METHOD_FAST`, then the memory required is at least  $(P*P*T + P*T + N*P)*b$ , where  $N$  is the number of observations.

## Summary Statistics Task Destructor

Task destructor is the `vslSSDeleteTask` routine intended to delete task objects and release memory.

### `vslSSDeleteTask`

*Destroys the task object and releases the memory.*

### Syntax

```
status = vslSSDeleteTask(&task);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name              | Type                       | Description                       |
|-------------------|----------------------------|-----------------------------------|
| <code>task</code> | <code>VSLSSTaskPtr*</code> | Descriptor of the task to destroy |

### Output Parameters

| Name                | Type             | Description                                                                                       |
|---------------------|------------------|---------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code> | Sets to <code>VSL_STATUS_OK</code> if the task is deleted; otherwise a non-zero code is returned. |

### Description

The `vslSSDeleteTask` routine deletes the task descriptor object, releases the memory allocated for the structure, and sets the task pointer to `NULL`. If `vslSSDeleteTask` fails to delete the task successfully, it returns an error code.

#### NOTE

Call of the destructor with the `NULL` pointer as the parameter results in termination of the function with no system crash.

## Summary Statistics Usage Examples

The following examples show various standard operations with Summary Statistics routines.

### Calculating Fixed Estimates for Fixed Data

The example shows recurrent calculation of the same estimates with a given set of variables for the complete life cycle of the task in the case of a variance-covariance matrix. The set of vector components to process remains unchanged, and the data comes in blocks. Before you call the `vslSSCompute` routine, initialize pointers to arrays for mean and covariance and set buffers.

```
...
double w[2];
double indices[DIM] = {1, 0, 1};

/* calculating mean for 1st and 3d random vector components */
```

```

/* Initialize parameters of the task */
p = DIM;
n = N;

xstorage = VSL_SS_MATRIX_STORAGE_ROWS;
covstorage = VSL_SS_MATRIX_STORAGE_FULL;

w[0] = 0.0; w[1] = 0.0;

for (i = 0; i < p; i++) mean[i] = 0.0;
for (i = 0; i < p*p; i++) cov[i] = 0.0;

status = vsldSSNewTask(&task, &p, &n, &xstorage, x, 0, indices);

status = vsldSSEditTask (task, VSL_SS_ED_ACCUM_WEIGHT, w);
status = vsldSSEditCovCor(task, mean, cov, &covstorage, 0, 0);

```

You can process data arrays that come in blocks as follows:

```

for (i = 0; i < num_of_blocks; i++)
{
 status = vsldSSCompute(task, VSL_SS_COV, VSL_SS_METHOD_FAST);
 /* Read new data block into array x */
}
...

```

### Calculating Different Estimates for Variable Data

The context of your calculation may change in the process of data analysis. The example below shows the data that comes in two blocks. You need to estimate a covariance matrix for the complete data, and the third central moment for the second block of the data using the weights that were accumulated for the previous datasets. The second block of the data is stored in another array. You can proceed as follows:

```

/* Set parameters for the task */
p = DIM;
n = N;
xstorage = VSL_SS_MATRIX_STORAGE_ROWS;
covstorage = VSL_SS_MATRIX_STORAGE_FULL;

w[0] = 0.0; w[1] = 0.0;

for (i = 0; i < p; i++) mean[i] = 0.0;
for (i = 0; i < p*p; i++) cov[i] = 0.0;

/* Create task */
status = vsldSSNewTask(&task, &p, &n, &xstorage, x1, 0, indices);

/* Initialize the task parameters */
status = vsldSSEditTask(task, VSL_SS_ED_ACCUM_WEIGHT, w);
status = vsldSSEditCovCor(task, mean, cov, &covstorage, 0, 0);

/* Calculate covariance for the x1 data */
status = vsldSSCompute(task, VSL_SS_COV, VSL_SS_METHOD_FAST);

/* Initialize array of the 3d central moments and pass the pointer to the task */
for (i = 0; i < p; i++) c3_m[i] = 0.0;

/* Modify task context */
status = vsldSSEditTask(task, VSL_SS_ED_3C_MOM, c3_m);

```

```

status = vsldSSEditTask(task, VSL_SS_ED_OBSERV, x2);

/* Calculate covariance for the x1 & x2 data block */
/* Calculate the 3d central moment for the 2nd data block using earlier accumulated weight */
status = vsldSSCompute(task, VSL_SS_COV|VSL_SS_3C_MOM, VSL_SS_METHOD_FAST);
...
status = vsldSSDeleteTask(&task);

```

Similarly, you can modify indices of the variables to be processed for the next data block.

## Summary Statistics Mathematical Notation and Definitions

The following notations are used in the mathematical definitions and the description of the Intel MKL Summary Statistics functions.

### Matrix and Weights of Observations

For a random  $p$ -dimensional vector  $\xi = (\xi_1, \dots, \xi_j, \dots, \xi_p)$ , this manual denotes the following:

- $(X)_i = (x_{ij})_{j=1..n}$  is the result of  $n$  independent observations for the  $i$ -th component  $\xi_i$  of the vector  $\xi$ .
- The two-dimensional array  $X = (x_{ij})_{n \times p}$  is the matrix of observations.
- The column  $[X]_j = (x_{ij})_{i=1..n}$  of the matrix  $X$  is the  $j$ -th observation of the random vector  $\xi$ .

Each observation  $[X]_j$  is assigned a non-negative weight  $w_j$ , where

- The vector  $(w_j)_{j=1..n}$  is a vector of weights corresponding to  $n$  observations of the random vector  $\xi$ .

- $$W = \sum_{i=1}^n w_i$$

is the accumulated weight corresponding to observations  $X$ .

### Vector of sample means

$$M(X) = (M_1(X), \dots, M_p(X)) \text{ with } M_i(X) = \frac{1}{w} \sum_{j=1}^n w_j x_{ij}$$

for all  $i = 1, \dots, p$ .

### Vector of sample partial sums

$$S(X) = (S_1(X), \dots, S_p(X)) \text{ with } S_i(X) = \sum_{j=1}^n w_j x_{ij}$$

for all  $i = 1, \dots, p$ .

### Vector of sample variances

$$V(X) = (V_1(X), \dots, V_p(X)) \text{ with } V_i(X) = \frac{1}{B} \sum_{j=1}^n w_j (x_{ij} - M_i(X))^2, B = W - \sum_{j=1}^n w_j^2 / W$$

for all  $i = 1, \dots, p$ .

### Vector of sample raw/algebraic moments of $k$ -th order, $k \geq 1$

$$R^{(k)}(X) = (R_1^{(k)}(X), \dots, R_p^{(k)}(X)) \text{ with } R_i^{(k)}(X) = \frac{1}{W} \sum_{j=1}^n w_j x_{ij}^k$$

for all  $i = 1, \dots, p$ .

**Vector of sample raw/algebraic partial sums of  $k$ -th order,  $k= 2, 3, 4$  (raw/algebraic partial sums of squares/cubes/fourth powers)**

$$S^k(X) = (S_1^k(X), \dots, S_p^k(X)) \text{ with } S_i^k(X) = \sum_{j=1}^n w_j x_{ij}^k$$

for all  $i = 1, \dots, p$ .

**Vector of sample central moments of the third and the fourth order**

$$C^{(k)}(X) = (C_1^{(k)}(X), \dots, C_p^{(k)}(X)) \text{ with } C_i^{(k)}(X) = \frac{1}{B} \sum_{j=1}^n w_j (x_{ij} - M_i(X))^k, B = \sum_{j=1}^n w_j$$

for all  $i = 1, \dots, p$  and  $k = 3, 4$ .

**Vector of sample central partial sums of  $k$ -th order,  $k= 2, 3, 4$  (central partial sums of squares/cubes/fourth powers)**

$$S^k(X) = (S_1^k(X), \dots, S_p^k(X)) \text{ with } S_i^k(X) = \sum_{j=1}^n w_j (x_{ij} - S_i(X))^k$$

for all  $i = 1, \dots, p$ .

**Vector of sample excess kurtosis values**

$$B(X) = (B_1(X), \dots, B_p(X)) \text{ with } B_i(X) = \frac{C_i^{(4)}(X)}{V_i^2(X)} - 3$$

for all  $i = 1, \dots, p$ .

**Vector of sample skewness values**

$$\Gamma(X) = (\Gamma_1(X), \dots, \Gamma_p(X)) \text{ with } \Gamma_i(X) = \frac{C_i^{(3)}(X)}{V_i^{1.5}(X)}$$

for all  $i = 1, \dots, p$ .

**Vector of sample variation coefficients**

$$VC(X) = (VC_1(X), \dots, VC_p(X)) \text{ with } VC_i(X) = \frac{V_i^{0.5}(X)}{M_i(X)}$$

for all  $i = 1, \dots, p$ .

**Matrix of order statistics**

Matrix  $Y = (Y_{ij})_{p \times n}$ , in which the  $i$ -th row  $(Y)_i = (Y_{ij})_{j=1..n}$  is obtained as a result of sorting in the ascending order of row  $(X)_i = (X_{ij})_{j=1..n}$  in the original matrix of observations.

**Vector of sample minimum values**

$$Min(X) = (Min_1(X), \dots, Min_p(X)), \text{ where } Min_i(X) = y_{i1}$$

for all  $i = 1, \dots, p$ .

**Vector of sample maximum values**

$$Max(X) = (Max_1(X), \dots, Max_p(X)), \text{ where } Max_i(X) = y_{in}$$

for all  $i = 1, \dots, p$ .

### Vector of sample median values

$$Med(X) = (Med_1(X), \dots, Med_p(X)), \text{ where } Med_i(X) = \begin{cases} y_{i, (n+1)/2}, & \text{if } n \text{ is odd} \\ (y_{i, n/2} + y_{i, n/2+1})/2, & \text{if } n \text{ is even} \end{cases}$$

for all  $i = 1, \dots, p$ .

### Vector of sample median absolute deviations

$$MDAD(X) = (MDAD_1(X), \dots, MDAD_p(X)), \text{ where } MDAD_i(X) = Med_i(Z) \text{ with } Z = (z_{ij})_{i=1\dots p, j=1\dots n'}$$

$$z_{ij} = |x_{ij} - Med_i(X)|$$

for all  $i = 1, \dots, p$ .

### Vector of sample mean absolute deviations

$$MNAD(X) = (MNAD_1(X), \dots, MNAD_p(X)), \text{ where } MNAD_i(X) = M_i(Z) \text{ with } Z = (z_{ij})_{i=1\dots p, j=1\dots n'}$$

$$z_{ij} = |x_{ij} - M_i(X)|$$

for all  $i = 1, \dots, p$ .

### Vector of sample quantile values

For a positive integer number  $q$  and  $k$  belonging to the interval  $[0, q-1]$ , point  $z_i$  is the  $k$ -th  $q$  quantile of the random variable  $\xi_i$  if  $P\{\xi_i \leq z_i\} \geq \beta$  and  $P\{\xi_i \leq z_i\} \geq 1 - \beta$ , where

- $P$  is the probability measure.
- $\beta = k/n$  is the quantile order.

The calculation of quantiles is as follows:

$j = [(n-1)\beta]$  and  $f = \{(n-1)\beta\}$  as integer and fractional parts of the number  $(n-1)\beta$ , respectively, and the vector of sample quantile values is

$$Q(X, \beta) = (Q_1(X, \beta), \dots, Q_p(X, \beta))$$

where

$$(Q_i(X, \beta) = Y_{i, j+1} + f(Y_{i, j+2} - Y_{i, j+1}))$$

for all  $i = 1, \dots, p$ .

### Variance-covariance matrix

$$C(X) = (c_{ij}(X))_{p \times p}$$

where

$$c_{ij}(X) = \frac{1}{B} \sum_{k=1}^n w_k (x_{ik} - M_i(X))(x_{jk} - M_j(X)), \quad B = W - \sum_{j=1}^n w_j^2 / W$$

### Cross-product matrix (matrix of cross-products and sums of squares)

$$CP(X) = (cp_{ij}(X))_{p \times p}$$

where

$$cp_{ij}(X) = \sum_{k=1}^n w_k (x_{ik} - M_i(X))(x_{jk} - M_j(X))$$

## Pooled and group variance-covariance matrices

The set  $N = \{1, \dots, n\}$  is partitioned into non-intersecting subsets

$$G_i, i = 1..g, N = \bigcup_{i=1}^g G_i$$

The observation  $[X]_j = (x_{ij})_{i=1..p}$  belongs to the group  $r$  if  $j \in G_r$ . One observation belongs to one group only. The group mean and variance-covariance matrices are calculated similarly to the formulas above:

$$M^{(r)}(X) = (M_1^{(r)}(X), \dots, M_p^{(r)}(X)) \text{ with } M_i^{(r)}(X) = \frac{1}{W^{(r)}} \sum_{j \in G_r} w_j x_{ij}, W^{(r)} = \sum_{j \in G_r} w_j$$

for all  $i = 1, \dots, p$ ,

$$C^{(r)}(X) = (c_{ij}^{(r)}(X))_{p \times p}$$

where

$$c_{ij}^{(r)}(X) = \frac{1}{B^{(r)}} \sum_{k \in G_r} w_k (x_{ik} - M_i^{(r)}(X))(x_{jk} - M_j^{(r)}(X)), B^{(r)} = W^{(r)} - \sum_{j \in G_r} w_j^2 / W^{(r)}$$

for all  $i = 1, \dots, p$  and  $j = 1, \dots, p$ .

A pooled variance-covariance matrix and a pooled mean are computed as weighted mean over group covariance matrices and group means, correspondingly:

$$M^{pooled}(X) = (M_1^{pooled}(X), \dots, M_p^{pooled}(X)) \text{ with } M_i^{pooled}(X) = \frac{1}{W^{(1)} + \dots + W^{(g)}} \sum_{r=1}^g W^{(r)} M_i^{(r)}(X)$$

for all  $i = 1, \dots, p$ ,

$$C^{pooled}(X) = (c_{ij}^{pooled}(X))_{p \times p}, c_{ij}^{pooled}(X) = \frac{1}{B^{(1)} + \dots + B^{(g)}} \sum_{r=1}^g B^{(r)} c_{ij}^{(r)}(X)$$

for all  $i = 1, \dots, p$  and  $j = 1, \dots, p$ .

## Correlation matrix

$$R(X) = (r_{ij}(X))_{p \times p}, \text{ where } r_{ij}(X) = \frac{c_{ij}}{\sqrt{c_{ii}c_{jj}}}$$

for all  $i = 1, \dots, p$  and  $j = 1, \dots, p$ .

## Partial variance-covariance matrix

For a random vector  $\xi$  partitioned into two components  $Z$  and  $Y$ , a variance-covariance matrix  $C$  describes the structure of dependencies in the vector  $\xi$ :

$$C(X) = \begin{pmatrix} C_Z(X) & C_{ZY}(X) \\ C_{YZ}(X) & C_Y(X) \end{pmatrix}.$$

The partial covariance matrix  $P(X) = (p_{ij}(X))_{k \times k}$  is defined as

$$P(X) = C_Y(X) - C_{YZ}(X)C_Z^{-1}C_{ZY}(X).$$

where  $k$  is the dimension of  $Y$ .

## Partial correlation matrix

The following is a partial correlation matrix for all  $i = 1, \dots, k$  and  $j = 1, \dots, k$ :

$$RP(X) = (rp_{ij}(X))_{k \times k}, \text{ where } rp_{ij}(X) = \frac{p_{ij}(X)}{\sqrt{p_{ii}(X)p_{jj}(X)}}$$

where

- $k$  is the dimension of  $Y$ .
- $\rho_{ij}(X)$  are elements of the partial variance-covariance matrix.

### Sorted dataset

Matrix  $Y = (y_{ij})_{p \times n}$ , in which the  $i$ -th row  $(Y)_i$  is obtained as a result of sorting in ascending order the row  $(X)_i = (x_{ij})_{j=1..n}$  in the original matrix of observations.



# Fourier Transform Functions

The general form of the discrete Fourier transform is

$$z_{k_1, k_2, \dots, k_d} = \sigma \times \sum_{j_d=0}^{n_d-1} \dots \sum_{j_2=0}^{n_2-1} \sum_{j_1=0}^{n_1-1} w_{j_1, j_2, \dots, j_d} \exp\left(\delta i 2\pi \sum_{l=1}^d j_l k_l / n_l\right)$$

for  $k_l = 0, \dots, n_l-1$  ( $l = 1, \dots, d$ ), where  $\sigma$  is a scale factor,  $\delta = -1$  for the forward transform, and  $\delta = +1$  for the inverse (backward) transform. In the forward transform, the input (periodic) sequence  $\{w_{j_1, j_2, \dots, j_d}\}$  belongs to the set of complex-valued sequences and real-valued sequences. Respective domains for the backward transform are represented by complex-valued sequences and complex-valued conjugate-even sequences.

The Intel® Math Kernel Library (Intel® MKL) provides an interface for computing a discrete Fourier transform through the fast Fourier transform algorithm. Prefixes `Dfti` in function names and `DFTI` in the names of configuration parameters stand for Discrete Fourier Transform Interface.

This chapter describes the following implementations of the fast Fourier transform functions available in Intel MKL:

- Fast Fourier transform (FFT) functions for single-processor or shared-memory systems (see [FFT Functions](#))
- [Cluster FFT functions](#) for distributed-memory architectures (available only for Intel® 64 and Intel® Many Integrated Core architectures)

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## NOTE

Intel MKL also supports the FFTW3\* interfaces to the fast Fourier transform functionality for shared memory paradigm (SMP) systems.

---

Both FFT and Cluster FFT functions compute an FFT in five steps:

1. Allocate a fresh descriptor for the problem with a call to the `DftiCreateDescriptor` or `DftiCreateDescriptorDM` function. The descriptor captures the configuration of the transform, such as the dimensionality (or rank), sizes, number of transforms, memory layout of the input/output data (defined by strides), and scaling factors. Many of the configuration settings are assigned default values in this call which you might need to modify in your application.
2. Optionally adjust the descriptor configuration with a call to the `DftiSetValue` or `DftiSetValueDM` function as needed. Typically, you must carefully define the data storage layout for an FFT or the data distribution among processes for a Cluster FFT. The configuration settings of the descriptor, such as the default values, can be obtained with the `DftiGetValue` or `DftiGetValueDM` function.
3. Commit the descriptor with a call to the `DftiCommitDescriptor` or `DftiCommitDescriptorDM` function, that is, make the descriptor ready for the transform computation. Once the descriptor is committed, the parameters of the transform, such as the type and number of transforms, strides and distances, the type and storage layout of the data, and so on, are "frozen" in the descriptor.
4. Compute the transform with a call to the `DftiComputeForward/DftiComputeBackward` or `DftiComputeForwardDM/DftiComputeBackwardDM` functions as many times as needed. Because the descriptor is defined and committed separately, all that the compute functions do is take the input and output data and compute the transform as defined. To modify any configuration parameters for another call to a compute function, use `DftiSetValue` followed by `DftiCommitDescriptor` (`DftiSetValueDM` followed by `DftiCommitDescriptorDM`) or create and commit another descriptor.
5. Deallocate the descriptor with a call to the `DftiFreeDescriptor` or `DftiFreeDescriptorDM` function. This returns the memory internally consumed by the descriptor to the operating system.

All the above functions return an integer status value, which is zero upon successful completion of the operation. You can interpret a non-zero status with the help of the [DftiErrorClass](#) or [DftiErrorMessage](#) function.

The FFT functions support lengths with arbitrary factors. You can improve performance of the Intel MKL FFT if the length of your data vector permits factorization into powers of optimized radices. See the *Intel MKL Developer Guide* for specific radices supported efficiently.

**NOTE**

The FFT functions assume the Cartesian representation of complex data (that is, the real and imaginary parts define a complex number). The Intel MKL Vector Mathematical Functions provide efficient tools for conversion to and from polar representation (see [Example "Conversion from Cartesian to polar representation of complex data"](#) and [Example "Conversion from polar to Cartesian representation of complex data"](#)).

**Optimization Notice**

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Notice revision #20110804

## FFT Functions

The fast Fourier transform function library of Intel MKL provides one-dimensional, two-dimensional, and multi-dimensional transforms (of up to seven dimensions) and offers both Fortran and C interfaces for all transform functions.

Table "FFT Functions in Intel MKL" lists FFT functions implemented in Intel MKL:

### FFT Functions in Intel MKL

| Function Name                        | Operation                                                                                     |
|--------------------------------------|-----------------------------------------------------------------------------------------------|
| Descriptor Manipulation Functions    |                                                                                               |
| <a href="#">DftiCreateDescriptor</a> | Allocates the descriptor data structure and initializes it with default configuration values. |
| <a href="#">DftiCommitDescriptor</a> | Performs all initialization for the actual FFT computation.                                   |
| <a href="#">DftiFreeDescriptor</a>   | Frees memory allocated for a descriptor.                                                      |
| <a href="#">DftiCopyDescriptor</a>   | Makes a copy of an existing descriptor.                                                       |
| FFT Computation Functions            |                                                                                               |
| <a href="#">DftiComputeForward</a>   | Computes the forward FFT.                                                                     |
| <a href="#">DftiComputeBackward</a>  | Computes the backward FFT.                                                                    |
| Descriptor Configuration Functions   |                                                                                               |
| <a href="#">DftiSetValue</a>         | Sets one particular configuration parameter with the specified configuration value.           |
| <a href="#">DftiGetValue</a>         | Gets the value of one particular configuration parameter.                                     |

| Function Name                 | Operation                                                       |
|-------------------------------|-----------------------------------------------------------------|
| Status Checking Functions     |                                                                 |
| <code>DftiErrorClass</code>   | Checks if the status reflects an error of a predefined class.   |
| <code>DftiErrorMessage</code> | Translates the numeric value of an error status into a message. |

## FFT Interface

The Intel MKL FFT functions are provided with the Fortran and C interfaces.

The materials presented in this chapter assume the availability of native complex types in C as they are specified in C9X.

To use the FFT functions, you need to include `mkl_dfti.h` in your C code.

The C interface provides the `DFTI_DESCRIPTOR_HANDLE` type, named constants of two enumeration types `DFTI_CONFIG_PARAM` and `DFTI_CONFIG_VALUE`, and functions, some of which accept different numbers of input arguments.

### NOTE

The current version of the library may not support some of the FFT functions or functionality described in the subsequent sections of this chapter. You can find the complete list of the implementation-specific exceptions in the Intel MKL Release Notes.

For the main categories of Intel MKL FFT functions, see [FFT Functions](#).

## Computing an FFT

You can find code examples that compute transforms in the [Fourier Transform Functions Code Examples](#) section in the Code Examples appendix.

Usually you can compute an FFT by five function calls (refer to the [usage model](#) for details). A single data structure, the descriptor, stores configuration parameters that can be changed independently.

The descriptor data structure, when created, contains information about the length and domain of the FFT to be computed, as well as the setting of several configuration parameters. Default settings for some of these parameters are as follows:

- Scale factor: none
- Number of data sets: one
- Data storage: contiguous
- Placement of results: in-place (the computed result overwrites the input data)

The default settings can be changed one at a time through the function `DftiSetValue` as illustrated in [Example "Changing Default Settings \(C\)"](#).

## Configuration Settings

Each of the configuration parameters is identified by a named constant in the `MKL_DFTI` module. These named constants have the enumeration type `DFTI_CONFIG_PARAM`.

All the Intel MKL FFT configuration parameters are readable. Some of them are read-only, while others can be set using the `DftiCreateDescriptor` or `DftiSetValue` function.

Values of the configuration parameters fall into the following groups:

- Values that have native data types. For example, the number of simultaneous transforms requested has an integer value, while the scale factor for a forward transform is a single-precision number.

- Values that are discrete in nature and are provided in the `MKL_DFTI` module as named constants. For example, the domain of the forward transform requires values to be named constants. The named constants for configuration values have the enumeration type `DFTI_CONFIG_VALUE`.

Table "Configuration Parameters" summarises the information on configuration parameters, along with their types and values. For more details of each configuration parameter, see the subsection describing this parameter.

## Configuration Parameters

| Configuration Parameter                                                                                               | Type/Value                                                                      | Comments                                                                                                                                                 |
|-----------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>Most common configuration parameters, no default, must be set explicitly by <code>DftiCreateDescriptor</code></i>  |                                                                                 |                                                                                                                                                          |
| <code>DFTI_PRECISION</code>                                                                                           | Named constant<br><code>DFTI_SINGLE</code> or<br><code>DFTI_DOUBLE</code>       | Precision of the computation.                                                                                                                            |
| <code>DFTI_FORWARD_DOMAIN</code>                                                                                      | Named constant<br><code>DFTI_COMPLEX</code> or<br><code>DFTI_REAL</code>        | Type of the transform.                                                                                                                                   |
| <code>DFTI_DIMENSION</code>                                                                                           | Integer scalar                                                                  | Dimension of the transform.                                                                                                                              |
| <code>DFTI_LENGTH</code>                                                                                              | Integer scalar/array                                                            | Lengths of each dimension.                                                                                                                               |
| <i>Common configuration parameters, settable by <code>DftiSetValue</code></i>                                         |                                                                                 |                                                                                                                                                          |
| <code>DFTI_PLACEMENT</code>                                                                                           | Named constant<br><code>DFTI_INPLACE</code> or<br><code>DFTI_NOT_INPLACE</code> | Defines whether the result overwrites the input data. Default value: <code>DFTI_INPLACE</code> .                                                         |
| <code>DFTI_FORWARD_SCALE</code>                                                                                       | Floating-point scalar                                                           | Scale factor for the forward transform.<br>Default value: 1.0.<br>Precision of the value should be the same as defined by <code>DFTI_PRECISION</code> .  |
| <code>DFTI_BACKWARD_SCALE</code>                                                                                      | Floating-point scalar                                                           | Scale factor for the backward transform.<br>Default value: 1.0.<br>Precision of the value should be the same as defined by <code>DFTI_PRECISION</code> . |
| <code>DFTI_NUMBER_OF_USER_THREADS</code>                                                                              | Integer scalar                                                                  | This configuration parameter is no longer used and kept for compatibility with previous versions of Intel MKL.                                           |
| <code>DFTI_THREAD_LIMIT</code>                                                                                        | Integer scalar                                                                  | Limits the number of threads for the <a href="#">DftiComputeForward</a> and <a href="#">DftiComputeBackward</a> .<br>Default value: 0.                   |
| <code>DFTI_DESCRIPTOR_NAME</code>                                                                                     | Character string                                                                | Assigns a name to a descriptor. Assumed length of the string is <code>DFTI_MAX_NAME_LENGTH</code> .<br>Default value: empty string.                      |
| <i>Data layout configuration parameters for single and multiple transforms. Settable by <code>DftiSetValue</code></i> |                                                                                 |                                                                                                                                                          |
| <code>DFTI_INPUT_STRIDES</code>                                                                                       | Integer array                                                                   | Defines the input data layout.                                                                                                                           |
| <code>DFTI_OUTPUT_STRIDES</code>                                                                                      | Integer array                                                                   | Defines the output data layout.                                                                                                                          |
| <code>DFTI_NUMBER_OF_TRANSFORMS</code>                                                                                | Integer scalar                                                                  | Number of transforms.                                                                                                                                    |

| Configuration Parameter                                                         | Type/Value                                                                                                                                                | Comments                                                                                                                          |
|---------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
|                                                                                 |                                                                                                                                                           | Default value: 1.                                                                                                                 |
| <code>DFTI_INPUT_DISTANCE</code>                                                | Integer scalar                                                                                                                                            | Defines the distance between input data sets for multiple transforms.                                                             |
|                                                                                 |                                                                                                                                                           | Default value: 0.                                                                                                                 |
| <code>DFTI_OUTPUT_DISTANCE</code>                                               | Integer scalar                                                                                                                                            | Defines the distance between output data sets for multiple transforms.                                                            |
|                                                                                 |                                                                                                                                                           | Default value: 0.                                                                                                                 |
| <code>DFTI_COMPLEX_STORAGE</code>                                               | Named constant<br><code>DFTI_COMPLEX_COMPLEX</code><br>X or <code>DFTI_REAL_REAL</code>                                                                   | Defines whether the real and imaginary parts of data for a complex transform are interleaved in one array or split in two arrays. |
|                                                                                 |                                                                                                                                                           | Default value: <code>DFTI_COMPLEX_COMPLEX</code> .                                                                                |
| <code>DFTI_REAL_STORAGE</code>                                                  | Named constant<br><code>DFTI_REAL_REAL</code>                                                                                                             | Defines how real data for a real transform is stored. Only the <code>DFTI_REAL_REAL</code> value is supported.                    |
| <code>DFTI_CONJUGATE_EVEN_STORAGE</code>                                        | Named constant<br><code>DFTI_COMPLEX_COMPLEX</code><br>X or<br><code>DFTI_COMPLEX_REAL</code>                                                             | Defines whether the complex data in the backward domain of a real transform is stored as complex elements or as real elements.    |
|                                                                                 |                                                                                                                                                           | For the default value, see the detailed description.                                                                              |
| <code>DFTI_PACKED_FORMAT</code>                                                 | Named constant<br><code>DFTI_CCE_FORMAT</code> ,<br><code>DFTI_CCS_FORMAT</code> ,<br><code>DFTI_PACK_FORMAT</code> , or<br><code>DFTI_PERM_FORMAT</code> | Defines the layout of real elements in the backward domain of a one-dimensional or two-dimensional real transform.                |
| <i>Advanced configuration parameters, settable by <code>DftiSetValue</code></i> |                                                                                                                                                           |                                                                                                                                   |
| <code>DFTI_WORKSPACE</code>                                                     | Named constant<br><code>DFTI_ALLOW</code> or<br><code>DFTI_AVOID</code>                                                                                   | Defines whether the library should prefer algorithms using additional memory.                                                     |
|                                                                                 |                                                                                                                                                           | Default value: <code>DFTI_ALLOW</code> .                                                                                          |
| <code>DFTI_ORDERING</code>                                                      | Named constant<br><code>DFTI_ORDERED</code> or<br><code>DFTI_BACKWARD_SCRAMBLED</code>                                                                    | Defines whether the result of a complex transform is ordered or permuted.                                                         |
|                                                                                 |                                                                                                                                                           | Default value: <code>DFTI_ORDERED</code> .                                                                                        |
| <i>Read-Only configuration parameters</i>                                       |                                                                                                                                                           |                                                                                                                                   |
| <code>DFTI_COMMIT_STATUS</code>                                                 | Named constant<br><code>DFTI_UNCOMMITTED</code> or<br><code>DFTI_COMMITTED</code>                                                                         | Readiness of the descriptor for computation.                                                                                      |
| <code>DFTI_VERSION</code>                                                       | String                                                                                                                                                    | Version of Intel MKL. Assumed length of the string is <code>DFTI_VERSION_LENGTH</code> .                                          |

**See Also**

Configuring and Computing an FFT in C/C++

## DFTI\_PRECISION

The configuration parameter `DFTI_PRECISION` denotes the floating-point precision in which the transform is to be carried out. A setting of `DFTI_SINGLE` stands for single precision, and a setting of `DFTI_DOUBLE` stands for double precision. The data must be presented in this precision, the computation is carried out in this precision, and the result is delivered in this precision.

`DFTI_PRECISION` does not have a default value. Set it explicitly by calling the `DftiCreateDescriptor` function.

To better understand configuration of the precision of transforms, you can refer to these examples in your Intel MKL directory:

```
./examples/dftc/source/basic_sp_complex_dft_1d.c
./examples/dftc/source/basic_dp_complex_dft_1d.c
```

### See Also

[DFTI\\_FORWARD\\_DOMAIN](#)  
[DFTI\\_DIMENSION, DFTI\\_LENGTHS](#)  
[DftiCreateDescriptor](#)

## DFTI\_FORWARD\_DOMAIN

The general form of a discrete Fourier transform is

$$z_{k_1, k_2, \dots, k_d} = \sigma \times \sum_{j_d=0}^{n_d-1} \dots \sum_{j_2=0}^{n_2-1} \sum_{j_1=0}^{n_1-1} w_{j_1, j_2, \dots, j_d} \exp\left(\delta i 2\pi \sum_{l=1}^d j_l k_l / n_l\right)$$

for  $k_l = 0, \dots, n_l-1$  ( $l = 1, \dots, d$ ), where  $\sigma$  is a scale factor,  $\delta = -1$  for the forward transform, and  $\delta = +1$  for the backward transform.

The Intel MKL implementation of the FFT algorithm, used for fast computation of discrete Fourier transforms, supports forward transforms on input sequences of two domains, as specified by the `DFTI_FORWARD_DOMAIN` configuration parameter: general complex-valued sequences (`DFTI_COMPLEX` domain) and general real-valued sequences (`DFTI_REAL` domain). The forward transform maps the forward domain to the corresponding backward domain, as shown in [Table "Correspondence of Forward and Backward Domain"](#).

The conjugate-even domain covers complex-valued sequences with the symmetry property:

$$x(k_1, k_2, \dots, k_d) = \text{conjugate}(x(n_1 - k_1, n_2 - k_2, \dots, n_d - k_d))$$

where the index arithmetic is performed modulo respective size, that is,

$$x(\dots, \text{expr}_s, \dots) \equiv x(\dots, \text{mod}(\text{expr}_s, n_s), \dots),$$

and therefore

$$x(\dots, n_s, \dots) \equiv x(\dots, 0, \dots).$$

Due to this property of conjugate-even sequences, only a part of such sequence is stored in the computer memory, as described in [DFTI\\_CONJUGATE\\_EVEN\\_STORAGE](#).

### Correspondence of Forward and Backward Domain

| Forward Domain                        | Implied Backward Domain               |
|---------------------------------------|---------------------------------------|
| Complex ( <code>DFTI_COMPLEX</code> ) | Complex ( <code>DFTI_COMPLEX</code> ) |
| Real ( <code>DFTI_REAL</code> )       | Conjugate-even                        |

`DFTI_FORWARD_DOMAIN` does not have a default value. Set it explicitly by calling the `DftiCreateDescriptor` function.

To better understand usage of the `DFTI_FORWARD_DOMAIN` configuration parameter, you can refer to these examples in your Intel MKL directory:

```
./examples/dftc/source/basic_sp_complex_dft_1d.c
./examples/dftc/source/basic_sp_real_dft_1d.c
```

### See Also

[DFTI\\_PRECISION](#)

[DFTI\\_DIMENSION](#), [DFTI\\_LENGTHS](#)

[DftiCreateDescriptor](#)

### DFTI\_DIMENSION, DFTI\_LENGTHS

The dimension of the transform is a positive integer value represented in an integer scalar of `MKL_LONG` data type. For a one-dimensional transform, the transform length is specified by a positive integer value represented in an integer scalar of `MKL_LONG` data type. For multi-dimensional ( $\geq 2$ ) transform, the lengths of each of the dimensions are supplied in an integer array (of `MKL_LONG` data type).

`DFTI_DIMENSION` and `DFTI_LENGTHS` do not have a default value. To set them, use the `DftiCreateDescriptor` function and not the `DftiSetValue` function.

To better understand usage of the `DFTI_DIMENSION` and `DFTI_LENGTHS` configuration parameters, you can refer to basic examples of one-, two-, and three-dimensional transforms in your Intel MKL directory. Naming conventions for the examples are self-explanatory. For example, refer to these examples of single-precision two-dimensional transforms:

```
./examples/dftc/source/basic_sp_real_dft_2d.c
./examples/dftc/source/basic_sp_complex_dft_2d.c
```

### See Also

[DFTI\\_FORWARD\\_DOMAIN](#)

[DFTI\\_PRECISION](#)

[DftiCreateDescriptor](#)

[DftiSetValue](#)

### DFTI\_PLACEMENT

By default, the computational functions overwrite the input data with the output result. That is, the default setting of the configuration parameter `DFTI_PLACEMENT` is `DFTI_INPLACE`. You can change that by setting it to `DFTI_NOT_INPLACE`.

---

**NOTE**

The data sets have no common elements.

---

To better understand usage of the `DFTI_PLACEMENT` configuration parameter, refer to the following example in your Intel MKL directory:

```
./examples/dftc/source/config_placement.c
```

### See Also

[DftiSetValue](#)

### DFTI\_FORWARD\_SCALE, DFTI\_BACKWARD\_SCALE

The forward transform and backward transform are each associated with a scale factor  $\sigma$  of its own having the default value of 1. You can specify the scale factors using one or both of the configuration parameters `DFTI_FORWARD_SCALE` and `DFTI_BACKWARD_SCALE`. For example, for a one-dimensional transform of length  $n$ , you can use the default scale of 1 for the forward transform and set the scale factor for the backward transform to be  $1/n$ , thus making the backward transform the inverse of the forward transform.

Set the scale factor configuration parameter using a real floating-point data type of the same precision as the value for `DFTI_PRECISION`.

**NOTE**

For inquiry of the scale factor with the `DftiGetValue` function, the `config_val` parameter must have the same floating-point precision as the descriptor.

**See Also**

[DftiSetValue](#)  
[DFTI\\_PRECISION](#)  
[DftiGetValue](#)

**DFTI\_NUMBER\_OF\_USER\_THREADS**

The `DFTI_NUMBER_OF_USER_THREADS` configuration parameter is no longer used and kept for compatibility with previous versions of Intel MKL.

**See Also**

[DftiSetValue](#)

**DFTI\_THREAD\_LIMIT**

In some situations you may need to limit the number of threads that the `DftiComputeForward` and `DftiComputeBackward` functions use. For example, if more than one thread calls Intel MKL, it might be important that the thread calling these functions does not oversubscribe computing resources (CPU cores). Similarly, a known limit of the maximum number of threads to be used in computations might help the `DftiCommitDescriptor` function to select a more optimal computation method.

Set the parameter `DFTI_THREAD_LIMIT` as follows:

- To a positive number, to specify the maximum number of threads to be used by the compute functions.
- To zero (the default value), to use the maximum number of threads permitted in Intel MKL FFT functions. See "Techniques to Set the Number of Threads" in the *Intel MKL Developer Guide* for more information.

On an attempt to set a negative value, the `DftiSetValue` function returns an error and does not update the descriptor.

The value of the `DFTI_THREAD_LIMIT` configuration parameter returned by the `DftiGetValue` function is defined as follows:

- 1 if Intel MKL runs in the sequential mode
- Depends of the commit status of the descriptor if Intel MKL runs in a threaded mode:

| Commit Status | Value                                                                                                                               |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------|
| Not committed | The value of <code>DFTI_THREAD_LIMIT</code> set in a previous call to the <code>DftiSetValue</code> function or the default value   |
| Committed     | The upper limit on the number of threads used by the <code>DftiComputeForward</code> and <code>DftiComputeBackward</code> functions |

To better understand usage of the `DFTI_THREAD_LIMIT` configuration parameter, refer to the following example in your Intel MKL directory:

```
./examples/dftc/source/config_thread_limit.c
```

**See Also**

[DftiGetValue](#)  
[DftiSetValue](#)  
[DftiCommitDescriptor](#)  
[DftiComputeForward](#)  
[DftiComputeBackward](#)  
[Threading Control](#)

DFTI\_COMMIT\_STATUS

DFTI\_INPUT\_STRIDES, DFTI\_OUTPUT\_STRIDES

The FFT interface provides configuration parameters that define the layout of multidimensional data in the computer memory. For  $d$ -dimensional data set  $X$  defined by dimensions  $N_1 \times N_2 \times \dots \times N_d$ , the layout describes where a particular element  $X(k_1, k_2, \dots, k_d)$  of the data set is located. The memory address of the element  $X(k_1, k_2, \dots, k_d)$  is expressed by the formula

$$\begin{aligned} \text{address of } X(k_1, k_2, \dots, k_d) &= \text{address of } X(0, 0, \dots, 0) + \text{offset} \\ &= \text{address of } X(0, 0, \dots, 0) + s_0 + k_1*s_1 + k_2*s_2 + \dots + k_d*s_d, \end{aligned}$$

where  $s_0$  is the displacement and  $s_1, \dots, s_d$  are generalized strides. The configuration parameters `DFTI_INPUT_STRIDES` and `DFTI_OUTPUT_STRIDES` enable you to get and set these values. The configuration value is an array of values  $(s_0, s_1, \dots, s_d)$  of `MKL_LONG` data type.

The offset is counted in elements of the data type defined by the descriptor configuration (rather than by the type of the variable passed to the computation functions). Specifically, the `DFTI_FORWARD_DOMAIN`, `DFTI_COMPLEX_STORAGE`, and `DFTI_CONJUGATE_EVEN_STORAGE` configuration parameters define the type of the elements as shown in [Table "Assumed Element Types of the Input/Output Data"](#):

**Assumed Element Types of the Input/Output Data**

| Descriptor Configuration                                                                                    | Element Type in the Forward Domain | Element Type in the Backward Domain |
|-------------------------------------------------------------------------------------------------------------|------------------------------------|-------------------------------------|
| <code>DFTI_FORWARD_DOMAIN=DFTI_COMPLEX</code><br><code>DFTI_COMPLEX_STORAGE=DFTI_COMPLEX_COMPLEX</code>     | Complex                            | Complex                             |
| <code>DFTI_FORWARD_DOMAIN=DFTI_COMPLEX</code><br><code>DFTI_COMPLEX_STORAGE=DFTI_REAL_REAL</code>           | Real                               | Real                                |
| <code>DFTI_FORWARD_DOMAIN=DFTI_REAL</code><br><code>DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_REAL</code>    | Real                               | Real                                |
| <code>DFTI_FORWARD_DOMAIN=DFTI_REAL</code><br><code>DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_COMPLEX</code> | Real                               | Complex                             |

The `DFTI_INPUT_STRIDES` configuration parameter defines the layout of the input data, while the element type is defined by the forward domain for the `DftiComputeForward` function and by the backward domain for the `DftiComputeBackward` function. The `DFTI_OUTPUT_STRIDES` configuration parameter defines the layout of the output data, while the element type is defined by the backward domain for the `DftiComputeForward` function and by the forward domain for `DftiComputeBackward` function.

**NOTE**

The `DFTI_INPUT_STRIDES` and `DFTI_OUTPUT_STRIDES` configuration parameters define the layout of input and output data, and not the forward-domain and backward-domain data. If the data layouts in forward domain and backward domain differ, set `DFTI_INPUT_STRIDES` and `DFTI_OUTPUT_STRIDES` explicitly and then commit the descriptor before calling computation functions.

For in-place transforms (`DFTI_PLACEMENT=DFTI_INPLACE`), the configuration set by `DFTI_OUTPUT_STRIDES` is ignored when the element types in the forward and backward domains are the same. If they are different, set `DFTI_OUTPUT_STRIDES` explicitly (even though the transform is in-place). Ensure a consistent configuration for in-place transforms, that is, the locations of the first elements on input and output must coincide in each dimension.

The FFT interface supports both positive and negative stride values. If you use negative strides, set the displacement of the data as follows:

$$s_0 = \sum_{i=1}^d (N_i - 1) \cdot \max(-s_i, 0).$$

The default setting of strides in a general multi-dimensional case assumes that the array that contains the data has no padding. The order of the strides depends on the programming language. For example:

```
MKL_LONG dims[] = { nd, ..., n2, n1 };
DftiCreateDescriptor(&hand, precision, domain, d, dims);
// The above call assumes data declaration: type X[nd]...[n2][n1]
// Default strides are { 0, nd*...*n2*n1, ..., n2*n1, n1, 1 }
```

Note that in case of a real FFT (`DFTI_FORWARD_DOMAIN=DFTI_REAL`), where different data layouts in the backward domain are available (see `DFTI_PACKED_FORMAT`), the default value of the strides is not intuitive for the recommended CCE format (configuration setting `DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_COMPLEX`). In case of an *in-place* real transform with the CCE format, set the strides explicitly, as follows:

```
MKL_LONG dims[] = { nd, ..., n2, n1 };
MKL_LONG rstrides[] = { 0, 2*nd*...*n2*(n1/2+1), ..., 2*n2*(n1/2+1), 2*(n1/2+1), 1 };
MKL_LONG cstrides[] = { 0, nd*...*n2*(n1/2+1), ..., n2*(n1/2+1), (n1/2+1), 1 };
DftiCreateDescriptor(&hand, precision, DFTI_REAL, d, dims);
DftiSetValue(hand, DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX);
// Set the strides appropriately for forward/backward transform
```

To better understand configuration of strides, you can also refer to these examples in your Intel MKL directory:

```
./examples/dftc/source/basic_sp_real_dft_2d.c
./examples/dftc/source/basic_sp_real_dft_3d.c
./examples/dftc/source/basic_dp_real_dft_2d.c
./examples/dftc/source/basic_dp_real_dft_3d.c
```

## See Also

[DFTI\\_FORWARD\\_DOMAIN](#)

[DFTI\\_PLACEMENT](#)

[DftiSetValue](#)

[DftiCommitDescriptor](#)

[DftiComputeForward](#)

[DftiComputeBackward](#)

## DFTI\_NUMBER\_OF\_TRANSFORMS

If you need to perform a large number of identical FFTs, you can do this in a single call to a `DftiCompute*` function with the value of the `DFTI_NUMBER_OF_TRANSFORMS` configuration parameter equal to the actual number of the transforms. The default value of this parameter is one. You can set this parameter to a positive integer value using the `MKL_LONG` data type. When setting the number of transforms to a value greater than one, you also need to specify the distance between the input and output data sets using one of the `DFTI_INPUT_DISTANCE` and `DFTI_OUTPUT_DISTANCE` configuration parameters or both.

- 
- The data sets to be transformed must not have common elements.
  - All the sets of data must be located within the same memory block.
- 

To better understand usage of the `DFTI_NUMBER_OF_TRANSFORMS` configuration parameter, refer to the following example in your Intel MKL directory:

```
./examples/dftc/source/config_number_of_transforms.c
```

## See Also

[FFT Computation Functions](#)

[DFTI\\_INPUT\\_DISTANCE](#), [DFTI\\_OUTPUT\\_DISTANCE](#)  
[DftiSetValue](#)

## [DFTI\\_INPUT\\_DISTANCE](#), [DFTI\\_OUTPUT\\_DISTANCE](#)

The FFT interface in Intel MKL enables computation of multiple transforms. To compute multiple transforms, you need to specify the data distribution of the multiple sets of data. The distance between the first data elements of consecutive data sets, [DFTI\\_INPUT\\_DISTANCE](#) for input data or [DFTI\\_OUTPUT\\_DISTANCE](#) for output data, specifies the distribution. The configuration setting is a value of [MKL\\_LONG](#) data type.

The default value for both configuration settings is one. You must set this parameter explicitly if the number of transforms is greater than one (see [DFTI\\_NUMBER\\_OF\\_TRANSFORMS](#)).

The distance is counted in elements of the data type defined by the descriptor configuration (rather than by the type of the variable passed to the computation functions). Specifically, the [DFTI\\_FORWARD\\_DOMAIN](#), [DFTI\\_COMPLEX\\_STORAGE](#), and [DFTI\\_CONJUGATE\\_EVEN\\_STORAGE](#) configuration parameters define the type of the elements as shown in [Table "Assumed Element Types of the Input/Output Data"](#).

---

### NOTE

The configuration parameters [DFTI\\_INPUT\\_DISTANCE](#) and [DFTI\\_OUTPUT\\_DISTANCE](#) define the distance within input and output data, and not within the forward-domain and backward-domain data. If the distances in the forward and backward domains differ, set [DFTI\\_INPUT\\_DISTANCE](#) and [DFTI\\_OUTPUT\\_DISTANCE](#) explicitly and then commit the descriptor before calling computation functions.

---

For in-place transforms ([DFTI\\_PLACEMENT=DFTI\\_INPLACE](#)), the configuration set by [DFTI\\_OUTPUT\\_DISTANCE](#) is ignored when the element types in the forward and backward domains are the same. If they are different, set [DFTI\\_OUTPUT\\_DISTANCE](#) explicitly (even though the transform is in-place). Ensure a consistent configuration for in-place transforms, that is, the locations of the data sets on input and output must coincide.

The following example illustrates setting of the [DFTI\\_INPUT\\_DISTANCE](#) configuration parameter:

```
MKL_LONG dims[] = { nd, ..., n2, n1 };
MKL_LONG distance = nd*...*n2*n1;
DftiCreateDescriptor(&hand, precision, DFTI_COMPLEX, d, dims);
DftiSetValue(hand, DFTI_NUMBER_OF_TRANSFORMS, (MKL_LONG)howmany);
DftiSetValue(hand, DFTI_INPUT_DISTANCE, distance);
```

To better understand configuration of the distances, you can also refer to the following example in your Intel MKL directory:

```
./examples/dftc/source/config_number_of_transforms.c
```

### See Also

[DFTI\\_PLACEMENT](#)  
[DftiSetValue](#)  
[DftiCommitDescriptor](#)  
[DftiComputeForward](#)  
[DftiComputeBackward](#)

## [DFTI\\_COMPLEX\\_STORAGE](#), [DFTI\\_REAL\\_STORAGE](#), [DFTI\\_CONJUGATE\\_EVEN\\_STORAGE](#)

Depending on the value of the [DFTI\\_FORWARD\\_DOMAIN](#) configuration parameter, the implementation of FFT supports several storage schemes for input and output data (see document [3] for the rationale behind the definition of the storage schemes). The data elements are placed within contiguous memory blocks, defined with generalized strides (see [DFTI\\_INPUT\\_STRIDES](#), [DFTI\\_OUTPUT\\_STRIDES](#)). For multiple transforms, all sets of data should be located within the same memory block, and the data sets should be placed at the same distance from each other (see [DFTI\\_NUMBER\\_OF\\_TRANSFORMS](#) and [DFTI\\_INPUT\\_DISTANCE](#), [DFTI\\_OUTPUT\\_DISTANCE](#)).

**TIP**

Avoid setting up multidimensional arrays with lists of pointers to one-dimensional arrays. Instead use a one-dimensional array with the explicit indexing to access the data elements.

FFT Examples demonstrate the usage of storage formats.

**DFTI\_COMPLEX\_STORAGE: storage schemes for a complex domain**

For the DFTI\_COMPLEX forward domain, both input and output sequences belong to a complex domain. In this case, the configuration parameter DFTI\_COMPLEX\_STORAGE can have one of the two values:

DFTI\_COMPLEX\_COMPLEX (default) or DFTI\_REAL\_REAL.

**NOTE**

In the Intel MKL FFT interface, storage schemes for a forward complex domain and the respective backward complex domain are the same.

With DFTI\_COMPLEX\_COMPLEX storage, complex-valued data sequences are referenced by a single complex parameter (array) AZ so that a complex-valued element  $z_{k_1, k_2, \dots, k_d}$  of the m-th d-dimensional sequence is located at  $AZ[m*distance + stride_0 + k_1*stride_1 + k_2*stride_2 + \dots + k_d*stride_d]$  as a structure consisting of the real and imaginary parts.

The following code illustrates usage of the DFTI\_COMPLEX\_COMPLEX storage:

```
complex *AZ = malloc(N1*N2*N3*M * sizeof(AZ[0]));
MKL_LONG ios[4], iodist; // input/output strides and distance
...
// on input: Z{k1,k2,k3,m}
// = AZ[ios[0] + k1*ios[1] + k2*ios[2] + k3*ios[3] + m*iodist]
status = DftiComputeForward(desc, AZ);
// on output: Z{k1,k2,k3,m}
// = AZ[ios[0] + k1*ios[1] + k2*ios[2] + k3*ios[3] + m*iodist]
```

With the DFTI\_REAL\_REAL storage, complex-valued data sequences are referenced by two real parameters AR and AI so that a complex-valued element  $z_{k_1, k_2, \dots, k_d}$  of the m-th sequence is computed as  $AR[m*distance + stride_0 + k_1*stride_1 + k_2*stride_2 + \dots + k_d*stride_d] + \sqrt{-1} * AI[m*distance + stride_0 + k_1*stride_1 + k_2*stride_2 + \dots + k_d*stride_d]$ .

The following code illustrates usage of the DFTI\_REAL\_REAL storage:

```
float *AR = malloc(N1*N2*N3*M * sizeof(AR[0]));
float *AI = malloc(N1*N2*N3*M * sizeof(AI[0]));
MKL_LONG ios[4], iodist; // input/output strides and distance
...
// on input: Z{k1,...,kd,m}
// = AR[ios[0] + k1*ios[1] + k2*ios[2] + k3*ios[3] + m*iodist]
// + I*AI[ios[0] + k1*ios[1] + k2*ios[2] + k3*ios[3] + m*iodist]
status = DftiComputeForward(desc, AR, AI);
// on output: Z{k1,...,kd,m}
// = AR[ios[0] + k1*ios[1] + k2*ios[2] + k3*ios[3] + m*iodist]
// + I*AI[ios[0] + k1*ios[1] + k2*ios[2] + k3*ios[3] + m*iodist]
```

**DFTI\_REAL\_STORAGE: storage schemes for a real domain**

The Intel MKL FFT interface supports only one configuration value for this storage scheme: DFTI\_REAL\_REAL.

With the DFTI\_REAL\_REAL storage, real-valued data sequences in a real domain are referenced by one real parameter AR so that real-valued element of the m-th sequence is located as  $AR[m*distance + stride_0 + k_1*stride_1 + k_2*stride_2 + \dots + k_d*stride_d]$ .

## DFTI\_CONJUGATE\_EVEN\_STORAGE: storage scheme for a conjugate-even domain

The Intel MKL FFT interface supports two configuration values for this parameter: `DFTI_COMPLEX_REAL` (default) and `DFTI_COMPLEX_COMPLEX`. In both cases, the conjugate-even symmetry of the data enables storing only about a half of the whole mathematical result, so that one part of it can be directly referenced in the memory while the other part can be reconstructed depending on the selected storage configuration.

With the `DFTI_COMPLEX_REAL` storage, the complex-valued data sequences in the conjugate-even domain can be reconstructed as described in section [DFTI\\_PACKED\\_FORMAT](#).

---

### Important

Although `DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_REAL` is the default setting for the `DFTI_REAL` forward domain, avoid using this storage scheme because it is supported only for one- and two-dimensional transforms and will be deprecated in future.

---

With the `DFTI_COMPLEX_COMPLEX` storage, the complex-valued data sequences in the conjugate-even domain are referenced by one complex parameter `AZ` so that a complex-valued element  $z_{k_1, k_2, \dots, k_d}$  of the  $m$ -th sequence can be referenced or reconstructed as described below.

---

### Important

Use the `DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_COMPLEX` configuration setting, which will become the default in future. This setting is supported for all transform ranks, provides a uniform pattern for reconstructing the entire conjugate-even sequence from the part of it that is actually stored, and is compatible with data layouts supported by other FFT libraries, such as FFTW. This storage scheme disregards the setting of `DFTI_PACKED_FORMAT`.

---

Consider a  $d$ -dimensional real-to-complex transform

$$Z_{k_1, k_2, \dots, k_d} \equiv \sum_{n_1=0}^{N_1-1} \dots \sum_{n_d=0}^{N_d-1} R_{n_1, n_2, \dots, n_d} e^{\frac{-2\pi i}{N_1} k_1 n_1} \dots e^{\frac{-2\pi i}{N_d} k_d n_d}$$

Because the input sequence  $R$  is real-valued, the mathematical result  $Z$  has conjugate-even symmetry:

$$z_{k_1, k_2, \dots, k_d} = \text{conjugate}(z_{N_1-k_1, N_2-k_2, \dots, N_d-k_d}),$$

where index arithmetic is performed modulo the length of the respective dimension. Obviously, the first element of the result is real-valued:

$$z_{0, 0, \dots, 0} = \text{conjugate}(z_{0, 0, \dots, 0}).$$

For dimensions with even lengths, the other elements are real-valued too. For example, if  $N_s$  is even,

$$z_{0, 0, \dots, N_s/2, 0, \dots, 0} = \text{conjugate}(z_{0, 0, \dots, N_s/2, 0, \dots, 0}).$$

With the conjugate-even symmetry, approximately a half of the result suffices to fully reconstruct it. For an arbitrary dimension  $h$ , it suffices to store elements  $z_{k_1, \dots, k_h, \dots, k_d}$  for the following indices:

- $k_h = 0, \dots, [N_h/2]$
- $k_i = 0, \dots, N_i - 1$ , where  $i = 1, \dots, d$  and  $i \neq h$

The symmetry property enables reconstructing the remaining elements: for  $k_h = [N_h/2] + 1, \dots, N_h - 1$ . In the Intel MKL FFT interface, the halved dimension is the last dimension.

The following code illustrates usage of the `DFTI_COMPLEX_COMPLEX` storage for a conjugate-even domain:

```
float *AR = malloc(N1*N2*M * sizeof(AR[0]));
complex *AZ = malloc(N1*(N2/2+1)*M * sizeof(AZ[0]));
MKL_LONG is[3], os[3], idist, odist; // input and output strides and distance
...
// on input: R{k1,k2,m}
// = AR[is[0] + k1*is[1] + k2*is[2] + m*idist]
status = DftiComputeForward(desc, R, C);
// on output:
// for k2=0..N2/2: Z{k1,k2,m} = AZ[os[0]+k1*os[1]+k2*os[2]+m*odist]
// for k2=N2/2+1..N2-1: Z{k1,k2,m} = conj(AZ[os[0]+(N1-k1)%N1*os[1]
// + (N2-k2)%N2*os[2]+m*odist])
```

For the backward transform, the input and output parameters and layouts exchange roles: set the strides describing the layout in the backward/forward domain as input/output strides, respectively. For example:

```
...
status = DftiSetValue(desc, DFTI_INPUT_STRIDES, fwd_domain_strides);
status = DftiSetValue(desc, DFTI_OUTPUT_STRIDES, bwd_domain_strides);
status = DftiCommitDescriptor(desc);
status = DftiComputeForward(desc, ...);
...
status = DftiSetValue(desc, DFTI_INPUT_STRIDES, bwd_domain_strides);
status = DftiSetValue(desc, DFTI_OUTPUT_STRIDES, fwd_domain_strides);
status = DftiCommitDescriptor(desc);
status = DftiComputeBackward(desc, ...);
```

---

### Important

For in-place transforms, ensure the first element of the input data has the same address as the first element of the output data for each dimension.

---

## See Also

[DftiSetValue](#)

## DFTI\_PACKED\_FORMAT

The result of the forward transform of real data is a conjugate-even sequence. Due to the symmetry property, only a part of the complex-valued sequence is stored in memory. The `DFTI_PACKED_FORMAT` configuration parameter defines how the data is packed. Possible values of `DFTI_PACKED_FORMAT` depend on the values of the `DFTI_CONJUGATE_EVEN_STORAGE` configuration parameter:

- `DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_COMPLEX`.

The only value of `DFTI_PACKED_FORMAT` can be `DFTI_CCE_FORMAT`. You can use this value with transforms of any dimension. For a description of the corresponding packed format, see [DFTI\\_CONJUGATE\\_EVEN\\_STORAGE](#).

- `DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_REAL`.

`DFTI_PACKED_FORMAT` can be `DFTI_CCS_FORMAT`, `DFTI_PACK_FORMAT`, or `DFTI_PERM_FORMAT`. You can use these values with one- and two-dimensional transforms only. The corresponding packed formats are described below.

---

### NOTE

Although `DFTI_CONJUGATE_EVEN_STORAGE=DFTI_COMPLEX_REAL` is the default setting for the `DFTI_REAL` forward domain, avoid using this storage scheme because it is supported only for one- and two-dimensional transforms, is incompatible with storage schemes of other FFT libraries, and will be deprecated in future.

---

### DFTI\_CCS\_FORMAT for One-dimensional Transforms

The following figure illustrates the storage of a one-dimensional (1D) size- $N$  conjugate-even sequence in a real array for the CCS, PACK, and PERM packed formats. The CCS format requires an array of size  $N+2$ , while the other formats require an array of size  $N$ . Zero-based indexing is used.

#### Storage of a 1D Size- $N$ Conjugate-even Sequence in a Real Array

|              | n= | 0              | 1              | 2              | 3              | ... | 2L-2             | 2L-1             | 2L             | 2L+1           | 2L+2     |
|--------------|----|----------------|----------------|----------------|----------------|-----|------------------|------------------|----------------|----------------|----------|
| CCS, N=2L    |    | R <sub>0</sub> | 0              | R <sub>1</sub> | I <sub>1</sub> | ... | R <sub>L-1</sub> | I <sub>L-1</sub> | R <sub>L</sub> | 0              |          |
| PACK, N=2L   |    | R <sub>0</sub> | R <sub>1</sub> | I <sub>1</sub> | R <sub>2</sub> | ... | I <sub>L-1</sub> | R <sub>L</sub>   |                |                |          |
| PERM, N=2L   |    | R <sub>0</sub> | R <sub>L</sub> | R <sub>1</sub> | I <sub>1</sub> | ... | R <sub>L-1</sub> | I <sub>L-1</sub> |                |                |          |
| CCS, N=2L+1  |    | R <sub>0</sub> | 0              | R <sub>1</sub> | I <sub>1</sub> | ... | R <sub>L-1</sub> | I <sub>L-1</sub> | R <sub>L</sub> | I <sub>L</sub> | not used |
| PACK, N=2L+1 |    | R <sub>0</sub> | R <sub>1</sub> | I <sub>1</sub> | R <sub>2</sub> | ... | I <sub>L-1</sub> | R <sub>L</sub>   | I <sub>L</sub> |                |          |
| PERM, N=2L+1 |    | R <sub>0</sub> | R <sub>1</sub> | I <sub>1</sub> | R <sub>2</sub> | ... | I <sub>L-1</sub> | R <sub>L</sub>   | I <sub>L</sub> |                |          |

The real and imaginary parts of the complex-valued conjugate-even sequence  $Z_k$  are located in a real-valued array AC as illustrated by figure "Storage of a 1D Size- $N$  Conjugate-even Sequence in a Real Array" and can be used to reconstruct the whole conjugate-even sequence as follows:

```
float *AR; // malloc(sizeof(float)*N)
float *AC; // malloc(sizeof(float)*(N+2))
...
status = DftiSetValue(desc, DFTI_PACKED_FORMAT, DFTI_CCS_FORMAT);
...
// on input: R{k} = AR[k]
status = DftiComputeForward(desc, AR, AC); // real-to-complex FFT
// on output:
// for k=0...N/2: Z{k} = AC[2*k+0] + I*AC[2*k+1]
// for k=N/2+1...N-1: Z{k} = AC[2*(N-k)%N + 0] - I*AC[2*(N-k)%N + 1]
```

**DFTI\_CCS\_FORMAT for Two-dimensional Transforms**

The following figure illustrates the storage of a two-dimensional (2D)  $M$ -by- $N$  conjugate-even sequence in a real array for the CCS packed format. This format requires an array of size  $(M+2)$ -by- $(N+2)$ . Row-major layout and zero-based indexing are used. Different colors mark logically separate parts of the result. "n/u" means "not used".

**Storage of a 2D  $M$ -by- $N$  Conjugate-even Sequence in a Real Array for the CCS Format**

|      | n=0                | 1   | 2                   | 3                   | ... | 2L-2                  | 2L-1                  | 2L                 | 2L+1 |
|------|--------------------|-----|---------------------|---------------------|-----|-----------------------|-----------------------|--------------------|------|
| m=0  | R <sub>0,0</sub>   | 0   | R <sub>0,1</sub>    | I <sub>0,1</sub>    | ... | R <sub>0,L-1</sub>    | I <sub>0,L-1</sub>    | R <sub>0,L</sub>   | 0    |
| 1    | 0                  | 0   | R <sub>1,1</sub>    | I <sub>1,1</sub>    | ... | R <sub>1,L-1</sub>    | I <sub>1,L-1</sub>    | 0                  | 0    |
| 2    | R <sub>1,0</sub>   | 0   | R <sub>2,1</sub>    | I <sub>2,1</sub>    | ... | R <sub>2,L-1</sub>    | I <sub>2,L-1</sub>    | R <sub>1,L</sub>   | 0    |
| 3    | I <sub>1,0</sub>   | 0   | R <sub>3,1</sub>    | I <sub>3,1</sub>    | ... | R <sub>3,L-1</sub>    | I <sub>3,L-1</sub>    | I <sub>1,L</sub>   | 0    |
| ...  | ...                | ... | ...                 | ...                 | ... | ...                   | ...                   | ...                | ...  |
| 2K-2 | R <sub>K-1,0</sub> | 0   | R <sub>2K-2,1</sub> | I <sub>2K-2,1</sub> | ... | R <sub>2K-2,L-1</sub> | I <sub>2K-2,L-1</sub> | R <sub>K-1,L</sub> | 0    |
| 2K-1 | I <sub>K-1,0</sub> | 0   | R <sub>2K-1,1</sub> | I <sub>2K-1,1</sub> | ... | R <sub>2K-1,L-1</sub> | I <sub>2K-1,L-1</sub> | I <sub>K-1,L</sub> | 0    |
| 2K   | R <sub>K,0</sub>   | 0   | n/u                 | n/u                 | ... | n/u                   | n/u                   | R <sub>K,L</sub>   | 0    |
| 2K+1 | 0                  | 0   | n/u                 | n/u                 | ... | n/u                   | n/u                   | 0                  | 0    |

CCS, M=2K, N=2L

|      | n=0                | 1   | 2                   | 3                   | ... | 2L-2                  | 2L-1                  | 2L                  | 2L+1                | 2L+2 |
|------|--------------------|-----|---------------------|---------------------|-----|-----------------------|-----------------------|---------------------|---------------------|------|
| m=0  | R <sub>0,0</sub>   | 0   | R <sub>0,1</sub>    | I <sub>0,1</sub>    | ... | R <sub>0,L-1</sub>    | I <sub>0,L-1</sub>    | R <sub>0,L</sub>    | I <sub>0,L</sub>    | n    |
| 1    | 0                  | 0   | R <sub>1,1</sub>    | I <sub>1,1</sub>    | ... | R <sub>1,L-1</sub>    | I <sub>1,L-1</sub>    | R <sub>1,L</sub>    | I <sub>1,L</sub>    | n    |
| 2    | R <sub>1,0</sub>   | 0   | R <sub>2,1</sub>    | I <sub>2,1</sub>    | ... | R <sub>2,L-1</sub>    | I <sub>2,L-1</sub>    | R <sub>2,L</sub>    | I <sub>2,L</sub>    | n    |
| 3    | I <sub>1,0</sub>   | 0   | R <sub>3,1</sub>    | I <sub>3,1</sub>    | ... | R <sub>3,L-1</sub>    | I <sub>3,L-1</sub>    | R <sub>3,L</sub>    | I <sub>3,L</sub>    | n    |
| ...  | ...                | ... | ...                 | ...                 | ... | ...                   | ...                   | ...                 | ...                 | ...  |
| 2K-2 | R <sub>K-1,0</sub> | 0   | R <sub>2K-2,1</sub> | I <sub>2K-2,1</sub> | ... | R <sub>2K-2,L-1</sub> | I <sub>2K-2,L-1</sub> | R <sub>2K-2,L</sub> | I <sub>2K-2,L</sub> | n    |
| 2K-1 | I <sub>K-1,0</sub> | 0   | R <sub>2K-1,1</sub> | I <sub>2K-1,1</sub> | ... | R <sub>2K-1,L-1</sub> | I <sub>2K-1,L-1</sub> | R <sub>2K-1,L</sub> | I <sub>2K-1,L</sub> | n    |
| 2K   | R <sub>K,0</sub>   | 0   | n/u                 | n/u                 | ... | n/u                   | n/u                   | n/u                 | n/u                 | n    |
| 2K+1 | 0                  | 0   | n/u                 | n/u                 | ... | n/u                   | n/u                   | n/u                 | n/u                 | n    |

CCS, M=2K, N=2L+1

|      | n=0                | 1   | 2                   | 3                   | ... | 2L-2                  | 2L-1                  | 2L                 | 2L+1 |
|------|--------------------|-----|---------------------|---------------------|-----|-----------------------|-----------------------|--------------------|------|
| m=0  | R <sub>0,0</sub>   | 0   | R <sub>0,1</sub>    | I <sub>0,1</sub>    | ... | R <sub>0,L-1</sub>    | I <sub>0,L-1</sub>    | R <sub>0,L</sub>   | 0    |
| 1    | 0                  | 0   | R <sub>1,1</sub>    | I <sub>1,1</sub>    | ... | R <sub>1,L-1</sub>    | I <sub>1,L-1</sub>    | 0                  | 0    |
| 2    | R <sub>1,0</sub>   | 0   | R <sub>2,1</sub>    | I <sub>2,1</sub>    | ... | R <sub>2,L-1</sub>    | I <sub>2,L-1</sub>    | R <sub>1,L</sub>   | 0    |
| 3    | I <sub>1,0</sub>   | 0   | R <sub>3,1</sub>    | I <sub>3,1</sub>    | ... | R <sub>3,L-1</sub>    | I <sub>3,L-1</sub>    | I <sub>1,L</sub>   | 0    |
| ...  | ...                | ... | ...                 | ...                 | ... | ...                   | ...                   | ...                | ...  |
| 2K-2 | R <sub>K-1,0</sub> | 0   | R <sub>2K-2,1</sub> | I <sub>2K-2,1</sub> | ... | R <sub>2K-2,L-1</sub> | I <sub>2K-2,L-1</sub> | R <sub>K-1,L</sub> | 0    |
| 2K-1 | I <sub>K-1,0</sub> | 0   | R <sub>2K-1,1</sub> | I <sub>2K-1,1</sub> | ... | R <sub>2K-1,L-1</sub> | I <sub>2K-1,L-1</sub> | I <sub>K-1,L</sub> | 0    |
| 2K   | R <sub>K,0</sub>   | 0   | R <sub>2K,1</sub>   | I <sub>2K,1</sub>   | ... | R <sub>2K,L-1</sub>   | I <sub>2K,L-1</sub>   | R <sub>K,L</sub>   | 0    |
| 2K+1 | I <sub>K,0</sub>   | 0   | n/u                 | n/u                 | ... | n/u                   | n/u                   | I <sub>K,L</sub>   | n/u  |
| 2K+2 | n/u                | n/u | n/u                 | n/u                 | ... | n/u                   | n/u                   | n/u                | n/u  |

CCS, M=2K+1, N=2L

|      | n=0                | 1   | 2                   | 3                   | ... | 2L-2                  | 2L-1                  | 2L                  | 2L+1                | 2L+2 |
|------|--------------------|-----|---------------------|---------------------|-----|-----------------------|-----------------------|---------------------|---------------------|------|
| m=0  | R <sub>0,0</sub>   | 0   | R <sub>0,1</sub>    | I <sub>0,1</sub>    | ... | R <sub>0,L-1</sub>    | I <sub>0,L-1</sub>    | R <sub>0,L</sub>    | I <sub>0,L</sub>    | 0    |
| 1    | 0                  | 0   | R <sub>1,1</sub>    | I <sub>1,1</sub>    | ... | R <sub>1,L-1</sub>    | I <sub>1,L-1</sub>    | R <sub>1,L</sub>    | I <sub>1,L</sub>    | 0    |
| 2    | R <sub>1,0</sub>   | 0   | R <sub>2,1</sub>    | I <sub>2,1</sub>    | ... | R <sub>2,L-1</sub>    | I <sub>2,L-1</sub>    | R <sub>2,L</sub>    | I <sub>2,L</sub>    | 0    |
| 3    | I <sub>1,0</sub>   | 0   | R <sub>3,1</sub>    | I <sub>3,1</sub>    | ... | R <sub>3,L-1</sub>    | I <sub>3,L-1</sub>    | R <sub>3,L</sub>    | I <sub>3,L</sub>    | 0    |
| ...  | ...                | ... | ...                 | ...                 | ... | ...                   | ...                   | ...                 | ...                 | ...  |
| 2K-2 | R <sub>K-1,0</sub> | 0   | R <sub>2K-2,1</sub> | I <sub>2K-2,1</sub> | ... | R <sub>2K-2,L-1</sub> | I <sub>2K-2,L-1</sub> | R <sub>2K-2,L</sub> | I <sub>2K-2,L</sub> | 0    |
| 2K-1 | I <sub>K-1,0</sub> | 0   | R <sub>2K-1,1</sub> | I <sub>2K-1,1</sub> | ... | R <sub>2K-1,L-1</sub> | I <sub>2K-1,L-1</sub> | R <sub>2K-1,L</sub> | I <sub>2K-1,L</sub> | 0    |
| 2K   | R <sub>K,0</sub>   | 0   | R <sub>2K,1</sub>   | I <sub>2K,1</sub>   | ... | R <sub>2K,L-1</sub>   | I <sub>2K,L-1</sub>   | R <sub>2K,L</sub>   | I <sub>2K,L</sub>   | 0    |
| 2K+1 | I <sub>K,0</sub>   | 0   | n/u                 | n/u                 | ... | n/u                   | n/u                   | n/u                 | n/u                 | 0    |
| 2K+2 | n/u                | n/u | n/u                 | n/u                 | ... | n/u                   | n/u                   | n/u                 | n/u                 | 0    |

CCS, M=2K+1, N=2L+1

The real and imaginary parts of the complex-valued conjugate-even sequence  $Z_{k_1, k_2}$  are located in a real-valued array AC as illustrated by figure "Storage of a 2D M-by-N Conjugate-even Sequence in a Real Array for the CCS Format" and can be used to reconstruct the whole sequence as follows:

```
float *AR; // malloc(sizeof(float)*N1*N2)
float *AC; // malloc(sizeof(float)*(N1+2)*(N2+2))
...
status = DftiSetValue(desc, DFTI_PACKED_FORMAT, DFTI_CCS_FORMAT);
...
// on input: R{k1,k2} = AR[(N2+2)*k1 + k2]
status = DftiComputeForward(desc, AR, AC); // real-to-complex FFT
// on output: Z{k1,k2} = re + I*im, where
// if (k1==0) {
// if (k2 <= N2/2) {
// re = AC[2*k2+0];
// im = AC[2*k2+1];
// } else {
// re = AC[2*(N2-k2)+0];
// im = -AC[2*(N2-k2)+1];
// }
// }
// else if (k2==0){
// if (k1 <= N1/2) {
// re = AC[(2*k1+0)*(N2+2)];
// im = AC[(2*k1+1)*(N2+2)];
// } else {
// re = AC[(2*(N1-k1)+0)*(N2+2)];
// im = -AC[(2*(N1-k1)+1)*(N2+2)];
// }
// }
// else if (k2 == N2-k2) {
// if (k1 <= N1/2) {
// re = AC[(2*k1+0)*(N2+2) + 2*(N2/2)];
// im = AC[(2*k1+1)*(N2+2) + 2*(N2/2)];
// } else{
// re = AC[(2*(N1-k1)+0)*(N2+2) + 2*(N2/2)];
// im = -AC[(2*(N1-k1)+1)*(N2+2) + 2*(N2/2)];
// }
// }
// else if (k2 <= N2/2) {
// re = AC[k1*(N2+2)+2*k2+0];
// im = AC[k1*(N2+2)+2*k2+1];
// } else {
// re = AC[(N1-k1)*(N2+2)+2*(N2-k2)+0];
// im = -AC[(N1-k1)*(N2+2)+2*(N2-k2)+1];
// }
```

## DFTI\_PACK\_FORMAT for One-dimensional Transforms

The real and imaginary parts of the complex-valued conjugate-even sequence  $Z_k$  are located in a real-valued array AC as illustrated by figure "Storage of a 1D Size-N Conjugate-even Sequence in a Real Array" and can be used to reconstruct the whole conjugate-even sequence as follows:

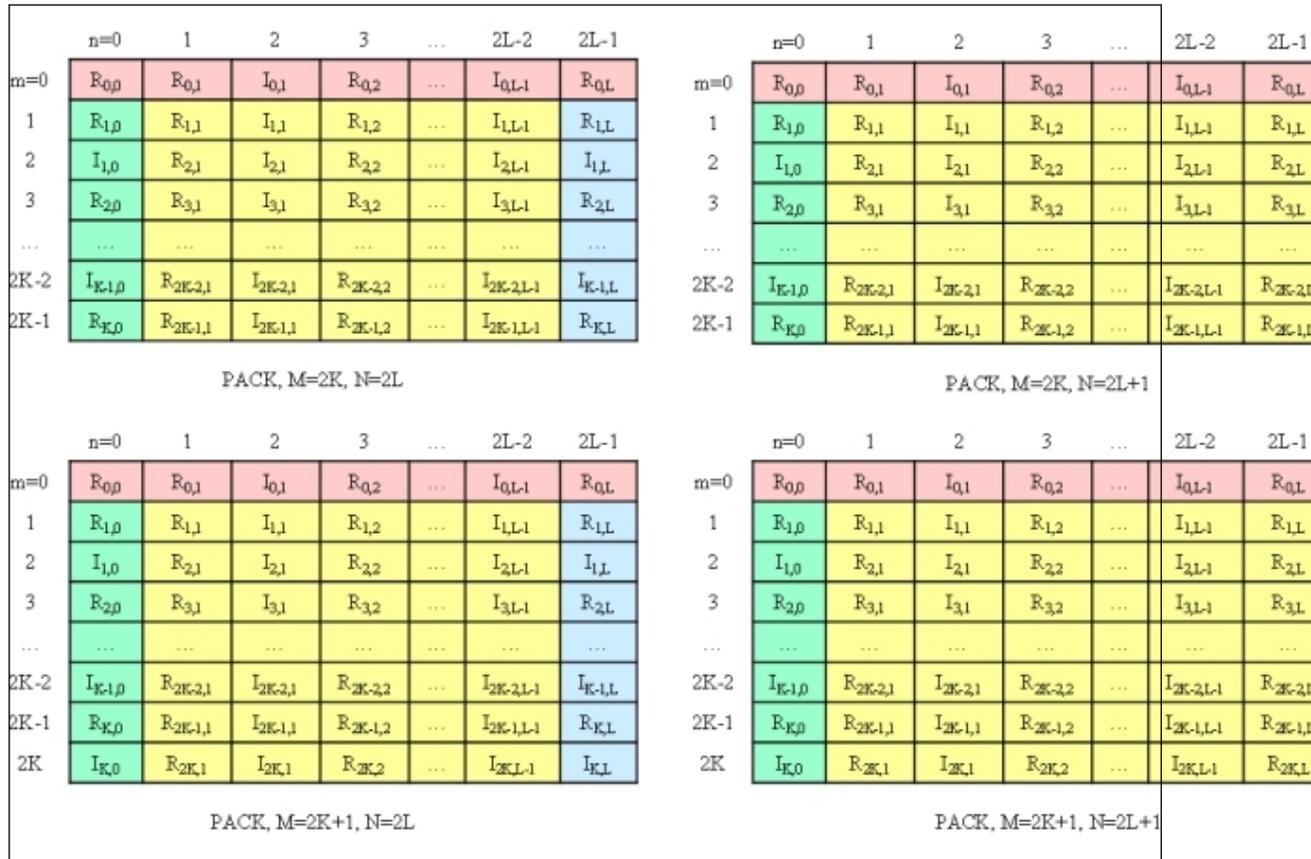
```
float *AR; // malloc(sizeof(float)*N)
float *AC; // malloc(sizeof(float)*N)
...
status = DftiSetValue(desc, DFTI_PACKED_FORMAT, DFTI_PACK_FORMAT);
...
// on input: R{k} = AR[k]
status = DftiComputeForward(desc, AR, AC); // real-to-complex FFT
// on output: Z{k} = re + I*im, where
// if (k == 0) {
// re = AC[0];
// im = 0;
// }
```

```
// } else if (k == N-k) {
// re = AC[2*k-1];
// im = 0;
// } else if (k <= N/2) {
// re = AC[2*k-1];
// im = AC[2*k-0];
// } else {
// re = AC[2*(N-k)-1];
// im = -AC[2*(N-k)-0];
// }
```

**DFTI\_PACK\_FORMAT for Two-dimensional Transforms**

The following figure illustrates the storage of a 2D  $M$ -by- $N$  conjugate-even sequence in a real array for the PACK packed format. This format requires an array of size  $M$ -by- $N$ . Row-major layout and zero-based indexing are used. Different colors mark logically separate parts of the result.

**Storage of a 2D  $M$ -by- $N$  Conjugate-even Sequence in a Real Array for the PACK Format**



The real and imaginary parts of the complex-valued conjugate-even sequence  $Z_{k_1,k_2}$  are located in a real-valued array AC as illustrated by figure "Storage of a 2D  $M$ -by- $N$  Conjugate-even Sequence in a Real Array for the PACK Format" and can be used to reconstruct the whole sequence as follows:

```
float *AR; // malloc(sizeof(float)*N1*N2)
float *AC; // malloc(sizeof(float)*N1*N2)
...
status = DftiSetValue(desc, DFTI_PACKED_FORMAT, DFTI_PACK_FORMAT);
...
// on input: R{k1,k2} = AR[N2*k1 + k2]
status = DftiComputeForward(desc, AR, AC); // real-to-complex FFT
// on output: Z{k1,k2} = re + I*im, where
```

```

// if (k1==0) {
// if (k2 == 0) {
// re = AC[0];
// im = 0;
// } else if (k2 == N2-k2) {
// re = AC[2*k2-1];
// im = 0;
// } else if (k2 <= N2/2) {
// re = AC[2*k2-1];
// im = AC[2*k2-0];
// } else {
// re = AC[2*(N2-k2)-1];
// im = -AC[2*(N2-k2)-0];
// }
// }
// else if (k2==0) {
// if (k1 == N1-k1) {
// re = AC[(N1-1)*N2];
// im = 0;
// } else if (k1 <= N1/2) {
// re = AC[(2*k1-1)*N2];
// im = AC[(2*k1-0)*N2];
// } else {
// re = AC[(2*(N1-k1)-1)*N2];
// im = -AC[(2*(N1-k1)-0)*N2];
// }
// }
// else if (k2 == N2-k2) {
// if (k1 == N1-k1) {
// re = AC[N1*N2 - 1];
// im = 0;
// } else if (k1 <= N1/2) {
// re = AC[(2*k1 - 1)*N2 + N2-1];
// im = AC[(2*k1 - 0)*N2 + N2-1];
// } else {
// re = AC[(2*(N1-k1) - 1)*N2 + N2-1];
// im = -AC[(2*(N1-k1) - 0)*N2 + N2-1];
// }
// }
// else if (k2 <= N2/2) {
// re = AC[k1*N2+2*k2-1];
// im = AC[k1*N2+2*k2-0];
// } else {
// re = AC[(N1-k1)*N2+2*(N2-k2) - 1];
// im = -AC[(N1-k1)*N2+2*(N2-k2) - 0];
// }

```

### DFTI\_PERM\_FORMAT for One-dimensional Transforms

The real and imaginary parts of the complex-valued conjugate-even sequence  $Z_k$  are located in real-valued array AC as illustrated by figure "Storage of a 1D Size- $N$  Conjugate-even Sequence in a Real Array" and can be used to reconstruct the whole conjugate-even sequence as follows:

```

float *AR; // malloc(sizeof(float)*N)
float *AC; // malloc(sizeof(float)*N)
...
status = DftiSetValue(desc, DFTI_PACKED_FORMAT, DFTI_PERM_FORMAT);
...
// on input: R{k} = AR[k]
status = DftiComputeForward(desc, AR, AC); // real-to-complex FFT
// on output: Z{k} = re + I*im, where
// if (k == 0) {
// re = AC[0];

```

```

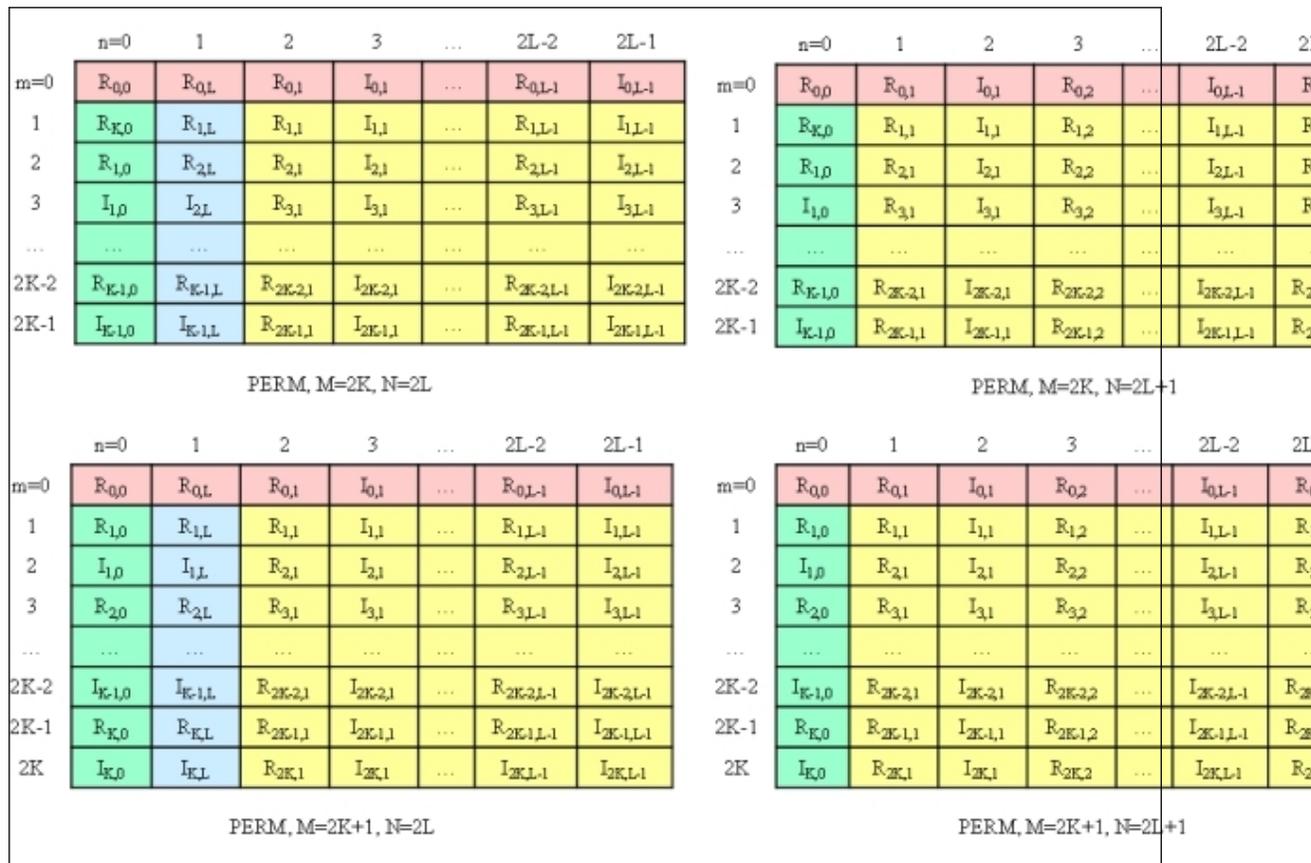
// im = 0;
// } else if (k == N-k) {
// re = AC[1];
// im = 0;
// } else if (k <= N/2) {
// re = AC[2*k+0 - N%2];
// im = AC[2*k+1 - N%2];
// } else {
// re = AC[2*(N-k)+0 - N%2];
// im = -AC[2*(N-k)+1 - N%2];
// }

```

### DFTI\_PERM\_FORMAT for Two-dimensional Transforms

The following figure illustrates the storage of a 2D  $M$ -by- $N$  conjugate-even sequence in a real array for the PERM packed format. This format requires an array of size  $M$ -by- $N$ . Row-major layout and zero-based indexing are used. Different colors mark logically separate parts of the result.

#### Storage of a 2D $M$ -by- $N$ Conjugate-Even Sequence in a Real Array for the PERM Format



The real and imaginary parts of the complex-valued conjugate-even sequence  $Z_{k_1,k_2}$  are located in real-valued array AC as illustrated by figure "Storage of a 2D  $M$ -by- $N$  Conjugate-even Sequence in a Real Array for the PERM Format" and can be used to reconstruct the whole sequence as follows:

```

float *AR; // malloc(sizeof(float)*N1*N2)
float *AC; // malloc(sizeof(float)*N1*N2)
...
status = DftiSetValue(desc, DFTI_PACKED_FORMAT, DFTI_PERM_FORMAT);
...
// on input: R{k1,k2} = AR[N2*k1 + k2]
status = DftiComputeForward(desc, AR, AC); // real-to-complex FFT

```

```
// on output: Z{k1,k2} = re + I*im, where
// if (k1==0) {
// if (k2 == 0) {
// re = AC[0];
// im = 0;
// } else if (k2 == N2-k2) {
// re = AC[1];
// im = 0;
// } else if (k2 <= N2/2) {
// re = AC[2*k2+0 - N2%2];
// im = AC[2*k2+1 - N2%2];
// } else {
// re = AC[2*(N2-k2)+0 - N2%2];
// im = -AC[2*(N2-k2)+1 - N2%2];
// }
// }
// else if (k2==0) {
// if (k1 == N1-k1) {
// re = AC[N2];
// im = 0;
// } else if (k1 <= N1/2) {
// re = AC[(2*k1+0 - N1%2)*N2];
// im = AC[(2*k1+1 - N1%2)*N2];
// } else {
// re = AC[(2*(N1-k1)+0 - N1%2)*N2];
// im = -AC[(2*(N1-k1)+1 - N1%2)*N2];
// }
// }
// else if (k2 == N2-k2) {
// if (k1 == N1-k1) {
// re = AC[N2 + 1];
// im = 0;
// } else if (k1 <= N1/2) {
// re = AC[(2*k1+0 - N1%2)*N2 + 1];
// im = AC[(2*k1+1 - N1%2)*N2 + 1];
// } else {
// re = AC[(2*(N1-k1)+0 - N1%2)*N2 + 1];
// im = -AC[(2*(N1-k1)+1 - N1%2)*N2 + 1];
// }
// }
// else if (k2 <= N2/2) {
// re = AC[k1*N2+2*k2+0 - N2%2];
// im = AC[k1*N2+2*k2+1 - N2%2];
// } else {
// re = AC[(N1-k1)*N2+2*(N2-k2)+0 - N2%2];
// im = -AC[(N1-k1)*N2+2*(N2-k2)+1 - N2%2];
// }
```

To better understand packed formats for two-dimensional transforms, refer to the following example in your Intel MKL directory:

```
./examples/dftc/source/config_conjugate_even_storage.c
```

## See Also

[DftiSetValue](#)

## DFTI\_WORKSPACE

The computation step for some FFT algorithms requires a scratch space for permutation or other purposes. To manage the use of the auxiliary storage, Intel MKL enables you to set the configuration parameter `DFTI_WORKSPACE` with the following values:

`DFTI_ALLOW` (default) Permits the use of the auxiliary storage.



## FFT Descriptor Manipulation Functions

This category contains the following functions: create a descriptor, commit a descriptor, copy a descriptor, and free a descriptor.

### DftiCreateDescriptor

*Allocates the descriptor data structure and initializes it with default configuration values.*

---

#### Syntax

```
status = DftiCreateDescriptor(&desc_handle, precision, forward_domain, dimension, length);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name                  | Type                                                                   | Description                                                                                                           |
|-----------------------|------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------|
| <i>precision</i>      | enum                                                                   | Precision of the transform: DFTI_SINGLE or DFTI_DOUBLE.                                                               |
| <i>forward_domain</i> | enum                                                                   | Forward domain of the transform: DFTI_COMPLEX or DFTI_REAL.                                                           |
| <i>dimension</i>      | MKL_LONG                                                               | Dimension of the transform.                                                                                           |
| <i>length</i>         | MKL_LONG if <i>dimension</i> = 1.<br>Array of type MKL_LONG otherwise. | Length of the transform for a one-dimensional transform. Lengths of each dimension for a multi-dimensional transform. |

#### Output Parameters

| Name               | Type                   | Description                 |
|--------------------|------------------------|-----------------------------|
| <i>desc_handle</i> | DFTI_DESCRIPTOR_HANDLE | FFT descriptor.             |
| <i>status</i>      | MKL_LONG               | Function completion status. |

#### Description

This function allocates memory for the descriptor data structure and instantiates it with all the default configuration settings for the precision, forward domain, dimension, and length of the desired transform. Because memory is allocated dynamically, the result is actually a pointer to the created descriptor. This function is slightly different from the "initialization" function that can be found in software packages or libraries that implement more traditional algorithms for computing an FFT. This function does not perform any significant computational work such as computation of twiddle factors. The function [DftiCommitDescriptor](#) does this work after the function [DftiSetValue](#) has set values of all necessary parameters.

The function returns zero when it completes successfully. See [Status Checking Functions](#) for more information on the returned status.

## Prototype

```

/* Note that the preprocessor definition provided below only illustrates
 * that the actual function called may be determined at compile time.
 * You can rely only on the declaration of the function.
 * For precise definition of the preprocessor macro, see the include/mkl_dfti.h
 * file in the Intel MKL directory.
 */
MKL_LONG DftiCreateDescriptor(DFTI_DESCRIPTOR_HANDLE * pHandle,
 enum DFTI_CONFIG_VALUE precision,
 enum DFTI_CONFIG_VALUE domain,
 MKL_LONG dimension, ... /* length/lengths */);

#define DftiCreateDescriptor(desc,prec,domain,dim,sizes) \
 ((prec)==DFTI_SINGLE && (dim)==1) ? \
 some_actual_function_s1d((desc),(domain),(MKL_LONG)(sizes)) : \
 ...

```

Variable *length/lengths* is interpreted as a scalar (MKL\_LONG) or an array (MKL\_LONG\*), depending on the value of parameter *dimension*. If the value of parameter *precision* is known at compile time, an optimizing compiler retains only the call to the respective specific function, thereby reducing the size of the statically linked application. Avoid direct calls to the specific functions used in the preprocessor macro definition, because their interface may change in future releases of the library. If the use of the macro is undesirable, you can safely undefine it after inclusion of the Intel MKL FFT header file, as follows:

```

#include "mkl_dfti.h"
#undef DftiCreateDescriptor

```

## See Also

[DFTI\\_PRECISION](#) configuration parameter

[DFTI\\_FORWARD\\_DOMAIN](#) configuration parameter

[DFTI\\_DIMENSION](#), [DFTI\\_LENGTHS](#) configuration parameters

[Configuration Parameters](#)

## DftiCommitDescriptor

*Performs all initialization for the actual FFT computation.*

## Syntax

```
status = DftiCommitDescriptor(desc_handle);
```

## Include Files

- mkl.h

## Input Parameters

| Name               | Type                   | Description     |
|--------------------|------------------------|-----------------|
| <i>desc_handle</i> | DFTI_DESCRIPTOR_HANDLE | FFT descriptor. |

## Output Parameters

| Name               | Type                   | Description             |
|--------------------|------------------------|-------------------------|
| <i>desc_handle</i> | DFTI_DESCRIPTOR_HANDLE | Updated FFT descriptor. |

| Name          | Type     | Description                 |
|---------------|----------|-----------------------------|
| <i>status</i> | MKL_LONG | Function completion status. |

### Description

This function completes initialization of a previously created descriptor, which is required before the descriptor can be used for FFT computations. Typically, committing the descriptor performs all initialization that is required for the actual FFT computation. The initialization done by the function may involve exploring different factorizations of the input length to find the optimal computation method.

If you call the `DftiSetValue` function to change configuration parameters of a committed descriptor (see [Descriptor Configuration Functions](#)), you must re-commit the descriptor before invoking a computation function. Typically, a committal function call is immediately followed by a computation function call (see [FFT Computation Functions](#)).

The function returns zero when it completes successfully. See [Status Checking Functions](#) for more information on the returned status.

### Prototype

```
MKL_LONG DftiCommitDescriptor(DFTI_DESCRIPTOR_HANDLE);
```

### DftiFreeDescriptor

*Frees the memory allocated for a descriptor.*

### Syntax

```
status = DftiFreeDescriptor(&desc_handle);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name               | Type                   | Description     |
|--------------------|------------------------|-----------------|
| <i>desc_handle</i> | DFTI_DESCRIPTOR_HANDLE | FFT descriptor. |

### Output Parameters

| Name               | Type                   | Description                                |
|--------------------|------------------------|--------------------------------------------|
| <i>desc_handle</i> | DFTI_DESCRIPTOR_HANDLE | Memory for the FFT descriptor is released. |
| <i>status</i>      | MKL_LONG               | Function completion status.                |

### Description

This function frees all memory allocated for a descriptor.

**NOTE**

Memory allocation/deallocation inside Intel MKL is managed by Intel MKL memory management software. So, even after successful completion of `FreeDescriptor`, the memory space may continue being allocated for the application because the memory management software sometimes does not return the memory space to the OS, but considers the space free and can reuse it for future memory allocation. See [Example "mkl\\_free\\_buffers Usage with FFT Functions"](#) in the description of the service function `FreeBuffers` on how to use Intel MKL memory management software and release memory to the OS.

The function returns zero when it completes successfully. See [Status Checking Functions](#) for more information on the returned status.

**Prototype**

```
MKL_LONG DftiFreeDescriptor(DFTI_DESCRIPTOR_HANDLE *);
```

**DftiCopyDescriptor**

*Makes a copy of an existing descriptor.*

**Syntax**

```
status = DftiCopyDescriptor(desc_handle_original, &desc_handle_copy);
```

**Include Files**

- `mkl.h`

**Input Parameters**

| Name                              | Type                                | Description                 |
|-----------------------------------|-------------------------------------|-----------------------------|
| <code>desc_handle_original</code> | <code>DFTI_DESCRIPTOR_HANDLE</code> | The FFT descriptor to copy. |

**Output Parameters**

| Name                          | Type                                | Description                     |
|-------------------------------|-------------------------------------|---------------------------------|
| <code>desc_handle_copy</code> | <code>DFTI_DESCRIPTOR_HANDLE</code> | The copy of the FFT descriptor. |
| <code>status</code>           | <code>MKL_LONG</code>               | Function completion status.     |

**Description**

This function makes a copy of an existing descriptor. The resulting descriptor `desc_handle_copy` and the existing descriptor `desc_handle_original` specify the same configuration of the transform, but do not have any memory areas in common ("deep copy").

The function returns zero when it completes successfully. See [Status Checking Functions](#) for more information on the returned status.

**Prototype**

```
MKL_LONG DftiCopyDescriptor(DFTI_DESCRIPTOR_HANDLE, DFTI_DESCRIPTOR_HANDLE *);
```

## FFT Descriptor Configuration Functions

This category contains the following functions: the value setting function `DftiSetValue` sets one particular configuration parameter to an appropriate value, and the value getting function `DftiGetValue` reads the value of one particular configuration parameter. While all configuration parameters are readable, you cannot set a few of them. Some of these contain fixed information of a particular implementation such as version number, or dynamic information, which is derived by the implementation during execution of one of the functions. See [Configuration Settings](#) for details.

### DftiSetValue

*Sets one particular configuration parameter with the specified configuration value.*

---

#### Syntax

```
status = DftiSetValue(desc_handle, config_param, config_val);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name                      | Type                                    | Description              |
|---------------------------|-----------------------------------------|--------------------------|
| <code>desc_handle</code>  | <code>DFTI_DESCRIPTOR_HANDLE</code>     | FFT descriptor.          |
| <code>config_param</code> | enum                                    | Configuration parameter. |
| <code>config_val</code>   | Depends on the configuration parameter. | Configuration value.     |

#### Output Parameters

| Name                     | Type                                | Description                 |
|--------------------------|-------------------------------------|-----------------------------|
| <code>desc_handle</code> | <code>DFTI_DESCRIPTOR_HANDLE</code> | Updated FFT descriptor.     |
| <code>status</code>      | <code>MKL_LONG</code>               | Function completion status. |

#### Description

This function sets one particular configuration parameter with the specified configuration value. Each configuration parameter is a named constant, and the configuration value must have the corresponding type, which can be a named constant or a native type. For available configuration parameters and the corresponding configuration values, see:

- `DFTI_PRECISION`
- `DFTI_FORWARD_DOMAIN`
- `DFTI_DIMENSION`, `DFTI_LENGTH`
- `DFTI_PLACEMENT`
- `DFTI_FORWARD_SCALE`, `DFTI_BACKWARD_SCALE`
- `DFTI_THREAD_LIMIT`
- `DFTI_INPUT_STRIDES`, `DFTI_OUTPUT_STRIDES`
- `DFTI_NUMBER_OF_TRANSFORMS`
- `DFTI_INPUT_DISTANCE`, `DFTI_OUTPUT_DISTANCE`

- `DFTI_COMPLEX_STORAGE`, `DFTI_REAL_STORAGE`, `DFTI_CONJUGATE_EVEN_STORAGE`
- `DFTI_PACKED_FORMAT`
- `DFTI_WORKSPACE`
- `DFTI_ORDERING`

The `DftiSetValue` function cannot be used to change configuration parameters `DFTI_FORWARD_DOMAIN`, `DFTI_PRECISION`, `DFTI_DIMENSION`, and `DFTI_LENGTHS`. Use the `DftiCreateDescriptor` function to set them.

Function calls needed to configure an FFT descriptor for a particular call to an FFT computation function are summarized in [Configuring and Computing an FFT in C C++](#).

The function returns zero when it completes successfully. See [Status Checking Functions](#) for more information on the returned status.

## Prototype

```
MKL_LONG DftiSetValue(DFTI_DESCRIPTOR_HANDLE, DFTI_CONFIG_PARAM , ...);
```

## See Also

[Configuration Settings](#) for more information on configuration parameters.

[DftiCreateDescriptor](#)

[DftiGetValue](#)

## DftiGetValue

*Gets the configuration value of one particular configuration parameter.*

## Syntax

```
status = DftiGetValue(desc_handle, config_param, &config_val);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                      | Type                                | Description                                                                                                                       |
|---------------------------|-------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|
| <code>desc_handle</code>  | <code>DFTI_DESCRIPTOR_HANDLE</code> | FFT descriptor.                                                                                                                   |
| <code>config_param</code> | enum                                | Configuration parameter. See <a href="#">Table "Configuration Parameters"</a> for allowable values of <code>config_param</code> . |

## Output Parameters

| Name                    | Type                                    | Description                 |
|-------------------------|-----------------------------------------|-----------------------------|
| <code>config_val</code> | Depends on the configuration parameter. | Configuration value.        |
| <code>status</code>     | <code>MKL_LONG</code>                   | Function completion status. |

## Description

This function gets the configuration value of one particular configuration parameter. Each configuration parameter is a named constant, and the configuration value must have the corresponding type, which can be a named constant or a native type. For available configuration parameters and the corresponding configuration values, see:

- `DFTI_PRECISION`
- `DFTI_FORWARD_DOMAIN`
- `DFTI_DIMENSION`, `DFTI_LENGTH`
- `DFTI_PLACEMENT`
- `DFTI_FORWARD_SCALE`, `DFTI_BACKWARD_SCALE`
- `DFTI_THREAD_LIMIT`
- `DFTI_INPUT_STRIDES`, `DFTI_OUTPUT_STRIDES`
- `DFTI_NUMBER_OF_TRANSFORMS`
- `DFTI_INPUT_DISTANCE`, `DFTI_OUTPUT_DISTANCE`
- `DFTI_COMPLEX_STORAGE`, `DFTI_REAL_STORAGE`, `DFTI_CONJUGATE_EVEN_STORAGE`
- `DFTI_PACKED_FORMAT`
- `DFTI_WORKSPACE`
- `DFTI_COMMIT_STATUS`
- `DFTI_ORDERING`

The function returns zero when it completes successfully. See [Status Checking Functions](#) for more information on the returned status.

## Prototype

```
MKL_LONG DftiGetValue(DFTI_DESCRIPTOR_HANDLE,
 DFTI_CONFIG_PARAM ,
 ...);
```

## See Also

[Configuration Settings](#) for more information on configuration parameters.

[DftiSetValue](#)

## FFT Computation Functions

This category contains the following functions: compute the forward transform and compute the backward transform.

### DftiComputeForward

*Computes the forward FFT.*

#### Syntax

```
status = DftiComputeForward(desc_handle, x_inout);
status = DftiComputeForward(desc_handle, x_in, y_out);
status = DftiComputeForward(desc_handle, xre_inout, xim_inout);
status = DftiComputeForward(desc_handle, xre_in, xim_in, yre_out, yim_out);
```

## Input Parameters

| Name                                              | Type                                                                                                                                                                     | Description                                                                                                                                                              |
|---------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>desc_handle</code>                          | <code>DFTI_DESCRIPTOR_HANDLE</code>                                                                                                                                      | FFT descriptor.                                                                                                                                                          |
| <code>x_inout, x_in</code>                        | Array of type <code>float</code> or <code>double</code> depending on the precision of the transform, specified in the <code>DFTI_PRECISION</code> configuration setting. | Data to be transformed in case of a real forward domain, specified in the <code>DFTI_FORWARD_DOMAIN</code> configuration setting.                                        |
| <code>xre_inout, xim_inout, xre_in, xim_in</code> | Array of type <code>float</code> or <code>double</code> depending on the precision of the transform.                                                                     | Real and imaginary parts of the data to be transformed in the case of a complex forward domain, specified in the <code>DFTI_FORWARD_DOMAIN</code> configuration setting. |

The suffix in parameter names corresponds to the value of the configuration parameter `DFTI_PLACEMENT` as follows:

- `_inout` to `DFTI_INPLACE`
- `_in` to `DFTI_NOT_INPLACE`

## Output Parameters

| Name                                                | Type                                                                                                 | Description                                                                                                                                                          |
|-----------------------------------------------------|------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>y_out</code>                                  | Array of type <code>float</code> or <code>double</code> depending on the precision of the transform. | The transformed data in case of a real backward domain, determined by the <code>DFTI_FORWARD_DOMAIN</code> configuration setting.                                    |
| <code>xre_inout, xim_inout, yre_out, yim_out</code> | Array of type <code>float</code> or <code>double</code> depending on the precision of the transform. | Real and imaginary parts of the transformed data in the case of a complex backward domain, determined by the <code>DFTI_FORWARD_DOMAIN</code> configuration setting. |
| <code>status</code>                                 | <code>MKL_LONG</code>                                                                                | Function completion status.                                                                                                                                          |

The suffix in parameter names corresponds to the value of the configuration parameter `DFTI_PLACEMENT` as follows:

- `_inout` to `DFTI_INPLACE`
- `_out` to `DFTI_NOT_INPLACE`

## Include Files

- `mkl.h`

## Description

The `DftiComputeForward` function accepts the descriptor handle parameter and one or more data parameters. Given a successfully configured and committed descriptor, this function computes the forward FFT, that is, the [transform](#) with the minus sign in the exponent,  $\delta = -1$ .

The `DFTI_COMPLEX_STORAGE`, `DFTI_REAL_STORAGE`, and `DFTI_CONJUGATE_EVEN_STORAGE` configuration parameters define the layout of the input and output data and must be properly set in a call to the `DftiSetValue` function.

The FFT descriptor must be properly configured prior to the function call. Function calls needed to configure an FFT descriptor for a particular call to an FFT computation function are summarized in [Configuring and Computing an FFT in C C++](#).

The number and types of the data parameters that the function requires may vary depending on the configuration of the descriptor. This variation is accommodated by variable parameters.

The function returns zero when it completes successfully. See [Status Checking Functions](#) for more information on the returned status.

## Prototype

```
MKL_LONG DftiComputeForward(DFTI_DESCRIPTOR_HANDLE, void*, ...);
```

## See Also

[Configuration Settings](#)

[DFTI\\_FORWARD\\_DOMAIN](#)

[DFTI\\_PLACEMENT](#)

[DFTI\\_PACKED\\_FORMAT](#)

[DFTI\\_COMPLEX\\_STORAGE](#), [DFTI\\_REAL\\_STORAGE](#), [DFTI\\_CONJUGATE\\_EVEN\\_STORAGE](#)

[DFTI\\_DIMENSION](#), [DFTI\\_LENGTHS](#)

[DFTI\\_INPUT\\_DISTANCE](#), [DFTI\\_OUTPUT\\_DISTANCE](#)

[DFTI\\_INPUT\\_STRIDES](#), [DFTI\\_OUTPUT\\_STRIDES](#)

[DftiComputeBackward](#)

[DftiSetValue](#)

## DftiComputeBackward

*Computes the backward FFT.*

## Syntax

```
status = DftiComputeBackward(desc_handle, x_inout);
```

```
status = DftiComputeBackward(desc_handle, y_in, x_out);
```

```
status = DftiComputeBackward(desc_handle, xre_inout, xim_inout);
```

```
status = DftiComputeBackward(desc_handle, yre_in, yim_in, xre_out, xim_out);
```

## Input Parameters

| Name                                              | Type                                                                                                                                                                     | Description                                                                                                                                                                |
|---------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>desc_handle</code>                          | <code>DFTI_DESCRIPTOR_HANDLE</code>                                                                                                                                      | FFT descriptor.                                                                                                                                                            |
| <code>x_inout, y_in</code>                        | Array of type <code>float</code> or <code>double</code> depending on the precision of the transform, specified in the <code>DFTI_PRECISION</code> configuration setting. | Data to be transformed in case of a real backward domain, determined by the <code>DFTI_FORWARD_DOMAIN</code> configuration setting.                                        |
| <code>xre_inout, xim_inout, yre_in, yim_in</code> | Array of type <code>float</code> or <code>double</code> depending on the precision of the transform.                                                                     | Real and imaginary parts of the data to be transformed in the case of a complex backward domain, determined by the <code>DFTI_FORWARD_DOMAIN</code> configuration setting. |

The suffix in parameter names corresponds to the value of the configuration parameter `DFTI_PLACEMENT` as follows:

- `_inout` to `DFTI_INPLACE`
- `_in` to `DFTI_NOT_INPLACE`

## Output Parameters

| Name                                                                                                | Type                                                                                                 | Description                                                                                                                                                        |
|-----------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x_out</code>                                                                                  | Array of type <code>float</code> or <code>double</code> depending on the precision of the transform. | The transformed data in case of a real forward domain, specified in the <code>DFTI_FORWARD_DOMAIN</code> configuration setting.                                    |
| <code>xre_inout</code> ,<br><code>xim_inout</code> ,<br><code>xre_out</code> , <code>xim_out</code> | Array of type <code>float</code> or <code>double</code> depending on the precision of the transform. | Real and imaginary parts of the transformed data in the case of a complex forward domain, specified in the <code>DFTI_FORWARD_DOMAIN</code> configuration setting. |
| <code>status</code>                                                                                 | <code>MKL_LONG</code>                                                                                | Function completion status.                                                                                                                                        |

The suffix in parameter names corresponds to the value of the configuration parameter `DFTI_PLACEMENT` as follows:

- `_inout` to `DFTI_INPLACE`
- `_out` to `DFTI_NOT_INPLACE`

## Include Files

- `mkl.h`

## Description

The function accepts the descriptor handle parameter and one or more data parameters. Given a successfully configured and committed descriptor, the `DftiComputeBackward` function computes the inverse FFT, that is, the [transform](#) with the plus sign in the exponent,  $\delta = +1$ .

The `DFTI_COMPLEX_STORAGE`, `DFTI_REAL_STORAGE`, and `DFTI_CONJUGATE_EVEN_STORAGE` configuration parameters define the layout of the input and output data and must be properly set in a call to the `DftiSetValue` function.

The FFT descriptor must be properly configured prior to the function call. Function calls needed to configure an FFT descriptor for a particular call to an FFT computation function are summarized in [Configuring and Computing an FFT in C C++](#).

The number and types of the data parameters that the function requires may vary depending on the configuration of the descriptor. This variation is accommodated by variable parameters.

The function returns zero when it completes successfully. See [Status Checking Functions](#) for more information on the returned status.

## Prototype

```
MKL_LONG DftiComputeBackward(DFTI_DESCRIPTOR_HANDLE, void *, ...);
```

## See Also

[Configuration Settings](#)  
[DFTI\\_FORWARD\\_DOMAIN](#)  
[DFTI\\_PLACEMENT](#)  
[DFTI\\_PACKED\\_FORMAT](#)

DFTI\_COMPLEX\_STORAGE, DFTI\_REAL\_STORAGE, DFTI\_CONJUGATE\_EVEN\_STORAGE  
 DFTI\_DIMENSION, DFTI\_LENGTHS  
 DFTI\_INPUT\_DISTANCE, DFTI\_OUTPUT\_DISTANCE  
 DFTI\_INPUT\_STRIDES, DFTI\_OUTPUT\_STRIDES  
 DftiComputeForward  
 DftiSetValue

### Configuring and Computing an FFT in C/C++

The table below summarizes information on configuring and computing an FFT in C/C++ for all kinds of transforms and possible combinations of input and output domains.

| FFT to Compute                                              | Input Data                  | Output Data                 | Required FFT Function Calls                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|-------------------------------------------------------------|-----------------------------|-----------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Complex-to-complex,<br>in-place,<br>forward or backward     | Interleaved complex numbers | Interleaved complex numbers | <pre> /* Configure a Descriptor */ status = DftiCreateDescriptor(&amp;hand, &lt;precision&gt;, DFTI_COMPLEX, &lt;dimension&gt;, &lt;sizes&gt;); status = DftiCommitDescriptor(hand);  /* Compute an FFT */ /* forward FFT */ status = DftiComputeForward(hand, X_inout); /* or backward FFT */ status = DftiComputeBackward(hand, X_inout);                     </pre>                                                                                              |
| Complex-to-complex,<br>out-of-place,<br>forward or backward | Interleaved complex numbers | Interleaved complex numbers | <pre> /* Configure a Descriptor */ status = DftiCreateDescriptor(&amp;hand, &lt;precision&gt;, DFTI_COMPLEX, &lt;dimension&gt;, &lt;sizes&gt;); status = DftiSetValue(hand, DFTI_PLACEMENT, DFTI_NOT_INPLACE); status = DftiCommitDescriptor(hand);  /* Compute an FFT */ /* forward FFT */ status = DftiComputeForward(hand, X_in, Y_out); /* or backward FFT */ status = DftiComputeBackward(hand, X_in, Y_out);                     </pre>                       |
| Complex-to-complex,<br>in-place,<br>forward or backward     | Split-complex numbers       | Split-complex numbers       | <pre> /* Configure a Descriptor */ status = DftiCreateDescriptor(&amp;hand, &lt;precision&gt;, DFTI_COMPLEX, &lt;dimension&gt;, &lt;sizes&gt;); status = DftiSetValue(hand, DFTI_COMPLEX_STORAGE, DFTI_REAL_REAL); status = DftiCommitDescriptor(hand);  /* Compute an FFT */ /* forward FFT */ status = DftiComputeForward(hand, Xre_inout, Xim_inout); /* or backward FFT */ status = DftiComputeBackward(hand, Xre_inout, Xim_inout);                     </pre> |

| FFT to Compute                                              | Input Data            | Output Data               | Required FFT Function Calls                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|-------------------------------------------------------------|-----------------------|---------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Complex-to-complex,<br>out-of-place,<br>forward or backward | Split-complex numbers | Split-complex numbers     | <pre> /* Configure a Descriptor */ status = DftiCreateDescriptor(&amp;hand, &lt;precision&gt;, DFTI_COMPLEX, &lt;dimension&gt;, &lt;sizes&gt;); status = DftiSetValue(hand, DFTI_COMPLEX_STORAGE, DFTI_REAL_REAL); status = DftiSetValue(hand, DFTI_PLACEMENT, DFTI_NOT_INPLACE); status = DftiCommitDescriptor(hand);  /* Compute an FFT */ /* forward FFT */ status = DftiComputeForward(hand, Xre_in, Xim_in, Yre_out, Yim_out); /* or backward FFT */ status = DftiComputeBackward(hand, Xre_in, Xim_in, Yre_out, Yim_out); </pre>                                                                                            |
| Real-to-complex,<br>in-place,<br>forward                    | Real numbers          | Numbers in the CCE format | <pre> /* Configure a Descriptor */ status = DftiCreateDescriptor(&amp;hand, &lt;precision&gt;, DFTI_REAL, &lt;dimension&gt;, &lt;sizes&gt;); status = DftiSetValue(hand, DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX); status = DftiSetValue(hand, DFTI_PACKED_FORMAT, DFTI_CCE_FORMAT); status = DftiSetValue(hand, DFTI_INPUT_STRIDES, &lt;real_strides&gt;); status = DftiSetValue(hand, DFTI_OUTPUT_STRIDES, &lt;complex_strides&gt;); status = DftiCommitDescriptor(hand);  /* Compute an FFT */ status = DftiComputeForward(hand, X_inout); </pre>                                                                    |
| Real-to-complex,<br>out-of-place,<br>forward                | Real numbers          | Numbers in the CCE format | <pre> /* Configure a Descriptor */ status = DftiCreateDescriptor(&amp;hand, &lt;precision&gt;, DFTI_REAL, &lt;dimension&gt;, &lt;sizes&gt;); status = DftiSetValue(hand, DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX); status = DftiSetValue(hand, DFTI_PACKED_FORMAT, DFTI_CCE_FORMAT); status = DftiSetValue(hand, DFTI_PLACEMENT, DFTI_NOT_INPLACE); status = DftiSetValue(hand, DFTI_INPUT_STRIDES, &lt;real_strides&gt;); status = DftiSetValue(hand, DFTI_OUTPUT_STRIDES, &lt;complex_strides&gt;); status = DftiCommitDescriptor(hand);  /* Compute an FFT */ status = DftiComputeForward(hand, X_in, Y_out); </pre> |

| FFT to Compute                                | Input Data                      | Output Data  | Required FFT Function Calls                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|-----------------------------------------------|---------------------------------|--------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Complex-to-real,<br>in-place,<br>backward     | Numbers in<br>the CCE<br>format | Real numbers | <pre> /* Configure a Descriptor */ status = DftiCreateDescriptor(&amp;hand, &lt;precision&gt;, DFTI_REAL, &lt;dimension&gt;, &lt;sizes&gt;); status = DftiSetValue(hand, DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX); status = DftiSetValue(hand, DFTI_PACKED_FORMAT, DFTI_CCE_FORMAT); status = DftiSetValue(hand, DFTI_INPUT_STRIDES, &lt;complex_strides&gt;); status = DftiSetValue(hand, DFTI_OUTPUT_STRIDES, &lt;real_strides&gt;); status = DftiCommitDescriptor(hand);  /* Compute an FFT */ status = DftiComputeBackward(hand, X_inout); </pre>                                                                    |
| Complex-to-real,<br>out-of-place,<br>backward | Numbers in<br>the CCE<br>format | Real numbers | <pre> /* Configure a Descriptor */ status = DftiCreateDescriptor(&amp;hand, &lt;precision&gt;, DFTI_REAL, &lt;dimension&gt;, &lt;sizes&gt;); status = DftiSetValue(hand, DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX); status = DftiSetValue(hand, DFTI_PLACEMENT, DFTI_NOT_INPLACE); status = DftiSetValue(hand, DFTI_PACKED_FORMAT, DFTI_CCE_FORMAT); status = DftiSetValue(hand, DFTI_INPUT_STRIDES, &lt;complex_strides&gt;); status = DftiSetValue(hand, DFTI_OUTPUT_STRIDES, &lt;real_strides&gt;); status = DftiCommitDescriptor(hand);  /* Compute an FFT */ status = DftiComputeBackward(hand, X_in, Y_out); </pre> |

You can find C programs that illustrate configuring and computing FFTs in the `examples/dftc/` subdirectory of your Intel MKL directory.

## Status Checking Functions

All of the descriptor manipulation, FFT computation, and descriptor configuration functions return an integer value denoting the status of the operation. The functions in this category check that status. The first function is a logical function that checks whether the status reflects an error of a predefined class, and the second is an error message function that returns a character string.

### DftiErrorClass

*Checks whether the status reflects an error of a predefined class.*

#### Syntax

```
predicate = DftiErrorClass(status, error_class);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name                     | Type                  | Description                           |
|--------------------------|-----------------------|---------------------------------------|
| <code>status</code>      | <code>MKL_LONG</code> | Completion status of an FFT function. |
| <code>error_class</code> | <code>MKL_LONG</code> | Predefined error class.               |

## Output Parameters

| Name                   | Type                  | Description         |
|------------------------|-----------------------|---------------------|
| <code>predicate</code> | <code>MKL_LONG</code> | Result of checking. |

## Description

The FFT interface in Intel MKL provides a set of predefined error classes listed in [Table "Predefined Error Classes"](#). They are named constants and have the type `MKL_LONG`.

### Predefined Error Classes

| Named Constants                              | Comments                                                                                                                                  |
|----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| <code>DFTI_NO_ERROR</code>                   | No error. The zero status belongs to this class.                                                                                          |
| <code>DFTI_MEMORY_ERROR</code>               | Usually associated with memory allocation                                                                                                 |
| <code>DFTI_INVALID_CONFIGURATION</code>      | Invalid settings of one or more configuration parameters                                                                                  |
| <code>DFTI_INCONSISTENT_CONFIGURATION</code> | Inconsistent configuration or input parameters                                                                                            |
| <code>DFTI_NUMBER_OF_THREADS_ERROR</code>    | Number of OMP threads in the computation function is not equal to the number of OMP threads in the initialization stage (commit function) |
| <code>DFTI_MULTITHREADED_ERROR</code>        | Usually associated with a value that OMP routines return in case of errors                                                                |
| <code>DFTI_BAD_DESCRIPTOR</code>             | Descriptor is unusable for computation                                                                                                    |
| <code>DFTI_UNIMPLEMENTED</code>              | Unimplemented legitimate settings; implementation dependent                                                                               |
| <code>DFTI_MKL_INTERNAL_ERROR</code>         | Internal library error                                                                                                                    |
| <code>DFTI_1D_LENGTH_EXCEEDS_INT32</code>    | Length of one of dimensions exceeds $2^{32} - 1$ (4 bytes).                                                                               |

The `DftiErrorClass` function returns a non-zero value if the status belongs to the predefined error class. To check whether a function call was successful, call `DftiErrorClass` with a specific error class. However, the zero value of the status belongs to the `DFTI_NO_ERROR` class and thus the zero status indicates successful completion of an operation. See [Example "Using Status Checking Functions"](#) for an illustration of correct use of the status checking functions.

#### NOTE

It is incorrect to directly compare a status with a predefined class.

## Prototype

```
MKL_LONG DftiErrorClass(MKL_LONG , MKL_LONG);
```

## DftiErrorMessage

*Generates an error message.*

### Syntax

```
error_message = DftiErrorMessage(status);
```

### Include Files

- mkl.h

### Input Parameters

| Name          | Type     | Description                      |
|---------------|----------|----------------------------------|
| <i>status</i> | MKL_LONG | Completion status of a function. |

### Output Parameters

| Name                 | Type          | Description                                  |
|----------------------|---------------|----------------------------------------------|
| <i>error_message</i> | Array of char | The character string with the error message. |

### Description

The error message function generates an error message character string. The function returns a pointer to a constant character string, that is, a character array with terminating '\0' character, and you do not need to free this pointer.

Example "Using Status Checking Function" shows how this function can be used.

### Prototype

```
char *DftiErrorMessage(MKL_LONG);
```

## Cluster FFT Functions

This section describes the cluster Fast Fourier Transform (FFT) functions implemented in Intel® MKL.

### NOTE

These functions are available only for Intel® 64 and Intel® Many Integrated Core architectures.

The cluster FFT function library was designed to perform fast Fourier transforms on a cluster, that is, a group of computers interconnected via a network. Each computer (node) in the cluster has its own memory and processor(s). Data interchanges between the nodes are provided by the network.

One or more processes may be running in parallel on each cluster node. To organize communication between different processes, the cluster FFT function library uses the Message Passing Interface (MPI). To avoid dependence on a specific MPI implementation (for example, MPICH, Intel® MPI, and others), the library works with MPI via a message-passing library for linear algebra called BLACS.

Cluster FFT functions of Intel MKL provide one-dimensional, two-dimensional, and multi-dimensional (up to the order of 7) functions and both Fortran and C interfaces for all transform functions.

To develop applications using the cluster FFT functions, you should have basic skills in MPI programming.

The interfaces for the Intel MKL cluster FFT functions are similar to the corresponding interfaces for the conventional Intel MKL [FFT functions](#), described earlier in this chapter. Refer there for details not explained in this section.

Table "Cluster FFT Functions in Intel MKL" lists cluster FFT functions implemented in Intel MKL:

**Cluster FFT Functions in Intel MKL**

| Function Name                       | Operation                                                                            |
|-------------------------------------|--------------------------------------------------------------------------------------|
| Descriptor Manipulation Functions   |                                                                                      |
| <code>DftiCreateDescriptorDM</code> | Allocates memory for the descriptor data structure and preliminarily initializes it. |
| <code>DftiCommitDescriptorDM</code> | Performs all initialization for the actual FFT computation.                          |
| <code>DftiFreeDescriptorDM</code>   | Frees memory allocated for a descriptor.                                             |
| FFT Computation Functions           |                                                                                      |
| <code>DftiComputeForwardDM</code>   | Computes the forward FFT.                                                            |
| <code>DftiComputeBackwardDM</code>  | Computes the backward FFT.                                                           |
| Descriptor Configuration Functions  |                                                                                      |
| <code>DftiSetValueDM</code>         | Sets one particular configuration parameter with the specified configuration value.  |
| <code>DftiGetValueDM</code>         | Gets the value of one particular configuration parameter.                            |

**Computing Cluster FFT**

The cluster FFT functions described later in this section are provided with Fortran and C interfaces.

Cluster FFT computation is performed by `DftiComputeForwardDM` and `DftiComputeBackwardDM` functions, called in a program using MPI, which will be referred to as MPI program. After an MPI program starts, a number of processes are created. MPI identifies each process by its rank. The processes are independent of one another and communicate via MPI. A function called in an MPI program is invoked in all the processes. Each process manipulates data according to its rank. Input or output data for a cluster FFT transform is a sequence of real or complex values. A cluster FFT computation function operates on the local part of the input data, that is, some part of the data to be operated in a particular process, as well as generates local part of the output data. While each process performs its part of computations, running in parallel and communicating through MPI, the processes perform the entire FFT computation. FFT computations using the Intel MKL cluster FFT functions are typically effected by a number of steps listed below:

1. Initiate MPI by calling `MPI_Init` (the function must be called prior to calling any FFT function and any MPI function).
2. Allocate memory for the descriptor and create it by calling `DftiCreateDescriptorDM`.
3. Specify one of several values of configuration parameters by one or more calls to `DftiSetValueDM`.
4. Obtain values of configuration parameters needed to create local data arrays; the values are retrieved by calling `DftiGetValueDM`.
5. Initialize the descriptor for the FFT computation by calling `DftiCommitDescriptorDM`.
6. Create arrays for local parts of input and output data and fill the local part of input data with values. (For more information, see [Distributing Data among Processes](#).)

7. Compute the transform by calling `DftiComputeForwardDM` or `DftiComputeBackwardDM`.
8. Gather local output data into the global array using MPI functions. (This step is optional because you may need to immediately employ the data differently.)
9. Release memory allocated for the descriptor by calling `DftiFreeDescriptorDM`.
10. Finalize communication through MPI by calling `MPI_Finalize` (the function must be called after the last call to a cluster FFT function and the last call to an MPI function).

Several code examples in the "Examples for Cluster FFT Functions" section in the Code Examples appendix illustrate cluster FFT computations.

## Distributing Data among Processes

The Intel MKL cluster FFT functions store all input and output multi-dimensional arrays (matrices) in one-dimensional arrays (vectors). The arrays are stored in the row-major order. For example, a two-dimensional matrix  $A$  of size  $(m,n)$  is stored in a vector  $B$  of size  $m*n$  so that

$$B[i*n+j]=A[i][j] \quad (i=0, \dots, m-1, j=0, \dots, n-1) .$$

---

### NOTE

Order of FFT dimensions is the same as the order of array dimensions in the programming language. For example, a 3-dimensional FFT with `Lengths=(m,n,l)` can be computed over an array `Ar[m][n][l]`.

---

All MPI processes involved in cluster FFT computation operate their own portions of data. These local arrays make up the virtual global array that the fast Fourier transform is applied to. It is your responsibility to properly allocate local arrays (if needed), fill them with initial data and gather resulting data into an actual global array or process the resulting data differently. To be able do this, see sections below on how the virtual global array is composed of the local ones.

## Multi-dimensional transforms

If the dimension of transform is greater than one, the cluster FFT function library splits data in the dimension whose index changes most slowly, so that the parts contain all elements with several consecutive values of this index. It is the first dimension in  $C$ . If the global array is two-dimensional, it gives each process several consecutive rows. Local arrays are placed in memory allocated for the virtual global array consecutively, in the order determined by process ranks. For example, in case of two processes, during the computation of a three-dimensional transform whose matrix has size  $(11,15,12)$ , the processes may store local arrays of sizes  $(6,15,12)$  and  $(5,15,12)$ , respectively.

If  $p$  is the number of MPI processes and the matrix of a transform to be computed has size  $(m,n,l)$ , each MPI process works with local data array of size  $(m_q, n, l)$ , where  $\sum m_q=m, q=0, \dots, p-1$ . Local input arrays must contain appropriate parts of the actual global input array, and then local output arrays will contain appropriate parts of the actual global output array. You can figure out which particular rows of the global array the local array must contain from the following configuration parameters of the cluster FFT interface: `CDFT_LOCAL_NX`, `CDFT_LOCAL_START_X`, and `CDFT_LOCAL_SIZE`. To retrieve values of the parameters, use the `DftiGetValueDM` function:

- `CDFT_LOCAL_NX` specifies how many rows of the global array the current process receives.
- `CDFT_LOCAL_START_X` specifies which row of the global input or output array corresponds to the first row of the local input or output array. If  $A$  is a global array and  $L$  is the appropriate local array, then

$$L[i][j][k]=A[i+cdft\_local\_start\_x][j][k], \text{ where } i=0, \dots, m_q-1, j=0, \dots, n-1, k=0, \dots, l-1.$$

Example "2D Out-of-place Cluster FFT Computation" in the Code Examples appendix shows how the data is distributed among processes for a two-dimensional cluster FFT computation.

## One-dimensional transforms

In this case, input and output data are distributed among processes differently and even the numbers of elements stored in a particular process before and after the transform may be different. Each local array stores a segment of consecutive elements of the appropriate global array. Such segment is determined by

the number of elements and a shift with respect to the first array element. So, to specify segments of the global input and output arrays that a particular process receives, *four* configuration parameters are needed: `CDFT_LOCAL_NX`, `CDFT_LOCAL_START_X`, `CDFT_LOCAL_OUT_NX`, and `CDFT_LOCAL_OUT_START_X`. Use the `DftiGetValueDM` function to retrieve their values. The meaning of the four configuration parameters depends upon the type of the transform, as shown in [Table "Data Distribution Configuration Parameters for 1D Transforms"](#):

#### Data Distribution Configuration Parameters for 1D Transforms

| Meaning of the Parameter           | Forward Transform                   | Backward Transform                  |
|------------------------------------|-------------------------------------|-------------------------------------|
| Number of elements in input array  | <code>CDFT_LOCAL_NX</code>          | <code>CDFT_LOCAL_OUT_NX</code>      |
| Elements shift in input array      | <code>CDFT_LOCAL_START_X</code>     | <code>CDFT_LOCAL_OUT_START_X</code> |
| Number of elements in output array | <code>CDFT_LOCAL_OUT_NX</code>      | <code>CDFT_LOCAL_NX</code>          |
| Elements shift in output array     | <code>CDFT_LOCAL_OUT_START_X</code> | <code>CDFT_LOCAL_START_X</code>     |

#### Memory size for local data

The memory size needed for local arrays cannot be just calculated from `CDFT_LOCAL_NX` (`CDFT_LOCAL_OUT_NX`), because the cluster FFT functions sometimes require allocating a little bit more memory for local data than just the size of the appropriate sub-array. The configuration parameter `CDFT_LOCAL_SIZE` specifies the size of the local input and output array in data elements. Each local input and output arrays must have size not less than `CDFT_LOCAL_SIZE*size_of_element`. Note that in the current implementation of the cluster FFT interface, data elements can be real or complex values, each complex value consisting of the real and imaginary parts. If you employ a user-defined workspace for in-place transforms (for more information, refer to [Table "Settable configuration Parameters"](#)), it must have the same size as the local arrays. [Example "1D In-place Cluster FFT Computations"](#) in the Code Examples appendix illustrates how the cluster FFT functions distribute data among processes in case of a one-dimensional FFT computation performed with a user-defined workspace.

#### Available Auxiliary Functions

If a global input array is located on one MPI process and you want to obtain its local parts or you want to gather the global output array on one MPI process, you can use functions `MKL_CDFT_ScatterData` and `MKL_CDFT_GatherData` to distribute or gather data among processes, respectively. These functions are defined in a file that is delivered with Intel MKL and located in the following subdirectory of the Intel MKL installation directory: `examples/cdftc/source/cdft_example_support.c`.

#### Restriction on Lengths of Transforms

The algorithm that the Intel MKL cluster FFT functions use to distribute data among processes imposes a restriction on lengths of transforms with respect to the number of MPI processes used for the FFT computation:

- For a multi-dimensional transform, lengths of the first two dimensions must be not less than the number of MPI processes.
- Length of a one-dimensional transform must be the product of two integers each of which is not less than the number of MPI processes.

Non-compliance with the restriction causes an error `CDFT_SPREAD_ERROR` (refer to [Error Codes](#) for details). To achieve the compliance, you can change the transform lengths and/or the number of MPI processes, which is specified at start of an MPI program. MPI-2 enables changing the number of processes during execution of an MPI program.

## Cluster FFT Interface

To use the cluster FFT functions, you need to access the header file `mkl_cdft.h` through "include".

The C interface provides a structure type `DFTI_DESCRIPTOR_DM_HANDLE` and a number of functions, some of which accept a different number of input arguments.

To provide communication between parallel processes through MPI, the following include statement must be present in your code:

- C/C++:  

```
#include "mpi.h"
```

There are three main categories of the cluster FFT functions in Intel MKL:

1. **Descriptor Manipulation** . There are three functions in this category. The `DftiCreateDescriptorDM` function creates an FFT descriptor whose storage is allocated dynamically. The `DftiCommitDescriptorDM` function "commits" the descriptor to all its settings. The `DftiFreeDescriptorDM` function frees up the memory allocated for the descriptor.
2. **FFT Computation** . There are two functions in this category. The `DftiComputeForwardDM` function performs the forward FFT computation, and the `DftiComputeBackwardDM` function performs the backward FFT computation.
3. **Descriptor Configuration** . There are two functions in this category. The `DftiSetValueDM` function sets one specific configuration value to one of the many configuration parameters. The `DftiGetValueDM` function gets the current value of any of these configuration parameters, all of which are readable. These parameters, though many, are handled one at a time.

## Cluster FFT Descriptor Manipulation Functions

There are three functions in this category: create a descriptor, commit a descriptor, and free a descriptor.

### DftiCreateDescriptorDM

*Allocates memory for the descriptor data structure and preliminarily initializes it.*

#### Syntax

```
status = DftiCreateDescriptorDM(comm, &handle, v1, v2, dim, size);
status = DftiCreateDescriptorDM(comm, &handle, v1, v2, dim, sizes);
```

#### Include Files

- `mkl_cdft.h`

#### Input Parameters

|                    |                                                                                                                                                           |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>comm</code>  | MPI communicator, e.g. <code>MPI_COMM_WORLD</code> .                                                                                                      |
| <code>v1</code>    | Precision of the transform.                                                                                                                               |
| <code>v2</code>    | Type of the forward domain. Must be <code>DFTI_COMPLEX</code> for complex-to-complex transforms or <code>DFTI_REAL</code> for real-to-complex transforms. |
| <code>dim</code>   | Dimension of the transform.                                                                                                                               |
| <code>size</code>  | Length of the transform in a one-dimensional case.                                                                                                        |
| <code>sizes</code> | Lengths of the transform in a multi-dimensional case.                                                                                                     |

## Output Parameters

*handle* Pointer to the descriptor handle of transform. If the function completes successfully, the pointer to the created handle is stored in the variable.

## Description

This function allocates memory in a particular MPI process for the descriptor data structure and instantiates it with default configuration settings with respect to the precision, domain, dimension, and length of the desired transform. The domain is understood to be the domain of the forward transform. The result is a pointer to the created descriptor. This function is slightly different from the "initialization" function `DftiCommitDescriptorDM` in a more traditional software packages or libraries used for computing the FFT. This function does not perform any significant computation work, such as twiddle factors computation, because the default configuration settings can still be changed using the function `DftiSetValueDM`.

The value of the parameter *vl* is specified through named constants `DFTI_SINGLE` and `DFTI_DOUBLE`. It corresponds to precision of input data, output data, and computation. A setting of `DFTI_SINGLE` indicates single-precision floating-point data type and a setting of `DFTI_DOUBLE` indicates double-precision floating-point data type.

The parameter *dim* is a simple positive integer indicating the dimension of the transform.

For one-dimensional transforms, length is a single integer value of the parameter *size* having type `MKL_LONG`; for multi-dimensional transforms, length is supplied with the parameter *sizes*, which is an array of integers having type `MKL_LONG`.

## Return Values

The function returns `DFTI_NO_ERROR` when completes successfully. In this case, the pointer to the created descriptor handle is stored in *handle*. If the function fails, it returns a value of another error class constant

## Prototype

```
MKL_LONG DftiCreateDescriptorDM(MPI_Comm,DFTI_DESCRIPTOR_DM_HANDLE*,
 enum DFTI_CONFIG_VALUE,enum DFTI_CONFIG_VALUE,MKL_LONG,...);
```

## DftiCommitDescriptorDM

*Performs all initialization for the actual FFT computation.*

## Syntax

```
status = DftiCommitDescriptorDM(handle);
```

## Include Files

- `mkl_cdft.h`

## Input Parameters

*handle* The descriptor handle. Must be valid, that is, created in a call to `DftiCreateDescriptorDM`.

## Description

The cluster FFT interface requires a function that completes initialization of a previously created descriptor before the descriptor can be used for FFT computations in a particular MPI process. The `DftiCommitDescriptorDM` function performs all initialization that facilitates the actual FFT computation. For the current implementation, it may involve exploring many different factorizations of the input length to search for highly efficient computation method.

Any changes of configuration parameters of a committed descriptor via the set value function (see [Descriptor Configuration Functions](#)) requires a re-committal of the descriptor before a computation function can be invoked. Typically, this committal function is called right before a computation function call (see [FFT Computation Functions](#)).

## Return Values

The function returns `DFTI_NO_ERROR` when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the [Error Codes](#) section).

## Prototype

```
MKL_LONG DftiCommitDescriptorDM(DFTI_DESCRIPTOR_DM_HANDLE handle);
```

## DftiFreeDescriptorDM

*Frees memory allocated for a descriptor.*

## Syntax

```
status = DftiFreeDescriptorDM(&handle);
```

## Include Files

- `mkl_cdft.h`

## Input Parameters

*handle* The descriptor handle. Must be valid, that is, created in a call to [DftiCreateDescriptorDM](#).

## Output Parameters

*handle* The descriptor handle. Memory allocated for the handle is released on output.

## Description

This function frees up all memory allocated for a descriptor in a particular MPI process. Call the `DftiFreeDescriptorDM` function to delete the descriptor handle. Upon successful completion of `DftiFreeDescriptorDM` the descriptor handle is no longer valid.

## Return Values

The function returns `DFTI_NO_ERROR` when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the [Error Codes](#) section).

## Prototype

```
MKL_LONG DftiFreeDescriptorDM(DFTI_DESCRIPTOR_DM_HANDLE *handle);
```

## Cluster FFT Computation Functions

There are two functions in this category: compute the forward transform and compute the backward transform.

### DftiComputeForwardDM

*Computes the forward FFT.*

#### Syntax

```
status = DftiComputeForwardDM(handle, in_X, out_X);
```

```
status = DftiComputeForwardDM(handle, in_out_X);
```

#### Include Files

- `mkl_cdft.h`

#### Input Parameters

|                             |                                                                                                                                                                        |
|-----------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>handle</code>         | The descriptor handle.                                                                                                                                                 |
| <code>in_X, in_out_X</code> | Local part of input data. Array of complex values. Refer to the <a href="#">Distributing Data among Processes</a> section on how to allocate and initialize the array. |

#### Output Parameters

|                              |                                                                                                                                                          |
|------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>out_X, in_out_X</code> | Local part of output data. Array of complex values. Refer to the <a href="#">Distributing Data among Processes</a> section on how to allocate the array. |
|------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|

#### Description

The `DftiComputeForwardDM` function computes the forward FFT. Forward FFT is the transform using the factor  $e^{-i2\pi/n}$ .

Before you call the function, the valid descriptor, created by `DftiCreateDescriptorDM`, must be configured and committed using the `DftiCommitDescriptorDM` function.

The computation is carried out by calling the `DftiComputeForward` function. So, the functions have very much in common, and details not explicitly mentioned below can be found in the description of `DftiComputeForward`.

Local part of input data, as well as local part of the output data, is an appropriate sequence of complex values (each complex value consists of two real numbers: real part and imaginary part) that a particular process stores. See the [Distributing Data Among Processes](#) section for details.

Refer to the [Configuration Settings](#) section for the list of configuration parameters that the descriptor passes to the function.

The configuration parameter `DFTI_PRECISION` determines the precision of input data, output data, and transform: a setting of `DFTI_SINGLE` indicates single-precision floating-point data type and a setting of `DFTI_DOUBLE` indicates double-precision floating-point data type.

The configuration parameter `DFTI_PLACEMENT` informs the function whether the computation should be in-place. If the value of this parameter is `DFTI_INPLACE` (default), you must call the function with two parameters, otherwise you must supply three parameters. If `DFTI_PLACEMENT = DFTI_INPLACE` and three parameters are supplied, then the third parameter is ignored.

## CAUTION

Even in case of an out-of-place transform, local array of input data *in\_X* may be changed. To save data, make its copy before calling `DftiComputeForwardDM`.

In case of an in-place transform, `DftiComputeForwardDM` dynamically allocates and deallocates a work buffer of the same size as the local input/output array requires.

## NOTE

You can specify your own workspace of the same size through the configuration parameter `CDFT_WORKSPACE` to avoid redundant memory allocation.

## Return Values

The function returns `DFTI_NO_ERROR` when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the [Error Codes](#) section).

## Prototype

```
MKL_LONG DftiComputeForwardDM(DFTI_DESCRIPTOR_DM_HANDLE handle, void *in_X, ...);
```

## DftiComputeBackwardDM

*Computes the backward FFT.*

### Syntax

```
status = DftiComputeBackwardDM(handle, in_X, out_X);
```

```
status = DftiComputeBackwardDM(handle, in_out_X);
```

### Include Files

- `mkl_cdft.h`

### Input Parameters

|                       |                                                                                                                                                                        |
|-----------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>handle</i>         | The descriptor handle.                                                                                                                                                 |
| <i>in_X, in_out_X</i> | Local part of input data. Array of complex values. Refer to the <a href="#">Distributing Data among Processes</a> section on how to allocate and initialize the array. |

### Output Parameters

|                        |                                                                                                                                                          |
|------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>out_X, in_out_X</i> | Local part of output data. Array of complex values. Refer to the <a href="#">Distributing Data among Processes</a> section on how to allocate the array. |
|------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|

### Description

The `DftiComputeBackwardDM` function computes the backward FFT. Backward FFT is the transform using the factor  $e^{i2\pi/n}$ .

Before you call the function, the valid descriptor, created by `DftiCreateDescriptorDM`, must be configured and committed using the `DftiCommitDescriptorDM` function.

The computation is carried out by calling the `DftiComputeBackward` function. So, the functions have very much in common, and details not explicitly mentioned below can be found in the description of `DftiComputeBackward`.

Local part of input data, as well as local part of the output data, is an appropriate sequence of complex values (each complex value consists of two real numbers: real part and imaginary part) that a particular process stores. See the [Distributing Data among Processes](#) section for details.

Refer to the [Configuration Settings](#) section for the list of configuration parameters that the descriptor passes to the function.

The configuration parameter `DFTI_PRECISION` determines the precision of input data, output data, and transform: a setting of `DFTI_SINGLE` indicates single-precision floating-point data type and a setting of `DFTI_DOUBLE` indicates double-precision floating-point data type.

The configuration parameter `DFTI_PLACEMENT` informs the function whether the computation should be in-place. If the value of this parameter is `DFTI_INPLACE` (default), you must call the function with two parameters, otherwise you must supply three parameters. If `DFTI_PLACEMENT = DFTI_INPLACE` and three parameters are supplied, then the third parameter is ignored.

---

### CAUTION

Even in case of an out-of-place transform, local array of input data `in_X` may be changed. To save data, make its copy before calling `DftiComputeBackwardDM`.

---

In case of an in-place transform, `DftiComputeBackwardDM` dynamically allocates and deallocates a work buffer of the same size as the local input/output array requires.

---

### NOTE

You can specify your own workspace of the same size through the configuration parameter `CDFT_WORKSPACE` to avoid redundant memory allocation.

---

## Return Values

The function returns `DFTI_NO_ERROR` when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the [Error Codes](#) section).

## Prototype

```
MKL_LONG DftiComputeBackwardDM(DFTI_DESCRIPTOR_DM_HANDLE handle, void *in_X,...);
```

## Cluster FFT Descriptor Configuration Functions

There are two functions in this category: the value setting function `DftiSetValueDM` sets one particular configuration parameter to an appropriate value, the value getting function `DftiGetValueDM` reads the value of one particular configuration parameter.

Some configuration parameters used by cluster FFT functions originate from the conventional FFT interface (see [Configuration Settings](#) subsection in the "FFT Functions" section for details).

Other configuration parameters are specific to the cluster FFT. Integer values of these parameters have type `MKL_LONG`. The exact type of the configuration parameters being floating-point scalars is `float` or `double`. The configuration parameters whose values are named constants have the `enum` type. They are defined in the `mkl_cdft.h` header file.

Names of the configuration parameters specific to the cluster FFT interface have prefix `CDFT`.

## DftiSetValueDM

Sets one particular configuration parameter with the specified configuration value.

### Syntax

```
status = DftiSetValueDM (handle, param, value);
```

### Include Files

- mkl\_cdft.h

### Input Parameters

|               |                                                                                                                                                                |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>handle</i> | The descriptor handle. Must be valid, that is, created in a call to <a href="#">DftiCreateDescriptorDM</a> .                                                   |
| <i>param</i>  | Name of a parameter to be set up in the descriptor handle. See <a href="#">Table "Settable Configuration Parameters"</a> for the list of available parameters. |
| <i>value</i>  | Value of the parameter.                                                                                                                                        |

### Description

This function sets one particular configuration parameter with the specified configuration value. The configuration parameter is one of the named constants listed in the table below, and the configuration value must have the corresponding type. See [Configuration Settings](#) for details of the meaning of each setting and for possible values of the parameters whose values are named constants.

### Settable Configuration Parameters

| Parameter Name      | Data Type                    | Description                                                                                     | Default Value                                                                                                                                                                                                                                 |
|---------------------|------------------------------|-------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| DFTI_FORWARD_SCALE  | Floating-point scalar        | Scale factor of forward transform.                                                              | 1.0                                                                                                                                                                                                                                           |
| DFTI_BACKWARD_SCALE | Floating-point scalar        | Scale factor of backward transform.                                                             | 1.0                                                                                                                                                                                                                                           |
| DFTI_PLACEMENT      | Named constant               | Placement of the computation result.                                                            | DFTI_INPLACE                                                                                                                                                                                                                                  |
| DFTI_ORDERING       | Named constant               | Scrambling of data order.                                                                       | DFTI_ORDERED                                                                                                                                                                                                                                  |
| CDFT_WORKSPACE      | Array of an appropriate type | Auxiliary buffer, a user-defined workspace. Enables saving memory during in-place computations. | NULL (allocate workspace dynamically).                                                                                                                                                                                                        |
| DFTI_PACKED_FORMAT  | Named constant               | Packed format, real data.                                                                       | <ul style="list-style-type: none"> <li>• DFTI_PERM_FORMAT — default and the only available value for one-dimensional transforms</li> <li>• DFTI_CCE_FORMAT — default and the only available value for multi-dimensional transforms</li> </ul> |

| Parameter Name | Data Type      | Description                                                                                                                                                                                                                                                                                                                                          | Default Value          |
|----------------|----------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|
| DFTI_TRANSPOSE | Named constant | This parameter determines how the output data is located for multi-dimensional transforms. If the parameter value is <code>DFTI_NONE</code> , the data is located in a usual manner described in this document. If the value is <code>DFTI_ALLOW</code> , the last (first) global transposition is not performed for a forward (backward) transform. | <code>DFTI_NONE</code> |

### Return Values

The function returns `DFTI_NO_ERROR` when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the [Error Codes](#) section).

### Prototype

```
MKL_LONG DftiSetValueDM(DFTI_DESCRIPTOR_DM_HANDLE handle, int param,...);
```

### DftiGetValueDM

*Gets the value of one particular configuration parameter.*

### Syntax

```
status = DftiGetValueDM(handle, param, &value);
```

### Include Files

- `mkl_cdft.h`

### Input Parameters

*handle* The descriptor handle. Must be valid, that is, created in a call to [DftiCreateDescriptorDM](#).

*param* Name of a parameter to be retrieved from the descriptor. See [Table "Retrievable Configuration Parameters"](#) for the list of available parameters.

### Output Parameters

*value* Value of the parameter.

### Description

This function gets the configuration value of one particular configuration parameter. The configuration parameter is one of the named constants listed in the table below, and the configuration value is the corresponding appropriate type, which can be a named constant or a native type. Possible values of the named constants can be found in [Table "Configuration Parameters"](#) and relevant subsections of the [Configuration Settings](#) section.

**Retrieval Configuration Parameters**

| Parameter Name         | Data Type                | Description                                                                                                                                                                                                |
|------------------------|--------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| DFTI_PRECISION         | Named constant           | Precision of computation, input data and output data.                                                                                                                                                      |
| DFTI_DIMENSION         | Integer scalar           | Dimension of the transform                                                                                                                                                                                 |
| DFTI_LENGTHS           | Array of integer values  | Array of lengths of the transform. Number of lengths corresponds to the dimension of the transform.                                                                                                        |
| DFTI_FORWARD_SCALE     | Floating-point scalar    | Scale factor of forward transform.                                                                                                                                                                         |
| DFTI_BACKWARD_SCALE    | Floating-point scalar    | Scale factor of backward transform.                                                                                                                                                                        |
| DFTI_PLACEMENT         | Named constant           | Placement of the computation result.                                                                                                                                                                       |
| DFTI_COMMIT_STATUS     | Named constant           | Shows whether descriptor has been committed.                                                                                                                                                               |
| DFTI_FORWARD_DOMAIN    | Named constant           | Forward domain of transforms, has the value of DFTI_COMPLEX or DFTI_REAL.                                                                                                                                  |
| DFTI_ORDERING          | Named constant           | Scrambling of data order.                                                                                                                                                                                  |
| CDFT_MPI_COMM          | Type of MPI communicator | MPI communicator used for transforms.                                                                                                                                                                      |
| CDFT_LOCAL_SIZE        | Integer scalar           | Necessary size of input, output, and buffer arrays in data elements.                                                                                                                                       |
| CDFT_LOCAL_X_START     | Integer scalar           | Row/element number of the global array that corresponds to the first row/element of the local array. For more information, see <a href="#">Distributing Data among Processes</a> .                         |
| CDFT_LOCAL_NX          | Integer scalar           | The number of rows/elements of the global array stored in the local array. For more information, see <a href="#">Distributing Data among Processes</a> .                                                   |
| CDFT_LOCAL_OUT_X_START | Integer scalar           | Element number of the appropriate global array that corresponds to the first element of the input or output local array in a 1D case. For details, see <a href="#">Distributing Data among Processes</a> . |
| CDFT_LOCAL_OUT_NX      | Integer scalar           | The number of elements of the appropriate global array that are stored in the input or output local array in a 1D case. For details, see <a href="#">Distributing Data among Processes</a> .               |

**Return Values**

The function returns DFTI\_NO\_ERROR when completes successfully. If the function fails, it returns a value of another error class constant (for the list of constants, refer to the [Error Codes](#) section).

## Prototype

```
MKL_LONG DftiGetValueDM(DFTI_DESCRIPTOR_DM_HANDLE handle, int param,...);
```

## Error Codes

All the cluster FFT functions return an integer value denoting the status of the operation. These values are identified by named constants. Each function returns `DFTI_NO_ERROR` if no errors were encountered during execution. Otherwise, a function generates an error code. In addition to FFT error codes, the cluster FFT interface has its own ones. Named constants specific to the cluster FFT interface have prefix "CDFT" in names. [Table "Error Codes that Cluster FFT Functions Return"](#) lists error codes that the cluster FFT functions may return.

### Error Codes that Cluster FFT Functions Return

| Named Constants                              | Comments                                                                                                                                   |
|----------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| <code>DFTI_NO_ERROR</code>                   | No error.                                                                                                                                  |
| <code>DFTI_MEMORY_ERROR</code>               | Usually associated with memory allocation.                                                                                                 |
| <code>DFTI_INVALID_CONFIGURATION</code>      | Invalid settings of one or more configuration parameters.                                                                                  |
| <code>DFTI_INCONSISTENT_CONFIGURATION</code> | Inconsistent configuration or input parameters.                                                                                            |
| <code>DFTI_NUMBER_OF_THREADS_ERROR</code>    | Number of OMP threads in the computation function is not equal to the number of OMP threads in the initialization stage (commit function). |
| <code>DFTI_MULTITHREADED_ERROR</code>        | Usually associated with a value that OMP routines return in case of errors.                                                                |
| <code>DFTI_BAD_DESCRIPTOR</code>             | Descriptor is unusable for computation.                                                                                                    |
| <code>DFTI_UNIMPLEMENTED</code>              | Unimplemented legitimate settings; implementation dependent.                                                                               |
| <code>DFTI_MKL_INTERNAL_ERROR</code>         | Internal library error.                                                                                                                    |
| <code>DFTI_1D_LENGTH_EXCEEDS_INT32</code>    | Length of one of dimensions exceeds $2^{32} - 1$ (4 bytes).                                                                                |
| <code>CDFT_SPREAD_ERROR</code>               | Data cannot be distributed (For more information, see <a href="#">Distributing Data among Processes.</a> )                                 |
| <code>CDFT_MPI_ERROR</code>                  | MPI error. Occurs when calling MPI.                                                                                                        |



# PBLAS Routines

This chapter describes the Intel® Math Kernel Library implementation of the PBLAS (Parallel Basic Linear Algebra Subprograms) routines from the ScaLAPACK package for distributed-memory architecture. PBLAS is intended for using in vector-vector, matrix-vector, and matrix-matrix operations to simplify the parallelization of linear codes. The design of PBLAS is as consistent as possible with that of the BLAS. The routine descriptions are arranged in several sections according to the PBLAS level of operation:

- [PBLAS Level 1 Routines](#) (distributed vector-vector operations)
- [PBLAS Level 2 Routines](#) (distributed matrix-vector operations)
- [PBLAS Level 3 Routines](#) (distributed matrix-matrix operations)

Each section presents the routine and function group descriptions in alphabetical order by the routine group name; for example, the `p?asum` group, the `p?axpy` group. The question mark in the group name corresponds to a character indicating the data type (`s`, `d`, `c`, and `z` or their combination); see [Routine Naming Conventions](#).

---

## NOTE

PBLAS routines are provided only with Intel® MKL versions for Linux\* and Windows\* OSs.

---

Generally, PBLAS runs on a network of computers using MPI as a message-passing layer and a set of prebuilt communication subprograms (BLACS), as well as a set of PBLAS optimized for the target architecture. The Intel MKL version of PBLAS is optimized for Intel® processors. For the detailed system and environment requirements see *Intel® MKL Release Notes* and *Intel® MKL Developer Guide*.

For full reference on PBLAS routines and related information, see [http://www.netlib.org/scalapack/html/pblas\\_qref.html](http://www.netlib.org/scalapack/html/pblas_qref.html).

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## PBLAS Routines Overview

The model of the computing environment for PBLAS is represented as a one-dimensional array of processes or also a two-dimensional process grid. To use PBLAS, all global matrices or vectors must be distributed on this array or grid prior to calling the PBLAS routines.

PBLAS uses the two-dimensional block-cyclic data distribution as a layout for dense matrix computations. This distribution provides good work balance between available processors, as well as gives the opportunity to use PBLAS Level 3 routines for optimal local computations. Information about the data distribution that is required to establish the mapping between each global array and its corresponding process and memory location is contained in the so called *array descriptor* associated with each global array. [Table "Content of the array descriptor for dense matrices"](#) gives an example of an array descriptor structure.

## Content of Array Descriptor for Dense Matrices

| Array Element # | Name         | Definition                                                                    |
|-----------------|--------------|-------------------------------------------------------------------------------|
| 1               | <i>dtype</i> | Descriptor type ( =1 for dense matrices)                                      |
| 2               | <i>ctxt</i>  | BLACS context handle for the process grid                                     |
| 3               | <i>m</i>     | Number of rows in the global array                                            |
| 4               | <i>n</i>     | Number of columns in the global array                                         |
| 5               | <i>mb</i>    | Row blocking factor                                                           |
| 6               | <i>nb</i>    | Column blocking factor                                                        |
| 7               | <i>rsrc</i>  | Process row over which the first row of the global array is distributed       |
| 8               | <i>csrc</i>  | Process column over which the first column of the global array is distributed |
| 9               | <i>lld</i>   | Leading dimension of the local array                                          |

The number of rows and columns of a global dense matrix that a particular process in a grid receives after data distributing is denoted by *LOCr()* and *LOCc()*, respectively. To compute these numbers, you can use the ScaLAPACK tool routine *numroc*.

After the block-cyclic distribution of global data is done, you may choose to perform an operation on a submatrix of the global matrix *A*, which is contained in the global subarray *sub(A)*, defined by the following 6 values (for dense matrices):

|              |                                                                          |
|--------------|--------------------------------------------------------------------------|
| <i>m</i>     | The number of rows of <i>sub(A)</i>                                      |
| <i>n</i>     | The number of columns of <i>sub(A)</i>                                   |
| <i>a</i>     | A pointer to the local array containing the entire global array <i>A</i> |
| <i>ia</i>    | The row index of <i>sub(A)</i> in the global array                       |
| <i>ja</i>    | The column index of <i>sub(A)</i> in the global array                    |
| <i>desca</i> | The array descriptor for the global array <i>A</i>                       |

Intel MKL provides the PBLAS routines with interface similar to the interface used in the Netlib PBLAS (see [http://www.netlib.org/scalapack/html/pblas\\_qref.html](http://www.netlib.org/scalapack/html/pblas_qref.html)).

## PBLAS Routine Naming Conventions

The naming convention for PBLAS routines is similar to that used for BLAS routines (see [Routine Naming Conventions in Chapter 2](#)). A general rule is that each routine name in PBLAS, which has a BLAS equivalent, is simply the BLAS name prefixed by initial letter *p* that stands for "parallel".

The Intel MKL PBLAS routine names have the following structure:

```
p <character> <name> <mod> ()
```

The *<character>* field indicates the Fortran data type:

|   |                           |
|---|---------------------------|
| s | real, single precision    |
| c | complex, single precision |
| d | real, double precision    |
| z | complex, double precision |
| i | integer                   |

Some routines and functions can have combined character codes, such as *sc* or *dz*.

For example, the function *pscasum* uses a complex input array and returns a real value.

The *<name>* field, in PBLAS level 1, indicates the operation type. For example, the PBLAS level 1 routines *p?dot*, *p?swap*, *p?copy* compute a vector dot product, vector swap, and a copy vector, respectively.

In PBLAS level 2 and 3, *<name>* reflects the matrix argument type:

|           |                   |
|-----------|-------------------|
| <i>ge</i> | general matrix    |
| <i>sy</i> | symmetric matrix  |
| <i>he</i> | Hermitian matrix  |
| <i>tr</i> | triangular matrix |

In PBLAS level 3, the *<name>=tran* indicates the transposition of the matrix.

The *<mod>* field, if present, provides additional details of the operation. The PBLAS level 1 names can have the following characters in the *<mod>* field:

|          |                     |
|----------|---------------------|
| <i>c</i> | conjugated vector   |
| <i>u</i> | unconjugated vector |

The PBLAS level 2 names can have the following additional characters in the *<mod>* field:

|           |                                                                    |
|-----------|--------------------------------------------------------------------|
| <i>mv</i> | matrix-vector product                                              |
| <i>sv</i> | solving a system of linear equations with matrix-vector operations |
| <i>r</i>  | rank-1 update of a matrix                                          |
| <i>r2</i> | rank-2 update of a matrix.                                         |

The PBLAS level 3 names can have the following additional characters in the *<mod>* field:

|            |                                                                    |
|------------|--------------------------------------------------------------------|
| <i>mm</i>  | matrix-matrix product                                              |
| <i>sm</i>  | solving a system of linear equations with matrix-matrix operations |
| <i>rk</i>  | rank- <i>k</i> update of a matrix                                  |
| <i>r2k</i> | rank- <i>2k</i> update of a matrix.                                |

The examples below show how to interpret PBLAS routine names:

|               |                                                                                                                                                                            |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>pddot</i>  | <i>&lt;p&gt;</i> <i>&lt;d&gt;</i> <i>&lt;dot&gt;</i> : double-precision real distributed vector-vector dot product                                                         |
| <i>pcdotc</i> | <i>&lt;p&gt;</i> <i>&lt;c&gt;</i> <i>&lt;dot&gt;</i> <i>&lt;c&gt;</i> : complex distributed vector-vector dot product, conjugated                                          |
| <i>pscsum</i> | <i>&lt;p&gt;</i> <i>&lt;sc&gt;</i> <i>&lt;asum&gt;</i> : sum of magnitudes of distributed vector elements, single precision real output and single precision complex input |
| <i>pcdotu</i> | <i>&lt;p&gt;</i> <i>&lt;c&gt;</i> <i>&lt;dot&gt;</i> <i>&lt;u&gt;</i> : distributed vector-vector dot product, unconjugated, complex                                       |
| <i>psgemv</i> | <i>&lt;p&gt;</i> <i>&lt;s&gt;</i> <i>&lt;ge&gt;</i> <i>&lt;mv&gt;</i> : distributed matrix-vector product, general matrix, single precision                                |
| <i>pztrmm</i> | <i>&lt;p&gt;</i> <i>&lt;z&gt;</i> <i>&lt;tr&gt;</i> <i>&lt;mm&gt;</i> : distributed matrix-matrix product, triangular matrix, double-precision complex.                    |

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## PBLAS Level 1 Routines

PBLAS Level 1 includes routines and functions that perform distributed vector-vector operations. [Table "PBLAS Level 1 Routine Groups and Their Data Types"](#) lists the PBLAS Level 1 routine groups and the data types associated with them.

### PBLAS Level 1 Routine Groups and Their Data Types

| Routine or Function Group | Data Types         | Description                                                                            |
|---------------------------|--------------------|----------------------------------------------------------------------------------------|
| <code>p?amax</code>       | s, d, c, z         | Calculates an index of the distributed vector element with maximum absolute value      |
| <code>p?asum</code>       | s, d, sc, dz       | Calculates sum of magnitudes of a distributed vector                                   |
| <code>p?axpy</code>       | s, d, c, z         | Calculates distributed vector-scalar product                                           |
| <code>p?copy</code>       | s, d, c, z         | Copies a distributed vector                                                            |
| <code>p?dot</code>        | s, d               | Calculates a dot product of two distributed real vectors                               |
| <code>p?dotc</code>       | c, z               | Calculates a dot product of two distributed complex vectors, one of them is conjugated |
| <code>p?dotu</code>       | c, z               | Calculates a dot product of two distributed complex vectors                            |
| <code>p?nrm2</code>       | s, d, sc, dz       | Calculates the 2-norm (Euclidean norm) of a distributed vector                         |
| <code>p?scal</code>       | s, d, c, z, cs, zd | Calculates a product of a distributed vector by a scalar                               |
| <code>p?swap</code>       | s, d, c, z         | Swaps two distributed vectors                                                          |

### `p?amax`

*Computes the global index of the element of a distributed vector with maximum absolute value.*

#### Syntax

```
void psamax (const MKL_INT *n , float *amax , MKL_INT *indx , const float *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);

void pdamax (const MKL_INT *n , double *amax , MKL_INT *indx , const double *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);
```

```

void pcamax (const MKL_INT *n , MKL_Complex8 *amax , MKL_INT *indx , const MKL_Complex8
*x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT
*incx);

void pzamax (const MKL_INT *n , MKL_Complex16 *amax , MKL_INT *indx , const
MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx);

```

### Include Files

- mkl\_pblas.h

### Description

The functions `p?amax` compute global index of the maximum element in absolute value of a distributed vector `sub(x)`,

where `sub(x)` denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix:ix+n-1, jx)$  if  $incx=1$ .

### Input Parameters

|                     |                                                                                                                                                                                 |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>      | (global) The length of distributed vector <code>sub(x)</code> , $n \geq 0$ .                                                                                                    |
| <code>x</code>      | (local)<br>Array, size $(jx-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector <code>sub(x)</code> .                                   |
| <code>ix, jx</code> | (global) The row and column indices in the distributed matrix <code>X</code> indicating the first row and the first column of the submatrix <code>sub(X)</code> , respectively. |
| <code>descx</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <code>X</code> .                                                                        |
| <code>incx</code>   | (global) Specifies the increment for the elements of <code>sub(x)</code> . Only two values are supported, namely 1 and $m_x$ . <code>incx</code> must not be zero.              |

### Output Parameters

|                   |                                                                                                                                     |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------------|
| <code>amax</code> | (global).<br>Maximum absolute value (magnitude) of elements of the distributed vector only in its scope.                            |
| <code>indx</code> | (global) The global index of the maximum element in absolute value of the distributed vector <code>sub(x)</code> only in its scope. |

### p?asum

*Computes the sum of magnitudes of elements of a distributed vector.*

### Syntax

```

void psasum (const MKL_INT *n , float *asum , const float *x , const MKL_INT *ix ,
const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);

void pdasum (const MKL_INT *n , double *asum , const double *x , const MKL_INT *ix ,
const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);

```

```
void pscasum (const MKL_INT *n , float *asum , const MKL_Complex8 *x , const MKL_INT
*ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);
```

```
void pdzasum (const MKL_INT *n , double *asum , const MKL_Complex16 *x , const MKL_INT
*ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);
```

## Include Files

- mkl\_pblas.h

## Description

The functions `p?asum` compute the sum of the magnitudes of elements of a distributed vector `sub(x)`, where `sub(x)` denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix: ix+n-1, jx)$  if  $incx= 1$ .

## Input Parameters

|                     |                                                                                                                                                                                 |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>      | (global) The length of distributed vector <code>sub(x)</code> , $n \geq 0$ .                                                                                                    |
| <code>x</code>      | (local)<br>Array, size $(jx-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector <code>sub(x)</code> .                                   |
| <code>ix, jx</code> | (global) The row and column indices in the distributed matrix <code>X</code> indicating the first row and the first column of the submatrix <code>sub(X)</code> , respectively. |
| <code>descx</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <code>X</code> .                                                                        |
| <code>incx</code>   | (global) Specifies the increment for the elements of <code>sub(x)</code> . Only two values are supported, namely 1 and $m_x$ . <code>incx</code> must not be zero.              |

## Output Parameters

|                   |                                                                                                                               |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------|
| <code>asum</code> | (local) and <code>p?casum</code> .<br>Contains the sum of magnitudes of elements of the distributed vector only in its scope. |
|-------------------|-------------------------------------------------------------------------------------------------------------------------------|

## p?axpy

*Computes a distributed vector-scalar product and adds the result to a distributed vector.*

---

## Syntax

```
void psaxpy (const MKL_INT *n , const float *a , const float *x , const MKL_INT *ix ,
const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , float *y , const
MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pdaxpy (const MKL_INT *n , const double *a , const double *x , const MKL_INT *ix ,
const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , double *y , const
MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pcaxpy (const MKL_INT *n , const MKL_Complex8 *a , const MKL_Complex8 *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx ,
MKL_Complex8 *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy);
```

```
void pzaxpy (const MKL_INT *n , const MKL_Complex16 *a , const MKL_Complex16 *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx ,
MKL_Complex16 *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy);
```

## Include Files

- mkl\_pblas.h

## Description

The `p?axpy` routines perform the following operation with distributed vectors:

```
sub(y) := sub(y) + a*sub(x)
```

where:

$a$  is a scalar;

`sub(x)` and `sub(y)` are  $n$ -element distributed vectors.

`sub(x)` denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix: ix+n-1, jx)$  if  $incx= 1$ ;

`sub(y)` denotes  $Y(iy, jy:jy+n-1)$  if  $incy=m_y$ , and  $Y(iy: iy+n-1, jy)$  if  $incy= 1$ .

## Input Parameters

|          |                                                                                                                                                                      |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $n$      | (global) The length of distributed vectors, $n \geq 0$ .                                                                                                             |
| $a$      | (local)<br>Specifies the scalar $a$ .                                                                                                                                |
| $x$      | (local)<br>Array, size $(jx-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector <code>sub(x)</code> .                        |
| $ix, jx$ | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix <code>sub(X)</code> , respectively. |
| $descx$  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                        |
| $incx$   | (global) Specifies the increment for the elements of <code>sub(x)</code> . Only two values are supported, namely 1 and $m_x$ . $incx$ must not be zero.              |
| $y$      | (local)<br>Array, size $(jy-1)*m_y + iy+(n-1)*abs(incy)$ .<br>This array contains the entries of the distributed vector <code>sub(y)</code> .                        |
| $iy, jy$ | (global) The row and column indices in the distributed matrix $Y$ indicating the first row and the first column of the submatrix <code>sub(Y)</code> , respectively. |
| $descy$  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $Y$ .                                                                        |
| $incy$   | (global) Specifies the increment for the elements of <code>sub(y)</code> . Only two values are supported, namely 1 and $m_y$ . $incy$ must not be zero.              |

## Output Parameters

$y$  Overwritten by  $\text{sub}(y) := \text{sub}(y) + a * \text{sub}(x)$ .

## p?copy

*Copies one distributed vector to another vector.*

### Syntax

```
void picopy (const MKL_INT *n , const MKL_INT *x , const MKL_INT *ix , const MKL_INT
*jx , const MKL_INT *descx , const MKL_INT *incx , MKL_INT *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pscopy (const MKL_INT *n , const float *x , const MKL_INT *ix , const MKL_INT
*jx , const MKL_INT *descx , const MKL_INT *incx , float *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pdcopy (const MKL_INT *n , const double *x , const MKL_INT *ix , const MKL_INT
*jx , const MKL_INT *descx , const MKL_INT *incx , double *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pccopy (const MKL_INT *n , const MKL_Complex8 *x , const MKL_INT *ix , const
MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , MKL_Complex8 *y , const
MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pzcopy (const MKL_INT *n , const MKL_Complex16 *x , const MKL_INT *ix , const
MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , MKL_Complex16 *y , const
MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

### Include Files

- mkl\_pblas.h

### Description

The p?copy routines perform a copy operation with distributed vectors defined as

```
sub(y) = sub(x),
```

where  $\text{sub}(x)$  and  $\text{sub}(y)$  are  $n$ -element distributed vectors.

$\text{sub}(x)$  denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix: ix+n-1, jx)$  if  $incx= 1$ ;

$\text{sub}(y)$  denotes  $Y(iy, jy:jy+n-1)$  if  $incy=m_y$ , and  $Y(iy: iy+n-1, jy)$  if  $incy= 1$ .

### Input Parameters

$n$  (global) The length of distributed vectors,  $n \geq 0$ .

$x$  (local)  
 Array, size  $(jx-1) * m_x + ix + (n-1) * \text{abs}(incx)$ .  
 This array contains the entries of the distributed vector  $\text{sub}(x)$ .

$ix, jx$  (global) The row and column indices in the distributed matrix  $X$  indicating the first row and the first column of the submatrix  $\text{sub}(X)$ , respectively.

|               |                                                                                                                                                                  |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                    |
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . <i>incx</i> must not be zero.         |
| <i>y</i>      | (local)<br>Array, size $(jy-1)*m_y + iy+(n-1)*\text{abs}(incy)$ .<br>This array contains the entries of the distributed vector $\text{sub}(y)$ .                 |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix $Y$ indicating the first row and the first column of the submatrix $\text{sub}(Y)$ , respectively. |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $Y$ .                                                                    |
| <i>incy</i>   | (global) Specifies the increment for the elements of $\text{sub}(y)$ . Only two values are supported, namely 1 and $m_y$ . <i>incy</i> must not be zero.         |

## Output Parameters

|          |                                                           |
|----------|-----------------------------------------------------------|
| <i>y</i> | Overwritten with the distributed vector $\text{sub}(x)$ . |
|----------|-----------------------------------------------------------|

## p?dot

Computes the dot product of two distributed real vectors.

### Syntax

```
void psdot (const MKL_INT *n , float *dot , const float *x , const MKL_INT *ix , const
MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const float *y , const
MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);

void pddot (const MKL_INT *n , double *dot , const double *x , const MKL_INT *ix ,
const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const double *y ,
const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

### Include Files

- `mkl_pblas.h`

### Description

The ?dot functions compute the dot product *dot* of two distributed real vectors defined as

$$\text{dot} = \text{sub}(x) \cdot \text{sub}(y)$$

where  $\text{sub}(x)$  and  $\text{sub}(y)$  are  $n$ -element distributed vectors.

$\text{sub}(x)$  denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix: ix+n-1, jx)$  if  $incx= 1$ ;

$\text{sub}(y)$  denotes  $Y(iy, jy:jy+n-1)$  if  $incy=m_y$ , and  $Y(iy: iy+n-1, jy)$  if  $incy= 1$ .

### Input Parameters

|          |                                                          |
|----------|----------------------------------------------------------|
| <i>n</i> | (global) The length of distributed vectors, $n \geq 0$ . |
|----------|----------------------------------------------------------|

|                     |                                                                                                                                                                                 |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>      | (local)<br>Array, size $(j_x-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector <code>sub(x)</code> .                                  |
| <code>ix, jx</code> | (global) The row and column indices in the distributed matrix <code>X</code> indicating the first row and the first column of the submatrix <code>sub(X)</code> , respectively. |
| <code>descx</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <code>X</code> .                                                                        |
| <code>incx</code>   | (global) Specifies the increment for the elements of <code>sub(x)</code> . Only two values are supported, namely 1 and $m_x$ . <code>incx</code> must not be zero.              |
| <code>y</code>      | (local)<br>Array, size $(j_y-1)*m_y + iy+(n-1)*abs(incy)$ .<br>This array contains the entries of the distributed vector <code>sub(y)</code> .                                  |
| <code>iy, jy</code> | (global) The row and column indices in the distributed matrix <code>Y</code> indicating the first row and the first column of the submatrix <code>sub(Y)</code> , respectively. |
| <code>descy</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <code>Y</code> .                                                                        |
| <code>incy</code>   | (global) Specifies the increment for the elements of <code>sub(y)</code> . Only two values are supported, namely 1 and $m_y$ . <code>incy</code> must not be zero.              |

## Output Parameters

|                  |                                                                                            |
|------------------|--------------------------------------------------------------------------------------------|
| <code>dot</code> | (local)<br>Dot product of <code>sub(x)</code> and <code>sub(y)</code> only in their scope. |
|------------------|--------------------------------------------------------------------------------------------|

## p?dotc

*Computes the dot product of two distributed complex vectors, one of them is conjugated.*

---

### Syntax

```
void pcdotc (const MKL_INT *n , MKL_Complex8 *dotc , const MKL_Complex8 *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const
MKL_Complex8 *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy);
```

```
void pzdorc (const MKL_INT *n , MKL_Complex16 *dotc , const MKL_Complex16 *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const
MKL_Complex16 *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy);
```

### Include Files

- `mk1_pblas.h`

## Description

The `p?dotc` functions compute the dot product `dotc` of two distributed vectors, with one vector conjugated:

```
dotc = conjg(sub(x)')*sub(y)
```

where `sub(x)` and `sub(y)` are  $n$ -element distributed vectors.

`sub(x)` denotes  $X(ix, jx:jx+n-1)$  if  $incx=m\_x$ , and  $X(ix: ix+n-1, jx)$  if  $incx= 1$ ;

`sub(y)` denotes  $Y(iy, jy:jy+n-1)$  if  $incy=m\_y$ , and  $Y(iy: iy+n-1, jy)$  if  $incy= 1$ .

## Input Parameters

|                     |                                                                                                                                                                      |
|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>      | (global) The length of distributed vectors, $n \geq 0$ .                                                                                                             |
| <code>x</code>      | (local)<br>Array, size $(jx-1)*m\_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector <code>sub(x)</code> .                       |
| <code>ix, jx</code> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix <code>sub(X)</code> , respectively. |
| <code>descx</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                        |
| <code>incx</code>   | (global) Specifies the increment for the elements of <code>sub(x)</code> . Only two values are supported, namely 1 and $m\_x$ . <code>incx</code> must not be zero.  |
| <code>y</code>      | (local)<br>Array, size $(jy-1)*m\_y + iy+(n-1)*abs(incy)$ .<br>This array contains the entries of the distributed vector <code>sub(y)</code> .                       |
| <code>iy, jy</code> | (global) The row and column indices in the distributed matrix $Y$ indicating the first row and the first column of the submatrix <code>sub(Y)</code> , respectively. |
| <code>descy</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $Y$ .                                                                        |
| <code>incy</code>   | (global) Specifies the increment for the elements of <code>sub(y)</code> . Only two values are supported, namely 1 and $m\_y$ . <code>incy</code> must not be zero.  |

## Output Parameters

|                   |                                                                                            |
|-------------------|--------------------------------------------------------------------------------------------|
| <code>dotc</code> | (local)<br>Dot product of <code>sub(x)</code> and <code>sub(y)</code> only in their scope. |
|-------------------|--------------------------------------------------------------------------------------------|

## p?dotu

Computes the dot product of two distributed complex vectors.

---

## Syntax

```
void pcdotu (const MKL_INT *n , MKL_Complex8 *dotu , const MKL_Complex8 *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const
MKL_Complex8 *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy);
```

```
void pzdotu (const MKL_INT *n , MKL_Complex16 *dotu , const MKL_Complex16 *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const
MKL_Complex16 *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy);
```

## Include Files

- mkl\_pblas.h

## Description

The `p?dotu` functions compute the dot product `dotu` of two distributed vectors defined as

$$\text{dotu} = \text{sub}(x)' * \text{sub}(y)$$

where `sub(x)` and `sub(y)` are  $n$ -element distributed vectors.

`sub(x)` denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix: ix+n-1, jx)$  if  $incx= 1$ ;

`sub(y)` denotes  $Y(iy, jy:jy+n-1)$  if  $incy=m_y$ , and  $Y(iy: iy+n-1, jy)$  if  $incy= 1$ .

## Input Parameters

|                     |                                                                                                                                                                      |
|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>      | (global) The length of distributed vectors, $n \geq 0$ .                                                                                                             |
| <code>x</code>      | (local)<br>Array, size $(jx-1) * m_x + ix + (n-1) * \text{abs}(incx)$ .<br>This array contains the entries of the distributed vector <code>sub(x)</code> .           |
| <code>ix, jx</code> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix <code>sub(X)</code> , respectively. |
| <code>descx</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                        |
| <code>incx</code>   | (global) Specifies the increment for the elements of <code>sub(x)</code> . Only two values are supported, namely 1 and $m_x$ . <code>incx</code> must not be zero.   |
| <code>y</code>      | (local)<br>Array, size $(jy-1) * m_y + iy + (n-1) * \text{abs}(incy)$ .<br>This array contains the entries of the distributed vector <code>sub(y)</code> .           |
| <code>iy, jy</code> | (global) The row and column indices in the distributed matrix $Y$ indicating the first row and the first column of the submatrix <code>sub(Y)</code> , respectively. |
| <code>descy</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $Y$ .                                                                        |
| <code>incy</code>   | (global) Specifies the increment for the elements of <code>sub(y)</code> . Only two values are supported, namely 1 and $m_y$ . <code>incy</code> must not be zero.   |

## Output Parameters

*dotu* (local)  
Dot product of  $\text{sub}(x)$  and  $\text{sub}(y)$  only in their scope.

## p?norm2

Computes the Euclidean norm of a distributed vector.

### Syntax

```
void psnorm2 (const MKL_INT *n , float *norm2 , const float *x , const MKL_INT *ix ,
const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);
void pdnorm2 (const MKL_INT *n , double *norm2 , const double *x , const MKL_INT *ix ,
const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);
void pscnorm2 (const MKL_INT *n , float *norm2 , const MKL_Complex8 *x , const MKL_INT
*ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);
void pdznorm2 (const MKL_INT *n , double *norm2 , const MKL_Complex16 *x , const MKL_INT
*ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);
```

### Include Files

- mkl\_pblas.h

### Description

The p?norm2 functions compute the Euclidean norm of a distributed vector  $\text{sub}(x)$ , where  $\text{sub}(x)$  is an  $n$ -element distributed vector.

$\text{sub}(x)$  denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix: ix+n-1, jx)$  if  $incx= 1$ .

### Input Parameters

*n* (global) The length of distributed vector  $\text{sub}(x)$ ,  $n \geq 0$ .

*x* (local)  
Array, size  $(jx-1)*m_x + ix+(n-1)*abs(incx)$ .  
This array contains the entries of the distributed vector  $\text{sub}(x)$ .

*ix, jx* (global) The row and column indices in the distributed matrix  $X$  indicating the first row and the first column of the submatrix  $\text{sub}(X)$ , respectively.

*descx* (global and local) array of dimension 9. The array descriptor of the distributed matrix  $X$ .

*incx* (global) Specifies the increment for the elements of  $\text{sub}(x)$ . Only two values are supported, namely 1 and  $m_x$ . *incx* must not be zero.

### Output Parameters

*norm2* (local) and pscnorm2.  
Contains the Euclidean norm of a distributed vector only in its scope.

## p?scal

Computes a product of a distributed vector by a scalar.

### Syntax

```
void pssscal (const MKL_INT *n , const float *a , float *x , const MKL_INT *ix , const
MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);

void pdscal (const MKL_INT *n , const double *a , double *x , const MKL_INT *ix , const
MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);

void pcscal (const MKL_INT *n , const MKL_Complex8 *a , MKL_Complex8 *x , const MKL_INT
*ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);

void pzscal (const MKL_INT *n , const MKL_Complex16 *a , MKL_Complex16 *x , const
MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);

void pcsscal (const MKL_INT *n , const float *a , MKL_Complex8 *x , const MKL_INT *ix ,
const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);

void pdzscal (const MKL_INT *n , const double *a , MKL_Complex16 *x , const MKL_INT
*ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx);
```

### Include Files

- mkl\_pblas.h

### Description

The p?scal routines multiplies a  $n$ -element distributed vector  $\text{sub}(x)$  by the scalar  $a$ :

$\text{sub}(x) = a * \text{sub}(x)$ ,

where  $\text{sub}(x)$  denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix:ix+n-1, jx)$  if  $incx=1$ .

### Input Parameters

|          |                                                                                                                                                                  |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $n$      | (global) The length of distributed vector $\text{sub}(x)$ , $n \geq 0$ .                                                                                         |
| $a$      | (global) and pcsscal<br>Specifies the scalar $a$ .                                                                                                               |
| $x$      | (local)<br>Array, size $(jx-1) * m_x + ix + (n-1) * \text{abs}(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .           |
| $ix, jx$ | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $\text{sub}(X)$ , respectively. |
| $descx$  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                    |
| $incx$   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . $incx$ must not be zero.              |

## Output Parameters

$x$  Overwritten by the updated distributed vector  $\text{sub}(x)$

## p?swap

*Swaps two distributed vectors.*

### Syntax

```
void psswap (const MKL_INT *n , float *x , const MKL_INT *ix , const MKL_INT *jx ,
const MKL_INT *descx , const MKL_INT *incx , float *y , const MKL_INT *iy , const
MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);

void pdswap (const MKL_INT *n , double *x , const MKL_INT *ix , const MKL_INT *jx ,
const MKL_INT *descx , const MKL_INT *incx , double *y , const MKL_INT *iy , const
MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);

void pcswap (const MKL_INT *n , MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT
*jx , const MKL_INT *descx , const MKL_INT *incx , MKL_Complex8 *y , const MKL_INT
*iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);

void pzswap (const MKL_INT *n , MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT
*jx , const MKL_INT *descx , const MKL_INT *incx , MKL_Complex16 *y , const MKL_INT
*iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

### Include Files

- mkl\_pblas.h

### Description

Given two distributed vectors  $\text{sub}(x)$  and  $\text{sub}(y)$ , the p?swap routines return vectors  $\text{sub}(y)$  and  $\text{sub}(x)$  swapped, each replacing the other.

Here  $\text{sub}(x)$  denotes  $X(ix, jx:jx+n-1)$  if  $incx=m_x$ , and  $X(ix:ix+n-1, jx)$  if  $incx=1$ ;

$\text{sub}(y)$  denotes  $Y(iy, jy:jy+n-1)$  if  $incy=m_y$ , and  $Y(iy:iy+n-1, jy)$  if  $incy=1$ .

### Input Parameters

|          |                                                                                                                                                                  |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $n$      | (global) The length of distributed vectors, $n \geq 0$ .                                                                                                         |
| $x$      | (local)<br>Array, size $(jx-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .                        |
| $ix, jx$ | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $\text{sub}(X)$ , respectively. |
| $descx$  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                    |
| $incx$   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . $incx$ must not be zero.              |
| $y$      | (local)                                                                                                                                                          |

Array, size  $(j_y-1)*m_y + i_y+(n-1)*abs(inc_y)$ .

This array contains the entries of the distributed vector  $sub(y)$ .

*i<sub>y</sub>*, *j<sub>y</sub>* (global) The row and column indices in the distributed matrix *Y* indicating the first row and the first column of the submatrix  $sub(Y)$ , respectively.

*descy* (global and local) array of dimension 9. The array descriptor of the distributed matrix *Y*.

*incy* (global) Specifies the increment for the elements of  $sub(y)$ . Only two values are supported, namely 1 and *m<sub>y</sub>*. *incy* must not be zero.

### Output Parameters

*x* Overwritten by distributed vector  $sub(y)$ .

*y* Overwritten by distributed vector  $sub(x)$ .

## PBLAS Level 2 Routines

This section describes PBLAS Level 2 routines, which perform distributed matrix-vector operations. [Table "PBLAS Level 2 Routine Groups and Their Data Types"](#) lists the PBLAS Level 2 routine groups and the data types associated with them.

### PBLAS Level 2 Routine Groups and Their Data Types

| Routine Groups          | Data Types | Description                                                                    |
|-------------------------|------------|--------------------------------------------------------------------------------|
| <a href="#">p?gemv</a>  | s, d, c, z | Matrix-vector product using a distributed general matrix                       |
| <a href="#">p?agemv</a> | s, d, c, z | Matrix-vector product using absolute values for a distributed general matrix   |
| <a href="#">p?ger</a>   | s, d       | Rank-1 update of a distributed general matrix                                  |
| <a href="#">p?gerc</a>  | c, z       | Rank-1 update (conjugated) of a distributed general matrix                     |
| <a href="#">p?geru</a>  | c, z       | Rank-1 update (unconjugated) of a distributed general matrix                   |
| <a href="#">p?hemv</a>  | c, z       | Matrix-vector product using a distributed Hermitian matrix                     |
| <a href="#">p?ahemv</a> | c, z       | Matrix-vector product using absolute values for a distributed Hermitian matrix |
| <a href="#">p?her</a>   | c, z       | Rank-1 update of a distributed Hermitian matrix                                |
| <a href="#">p?her2</a>  | c, z       | Rank-2 update of a distributed Hermitian matrix                                |
| <a href="#">p?symv</a>  | s, d       | Matrix-vector product using a distributed symmetric matrix                     |
| <a href="#">p?asymv</a> | s, d       | Matrix-vector product using absolute values for a distributed symmetric matrix |
| <a href="#">p?syr</a>   | s, d       | Rank-1 update of a distributed symmetric matrix                                |
| <a href="#">p?syr2</a>  | s, d       | Rank-2 update of a distributed symmetric matrix                                |

| Routine Groups          | Data Types | Description                                                                                   |
|-------------------------|------------|-----------------------------------------------------------------------------------------------|
| <a href="#">p?trmv</a>  | s, d, c, z | Distributed matrix-vector product using a triangular matrix                                   |
| <a href="#">p?atrmv</a> | s, d, c, z | Distributed matrix-vector product using absolute values for a triangular matrix               |
| <a href="#">p?trsv</a>  | s, d, c, z | Solves a system of linear equations whose coefficients are in a distributed triangular matrix |

## [p?gemv](#)

*Computes a distributed matrix-vector product using a general matrix.*

### Syntax

```
void psgemv (const char *trans , const MKL_INT *m , const MKL_INT *n , const float
*alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , const float *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT
*descx , const MKL_INT *incx , const float *beta , float *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pdgemv (const char *trans , const MKL_INT *m , const MKL_INT *n , const double
*alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , const double *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT
*descx , const MKL_INT *incx , const double *beta , double *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pcgemv (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx ,
const MKL_INT *descx , const MKL_INT *incx , const MKL_Complex8 *beta , MKL_Complex8
*y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT
*incy);
```

```
void pzgemv (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx ,
const MKL_INT *descx , const MKL_INT *incx , const MKL_Complex16 *beta , MKL_Complex16
*y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT
*incy);
```

### Include Files

- `mkl_pblas.h`

### Description

The `p?gemv` routines perform a distributed matrix-vector operation defined as

```
sub(y) := alpha*sub(A)*sub(x) + beta*sub(y),
```

or

```
sub(y) := alpha*sub(A)'*sub(x) + beta*sub(y),
```

or

```
sub(y) := alpha*conjg(sub(A)')*sub(x) + beta*sub(y),
```

where

*alpha* and *beta* are scalars,

*sub(A)* is a *m*-by-*n* submatrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$ ,

*sub(x)* and *sub(y)* are subvectors.

When *trans* = 'N' or 'n', *sub(x)* denotes  $X(\text{ix}, \text{jx}:\text{jx}+n-1)$  if *incx* = *m\_x*, and  $X(\text{ix}:\text{ix}+n-1, \text{jx})$  if *incx* = 1, *sub(y)* denotes  $Y(\text{iy}, \text{jy}:\text{jy}+m-1)$  if *incy* = *m\_y*, and  $Y(\text{iy}:\text{iy}+m-1, \text{jy})$  if *incy* = 1.

When *trans* = 'T' or 't', or 'C', or 'c', *sub(x)* denotes  $X(\text{ix}, \text{jx}:\text{jx}+m-1)$  if *incx* = *m\_x*, and  $X(\text{ix}:\text{ix}+m-1, \text{jx})$  if *incx* = 1, *sub(y)* denotes  $Y(\text{iy}, \text{jy}:\text{jy}+n-1)$  if *incy* = *m\_y*, and  $Y(\text{iy}:\text{iy}+m-1, \text{jy})$  if *incy* = 1.

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>trans</i>  | (global) Specifies the operation:<br>if <i>trans</i> = 'N' or 'n', then $\text{sub}(y) := \text{alpha} * \text{sub}(A)' * \text{sub}(x) + \text{beta} * \text{sub}(y)$ ;<br>if <i>trans</i> = 'T' or 't', then $\text{sub}(y) := \text{alpha} * \text{sub}(A)' * \text{sub}(x) + \text{beta} * \text{sub}(y)$ ;<br>if <i>trans</i> = 'C' or 'c', then $\text{sub}(y) := \text{alpha} * \text{conjg}(\text{sub}A)' * \text{sub}(x) + \text{beta} * \text{sub}(y)$ . |
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrix <i>sub(A)</i> , $m \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                       |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix <i>sub(A)</i> , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                    |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>a</i>      | (local)<br>Array, size $(\text{lld}_a, \text{LOCq}(\text{ja}+n-1))$ . Before entry this array must contain the local pieces of the distributed matrix <i>sub(A)</i> .                                                                                                                                                                                                                                                                                              |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively.                                                                                                                                                                                                                                                                                                |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                 |
| <i>x</i>      | (local)<br>Array, size $(\text{jx}-1) * m_x + \text{ix} + (n-1) * \text{abs}(\text{incx})$ when <i>trans</i> = 'N' or 'n', and $(\text{jx}-1) * m_x + \text{ix} + (m-1) * \text{abs}(\text{incx})$ otherwise.<br>This array contains the entries of the distributed vector <i>sub(x)</i> .                                                                                                                                                                         |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix <i>X</i> indicating the first row and the first column of the submatrix <i>sub(x)</i> , respectively.                                                                                                                                                                                                                                                                                                |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                                                                                                                                                                                                                                                                                                                 |

|               |                                                                                                                                                                                                                                        |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . <i>incx</i> must not be zero.                                                                               |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> . When <i>beta</i> is set to zero, then $\text{sub}(y)$ need not be set on input.                                                                                                         |
| <i>y</i>      | (local)<br>Array, size $(jy-1)*m_y + iy+(m-1)*\text{abs}(incy)$ when <i>trans</i> = 'N' or 'n', and $(jy-1)*m_y + iy+(n-1)*\text{abs}(incy)$ otherwise.<br>This array contains the entries of the distributed vector $\text{sub}(y)$ . |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix Y indicating the first row and the first column of the submatrix $\text{sub}(y)$ , respectively.                                                                         |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix Y.                                                                                                                                             |
| <i>incy</i>   | (global) Specifies the increment for the elements of $\text{sub}(y)$ . Only two values are supported, namely 1 and $m_y$ . <i>incy</i> must not be zero.                                                                               |

## Output Parameters

|          |                                                                 |
|----------|-----------------------------------------------------------------|
| <i>y</i> | Overwritten by the updated distributed vector $\text{sub}(y)$ . |
|----------|-----------------------------------------------------------------|

## p?agemv

*Computes a distributed matrix-vector product using absolute values for a general matrix.*

### Syntax

```
void psagemv (const char *trans , const MKL_INT *m , const MKL_INT *n , const float
*alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , const float *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT
*descx , const MKL_INT *incx , const float *beta , float *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pdagemv (const char *trans , const MKL_INT *m , const MKL_INT *n , const double
*alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , const double *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT
*descx , const MKL_INT *incx , const double *beta , double *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pcagemv (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx ,
const MKL_INT *descx , const MKL_INT *incx , const MKL_Complex8 *beta , MKL_Complex8
*y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT
*incy);
```

```
void pzagemv (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx ,
```

```
const MKL_INT *descx , const MKL_INT *incx , const MKL_Complex16 *beta , MKL_Complex16
*y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT
*incy);
```

## Include Files

- mkl\_pblas.h

## Description

The `p?agemv` routines perform a distributed matrix-vector operation defined as

```
sub(y) := abs(alpha)*abs(sub(A)')*abs(sub(x)) + abs(beta*sub(y)),
```

or

```
sub(y) := abs(alpha)*abs(sub(A)')*abs(sub(x)) + abs(beta*sub(y)),
```

or

```
sub(y) := abs(alpha)*abs(conjg(sub(A)'))*abs(sub(x)) + abs(beta*sub(y)),
```

where

*alpha* and *beta* are scalars,

*sub(A)* is a *m*-by-*n* submatrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$ ,

*sub(x)* and *sub(y)* are subvectors.

When *trans* = 'N' or 'n',

*sub(x)* denotes  $X(\text{ix}:\text{ix}, \text{jx}:\text{jx}+n-1)$  if *incx* = *m<sub>x</sub>*, and

$X(\text{ix}:\text{ix}+n-1, \text{jx}:\text{jx})$  if *incx* = 1,

*sub(y)* denotes  $Y(\text{iy}:\text{iy}, \text{jy}:\text{jy}+m-1)$  if *incy* = *m<sub>y</sub>*, and

$Y(\text{iy}:\text{iy}+m-1, \text{jy}:\text{jy})$  if *incy* = 1.

When *trans* = 'T' or 't', or 'C', or 'c',

*sub(x)* denotes  $X(\text{ix}:\text{ix}, \text{jx}:\text{jx}+m-1)$  if *incx* = *m<sub>x</sub>*, and

$X(\text{ix}:\text{ix}+m-1, \text{jx}:\text{jx})$  if *incx* = 1,

*sub(y)* denotes  $Y(\text{iy}:\text{iy}, \text{jy}:\text{jy}+n-1)$  if *incy* = *m<sub>y</sub>*, and

$Y(\text{iy}:\text{iy}+m-1, \text{jy}:\text{jy})$  if *incy* = 1.

## Input Parameters

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>trans</i> | (global) Specifies the operation:<br>if <i>trans</i> = 'N' or 'n', then $\text{sub}(y) :=  \alpha  *  \text{sub}(A)  *  \text{sub}(x)  +  \beta * \text{sub}(y) $<br>if <i>trans</i> = 'T' or 't', then $\text{sub}(y) :=  \alpha  *  \text{sub}(A)'  *  \text{sub}(x)  +  \beta * \text{sub}(y) $<br>if <i>trans</i> = 'C' or 'c', then $\text{sub}(y) :=  \alpha  *  \text{sub}(A)'  *  \text{sub}(x)  +  \beta * \text{sub}(y) $ . |
| <i>m</i>     | (global) Specifies the number of rows of the distributed matrix <i>sub(A)</i> , $m \geq 0$ .                                                                                                                                                                                                                                                                                                                                          |

|               |                                                                                                                                                                                                                                          |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                        |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                          |
| <i>a</i>      | (local)<br>Array, size $(lld\_a, \text{LOCq}(ja+n-1))$ . Before entry this array must contain the local pieces of the distributed matrix $\text{sub}(A)$ .                                                                               |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                    |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                       |
| <i>x</i>      | (local)<br>Array, size $(jx-1)*m\_x + ix+(n-1)*\text{abs}(incx)$ when <i>trans</i> = 'N' or 'n', and $(jx-1)*m\_x + ix+(m-1)*\text{abs}(incx)$ otherwise.<br>This array contains the entries of the distributed vector $\text{sub}(x)$ . |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix <i>X</i> indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively.                                                                    |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                                                                                       |
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m\_x$ . <i>incx</i> must not be zero.                                                                                |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> . When <i>beta</i> is set to zero, then $\text{sub}(y)$ need not be set on input.                                                                                                           |
| <i>y</i>      | (local)<br>Array, size $(jy-1)*m\_y + iy+(m-1)*\text{abs}(incy)$ when <i>trans</i> = 'N' or 'n', and $(jy-1)*m\_y + iy+(n-1)*\text{abs}(incy)$ otherwise.<br>This array contains the entries of the distributed vector $\text{sub}(y)$ . |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix <i>Y</i> indicating the first row and the first column of the submatrix $\text{sub}(y)$ , respectively.                                                                    |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>Y</i> .                                                                                                                                       |
| <i>incy</i>   | (global) Specifies the increment for the elements of $\text{sub}(y)$ . Only two values are supported, namely 1 and $m\_y$ . <i>incy</i> must not be zero.                                                                                |

## Output Parameters

|          |                                                                 |
|----------|-----------------------------------------------------------------|
| <i>y</i> | Overwritten by the updated distributed vector $\text{sub}(y)$ . |
|----------|-----------------------------------------------------------------|

## p?ger

Performs a rank-1 update of a distributed general matrix.

### Syntax

```
void psgger (const MKL_INT *m , const MKL_INT *n , const float *alpha , const float *x ,
const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx ,
const float *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy , float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca);
```

```
void pdger (const MKL_INT *m , const MKL_INT *n , const double *alpha , const double
*x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT
*incx , const double *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT
*descy , const MKL_INT *incy , double *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca);
```

### Include Files

- mkl\_pblas.h

### Description

The p?ger routines perform a distributed matrix-vector operation defined as

$$\text{sub}(A) := \alpha * \text{sub}(x) * \text{sub}(y)' + \text{sub}(A),$$

where:

*alpha* is a scalar,

sub(A) is a *m*-by-*n* distributed general matrix, sub(A)=A(*ia:ia+m-1*, *ja:ja+n-1*),

sub(x) is an *m*-element distributed vector, sub(y) is an *n*-element distributed vector,

sub(x) denotes X(*ix*, *jx:jx+m-1*) if *incx* = *m\_x*, and X(*ix: ix+m-1*, *jx*) if *incx* = 1,

sub(y) denotes Y(*iy*, *jy:jy+n-1*) if *incy* = *m\_y*, and Y(*iy: iy+n-1*, *jy*) if *incy* = 1.

### Input Parameters

|                       |                                                                                                                                                                                      |
|-----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>m</i>              | (global) Specifies the number of rows of the distributed matrix sub(A), <i>m</i> ≥ 0.                                                                                                |
| <i>n</i>              | (global) Specifies the number of columns of the distributed matrix sub(A), <i>n</i> ≥ 0.                                                                                             |
| <i>alpha</i>          | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                      |
| <i>x</i>              | (local)<br>Array, size at least ( <i>jx-1</i> ) * <i>m_x</i> + <i>ix</i> + ( <i>m-1</i> ) * abs( <i>incx</i> ).<br>This array contains the entries of the distributed vector sub(x). |
| <i>ix</i> , <i>jx</i> | (global) The row and column indices in the distributed matrix X indicating the first row and the first column of the submatrix sub(x), respectively.                                 |

|               |                                                                                                                                                                     |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                  |
| <i>incx</i>   | (global) Specifies the increment for the elements of <i>sub(x)</i> . Only two values are supported, namely 1 and <i>m_x</i> . <i>incx</i> must not be zero.         |
| <i>y</i>      | (local)<br>Array, size at least $(jy-1)*m_y + iy+(n-1)*abs(incy)$ .<br>This array contains the entries of the distributed vector <i>sub(y)</i> .                    |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix <i>Y</i> indicating the first row and the first column of the submatrix <i>sub(y)</i> , respectively. |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>Y</i> .                                                                  |
| <i>incy</i>   | (global) Specifies the increment for the elements of <i>sub(y)</i> . Only two values are supported, namely 1 and <i>m_y</i> . <i>incy</i> must not be zero.         |
| <i>a</i>      | (local)<br>Array, size $(lld_a, LOCq(ja+n-1))$ .<br>Before entry this array contains the local pieces of the distributed matrix <i>sub(A)</i> .                     |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                  |

## Output Parameters

*a* Overwritten by the updated distributed matrix *sub(A)*.

## p?gerc

*Performs a rank-1 update (conjugated) of a distributed general matrix.*

### Syntax

```
void pcgerc (const MKL_INT *m , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex8 *y , const MKL_INT *iy , const MKL_INT *jy , const
MKL_INT *descy , const MKL_INT *incy , MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca);
```

```
void pzgerc (const MKL_INT *m , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex16 *y , const MKL_INT *iy , const MKL_INT *jy , const
MKL_INT *descy , const MKL_INT *incy , MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca);
```

## Include Files

- `mkl_pblas.h`

## Description

The `p?gerc` routines perform a distributed matrix-vector operation defined as

$$\text{sub}(A) := \alpha * \text{sub}(x) * \text{conjg}(\text{sub}(y)') + \text{sub}(A),$$

where:

*alpha* is a scalar,

*sub(A)* is a *m*-by-*n* distributed general matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+n-1)$ ,

*sub(x)* is an *m*-element distributed vector, *sub(y)* is an *n*-element distributed vector,

*sub(x)* denotes  $X(\text{ix}, \text{jx}:\text{jx}+m-1)$  if *incx* = *m\_x*, and  $X(\text{ix}:\text{ix}+m-1, \text{jx})$  if *incx* = 1,

*sub(y)* denotes  $Y(\text{iy}, \text{jy}:\text{jy}+n-1)$  if *incy* = *m\_y*, and  $Y(\text{iy}:\text{iy}+n-1, \text{jy})$  if *incy* = 1.

## Input Parameters

|               |                                                                                                                                                                                    |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrix <i>sub(A)</i> , $m \geq 0$ .                                                                                       |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix <i>sub(A)</i> , $n \geq 0$ .                                                                                    |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                    |
| <i>x</i>      | (local)<br>Array, size at least $(\text{jx}-1) * m_x + \text{ix} + (n-1) * \text{abs}(\text{incx})$ .<br>This array contains the entries of the distributed vector <i>sub(x)</i> . |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix <i>X</i> indicating the first row and the first column of the submatrix <i>sub(x)</i> , respectively.                |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                                 |
| <i>incx</i>   | (global) Specifies the increment for the elements of <i>sub(x)</i> . Only two values are supported, namely 1 and <i>m_x</i> . <i>incx</i> must not be zero.                        |
| <i>y</i>      | (local)<br>Array, size at least $(\text{jy}-1) * m_y + \text{iy} + (n-1) * \text{abs}(\text{incy})$ .<br>This array contains the entries of the distributed vector <i>sub(y)</i> . |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix <i>Y</i> indicating the first row and the first column of the submatrix <i>sub(y)</i> , respectively.                |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>Y</i> .                                                                                 |
| <i>incy</i>   | (global) Specifies the increment for the elements of <i>sub(y)</i> . Only two values are supported, namely 1 and <i>m_y</i> . <i>incy</i> must not be zero.                        |

|               |                                                                                                                                                           |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>      | (local)<br>Array, size at least $(lld\_a, LOCq(ja+n-1))$ . Before entry this array contains the local pieces of the distributed matrix $sub(A)$ .         |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix $A$ indicating the first row and the first column of the submatrix $sub(A)$ , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $A$ .                                                             |

## Output Parameters

|          |                                                          |
|----------|----------------------------------------------------------|
| <i>a</i> | Overwritten by the updated distributed matrix $sub(A)$ . |
|----------|----------------------------------------------------------|

## p?geru

*Performs a rank-1 update (unconjugated) of a distributed general matrix.*

### Syntax

```
void pcgeru (const MKL_INT *m , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex8 *y , const MKL_INT *iy , const MKL_INT *jy , const
MKL_INT *descy , const MKL_INT *incy , MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca);
```

```
void pzgeru (const MKL_INT *m , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex16 *y , const MKL_INT *iy , const MKL_INT *jy , const
MKL_INT *descy , const MKL_INT *incy , MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca);
```

### Include Files

- mkl\_pblas.h

### Description

The p?geru routines perform a matrix-vector operation defined as

$$sub(A) := alpha * sub(x) * sub(y)' + sub(A),$$

where:

*alpha* is a scalar,

$sub(A)$  is a  $m$ -by- $n$  distributed general matrix,  $sub(A) = A(ia:ia+m-1, ja:ja+n-1)$ ,

$sub(x)$  is an  $m$ -element distributed vector,  $sub(y)$  is an  $n$ -element distributed vector,

$sub(x)$  denotes  $X(ix, jx:jx+m-1)$  if  $incx = m_x$ , and  $X(ix:ix+m-1, jx)$  if  $incx = 1$ ,

$sub(y)$  denotes  $Y(iy, jy:jy+n-1)$  if  $incy = m_y$ , and  $Y(iy:iy+n-1, jy)$  if  $incy = 1$ .

## Input Parameters

|               |                                                                                                                                                                       |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrix $\text{sub}(A)$ , $m \geq 0$ .                                                                        |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                     |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                       |
| <i>x</i>      | (local)<br>Array, size at least $(j_x-1)*m_x + i_x + (n-1)*\text{abs}(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .         |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix <i>X</i> indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively. |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                    |
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . <i>incx</i> must not be zero.              |
| <i>y</i>      | (local)<br>Array, size at least $(j_y-1)*m_y + i_y + (n-1)*\text{abs}(incy)$ .<br>This array contains the entries of the distributed vector $\text{sub}(y)$ .         |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix <i>Y</i> indicating the first row and the first column of the submatrix $\text{sub}(y)$ , respectively. |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>Y</i> .                                                                    |
| <i>incy</i>   | (global) Specifies the increment for the elements of $\text{sub}(y)$ . Only two values are supported, namely 1 and $m_y$ . <i>incy</i> must not be zero.              |
| <i>a</i>      | (local)<br>Array, size at least $(lld_a, \text{LOCq}(ja+n-1))$ . Before entry this array contains the local pieces of the distributed matrix $\text{sub}(A)$ .        |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                    |

## Output Parameters

|          |                                                                 |
|----------|-----------------------------------------------------------------|
| <i>a</i> | Overwritten by the updated distributed matrix $\text{sub}(A)$ . |
|----------|-----------------------------------------------------------------|

## p?hemv

Computes a distributed matrix-vector product using a Hermitian matrix.

### Syntax

```
void pchemv (const char *uplo , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const
MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex8 *beta , MKL_Complex8 *y , const MKL_INT *iy , const
MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pzhemv (const char *uplo , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const
MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex16 *beta , MKL_Complex16 *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

### Include Files

- mkl\_pblas.h

### Description

The p?hemv routines perform a distributed matrix-vector operation defined as

$$\text{sub}(y) := \alpha * \text{sub}(A) * \text{sub}(x) + \beta * \text{sub}(y),$$

where:

*alpha* and *beta* are scalars,

*sub(A)* is a *n*-by-*n* Hermitian distributed matrix,  $\text{sub}(A) = A(ia:ia+n-1, ja:ja+n-1)$ ,

*sub(x)* and *sub(y)* are distributed vectors.

*sub(x)* denotes  $X(ix, jx:jx+n-1)$  if *incx* = *m<sub>x</sub>*, and  $X(ix:ix+n-1, jx)$  if *incx* = 1,

*sub(y)* denotes  $Y(iy, jy:jy+n-1)$  if *incy* = *m<sub>y</sub>*, and  $Y(iy:iy+n-1, jy)$  if *incy* = 1.

### Input Parameters

|              |                                                                                                                                                                                                                                                                                                                          |
|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>  | (global) Specifies whether the upper or lower triangular part of the Hermitian distributed matrix <i>sub(A)</i> is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the <i>sub(A)</i> is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the <i>sub(A)</i> is used. |
| <i>n</i>     | (global) Specifies the order of the distributed matrix <i>sub(A)</i> , $n \geq 0$ .                                                                                                                                                                                                                                      |
| <i>alpha</i> | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                          |
| <i>a</i>     | (local)<br>Array, size ( <i>lld<sub>a</sub></i> , <i>LOCq(ja+n-1)</i> ). This array contains the local pieces of the distributed matrix <i>sub(A)</i> .                                                                                                                                                                  |

Before entry when  $uplo = 'U'$  or  $'u'$ , the  $n$ -by- $n$  upper triangular part of the distributed matrix  $sub(A)$  must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of  $sub(A)$  is not referenced, and when  $uplo = 'L'$  or  $'l'$ , the  $n$ -by- $n$  lower triangular part of the distributed matrix  $sub(A)$  must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of  $sub(A)$  is not referenced.

|               |                                                                                                                                                           |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix $A$ indicating the first row and the first column of the submatrix $sub(A)$ , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $A$ .                                                             |
| <i>x</i>      | (local)<br>Array, size at least $(jx-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector $sub(x)$ .               |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $sub(x)$ , respectively. |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                             |
| <i>incx</i>   | (global) Specifies the increment for the elements of $sub(x)$ . Only two values are supported, namely 1 and $m_x$ . $incx$ must not be zero.              |
| <i>beta</i>   | (global)<br>Specifies the scalar $beta$ . When $beta$ is set to zero, then $sub(y)$ need not be set on input.                                             |
| <i>y</i>      | (local)<br>Array, size at least $(jy-1)*m_y + iy+(n-1)*abs(incy)$ .<br>This array contains the entries of the distributed vector $sub(y)$ .               |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix $Y$ indicating the first row and the first column of the submatrix $sub(y)$ , respectively. |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $Y$ .                                                             |
| <i>incy</i>   | (global) Specifies the increment for the elements of $sub(y)$ . Only two values are supported, namely 1 and $m_y$ . $incy$ must not be zero.              |

## Output Parameters

|          |                                                          |
|----------|----------------------------------------------------------|
| <i>y</i> | Overwritten by the updated distributed vector $sub(y)$ . |
|----------|----------------------------------------------------------|

## p?ahemv

Computes a distributed matrix-vector product using absolute values for a Hermitian matrix.

## Syntax

```
void pcahemv (const char *uplo , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const
MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex8 *beta , MKL_Complex8 *y , const MKL_INT *iy , const
MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pzahemv (const char *uplo , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const
MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex16 *beta , MKL_Complex16 *y , const MKL_INT *iy ,
const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

## Include Files

- mkl\_pblas.h

## Description

The p?ahemv routines perform a distributed matrix-vector operation defined as

$$\text{sub}(y) := \text{abs}(\alpha) * \text{abs}(\text{sub}(A)) * \text{abs}(\text{sub}(x)) + \text{abs}(\beta * \text{sub}(y)),$$

where:

*alpha* and *beta* are scalars,

*sub(A)* is a *n*-by-*n* Hermitian distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ ,

*sub(x)* and *sub(y)* are distributed vectors.

*sub(x)* denotes  $X(\text{ix}, \text{jx}:\text{jx}+n-1)$  if *incx* = *m<sub>x</sub>*, and  $X(\text{ix}:\text{ix}+n-1, \text{jx})$  if *incx* = 1,

*sub(y)* denotes  $Y(\text{iy}, \text{jy}:\text{jy}+n-1)$  if *incy* = *m<sub>y</sub>*, and  $Y(\text{iy}:\text{iy}+n-1, \text{jy})$  if *incy* = 1.

## Input Parameters

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>  | (global) Specifies whether the upper or lower triangular part of the Hermitian distributed matrix <i>sub(A)</i> is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the <i>sub(A)</i> is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the <i>sub(A)</i> is used.                                                                                                                                                                                                                    |
| <i>n</i>     | (global) Specifies the order of the distributed matrix <i>sub(A)</i> , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| <i>alpha</i> | (global)<br><br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| <i>a</i>     | (local)<br><br>Array, size ( <i>lld<sub>a</sub></i> , LOCq( <i>ja</i> + <i>n</i> -1)). This array contains the local pieces of the distributed matrix <i>sub(A)</i> .<br><br>Before entry when <i>uplo</i> = 'U' or 'u', the <i>n</i> -by- <i>n</i> upper triangular part of the distributed matrix <i>sub(A)</i> must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of <i>sub(A)</i> is not referenced, and when <i>uplo</i> = 'L' or 'l', the <i>n</i> -by- <i>n</i> lower |

triangular part of the distributed matrix  $\text{sub}(A)$  must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of  $\text{sub}(A)$  is not referenced.

|               |                                                                                                                                                                  |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix $A$ indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $A$ .                                                                    |
| <i>x</i>      | (local)<br>Array, size at least $(j_x-1)*m_x + i_x + (n-1)*\text{abs}(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .    |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively. |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                    |
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . <i>incx</i> must not be zero.         |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> . When <i>beta</i> is set to zero, then $\text{sub}(y)$ need not be set on input.                                   |
| <i>y</i>      | (local)<br>Array, size at least $(j_y-1)*m_y + i_y + (n-1)*\text{abs}(incy)$ .<br>This array contains the entries of the distributed vector $\text{sub}(y)$ .    |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix $Y$ indicating the first row and the first column of the submatrix $\text{sub}(y)$ , respectively. |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $Y$ .                                                                    |
| <i>incy</i>   | (global) Specifies the increment for the elements of $\text{sub}(y)$ . Only two values are supported, namely 1 and $m_y$ . <i>incy</i> must not be zero.         |

## Output Parameters

|          |                                                                 |
|----------|-----------------------------------------------------------------|
| <i>y</i> | Overwritten by the updated distributed vector $\text{sub}(y)$ . |
|----------|-----------------------------------------------------------------|

## p?her

Performs a rank-1 update of a distributed Hermitian matrix.

---

## Syntax

```
void pcher (const char *uplo , const MKL_INT *n , const float *alpha , const
MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca);
```

```
void pzher (const char *uplo , const MKL_INT *n , const double *alpha , const
MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const
MKL_INT *desca);
```

## Include Files

- mkl\_pblas.h

## Description

The `p?her` routines perform a distributed matrix-vector operation defined as

$$\text{sub}(A) := \alpha * \text{sub}(x) * \text{conjg}(\text{sub}(x)') + \text{sub}(A),$$

where:

$\alpha$  is a real scalar,

$\text{sub}(A)$  is a  $n$ -by- $n$  distributed Hermitian matrix,  $\text{sub}(A) = A(ia:ia+n-1, ja:ja+n-1)$ ,

$\text{sub}(x)$  is distributed vector.

$\text{sub}(x)$  denotes  $X(ix, jx:jx+n-1)$  if  $incx = m_x$ , and  $X(ix:ix+n-1, jx)$  if  $incx = 1$ .

## Input Parameters

|                     |                                                                                                                                                                                                                                                                                                                                              |
|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>uplo</code>   | (global) Specifies whether the upper or lower triangular part of the Hermitian distributed matrix $\text{sub}(A)$ is used:<br><br>If <code>uplo = 'U' or 'u'</code> , then the upper triangular part of the $\text{sub}(A)$ is used.<br><br>If <code>uplo = 'L' or 'l'</code> , then the low triangular part of the $\text{sub}(A)$ is used. |
| <code>n</code>      | (global) Specifies the order of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                                                                                                                                        |
| <code>alpha</code>  | (global)<br>Specifies the scalar $\alpha$ .                                                                                                                                                                                                                                                                                                  |
| <code>x</code>      | (local)<br>Array, size at least $(jx-1)*m_x + ix+(n-1)*\text{abs}(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .                                                                                                                                                                                    |
| <code>ix, jx</code> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively.                                                                                                                                                                             |
| <code>descx</code>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                                                                                                                                                                                                |
| <code>incx</code>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . $incx$ must not be zero.                                                                                                                                                                                          |

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>      | (local)<br><br>Array, size $(lld\_a, LOCq(ja+n-1))$ . This array contains the local pieces of the distributed matrix $sub(A)$ .<br><br>Before entry with <i>uplo</i> = 'U' or 'u', the <i>n</i> -by- <i>n</i> upper triangular part of the distributed matrix $sub(A)$ must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of $sub(A)$ is not referenced, and with <i>uplo</i> = 'L' or 'l', the <i>n</i> -by- <i>n</i> lower triangular part of the distributed matrix $sub(A)$ must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of $sub(A)$ is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $sub(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

## Output Parameters

|          |                                                                                                                                                                                                                                                                                                                                                        |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i> | With <i>uplo</i> = 'U' or 'u', the upper triangular part of the array <i>a</i> is overwritten by the upper triangular part of the updated distributed matrix $sub(A)$ .<br><br>With <i>uplo</i> = 'L' or 'l', the lower triangular part of the array <i>a</i> is overwritten by the lower triangular part of the updated distributed matrix $sub(A)$ . |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## p?her2

Performs a rank-2 update of a distributed Hermitian matrix.

### Syntax

```
void pcher2 (const char *uplo , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex8 *y , const MKL_INT *iy , const MKL_INT *jy , const
MKL_INT *descy , const MKL_INT *incy , MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca);
```

```
void pzher2 (const char *uplo , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx , const MKL_Complex16 *y , const MKL_INT *iy , const MKL_INT *jy , const
MKL_INT *descy , const MKL_INT *incy , MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca);
```

### Include Files

- mkl\_pblas.h

### Description

The p?her2 routines perform a distributed matrix-vector operation defined as

$$sub(A) := alpha * sub(x) * conj(sub(y)') + conj(alpha) * sub(y) * conj(sub(x)') + sub(A),$$

where:

*alpha* is a scalar,

$\text{sub}(A)$  is a  $n$ -by- $n$  distributed Hermitian matrix,  $\text{sub}(A)=A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ ,

$\text{sub}(x)$  and  $\text{sub}(y)$  are distributed vectors.

$\text{sub}(x)$  denotes  $X(\text{ix}, \text{jx}:\text{jx}+n-1)$  if  $\text{incx} = m_x$ , and  $X(\text{ix}:\text{ix}+n-1, \text{jx})$  if  $\text{incx} = 1$ ,

$\text{sub}(y)$  denotes  $Y(\text{iy}, \text{jy}:\text{jy}+n-1)$  if  $\text{incy} = m_y$ , and  $Y(\text{iy}:\text{iy}+n-1, \text{jy})$  if  $\text{incy} = 1$ .

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | (global) Specifies whether the upper or lower triangular part of the distributed Hermitian matrix $\text{sub}(A)$ is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the $\text{sub}(A)$ is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the $\text{sub}(A)$ is used. |
| <i>n</i>      | (global) Specifies the order of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                                                                                                                          |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                |
| <i>x</i>      | (local)<br>Array, size at least $(\text{jx}-1)*m_x + \text{ix}+(n-1)*\text{abs}(\text{incx})$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .                                                                                                                                                 |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively.                                                                                                                                                               |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                                                                                                                                                                                  |
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . <i>incx</i> must not be zero.                                                                                                                                                                       |
| <i>y</i>      | (local)<br>Array, size at least $(\text{jy}-1)*m_y + \text{iy}+(n-1)*\text{abs}(\text{incy})$ .<br>This array contains the entries of the distributed vector $\text{sub}(y)$ .                                                                                                                                                 |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix $Y$ indicating the first row and the first column of the submatrix $\text{sub}(y)$ , respectively.                                                                                                                                                               |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $Y$ .                                                                                                                                                                                                                                  |
| <i>incy</i>   | (global) Specifies the increment for the elements of $\text{sub}(y)$ . Only two values are supported, namely 1 and $m_y$ . <i>incy</i> must not be zero.                                                                                                                                                                       |
| <i>a</i>      | (local)<br>Array, size $(\text{lld}_a, \text{LOCq}(\text{ja}+n-1))$ . This array contains the local pieces of the distributed matrix $\text{sub}(A)$ .                                                                                                                                                                         |

Before entry with `uplo = 'U'` or `'u'`, the  $n$ -by- $n$  upper triangular part of the distributed matrix `sub(A)` must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of `sub(A)` is not referenced, and with `uplo = 'L'` or `'l'`, the  $n$ -by- $n$  lower triangular part of the distributed matrix `sub(A)` must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of `sub(A)` is not referenced.

`ia, ja` (global) The row and column indices in the distributed matrix  $A$  indicating the first row and the first column of the submatrix `sub(A)`, respectively.

`desca` (global and local) array of dimension 9. The array descriptor of the distributed matrix  $A$ .

## Output Parameters

`a` With `uplo = 'U'` or `'u'`, the upper triangular part of the array  $a$  is overwritten by the upper triangular part of the updated distributed matrix `sub(A)`.

With `uplo = 'L'` or `'l'`, the lower triangular part of the array  $a$  is overwritten by the lower triangular part of the updated distributed matrix `sub(A)`.

## p?symv

*Computes a distributed matrix-vector product using a symmetric matrix.*

### Syntax

```
void pssymv (const char *uplo , const MKL_INT *n , const float *alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const float *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const float *beta , float *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pdsymv (const char *uplo , const MKL_INT *n , const double *alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const double *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const double *beta , double *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

### Include Files

- `mkl_pblas.h`

### Description

The `p?symv` routines perform a distributed matrix-vector operation defined as

$$\text{sub}(y) := \alpha * \text{sub}(A) * \text{sub}(x) + \beta * \text{sub}(y),$$

where:

$\alpha$  and  $\beta$  are scalars,

`sub(A)` is a  $n$ -by- $n$  symmetric distributed matrix, `sub(A)=A(ia:ia+n-1, ja:ja+n-1)`,

$\text{sub}(x)$  and  $\text{sub}(y)$  are distributed vectors.

$\text{sub}(x)$  denotes  $X(ix, jx:jx+n-1)$  if  $incx = m_x$ , and  $X(ix:ix+n-1, jx)$  if  $incx = 1$ ,

$\text{sub}(y)$  denotes  $Y(iy, jy:jy+n-1)$  if  $incy = m_y$ , and  $Y(iy:iy+n-1, jy)$  if  $incy = 1$ .

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | (global) Specifies whether the upper or lower triangular part of the symmetric distributed matrix $\text{sub}(A)$ is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the $\text{sub}(A)$ is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the $\text{sub}(A)$ is used.                                                                                                                                                                                                                                                                                                                                                                            |
| <i>n</i>      | (global) Specifies the order of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>a</i>      | (local)<br>Array, size $(lld_a, LOCq(ja+n-1))$ . This array contains the local pieces of the distributed matrix $\text{sub}(A)$ .<br><br>Before entry when <i>uplo</i> = 'U' or 'u', the $n$ -by- $n$ upper triangular part of the distributed matrix $\text{sub}(A)$ must contain the upper triangular part of the symmetric distributed matrix and the strictly lower triangular part of $\text{sub}(A)$ is not referenced, and when <i>uplo</i> = 'L' or 'l', the $n$ -by- $n$ lower triangular part of the distributed matrix $\text{sub}(A)$ must contain the lower triangular part of the symmetric distributed matrix and the strictly upper triangular part of $\text{sub}(A)$ is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix $A$ indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $A$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>x</i>      | (local)<br>Array, size at least $(jx-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . <i>incx</i> must not be zero.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> . When <i>beta</i> is set to zero, then $\text{sub}(y)$ need not be set on input.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |

|               |                                                                                                                                                                |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>y</i>      | (local)<br>Array, size at least $(j_y-1)*m_y + i_y+(n-1)*abs(incy)$ .<br>This array contains the entries of the distributed vector $sub(y)$ .                  |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix <i>Y</i> indicating the first row and the first column of the submatrix $sub(y)$ , respectively. |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>Y</i> .                                                             |
| <i>incy</i>   | (global) Specifies the increment for the elements of $sub(y)$ . Only two values are supported, namely 1 and $m_y$ . <i>incy</i> must not be zero.              |

## Output Parameters

|          |                                                          |
|----------|----------------------------------------------------------|
| <i>y</i> | Overwritten by the updated distributed vector $sub(y)$ . |
|----------|----------------------------------------------------------|

## p?asymv

*Computes a distributed matrix-vector product using absolute values for a symmetric matrix.*

### Syntax

```
void psasymv (const char *uplo , const MKL_INT *n , const float *alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const float *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const float *beta , float *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pdasymv (const char *uplo , const MKL_INT *n , const double *alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const double *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const double *beta , double *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

### Include Files

- mkl\_pblas.h

### Description

The p?symv routines perform a distributed matrix-vector operation defined as

$$sub(y) := abs(alpha)*abs(sub(A))*abs(sub(x)) + abs(beta*sub(y)),$$

where:

*alpha* and *beta* are scalars,

*sub(A)* is a *n*-by-*n* symmetric distributed matrix,  $sub(A)=A(ia:ia+n-1, ja:ja+n-1)$ ,

*sub(x)* and *sub(y)* are distributed vectors.

*sub(x)* denotes  $X(ix, jx:jx+n-1)$  if *incx* =  $m_x$ , and  $X(ix: ix+n-1, jx)$  if *incx* = 1,

*sub(y)* denotes  $Y(iy, jy:jy+n-1)$  if *incy* =  $m_y$ , and  $Y(iy: iy+n-1, jy)$  if *incy* = 1.

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | (global) Specifies whether the upper or lower triangular part of the symmetric distributed matrix $\text{sub}(A)$ is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the $\text{sub}(A)$ is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the $\text{sub}(A)$ is used.                                                                                                                                                                                                                                                                                                                                                                             |
| <i>n</i>      | (global) Specifies the order of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <i>a</i>      | (local)<br>Array, size $(lld\_a, LOCq(ja+n-1))$ . This array contains the local pieces of the distributed matrix $\text{sub}(A)$ .<br><br>Before entry when <i>uplo</i> = 'U' or 'u', the $n$ -by- $n$ upper triangular part of the distributed matrix $\text{sub}(A)$ must contain the upper triangular part of the symmetric distributed matrix and the strictly lower triangular part of $\text{sub}(A)$ is not referenced, and when <i>uplo</i> = 'L' or 'l', the $n$ -by- $n$ lower triangular part of the distributed matrix $\text{sub}(A)$ must contain the lower triangular part of the symmetric distributed matrix and the strictly upper triangular part of $\text{sub}(A)$ is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix $A$ indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $A$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <i>x</i>      | (local)<br>Array, size at least $(jx-1)*m\_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m\_x$ . <i>incx</i> must not be zero.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> . When <i>beta</i> is set to zero, then $\text{sub}(y)$ need not be set on input.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>y</i>      | (local)<br>Array, size at least $(jy-1)*m\_y + iy+(n-1)*abs(incy)$ .<br>This array contains the entries of the distributed vector $\text{sub}(y)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

|               |                                                                                                                                                                       |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix <i>Y</i> indicating the first row and the first column of the submatrix <i>sub(y)</i> , respectively.   |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>Y</i> .                                                                    |
| <i>incy</i>   | (global) Specifies the increment for the elements of <i>sub(y)</i> . Only two values are supported, namely 1 and <i>m<sub>y</sub></i> . <i>incy</i> must not be zero. |

## Output Parameters

|          |                                                               |
|----------|---------------------------------------------------------------|
| <i>y</i> | Overwritten by the updated distributed vector <i>sub(y)</i> . |
|----------|---------------------------------------------------------------|

## p?syr

Performs a rank-1 update of a distributed symmetric matrix.

### Syntax

```
void pssyr (const char *uplo , const MKL_INT *n , const float *alpha , const float *x ,
const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx ,
float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca);

void pdsyr (const char *uplo , const MKL_INT *n , const double *alpha , const double
*x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT
*incx , double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca);
```

### Include Files

- `mkl_pblas.h`

### Description

The `p?syr` routines perform a distributed matrix-vector operation defined as

$$\text{sub}(A) := \alpha * \text{sub}(x) * \text{sub}(x)' + \text{sub}(A),$$

where:

*alpha* is a scalar,

*sub(A)* is a *n*-by-*n* distributed symmetric matrix, *sub(A)* = *A*(*ia:ia+n-1, ja:ja+n-1*),

*sub(x)* is distributed vector.

*sub(x)* denotes *X*(*ix, jx:jx+n-1*) if *incx* = *m<sub>x</sub>*, and *X*(*ix: ix+n-1, jx*) if *incx* = 1,

### Input Parameters

|             |                                                                                                                                                                                                                                                                                                                          |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i> | (global) Specifies whether the upper or lower triangular part of the symmetric distributed matrix <i>sub(A)</i> is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the <i>sub(A)</i> is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the <i>sub(A)</i> is used. |
| <i>n</i>    | (global) Specifies the order of the distributed matrix <i>sub(A)</i> , <i>n</i> ≥ 0.                                                                                                                                                                                                                                     |

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <i>x</i>      | (local)<br>Array, size at least $(j_x-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector $sub(x)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix <i>X</i> indicating the first row and the first column of the submatrix $sub(x)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| <i>incx</i>   | (global) Specifies the increment for the elements of $sub(x)$ . Only two values are supported, namely 1 and $m_x$ . <i>incx</i> must not be zero.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <i>a</i>      | (local)<br>Array, size $(lld_a, LOCq(ja+n-1))$ . This array contains the local pieces of the distributed matrix $sub(A)$ .<br><br>Before entry with <i>uplo</i> = 'U' or 'u', the <i>n</i> -by- <i>n</i> upper triangular part of the distributed matrix $sub(A)$ must contain the upper triangular part of the symmetric distributed matrix and the strictly lower triangular part of $sub(A)$ is not referenced, and with <i>uplo</i> = 'L' or 'l', the <i>n</i> -by- <i>n</i> lower triangular part of the distributed matrix $sub(A)$ must contain the lower triangular part of the symmetric distributed matrix and the strictly upper triangular part of $sub(A)$ is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $sub(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |

## Output Parameters

|          |                                                                                                                                                                                                                                                                                                                                                        |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i> | With <i>uplo</i> = 'U' or 'u', the upper triangular part of the array <i>a</i> is overwritten by the upper triangular part of the updated distributed matrix $sub(A)$ .<br><br>With <i>uplo</i> = 'L' or 'l', the lower triangular part of the array <i>a</i> is overwritten by the lower triangular part of the updated distributed matrix $sub(A)$ . |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## p?syr2

Performs a rank-2 update of a distributed symmetric matrix.

---

## Syntax

```
void pssyr2 (const char *uplo , const MKL_INT *n , const float *alpha , const float
*x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT
*incx , const float *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT
*descy , const MKL_INT *incy , float *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca);
```

```
void pdsyr2 (const char *uplo , const MKL_INT *n , const double *alpha , const double
*x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT
*incx , const double *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT
*descy , const MKL_INT *incy , double *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca);
```

## Include Files

- mkl\_pblas.h

## Description

The p?syr2 routines perform a distributed matrix-vector operation defined as

$$\text{sub}(A) := \alpha * \text{sub}(x) * \text{sub}(y)' + \alpha * \text{sub}(y) * \text{sub}(x)' + \text{sub}(A),$$

where:

*alpha* is a scalar,

*sub(A)* is a *n*-by-*n* distributed symmetric matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ ,

*sub(x)* and *sub(y)* are distributed vectors.

*sub(x)* denotes  $X(\text{ix}, \text{jx}:\text{jx}+n-1)$  if *incx* = *m<sub>x</sub>*, and  $X(\text{ix}:\text{ix}+n-1, \text{jx})$  if *incx* = 1,

*sub(y)* denotes  $Y(\text{iy}, \text{jy}:\text{jy}+n-1)$  if *incy* = *m<sub>y</sub>*, and  $Y(\text{iy}:\text{iy}+n-1, \text{jy})$  if *incy* = 1.

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                          |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | (global) Specifies whether the upper or lower triangular part of the distributed symmetric matrix <i>sub(A)</i> is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the <i>sub(A)</i> is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the <i>sub(A)</i> is used. |
| <i>n</i>      | (global) Specifies the order of the distributed matrix <i>sub(A)</i> , $n \geq 0$ .                                                                                                                                                                                                                                      |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                          |
| <i>x</i>      | (local)<br>Array, size at least $(\text{jx}-1) * m_x + \text{ix} + (n-1) * \text{abs}(\text{incx})$ .<br>This array contains the entries of the distributed vector <i>sub(x)</i> .                                                                                                                                       |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix <i>X</i> indicating the first row and the first column of the submatrix <i>sub(x)</i> , respectively.                                                                                                                                                      |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                                                                                                                                                                       |

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m_x$ . <i>incx</i> must not be zero.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>y</i>      | (local)<br>Array, size at least $(jy-1)*m_y + iy + (n-1)*\text{abs}(incy)$ .<br>This array contains the entries of the distributed vector $\text{sub}(y)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix <i>Y</i> indicating the first row and the first column of the submatrix $\text{sub}(y)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <i>descy</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>Y</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>incy</i>   | (global) Specifies the increment for the elements of $\text{sub}(y)$ . Only two values are supported, namely 1 and $m_y$ . <i>incy</i> must not be zero.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>a</i>      | (local)<br>Array, size $(lld_a, \text{LOCq}(ja+n-1))$ . This array contains the local pieces of the distributed matrix $\text{sub}(A)$ .<br><br>Before entry with <i>uplo</i> = 'U' or 'u', the <i>n</i> -by- <i>n</i> upper triangular part of the distributed matrix $\text{sub}(A)$ must contain the upper triangular part of the distributed symmetric matrix and the strictly lower triangular part of $\text{sub}(A)$ is not referenced, and with <i>uplo</i> = 'L' or 'l', the <i>n</i> -by- <i>n</i> lower triangular part of the distributed matrix $\text{sub}(A)$ must contain the lower triangular part of the distributed symmetric matrix and the strictly upper triangular part of $\text{sub}(A)$ is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |

## Output Parameters

|          |                                                                                                                                                                                                                                                                                                                                                                      |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i> | With <i>uplo</i> = 'U' or 'u', the upper triangular part of the array <i>a</i> is overwritten by the upper triangular part of the updated distributed matrix $\text{sub}(A)$ .<br><br>With <i>uplo</i> = 'L' or 'l', the lower triangular part of the array <i>a</i> is overwritten by the lower triangular part of the updated distributed matrix $\text{sub}(A)$ . |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## p?trmv

Computes a distributed matrix-vector product using a triangular matrix.

### Syntax

```
void pstrmv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca ,
float *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT
*incx);
```

```
void pdtrmv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca ,
double *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx);
```

```
void pctrmv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT
*descx , const MKL_INT *incx);
```

```
void pztrmv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT
*descx , const MKL_INT *incx);
```

## Include Files

- mkl\_pblas.h

## Description

The `p?trmv` routines perform one of the following distributed matrix-vector operations defined as

$\text{sub}(x) := \text{sub}(A) * \text{sub}(x)$ , or  $\text{sub}(x) := \text{sub}(A)' * \text{sub}(x)$ , or  $\text{sub}(x) := \text{conjg}(\text{sub}(A)') * \text{sub}(x)$ ,

where:

$\text{sub}(A)$  is a  $n$ -by- $n$  unit, or non-unit, upper or lower triangular distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ ,

$\text{sub}(x)$  is an  $n$ -element distributed vector.

$\text{sub}(x)$  denotes  $X(\text{ix}, \text{jx}:\text{jx}+n-1)$  if  $\text{incx} = m_x$ , and  $X(\text{ix}:\text{ix}+n-1, \text{jx})$  if  $\text{incx} = 1$ ,

## Input Parameters

|              |                                                                                                                                                                                                                                                                                                                                                                                      |
|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>  | (global) Specifies whether the distributed matrix $\text{sub}(A)$ is upper or lower triangular:<br>if <i>uplo</i> = 'U' or 'u', then the matrix is upper triangular;<br>if <i>uplo</i> = 'L' or 'l', then the matrix is low triangular.                                                                                                                                              |
| <i>trans</i> | (global) Specifies the form of $\text{op}(\text{sub}(A))$ used in the matrix equation:<br>if <i>transa</i> = 'N' or 'n', then $\text{sub}(x) := \text{sub}(A) * \text{sub}(x)$ ;<br>if <i>transa</i> = 'T' or 't', then $\text{sub}(x) := \text{sub}(A)' * \text{sub}(x)$ ;<br>if <i>transa</i> = 'C' or 'c', then $\text{sub}(x) := \text{conjg}(\text{sub}(A)') * \text{sub}(x)$ . |
| <i>diag</i>  | (global) Specifies whether the matrix $\text{sub}(A)$ is unit triangular:<br>if <i>diag</i> = 'U' or 'u' then the matrix is unit triangular;<br>if <i>diag</i> = 'N' or 'n', then the matrix is not unit triangular.                                                                                                                                                                 |
| <i>n</i>     | (global) Specifies the order of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                                                                                                                                                                                |
| <i>a</i>     | (local)<br>Array, size at least $(\text{lld}_a, \text{LOCq}(1, \text{ja}+n-1))$ .                                                                                                                                                                                                                                                                                                    |

Before entry with *uplo* = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix *sub(A)*, and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix *sub(A)* is not referenced.

Before entry with *uplo* = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix *sub(A)*, and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix *sub(A)* is not referenced.

When *diag* = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix *sub(A)* are not referenced either, but are assumed to be unity.

|               |                                                                                                                                                                     |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                  |
| <i>x</i>      | (local)<br>Array, size at least $(jx-1)*m_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector <i>sub(x)</i> .                    |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix <i>X</i> indicating the first row and the first column of the submatrix <i>sub(x)</i> , respectively. |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                  |
| <i>incx</i>   | (global) Specifies the increment for the elements of <i>sub(x)</i> . Only two values are supported, namely 1 and <i>m_x</i> . <i>incx</i> must not be zero.         |

## Output Parameters

|          |                                                                   |
|----------|-------------------------------------------------------------------|
| <i>x</i> | Overwritten by the transformed distributed vector <i>sub(x)</i> . |
|----------|-------------------------------------------------------------------|

## p?atrmv

Computes a distributed matrix-vector product using absolute values for a triangular matrix.

### Syntax

```
void psatrmv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const float *alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const float *x , const MKL_INT *ix , const MKL_INT *jx , const
MKL_INT *descx , const MKL_INT *incx , const float *beta , float *y , const MKL_INT
*iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pdatrmv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const double *alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const double *x , const MKL_INT *ix , const MKL_INT *jx , const
MKL_INT *descx , const MKL_INT *incx , const double *beta , double *y , const MKL_INT
*iy , const MKL_INT *jy , const MKL_INT *descy , const MKL_INT *incy);
```

```
void pcatrmv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8 *x , const MKL_INT *ix , const
MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const MKL_Complex8 *beta ,
MKL_Complex8 *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy);
```

```
void pzatrmv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16 *x , const MKL_INT *ix , const
MKL_INT *jx , const MKL_INT *descx , const MKL_INT *incx , const MKL_Complex16 *beta ,
MKL_Complex16 *y , const MKL_INT *iy , const MKL_INT *jy , const MKL_INT *descy , const
MKL_INT *incy);
```

## Include Files

- mkl\_pblas.h

## Description

The `p?atrmv` routines perform one of the following distributed matrix-vector operations defined as

$\text{sub}(y) := \text{abs}(\alpha) * \text{abs}(\text{sub}(A)) * \text{abs}(\text{sub}(x)) + \text{abs}(\beta * \text{sub}(y))$ , or

$\text{sub}(y) := \text{abs}(\alpha) * \text{abs}(\text{sub}(A)') * \text{abs}(\text{sub}(x)) + \text{abs}(\beta * \text{sub}(y))$ , or

$\text{sub}(y) := \text{abs}(\alpha) * \text{abs}(\text{conjg}(\text{sub}(A)')) * \text{abs}(\text{sub}(x)) + \text{abs}(\beta * \text{sub}(y))$ ,

where:

*alpha* and *beta* are scalars,

*sub(A)* is a *n*-by-*n* unit, or non-unit, upper or lower triangular distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ ,

*sub(x)* is an *n*-element distributed vector.

*sub(x)* denotes  $X(\text{ix}, \text{jx}:\text{jx}+n-1)$  if *incx* = *m\_x*, and  $X(\text{ix}:\text{ix}+n-1, \text{jx})$  if *incx* = 1.

## Input Parameters

*uplo* (global) Specifies whether the distributed matrix *sub(A)* is upper or lower triangular:

if *uplo* = 'U' or 'u', then the matrix is upper triangular;

if *uplo* = 'L' or 'l', then the matrix is low triangular.

*trans* (global) Specifies the form of  $\text{op}(\text{sub}(A))$  used in the matrix equation:

if *trans* = 'N' or 'n', then  $\text{sub}(y) := |\alpha| * |\text{sub}(A)| * |\text{sub}(x)| + |\beta * \text{sub}(y)|$ ;

if *trans* = 'T' or 't', then  $\text{sub}(y) := |\alpha| * |\text{sub}(A)'| * |\text{sub}(x)| + |\beta * \text{sub}(y)|$ ;

if *trans* = 'C' or 'c', then  $\text{sub}(y) := |\alpha| * |\text{conjg}(\text{sub}(A)')| * |\text{sub}(x)| + |\beta * \text{sub}(y)|$ .

*diag* (global) Specifies whether the matrix *sub(A)* is unit triangular:

if *diag* = 'U' or 'u' then the matrix is unit triangular;

if *diag* = 'N' or 'n', then the matrix is not unit triangular.

---

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>      | (global) Specifies the order of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| <i>a</i>      | (local)<br>Array, size at least $(lld\_a, LOCq(1, ja+n-1))$ .<br><br>Before entry with <i>uplo</i> = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix $\text{sub}(A)$ , and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix $\text{sub}(A)$ is not referenced.<br><br>Before entry with <i>uplo</i> = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix $\text{sub}(A)$ , and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix $\text{sub}(A)$ is not referenced.<br><br>When <i>diag</i> = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix $\text{sub}(A)$ are not referenced either, but are assumed to be unity. |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <i>x</i>      | (local)<br>Array, size at least $(jx-1)*m\_x + ix+(n-1)*abs(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix <i>X</i> indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>X</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <i>incx</i>   | (global) Specifies the increment for the elements of $\text{sub}(x)$ . Only two values are supported, namely 1 and $m\_x$ . <i>incx</i> must not be zero.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> . When <i>beta</i> is set to zero, then $\text{sub}(y)$ need not be set on input.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <i>y</i>      | (local)<br>Array, size $(jy-1)*m\_y + iy+(m-1)*abs(incy)$ when <i>trans</i> = 'N' or 'n', and $(jy-1)*m\_y + iy+(n-1)*abs(incy)$ otherwise.<br>This array contains the entries of the distributed vector $\text{sub}(y)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <i>iy, jy</i> | (global) The row and column indices in the distributed matrix <i>Y</i> indicating the first row and the first column of the submatrix $\text{sub}(y)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |

|              |                                                                                                                                                                       |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>descy</i> | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>Y</i> .                                                                    |
| <i>incy</i>  | (global) Specifies the increment for the elements of <i>sub(y)</i> . Only two values are supported, namely 1 and <i>m<sub>y</sub></i> . <i>incy</i> must not be zero. |

## Output Parameters

|          |                                                                   |
|----------|-------------------------------------------------------------------|
| <i>y</i> | Overwritten by the transformed distributed vector <i>sub(y)</i> . |
|----------|-------------------------------------------------------------------|

## p?trsv

*Solves a system of linear equations whose coefficients are in a distributed triangular matrix.*

### Syntax

```
void pstrsv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca ,
float *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const MKL_INT
*incx);
```

```
void pdtrsv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca ,
double *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT *descx , const
MKL_INT *incx);
```

```
void pctrsv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , MKL_Complex8 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT
*descx , const MKL_INT *incx);
```

```
void pztrsv (const char *uplo , const char *trans , const char *diag , const MKL_INT
*n , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , MKL_Complex16 *x , const MKL_INT *ix , const MKL_INT *jx , const MKL_INT
*descx , const MKL_INT *incx);
```

### Include Files

- mkl\_pblas.h

### Description

The p?trsv routines solve one of the systems of equations:

$$\text{sub}(A) * \text{sub}(x) = b, \text{ or } \text{sub}(A)' * \text{sub}(x) = b, \text{ or } \text{conjg}(\text{sub}(A)') * \text{sub}(x) = b,$$

where:

*sub(A)* is a *n*-by-*n* unit, or non-unit, upper or lower triangular distributed matrix, *sub(A)* = *A(ia:ia+n-1, ja:ja+n-1)*,

*b* and *sub(x)* are *n*-element distributed vectors,

*sub(x)* denotes *X(ix, jx:jx+n-1)* if *incx* = *m<sub>x</sub>*, and *X(ix: ix+n-1, jx)* if *incx* = 1,.

The routine does not test for singularity or near-singularity. Such tests must be performed before calling this routine.

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | (global) Specifies whether the distributed matrix $\text{sub}(A)$ is upper or lower triangular:<br>if <i>uplo</i> = 'U' or 'u', then the matrix is upper triangular;<br>if <i>uplo</i> = 'L' or 'l', then the matrix is low triangular.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <i>trans</i>  | (global) Specifies the form of the system of equations:<br>if <i>transa</i> = 'N' or 'n', then $\text{sub}(A) * \text{sub}(x) = b$ ;<br>if <i>transa</i> = 'T' or 't', then $\text{sub}(A)' * \text{sub}(x) = b$ ;<br>if <i>transa</i> = 'C' or 'c', then $\text{conjg}(\text{sub}(A)') * \text{sub}(x) = b$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <i>diag</i>   | (global) Specifies whether the matrix $\text{sub}(A)$ is unit triangular:<br>if <i>diag</i> = 'U' or 'u' then the matrix is unit triangular;<br>if <i>diag</i> = 'N' or 'n', then the matrix is not unit triangular.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| <i>n</i>      | (global) Specifies the order of the distributed matrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>a</i>      | (local)<br>Array, size at least $(lld\_a, LOCq(1, ja+n-1))$ .<br>Before entry with <i>uplo</i> = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix $\text{sub}(A)$ , and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix $\text{sub}(A)$ is not referenced.<br>Before entry with <i>uplo</i> = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix $\text{sub}(A)$ , and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix $\text{sub}(A)$ is not referenced .<br>When <i>diag</i> = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix $\text{sub}(A)$ are not referenced either, but are assumed to be unity. |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix $A$ indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $A$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <i>x</i>      | (local)<br>Array, size at least $(jx-1) * m\_x + ix + (n-1) * \text{abs}(incx)$ .<br>This array contains the entries of the distributed vector $\text{sub}(x)$ . Before entry, $\text{sub}(x)$ must contain the $n$ -element right-hand side distributed vector $b$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>ix, jx</i> | (global) The row and column indices in the distributed matrix $X$ indicating the first row and the first column of the submatrix $\text{sub}(x)$ , respectively.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>descx</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $X$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |

*incx* (global) Specifies the increment for the elements of  $\text{sub}(x)$ . Only two values are supported, namely 1 and  $m_x$ . *incx* must not be zero.

## Output Parameters

*x* Overwritten with the solution vector.

## PBLAS Level 3 Routines

The PBLAS Level 3 routines perform distributed matrix-matrix operations. [Table "PBLAS Level 3 Routine Groups and Their Data Types"](#) lists the PBLAS Level 3 routine groups and the data types associated with them.

### PBLAS Level 3 Routine Groups and Their Data Types

| Routine Group           | Data Types | Description                                                         |
|-------------------------|------------|---------------------------------------------------------------------|
| <a href="#">p?geadd</a> | s, d, c, z | Distributed matrix-matrix sum of general matrices                   |
| <a href="#">p?tradd</a> | s, d, c, z | Distributed matrix-matrix sum of triangular matrices                |
| <a href="#">p?gemm</a>  | s, d, c, z | Distributed matrix-matrix product of general matrices               |
| <a href="#">p?hemm</a>  | c, z       | Distributed matrix-matrix product, one matrix is Hermitian          |
| <a href="#">p?herk</a>  | c, z       | Rank-k update of a distributed Hermitian matrix                     |
| <a href="#">p?her2k</a> | c, z       | Rank-2k update of a distributed Hermitian matrix                    |
| <a href="#">p?symm</a>  | s, d, c, z | Matrix-matrix product of distributed symmetric matrices             |
| <a href="#">p?syrk</a>  | s, d, c, z | Rank-k update of a distributed symmetric matrix                     |
| <a href="#">p?syr2k</a> | s, d, c, z | Rank-2k update of a distributed symmetric matrix                    |
| <a href="#">p?tran</a>  | s, d       | Transposition of a real distributed matrix                          |
| <a href="#">p?tranc</a> | c, z       | Transposition of a complex distributed matrix (conjugated)          |
| <a href="#">p?tranu</a> | c, z       | Transposition of a complex distributed matrix                       |
| <a href="#">p?trmm</a>  | s, d, c, z | Distributed matrix-matrix product, one matrix is triangular         |
| <a href="#">p?trsm</a>  | s, d, c, z | Solution of a distributed matrix equation, one matrix is triangular |

### [p?geadd](#)

*Performs sum operation for two distributed general matrices.*

#### Syntax

```
void psgeadd (const char *trans , const MKL_INT *m , const MKL_INT *n , const float
*alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , const float *beta , float *c , const MKL_INT *ic , const MKL_INT *jc , const
MKL_INT *descc);
```

```
void pdgeadd (const char *trans , const MKL_INT *m , const MKL_INT *n , const double
*alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT
*desca , const double *beta , double *c , const MKL_INT *ic , const MKL_INT *jc ,
const MKL_INT *desc);
```

```
void pcgeadd (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const MKL_Complex8 *beta , MKL_Complex8 *c , const MKL_INT *ic ,
const MKL_INT *jc , const MKL_INT *desc);
```

```
void pzgeadd (const char *trans , const MKL_INT *m , const MKL_INT *n , const
MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const MKL_Complex16 *beta , MKL_Complex16 *c , const MKL_INT
*ic , const MKL_INT *jc , const MKL_INT *desc);
```

## Include Files

- mkl\_pblas.h

## Description

The `p?geadd` routines perform sum operation for two distributed general matrices. The operation is defined as

$$\text{sub}(C) := \text{beta} * \text{sub}(C) + \text{alpha} * \text{op}(\text{sub}(A)),$$

where:

$\text{op}(x)$  is one of  $\text{op}(x) = x$ , or  $\text{op}(x) = x'$ ,

$\text{alpha}$  and  $\text{beta}$  are scalars,

$\text{sub}(C)$  is an  $m$ -by- $n$  distributed matrix,  $\text{sub}(C) = C(\text{ic}:\text{ic}+m-1, \text{jc}:\text{jc}+n-1)$ .

$\text{sub}(A)$  is a distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+m-1)$ .

## Input Parameters

|              |                                                                                                                                                                                                                                                                                                 |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>trans</i> | (global) Specifies the operation:<br>if <i>trans</i> = 'N' or 'n', then $\text{op}(\text{sub}(A)) := \text{sub}(A)$ ;<br>if <i>trans</i> = 'T' or 't', then $\text{op}(\text{sub}(A)) := \text{sub}(A)'$ ;<br>if <i>trans</i> = 'C' or 'c', then $\text{op}(\text{sub}(A)) := \text{sub}(A)'$ . |
| <i>m</i>     | (global) Specifies the number of rows of the distributed matrix $\text{sub}(C)$ and the number of columns of the submatrix $\text{sub}(A)$ , $m \geq 0$ .                                                                                                                                       |
| <i>n</i>     | (global) Specifies the number of columns of the distributed matrix $\text{sub}(C)$ and the number of rows of the submatrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                       |
| <i>alpha</i> | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                 |
| <i>a</i>     | (local)<br>Array, size $(\text{lld}_a, \text{LOCq}(\text{ja}+m-1))$ . This array contains the local pieces of the distributed matrix $\text{sub}(A)$ .                                                                                                                                          |

|               |                                                                                                                                                                     |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                  |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .<br>When <i>beta</i> is equal to zero, then <i>sub(C)</i> need not be set on input.                                   |
| <i>c</i>      | (local)<br>Array, size ( <i>lld_c</i> , <i>LOCq(jc+n-1)</i> ).<br>This array contains the local pieces of the distributed matrix <i>sub(C)</i> .                    |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix <i>C</i> indicating the first row and the first column of the submatrix <i>sub(C)</i> , respectively. |
| <i>descc</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>C</i> .                                                                  |

## Output Parameters

|          |                                       |
|----------|---------------------------------------|
| <i>c</i> | Overwritten by the updated submatrix. |
|----------|---------------------------------------|

## p?tradd

*Performs sum operation for two distributed triangular matrices.*

---

### Syntax

```
void pstradd (const char *uplo , const char *trans , const MKL_INT *m , const MKL_INT *n , const float *alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const float *beta , float *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

```
void pdtradd (const char *uplo , const char *trans , const MKL_INT *m , const MKL_INT *n , const double *alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const double *beta , double *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

```
void ptradd (const char *uplo , const char *trans , const MKL_INT *m , const MKL_INT *n , const MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8 *beta , MKL_Complex8 *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

```
void pztradd (const char *uplo , const char *trans , const MKL_INT *m , const MKL_INT *n , const MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16 *beta , MKL_Complex16 *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

### Include Files

- `mkl_pblas.h`

## Description

The `p?tradd` routines perform sum operation for two distributed triangular matrices. The operation is defined as

$$\text{sub}(C) := \text{beta} * \text{sub}(C) + \text{alpha} * \text{op}(\text{sub}(A)),$$

where:

$\text{op}(x)$  is one of  $\text{op}(x) = x$ , or  $\text{op}(x) = x'$ , or  $\text{op}(x) = \text{conjg}(x')$ .

$\text{alpha}$  and  $\text{beta}$  are scalars,

$\text{sub}(C)$  is an  $m$ -by- $n$  distributed matrix,  $\text{sub}(C) = C(\text{ic}:\text{ic}+m-1, \text{jc}:\text{jc}+n-1)$ .

$\text{sub}(A)$  is a distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+m-1)$ .

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                               |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | (global) Specifies whether the distributed matrix $\text{sub}(C)$ is upper or lower triangular:<br>if <i>uplo</i> = 'U' or 'u', then the matrix is upper triangular;<br>if <i>uplo</i> = 'L' or 'l', then the matrix is low triangular.                                                                       |
| <i>trans</i>  | (global) Specifies the operation:<br>if <i>trans</i> = 'N' or 'n', then $\text{op}(\text{sub}(A)) := \text{sub}(A)$ ;<br>if <i>trans</i> = 'T' or 't', then $\text{op}(\text{sub}(A)) := \text{sub}(A)'$ ;<br>if <i>trans</i> = 'C' or 'c', then $\text{op}(\text{sub}(A)) := \text{conjg}(\text{sub}(A)')$ . |
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrix $\text{sub}(C)$ and the number of columns of the submatrix $\text{sub}(A)$ , $m \geq 0$ .                                                                                                                                                     |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix $\text{sub}(C)$ and the number of rows of the submatrix $\text{sub}(A)$ , $n \geq 0$ .                                                                                                                                                     |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                               |
| <i>a</i>      | (local)<br>Array, size $(\text{lld}_a, \text{LOCq}(\text{ja}+m-1))$ . This array contains the local pieces of the distributed matrix $\text{sub}(A)$ .                                                                                                                                                        |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix $A$ indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                                                                                              |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $A$ .                                                                                                                                                                                                                 |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .<br>When <i>beta</i> is equal to zero, then $\text{sub}(C)$ need not be set on input.                                                                                                                                                                           |
| <i>c</i>      | (local)<br>Array, size $(\text{lld}_c, \text{LOCq}(\text{jc}+n-1))$ .                                                                                                                                                                                                                                         |

This array contains the local pieces of the distributed matrix  $\text{sub}(C)$ .

*ic, jc* (global) The row and column indices in the distributed matrix  $C$  indicating the first row and the first column of the submatrix  $\text{sub}(C)$ , respectively.

*desc* (global and local) array of dimension 9. The array descriptor of the distributed matrix  $C$ .

## Output Parameters

*c* Overwritten by the updated submatrix.

## p?gemm

*Computes a scalar-matrix-matrix product and adds the result to a scalar-matrix product for distributed matrices.*

### Syntax

```
void psgemm (const char *transa , const char *transb , const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , const float *alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const float *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb , const float *beta , float *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *desc);
```

```
void pdgemm (const char *transa , const char *transb , const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , const double *alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const double *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb , const double *beta , double *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *desc);
```

```
void pcgemm (const char *transa , const char *transb , const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , const MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8 *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb , const MKL_Complex8 *beta , MKL_Complex8 *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *desc);
```

```
void pzgemm (const char *transa , const char *transb , const MKL_INT *m , const MKL_INT *n , const MKL_INT *k , const MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16 *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb , const MKL_Complex16 *beta , MKL_Complex16 *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *desc);
```

### Include Files

- mkl\_pblas.h

### Description

The `p?gemm` routines perform a matrix-matrix operation with general distributed matrices. The operation is defined as

$$\text{sub}(C) := \alpha * \text{op}(\text{sub}(A)) * \text{op}(\text{sub}(B)) + \beta * \text{sub}(C),$$

where:

$\text{op}(x)$  is one of  $\text{op}(x) = x$ , or  $\text{op}(x) = x'$ ,

*alpha* and *beta* are scalars,

$\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+k-1)$ ,  $\text{sub}(B) = B(\text{ib}:\text{ib}+k-1, \text{jb}:\text{jb}+n-1)$ , and  $\text{sub}(C) = C(\text{ic}:\text{ic}+m-1, \text{jc}:\text{jc}+n-1)$ , are distributed matrices.

### Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                            |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>transa</i> | (global) Specifies the form of $\text{op}(\text{sub}(A))$ used in the matrix multiplication:<br>if <i>transa</i> = 'N' or 'n', then $\text{op}(\text{sub}(A)) = \text{sub}(A)$ ;<br>if <i>transa</i> = 'T' or 't', then $\text{op}(\text{sub}(A)) = \text{sub}(A)'$ ;<br>if <i>transa</i> = 'C' or 'c', then $\text{op}(\text{sub}(A)) = \text{sub}(A)'$ . |
| <i>transb</i> | (global) Specifies the form of $\text{op}(\text{sub}(B))$ used in the matrix multiplication:<br>if <i>transb</i> = 'N' or 'n', then $\text{op}(\text{sub}(B)) = \text{sub}(B)$ ;<br>if <i>transb</i> = 'T' or 't', then $\text{op}(\text{sub}(B)) = \text{sub}(B)'$ ;<br>if <i>transb</i> = 'C' or 'c', then $\text{op}(\text{sub}(B)) = \text{sub}(B)'$ . |
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrices $\text{op}(\text{sub}(A))$ and $\text{sub}(C)$ , $m \geq 0$ .                                                                                                                                                                                                                            |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrices $\text{op}(\text{sub}(B))$ and $\text{sub}(C)$ , $n \geq 0$ .<br><br>The value of <i>n</i> must be at least zero.                                                                                                                                                                     |
| <i>k</i>      | (global) Specifies the number of columns of the distributed matrix $\text{op}(\text{sub}(A))$ and the number of rows of the distributed matrix $\text{op}(\text{sub}(B))$ .<br><br>The value of <i>k</i> must be greater than or equal to 0.                                                                                                               |
| <i>alpha</i>  | (global)<br><br>Specifies the scalar <i>alpha</i> .<br><br>When <i>alpha</i> is equal to zero, then the local entries of the arrays <i>a</i> and <i>b</i> corresponding to the entries of the submatrices $\text{sub}(A)$ and $\text{sub}(B)$ respectively need not be set on input.                                                                       |
| <i>a</i>      | (local)<br><br>Array, size <i>lld_a</i> by <i>kla</i> , where <i>kla</i> is $\text{LOCc}(\text{ja}+k-1)$ when <i>transa</i> = 'N' or 'n', and is $\text{LOCq}(\text{ja}+m-1)$ otherwise. Before entry this array must contain the local pieces of the distributed matrix $\text{sub}(A)$ .                                                                 |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively                                                                                                                                                                                       |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                         |
| <i>b</i>      | (local)<br><br>Array, size <i>lld_b</i> by <i>klb</i> , where <i>klb</i> is $\text{LOCc}(\text{jb}+n-1)$ when <i>transb</i> = 'N' or 'n', and is $\text{LOCq}(\text{jb}+k-1)$ otherwise. Before entry this array must contain the local pieces of the distributed matrix $\text{sub}(B)$ .                                                                 |

|               |                                                                                                                                                                      |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ib, jb</i> | (global) The row and column indices in the distributed matrix <i>B</i> indicating the first row and the first column of the submatrix $\text{sub}(B)$ , respectively |
| <i>descb</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>B</i> .                                                                   |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .<br>When <i>beta</i> is equal to zero, then $\text{sub}(C)$ need not be set on input.                                  |
| <i>c</i>      | (local)<br>Array, size $(\text{lld}_a, \text{LOCq}(jc+n-1))$ . Before entry this array must contain the local pieces of the distributed matrix $\text{sub}(C)$ .     |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix <i>C</i> indicating the first row and the first column of the submatrix $\text{sub}(C)$ , respectively |
| <i>desc</i>   | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>C</i> .                                                                   |

## Output Parameters

|          |                                                                                                                                                                |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>c</i> | Overwritten by the <i>m</i> -by- <i>n</i> distributed matrix<br>$\alpha * \text{op}(\text{sub}(A)) * \text{op}(\text{sub}(B)) + \text{beta} * \text{sub}(C)$ . |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|

## p?hemm

Performs a scalar-matrix-matrix product (one matrix operand is Hermitian) and adds the result to a scalar-matrix product.

### Syntax

```
void pchemm (const char *side , const char *uplo , const MKL_INT *m , const MKL_INT
*n , const MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8 *b , const MKL_INT *ib , const
MKL_INT *jb , const MKL_INT *descb , const MKL_Complex8 *beta , MKL_Complex8 *c , const
MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

```
void pzhemm (const char *side , const char *uplo , const MKL_INT *m , const MKL_INT
*n , const MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16 *b , const MKL_INT *ib , const
MKL_INT *jb , const MKL_INT *descb , const MKL_Complex16 *beta , MKL_Complex16 *c ,
const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

### Include Files

- mkl\_pblas.h

### Description

The p?hemm routines perform a matrix-matrix operation with distributed matrices. The operation is defined as

```
sub(C) := alpha * sub(A) * sub(B) + beta * sub(C) ,
```

or

```
sub(C) := alpha*sub(B)*sub(A) + beta*sub(C),
```

where:

*alpha* and *beta* are scalars,

*sub(A)* is a Hermitian distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+m-1)$ , if *side* = 'L', and  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ , if *side* = 'R'.

*sub(B)* and *sub(C)* are *m*-by-*n* distributed matrices.

$\text{sub}(B) = B(\text{ib}:\text{ib}+m-1, \text{jb}:\text{jb}+n-1)$ ,  $\text{sub}(C) = C(\text{ic}:\text{ic}+m-1, \text{jc}:\text{jc}+n-1)$ .

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>side</i>   | (global) Specifies whether the Hermitian distributed matrix <i>sub(A)</i> appears on the left or right in the operation:<br><br>if <i>side</i> = 'L' or 'l', then $\text{sub}(C) := \alpha * \text{sub}(A) * \text{sub}(B) + \beta * \text{sub}(C)$ ;<br><br>if <i>side</i> = 'R' or 'r', then $\text{sub}(C) := \alpha * \text{sub}(B) * \text{sub}(A) + \beta * \text{sub}(C)$ .                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>uplo</i>   | (global) Specifies whether the upper or lower triangular part of the Hermitian distributed matrix <i>sub(A)</i> is used:<br><br>if <i>uplo</i> = 'U' or 'u', then the upper triangular part is used;<br><br>if <i>uplo</i> = 'L' or 'l', then the lower triangular part is used.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>m</i>      | (global) Specifies the number of rows of the distribute submatrix <i>sub(C)</i> , $m \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <i>n</i>      | (global) Specifies the number of columns of the distribute submatrix <i>sub(C)</i> , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <i>a</i>      | (local)<br>Array, size $(\text{lld}_a, \text{LOCq}(\text{ja}+\text{na}-1))$ .<br><br>Before entry this array must contain the local pieces of the symmetric distributed matrix <i>sub(A)</i> , such that when <i>uplo</i> = 'U' or 'u', the <i>na</i> -by- <i>na</i> upper triangular part of the distributed matrix <i>sub(A)</i> must contain the upper triangular part of the Hermitian distributed matrix and the strictly lower triangular part of <i>sub(A)</i> is not referenced, and when <i>uplo</i> = 'L' or 'l', the <i>na</i> -by- <i>na</i> lower triangular part of the distributed matrix <i>sub(A)</i> must contain the lower triangular part of the Hermitian distributed matrix and the strictly upper triangular part of <i>sub(A)</i> is not referenced. |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |

|               |                                                                                                                                                                |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>b</i>      | (local)<br>Array, size $(lld\_b, LOCq(jb+n-1))$ . Before entry this array must contain the local pieces of the distributed matrix $sub(B)$ .                   |
| <i>ib, jb</i> | (global) The row and column indices in the distributed matrix <i>B</i> indicating the first row and the first column of the submatrix $sub(B)$ , respectively. |
| <i>descb</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>B</i> .                                                             |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .<br>When <i>beta</i> is set to zero, then $sub(C)$ need not be set on input.                                     |
| <i>c</i>      | (local)<br>Array, size $(lld\_c, LOCq(jc+n-1))$ . Before entry this array must contain the local pieces of the distributed matrix $sub(C)$ .                   |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix <i>C</i> indicating the first row and the first column of the submatrix $sub(C)$ , respectively  |
| <i>desc</i>   | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>C</i> .                                                             |

## Output Parameters

|          |                                                                       |
|----------|-----------------------------------------------------------------------|
| <i>c</i> | Overwritten by the <i>m</i> -by- <i>n</i> updated distributed matrix. |
|----------|-----------------------------------------------------------------------|

## p?herk

Performs a rank-*k* update of a distributed Hermitian matrix.

### Syntax

```
void pcherk (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT *k , const float *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const float *beta , MKL_Complex8 *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *desc) ;
```

```
void pzherk (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT *k , const double *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const double *beta , MKL_Complex16 *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *desc) ;
```

### Include Files

- mkl\_pblas.h

### Description

The p?herk routines perform a distributed matrix-matrix operation defined as

```
sub(C) := alpha * sub(A) * conjg(sub(A)') + beta * sub(C) ,
```

or

```
sub(C) := alpha*conjg(sub(A)')*sub(A) + beta*sub(C),
```

where:

*alpha* and *beta* are scalars,

*sub(C)* is an *n*-by-*n* Hermitian distributed matrix,  $\text{sub}(C) = C(\text{ic}:\text{ic}+n-1, \text{jc}:\text{jc}+n-1)$ .

*sub(A)* is a distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+k-1)$ , if *trans* = 'N' or 'n', and  $\text{sub}(A) = A(\text{ia}:\text{ia}+k-1, \text{ja}:\text{ja}+n-1)$  otherwise.

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                     |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | (global) Specifies whether the upper or lower triangular part of the Hermitian distributed matrix <i>sub(C)</i> is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the <i>sub(C)</i> is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the <i>sub(C)</i> is used.                            |
| <i>trans</i>  | (global) Specifies the operation:<br><br>if <i>trans</i> = 'N' or 'n', then $\text{sub}(C) := \text{alpha} * \text{sub}(A) * \text{conjg}(\text{sub}(A)') + \text{beta} * \text{sub}(C)$ ;<br><br>if <i>trans</i> = 'C' or 'c', then $\text{sub}(C) := \text{alpha} * \text{conjg}(\text{sub}(A)') * \text{sub}(A) + \text{beta} * \text{sub}(C)$ . |
| <i>n</i>      | (global) Specifies the order of the distributed matrix <i>sub(C)</i> , $n \geq 0$ .                                                                                                                                                                                                                                                                 |
| <i>k</i>      | (global) On entry with <i>trans</i> = 'N' or 'n', <i>k</i> specifies the number of columns of the distributed matrix <i>sub(A)</i> , and on entry with <i>trans</i> = 'T' or 't' or 'C' or 'c', <i>k</i> specifies the number of rows of the distributed matrix <i>sub(A)</i> , $k \geq 0$ .                                                        |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                     |
| <i>a</i>      | (local)<br>Array, size ( <i>lld_a</i> , <i>kla</i> ), where <i>kla</i> is $\text{LOCq}(\text{ja}+k-1)$ when <i>trans</i> = 'N' or 'n', and is $\text{LOCq}(\text{ja}+n-1)$ otherwise. Before entry with <i>trans</i> = 'N' or 'n', this array contains the local pieces of the distributed matrix <i>sub(A)</i> .                                   |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively.                                                                                                                                                                                 |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                  |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .                                                                                                                                                                                                                                                                                                      |
| <i>c</i>      | (local)<br>Array, size ( <i>lld_c</i> , $\text{LOCq}(\text{jc}+n-1)$ ).                                                                                                                                                                                                                                                                             |

Before entry with `uplo = 'U' or 'u'`, this array contains  $n$ -by- $n$  upper triangular part of the symmetric distributed matrix `sub(C)` and its strictly lower triangular part is not referenced.

Before entry with `uplo = 'L' or 'l'`, this array contains  $n$ -by- $n$  lower triangular part of the symmetric distributed matrix `sub(C)` and its strictly upper triangular part is not referenced.

`ic, jc` (global) The row and column indices in the distributed matrix `C` indicating the first row and the first column of the submatrix `sub(C)`, respectively.

`desc` (global and local) array of dimension 9. The array descriptor of the distributed matrix `C`.

## Output Parameters

`c` With `uplo = 'U' or 'u'`, the upper triangular part of `sub(C)` is overwritten by the upper triangular part of the updated distributed matrix.

With `uplo = 'L' or 'l'`, the lower triangular part of `sub(C)` is overwritten by the upper triangular part of the updated distributed matrix.

## p?her2k

Performs a rank-2k update of a Hermitian distributed matrix.

### Syntax

```
void pcher2k (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT *k , const MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8 *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb , const float *beta , MKL_Complex8 *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *desc) ;
```

```
void pzher2k (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT *k , const MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16 *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb , const double *beta , MKL_Complex16 *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *desc) ;
```

### Include Files

- `mkl_pblas.h`

### Description

The `p?her2k` routines perform a distributed matrix-matrix operation defined as

$$\text{sub}(C) := \alpha * \text{sub}(A) * \text{conjg}(\text{sub}(B)') + \text{conjg}(\alpha) * \text{sub}(B) * \text{conjg}(\text{sub}(A)') + \beta * \text{sub}(C),$$

or

$$\text{sub}(C) := \alpha * \text{conjg}(\text{sub}(A)') * \text{sub}(A) + \text{conjg}(\alpha) * \text{conjg}(\text{sub}(B)') * \text{sub}(B) + \beta * \text{sub}(C),$$

where:

*alpha* and *beta* are scalars,

`sub(C)` is an  $n$ -by- $n$  Hermitian distributed matrix, `sub(C) = C(ic:ic+n-1, jc:jc+n-1)`.

$\text{sub}(A)$  is a distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+k-1)$ , if  $\text{trans} = 'N'$  or  $'n'$ , and  $\text{sub}(A) = A(\text{ia}:\text{ia}+k-1, \text{ja}:\text{ja}+n-1)$  otherwise.

$\text{sub}(B)$  is a distributed matrix,  $\text{sub}(B) = B(\text{ib}:\text{ib}+n-1, \text{jb}:\text{jb}+k-1)$ , if  $\text{trans} = 'N'$  or  $'n'$ , and  $\text{sub}(B) = B(\text{ib}:\text{ib}+k-1, \text{jb}:\text{jb}+n-1)$  otherwise.

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | (global) Specifies whether the upper or lower triangular part of the Hermitian distributed matrix $\text{sub}(C)$ is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the $\text{sub}(C)$ is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the $\text{sub}(C)$ is used.                                                                                                                                          |
| <i>trans</i>  | (global) Specifies the operation:<br><br>if <i>trans</i> = 'N' or 'n', then $\text{sub}(C) := \alpha * \text{sub}(A) * \text{conjg}(\text{sub}(B)') + \text{conjg}(\alpha) * \text{sub}(B) * \text{conjg}(\text{sub}(A)') + \beta * \text{sub}(C)$ ;<br><br>if <i>trans</i> = 'C' or 'c', then $\text{sub}(C) := \alpha * \text{conjg}(\text{sub}(A)') * \text{sub}(A) + \text{conjg}(\alpha) * \text{conjg}(\text{sub}(B)') * \text{sub}(B) + \beta * \text{sub}(C)$ . |
| <i>n</i>      | (global) Specifies the order of the distributed matrix $\text{sub}(C)$ , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>k</i>      | (global) On entry with <i>trans</i> = 'N' or 'n', <i>k</i> specifies the number of columns of the distributed matrices $\text{sub}(A)$ and $\text{sub}(B)$ , and on entry with <i>trans</i> = 'C' or 'c', <i>k</i> specifies the number of rows of the distributed matrices $\text{sub}(A)$ and $\text{sub}(B)$ , $k \geq 0$ .                                                                                                                                          |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                         |
| <i>a</i>      | (local)<br>Array, size $(\text{lld}_a, \text{kla})$ , where <i>kla</i> is $\text{LOCq}(\text{ja}+k-1)$ when <i>trans</i> = 'N' or 'n', and is $\text{LOCq}(\text{ja}+n-1)$ otherwise. Before entry with <i>trans</i> = 'N' or 'n', this array contains the local pieces of the distributed matrix $\text{sub}(A)$ .                                                                                                                                                     |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                                                                                                                                                                                                                                                   |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                      |
| <i>b</i>      | (local)<br>Array, size $(\text{lld}_b, \text{k1b})$ , where <i>k1b</i> is $\text{LOCq}(\text{jb}+k-1)$ when <i>trans</i> = 'N' or 'n', and is $\text{LOCq}(\text{jb}+n-1)$ otherwise. Before entry with <i>trans</i> = 'N' or 'n', this array contains the local pieces of the distributed matrix $\text{sub}(B)$ .                                                                                                                                                     |
| <i>ib, jb</i> | (global) The row and column indices in the distributed matrix <i>B</i> indicating the first row and the first column of the submatrix $\text{sub}(B)$ , respectively.                                                                                                                                                                                                                                                                                                   |
| <i>descb</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>B</i> .                                                                                                                                                                                                                                                                                                                                                                      |
| <i>beta</i>   | (global)                                                                                                                                                                                                                                                                                                                                                                                                                                                                |

Specifies the scalar *beta*.

*c*

(local)

Array, size (*lld\_c*, *LOCq(jc+n-1)*).

Before entry with *uplo* = 'U' or 'u', this array contains *n*-by-*n* upper triangular part of the symmetric distributed matrix *sub(C)* and its strictly lower triangular part is not referenced.

Before entry with *uplo* = 'L' or 'l', this array contains *n*-by-*n* lower triangular part of the symmetric distributed matrix *sub(C)* and its strictly upper triangular part is not referenced.

*ic, jc*

(global) The row and column indices in the distributed matrix *C* indicating the first row and the first column of the submatrix *sub(C)*, respectively.

*descc*

(global and local) array of dimension 9. The array descriptor of the distributed matrix *C*.

## Output Parameters

*c*

With *uplo* = 'U' or 'u', the upper triangular part of *sub(C)* is overwritten by the upper triangular part of the updated distributed matrix.

With *uplo* = 'L' or 'l', the lower triangular part of *sub(C)* is overwritten by the upper triangular part of the updated distributed matrix.

## p?symm

*Performs a scalar-matrix-matrix product (one matrix operand is symmetric) and adds the result to a scalar-matrix product for distribute matrices.*

### Syntax

```
void pssymm (const char *side , const char *uplo , const MKL_INT *m , const MKL_INT
*n , const float *alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const float *b , const MKL_INT *ib , const MKL_INT *jb , const
MKL_INT *descb , const float *beta , float *c , const MKL_INT *ic , const MKL_INT
*jc , const MKL_INT *descc);
```

```
void pdsymm (const char *side , const char *uplo , const MKL_INT *m , const MKL_INT
*n , const double *alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const double *b , const MKL_INT *ib , const MKL_INT *jb , const
MKL_INT *descb , const double *beta , double *c , const MKL_INT *ic , const MKL_INT
*jc , const MKL_INT *descc);
```

```
void pcsymm (const char *side , const char *uplo , const MKL_INT *m , const MKL_INT
*n , const MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8 *b , const MKL_INT *ib , const
MKL_INT *jb , const MKL_INT *descb , const MKL_Complex8 *beta , MKL_Complex8 *c , const
MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

```
void pzsymm (const char *side , const char *uplo , const MKL_INT *m , const MKL_INT
*n , const MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16 *b , const MKL_INT *ib , const
MKL_INT *jb , const MKL_INT *descb , const MKL_Complex16 *beta , MKL_Complex16 *c ,
const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

## Include Files

- mkl\_pblas.h

## Description

The `p?symm` routines perform a matrix-matrix operation with distributed matrices. The operation is defined as

```
sub(C) := alpha*sub(A)*sub(B) + beta*sub(C) ,
```

or

```
sub(C) := alpha*sub(B)*sub(A) + beta*sub(C) ,
```

where:

*alpha* and *beta* are scalars,

*sub(A)* is a symmetric distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+m-1)$ , if *side* = 'L', and  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ , if *side* = 'R'.

*sub(B)* and *sub(C)* are *m*-by-*n* distributed matrices.

$\text{sub}(B) = B(\text{ib}:\text{ib}+m-1, \text{jb}:\text{jb}+n-1)$ ,  $\text{sub}(C) = C(\text{ic}:\text{ic}+m-1, \text{jc}:\text{jc}+n-1)$ .

## Input Parameters

|              |                                                                                                                                                                                                                                                                                                                                                                                                    |
|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>side</i>  | (global) Specifies whether the symmetric distributed matrix <i>sub(A)</i> appears on the left or right in the operation:<br>if <i>side</i> = 'L' or 'l', then $\text{sub}(C) := \text{alpha} * \text{sub}(A) * \text{sub}(B) + \text{beta} * \text{sub}(C)$ ;<br>if <i>side</i> = 'R' or 'r', then $\text{sub}(C) := \text{alpha} * \text{sub}(B) * \text{sub}(A) + \text{beta} * \text{sub}(C)$ . |
| <i>uplo</i>  | (global) Specifies whether the upper or lower triangular part of the symmetric distributed matrix <i>sub(A)</i> is used:<br>if <i>uplo</i> = 'U' or 'u', then the upper triangular part is used;<br>if <i>uplo</i> = 'L' or 'l', then the lower triangular part is used.                                                                                                                           |
| <i>m</i>     | (global) Specifies the number of rows of the distribute submatrix <i>sub(C)</i> , $m \geq 0$ .                                                                                                                                                                                                                                                                                                     |
| <i>n</i>     | (global) Specifies the number of columns of the distribute submatrix <i>sub(C)</i> , $m \geq 0$ .                                                                                                                                                                                                                                                                                                  |
| <i>alpha</i> | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                    |
| <i>a</i>     | (local)<br>Array, size $(\text{lld}_a, \text{LOCq}(\text{ja}+\text{na}-1))$ .                                                                                                                                                                                                                                                                                                                      |

Before entry this array must contain the local pieces of the symmetric distributed matrix  $\text{sub}(A)$ , such that when  $\text{uplo} = 'U'$  or  $'u'$ , the  $na$ -by- $na$  upper triangular part of the distributed matrix  $\text{sub}(A)$  must contain the upper triangular part of the symmetric distributed matrix and the strictly lower triangular part of  $\text{sub}(A)$  is not referenced, and when  $\text{uplo} = 'L'$  or  $'l'$ , the  $na$ -by- $na$  lower triangular part of the distributed matrix  $\text{sub}(A)$  must contain the lower triangular part of the symmetric distributed matrix and the strictly upper triangular part of  $\text{sub}(A)$  is not referenced.

|               |                                                                                                                                                                  |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix $A$ indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $A$ .                                                                    |
| <i>b</i>      | (local)<br>Array, size $(\text{lld}_b, \text{LOCq}(jb+n-1))$ . Before entry this array must contain the local pieces of the distributed matrix $\text{sub}(B)$ . |
| <i>ib, jb</i> | (global) The row and column indices in the distributed matrix $B$ indicating the first row and the first column of the submatrix $\text{sub}(B)$ , respectively. |
| <i>descb</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $B$ .                                                                    |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .<br>When <i>beta</i> is set to zero, then $\text{sub}(C)$ need not be set on input.                                |
| <i>c</i>      | (local)<br>Array, size $(\text{lld}_c, \text{LOCq}(jc+n-1))$ . Before entry this array must contain the local pieces of the distributed matrix $\text{sub}(C)$ . |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix $C$ indicating the first row and the first column of the submatrix $\text{sub}(C)$ , respectively. |
| <i>descc</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix $C$ .                                                                    |

## Output Parameters

|          |                                                 |
|----------|-------------------------------------------------|
| <i>c</i> | Overwritten by the $m$ -by- $n$ updated matrix. |
|----------|-------------------------------------------------|

## p?syrk

Performs a rank- $k$  update of a symmetric distributed matrix.

### Syntax

```
void pssyrk (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT
*k , const float *alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const float *beta , float *c , const MKL_INT *ic , const MKL_INT
*jc , const MKL_INT *descc);
```

```

void pdsyrk (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT
*k , const double *alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const double *beta , double *c , const MKL_INT *ic , const
MKL_INT *jc , const MKL_INT *descc);

void pcsyrk (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT
*k , const MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8 *beta , MKL_Complex8 *c , const
MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);

void pzsyk (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT
*k , const MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16 *beta , MKL_Complex16 *c ,
const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);

```

## Include Files

- mkl\_pblas.h

## Description

The p?syrk routines perform a distributed matrix-matrix operation defined as

$$\text{sub}(C) := \alpha * \text{sub}(A) * \text{sub}(A)' + \beta * \text{sub}(C),$$

or

$$\text{sub}(C) := \alpha * \text{sub}(A)' * \text{sub}(A) + \beta * \text{sub}(C),$$

where:

*alpha* and *beta* are scalars,

*sub(C)* is an *n*-by-*n* symmetric distributed matrix,  $\text{sub}(C) = C(ic:ic+n-1, jc:jc+n-1)$ .

*sub(A)* is a distributed matrix,  $\text{sub}(A) = A(ia:ia+n-1, ja:ja+k-1)$ , if *trans* = 'N' or 'n', and  $\text{sub}(A) = A(ia:ia+k-1, ja:ja+n-1)$  otherwise.

## Input Parameters

|              |                                                                                                                                                                                                                                                                                                                          |
|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>  | (global) Specifies whether the upper or lower triangular part of the symmetric distributed matrix <i>sub(C)</i> is used:<br><br>If <i>uplo</i> = 'U' or 'u', then the upper triangular part of the <i>sub(C)</i> is used.<br><br>If <i>uplo</i> = 'L' or 'l', then the low triangular part of the <i>sub(C)</i> is used. |
| <i>trans</i> | (global) Specifies the operation:<br><br>if <i>trans</i> = 'N' or 'n', then $\text{sub}(C) := \alpha * \text{sub}(A) * \text{sub}(A)' + \beta * \text{sub}(C)$ ;<br><br>if <i>trans</i> = 'T' or 't', then $\text{sub}(C) := \alpha * \text{sub}(A)' * \text{sub}(A) + \beta * \text{sub}(C)$ .                          |
| <i>n</i>     | (global) Specifies the order of the distributed matrix <i>sub(C)</i> , $n \geq 0$ .                                                                                                                                                                                                                                      |
| <i>k</i>     | (global) On entry with <i>trans</i> = 'N' or 'n', <i>k</i> specifies the number of columns of the distributed matrix <i>sub(A)</i> , and on entry with <i>trans</i> = 'T' or 't', <i>k</i> specifies the number of rows of the distributed matrix <i>sub(A)</i> , $k \geq 0$ .                                           |

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <i>a</i>      | (local)<br>Array, size $(l1d\_a, kla)$ , where <i>kla</i> is $LOCq(ja+k-1)$ when <i>trans</i> = 'N' or 'n', and is $LOCq(ja+n-1)$ otherwise. Before entry with <i>trans</i> = 'N' or 'n', this array contains the local pieces of the distributed matrix $sub(A)$ .                                                                                                                                                                                                                             |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $sub(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                  |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                                              |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <i>c</i>      | (local)<br>Array, size $(l1d\_c, LOCq(jc+n-1))$ .<br>Before entry with <i>uplo</i> = 'U' or 'u', this array contains <i>n</i> -by- <i>n</i> upper triangular part of the symmetric distributed matrix $sub(C)$ and its strictly lower triangular part is not referenced.<br>Before entry with <i>uplo</i> = 'L' or 'l', this array contains <i>n</i> -by- <i>n</i> lower triangular part of the symmetric distributed matrix $sub(C)$ and its strictly upper triangular part is not referenced. |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix <i>C</i> indicating the first row and the first column of the submatrix $sub(C)$ , respectively.                                                                                                                                                                                                                                                                                                                                  |
| <i>descc</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>C</i> .                                                                                                                                                                                                                                                                                                                                                                                              |

## Output Parameters

|          |                                                                                                                                                                                                                                                                                                            |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>c</i> | With <i>uplo</i> = 'U' or 'u', the upper triangular part of $sub(C)$ is overwritten by the upper triangular part of the updated distributed matrix.<br>With <i>uplo</i> = 'L' or 'l', the lower triangular part of $sub(C)$ is overwritten by the upper triangular part of the updated distributed matrix. |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## p?syr2k

Performs a rank-2k update of a symmetric distributed matrix.

---

### Syntax

```
void pssyr2k (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT *k , const float *alpha , const float *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const float *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb , const float *beta , float *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

```

void pdsyr2k (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT
*k , const double *alpha , const double *a , const MKL_INT *ia , const MKL_INT *ja ,
const MKL_INT *desca , const double *b , const MKL_INT *ib , const MKL_INT *jb , const
MKL_INT *descb , const double *beta , double *c , const MKL_INT *ic , const MKL_INT
*jc , const MKL_INT *descc);

void pcsyr2k (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT
*k , const MKL_Complex8 *alpha , const MKL_Complex8 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex8 *b , const MKL_INT *ib , const
MKL_INT *jb , const MKL_INT *descb , const MKL_Complex8 *beta , MKL_Complex8 *c , const
MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);

void pzsyr2k (const char *uplo , const char *trans , const MKL_INT *n , const MKL_INT
*k , const MKL_Complex16 *alpha , const MKL_Complex16 *a , const MKL_INT *ia , const
MKL_INT *ja , const MKL_INT *desca , const MKL_Complex16 *b , const MKL_INT *ib , const
MKL_INT *jb , const MKL_INT *descb , const MKL_Complex16 *beta , MKL_Complex16 *c ,
const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);

```

## Include Files

- mkl\_pblas.h

## Description

The `p?syr2k` routines perform a distributed matrix-matrix operation defined as

$$\text{sub}(C) := \alpha \text{sub}(A) * \text{sub}(B) + \alpha \text{sub}(B) * \text{sub}(A) + \beta \text{sub}(C),$$

or

$$\text{sub}(C) := \alpha \text{sub}(A) ' * \text{sub}(B) + \alpha \text{sub}(B) ' * \text{sub}(A) + \beta \text{sub}(C),$$

where:

*alpha* and *beta* are scalars,

*sub(C)* is an *n*-by-*n* symmetric distributed matrix,  $\text{sub}(C) = C(ic:ic+n-1, jc:jc+n-1)$ .

*sub(A)* is a distributed matrix,  $\text{sub}(A) = A(ia:ia+n-1, ja:ja+k-1)$ , if *trans* = 'N' or 'n', and  $\text{sub}(A) = A(ia:ia+k-1, ja:ja+n-1)$  otherwise.

*sub(B)* is a distributed matrix,  $\text{sub}(B) = B(ib:ib+n-1, jb:jb+k-1)$ , if *trans* = 'N' or 'n', and  $\text{sub}(B) = B(ib:ib+k-1, jb:jb+n-1)$  otherwise.

## Input Parameters

*uplo* (global) Specifies whether the upper or lower triangular part of the symmetric distributed matrix *sub(C)* is used:

If *uplo* = 'U' or 'u', then the upper triangular part of the *sub(C)* is used.

If *uplo* = 'L' or 'l', then the low triangular part of the *sub(C)* is used.

*trans* (global) Specifies the operation:

if *trans* = 'N' or 'n', then  $\text{sub}(C) := \alpha \text{sub}(A) * \text{sub}(B) + \alpha \text{sub}(B) * \text{sub}(A) + \beta \text{sub}(C);$

if *trans* = 'T' or 't', then  $\text{sub}(C) := \alpha \text{sub}(B) ' * \text{sub}(A) + \alpha \text{sub}(A) ' * \text{sub}(B) + \beta \text{sub}(C).$

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|---------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>      | (global) Specifies the order of the distributed matrix $\text{sub}(C)$ , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                            |
| <i>k</i>      | (global) On entry with $\text{trans} = 'N'$ or $'n'$ , $k$ specifies the number of columns of the distributed matrices $\text{sub}(A)$ and $\text{sub}(B)$ , and on entry with $\text{trans} = 'T'$ or $'t'$ , $k$ specifies the number of rows of the distributed matrices $\text{sub}(A)$ and $\text{sub}(B)$ , $k \geq 0$ .                                                                                                                                                                                   |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <i>a</i>      | (local)<br>Array, size $(\text{lld}_a, \text{kla})$ , where $\text{kla}$ is $\text{LOCq}(ja+k-1)$ when $\text{trans} = 'N'$ or $'n'$ , and is $\text{LOCq}(ja+n-1)$ otherwise. Before entry with $\text{trans} = 'N'$ or $'n'$ , this array contains the local pieces of the distributed matrix $\text{sub}(A)$ .                                                                                                                                                                                                |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively.                                                                                                                                                                                                                                                                                                                                            |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                                                                                                                                                                                                                                                                                                                                                               |
| <i>b</i>      | (local)<br>Array, size $(\text{lld}_b, \text{k1b})$ , where $\text{k1b}$ is $\text{LOCq}(jb+k-1)$ when $\text{trans} = 'N'$ or $'n'$ , and is $\text{LOCq}(jb+n-1)$ otherwise. Before entry with $\text{trans} = 'N'$ or $'n'$ , this array contains the local pieces of the distributed matrix $\text{sub}(B)$ .                                                                                                                                                                                                |
| <i>ib, jb</i> | (global) The row and column indices in the distributed matrix <i>B</i> indicating the first row and the first column of the submatrix $\text{sub}(B)$ , respectively.                                                                                                                                                                                                                                                                                                                                            |
| <i>descb</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>B</i> .                                                                                                                                                                                                                                                                                                                                                                                                               |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>c</i>      | (local)<br>Array, size $(\text{lld}_c, \text{LOCq}(jc+n-1))$ .<br>Before entry with $\text{uplo} = 'U'$ or $'u'$ , this array contains $n$ -by- $n$ upper triangular part of the symmetric distributed matrix $\text{sub}(C)$ and its strictly lower triangular part is not referenced.<br>Before entry with $\text{uplo} = 'L'$ or $'l'$ , this array contains $n$ -by- $n$ lower triangular part of the symmetric distributed matrix $\text{sub}(C)$ and its strictly upper triangular part is not referenced. |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix <i>C</i> indicating the first row and the first column of the submatrix $\text{sub}(C)$ , respectively.                                                                                                                                                                                                                                                                                                                                            |
| <i>desc</i>   | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>C</i> .                                                                                                                                                                                                                                                                                                                                                                                                               |

## Output Parameters

*c* With *uplo* = 'U' or 'u', the upper triangular part of *sub(C)* is overwritten by the upper triangular part of the updated distributed matrix.  
 With *uplo* = 'L' or 'l', the lower triangular part of *sub(C)* is overwritten by the upper triangular part of the updated distributed matrix.

## p?tran

*Transposes a real distributed matrix.*

### Syntax

```
void pstran (const MKL_INT *m , const MKL_INT *n , const float *alpha , const float
*a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const float
*beta , float *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

```
void pdtran (const MKL_INT *m , const MKL_INT *n , const double *alpha , const double
*a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const double
*beta , double *c , const MKL_INT *ic , const MKL_INT *jc , const MKL_INT *descc);
```

### Include Files

- mkl\_pblas.h

### Description

The *p?tran* routines transpose a real distributed matrix. The operation is defined as

$$\text{sub}(C) := \text{beta} * \text{sub}(C) + \text{alpha} * \text{sub}(A)'$$

where:

*alpha* and *beta* are scalars,

*sub(C)* is an *m*-by-*n* distributed matrix,  $\text{sub}(C) = C(\text{ic}:\text{ic}+m-1, \text{jc}:\text{jc}+n-1)$ .

*sub(A)* is a distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+m-1)$ .

### Input Parameters

|               |                                                                                                                                                                     |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrix <i>sub(C)</i> , $m \geq 0$ .                                                                        |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix <i>sub(C)</i> , $n \geq 0$ .                                                                     |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                     |
| <i>a</i>      | (local)<br>Array, size $(\text{lld}_a, \text{LOCq}(\text{ja}+m-1))$ . This array contains the local pieces of the distributed matrix <i>sub(A)</i> .                |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively. |

|               |                                                                                                                                                                     |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                  |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .<br>When <i>beta</i> is equal to zero, then <i>sub(C)</i> need not be set on input.                                   |
| <i>c</i>      | (local)<br>Array, size ( <i>lld_c</i> , <i>LOCq(jc+n-1)</i> ).<br>This array contains the local pieces of the distributed matrix <i>sub(C)</i> .                    |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix <i>C</i> indicating the first row and the first column of the submatrix <i>sub(C)</i> , respectively. |
| <i>descc</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>C</i> .                                                                  |

## Output Parameters

|          |                                       |
|----------|---------------------------------------|
| <i>c</i> | Overwritten by the updated submatrix. |
|----------|---------------------------------------|

## p?tranu

*Transposes a distributed complex matrix.*

### Syntax

```
void pctranu (const MKL_INT *m , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const
MKL_Complex8 *beta , MKL_Complex8 *c , const MKL_INT *ic , const MKL_INT *jc , const
MKL_INT *descc);
```

```
void pztranu (const MKL_INT *m , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const
MKL_Complex16 *beta , MKL_Complex16 *c , const MKL_INT *ic , const MKL_INT *jc , const
MKL_INT *descc);
```

### Include Files

- `mk1_pblas.h`

### Description

The `p?tranu` routines transpose a complex distributed matrix. The operation is defined as

$$\text{sub}(C) := \text{beta} * \text{sub}(C) + \text{alpha} * \text{sub}(A)'$$

where:

*alpha* and *beta* are scalars,

*sub(C)* is an *m*-by-*n* distributed matrix,  $\text{sub}(C) = C(\text{ic}:\text{ic}+m-1, \text{jc}:\text{jc}+n-1)$ .

*sub(A)* is a distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+m-1)$ .

## Input Parameters

|               |                                                                                                                                                                       |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrix $\text{sub}(C)$ , $m \geq 0$ .                                                                        |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix $\text{sub}(C)$ , $n \geq 0$ .                                                                     |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                       |
| <i>a</i>      | (local)<br>Array, size $(l1d\_a, \text{LOCq}(ja+m-1))$ . This array contains the local pieces of the distributed matrix $\text{sub}(A)$ .                             |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix $\text{sub}(A)$ , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                    |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .<br>When <i>beta</i> is equal to zero, then $\text{sub}(C)$ need not be set on input.                                   |
| <i>c</i>      | (local)<br>Array, size $(l1d\_c, \text{LOCq}(jc+n-1))$ .<br>This array contains the local pieces of the distributed matrix $\text{sub}(C)$ .                          |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix <i>C</i> indicating the first row and the first column of the submatrix $\text{sub}(C)$ , respectively. |
| <i>descc</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>C</i> .                                                                    |

## Output Parameters

|          |                                       |
|----------|---------------------------------------|
| <i>c</i> | Overwritten by the updated submatrix. |
|----------|---------------------------------------|

## p?tranc

Transposes a complex distributed matrix, conjugated.

### Syntax

```
void pctranc (const MKL_INT *m , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const
MKL_Complex8 *beta , MKL_Complex8 *c , const MKL_INT *ic , const MKL_INT *jc , const
MKL_INT *descc);
```

```
void pztranc (const MKL_INT *m , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , const
MKL_Complex16 *beta , MKL_Complex16 *c , const MKL_INT *ic , const MKL_INT *jc , const
MKL_INT *descc);
```

## Include Files

- `mkl_pblas.h`

## Description

The `p?tranc` routines transpose a complex distributed matrix. The operation is defined as

```
sub(C) := beta*sub(C) + alpha*conjg(sub(A)'),
```

where:

*alpha* and *beta* are scalars,

*sub(C)* is an *m*-by-*n* distributed matrix,  $sub(C) = C(ic:ic+m-1, jc:jc+n-1)$ .

*sub(A)* is a distributed matrix,  $sub(A) = A(ia:ia+n-1, ja:ja+m-1)$ .

## Input Parameters

|               |                                                                                                                                                                     |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrix <i>sub(C)</i> , $m \geq 0$ .                                                                        |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix <i>sub(C)</i> , $n \geq 0$ .                                                                     |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .                                                                                                                     |
| <i>a</i>      | (local)<br>Array, size $(lld\_a, LOCq(ja+m-1))$ . This array contains the local pieces of the distributed matrix <i>sub(A)</i> .                                    |
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively. |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                  |
| <i>beta</i>   | (global)<br>Specifies the scalar <i>beta</i> .<br>When <i>beta</i> is equal to zero, then <i>sub(C)</i> need not be set on input.                                   |
| <i>c</i>      | (local)<br>Array, size $(lld\_c, LOCq(jc+n-1))$ .<br>This array contains the local pieces of the distributed matrix <i>sub(C)</i> .                                 |
| <i>ic, jc</i> | (global) The row and column indices in the distributed matrix <i>C</i> indicating the first row and the first column of the submatrix <i>sub(C)</i> , respectively. |
| <i>desc</i>   | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>C</i> .                                                                  |

## Output Parameters

|          |                                       |
|----------|---------------------------------------|
| <i>c</i> | Overwritten by the updated submatrix. |
|----------|---------------------------------------|

## p?trmm

Computes a scalar-matrix-matrix product (one matrix operand is triangular) for distributed matrices.

### Syntax

```
void pstrmm (const char *side , const char *uplo , const char *transa , const char
*diag , const MKL_INT *m , const MKL_INT *n , const float *alpha , const float *a ,
const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , float *b , const MKL_INT
*ib , const MKL_INT *jb , const MKL_INT *descb);
```

```
void pdtrmm (const char *side , const char *uplo , const char *transa , const char
*diag , const MKL_INT *m , const MKL_INT *n , const double *alpha , const double *a ,
const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , double *b , const
MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb);
```

```
void pctrmm (const char *side , const char *uplo , const char *transa , const char
*diag , const MKL_INT *m , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca ,
MKL_Complex8 *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb);
```

```
void pztrmm (const char *side , const char *uplo , const char *transa , const char
*diag , const MKL_INT *m , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca ,
MKL_Complex16 *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb);
```

### Include Files

- mkl\_pblas.h

### Description

The p?trmm routines perform a matrix-matrix operation using triangular matrices. The operation is defined as

$$\text{sub}(B) := \alpha * \text{op}(\text{sub}(A)) * \text{sub}(B)$$

or

$$\text{sub}(B) := \alpha * \text{sub}(B) * \text{op}(\text{sub}(A))$$

where:

$\alpha$  is a scalar,

$\text{sub}(B)$  is an  $m$ -by- $n$  distributed matrix,  $\text{sub}(B) = B(ib:ib+m-1, jb:jb+n-1)$ .

$A$  is a unit, or non-unit, upper or lower triangular distributed matrix,  $\text{sub}(A) = A(ia:ia+m-1, ja:ja+m-1)$ , if  $side = 'L'$  or  $'l'$ , and  $\text{sub}(A) = A(ia:ia+n-1, ja:ja+n-1)$ , if  $side = 'R'$  or  $'r'$ .

$\text{op}(\text{sub}(A))$  is one of  $\text{op}(\text{sub}(A)) = \text{sub}(A)$ , or  $\text{op}(\text{sub}(A)) = \text{sub}(A)'$ , or  $\text{op}(\text{sub}(A)) = \text{conjg}(\text{sub}(A)')$ .

### Input Parameters

*side* (global) Specifies whether  $\text{op}(\text{sub}(A))$  appears on the left or right of  $\text{sub}(B)$  in the operation:  
 if  $side = 'L'$  or  $'l'$ , then  $\text{sub}(B) := \alpha * \text{op}(\text{sub}(A)) * \text{sub}(B)$ ;  
 if  $side = 'R'$  or  $'r'$ , then  $\text{sub}(B) := \alpha * \text{sub}(B) * \text{op}(\text{sub}(A))$ .

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>uplo</i>   | <p>(global) Specifies whether the distributed matrix <math>\text{sub}(A)</math> is upper or lower triangular:</p> <p>if <i>uplo</i> = 'U' or 'u', then the matrix is upper triangular;</p> <p>if <i>uplo</i> = 'L' or 'l', then the matrix is low triangular.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>transa</i> | <p>(global) Specifies the form of <math>\text{op}(\text{sub}(A))</math> used in the matrix multiplication:</p> <p>if <i>transa</i> = 'N' or 'n', then <math>\text{op}(\text{sub}(A)) = \text{sub}(A)</math>;</p> <p>if <i>transa</i> = 'T' or 't', then <math>\text{op}(\text{sub}(A)) = \text{sub}(A)'</math>;</p> <p>if <i>transa</i> = 'C' or 'c', then <math>\text{op}(\text{sub}(A)) = \text{conjg}(\text{sub}(A)')</math>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <i>diag</i>   | <p>(global) Specifies whether the matrix <math>\text{sub}(A)</math> is unit triangular:</p> <p>if <i>diag</i> = 'U' or 'u' then the matrix is unit triangular;</p> <p>if <i>diag</i> = 'N' or 'n', then the matrix is not unit triangular.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <i>m</i>      | <p>(global) Specifies the number of rows of the distributed matrix <math>\text{sub}(B)</math>, <math>m \geq 0</math>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| <i>n</i>      | <p>(global) Specifies the number of columns of the distributed matrix <math>\text{sub}(B)</math>, <math>n \geq 0</math>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <i>alpha</i>  | <p>(global)</p> <p>Specifies the scalar <i>alpha</i>.</p> <p>When <i>alpha</i> is zero, then the array <i>b</i> need not be set before entry.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>a</i>      | <p>(local)</p> <p>Array, size <i>lld_a</i> by <i>ka</i>, where <i>ka</i> is at least <math>\text{LOCq}(1, ja+m-1)</math> when <i>side</i> = 'L' or 'l' and is at least <math>\text{LOCq}(1, ja+n-1)</math> when <i>side</i> = 'R' or 'r'.</p> <p>Before entry with <i>uplo</i> = 'U' or 'u', this array contains the local entries corresponding to the entries of the upper triangular distributed matrix <math>\text{sub}(A)</math>, and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix <math>\text{sub}(A)</math> is not referenced.</p> <p>Before entry with <i>uplo</i> = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix <math>\text{sub}(A)</math>, and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix <math>\text{sub}(A)</math> is not referenced .</p> <p>When <i>diag</i> = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix <math>\text{sub}(A)</math> are not referenced either, but are assumed to be unity.</p> |
| <i>ia, ja</i> | <p>(global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <math>\text{sub}(A)</math>, respectively.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>desca</i>  | <p>(global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <i>b</i>      | <p>(local)</p> <p>Array, size (<i>lld_b</i>, <math>\text{LOCq}(1, jb+n-1)</math>).</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

Before entry, this array contains the local pieces of the distributed matrix  $\text{sub}(B)$ .

*ib, jb* (global) The row and column indices in the distributed matrix  $B$  indicating the first row and the first column of the submatrix  $\text{sub}(B)$ , respectively.

*descb* (global and local) array of dimension 9. The array descriptor of the distributed matrix  $B$ .

## Output Parameters

*b* Overwritten by the transformed distributed matrix.

## p?trsm

*Solves a distributed matrix equation (one matrix operand is triangular).*

### Syntax

```
void pstrsm (const char *side , const char *uplo , const char *transa , const char
*diag , const MKL_INT *m , const MKL_INT *n , const float *alpha , const float *a ,
const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , float *b , const MKL_INT
*ib , const MKL_INT *jb , const MKL_INT *descb);
```

```
void pdtrsm (const char *side , const char *uplo , const char *transa , const char
*diag , const MKL_INT *m , const MKL_INT *n , const double *alpha , const double *a ,
const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca , double *b , const
MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb);
```

```
void pctrsm (const char *side , const char *uplo , const char *transa , const char
*diag , const MKL_INT *m , const MKL_INT *n , const MKL_Complex8 *alpha , const
MKL_Complex8 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca ,
MKL_Complex8 *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb);
```

```
void pztrsm (const char *side , const char *uplo , const char *transa , const char
*diag , const MKL_INT *m , const MKL_INT *n , const MKL_Complex16 *alpha , const
MKL_Complex16 *a , const MKL_INT *ia , const MKL_INT *ja , const MKL_INT *desca ,
MKL_Complex16 *b , const MKL_INT *ib , const MKL_INT *jb , const MKL_INT *descb);
```

### Include Files

- mkl\_pblas.h

### Description

The `p?trsm` routines solve one of the following distributed matrix equations:

$$\text{op}(\text{sub}(A)) * X = \alpha * \text{sub}(B),$$

or

$$X * \text{op}(\text{sub}(A)) = \alpha * \text{sub}(B),$$

where:

$\alpha$  is a scalar,

$X$  and  $\text{sub}(B)$  are  $m$ -by- $n$  distributed matrices,  $\text{sub}(B) = B(ib:ib+m-1, jb:jb+n-1)$ ;

$A$  is a unit, or non-unit, upper or lower triangular distributed matrix,  $\text{sub}(A) = A(\text{ia}:\text{ia}+m-1, \text{ja}:\text{ja}+m-1)$ , if  $\text{side} = 'L'$  or  $'l'$ , and  $\text{sub}(A) = A(\text{ia}:\text{ia}+n-1, \text{ja}:\text{ja}+n-1)$ , if  $\text{side} = 'R'$  or  $'r'$ ;

$\text{op}(\text{sub}(A))$  is one of  $\text{op}(\text{sub}(A)) = \text{sub}(A)$ , or  $\text{op}(\text{sub}(A)) = \text{sub}(A)'$ , or  $\text{op}(\text{sub}(A)) = \text{conjg}(\text{sub}(A)')$ .

The distributed matrix  $\text{sub}(B)$  is overwritten by the solution matrix  $X$ .

## Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>side</i>   | (global) Specifies whether $\text{op}(\text{sub}(A))$ appears on the left or right of $X$ in the equation:<br>if $\text{side} = 'L'$ or $'l'$ , then $\text{op}(\text{sub}(A)) * X = \text{alpha} * \text{sub}(B)$ ;<br>if $\text{side} = 'R'$ or $'r'$ , then $X * \text{op}(\text{sub}(A)) = \text{alpha} * \text{sub}(B)$ .                                                                                                                                                                                                                                                 |
| <i>uplo</i>   | (global) Specifies whether the distributed matrix $\text{sub}(A)$ is upper or lower triangular:<br>if $\text{uplo} = 'U'$ or $'u'$ , then the matrix is upper triangular;<br>if $\text{uplo} = 'L'$ or $'l'$ , then the matrix is low triangular.                                                                                                                                                                                                                                                                                                                              |
| <i>transa</i> | (global) Specifies the form of $\text{op}(\text{sub}(A))$ used in the matrix equation:<br>if $\text{transa} = 'N'$ or $'n'$ , then $\text{op}(\text{sub}(A)) = \text{sub}(A)$ ;<br>if $\text{transa} = 'T'$ or $'t'$ , then $\text{op}(\text{sub}(A)) = \text{sub}(A)'$ ;<br>if $\text{transa} = 'C'$ or $'c'$ , then $\text{op}(\text{sub}(A)) = \text{conjg}(\text{sub}(A)')$ .                                                                                                                                                                                              |
| <i>diag</i>   | (global) Specifies whether the matrix $\text{sub}(A)$ is unit triangular:<br>if $\text{diag} = 'U'$ or $'u'$ then the matrix is unit triangular;<br>if $\text{diag} = 'N'$ or $'n'$ , then the matrix is not unit triangular.                                                                                                                                                                                                                                                                                                                                                  |
| <i>m</i>      | (global) Specifies the number of rows of the distributed matrix $\text{sub}(B)$ , $m \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <i>n</i>      | (global) Specifies the number of columns of the distributed matrix $\text{sub}(B)$ , $n \geq 0$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <i>alpha</i>  | (global)<br>Specifies the scalar <i>alpha</i> .<br>When <i>alpha</i> is zero, then <i>a</i> is not referenced and <i>b</i> need not be set before entry.                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <i>a</i>      | (local)<br>Array, size <i>lld_a</i> by <i>ka</i> , where <i>ka</i> is at least $\text{LOCq}(1, \text{ja}+m-1)$ when $\text{side} = 'L'$ or $'l'$ and is at least $\text{LOCq}(1, \text{ja}+n-1)$ when $\text{side} = 'R'$ or $'r'$ .<br>Before entry with $\text{uplo} = 'U'$ or $'u'$ , this array contains the local entries corresponding to the entries of the upper triangular distributed matrix $\text{sub}(A)$ , and the local entries corresponding to the entries of the strictly lower triangular part of the distributed matrix $\text{sub}(A)$ is not referenced. |

Before entry with *uplo* = 'L' or 'l', this array contains the local entries corresponding to the entries of the lower triangular distributed matrix *sub(A)*, and the local entries corresponding to the entries of the strictly upper triangular part of the distributed matrix *sub(A)* is not referenced .

When *diag* = 'U' or 'u', the local entries corresponding to the diagonal elements of the submatrix *sub(A)* are not referenced either, but are assumed to be unity.

|               |                                                                                                                                                                           |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ia, ja</i> | (global) The row and column indices in the distributed matrix <i>A</i> indicating the first row and the first column of the submatrix <i>sub(A)</i> , respectively.       |
| <i>desca</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>A</i> .                                                                        |
| <i>b</i>      | (local)<br>Array, size ( <i>lld_b</i> , <i>LOCq</i> (1, <i>jb+n-1</i> )).<br>Before entry, this array contains the local pieces of the distributed matrix <i>sub(B)</i> . |
| <i>ib, jb</i> | (global) The row and column indices in the distributed matrix <i>B</i> indicating the first row and the first column of the submatrix <i>sub(B)</i> , respectively.       |
| <i>descb</i>  | (global and local) array of dimension 9. The array descriptor of the distributed matrix <i>B</i> .                                                                        |

### Output Parameters

|          |                                                           |
|----------|-----------------------------------------------------------|
| <i>b</i> | Overwritten by the solution distributed matrix <i>X</i> . |
|----------|-----------------------------------------------------------|



# Partial Differential Equations Support

11

The Intel® Math Kernel Library (Intel® MKL) provides tools for solving Partial Differential Equations (PDE). These tools are Trigonometric Transform interface routines (see [Trigonometric Transform Routines](#)) and Poisson Solver (see [Fast Poisson Solver Routines](#)).

Poisson Solver is designed for fast solving of simple Helmholtz, Poisson, and Laplace problems. The solver is based on the Trigonometric Transform interface, which is, in turn, based on the Intel MKL Fast Fourier Transform (FFT) interface (refer to [Fourier Transform Functions](#)), optimized for Intel® processors.

Direct use of the Trigonometric Transform routines may be helpful to those who have already implemented their own solvers similar to the Intel MKL Poisson Solver. As it may be hard enough to modify the original code so as to make it work with Poisson Solver, you are encouraged to use fast (staggered) sine/cosine transforms implemented in the Trigonometric Transform interface to improve performance of your solver.

Both Trigonometric Transform and Poisson Solver routines can be called from C and Fortran.

## Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

## Trigonometric Transform Routines

In addition to the Fast Fourier Transform (FFT) interface, described in chapter "[Fast Fourier Transforms](#)", Intel® MKL supports the Real Discrete Trigonometric Transforms (sometimes called real-to-real Discrete Fourier Transforms) interface. In this document, the interface is referred to as TT interface. It implements a group of routines (TT routines) used to compute sine/cosine, staggered sine/cosine, and twice staggered sine/cosine transforms (referred to as staggered2 sine/cosine transforms, for brevity). The TT interface provides much flexibility of use: you can adjust routines to your particular needs at the cost of manually tuning routine parameters or just call routines with default parameter values. The current Intel MKL implementation of the TT interface can be used in solving partial differential equations and contains routines that are helpful for Fast Poisson and similar solvers.

For the list of Trigonometric Transforms currently implemented in Intel MKL TT interface, see [Transforms Implemented](#).

If you have got used to the FFTW interface ([www.fftw.org](http://www.fftw.org)), you can call the TT interface functions through real-to-real FFTW to Intel MKL wrappers without changing FFTW function calls in your code (refer to [FFTW to Intel® MKL Wrappers for FFTW 3.x](#) for details). However, you are strongly encouraged to use the native TT interface for better performance. Another reason why you should use the wrappers cautiously is that TT and the real-to-real FFTW interfaces are not fully compatible and some features of the real-to-real FFTW, such as strides and multidimensional transforms, are not available through wrappers.

## Trigonometric Transforms Implemented

TT routines allow computing the following transforms:

## Forward sine transform

$$F(k) = \frac{2}{n} \sum_{i=1}^{n-1} f(i) \sin \frac{ki\pi}{n}, k = 1, \dots, n-1$$

## Backward sine transform

$$f(i) = \sum_{k=1}^{n-1} F(k) \sin \frac{ki\pi}{n}, i = 1, \dots, n-1$$

## Forward staggered sine transform

$$F(k) = \frac{1}{n} \sin \frac{(2k-1)\pi}{2} f(n) + \frac{2}{n} \sum_{i=1}^{n-1} f(i) \sin \frac{(2k-1)i\pi}{2n}, k = 1, \dots, n$$

## Backward staggered sine transform

$$f(i) = \sum_{k=1}^n F(k) \sin \frac{(2k-1)i\pi}{2n}, i = 1, \dots, n$$

## Forward staggered2 sine transform

$$F(k) = \frac{2}{n} \sum_{i=1}^n f(i) \sin \frac{(2k-1)(2i-1)\pi}{4n}, k = 1, \dots, n$$

## Backward staggered2 sine transform

$$f(i) = \sum_{k=1}^n F(k) \sin \frac{(2k-1)(2i-1)\pi}{4n}, i = 1, \dots, n$$

## Forward cosine transform

$$F(k) = \frac{1}{n} [f(0) + f(n) \cos k\pi] + \frac{2}{n} \sum_{i=1}^{n-1} f(i) \cos \frac{ki\pi}{n}, k = 0, \dots, n$$

## Backward cosine transform

$$f(i) = \frac{1}{2} [F(0) + F(n) \cos i\pi] + \sum_{k=1}^{n-1} F(k) \cos \frac{ki\pi}{n}, i = 0, \dots, n$$

## Forward staggered cosine transform

$$F(k) = \frac{1}{n} f(0) + \frac{2}{n} \sum_{i=1}^{n-1} f(i) \cos \frac{(2k+1)i\pi}{2n}, k = 0, \dots, n-1$$

## Backward staggered cosine transform

$$f(i) = \sum_{k=0}^{n-1} F(k) \cos \frac{(2k+1)i\pi}{2n}, i = 0, \dots, n-1$$

## Forward staggered2 cosine transform

$$F(k) = \frac{2}{n} \sum_{i=1}^n f(i) \cos \frac{(2k-1)(2i-1)\pi}{4n}, k = 1, \dots, n$$

## Backward staggered2 cosine transform

$$f(i) = \sum_{k=1}^n F(k) \cos \frac{(2k-1)(2i-1)\pi}{4n}, i = 1, \dots, n$$

**NOTE**

The size of the transform  $n$  can be any integer greater or equal to 2.

**Sequence of Invoking TT Routines**

Computation of a transform using TT interface is conceptually divided into four steps, each of which is performed via a dedicated routine. [Table "TT Interface Routines"](#) lists the routines and briefly describes their purpose and use.

Most TT routines have versions operating with single-precision and double-precision data. Names of such routines begin respectively with "s" and "d". The wildcard "?" stands for either of these symbols in routine names.

**TT Interface Routines**

| Routine                                                                         | Description                                                                                                                                          |
|---------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>?_init_trig_transform</code>                                              | Initializes basic data structures of Trigonometric Transforms.                                                                                       |
| <code>?_commit_trig_transform</code>                                            | Checks consistency and correctness of user-defined data and creates a data structure to be used by Intel MKL FFT interface <sup>1</sup> .            |
| <code>?_forward_trig_transform</code><br><code>?_backward_trig_transform</code> | Computes a forward/backward Trigonometric Transform of a specified type using the appropriate formula (see <a href="#">Transforms Implemented</a> ). |
| <code>free_trig_transform</code>                                                | Releases the memory used by a data structure needed for calling FFT interface <sup>1</sup> .                                                         |

<sup>1</sup>TT routines call Intel MKL FFT interface for better performance.

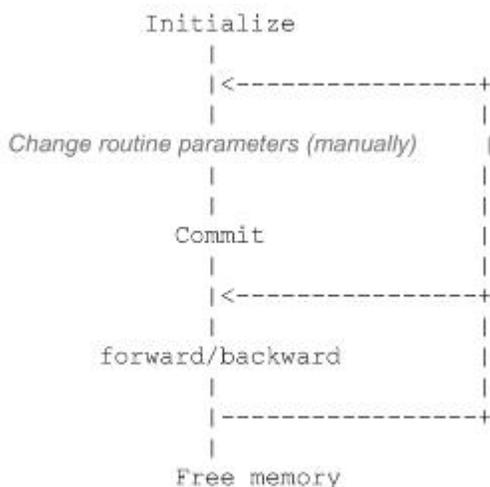
To find a transformed vector for a particular input vector only once, the Intel MKL TT interface routines are normally invoked in the order in which they are listed in [Table "TT Interface Routines"](#).

**NOTE**

Though the order of invoking TT routines may be changed, it is highly recommended to follow the above order of routine calls.

The diagram in [Figure "Typical Order of Invoking TT Interface Routines"](#) indicates the typical order in which TT interface routines can be invoked in a general case (prefixes and suffixes in routine names are omitted).

**Typical Order of Invoking TT Interface Routines**



A general scheme of using TT routines for double-precision computations is shown below. A similar scheme holds for single-precision computations with the only difference in the initial letter of routine names.

```
...
 d_init_trig_transform(&n, &tt_type, ipar, dpar, &ir);
/* Change parameters in ipar if necessary. */
/* Note that the result of the Transform will be in f. If you want to preserve the data stored in
f,
save it to another location before the function call below */
 d_commit_trig_transform(f, &handle, ipar, dpar, &ir);
 d_forward_trig_transform(f, &handle, ipar, dpar, &ir);
 d_backward_trig_transform(f, &handle, ipar, dpar, &ir);
 free_trig_transform(&handle, ipar, &ir);
/* here the user may clean the memory used by f, dpar, ipar */
...
```

You can find examples of code that uses TT interface routines to solve one-dimensional Helmholtz problem in the `examples\pdetttc\source` folder in your Intel MKL directory.

## Trigonometric Transform Interface Description

All types in this documentation are either standard C types `float` and `double` or `MKL_INT` integer type. For more information on the C types, refer to [C Datatypes Specific to Intel MKL](#) and the *Intel(R) MKL Developer Guide*. To better understand usage of the types, see examples in the `examples\pdetttc\source` folder in your Intel MKL directory.

### Routine Options

All TT routines use parameters to pass various options to one another. These parameters are arrays `ipar`, `dpar` and `spar`. Values for these parameters should be specified very carefully (see [Common Parameters](#)). You can change these values during computations to meet your needs.

---

#### WARNING

To avoid failure or incorrect results, you must provide correct and consistent parameters to the routines.

---

### User Data Arrays

TT routines take arrays of user data as input. For example, user arrays are passed to the routine `d_forward_trig_transform` to compute a forward Trigonometric Transform. To minimize storage requirements and improve the overall run-time efficiency, Intel MKL TT routines do not make copies of user input arrays.

---

#### NOTE

If you need a copy of your input data arrays, you must save them yourself.

---

For better performance, align your data arrays as recommended in the *Intel MKL Developer Guide* (search the document for coding techniques to improve performance).

## TT Routines

The section gives detailed description of TT routines, their syntax, parameters and values they return. Double-precision and single-precision versions of the same routine are described together.

TT routines call Intel MKL FFT interface (described in section "[FFT Functions](#)" in chapter "Fast Fourier Transforms"), which enhances performance of the routines.

## ?\_init\_trig\_transform

Initializes basic data structures of a Trigonometric Transform.

---

### Syntax

```
void d_init_trig_transform(MKL_INT *n, MKL_INT *tt_type, MKL_INT ipar[], double dpar[], MKL_INT *stat);
```

```
void s_init_trig_transform(MKL_INT *n, MKL_INT *tt_type, MKL_INT ipar[], float spar[], MKL_INT *stat);
```

### Include Files

- mkl.h

### Input Parameters

|                |                                                                                                                                                                                                                                                                                                                                                         |
|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>       | MKL_INT*. Contains the size of the problem, which should be a positive integer greater than 1. Note that data vector of the transform, which other TT routines will use, must have size $n+1$ for all but staggered2 transforms. Staggered2 transforms require the vector of size $n$ .                                                                 |
| <i>tt_type</i> | MKL_INT*. Contains the type of transform to compute, defined via a set of named constants. The following constants are available in the current implementation of TT interface: MKL_SINE_TRANSFORM, MKL_STAGGERED_SINE_TRANSFORM, MKL_STAGGERED2_SINE_TRANSFORM; MKL_COSINE_TRANSFORM, MKL_STAGGERED_COSINE_TRANSFORM, MKL_STAGGERED2_COSINE_TRANSFORM. |

### Output Parameters

|             |                                                                                                                                                     |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> | MKL_INT array of size 128. Contains integer data needed for Trigonometric Transform computations.                                                   |
| <i>dpar</i> | double array of size $5n/2+2$ . Contains double-precision data needed for Trigonometric Transform computations.                                     |
| <i>spar</i> | float array of size $5n/2+2$ . Contains single-precision data needed for Trigonometric Transform computations.                                      |
| <i>stat</i> | MKL_INT*. Contains the routine completion status, which is also written to <i>ipar</i> [6]. The status should be 0 to proceed to other TT routines. |

### Description

The `?_init_trig_transform` routine initializes basic data structures for Trigonometric Transforms of appropriate precision. After a call to `?_init_trig_transform`, all subsequently invoked TT routines use values of *ipar* and *dpar* (*spar*) array parameters returned by `?_init_trig_transform`. The routine initializes the entire array *ipar*. In the *dpar* or *spar* array, `?_init_trig_transform` initializes elements that do not depend upon the type of transform. For a detailed description of arrays *ipar*, *dpar* and *spar*, refer to the [Common Parameters](#) section. You can skip a call to the initialization routine in your code. For more information, see [Caveat on Parameter Modifications](#).

## Return Values

`stat= 0`

The routine successfully completed the task. In general, to proceed with computations, the routine should complete with this `stat` value.

`stat= -99999`

The routine failed to complete the task.

## ?\_commit\_trig\_transform

*Checks consistency and correctness of user's data as well as initializes certain data structures required to perform the Trigonometric Transform.*

---

## Syntax

```
void d_commit_trig_transform(double f[], DFTI_DESCRIPTOR_HANDLE *handle, MKL_INT ipar[], double dpar[], MKL_INT *stat);
```

```
void s_commit_trig_transform(float f[], DFTI_DESCRIPTOR_HANDLE *handle, MKL_INT ipar[], float spar[], MKL_INT *stat);
```

## Include Files

- `mkl.h`

## Input Parameters

|                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>f</code>    | <p>double for <code>d_commit_trig_transform</code>,<br/>float for <code>s_commit_trig_transform</code>,<br/>array of size <math>n</math> for staggered2 transforms and of size <math>n+1</math> for all other transforms, where <math>n</math> is the size of the problem. Contains data vector to be transformed. Note that the following values should be 0.0 up to rounding errors:</p> <ul style="list-style-type: none"> <li>• <math>f[0]</math> and <math>f[n]</math> for sine transforms</li> <li>• <math>f[n]</math> for staggered cosine transforms</li> <li>• <math>f[0]</math> for staggered sine transforms.</li> </ul> <p>Otherwise, the routine will produce a warning, and the result of the computations for sine transforms may be wrong. These restrictions meet the requirements of the Intel MKL Poisson Solver, which the TT interface is primarily designed for (for details, see <a href="#">Fast Poisson Solver Routines</a>).</p> |
| <code>ipar</code> | MKL_INT array of size 128. Contains integer data needed for Trigonometric Transform computations.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <code>dpar</code> | double array of size $5n/2+2$ . Contains double-precision data needed for Trigonometric Transform computations. The routine initializes most elements of this array.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <code>spar</code> | float array of size $5n/2+2$ . Contains single-precision data needed for Trigonometric Transform computations. The routine initializes most elements of this array.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

## Output Parameters

|               |                                                                                                                                                                                     |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>handle</i> | DFTI_DESCRIPTOR_HANDLE*. The data structure used by Intel MKL FFT interface (for details, refer to section " <a href="#">FFT Functions</a> " in chapter "Fast Fourier Transforms"). |
| <i>ipar</i>   | Contains integer data needed for Trigonometric Transform computations. On output, <i>ipar[6]</i> is updated with the <i>stat</i> value.                                             |
| <i>dpar</i>   | Contains double-precision data needed for Trigonometric Transform computations. On output, the entire array is initialized.                                                         |
| <i>spar</i>   | Contains single-precision data needed for Trigonometric Transform computations. On output, the entire array is initialized.                                                         |
| <i>stat</i>   | MKL_INT*. Contains the routine completion status, which is also written to <i>ipar[6]</i> .                                                                                         |

## Description

The routine `?_commit_trig_transform` checks consistency and correctness of the parameters to be passed to the transform routines `?_forward_trig_transform` and/or `?_backward_trig_transform`. The routine also initializes the following data structures: *handle*, *dpar* in case of `d_commit_trig_transform`, and *spar* in case of `s_commit_trig_transform`. The `?_commit_trig_transform` routine initializes only those elements of *dpar* or *spar* that depend upon the type of transform, defined in the `?_init_trig_transform` routine and passed to `?_commit_trig_transform` with the *ipar* array. The size of the problem *n*, which determines sizes of the array parameters, is also passed to the routine with the *ipar* array and defined in the previously called `?_init_trig_transform` routine. For a detailed description of arrays *ipar*, *dpar* and *spar*, refer to the [Common Parameters](#) section. The routine performs only a basic check for correctness and consistency of the parameters. If you are going to modify parameters of TT routines, see the [Caveat on Parameter Modifications](#) section. Unlike `?_init_trig_transform`, you must call the `?_commit_trig_transform` routine in your code.

## Return Values

|                    |                                                                                                                                                                                                                                               |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>stat</i> = 11   | The routine produced some warnings and made some changes in the parameters to achieve their correctness and/or consistency. You may proceed with computations by assigning <i>ipar[6]</i> =0 if you are sure that the parameters are correct. |
| <i>stat</i> = 10   | The routine made some changes in the parameters to achieve their correctness and/or consistency. You may proceed with computations by assigning <i>ipar[6]</i> =0 if you are sure that the parameters are correct.                            |
| <i>stat</i> = 1    | The routine produced some warnings. You may proceed with computations by assigning <i>ipar[6]</i> =0 if you are sure that the parameters are correct.                                                                                         |
| <i>stat</i> = 0    | The routine completed the task normally.                                                                                                                                                                                                      |
| <i>stat</i> = -100 | The routine stopped for any of the following reasons: <ul style="list-style-type: none"> <li>• An error in the user's data was encountered.</li> </ul>                                                                                        |

- Data in *ipar*, *dpar* or *spar* parameters became incorrect and/or inconsistent as a result of modifications.

*stat*= -1000

The routine stopped because of an FFT interface error.

*stat*= -10000

The routine stopped because the initialization failed to complete or the parameter *ipar*[0] was altered by mistake.

---

## NOTE

Although positive values of *stat* usually indicate minor problems with the input data and Trigonometric Transform computations can be continued, you are highly recommended to investigate the problem first and achieve *stat*=0.

---

## ?\_forward\_trig\_transform

Computes the forward Trigonometric Transform of type specified by the parameter.

---

### Syntax

```
void d_forward_trig_transform(double f[], DFTI_DESCRIPTOR_HANDLE *handle, MKL_INT ipar[], double dpar[], MKL_INT *stat);
```

```
void s_forward_trig_transform(float f[], DFTI_DESCRIPTOR_HANDLE *handle, MKL_INT ipar[], float spar[], MKL_INT *stat);
```

### Include Files

- mkl.h

### Input Parameters

*f* double for `d_forward_trig_transform`,  
float for `s_forward_trig_transform`,  
array of size *n* for staggered2 transforms and of size *n*+1 for all other transforms, where *n* is the size of the problem. On input, contains data vector to be transformed. Note that the following values should be 0.0 up to rounding errors:

- *f*[0] and *f*[*n*] for sine transforms
- *f*[*n*] for staggered cosine transforms
- *f*[0] for staggered sine transforms.

Otherwise, the routine will produce a warning, and the result of the computations for sine transforms may be wrong. The above restrictions meet the requirements of the Intel MKL Poisson Solver, which the TT interface is primarily designed for (for details, see [Fast Poisson Solver Routines](#)).

*handle* DFTI\_DESCRIPTOR\_HANDLE\*. The data structure used by Intel MKL FFT interface (for details, see [FFT Functions](#)).

---

|             |                                                                                                                 |
|-------------|-----------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> | MKL_INT array of size 128. Contains integer data needed for Trigonometric Transform computations.               |
| <i>dpar</i> | double array of size $5n/2+2$ . Contains double-precision data needed for Trigonometric Transform computations. |
| <i>spar</i> | float array of size $5n/2+2$ . Contains single-precision data needed for Trigonometric Transform computations.  |

## Output Parameters

|             |                                                                                                                                          |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------|
| <i>f</i>    | Contains the transformed vector on output.                                                                                               |
| <i>ipar</i> | Contains integer data needed for Trigonometric Transform computations. On output, <i>ipar</i> [6] is updated with the <i>stat</i> value. |
| <i>stat</i> | MKL_INT*. Contains the routine completion status, which is also written to <i>ipar</i> [6].                                              |

## Description

The routine computes the forward Trigonometric Transform of type defined in the `?_init_trig_transform` routine and passed to `?_forward_trig_transform` with the *ipar* array. The size of the problem *n*, which determines sizes of the array parameters, is also passed to the routine with the *ipar* array and defined in the previously called `?_init_trig_transform` routine. The other data that facilitates the computation is created by `?_commit_trig_transform` and supplied in *dpar* or *spar*. For a detailed description of arrays *ipar*, *dpar* and *spar*, refer to the [Common Parameters](#) section. The routine has a commit step, which calls the `?_commit_trig_transform` routine. The transform is computed according to formulas given in the [Transforms Implemented](#) section. The routine replaces the input vector *f* with the transformed vector.

---

### NOTE

If you need a copy of the data vector *f* to be transformed, make the copy before calling the `?_forward_trig_transform` routine.

---

## Return Values

|                      |                                                                                                                                                                                                                                                                                                 |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>stat</i> = 0      | The routine completed the task normally.                                                                                                                                                                                                                                                        |
| <i>stat</i> = -100   | The routine stopped for any of the following reasons: <ul style="list-style-type: none"> <li>• An error in the user's data was encountered.</li> <li>• Data in <i>ipar</i>, <i>dpar</i> or <i>spar</i> parameters became incorrect and/or inconsistent as a result of modifications.</li> </ul> |
| <i>stat</i> = -1000  | The routine stopped because of an FFT interface error.                                                                                                                                                                                                                                          |
| <i>stat</i> = -10000 | The routine stopped because its commit step failed to complete or the parameter <i>ipar</i> [0] was altered by mistake.                                                                                                                                                                         |

## ?\_backward\_trig\_transform

Computes the backward Trigonometric Transform of type specified by the parameter.

### Syntax

```
void d_backward_trig_transform(double f[], DFTI_DESCRIPTOR_HANDLE *handle, MKL_INT ipar[], double dpar[], MKL_INT *stat);
```

```
void s_backward_trig_transform(float f[], DFTI_DESCRIPTOR_HANDLE *handle, MKL_INT ipar[], float spar[], MKL_INT *stat);
```

### Include Files

- mkl.h

### Input Parameters

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>f</i>      | double for <code>d_backward_trig_transform</code> ,<br>float for <code>s_backward_trig_transform</code> ,<br>array of size $n$ for staggered2 transforms and of size $n+1$ for all other transforms, where $n$ is the size of the problem. On input, contains data vector to be transformed. Note that the following values should be 0.0 up to rounding errors: <ul style="list-style-type: none"> <li>• <math>f[0]</math> and <math>f[n]</math> for sine transforms</li> <li>• <math>f[n]</math> for staggered cosine transforms</li> <li>• <math>f[0]</math> for staggered sine transforms.</li> </ul> Otherwise, the routine will produce a warning, and the result of the computations for sine transforms may be wrong. The above restrictions meet the requirements of the Intel MKL Poisson Solver, which the TT interface is primarily designed for (for details, see <a href="#">Fast Poisson Solver Routines</a> ). |
| <i>handle</i> | DFTI_DESCRIPTOR_HANDLE*. The data structure used by Intel MKL FFT interface (for details, see <a href="#">FFT Functions</a> ).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <i>ipar</i>   | MKL_INT array of size 128. Contains integer data needed for Trigonometric Transform computations.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <i>dpar</i>   | double array of size $5n/2+2$ . Contains double-precision data needed for Trigonometric Transform computations.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <i>spar</i>   | float array of size $5n/2+2$ . Contains single-precision data needed for Trigonometric Transform computations.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |

### Output Parameters

|             |                                                                                                                                          |
|-------------|------------------------------------------------------------------------------------------------------------------------------------------|
| <i>f</i>    | Contains the transformed vector on output.                                                                                               |
| <i>ipar</i> | Contains integer data needed for Trigonometric Transform computations. On output, <i>ipar</i> [6] is updated with the <i>stat</i> value. |
| <i>stat</i> | MKL_INT*. Contains the routine completion status, which is also written to <i>ipar</i> [6].                                              |

## Description

The routine computes the backward Trigonometric Transform of type defined in the `?_init_trig_transform` routine and passed to `?_backward_trig_transform` with the `ipar` array. The size of the problem  $n$ , which determines sizes of the array parameters, is also passed to the routine with the `ipar` array and defined in the previously called `?_init_trig_transform` routine. The other data that facilitates the computation is created by `?_commit_trig_transform` and supplied in `dpar` or `spar`. For a detailed description of arrays `ipar`, `dpar` and `spar`, refer to the [Common Parameters](#) section. The routine has a commit step, which calls the `?_commit_trig_transform` routine. The transform is computed according to formulas given in the [Transforms Implemented](#) section. The routine replaces the input vector  $f$  with the transformed vector.

---

### NOTE

If you need a copy of the data vector  $f$  to be transformed, make the copy before calling the `?_backward_trig_transform` routine.

---

## Return Values

|                           |                                                                                                                                                                                                                                                                                                                   |
|---------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>stat= 0</code>      | The routine completed the task normally.                                                                                                                                                                                                                                                                          |
| <code>stat= -100</code>   | The routine stopped for any of the following reasons: <ul style="list-style-type: none"> <li>• An error in the user's data was encountered.</li> <li>• Data in <code>ipar</code>, <code>dpar</code> or <code>spar</code> parameters became incorrect and/or inconsistent as a result of modifications.</li> </ul> |
| <code>stat= -1000</code>  | The routine stopped because of an FFT interface error.                                                                                                                                                                                                                                                            |
| <code>stat= -10000</code> | The routine stopped because its commit step failed to complete or the parameter <code>ipar[0]</code> was altered by mistake.                                                                                                                                                                                      |

## free\_trig\_transform

*Cleans the memory allocated for the data structure used by the FFT interface.*

---

## Syntax

```
void free_trig_transform(DFTI_DESCRIPTOR_HANDLE *handle, MKL_INT ipar[], MKL_INT *stat);
```

## Include Files

- `mkl.h`

## Input Parameters

|                     |                                                                                                                                                                                     |
|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ipar</code>   | MKL_INT array of size 128. Contains integer data needed for Trigonometric Transform computations.                                                                                   |
| <code>handle</code> | DFTI_DESCRIPTOR_HANDLE*. The data structure used by Intel MKL FFT interface (for details, refer to section " <a href="#">FFT Functions</a> " in chapter "Fast Fourier Transforms"). |

## Output Parameters

|               |                                                                                                                                         |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <i>handle</i> | The data structure used by Intel MKL FFT interface. Memory allocated for the structure is released on output.                           |
| <i>ipar</i>   | Contains integer data needed for Trigonometric Transform computations. On output, <i>ipar[6]</i> is updated with the <i>stat</i> value. |
| <i>stat</i>   | MKL_INT*. Contains the routine completion status, which is also written to <i>ipar[6]</i> .                                             |

## Description

The `free_trig_transform` routine cleans the memory used by the *handle* structure, needed for Intel MKL FFT functions. To release the memory allocated for other parameters, include cleaning of the memory in your code.

## Return Values

|                      |                                                        |
|----------------------|--------------------------------------------------------|
| <i>stat</i> = 0      | The routine completed the task normally.               |
| <i>stat</i> = -1000  | The routine stopped because of an FFT interface error. |
| <i>stat</i> = -99999 | The routine failed to complete the task.               |

## Common Parameters of the Trigonometric Transforms

This section provides description of array parameters that hold TT routine options: *ipar*, *dpar* and *spar*.

---

### NOTE

Initial values are assigned to the array parameters by the appropriate `?_init_trig_transform` and `?_commit_trig_transform` routines.

---

|             |                                                                                                                                                                                   |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> | MKL_INT array of size 128, holds integer data needed for Trigonometric Transform computations. Its elements are described in <a href="#">Table "Elements of the ipar Array"</a> : |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## Elements of the ipar Array

| Index | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0     | Contains the size of the problem to solve. The <code>?_init_trig_transform</code> routine sets <i>ipar[0]=n</i> , and all subsequently called TT routines use <i>ipar[0]</i> as the size of the transform.                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| 1     | Contains error messaging options: <ul style="list-style-type: none"> <li><i>ipar[1]=-1</i> indicates that all error messages will be printed to the file <code>MKL_Trig_Transforms_log.txt</code> in the folder from which the routine is called. If the file does not exist, the routine tries to create it. If the attempt fails, the routine prints information that the file cannot be created to the standard output device.</li> <li><i>ipar[1]=0</i> indicates that no error messages will be printed.</li> <li><i>ipar[1]=1</i> (default) indicates that all error messages will be printed to the preconnected default output device (usually, screen).</li> </ul> |

| Index          | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2              | <p>In case of errors, each TT routine assigns a non-zero value to <i>stat</i> regardless of the <i>ipar[1]</i> setting.</p> <p>Contains warning messaging options:</p> <ul style="list-style-type: none"> <li>• <i>ipar[2]=-1</i> indicates that all warning messages will be printed to the file <code>MKL_Trig_Transforms_log.txt</code> in the directory from which the routine is called. If the file does not exist, the routine tries to create it. If the attempt fails, the routine prints information that the file cannot be created to the standard output device.</li> <li>• <i>ipar[2]=0</i> indicates that no warning messages will be printed.</li> <li>• <i>ipar[2]=1</i> (default) indicates that all warning messages will be printed to the preconnected default output device (usually, screen).</li> </ul> <p>In case of warnings, the <i>stat</i> parameter will acquire a non-zero value regardless of the <i>ipar[2]</i> setting.</p> |
| 3 through 4    | Reserved for future use.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 5              | Contains the type of the transform. The <code>?_init_trig_transform</code> routine sets <i>ipar[5]=tt_type</i> , and all subsequently called TT routines use <i>ipar[5]</i> as the type of the transform.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| 6              | Contains the <i>stat</i> value returned by the last completed TT routine. Used to check that the previous call to a TT routine completed with <i>stat</i> =0.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 7              | <p>Informs the <code>?_commit_trig_transform</code> routines whether to initialize data structures <i>dpar</i> (<i>spar</i>) and <i>handle</i>. <i>ipar[7]=0</i> indicates that the routine should skip the initialization and only check correctness and consistency of the parameters. Otherwise, the routine initializes the data structures. The default value is 1.</p> <p>The possibility to check correctness and consistency of input data without initializing data structures <i>dpar</i>, <i>spar</i> and <i>handle</i> enables avoiding performance losses in a repeated use of the same transform for different data vectors. Note that you can benefit from the opportunity that <i>ipar[7]</i> gives only if you are sure to have supplied proper tolerance value in the <i>dpar</i> or <i>spar</i> array. Otherwise, avoid tuning this parameter.</p>                                                                                         |
| 8              | <p>Contains message style options for TT routines. Specifically, if <i>ipar[8]</i> is non-zero, TT routines print the messages in C-style notations. The default value is 1.</p> <p>When specifying message style options, be aware that by default, numbering of elements in C arrays starts at 0. For example, "<i>parameter ipar[0]=3 should be an even integer</i>" is a C-style message. The use of <i>ipar[8]</i> enables you to view messages in a more convenient style.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| 9              | Specifies the number of OpenMP threads to run TT routines in the OpenMP environment of the Intel MKL Poisson Solver. The default value is 1. You are highly recommended not to alter this value. See also <a href="#">Caveat on Parameter Modifications</a> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| 10             | Specifies the mode of compatibility with FFTW. The default value is 0. Set the value to 1 to invoke compatibility with FFTW. In the latter case, results will not be normalized, because FFTW does not do this. It is highly recommended not to alter this value, but rather use real-to-real FFTW to MKL wrappers, described in <a href="#">FFTW to Intel® MKL Wrappers for FFTW 3.x</a> . See also <a href="#">Caveat on Parameter Modifications</a> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 11 through 127 | Reserved for future use.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |

**NOTE**

While you can declare the *ipar* array as `MKL_INT ipar[11]`, for future compatibility you should declare *ipar* as `MKL_INT ipar[128]`.

Arrays *dpar* and *spar* are the same except in the data precision:

- dpar*                      double array of size  $5n/2+2$ , holds data needed for double-precision routines to perform TT computations. This array is initialized in the `d_init_trig_transform` and `d_commit_trig_transform` routines.
- spar*                      float array of size  $5n/2+2$ , holds data needed for single-precision routines to perform TT computations. This array is initialized in the `s_init_trig_transform` and `s_commit_trig_transform` routines.

As *dpar* and *spar* have similar elements in respective positions, the elements are described together in [Table "Elements of the dpar and spar Arrays"](#):

**Elements of the dpar and spar Arrays**

| Index              | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0                  | Contains the first absolute tolerance used by the appropriate <code>?_commit_trig_transform</code> routine. For a staggered cosine or a sine transform, $f[n]$ should be equal to 0.0 and for a staggered sine or a sine transform, $f[0]$ should be equal to 0.0. The <code>?_commit_trig_transform</code> routine checks whether absolute values of these parameters are below $dpar[0]*n$ or $spar[0]*n$ , depending on the routine precision. To suppress warnings resulting from tolerance checks, set $dpar[0]$ or $spar[0]$ to a sufficiently large number.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 1                  | Reserved for future use.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 2 through $5n/2+1$ | Contain tabulated values of trigonometric functions. Contents of the elements depend upon the type of transform <i>tt_type</i> , set up in the <code>?_commit_trig_transform</code> routine: <ul style="list-style-type: none"> <li>• If <i>tt_type</i>=<code>MKL_SINE_TRANSFORM</code>, the transform uses only the first <math>n/2</math> array elements, which contain tabulated sine values.</li> <li>• If <i>tt_type</i>=<code>MKL_STAGGERED_SINE_TRANSFORM</code>, the transform uses only the first <math>3n/2</math> array elements, which contain tabulated sine and cosine values.</li> <li>• If <i>tt_type</i>=<code>MKL_STAGGERED2_SINE_TRANSFORM</code>, the transform uses all the <math>5n/2</math> array elements, which contain tabulated sine and cosine values.</li> <li>• If <i>tt_type</i>=<code>MKL_COSINE_TRANSFORM</code>, the transform uses only the first <math>n</math> array elements, which contain tabulated cosine values.</li> <li>• If <i>tt_type</i>=<code>MKL_STAGGERED_COSINE_TRANSFORM</code>, the transform uses only the first <math>3n/2</math> elements, which contain tabulated sine and cosine values.</li> <li>• If <i>tt_type</i>=<code>MKL_STAGGERED2_COSINE_TRANSFORM</code>, the transform uses all the <math>5n/2</math> elements, which contain tabulated sine and cosine values.</li> </ul> |

**NOTE**

To save memory, you can define the array size depending upon the type of transform.

**Caveat on Parameter Modifications**

Flexibility of the TT interface enables you to skip a call to the `?_init_trig_transform` routine and to initialize the basic data structures explicitly in your code. You may also need to modify the contents of *ipar*, *dpar* and *spar* arrays after initialization. When doing so, provide correct and consistent data in the arrays.

Mistakenly altered arrays cause errors or wrong computation. You can perform a basic check for correctness and consistency of parameters by calling the `?_commit_trig_transform` routine; however, this does not ensure the correct result of a transform but only reduces the chance of errors or wrong results.

**NOTE**

To supply correct and consistent parameters to TT routines, you should have considerable experience in using the TT interface and good understanding of elements that the `ipar`, `spar` and `dpar` arrays contain and dependencies between values of these elements.

However, in rare occurrences, even advanced users might fail to compute a transform using TT routines after the parameter modifications. In cases like these, refer for technical support at <http://www.intel.com/software/products/support/>.

**WARNING**

The only way that ensures proper computation of the Trigonometric Transforms is to follow a typical sequence of invoking the routines and not change the default set of parameters. So, avoid modifications of `ipar`, `dpar` and `spar` arrays unless a strong need arises.

**Optimization Notice**

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

**Trigonometric Transform Implementation Details**

Several aspects of the Intel MKL TT interface are platform-specific and language-specific. To promote portability across platforms and ease of use across different languages, Intel MKL provides you with the TT language-specific header file to include in your code:

- `mkl_trig_transforms.h`, to be used together with `mkl_dfti.h`.

**NOTE**

- Use of the Intel MKL TT software without including the above language-specific header files is not supported.

**Header File**

The header file below defines the following function prototypes:

```
void d_init_trig_transform(MKL_INT *, MKL_INT *, MKL_INT *, double *, MKL_INT *);
void d_commit_trig_transform(double *, DFTI_DESCRIPTOR_HANDLE *, MKL_INT *, double *, MKL_INT *);
void d_forward_trig_transform(double *, DFTI_DESCRIPTOR_HANDLE *, MKL_INT *, double *, MKL_INT *);
void d_backward_trig_transform(double *, DFTI_DESCRIPTOR_HANDLE *, MKL_INT *, double *, MKL_INT *);

void s_init_trig_transform(MKL_INT *, MKL_INT *, MKL_INT *, float *, MKL_INT *);
void s_commit_trig_transform(float *, DFTI_DESCRIPTOR_HANDLE *, MKL_INT *, float *, MKL_INT *);
```

```
void s_forward_trig_transform(float *, DFTI_DESCRIPTOR_HANDLE *, MKL_INT *, float *, MKL_INT *);
void s_backward_trig_transform(float *, DFTI_DESCRIPTOR_HANDLE *, MKL_INT *, float *, MKL_INT *);

void free_trig_transform(DFTI_DESCRIPTOR_HANDLE *, MKL_INT *, MKL_INT *);
```

## Fast Poisson Solver Routines

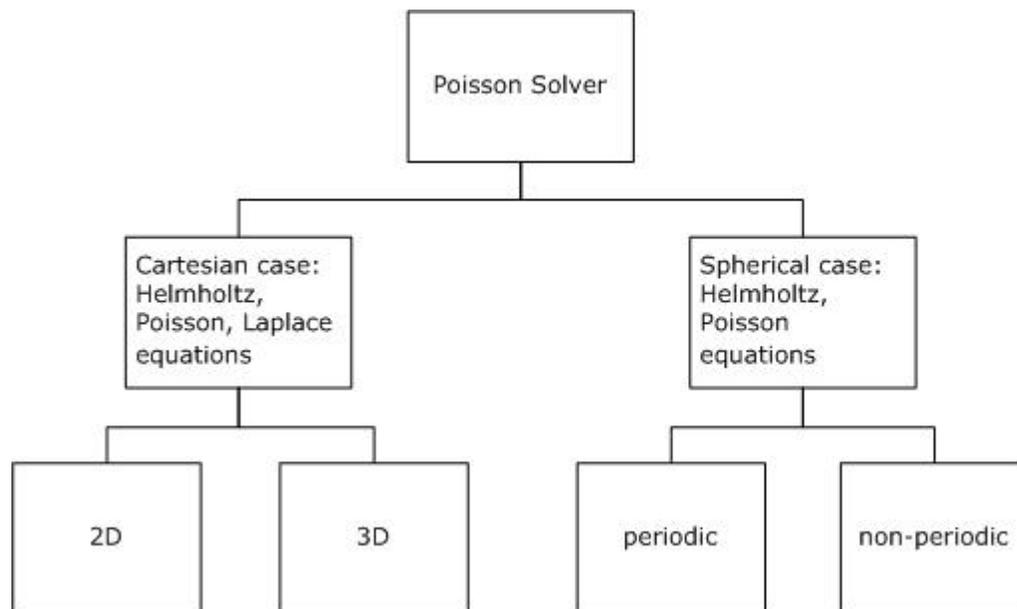
In addition to the Real Discrete Trigonometric Transforms (TT) interface (refer to [Trigonometric Transform Routines](#)), Intel MKL supports the the Poisson Solver interface. This interface implements a group of routines (Poisson Solver routines) used to compute a solution of Laplace, Poisson, and Helmholtz problems of a special kind using discrete Fourier transforms. Laplace and Poisson problems are special cases of a more general Helmholtz problem. The problems that are solved by the Poisson Solver interface are defined more exactly in the [Poisson Solver Implementation](#) subsection. The Poisson Solver interface provides much flexibility of use: you can call routines with the default parameter values or adjust routines to your particular needs by manually tuning routine parameters. You can adjust the style of error and warning messages to a C notation by setting up a dedicated parameter. This adds convenience to debugging, because you can read information in the way that is natural for your code. The Intel MKL Poisson Solver interface currently contains only routines that implement the following solvers:

- Fast Laplace, Poisson and Helmholtz solvers in a Cartesian coordinate system
- Fast Poisson and Helmholtz solvers in a spherical coordinate system.

### Poisson Solver Implementation

Poisson Solver routines enable approximate solving of certain two-dimensional and three-dimensional problems. [Figure "Structure of the Poisson Solver"](#) shows the general structure of the Poisson Solver.

#### Structure of the Poisson Solver



#### NOTE

Although in the Cartesian case, both periodic and non-periodic solvers are also supported, they use the same interfaces.

Sections below provide details of the problems that can be solved using Intel MKL Poisson Solver.

### Two-Dimensional Problems

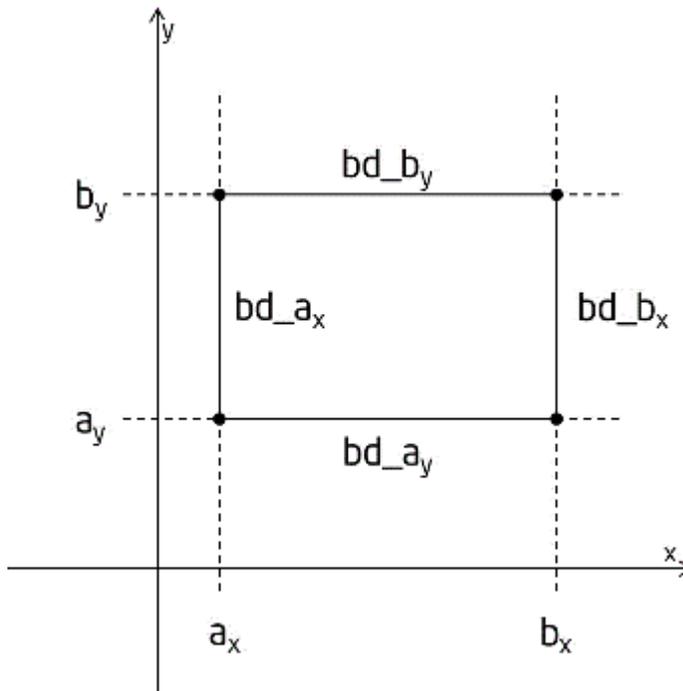
#### Notational Conventions

The Poisson Solver interface description uses the following notation for boundaries of a rectangular domain  $a_x < x < b_x, a_y < y < b_y$  on a Cartesian plane:

$$bd\_a_x = \{x = a_x, a_y \leq y \leq b_y\}, \quad bd\_b_x = \{x = b_x, a_y \leq y \leq b_y\}$$

$$bd\_a_y = \{a_x \leq x \leq b_x, y = a_y\}, \quad bd\_b_y = \{a_x \leq x \leq b_x, y = b_y\}.$$

The following figure shows these boundaries:



The wildcard "+" may stand for any of the symbols  $a_x, b_x, a_y, b_y$ , so  $bd\_+$  denotes any of the above boundaries.

The Poisson Solver interface description uses the following notation for boundaries of a rectangular domain  $a_\varphi < \varphi < b_\varphi, a_\theta < \theta < b_\theta$  on a sphere  $0 \leq \varphi \leq 2\pi, 0 \leq \theta \leq \pi$ :

$$bd\_a_\varphi = \{\varphi = a_\varphi, a_\theta \leq \theta \leq b_\theta\}, \quad bd\_b_\varphi = \{\varphi = b_\varphi, a_\theta \leq \theta \leq b_\theta\},$$

$$bd\_a_\theta = \{a_\varphi \leq \varphi \leq b_\varphi, \theta = a_\theta\}, \quad bd\_b_\theta = \{a_\varphi \leq \varphi \leq b_\varphi, \theta = b_\theta\}.$$

The wildcard "~" may stand for any of the symbols  $a_\varphi, b_\varphi, a_\theta, b_\theta$ , so  $bd\_~$  denotes any of the above boundaries.

#### Two-dimensional Helmholtz problem on a Cartesian plane

The two-dimensional (2D) Helmholtz problem is to find an approximate solution of the Helmholtz equation

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + qu = f(x, y), \quad q = const \geq 0$$

in a rectangle, that is, a rectangular domain  $a_x < x < b_x, a_y < y < b_y$ , with one of the following boundary conditions on each boundary  $bd\_+$ :

- The Dirichlet boundary condition

$$u(x, y) = G(x, y)$$

- The Neumann boundary condition

$$\frac{\partial u}{\partial n}(x, y) = g(x, y)$$

where

$$n = -x \text{ on } bd\_a_x, n = x \text{ on } bd\_b_x,$$

$$n = -y \text{ on } bd\_a_y, n = y \text{ on } bd\_b_y.$$

- Periodic boundary conditions

$$u(a_x, y) = u(b_x, y), \frac{\partial}{\partial x} u(a_x, y) = \frac{\partial}{\partial x} u(b_x, y),$$

$$u(x, a_y) = u(x, b_y), \frac{\partial}{\partial y} u(x, a_y) = \frac{\partial}{\partial y} u(x, b_y).$$

## Two-dimensional Poisson problem on a Cartesian plane

The Poisson problem is a special case of the Helmholtz problem, when  $q=0$ . The 2D Poisson problem is to find an approximate solution of the Poisson equation

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

in a rectangle  $a_x < x < b_x, a_y < y < b_y$  with the Dirichlet, Neumann, or periodic boundary conditions on each boundary  $bd\_+$ . In case of a problem with the Neumann boundary condition on the entire boundary, you can find the solution of the problem only up to a constant. In this case, the Poisson Solver will compute the solution that provides the minimal Euclidean norm of a residual.

## Two-dimensional (2D) Laplace problem on a Cartesian plane

The Laplace problem is a special case of the Helmholtz problem, when  $q=0$  and  $f(x, y)=0$ . The 2D Laplace problem is to find an approximate solution of the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in a rectangle  $a_x < x < b_x, a_y < y < b_y$  with the Dirichlet, Neumann, or periodic boundary conditions on each boundary  $bd\_+$ .

## Helmholtz problem on a sphere

The Helmholtz problem on a sphere is to find an approximate solution of the Helmholtz equation

$$-\Delta_s u + qu = f, q = const \geq 0,$$

$$\Delta_s = \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right)$$

in a domain bounded by angles  $a_\phi \leq \phi \leq b_\phi, a_\theta \leq \theta \leq b_\theta$  (spherical rectangle), with boundary conditions for particular domains listed in [Table "Details of Helmholtz Problem on a Sphere"](#).

## Details of Helmholtz Problem on a Sphere

| Domain on a sphere | Boundary condition                                                 | Periodic/non-periodic case |
|--------------------|--------------------------------------------------------------------|----------------------------|
|                    | Homogeneous Dirichlet boundary conditions on each boundary $bd\_~$ | <i>non-periodic</i>        |

| Domain on a sphere                                                                             | Boundary condition                                                                                                                                         | Periodic/non-periodic case |
|------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|
| Rectangular, that is, $b_\phi - a_\phi < 2\pi$ and $b_\theta - a_\theta < \pi$                 |                                                                                                                                                            |                            |
| Where $a_\phi = 0$ , $b_\phi = 2\pi$ , and $b_\theta - a_\theta < \pi$                         | Homogeneous Dirichlet boundary conditions on the boundaries $bd_{a_\theta}$ and $bd_{b_\theta}$                                                            | <i>periodic</i>            |
| Entire sphere, that is, $a_\phi = 0$ , $b_\phi = 2\pi$ , $a_\theta = 0$ , and $b_\theta = \pi$ | Boundary condition $\left(\sin\theta \frac{\partial u}{\partial \theta}\right) = 0$ at $\theta \rightarrow 0$ and $\theta \rightarrow \pi$<br>at the poles | <i>periodic</i>            |

**Poisson problem on a sphere**

The Poisson problem is a special case of the Helmholtz problem, when  $q=0$ . The Poisson problem on a sphere is to find an approximate solution of the Poisson equation

$$-\Delta_s u = f, \Delta_s = \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial \phi^2} + \frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial}{\partial \theta} \right)$$

in a spherical rectangle  $a_\phi \leq \phi \leq b_\phi$ ,  $a_\theta \leq \theta \leq b_\theta$  in cases listed in [Table "Details of Helmholtz Problem on a Sphere"](#). The solution to the Poisson problem on the entire sphere can be found up to a constant only. In this case, Poisson Solver will compute the solution that provides the minimal Euclidean norm of a residual.

**Approximation of 2D problems**

To find an approximate solution for any of the 2D problems, in the rectangular domain a uniform mesh can be defined for the Cartesian case as:

$$\{x_i = a_x + ih_x, y_j = a_y + jh_y\},$$

$$i = 0, \dots, n_x, j = 0, \dots, n_y, h_x = \frac{b_x - a_x}{n_x}, h_y = \frac{b_y - a_y}{n_y}$$

and for the spherical case as:

$$\{\phi_i = a_\phi + ih_\phi, \theta_j = a_\theta + jh_\theta\},$$

$$i = 0, \dots, n_\phi, j = 0, \dots, n_\theta, h_\phi = \frac{b_\phi - a_\phi}{n_\phi}, h_\theta = \frac{b_\theta - a_\theta}{n_\theta}.$$

The Poisson Solver uses the standard five-point finite difference approximation on this mesh to compute the approximation to the solution:

- In the Cartesian case, the values of the approximate solution will be computed in the mesh points  $(x_i, y_j)$  provided that you can supply the values of the right-hand side  $f(x, y)$  in these points and the values of the appropriate boundary functions  $G(x, y)$  and/or  $g(x, y)$  in the mesh points laying on the boundary of the rectangular domain.
- In the spherical case, the values of the approximate solution will be computed in the mesh points  $(\phi_i, \theta_j)$  provided that you can supply the values of the right-hand side  $f(\phi, \theta)$  in these points.

**NOTE**

The number of mesh intervals  $n_\phi$  in the  $\phi$  direction of a spherical mesh must be even in the periodic case. The Poisson Solver does not support spherical meshes that do not meet this condition.

**Three-Dimensional Problems**

**Notational Conventions**

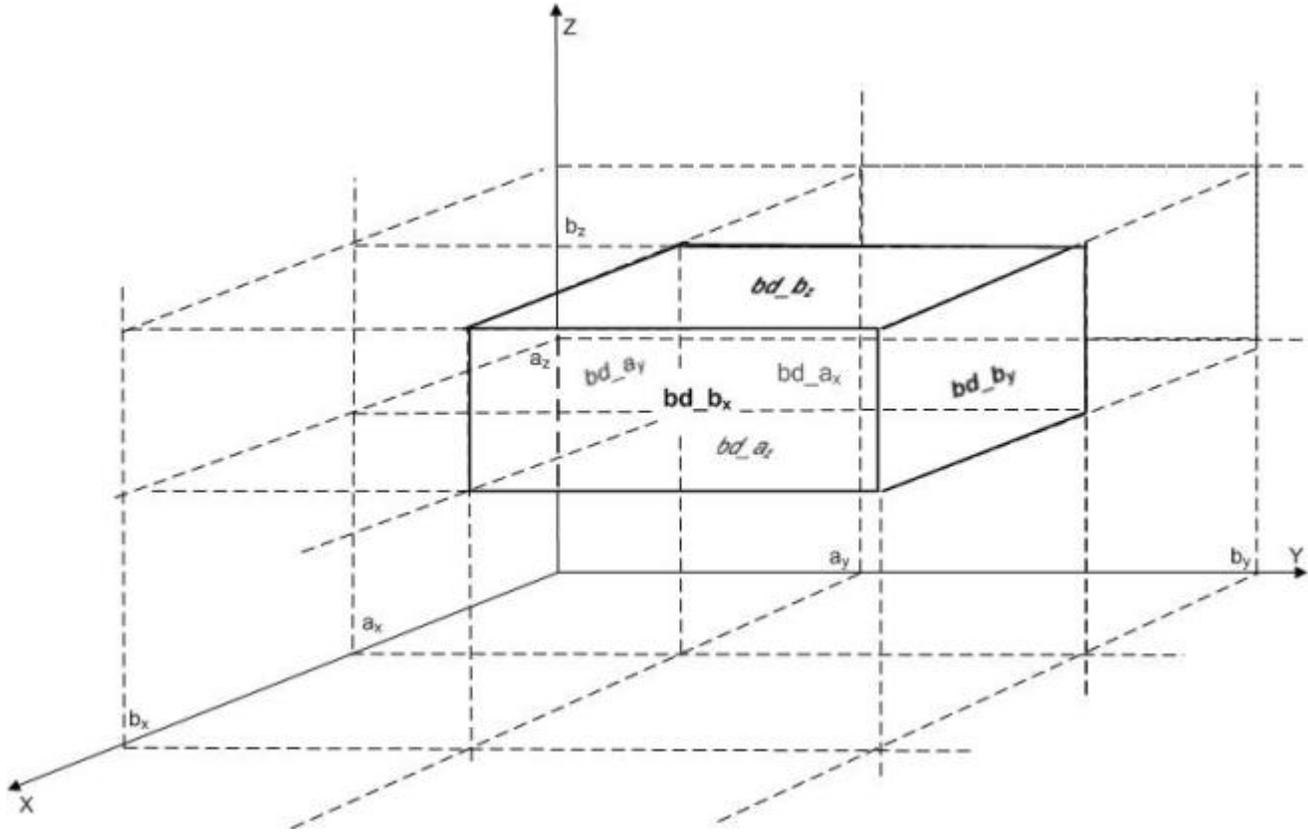
The Poisson Solver interface description uses the following notation for boundaries of a parallelepiped domain  $a_x < x < b_x, a_y < y < b_y, a_z < z < b_z$ :

$$bd\_a_x = \{x = a_x, a_y \leq y \leq b_y, a_z \leq z \leq b_z\}, bd\_b_x = \{x = b_x, a_y \leq y \leq b_y, a_z \leq z \leq b_z\},$$

$$bd\_a_y = \{a_x \leq x \leq b_x, y = a_y, a_z \leq z \leq b_z\}, bd\_b_y = \{a_x \leq x \leq b_x, y = b_y, a_z \leq z \leq b_z\},$$

$$bd\_a_z = \{a_x \leq x \leq b_x, a_y \leq y \leq b_y, z = a_z\}, bd\_b_z = \{a_x \leq x \leq b_x, a_y \leq y \leq b_y, z = b_z\}.$$

The following figure shows these boundaries:



The wildcard "+" may stand for any of the symbols  $a_x, b_x, a_y, b_y, a_z, b_z$ , so  $bd\_+$  denotes any of the above boundaries.

### Three-dimensional (3D) Helmholtz problem

The 3D Helmholtz problem is to find an approximate solution of the Helmholtz equation

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} + qu = f(x, y, z), \quad q = const \geq 0$$

in a parallelepiped, that is, a parallelepiped domain  $a_x < x < b_x, a_y < y < b_y, a_z < z < b_z$ , with one of the following boundary conditions on each boundary  $bd\_+$ :

- The Dirichlet boundary condition

$$u(x, y, z) = G(x, y, z)$$

- The Neumann boundary condition

$$\frac{\partial u}{\partial n}(x, y, z) = g(x, y, z)$$

where

$$n = -x \text{ on } bd\_a_x, n = x \text{ on } bd\_b_x,$$

$$n = -y \text{ on } bd_{a_y}, n = y \text{ on } bd_{b_y},$$

$$n = -z \text{ on } bd_{a_z}, n = z \text{ on } bd_{b_z}.$$

- Periodic boundary conditions

$$u(a_x, y, z) = u(b_x, y, z), \quad \frac{\partial}{\partial x} u(a_x, y, z) = \frac{\partial}{\partial x} u(b_x, y, z),$$

$$u(x, a_y, z) = u(x, b_y, z), \quad \frac{\partial}{\partial y} u(x, a_y, z) = \frac{\partial}{\partial y} u(x, b_y, z),$$

$$u(x, y, a_z) = u(x, y, b_z), \quad \frac{\partial}{\partial z} u(x, y, a_z) = \frac{\partial}{\partial z} u(x, y, b_z).$$

### Three-dimensional (3D) Poisson problem

The Poisson problem is a special case of the Helmholtz problem, when  $q=0$ . The 3D Poisson problem is to find an approximate solution of the Poisson equation

$$-\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2} = f(x, y, z)$$

in a parallelepiped  $a_x < x < b_x$ ,  $a_y < y < b_y$ ,  $a_z < z < b_z$  with the Dirichlet, Neumann, or periodic boundary conditions on each boundary  $bd_{\pm}$ .

### Three-dimensional (3D) Laplace problem

The Laplace problem is a special case of the Helmholtz problem, when  $q=0$  and  $f(x, y, z)=0$ . The 3D Laplace problem is to find an approximate solution of the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

in a parallelepiped  $a_x < x < b_x$ ,  $a_y < y < b_y$ ,  $a_z < z < b_z$  with the Dirichlet, Neumann, or periodic boundary conditions on each boundary  $bd_{\pm}$ .

### Approximation of 3D problems

To find an approximate solution for each of the 3D problems, a uniform mesh can be defined in the parallelepiped domain as:

$$\{x_i = a_x + ih_x, y_j = a_y + jh_y, z_k = a_z + kh_z\},$$

where

$$i = 0, \dots, n_x, \quad j = 0, \dots, n_y, \quad k = 0, \dots, n_z,$$

$$h_x = \frac{b_x - a_x}{n_x}, \quad h_y = \frac{b_y - a_y}{n_y}, \quad h_z = \frac{b_z - a_z}{n_z}.$$

The Poisson Solver uses the standard seven-point finite difference approximation on this mesh to compute the approximation to the solution. The values of the approximate solution will be computed in the mesh points  $(x_i, y_j, z_k)$ , provided that you can supply the values of the right-hand side  $f(x, y, z)$  in these points and the values of the appropriate boundary functions  $G(x, y, z)$  and/or  $g(x, y, z)$  in the mesh points laying on the boundary of the parallelepiped domain.

## Sequence of Invoking Poisson Solver Routines

### NOTE

This description always shows the solution process for the Helmholtz problem, because Fast Poisson Solvers and Fast Laplace Solvers are special cases of Fast Helmholtz Solvers (see [Poisson Solver Implementation](#)).

The Poisson Solver interface enables you to compute a solution of the Helmholtz problem in four steps. Each step is performed by a dedicated routine. [Table "Poisson Solver Interface Routines"](#) lists the routines and briefly describes their purpose.

Most Poisson Solver routines have versions operating with single-precision and double-precision data. Names of such routines begin respectively with "s" and "d". The wildcard "?" stands for either of these symbols in routine names. The routines for the Cartesian coordinate system have 2D and 3D versions. Their names end respectively in "2D" and "3D". The routines for spherical coordinate system have periodic and non-periodic versions. Their names end respectively in "p" and "np".

### Poisson Solver Interface Routines

| Routine                                                                                 | Description                                                                                                                                                             |
|-----------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np</code>         | Initializes basic data structures for Fast Helmholtz Solver in the 2D/3D/periodic/non-periodic case, respectively.                                                      |
| <code>?_commit_Helmholtz_2D/?_commit_Helmholtz_3D/?_commit_sph_p/?_commit_sph_np</code> | Checks consistency and correctness of input data and initializes data structures for the solver, including those used by the Intel MKL FFT interface <sup>1</sup> .     |
| <code>?_Helmholtz_2D/?_Helmholtz_3D/?_sph_p/?_sph_np</code>                             | Computes an approximate solution of the 2D/3D/periodic/non-periodic Helmholtz problem (see <a href="#">Poisson Solver Implementation</a> ) specified by the parameters. |
| <code>free_Helmholtz_2D/free_Helmholtz_3D/free_sph_p/free_sph_np</code>                 | Releases the memory used by the data structures needed for calling the Intel MKL FFT interface <sup>1</sup> .                                                           |

<sup>1</sup> Poisson Solver routines call the Intel MKL FFT interface for better performance.

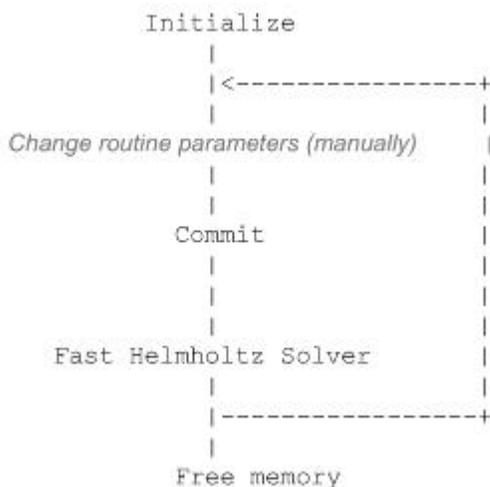
To find an approximate solution of Helmholtz problem only once, the Intel MKL Poisson Solver interface routines are normally invoked in the order in which they are listed in [Table "Poisson Solver Interface Routines"](#).

### NOTE

Though the order of invoking Poisson Solver routines may be changed, it is highly recommended to follow the above order of routine calls.

The diagram in [Figure "Typical Order of Invoking Poisson Solver Routines"](#) indicates the typical order in which Poisson Solver routines can be invoked in a general case.

### Typical Order of Invoking Poisson Solver Routines



A general scheme of using Poisson Solver routines for double-precision computations in a 3D Cartesian case is shown below. You can change this scheme to a scheme for single-precision computations by changing the initial letter of the Poisson Solver routine names from "d" to "s". You can also change the scheme below from the 3D to 2D case by changing the ending of the Poisson Solver routine names.

```

...
d_init_Helmholtz_3D(&ax, &bx, &ay, &by, &az, &bz, &nx, &ny, &nz, Bctype, ipar, dpar, &stat);
/* change parameters in ipar and/or dpar if necessary. */
/* note that the result of the Fast Helmholtz Solver will be in f. If you want to keep the data
that is stored in f, save it to another location before the function call below */
d_commit_Helmholtz_3D(f, bd_ax, bd_bx, bd_ay, bd_by, bd_az, bd_bz, &xhandle, &yhandle, ipar, dpar,
&stat);
d_Helmholtz_3D(f, bd_ax, bd_bx, bd_ay, bd_by, bd_az, bd_bz, &xhandle, &yhandle, ipar, dpar, &stat);
free_Helmholtz_3D (&xhandle, &yhandle, ipar, &stat);
/* here you may clean the memory used by f, dpar, ipar */
...

```

A general scheme of using Poisson Solver routines for double-precision computations in a spherical periodic case is shown below. You can change this scheme to a scheme for single-precision computations by changing the initial letter of the Poisson Solver routine names from "d" to "s". You can also change the scheme below to a scheme for a non-periodic case by changing the ending of the Poisson Solver routine names from "p" to "np".

```

...
d_init_sph_p(&ap, &bp, &at, &bt, &np, &nt, &q, ipar, dpar, &stat);
/* change parameters in ipar and/or dpar if necessary. */
/* note that the result of the Fast Helmholtz Solver will be in f. If you want to keep the data
that is stored in f, save it to another location before the function call below */
d_commit_sph_p(f, &handle_s, &handle_c, ipar, dpar, &stat);
d_sph_p(f, &handle_s, &handle_c, ipar, dpar, &stat);
free_sph_p(&handle_s, &handle_c, ipar, &stat);
/* here you may clean the memory used by f, dpar, ipar */
...

```

You can find examples of code that uses Poisson Solver routines to solve Helmholtz problem (in both Cartesian and spherical cases) in the `examples\pdepoissonc\source` folder in your Intel MKL directory.

## Fast Poisson Solver Interface Description

All numerical types in this section are either standard C types `float` and `double` or `MKL_INT` integer type. For more information on the C types, refer to [C Datatypes Specific to Intel MKL](#) and the *Intel(R) MKL Developer Guide*. To better understand usage of the types, see examples in the `examples\pdepoissonc\source` folder in your Intel MKL directory.

### Routine Options

All Poisson Solver routines use parameters for passing various options to the routines. These parameters are arrays `ipar`, `dpar`, and `spar`. Values for these parameters should be specified very carefully (see [Common Parameters](#)). You can change these values during computations to meet your needs. For more details, see the descriptions of specific routines.

---

**WARNING**

To avoid failure or incorrect results, you must provide correct and consistent parameters to the routines.

---

### User Data Arrays

Poisson Solver routines take arrays of user data as input. For example, the `d_Helmholtz_3D` routine takes user arrays to compute an approximate solution to the 3D Helmholtz problem. To minimize storage requirements and improve the overall run-time efficiency, Intel MKL Poisson Solver routines do not make copies of user input arrays.

---

**NOTE**

If you need a copy of your input data arrays, you must save them yourself.

---

For better performance, align your data arrays as recommended in the *Intel MKL Developer Guide* (search the document for coding techniques to improve performance).

## Routines for the Cartesian Solver

The section describes Poisson Solver routines for the Cartesian case, their syntax, parameters, and return values. All flavors of the same routine are described together: single- and double-precision and 2D and 3D.

---

**NOTE**

Some of the routine parameters are used only in the 3D Fast Helmholtz Solver.

---

Poisson Solver routines call Intel MKL FFT routines (described in section "[FFT Functions](#)" in chapter "Fast Fourier Transforms"), which enhance performance of the Poisson Solver routines.

### ?\_init\_Helmholtz\_2D/?\_init\_Helmholtz\_3D

*Initializes basic data structures of the Fast 2D/3D Helmholtz Solver.*

---

#### Syntax

```
void d_init_Helmholtz_2D (const double * ax, const double * bx, const double * ay,
const double * by, const MKL_INT * nx, const MKL_INT * ny, const char * BCtype, const
double * q, MKL_INT * ipar, double * dpar, MKL_INT * stat);
```

```
void s_init_Helmholtz_2D (const float * ax, const float * bx, const float * ay, const
float * by, const MKL_INT * nx, const MKL_INT * ny, const char * Bctype, const float *
q, MKL_INT * ipar, float * spar, MKL_INT * stat);
```

```
void d_init_Helmholtz_3D (const double * ax, const double * bx, const double * ay,
const double * by, const double * az, const double * bz, const MKL_INT * nx, const
MKL_INT * ny, const MKL_INT * nz, const char * Bctype, const double * q, MKL_INT
*ipar, double * dpar, MKL_INT * stat);
```

```
void s_init_Helmholtz_3D (const float * ax, const float * bx, const float * ay, const
float * by, const float * az, const float * bz, const MKL_INT * nx, const MKL_INT *
ny, const MKL_INT * nz, const char * Bctype, const float * q, MKL_INT * ipar, float *
spar, MKL_INT * stat);
```

## Include Files

- mkl.h

## Input Parameters

|           |                                                                                                                                                                                                                       |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ax</i> | double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D,<br>float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D.<br>The coordinate of the leftmost boundary of the domain along the x-axis.                                |
| <i>bx</i> | double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D,<br>float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D.<br>The coordinate of the rightmost boundary of the domain along the x-axis.                               |
| <i>ay</i> | double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D,<br>float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D.<br>The coordinate of the leftmost boundary of the domain along the y-axis.                                |
| <i>by</i> | double* for d_init_Helmholtz_2D/d_init_Helmholtz_3D,<br>float* for s_init_Helmholtz_2D/s_init_Helmholtz_3D.<br>The coordinate of the rightmost boundary of the domain along the y-axis.                               |
| <i>az</i> | double* for d_init_Helmholtz_3D,<br>float* for s_init_Helmholtz_3D.<br>The coordinate of the leftmost boundary of the domain along the z-axis.<br>This parameter is needed only for the ?_init_Helmholtz_3D routine.  |
| <i>bz</i> | double* for d_init_Helmholtz_3D,<br>float* for s_init_Helmholtz_3D.<br>The coordinate of the rightmost boundary of the domain along the z-axis.<br>This parameter is needed only for the ?_init_Helmholtz_3D routine. |
| <i>nx</i> | MKL_INT*. The number of mesh intervals along the x-axis.                                                                                                                                                              |
| <i>ny</i> | MKL_INT*. The number of mesh intervals along the y-axis.                                                                                                                                                              |

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>nz</i>     | MKL_INT*. The number of mesh intervals along the z-axis. This parameter is needed only for the <code>?_init_Helmholtz_3D</code> routine.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>BCtype</i> | char*. Contains the type of boundary conditions on each boundary. Must contain four characters for <code>?_init_Helmholtz_2D</code> and six characters for <code>?_init_Helmholtz_3D</code> . Each of the characters can be 'N' (Neumann boundary condition), 'D' (Dirichlet boundary condition), or 'P' (periodic boundary conditions). Specify the types of boundary conditions for the boundaries in the following order: <i>bd_ax</i> , <i>bd_bx</i> , <i>bd_ay</i> , <i>bd_by</i> , <i>bd_az</i> , and <i>bd_bz</i> . Specify periodic boundary conditions on the respective boundaries in pairs (for example, 'PPDD' or 'NNPP' in the 2D case). The types of boundary conditions for the last two boundaries are needed only in the 3D case. |
| <i>q</i>      | double* for <code>d_init_Helmholtz_2D/d_init_Helmholtz_3D</code> ,<br>float* for <code>s_init_Helmholtz_2D/s_init_Helmholtz_3D</code> .<br><br>The constant Helmholtz coefficient. Note that to solve Poisson or Laplace problem, you should set the value of <i>q</i> to 0.                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |

## Output Parameters

|             |                                                                                                                                                                                                                                               |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> | MKL_INT array of size 128. Contains integer data to be used by Fast Helmholtz Solver (for details, refer to <a href="#">ipar</a> ).                                                                                                           |
| <i>dpar</i> | double array of size $5 * n_x / 2 + 7$ in the 2D case or $5 * (n_x + n_y) / 2 + 9$ in the 3D case. Contains double-precision data to be used by Fast Helmholtz Solver (for details, refer to <a href="#">dpar</a> and <a href="#">spar</a> ). |
| <i>spar</i> | float array of size $5 * n_x / 2 + 7$ in the 2D case or $5 * (n_x + n_y) / 2 + 9$ in the 3D case. Contains single-precision data to be used by Fast Helmholtz Solver (for details, refer to <a href="#">dpar</a> and <a href="#">spar</a> ).  |
| <i>stat</i> | MKL_INT*. Routine completion status, which is also written to <code>ipar[0]</code> . Continue to call other Poisson Solver routines only if the status is 0.                                                                                  |

## Description

The `?_init_Helmholtz_2D/?_init_Helmholtz_3D` routines initialize basic data structures for Poisson Solver computations of the appropriate precision. All routines invoked after a call to a `?_init_Helmholtz_2D/?_init_Helmholtz_3D` routine use values of the *ipar*, *dpar* and *spar* array parameters returned by the routine. Detailed description of the array parameters can be found in [Common Parameters](#).

---

### CAUTION

Data structures initialized and created by 2D flavors of the routine cannot be used by 3D flavors of any Poisson Solver routines, and vice versa.

---

You can skip calls to these routines in your code. However, see [Caveat on Parameter Modifications](#) for information on initializing the data structures.

## Return Values

`stat= 0`

The routine successfully completed the task. In general, to proceed with computations, the routine should complete with this `stat` value.

`stat= -99999`

The routine failed to complete the task because of a fatal error.

## `_commit_Helmholtz_2D/?_commit_Helmholtz_3D`

*Checks consistency and correctness of input data and initializes certain data structures required to solve 2D/3D Helmholtz problem.*

## Syntax

```
void d_commit_Helmholtz_2D (double * f, const double * bd_ax, const double * bd_bx,
const double * bd_ay, const double * bd_by, DFTI_DESCRIPTOR_HANDLE * xhandle, MKL_INT *
ipar, double * dpar, MKL_INT * stat);
```

```
void s_commit_Helmholtz_2D (float * f, const float * bd_ax, const float * bd_bx, const
float * bd_ay, const float * bd_by, DFTI_DESCRIPTOR_HANDLE * xhandle, MKL_INT * ipar,
float * spar, MKL_INT * stat);
```

```
void d_commit_Helmholtz_3D (double * f, const double * bd_ax, const double * bd_bx,
const double * bd_ay, const double * bd_by, const double * bd_az, const double * bd_bz,
DFTI_DESCRIPTOR_HANDLE * xhandle, DFTI_DESCRIPTOR_HANDLE * yhandle, MKL_INT * ipar,
double * dpar, MKL_INT * stat);
```

```
void s_commit_Helmholtz_3D (float * f, const float * bd_ax, const float * bd_bx, const
float * bd_ay, const float * bd_by, const float * bd_az, const float * bd_bz,
DFTI_DESCRIPTOR_HANDLE * xhandle, DFTI_DESCRIPTOR_HANDLE * yhandle, MKL_INT * ipar,
float * spar, MKL_INT * stat);
```

## Include Files

- `mk1.h`

## Input Parameters

`f` double\* for `d_commit_Helmholtz_2D/d_commit_Helmholtz_3D`,  
float\* for `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D`.

Contains the right-hand side of the problem packed in a single vector:

- 2D problem: The size of the vector for the is  $(n_x+1)*(n_y+1)$ . The value of the right-hand side in the mesh point  $(i, j)$  is stored in  $f[i+j*(n_x+1)]$ .
- 3D problem: The size of the vector for the is  $(n_x+1)*(n_y+1)*(n_z+1)$ . The value of the right-hand side in the mesh point  $(i, j, k)$  is stored in  $f[i+j*(n_x+1)+k*(n_x+1)*(n_y+1)]$ .

Note that to solve the Laplace problem, you should set all the elements of the array `f` to 0.

Note also that the array `f` may be altered by the routine. To preserve the `f` vector, save it to another memory location.

|              |                                                                                                                                                                                                                                                                                                                                                                              |
|--------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i>  | <p>MKL_INT array of size 128. Contains integer data to be used by the Fast Helmholtz Solver (for details, refer to <a href="#">ipar</a>).</p>                                                                                                                                                                                                                                |
| <i>dpar</i>  | <p>double array of size depending on the dimension of the problem:</p> <ul style="list-style-type: none"> <li>• 2D problem: <math>5 \cdot n_x / 2 + 7</math></li> <li>• 3D problem: <math>5 \cdot (n_x + n_y) / 2 + 9</math></li> </ul> <p>Contains double-precision data to be used by the Fast Helmholtz Solver (for details, refer to <a href="#">dpar and spar</a>).</p> |
| <i>spar</i>  | <p>float array of size depending on the dimension of the problem:</p> <ul style="list-style-type: none"> <li>• 2D problem: <math>5 \cdot n_x / 2 + 7</math></li> <li>• 3D problem: <math>5 \cdot (n_x + n_y) / 2 + 9</math></li> </ul> <p>Contains single-precision data to be used by the Fast Helmholtz Solver (for details, refer to <a href="#">dpar and spar</a>).</p>  |
| <i>bd_ax</i> | <p>double* for d_commit_Helmholtz_2D/d_commit_Helmholtz_3D,<br/>float* for s_commit_Helmholtz_2D/s_commit_Helmholtz_3D.</p> <p>Contains values of the boundary condition on the leftmost boundary of the domain along the x-axis (for more information, refer to <a href="#">a detailed description of bd_ax</a>).</p>                                                       |
| <i>bd_bx</i> | <p>double* for d_commit_Helmholtz_2D/d_commit_Helmholtz_3D,<br/>float* for s_commit_Helmholtz_2D/s_commit_Helmholtz_3D.</p> <p>Contains values of the boundary condition on the rightmost boundary of the domain along the x-axis (for more information, refer to <a href="#">a detailed description of bd_bx</a>).</p>                                                      |
| <i>bd_ay</i> | <p>double* for d_commit_Helmholtz_2D/d_commit_Helmholtz_3D,<br/>float* for s_commit_Helmholtz_2D/s_commit_Helmholtz_3D.</p> <p>Contains values of the boundary condition on the leftmost boundary of the domain along the y-axis (for more information, refer to <a href="#">a detailed description of bd_ay</a>).</p>                                                       |
| <i>bd_by</i> | <p>double* for d_commit_Helmholtz_2D/d_commit_Helmholtz_3D,<br/>float* for s_commit_Helmholtz_2D/s_commit_Helmholtz_3D.</p> <p>Contains values of the boundary condition on the rightmost boundary of the domain along the y-axis (for more information, refer to <a href="#">a detailed description of bd_by</a>).</p>                                                      |
| <i>bd_az</i> | <p>double* for d_commit_Helmholtz_3D,<br/>float* for s_commit_Helmholtz_3D.</p> <p>Used only by ?_commit_Helmholtz_3D. Contains values of the boundary condition on the leftmost boundary of the domain along the z-axis (for more information, refer to <a href="#">a detailed description of bd_az</a>).</p>                                                               |
| <i>bd_bz</i> | <p>double* for d_commit_Helmholtz_3D,<br/>float* for s_commit_Helmholtz_3D.</p>                                                                                                                                                                                                                                                                                              |

Used only by `?_commit_Helmholtz_3D`. Contains values of the boundary condition on the rightmost boundary of the domain along the z-axis (for more information, refer to [a detailed description of `bd\_bz`](#)).

## Output Parameters

|                               |                                                                                                                                                                                                                                                                              |
|-------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>f</code>                | Contains right-hand side of the problem, possibly altered on output.                                                                                                                                                                                                         |
| <code>ipar</code>             | Contains integer data to be used by Fast Helmholtz Solver. Modified on output as explained in <a href="#">ipar</a> .                                                                                                                                                         |
| <code>dpar</code>             | Contains double-precision data to be used by Fast Helmholtz Solver. Modified on output as explained in <a href="#">dpar</a> and <a href="#">spar</a> .                                                                                                                       |
| <code>spar</code>             | Contains single-precision data to be used by Fast Helmholtz Solver. Modified on output as explained in <a href="#">dpar</a> and <a href="#">spar</a> .                                                                                                                       |
| <code>xhandle, yhandle</code> | <code>DFTI_DESCRIPTOR_HANDLE*</code> . Data structures used by the Intel MKL FFT interface (for details, refer to section " <a href="#">FFT Functions</a> " in chapter "Fast Fourier Transforms"). <code>yhandle</code> is used only by <code>?_commit_Helmholtz_3D</code> . |
| <code>stat</code>             | <code>MKL_INT*</code> . Routine completion status, which is also written to <code>ipar[0]</code> . Continue to call other Poisson Solver routines only if the status is 0.                                                                                                   |

## Description

The `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routines check the consistency and correctness of the parameters to be passed to the solver routines `?_Helmholtz_2D/?_Helmholtz_3D`. They also initialize the `xhandle` and `yhandle` data structures, `ipar` array, and `dpar` or `spar` array, depending upon the routine precision. Refer to [Common Parameters](#) to find out which particular array elements the `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routines initialize and to what values these elements are initialized.

The routines perform only a basic check for correctness and consistency. If you are going to modify parameters of Poisson Solver routines, see [Caveat on Parameter Modifications](#). Unlike `?_init_Helmholtz_2D/?_init_Helmholtz_3D`, you must call the `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routines in your code. Values of `ax`, `bx`, `ay`, `by`, `az`, and `bz` are passed to the routines with the `spar/dpar` array, and values of `nx`, `ny`, `nz`, and `BCTYPE` are passed with the `ipar` array.

## Return Values

|                           |                                                                                                                                                                                 |
|---------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>stat= 1</code>      | The routine completed without errors but with warnings.                                                                                                                         |
| <code>stat= 0</code>      | The routine successfully completed the task.                                                                                                                                    |
| <code>stat= -100</code>   | The routine stopped because an error in the input data was found, or the data in the <code>dpar</code> , <code>spar</code> , or <code>ipar</code> array was altered by mistake. |
| <code>stat= -1000</code>  | The routine stopped because of an Intel MKL FFT or TT interface error.                                                                                                          |
| <code>stat= -10000</code> | The routine stopped because the initialization failed to complete or the parameter <code>ipar[0]</code> was altered by mistake.                                                 |

`stat= -99999`

The routine failed to complete the task because of a fatal error.

## ?\_Helmholtz\_2D/?\_Helmholtz\_3D

Computes the solution of the 2D/3D Helmholtz problem specified by the parameters.

### Syntax

```
void d_Helmholtz_2D (double * f, const double * bd_ax, const double * bd_bx, const
double * bd_ay, const double *bd_by, DFTI_DESCRIPTOR_HANDLE * xhandle, MKL_INT * ipar,
const double * dpar, MKL_INT * stat);

void s_Helmholtz_2D (float * f, const float * bd_ax, const float * bd_bx, const float *
bd_ay, const float * bd_by, DFTI_DESCRIPTOR_HANDLE * xhandle, MKL_INT * ipar, const
float * spar, MKL_INT * stat);

void d_Helmholtz_3D (double * f, const double * bd_ax, const double * bd_bx, const
double * bd_ay, const double *bd_by, const double * bd_az, const double * bd_bz,
DFTI_DESCRIPTOR_HANDLE * xhandle, DFTI_DESCRIPTOR_HANDLE * yhandle, MKL_INT * ipar,
const double * dpar, MKL_INT * stat);

void s_Helmholtz_3D (float * f, const float * bd_ax, const float * bd_bx, const float *
bd_ay, const float * bd_by, const float * bd_az, const float * bd_bz,
DFTI_DESCRIPTOR_HANDLE * xhandle, DFTI_DESCRIPTOR_HANDLE * yhandle, MKL_INT * ipar,
const float * spar, MKL_INT * stat);
```

### Include Files

- `mkl.h`

### Input Parameters

`f` double\* for `d_Helmholtz_2D/d_Helmholtz_3D`,  
float\* for `s_Helmholtz_2D/s_Helmholtz_3D`.

Contains the right-hand side of the problem packed in a single vector and modified by the appropriate `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routine. Note that an attempt to substitute the original right-hand side vector, which was passed to the `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routine, at this point results in an incorrect solution.

- 2D problem: the size of the vector is  $(n_x+1)*(n_y+1)$ . The value of the modified right-hand side in the mesh point  $(i, j)$  is stored in  $f[i+j*(n_x+1)]$ .
- 3D problem: the size of the vector is  $(n_x+1)*(n_y+1)*(n_z+1)$ . The value of the modified right-hand side in the mesh point  $(i, j, k)$  is stored in  $f[i+j*(n_x+1)+k*(n_x+1)*(n_y+1)]$ .

`xhandle, yhandle` DFTI\_DESCRIPTOR\_HANDLE\*. Data structures used by the Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). `yhandle` is used only by `?_Helmholtz_3D`.

|              |                                                                                                                                                                                                                                                                                                                                                                          |
|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i>  | MKL_INT array of size 128. Contains integer data to be used by Fast Helmholtz Solver (for details, refer to <a href="#">ipar</a> ).                                                                                                                                                                                                                                      |
| <i>dpar</i>  | <p>double array of size depending on the dimension of the problem:</p> <ul style="list-style-type: none"> <li>• 2D problem: <math>5 \cdot n_x / 2 + 7</math></li> <li>• 3D problem: <math>5 \cdot (n_x + n_y) / 2 + 9</math></li> </ul> <p>Contains double-precision data to be used by Fast Helmholtz Solver (for details, refer to <a href="#">dpar and spar</a>).</p> |
| <i>spar</i>  | <p>float array of size depending on the dimension of the problem:</p> <ul style="list-style-type: none"> <li>• 2D problem: <math>5 \cdot n_x / 2 + 7</math></li> <li>• 3D problem: <math>5 \cdot (n_x + n_y) / 2 + 9</math></li> </ul> <p>Contains single-precision data to be used by Fast Helmholtz Solver (for details, refer to <a href="#">dpar and spar</a>).</p>  |
| <i>bd_ax</i> | <p>double* for d_Helmholtz_2D/d_Helmholtz_3D,<br/>float* for s_Helmholtz_2D/s_Helmholtz_3D.</p> <p>Contains values of the boundary condition on the leftmost boundary of the domain along the x-axis (for more information, refer to <a href="#">a detailed description of bd_ax</a>).</p>                                                                               |
| <i>bd_bx</i> | <p>double* for d_Helmholtz_2D/d_Helmholtz_3D,<br/>float* for s_Helmholtz_2D/s_Helmholtz_3D.</p> <p>Contains values of the boundary condition on the rightmost boundary of the domain along the x-axis (for more information, refer to <a href="#">a detailed description of bd_bx</a>).</p>                                                                              |
| <i>bd_ay</i> | <p>double* for d_Helmholtz_2D/d_Helmholtz_3D,<br/>float* for s_Helmholtz_2D/s_Helmholtz_3D.</p> <p>Contains values of the boundary condition on the leftmost boundary of the domain along the y-axis for more information, refer to <a href="#">a detailed description of bd_ay</a>).</p>                                                                                |
| <i>bd_by</i> | <p>double* for d_Helmholtz_2D/d_Helmholtz_3D,<br/>float* for s_Helmholtz_2D/s_Helmholtz_3D.</p> <p>Contains values of the boundary condition on the rightmost boundary of the domain along the y-axis (for more information, refer to <a href="#">a detailed description of bd_by</a>).</p>                                                                              |
| <i>bd_az</i> | <p>double* for d_Helmholtz_3D,<br/>float* for s_Helmholtz_3D.</p> <p>Used only by ?_Helmholtz_3D. Contains values of the boundary condition on the leftmost boundary of the domain along the z-axis (for more information, refer to <a href="#">a detailed description of bd_az</a>).</p>                                                                                |
| <i>bd_bz</i> | <p>double* for d_Helmholtz_3D,<br/>float* for s_Helmholtz_3D.</p>                                                                                                                                                                                                                                                                                                        |

Used only by `?_Helmholtz_3D`. Contains values of the boundary condition on the rightmost boundary of the domain along the z-axis (for more information, refer to [a detailed description of `bd\_bz`](#)).

---

**NOTE**

To avoid incorrect computation results, do not change arrays `bd_ax`, `bd_bx`, `bd_ay`, `bd_by`, `bd_az`, `bd_bz` between a call to the `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routine and a subsequent call to the appropriate `?_Helmholtz_2D/?_Helmholtz_3D` routine.

---

## Output Parameters

|                               |                                                                                                                                                              |
|-------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>f</code>                | On output, contains the approximate solution to the problem packed the same way as the right-hand side of the problem was packed on input.                   |
| <code>xhandle, yhandle</code> | Data structures used by the Intel MKL FFT interface. Although the addresses do not change, the structures are modified on output.                            |
| <code>ipar</code>             | Contains integer data to be used by Fast Helmholtz Solver. Modified on output as explained in <a href="#">ipar</a> .                                         |
| <code>stat</code>             | MKL_INT*. Routine completion status, which is also written to <code>ipar[0]</code> . Continue to call other Poisson Solver routines only if the status is 0. |

## Description

The `?_Helmholtz_2D/?_Helmholtz_3D` routines compute the approximate solution of the Helmholtz problem defined in the previous calls to the corresponding initialization and commit routines. The solution is computed according to formulas given in the [Poisson Solver Implementation](#) section. The `f` parameter, which initially holds the packed vector of the right-hand side of the problem, is replaced by the computed solution packed in the same way. Values of `ax`, `bx`, `ay`, `by`, `az`, and `bz` are passed to the routines with the `spar/dpar` array, and values of `nx`, `ny`, `nz`, and `Bctype` are passed with the `ipar` array.

## Return Values

|                          |                                                                                                                                                                                |
|--------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>stat= 1</code>     | The routine completed without errors but with some warnings.                                                                                                                   |
| <code>stat= 0</code>     | The routine successfully completed the task.                                                                                                                                   |
| <code>stat= -2</code>    | The routine stopped because division by zero occurred. It usually happens if the data in the <code>dpar</code> or <code>spar</code> array was altered by mistake.              |
| <code>stat= -3</code>    | The routine stopped because the sufficient memory was unavailable for the computations.                                                                                        |
| <code>stat= -100</code>  | The routine stopped because an error in the input data was found or the data in the <code>dpar</code> , <code>spar</code> , or <code>ipar</code> array was altered by mistake. |
| <code>stat= -1000</code> | The routine stopped because of the Intel MKL FFT or TT interface error.                                                                                                        |

`stat= -10000`

The routine stopped because the initialization failed to complete or the parameter `ipar[0]` was altered by mistake.

`stat= -99999`

The routine failed to complete the task because of a fatal error.

## free\_Helmholtz\_2D/free\_Helmholtz\_3D

Releases the memory allocated for the data structures used by the FFT interface.

### Syntax

```
void free_Helmholtz_2D(DFTI_DESCRIPTOR_HANDLE* xhandle, MKL_INT* ipar, MKL_INT* stat);
void free_Helmholtz_3D(DFTI_DESCRIPTOR_HANDLE* xhandle, DFTI_DESCRIPTOR_HANDLE*
yhandle, MKL_INT* ipar, MKL_INT* stat);
```

### Include Files

- `mkl.h`

### Input Parameters

`xhandle, yhandle`

`DFTI_DESCRIPTOR_HANDLE*`. Data structures used by the Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). The structure `yhandle` is used only by `free_Helmholtz_3D`.

`ipar`

`MKL_INT` array of size 128. Contains integer data used by Fast Helmholtz Solver (for details, refer to `ipar`).

### Output Parameters

`xhandle, yhandle`

Data structures used by the Intel MKL FFT interface. Memory allocated for the structures is released on output.

`ipar`

Contains integer data used by Fast Helmholtz Solver. On output, the status of the routine call is written to `ipar[0]`.

`stat`

`MKL_INT*`. Routine completion status, which is also written to `ipar[0]`.

### Description

The `free_Helmholtz_2D-free_Helmholtz_3D` routine releases the memory used by the `xhandle` and `yhandle` structures, which are needed for calling the Intel MKL FFT functions. To release memory allocated for other parameters, include memory release statements in your code.

### Return Values

`stat= 0`

The routine successfully completed the task.

`stat= -1000`

The routine stopped because of an Intel MKL FFT or TT interface error.

`stat= -99999`

The routine failed to complete the task because of a fatal error.

## Routines for the Spherical Solver

The section describes Poisson Solver routines for the spherical case, their syntax, parameters, and return values. All flavors of the same routine are described together: single- and double-precision and periodic (having names ending in "p") and non-periodic (having names ending in "np").

These Poisson Solver routines also call the Intel MKL FFT routines (described in section "FFT Functions" in chapter "Fast Fourier Transforms"), which enhance the performance of the Poisson Solver routines.

### ?\_init\_sph\_p/?\_init\_sph\_np

*Initializes basic data structures of the periodic and non-periodic Fast Helmholtz Solver on a sphere.*

---

#### Syntax

```
void d_init_sph_p (const double * ap, const double * at, const double * bp, const
double * bt, const MKL_INT * np, const MKL_INT * nt, const double * q, MKL_INT * ipar,
double * dpar, MKL_INT * stat);
```

```
void s_init_sph_p (const float * ap, const float * at, const float * bp, const float *
bt, const MKL_INT * np, const MKL_INT * nt, const float * q, MKL_INT * ipar, float *
spar, MKL_INT * stat);
```

```
void d_init_sph_np (const double * ap, const double * at, const double * bp, const
double * bt, const MKL_INT * np, const MKL_INT * nt, const double * q, MKL_INT * ipar,
double * dpar, MKL_INT * stat);
```

```
void s_init_sph_np (const float * ap, const float * at, const float * bp, const float *
bt, const MKL_INT * np, const MKL_INT * nt, const float * q, MKL_INT * ipar, float *
spar, MKL_INT * stat);
```

#### Include Files

- `mkl.h`

#### Input Parameters

|                 |                                                                                                                                                                                                                                |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ap</code> | <p>double* for <code>d_init_sph_p/d_init_sph_np</code>,<br/>float* for <code>s_init_sph_p/s_init_sph_np</code>.</p> <p>The coordinate (angle) of the leftmost boundary of the domain along the <math>\varphi</math>-axis.</p>  |
| <code>bp</code> | <p>double* for <code>d_init_sph_p/d_init_sph_np</code>,<br/>float* for <code>s_init_sph_p/s_init_sph_np</code>.</p> <p>The coordinate (angle) of the rightmost boundary of the domain along the <math>\varphi</math>-axis.</p> |
| <code>at</code> | <p>double* for <code>d_init_sph_p/d_init_sph_np</code>,<br/>float* for <code>s_init_sph_p/s_init_sph_np</code>.</p>                                                                                                            |

|           |                                                                                                                                                                                                                        |
|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|           | The coordinate (angle) of the leftmost boundary of the domain along the $\theta$ -axis.                                                                                                                                |
| <i>bt</i> | double* for <code>d_init_sph_p/d_init_sph_np</code> ,<br>float* for <code>s_init_sph_p/s_init_sph_np</code> .<br><br>The coordinate (angle) of the rightmost boundary of the domain along the $\theta$ -axis.          |
| <i>np</i> | MKL_INT*. The number of mesh intervals along the $\varphi$ -axis. Must be even in the periodic case.                                                                                                                   |
| <i>nt</i> | MKL_INT*. The number of mesh intervals along the $\theta$ -axis.                                                                                                                                                       |
| <i>q</i>  | double* for <code>d_init_sph_p/d_init_sph_np</code> ,<br>float* for <code>s_init_sph_p/s_init_sph_np</code> .<br><br>The constant Helmholtz coefficient. To solve the Poisson problem, set the value of <i>q</i> to 0. |

## Output Parameters

|             |                                                                                                                                                                                              |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ipar</i> | MKL_INT array of size 128. Contains integer data to be used by Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">ipar</a> ).                                              |
| <i>dpar</i> | double array of size $5*np/2+nt+10$ . Contains double-precision data to be used by Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">dpar</a> and <a href="#">spar</a> ). |
| <i>spar</i> | float array of size $5*np/2+nt+10$ . Contains single-precision data to be used by Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">dpar</a> and <a href="#">spar</a> ).  |
| <i>stat</i> | MKL_INT*. Routine completion status, which is also written to <code>ipar[0]</code> . Continue to call other Poisson Solver routines only if the status is 0.                                 |

## Description

The `?_init_sph_p/?_init_sph_np` routines initialize basic data structures for Poisson Solver computations. All routines invoked after a call to a `?_init_Helmholtz_2D/?_init_Helmholtz_3D` routine use values of the *ipar*, *dpar*, and *spar* array parameters returned by the routine. A detailed description of the array parameters can be found in [Common Parameters](#).

---

### CAUTION

Data structures initialized and created by periodic flavors of the routine cannot be used by non-periodic flavors of any Poisson Solver routines for Helmholtz Solver on a sphere, and vice versa.

---

You can skip calls to these routines in your code. However, see [Caveat on Parameter Modifications](#) for information on initializing the data structures.

## Return Values

|                           |                                                                                                                                                       |
|---------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>stat= 0</code>      | The routine successfully completed the task. In general, to proceed with computations, the routine should complete with this <code>stat</code> value. |
| <code>stat= -99999</code> | The routine failed to complete the task because of fatal error.                                                                                       |

## ?\_commit\_sph\_p/?\_commit\_sph\_np

*Checks consistency and correctness of input data and initializes certain data structures required to solve the periodic/non-periodic Helmholtz problem on a sphere.*

## Syntax

```
void d_commit_sph_p(double* f, DFTI_DESCRIPTOR_HANDLE* handle_s,
DFTI_DESCRIPTOR_HANDLE* handle_c, MKL_INT* ipar, double* dpar, MKL_INT* stat);
void s_commit_sph_p(float* f, DFTI_DESCRIPTOR_HANDLE* handle_s, DFTI_DESCRIPTOR_HANDLE*
handle_c, MKL_INT* ipar, float* spar, MKL_INT* stat);
void d_commit_sph_np(double* f, DFTI_DESCRIPTOR_HANDLE* handle, MKL_INT* ipar, double*
dpar, MKL_INT* stat);
void s_commit_sph_np(float* f, DFTI_DESCRIPTOR_HANDLE* handle, MKL_INT* ipar, float*
spar, MKL_INT* stat);
```

## Include Files

- `mkl.h`

## Input Parameters

|                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>f</code>    | <p>double* for <code>d_commit_sph_p/d_commit_sph_np</code>,<br/>float* for <code>s_commit_sph_p/s_commit_sph_np</code>.</p> <p>Contains the right-hand side of the problem packed in a single vector. The size of the vector is <math>(np+1)*(nt+1)</math> and value of the right-hand side in the mesh point <math>(i, j)</math> is stored in <math>f[i+j*(np+1)]</math>.</p> <p>Note that the array <code>f</code> may be altered by the routine. Save this vector to another memory location if you want to preserve it.</p> |
| <code>ipar</code> | MKL_INT array of size 128. Contains integer data to be used by the Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">ipar</a> ).                                                                                                                                                                                                                                                                                                                                                                             |
| <code>dpar</code> | double array of size $5*np/2+nt+10$ . Contains double-precision data to be used by the Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">dpar</a> and <a href="#">spar</a> ).                                                                                                                                                                                                                                                                                                                                |
| <code>spar</code> | float array of size $5*np/2+nt+10$ . Contains single-precision data to be used by the Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">dpar</a> and <a href="#">spar</a> ).                                                                                                                                                                                                                                                                                                                                 |

## Output Parameters

|                                   |                                                                                                                                                                                                                                                                                                                                        |
|-----------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>f</i>                          | Contains the right-hand side of the problem, possibly altered on output.                                                                                                                                                                                                                                                               |
| <i>ipar</i>                       | Contains integer data to be used by the Fast Helmholtz Solver on a sphere. Modified on output as explained in <a href="#">ipar</a> .                                                                                                                                                                                                   |
| <i>dpar</i>                       | Contains double-precision data to be used by the Fast Helmholtz Solver on a sphere. Modified on output as explained in <a href="#">dpar</a> and <a href="#">spar</a> .                                                                                                                                                                 |
| <i>spar</i>                       | Contains single-precision data to be used by the Fast Helmholtz Solver on a sphere. Modified on output as explained in <a href="#">dpar</a> and <a href="#">spar</a> .                                                                                                                                                                 |
| <i>handle_s, handle_c, handle</i> | DFTI_DESCRIPTOR_HANDLE*. Data structures used by the Intel MKL FFT interface (for details, refer to section " <a href="#">FFT Functions</a> " in chapter "Fast Fourier Transforms"). <i>handle_s</i> and <i>handle_c</i> are used only in <code>?_commit_sph_p</code> and <i>handle</i> is used only in <code>?_commit_sph_np</code> . |
| <i>stat</i>                       | MKL_INT*. Routine completion status, which is also written to <i>ipar</i> [0]. Continue to call other Poisson Solver routines only if the status is 0.                                                                                                                                                                                 |

## Description

The `?_commit_sph_p/?_commit_sph_np` routines check consistency and correctness of the parameters to be passed to the solver routines `?_sph_p/?_sph_np`, respectively. They also initialize certain data structures. The routine `?_commit_sph_p` initializes structures *handle\_s* and *handle\_c*, and `?_commit_sph_np` initializes *handle*. The routines also initialize the *ipar* array and *dpar* or *spar* array, depending upon the routine precision. Refer to [Common Parameters](#) to find out which particular array elements the `?_commit_sph_p/?_commit_sph_np` routines initialize and to what values these elements are initialized.

The routines perform only a basic check for correctness and consistency. If you are going to modify parameters of Poisson Solver routines, see [Caveat on Parameter Modifications](#). Unlike `?_init_sph_p/?_init_sph_np`, you must call the `?_commit_sph_p/?_commit_sph_np` routines. Values of *np* and *nt* are passed to each of the routines with the *ipar* array.

## Return Values

|                      |                                                                                                                                                              |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>stat</i> = 1      | The routine completed without errors but with warnings.                                                                                                      |
| <i>stat</i> = 0      | The routine successfully completed the task.                                                                                                                 |
| <i>stat</i> = -100   | The routine stopped because an error in the input data was found or the data in the <i>dpar</i> , <i>spar</i> , or <i>ipar</i> array was altered by mistake. |
| <i>stat</i> = -1000  | The routine stopped because of an Intel MKL FFT or TT interface error.                                                                                       |
| <i>stat</i> = -10000 | The routine stopped because the initialization failed to complete or the parameter <i>ipar</i> [0] was altered by mistake.                                   |
| <i>stat</i> = -99999 | The routine failed to complete the task because of a fatal error.                                                                                            |

## ?\_sph\_p/?\_sph\_np

Computes the solution of the spherical Helmholtz problem specified by the parameters.

### Syntax

```
void d_sph_p(double* f, DFTI_DESCRIPTOR_HANDLE* handle_s, DFTI_DESCRIPTOR_HANDLE* handle_c, MKL_INT* ipar, double* dpar, MKL_INT* stat);
```

```
void s_sph_p(float* f, DFTI_DESCRIPTOR_HANDLE* handle_s, DFTI_DESCRIPTOR_HANDLE* handle_c, MKL_INT* ipar, float* spar, MKL_INT* stat);
```

```
void d_sph_np(double* f, DFTI_DESCRIPTOR_HANDLE* handle, MKL_INT* ipar, double* dpar, MKL_INT* stat);
```

```
void s_sph_np(float* f, DFTI_DESCRIPTOR_HANDLE* handle, MKL_INT* ipar, float* spar, MKL_INT* stat);
```

### Include Files

- mkl.h

### Input Parameters

|                                   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|-----------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>f</i>                          | double* for d_sph_p/d_sph_np,<br>float* for s_sph_p/s_sph_np.<br><br>Contains the right-hand side of the problem packed in a single vector and modified by the appropriate ?_commit_sph_p/?_commit_sph_np routine. Note that an attempt to substitute the original right-hand side vector, which was passed to the ?_commit_sph_p/?_commit_sph_np routine, at this point results in an incorrect solution.<br><br>The size of the vector is $(np+1)*(nt+1)$ and the value of the modified right-hand side in the mesh point $(i, j)$ is stored in $f[i+j*(np+1)]$ . |
| <i>handle_s, handle_c, handle</i> | DFTI_DESCRIPTOR_HANDLE*. Data structures used by Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). <i>handle_s</i> and <i>handle_c</i> are used only in ?_sph_p and <i>handle</i> is used only in ?_sph_np.                                                                                                                                                                                                                                                                                             |
| <i>ipar</i>                       | MKL_INT array of size 128. Contains integer data to be used by the Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">ipar</a> ).                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <i>dpar</i>                       | double array of size $5*np/2+nt+10$ . Contains double-precision data to be used by the Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">dpar and spar</a> ).                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>spar</i>                       | float array of size $5*np/2+nt+10$ . Contains single-precision data to be used by the Fast Helmholtz Solver on a sphere (for details, refer to <a href="#">dpar and spar</a> ).                                                                                                                                                                                                                                                                                                                                                                                     |

### Output Parameters

|          |                                                                                                                                            |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------|
| <i>f</i> | On output, contains the approximate solution to the problem packed the same way as the right-hand side of the problem was packed on input. |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------|

|                                                      |                                                                                                                                                                        |
|------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>handle_s</i> , <i>handle_c</i> ,<br><i>handle</i> | Data structures used by the Intel MKL FFT interface.                                                                                                                   |
| <i>ipar</i>                                          | Contains integer data to be used by the Fast Helmholtz Solver on a sphere. Modified on output as explained in <a href="#">ipar</a> .                                   |
| <i>dpar</i>                                          | Contains double-precision data to be used by the Fast Helmholtz Solver on a sphere. Modified on output as explained in <a href="#">dpar</a> and <a href="#">spar</a> . |
| <i>spar</i>                                          | Contains single-precision data to be used by the Fast Helmholtz Solver on a sphere. Modified on output as explained in <a href="#">dpar</a> and <a href="#">spar</a> . |
| <i>stat</i>                                          | MKL_INT*. Routine completion status, which is also written to <i>ipar[0]</i> . Continue to call other Poisson Solver routines only if the status is 0.                 |

## Description

The *sph\_p/sph\_np* routines compute the approximate solution on a sphere of the Helmholtz problem defined in the previous calls to the corresponding initialization and commit routines. The solution is computed according to the formulas given in the [Poisson Solver Implementation](#) section. The *f* parameter, which initially holds the packed vector of the right-hand side of the problem, is replaced by the computed solution packed in the same way. Values of *np* and *nt* are passed to each of the routines with the *ipar* array.

## Return Values

|                      |                                                                                                                                                              |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>stat</i> = 1      | The routine completed without errors but with warnings.                                                                                                      |
| <i>stat</i> = 0      | The routine successfully completed the task.                                                                                                                 |
| <i>stat</i> = -2     | The routine stopped because division by zero occurred. It usually happens if the data in the <i>dpar</i> or <i>spar</i> array was altered by mistake.        |
| <i>stat</i> = -3     | The routine stopped because the memory was insufficient to complete the computations.                                                                        |
| <i>stat</i> = -100   | The routine stopped because an error in the input data was found or the data in the <i>dpar</i> , <i>spar</i> , or <i>ipar</i> array was altered by mistake. |
| <i>stat</i> = -1000  | The routine stopped because of an Intel MKL FFT or TT interface error.                                                                                       |
| <i>stat</i> = -10000 | The routine stopped because the initialization failed to complete or the parameter <i>ipar[0]</i> was altered by mistake.                                    |
| <i>stat</i> = -99999 | The routine failed to complete the task because of a fatal error.                                                                                            |

## **free\_sph\_p/free\_sph\_np**

*Releases the memory allocated for the data structures used by the FFT interface.*

---

## Syntax

```
void free_sph_p(DFTI_DESCRIPTOR_HANDLE* handle_s, DFTI_DESCRIPTOR_HANDLE* handle_c,
MKL_INT* ipar, MKL_INT* stat);
```

```
void free_sph_np(DFTI_DESCRIPTOR_HANDLE* handle, MKL_INT* ipar, MKL_INT* stat);
```

## Include Files

- mkl.h

## Input Parameters

*handle\_s*, *handle\_c*,  
*handle*                    DFTI\_DESCRIPTOR\_HANDLE\*. Data structures used by the Intel MKL FFT interface (for details, refer to section "FFT Functions" in chapter "Fast Fourier Transforms"). The structures *handle\_s* and *handle\_c* are used only in *free\_sph\_p*, and *handle* is used only in *free\_sph\_np*.

*ipar*                      MKL\_INT array of size 128. Contains integer data to be used by Fast Helmholtz Solver on a sphere (for details, refer to [ipar](#)).

## Output Parameters

*handle\_s*, *handle\_c*,  
*handle*                    Data structures used by the Intel MKL FFT interface. Memory allocated for the structures is released on output.

*ipar*                      Contains integer data to be used by Fast Helmholtz Solver on a sphere. On output, the status of the routine call is written to *ipar[0]*.

*stat*                      MKL\_INT\*. Routine completion status, which is also written to *ipar[0]*.

## Description

The *free\_sph\_p/free\_sph\_np* routine releases the memory used by the *handle\_s*, *handle\_c* or *handle* structures, needed for calling the Intel MKL FFT functions. To release memory allocated for other parameters, include memory release statements in your code.

## Return Values

|                      |                                                                        |
|----------------------|------------------------------------------------------------------------|
| <i>stat</i> = 0      | The routine successfully completed the task.                           |
| <i>stat</i> = -1000  | The routine stopped because of an Intel MKL FFT or TT interface error. |
| <i>stat</i> = -99999 | The routine failed to complete the task because of a fatal error.      |

## Common Parameters for the Poisson Solver

### *ipar*

*ipar*                      MKL\_INT array of size 128, holds integer data needed for Fast Helmholtz Solver (both for Cartesian and spherical coordinate systems). Its elements are described in [Table "Elements of the ipar Array"](#):

**NOTE**

Initial values are assigned to the array parameters by the appropriate `?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np` and `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D/?_commit_sph_p/?_commit_sph_np` routines.

**Elements of the ipar Array**

| Index | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|-------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0     | <p>Contains status value of the last Poisson Solver routine called. In general, it should be 0 on exit from a routine to proceed with the Fast Helmholtz Solver. The element has no predefined values. This element can also be used to inform the <code>?_commit_Helmholtz_2D/?_commit_Helmholtz_3D/?_commit_sph_p/?_commit_sph_np</code> routines of how the Commit step of the computation should be carried out (see <a href="#">Figure "Typical Order of Invoking Poisson Solver Routines"</a>). A non-zero value of <code>ipar[0]</code> with decimal representation</p> $\overline{abc} = 100a + 10b + c$ <p>=100a+10b+c, where each of <i>a</i>, <i>b</i>, and <i>c</i> is equal to 0 or 9, indicates that some parts of the Commit step should be omitted.</p> <ul style="list-style-type: none"> <li>• If <i>c</i>=9, the routine omits checking of parameters and initialization of the data structures.</li> <li>• If <i>b</i>=9,             <ul style="list-style-type: none"> <li>• In the Cartesian case, the routine omits the adjustment of the right-hand side vector <i>f</i> to the Neumann boundary condition (multiplication of boundary values by 0.5 as well as incorporation of the boundary function <i>g</i>) and/or the Dirichlet boundary condition (setting boundary values to 0 as well as incorporation of the boundary function <i>G</i>).</li> <li>• For the Helmholtz solver on a sphere, the routine omits computation of the spherical weights for the <code>dpar/spar</code> array.</li> </ul> </li> <li>• If <i>a</i>=9, the routine omits the normalization of the right-hand side vector <i>f</i>. Depending on the solver, the normalization means:             <ul style="list-style-type: none"> <li>• 2D Cartesian case: multiplication by <math>h_y^2</math>, where <math>h_y</math> is the mesh size in the <i>y</i> direction (for details, see <a href="#">Poisson Solver Implementation</a>).</li> <li>• 3D (Cartesian) case: multiplication by <math>h_z^2</math>, where <math>h_z</math> is the mesh size in the <i>z</i> direction.</li> <li>• Helmholtz solver on a sphere: multiplication by <math>h_\theta^2</math>, where <math>h_\theta</math> is the mesh size in the <math>\theta</math> direction (for details, see <a href="#">Poisson Solver Implementation</a>).</li> </ul> </li> </ul> <p>Using <code>ipar[0]</code> you can adjust the routine to your needs and improve efficiency in solving multiple Helmholtz problems that differ only in the right-hand side. You must be cautious when using this method, because any misunderstanding of the commit process may cause incorrect results or program failure (see also <a href="#">Caveat on Parameter Modifications</a>).</p> |
| 1     | <p>Contains error messaging options:</p> <ul style="list-style-type: none"> <li>• <code>ipar[1]=-1</code> indicates that all error messages are printed to the <code>MKL_Poisson_Library_log.txt</code> file in the folder from which the routine is called. If the file does not exist, the routine tries to create it. If the attempt fails, the routine prints information that the file cannot be created to the standard output device (usually, screen).</li> <li>• <code>ipar[1]=0</code> indicates that no error messages will be printed.</li> <li>• <code>ipar[1]=1</code> is the default value. It indicates that all error messages are printed to the standard output device.</li> </ul> <p>In case of errors, the <code>stat</code> parameter contains a non-zero value on exit from a routine regardless of the <code>ipar[1]</code> setting.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| 2     | <p>Contains warning messaging options:</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

| Index                                                       | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|-------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                             | <ul style="list-style-type: none"> <li>• <i>ipar[2]=-1</i> indicates that all warning messages are printed to the <code>MKL_Poisson_Library_log.txt</code> file in the directory from which the routine is called. If the file does not exist, the routine tries to create it. If the attempt fails, the routine prints information that the file cannot be created to the standard output device.</li> <li>• <i>ipar[2]=0</i> indicates that no warning messages will be printed.</li> <li>• <i>ipar[2]=1</i> is the default value. It indicates that all warning messages are printed to the standard output device.</li> </ul> <p>In case of warnings, the <i>stat</i> parameter contains a non-zero value on exit from a routine regardless of the <i>ipar[2]</i> setting.</p> |
| 3                                                           | Unused.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| Parameters 4 through 9 are used only in the Cartesian case. |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| 4                                                           | Takes this value: <ul style="list-style-type: none"> <li>• 2, if <i>BCtype[0]='P'</i></li> <li>• 1, if <i>BCtype[0]='N'</i></li> <li>• 0, if <i>BCtype[0]='D'</i></li> <li>• -1, otherwise</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 5                                                           | Takes this value: <ul style="list-style-type: none"> <li>• 2, if <i>BCtype[1]='P'</i></li> <li>• 1, if <i>BCtype[1]='N'</i></li> <li>• 0, if <i>BCtype[1]='D'</i></li> <li>• -1, otherwise</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 6                                                           | Takes this value: <ul style="list-style-type: none"> <li>• 2, if <i>BCtype[2]='P'</i></li> <li>• 1, if <i>BCtype[2]='N'</i></li> <li>• 0, if <i>BCtype[2]='D'</i></li> <li>• -1, otherwise</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 7                                                           | Takes this value: <ul style="list-style-type: none"> <li>• 2, if <i>BCtype[3]='P'</i></li> <li>• 1, if <i>BCtype[3]='N'</i></li> <li>• 0, if <i>BCtype[3]='D'</i></li> <li>• -1, otherwise</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 8                                                           | Takes this value: <ul style="list-style-type: none"> <li>• 2, if <i>BCtype[4]='P'</i></li> <li>• 1, if <i>BCtype[4]='N'</i></li> <li>• 0, if <i>BCtype[4]='D'</i></li> <li>• -1, otherwise</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| 9                                                           | Takes this value: <ul style="list-style-type: none"> <li>• 2, if <i>BCtype[5]='P'</i></li> <li>• 1, if <i>BCtype[5]='N'</i></li> <li>• 0, if <i>BCtype[5]='D'</i></li> <li>• -1, otherwise</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

| Index | Description                                                                                                                                                                                                                                                                                         |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10    | Takes the value of <ul style="list-style-type: none"> <li><math>n_x</math>, that is, the number of intervals along the <math>x</math>-axis, in the Cartesian case.</li> <li><math>n_p</math>, that is, the number of intervals along the <math>\phi</math>-axis, in the spherical case.</li> </ul>  |
| 11    | Takes the value of <ul style="list-style-type: none"> <li><math>n_y</math>, that is, the number of intervals along the <math>y</math>-axis, in the Cartesian case</li> <li><math>n_t</math>, that is, the number of intervals along the <math>\theta</math>-axis, in the spherical case.</li> </ul> |
| 12    | Takes the value of $n_z$ , the number of intervals along the $z$ -axis. This parameter is used only in the 3D case (Cartesian).                                                                                                                                                                     |
| 13    | Has the value of 6, which specifies the internal partitioning of the $dpar/spar$ array.                                                                                                                                                                                                             |
| 14    | Takes the value of $ipar[13]+ipar[10]+1$ , which specifies the internal partitioning of the $dpar/spar$ array.                                                                                                                                                                                      |

The values of  $ipar[15] - ipar[20]$  depend on the dimension of the problem for the Cartesian solver or on whether the solver on a sphere is periodic.

|    | Cartesian Solver                                                                                                   |                                                                                                                    | Spherical Solver                                                                                                   |                                                                                                                    |
|----|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
|    | 2D case                                                                                                            | 3D case                                                                                                            | Periodic case                                                                                                      | Non-periodic case                                                                                                  |
| 15 | Unused                                                                                                             | Takes the value of $ipar[14]+1$ , which specifies the internal partitioning of the $dpar/spar$ array.              |                                                                                                                    |                                                                                                                    |
| 16 | Unused                                                                                                             | Takes the value of $ipar[14]+ipar[11]+1$ , which specifies the internal partitioning of the $dpar/spar$ array.     |                                                                                                                    |                                                                                                                    |
| 17 | Takes the value of $ipar[14]+1$ , which specifies the internal partitioning of the $dpar/spar$ array.              | Takes the value of $ipar[16]+1$ , which specifies the internal partitioning of the $dpar/spar$ array.              |                                                                                                                    |                                                                                                                    |
| 18 | Takes the value of $ipar[14]+3*ipar[10]/2+1$ , which specifies the internal partitioning of the $dpar/spar$ array. | Takes the value of $ipar[16]+3*ipar[10]/2+1$ , which specifies the internal partitioning of the $dpar/spar$ array. | Takes the value of $ipar[16]+3*ipar[10]/4+1$ , which specifies the internal partitioning of the $dpar/spar$ array. | Takes the value of $ipar[16]+3*ipar[10]/2+1$ , which specifies the internal partitioning of the $dpar/spar$ array. |
| 19 | Takes the value of $ipar[18]+1$ , which specifies the internal partitioning of the $dpar/spar$ array.              | Takes the value of $ipar[18]+1$ , which specifies the internal partitioning of the $dpar/spar$ array.              |                                                                                                                    | Unused                                                                                                             |
| 20 | Takes the value of $ipar[19]+3*ipar[12]/4$ , which specifies the internal partitioning of the $dpar/spar$ array.   | Takes the value of $ipar[18]+3*ipar[11]/2+1$ , which specifies the internal partitioning of the $dpar/spar$ array. | Takes the value of $ipar[18]+3*ipar[10]/4+1$ , which specifies the internal partitioning of the $dpar/spar$ array. | Unused                                                                                                             |

The values of  $ipar[21] - ipar[119]$  are assigned regardless of the dimension of the problem for the Cartesian solver or of whether the solver on a sphere is periodic.

| Index                 | Description                                                                                                                                                                                                                                                                                                                            |
|-----------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 21                    | Contains message style options. Specifically: <ul style="list-style-type: none"> <li><code>ipar[21]=1</code> (default) indicates that Poisson Solver routines print the messages in C-style notations.</li> </ul>                                                                                                                      |
| 22                    | Contains the number of OpenMP threads to be used for computations in a multithreaded environment. The default value is 1 in the serial mode, and the result returned by the <code>mkl_get_max_threads</code> function otherwise.                                                                                                       |
| 23                    | Takes the value of <code>ipar[18]+1</code> , which specifies the internal partitioning of the <code>dpar/spar</code> array in the periodic Cartesian case.                                                                                                                                                                             |
| 24                    | Takes the value of <code>ipar[23]+3*ipar[12]/4</code> , which specifies the internal partitioning of the <code>dpar/spar</code> array in the periodic Cartesian case.                                                                                                                                                                  |
| 25                    | Takes the value of <code>ipar[20]+1</code> , which specifies the internal partitioning of the <code>dpar/spar</code> array in the periodic 3D Cartesian case.                                                                                                                                                                          |
| 26                    | Takes the value of <code>ipar[25]+3*ipar[13]/4</code> , which specifies the internal partitioning of the <code>dpar/spar</code> array in the periodic 3D Cartesian case.                                                                                                                                                               |
| 27<br>through<br>39   | Unused.                                                                                                                                                                                                                                                                                                                                |
| 40<br>through<br>59   | Contain the first twenty elements of the <code>ipar</code> array of the first Trigonometric Transform that the solver uses. (For details, see <a href="#">Common Parameters</a> in the "Trigonometric Transform Routines" section.)                                                                                                    |
| 60<br>through<br>79   | Contain the first twenty elements of the <code>ipar</code> array of the second Trigonometric Transform that the 3D Cartesian and periodic spherical solvers use. (For details, see <a href="#">Common Parameters</a> in the "Trigonometric Transform Routines" section.)                                                               |
| 80<br>through<br>99   | Contain the first twenty elements of the <code>ipar</code> array of the third Trigonometric Transform that the solver uses in case of periodic boundary conditions along the <i>x</i> -axis. (For details, see <a href="#">Common Parameters</a> in the "Trigonometric Transform Routines" section.)                                   |
| 100<br>through<br>119 | Contain the first twenty elements of the <code>ipar</code> array of the fourth Trigonometric Transform used by periodic spherical solvers and 3D Cartesian solvers with periodic boundary conditions along the <i>y</i> -axis. (For details, see <a href="#">Common Parameters</a> in the "Trigonometric Transform Routines" section.) |

**NOTE**

While you can declare the `ipar` array as `MKL_INT ipar[120]`, for future compatibility you should declare `ipar` as `MKL_INT ipar[128]`.

**dpar and spar**

Arrays `dpar` and `spar` are the same except in the data precision:

- `dpar` Holds data needed for double-precision Fast Helmholtz Solver computations.
- For the Cartesian solver, `double` array of size  $5 \cdot n_x / 2 + 7$  in the 2D case or  $5 \cdot (n_x + n_y) / 2 + 9$  in the 3D case; initialized in the `d_init_Helmholtz_2D/`, `d_init_Helmholtz_3D` and `d_commit_Helmholtz_2D/`, `d_commit_Helmholtz_3D` routines.

- For the spherical solver, `double` array of size  $5*nx/2+nt+10$ ; initialized in the `d_init_sph_p/d_init_sph_np` and `d_commit_sph_p/d_commit_sph_np` routines.

*spar*

Holds data needed for single-precision Fast Helmholtz Solver computations.

- For the Cartesian solver, `float` array of size  $5*nx/2+7$  in the 2D case or  $5*(nx+ny)/2+9$  in the 3D case; initialized in the `s_init_Helmholtz_2D/s_init_Helmholtz_3D` and `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D` routines.
- For the spherical solver, `float` array of size  $5*nx/2+nt+10$ ; initialized in the `s_init_sph_p/s_init_sph_np` and `s_commit_sph_p/s_commit_sph_np` routines.

Because *dpar* and *spar* have similar elements in each position, the elements are described together in [Table "Elements of the dpar and spar Arrays"](#):

**Elements of the dpar and spar Arrays**

| Index | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0     | <p>In the Cartesian case, contains the length of the interval along the x-axis right after a call to the <code>?_init_Helmholtz_2D/?_init_Helmholtz_3D</code> routine or the mesh size <math>h_x</math> in the x direction (for details, see <a href="#">Poisson Solver Implementation</a>) after a call to the <code>?_commit_Helmholtz_2D/?_commit_Helmholtz_3D</code> routine.</p> <p>In the spherical case, contains the length of the interval along the <math>\varphi</math>-axis right after a call to the <code>?_init_sph_p/?_init_sph_np</code> routine or the mesh size <math>h_\varphi</math> in the <math>\varphi</math> direction (for details, see <a href="#">Poisson Solver Implementation</a>) after a call to the <code>?_commit_sph_p/?_commit_sph_np</code> routine.</p> |
| 1     | <p>In the Cartesian case, contains the length of the interval along the y-axis right after a call to the <code>?_init_Helmholtz_2D/?_init_Helmholtz_3D</code> routine or the mesh size <math>h_y</math> in the y direction (for details, see <a href="#">Poisson Solver Implementation</a>) after a call to the <code>?_commit_Helmholtz_2D/?_commit_Helmholtz_3D</code> routine.</p> <p>In the spherical case, contains the length of the interval along the <math>\theta</math>-axis right after a call to the <code>?_init_sph_p/?_init_sph_np</code> routine or the mesh size <math>h_\theta</math> in the <math>\theta</math> direction (for details, see <a href="#">Poisson Solver Implementation</a>) after a call to the <code>?_commit_sph_p/?_commit_sph_np</code> routine.</p>    |
| 2     | <p>In the Cartesian case, contains the length of the interval along the z-axis right after a call to the <code>?_init_Helmholtz_2D/?_init_Helmholtz_3D</code> routine or the mesh size <math>h_z</math> in the z direction (for details, see <a href="#">Poisson Solver Implementation</a>) after a call to the <code>?_commit_Helmholtz_2D/?_commit_Helmholtz_3D</code> routine. In the Cartesian solver, this parameter is used only in the 3D case.</p> <p>In the spherical solver, contains the coordinate of the leftmost boundary along the <math>\theta</math>-axis after a call to the <code>?_init_sph_p/?_init_sph_np</code> routine.</p>                                                                                                                                           |
| 3     | <p>Contains the value of the coefficient <math>q</math> after a call to the <code>?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np</code> routine.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 4     | <p>Contains the tolerance parameter after a call to the <code>?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np</code> routine.</p> <ul style="list-style-type: none"> <li>• In the Cartesian case, this value is used only for the pure Neumann boundary conditions (<code>BCtype="NNNN"</code> in the 2D case; <code>BCtype="NNNNNN"</code> in the 3D case). This is a special case, because the right-hand side of the problem cannot be arbitrary if the coefficient <math>q</math> is zero. The Poisson Solver verifies that the classical solution exists (up to rounding errors) using this tolerance. In any case, the Poisson Solver computes the normal solution, that is, the solution that has the</li> </ul>                                                    |

| Index                                                         | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|---------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                               | <p>minimal Euclidean norm of residual. Nevertheless, the <code>?_Helmholtz_2D/?_Helmholtz_3D</code> routine informs you that the solution may not exist in a classical sense (up to rounding errors).</p> <ul style="list-style-type: none"> <li>In the spherical case, the value is used for the special case of a periodic problem on the entire sphere. This special case is similar to the Cartesian case with pure Neumann boundary conditions. Here the Poisson Solver computes the normal solution as well. The parameter is also used to detect whether the problem is periodic up to rounding errors.</li> </ul> <p>The default value for this parameter is 1.0E-10 in case of double-precision computations or 1.0E-4 in case of single-precision computations. You can increase the value of the tolerance, for instance, to avoid the warnings that may appear.</p> |
| <code>ipar[13]-1</code><br>through<br><code>ipar[14]-1</code> | <p>In the Cartesian case, contain the spectrum of the one-dimensional (1D) problem along the <math>x</math>-axis after a call to the <code>?_commit_Helmholtz_2D/?_commit_Helmholtz_3D</code> routine.</p> <p>In the spherical case, contains the spectrum of the 1D problem along the <math>\varphi</math>-axis after a call to the <code>?_commit_sph_p/?_commit_sph_np</code> routine.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <code>ipar[15]-1</code><br>through<br><code>ipar[16]-1</code> | <p>In the Cartesian case, contain the spectrum of the 1D problem along the <math>y</math>-axis after a call to the <code>?_commit_Helmholtz_3D</code> routine. These elements are used only in the 3D case.</p> <p>In the spherical case, contains the spherical weights after a call to the <code>?_commit_sph_p/?_commit_sph_np</code> routine.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <code>ipar[17]-1</code><br>through<br><code>ipar[18]-1</code> | <p>Take the values of the (staggered) sine/cosine in the mesh points:</p> <ul style="list-style-type: none"> <li>along the <math>x</math>-axis after a call to the <code>?_commit_Helmholtz_2D/?_commit_Helmholtz_3D</code> routine for a Cartesian solver</li> <li>along the <math>\varphi</math>-axis after a call to the <code>?_commit_sph_p/?_commit_sph_np</code> routine for a spherical solver.</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <code>ipar[19]-1</code><br>through<br><code>ipar[20]-1</code> | <p>Take the values of the (staggered) sine/cosine in the mesh points:</p> <ul style="list-style-type: none"> <li>along the <math>y</math>-axis after a call to the <code>?_commit_Helmholtz_3D</code> routine for a Cartesian 3D solver</li> <li>along the <math>\varphi</math>-axis after a call to the <code>?_commit_sph_p</code> routine for a spherical periodic solver.</li> </ul> <p>These elements are not used in the 2D Cartesian case and in the non-periodic spherical case.</p>                                                                                                                                                                                                                                                                                                                                                                                    |
| <code>ipar[23]-1</code><br>through<br><code>ipar[24]-1</code> | <p>Take the values of the (staggered) sine/cosine in the mesh points along the <math>x</math>-axis after a call to the <code>?_commit_Helmholtz_2D/?_commit_Helmholtz_3D</code> routine. These elements are used only in the periodic Cartesian case.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <code>ipar[25]-1</code><br>through<br><code>ipar[26]-1</code> | <p>Take the values of the (staggered) sine/cosine in the mesh points along the <math>x</math>-axis after a call to the <code>?_commit_Helmholtz_3D</code> routine. These elements are used only in the periodic 3D Cartesian case.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |

**NOTE**

You may define the array size depending upon the type of the problem to solve.

## Caveat on Parameter Modifications

Flexibility of the Poisson Solver interface enables you to skip calls to the `?_init_Helmholtz_2D/?_init_Helmholtz_3D/?_init_sph_p/?_init_sph_np` routine and to initialize the basic data structures explicitly in your code. You may also need to modify contents of the `ipar`, `dpar`, and `spar` arrays after initialization. When doing so, provide correct and consistent data in the arrays. Mistakenly altered arrays cause errors or incorrect results. You can perform a basic check for correctness and consistency of parameters by calling the `?_commit_Helmholtz_2D/?_commit_Helmholtz_3D` routine; however, this does not ensure the correct solution but only reduces the chance of errors or wrong results.

---

### NOTE

To supply correct and consistent parameters to Poisson Solver routines, you should have considerable experience in using the Poisson Solver interface and good understanding of the solution process, as well as elements contained in the `ipar`, `spar`, and `dpar` arrays and dependencies between values of these elements.

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In rare occurrences when you fail in tuning parameters for the Fast Helmholtz Solver, refer for technical support at <http://www.intel.com/software/products/support/>.

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### WARNING

The only way that ensures a proper solution of a Helmholtz problem is to follow a typical sequence of invoking the routines and not change the default set of parameters. So, avoid modifications of `ipar`, `dpar`, and `spar` arrays unless it is necessary.

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## Parameters That Define Boundary Conditions

Poisson Solver routines for the Cartesian solver use the following common parameters to define the boundary conditions.

### Parameters to Define Boundary Conditions for the Cartesian Solver

| Parameter          | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|--------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>bd_ax</code> | <p>double* for <code>d_commit_Helmholtz_2D/d_commit_Helmholtz_3D</code> and <code>d_Helmholtz_2D/d_Helmholtz_3D</code>,</p> <p>float* for <code>s_commit_Helmholtz_2D/s_commit_Helmholtz_3D</code> and <code>s_Helmholtz_2D/s_Helmholtz_3D</code>.</p> <p>Contains values of the boundary condition on the leftmost boundary of the domain along the <math>x</math>-axis.</p> <ul style="list-style-type: none"> <li>2D problem: the size of the array is <math>n_y+1</math>. Its contents depend on the boundary conditions as follows: <ul style="list-style-type: none"> <li>Dirichlet boundary condition (value of <code>BCtype[0]</code> is 'D'): values of the function <math>G(ax, y_j)</math>, <math>j=0, \dots, n_y</math>.</li> <li>Neumann boundary condition (value of <code>BCtype[0]</code> is 'N'): values of the function <math>g(ax, y_j)</math>, <math>j=0, \dots, n_y</math>.</li> </ul> <p>The value corresponding to the index <math>j</math> is placed in <code>bd_ax[j]</code>.</p> </li> <li>3D problem: the size of the array is <math>(n_y+1)*(n_z+1)</math>. Its contents depend on the boundary conditions as follows: <ul style="list-style-type: none"> <li>Dirichlet boundary condition (value of <code>BCtype[0]</code> is 'D'): values of the function <math>G(ax, y_j, z_k)</math>, <math>j=0, \dots, n_y</math>, <math>k=0, \dots, n_z</math>.</li> <li>Neumann boundary condition (value of <code>BCtype[0]</code> is 'N'): the values of the function <math>g(ax, y_j, z_k)</math>, <math>j=0, \dots, n_y</math>, <math>k=0, \dots, n_z</math>.</li> </ul> </li> </ul> |

---

| Parameter | Description |
|-----------|-------------|
|-----------|-------------|

---

The values are packed in the array so that the value corresponding to indices  $(j, k)$  is placed in  $bd\_ax[j+k*(ny+1)]$ .

For periodic boundary conditions (the value of  $BCtype[0]$  is 'P'), this parameter is not used, so it can accept a dummy pointer.

*bd\_bx* double\* for `d_commit_Helmholtz_2D/d_commit_Helmholtz_3D` and `d_Helmholtz_2D/d_Helmholtz_3D`,

float\* for `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D` and `s_Helmholtz_2D/s_Helmholtz_3D`.

Contains values of the boundary condition on the rightmost boundary of the domain along the x-axis.

- 2D problem: the size of the array is  $ny+1$ . Its contents depend on the boundary conditions as follows:
  - Dirichlet boundary condition (value of  $BCtype[1]$  is 'D'): values of the function  $G(bx, y_j), j=0, \dots, ny$ .
  - Neumann boundary condition (value of  $BCtype[1]$  is 'N'): values of the function  $g(bx, y_j), j=0, \dots, ny$ .

The value corresponding to the index  $j$  is placed in  $bd\_bx[j]$ .

- 3D problem: the size of the array is  $(ny+1)*(nz+1)$ . Its contents depend on the boundary conditions as follows:
  - Dirichlet boundary condition (value of  $BCtype[1]$  is 'D'): values of the function  $G(bx, y_j, z_k), j=0, \dots, ny, k=0, \dots, nz$ .
  - Neumann boundary condition (value of  $BCtype[1]$  is 'N'): values of the function  $g(bx, y_j, z_k), j=0, \dots, ny, k=0, \dots, nz$ .

The values are packed in the array so that the value corresponding to indices  $(j, k)$  is placed in  $bd\_bx[j+k*(ny+1)]$ .

For periodic boundary conditions (the value of  $BCtype[1]$  is 'P'), this parameter is not used, so it can accept a dummy pointer.

*bd\_ay* double\* for `d_commit_Helmholtz_2D/d_commit_Helmholtz_3D` and `d_Helmholtz_2D/d_Helmholtz_3D`,

float\* for `s_commit_Helmholtz_2D/s_commit_Helmholtz_3D` and `s_Helmholtz_2D/s_Helmholtz_3D`.

Contains values of the boundary condition on the leftmost boundary of the domain along the y-axis.

- 2D problem: the size of the array is  $nx+1$ . Its contents depend on the boundary conditions as follows:
  - Dirichlet boundary condition (value of  $BCtype[2]$  is 'D'): values of the function  $G(x_i, ay), i=0, \dots, nx$ .
  - Neumann boundary condition (value of  $BCtype[2]$  is 'N'): values of the function  $g(x_i, ay), i=0, \dots, nx$ .

The value corresponding to the index  $i$  is placed in  $bd\_ay[i]$ .

- 3D problem: the size of the array is  $(nx+1)*(nz+1)$ . Its contents depend on the boundary conditions as follows:
  - Dirichlet boundary condition (value of  $BCtype[2]$  is 'D'): values of the function  $G(x_i, ay, z_k), i=0, \dots, nx, k=0, \dots, nz$ .
  - Neumann boundary condition (value of  $BCtype[2]$  is 'N'): values of the function  $g(x_i, ay, z_k), i=0, \dots, nx, k=0, \dots, nz$ .

| Parameter    | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|              | <p>The values are packed in the array so that the value corresponding to indices <math>(i, k)</math> is placed in <math>bd\_ay[i+k*(nx+1)]</math>.</p> <p>For periodic boundary conditions (the value of <math>BCtype[2]</math> is 'P'), this parameter is not used, so it can accept a dummy pointer.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <i>bd_by</i> | <p>double* for <code>d_commit_Helmholtz_2D/d_commit_Helmholtz_3D</code> and <code>d_Helmholtz_2D/d_Helmholtz_3D</code>,</p> <p>float* for <code>s_commit_Helmholtz_2D/s_commit_Helmholtz_3D</code> and <code>s_Helmholtz_2D/s_Helmholtz_3D</code>.</p> <p>Contains values of the boundary condition on the rightmost boundary of the domain along the y-axis.</p> <ul style="list-style-type: none"> <li>• 2D problem: the size of the array is <math>nx+1</math>. Its contents depend on the boundary conditions as follows: <ul style="list-style-type: none"> <li>• Dirichlet boundary condition (value of <math>BCtype[3]</math> is 'D'): values of the function <math>G(x_i, by)</math>, <math>i=0, \dots, nx</math>.</li> <li>• Neumann boundary condition (value of <math>BCtype[3]</math> is 'N'): values of the function <math>g(x_i, by)</math>, <math>i=0, \dots, nx</math>.</li> </ul> <p>The value corresponding to the index <math>i</math> is placed in <math>bd\_by[i]</math>.</p> </li> <li>• 3D problem: the size of the array is <math>(nx+1)*(nz+1)</math>. Its contents depend on the boundary conditions as follows: <ul style="list-style-type: none"> <li>• Dirichlet boundary condition (value of <math>BCtype[3]</math> is 'D'): values of the function <math>G(x_i, by, z_k)</math>, <math>i=0, \dots, nx, k=0, \dots, nz</math>.</li> <li>• Neumann boundary condition (value of <math>BCtype[3]</math> is 'N'): values of the function <math>g(x_i, by, z_k)</math>, <math>i=0, \dots, nx, k=0, \dots, nz</math>.</li> </ul> <p>The values are packed in the array so that the value corresponding to indices <math>(i, k)</math> is placed in <math>bd\_by[i+k*(nx+1)]</math>.</p> </li> </ul> <p>For periodic boundary conditions (the value of <math>BCtype[3]</math> is 'P'), this parameter is not used, so it can accept a dummy pointer.</p> |
| <i>bd_az</i> | <p>double* for <code>d_commit_Helmholtz_3D</code> and <code>d_Helmholtz_3D</code>,</p> <p>float* for <code>s_commit_Helmholtz_3D</code> and <code>s_Helmholtz_3D</code>.</p> <p>Used only by <code>?_commit_Helmholtz_3D</code> and <code>?_Helmholtz_3D</code>. Contains values of the boundary condition on the leftmost boundary of the domain along the z-axis.</p> <p>The size of the array is <math>(nx+1)*(ny+1)</math>. Its contents depend on the boundary conditions as follows:</p> <ul style="list-style-type: none"> <li>• Dirichlet boundary condition (value of <math>BCtype[4]</math> is 'D'): values of the function <math>G(x_i, y_j, az)</math>, <math>i=0, \dots, nx, j=0, \dots, ny</math>.</li> <li>• Neumann boundary condition (value of <math>BCtype[4]</math> is 'N'), values of the function <math>g(x_i, y_j, az)</math>, <math>i=0, \dots, nx, j=0, \dots, ny</math>.</li> </ul> <p>The values are packed in the array so that the value corresponding to indices <math>(i, j)</math> is placed in <math>bd\_az[i+j*(nx+1)]</math>.</p> <p>For periodic boundary conditions (the value of <math>BCtype[4]</math> is 'P'), this parameter is not used, so it can accept a dummy pointer.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| <i>bd_bz</i> | <p>double* for <code>d_commit_Helmholtz_3D</code> and <code>d_Helmholtz_3D</code>,</p> <p>float* for <code>s_commit_Helmholtz_3D</code> and <code>s_Helmholtz_3D</code>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |

| Parameter | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|           | <p>Used only by <code>?_commit_Helmholtz_3D</code> and <code>?_Helmholtz_3D</code>. Contains values of the boundary condition on the rightmost boundary of the domain along the z-axis.</p> <p>The size of the array is <math>(nx+1)*(ny+1)</math>. Its contents depend on the boundary conditions as follows:</p> <ul style="list-style-type: none"> <li>• Dirichlet boundary condition (value of <code>BCtype[5]</code> is 'D'): values of the function <math>G(x_i, y_j, bz)</math>, <math>i=0, \dots, nx, j=0, \dots, ny</math>.</li> <li>• Neumann boundary condition (value of <code>BCtype[5]</code> is 'N'): values of the function <math>g(x_i, y_j, bz)</math>, <math>i=0, \dots, nx, j=0, \dots, ny</math>.</li> </ul> <p>The values are packed in the array so that the value corresponding to indices <math>(i, j)</math> is placed in <code>bd_bz[i+j*(nx+1)]</code>.</p> <p>For periodic boundary conditions (the value of <code>BCtype[5]</code> is 'P'), this parameter is not used, so it can accept a dummy pointer.</p> |

### See Also

[\\_commit\\_Helmholtz\\_2D/?\\_commit\\_Helmholtz\\_3D](#)  
[?\\_Helmholtz\\_2D/?\\_Helmholtz\\_3D](#)

### Poisson Solver Implementation Details

Several aspects of the Intel MKL Poisson Solver interface are platform-specific and language-specific. To promote portability of the Intel MKL Poisson Solver interface across platforms and ease of use across different languages, Intel MKL provides you with the Poisson Solver language-specific header file to include in your code:

- `mkl_poisson.h`, to be used together with `mkl_dfti.h`.

#### NOTE

- Use of the Intel MKL Poisson Solver software without including the above language-specific header files is not supported.

### Header File

The header file defines the function prototypes for the Cartesian and spherical solver available in specific function descriptions.

# Nonlinear Optimization Problem Solvers

# 12

Intel® Math Kernel Library (Intel® MKL) provides tools for solving nonlinear least squares problems using the Trust-Region (TR) algorithms. The solver routines are grouped according to their purpose as follows:

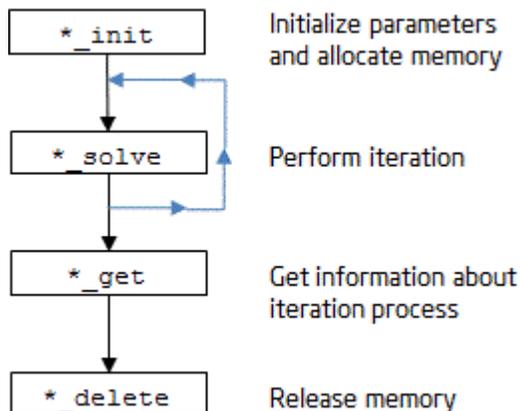
- [Nonlinear Least Squares Problem without Constraints](#)
- [Nonlinear Least Squares Problem with Linear \(Boundary\) Constraints](#)
- [Jacobian Matrix Calculation Routines](#)

For more information on the key concepts required to understand the use of the Intel MKL nonlinear least squares problem solver routines, see [\[Conn00\]](#).

## Nonlinear Solver Organization and Implementation

The Intel MKL solver routines for nonlinear least squares problems use reverse communication interfaces (RCI). That means you need to provide the solver with information required for the iteration process, for example, the corresponding Jacobian matrix, or values of the objective function. RCI removes the dependency of the solver on specific implementation of the operations. However, it does require that you organize a computational loop.

### Typical order for invoking RCI solver routines



The nonlinear least squares problem solver routines, or Trust-Region (TR) solvers, are implemented with threading support. You can manage the threads using [Threading Control Functions](#). The TR solvers use BLAS and LAPACK routines, and offer the same parallelism as those domains. The `?jacobi` and `?jacobix` routines of Jacobian matrix calculations are parallel. These routines (`?jacobi` and `?jacobix`) make calls to the user-supplied functions with different  $x$  parameters for multiple threads.

### Memory Allocation and Handles

To make the TR solver routines easy to use, you are not required to allocate temporary working storage. The solver allocates any required memory. To allow multiple users to access the solver simultaneously, the solver keeps track of the storage allocated for a particular application by using a data object called a *handle*. Each TR solver routine creates, uses, or deletes a handle. To declare a handle, include `mk1.h`.

Declare a handle as one of the following:

```
#include "mkl.h"
_TRNSP_HANDLE_t handle;
```

or

```
_TRNSPBC_HANDLE_t handle;
```

The first declaration is used for nonlinear least squares problems without boundary constraints, and the second is used for nonlinear least squares problems with boundary constraints.

### Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

## Nonlinear Solver Routine Naming Conventions

The TR routine names have the following structure:

```
<character><name>_<action>()
```

where

- *<character>* indicates the data type:

|   |        |
|---|--------|
| s | float  |
| d | double |

- *<name>* indicates the task type:

|          |                                                              |
|----------|--------------------------------------------------------------|
| trnlsp   | nonlinear least squares problem without constraints          |
| trnlspbc | nonlinear least squares problem with boundary constraints    |
| jacobi   | computation of the Jacobian matrix using central differences |

- *<action>* indicates an action on the task:

|        |                                                                                                      |
|--------|------------------------------------------------------------------------------------------------------|
| init   | initializes the solver                                                                               |
| check  | checks correctness of the input parameters                                                           |
| solve  | solves the problem                                                                                   |
| get    | retrieves the number of iterations, the stop criterion, the initial residual, and the final residual |
| delete | releases the allocated data                                                                          |

## Nonlinear Least Squares Problem without Constraints

The nonlinear least squares problem without constraints can be described as follows:

$$\min_{x \in \mathbb{R}^n} \|F(x)\|_2^2 = \min_{x \in \mathbb{R}^n} \|y - f(x)\|_2^2, y \in \mathbb{R}^m, x \in \mathbb{R}^n, f: \mathbb{R}^n \rightarrow \mathbb{R}^m, m \geq n,$$

where

$F(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a twice differentiable function in  $\mathbb{R}^n$ .

Solving a nonlinear least squares problem means searching for the best approximation to the vector  $y$  with the model function  $f_i(x)$  and nonlinear variables  $x$ . The best approximation means that the sum of squares of residuals  $y_i - f_i(x)$  is the minimum.

See usage examples in the `examples\solverc\source` folder of your Intel MKL directory. Specifically, see `ex_nlsqp_c.c`.

### RCI TR Routines

| Routine Name                | Operation                                                                                 |
|-----------------------------|-------------------------------------------------------------------------------------------|
| <code>?trnlsp_init</code>   | Initializes the solver.                                                                   |
| <code>?trnlsp_check</code>  | Checks correctness of the input parameters.                                               |
| <code>?trnlsp_solve</code>  | Solves a nonlinear least squares problem using the Trust-Region algorithm.                |
| <code>?trnlsp_get</code>    | Retrieves the number of iterations, stop criterion, initial residual, and final residual. |
| <code>?trnlsp_delete</code> | Releases allocated data.                                                                  |

### ?trnlsp\_init

Initializes the solver of a nonlinear least squares problem.

#### Syntax

```
MKL_INT strnlsp_init (_TRNSP_HANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m,
const float* x, const float* &eps, const MKL_INT* &iter1, const MKL_INT* &iter2, const
float* &rs);
```

```
MKL_INT dtrnlsp_init (_TRNSP_HANDLE_t* &handle, MKL_INT* &n, const MKL_INT* &m, const
double* x, const double* &eps, const MKL_INT* &iter1, const MKL_INT* &iter2, const
double* &rs);
```

#### Include Files

- `mkl.h`

#### Description

The `?trnlsp_init` routine initializes the solver.

After initialization, all subsequent invocations of the `?trnlsp_solve` routine should use the values of the `handle` returned by `?trnlsp_init`.

The `eps` array contains a number indicating the stopping criteria:

| <i>eps</i> Value | Description                                                                                                                 |
|------------------|-----------------------------------------------------------------------------------------------------------------------------|
| 0                | $\Delta < eps[0]$                                                                                                           |
| 1                | $  F(x)  _2 < eps[1]$                                                                                                       |
| 2                | The Jacobian matrix is singular.<br>$  J(x)_{[m*(j-1)...m*j-1]}  _2 < eps[2], j = 1, \dots, n$                              |
| 3                | $  s  _2 < eps[3]$                                                                                                          |
| 4                | $  F(x)  _2 -   F(x) - J(x)s  _2 < eps[4]$                                                                                  |
| 5                | The trial step precision. If $eps[5] = 0$ , then the trial step meets the required precision ( $\leq 1.0 \cdot 10^{-10}$ ). |

**Note:**

- $J(x)$  is the Jacobian matrix.
- $\Delta$  is the trust-region area.
- $F(x)$  is the value of the functional.
- $s$  is the trial step.

### Input Parameters

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>     | Length of $x$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>m</i>     | Length of $F(x)$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <i>x</i>     | Array of size $n$ . Initial guess.                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| <i>eps</i>   | Array of size 6; contains stopping criteria. See the values in the Description section.                                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>iter1</i> | Specifies the maximum number of iterations.                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <i>iter2</i> | Specifies the maximum number of iterations of trial step calculation.                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <i>rs</i>    | Definition of initial size of the trust region (boundary of the trial step). The recommend minimum value is 0.1, and the recommended maximum value is 100.0. Based on your knowledge of the objective function and initial guess you can increase or decrease the initial trust region. It can influence the iteration process, for example, the direction of the iteration process and the number of iterations. If you set <i>rs</i> to 0.0, the solver uses the default value, which is 100.0. |

### Output Parameters

|               |                                                                                                                                                                                                                                                                                                                                               |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>handle</i> | Type <code>_TRNSP_HANDLE_t</code> .                                                                                                                                                                                                                                                                                                           |
| <i>res</i>    | Indicates task completion status. <ul style="list-style-type: none"> <li>• <math>res = TR\_SUCCESS</math> - the routine completed the task normally.</li> <li>• <math>res = TR\_INVALID\_OPTION</math> - there was an error in the input parameters.</li> <li>• <math>res = TR\_OUT\_OF\_MEMORY</math> - there was a memory error.</li> </ul> |

TR\_SUCCESS, TR\_INVALID\_OPTION, and TR\_OUT\_OF\_MEMORY are defined in the mkl\_rci.h include file.

**See Also**

?trnlsp\_solve

**?trnlsp\_check**

Checks the correctness of handle and arrays containing Jacobian matrix, objective function, and stopping criteria.

**Syntax**

```
MKL_INT strnlsp_check (_TRNSP_HANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m,
const float* fjac, const float* fvec, const float* eps, MKL_INT* info);
```

```
MKL_INT dtrnlsp_check (_TRNSP_HANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m,
const double* fjac, const double* fvec, const double* eps, MKL_INT* info);
```

**Include Files**

- mkl.h

**Description**

The ?trnlsp\_check routine checks the arrays passed into the solver as input parameters. If an array contains any INF or NaN values, the routine sets the flag in output array info (see the description of the values returned in the Output Parameters section for the info array).

**Input Parameters**

|               |                                                                                                              |
|---------------|--------------------------------------------------------------------------------------------------------------|
| <i>handle</i> | Type <code>_TRNSPBC_HANDLE_t</code> .                                                                        |
| <i>n</i>      | Length of <i>x</i> .                                                                                         |
| <i>m</i>      | Length of <i>F(x)</i> .                                                                                      |
| <i>fjac</i>   | Array of size <i>m</i> by <i>n</i> . Contains the Jacobian matrix of the function.                           |
| <i>fvec</i>   | Array of size <i>m</i> . Contains the function values at <i>x</i> , where $fvec[i] = (y_i - \hat{f}_i(x))$ . |
| <i>eps</i>    | Array of size 6; contains stopping criteria. See the values in the Description section of the ?trnlsp_init.  |

**Output Parameters**

*info* Array of size 6.  
Results of input parameter checking:

| Parameter       | Used for                | Value | Description                  |
|-----------------|-------------------------|-------|------------------------------|
| <i>info</i> [0] | Flags for <i>handle</i> | 0     | The handle is valid.         |
|                 |                         | 1     | The handle is not allocated. |

| Parameter       | Used for              | Value | Description                                                       |
|-----------------|-----------------------|-------|-------------------------------------------------------------------|
| <i>info</i> [1] | Flags for <i>fjac</i> | 0     | The <i>fjac</i> array is valid.                                   |
|                 |                       | 1     | The <i>fjac</i> array is not allocated                            |
|                 |                       | 2     | The <i>fjac</i> array contains NaN.                               |
|                 |                       | 3     | The <i>fjac</i> array contains Inf.                               |
| <i>info</i> [2] | Flags for <i>fvec</i> | 0     | The <i>fvec</i> array is valid.                                   |
|                 |                       | 1     | The <i>fvec</i> array is not allocated                            |
|                 |                       | 2     | The <i>fvec</i> array contains NaN.                               |
|                 |                       | 3     | The <i>fvec</i> array contains Inf.                               |
| <i>info</i> [3] | Flags for <i>eps</i>  | 0     | The <i>eps</i> array is valid.                                    |
|                 |                       | 1     | The <i>eps</i> array is not allocated                             |
|                 |                       | 2     | The <i>eps</i> array contains NaN.                                |
|                 |                       | 3     | The <i>eps</i> array contains Inf.                                |
|                 |                       | 4     | The <i>eps</i> array contains a value less than or equal to zero. |

*res*

Information about completion of the task.

*res* = TR\_SUCCESS - the routine completed the task normally.

TR\_SUCCESS is defined in the `mkl_rci.h` include file.

## ?trnlsp\_solve

Solves a nonlinear least squares problem using the TR algorithm.

### Syntax

```
MKL_INT strnlsp_solve (_TRNSP_HANDLE_t* &handle, float* fvec, float* fjac, MKL_INT* &RCI_Request);
```

```
MKL_INT dtrnlsp_solve (_TRNSP_HANDLE_t* &handle, double* fvec, double* fjac, MKL_INT* &RCI_Request);
```

### Include Files

- `mkl.h`

### Description

The ?trnlsp\_solve routine uses the TR algorithm to solve nonlinear least squares problems.

The problem is stated as follows:

$$\min_{x \in \mathbb{R}^n} \|F(x)\|_2^2 = \min_{x \in \mathbb{R}^n} \|y - f(x)\|_2^2, y \in \mathbb{R}^m, x \in \mathbb{R}^n, f: \mathbb{R}^n \rightarrow \mathbb{R}^m, m \geq n,$$

where

- $F(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$
- $m \geq n$

From a current point  $x_{current}$ , the algorithm uses the trust-region approach:

$$\min_{x \in \mathbb{R}^n} \left\| F(x_{current}) + J(x_{current})(x_{new} - x_{current}) \right\|_2^2 \quad \text{subject to} \quad \|x_{new} - x_{current}\| \leq \Delta_{current}$$

to get  $x_{new} = x_{current} + s$  that satisfies

$$\min_{s \in \mathbb{R}^n} \left\| J^T(x)J(x)s + J^T F(x) \right\|_2^2$$

where

- $J(x)$  is the Jacobian matrix
- $s$  is the trial step
- $\|s\|_2 \leq \Delta_{current}$

The *RCI\_Request* parameter provides additional information:

| <i>RCI_Request</i> Value | Description                                                                                                             |
|--------------------------|-------------------------------------------------------------------------------------------------------------------------|
| 2                        | Request to calculate the Jacobian matrix and put the result into <i>fjac</i>                                            |
| 1                        | Request to recalculate the function at vector $x$ and put the result into <i>fvec</i>                                   |
| 0                        | One successful iteration step on the current trust-region radius (that does not mean that the value of $x$ has changed) |
| -1                       | The algorithm has exceeded the maximum number of iterations                                                             |
| -2                       | $\Delta < eps[0]$                                                                                                       |
| -3                       | $\ F(x)\ _2 < eps[1]$                                                                                                   |
| -4                       | The Jacobian matrix is singular.<br>$\ J(x)_{[m*(j-1)...m*j-1]}\ _2 < eps[2], j = 1, \dots, n$                          |
| -5                       | $\ s\ _2 < eps[3]$                                                                                                      |
| -6                       | $\ F(x)\ _2 - \ F(x) - J(x)s\ _2 < eps[4]$                                                                              |

Note:

- $J(x)$  is the Jacobian matrix.
- $\Delta$  is the trust-region area.
- $F(x)$  is the value of the functional.
- $s$  is the trial step.

## Input Parameters

|               |                                                                                                        |
|---------------|--------------------------------------------------------------------------------------------------------|
| <i>handle</i> | Type <code>_TRNSP_HANDLE_t</code> .                                                                    |
| <i>fvec</i>   | Array of size <i>m</i> . Contains the function values at <i>x</i> , where $fvec[i] = (y_i - f_i(x))$ . |
| <i>fjac</i>   | Array of size <i>m</i> by <i>n</i> . Contains the Jacobian matrix of the function.                     |

## Output Parameters

|                    |                                                                                                                                                                                                                    |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>fvec</i>        | Array of size <i>m</i> . Updated function evaluated at <i>x</i> .                                                                                                                                                  |
| <i>RCI_Request</i> | <p>Informs about the task stage.</p> <p>See the Description section for the parameter values and their meaning.</p>                                                                                                |
| <i>res</i>         | <p>Indicates the task completion.</p> <p><i>res</i> = <code>TR_SUCCESS</code> - the routine completed the task normally.</p> <p><code>TR_SUCCESS</code> is defined in the <code>mkl_rci.h</code> include file.</p> |

## ?trnlsp\_get

Retrieves the number of iterations, stop criterion, initial residual, and final residual.

### Syntax

```
MKL_INT strnlsp_get(_TRNSP_HANDLE_t* &handle, MKL_INT* &iter, MKL_INT* &st_cr, float* &r1, float* &r2);
```

```
MKL_INT dtrnlsp_get (_TRNSP_HANDLE_t* &handle, MKL_INT* &iter, MKL_INT* &st_cr, double* &r1, double* &r2);
```

### Include Files

- `mkl.h`

### Description

The routine retrieves the current number of iterations, the stop criterion, the initial residual, and final residual.

The initial residual is the value of the functional  $(\|y - f(x)\|)$  of the initial *x* values provided by the user.

The final residual is the value of the functional  $(\|y - f(x)\|)$  of the final *x* resulting from the algorithm operation.

The *st\_cr* parameter contains a number indicating the stop criterion:

| <i>st_cr</i> Value | Description                                                 |
|--------------------|-------------------------------------------------------------|
| 1                  | The algorithm has exceeded the maximum number of iterations |
| 2                  | $\Delta < eps[0]$                                           |
| 3                  | $\ F(x)\ _2 < eps[1]$                                       |
| 4                  | The Jacobian matrix is singular.                            |

| <i>st_cr</i> Value | Description                                                |
|--------------------|------------------------------------------------------------|
|                    | $  J(x)_{[m*(j-1)...m*j-1]}  _2 < eps[2], j = 1, \dots, n$ |
| 5                  | $  s  _2 < eps[3]$                                         |
| 6                  | $  F(x)  _2 -   F(x) - J(x)s  _2 < eps[4]$                 |

Note:

- $J(x)$  is the Jacobian matrix.
- $\Delta$  is the trust-region area.
- $F(x)$  is the value of the functional.
- $s$  is the trial step.

### Input Parameters

*handle* Type `_TRNSP_HANDLE_t`.

### Output Parameters

*iter* Contains the current number of iterations.

*st\_cr* Contains the stop criterion.  
See the Description section for the parameter values and their meanings.

*r1* Contains the residual,  $(||y - f(x)||)$  given the initial  $x$ .

*r2* Contains the final residual, that is, the value of the functional  $(||y - f(x)||)$  of the final  $x$  resulting from the algorithm operation.

*res* Indicates the task completion.  
`res = TR_SUCCESS` - the routine completed the task normally.  
`TR_SUCCESS` is defined in the `mkl_rci.h` include file.

### ?trnlsp\_delete

*Releases allocated data.*

#### Syntax

```
MKL_INT strnlsp_delete(_TRNSP_HANDLE_t* &handle);
MKL_INT dtrnlsp_delete(_TRNSP_HANDLE_t* &handle);
```

#### Include Files

- `mkl.h`

#### Description

The `?trnlsp_delete` routine releases all memory allocated for the handle.

This routine flags memory as not used, but to actually release all memory you must call the support function [mkl\\_free\\_buffers](#).

## Input Parameters

*handle* Type `_TRNSP_HANDLE_t`.

## Output Parameters

*res* Indicates the task completion.  
*res* = `TR_SUCCESS` means the routine completed the task normally.  
`TR_SUCCESS` is defined in the `mkl_rci.h` include file.

# Nonlinear Least Squares Problem with Linear (Bound) Constraints

The nonlinear least squares problem with linear bound constraints is very similar to the [nonlinear least squares problem without constraints](#) but it has the following constraints:

$$l_i \leq x_i \leq u_i, i = 1, \dots, n, \quad l, u \in R^n.$$

See usage examples in the `examples\solverc\source` folder of your Intel MKL directory. Specifically, see `ex_nlsqp_bc_c.c`.

## RCI TR Routines for Problem with Bound Constraints

| Routine Name                  | Operation                                                                                 |
|-------------------------------|-------------------------------------------------------------------------------------------|
| <code>?trnlspsc_init</code>   | Initializes the solver.                                                                   |
| <code>?trnlspsc_check</code>  | Checks correctness of the input parameters.                                               |
| <code>?trnlspsc_solve</code>  | Solves a nonlinear least squares problem using RCI and the Trust-Region algorithm.        |
| <code>?trnlspsc_get</code>    | Retrieves the number of iterations, stop criterion, initial residual, and final residual. |
| <code>?trnlspsc_delete</code> | Releases allocated data.                                                                  |

### ?trnlspsc\_init

*Initializes the solver of nonlinear least squares problem with linear (boundary) constraints.*

#### Syntax

```
MKL_INT strnlspsc_init (_TRNSPBC_HANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m, const float* x, const float* LW, const float* UP, const float* eps, const MKL_INT* &iter1, const MKL_INT* &iter2, const float* &rs);

MKL_INT dtrnlspsc_init (_TRNSPBC_HANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m, const double* x, const double* LW, const double* UP, const double* eps, const MKL_INT* &iter1, const MKL_INT* &iter2, const double* &rs);
```

## Include Files

- `mkl.h`

## Description

The `?trnlspbc_init` routine initializes the solver.

After initialization all subsequent invocations of the `?trnlspbc_solve` routine should use the values of the handle returned by `?trnlspbc_init`.

The `eps` array contains a number indicating the stopping criteria:

| <code>eps</code> Value | Description                                                                                                                 |
|------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| 0                      | $\Delta < eps[0]$                                                                                                           |
| 1                      | $\ F(x)\ _2 < eps[1]$                                                                                                       |
| 2                      | The Jacobian matrix is singular.<br>$\ J(x)_{[m*(j-1)...m*j-1]}\ _2 < eps[2], j = 1, \dots, n$                              |
| 3                      | $\ s\ _2 < eps[3]$                                                                                                          |
| 4                      | $\ F(x)\ _2 - \ F(x) - J(x)s\ _2 < eps[4]$                                                                                  |
| 5                      | The trial step precision. If $eps[5] = 0$ , then the trial step meets the required precision ( $\leq 1.0 \cdot 10^{-10}$ ). |

Note:

- $J(x)$  is the Jacobian matrix.
- $\Delta$  is the trust-region area.
- $F(x)$  is the value of the functional.
- $s$  is the trial step.

## Input Parameters

|                    |                                                                                         |
|--------------------|-----------------------------------------------------------------------------------------|
| <code>n</code>     | Length of $x$ .                                                                         |
| <code>m</code>     | Length of $F(x)$ .                                                                      |
| <code>x</code>     | Array of size $n$ . Initial guess.                                                      |
| <code>LW</code>    | Array of size $n$ .<br>Contains low bounds for $x$ ( $lw_i < x_i$ ).                    |
| <code>UP</code>    | Array of size $n$ .<br>Contains upper bounds for $x$ ( $up_i > x_i$ ).                  |
| <code>eps</code>   | Array of size 6; contains stopping criteria. See the values in the Description section. |
| <code>iter1</code> | Specifies the maximum number of iterations.                                             |
| <code>iter2</code> | Specifies the maximum number of iterations of trial step calculation.                   |

*rs* Definition of initial size of the trust region (boundary of the trial step). The recommended minimum value is 0.1, and the recommended maximum value is 100.0. Based on your knowledge of the objective function and initial guess you can increase or decrease the initial trust region. It can influence the iteration process, for example, the direction of the iteration process and the number of iterations. If you set *rs* to 0.0, the solver uses the default value, which is 100.0.

## Output Parameters

*handle* Type `_TRNSPBC_HANDLE_t`.

*res* Informs about the task completion.

- *res* = `TR_SUCCESS` - the routine completed the task normally.
- *res* = `TR_INVALID_OPTION` - there was an error in the input parameters.
- *res* = `TR_OUT_OF_MEMORY` - there was a memory error.

`TR_SUCCESS`, `TR_INVALID_OPTION`, and `TR_OUT_OF_MEMORY` are defined in the `mkl_rci.h` include file.

## ?trnlspsc\_check

*Checks the correctness of handle and arrays containing Jacobian matrix, objective function, lower and upper bounds, and stopping criteria.*

---

### Syntax

```
MKL_INT strnlspsc_check (_TRNSPBC_HANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m, const float* fjac, const float* fvec, const float* LW, const float* UP, const float* eps, MKL_INT* info);
```

```
MKL_INT dtrnlspsc_check (_TRNSPBC_HANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m, const double* fjac, const double* fvec, const double* LW, const double* UP, const double* eps, MKL_INT* info);
```

### Include Files

- `mkl.h`

### Description

The `?trnlspsc_check` routine checks the arrays passed into the solver as input parameters. If an array contains any `INF` or `NaN` values, the routine sets the flag in output array *info* (see the description of the values returned in the Output Parameters section for the *info* array).

### Input Parameters

*handle* Type `_TRNSPBC_HANDLE_t`.

*n* Length of *x*.

*m* Length of *F(x)*.

*fjac* Array of size *m* by *n*. Contains the Jacobian matrix of the function.

|             |                                                                                                                                |
|-------------|--------------------------------------------------------------------------------------------------------------------------------|
| <i>fvec</i> | Array of size $m$ . Contains the function values at $x$ , where $fvec[i] = (y_i - f_i(x))$ .                                   |
| <i>LW</i>   | Array of size $n$ .<br>Contains low bounds for $x$ ( $lw_i < x_i$ ).                                                           |
| <i>UP</i>   | Array of size $n$ .<br>Contains upper bounds for $x$ ( $up_i > x_i$ ).                                                         |
| <i>eps</i>  | Array of size 6; contains stopping criteria. See the values in the Description section of the <a href="#">?trnlspbc_init</a> . |

## Output Parameters

*info* Array of size 6.  
Results of input parameter checking:

| Parameter       | Used for                | Value | Description                                      |
|-----------------|-------------------------|-------|--------------------------------------------------|
| <i>info</i> [0] | Flags for <i>handle</i> | 0     | The handle is valid.                             |
|                 |                         | 1     | The handle is not allocated.                     |
| <i>info</i> [1] | Flags for <i>fjac</i>   | 0     | The <i>fjac</i> array is valid.                  |
|                 |                         | 1     | The <i>fjac</i> array is not allocated           |
|                 |                         | 2     | The <i>fjac</i> array contains NaN.              |
|                 |                         | 3     | The <i>fjac</i> array contains Inf.              |
| <i>info</i> [2] | Flags for <i>fvec</i>   | 0     | The <i>fvec</i> array is valid.                  |
|                 |                         | 1     | The <i>fvec</i> array is not allocated           |
|                 |                         | 2     | The <i>fvec</i> array contains NaN.              |
|                 |                         | 3     | The <i>fvec</i> array contains Inf.              |
| <i>info</i> [3] | Flags for <i>LW</i>     | 0     | The <i>LW</i> array is valid.                    |
|                 |                         | 1     | The <i>LW</i> array is not allocated             |
|                 |                         | 2     | The <i>LW</i> array contains NaN.                |
|                 |                         | 3     | The <i>LW</i> array contains Inf.                |
|                 |                         | 4     | The lower bound is greater than the upper bound. |
| <i>info</i> [4] | Flags for <i>up</i>     | 0     | The <i>up</i> array is valid.                    |
|                 |                         | 1     | The <i>up</i> array is not allocated             |
|                 |                         | 2     | The <i>up</i> array contains NaN.                |

| Parameter       | Used for             | Value | Description                                                       |
|-----------------|----------------------|-------|-------------------------------------------------------------------|
|                 |                      | 3     | The <i>up</i> array contains Inf.                                 |
|                 |                      | 4     | The upper bound is less than the lower bound.                     |
| <i>info</i> [5] | Flags for <i>eps</i> | 0     | The <i>eps</i> array is valid.                                    |
|                 |                      | 1     | The <i>eps</i> array is not allocated                             |
|                 |                      | 2     | The <i>eps</i> array contains NaN.                                |
|                 |                      | 3     | The <i>eps</i> array contains Inf.                                |
|                 |                      | 4     | The <i>eps</i> array contains a value less than or equal to zero. |

*res*

Information about completion of the task.

*res* = TR\_SUCCESS - the routine completed the task normally.

TR\_SUCCESS is defined in the `mkl_rci.h` include file.

## ?trnlspbc\_solve

Solves a nonlinear least squares problem with linear (bound) constraints using the Trust-Region algorithm.

### Syntax

```
MKL_INT strnlspbc_solve (_TRNSPBC_HANDLE_t* &handle, float* fvec, float* fjac, MKL_INT* &RCI_Request);
```

```
MKL_INT dtrnlspbc_solve (_TRNSPBC_HANDLE_t* &handle, double* fvec, double* fjac, MKL_INT* &RCI_Request);
```

### Include Files

- `mkl.h`

### Description

The `?trnlspbc_solve` routine, based on RCI, uses the Trust-Region algorithm to solve nonlinear least squares problems with linear (bound) constraints. The problem is stated as follows:

$$\min_{x \in \mathbb{R}^n} \|F(x)\|_2^2 = \min_{x \in \mathbb{R}^n} \|y - f(x)\|_2^2, y \in \mathbb{R}^m, x \in \mathbb{R}^n, f: \mathbb{R}^n \rightarrow \mathbb{R}^m, m \geq n$$

where

$$l_i \leq x_i \leq u_i$$

$$i = 1, \dots, n.$$

The `RCI_Request` parameter provides additional information:

| <i>RCI_Request</i> Value | Description                                                                                                                  |
|--------------------------|------------------------------------------------------------------------------------------------------------------------------|
| 2                        | Request to calculate the Jacobian matrix and put the result into <i>fjac</i>                                                 |
| 1                        | Request to recalculate the function at vector <i>x</i> and put the result into <i>fvec</i>                                   |
| 0                        | One successful iteration step on the current trust-region radius (that does not mean that the value of <i>x</i> has changed) |
| -1                       | The algorithm has exceeded the maximum number of iterations                                                                  |
| -2                       | $\Delta < eps[0]$                                                                                                            |
| -3                       | $  F(x)  _2 < eps[1]$                                                                                                        |
| -4                       | The Jacobian matrix is singular.<br>$  J(x)_{[m*(j-1)...m*j-1]}  _2 < eps[2], j = 1, \dots, n$                               |
| -5                       | $  s  _2 < eps[3]$                                                                                                           |
| -6                       | $  F(x)  _2 -   F(x) - J(x)s  _2 < eps[4]$                                                                                   |

Note:

- $J(x)$  is the Jacobian matrix.
- $\Delta$  is the trust-region area.
- $F(x)$  is the value of the functional.
- $s$  is the trial step.

### Input Parameters

|               |                                                                                              |
|---------------|----------------------------------------------------------------------------------------------|
| <i>handle</i> | Type <code>_TRNSPBC_HANDLE_t</code> .                                                        |
| <i>fvec</i>   | Array of size $m$ . Contains the function values at $x$ , where $fvec[i] = (y_i - f_i(x))$ . |
| <i>fjac</i>   | Array of size $m$ by $n$ . Contains the Jacobian matrix of the function.                     |

### Output Parameters

|                    |                                                                                                                                                                                                                       |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>fvec</i>        | Array of size $m$ . Updated function evaluated at $x$ .                                                                                                                                                               |
| <i>RCI_Request</i> | <p>Informs about the task stage.</p> <p>See the Description section for the parameter values and their meaning.</p>                                                                                                   |
| <i>res</i>         | <p>Informs about the task completion.</p> <p><math>res = TR\_SUCCESS</math> means the routine completed the task normally.</p> <p><code>TR\_SUCCESS</code> is defined in the <code>mkl_rci.h</code> include file.</p> |

### ?trnlspsc\_get

Retrieves the number of iterations, stop criterion, initial residual, and final residual.



`res = TR_SUCCESS` - the routine completed the task normally.

`TR_SUCCESS` is defined in the `mkl_rci.h` include file.

## ?trnlspsc\_delete

*Releases allocated data.*

### Syntax

```
MKL_INT strnlspsc_delete (_TRNSPBC_HANDLE_t* &handle);
```

```
MKL_INT dtrnlspsc_delete (_TRNSPBC_HANDLE_t* &handle);
```

### Include Files

- `mkl.h`

### Description

The `?trnlspsc_delete` routine releases all memory allocated for the handle.

#### NOTE

This routine flags memory as not used, but to actually release all memory you must call the support function `mkl_free_buffers`.

### Input Parameters

`handle` Type `_TRNSPBC_HANDLE_t`.

### Output Parameters

`res` Informs about the task completion.  
`res = TR_SUCCESS` means the routine completed the task normally.  
`TR_SUCCESS` is defined in the `mkl_rci.h` include file.

## Jacobian Matrix Calculation Routines

This section describes routines that compute the Jacobian matrix using the central difference algorithm. Jacobian matrix calculation is required to solve a nonlinear least squares problem and systems of nonlinear equations (with or without linear bound constraints). Routines for calculation of the Jacobian matrix have the "Black-Box" interfaces, where you pass the objective function via parameters. Your objective function must have a fixed interface.

### Jacobian Matrix Calculation Routines

| Routine Name                | Operation                                                                                                |
|-----------------------------|----------------------------------------------------------------------------------------------------------|
| <code>?jacobi_init</code>   | Initializes the solver.                                                                                  |
| <code>?jacobi_solve</code>  | Computes the Jacobian matrix of the function on the basis of RCI using the central difference algorithm. |
| <code>?jacobi_delete</code> | Removes data.                                                                                            |

| Routine Name          | Operation                                                                                                                                 |
|-----------------------|-------------------------------------------------------------------------------------------------------------------------------------------|
| <code>?jacobi</code>  | Computes the Jacobian matrix of the <code>fcn</code> function using the central difference algorithm.                                     |
| <code>?jacobix</code> | Presents an alternative interface for the <code>?jacobi</code> function enabling you to pass additional data into the objective function. |

## `?jacobi_init`

Initializes the solver for Jacobian calculations.

### Syntax

```
MKL_INT sjacobi_init (_JACOBIHANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m, const float* x, const float* fjac, const float* &eps);
```

```
MKL_INT djacobi_init (_JACOBIHANDLE_t* &handle, const MKL_INT* &n, const MKL_INT* &m, const double* x, const double* fjac, const double* &eps);
```

### Include Files

- `mkl.h`

### Description

The routine initializes the solver.

### Input Parameters

|                   |                                                                          |
|-------------------|--------------------------------------------------------------------------|
| <code>n</code>    | Length of $x$ .                                                          |
| <code>m</code>    | Length of $F$ .                                                          |
| <code>x</code>    | Array of size $n$ . Vector, at which the function is evaluated.          |
| <code>eps</code>  | Precision of the Jacobian matrix calculation.                            |
| <code>fjac</code> | Array of size $m$ by $n$ . Contains the Jacobian matrix of the function. |

### Output Parameters

|                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>handle</code> | Data object of the <code>_JACOBIHANDLE_t</code> type.                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <code>res</code>    | Indicates task completion status. <ul style="list-style-type: none"> <li>• <code>res = TR_SUCCESS</code> - the routine completed the task normally.</li> <li>• <code>res = TR_INVALID_OPTION</code> - there was an error in the input parameters.</li> <li>• <code>res = TR_OUT_OF_MEMORY</code> - there was a memory error.</li> </ul> <p><code>TR_SUCCESS</code>, <code>TR_INVALID_OPTION</code>, and <code>TR_OUT_OF_MEMORY</code> are defined in the <code>mkl_rci.h</code> include file.</p> |

## `?jacobi_solve`

Computes the Jacobian matrix of the function using RCI and the central difference algorithm.

## Syntax

```
MKL_INT sjacobi_solve (_JACOBI_MATRIX_HANDLE_t* &handle, float* f1, float* f2, MKL_INT*
&RCI_Request);
```

```
MKL_INT djacobi_solve (_JACOBI_MATRIX_HANDLE_t* &handle, double* f1, double* f2,
MKL_INT* &RCI_Request);
```

## Include Files

- mkl.h

## Description

The `?jacobi_solve` routine computes the Jacobian matrix of the function using RCI and the central difference algorithm.

See usage examples in the `examples\solverc\source` folder of your Intel MKL directory. Specifically, see `sjacobi_rci_c.c` and `djacobi_rci_c.c`.

## Input Parameters

|                    |                                                                |
|--------------------|----------------------------------------------------------------|
| <i>handle</i>      | Type <code>_JACOBI_MATRIX_HANDLE_t</code> .                    |
| <i>RCI_Request</i> | Set to 0 before the first call to <code>?jacobi_solve</code> . |

## Output Parameters

|                    |                                                                                                                                                                                                                                                                                                                                                                                                           |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>f1</i>          | Contains the updated function values at $x + \text{eps}$ .                                                                                                                                                                                                                                                                                                                                                |
| <i>f2</i>          | Array of size $m$ . Contains the updated function values at $x - \text{eps}$ .                                                                                                                                                                                                                                                                                                                            |
| <i>RCI_Request</i> | Provides information about the task completion. When equal to 0, the task has completed successfully.<br><br><i>RCI_Request</i> = 1 indicates that you should compute the function values at the current $x$ point and put the results into <i>f1</i> .<br><br><i>RCI_Request</i> = 2 indicates that you should compute the function values at the current $x$ point and put the results into <i>f2</i> . |
| <i>res</i>         | Indicates the task completion status. <ul style="list-style-type: none"> <li>• <i>res</i> = <code>TR_SUCCESS</code> - the routine completed the task normally.</li> <li>• <i>res</i> = <code>TR_INVALID_OPTION</code> - there was an error in the input parameters.</li> </ul> <p><code>TR_SUCCESS</code> and <code>TR_INVALID_OPTION</code> are defined in the <code>mkl_rci.h</code> include file.</p>  |

## See Also

[?jacobi\\_init](#)

## ?jacobi\_delete

*Releases allocated data.*

---

## Syntax

```
MKL_INT sjacobi_delete (_JACOBI_MATRIX_HANDLE_t* &handle);
```



| Parameter         | Type | Description                                                                                                                     |
|-------------------|------|---------------------------------------------------------------------------------------------------------------------------------|
| $m$               |      | Length of $f$                                                                                                                   |
| $n$               |      | Length of $x$                                                                                                                   |
| $x$               |      | Array of size $n$ . Vector, at which the function is evaluated. The <code>fcn</code> function should not change this parameter. |
| Output Parameters |      |                                                                                                                                 |
| $f$               |      | Array of size $m$ ; contains the function values at $x$ .                                                                       |

You need to declare `fcn` as `extern` in the calling program.

|       |                                                                |
|-------|----------------------------------------------------------------|
| $n$   | Length of $X$ .                                                |
| $m$   | Length of $F$ .                                                |
| $x$   | Array of size $n$ . Vector at which the function is evaluated. |
| $eps$ | Precision of the Jacobian matrix calculation.                  |

## Output Parameters

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $fjac$ | Array of size $m$ by $n$ . Contains the Jacobian matrix of the function.                                                                                                                                                                                                                                                                                                                                                                                                                          |
| $res$  | Indicates task completion status. <ul style="list-style-type: none"> <li><math>res = TR\_SUCCESS</math> - the routine completed the task normally.</li> <li><math>res = TR\_INVALID\_OPTION</math> - there was an error in the input parameters.</li> <li><math>res = TR\_OUT\_OF\_MEMORY</math> - there was a memory error.</li> </ul> <p><code>TR_SUCCESS</code>, <code>TR_INVALID_OPTION</code>, and <code>TR_OUT_OF_MEMORY</code> are defined in the <code>mkl_rci.h</code> include file.</p> |

## See Also

[?jacobix](#)

## ?jacobix

Alternative interface for `?jacobi` function for passing additional data into the objective function.

### Syntax

```
MKL_INT sjacobix (USRFCNXS fcn, const MKL_INT* &n, const MKL_INT * &m, float* fjac,
float* x, float* &eps, void* user_data);
```

```
MKL_INT djacobix (USRFCNXD fcn, const MKL_INT* &n, const MKL_INT * &m, double* fjac,
double* x, double* &eps, void* user_data);
```

### Include Files

- `mkl.h`

## Description

The `?jacobix` routine presents an alternative interface for the `?jacobi` function that enables you to pass additional data into the objective function `fcn`.

See calling and usage examples in the `examples\solverc\source` folder of your Intel MKL directory. Specifically, see `ex_nlsqp_c_x.c` and `ex_nlsqp_bc_c_x.c`.

## Input Parameters

`fcn`

User-supplied subroutine to evaluate the function that defines the least squares problem. Call `fcn (&m, &n, x, f, user_data)` with the following parameters:

| Parameter              | Description                                                                                                                                |
|------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Input Parameters       |                                                                                                                                            |
| <code>m</code>         | Length of <code>f</code>                                                                                                                   |
| <code>n</code>         | Length of <code>x</code>                                                                                                                   |
| <code>x</code>         | Array of size <code>n</code> . Vector, at which the function is evaluated. The <code>fcn</code> function should not change this parameter. |
| <code>user_data</code> | Pointer to your additional data, if any. Otherwise, a dummy argument.                                                                      |
| Output Parameters      |                                                                                                                                            |
| <code>f</code>         | Array of size <code>m</code> ; contains the function values at <code>x</code> .                                                            |

You need to declare `fcn` as `extern` in the calling program.

`n`

Length of `X`.

`m`

Length of `F`.

`x`

Array of size `n`. Vector at which the function is evaluated.

`eps`

Precision of the Jacobian matrix calculation.

`user_data`

Pointer to your additional data. If there is no additional data, this is a dummy argument.

## Output Parameters

`fjac`

Array of size `m` by `n`). Contains the Jacobian matrix of the function.

`res`

Indicates task completion status.

- `res = TR_SUCCESS` - the routine completed the task normally.
- `res = TR_INVALID_OPTION` - there was an error in the input parameters.
- `res = TR_OUT_OF_MEMORY` - there was a memory error.

`TR_SUCCESS`, `TR_INVALID_OPTION`, and `TR_OUT_OF_MEMORY` are defined in the `mkl_rci.h` include file.

---

**See Also**

?jacobi



# Support Functions

Intel® Math Kernel Library (Intel® MKL) support functions are subdivided into the following groups according to their purpose:

Version Information

Threading Control

Error Handling

Character Equality Testing

Timing

Memory Management

Single Dynamic Library Control

Intel® Many Integrated Core (Intel® MIC) Architecture Support

Conditional Numerical Reproducibility Control

Miscellaneous

The following table lists Intel MKL support functions.

## Intel MKL Support Functions

| Function Name                           | Operation                                                                                                                      |
|-----------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|
| Version Information                     |                                                                                                                                |
| <code>mkl_get_version</code>            | Returns the Intel MKL version.                                                                                                 |
| <code>mkl_get_version_string</code>     | Returns the Intel MKL version in a character string.                                                                           |
| Threading Control                       |                                                                                                                                |
| <code>mkl_set_num_threads</code>        | Specifies the number of OpenMP* threads to use.                                                                                |
| <code>mkl_domain_set_num_threads</code> | Specifies the number of OpenMP* threads for a particular function domain.                                                      |
| <code>mkl_set_num_threads_local</code>  | Specifies the number of OpenMP* threads for all Intel MKL functions on the current execution thread.                           |
| <code>mkl_set_dynamic</code>            | Enables Intel MKL to dynamically change the number of OpenMP* threads.                                                         |
| <code>mkl_get_max_threads</code>        | Gets the number of OpenMP* threads targeted for parallelism.                                                                   |
| <code>mkl_domain_get_max_threads</code> | Gets the number of OpenMP* threads targeted for parallelism for a particular function domain.                                  |
| <code>mkl_get_dynamic</code>            | Determines whether Intel MKL is enabled to dynamically change the number of OpenMP* threads.                                   |
| <code>mkl_set_num_stripes</code>        | Specifies the number of partitions along the leading dimension of the output matrix for parallel <code>?gemm</code> functions. |

| Function Name                          | Operation                                                                                                                       |
|----------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| <code>mkl_get_num_stripes</code>       | Gets the number of partitions along the leading dimension of the output matrix for parallel ?gemm functions.                    |
| <b>Error Handling</b>                  |                                                                                                                                 |
| <code>xerbla</code>                    | Error handling function called by BLAS, LAPACK, Vector Math, and Vector Statistics functions.                                   |
| <code>pxerbla</code>                   | Handles error conditions for the ScaLAPACK routines.                                                                            |
| <code>LAPACKE_xerbla</code>            | Error handling function called by the C interface to LAPACK functions.                                                          |
| <code>mkl_set_exit_handler</code>      | Sets the custom handler of fatal errors.                                                                                        |
| <b>Character Equality Testing</b>      |                                                                                                                                 |
| <code>lsame</code>                     | Tests two characters for equality regardless of the case.                                                                       |
| <code>lsamen</code>                    | Tests two character strings for equality regardless of the case.                                                                |
| <b>Timing</b>                          |                                                                                                                                 |
| <code>second/dsecnd</code>             | Returns elapsed time in seconds. Use to estimate real time between two calls to this function.                                  |
| <code>mkl_get_cpu_clocks</code>        | Returns elapsed CPU clocks.                                                                                                     |
| <code>mkl_get_cpu_frequency</code>     | Returns CPU frequency value in GHz.                                                                                             |
| <code>mkl_get_max_cpu_frequency</code> | Returns the maximum CPU frequency value in GHz.                                                                                 |
| <code>mkl_get_clocks_frequency</code>  | Returns the frequency value in GHz based on constant-rate Time Stamp Counter.                                                   |
| <b>Memory Management</b>               |                                                                                                                                 |
| <code>mkl_free_buffers</code>          | Frees unused memory allocated by the Intel MKL Memory Allocator.                                                                |
| <code>mkl_thread_free_buffers</code>   | Frees unused memory allocated by the Intel MKL Memory Allocator in the current thread.                                          |
| <code>mkl_mem_stat</code>              | Reports the status of the Intel MKL Memory Allocator.                                                                           |
| <code>mkl_peak_mem_usage</code>        | Reports the peak memory allocated by the Intel MKL Memory Allocator.                                                            |
| <code>mkl_disable_fast_mm</code>       | Turns off the Intel MKL Memory Allocator for Intel MKL functions to directly use the system <code>malloc/free</code> functions. |
| <code>mkl_malloc</code>                | Allocates an aligned memory buffer.                                                                                             |
| <code>mkl_calloc</code>                | Allocates and initializes an aligned memory buffer.                                                                             |
| <code>mkl_realloc</code>               | Changes the size of memory buffer allocated by <code>mkl_malloc/mkl_calloc</code> .                                             |
| <code>mkl_free</code>                  | Frees the aligned memory buffer allocated by <code>mkl_malloc/mkl_calloc</code> .                                               |

| Function Name                               | Operation                                                                                                                                                                                |
|---------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mkl_set_memory_limit</code>           | On Linux, sets the limit of memory that Intel MKL can allocate for a specified type of memory.                                                                                           |
| Single Dynamic Library (SDL) Control        |                                                                                                                                                                                          |
| <code>mkl_set_interface_layer</code>        | Sets the interface layer for Intel MKL at run time.                                                                                                                                      |
| <code>mkl_set_threading_layer</code>        | Sets the threading layer for Intel MKL at run time.                                                                                                                                      |
| <code>mkl_set_xerbla</code>                 | Replaces the error handling routine. Use with the Single Dynamic Library on Windows*.                                                                                                    |
| <code>mkl_set_progress</code>               | Replaces the progress information routine.                                                                                                                                               |
| <code>mkl_set_pardiso_pivot</code>          | Replaces the routine handling Intel MKL PARDISO pivots with a user-defined routine. Use with the Single Dynamic Library (SDL).                                                           |
| Intel MIC Architecture Support              |                                                                                                                                                                                          |
| <code>mkl_mic_enable</code>                 | Enables Automatic Offload mode.                                                                                                                                                          |
| <code>mkl_mic_disable</code>                | Disables Automatic Offload mode.                                                                                                                                                         |
| <code>mkl_mic_get_device_count</code>       | Returns the number of Intel® Xeon Phi™ coprocessors on the system when called on the host CPU.                                                                                           |
| <code>mkl_mic_set_workdivision</code>       | For computations in the Automatic Offload mode, sets the fraction of the work for the specified coprocessor or host CPU to do.                                                           |
| <code>mkl_mic_get_workdivision</code>       | For computations in the Automatic Offload mode, retrieves the fraction of the work for the specified coprocessor or host CPU to do.                                                      |
| <code>mkl_mic_set_max_memory</code>         | Sets the maximum amount of coprocessor memory reserved for Automatic Offload computations.                                                                                               |
| <code>mkl_mic_free_memory</code>            | Frees the coprocessor memory reserved for Automatic Offload computations.                                                                                                                |
| <code>mkl_mic_register_memory</code>        | Enables/disables the <code>mkl_malloc</code> function running in Automatic Offload mode to register allocated memory.                                                                    |
| <code>mkl_mic_set_device_num_threads</code> | Sets the maximum number of OpenMP* threads to use on an Intel Xeon Phi coprocessor for the Automatic Offload computations.                                                               |
| <code>mkl_mic_set_resource_limit</code>     | For computations in the Automatic Offload mode, sets the maximum fraction of available Intel Xeon Phi coprocessor computational resources (cores) that the calling process can use.      |
| <code>mkl_mic_get_resource_limit</code>     | For computations in the Automatic Offload mode, retrieves the maximum fraction of available Intel Xeon Phi coprocessor computational resources (cores) that the calling process can use. |
| <code>mkl_mic_set_offload_report</code>     | Turns on/off reporting of Automatic Offload profiling.                                                                                                                                   |
| <code>mkl_mic_set_flags</code>              | Sets flags to control the behavior of computations in the Automatic Offload mode.                                                                                                        |
| <code>mkl_mic_get_flags</code>              | Retrieves flags that control the behavior of computations in the Automatic Offload mode.                                                                                                 |
| <code>mkl_mic_get_status</code>             | For the Automatic Offload mode, returns the status of the latest call to an Intel MKL function.                                                                                          |

| Function Name                                       | Operation                                                                                                                   |
|-----------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| <code>mkl_mic_clear_status</code>                   | For the Automatic Offload mode, clears the status of the latest call to an Intel MKL function.                              |
| <code>mkl_mic_get_meminfo</code>                    | Retrieves the amount of total and free memory for the specified coprocessor or host CPU.                                    |
| <code>mkl_mic_get_cpuinfo</code>                    | Retrieves the number of cores, hardware threads, and frequency for the specified coprocessor or host CPU.                   |
| Conditional Numerical Reproducibility (CNR) Control |                                                                                                                             |
| <code>mkl_cbwr_set</code>                           | Configures the CNR mode of Intel MKL.                                                                                       |
| <code>mkl_cbwr_get</code>                           | Returns the current CNR settings.                                                                                           |
| <code>mkl_cbwr_get_auto_branch</code>               | Automatically detects the CNR code branch for your platform.                                                                |
| Miscellaneous                                       |                                                                                                                             |
| <code>mkl_progress</code>                           | Provides progress information.                                                                                              |
| <code>mkl_enable_instructions</code>                | Enables dispatching for new Intel® architectures or restricts the set of Intel® instruction sets available for dispatching. |
| <code>mkl_set_env_mode</code>                       | Sets up the mode that ignores environment settings specific to Intel MKL.                                                   |
| <code>mkl_verbose</code>                            | Enables or disables Intel MKL Verbose mode.                                                                                 |
| <code>mkl_set_mpi</code>                            | Sets the implementation of the message-passing interface to be used by Intel MKL.                                           |
| <code>mkl_finalize</code>                           | Terminates Intel MKL execution environment and frees resources allocated by the library.                                    |

### Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

## Version Information

Intel® MKL provides two methods for extracting information about the library version number:

- extracting a version string using the `mkl_get_version_string` function
- using the `mkl_get_version` function to obtain an `MKLVersion` structure that contains the version information

A makefile is also provided to automatically build the examples and output summary files containing the version information for the current library.

## mkl\_get\_version

Returns the Intel MKL version.

### Syntax

```
void mkl_get_version(MKLVersion* pVersion);
```

### Include Files

- mkl.h

### Output Parameters

*pVersion*                      Pointer to the `MKLVersion` structure.

### Description

The `mkl_get_version` function collects information about the active C version of the Intel MKL software and returns this information in a structure of `MKLVersion` type by the *pVersion* address. The `MKLVersion` structure type is defined in the `mkl_types.h` file. The following fields of the `MKLVersion` structure are available:

|                            |                                                                                                                                                                                                                                                                                                                         |
|----------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>MajorVersion</code>  | is the major number of the current library version.                                                                                                                                                                                                                                                                     |
| <code>MinorVersion</code>  | is the minor number of the current library version.                                                                                                                                                                                                                                                                     |
| <code>UpdateVersion</code> | is the update number of the current library version.                                                                                                                                                                                                                                                                    |
| <code>ProductStatus</code> | is the status of the current library version. Possible variants are "Beta" or "Product".                                                                                                                                                                                                                                |
| <code>Build</code>         | is the string that contains the build date and the internal build number.                                                                                                                                                                                                                                               |
| <code>Platform</code>      | is the string that contains the current architecture. Possible variants are "IA-32 architecture" or "Intel(R) 64 architecture".                                                                                                                                                                                         |
| <code>Processor</code>     | is the processor optimization. Normally it is targeted for the processor installed on your system and based on the detection of the Intel MKL library that is optimal for the installed processor. In the Conditional Numerical Reproducibility (CNR) mode, the processor optimization matches the selected CNR branch. |

#### Optimization Notice

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## mkl\_get\_version Usage

---

```

#include <stdio.h>
#include <stdlib.h>
#include "mkl.h"

int main(void)
{
 MKLVersion Version;

 mkl_get_version(&Version);

 printf("Major version: %d\n",Version.MajorVersion);
 printf("Minor version: %d\n",Version.MinorVersion);
 printf("Update version: %d\n",Version.UpdateVersion);
 printf("Product status: %s\n",Version.ProductStatus);
 printf("Build: %s\n",Version.Build);
 printf("Platform: %s\n",Version.Platform);
 printf("Processor optimization: %s\n",Version.Processor);
 printf("=====\n");
 printf("\n");

 return 0;
}

```

### Output:

|                        |                                |
|------------------------|--------------------------------|
| Major Version          | 11                             |
| Minor Version          | 0                              |
| Update Version         | 2                              |
| Product status         | Product                        |
| Build                  | 20121113                       |
| Platform               | Intel(R) 64 architecture       |
| Processor optimization | Intel(R) Core(TM) i7 Processor |

### See Also

[Conditional Numerical Reproducibility Control](#)

## mkl\_get\_version\_string

*Returns the Intel MKL version in a character string.*

### Syntax

```
void mkl_get_version_string (char* buf, int len);
```

### Include Files

- mkl.h

## Output Parameters

| Name             | Type               | Description                 |
|------------------|--------------------|-----------------------------|
| <code>buf</code> | <code>char*</code> | Source string               |
| <code>len</code> | <code>int</code>   | Length of the source string |

## Description

The function returns a string that contains the Intel MKL version.

For usage details, see the code example below:

## Example

```
#include <stdio.h>
#include "mkl.h"

int main(void)
{
 int len=198;
 char buf[198];
 mkl_get_version_string(buf, len);
 printf("%s\n",buf);
 printf("\n");
 return 0;
}
```

## Threading Control

Intel® MKL provides functions for OpenMP\* threading control, discussed in this section.

### Important

If Intel MKL operates within the Intel® Threading Building Blocks (Intel® TBB) execution environment, the environment variables for OpenMP\* threading control, such as `OMP_NUM_THREADS`, and Intel MKL functions discussed in this section have no effect. If the Intel TBB threading technology is used, control the number of threads through the Intel TBB application programming interface. Read the documentation for the `tbb::task_scheduler_init` class at <https://www.threadingbuildingblocks.org/documentation> to find out how to specify the number of Intel TBB threads.

If Intel® MKL operates within an OpenMP\* execution environment, you can control the number of threads for Intel MKL using OpenMP\* run-time library routines and environment variables (see the OpenMP\* specification for details). Additionally Intel MKL provides *optional* threading control functions and environment variables that enable you to specify the number of threads for Intel MKL and to control dynamic adjustment of the number of threads *independently* of the OpenMP\* settings. The settings made with the Intel MKL threading control functions and environment variables do not affect OpenMP\* settings but take precedence over them.

If none of the threading control functions is used, Intel MKL environment variables may control Intel MKL threading. For details of those environment variables, see the *Intel MKL Developer Guide*.

You can specify the number of threads for Intel MKL function domains with the `mkl_set_num_threads` or `mkl_domain_set_num_threads` function. While `mkl_set_num_threads` specifies the number of threads for the entire Intel MKL, `mkl_domain_set_num_threads` does it for a specific function domain. The following table lists the function domains that support independent threading control. The table also provides named constants to pass to threading control functions as a parameter that specifies the function domain.

**Intel MKL Function Domains**

| Function Domain                                                                                             | Named Constant     |
|-------------------------------------------------------------------------------------------------------------|--------------------|
| Basic Linear Algebra Subroutines (BLAS)                                                                     | MKL_DOMAIN_BLAS    |
| Fast Fourier Transform (FFT) functions, except Cluster FFT functions                                        | MKL_DOMAIN_FFT     |
| Vector Math (VM) functions                                                                                  | MKL_DOMAIN_VML     |
| Parallel Direct Solver (PARDISO) functions                                                                  | MKL_DOMAIN_PARDISO |
| All Intel MKL functions except the functions from the domains where the number of threads is set explicitly | MKL_DOMAIN_ALL     |

**WARNING**

Do not increase the number of OpenMP threads used for Intel MKL PARDISO between the first call to `pardiso` and the factorization or solution phase. Because the minimum amount of memory required for out-of-core execution depends on the number of OpenMP threads, increasing it after the initial call can cause incorrect results.

Both `mkl_set_num_threads` and `mkl_domain_set_num_threads` functions set the number of threads for all subsequent calls to Intel MKL from all applications threads. Use the `mkl_set_num_threads_local` function to specify different numbers of threads for Intel MKL on different execution threads of your application. The thread-local settings take precedence over the global settings. However, the thread-local settings may have undesirable side effects (see the description of the `mkl_set_num_threads_local` function for details).

By default, Intel MKL can adjust the specified number of threads dynamically. For example, Intel MKL may use fewer threads if the size of the computation is not big enough or not create parallel regions when running within an OpenMP\* parallel region. Although Intel MKL may actually use a different number of threads from the number specified, the library does not create parallel regions with more threads than specified. If dynamic adjustment of the number of threads is disabled, Intel MKL attempts to use the specified number of threads in internal parallel regions (for more information, see the *Intel MKL Developer Guide*). Use the `mkl_set_dynamic` function to control dynamic adjustment of the number of threads.

**mkl\_set\_num\_threads**

*Specifies the number of OpenMP\* threads to use.*

**Syntax**

```
void mkl_set_num_threads(int nt);
```

**Include Files**

- `mkl.h`

**Input Parameters**

| Name            | Type             | Description                                                                                                                     |
|-----------------|------------------|---------------------------------------------------------------------------------------------------------------------------------|
| <code>nt</code> | <code>int</code> | <code>nt &gt; 0</code> - The number of threads suggested by the user.<br><code>nt ≤ 0</code> - Invalid value, which is ignored. |

**Description**

This function enables you to specify how many OpenMP threads Intel MKL should use for internal parallel regions. If this number is not set (default), Intel MKL functions use the default number of threads for the OpenMP run-time library. The specified number of threads applies:

- To all Intel MKL functions except the functions from the domains where the number of threads is set with [mkl\\_domain\\_set\\_num\\_threads](#)
- To all execution threads except the threads where the number of threads is set with [mkl\\_set\\_num\\_threads\\_local](#)

The number specified is a hint, and Intel MKL may actually use a smaller number.

---

**NOTE**

This function takes precedence over the `MKL_NUM_THREADS` environment variable.

---

## Example

```
#include "mkl.h"
...
mkl_set_num_threads(4);
my_compute_using_mkl(); // Intel MKL uses up to 4 OpenMP threads
```

## mkl\_domain\_set\_num\_threads

*Specifies the number of OpenMP\* threads for a particular function domain.*

---

### Syntax

```
int mkl_domain_set_num_threads (int nt, int domain);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name          | Type             | Description                                                                                                                                                                                                             |
|---------------|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>nt</i>     | <code>int</code> | <p><i>nt</i> &gt; 0 - The number of threads suggested by the user.</p> <p><i>nt</i> = 0 - The default number of threads for the OpenMP run-time library.</p> <p><i>nt</i> &lt; 0 - Invalid value, which is ignored.</p> |
| <i>domain</i> | <code>int</code> | The named constant that defines the targeted domain.                                                                                                                                                                    |

### Description

This function specifies how many OpenMP threads a particular function domain of Intel MKL should use. If this number is not set (default) or if it is set to zero in a call to this function, Intel MKL uses the default number of threads for the OpenMP run-time library. The number of threads specified applies to the specified function domain on all execution threads except the threads where the number of threads is set with [mkl\\_set\\_num\\_threads\\_local](#). For a list of supported values of the *domain* argument, see [Table "Intel MKL Function Domains"](#).

The number of threads specified is only a hint, and Intel MKL may actually use a smaller number.

---

**NOTE**

This function takes precedence over the `MKL_DOMAIN_NUM_THREADS` environment variable.

---

## Return Values

| Name              | Type             | Description                                                                                                                |
|-------------------|------------------|----------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | <code>int</code> | 1 - Indicates no error, execution is successful.<br>0 - Indicates a failure, possibly because of invalid input parameters. |

## Example

```
#include "mkl.h"
...
mkl_domain_set_num_threads(4, MKL_DOMAIN_BLAS);
my_compute_using_mkl_blas(); // Intel MKL BLAS functions use up to 4 threads
my_compute_using_mkl_dft(); // Intel MKL FFT functions use the default number of threads
```

## mkl\_set\_num\_threads\_local

*Specifies the number of OpenMP\* threads for all Intel MKL functions on the current execution thread.*

### Syntax

```
int mkl_set_num_threads_local(int nt);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name            | Type             | Description                                                                                                                                                                                                              |
|-----------------|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>nt</code> | <code>int</code> | <code>nt &gt; 0</code> - The number of threads for Intel MKL functions to use on the current execution thread.<br><code>nt = 0</code> - A request to reset the thread-local number of threads and use the global number. |

### Description

This function sets the number of OpenMP threads that Intel MKL functions should request for parallel computation. The number of threads is thread-local, which means that it only affects the current execution thread of the application. If the thread-local number is not set or if this number is set to zero in a call to this function, Intel MKL functions use the global number of threads. You can set the global number of threads using the [mkl\\_set\\_num\\_threads](#) or [mkl\\_domain\\_set\\_num\\_threads](#) function.

The thread-local number of threads takes precedence over the global number: if the thread-local number is non-zero, changes to the global number of threads have no effect on the current thread.

#### CAUTION

If your application is threaded with OpenMP\* and parallelization of Intel MKL is based on nested OpenMP parallelism, different OpenMP parallel regions reuse OpenMP threads. Therefore a thread-local setting in one OpenMP parallel region may continue to affect not only the master thread after the parallel region ends, but also subsequent parallel regions. To avoid performance implications of this side effect, reset the thread-local number of threads before leaving the OpenMP parallel region (see [Examples](#) for how to do it).

## Return Values

| Name                 | Type             | Description                                                                                                                                     |
|----------------------|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>save_nt</code> | <code>int</code> | The value of the thread-local number of threads that was used before this function call. Zero means that the global number of threads was used. |

## Examples

This example shows how to avoid the side effect of a thread-local number of threads by reverting to the global setting:

```
#include "omp.h"
#include "mkl.h"
...
mkl_set_num_threads(16);
my_compute_using_mkl(); // Intel MKL functions use up to 16 threads
#pragma omp parallel num_threads(2)
{
 if (0 == omp_get_thread_num())
 mkl_set_num_threads_local(4);
 else
 mkl_set_num_threads_local(12);

 my_compute_using_mkl(); // Intel MKL functions use up to 4 threads on thread 0
 // and up to 12 threads on thread 1
}
my_compute_using_mkl(); // Intel MKL functions use up to 4 threads (!)
mkl_set_num_threads_local(0); // make master thread use global setting
my_compute_using_mkl(); // Intel MKL functions use up to 16 threads
```

This example shows how to avoid the side effect of a thread-local number of threads by saving and restoring the existing setting:

```
#include "mkl.h"
void my_compute(int nt)
{
 int save = mkl_set_num_threads_local(nt); // save the Intel MKL number of threads
 my_compute_using_mkl(); // Intel MKL functions use up to nt threads on this thread
 mkl_set_num_threads_local(save); // restore the Intel MKL number of threads
}
```

## **mkl\_set\_dynamic**

*Enables Intel MKL to dynamically change the number of OpenMP\* threads.*

### Syntax

```
void mkl_set_dynamic (int flag);
```

### Include Files

- `mkl.h`

## Input Parameters

| Name        | Type | Description                                                                                                                                                                      |
|-------------|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>flag</i> | int  | <p><i>flag</i> = 0 - Requests disabling dynamic adjustment of the number of threads.</p> <p><i>flag</i> ≠ 0 - Requests enabling dynamic adjustment of the number of threads.</p> |

## Description

This function indicates whether Intel MKL can dynamically change the number of OpenMP threads or should avoid doing this. The setting applies to all Intel MKL functions on all execution threads. This function takes precedence over the `MKL_DYNAMIC` environment variable.

Dynamic adjustment of the number of threads is enabled by default. Specifically, Intel MKL may use fewer threads in parallel regions than the number returned by the [mkl\\_get\\_max\\_threads](#) function. Disabling dynamic adjustment of the number of threads does not ensure that Intel MKL actually uses the specified number of threads, although the library attempts to use that number.

### TIP

If you call Intel MKL from within an OpenMP parallel region and want to create internal parallel regions, either disable dynamic adjustment of the number of threads or set the thread-local number of threads (see [mkl\\_set\\_num\\_threads\\_local](#) for how to do it).

## Example

```
#include "mkl.h"
...
mkl_set_num_threads(8);
#pragma omp parallel
{
 my_compute_with_mkl(); // Intel MKL uses 1 thread, being called from OpenMP parallel region
 mkl_set_dynamic(0); // disable adjustment of the number of threads
 my_compute_with_mkl(); // Intel MKL uses 8 threads
}
```

## mkl\_get\_max\_threads

*Gets the number of OpenMP\* threads targeted for parallelism.*

### Syntax

```
int mkl_get_max_threads (void);
```

### Include Files

- `mkl.h`

### Description

This function returns the number of OpenMP threads for Intel MKL to use in internal parallel regions. This number depends on whether dynamic adjustment of the number of threads by Intel MKL is disabled (by an environment setting or in a function call):

- If the dynamic adjustment is disabled, the function inspects the environment settings and return values of the function calls below in the order they are listed until it finds a non-zero value:

- A call to [mkl\\_set\\_num\\_threads\\_local](#)
- The last of the calls to [mkl\\_set\\_num\\_threads](#) or [mkl\\_domain\\_set\\_num\\_threads](#)( ..., MKL\_DOMAIN\_ALL)
- The MKL\_DOMAIN\_NUM\_THREADS environment variable with the MKL\_DOMAIN\_ALL tag
- The MKL\_NUM\_THREADS environment variable
- A call to [omp\\_set\\_num\\_threads](#)
- The OMP\_NUM\_THREADS environment variable
- If the dynamic adjustment is enabled, the function returns the number of physical cores on your system.

The number of threads returned by this function is a hint, and Intel MKL may actually use a different number.

## Return Values

| Name      | Type | Description                                                                                |
|-----------|------|--------------------------------------------------------------------------------------------|
| <i>nt</i> | int  | The maximum number of threads for Intel MKL functions to use in internal parallel regions. |

## Example

```
#include "mkl.h"
...
if (1 == mkl_get_max_threads()) puts("Intel MKL does not employ threading");
```

## See Also

[mkl\\_set\\_dynamic](#)  
[mkl\\_get\\_dynamic](#)

## mkl\_domain\_get\_max\_threads

*Gets the number of OpenMP\* threads targeted for parallelism for a particular function domain.*

## Syntax

```
int mkl_domain_get_max_threads (int domain);
```

## Include Files

- mkl.h

## Input Parameters

| Name          | Type | Description                                          |
|---------------|------|------------------------------------------------------|
| <i>domain</i> | int  | The named constant that defines the targeted domain. |

## Description

Computational functions of the Intel MKL function domain defined by the *domain* parameter use the value returned by this function as a limit of the number of OpenMP threads they should request for parallel computations. The [mkl\\_domain\\_get\\_max\\_threads](#) function returns the thread-local number of threads or, if that value is zero or not set, the global number of threads. To determine this number, the function inspects the environment settings and return values of the function calls below in the order they are listed until it finds a non-zero value:

- A call to [mkl\\_set\\_num\\_threads\\_local](#)
- The last of the calls to [mkl\\_set\\_num\\_threads](#) or [mkl\\_domain\\_set\\_num\\_threads](#)( ..., MKL\_DOMAIN\_ALL)
- A call to [mkl\\_domain\\_set\\_num\\_threads](#)( ..., *domain*)

- The `MKL_DOMAIN_NUM_THREADS` environment variable with the `MKL_DOMAIN_ALL` tag
- The `MKL_DOMAIN_NUM_THREADS` environment variable (with the specific domain tag)
- The `MKL_NUM_THREADS` environment variable
- A call to `omp_set_num_threads`
- The `OMP_NUM_THREADS` environment variable

Actual number of threads used by the Intel MKL computational functions may vary depending on the problem size and on whether dynamic adjustment of the number of threads is enabled (see the description of [mkl\\_set\\_dynamic](#)). For a list of supported values of the `domain` argument, see [Table "Intel MKL Function Domains"](#).

## Return Values

| Name            | Type             | Description                                                                                                                                                                                                                                              |
|-----------------|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>nt</code> | <code>int</code> | The maximum number of threads for Intel MKL functions from a given domain to use in internal parallel regions.<br><br>If an invalid value of <code>domain</code> is supplied, the function returns the number of threads for <code>MKL_DOMAIN_ALL</code> |

## Example

```
#include "mkl.h"
...
if (1 < mkl_domain_get_max_threads(MKL_DOMAIN_BLAS))
puts("Intel MKL BLAS functions employ threading");
```

## [mkl\\_get\\_dynamic](#)

*Determines whether Intel MKL is enabled to dynamically change the number of OpenMP\* threads.*

### Syntax

```
int mkl_get_dynamic(void);
```

### Include Files

- `mkl.h`

### Description

This function returns the status of dynamic adjustment of the number of OpenMP\* threads. To determine this status, the function inspects the return value of the following function call and if it is undefined, inspects the environment setting below:

- A call to [mkl\\_set\\_dynamic](#)
- The `MKL_DYNAMIC` environment variable

---

#### NOTE

Dynamic adjustment of the number of threads is enabled by default.

---

The dynamic adjustment works as follows. Suppose that the [mkl\\_get\\_max\\_threads](#) function returns the number of threads equal to  $N$ . If dynamic adjustment is enabled, Intel MKL may request up to  $N$  threads, depending on the size of the problem. If dynamic adjustment is disabled, Intel MKL requests exactly  $N$

threads for internal parallel regions (provided it uses a threaded algorithm with at least  $N$  computations that can be done in parallel). However, the OpenMP\* run-time library may be configured to supply fewer threads than Intel MKL requests, depending on the OpenMP\* setting of dynamic adjustment.

## Return Values

| Name             | Type             | Description                                                                                                                 |
|------------------|------------------|-----------------------------------------------------------------------------------------------------------------------------|
| <code>ret</code> | <code>int</code> | 0 - Dynamic adjustment of the number of threads is disabled.<br>1 - Dynamic adjustment of the number of threads is enabled. |

## Example

```
#include "mkl.h"
...
int nt = mkl_get_max_threads();
if (1 == mkl_get_dynamic())
printf("Intel MKL may use less than %i threads for a large problem", nt);
else
printf("Intel MKL should use %i threads for a large problem", nt);
```

## mkl\_set\_num\_stripes

*Specifies the number of partitions along the leading dimension of the output matrix for parallel ?gemm functions.*

### Syntax

```
void mkl_set_num_stripes(int ns);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name            | Type             | Description                                                                                                                                                              |
|-----------------|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ns</code> | <code>int</code> | $ns > 0$ - Specifies the number of partitions to use.<br>$ns = 0$ - Instructs Intel MKL to use the default partitioning algorithm.<br>$ns < 0$ - Invalid value; ignored. |

### Description

This function enables you to specify the number of stripes, or partitions along the leading dimension of the output matrix, for parallel ?gemm functions. If this number is not set (default) or if it is set to zero, Intel MKL ?gemm functions use the default partitioning algorithm. The specified number of partitions only applies to ?gemm functions.

The number specified is a hint, and Intel MKL may actually use a smaller number.

#### NOTE

This function takes precedence over the `MKL_NUM_STRIPES` environment variable.

## Example

```
#include "mkl.h"
...
mkl_set_num_stripes(4);
dgemm(...); // Intel MKL uses up to 4 stripes for dgemm
```

## See Also

[mkl\\_get\\_num\\_stripes](#)

## mkl\_get\_num\_stripes

*Gets the number of partitions along the leading dimension of the output matrix for parallel ?gemm functions.*

## Syntax

```
int mkl_get_num_stripes(void);
```

## Include Files

- mkl.h

## Description

This function returns the number of stripes, that is, partitions along the leading dimension of the output matrix, for parallel ?gemm functions. The number of partitions only applies to ?gemm functions.

The number returned is a hint, and Intel MKL may actually use a smaller number.

## Return Values

| Name      | Type | Description                                                 |
|-----------|------|-------------------------------------------------------------|
| <i>ns</i> | int  | The number of stripes for Intel MKL ?gemm functions to use. |

## Example

```
#include "mkl.h"
...
int ns = mkl_get_num_stripes();
if (ns > 0) printf("Intel MKL uses %d number of stripes\n", ns);
```

## See Also

[mkl\\_set\\_num\\_stripes](#)

# Error Handling

## Error Handling for Linear Algebra Routines

### xerbla

*Error handling function called by BLAS, LAPACK, Vector Math, and Vector Statistics functions.*

## Syntax

```
void xerbla(const char * sname, const int* info, const int len);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name               | Type                     | Description                                                                                          |
|--------------------|--------------------------|------------------------------------------------------------------------------------------------------|
| <code>sname</code> | <code>const char*</code> | The name of the routine that called <code>xerbla</code>                                              |
| <code>info</code>  | <code>const int*</code>  | The position of the invalid parameter in the parameter list of the calling function or an error code |
| <code>len</code>   | <code>const int</code>   | Length of the source string                                                                          |

## Description

The `xerbla` function is an error handler for Intel MKL BLAS, LAPACK, Vector Math, and Vector Statistics functions. These functions call `xerbla` if an issue is encountered on entry or during the function execution.

`xerbla` operates as follows:

1. Prints a message that depends on the value of the `info` parameter as explained in the [Error Messages Printed by `xerbla`](#) table.

---

### NOTE

A specific message can differ from the listed messages in numeric values and/or function names.

---

2. Returns to the calling application.

Comments in the Netlib LAPACK reference code ([http://www.netlib.org/lapack/explore-html/d1/dc0/\\_b\\_l\\_a\\_s\\_2\\_s\\_r\\_c\\_2xerbla\\_8f.html](http://www.netlib.org/lapack/explore-html/d1/dc0/_b_l_a_s_2_s_r_c_2xerbla_8f.html)) suggest this behavior although the LAPACK User's Guide recommends that the execution should stop when an error occurs.

## Error Messages Printed by `xerbla`

| Value of <code>info</code> | Error Message                                                                  |
|----------------------------|--------------------------------------------------------------------------------|
| 1001                       | Intel MKL ERROR: Incompatible optional parameters on entry to DGEMM.           |
| 1212                       | Intel MKL INTERNAL ERROR: Issue accessing coprocessor in function CGEEV.       |
| 1000 or 1089               | Intel MKL INTERNAL ERROR: Insufficient workspace available in function CGELSD. |
| < 0                        | Intel MKL INTERNAL ERROR: Condition 1 detected in function DLASD8.             |
| Other                      | Intel MKL ERROR: Parameter 6 was incorrect on entry to DGEMM.                  |

Note that `xerbla` is an internal function. You can change or disable printing of an error message by providing your own `xerbla` function. The following examples illustrate usage of `xerbla`.

## Example

```
void xerbla(char* sname, int* info, int len){
// sname - name of the function that called xerbla
// info - position of the invalid parameter in the parameter list
```

```
// len - length of the name in bytes
printf("\nXERBLA is called :%s: %d\n",srname,*info);
}
```

## See Also

[mkl\\_set\\_xerbla](#)

## pxerbla

*Error handling routine called by ScaLAPACK routines.*

## Syntax

```
void pxerbla (MKL_INT* ictxt, char* srname, MKL_INT* info, MKL_INT srname_len);
```

## Include Files

- `mkl_scalapack.h`

## Input Parameters

|                   |                      |                                                                                                         |
|-------------------|----------------------|---------------------------------------------------------------------------------------------------------|
| <i>ictxt</i>      | (local)<br>MKL_INT*  | The BLACS context handle, indicating the global context of the operation. The context itself is global. |
| <i>srname</i>     | (global)<br>char*    | The name of the routine that called <code>pxerbla</code> .                                              |
| <i>info</i>       | (global)<br>MKL_INT* | The position of the invalid parameter in the parameter list of the calling routine.                     |
| <i>srname_len</i> | (global)<br>MKL_INT  | The length of the calling routine name.                                                                 |

## Description

This routine is an error handler for the *ScaLAPACK* routines. It is called if an input parameter has an invalid value. A message is printed and program execution continues. For *ScaLAPACK* driver and computational routines, a `RETURN` statement is issued following the call to `pxerbla`.

Control returns to the higher-level calling routine, and you can determine how the program should proceed. However, in the specialized low-level *ScaLAPACK* routines (auxiliary routines that are Level 2 equivalents of computational routines), the call to `pxerbla()` is immediately followed by a call to `BLACS_ABORT()` to terminate program execution since recovery from an error at this level in the computation is not possible.

It is always good practice to check for a non-zero value of *info* on return from a *ScaLAPACK* routine. Installers may consider modifying this routine in order to call system-specific exception-handling facilities.

## LAPACKE\_xerbla

Error handling function called by the C interface to LAPACK functions.

---

### Syntax

```
void LAPACKE_xerbla(const char * name, lapack_int info);
```

### Include Files

- mkl.h

### Input Parameters

| Name        | Type        | Description                                                                                          |
|-------------|-------------|------------------------------------------------------------------------------------------------------|
| <i>name</i> | const char* | The name of the routine that called LAPACKE_xerbla                                                   |
| <i>info</i> | lapack_int  | The position of the invalid parameter in the parameter list of the calling function or an error code |

### Description

The LAPACKE\_xerbla function is an error handler for Intel MKL LAPACKE functions (the C interface to LAPACK functionality). If a LAPACKE function encounters an issue on entry or during the function execution, it calls LAPACKE\_xerbla to print an error message and return an error code.

---

**NOTE**

The LAPACKE\_xerbla routine does not replace the xerbla routine. For instance, if an issue occurs when a LAPACK function is called by a LAPACKE function, the LAPACK function calls xerbla.

---

### Error Messages Printed by LAPACKE\_xerbla

---

| Value of <i>info</i>          | Example Error Message                                        |
|-------------------------------|--------------------------------------------------------------|
| LAPACK_WORK_MEMORY_ERROR      | Not enough memory to allocate work array in LAPACKE_dgees    |
| LAPACK_TRANSPOSE_MEMORY_ERROR | Not enough memory to transpose matrix in LAPACKE_dgetrf_work |
| < 0                           | Wrong parameter 1 in LAPACKE_dgetrf                          |

---



---

**NOTE**

LAPACKE\_xerbla is an internal function. You can change or disable printing of an error message by providing your own LAPACKE\_xerbla function. Intel MKL does not provide functionality for dynamic replacement of LAPACKE\_xerbla.

---

### See Also

xerbla

### Handling Fatal Errors

A fatal error is a circumstance under which Intel MKL cannot continue the computation. For example, a fatal error occurs when Intel MKL cannot load a dynamic library or confronts an unsupported CPU type. In case of a fatal error, the default Intel MKL behavior is to print an explanatory message to the console and call an

internal function that terminates the application with a call to the system `exit()` function. Intel MKL enables you to override this behavior by setting a custom handler of fatal errors. The custom error handler can be configured to throw a C++ exception, set a global variable indicating the failure, or otherwise handle cannot-continue situations. It is not necessary for the custom error handler to call the system `exit()` function. Once execution of the error handler completes, a call to Intel MKL returns to the calling program without performing any computations and leaves no memory allocated by Intel MKL and no thread synchronization pending on return.

To specify a custom fatal error handler, call the `mkl_set_exit_handler` function.

## `mkl_set_exit_handler`

*Sets the custom handler of fatal errors.*

---

### Syntax

```
int mkl_set_exit_handler (MKLExitHandler myexit);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                | Prototype                             | Description               |
|---------------------|---------------------------------------|---------------------------|
| <code>myexit</code> | <code>void (*myexit)(int why);</code> | The error handler to set. |

### Description

This function sets the custom handler of fatal errors. If the input parameter is `NULL`, the system `exit()` function is set.

The following example shows how to use a custom handler of fatal errors in your C++ application:

```
#include "mkl.h"
void my_exit(int why){
 throw my_exception();
}
int ComputationFunction()
{
 mkl_set_exit_handler(my_exit);
 try {
 compute_using_mkl();
 }
 catch (const my_exception& e) {
 handle_exception();
 }
}
```

---

## Character Equality Testing

---

### `lsame`

*Tests two characters for equality regardless of the case.*

---

## Syntax

```
int lsame(const char* ca, const char* cb, int lca, int lcb);
```

## Include Files

- mkl.h

## Input Parameters

| Name            | Type        | Description                                           |
|-----------------|-------------|-------------------------------------------------------|
| <i>ca, cb</i>   | const char* | Pointers to the single characters to be compared      |
| <i>lca, lcb</i> | int         | Lengths of the input character strings, equal to one. |

## Description

This logical function checks whether two characters are equal regardless of the case.

## Return Values

| Name       | Type | Description                                                                                                                                                                                                                                                  |
|------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>val</i> | int  | Result of the comparison: <ul style="list-style-type: none"> <li>• a non-zero value if <i>ca</i> is the same letter as <i>cb</i>, maybe except for the case.</li> <li>• zero if <i>ca</i> and <i>cb</i> are different letters for whatever cases.</li> </ul> |

## lsamen

*Tests two character strings for equality regardless of the case.*

---

## Syntax

```
MKL_INT lsamen(const MKL_INT* n, const char* ca, const char* cb);
```

## Include Files

- mkl.h

## Input Parameters

| Name          | Type           | Description                                                                                                                       |
|---------------|----------------|-----------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>      | const MKL_INT* | Pointer to the number of characters in <i>ca</i> and <i>cb</i> to be compared.                                                    |
| <i>ca, cb</i> | const char*    | Character strings of length at least <i>n</i> to be compared. Only the first <i>n</i> characters of each string will be accessed. |

## Description

This logical function tests whether the first *n* letters of one string are the same as the first *n* letters of the other string, regardless of the case.

## Return Values

| Name             | Type                 | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|------------------|----------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>val</code> | <code>MKL_INT</code> | Result of the comparison: <ul style="list-style-type: none"><li>• a non-zero value if the first <math>n</math> letters in <math>ca</math> and <math>cb</math> character strings are equal, maybe except for the case, or if the length of character string <math>ca</math> or <math>cb</math> is less than <math>n</math>.</li><li>• zero if the first <math>n</math> letters in <math>ca</math> and <math>cb</math> character strings are different for whatever cases.</li></ul> |

## Timing

---

### `second/dsecnd`

Returns elapsed time in seconds. Use to estimate real time between two calls to this function.

---

#### Syntax

```
float second(void);
double dsecnd(void);
```

#### Include Files

- `mkl.h`

#### Description

The `second/dsecnd` function returns time in seconds to be used to estimate real time between two calls to the function. The difference between these functions is in the precision of the floating-point type of the result: while `second` returns the single-precision type, `dsecnd` returns the double-precision type.

Use these functions to measure durations. To do this, call each of these functions twice. For example, to measure performance of a routine, call the appropriate function directly before a call to the routine to be measured, and then after the call of the routine. The difference between the returned values shows real time spent in the routine.

Initializations may take some time when the `second/dsecnd` function runs for the first time. To eliminate the effect of this extra time on your measurements, make the first call to `second/dsecnd` in advance.

Do not use `second` to measure short time intervals because the single-precision format is not capable of holding sufficient timer precision.

#### Return Values

| Name             | Type                                                                                      | Description                  |
|------------------|-------------------------------------------------------------------------------------------|------------------------------|
| <code>val</code> | <code>float</code> for <code>second</code><br><code>double</code> for <code>dsecnd</code> | Elapsed real time in seconds |

### `mkl_get_cpu_clocks`

Returns elapsed CPU clocks.

---

## Syntax

```
void mkl_get_cpu_clocks (unsigned MKL_INT64 *clocks);
```

## Include Files

- mkl.h

## Output Parameters

| Name                | Type               | Description        |
|---------------------|--------------------|--------------------|
| <code>clocks</code> | unsigned MKL_INT64 | Elapsed CPU clocks |

## Description

The `mkl_get_cpu_clocks` function returns the elapsed CPU clocks.

This may be useful when timing short intervals with high resolution. The `mkl_get_cpu_clocks` function is also applied in pairs like `second/dsecond`. Note that out-of-order code execution on IA-32 or Intel® 64 architecture processors may disturb the exact elapsed CPU clocks value a little bit, which may be important while measuring extremely short time intervals.

### Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

## `mkl_get_cpu_frequency`

*Returns the current CPU frequency value in GHz.*

## Syntax

```
double mkl_get_cpu_frequency(void);
```

## Include Files

- mkl.h

## Description

The function `mkl_get_cpu_frequency` returns the current CPU frequency in GHz.

### NOTE

The returned value may vary from run to run if power management or Intel® Turbo Boost Technology is enabled.

## Return Values

| Name        | Type   | Description                        |
|-------------|--------|------------------------------------|
| <i>freq</i> | double | Current CPU frequency value in GHz |

## mkl\_get\_max\_cpu\_frequency

Returns the maximum CPU frequency value in GHz.

---

### Syntax

```
double mkl_get_max_cpu_frequency(void);
```

### Include Files

- mkl.h

### Description

The function `mkl_get_max_cpu_frequency` returns the maximum CPU frequency in GHz.

## Return Values

| Name        | Type   | Description                        |
|-------------|--------|------------------------------------|
| <i>freq</i> | double | Maximum CPU frequency value in GHz |

## mkl\_get\_clocks\_frequency

Returns the frequency value in GHz based on constant-rate Time Stamp Counter.

---

### Syntax

```
double mkl_get_clocks_frequency (void);
```

### Include Files

- mkl.h

### Description

The function `mkl_get_clocks_frequency` returns the CPU frequency value (in GHz) based on constant-rate Time Stamp Counter (TSC). Use of the constant-rate TSC ensures that each clock tick is constant even if the CPU frequency changes. Therefore, the returned frequency is constant.

---

#### NOTE

Obtaining the frequency may take some time when `mkl_get_clocks_frequency` is called for the first time. The same holds for functions `second/dsecnd`, which call `mkl_get_clocks_frequency`.

---

## Return Values

| Name        | Type   | Description            |
|-------------|--------|------------------------|
| <i>freq</i> | double | Frequency value in GHz |

**See Also**

second/dsecnd

## Memory Management

This section describes the Intel MKL memory functions. See the *Intel® MKL Developer Guide* for more memory usage information.

### mkl\_free\_buffers

*Frees unused memory allocated by the Intel MKL Memory Allocator.*

#### Syntax

```
void mkl_free_buffers (void);
```

#### Include Files

- mkl.h

#### Description

To improve performance of Intel MKL, the Memory Allocator uses per-thread memory pools where buffers may be collected for fast reuse. The `mkl_free_buffers` function frees unused memory allocated by the Memory Allocator.

See the *Intel MKL Developer Guide* for details.

You should call `mkl_free_buffers` after the last call to Intel MKL functions. In large applications, if you suspect that the memory may get insufficient, you may call this function earlier, but anticipate a drop in performance that may occur due to reallocation of buffers for subsequent calls to Intel MKL functions.

In a threaded application, avoid calling `mkl_free_buffers` from each thread because the function has a global effect. Call `mkl_thread_free_buffers` instead.

#### Usage of mkl\_free\_buffers with FFT Functions

```
DFTI_DESCRIPTOR_HANDLE hand1;
DFTI_DESCRIPTOR_HANDLE hand2;
void mkl_free_buffers(void);
.
/* Using MKL FFT */
Status = DftiCreateDescriptor(&hand1, DFTI_SINGLE, DFTI_COMPLEX, dim, m1);
Status = DftiCommitDescriptor(hand1);
Status = DftiComputeForward(hand1, s_array1);
.
Status = DftiCreateDescriptor(&hand2, DFTI_SINGLE, DFTI_COMPLEX, dim, m2);
Status = DftiCommitDescriptor(hand2);
.
Status = DftiFreeDescriptor(&hand1);
/* Do not call mkl_free_buffers() here because the hand2 descriptor will be corrupted! */
.
Status = DftiComputeBackward(hand2, s_array2));
Status = DftiFreeDescriptor(&hand2);
/* Here you finish using Intel MKL FFT */
/* Memory leak will be triggered by any memory control tool */
/* Use mkl_free_buffers() to avoid memory leaking */
mkl_free_buffers();
```

## **mkl\_thread\_free\_buffers**

*Frees unused memory allocated by the Intel MKL Memory Allocator in the current thread.*

---

### **Syntax**

```
void mkl_thread_free_buffers (void);
```

### **Include Files**

- `mkl.h`

### **Description**

To improve performance of Intel MKL, the Memory Allocator uses per-thread memory pools where buffers may be collected for fast reuse. The `mkl_thread_free_buffers` function frees unused memory allocated by the Memory Allocator in the current thread only.

You should call `mkl_thread_free_buffers` after the last call to Intel MKL functions in the current thread. In large applications, if you suspect that the memory may get insufficient, you may call this function earlier, but anticipate a drop in performance that may occur due to reallocation of buffers for subsequent calls to Intel MKL functions.

### **See Also**

[mkl\\_free\\_buffers](#)

## **mkl\_disable\_fast\_mm**

*Turns off the Intel MKL Memory Allocator for Intel MKL functions to directly use the system `malloc/free` functions.*

---

### **Syntax**

```
int mkl_disable_fast_mm (void);
```

### **Include Files**

- `mkl.h`

### **Description**

The `mkl_disable_fast_mm` function turns the Intel MKL Memory Allocator off for Intel MKL functions to directly use the system `malloc/free` functions. Intel MKL Memory Allocator uses per-thread memory pools where buffers may be collected for fast reuse. The Memory Allocator is turned on by default for better performance. To turn it off, you can use the `mkl_disable_fast_mm` function or the `MKL_DISABLE_FAST_MM` environment variable (See the *Intel MKL Developer Guide* for details.) Call `mkl_disable_fast_mm` before calling any Intel MKL functions that require allocation of memory buffers.

---

#### **NOTE**

Turning the Memory Allocator off negatively impacts performance of some Intel MKL routines, especially for small problem sizes.

---

## Return Values

| Name      | Type | Description                                                                                          |
|-----------|------|------------------------------------------------------------------------------------------------------|
| <i>mm</i> | int  | 1 - The Memory Allocator is successfully turned off.<br>0 - Turning the Memory Allocator off failed. |

## mkl\_mem\_stat

*Reports the status of the Intel MKL Memory Allocator.*

### Syntax

```
MKL_INT64 mkl_mem_stat (int* AllocatedBuffers);
```

### Include Files

- `mkl.h`

### Output Parameters

| Name                    | Type | Description                                   |
|-------------------------|------|-----------------------------------------------|
| <i>AllocatedBuffers</i> | int  | The number of buffers allocated by Intel MKL. |

### Description

The function returns the number of buffers allocated by Intel MKL and the amount of memory in these buffers. Intel MKL can allocate the memory buffers internally or in a call to [mkl\\_malloc/mkl\\_calloc](#). If no buffers are allocated at the moment, the `mkl_mem_stat` function returns 0. Call `mkl_mem_stat` to check the Intel MKL memory status.

---

#### NOTE

If you free all the memory allocated in calls to `mkl_malloc` or `mkl_calloc` and then call `mkl_free_buffers`, a subsequent call to `mkl_mem_stat` normally returns 0.

---

## Return Values

| Name                  | Type      | Description                                |
|-----------------------|-----------|--------------------------------------------|
| <i>AllocatedBytes</i> | MKL_INT64 | The amount of allocated memory (in bytes). |

### See Also

[Usage Example for the Memory Functions](#)

## mkl\_peak\_mem\_usage

*Reports the peak memory allocated by the Intel MKL Memory Allocator.*

### Syntax

```
MKL_INT64 mkl_peak_mem_usage (intmode);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name              | Type             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-------------------|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mode</code> | <code>int</code> | Requested mode of the function's operation.<br>Possible values: <ul style="list-style-type: none"> <li>• <code>MKL_PEAK_MEM_ENABLE</code> - start gathering the peak memory data</li> <li>• <code>MKL_PEAK_MEM_DISABLE</code> - stop gathering the peak memory data</li> <li>• <code>MKL_PEAK_MEM</code> - return the peak memory</li> <li>• <code>MKL_PEAK_MEM_RESET</code> - return the peak memory and reset the counter to start gathering the peak memory data from scratch</li> </ul> |

## Description

The `mk1_peak_mem_usage` function reports the peak memory allocated by the Intel MKL Memory Allocator.

Gathering the peak memory data is turned off by default. If you need to know the peak memory, explicitly turn the data gathering mode on by calling the function with the `MKL_PEAK_MEM_ENABLE` value of the parameter. Use the `MKL_PEAK_MEM` and `MKL_PEAK_MEM_RESET` values only when the data gathering mode is turned on. Otherwise the function returns -1. The data gathering mode leads to performance degradation, so when the mode is turned on, you can turn it off by calling the function with the `MKL_PEAK_MEM_DISABLE` value of the parameter.

- 
- If Intel MKL is running in a threaded mode, the `mk1_peak_mem_usage` function may return different amounts of memory from run to run.
  - The function reports the peak memory for the entire application, not just for the calling thread.
- 

## Return Values

| Name                        | Type                   | Description                                                                           |
|-----------------------------|------------------------|---------------------------------------------------------------------------------------|
| <code>AllocatedBytes</code> | <code>MKL_INT64</code> | The peak memory allocated by the Memory Allocator (in bytes) or -1 in case of errors. |

## See Also

[Usage Example for the Memory Functions](#)

## `mk1_malloc`

*Allocates an aligned memory buffer.*

---

## Syntax

```
void* mk1_malloc (size_t alloc_size, int alignment);
```

## Include Files

- `mk1.h`

## Input Parameters

| Name                    | Type                | Description                         |
|-------------------------|---------------------|-------------------------------------|
| <code>alloc_size</code> | <code>size_t</code> | Size of the buffer to be allocated. |
| <code>alignment</code>  | <code>int</code>    | Alignment of the buffer.            |

## Description

The function allocates an `alloc_size`-byte buffer aligned on the `alignment`-byte boundary.

If `alignment` is not a power of 2, the 32-byte alignment is used.

## Return Values

| Name               | Type               | Description                                                                                                 |
|--------------------|--------------------|-------------------------------------------------------------------------------------------------------------|
| <code>a_ptr</code> | <code>void*</code> | Pointer to the allocated buffer if <code>alloc_size</code> $\geq$ 1, NULL if <code>alloc_size</code> $<$ 1. |

## See Also

[mkl\\_free](#)

[Usage Example for the Memory Functions](#)

## mkl\_calloc

*Allocates and initializes an aligned memory buffer.*

## Syntax

```
void* mkl_calloc (size_t num, size_t size, int alignment);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                   | Type                | Description                                           |
|------------------------|---------------------|-------------------------------------------------------|
| <code>num</code>       | <code>size_t</code> | The number of elements in the buffer to be allocated. |
| <code>size</code>      | <code>size_t</code> | The size of the element.                              |
| <code>alignment</code> | <code>int</code>    | Alignment of the buffer.                              |

## Description

The function allocates a `num*size`-byte buffer, aligned on the `alignment`-byte boundary, and initializes the buffer with zeros.

If `alignment` is not a power of 2, the 64-byte alignment is used.

## Return Values

| Name               | Type               | Description                                                    |
|--------------------|--------------------|----------------------------------------------------------------|
| <code>a_ptr</code> | <code>void*</code> | Pointer to the allocated buffer if <code>size</code> $\geq$ 1, |

---

| Name | Type | Description              |
|------|------|--------------------------|
|      |      | NULL if <i>size</i> < 1. |

### See Also

[mkl\\_malloc](#)

[mkl\\_realloc](#)

[mkl\\_free](#)

[Usage Example for the Memory Functions](#)

## mkl\_realloc

*Changes the size of memory buffer allocated by mkl\_malloc/mkl\_calloc.*

---

### Syntax

```
void* mkl_realloc (void *ptr, size_t size);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name        | Type   | Description                                                                                                                  |
|-------------|--------|------------------------------------------------------------------------------------------------------------------------------|
| <i>ptr</i>  | void*  | Pointer to the memory buffer allocated by the <code>mkl_malloc</code> or <code>mkl_calloc</code> function or a NULL pointer. |
| <i>size</i> | size_t | New size of the buffer.                                                                                                      |

### Description

The function changes the size of the memory buffer allocated by the `mkl_malloc` or `mkl_calloc` function to *size* bytes. The first bytes of the returned buffer up to the minimum of the old and new sizes keep the content of the input buffer. The returned memory buffer can have a different location than the input one. If *ptr* is NULL, the function works as `mkl_malloc`.

### Return Values

| Name         | Type  | Description                                                                                                                                                           |
|--------------|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a_ptr</i> | void* | <ul style="list-style-type: none"><li>• Pointer to the re-allocated buffer if re-allocation is successful.</li><li>• NULL if re-allocation is unsuccessful.</li></ul> |

### See Also

[mkl\\_malloc](#)

[mkl\\_calloc](#)

[mkl\\_free](#)

[Usage Example for the Memory Functions](#)

## mkl\_free

*Frees the aligned memory buffer allocated by mkl\_malloc/mkl\_calloc.*

---

### Syntax

```
void mkl_free (void *a_ptr);
```

### Include Files

- mkl.h

### Input Parameters

| Name               | Type               | Description                        |
|--------------------|--------------------|------------------------------------|
| <code>a_ptr</code> | <code>void*</code> | Pointer to the buffer to be freed. |

### Description

The function frees the buffer pointed by `a_ptr` and allocated by the `mkl_malloc()` or `mkl_calloc()` function and does nothing if `a_ptr` is `NULL`.

### See Also

[mkl\\_malloc](#)

[mkl\\_calloc](#)

[Usage Example for the Memory Functions](#)

## mkl\_set\_memory\_limit

*On Linux, sets the limit of memory that Intel MKL can allocate for a specified type of memory.*

---

### Syntax

```
int mkl_set_memory_limit (int mem_type, size_t limit);
```

### Include Files

- mkl.h

### Input Parameters

| Name                  | Type                | Description                                                                                                                                                                                                                                                                                                                                                                                     |
|-----------------------|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mem_type</code> | <code>int</code>    | Type of memory to limit. Possible values:<br><br>MKL_MEM_MCDRAM - Multi-Channel Dynamic Random Access Memory (MCDRAM) (for more details, see <a href="https://software.intel.com/en-us/articles/mcdram-high-bandwidth-memory-on-knights-landing-analysis-methods-tools">https://software.intel.com/en-us/articles/mcdram-high-bandwidth-memory-on-knights-landing-analysis-methods-tools</a> ). |
| <code>limit</code>    | <code>size_t</code> | Memory limit in megabytes.                                                                                                                                                                                                                                                                                                                                                                      |

### Description

This function sets the limit for the amount of memory that Intel MKL can allocate for the specified memory type. The limit bounds both internal allocations (inside Intel MKL computation routines) and external allocations (in a call to `mkl_malloc`, `mkl_calloc`, or `mkl_realloc`). By default no limit is set for memory allocation.

Call `mkl_set_memory_limit` at most once, prior to calling any other Intel MKL function in your application except `mkl_set_interface_layer` and `mkl_set_threading_layer`.

- Allocation in MCDRAM requires `libmemkind` and `libjemalloc` dynamic libraries which are a part of Intel® Manycore Platform Software Package (Intel® MPSP) for Linux\*.
- The `mkl_set_memory_limit` function takes precedence over the `MKL_FAST_MEMORY_LIMIT` environment variable.

## Return Values

| Type | Description                                                                                                                                     |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| int  | Status of the function completion: <ul style="list-style-type: none"> <li>• 1 - the limit is set</li> <li>• 0 - the limit is not set</li> </ul> |

## See Also

[mkl\\_malloc](#)

[mkl\\_calloc](#)

[mkl\\_realloc](#)

[Usage Example for the Memory Functions](#)

## Usage Example for the Memory Functions

```
#include <stdio.h>
#include <mkl.h>

int main(void) {
 double *a, *b, *c;
 int n, i;
 double alpha, beta;
 MKL_INT64 AllocatedBytes;
 int N_AllocatedBuffers;

 alpha = 1.1; beta = -1.2;
 n = 1000;
 mkl_peak_mem_usage(MKL_PEAK_MEM_ENABLE);
 a = (double*)mkl_malloc(n*n*sizeof(double), 64);
 b = (double*)mkl_malloc(n*n*sizeof(double), 64);
 c = (double*)mkl_calloc(n*n, sizeof(double), 64);
 for (i=0; i<(n*n); i++) {
 a[i] = (double)(i+1);
 b[i] = (double)(-i-1);
 }

 dgemm("N", "N", &n, &n, &n, &alpha, a, &n, b, &n, &beta, c, &n);
 AllocatedBytes = mkl_mem_stat(&N_AllocatedBuffers);
 printf("\nDGEMM uses %d bytes in %d buffers", AllocatedBytes, N_AllocatedBuffers);

 mkl_free_buffers();
 mkl_free(a);
 mkl_free(b);
 mkl_free(c);

 AllocatedBytes = mkl_mem_stat(&N_AllocatedBuffers);
```

```

if (AllocatedBytes > 0) {
 printf("\nMKL memory leak!");
 printf("\nAfter mkl_free_buffers there are %d bytes in %d buffers",
 AllocatedBytes, N_AllocatedBuffers);
}
printf("\nPeak memory allocated by Intel MKL memory allocator %d bytes. Start to count new
memory peak",
 mkl_peak_mem_usage(MKL_PEAK_MEM_RESET));
a = (double*)mkl_malloc(n*n*sizeof(double), 64);
a = (double*)mkl_realloc(a, 2*n*n*sizeof(double));
mkl_free(a);
printf("\nPeak memory allocated by Intel MKL memory allocator after reset of peak memory counter
%d bytes\n",
 mkl_peak_mem_usage(MKL_PEAK_MEM));

return 0;
}

```

## Single Dynamic Library Control

Intel® MKL provides the Single Dynamic Library (SDL), which enables setting the interface and threading layer for Intel MKL at run time. See *Intel® MKL Developer Guide* for details of SDL and layered model concept. This section describes the functions supporting SDL.

### mkl\_set\_interface\_layer

*Sets the interface layer for Intel MKL at run time. Use with the Single Dynamic Library.*

#### Syntax

```
int mkl_set_interface_layer (int required_interface);
```

#### Include Files

- mkl.h

#### Input Parameters

| Name                            | Type | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|---------------------------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>required_interface</code> | int  | <p>Determines the interface layer. Possible values depend on the system architecture. Some of the values are only available on Linux* OS:</p> <ul style="list-style-type: none"> <li>• Intel® 64 architecture: <ul style="list-style-type: none"> <li>MKL_INTERFACE_LP64 for the Intel LP64 interface.</li> <li>MKL_INTERFACE_ILP64 for the Intel ILP64 interface.</li> <li>MKL_INTERFACE_LP64+MKL_INTERFACE_GNU for the GNU* LP64 interface on Linux OS.</li> <li>MKL_INTERFACE_ILP64+MKL_INTERFACE_GNU for the GNU ILP64 interface on Linux OS.</li> </ul> </li> <li>• IA-32 architecture: <ul style="list-style-type: none"> <li>MKL_INTERFACE_LP64 for the Intel interface on Linux OS.</li> </ul> </li> </ul> |

| Name | Type | Description                                                                                     |
|------|------|-------------------------------------------------------------------------------------------------|
|      |      | MKL_INTERFACE_LP64+MKL_INTERFACE_GNU or<br>MKL_INTERFACE_GNU for the GNU interface on Linux OS. |

## Description

If you are using the Single Dynamic Library (SDL), the `mkl_set_interface_layer` function sets the specified interface layer for Intel MKL at run time.

Call this function prior to calling any other Intel MKL function in your application except `mkl_set_threading_layer`. You can call `mkl_set_interface_layer` and `mkl_set_threading_layer` in any order.

The `mkl_set_interface_layer` function takes precedence over the `MKL_INTERFACE_LAYER` environment variable.

See *Intel MKL Developer Guide* for the layered model concept and usage details of the SDL.

## Return Values

| Type             | Description                                                                                                                                                                                                                                                                                                                                                                                                        |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>int</code> | <ul style="list-style-type: none"> <li>Current interface layer if it is set in a call to <code>mkl_set_interface_layer</code> or specified by environment variables or defaults.</li> </ul> <p>Possible values are specified in <a href="#">Input Parameters</a>.</p> <ul style="list-style-type: none"> <li>-1, if the layer was not specified prior to the call and the input parameter is incorrect.</li> </ul> |

## `mkl_set_threading_layer`

*Sets the threading layer for Intel MKL at run time. Use with the Single Dynamic Library (SDL).*

---

## Syntax

```
int mkl_set_threading_layer (int required_threading);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                            | Type             | Description                                                                                                                                                                                                                                                                         |
|---------------------------------|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>required_threading</code> | <code>int</code> | Determines the threading layer. Possible values:<br><code>MKL_THREADING_INTEL</code> for Intel threading.<br><code>MKL_THREADING_SEQUENTIAL</code> for the sequential mode of Intel MKL.<br><code>MKL_THREADING_TBB</code> for threading with the Intel® Threading Building Blocks. |

| Name | Type | Description                                                                                                                                                                    |
|------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|      |      | MKL_THREADING_PGI for PGI threading on Windows* or Linux* operating system only. Do not use this value with the SDL for Intel® Many Integrated Core (Intel® MIC) Architecture. |
|      |      | MKL_THREADING_GNU for GNU threading on Linux* operating system only. Do not use this value with the SDL for Intel MIC Architecture.                                            |

## Description

If you are using the Single Dynamic Library (SDL), the `mkl_set_threading_layer` function sets the specified threading layer for Intel MKL at run time.

Call this function prior to calling any other Intel MKL function in your application except `mkl_set_interface_layer`.

You can call `mkl_set_threading_layer` and `mkl_set_interface_layer` in any order.

The `mkl_set_threading_layer` function takes precedence over the `MKL_THREADING_LAYER` environment variable.

See *Intel MKL Developer Guide* for the layered model concept and usage details of the SDL.

## Return Values

| Type             | Description                                                                                                                                                                                                                                                                                                                                                        |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>int</code> | <ul style="list-style-type: none"> <li>Current threading layer if it is set in a call to <code>mkl_set_threading_layer</code> or specified by environment variables or defaults. Possible values are specified in <a href="#">Input Parameters</a>.</li> <li>-1, if the layer was not specified prior to the call and the input parameter is incorrect.</li> </ul> |

## mkl\_set\_xerbla

*Replaces the error handling routine. Use with the Single Dynamic Library on Windows\*.*

## Syntax

```
XerblaEntry mkl_set_xerbla (XerblaEntry new_xerbla_ptr);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                        | Type                     | Description                                       |
|-----------------------------|--------------------------|---------------------------------------------------|
| <code>new_xerbla_ptr</code> | <code>XerblaEntry</code> | Pointer to the error handling routine to be used. |

## Description

If you are linking with the Single Dynamic Library (SDL) `mkl_rt.lib` on Windows\*, the `mkl_set_xerbla` function replaces the error handling routine that is called by Intel MKL functions with the routine specified by the parameter.

See *Intel MKL Developer Guide* for details of SDL.

## Return Values

The function returns the pointer to the replaced error handling routine.

## See Also

[xerbla](#)

## **mkl\_set\_progress**

*Replaces the progress information routine.*

---

### Syntax

```
ProgressEntry mkl_set_progress (ProgressEntry new_progress_ptr);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                          | Type                       | Description                                             |
|-------------------------------|----------------------------|---------------------------------------------------------|
| <code>new_progress_ptr</code> | <code>ProgressEntry</code> | Pointer to the progress information routine to be used. |

### Description

The `mkl_set_progress` function replaces the currently used progress information routine with the routine specified by the parameter.

Usually a user-supplied `mkl_progress` function redefines the default `mkl_progress` function automatically. However, you must call `mkl_set_progress` to replace the default `mkl_progress` on Windows\* in any of the following cases:

- You are using the Single Dynamic Library (SDL) `mkl_rt.lib`.
- You link dynamically with ScaLAPACK.

See *Intel MKL Developer Guide* for details of SDL.

### Return Values

The function returns the pointer to the replaced progress information routine.

## See Also

[mkl\\_progress](#)

## **mkl\_set\_pardiso\_pivot**

*Replaces the routine handling Intel MKL PARDISO pivots with a user-defined routine. Use with the Single Dynamic Library (SDL).*

---

### Syntax

```
PardisopivotEntry mkl_set_pardiso_pivot (PardisopivotEntry new_pardiso_pivot_ptr);
```

### Include Files

- `mkl.h`

## Input Parameters

| Name                               | Type                           | Description                                      |
|------------------------------------|--------------------------------|--------------------------------------------------|
| <code>new_pardiso_pivot_ptr</code> | <code>PardisoPivotEntry</code> | Pointer to the pivot setting routine to be used. |

## Description

If you are using the Single Dynamic Library (SDL), the `mkl_set_pardiso_pivot` function replaces the pivot setting routine that is called by Intel MKL functions with the routine specified by the parameter.

See *Intel MKL Developer Guide* for usage details of the SDL.

## Return Values

| Type                           | Description                                    |
|--------------------------------|------------------------------------------------|
| <code>PardisoPivotEntry</code> | Pointer to the replaced pivot setting routine. |

## See Also

[mkl\\_pardiso\\_pivot](#)

# Intel Many Integrated Core Architecture Support

This section describes Intel MKL functions to support the use of Intel Xeon Phi coprocessors and especially Automatic Offload mode, when the computations are automatically offloaded to the coprocessors.

The `mkl_mic_enable` function enables Intel MKL to offload computations to Intel Xeon Phi coprocessors automatically, while the `mkl_mic_disable` function disables automatic offloading.

### Important

Automatic Offload supports only OpenMP\* threaded Intel MKL.

Optional work-division control functions enable you to specify the fractional amount of work to distribute between the host CPU and the coprocessors in the Automatic Offload mode. Work division is a fractional measure ranging from 0.0 to 1.0. For example, setting work division for the host CPU to 0.5 means to keep half of the computational work on the host CPU and move half to the coprocessor(s). Setting work division to 0.25 for a coprocessor means to offload a quarter of the computational work to this coprocessor while leaving the rest on the host CPU.

Other functions enable more control over computational resources of Intel Xeon Phi coprocessors.

### NOTE

The support functions for Intel MIC Architecture take precedence over the respective environment variables.

### Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

**Optimization Notice**

Notice revision #20110804

**mkl\_mic\_enable***Enables Automatic Offload mode.***Syntax**

```
int mkl_mic_enable();
```

**Include Files**

- mkl.h

**Description**

The `mkl_mic_enable` function enables Automatic Offload mode and initializes Intel Xeon Phi coprocessors available on your system. This function takes precedence over the `MKL_MIC_ENABLE` environment variable. Unlike the function, the environment variable enables Automatic Offload mode, but the initialization is delayed until a call to an Intel MKL function supporting Automatic Offload. For functions supporting Automatic Offload, see the *Intel MKL Release Notes*.

**Return Values**

| Name              | Type             | Description                                                                                                                                       |
|-------------------|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | <code>int</code> | Result status:<br>= 0 Indicates that Automatic Offload mode is successfully enabled.<br>< 0 Indicates a failure to enable Automatic Offload mode. |

**mkl\_mic\_disable***Disables Automatic Offload mode.***Syntax**

```
int mkl_mic_disable();
```

**Include Files**

- mkl.h

**Description**

The `mkl_mic_disable` function disables Automatic Offload mode. To enable Automatic Offload mode back, use the `mkl_mic_enable` function.

**NOTE**

If Automatic Offload mode is not enabled, a call to `mkl_mic_disable` completes successfully if Intel Xeon Phi coprocessors are available on your system.

## Return Values

| Name              | Type             | Description                                                                                                                                                           |
|-------------------|------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | <code>int</code> | <p>Result status:</p> <p>= 0 Indicates that Automatic Offload mode is successfully disabled.</p> <p>&lt; 0 Indicates a failure to disable Automatic Offload mode.</p> |

## `mkl_mic_get_device_count`

Returns the number of Intel Xeon Phi coprocessors on the system when called on the host CPU.

### Syntax

```
int mkl_mic_get_device_count();
```

### Include Files

- `mkl.h`

### Description

The `mkl_get_device_count` function returns the number of Intel Xeon Phi coprocessors on your system. You may need this number to specify a custom work-division scheme.

---

#### CAUTION

Call this function only on the host CPU.

---

## Return Values

| Name                  | Type             | Description                                                                                       |
|-----------------------|------------------|---------------------------------------------------------------------------------------------------|
| <code>ndevices</code> | <code>int</code> | The number of Intel Xeon Phi coprocessors available on the system. Equals zero if there are none. |

### See Also

[mkl\\_mic\\_enable](#)

## `mkl_mic_set_workdivision`

For computations in the Automatic Offload mode, sets the fraction of the work for the specified coprocessor or host CPU to do.

### Syntax

```
int mkl_mic_set_workdivision (MKL_MIC_TARGET_TYPE target_type, int target_number, double wd);
```

### Include Files

- `mkl.h`

## Input Parameters

| Name                       | Type                             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|----------------------------|----------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>target_type</code>   | <code>MKL_MIC_TARGET_TYPE</code> | Type of the target device. Use one of the following values: <ul style="list-style-type: none"> <li><code>MKL_TARGET_HOST</code> - host CPU</li> <li><code>MKL_TARGET_MIC</code> - Intel Xeon Phi coprocessor, default</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <code>target_number</code> | <code>int</code>                 | The device to set the fraction of work for. Takes the following values: <ul style="list-style-type: none"> <li><math>\geq 0</math>. Specifies execution on a specific coprocessor. The coprocessor is determined by <code>target_number</code> modulo the number of Intel Xeon Phi coprocessors on the system as returned by <code>mkl_mic_get_device_count()</code>. For example: for a system with 4 Intel Xeon Phi coprocessors, <code>target_number = 6</code> determines the coprocessor number 2.</li> <li><math>&lt; 0</math>. Reserved.</li> </ul> <p>If <code>target_type = MKL_TARGET_HOST</code>, the function ignores the <code>target_number</code> parameter, which may have any value.</p> |
| <code>wd</code>            | <code>double</code>              | The fractional amount of the work that the specified device should do, where $0.0 \leq wd \leq 1.0$ .<br>Specifying <code>MKL_MIC_AUTO_WORKDIVISION</code> for <code>wd</code> indicates that Intel MKL should determine the amount of the work for the specified device.                                                                                                                                                                                                                                                                                                                                                                                                                                 |

## Description

If you are using Intel MKL in Automatic Offload mode, the `mkl_mic_set_workdivision` function specifies how much work each Intel Xeon Phi coprocessor or the host CPU should do. This function takes precedence over the `MKL_HOST_WORKDIVISION`, `MKL_MIC_WORKDIVISION`, and `MKL_MIC_<number>_WORKDIVISION` environment variables (see the *Intel MKL Developer Guide* for details).

- 
- Intel MKL interprets the fraction of work set by the `mkl_mic_set_workdivision` function as guidance toward dividing work between coprocessors, but the library may choose a different work division if necessary.
  - Intel MKL resolves the collision that arises if the sum of all the work-division fractions does not equal one.
  - For LAPACK routines, setting the fraction of work to any value other than 0.0 enables the specified processor for Automatic Offload mode. However Intel MKL LAPACK does not use the value specified to divide the workload. For example, setting the fraction to 0.5 has the same effect as setting the fraction to 1.0.
-

## Return Values

| Name              | Type             | Description                                                                                                                                                                           |
|-------------------|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | <code>int</code> | <p>Result status:</p> <ul style="list-style-type: none"> <li>= 0 Indicates that the fraction is successfully set.</li> <li>&lt; 0 Indicates a failure to set the fraction.</li> </ul> |

## See Also

[mkl\\_mic\\_get\\_device\\_count](#)  
[mkl\\_mic\\_enable](#)  
[mkl\\_mic\\_get\\_workdivision](#)  
[mkl\\_mic\\_get\\_cpuserinfo](#)

## mkl\_mic\_get\_workdivision

For computations in the Automatic Offload mode, retrieves the fraction of the work for the specified coprocessor or host CPU to do.

## Syntax

```
int mkl_mic_get_workdivision (MKL_MIC_TARGET_TYPE target_type, int target_number,
double *wd);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                       | Type                             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|----------------------------|----------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>target_type</code>   | <code>MKL_MIC_TARGET_TYPE</code> | <p>Type of the target device. Use one of the following values:</p> <ul style="list-style-type: none"> <li>• <code>MKL_TARGET_HOST</code> - host CPU</li> <li>• <code>MKL_TARGET_MIC</code> - Intel Xeon Phi coprocessor, default</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <code>target_number</code> | <code>int</code>                 | <p>The device to retrieve the fraction of work for. Takes the following values:</p> <ul style="list-style-type: none"> <li>• <math>\geq 0</math>. Specifies execution on a specific coprocessor. The coprocessor is determined by <code>target_number</code> modulo the number of Intel Xeon Phi coprocessors on the system as returned by <code>mkl_mic_get_device_count()</code>. For example: for a system with 4 Intel Xeon Phi coprocessors, <code>target_number = 6</code> determines the coprocessor number 2.</li> <li>• <math>&lt; 0</math>. Reserved.</li> </ul> <p>If <code>target_type = MKL_TARGET_HOST</code>, the function ignores the <code>target_number</code> parameter, which may have any value.</p> |

## Output Parameters

| Name            | Type   | Description                                                                                                                                                                                                                                               |
|-----------------|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>wd</code> | double | The fractional amount of the work that the specified device should do, where $0.0 \leq wd \leq 1.0$ .<br><br>MKL_MIC_AUTO_WORKDIVISION set for <code>wd</code> indicates that Intel MKL should determine the amount of the work for the specified device. |

## Description

If you are using Intel MKL in the Automatic Offload mode, the `mkl_mic_get_workdivision` function provides you with the amount of the work that the specified coprocessor or host CPU is configured to do.

## Return Values

| Name              | Type | Description                                                                                                                      |
|-------------------|------|----------------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | int  | Result status:<br>= 0 Indicates that the fraction is successfully returned.<br>< 0 Indicates a failure to retrieve the fraction. |

## See Also

[mkl\\_mic\\_get\\_device\\_count](#)  
[mkl\\_mic\\_enable](#)  
[mkl\\_mic\\_set\\_workdivision](#)

## mkl\_mic\_set\_max\_memory

*Sets the maximum amount of Intel Xeon Phi coprocessor memory reserved for the Automatic Offload computations.*

---

## Syntax

```
int mkl_mic_set_max_memory (MKL_MIC_TARGET_TYPE target_type, int target_number, size_t mem_size);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                       | Type                | Description                                                                                                                     |
|----------------------------|---------------------|---------------------------------------------------------------------------------------------------------------------------------|
| <code>target_type</code>   | MKL_MIC_TARGET_TYPE | Type of the target device. Use the value of MKL_TARGET_MIC - Intel Xeon Phi coprocessor, default.                               |
| <code>target_number</code> | int                 | The coprocessor number for which the maximum memory used for Automatic Offload computations is set. Takes the following values: |

| Name                  | Type                | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|-----------------------|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mem_size</code> | <code>size_t</code> | <ul style="list-style-type: none"> <li><math>\geq 0</math>. Specifies execution on a specific coprocessor. The coprocessor is determined by <code>target_number</code> modulo the number of Intel Xeon Phi coprocessors on the system as returned by <code>mkl_mic_get_device_count()</code>. For example: for a system with 4 Intel Xeon Phi coprocessors, <code>target_number = 6</code> determines the coprocessor number 2.</li> <li><math>&lt; 0</math>. Reserved.</li> </ul> <p>For the <code>target_number</code> device, the amount of memory in kilobytes to reserve for Automatic Offload computations. Intel MKL attempts to not exceed the specified memory size for Automatic Offload computations.</p> |

## Description

The `mkl_mic_set_max_memory` function enables you to limit coprocessor memory used by the Automatic Offload computations. Intel MKL reserves the specified memory on Intel Xeon Phi coprocessors. This can improve the performance of Automatic Offload computations by reducing the cost of buffer initialization and data transfer. The specified memory is reserved for the calling process, and the threads of a process share the specified memory. Intel MKL allocates additional memory for each process that performs Automatic Offload computations.

This function takes precedence over the `MKL_MIC_MAX_MEMORY` and `MKL_MIC_<number>_MAX_MEMORY` environment variables (see the *Intel MKL Developer Guide* for details).

- 
- Call `mkl_mic_set_max_memory` before any Intel MKL functions that do Automatic Offload computations.
  - Use `mkl_mic_free_memory` to free the coprocessor memory reserved for the Automatic Offload computations.
- 

## Return Values

| Name              | Type             | Description                                                                                                                                                                                                                    |
|-------------------|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | <code>int</code> | <p>Result status:</p> <ul style="list-style-type: none"> <li><code>= 0</code> Indicates the coprocessor memory reserved for Automatic Offload is set successfully.</li> <li><code>&lt; 0</code> Indicates an error.</li> </ul> |

## See Also

[mkl\\_mic\\_free\\_memory](#)  
[mkl\\_mic\\_enable](#)  
[mkl\\_mic\\_get\\_meminfo](#)

## `mkl_mic_free_memory`

*Frees the coprocessor memory reserved for the Automatic Offload computations.*

---

## Syntax

```
int mkl_mic_free_memory (MKL_MIC_TARGET_TYPE target_type, int target_number);
```

## Include Files

- mkl.h

## Input Parameters

| Name                 | Type                | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|----------------------|---------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>target_type</i>   | MKL_MIC_TARGET_TYPE | Type of the target device. Use the value of MKL_TARGET_MIC - Intel Xeon Phi coprocessor, default.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>target_number</i> | int                 | The coprocessor number for which the memory reserved for Automatic Offload computations is freed. Takes the following values: <ul style="list-style-type: none"> <li>• <math>\geq 0</math>. Specifies execution on a specific coprocessor. The coprocessor is determined by <i>target_number</i> modulo the number of Intel Xeon Phi coprocessors on the system as returned by <code>mkl_mic_get_device_count()</code>. For example: for a system with 4 Intel Xeon Phi coprocessors, <i>target_number</i> = 6 determines the coprocessor number 2.</li> <li>• <math>&lt; 0</math>. Reserved.</li> </ul> |

## Description

The `mkl_mic_free_memory` function frees the coprocessor memory reserved for the Automatic Offload computations. If you call `mkl_mic_set_max_memory` to specify the maximum coprocessor memory for Automatic Offload computations, Intel MKL reserves and reuses the specified coprocessor memory during multiple Automatic Offload calls. You can reclaim the coprocessor memory by calling `mkl_mic_free_memory`.

- 
- Currently, Intel MKL reserves the coprocessor memory only if the `mkl_mic_set_max_memory` function is called. Therefore, `mkl_mic_free_memory` has no effect unless there is a prior call to the `mkl_mic_set_max_memory` function.
  - If you do not call `mkl_mic_free_memory`, Intel MKL frees the coprocessor memory at the program exit.
- 

## Return Values

| Name        | Type | Description                                                                                                                                                                                     |
|-------------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ierr</i> | int  | Result status: <ul style="list-style-type: none"> <li>= 0 Indicates the coprocessor memory reserved for Automatic Offload is freed successfully.</li> <li>&lt; 0 Indicates an error.</li> </ul> |

## See Also

[mkl\\_mic\\_set\\_max\\_memory](#)  
[mkl\\_mic\\_enable](#)

## **mkl\_mic\_register\_memory**

*Enables/disables the `mkl_malloc` function running in Automatic Offload mode to register allocated memory.*

### **Syntax**

```
void mkl_mic_register_memory (int control);
```

### **Include Files**

- `mkl.h`

### **Input Parameters**

| <b>Name</b>          | <b>Type</b>      | <b>Description</b>                                                                                                                                                                           |
|----------------------|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>control</code> | <code>int</code> | Desired behavior of <code>mkl_malloc</code> .<br>Possible values:<br>0 - not register allocated memory, default.<br>1 - register allocated memory if Automatic Offload (AO) mode is enabled. |

### **Description**

The `mkl_mic_register_memory` function enables or disables the `mkl_malloc` function to register allocated memory when `mkl_malloc` runs in AO mode.

Registration of memory may reduce the overhead introduced by the operating system during data transfers between Intel Xeon Phi coprocessors and the host CPU.

This function takes precedence over the `MKL_MIC_REGISTER_MEMORY` environment variable.

If AO mode is disabled, the function has no effect.

### **See Also**

[mkl\\_malloc](#) Allocates an aligned memory buffer.

[mkl\\_mic\\_enable](#) Enables Automatic Offload mode.

## **mkl\_mic\_set\_device\_num\_threads**

*Sets the maximum number of OpenMP\* threads to use on an Intel Xeon Phi coprocessor for the Automatic Offload computations.*

### **Syntax**

```
int mkl_mic_set_device_num_threads (MKL_MIC_TARGET_TYPE target_type, int target_number, int num_threads);
```

### **Include Files**

- `mkl.h`

### **Input Parameters**

| <b>Name</b>              | <b>Type</b>                      | <b>Description</b>                          |
|--------------------------|----------------------------------|---------------------------------------------|
| <code>target_type</code> | <code>MKL_MIC_TARGET_TYPE</code> | Type of the target device. Use the value of |

| Name                       | Type | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|----------------------------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>target_number</code> | int  | <p>MKL_TARGET_MIC - Intel Xeon Phi coprocessor, default.</p> <p>The coprocessor number for which the maximum number of threads to be used for Automatic Offload computations is set. Takes the following values:</p> <ul style="list-style-type: none"> <li>• <math>\geq 0</math>. Specifies execution on a specific coprocessor. The coprocessor is determined by <code>target_number</code> modulo the number of Intel Xeon Phi coprocessors on the system as returned by <code>mkl_mic_get_device_count()</code>. For example: for a system with 4 Intel Xeon Phi coprocessors, <code>target_number = 6</code> determines the coprocessor number 2.</li> <li>• <math>&lt; 0</math>. Reserved.</li> </ul> |
| <code>num_threads</code>   | int  | <p>The number of threads to use for Automatic Offload computations on the <code>target_number</code> coprocessor. Must be greater than 0.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |

## Description

The `mkl_mic_set_device_num_threads` function enables you to limit the number of OpenMP threads to use for Automatic Offload computations on a specific Intel Xeon Phi coprocessor.

This function takes precedence over the `MIC_OMP_NUM_THREADS` and `MKL_MIC_<number>_OMP_NUM_THREADS` environment variables (see the *Intel MKL Developer Guide* for details).

Unless the maximum number of threads is set by the environment variables or this function, Intel MKL uses all coprocessor cores.

Call `mkl_mic_set_device_num_threads` before initialization of Automatic Offload mode. Otherwise the function returns -1 without setting the number of threads.

---

### NOTE

Automatic Offload mode can be initialized as follows:

- *explicitly*, in a call to the `mkl_mic_enable` function.
  - *implicitly*, in the first call to an Intel MKL function that does Automatic Offload computations (for example: `?GEMM`). See the *Intel MKL Release Notes* for which functions support Automatic Offload mode.
- 

The `mkl_mic_set_device_num_threads` function sets the maximum number of threads to use for the Automatic Offload computations on Intel Xeon Phi coprocessors only. To control host CPU threading, use general threading control functions and see "Using Additional Threading Control" in the *Intel MKL Developer Guide* for more information.

## Return Values

| Name              | Type | Description    |
|-------------------|------|----------------|
| <code>ierr</code> | int  | Result status: |

| Name | Type | Description                                                   |
|------|------|---------------------------------------------------------------|
|      |      | = 0 Indicates that the number of threads is set successfully. |
|      |      | < 0 Indicates a failure to set the number of threads.         |

### See Also

[mkl\\_mic\\_enable](#)  
Threading Control

### [mkl\\_mic\\_set\\_resource\\_limit](#)

*For computations in the Automatic Offload mode, sets the maximum fraction of available Intel Xeon Phi coprocessor computational resources (cores) that the calling process can use.*

### Syntax

```
int mkl_mic_set_resource_limit (double fraction);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name            | Type   | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|-----------------|--------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>fraction</i> | double | <p>The fractional amount of Intel Xeon Phi coprocessor computational resources that the calling process can use for Automatic Offload (AO).</p> <p>Possible values: <math>0.0 \leq \textit{fraction} \leq 1.0</math>.</p> <p>Special values:</p> <ul style="list-style-type: none"> <li>• 0.0 - you need to manage the coprocessors explicitly using the <a href="#">mkl_mic_set_device_num_threads</a> function or the <code>MIC_KMP_AFFINITY</code> environment variable. Default.</li> <li>• <code>MKL_MPI_PPN</code> constant - enables a fully automated mode for message-passing interface (MPI) applications. In this mode, Intel MKL tries to read the number of MPI processes per node (<i>ppn</i>) from the environment variables passed to the process by MPI. If this attempt is successful, Intel MKL sets the actual value of the fractional amount to <math>1.0/\textit{ppn}</math>.</li> </ul> |

#### NOTE

For Intel® MPI Library, Open MPI, and IBM Platform MPI, Intel MKL automatically detects *ppn*. For other MPI implementations, use the `MKL_MPI_PPN` environment variable to set *ppn*.

## Description

If you are using Intel MKL in the AO mode, the `mkl_mic_set_resource_limit` function specifies how much of the computational resources of Intel Xeon Phi coprocessors can be used by the calling process. Use this function if you need to share coprocessor cores automatically across multiple processes that call Intel MKL in the AO mode. For example, this might be useful in MPI applications.

You can also enable this functionality using the `MKL_MIC_RESOURCE_LIMIT` environment variable (see the *Intel MKL Developer Guide* for details), but the `mkl_mic_set_resource_limit` function take precedence over the environment variable.

If *fraction* is set to a valid non-zero value, Intel MKL enables automatic reservation of Intel Xeon Phi coprocessor cores.

Actual reservation is made during a call to an Intel MKL AO function and works as follows:

1. Intel MKL converts *fraction* to a number of Intel Xeon Phi coprocessor cores and tries to find cores that are not reserved by other processes.

The number of available cores can be less than the requested number. Intel MKL considers cores of all available coprocessors as a single space and can find cores on different coprocessors. However, Intel MKL tends to reserve as many cores as were requested and to reserve cores compactly (using as few coprocessors as possible).

2. Intel MKL exclusively reserves cores found in the previous step.

Intel MKL makes reservation safely across a persistent shared memory region using semaphores.

3. Intel MKL performs AO computations on reserved cores only.

You do not need to set any threading parameters for Intel Xeon Phi coprocessors, such as `MIC_OMP_NUM_THREADS` or `MIC_KMP_AFFINITY` (these settings are ignored when automatic resource sharing is enabled).

4. Upon completion of an AO call, Intel MKL releases reserved cores.

Be aware of the following features of the automatic reservation:

- Execution of Intel MKL AO functions falls back to the host if Intel MKL fails to reserve the minimum number of Intel Xeon Phi coprocessor cores that provide a benefit over executing on the host.
- The last core of Intel Xeon Phi coprocessors cannot be reserved because it is used for system processes.
- Reservation works only within a space of a single user. In other words, Intel MKL cannot share Intel Xeon Phi coprocessor resources across processes from different users.

### Optimization Notice

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## Examples

1. One Intel Xeon Phi coprocessor with 61 cores is available on the system. One process calls `mkl_mic_set_resource_limit(1.0)` and then calls `dgemm` in AO mode. As a result, 60 Intel Xeon Phi coprocessor cores are reserved at the beginning of AO `dgemm`. Then `mkl_mic_set_resource_limit(0.3)` is invoked and AO `dgemm` is called one more time. As a result,

`dgemm` is run on 18 Intel Xeon Phi coprocessor cores. During this run of AO `dgemm`, another process calls `mkl_mic_set_resource_limit(1.0)` and invokes AO `dgetrf`. Because 18 cores are reserved by the first process, only 42 cores are reserved for AO `dgetrf`.

2. Two Intel Xeon Phi coprocessors with 61 cores each are available on the system. Three processes simultaneously call `mkl_mic_set_resource_limit(0.34)` and then call `dpotrf` in AO mode. As a result, one process receives 40 cores from coprocessor 1, another process receives 40 cores from coprocessor 2, and the remaining process is given 20 cores from each of the two coprocessors for a total of 40 cores.
3. Two Intel Xeon Phi coprocessors with 58 cores each are available on the system. The user sets `MKL_MIC_PPN=4` and runs an MPI application with 4 MPI ranks. The following sequence of functions is called in each MPI process:

```
mkl_mic_set_workdivision(MKL_TARGET_MIC, 2, 0.0);
mkl_mic_set_resource_limit(MKL_MIC_PPN);
MPI_Init(...);
//calls to Intel MKL AO functions;
MPI_Finalize(...);
```

As a result, actual *fraction* is set to 0.25 for each MPI process, and each MPI process receives 14 cores on Intel Xeon Phi coprocessor 1 (because the user excluded co-processor 2 from computations by setting zero workdivision for it).

## Return Values

| Name              | Type             | Description                                                                                                            |
|-------------------|------------------|------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | <code>int</code> | Result status:<br>= 0 Indicates that the fraction is set successfully.<br>< 0 Indicates a failure to set the fraction. |

## See Also

[mkl\\_mic\\_get\\_resource\\_limit](#)

[mkl\\_mic\\_enable](#)

[mkl\\_mic\\_get\\_device\\_count](#)

## mkl\_mic\_get\_resource\_limit

*For computations in the Automatic Offload mode, retrieves the maximum fraction of available Intel Xeon Phi coprocessor computational resources (cores) that the calling process can use.*

## Syntax

```
int mkl_mic_get_resource_limit (double* fraction);
```

## Include Files

- `mkl.h`

## Output Parameters

| Name                  | Type                 | Description                                                                                                                              |
|-----------------------|----------------------|------------------------------------------------------------------------------------------------------------------------------------------|
| <code>fraction</code> | <code>double*</code> | The fractional amount of Intel Xeon Phi coprocessor computational resources that the calling process can use for Automatic Offload (AO). |

| Name | Type | Description                                     |
|------|------|-------------------------------------------------|
|      |      | Possible values: $0.0 \leq fraction \leq 1.0$ . |

## Description

If you are using Intel MKL in the AO mode, the `mkl_mic_get_resource_limit` function retrieves the fractional amount of Intel Xeon Phi coprocessor computational resources that can be used by the calling process.

## Return Values

| Name              | Type             | Description                                                                                                                    |
|-------------------|------------------|--------------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | <code>int</code> | Result status:<br>= 0 Indicates that the fraction is successfully returned.<br>< 0 Indicates a failure to return the fraction. |

## See Also

- [mkl\\_mic\\_set\\_resource\\_limit](#)
- [mkl\\_mic\\_enable](#)
- [mkl\\_mic\\_get\\_device\\_count](#)
- [mkl\\_mic\\_get\\_workdivision](#)

## **mkl\_mic\_set\_offload\_report**

*Turns on/off reporting of Automatic Offload profiling.*

## Syntax

```
int mkl_mic_set_offload_report (int enabled);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                 | Type             | Description                                                                                                                                                                                                                                                                                                |
|----------------------|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>enabled</code> | <code>int</code> | This parameter specifies whether to turn the reporting on or off. Takes values with boolean semantics: <ul style="list-style-type: none"> <li>• 0 - Reporting of Automatic Offload profiling should be turned off.</li> <li>• 1 - Reporting of Automatic Offload profiling should be turned on.</li> </ul> |

## Description

If the `OFFLOAD_REPORT` environment variable has the value of 1 or 2, the `mkl_mic_set_offload_report` function turns on/off reporting of the Automatic Offload profiling at run time. If the `OFFLOAD_REPORT` environment variable is not set or is set to a value different from 1 and 2, the function has no effect. If `OFFLOAD_REPORT` is set to 1 or 2, the reporting is turned on at a program startup. The `mkl_mic_set_offload_report` function does not change the reporting level or the value of the environment variable. Instead, it determines which Intel MKL functions called in the Automatic Offload mode produce profiling reports.

For details of the reporting level and `OFFLOAD_REPORT` environment variable, see the *Intel MKL Developer Guide*.

## Return Values

| Name               | Type             | Description                                                             |
|--------------------|------------------|-------------------------------------------------------------------------|
| <code>iprev</code> | <code>int</code> | The previous value that determined whether the reporting was on or off. |

## `mkl_mic_set_flags`

Sets flags to control the behavior of computations in the Automatic Offload mode.

### Syntax

```
int mkl_mic_set_flags (int flags);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name               | Type             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|--------------------|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>flags</code> | <code>int</code> | <p>The settings requested:</p> <ul style="list-style-type: none"> <li>• <code>MKL_MIC_DEFAULT_FLAGS</code> - The default Automatic Offload (AO) mode, when computations fall back to host in the case of any AO error, such as lack of memory or other resources on the Intel Xeon Phi coprocessor.</li> <li>• <code>MKL_MIC_DISABLE_HOST_FALLBACK</code> - The mode when computations do not fall back to host in the case of an AO error, but the AO function immediately returns control to the calling program without completing the computations.</li> </ul> |

### Description

This function sets flags to control the behavior of computations in the AO mode. Depending on the flags set, the AO program may silently return to the calling program in the case of an AO error. To check whether the AO call is successful, call the `mkl_mic_get_status` function and check whether the return value is non-negative. For more details of `mkl_mic_get_status` and in particular, for a sequence of function calls that enables checking the status of an AO call, see [mkl\\_mic\\_get\\_status](#).

### Return Values

| Name                   | Type             | Description               |
|------------------------|------------------|---------------------------|
| <code>old_flags</code> | <code>int</code> | The previously set flags. |

### See Also

[mkl\\_mic\\_get\\_flags](#)

## mkl\_mic\_get\_flags

Retrieves flags that control the behavior of computations in the Automatic Offload mode.

---

### Syntax

```
int mkl_mic_get_flags (void);
```

### Include Files

- mkl.h

### Description

This function retrieves flags that control the behavior of computations in the Automatic Offload (AO) mode. These flags are either the default flags for the AO mode or the flags set in a call to the `mkl_mic_set_flags` function.

---

#### NOTE

Depending on the flags set, the AO program may silently return to the calling program in the case of an AO error. To check whether the AO call is successful, call the `mkl_mic_get_status` function and check whether the return value is non-negative. For more details of `mkl_mic_get_status` and in particular, for a sequence of function calls that enables checking the status of an AO call, see [mkl\\_mic\\_get\\_status](#).

---

### Return Values

| Name               | Type             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|--------------------|------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>flags</code> | <code>int</code> | The flags returned, which can be one of the following: <ul style="list-style-type: none"><li>• <code>MKL_MIC_DEFAULT_FLAGS</code> - The default AO mode, when computations fall back to host in the case of any AO error, such as lack of memory or other resources on the Intel Xeon Phi coprocessor.</li><li>• <code>MKL_MIC_DISABLE_HOST_FALLBACK</code> - The mode when computations do not fall back to host in the case of an AO error, but the AO function immediately returns control to the calling program without completing the computations.</li></ul> |

### See Also

[mkl\\_mic\\_set\\_flags](#)

## mkl\_mic\_get\_status

For the Automatic Offload mode, returns the status of the latest call to an Intel MKL function.

---

### Syntax

```
int mkl_mic_get_status (void);
```

## Include Files

- `mkl.h`

## Description

This function returns the status of the latest call to an Intel MKL function done in the Automatic Offload (AO) mode. The sign of the returned value characterizes the status of the AO computations at a high level:

- = 0 - The computations completed successfully using AO
- > 0 - The computations were done without using AO
- < 0 - The computations were not completed at all

In an AO call to an Intel MKL function, offloading computations as expected may not be possible for various reasons, such as:

- Lack of available memory or computation cores on an Intel Xeon Phi coprocessor
- Incorrect environment settings (which result in a failure to locate or load a coprocessor driver or libraries)
- Communication error during data transfer to or from a coprocessor (for example, a remote process dies unexpectedly)
- Other unexpected error

The status returned helps you to find out the reason of offload failure.

Because the status is a thread-local value, to be able to check the status of an Intel MKL AO function, the following sequence of function calls in the same thread is required:

1. `mkl_mic_clear_status`
2. Intel MKL function
3. `mkl_mic_get_status`

## Return Values

| Name                | Type             |
|---------------------|------------------|
| <code>status</code> | <code>int</code> |

### Description

The thread-local value of the status:

- `MKL_MIC_SUCCESS` (0) - The computations successfully completed with offloading to Intel Xeon Phi coprocessors.
- `MKL_MIC_NOT_IMPL` (1) - The computations successfully completed only on the host because AO is not implemented for this function or for the combination of input parameters of the function.
- `MKL_MIC_HOST_FALLBACK` (2) - The computations successfully completed only on the host because AO could not start. Possible reasons: the environment path does not include required system libraries, resources are insufficient, memory allocation failed on a coprocessor, and so on.
- `MKL_MIC_DISABLED` (3) - The computations successfully completed only on the host because AO is disabled.
- `MKL_MIC_FAILED` (-1) - The computations were not completed. AO could start, but an error occurred during the computations. Note that the function output data may be corrupted in this case.
- `MKL_MIC_HOST_FALLBACK_DISABLED` (-2) - The computations were not completed because AO could not start. Possible reasons: the environment path does not

| Name | Type | Description                                                                                                                                                                                                                                                                                                                                                                                                                 |
|------|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|      |      | include required system libraries, resources are insufficient, memory allocation failed on a coprocessor, and so on. Unlike <code>MKL_MIC_HOST_FALLBACK</code> , the <code>mkl_mic_get_status</code> function can return this status only if the <code>mkl_mic_set_flags</code> function disabled falling back to host. Unlike for <code>MKL_MIC_FAILED</code> status, the function input and output data remain untouched. |

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#### See Also

[mkl\\_mic\\_set\\_flags](#)  
[mkl\\_mic\\_clear\\_status](#)

### **mkl\_mic\_clear\_status**

*For the Automatic Offload mode, clears the status of the latest call to an Intel MKL function.*

#### Syntax

```
int mkl_mic_clear_status (void);
```

#### Include Files

- `mkl.h`

#### Description

This function clears the status of the latest call to an Intel MKL function done in the Automatic Offload (AO) mode. Call `mkl_mic_clear_status` before calling an Intel MKL function in the AO mode to be able to check the status of the AO call. Because the status is a thread-local value, getting the status for an Intel MKL function requires the following sequence of function calls in the same thread:

1. `mkl_mic_clear_status`
2. Intel MKL function
3. `mkl_mic_get_status`

#### Optimization Notice

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**Optimization Notice**

dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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**See Also**

[mkl\\_mic\\_get\\_status](#)

**mkl\_mic\_get\_meminfo**

*Retrieves the amount of total and free memory for the specified coprocessor or host CPU.*

**Syntax**

```
int mkl_mic_get_meminfo (MKL_MIC_TARGET_TYPE target_type, int target_number, int* totalmem, int* freemem);
```

**Include Files**

- `mkl.h`

**Input Parameters**

| Name                       | Type                             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|----------------------------|----------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>target_type</code>   | <code>MKL_MIC_TARGET_TYPE</code> | Type of the target device. Use one of the following values: <ul style="list-style-type: none"> <li>• <code>MKL_TARGET_HOST</code> - host CPU</li> <li>• <code>MKL_TARGET_MIC</code> - Intel Xeon Phi coprocessor, default</li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <code>target_number</code> | <code>int</code>                 | The device to retrieve the memory information for. Takes the following values: <ul style="list-style-type: none"> <li>• <math>\geq 0</math>. Specifies execution on a specific coprocessor. The coprocessor is determined by <code>target_number</code> modulo the number of Intel Xeon Phi coprocessors on the system as returned by <code>mkl_mic_get_device_count()</code>. For example: for a system with 4 Intel Xeon Phi coprocessors, <code>target_number = 6</code> determines the coprocessor number 2.</li> <li>• <math>&lt; 0</math>. Reserved.</li> </ul> <p>If <code>target_type = MKL_TARGET_HOST</code>, the function ignores the <code>target_number</code> parameter, which may have any value.</p> |

## Output Parameters

| Name                  | Type              | Description                                                            |
|-----------------------|-------------------|------------------------------------------------------------------------|
| <code>totalmem</code> | <code>int*</code> | The total amount of memory on the target device in kilobytes.          |
| <code>freemem</code>  | <code>int*</code> | The amount of free memory available on the target device in kilobytes. |

## Description

Use this function to retrieve the amount of total and free memory (in kilobytes) available on an offload device or host CPU.

## Return Values

| Name              | Type             | Description                                                                                                                                                       |
|-------------------|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ierr</code> | <code>int</code> | Result status:<br>= 0 Indicates that the memory information for the target device is successfully returned.<br>< 0 Indicates a failure to return the information. |

## See Also

[mkl\\_mic\\_set\\_max\\_memory](#)  
[mkl\\_mic\\_get\\_cpuserinfo](#)

## mkl\_mic\_get\_cpuserinfo

*Retrieves the number of cores, hardware threads, and frequency for the specified coprocessor or host CPU.*

## Syntax

```
int mkl_mic_get_cpuserinfo (MKL_MIC_TARGET_TYPE target_type, int target_number, int* ncores, int* nthreads, double* freq);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                       | Type                             | Description                                                                                                                                                                                                                          |
|----------------------------|----------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>target_type</code>   | <code>MKL_MIC_TARGET_TYPE</code> | Type of the target device. Use one of the following values: <ul style="list-style-type: none"> <li>• <code>MKL_TARGET_HOST</code> - host CPU</li> <li>• <code>MKL_TARGET_MIC</code> - Intel Xeon Phi coprocessor, default</li> </ul> |
| <code>target_number</code> | <code>int</code>                 | The device to retrieve the information for. Takes the following values:                                                                                                                                                              |

| Name | Type | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|------|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|      |      | <ul style="list-style-type: none"> <li>• <math>\geq 0</math>. Specifies execution on a specific coprocessor. The coprocessor is determined by <i>target_number</i> modulo the number of Intel Xeon Phi coprocessors on the system as returned by <code>mkl_mic_get_device_count()</code>. For example: for a system with 4 Intel Xeon Phi coprocessors, <i>target_number</i> = 6 determines the coprocessor number 2.</li> <li>• <math>&lt; 0</math>. Reserved.</li> </ul> <p>If <i>target_type</i> = <code>MKL_TARGET_HOST</code>, the function ignores the <i>target_number</i> parameter, which may have any value.</p> |

### Output Parameters

| Name            | Type    | Description                                          |
|-----------------|---------|------------------------------------------------------|
| <i>ncores</i>   | int*    | The number of physical cores on the target device.   |
| <i>nthreads</i> | int*    | The number of hardware threads on the target device. |
| <i>freq</i>     | double* | The frequency in Hz of the target device.            |

### Description

Use this function to retrieve the number of cores, hardware threads, and frequency for the host CPU or an Intel Xeon Phi coprocessor.

### Return Values

| Name        | Type | Description                                                                                                                                                                                                        |
|-------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ierr</i> | int  | Result status: <ul style="list-style-type: none"> <li>= 0 Indicates that the information for the target device is successfully returned.</li> <li>&lt; 0 Indicates a failure to return the information.</li> </ul> |

### See Also

[mkl\\_mic\\_get\\_meminfo](#)  
[mkl\\_mic\\_get\\_device\\_count](#)  
[mkl\\_mic\\_set\\_resource\\_limit](#)  
[mkl\\_mic\\_set\\_workdivision](#)

## Conditional Numerical Reproducibility Control

The CNR mode of Intel MKL ensures bitwise reproducible results from run to run of Intel MKL functions on a fixed number of threads for a specific Intel instruction set architecture (ISA) under the following conditions:

- Calls to Intel MKL occur in a single executable
- The number of computational threads used by the library does not change in the run

Intel MKL offers both functions and environment variables to support conditional numerical reproducibility. See the *Intel MKL Developer Guide* for more information on bitwise reproducible results of computations and for details about the environment variables.

The support functions enable you to configure the CNR mode and also provide information on the current and optimal CNR branch on your system. [Usage Examples for CNR Support Functions](#) illustrate usage of these functions.

---

### Important

Call the functions that define the behavior of CNR before any of the math library functions that they control.

---

Intel MKL provides named constants for use as input and output parameters of the functions instead of integer values. See [Named Constants for CNR Control](#) for a list of the named constants.

Although you can configure the CNR mode using either the support functions or the environment variables, the functions offer more flexible configuration and control than the environment variables. Settings specified by the functions take precedence over the settings specified by the environment variables.

Use Intel MKL in the CNR mode only in case a need for bitwise reproducible results is critical. Otherwise, run Intel MKL as usual to avoid performance degradation.

While you can supply unaligned input and output data to Intel MKL functions running in the CNR mode, use of aligned data is recommended. Refer to [Reproducibility Conditions](#) for more details.

### Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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## [mkl\\_cbwr\\_set](#)

*Configures the CNR mode of Intel MKL.*

---

### Syntax

```
int mkl_cbwr_set (int setting);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                 | Type             | Description                                                                                                                     |
|----------------------|------------------|---------------------------------------------------------------------------------------------------------------------------------|
| <code>setting</code> | <code>int</code> | CNR branch to set. See <a href="#">Named Constants for CNR Control</a> for a list of named constants that specify the settings. |

### Description

The `mkl_cbwr_set` function configures the CNR mode. In this release, it sets the CNR branch and turns on the CNR mode.

**NOTE**

Settings specified by the `mkl_cbwr_set` function take precedence over the settings specified by the `MKL_CBWR` environment variable.

**Return Values**

| Name                | Type             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|---------------------|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code> | <p>The status of the function completion:</p> <ul style="list-style-type: none"> <li><code>MKL_CBWR_SUCCESS</code> - the function completed successfully.</li> <li><code>MKL_CBWR_ERR_INVALID_INPUT</code> - an invalid setting is requested.</li> <li><code>MKL_CBWR_ERR_UNSUPPORTED_BRANCH</code> - the input value of the branch does not match the instruction set architecture (ISA) of your system. See <a href="#">Named Constants for CNR Control</a> for more details.</li> <li><code>MKL_CBWR_ERR_MODE_CHANGE_FAILURE</code> - the <code>mkl_cbwr_set</code> function requested to change the current CNR branch after a call to some Intel MKL function other than a CNR function.</li> </ul> |

**See Also**

[Usage Examples for CNR Support Functions](#)

**mkl\_cbwr\_get**

*Returns the current CNR settings.*

**Syntax**

```
int mkl_cbwr_get (int option);
```

**Include Files**

- `mkl.h`

**Input Parameters**

| Name                | Type             | Description                                                                                                                                                                                                                                                                                                            |
|---------------------|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>option</code> | <code>int</code> | <p>Specifies what kind of settings is requested. Named constants define possible values of <code>option</code>:</p> <ul style="list-style-type: none"> <li><code>MKL_CBWR_BRANCH</code> - indicates the setting of the CNR branch.</li> <li><code>MKL_CBWR_ALL</code> - indicates all CNR-related settings.</li> </ul> |

**NOTE**

This release supports no CNR settings other than branch.

## Description

The `mkl_cbwr_get` function returns the requested CNR settings. If the function completes successfully and the CNR mode is turned on, `mkl_cbwr_get` returns the *specific* CNR branch.

### NOTE

To turn the CNR mode on, use the `mkl_cbwr_set` function or environment variables. For more details, see the *Intel MKL Developer Guide*.

## Return Values

| Name                 | Type             | Description                                                                                                                                                                                                                                                                      |
|----------------------|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>setting</code> | <code>int</code> | Requested CNR settings. See <a href="#">Named Constants for CNR Control</a> for a list of named constants that specify the settings.<br><br>If the value of the <code>option</code> parameter is not permitted, contains the <code>MKL_CBWR_ERR_INVALID_INPUT</code> error code. |

## See Also

[Usage Examples for CNR Support Functions](#)  
[mkl\\_cbwr\\_set](#)

## mkl\_cbwr\_get\_auto\_branch

*Automatically detects the CNR code branch for your platform.*

## Syntax

```
int mkl_cbwr_get_auto_branch (void);
```

## Include Files

- `mkl.h`

## Description

The `mkl_cbwr_get_auto_branch` function uses a run-time CPU check to return a CNR branch that is optimized for the processor where the program is currently running.

### Optimization Notice

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## Return Values

| Name                 | Type             | Description                                                                                                                      |
|----------------------|------------------|----------------------------------------------------------------------------------------------------------------------------------|
| <code>setting</code> | <code>int</code> | Automatically detected CNR branch. May be any <i>specific</i> branch listed in <a href="#">Named Constants for CNR Control</a> . |

## See Also

[Usage Examples for CNR Support Functions](#)

## Named Constants for CNR Control

Intel MKL provides reproducible results for a certain code branch, determined by the instruction set architecture (ISA). To define CNR code branches, use the following named constants as input/output for conditional numerical reproducibility support functions. Pass named constants to the functions instead of their values.

| Named Constant                   | Value | Description                                                                                                                                                                                            |
|----------------------------------|-------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>MKL_CBWR_AUTO</code>       | 2     | CNR mode uses the standard ISA-based dispatching model while ensuring fixed cache sizes, deterministic reductions, and static scheduling<br><br><b>CNR mode uses the branch for the following ISA:</b> |
| <code>MKL_CBWR_COMPATIBLE</code> | 3     | Intel® Streaming SIMD Extensions 2 (Intel® SSE2) without <code>rcpps/rsqrtps</code> instructions                                                                                                       |
| <code>MKL_CBWR_SSE2</code>       | 4     | Intel SSE2                                                                                                                                                                                             |
| <code>MKL_CBWR_SSE3</code>       | 5     | DEPRECATED. Intel® Streaming SIMD Extensions 3 (Intel® SSE3). This setting is kept for backward compatibility and is equivalent to <code>MKL_CBWR_SSE2</code> .                                        |
| <code>MKL_CBWR_SSSE3</code>      | 6     | Supplemental Streaming SIMD Extensions 3 (SSSE3)                                                                                                                                                       |
| <code>MKL_CBWR_SSE4_1</code>     | 7     | Intel® Streaming SIMD Extensions 4-1 (SSE4-1)                                                                                                                                                          |
| <code>MKL_CBWR_SSE4_2</code>     | 8     | Intel® Streaming SIMD Extensions 4-2 (SSE4-2)                                                                                                                                                          |
| <code>MKL_CBWR_AVX</code>        | 9     | Intel® Advanced Vector Extensions (Intel® AVX)                                                                                                                                                         |
| <code>MKL_CBWR_AVX2</code>       | 10    | Intel® Advanced Vector Extensions 2 (Intel® AVX2)                                                                                                                                                      |

When specifying the CNR branch with the named constants, be aware of the following:

- Reproducible results are provided under [Reproducibility Conditions](#).
- Settings other than `MKL_CBWR_AUTO` or `MKL_CBWR_COMPATIBLE` are available only for Intel processors.
- Intel and Intel compatible CPUs have a few instructions, such as approximation instructions `rcpps/rsqrtps`, that may return different results. Setting the branch to `MKL_CBWR_COMPATIBLE` ensures that Intel MKL does not use these instructions and forces a single Intel SSE2 only code path to be executed.

### Optimization Notice

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optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

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**See Also**

[Usage Examples for CNR Support Functions](#)

**Reproducibility Conditions**

To get reproducible results from run to run, ensure that the number of threads is fixed and constant. Specifically:

- If you are running your program with OpenMP\* parallelization on different processors, explicitly specify the number of threads.
- To ensure that your application has deterministic behavior with OpenMP\* parallelization and does not adjust the number of threads dynamically at run time, set `MKL_DYNAMIC` and `OMP_DYNAMIC` to `FALSE`. This is especially needed if you are running your program on different systems.
- If you are running your program with the Intel® Threading Building Blocks parallelization, numerical reproducibility is not guaranteed.
 

---

  - As usual, you should align your data, even in CNR mode, to obtain the best possible performance. While CNR mode also fully supports unaligned input and output data, the use of it might reduce the performance of some Intel MKL functions on earlier Intel processors. To ensure proper alignment of arrays, allocate memory for them using `mkl_malloc/mkl_calloc`.
  - Conditional Numerical Reproducibility does not ensure that bitwise-identical NaN values are generated when the input data contains NaN values.
  - If dynamic memory allocation fails on one run but succeeds on another run, you may fail to get reproducible results between these two runs.

---

**See Also**

[mkl\\_malloc](#)

[mkl\\_calloc](#)

**Usage Examples for CNR Support Functions**

The following examples illustrate usage of support functions for conditional numerical reproducibility.

**Setting Automatically Detected CNR Branch**

```
#include <mkl.h>
int main(void) {
 int my_cbwr_branch;
 /* Find the available MKL_CBWR_BRANCH automatically */
 my_cbwr_branch = mkl_cbwr_get_auto_branch();
 /* User code without Intel MKL calls */
 /* Piece of the code where CNR of Intel MKL is needed */
 /* The performance of Intel MKL functions might be reduced for CNR mode */
 if (mkl_cbwr_set(my_cbwr_branch) != MKL_CBWR_SUCCESS) {
 printf("Error in setting MKL_CBWR_BRANCH! Aborting...\n");
 return;
 }
}
```

```

}
/* CNR calls to Intel MKL + any other code */
}

```

## Use of the `mkl_cbwr_get` Function

```

#include <mkl.h>
int main(void) {
 int my_cbwr_branch;
 /* Piece of the code where CNR of Intel MKL is analyzed */
 my_cbwr_branch = mkl_cbwr_get(MKL_CBWR_BRANCH);
 switch (my_cbwr_branch) {
 case MKL_CBWR_AUTO:
 /* actions in case of automatic mode */
 break;
 case MKL_CBWR_SSSE3:
 /* actions for SSSE3 code */
 break;
 default:
 /* all other cases */
 }
 /* User code */
}

```

## Miscellaneous

---

### `mkl_progress`

*Provides progress information.*

#### Syntax

```
int mkl_progress (int* thread_process, int* step, char* stage, int lstage);
```

#### Include Files

- `mkl.h`

#### Input Parameters

| Name                        | Type                    | Description                                                                                                                                                                                                                                                                                                                                                              |
|-----------------------------|-------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>thread_process</code> | <code>const int*</code> | Indicates the number of thread or process the progress routine is called from: <ul style="list-style-type: none"> <li>• The thread number for non-cluster components linked with OpenMP threading layer</li> <li>• Zero for non-cluster components linked with sequential threading layer</li> <li>• The process number (MPI rank) for non-cluster components</li> </ul> |
| <code>step</code>           | <code>const int*</code> | Pointer to the linear progress indicator that shows the amount of work done. Increases from 0 to the linear size of the problem during the computation.                                                                                                                                                                                                                  |

| Name          | Type        | Description                                                                                                          |
|---------------|-------------|----------------------------------------------------------------------------------------------------------------------|
| <i>stage</i>  | const char* | Message indicating the name of the routine or the name of the computation stage the progress routine is called from. |
| <i>lstage</i> | int         | The length of a stage string excluding the trailing NULL character.                                                  |

## Description

The `mkl_progress` function is intended to track progress of a lengthy computation and/or interrupt the computation. By default this routine does nothing but the user application can redefine it to obtain the computation progress information. You can set it to perform certain operations during the routine computation, for instance, to print a progress indicator. A non-zero return value may be supplied by the redefined function to break the computation.

Some Intel MKL functions from LAPACK, ScaLAPACK, DSS/PARDISO, and Parallel Direct Sparse Solver for Clusters regularly call the `mkl_progress` function during the computation. Refer to the description of a specific function from those domains to see whether the function supports this feature or not.

If a LAPACK function returns `info=-1002`, the function was interrupted by `mkl_progress`. Because ScaLAPACK does not support interruption of the computation, Intel MKL ignores any value returned by `mkl_progress`.

While a user-supplied `mkl_progress` function usually redefines the default `mkl_progress` function automatically, some configurations require calling the `mkl_set_progress` function to replace the default `mkl_progress` function. Call `mkl_set_progress` to replace the default `mkl_progress` on Windows\* in any of the following cases:

- You are using the Single Dynamic Library (SDL) `mkl_rt.lib`.
- You link dynamically with ScaLAPACK.

---

### WARNING

The `mkl_progress` function only supports OpenMP\* threading and sequential execution.

---

## Return Values

| Name            | Type | Description                                                                                                         |
|-----------------|------|---------------------------------------------------------------------------------------------------------------------|
| <i>stopflag</i> | int  | The stopping flag. A non-zero flag forces the routine to be interrupted. The zero flag is the default return value. |

## Example

The following example prints the progress information to the standard output device:

```
#include <stdio.h>
#include <string.h>
#define BUFLLEN 16
int mkl_progress(int* thread_process, int* step, char* stage, int lstage)
{
 char buf[BUFLLEN];
 if(lstage >= BUFLLEN) lstage = BUFLLEN-1;
 strncpy(buf, stage, lstage);
 buf[lstage] = '\0';
 printf("In thread %i, at stage %s, steps passed %i\n", *thread_process, buf, *step);
 return 0;
}
```

## mkl\_enable\_instructions

Enables dispatching for new Intel® architectures or restricts the set of Intel® instruction sets available for dispatching.

### Syntax

```
int mkl_enable_instructions (int isa);
```

### Include Files

- mkl.h

### Input Parameters

| Name             | Type             | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|------------------|------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>isa</code> | <code>int</code> | <p>The latest Intel® instruction-set architecture (ISA) for Intel MKL to dispatch.</p> <p>Possible values, in the order from the latest to the earliest instruction set:</p> <ul style="list-style-type: none"> <li><code>MKL_ENABLE_AVX512</code> Intel® Advanced Vector Extensions 512 (Intel® AVX-512) on Intel® Xeon® processors.</li> <li><code>MKL_ENABLE_AVX512_MIC</code> Intel AVX-512 on Intel® Xeon Phi™ processors and coprocessors.</li> <li><code>MKL_ENABLE_AVX2</code> Intel® Advanced Vector Extensions 2 (Intel® AVX2).</li> <li><code>MKL_ENABLE_AVX</code> Intel® Advanced Vector Extensions (Intel® AVX).</li> <li><code>MKL_ENABLE_SSE4_2</code> Intel® Streaming SIMD Extensions 4-2 (Intel® SSE4-2).</li> </ul> |

### Description

Intel MKL does run-time processor dispatching to identify appropriate internal code paths to traverse for Intel MKL functions called by the application. The `mkl_enable_instructions` function controls the behavior of the dispatcher to do either of the following:

- Enable dispatching for new Intel architectures.

Intel MKL does not dispatch instruction sets that do not have silicon available at time of the product launch. Call `mkl_enable_instructions` to enable dispatching the code path for such an ISA in a simulator environment or on hardware that supports this ISA.

- Restrict the set of Intel instruction sets available for dispatching.

Call `mkl_enable_instructions` to restrict dispatching to code paths for earlier ISA. For example, if the hardware supports Intel AVX, a call to `mkl_enable_instructions` with the `MKL_ENABLE_SSE4_2` parameter forces the dispatcher to use the Intel SSE4-2 code path.

If the system does not support the instruction set specified by the `isa` parameter or the system is based on a non-Intel architecture, `mkl_enable_instructions` does nothing and returns zero.

Settings specified by the `mkl_enable_instructions` function set an upper limit to settings specified by the `mkl_cbwr_set` function.

You can use the `MKL_ENABLE_INSTRUCTIONS` environment variable instead of calling `mkl_enable_instructions` (for more details, see the *Intel MKL Developer Guide*), but the settings specified by the function take precedence over the settings specified by the environment variable.

## Return Values

| Name             | Type             | Description                                                                                                                                                                                                                                                                                                                                                                                              |
|------------------|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>irc</code> | <code>int</code> | <p>Function completion status:</p> <p>1 - Intel MKL dispatches the code path for the specified ISA by default.</p> <p>0 - The request is rejected. Usually this occurs if <code>mkl_enable_instructions</code> was called:</p> <ul style="list-style-type: none"> <li>• After another Intel MKL function</li> <li>• On a non-Intel architecture</li> <li>• With an incompatible ISA specified</li> </ul> |

### Optimization Notice

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## `mkl_set_env_mode`

*Sets up the mode that ignores environment settings specific to Intel MKL.*

### Syntax

```
int mkl_set_env_mode(int mode);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name              | Type             | Description                                                                                                                                                                                                                        |
|-------------------|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>mode</code> | <code>int</code> | <p>Specifies what mode to set. For details, see <a href="#">Description</a>. Possible values:</p> <ul style="list-style-type: none"> <li>• 0 - Do nothing.</li> </ul> <p>Use this value to query the current environment mode.</p> |

| Name | Type | Description                                                                                                               |
|------|------|---------------------------------------------------------------------------------------------------------------------------|
|      |      | <ul style="list-style-type: none"> <li>1 - Make Intel MKL ignore environment settings specific to the library.</li> </ul> |

## Description

In the default *environment mode*, Intel MKL can control its behavior using environment variables for threading, memory management, Conditional Numerical Reproducibility, automatic offload, and so on. The `mkl_set_env_mode` function sets up the environment mode that ignores all settings specified by Intel MKL environment variables except `MIC_LD_LIBRARY_PATH` and `MKLROOT`.

## Return Values

| Name                      | Type             | Description                                                                                                                                                                            |
|---------------------------|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>current_mode</code> | <code>int</code> | Environment mode that was used before the function call: <ul style="list-style-type: none"> <li>0 - Default</li> <li>1 - Ignore environment settings specific to Intel MKL.</li> </ul> |

## `mkl_verbose`

*Enables or disables Intel MKL Verbose mode.*

## Syntax

```
int mkl_verbose (int enable);
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                | Type             | Description                                                                                                                                                                                                                                                                    |
|---------------------|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>enable</code> | <code>int</code> | Desired state of the Intel MKL Verbose mode. Indicates whether printing Intel MKL function call information should be turned on or off. Possible values: <ul style="list-style-type: none"> <li>0 - disable the Verbose mode.</li> <li>1 - enable the Verbose mode.</li> </ul> |

## Description

This function enables or disables the Intel MKL Verbose mode, in which computational functions print call description information. For details of the Verbose mode, see the *Intel MKL Developer Guide*, available in the Intel® Software Documentation Library.

---

### NOTE

The setting for the Verbose mode specified by the `mkl_verbose` function takes precedence over the setting specified by the `MKL_VERBOSE` environment variable.

---

## Return Values

| Name                | Type             | Description                                                                                                                                                                                                                                                                                                                                          |
|---------------------|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code> | <ul style="list-style-type: none"> <li>If the requested operation completed successfully, contains previous state of the verbose mode: <ul style="list-style-type: none"> <li>0 - disabled</li> <li>1 - enabled</li> </ul> </li> <li>If the function failed to complete the operation because of an incorrect input parameter, equals -1.</li> </ul> |

## See Also

[Intel Software Documentation Library](#)

## mkl\_set\_mpi

Sets the implementation of the message-passing interface to be used by Intel MKL.

### Syntax

```
int mkl_set_mpi (int vendor, const char *custom_library_name);
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                             | Type                      | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------------------------------|---------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>vendor</code>              | <code>int</code>          | <p>Specifies the implementation of the message-passing interface (MPI) to use:</p> <p>Possible values:</p> <ul style="list-style-type: none"> <li>• <code>MKL_BLACS_CUSTOM</code> - a custom MPI library. Requires a prebuilt custom MPI BLACS library.</li> <li>• <code>MKL_BLACS_MS_MPI</code> - Microsoft MPI library.</li> <li>• <code>MKL_BLACS_INTELMPI</code> - Intel® MPI library.</li> <li>• <code>MKL_BLACS_MPICH</code> - MPICH MPI library.</li> </ul> |
| <code>custom_library_name</code> | <code>const char *</code> | <p>The filename (without a directory name) of the custom BLACS dynamic library to use. This library must be located in the directory with your application executable or with Intel MKL dynamic libraries. Can be <code>NULL</code> or an empty string.</p>                                                                                                                                                                                                        |

### Description

Call this function to set the MPI implementation to be used by Intel MKL on Windows\* OS when dynamic Intel MKL libraries are used. For all other configurations, the function returns an error indicating that you cannot set the MPI implementation. You can specify your own prebuilt dynamic BLACS library for a custom MPI by setting `vendor` to `MKL_BLACS_CUSTOM` and optionally passing the name of the custom BLACS dynamic library. If the `custom_library_path` parameter is `NULL` or an empty string, Intel MKL uses the default platform-specific library name: `mkl_blacs_custom_lp64.dll` or `mkl_blacs_custom_ilp64.dll`, depending on whether the BLACS interface linked against your application is LP64 or ILP64.

## Return Values

| Name                | Type             | Description                                                                                                                                                                                                                                                                                                                  |
|---------------------|------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code> | <p>The return status:</p> <ul style="list-style-type: none"> <li>• 0 - The function completed successfully.</li> <li>• -1 - The <code>vendor</code> parameter is invalid.</li> <li>• -2 - The <code>custom_library_name</code> parameter is invalid.</li> <li>• -3 - The MPI library cannot be set at this point.</li> </ul> |

### Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

## mkl\_finalize

*Terminates Intel MKL execution environment and frees resources allocated by the library.*

### Syntax

```
void mkl_finalize(void);
```

### Include Files

- `mkl.h`

### Description

This function frees resources allocated by Intel MKL. Once this function is called, the application can no longer call Intel MKL functions other than `mkl_finalize`.

In particular, the `mkl_finalize` function enables you to free resources when a third-party shared library is statically linked to Intel MKL. To avoid resource leaks that may happen when a shared library is loaded and unloaded multiple times, call `mkl_finalize` each time the library is unloaded. The recommended method to do this depends on the operating system:

- On Linux\* or macOS\*, place the call into a shared library destructor.
- On Windows\*, call `mkl_finalize` from the `DLL_PROCESS_DETACH` handler of `DllMain`.

### NOTE

Intel MKL shared libraries automatically perform finalization when they are unloaded. If an application is statically linked to Intel MKL, the operating system frees all resources allocated by Intel MKL during termination of the process associated with the application.



# BLACS Routines

This chapter describes the Intel® Math Kernel Library implementation of FORTRAN 77 routines from the BLACS (Basic Linear Algebra Communication Subprograms) package. These routines are used to support a linear algebra oriented message passing interface that may be implemented efficiently and uniformly across a large range of distributed memory platforms.

The BLACS routines make linear algebra applications both easier to program and more portable. For this purpose, they are used in Intel MKL intended for the Linux\* and Windows\* OSs as the communication layer of ScaLAPACK and Cluster FFT.

On computers, a linear algebra matrix is represented by a two dimensional array (2D array), and therefore the BLACS operate on 2D arrays. See description of the basic [matrix shapes](#) in a special section.

The BLACS routines implemented in Intel MKL are of four categories:

- Combines
- Point to Point Communication
- Broadcast
- Support.

The [Combines](#) take data distributed over processes and combine the data to produce a result. The [Point to Point](#) routines are intended for point-to-point communication and [Broadcast](#) routines send data possessed by one process to all processes within a scope.

The [Support routines](#) perform distinct tasks that can be used for initialization, destruction, information, and miscellaneous tasks.

## Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

## Matrix Shapes

The BLACS routines recognize the two most common classes of matrices for dense linear algebra. The first of these classes consists of general rectangular matrices, which in machine storage are 2D arrays consisting of  $m$  rows and  $n$  columns, with a leading dimension,  $lda$ , that determines the distance between successive columns in memory.

The *general rectangular* matrices take the following parameters as input when determining what array to operate on:

- |     |                                                                                                                                       |
|-----|---------------------------------------------------------------------------------------------------------------------------------------|
| $m$ | (input) INTEGER. The number of matrix rows to be operated on.                                                                         |
| $n$ | (input) INTEGER. The number of matrix columns to be operated on.                                                                      |
| $a$ | (input/output) TYPE (depends on routine), array of dimension $(lda, n)$ .<br>A pointer to the beginning of the (sub)array to be sent. |

*lda* (input) INTEGER. The distance between two elements in matrix row.

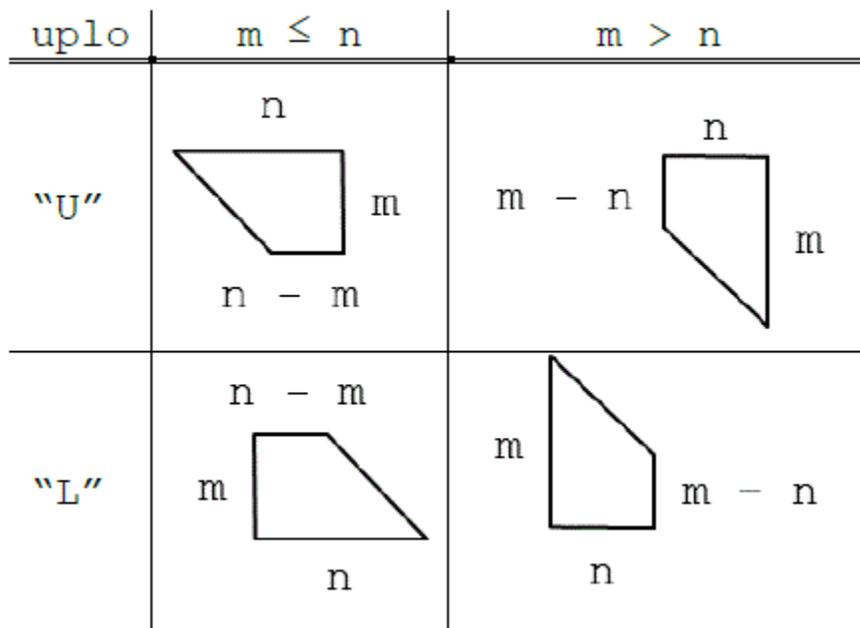
The second class of matrices recognized by the BLACS are *trapezoidal* matrices (triangular matrices are a sub-class of trapezoidal). Trapezoidal arrays are defined by *m*, *n*, and *lda*, as above, but they have two additional parameters as well. These parameters are:

*uplo* (input) CHARACTER\*1 . Indicates whether the matrix is upper or lower trapezoidal, as discussed below.

*diag* (input) CHARACTER\*1 . Indicates whether the diagonal of the matrix is unit diagonal (will not be operated on) or otherwise (will be operated on).

The shape of the trapezoidal arrays is determined by these parameters as follows:

**Trapezoidal Arrays Shapes**



The packing of arrays, if required, so that they may be sent efficiently is hidden, allowing the user to concentrate on the logical matrix, rather than on how the data is organized in the system memory.

**Repeatability and Coherence**

Floating point computations are not exact on almost all modern architectures. This lack of precision is particularly problematic in parallel operations. Since floating point computations are inexact, algorithms are classified according to whether they are *repeatable* and to what degree they guarantee *coherence*.

- Repeatable: a routine is repeatable if it is guaranteed to give the same answer if called multiple times with the same parallel configuration and input.
- Coherent: a routine is coherent if all processes selected to receive the answer get identical results.

**NOTE**

Repeatability and coherence do not effect correctness. A routine may be both incoherent and non-repeatable, and still give correct output. But inaccuracies in floating point calculations may cause the routine to return differing values, all of which are equally valid.

## Repeatability

Because the precision of floating point arithmetic is limited, it is not truly associative:  $(a + b) + c$  might not be the same as  $a + (b + c)$ . The lack of exact arithmetic can cause problems whenever the possibility for reordering of floating point calculations exists. This problem becomes prevalent in parallel computing due to race conditions in message passing. For example, consider a routine which sums numbers stored on different processes. Assume this routine runs on four processes, with the numbers to be added being the process numbers themselves. Therefore, process 0 has the value 0:0, process 1 has the value 1:0, and so on.

One algorithm for the computation of this result is to have all processes send their process numbers to process 0; process 0 adds them up, and sends the result back to all processes. So, process 0 would add a number to 0:0 in the first step. If receiving the process numbers is ordered so that process 0 always receives the message from process 1 first, then 2, and finally 3, this results in a repeatable algorithm, which evaluates the expression  $((0:0 + 1:0) + 2:0) + 3:0$ .

However, to get the best parallel performance, it is better not to require a particular ordering, and just have process 0 add the first available number to its value and continue to do so until all numbers have been added in. Using this method, a race condition occurs, because the order of the operation is determined by the order in which process 0 receives the messages, which can be effected by any number of things. This implementation is not repeatable, because the answer can vary between invocations, even if the input is the same. For instance, one run might produce the sequence  $((0:0 + 1:0) + 2:0) + 3:0$ , while a subsequent run could produce  $((0:0 + 2:0) + 1:0) + 3:0$ . Both of these results are correct summations of the given numbers, but because of floating point roundoff, they might be different.

## Coherence

A routine produces coherent output if all processes are guaranteed to produce the exact same results. Obviously, almost no algorithm involving communication is coherent if communication can change the values being communicated. Therefore, if the parallel system being studied cannot guarantee that communication between processes preserves values, no routine is guaranteed to produce coherent results.

If communication is assumed to be coherent, there are still various levels of coherent algorithms. Some algorithms guarantee coherence only if floating point operations are done in the exact same order on every node. This is *homogeneous coherence*: the result will be coherent if the parallel machine is homogeneous in its handling of floating point operations.

A stronger assertion of coherence is *heterogeneous coherence*, which does not require all processes to have the same handling of floating point operations.

In general, a routine that is homogeneous coherent performs computations redundantly on all nodes, so that all processes get the same answer only if all processes perform arithmetic in the exact same way, whereas a routine which is heterogeneous coherent is usually constrained to having one process calculate the final result, and broadcast it to all other processes.

## Example of Incoherence

An incoherent algorithm is one which does not guarantee that all processes get the same result even on a homogeneous system with coherent communication. The previous example of summing the process numbers demonstrates this kind of behavior. One way to perform such a sum is to have every process broadcast its number to all other processes. Each process then adds these numbers, starting with its own. The calculations performed by each process receives would then be:

- Process 0 :  $((0:0 + 1:0) + 2:0) + 3:0$
- Process 1 :  $((1:0 + 2:0) + 3:0) + 0:0$
- Process 2 :  $((2:0 + 3:0) + 0:0) + 1:0$
- Process 3 :  $((3:0 + 0:0) + 1:0) + 0:0$

All of these results are equally valid, and since all the results might be different from each other, this algorithm is incoherent. Notice, however, that this algorithm is repeatable: each process will get the same result if the algorithm is called again on the same data.

## Example of Homogeneous Coherence

Another way to perform this summation is for all processes to send their data to all other processes, and to ensure the result is not incoherent, enforce the ordering so that the calculation each node performs is  $((0:0 + 1:0) + 2:0) + 3:0$ . This answer is the same for all processes only if all processes do the floating point arithmetic in the same way. Otherwise, each process may make different floating point errors during the addition, leading to incoherence of the output. Notice that since there is a specific ordering to the addition, this algorithm is repeatable.

## Example of Heterogeneous Coherence

In the final example, all processes send the result to process 0, which adds the numbers and broadcasts the result to the rest of the processes. Since one process does all the computation, it can perform the operations in any order and it will give coherent results as long as communication is itself coherent. If a particular order is not forced on the the addition, the algorithm will not be repeatable. If a particular order is forced, it will be repeatable.

## Summary

Repeatability and coherence are separate issues which may occur in parallel computations. These concepts may be summarized as:

- **Repeatability:** The routine will yield the exact same result if it run multiple times on an identical problem. Each process may get a different result than the others (i.e., repeatability does not imply coherence), but that value will not change if the routine is invoked multiple times.
- **Homogeneous coherence:** All processes selected to possess the result will receive the exact same answer if:
  - Communication does not change the value of the communicated data.
  - All processes perform floating point arithmetic exactly the same.
- **Heterogeneous coherence:** All processes will receive the exact same answer if communication does not change the value of the communicated data.

In general, lack of the associative property for floating point calculations may cause both incoherence and non-repeatability. Algorithms that rely on redundant computations are at best homogeneous coherent, and algorithms in which one process broadcasts the result are heterogeneous coherent. Repeatability does not imply coherence, nor does coherence imply repeatability.

Since these issues do not effect the correctness of the answer, they can usually be ignored. However, in very specific situations, these issues may become very important. A stopping criteria should not be based on incoherent results, for instance. Also, a user creating and debugging a parallel program may wish to enforce repeatability so the exact same program sequence occurs on every run.

In the BLACS, coherence and repeatability apply only in the context of the combine operations. As mentioned above, it is possible to have communication which is incoherent (for instance, two machines which store floating point numbers differently may easily produce incoherent communication, since a number stored on machine A may not have a representation on machine B). However, the BLACS cannot control this issue. Communication is assumed to be coherent, which for communication implies that it is also repeatable.

For combine operations, the BLACS allow you to set flags indicating that you would like combines to be repeatable and/or heterogeneous coherent (see [blacs\\_get](#) and [blacs\\_set](#) for details on setting these flags).

If the BLACS are instructed to guarantee heterogeneous coherency, the BLACS restrict the topologies which can be used so that one process calculates the final result of the combine, and if necessary, broadcasts the answer to all other processes.

If the BLACS are instructed to guarantee repeatability, orderings will be enforced in the topologies which are selected. This may result in loss of performance which can range from negligible to serious depending on the application.

A couple of additional notes are in order. Incoherence and nonrepeatability can arise as a result of floating point errors, as discussed previously. This might lead you to suspect that integer calculations are always repeatable and coherent, since they involve exact arithmetic. This is true if overflow is ignored. With overflow taken into consideration, even integer calculations can display incoherence and non-repeatability. Therefore, if the repeatability or coherence flags are set, the BLACS treats integer combines the same as floating point combines in enforcing repeatability and coherence guards.

By their nature, maximization and minimization should always be repeatable. In the complex precisions, however, the real and imaginary parts must be combined in order to obtain a magnitude value used to do the comparison (this is typically  $|r| + |i|$  or  $\text{sqr}(r^2 + i^2)$ ). This allows for the possibility of heterogeneous incoherence. The BLACS therefore restrict which topologies are used for maximization and minimization in the complex routines when the heterogeneous coherence flag is set.

## BLACS Combine Operations

This section describes BLACS routines that combine the data to produce a result.

In a combine operation, each participating process contributes data that is combined with other processes' data to produce a result. This result can be given to a particular process (called the *destination* process), or to all participating processes. If the result is given to only one process, the operation is referred to as a *leave-on-one* combine, and if the result is given to all participating processes the operation is referenced as a *leave-on-all* combine.

At present, three kinds of combines are supported. They are:

- element-wise summation
- element-wise absolute value maximization
- element-wise absolute value minimization

of general rectangular arrays.

Note that a combine operation combines data between processes. By definition, a combine performed across a scope of only one process does not change the input data. This is why the operations (*max/min/sum*) are specified as *element-wise*. Element-wise indicates that each element of the input array will be combined with the corresponding element from all other processes' arrays to produce the result. Thus, a 4 x 2 array of inputs produces a 4 x 2 answer array.

When the *max/min* comparison is being performed, absolute value is used. For example, -5 and 5 are equivalent. However, the returned value is unchanged; that is, it is not the absolute value, but is a signed value instead. Therefore, if you performed a BLACS absolute value maximum combine on the numbers -5, 3, 1, 8 the result would be -8.

The initial symbol ? in the routine names below masks the data type:

|   |                           |
|---|---------------------------|
| i | integer                   |
| s | single precision real     |
| d | double precision real     |
| c | single precision complex  |
| z | double precision complex. |

### BLACS Combines

| Routine name           | Results of operation                                                                                |
|------------------------|-----------------------------------------------------------------------------------------------------|
| <a href="#">gamx2d</a> | Entries of result matrix will have the value of the greatest absolute value found in that position. |
| <a href="#">gamn2d</a> | Entries of result matrix will have the value of the smallest absolute value found in that position. |

| Routine name        | Results of operation                                               |
|---------------------|--------------------------------------------------------------------|
| <code>gsum2d</code> | Entries of result matrix will have the summation of that position. |

## ?gamx2d

*Performs element-wise absolute value maximization.*

### Syntax

```
call igamx2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
call sgamx2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
call dgamx2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
call cgamx2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
call zgamx2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
```

### Input Parameters

|                      |                                                                                                                                                                                                                                           |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>icontxt</code> | INTEGER. Integer handle that indicates the context.                                                                                                                                                                                       |
| <code>scope</code>   | CHARACTER*1. Indicates what scope the combine should proceed on. Limited to ROW, COLUMN, or ALL.                                                                                                                                          |
| <code>top</code>     | CHARACTER*1. Communication pattern to use during the combine operation.                                                                                                                                                                   |
| <code>m</code>       | INTEGER. The number of matrix rows to be combined.                                                                                                                                                                                        |
| <code>n</code>       | INTEGER. The number of matrix columns to be combined.                                                                                                                                                                                     |
| <code>a</code>       | TYPE array ( <code>lda</code> , <code>n</code> ). Matrix to be compared with to produce the maximum.                                                                                                                                      |
| <code>lda</code>     | INTEGER. The leading dimension of the matrix <i>A</i> , that is, the distance between two successive elements in a matrix row.                                                                                                            |
| <code>rcflag</code>  | INTEGER.<br>If <code>rcflag = -1</code> , the arrays <code>ra</code> and <code>ca</code> are not referenced and need not exist. Otherwise, <code>rcflag</code> indicates the leading dimension of these arrays, and so must be $\geq m$ . |
| <code>rdest</code>   | INTEGER.<br>The process row coordinate of the process that should receive the result. If <code>rdest</code> or <code>cdest = -1</code> , all processes within the indicated scope receive the answer.                                     |
| <code>cdest</code>   | INTEGER.<br>The process column coordinate of the process that should receive the result. If <code>rdest</code> or <code>cdest = -1</code> , all processes within the indicated scope receive the answer.                                  |

## Output Parameters

|           |                                                                                                                                                                                                                                                                                                                                                                                                      |
|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>  | TYPE array ( <i>lda</i> , <i>n</i> ). Contains the result if this process is selected to receive the answer, or intermediate results if the process is not selected to receive the result.                                                                                                                                                                                                           |
| <i>ra</i> | INTEGER array ( <i>rcflag</i> , <i>n</i> ).<br>If <i>rcflag</i> = -1, this array will not be referenced, and need not exist. Otherwise, it is an integer array (of size at least <i>rcflag</i> × <i>n</i> ) indicating the row index of the process that provided the maximum. If the calling process is not selected to receive the result, this array will contain intermediate (useless) results. |
| <i>ca</i> | INTEGER array ( <i>rcflag</i> , <i>n</i> ).<br>If <i>rcflag</i> = -1, this array will not be referenced, and need not exist. Otherwise, it is an integer array (of size at least <i>rcflag</i> × <i>n</i> ) indicating the row index of the process that provided the maximum. If the calling process is not selected to receive the result, this array will contain intermediate (useless) results. |

## Description

This routine performs element-wise absolute value maximization, that is, each element of matrix *A* is compared with the corresponding element of the other process's matrices. Note that the value of *A* is returned, but the absolute value is used to determine the maximum (the 1-norm is used for complex numbers). Combines may be globally-blocking, so they must be programmed as if no process returns until all have called the routine.

## See Also

[Examples of BLACS Routines Usage](#)

## ?gamn2d

Performs element-wise absolute value minimization.

## Syntax

```
call igamn2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
call sgamn2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
call dgamn2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
call cgamn2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
call zgamn2d(icontxt, scope, top, m, n, a, lda, ra, ca, rcflag, rdest, cdest)
```

## Input Parameters

|                |                                                                                                  |
|----------------|--------------------------------------------------------------------------------------------------|
| <i>icontxt</i> | INTEGER. Integer handle that indicates the context.                                              |
| <i>scope</i>   | CHARACTER*1. Indicates what scope the combine should proceed on. Limited to ROW, COLUMN, or ALL. |
| <i>top</i>     | CHARACTER*1. Communication pattern to use during the combine operation.                          |
| <i>m</i>       | INTEGER. The number of matrix rows to be combined.                                               |

|               |                                                                                                                                                                                                                      |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>      | INTEGER. The number of matrix columns to be combined.                                                                                                                                                                |
| <i>a</i>      | TYPE array ( <i>lda</i> , <i>n</i> ). Matrix to be compared with to produce the minimum.                                                                                                                             |
| <i>lda</i>    | INTEGER. The leading dimension of the matrix <i>A</i> , that is, the distance between two successive elements in a matrix row.                                                                                       |
| <i>rcflag</i> | INTEGER.<br><br>If <i>rcflag</i> = -1, the arrays <i>ra</i> and <i>ca</i> are not referenced and need not exist. Otherwise, <i>rcflag</i> indicates the leading dimension of these arrays, and so must be $\geq m$ . |
| <i>rdest</i>  | INTEGER.<br><br>The process row coordinate of the process that should receive the result. If <i>rdest</i> or <i>cdest</i> = -1, all processes within the indicated scope receive the answer.                         |
| <i>cdest</i>  | INTEGER.<br><br>The process column coordinate of the process that should receive the result. If <i>rdest</i> or <i>cdest</i> = -1, all processes within the indicated scope receive the answer.                      |

## Output Parameters

|           |                                                                                                                                                                                                                                                                                                                                                                                                          |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i>  | TYPE array ( <i>lda</i> , <i>n</i> ). Contains the result if this process is selected to receive the answer, or intermediate results if the process is not selected to receive the result.                                                                                                                                                                                                               |
| <i>ra</i> | INTEGER array ( <i>rcflag</i> , <i>n</i> ).<br><br>If <i>rcflag</i> = -1, this array will not be referenced, and need not exist. Otherwise, it is an integer array (of size at least <i>rcflag</i> × <i>n</i> ) indicating the row index of the process that provided the minimum. If the calling process is not selected to receive the result, this array will contain intermediate (useless) results. |
| <i>ca</i> | INTEGER array ( <i>rcflag</i> , <i>n</i> ).<br><br>If <i>rcflag</i> = -1, this array will not be referenced, and need not exist. Otherwise, it is an integer array (of size at least <i>rcflag</i> × <i>n</i> ) indicating the row index of the process that provided the minimum. If the calling process is not selected to receive the result, this array will contain intermediate (useless) results. |

## Description

This routine performs element-wise absolute value minimization, that is, each element of matrix *A* is compared with the corresponding element of the other process's matrices. Note that the value of *A* is returned, but the absolute value is used to determine the minimum (the 1-norm is used for complex numbers). Combines may be globally-blocking, so they must be programmed as if no process returns until all have called the routine.

## See Also

[Examples of BLACS Routines Usage](#)

## ?gsum2d

*Performs element-wise summation.*

---

### Syntax

```
call igsum2d(icontxt, scope, top, m, n, a, lda, rdest, cdest)
call sgsum2d(icontxt, scope, top, m, n, a, lda, rdest, cdest)
call dgsum2d(icontxt, scope, top, m, n, a, lda, rdest, cdest)
call cgsum2d(icontxt, scope, top, m, n, a, lda, rdest, cdest)
call zgsum2d(icontxt, scope, top, m, n, a, lda, rdest, cdest)
```

### Input Parameters

|                |                                                                                                                                                                                             |
|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>icontxt</i> | INTEGER. Integer handle that indicates the context.                                                                                                                                         |
| <i>scope</i>   | CHARACTER*1. Indicates what scope the combine should proceed on. Limited to ROW, COLUMN, or ALL.                                                                                            |
| <i>top</i>     | CHARACTER*1. Communication pattern to use during the combine operation.                                                                                                                     |
| <i>m</i>       | INTEGER. The number of matrix rows to be combined.                                                                                                                                          |
| <i>n</i>       | INTEGER. The number of matrix columns to be combined.                                                                                                                                       |
| <i>a</i>       | TYPE array ( <i>lda</i> , <i>n</i> ). Matrix to be added to produce the sum.                                                                                                                |
| <i>lda</i>     | INTEGER. The leading dimension of the matrix <i>A</i> , that is, the distance between two successive elements in a matrix row.                                                              |
| <i>rdest</i>   | INTEGER.<br>The process row coordinate of the process that should receive the result. If <i>rdest</i> or <i>cdest</i> = -1, all processes within the indicated scope receive the answer.    |
| <i>cdest</i>   | INTEGER.<br>The process column coordinate of the process that should receive the result. If <i>rdest</i> or <i>cdest</i> = -1, all processes within the indicated scope receive the answer. |

### Output Parameters

|          |                                                                                                                                                                                            |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>a</i> | TYPE array ( <i>lda</i> , <i>n</i> ). Contains the result if this process is selected to receive the answer, or intermediate results if the process is not selected to receive the result. |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

### Description

This routine performs element-wise summation, that is, each element of matrix *A* is summed with the corresponding element of the other process's matrices. Combines may be globally-blocking, so they must be programmed as if no process returns until all have called the routine.

### See Also

[Examples of BLACS Routines Usage](#)

## BLACS Point To Point Communication

This section describes BLACS routines for point to point communication.

Point to point communication requires two complementary operations. The *send* operation produces a message that is then consumed by the *receive* operation. These operations have various resources associated with them. The main such resource is the buffer that holds the data to be sent or serves as the area where the incoming data is to be received. The level of *blocking* indicates what correlation the return from a send/receive operation has with the availability of these resources and with the status of message.

### Non-blocking

The return from the *send* or *receive* operations does not imply that the resources may be reused, that the message has been sent/received or that the complementary operation has been called. Return means only that the send/receive has been started, and will be completed at some later date. Polling is required to determine when the operation has finished.

In non-blocking message passing, the concept of *communication/computation overlap* (abbreviated C/C overlap) is important. If a system possesses C/C overlap, independent computation can occur at the same time as communication. That means a nonblocking operation can be posted, and unrelated work can be done while the message is sent/received in parallel. If C/C overlap is not present, after returning from the routine call, computation will be interrupted at some later date when the message is actually sent or received.

### Locally-blocking

Return from the *send* or *receive* operations indicates that the resources may be reused. However, since this only depends on local information, it is unknown whether the complementary operation has been called. There are no locally-blocking receives: the send must be completed before the receive buffer is available for re-use.

If a receive has not been posted at the time a locally-blocking send is issued, buffering will be required to avoid losing the message. Buffering can be done on the sending process, the receiving process, or not done at all, losing the message.

### Globally-blocking

Return from a globally-blocking procedure indicates that the operation resources may be reused, and that complement of the operation has at least been posted. Since the receive has been posted, there is no buffering required for globally-blocking sends: the message is always sent directly into the user's receive buffer.

Almost all processors support non-blocking communication, as well as some other level of blocking sends. What level of blocking the send possesses varies between platforms. For instance, the Intel® processors support locally-blocking sends, with buffering done on the receiving process. This is a very important distinction, because codes written assuming locally-blocking sends will hang on platforms with globally-blocking sends. Below is a simple example of how this can occur:

```
IAM = MY_PROCESS_ID()
IF (IAM .EQ. 0) THEN
 SEND TO PROCESS 1
 RECV FROM PROCESS 1
ELSE IF (IAM .EQ. 1) THEN
 SEND TO PROCESS 0
 RECV FROM PROCESS 0
END IF
```

If the send is globally-blocking, process 0 enters the send, and waits for process 1 to start its receive before continuing. In the meantime, process 1 starts to send to 0, and waits for 0 to receive before continuing. Both processes are now waiting on each other, and the program will never continue.

The solution for this case is obvious. One of the processes simply reverses the order of its communication calls and the hang is avoided. However, when the communication is not just between two processes, but rather involves a hierarchy of processes, determining how to avoid this kind of difficulty can become problematic.

For this reason, it was decided the BLACS would support locally-blocking sends. On systems natively supporting globally-blocking sends, non-blocking sends coupled with buffering is used to simulate locally-blocking sends. The BLACS support globally-blocking receives.

In addition, the BLACS specify that point to point messages between two given processes will be strictly ordered. If process 0 sends three messages (label them *A*, *B*, and *C*) to process 1, process 1 must receive *A* before it can receive *B*, and message *C* can be received only after both *A* and *B*. The main reason for this restriction is that it allows for the computation of message identifiers.

Note, however, that messages from different processes are not ordered. If processes 0, . . . , 3 send messages *A*, . . . , *D* to process 4, process 4 may receive these messages in any order that is convenient.

### Convention

The convention used in the communication routine names follows the template `?xxyy2d`, where the letter in the ? position indicates the data type being sent, `xx` is replaced to indicate the shape of the matrix, and the `yy` positions are used to indicate the type of communication to perform:

|                 |                                                                        |
|-----------------|------------------------------------------------------------------------|
| <code>i</code>  | integer                                                                |
| <code>s</code>  | single precision real                                                  |
| <code>d</code>  | double precision real                                                  |
| <code>c</code>  | single precision complex                                               |
| <code>z</code>  | double precision complex                                               |
| <code>ge</code> | The data to be communicated is stored in a general rectangular matrix. |
| <code>tr</code> | The data to be communicated is stored in a trapezoidal matrix.         |
| <code>sd</code> | Send. One process sends to another.                                    |
| <code>rv</code> | Receive. One process receives from another.                            |

### BLACS Point To Point Communication

| Routine name        | Operation performed                                               |
|---------------------|-------------------------------------------------------------------|
| <code>gesd2d</code> | Take the indicated matrix and send it to the destination process. |
| <code>trsd2d</code> |                                                                   |
| <code>gerv2d</code> | Receive a message from the process into the matrix.               |
| <code>trrv2d</code> |                                                                   |

As a simple example, the pseudo code given above is rewritten below in terms of the BLACS. It is further specified that the data being exchanged is the double precision vector *X*, which is 5 elements long.

```
CALL GRIDINFO(NPROW, NPCOL, MYPROW, MYPCOL)
IF (MYPROW.EQ.0 .AND. MYPCOL.EQ.0) THEN
 CALL DGESD2D(5, 1, X, 5, 1, 0)
 CALL DGERV2D(5, 1, X, 5, 1, 0)
ELSE IF (MYPROW.EQ.1 .AND. MYPCOL.EQ.0) THEN
```

```
CALL DGESD2D(5, 1, X, 5, 0, 0)
CALL DGERV2D(5, 1, X, 5, 0, 0)
END IF
```

## ?gesd2d

*Takes a general rectangular matrix and sends it to the destination process.*

### Syntax

```
call igesd2d(icontxt, m, n, a, lda, rdest, cdest)
call sgesd2d(icontxt, m, n, a, lda, rdest, cdest)
call dgesd2d(icontxt, m, n, a, lda, rdest, cdest)
call cgesd2d(icontxt, m, n, a, lda, rdest, cdest)
call zgesd2d(icontxt, m, n, a, lda, rdest, cdest)
```

### Input Parameters

|                     |                                                                                  |
|---------------------|----------------------------------------------------------------------------------|
| <i>icontxt</i>      | INTEGER. Integer handle that indicates the context.                              |
| <i>m, n, a, lda</i> | Describe the matrix to be sent. See <a href="#">Matrix Shapes</a> for details.   |
| <i>rdest</i>        | INTEGER.<br>The process row coordinate of the process to send the message to.    |
| <i>cdest</i>        | INTEGER.<br>The process column coordinate of the process to send the message to. |

### Description

This routine takes the indicated general rectangular matrix and sends it to the destination process located at {RDEST, CDEST} in the process grid. Return from the routine indicates that the buffer (the matrix A) may be reused. The routine is locally-blocking, that is, it will return even if the corresponding receive is not posted.

### See Also

[Examples of BLACS Routines Usage](#)

## ?trsd2d

*Takes a trapezoidal matrix and sends it to the destination process.*

### Syntax

```
call itrdsd2d(icontxt, uplo, diag, m, n, a, lda, rdest, cdest)
call strsd2d(icontxt, uplo, diag, m, n, a, lda, rdest, cdest)
call dtrsd2d(icontxt, uplo, diag, m, n, a, lda, rdest, cdest)
call ctrsd2d(icontxt, uplo, diag, m, n, a, lda, rdest, cdest)
call ztrsd2d(icontxt, uplo, diag, m, n, a, lda, rdest, cdest)
```

## Input Parameters

|                                 |                                                                                  |
|---------------------------------|----------------------------------------------------------------------------------|
| <i>icontxt</i>                  | INTEGER. Integer handle that indicates the context.                              |
| <i>uplo, diag, m, n, a, lda</i> | Describe the matrix to be sent. See <a href="#">Matrix Shapes</a> for details.   |
| <i>rdest</i>                    | INTEGER.<br>The process row coordinate of the process to send the message to.    |
| <i>cdest</i>                    | INTEGER.<br>The process column coordinate of the process to send the message to. |

## Description

This routine takes the indicated trapezoidal matrix and sends it to the destination process located at {RDEST, CDEST} in the process grid. Return from the routine indicates that the buffer (the matrix *A*) may be reused. The routine is locally-blocking, that is, it will return even if the corresponding receive is not posted.

## ?gerv2d

*Receives a message from the process into the general rectangular matrix.*

## Syntax

```
call igerv2d(icontxt, m, n, a, lda, rsrc, csrc)
call sgerv2d(icontxt, m, n, a, lda, rsrc, csrc)
call dgerv2d(icontxt, m, n, a, lda, rsrc, csrc)
call cgerv2d(icontxt, m, n, a, lda, rsrc, csrc)
call zgerv2d(icontxt, m, n, a, lda, rsrc, csrc)
```

## Input Parameters

|                  |                                                                                |
|------------------|--------------------------------------------------------------------------------|
| <i>icontxt</i>   | INTEGER. Integer handle that indicates the context.                            |
| <i>m, n, lda</i> | Describe the matrix to be sent. See <a href="#">Matrix Shapes</a> for details. |
| <i>rsrc</i>      | INTEGER.<br>The process row coordinate of the source of the message.           |
| <i>csrc</i>      | INTEGER.<br>The process column coordinate of the source of the message.        |

## Output Parameters

|          |                                                                        |
|----------|------------------------------------------------------------------------|
| <i>a</i> | An array of dimension $(lda, n)$ to receive the incoming message into. |
|----------|------------------------------------------------------------------------|

## Description

This routine receives a message from process {RSRC, CSRC} into the general rectangular matrix *A*. This routine is globally-blocking, that is, return from the routine indicates that the message has been received into *A*.

## See Also

Examples of BLACS Routines Usage

## ?trrv2d

Receives a message from the process into the trapezoidal matrix.

---

### Syntax

```
call itrrv2d(ictxt, uplo, diag, m, n, a, lda, rsrc, csrc)
call strrv2d(ictxt, uplo, diag, m, n, a, lda, rsrc, csrc)
call dtrrv2d(ictxt, uplo, diag, m, n, a, lda, rsrc, csrc)
call ctrrv2d(ictxt, uplo, diag, m, n, a, lda, rsrc, csrc)
call ztrrv2d(ictxt, uplo, diag, m, n, a, lda, rsrc, csrc)
```

### Input Parameters

|                              |                                                                                |
|------------------------------|--------------------------------------------------------------------------------|
| <i>ictxt</i>                 | INTEGER. Integer handle that indicates the context.                            |
| <i>uplo, diag, m, n, lda</i> | Describe the matrix to be sent. See <a href="#">Matrix Shapes</a> for details. |
| <i>rsrc</i>                  | INTEGER.<br>The process row coordinate of the source of the message.           |
| <i>csrc</i>                  | INTEGER.<br>The process column coordinate of the source of the message.        |

### Output Parameters

|          |                                                                        |
|----------|------------------------------------------------------------------------|
| <i>a</i> | An array of dimension $(lda, n)$ to receive the incoming message into. |
|----------|------------------------------------------------------------------------|

### Description

This routine receives a message from process {RSRC, CSRC} into the trapezoidal matrix A. This routine is globally-blocking, that is, return from the routine indicates that the message has been received into A.

## BLACS Broadcast Routines

---

This section describes BLACS broadcast routines.

A broadcast sends data possessed by one process to all processes within a scope. Broadcast, much like point to point communication, has two complementary operations. The process that owns the data to be broadcast issues a *broadcast/send*. All processes within the same scope must then issue the complementary *broadcast/receive*.

The BLACS define that both broadcast/send and broadcast/receive are *globally-blocking*. Broadcasts/receives cannot be locally-blocking since they must post a receive. Note that receives cannot be locally-blocking. When a given process can leave, a broadcast/receive operation is topology dependent, so, to avoid a hang as topology is varied, the broadcast/receive must be treated as if no process can leave until all processes have called the operation.

Broadcast/sends could be defined to be *locally-blocking*. Since no information is being received, as long as locally-blocking point to point sends are used, the broadcast/send will be locally blocking. However, defining one process within a scope to be locally-blocking while all other processes are globally-blocking adds little to the programmability of the code. On the other hand, leaving the option open to have globally-blocking broadcast/sends may allow for optimization on some platforms.

The fact that broadcasts are defined as globally-blocking has several important implications. The first is that scoped operations (broadcasts or combines) must be strictly ordered, that is, all processes within a scope must agree on the order of calls to separate scoped operations. This constraint falls in line with that already in place for the computation of message IDs, and is present in point to point communication as well.

A less obvious result is that scoped operations with `SCOPE = 'ALL'` must be ordered with respect to any other scoped operation. This means that if there are two broadcasts to be done, one along a column, and one involving the entire process grid, all processes within the process column issuing the column broadcast must agree on which broadcast will be performed first.

The convention used in the communication routine names follows the template `?xxyy2d`, where the letter in the `?` position indicates the data type being sent, `xx` is replaced to indicate the shape of the matrix, and the `yy` positions are used to indicate the type of communication to perform:

|    |                                                                                                |
|----|------------------------------------------------------------------------------------------------|
| i  | integer                                                                                        |
| s  | single precision real                                                                          |
| d  | double precision real                                                                          |
| c  | single precision complex                                                                       |
| z  | double precision complex                                                                       |
| ge | The data to be communicated is stored in a general rectangular matrix.                         |
| tr | The data to be communicated is stored in a trapezoidal matrix.                                 |
| bs | Broadcast/send. A process begins the broadcast of data within a scope.                         |
| br | Broadcast/receive A process receives and participates in the broadcast of data within a scope. |

### BLACS Broadcast Routines

| Routine name           | Operation performed                                   |
|------------------------|-------------------------------------------------------|
| <a href="#">gebs2d</a> | Start a broadcast along a scope.                      |
| <a href="#">trbs2d</a> |                                                       |
| <a href="#">gebr2d</a> | Receive and participate in a broadcast along a scope. |
| <a href="#">trbr2d</a> |                                                       |

#### Optimization Notice

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Notice revision #20110804

## ?gebs2d

Starts a broadcast along a scope for a general rectangular matrix.

---

### Syntax

```
call igebs2d(icontxt, scope, top, m, n, a, lda)
call sgebs2d(icontxt, scope, top, m, n, a, lda)
call dgebs2d(icontxt, scope, top, m, n, a, lda)
call cgebs2d(icontxt, scope, top, m, n, a, lda)
call zgebs2d(icontxt, scope, top, m, n, a, lda)
```

### Input Parameters

|                     |                                                                                                          |
|---------------------|----------------------------------------------------------------------------------------------------------|
| <i>icontxt</i>      | INTEGER. Integer handle that indicates the context.                                                      |
| <i>scope</i>        | CHARACTER*1. Indicates what scope the broadcast should proceed on. Limited to 'Row', 'Column', or 'All'. |
| <i>top</i>          | CHARACTER*1. Indicates the communication pattern to use for the broadcast.                               |
| <i>m, n, a, lda</i> | Describe the matrix to be sent. See <a href="#">Matrix Shapes</a> for details.                           |

### Description

This routine starts a broadcast along a scope. All other processes within the scope must call broadcast/receive for the broadcast to proceed. At the end of a broadcast, all processes within the scope will possess the data in the general rectangular matrix *A*.

Broadcasts may be globally-blocking. This means no process is guaranteed to return from a broadcast until all processes in the scope have called the appropriate routine (broadcast/send or broadcast/receive).

### See Also

[Examples of BLACS Routines Usage](#)

## ?trbs2d

Starts a broadcast along a scope for a trapezoidal matrix.

---

### Syntax

```
call itrbs2d(icontxt, scope, top, uplo, diag, m, n, a, lda)
call strbs2d(icontxt, scope, top, uplo, diag, m, n, a, lda)
call dtrbs2d(icontxt, scope, top, uplo, diag, m, n, a, lda)
call ctrbs2d(icontxt, scope, top, uplo, diag, m, n, a, lda)
call ztrbs2d(icontxt, scope, top, uplo, diag, m, n, a, lda)
```

### Input Parameters

|                |                                                     |
|----------------|-----------------------------------------------------|
| <i>icontxt</i> | INTEGER. Integer handle that indicates the context. |
|----------------|-----------------------------------------------------|

|                                 |                                                                                                          |
|---------------------------------|----------------------------------------------------------------------------------------------------------|
| <i>scope</i>                    | CHARACTER*1. Indicates what scope the broadcast should proceed on. Limited to 'Row', 'Column', or 'All'. |
| <i>top</i>                      | CHARACTER*1. Indicates the communication pattern to use for the broadcast.                               |
| <i>uplo, diag, m, n, a, lda</i> | Describe the matrix to be sent. See <a href="#">Matrix Shapes</a> for details.                           |

## Description

This routine starts a broadcast along a scope. All other processes within the scope must call broadcast/receive for the broadcast to proceed. At the end of a broadcast, all processes within the scope will possess the data in the trapezoidal matrix *A*.

Broadcasts may be globally-blocking. This means no process is guaranteed to return from a broadcast until all processes in the scope have called the appropriate routine (broadcast/send or broadcast/receive).

## ?gebr2d

*Receives and participates in a broadcast along a scope for a general rectangular matrix.*

---

## Syntax

```
call igebr2d(icontxt, scope, top, m, n, a, lda, rsrc, csrc)
call sgebr2d(icontxt, scope, top, m, n, a, lda, rsrc, csrc)
call dgebr2d(icontxt, scope, top, m, n, a, lda, rsrc, csrc)
call cgebr2d(icontxt, scope, top, m, n, a, lda, rsrc, csrc)
call zgebr2d(icontxt, scope, top, m, n, a, lda, rsrc, csrc)
```

## Input Parameters

|                  |                                                                                                          |
|------------------|----------------------------------------------------------------------------------------------------------|
| <i>icontxt</i>   | INTEGER. Integer handle that indicates the context.                                                      |
| <i>scope</i>     | CHARACTER*1. Indicates what scope the broadcast should proceed on. Limited to 'Row', 'Column', or 'All'. |
| <i>top</i>       | CHARACTER*1. Indicates the communication pattern to use for the broadcast.                               |
| <i>m, n, lda</i> | Describe the matrix to be sent. See <a href="#">Matrix Shapes</a> for details.                           |
| <i>rsrc</i>      | INTEGER.<br>The process row coordinate of the process that called broadcast/send.                        |
| <i>csrc</i>      | INTEGER.<br>The process column coordinate of the process that called broadcast/send.                     |

## Output Parameters

|          |                                                                        |
|----------|------------------------------------------------------------------------|
| <i>a</i> | An array of dimension $(lda, n)$ to receive the incoming message into. |
|----------|------------------------------------------------------------------------|

## Description

This routine receives and participates in a broadcast along a scope. At the end of a broadcast, all processes within the scope will possess the data in the general rectangular matrix  $A$ . Broadcasts may be globally-blocking. This means no process is guaranteed to return from a broadcast until all processes in the scope have called the appropriate routine (broadcast/send or broadcast/receive).

## See Also

[Examples of BLACS Routines Usage](#)

## ?trbr2d

*Receives and participates in a broadcast along a scope for a trapezoidal matrix.*

## Syntax

```
call itrbr2d(icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc)
call strbr2d(icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc)
call dtrbr2d(icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc)
call ctrbr2d(icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc)
call ztrbr2d(icontxt, scope, top, uplo, diag, m, n, a, lda, rsrc, csrc)
```

## Input Parameters

|                              |                                                                                                          |
|------------------------------|----------------------------------------------------------------------------------------------------------|
| <i>icontxt</i>               | INTEGER. Integer handle that indicates the context.                                                      |
| <i>scope</i>                 | CHARACTER*1. Indicates what scope the broadcast should proceed on. Limited to 'Row', 'Column', or 'All'. |
| <i>top</i>                   | CHARACTER*1. Indicates the communication pattern to use for the broadcast.                               |
| <i>uplo, diag, m, n, lda</i> | Describe the matrix to be sent. See <a href="#">Matrix Shapes</a> for details.                           |
| <i>rsrc</i>                  | INTEGER.<br>The process row coordinate of the process that called broadcast/send.                        |
| <i>csrc</i>                  | INTEGER.<br>The process column coordinate of the process that called broadcast/send.                     |

## Output Parameters

|          |                                                                        |
|----------|------------------------------------------------------------------------|
| <i>a</i> | An array of dimension $(lda, n)$ to receive the incoming message into. |
|----------|------------------------------------------------------------------------|

## Description

This routine receives and participates in a broadcast along a scope. At the end of a broadcast, all processes within the scope will possess the data in the trapezoidal matrix  $A$ . Broadcasts may be globally-blocking. This means no process is guaranteed to return from a broadcast until all processes in the scope have called the appropriate routine (broadcast/send or broadcast/receive).



## blacs\_setup

*Allocates virtual machine and spawns processes.*

---

### Syntax

```
call blacs_setup(mypnum, nprocs)
```

### Input Parameters

*nprocs* INTEGER. On the process spawned from the keyboard rather than from `pvmspawn`, this parameter indicates the number of processes to create when building the virtual machine.

### Output Parameters

*mypnum* INTEGER. An integer between 0 and (*nprocs* - 1) that uniquely identifies each process.

*nprocs* INTEGER. For all processes other than spawned from the keyboard, this parameter means the number of processes available for BLACS use.

### Description

This routine only accomplishes meaningful work in the PVM BLACS. On all other platforms, it is functionally equivalent to `blacs_pinfo`. The BLACS assume a static system, that is, the given number of processes does not change. PVM supplies a dynamic system, allowing processes to be added to the system on the fly.

`blacs_setup` is used to allocate the virtual machine and spawn off processes. It reads in a file called `blacs_setup.dat`, in which the first line must be the name of your executable. The second line is optional, but if it exists, it should be a PVM spawn flag. Legal values at this time are 0 (`PvmTaskDefault`), 4 (`PvmTaskDebug`), 8 (`PvmTaskTrace`), and 12 (`PvmTaskDebug + PvmTaskTrace`). The primary reason for this line is to allow the user to easily turn on and off PVM debugging. Additional lines, if any, specify what machines should be added to the current configuration before spawning *nprocs*-1 processes to the machines in a round robin fashion.

*nprocs* is input on the process which has no PVM parent (that is, *mypnum*=0), and both parameters are output for all processes. So, on PVM systems, the call to `blacs_pinfo` informs you that the virtual machine has not been set up, and a call to `blacs_setup` then sets up the machine and returns the real values for *mypnum* and *nprocs*.

Note that if the file `blacs_setup.dat` does not exist, the BLACS prompt the user for the executable name, and processes are spawned to the current PVM configuration.

### See Also

[Examples of BLACS Routines Usage](#)

## blacs\_get

*Gets values that BLACS use for internal defaults.*

---

### Syntax

```
call blacs_get(icontxt, what, val)
```

### Input Parameters

*icontxt* INTEGER. On values of *what* that are tied to a particular context, this parameter is the integer handle indicating the context. Otherwise, ignored.

*what* INTEGER. Indicates what BLACS internal(s) should be returned in *val*. Present options are:

- *what* = 0 : Handle indicating default system context.
- *what* = 1 : The BLACS message ID range.
- *what* = 2 : The BLACS debug level the library was compiled with.
- *what* = 10 : Handle indicating the system context used to define the BLACS context whose handle is *icontxt*.
- *what* = 11 : Number of rings multiring broadcast topology is presently using.
- *what* = 12 : Number of branches general tree broadcast topology is presently using.
- *what* = 13 : Number of rings multiring combine topology is presently using.
- *what* = 14 : Number of branches general tree combine topology is presently using.
- *what* = 15 : Whether topologies are forced to be repeatable or not. A non-zero return value indicates that topologies are being forced to be repeatable. See [Repeatability and Coherence](#) for more information about repeatability.
- *what* = 16 : Whether topologies are forced to be heterogenous coherent or not. A non-zero return value indicates that topologies are being forced to be heterogenous coherent. See [Repeatability and Coherence](#) for more information about coherence.

## Output Parameters

*val* INTEGER. The value of the BLACS internal.

## Description

This routine gets the values that the BLACS are using for internal defaults. Some values are tied to a BLACS context, and some are more general. The most common use is in retrieving a default system context for input into `blacs_gridinit` or `blacs_gridmap`.

Some systems, such as MPI\*, supply their own version of context. For those users who mix system code with BLACS code, a BLACS context should be formed in reference to a system context. Thus, the grid creation routines take a system context as input. If you wish to have strictly portable code, you may use `blacs_get` to retrieve a default system context that will include all available processes. This value is not tied to a BLACS context, so the parameter *icontxt* is unused.

`blacs_get` returns information on three quantities that are tied to an individual BLACS context, which is passed in as *icontxt*. The information that may be retrieved is:

- The handle of the system context upon which this BLACS context was defined
- The number of rings for TOP = 'M' (multiring broadcast/combine)
- The number of branches for TOP = 'T' (general tree broadcast/general tree gather).
- Whether topologies are being forced to be repeatable or heterogenous coherent.

## See Also

[Examples of BLACS Routines Usage](#)

## `blacs_set`

*Sets values that BLACS use for internal defaults.*

---

## Syntax

call `blacs_set( ictxt, what, val )`

## Input Parameters

|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ictxt</i> | INTEGER. For values of <i>what</i> that are tied to a particular context, this parameter is the integer handle indicating the context. Otherwise, ignored.                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| <i>what</i>  | <p>INTEGER. Indicates what BLACS internal(s) should be set. Present values are:</p> <ul style="list-style-type: none"> <li>• 1 = Set the BLACS message ID range</li> <li>• 11 = Number of rings for multiring broadcast topology to use</li> <li>• 12 = Number of branches for general tree broadcast topology to use</li> <li>• 13 = Number of rings for multiring combine topology to use</li> <li>• 14 = Number of branches for general tree combine topology to use</li> <li>• 15 = Force topologies to be repeatable or not</li> <li>• 16 = Force topologies to be heterogenous coherent or not</li> </ul> |
| <i>val</i>   | INTEGER. Array of dimension (*). Indicates the value(s) the internals should be set to. The specific meanings depend on <i>what</i> values, as discussed in Description below.                                                                                                                                                                                                                                                                                                                                                                                                                                  |

## Description

This routine sets the BLACS internal defaults depending on *what* values:

|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>what</i> = 1  | <p>Setting the BLACS message ID range.</p> <p>If you wish to mix the BLACS with other message-passing packages, restrict the BLACS to a certain message ID range not to be used by the non-BLACS routines. The message ID range must be set before the first call to <code>blacs_gridinit</code> or <code>blacs_gridmap</code>. Subsequent calls will have no effect. Because the message ID range is not tied to a particular context, the parameter <i>ictxt</i> is ignored, and <i>val</i> is defined as:</p> <p>VAL (input) INTEGER array of dimension (2)</p> <p>VAL(1) : The smallest message ID (also called message type or message tag) the BLACS should use.</p> <p>VAL(2) : The largest message ID (also called message type or message tag) the BLACS should use.</p> |
| <i>what</i> = 11 | <p>Set number of rings for TOP = 'M' (multiring broadcast). This quantity is tied to a context, so <i>ictxt</i> is used, and <i>val</i> is defined as:</p> <p>VAL (input) INTEGER array of dimension (1)</p> <p>VAL(1) : The number of rings for multiring topology to use.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <i>what</i> = 12 | <p>Set number of branches for TOP = 'T' (general tree broadcast). This quantity is tied to a context, so <i>ictxt</i> is used, and <i>val</i> is defined as:</p> <p>VAL (input) INTEGER array of dimension (1)</p> <p>VAL(1) : The number of branches for general tree topology to use.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |

|                  |                                                                                                                                                                                                                                                                                                                                                                                                                                |
|------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>what</i> = 13 | <p>Set number of rings for TOP = 'M' (multiring combine). This quantity is tied to a context, so <i>icontxt</i> is used, and <i>val</i> is defined as:</p> <p>VAL (input) INTEGER array of dimension (1)</p> <p>VAL(1) : The number of rings for multiring topology to use.</p>                                                                                                                                                |
| <i>what</i> = 14 | <p>Set number of branches for TOP = 'T' (general tree gather). This quantity is tied to a context, so <i>icontxt</i> is used, and <i>val</i> is defined as:</p> <p>VAL (input) INTEGER array of dimension (1)</p> <p>VAL(1) : The number of branches for general tree topology to use.</p>                                                                                                                                     |
| <i>what</i> = 15 | <p>Force topologies to be repeatable or not (see <a href="#">Repeatability and Coherence</a> for more information about repeatability).</p> <p>VAL (input) INTEGER array of dimension (1)</p> <p>VAL(1) = 0 (default)      Topologies are not required to be repeatable.</p> <p>VAL(1) ≠ 0                All used topologies are required to be repeatable, which might degrade performance.</p>                              |
| <i>what</i> = 16 | <p>Force topologies to be heterogenous coherent or not (see <a href="#">Repeatability and Coherence</a> for more information about coherence).</p> <p>VAL (input) INTEGER array of dimension (1)</p> <p>VAL(1) = 0 (default)      Topologies are not required to be heterogenous coherent.</p> <p>VAL(1) ≠ 0                All used topologies are required to be heterogenous coherent, which might degrade performance.</p> |

## blacs\_gridinit

Assigns available processes into BLACS process grid.

### Syntax

call blacs\_gridinit( *icontxt*, *layout*, *nprow*, *npcol* )

### Input Parameters

|                |                                                                                                                                                                                                                                                                             |
|----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>icontxt</i> | <p>INTEGER. Integer handle indicating the system context to be used in creating the BLACS context. Call <code>blacs_get</code> to obtain a default system context.</p>                                                                                                      |
| <i>layout</i>  | <p>CHARACTER*1. Indicates how to map processes to BLACS grid. Options are:</p> <ul style="list-style-type: none"> <li>• 'R' : Use row-major natural ordering</li> <li>• 'C' : Use column-major natural ordering</li> <li>• ELSE : Use row-major natural ordering</li> </ul> |
| <i>nprow</i>   | <p>INTEGER. Indicates how many process rows the process grid should contain.</p>                                                                                                                                                                                            |

*npcol* INTEGER. Indicates how many process columns the process grid should contain.

## Output Parameters

*icontxt* INTEGER. Integer handle to the created BLACS context.

## Description

All BLACS codes must call this routine, or its sister routine `blacs_gridmap`. These routines take the available processes, and assign, or map, them into a BLACS process grid. In other words, they establish how the BLACS coordinate system maps into the native machine process numbering system. Each BLACS grid is contained in a context, so that it does not interfere with distributed operations that occur within other grids/contexts. These grid creation routines may be called repeatedly to define additional contexts/grids.

The creation of a grid requires input from all processes that are defined to be in this grid. Processes belonging to more than one grid have to agree on which grid formation will be serviced first, much like the globally blocking sum or broadcast.

These grid creation routines set up various internals for the BLACS, and one of them must be called before any calls are made to the non-initialization BLACS.

Note that these routines map already existing processes to a grid: the processes are not created dynamically. On most parallel machines, the processes are "created" when you run your executable. When using the PVM BLACS, if the virtual machine has not been set up yet, the routine `blacs_setup` should be used to create the virtual machine.

This routine creates a simple  $nprow \times npcold$  process grid. This process grid uses the first  $nprow * npcold$  processes, and assigns them to the grid in a row- or column-major natural ordering. If these process-to-grid mappings are unacceptable, call `blacs_gridmap`.

## See Also

[Examples of BLACS Routines Usage](#)

[blacs\\_get](#)

[blacs\\_gridmap](#)

[blacs\\_setup](#)

## **blacs\_gridmap**

*Maps available processes into BLACS process grid.*

---

## Syntax

```
call blacs_gridmap(icontxt, usermap, ldumap, nprow, npcold)
```

## Input Parameters

*icontxt* INTEGER. Integer handle indicating the system context to be used in creating the BLACS context. Call `blacs_get` to obtain a default system context.

*usermap* INTEGER. Array, dimension  $(ldumap, npcold)$ , indicating the process-to-grid mapping.

*ldumap* INTEGER. Leading dimension of the 2D array *usermap*.  $ldumap \geq nprow$ .

*nprow* INTEGER. Indicates how many process rows the process grid should contain.

*npcol* INTEGER. Indicates how many process columns the process grid should contain.

## Output Parameters

*icontxt* INTEGER. Integer handle to the created BLACS context.

## Description

All BLACS codes must call this routine, or its sister routine `blacs_gridinit`. These routines take the available processes, and assign, or map, them into a BLACS process grid. In other words, they establish how the BLACS coordinate system maps into the native machine process numbering system. Each BLACS grid is contained in a context, so that it does not interfere with distributed operations that occur within other grids/contexts. These grid creation routines may be called repeatedly to define additional contexts/grids.

The creation of a grid requires input from all processes that are defined to be in this grid. Processes belonging to more than one grid have to agree on which grid formation will be serviced first, much like the globally blocking sum or broadcast.

These grid creation routines set up various internals for the BLACS, and one of them must be called before any calls are made to the non-initialization BLACS.

Note that these routines map already existing processes to a grid: the processes are not created dynamically. On most parallel machines, the processes are actual processors (hardware), and they are "created" when you run your executable. When using the PVM BLACS, if the virtual machine has not been set up yet, the routine `blacs_setup` should be used to create the virtual machine.

This routine allows the user to map processes to the process grid in an arbitrary manner. `usermap(i,j)` holds the process number of the process to be placed in  $\{i, j\}$  of the process grid. On most distributed systems, this process number is a machine defined number between  $0 \dots nprow-1$ . For PVM, these node numbers are the PVM TIDS (Task IDs). The `blacs_gridmap` routine is intended for an experienced user. The `blacs_gridinit` routine is much simpler. `blacs_gridinit` simply performs a `gridmap` where the first  $nprow * npcold$  processes are mapped into the current grid in a row-major natural ordering. If you are an experienced user, `blacs_gridmap` allows you to take advantage of your system's actual layout. That is, you can map nodes that are physically connected to be neighbors in the BLACS grid, etc. The `blacs_gridmap` routine also opens the way for *multigridding*: you can separate your nodes into arbitrary grids, join them together at some later date, and then re-split them into new grids. `blacs_gridmap` also provides the ability to make arbitrary grids or subgrids (for example, a "nearest neighbor" grid), which can greatly facilitate operations among processes that do not fall on a row or column of the main process grid.

## See Also

[Examples of BLACS Routines Usage](#)

[blacs\\_get](#)

[blacs\\_gridinit](#)

[blacs\\_setup](#)

## Destruction Routines

This section describes BLACS routines that destroy grids, abort processes, and free resources.

### BLACS Destruction Routines

| Routine name                   | Operation performed    |
|--------------------------------|------------------------|
| <a href="#">blacs_freebuff</a> | Frees BLACS buffer.    |
| <a href="#">blacs_gridexit</a> | Frees a BLACS context. |

| Routine name             | Operation performed                                         |
|--------------------------|-------------------------------------------------------------|
| <code>blacs_abort</code> | Aborts all processes.                                       |
| <code>blacs_exit</code>  | Frees all BLACS contexts and releases all allocated memory. |

## **blacs\_freebuff**

*Frees BLACS buffer.*

### **Syntax**

call `blacs_freebuff( ictxt, wait )`

### **Input Parameters**

|                    |                                                                                                                                                                                                                 |
|--------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ictxt</code> | INTEGER. Integer handle that indicates the BLACS context.                                                                                                                                                       |
| <code>wait</code>  | INTEGER. Parameter indicating whether to wait for non-blocking operations or not. If equals 0, the operations should not be waited for; free only unused buffers. Otherwise, wait in order to free all buffers. |

### **Description**

This routine releases the BLACS buffer.

The BLACS have at least one internal buffer that is used for packing messages. The number of internal buffers depends on what platform you are running the BLACS on. On systems where memory is tight, keeping this buffer or buffers may become expensive. Call `freebuff` to release the buffer. However, the next call of a communication routine that requires packing reallocates the buffer.

The `wait` parameter determines whether the BLACS should wait for any non-blocking operations to be completed or not. If `wait = 0`, the BLACS free any buffers that can be freed without waiting. If `wait` is not 0, the BLACS free all internal buffers, even if non-blocking operations must be completed first.

## **blacs\_gridexit**

*Frees a BLACS context.*

### **Syntax**

call `blacs_gridexit( ictxt )`

### **Input Parameters**

|                    |                                                                       |
|--------------------|-----------------------------------------------------------------------|
| <code>ictxt</code> | INTEGER. Integer handle that indicates the BLACS context to be freed. |
|--------------------|-----------------------------------------------------------------------|

### **Description**

This routine frees a BLACS context.

Release the resources when contexts are no longer needed. After freeing a context, the context no longer exists, and its handle may be re-used if new contexts are defined.

## **blacs\_abort**

*Aborts all processes.*

### **Syntax**

call `blacs_abort( ictxt, errornum )`

## Input Parameters

|                 |                                                                         |
|-----------------|-------------------------------------------------------------------------|
| <i>icontxt</i>  | INTEGER. Integer handle that indicates the BLACS context to be aborted. |
| <i>errornum</i> | INTEGER. User-defined integer error number.                             |

## Description

This routine aborts all the BLACS processes, not only those confined to a particular context.

Use `blacs_abort` to abort all the processes in case of a serious error. Note that both parameters are input, but the routine uses them only in printing out the error message. The context handle passed in is not required to be a valid context handle.

## `blacs_exit`

*Frees all BLACS contexts and releases all allocated memory.*

---

## Syntax

```
call blacs_exit(continue)
```

## Input Parameters

|                 |                                                                                                                                                                                                                                                                         |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>continue</i> | INTEGER. Flag indicating whether message passing continues after the BLACS are done. If <i>continue</i> is non-zero, the user is assumed to continue using the machine after completing the BLACS. Otherwise, no message passing is assumed after calling this routine. |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

## Description

This routine frees all BLACS contexts and releases all allocated memory.

This routine should be called when a process has finished all use of the BLACS. The *continue* parameter indicates whether the user will be using the underlying communication platform after the BLACS are finished. This information is most important for the PVM BLACS. If *continue* is set to 0, then `pvm_exit` is called; otherwise, it is not called. Setting *continue* not equal to 0 indicates that explicit PVM `send/recvs` will be called after the BLACS routines are used. Make sure your code calls `pvm_exit`. PVM users should either call `blacs_exit` or explicitly call `pvm_exit` to avoid PVM problems.

## See Also

[Examples of BLACS Routines Usage](#)

## Informational Routines

This section describes BLACS routines that return information involving the process grid.

### BLACS Informational Routines

---

| Routine name                | Operation performed                                                   |
|-----------------------------|-----------------------------------------------------------------------|
| <code>blacs_gridinfo</code> | Returns information on the current grid.                              |
| <code>blacs_pnum</code>     | Returns the system process number of the process in the process grid. |
| <code>blacs_pcoord</code>   | Returns the row and column coordinates in the process grid.           |

---



## Syntax

```
call blacs_pcoord(icontxt, pnum, prow, pcol)
```

## Input Parameters

*icontxt*                                    INTEGER. Integer handle that indicates the context.

*pnum*                                        INTEGER. Process number the coordinates of which are to be determined. This parameter stand for the process number of the underlying machine, that is, it is a `tid` for PVM.

## Output Parameters

*prow*                                        INTEGER. Row coordinates of the *pnum* process in the BLACS grid.

*pcol*                                        INTEGER. Column coordinates of the *pnum* process in the BLACS grid.

## Description

Given the system process number, this function returns the row and column coordinates in the BLACS process grid.

## See Also

Examples of BLACS Routines Usage

## Miscellaneous Routines

This section describes `blacs_barrier` routine.

### BLACS Informational Routines

| Routine name               | Operation performed                                                                                    |
|----------------------------|--------------------------------------------------------------------------------------------------------|
| <code>blacs_barrier</code> | Holds up execution of all processes within the indicated scope until they have all called the routine. |

## `blacs_barrier`

*Holds up execution of all processes within the indicated scope.*

## Syntax

```
call blacs_barrier(icontxt, scope)
```

## Input Parameters

*icontxt*                                    INTEGER. Integer handle that indicates the context.

*scope*                                        CHARACTER\*1. Parameter that indicates whether a process row (*scope*='R'), column ('C'), or entire grid ('A') will participate in the barrier.

## Description

This routine holds up execution of all processes within the indicated scope until they have all called the routine.

## **Examples of BLACS Routines Usage**

---

# Data Fitting Functions

Data Fitting functions in Intel® MKL provide spline-based interpolation capabilities that you can use to approximate functions, function derivatives or integrals, and perform cell search operations.

The Data Fitting component is task based. The task is a data structure or descriptor that holds the parameters related to a specific Data Fitting operation. You can modify the task parameters using the task editing functionality of the library.

For definition of the implemented operations, see [Mathematical Conventions](#).

Data Fitting routines use the following workflow to process a task:

1. Create a task or multiple tasks.
2. Modify the task parameters.
3. Perform a Data Fitting computation.
4. Destroy the task or tasks.

All Data Fitting functions fall into the following categories:

[Task Creation and Initialization Routines](#) - routines that create a new Data Fitting task descriptor and initialize the most common parameters, such as partition of the interpolation interval, values of the vector-valued function, and the parameters describing their structure.

[Task Configuration Routines](#) - routines that set, modify, or query parameters in an existing Data Fitting task.

[Computational Routines](#) - routines that perform Data Fitting computations, such as construction of a spline, interpolation, computation of derivatives and integrals, and search.

[Task Destructors](#) - routines that delete Data Fitting task descriptors and deallocate resources.

You can access the Data Fitting routines through the Fortran and C89/C99 language interfaces. You can also use the C89 interface with more recent versions of C/C++, or the Fortran 90 interface with programs written in Fortran 95.

The `$(MKL)/include` directory of the Intel® MKL contains the following Data Fitting header files:

- `mkl_df.h`

You can find examples that demonstrate usage of Data Fitting routines in the `$(MKL)/examples/datafittingc` directory .

## Data Fitting Function Naming Conventions

The interface of the Data Fitting functions, types, and constants are case-sensitive and can be in lowercase, uppercase, and mixed case.

The names of all routines have the following structure:

```
df[datatype]<base_name>
```

where

- `df` is a prefix indicating that the routine belongs to the Data Fitting component of Intel MKL.
- `[datatype]` field specifies the type of the input and/or output data and can be `s` (for the single precision real type), `d` (for the double precision real type), or `i` (for the integer type). This field is omitted in the names of the routines that are not data type dependent.
- `<base_name>` field specifies the functionality the routine performs. For example, this field can be `NewTask1D`, `Interpolate1D`, or `DeleteTask`

## Data Fitting Function Data Types

The Data Fitting component provides routines for processing single and double precision real data types. The results of cell search operations are returned as a generic integer data type.

All Data Fitting routines use the following data type:

| Type      | Data Object       |
|-----------|-------------------|
| DFTaskPtr | Pointer to a task |

**NOTE**

The actual size of the generic integer type is platform-dependent. Before compiling your application, you need to set an appropriate byte size for integers. For details, see section *Using the ILP64 Interface vs. LP64 Interface* of the Intel® MKL Developer Guide.

## Mathematical Conventions for Data Fitting Functions

This section explains the notation used for Data Fitting function descriptions. Spline notations are based on the terminology and definitions of [deBoor2001]. The Subbotin quadratic spline definition follows the conventions of [StechSub76]. The quasi-uniform partition definition is based on [Schumaker2007].

### Mathematical Notation in the Data Fitting Component

| Concept                                                                                                                                                                                                                                                        | Mathematical Notation                                                                                                                                                                                                                                                                                                                                                                                                          |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Partition of interpolation interval $[a, b]$ , where <ul style="list-style-type: none"> <li><math>x_i</math> denotes breakpoints.</li> <li><math>[x_i, x_{i+1})</math> denotes a sub-interval (cell) of size <math>\Delta_i = x_{i+1} - x_i</math>.</li> </ul> | $\{x_i\}_{i=1, \dots, n}$ , where $a = x_1 < x_2 < \dots < x_n = b$                                                                                                                                                                                                                                                                                                                                                            |
| Quasi-uniform partition of interpolation interval $[a, b]$                                                                                                                                                                                                     | Partition $\{x_i\}_{i=1, \dots, n}$ which meets the constraint with a constant $C$ defined as $1 \leq M / m \leq C,$ where <ul style="list-style-type: none"> <li><math>M = \max_{i=1, \dots, n-1} (\Delta_i)</math></li> <li><math>m = \min_{i=1, \dots, n-1} (\Delta_i)</math></li> <li><math>\Delta_i = x_{i+1} - x_i</math></li> </ul>                                                                                     |
| Vector-valued function of dimension $p$ being fit                                                                                                                                                                                                              | $f(x) = (f_1(x), \dots, f_p(x))$                                                                                                                                                                                                                                                                                                                                                                                               |
| Piecewise polynomial (PP) function $f$ of order $k+1$                                                                                                                                                                                                          | $f(x) = P_i(x)$ , if $x \in [x_i, x_{i+1})$ , $i = 1, \dots, n-1$ where <ul style="list-style-type: none"> <li><math>\{x_i\}_{i=1, \dots, n}</math> is a strictly increasing sequence of breakpoints.</li> <li><math>P_i(x) = c_{i,0} + c_{i,1}(x - x_i) + \dots + c_{i,k}(x - x_i)^k</math> is a polynomial of degree <math>k</math> (order <math>k+1</math>) over the interval <math>x \in [x_i, x_{i+1})</math>.</li> </ul> |
| Function $p$ agrees with function $f$ at the points $\{x_i\}_{i=1, \dots, n}$ .                                                                                                                                                                                | For every point $\zeta$ in sequence $\{x_i\}_{i=1, \dots, n}$ that occurs $m$ times, the equality $p^{(i-1)}(\zeta) = f^{(i-1)}(\zeta)$ holds for all $i = 1, \dots, m$ , where $p^{(i)}(t)$ is the derivative of the $i$ -th order.                                                                                                                                                                                           |

| Concept                                                                                                                                                                                                      | Mathematical Notation                                                                                                                                                                            |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| The $k$ -th divided difference of function $f$ at points $x_i, \dots, x_{i+k}$ . This difference is the leading coefficient of the polynomial of order $k+1$ that agrees with $f$ at $x_i, \dots, x_{i+k}$ . | $[x_i, \dots, x_{i+k}] f$<br>In particular, <ul style="list-style-type: none"> <li><math>[x_1] f = f(x_1)</math></li> <li><math>[x_1, x_2] f = (f(x_1) - f(x_2)) / (x_1 - x_2)</math></li> </ul> |
| A $k$ -order derivative of interpolant $f(x)$ at interpolation site $\tau$ .                                                                                                                                 | $f^{(k)}(\tau)$                                                                                                                                                                                  |

**Interpolants to the Function  $f$  at  $x_1, \dots, x_n$  and Boundary Conditions**

| Concept                                  | Mathematical Notation                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Linear interpolant                       | $P_i(x) = c_{1,i} + c_{2,i}(x - x_i)$ ,<br>where <ul style="list-style-type: none"> <li><math>x \in [x_i, x_{i+1})</math></li> <li><math>c_{1,i} = f(x_i)</math></li> <li><math>c_{2,i} = [x_i, x_{i+1}] f</math></li> <li><math>i = 1, \dots, n-1</math></li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| Piecewise parabolic interpolant          | $P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2, x \in [x_i, x_{i+1})$<br>Coefficients $c_{1,i}$ , $c_{2,i}$ , and $c_{3,i}$ depend on the conditions: <ul style="list-style-type: none"> <li><math>P_i(x_i) = f(x_i)</math></li> <li><math>P_i(x_{i+1}) = f(x_{i+1})</math></li> <li><math>P_i((x_{i+1} + x_i) / 2) = v_{i+1}</math></li> </ul> where parameter $v_{i+1}$ depends on the interpolant being continuously differentiable:<br>$P_{i-1}^{(1)}(x_i) = P_i^{(1)}(x_i)$                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| Piecewise parabolic Subbotin interpolant | $P(x) = P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + d_{3,i}((x - t_i)_+)^2$ ,<br>where <ul style="list-style-type: none"> <li><math>x \in [t_i, t_{i+1})</math></li> <li><math>\{t_i\}_{i=1, \dots, n+1}</math> is a sequence of knots such that                             <ul style="list-style-type: none"> <li><math>t_1 = x_1, t_{n+1} = x_n</math></li> <li><math>t_i \in (x_{i-1}, x_i), i = 2, \dots, n</math></li> </ul> </li> <li><math>x_+ = f(x) = \begin{cases} 0, &amp; \text{if } x &lt; 0 \\ x, &amp; \text{if } x \geq 0 \end{cases}</math></li> </ul> Coefficients $c_{1,i}$ , $c_{2,i}$ , $c_{3,i}$ , and $d_{3,i}$ depend on the following conditions: <ul style="list-style-type: none"> <li><math>P_i(x_i) = f(x_i), P_i(x_{i+1}) = f(x_{i+1})</math></li> <li><math>P(x)</math> is a continuously differentiable polynomial of the second degree on <math>[t_i, t_{i+1}), i = 1, \dots, n</math>.</li> </ul> |
| Piecewise cubic Hermite interpolant      | $P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3$ ,<br>where <ul style="list-style-type: none"> <li><math>x \in [x_i, x_{i+1})</math></li> <li><math>c_{1,i} = f(x_i)</math></li> <li><math>c_{2,i} = s_i</math></li> <li><math>c_{3,i} = ([x_i, x_{i+1}] f - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)</math></li> <li><math>c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}] f) / (\Delta x_i)^2</math></li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

| Concept                                       | Mathematical Notation                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-----------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                               | <ul style="list-style-type: none"> <li><math>i = 1, \dots, n-1</math></li> <li><math>s_i = f^{(1)}(x_i)</math></li> </ul>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Piecewise cubic Bessel interpolant            | $P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3,$ <p>where</p> <ul style="list-style-type: none"> <li><math>x \in [x_i, x_{i+1})</math></li> <li><math>c_{1,i} = f(x_i)</math></li> <li><math>c_{2,i} = s_i</math></li> <li><math>c_{3,i} = ([x_i, x_{i+1}]f - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)</math></li> <li><math>c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}]f) / (\Delta x_i)^2</math></li> <li><math>i = 1, \dots, n-1</math></li> <li><math>s_i = (\Delta x_i[x_{i-1}, x_i]f + \Delta x_{i-1}[x_i, x_{i+1}]f) / (\Delta x_i + \Delta x_{i+1})</math></li> </ul>                                                       |
| Piecewise cubic co-monotone Hyman interpolant | $P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3,$ <p>where</p> <ul style="list-style-type: none"> <li><math>x \in [x_i, x_{i+1})</math></li> <li><math>c_{1,i} = f(x_i)</math></li> <li><math>c_{2,i} = s_i</math></li> <li><math>c_{3,i} = ([x_i, x_{i+1}]f - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)</math></li> <li><math>c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}]f) / (\Delta x_i)^2</math></li> <li><math>i = 1, \dots, n-1</math></li> <li><math>s_i</math> is set similarly to the Bessel cubic spline, but modified to preserve the monotonicity of the input data on the current interpolation sub-interval.</li> </ul> |
| Piecewise cubic Akima interpolant             | $P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3,$ <p>where</p> <ul style="list-style-type: none"> <li><math>x \in [x_i, x_{i+1})</math></li> <li><math>c_{1,i} = f(x_i)</math></li> <li><math>c_{2,i} = s_i</math></li> <li><math>c_{3,i} = ([x_i, x_{i+1}]f - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)</math></li> <li><math>c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}]f) / (\Delta x_i)^2</math></li> <li><math>i = 1, \dots, n-1</math></li> <li><math>s_i = (w_{i+1}[x_{i-1}, x_i]f + w_{i-1}[x_i, x_{i+1}]f) / (w_{i+1} + w_{i-1}),</math></li> </ul> <p>where</p> $w_i =  [x_i, x_{i+1}]f - [x_{i-1}, x_i]f $                 |
| Piecewise natural cubic interpolant           | $P_i(x) = c_{1,i} + c_{2,i}(x - x_i) + c_{3,i}(x - x_i)^2 + c_{4,i}(x - x_i)^3,$ <p>where</p> <ul style="list-style-type: none"> <li><math>x \in [x_i, x_{i+1})</math></li> <li><math>c_{1,i} = f(x_i)</math></li> <li><math>c_{2,i} = s_i</math></li> <li><math>c_{3,i} = ([x_i, x_{i+1}]f - s_i) / (\Delta x_i) - c_{4,i}(\Delta x_i)</math></li> <li><math>c_{4,i} = (s_i + s_{i+1} - 2[x_i, x_{i+1}]f) / (\Delta x_i)^2</math></li> <li><math>i = 1, \dots, n-1</math></li> <li>Parameter <math>s_i</math> depends on the condition that the interpolant is twice continuously differentiable: <math>P_{i-1}^{(2)}(x_i) = P_i^{(2)}(x_i)</math>.</li> </ul>     |
| Not-a-knot boundary condition.                | Parameters $s_1$ and $s_n$ provide $P_1 = P_2$ and $P_{n-1} = P_n$ , so that the first and the last interior breakpoints are inactive.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |

| Concept                                                                           | Mathematical Notation                                                                                                                                                                             |
|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Free-end boundary condition.                                                      | $f''(x_1) = f''(x_n) = 0$                                                                                                                                                                         |
| Look-up interpolator for discrete set of points $(x_1, y_1), \dots, (x_n, y_n)$ . | $y(x) = \begin{cases} y_1, & \text{if } x = x_1 \\ y_2, & \text{if } x = x_2 \\ \dots & \\ y_n, & \text{if } x = x_n \\ \text{error,} & \text{otherwise} \end{cases}$                             |
| Step-wise constant continuous right interpolator.                                 | $y(x) = \begin{cases} y_1, & \text{if } x_1 \leq x < x_2 \\ y_2, & \text{if } x_2 \leq x < x_3 \\ \dots & \\ y_{n-1}, & \text{if } x_{n-1} \leq x < x_n \\ y_n, & \text{if } x = x_n \end{cases}$ |
| Step-wise constant continuous left interpolator.                                  | $y(x) = \begin{cases} y_1, & \text{if } x = x_1 \\ y_2, & \text{if } x_1 < x \leq x_2 \\ y_3, & \text{if } x_2 < x \leq x_3 \\ \dots & \\ y_n, & \text{if } x_{n-1} < x \leq x_n \end{cases}$     |

## Data Fitting Usage Model

Consider an algorithm that uses the Data Fitting functions. Typically, such algorithms consist of four steps or stages:

1. Create a task. You can call the Data Fitting function several times to create multiple tasks.

```
status = dfdNewTask1D(&task, nx, x, xhint, ny, y, yhint);
```

2. Modify the task parameters.

```
status = dfdEditPPSpline1D(task, s_order, c_type, bc_type, bc, ic_type, ic,
scoeff, scoeffhint);
```

3. Perform Data Fitting spline-based computations. You may reiterate steps 2-3 as needed.

```
status = dfdInterpolate1D(task, estimate, method, nsite, site, sitehint, ndorder,
dorder, datahint, r, rhint, cell);
```

4. Destroy the task or tasks.

```
status = dfDeleteTask(&task);
```

### See Also

[Data Fitting Usage Examples](#)

## Data Fitting Usage Examples

The examples below illustrate several operations that you can perform with Data Fitting routines.

You can get source code for similar examples in the `.\examples\datafittingc` subdirectory of the Intel MKL installation directory.

The following example demonstrates the construction of a linear spline using Data Fitting routines. The spline approximates a scalar function defined on non-uniform partition. The coefficients of the spline are returned as a one-dimensional array:

### Example of Linear Spline Construction

```
#include "mkl.h"
#define N 500 /* Size of partition, number of breakpoints */
#define SPLINE_ORDER DF_PP_LINEAR /* Linear spline to construct */

int main()
{
 int status; /* Status of a Data Fitting operation */
 DFTaskPtr task; /* Data Fitting operations are task based */

 /* Parameters describing the partition */
 MKL_INT nx; /* The size of partition x */
 double x[N]; /* Partition x */
 MKL_INT xhint; /* Additional information about the structure of breakpoints */

 /* Parameters describing the function */
 MKL_INT ny; /* Function dimension */
 double y[N]; /* Function values at the breakpoints */
 MKL_INT yhint; /* Additional information about the function */

 /* Parameters describing the spline */
 MKL_INT s_order; /* Spline order */
 MKL_INT s_type; /* Spline type */
 MKL_INT ic_type; /* Type of internal conditions */
 double* ic; /* Array of internal conditions */
 MKL_INT bc_type; /* Type of boundary conditions */
 double* bc; /* Array of boundary conditions */

 double scoeff[(N-1)*SPLINE_ORDER]; /* Array of spline coefficients */
 MKL_INT scoeffhint; /* Additional information about the coefficients */

 /* Initialize the partition */
 nx = N;
 /* Set values of partition x */
 ...
 xhint = DF_NO_HINT; /* No additional information about the function is provided.
 By default, the partition is non-uniform. */
 /* Initialize the function */
 ny = 1; /* The function is scalar. */

 /* Set function values */
 ...
 yhint = DF_NO_HINT; /* No additional information about the function is provided. */

 /* Create a Data Fitting task */
 status = dfdNewTask1D(&task, nx, x, xhint, ny, y, yhint);

 /* Check the Data Fitting operation status */
 ...

 /* Initialize spline parameters */
 s_order = DF_PP_LINEAR; /* Spline is of the second order. */
 s_type = DF_PP_DEFAULT; /* Spline is of the default type. */

 /* Define internal conditions for linear spline construction (none in this example) */
 ic_type = DF_NO_IC;
```

```

ic = NULL;

/* Define boundary conditions for linear spline construction (none in this example) */
bc_type = DF_NO_BC;
bc = NULL;
scoeffhint = DF_NO_HINT; /* No additional information about the spline. */

/* Set spline parameters in the Data Fitting task */
status = dfdEditPPSpline1D(task, s_order, s_type, bc_type, bc, ic_type,
 ic, scoeff, scoeffhint);

/* Check the Data Fitting operation status */
...

/* Use a standard computation method to construct a linear spline: */
/* $P_i(x) = c_{i,0} + c_{i,1}(x - x_i)$, $i=0, \dots, N-2$ */
/* The library packs spline coefficients to array scoeff. */
/* $scoeff[2*i+0]=c_{i,0}$ and $scoeff[2*i+1]=c_{i,1}$, $i=0, \dots, N-2$ */
status = dfdConstruct1D(task, DF_PP_SPLINE, DF_METHOD_STD);

/* Check the Data Fitting operation status */
...

/* Process spline coefficients */
...

/* Deallocate Data Fitting task resources */
status = dfDeleteTask(&task);

/* Check the Data Fitting operation status */
...
return 0 ;
}

```

The following example demonstrates cubic spline-based interpolation using Data Fitting routines. In this example, a scalar function defined on non-uniform partition is approximated by Bessel cubic spline using not-a-knot boundary conditions. Once the spline is constructed, you can use the spline to compute spline values at the given sites. Computation results are packed by the Data Fitting routine in row-major format.

### Example of Cubic Spline-Based Interpolation

```

#include "mkl.h"

#define NX 100 /* Size of partition, number of breakpoints */
#define NSITE 1000 /* Number of interpolation sites */
#define SPLINE_ORDER DF_PP_CUBIC /* A cubic spline to construct */

int main()
{
 int status; /* Status of a Data Fitting operation */
 DFTaskPtr task; /* Data Fitting operations are task based */

 /* Parameters describing the partition */
 MKL_INT nx; /* The size of partition x */
 double x[NX]; /* Partition x */
 MKL_INT xhint; /* Additional information about the structure of breakpoints */

 /* Parameters describing the function */
 MKL_INT ny; /* Function dimension */
 double y[NX]; /* Function values at the breakpoints */
}

```

```

MKL_INT yhint; /* Additional information about the function */

/* Parameters describing the spline */
MKL_INT s_order; /* Spline order */
MKL_INT s_type; /* Spline type */
MKL_INT ic_type; /* Type of internal conditions */
double* ic; /* Array of internal conditions */
MKL_INT bc_type; /* Type of boundary conditions */
double* bc; /* Array of boundary conditions */

double scoeff[(NX-1)* SPLINE_ORDER]; /* Array of spline coefficients */
MKL_INT scoeffhint; /* Additional information about the coefficients */

/* Parameters describing interpolation computations */
MKL_INT nsite; /* Number of interpolation sites */
double site[NSITE]; /* Array of interpolation sites */
MKL_INT sitehint; /* Additional information about the structure of
 interpolation sites */

MKL_INT ndorder, dorder; /* Parameters defining the type of interpolation */

double* datahint; /* Additional information on partition and interpolation sites */

double r[NSITE]; /* Array of interpolation results */
MKL_INT rhint; /* Additional information on the structure of the results */
MKL_INT* cell; /* Array of cell indices */

/* Initialize the partition */
nx = NX;

/* Set values of partition x */
...
xhint = DF_NON_UNIFORM_PARTITION; /* The partition is non-uniform. */

/* Initialize the function */
ny = 1; /* The function is scalar. */

/* Set function values */
...
yhint = DF_NO_HINT; /* No additional information about the function is provided. */

/* Create a Data Fitting task */
status = dfdNewTask1D(&task, nx, x, xhint, ny, y, yhint);

/* Check the Data Fitting operation status */
...

/* Initialize spline parameters */
s_order = DF_PP_CUBIC; /* Spline is of the fourth order (cubic spline). */
s_type = DF_PP_BESSEL; /* Spline is of the Bessel cubic type. */

/* Define internal conditions for cubic spline construction (none in this example) */
ic_type = DF_NO_IC;
ic = NULL;

/* Use not-a-knot boundary conditions. In this case, the is first and the last
interior breakpoints are inactive, no additional values are provided. */
bc_type = DF_BC_NOT_A_KNOT;
bc = NULL;

```

```

scoeffhint = DF_NO_HINT; /* No additional information about the spline. */

/* Set spline parameters in the Data Fitting task */
status = dfdEditPPSpline1D(task, s_order, s_type, bc_type, bc, ic_type,
 ic, scoeff, scoeffhint);

/* Check the Data Fitting operation status */
...

/* Use a standard method to construct a cubic Bessel spline: */
/* $P_i(x) = c_{i,0} + c_{i,1}(x - x_i) + c_{i,2}(x - x_i)^2 + c_{i,3}(x - x_i)^3$, */
/* The library packs spline coefficients to array scoeff: */
/* scoeff[4*i+0] = ci,0, scoef[4*i+1] = ci,1, */
/* scoeff[4*i+2] = ci,2, scoef[4*i+3] = ci,3, */
/* i=0,...,N-2 */
status = dfdConstruct1D(task, DF_PP_SPLINE, DF_METHOD_STD);

/* Check the Data Fitting operation status */
...

/* Initialize interpolation parameters */
nsite = NSITE;

/* Set site values */
...

sitehint = DF_NON_UNIFORM_PARTITION; /* Partition of sites is non-uniform */

/* Request to compute spline values */
ndorder = 1;
dorder = 1;
datahint = DF_NO_APRIORI_INFO; /* No additional information about breakpoints or
 sites is provided. */
rhint = DF_MATRIX_STORAGE_ROWS; /* The library packs interpolation results
 in row-major format. */
cell = NULL; /* Cell indices are not required. */

/* Solve interpolation problem using the default method: compute the spline values
 at the points site(i), i=0,..., nsite-1 and place the results to array r */
status = dfdInterpolate1D(task, DF_INTERP, DF_METHOD_STD, nsite, site,
 sitehint, ndorder, &dorder, datahint, r, rhint, cell);

/* Check Data Fitting operation status */
...

/* De-allocate Data Fitting task resources */
status = dfDeleteTask(&task);
/* Check Data Fitting operation status */
...
return 0;
}

```

The following example demonstrates how to compute indices of cells containing given sites. This example uses uniform partition presented with two boundary points. The sites are in the ascending order.

### Example of Cell Search

```

#include "mkl.h"

#define NX 100 /* Size of partition, number of breakpoints */

```

```

#define NSITE 1000 /* Number of interpolation sites */

int main()
{
 int status; /* Status of a Data Fitting operation */
 DFTaskPtr task; /* Data Fitting operations are task based */

 /* Parameters describing the partition */
 MKL_INT nx; /* The size of partition x */
 float x[2]; /* Partition x is uniform and holds endpoints
 of interpolation interval [a, b] */
 MKL_INT xhint; /* Additional information about the structure of breakpoints */

 /* Parameters describing the function */
 MKL_INT ny; /* Function dimension */
 float *y; /* Function values at the breakpoints */
 MKL_INT yhint; /* Additional information about the function */

 /* Parameters describing cell search */
 MKL_INT nsite; /* Number of interpolation sites */
 float site[NSITE]; /* Array of interpolation sites */
 MKL_INT sitehint; /* Additional information about the structure of sites */

 float* datahint; /* Additional information on partition and interpolation sites */

 MKL_INT cell[NSITE]; /* Array for cell indices */

 /* Initialize a uniform partition */
 nx = NX;
 /* Set values of partition x: for uniform partition, */
 /* provide end-points of the interpolation interval [-1.0,1.0] */
 x[0] = -1.0f; x[1] = 1.0f;
 xhint = DF_UNIFORM_PARTITION; /* Partition is uniform */

 /* Initialize function parameters */
 /* In cell search, function values are not necessary and are set to zero/NULL values */
 ny = 0;
 y = NULL;
 yhint = DF_NO_HINT;

 /* Create a Data Fitting task */
 status = dfsNewTask1D(&task, nx, x, xhint, ny, y, yhint);

 /* Check Data Fitting operation status */
 ...

 /* Initialize interpolation (cell search) parameters */
 nsite = NSITE;

 /* Set sites in the ascending order */
 ...
 sitehint = DF_SORTED_DATA; /* Sites are provided in the ascending order. */
 datahint = DF_NO_APRIORI_INFO; /* No additional information
 about breakpoints/sites is provided.*/

 /* Use a standard method to compute indices of the cells that contain
 interpolation sites. The library places the index of the cell containing
 site(i) to the cell(i), i=0,...,nsite-1 */
 status = dfsSearchCells1D(task, DF_METHOD_STD, nsite, site, sitehint,

```

```

 datahint, cell);
/* Check Data Fitting operation status */
...

/* Process cell indices */
...

/* Deallocate Data Fitting task resources */
status = dfDeleteTask(&task);

/* Check Data Fitting operation status */
...
return 0;
}

```

## Data Fitting Function Task Status and Error Reporting

The Data Fitting routines report a task status through integer values. Negative status values indicate errors, while positive values indicate warnings. An error can be caused by invalid parameter values or a memory allocation failure.

The status codes have symbolic names predefined in the header file as macros via the `#define` statements.

If no error occurred, the function returns the `DF_STATUS_OK` code defined as zero:

```
#define DF_STATUS_OK 0
```

In case of an error, the function returns a non-zero error code that specifies the origin of the failure. Header files define the following status codes:

### Status Codes in the Data Fitting Component

| Status Code                                                                          | Description                                             |
|--------------------------------------------------------------------------------------|---------------------------------------------------------|
| <b>Common Status Codes</b>                                                           |                                                         |
| <code>DF_STATUS_OK</code>                                                            | Operation completed successfully.                       |
| <code>DF_ERROR_NULL_TASK</code>                                                      | Data Fitting task is a <code>NULL</code> pointer.       |
| <code>DF_ERROR_MEM_FAILURE</code>                                                    | Memory allocation failure.                              |
| <code>DF_ERROR_METHOD_NOT_SUPPORTED</code>                                           | Requested method is not supported.                      |
| <code>DF_ERROR_COMP_TYPE_NOT_SUPPORTED</code>                                        | Requested computation type is not supported.            |
| <code>DF_ERROR_NULL_PTR</code>                                                       | Pointer to parameter is null.                           |
| <b>Data Fitting Task Creation and Initialization, and Generic Editing Operations</b> |                                                         |
| <code>DF_ERROR_BAD_NX</code>                                                         | Invalid number of breakpoints.                          |
| <code>DF_ERROR_BAD_X</code>                                                          | Array of breakpoints is invalid.                        |
| <code>DF_ERROR_BAD_X_HINT</code>                                                     | Invalid hint describing the structure of the partition. |
| <code>DF_ERROR_BAD_NY</code>                                                         | Invalid dimension of vector-valued function $y$ .       |
| <code>DF_ERROR_BAD_Y</code>                                                          | Array of function values is invalid.                    |
| <code>DF_ERROR_BAD_Y_HINT</code>                                                     | Invalid flag describing the structure of function $y$   |

### Data Fitting Task-Specific Editing Operations

| Status Code                                | Description                                                                                                                                                                   |
|--------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| DF_ERROR_BAD_SPLINE_ORDER                  | Invalid spline order.                                                                                                                                                         |
| DF_ERROR_BAD_SPLINE_TYPE                   | Invalid spline type.                                                                                                                                                          |
| DF_ERROR_BAD_IC_TYPE                       | Type of internal conditions used for spline construction is invalid.                                                                                                          |
| DF_ERROR_BAD_IC                            | Array of internal conditions for spline construction is not defined.                                                                                                          |
| DF_ERROR_BAD_BC_TYPE                       | Type of boundary conditions used in spline construction is invalid.                                                                                                           |
| DF_ERROR_BAD_BC                            | Array of boundary conditions for spline construction is not defined.                                                                                                          |
| DF_ERROR_BAD_PP_COEFF                      | Array of piecewise polynomial spline coefficients is not defined.                                                                                                             |
| DF_ERROR_BAD_PP_COEFF_HINT                 | Invalid flag describing the structure of the piecewise polynomial spline coefficients.                                                                                        |
| DF_ERROR_BAD_PERIODIC_VAL                  | Function values at the endpoints of the interpolation interval are not equal as required in periodic boundary conditions.                                                     |
| DF_ERROR_BAD_DATA_ATTR                     | Invalid attribute of the pointer to be set or modified in Data Fitting task descriptor with the <code>df?</code> <code>EditIdxPtr</code> task editor.                         |
| DF_ERROR_BAD_DATA_IDX                      | Index of the pointer to be set or modified in the Data Fitting task descriptor with the <code>df?</code> <code>EditIdxPtr</code> task editor is out of the pre-defined range. |
| <b>Data Fitting Computation Operations</b> |                                                                                                                                                                               |
| DF_ERROR_BAD_NSITE                         | Invalid number of interpolation sites.                                                                                                                                        |
| DF_ERROR_BAD_SITE                          | Array of interpolation sites is not defined.                                                                                                                                  |
| DF_ERROR_BAD_SITE_HINT                     | Invalid flag describing the structure of interpolation sites.                                                                                                                 |
| DF_ERROR_BAD_NDORDER                       | Invalid size of the array defining derivative orders to be computed at interpolation sites.                                                                                   |
| DF_ERROR_BAD_DORDER                        | Array defining derivative orders to be computed at interpolation sites is not defined.                                                                                        |
| DF_ERROR_BAD_DATA_HINT                     | Invalid flag providing additional information about partition or interpolation sites.                                                                                         |
| DF_ERROR_BAD_INTERP                        | Array of spline-based interpolation results is not defined.                                                                                                                   |
| DF_ERROR_BAD_INTERP_HINT                   | Invalid flag defining the structure of spline-based interpolation results.                                                                                                    |
| DF_ERROR_BAD_CELL_IDX                      | Array of indices of partition cells containing interpolation sites is not defined.                                                                                            |
| DF_ERROR_BAD_NLIM                          | Invalid size of arrays containing integration limits.                                                                                                                         |

| Status Code                     | Description                                                                            |
|---------------------------------|----------------------------------------------------------------------------------------|
| DF_ERROR_BAD_LLIM               | Array of the left-side integration limits is not defined.                              |
| DF_ERROR_BAD_RLIM               | Array of the right-side integration limits is not defined.                             |
| DF_ERROR_BAD_INTEGR             | Array of spline-based integration results is not defined.                              |
| DF_ERROR_BAD_INTEGR_HINT        | Invalid flag providing the structure of the array of spline-based integration results. |
| DF_ERROR_BAD_LOOKUP_INTERP_SITE | Bad site provided for interpolation with look-up interpolator.                         |

**NOTE**

The routine that estimates piecewise polynomial cubic spline coefficients can return internal error codes related to the specifics of the implementation. Such error codes indicate invalid input data or other issues unrelated to Data Fitting routines.

## Data Fitting Task Creation and Initialization Routines

Task creation and initialization routines are functions used to create a new task descriptor and initialize its parameters. The Data Fitting component provides the `df?NewTask1D` routine that creates and initializes a new task descriptor for a one-dimensional Data Fitting task.

**df?NewTask1D**

*Creates and initializes a new task descriptor for a one-dimensional Data Fitting task.*

**Syntax**

```
status = dfsNewTask1D(&task, nx, x, xhint, ny, y, yhint)
```

```
status = dfdNewTask1D(&task, nx, x, xhint, ny, y, yhint)
```

**Include Files**

- `mkl.h`

**Input Parameters**

| Name            | Type                                                                                                                | Description                                                                                                                                                                                                                                                                                                                                         |
|-----------------|---------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>nx</code> | <code>const MKL_INT</code>                                                                                          | Number of breakpoints representing partition of interpolation interval $[a, b]$ .                                                                                                                                                                                                                                                                   |
| <code>x</code>  | <code>const float*</code> for <code>dfsNewTask1D</code><br><code>const double*</code> for <code>dfdNewTask1D</code> | One-dimensional array containing the strictly sorted breakpoints from interpolation interval $[a, b]$ . The structure of the array is defined by parameter <code>xhint</code> : <ul style="list-style-type: none"> <li>• If partition is non-uniform or quasi-uniform, the array should contain <code>nx</code> strictly ordered values.</li> </ul> |

| Name                                                                                                         | Type                                                        | Description                                                                                                                                                                                                                                                                                                                       |
|--------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                                                                                              |                                                             | <ul style="list-style-type: none"> <li>If partition is uniform, the array should contain two entries that represent endpoints of interpolation interval <math>[a, b]</math>.</li> </ul>                                                                                                                                           |
| <hr/>                                                                                                        |                                                             |                                                                                                                                                                                                                                                                                                                                   |
| <b>CAUTION</b>                                                                                               |                                                             |                                                                                                                                                                                                                                                                                                                                   |
| The array must be strictly sorted. If it is unordered, the results of data fitting routines are not correct. |                                                             |                                                                                                                                                                                                                                                                                                                                   |
| <hr/>                                                                                                        |                                                             |                                                                                                                                                                                                                                                                                                                                   |
| <i>xhint</i>                                                                                                 | const MKL_INT                                               | A flag describing the structure of partition <i>x</i> . For the list of possible values of <i>xhint</i> , see table "Hint Values for Partition <i>x</i> ". If you set the flag to the DF_NO_HINT value, the library interprets the partition as non-uniform.                                                                      |
| <i>ny</i>                                                                                                    | const MKL_INT                                               | Dimension of vector-valued function <i>y</i> .                                                                                                                                                                                                                                                                                    |
| <i>y</i>                                                                                                     | const float* for dfsNewTask<br>const double* for dfdNewTask | Vector-valued function <i>y</i> , array of size $n_x \times n_y$ .<br>The storage format of function values in the array is defined by the value of flag <i>yhint</i> .                                                                                                                                                           |
| <i>yhint</i>                                                                                                 | const MKL_INT                                               | A flag describing the structure of array <i>y</i> . Valid hint values are listed in table "Hint Values for Vector-Valued Function <i>y</i> ". If you set the flag to the DF_NO_HINT value, the library assumes that all <i>ny</i> coordinates of the vector-valued function <i>y</i> are provided and stored in row-major format. |

### Output Parameters

| Name          | Type      | Description                                                                                                                                                                                                                                            |
|---------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>   | DFTaskPtr | Descriptor of the task.                                                                                                                                                                                                                                |
| <i>status</i> | int       | Status of the routine: <ul style="list-style-type: none"> <li>DF_STATUS_OK if the task is created successfully.</li> <li>Non-zero error code if the task creation failed. See "Task Status and Error Reporting" for error code definitions.</li> </ul> |

### Description

The `df?NewTask1D` routine creates and initializes a new Data Fitting task descriptor with user-specified parameters for a one-dimensional Data Fitting task. The *x* and *nx* parameters representing the partition of interpolation interval  $[a, b]$  are mandatory. If you provide invalid values for these parameters, such as a NULL pointer *x* or the number of breakpoints smaller than two, the routine does not create the Data Fitting task and returns an error code.

If you provide a vector-valued function *y*, make sure that the function dimension *ny* and the array of function values *y* are both valid. If any of these parameters are invalid, the routine does not create the Data Fitting task and returns an error code.

If you store coordinates of the vector-valued function *y* in non-contiguous memory locations, you can set the *yhint* flag to `DF_1ST_COORDINATE`, and pass only the first coordinate of the function into the task creation routine. After successful creation of the Data Fitting task, you can pass the remaining coordinates using the `df?EditIdxPtr` task editor.

If the routine fails to create the task descriptor, it returns a `NULL` task pointer.

The routine supports the following hint values for partition  $x$ :

#### Hint Values for Partition $x$

| Value                                   | Description                                                               |
|-----------------------------------------|---------------------------------------------------------------------------|
| <code>DF_NON_UNIFORM_PARTITION</code>   | Partition is non-uniform.                                                 |
| <code>DF_QUASI_UNIFORM_PARTITION</code> | Partition is quasi-uniform.                                               |
| <code>DF_UNIFORM_PARTITION</code>       | Partition is uniform.                                                     |
| <code>DF_NO_HINT</code>                 | No hint is provided. By default, partition is interpreted as non-uniform. |

The routine supports the following hint values for the vector-valued function:

#### Hint Values for Vector-Valued Function $y$

| Value                               | Description                                                                                                                 |
|-------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| <code>DF_MATRIX_STORAGE_ROWS</code> | Data is stored in row-major format according to C conventions.                                                              |
| <code>DF_MATRIX_STORAGE_COLS</code> | Data is stored in column-major format according to Fortran conventions.                                                     |
| <code>DF_1ST_COORDINATE</code>      | The first coordinate of vector-valued data is provided.                                                                     |
| <code>DF_NO_HINT</code>             | No hint is provided. By default, the coordinates of vector-valued function $y$ are provided and stored in row-major format. |

#### NOTE

You must preserve the arrays  $x$  (breakpoints) and  $y$  (vector-valued functions) through the entire workflow of the Data Fitting computations for a task, as the task stores the addresses of the arrays for spline-based computations.

## Task Configuration Routines

In order to configure tasks, you can use task editors and task query routines.

Task editors initialize or change the predefined Data Fitting task parameters. You can use task editors to initialize or modify pointers to arrays or parameter values.

Task editors can be task-specific or generic. Task-specific editors can modify more than one parameter related to a specific task. Generic editors modify a single parameter at a time.

The Data Fitting component of Intel MKL provides the following task editors:

#### Data Fitting Task Editors

| Editor                                          | Description                                            | Type          |
|-------------------------------------------------|--------------------------------------------------------|---------------|
| <code>df?</code><br><code>EditPPSpline1D</code> | Changes parameters of the piecewise polynomial spline. | Task-specific |
| <code>df?EditPtr</code>                         | Changes a pointer in the task descriptor.              | Generic       |
| <code>dfiEditVal</code>                         | Changes a value in the task descriptor.                | Generic       |

| Editor                     | Description                                                                                                         | Type    |
|----------------------------|---------------------------------------------------------------------------------------------------------------------|---------|
| <code>df?EditIdxPtr</code> | Changes a coordinate of data represented in matrix format, such as a vector-valued function or spline coefficients. | Generic |

Task query routines are used to read the predefined Data Fitting task parameters. You can use task query routines to read the values of pointers or parameters.

Task query routines are generic (not task-specific), allowing you to read a single parameter at a time.

The Data Fitting component of the Intel MKL provides the following task query routines:

### Data Fitting Task Query Routines

| Editor                      | Description                                                                                                         | Type    |
|-----------------------------|---------------------------------------------------------------------------------------------------------------------|---------|
| <code>df?QueryPtr</code>    | Queries a pointer in the task descriptor.                                                                           | Generic |
| <code>dfiQueryVal</code>    | Queries a value in the task descriptor.                                                                             | Generic |
| <code>df?QueryIdxPtr</code> | Queries a coordinate of data represented in matrix format, such as a vector-valued function or spline coefficients. | Generic |

### `df?EditPPSpline1D`

*Modifies parameters representing a spline in a Data Fitting task descriptor.*

### Syntax

```
status = dfsEditPPSpline1D(task, s_order, s_type, bc_type, bc, ic_type, ic, scoeff, scoeffhint)
```

```
status = dfdEditPPSpline1D(task, s_order, s_type, bc_type, bc, ic_type, ic, scoeff, scoeffhint)
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                 | Type                       | Description                                                                                                                                                        |
|----------------------|----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>task</code>    | <code>DFTaskPtr</code>     | Descriptor of the task.                                                                                                                                            |
| <code>s_order</code> | <code>const MKL_INT</code> | Spline order. The parameter takes one of the values described in table " <a href="#">Spline Orders Supported by Data Fitting Functions</a> ".                      |
| <code>s_type</code>  | <code>const MKL_INT</code> | Spline type. The parameter takes one of the values described in table " <a href="#">Spline Types Supported by Data Fitting Functions</a> ".                        |
| <code>bc_type</code> | <code>const MKL_INT</code> | Type of boundary conditions. The parameter takes one of the values described in table " <a href="#">Boundary Conditions Supported by Data Fitting Functions</a> ". |

| Name              | Type                                                                                | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|-------------------|-------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>bc</i>         | const float* for<br>dfsEditPPSpline1D<br><br>const double* for<br>dfdEditPPSpline1D | Pointer to boundary conditions. The size of the array is defined by the value of parameter <i>bc_type</i> : <ul style="list-style-type: none"> <li>• If you set free-end or not-a-knot boundary conditions, pass the NULL pointer to this parameter.</li> <li>• If you combine boundary conditions at the endpoints of the interpolation interval, pass an array of two elements.</li> <li>• If you set a boundary condition for the default quadratic spline or a periodic condition for Hermite or the default cubic spline, pass an array of one element.</li> </ul>                                                                                                                                                                                                                                  |
| <i>ic_type</i>    | const MKL_INT                                                                       | Type of internal conditions. The parameter takes one of the values described in table " <a href="#">Internal Conditions Supported by Data Fitting Functions</a> ".                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| <i>ic</i>         | const float* for<br>dfsEditPPSpline1D<br><br>const double* for<br>dfdEditPPSpline1D | A non-NULL pointer to the array of internal conditions. The size of the array is defined by the value of parameter <i>ic_type</i> : <ul style="list-style-type: none"> <li>• If you set first derivatives or second derivatives internal conditions (<i>ic_type</i>=DF_IC_1ST_DER or <i>ic_type</i>=DF_IC_2ND_DER), pass an array of <i>n-1</i> derivative values at the internal points of the interpolation interval.</li> <li>• If you set the knot values internal condition for Subbotin spline (<i>ic_type</i>=DF_IC_Q_KNOT) and the knot partition is non-uniform, pass an array of <i>n+1</i> elements.</li> <li>• If you set the knot values internal condition for Subbotin spline (<i>ic_type</i>=DF_IC_Q_KNOT) and the knot partition is uniform, pass an array of four elements.</li> </ul> |
| <i>scoeff</i>     | const float* for<br>dfsEditPPSpline1D<br><br>const double* for<br>dfdEditPPSpline1D | Spline coefficients. An array of size <i>ny*s_order*(nx-1)</i> . The storage format of the coefficients in the array is defined by the value of flag <i>scoeffhint</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <i>scoeffhint</i> | const MKL_INT                                                                       | A flag describing the structure of the array of spline coefficients. For valid hint values, see table " <a href="#">Hint Values for Spline Coefficients</a> ". The library stores the coefficients in row-major format. The default value is DF_NO_HINT.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |

## Output Parameters

| Name          | Type | Description                                                                                                                                                                                                                                                                                |
|---------------|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>status</i> | int  | Status of the routine: <ul style="list-style-type: none"> <li>• DF_STATUS_OK if the routine execution completed successfully.</li> <li>• Non-zero error code if the routine execution failed. See "<a href="#">Task Status and Error Reporting</a>" for error code definitions.</li> </ul> |

## Description

The editor modifies parameters that describe the order, type, boundary conditions, internal conditions, and coefficients of a spline. The spline order definition is provided in the "[Mathematical Conventions](#)" section. You can set the spline order to any value supported by Data Fitting functions. The table below lists the available values:

### Spline Orders Supported by the Data Fitting Functions

| Order           | Description                                                                            |
|-----------------|----------------------------------------------------------------------------------------|
| DF_PP_STD       | Artificial value. Use this value for look-up and step-wise constant interpolants only. |
| DF_PP_LINEAR    | Piecewise polynomial spline of the second order (linear spline).                       |
| DF_PP_QUADRATIC | Piecewise polynomial spline of the third order (quadratic spline).                     |
| DF_PP_CUBIC     | Piecewise polynomial spline of the fourth order (cubic spline).                        |

To perform computations with a spline not supported by Data Fitting routines, set the parameter defining the spline order and pass the spline coefficients to the library in the supported format. For format description, see figure "[Row-major Coefficient Storage Format](#)".

The table below lists the supported spline types:

### Spline Types Supported by Data Fitting Functions

| Type                             | Description                                                                                     |
|----------------------------------|-------------------------------------------------------------------------------------------------|
| DF_PP_DEFAULT                    | The default spline type. You can use this type with linear, quadratic, or user-defined splines. |
| DF_PP_SUBBOTIN                   | Quadratic splines based on Subbotin algorithm, [ <a href="#">StechSub76</a> ].                  |
| DF_PP_NATURAL                    | Natural cubic spline.                                                                           |
| DF_PP_HERMITE                    | Hermite cubic spline.                                                                           |
| DF_PP_BESSEL                     | Bessel cubic spline.                                                                            |
| DF_PP_HYMAN                      | Co-monotone Hyman cubic spline, [ <a href="#">Hyman83</a> ].                                    |
| DF_PP_AKIMA                      | Akima cubic spline.                                                                             |
| DF_LOOKUP_INTERPOLANT            | Look-up interpolant.                                                                            |
| DF_CR_STEPWISE_CONST_INTERPOLANT | Continuous right step-wise constant interpolant.                                                |
| DF_CL_STEPWISE_CONST_INTERPOLANT | Continuous left step-wise constant interpolant.                                                 |

If you perform computations with look-up or step-wise constant interpolants, set the spline order to the DF\_PP\_STD value.

Construction of specific splines may require boundary or internal conditions. To compute coefficients of such splines, you should pass boundary or internal conditions to the library by specifying the type of the conditions and providing the necessary values. For splines that do not require additional conditions, such as linear splines, set condition types to DF\_NO\_BC and DF\_NO\_IC, and pass NULL pointers to the conditions. The table below defines the supported boundary conditions:

**Boundary Conditions Supported by Data Fitting Functions**

| Boundary Condition  | Description                                  | Spline                                                                       |
|---------------------|----------------------------------------------|------------------------------------------------------------------------------|
| DF_NO_BC            | No boundary conditions provided.             | All                                                                          |
| DF_BC_NOT_A_KNOT    | Not-a-knot boundary conditions.              | Akima, Bessel, co-monotone Hyman, Hermite, natural cubic                     |
| DF_BC_FREE_END      | Free-end boundary conditions.                | Akima, Bessel, co-monotone Hyman, Hermite, natural cubic, quadratic Subbotin |
| DF_BC_1ST_LEFT_DER  | The first derivative at the left endpoint.   | Akima, Bessel, co-monotone Hyman, Hermite, natural cubic, quadratic Subbotin |
| DF_BC_1ST_RIGHT_DER | The first derivative at the right endpoint.  | Akima, Bessel, co-monotone Hyman, Hermite, natural cubic, quadratic Subbotin |
| DF_BC_2ST_LEFT_DER  | The second derivative at the left endpoint.  | Akima, Bessel, co-monotone Hyman, Hermite, natural cubic, quadratic Subbotin |
| DF_BC_2ND_RIGHT_DER | The second derivative at the right endpoint. | Akima, Bessel, co-monotone Hyman, Hermite, natural cubic, quadratic Subbotin |
| DF_BC_PERIODIC      | Periodic boundary conditions.                | Linear, all cubic splines                                                    |
| DF_BC_Q_VAL         | Function value at point $(x_0 + x_1)/2$      | Default quadratic                                                            |

**NOTE**

To construct a natural cubic spline, pass these settings to the editor:

- DF\_PP\_CUBIC as the spline order,
- DF\_PP\_NATURAL as the spline type, and
- DF\_BC\_FREE\_END as the boundary condition.

To construct a cubic spline with other boundary conditions, pass these settings to the editor:

- DF\_PP\_CUBIC as the spline order,
- DF\_PP\_NATURAL as the spline type, and
- the required type of boundary condition.

For a co-monotone Hyman cubic spline, you should set the first derivatives as boundary conditions to zeros, otherwise co-monotonicity can be broken on the leftmost and rightmost sub-intervals of the interpolation interval. You can also use first or second divided difference formulas for the first derivatives as boundary conditions. If the sign of the second divided difference differs from the sign of the respective first divided difference, replace the second divided difference with the first one or set the corresponding derivative to zero as the boundary condition. If the co-monotonicity criteria ([Fritsch80]) is not met, Intel MKL can return unexpected results.

For Akima, Hermite, Bessel, co-monotone Hyman, and default cubic splines use the corresponding type defined in [Table Spline Types Supported by Data Fitting Functions](#).

You can combine the values of boundary conditions with a bitwise OR operation. This permits you to pass combinations of first and second derivatives at the endpoints of the interpolation interval into the library. To pass a first derivative at the left endpoint and a second derivative at the right endpoint, set the boundary conditions to `DF_BC_1ST_LEFT_DER OR DF_BC_2ND_RIGHT_DER`.

You should pass the combined boundary conditions as an array of two elements. The first entry of the array contains the value of the boundary condition for the left endpoint of the interpolation interval, and the second entry - for the right endpoint. Pass other boundary conditions as arrays of one element.

For the conditions defined as a combination of valid values, the library applies the following rules to identify the boundary condition type:

- If not required for spline construction, the value of boundary conditions is ignored.
- Not-a-knot condition has the highest priority. If set, other boundary conditions are ignored.
- Free-end condition has the second priority after the not-a-knot condition. If set, other boundary conditions are ignored.
- Periodic boundary condition has the next priority after the free-end condition.
- The first derivative has higher priority than the second derivative at the right and left endpoints.

If you set the periodic boundary condition, make sure that function values at the endpoints of the interpolation interval are identical. Otherwise, the library returns an error code. The table below specifies the values to be provided for each type of spline if the periodic boundary condition is set.

### Boundary Requirements for Periodic Conditions

| Spline Type             | Periodic Boundary Condition Support | Boundary Value    |
|-------------------------|-------------------------------------|-------------------|
| Linear                  | Yes                                 | Not required      |
| Default quadratic       | No                                  |                   |
| Subbotin quadratic      | No                                  |                   |
| Natural cubic           | Yes                                 | Not required      |
| Bessel                  | Yes                                 | Not required      |
| Co-monotone Hyman cubic | Yes                                 | Not required      |
| Akima                   | Yes                                 | Not required      |
| Hermite cubic           | Yes                                 | First derivative  |
| Default cubic           | Yes                                 | Second derivative |

Internal conditions supported in the Data Fitting domain that you can use for the `ic_type` parameter are the following:

### Internal Conditions Supported by Data Fitting Functions

| Internal Condition         | Description                                                                                                                                                     | Spline        |
|----------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|
| <code>DF_NO_IC</code>      | No internal conditions provided.                                                                                                                                |               |
| <code>DF_IC_1ST_DER</code> | Array of first derivatives of size $n-2$ , where $n$ is the number of breakpoints. Derivatives are applicable to each coordinate of the vector-valued function. | Hermite cubic |

| Internal Condition | Description                                                                                                                                                      | Spline             |
|--------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|
| DF_IC_2ND_DER      | Array of second derivatives of size $n-2$ , where $n$ is the number of breakpoints. Derivatives are applicable to each coordinate of the vector-valued function. | Default cubic      |
| DF_IC_Q_KNOT       | Knot array of size $n+1$ , where $n$ is the number of breakpoints.                                                                                               | Subbotin quadratic |

To construct a Subbotin quadratic spline, you have three options to get the array of knots in the library:

- If you do not provide the knots, the library uses the default values of knots  $t = \{t_i\}$ ,  $i = 0, \dots, n$  according to the rule:

$$t_0 = x_0, t_n = x_{n-1}, t_i = (x_i + x_{i-1})/2, i = 1, \dots, n - 1.$$

- If you provide the knots in an array of size  $n + 1$ , the knots form a non-uniform partition. Make sure that the knot values you provide meet the following conditions:

$$t_0 = x_0, t_n = x_{n-1}, t_i \in (x_{i-1}, x_i), i = 1, \dots, n - 1.$$

- If you provide the knots in an array of size 4, the knots form a uniform partition

$$t_0 = x_0, t_1 = l, t_2 = r, t_3 = x_{n-1}, \text{ where } l \in (x_0, x_1) \text{ and } r \in (x_{n-2}, x_{n-1}).$$

In this case, you need to set the value of the `ic_type` parameter holding the type of internal conditions to `DF_IC_Q_KNOT` OR `DF_UNIFORM_PARTITION`.

#### NOTE

Since the partition is uniform, perform an OR operation with the `DF_UNIFORM_PARTITION` partition hint value described in [Table Hint Values for Partition x](#).

For computations based on look-up and step-wise constant interpolants, you can avoid calling the `df?EditPPSpline1D` editor and directly call one of the routines for spline-based computation of spline values, derivatives, or integrals. For example, you can call the `df?Construct1D` routine to construct the required spline with the given attributes, such as order or type.

The memory location of the spline coefficients is defined by the `scoeff` parameter. Make sure that the size of the array is sufficient to hold  $ny * s\_order * (nx-1)$  values.

The `df?EditPPSpline1D` routine supports the following hint values for spline coefficients:

#### Hint Values for Spline Coefficients

| Order             | Description                                                                                      |
|-------------------|--------------------------------------------------------------------------------------------------|
| DF_1ST_COORDINATE | The first coordinate of vector-valued data is provided.                                          |
| DF_NO_HINT        | No hint is provided. By default, all sets of spline coefficients are stored in row-major format. |

The coefficients for all coordinates of the vector-valued function are packed in memory one by one in successive order, from function  $y_1$  to function  $y_{ny}$ .

Within each coordinate, the library stores the coefficients as an array, in row-major format:

$$c_{1,0}, c_{1,1}, \dots, c_{1,k}, c_{2,0}, c_{2,1}, \dots, c_{2,k}, \dots, c_{n-1,0}, c_{n-1,1}, \dots, c_{n-1,k}$$

Mapping of the coefficients to storage in the `scoeff` array is described below, where  $c_{i,j}$  is the  $j$ th coefficient of the function

$$P_i(x) = c_{i,0} + c_{i,1}(x - x_i) + \dots + c_{i,k}(x - x_i)^k$$

See [Mathematical Conventions](#) for more details on nomenclature and interpolants.

### Row-major Coefficient Storage Format

$$\begin{array}{l}
 P_1(x) = \vec{c}_{1,0} + \vec{c}_{1,1}(x - x_1) + \dots + \vec{c}_{1,k}(x - x_1)^k \\
 P_2(x) = \vec{c}_{2,0} + \vec{c}_{2,1}(x - x_2) + \dots + \vec{c}_{2,k}(x - x_2)^k \\
 \vdots \\
 P_{n-1}(x) = \vec{c}_{n-1,0} + \vec{c}_{n-1,1}(x - x_{n-1}) + \dots + \vec{c}_{n-1,k}(x - x_{n-1})^k
 \end{array}$$

If you store splines corresponding to different coordinates of the vector-valued function at non-contiguous memory locations, do the following:

1. Set the *scoeffhint* flag to `DF_1ST_COORDINATE` and provide the spline for the first coordinate.
2. Pass the spline coefficients for the remaining coordinates into the Data Fitting task using the `df?EditIdxPtr` task editor.

Using the `df?EditPPSpline1D` task editor, you can provide to the Data Fitting task an already constructed spline that you want to use in computations. To ensure correct interpretation of the memory content, you should set the following parameters:

- Spline order and type, if appropriate. If the spline is not supported by the library, set the *s\_type* parameter to `DF_PP_DEFAULT`.
- Pointer to the array of spline coefficients in row-major format.
- The *scoeffhint* parameter describing the structure of the array:
  - Set the *scoeffhint* flag to the `DF_1ST_COORDINATE` value to pass spline coefficients stored at different memory locations. In this case, you can set the parameters that describe boundary and internal conditions to zero.
  - Use the default value `DF_NO_HINT` for all other cases.

Before passing an already constructed spline into the library, you should call the `dfiEditVal` task editor to provide the dimension of the spline `DF_NY`. See table "[Parameters Supported by the dfiEditVal Task Editor](#)" for details.

After you provide the spline to the Data Fitting task, you can run computations that use this spline.

**NOTE**

You must preserve the arrays *bc* (boundary conditions), *ic* (internal conditions), and *scoeff* (spline coefficients) through the entire workflow of the Data Fitting computations for a task, as the task stores the addresses of the arrays for spline-based computations.

**df?EditPtr**

Modifies a pointer to an array held in a Data Fitting task descriptor.

---

**Syntax**

```
status = dfsEditPtr(task, ptr_attr, ptr)
```

```
status = dfdEditPtr(task, ptr_attr, ptr)
```

**Include Files**

- mkl.h

**Input Parameters**

| Name            | Type                                                        | Description                                                                                                                                |
|-----------------|-------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>     | DFTaskPtr                                                   | Descriptor of the task.                                                                                                                    |
| <i>ptr_attr</i> | const MKL_INT                                               | The parameter to change. For details, see the <i>Pointer Attribute</i> column in table "Pointers Supported by the df?EditPtr Task Editor". |
| <i>ptr</i>      | const float* for dfsEditPtr<br>const double* for dfdEditPtr | New pointer. For details, see the <i>Purpose</i> column in table "Pointers Supported by the df?EditPtr Task Editor".                       |

**Output Parameters**

| Name          | Type | Description                                                                                                                                                                                                                                          |
|---------------|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>status</i> | int  | Status of the routine: <ul style="list-style-type: none"> <li>• DF_STATUS_OK if the routine execution completed successfully.</li> <li>• Non-zero error code otherwise. See "Task Status and Error Reporting" for error code definitions.</li> </ul> |

**Description**

The df?EditPtr editor replaces the pointer of type *ptr\_attr* stored in a Data Fitting task descriptor with a new pointer *ptr*. The table below describes types of pointers supported by the editor:

**Pointers Supported by the df?EditPtr Task Editor**

| Pointer Attribute | Purpose                                                                                                                                                                                                                                                                                                    |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| DF_X              | Partition $x$ of the interpolation interval, an array of strictly sorted breakpoints. <div style="border: 1px solid black; padding: 5px; margin-top: 10px;"> <p><b>CAUTION</b><br/>The array must be strictly sorted. If it is unordered, the results of data fitting routines are not correct.</p> </div> |
| DF_Y              | Vector-valued function $y$                                                                                                                                                                                                                                                                                 |

| Pointer Attribute | Purpose                                                                                                                                          |
|-------------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| DF_IC             | Internal conditions for spline construction. For details, see table " <a href="#">Internal Conditions Supported by Data Fitting Functions</a> ". |
| DF_BC             | Boundary conditions for spline construction. For details, see table " <a href="#">Boundary Conditions Supported by Data Fitting Functions</a> ". |
| DF_PP_SCOEFF      | Spline coefficients                                                                                                                              |

You can use `df?EditPtr` to modify different types of pointers including pointers to the vector-valued function and spline coefficients stored in contiguous memory. Use the `df?EditIdxPtr` editor if you need to modify pointers to coordinates of the vector-valued function or spline coefficients stored at non-contiguous memory locations.

If you modify a partition of the interpolation interval, then you should call the `dfiEditVal` task editor with the corresponding value of `DF_XHINT`, even if the structure of the partition remains the same.

If you pass a `NULL` pointer to the `df?EditPtr` task editor, the task remains unchanged and the routine returns an error code. For the predefined error codes, please see "[Task Status and Error Reporting](#)".

### NOTE

You must preserve the arrays  $x$  (breakpoints),  $y$  (vector-valued functions),  $bc$  (boundary conditions),  $ic$  (internal conditions), and  $scoeff$  (spline coefficients) through the entire workflow of the Data Fitting computations which use those arrays, as the task stores the addresses of the arrays for spline-based computations.

### `dfiEditVal`

Modifies a parameter value in a Data Fitting task descriptor.

### Syntax

```
status = dfiEditVal(task, val_attr, val)
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                  | Type                       | Description                                                                                                |
|-----------------------|----------------------------|------------------------------------------------------------------------------------------------------------|
| <code>task</code>     | <code>DFTaskPtr</code>     | Descriptor of the task.                                                                                    |
| <code>val_attr</code> | <code>const MKL_INT</code> | The parameter to change. See table " <a href="#">Parameters Supported by the dfiEditVal Task Editor</a> ". |
| <code>val</code>      | <code>const MKL_INT</code> | A new parameter value. See table " <a href="#">Parameters Supported by the dfiEditVal Task Editor</a> ".   |

### Output Parameters

| Name                | Type             | Description            |
|---------------------|------------------|------------------------|
| <code>status</code> | <code>int</code> | Status of the routine: |

| Name | Type | Description                                                                                                                                                                                                                               |
|------|------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|      |      | <ul style="list-style-type: none"> <li>DF_STATUS_OK if the routine execution completed successfully.</li> <li>Non-zero error code otherwise. See <a href="#">"Task Status and Error Reporting"</a> for error code definitions.</li> </ul> |

## Description

The `dfiEditVal` task editor replaces the parameter of type `val_attr` stored in a Data Fitting task descriptor with a new value `val`. The table below describes valid types of parameter `val_attr` supported by the editor:

### Parameters Supported by the `dfiEditVal` Task Editor

| Parameter Attribute | Purpose                                                                                                                                                                        |
|---------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| DF_NX               | Number of breakpoints                                                                                                                                                          |
| DF_XHINT            | A flag describing the structure of partition. See table <a href="#">"Hint Values for Partition x"</a> for the list of available values.                                        |
| DF_NY               | Dimension of the vector-valued function                                                                                                                                        |
| DF_YHINT            | A flag describing the structure of the vector-valued function. See table <a href="#">"Hint Values for Vector Function y"</a> for the list of available values.                 |
| DF_SPLINE_ORDER     | Spline order. See table <a href="#">"Spline Orders Supported by Data Fitting Functions"</a> for the list of available values.                                                  |
| DF_SPLINE_TYPE      | Spline type. See table <a href="#">"Spline Types Supported by Data Fitting Functions"</a> for the list of available values.                                                    |
| DF_BC_TYPE          | Type of boundary conditions used in spline construction. See table <a href="#">"Boundary Conditions Supported by Data Fitting Functions"</a> for the list of available values. |
| DF_IC_TYPE          | Type of internal conditions used in spline construction. See table <a href="#">"Internal Conditions Supported by Data Fitting Functions"</a> for the list of available values. |
| DF_PP_COEFF_HINT    | A flag describing the structure of spline coefficients. See table <a href="#">"Hint Values for Spline Coefficients"</a> for the list of available values.                      |
| DF_CHECK_FLAG       | A flag which controls checking of Data Fitting parameters. See table <a href="#">"Possible Values for the DF_CHECK_FLAG Parameter"</a> for the list of available values.       |

If you pass a zero value for the parameter describing the size of the arrays that hold coefficients for a partition, a vector-valued function, or a spline, the parameter held in the Data fitting task remains unchanged and the routine returns an error code. For the predefined error codes, see ["Task Status and Error Reporting"](#).

### Possible Values for the `DF_CHECK_FLAG` Parameter

| Value                | Description                                                                                 |
|----------------------|---------------------------------------------------------------------------------------------|
| DF_ENABLE_CHECK_FLAG | Checks the correctness of parameters of Data Fitting computational routines (default mode). |

| Value                 | Description                                                                                |
|-----------------------|--------------------------------------------------------------------------------------------|
| DF_DISABLE_CHECK_FLAG | Disables checking of the correctness of parameters of Data Fitting computational routines. |

Use `DF_CHECK_FLAG` for `val_attr` in order to control validation of parameters of Data Fitting computational routines such as `df?Construct1D`, `df?Interpolate1D/df?InterpolateEx1D`, and `df?SearchCells1D/df?SearchCellsEx1D`, which can perform better with a small number of interpolation sites or integration limits (fewer than one dozen). The default mode, with checking of parameters enabled, should be used as you develop a Data Fitting-based application. After you complete development you can disable parameter checking in order to improve the performance of your application.

If you modify the parameter describing dimensions of the arrays that hold the vector-valued function or spline coefficients in contiguous memory, you should call the `df?EditPtr` task editor with the corresponding pointers to the vector-valued function or spline coefficients even when this pointer remains unchanged. Call the `df?EditIdxPtr` editor if those arrays are stored in non-contiguous memory locations.

You must call the `dfiEditVal` task editor to edit the structure of the partition `DF_XHINT` every time you modify a partition using `df?EditPtr`, even if the structure of the partition remains the same.

### df?EditIdxPtr

*Modifies a pointer to the memory representing a coordinate of the data stored in matrix format.*

### Syntax

```
status = dfsEditIdxPtr(task, ptr_attr, idx, ptr)
```

```
status = dfdEditIdxPtr(task, ptr_attr, idx, ptr)
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                  | Type                                                                                                                  | Description                                                                                                                                                              |
|-----------------------|-----------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>task</code>     | <code>DFTaskPtr</code>                                                                                                | Descriptor of the task.                                                                                                                                                  |
| <code>ptr_attr</code> | <code>const MKL_INT</code>                                                                                            | Type of the data to be modified. The parameter takes one of the values described in "Data Attributes Supported by the <code>df?EditIdxPtr</code> Task Editor".           |
| <code>idx</code>      | <code>const MKL_INT</code>                                                                                            | Index of the coordinate whose pointer is to be modified.                                                                                                                 |
| <code>ptr</code>      | <code>const float*</code> for <code>dfsEditIdxPtr</code><br><code>const double*</code> for <code>dfdEditIdxPtr</code> | Pointer to the data that holds values of coordinate <code>idx</code> . For details, see table "Data Attributes Supported by the <code>df?EditIdxPtr</code> Task Editor". |

### Output Parameters

| Name                | Type             | Description            |
|---------------------|------------------|------------------------|
| <code>status</code> | <code>int</code> | Status of the routine: |

| Name | Type | Description                                                                                                                                                                                                                                                |
|------|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|      |      | <ul style="list-style-type: none"> <li>• <code>DF_STATUS_OK</code> if the routine execution completed successfully.</li> <li>• Non-zero error code otherwise. See "<a href="#">Task Status and Error Reporting</a>" for error code definitions.</li> </ul> |

## Description

The routine modifies a pointer to the array that holds the `idx` coordinate of vector-valued function `y` or the pointer to the array of spline coefficients corresponding to the given coordinate.

You can use the editor if you need to pass into a Data Fitting task or modify the pointer to coordinates of the vector-valued function or spline coefficients held at non-contiguous memory locations. Do not use the editor for coordinates at contiguous memory locations in row-major format.

Before calling this editor, make sure that you have created and initialized the task using a task creation function or a relevant editor such as the generic or specific `df?EditPPSpline1D` editor.

## Data Attributes Supported by the `df?EditIdxPtr` Task Editor

| Data Attribute            | Description                              |
|---------------------------|------------------------------------------|
| <code>DF_Y</code>         | Vector-valued function <code>y</code>    |
| <code>DF_PP_SCOEFF</code> | Piecewise polynomial spline coefficients |

When using `df?EditIdxPtr`, you might receive an error code in the following cases:

- You passed an unsupported parameter value into the editor.
- The value of the index exceeds the predefined value that equals the dimension `ny` of the vector-valued function.
- You pass a `NULL` pointer to the editor. In this case, the task remains unchanged.
- You pass a pointer to the `idx` coordinate of the vector-valued function you provided to contiguous memory in column-major format.

The code example below demonstrates how to use the editor for providing values of a vector-valued function stored in two non-contiguous arrays:

```
#define NX 1000 /* number of break points */
#define NY 2 /* dimension of vector-valued function */
int main()
{
 DFTaskPtr task;
 double x[NX];
 double y1[NX], y2[NX]; /* vector-valued function is stored as two arrays */
 /* Provide first coordinate of two-dimensional function y into creation routine */
 status = dfdNewTask1D(&task, NX, x, DF_NON_UNIFORM_PARTITION, NY, y1,
 DF_1ST_COORDINATE);
 /* Provide second coordiante of two-dimensional function */
 status = dfdEditIdxPtr(task, DF_Y, 1, y2);
 ...
}
```

## `df?QueryPtr`

*Reads a pointer to an array held in a Data Fitting task descriptor.*

## Syntax

```
status = dfsQueryPtr(task, ptr_attr, ptr)
```

```
status = dfdQueryPtr(task, ptr_attr, ptr)
```

## Include Files

- mkl.h

## Input Parameters

| Name            | Type          | Description                                                                                                                                                                                                                           |
|-----------------|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>     | DFTaskPtr     | Descriptor of the task.                                                                                                                                                                                                               |
| <i>ptr_attr</i> | const MKL_INT | The parameter to query. The query routine supports pointer attributes described in the table " <a href="#">Pointers Supported by the df?EditPtr Task Editor</a> ". For details, see the <i>Pointer Attribute</i> column in the table. |

## Output Parameters

| Name          | Type                                                | Description                                                                                                                                                                                                                                                          |
|---------------|-----------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>ptr</i>    | float** for dfsQueryPtr<br>double** for dfdQueryPtr | Pointer to array returned by the query routine. For details, see the <i>Purpose</i> column in table " <a href="#">Pointers Supported by the df?EditPtr Task Editor</a> ".                                                                                            |
| <i>status</i> | int                                                 | Status of the routine: <ul style="list-style-type: none"> <li>• DF_STATUS_OK if the routine execution completed successfully.</li> <li>• Non-zero error code otherwise. See "<a href="#">Task Status and Error Reporting</a>" for error code definitions.</li> </ul> |

## Description

The `df?QueryPtr` routine returns the pointer of type `ptr_attr` stored in a Data Fitting task descriptor as parameter `ptr`. Attributes of the pointers supported by the query function are identical to those supported by the editor `df?EditPtr` editor in the table "[Pointers Supported by the df?EditPtr Task Editor](#)".

You can use `df?QueryPtr` to read different types of pointers including pointers to the vector-valued function and spline coefficients stored in contiguous memory.

## `dfiQueryVal`

*Reads a parameter value in a Data Fitting task descriptor.*

---

## Syntax

```
status = dfiQueryVal(task, val_attr, val)
```

## Include Files

- mkl.h

## Input Parameters

| Name                  | Type                       | Description                                                                                                                                                                   |
|-----------------------|----------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>task</code>     | <code>DFTaskPtr</code>     | Descriptor of the task.                                                                                                                                                       |
| <code>val_attr</code> | <code>const MKL_INT</code> | The parameter to query. The query function supports the parameter attributes described in <a href="#">"Parameters Supported by the <code>dfiEditVal</code> Task Editor"</a> . |

## Output Parameters

| Name                | Type                 | Description                                                                                                                                                                                                                                                                       |
|---------------------|----------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>val</code>    | <code>MKL_INT</code> | The parameter value returned by the query function. See table <a href="#">"Parameters Supported by the <code>dfiEditVal</code> Task Editor"</a> .                                                                                                                                 |
| <code>status</code> | <code>int</code>     | Status of the routine: <ul style="list-style-type: none"> <li>• <code>DF_STATUS_OK</code> if the routine execution completed successfully.</li> <li>• Non-zero error code otherwise. See <a href="#">"Task Status and Error Reporting"</a> for error code definitions.</li> </ul> |

## Description

The `dfiQueryVal` routine returns a parameter of type `val_attr` stored in a Data Fitting task descriptor as parameter `val`. The query function supports the parameter attributes described in ["Parameters Supported by the `dfiEditVal` Task Editor"](#).

### `df?QueryIdxPtr`

*Reads a pointer to the memory representing a coordinate of the data stored in matrix format.*

## Syntax

```
status = dfsQueryIdxPtr(task, ptr_attr, idx, ptr)
```

```
status = dfdQueryIdxPtr(task, ptr_attr, idx, ptr)
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                  | Type                       | Description                                                                                                                                                                    |
|-----------------------|----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>task</code>     | <code>DFTaskPtr</code>     | Descriptor of the task.                                                                                                                                                        |
| <code>ptr_attr</code> | <code>const MKL_INT</code> | Pointer attribute to query. The parameter takes one of the attributes described in <a href="#">"Data Attributes Supported by the <code>df?EditIdxPtr</code> Task Editor"</a> . |
| <code>idx</code>      | <code>const MKL_INT</code> | Index of the coordinate of the pointer to query.                                                                                                                               |

## Output Parameters

| Name                | Type                                                                                                        | Description                                                                                                                                                                                                                                                   |
|---------------------|-------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>ptr</code>    | <code>float*</code> for <code>dfsQueryIdxPtr</code><br><code>double*</code> for <code>dfdQueryIdxPtr</code> | Pointer to the data that holds values of coordinate <code>idx</code> returned. For details, see table "Data Attributes Supported by the <code>df?EditIdxPtr</code> Task Editor".                                                                              |
| <code>status</code> | <code>int</code>                                                                                            | Status of the routine: <ul style="list-style-type: none"> <li><code>DF_STATUS_OK</code> if the routine execution completed successfully.</li> <li>Non-zero error code otherwise. See "Task Status and Error Reporting" for error code definitions.</li> </ul> |

## Description

The routine returns a pointer to the array that holds the `idx` coordinate of vector-valued function `y` or the pointer to the array of spline coefficients corresponding to the given coordinate.

You can use the query routine if you need the pointer to coordinates of the vector-valued function or spline coefficients held at non-contiguous memory locations or at a contiguous memory location in row-major format (the default storage format for spline coefficients).

Before calling this query routine, make sure that you have created and initialized the task using a task creation function or a relevant editor such as the generic or specific `df?EditPPSpline1D` editor.

When using `df?QueryIdxPtr`, you might receive an error code in the following cases:

- You passed an unsupported parameter value into the editor.
- The value of the index exceeds the predefined value that equals the dimension `ny` of the vector-valued function.
- You request the pointer to the `idx` coordinate of the vector-valued function you provided to contiguous memory in column-major format.

## Data Fitting Computational Routines

Data Fitting computational routines are functions used to perform spline-based computations, such as:

- spline construction
- computation of values, derivatives, and integrals of the predefined order
- cell search

Once you create a Data Fitting task and initialize the required parameters, you can call computational routines as many times as necessary.

The table below lists the available computational routines:

### Data Fitting Computational Routines

| Routine                         | Description                                                                   |
|---------------------------------|-------------------------------------------------------------------------------|
| <code>df?Construct1D</code>     | Constructs a spline for a one-dimensional Data Fitting task.                  |
| <code>df?Interpolate1D</code>   | Computes spline values and derivatives.                                       |
| <code>df?InterpolateEx1D</code> | Computes spline values and derivatives by calling user-provided interpolants. |
| <code>df?Integrate1D</code>     | Computes spline-based integrals.                                              |

| Routine                            | Description                                                                                    |
|------------------------------------|------------------------------------------------------------------------------------------------|
| <a href="#">df?IntegrateEx1D</a>   | Computes spline-based integrals by calling user-provided integrators.                          |
| <a href="#">df?SearchCells1D</a>   | Finds indices of cells containing interpolation sites.                                         |
| <a href="#">df?SearchCellsEx1D</a> | Finds indices of cells containing interpolation sites by calling user-provided cell searchers. |

If a Data Fitting computation completes successfully, the computational routines return the `DF_STATUS_OK` code. If an error occurs, the routines return an error code specifying the origin of the failure. Some possible errors are the following:

- The task pointer is `NULL`.
- Memory allocation failed.
- The computation failed for another reason.

For the list of available status codes, see "[Task Status and Error Reporting](#)".

---

#### NOTE

Data Fitting computational routines do not control errors for floating-point conditions, such as overflow, gradual underflow, or operations with Not a Number (NaN) values.

---

## [df?Construct1D](#)

---

### Syntax

Constructs a spline of the given type.

```
status = dfsConstruct1D(task, s_format, method)
```

```
status = dfdConstruct1D(task, s_format, method)
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                  | Type                       | Description                                                              |
|-----------------------|----------------------------|--------------------------------------------------------------------------|
| <code>task</code>     | <code>DFTaskPtr</code>     | Descriptor of the task.                                                  |
| <code>s_format</code> | <code>const MKL_INT</code> | Spline format. The supported value is <code>DF_PP_SPLINE</code> .        |
| <code>method</code>   | <code>const MKL_INT</code> | Construction method. The supported value is <code>DF_METHOD_STD</code> . |

### Output Parameters

| Name                | Type             | Description                                                                                                                                           |
|---------------------|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code> | Status of the routine: <ul style="list-style-type: none"> <li>• <code>DF_STATUS_OK</code> if the routine execution completed successfully.</li> </ul> |

| Name | Type | Description                                                                                                                                                                              |
|------|------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|      |      | <ul style="list-style-type: none"> <li>Non-zero error code if the routine execution failed. See <a href="#">"Task Status and Error Reporting"</a> for error code definitions.</li> </ul> |

### Description

Before calling `df?Construct1D`, you need to create and initialize the task, and set the parameters representing the spline. Then you can call the `df?Construct1D` routine to construct the spline. The format of the spline is defined by parameter `s_format`. The method for spline construction is defined by parameter `method`. Upon successful construction, the spline coefficients are available in the user-provided memory location in the format you set through the Data Fitting editor. For the available storage formats, see table ["Hint Values for Spline Coefficients"](#).

### `df?Interpolate1D/df?InterpolateEx1D`

*Runs data fitting computations.*

### Syntax

```
status = dfsInterpolate1D(task, type, method, nsite, site, sitehint, ndorder, dorder, datahint, r, rhint, cell)
```

```
status = dfdInterpolate1D(task, type, method, nsite, site, sitehint, ndorder, dorder, datahint, r, rhint, cell)
```

```
status = dfsInterpolateEx1D(task, type, method, nsite, site, sitehint, ndorder, dorder, datahint, r, rhint, cell, le_cb, le_params, re_cb, re_params, i_cb, i_params, search_cb, search_params)
```

```
status = dfdInterpolateEx1D(task, type, method, nsite, site, sitehint, ndorder, dorder, datahint, r, rhint, cell, le_cb, le_params, re_cb, re_params, i_cb, i_params, search_cb, search_params)
```

### Include Files

- `mkl.h`

### Input Parameters

| Name                | Type                                                                           | Description                                                                                                                                                                                                                                                        |
|---------------------|--------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>task</code>   | <code>DFTaskPtr</code>                                                         | Descriptor of the task.                                                                                                                                                                                                                                            |
| <code>type</code>   | <code>const MKL_INT</code>                                                     | Type of spline-based computations. The parameter takes one or more values combined with an OR operation. For the list of possible values, see table <a href="#">"Computation Types Supported by the <code>df?Interpolate1D/df?Interpolate1D</code> Routines"</a> . |
| <code>method</code> | <code>const MKL_INT</code>                                                     | Computation method. The supported value is <code>DF_METHOD_PP</code> .                                                                                                                                                                                             |
| <code>nsite</code>  | <code>const MKL_INT</code>                                                     | Number of interpolation sites.                                                                                                                                                                                                                                     |
| <code>site</code>   | <code>const float*</code> for <code>dfsInterpolate1D/dfsInterpolateEx1D</code> | Array of interpolation sites of size <code>nsite</code> . The structure of the array is defined by the <code>sitehint</code> parameter:                                                                                                                            |

| Name             | Type                                                                                                                            | Description                                                                                                                                                                                                                                                                                                                                                                                                                            |
|------------------|---------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                  | const double* for<br>dfdInterpolate1D/<br>dfdInterpolateEx1D                                                                    | <ul style="list-style-type: none"> <li>If sites form a non-uniform partition, the array should contain <i>nsite</i> values.</li> <li>If sites form a uniform partition, the array should contain two entries that represent the left and the right interpolation sites. The first entry of the array contains the left-most interpolation point. The second entry of the array contains the right-most interpolation point.</li> </ul> |
| <i>sit hint</i>  | const MKL_INT                                                                                                                   | A flag describing the structure of the interpolation sites. For the list of possible values of <i>sit hint</i> , see table " <a href="#">Hint Values for Interpolation Sites</a> ". If you set the flag to DF_NO_HINT, the library interprets the site-defined partition as non-uniform.                                                                                                                                               |
| <i>nd order</i>  | const MKL_INT                                                                                                                   | Maximal derivative order increased by one to be computed at interpolation sites.                                                                                                                                                                                                                                                                                                                                                       |
| <i>d order</i>   | const MKL_INT*                                                                                                                  | Array of size <i>nd order</i> that defines the order of the derivatives to be computed at the interpolation sites. If all the elements in <i>d order</i> are zero, the library computes the spline values only. If you do not need interpolation computations, set <i>nd order</i> to zero and pass a NULL pointer to <i>d order</i> .                                                                                                 |
| <i>data hint</i> | const float* for<br>dfsInterpolate1D/<br>dfsInterpolateEx1D<br><br>const double* for<br>dfdInterpolate1D/<br>dfdInterpolateEx1D | Array that contains additional information about the structure of partition <i>x</i> and interpolation sites. This data helps to speed up the computation. If you provide a NULL pointer, the routine uses the default settings for computations. For details on the <i>data hint</i> array, see table " <a href="#">Structure of the data hint Array</a> ".                                                                           |
| <i>r</i>         | float* for<br>dfsInterpolate1D/<br>dfsInterpolateEx1D<br><br>double* for<br>dfdInterpolate1D/<br>dfdInterpolateEx1D             | Array for results. If you do not need spline-based interpolation, set this pointer to NULL.                                                                                                                                                                                                                                                                                                                                            |
| <i>rh int</i>    | const MKL_INT                                                                                                                   | A flag describing the structure of the results. For the list of possible values of <i>rh int</i> , see table " <a href="#">Hint Values for the rh int Parameter</a> ". If you set the flag to DF_NO_HINT, the library stores the result in row-major format.                                                                                                                                                                           |
| <i>cell</i>      | MKL_INT*                                                                                                                        | Array of cell indices in partition <i>x</i> that contain the interpolation sites. Provide this parameter as input if <i>type</i> is DF_INTERP_USER_CELL. If you do not need cell indices, set this parameter to NULL.                                                                                                                                                                                                                  |
| <i>le_cb</i>     | const dfsInterpCallback for<br>dfsInterpolateEx1D<br><br>const dfdInterpCallback for<br>dfdInterpolateEx1D                      | User-defined callback function for extrapolation at the sites to the left of the interpolation interval.<br><br>Set to NULL if you are not supplying a callback function.                                                                                                                                                                                                                                                              |

| Name                 | Type                                                                                                                       | Description                                                                                                                                                                                                                         |
|----------------------|----------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>le_params</i>     | const void*                                                                                                                | <p>Pointer to additional user-defined parameters passed by the library to the <i>le_cb</i> function.</p> <p>Set to <code>NULL</code> if there are no additional parameters or if you are not supplying a callback function.</p>     |
| <i>re_cb</i>         | constdfsInterpCallBack<br>for dfsInterpolateEx1D<br><br>constdfdInterpCallBack<br>for dfdInterpolateEx1D                   | <p>User-defined callback function for extrapolation at the sites to the right of the interpolation interval.</p> <p>Set to <code>NULL</code> if you are not supplying a callback function.</p>                                      |
| <i>re_params</i>     | const void*                                                                                                                | <p>Pointer to additional user-defined parameters passed by the library to the <i>re_cb</i> function.</p> <p>Set to <code>NULL</code> if there are no additional parameters or if you are not supplying a callback function.</p>     |
| <i>i_cb</i>          | constdfsInterpCallBack<br>for dfsInterpolateEx1D<br><br>constdfdInterpCallBack<br>for dfdInterpolateEx1D                   | <p>User-defined callback function for interpolation within the interpolation interval.</p> <p>Set to <code>NULL</code> if you are not supplying a callback function.</p>                                                            |
| <i>i_params</i>      | const void*                                                                                                                | <p>Pointer to additional user-defined parameters passed by the library to the <i>i_cb</i> function.</p> <p>Set to <code>NULL</code> if there are no additional parameters or if you are not supplying a callback function.</p>      |
| <i>search_cb</i>     | constdfsSearchCellsCal<br>lBack for<br>dfsInterpolateEx1D<br><br>constdfdSearchCellsCal<br>lBack for<br>dfdInterpolateEx1D | <p>User-defined callback function for computing indices of cells that can contain interpolation sites.</p> <p>Set to <code>NULL</code> if you are not supplying a callback function.</p>                                            |
| <i>search_params</i> | const void*                                                                                                                | <p>Pointer to additional user-defined parameters passed by the library to the <i>search_cb</i> function.</p> <p>Set to <code>NULL</code> if there are no additional parameters or if you are not supplying a callback function.</p> |

## Output Parameters

| Name          | Type | Description                                                                                                                                                                                                                                                                                                    |
|---------------|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>status</i> | int  | <p>Status of the routine:</p> <ul style="list-style-type: none"> <li>• <code>DF_STATUS_OK</code> if the routine execution completed successfully.</li> <li>• Non-zero error code if the routine execution failed. See <a href="#">"Task Status and Error Reporting"</a> for error code definitions.</li> </ul> |
| <i>r</i>      |      | Contains results of computations at the interpolation sites.                                                                                                                                                                                                                                                   |

| Name              | Type | Description                                                                                                                                   |
|-------------------|------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| <code>cell</code> |      | Array of cell indices in partition $x$ that contain the interpolation sites, which is computed if <code>type</code> is <code>DF_CELL</code> . |

## Description

The `df?Interpolate1D/df?InterpolateEx1D` routine performs spline-based computations with user-defined settings. The routine supports two types of computations for interpolation sites provided in array `site`:

### Computation Types Supported by the `df?Interpolate1D/df?InterpolateEx1D` Routines

| Type                             | Description                                                                                                                                                                                                                                                                                                                                  |
|----------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>DF_INTERP</code>           | Compute derivatives of predefined order. The derivative of the zero order is the spline value.                                                                                                                                                                                                                                               |
| <code>DF_INTERP_USER_CELL</code> | <p>Compute derivatives of predefined order given user-provided cell indices. The derivative of the zero order is the spline value.</p> <p>For this type of the computations you should provide a valid <code>cell</code> array, which holds the indices of cells in the <code>site</code> array containing relevant interpolation sites.</p> |
| <code>DF_CELL</code>             | Compute indices of cells in partition $x$ that contain the sites.                                                                                                                                                                                                                                                                            |

If the indices of cells which contain interpolation types are available before the call to `df?`

`Interpolate1D/df?InterpolateEx1D`, you can improve performance by using the `DF_INTERP_USER_CELL` computation type.

#### NOTE

If you pass any combination of `DF_INTERP`, `DF_INTERP_USER_CELL`, and `DF_CELL` computation types to the routine, the library uses the `DF_INTERP_USER_CELL` computation mode.

If you specify `DF_INTERP_USER_CELL` computation mode and a user-defined callback function for computing cell indices to `df?InterpolateEx1D`, the library uses the `DF_INTERP_USER_CELL` computation mode, and the call-back function is not called.

If the sites do not belong to interpolation interval  $[a, b]$ , the library uses:

- polynomial  $P_0$  of the spline constructed on interval  $[x_0, x_1]$  for computations at the sites to the left of  $a$ .
- polynomial  $P_{n-2}$  of the spline constructed on interval  $[x_{n-2}, x_{n-1}]$  for computations at the sites to the right of  $b$ .

Interpolation sites support the following hints:

### Hint Values for Interpolation Sites

| Value                                 | Description                                                                               |
|---------------------------------------|-------------------------------------------------------------------------------------------|
| <code>DF_NON_UNIFORM_PARTITION</code> | Partition is non-uniform.                                                                 |
| <code>DF_UNIFORM_PARTITION</code>     | Partition is uniform.                                                                     |
| <code>DF_SORTED_DATA</code>           | Interpolation sites are sorted in the ascending order and define a non-uniform partition. |

| Value      | Description                                                                                                  |
|------------|--------------------------------------------------------------------------------------------------------------|
| DF_NO_HINT | No hint is provided. By default, the partition defined by interpolation sites is interpreted as non-uniform. |

**NOTE**

If you pass a sorted array of interpolation sites to the Intel MKL, set the *sitehint* parameter to the DF\_SORTED\_DATA value. The library uses this information when choosing the search algorithm and ignores any other data hints about the structure of the interpolation sites.

Data Fitting computation routines can use the following hints to speed up the computation:

- DF\_UNIFORM\_PARTITION describes the structure of breakpoints and the interpolation sites.
- DF\_QUASI\_UNIFORM\_PARTITION describes the structure of breakpoints.

Pass the above hints to the library when appropriate.

For spline-based interpolation, you should set the derivatives whose values are required for the computation. You can provide the derivatives by setting the *dorder* array of size *ndorder* as follows:

$$dorder[i] = \begin{cases} 1, & \text{if derivative of the } i\text{-th order is required} \\ 0, & \text{otherwise} \end{cases} \quad i = 0, \dots, ndorder - 1$$

Orders of derivatives  $i_d (d = 0, 1, \dots, nder - 1)$ , corresponding to non-zero derivatives to be calculated, form the array  $\{i_d\}$  of length  $nder \leq ndorder$ .

The storage format for the interpolation results is specified using the *rhint* parameter values. For each storage format, Table [Hint Values for the \*rhint\* Parameter](#) describes how to get the result  $R(j, s, i_d)$  from array *r*, for function index  $j (0 \leq j \leq yn - 1)$ , site number  $s (0 \leq s \leq nsite - 1)$ , and derivative index  $i_d (0 \leq d \leq nder - 1)$ , where *yn* is the number of functions, *nsite* is the number of sites, and *nder* is the total number of non-zero derivatives for interpolation. The array *r* can be either a one-dimensional array of size  $ny * nder * nsite$  or a three-dimensional array with the dimensions described in the table.

**Hint Values for the *rhint* Parameter**

| Value                                                              | Location of $R(j, s, i_d)$ , One-dimensional Array Storage | Location of $R(j, s, i_d)$ , Three-dimensional Array Storage    |
|--------------------------------------------------------------------|------------------------------------------------------------|-----------------------------------------------------------------|
| DF_MATRIX_STORAGE_FU<br>NCS_SITES_DERS<br>(DF_MATRIX_STORAGE_ROWS) | $r[d + nder * (s + nsite * j)]$                            | $r[j][s][d]$<br><br><i>r</i> declared as $r[ny][nsite][nder]$ . |
| DF_MATRIX_STORAGE_FU<br>NCS_DERS_SITES<br>(DF_MATRIX_STORAGE_COLS) | $r[s + nsite * (d + nder * j)]$                            | $r[j][d][s]$<br><br><i>r</i> declared as $r[ny][nder][nsite]$ . |
| DF_MATRIX_STORAGE_SI<br>TES_FUNCS_DERS                             | $r[d + nder * (j + ny * s)]$                               | $r[s][j][d]$<br><br><i>r</i> declared as $r[nsite][ny][nder]$ . |
| DF_MATRIX_STORAGE_SI<br>TES_DERS_FUNCS                             | $r[j + ny * (d + nder * s)]$                               | $r[s][d][j]$<br><br><i>r</i> declared as $r[nsite][nder][ny]$ . |

| Value      | Location of $R(j, s, i_d)$ , One-dimensional Array Storage                                                     | Location of $R(j, s, i_d)$ , Three-dimensional Array Storage |
|------------|----------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------|
| DF_NO_HINT | No hint is provided. By default, the results are stored as in $rhint =$<br>DF_MATRIX_STORAGE_FUNCS_SITES_DERS. |                                                              |

The following figures show the structure of the storage formats. Each shows sequential memory layout line by line, left to right.

- Storage in  $r$  for  $rhint =$  DF\_MATRIX\_STORAGE\_FUNCS\_SITES\_DERS (DF\_MATRIX\_STORAGE\_ROWS) :

|                        |                        |     |                               |
|------------------------|------------------------|-----|-------------------------------|
| $R(0, 0, i_0)$         | $R(0, 0, i_1)$         | ... | $R(0, 0, i_{nder-1})$         |
| $R(0, 1, i_0)$         | $R(0, 1, i_1)$         | ... | $R(0, 1, i_{nder-1})$         |
| ...                    | ...                    | ... | ...                           |
| $R(0, nsite - 1, i_0)$ | $R(0, nsite - 1, i_1)$ | ... | $R(0, nsite - 1, i_{nder-1})$ |
| $R(1, 0, i_0)$         | $R(1, 0, i_1)$         | ... | $R(1, 0, i_{nder-1})$         |
| $R(1, 1, i_0)$         | $R(1, 1, i_1)$         | ... | $R(1, 1, i_{nder-1})$         |
| ...                    | ...                    | ... | ...                           |
| $R(1, nsite - 1, i_0)$ | $R(1, nsite - 1, i_1)$ | ... | $R(1, nsite - 1, i_{nder-1})$ |
| ...                    | ...                    | ... | ...                           |

- Storage in  $r$  for  $rhint =$  DF\_MATRIX\_STORAGE\_FUNCS\_DERS\_SITES (DF\_MATRIX\_STORAGE\_COLS) :

|                       |                       |     |                               |
|-----------------------|-----------------------|-----|-------------------------------|
| $R(0, 0, i_0)$        | $R(0, 1, i_0)$        | ... | $R(0, nsite - 1, i_0)$        |
| $R(0, 0, i_1)$        | $R(0, 1, i_1)$        | ... | $R(0, nsite - 1, i_1)$        |
| ...                   | ...                   | ... | ...                           |
| $R(0, 0, i_{nder-1})$ | $R(0, 1, i_{nder-1})$ | ... | $R(0, nsite - 1, i_{nder-1})$ |
| $R(1, 0, i_0)$        | $R(1, 1, i_0)$        | ... | $R(1, nsite - 1, i_0)$        |
| $R(1, 0, i_1)$        | $R(1, 1, i_1)$        | ... | $R(1, nsite - 1, i_1)$        |
| ...                   | ...                   | ... | ...                           |
| $R(1, 0, i_{nder-1})$ | $R(1, 1, i_{nder-1})$ | ... | $R(1, nsite - 1, i_{nder-1})$ |
| ...                   | ...                   | ... | ...                           |

- Storage in  $r$  for  $rhint =$  DF\_MATRIX\_STORAGE\_SITES\_FUNCS\_DERS :

$$\begin{array}{cccc}
 R(0, 0, i_0) & R(0, 0, i_1) & \dots & R(0, 0, i_{nder-1}) \\
 R(1, 0, i_0) & R(1, 0, i_1) & \dots & R(1, 0, i_{nder-1}) \\
 \dots & \dots & \dots & \dots \\
 R(ny-1, 0, i_0) & R(ny-1, 0, i_1) & \dots & R(ny-1, 0, i_{nder-1}) \\
 \\
 R(0, 1, i_0) & R(0, 1, i_1) & \dots & R(0, 1, i_{nder-1}) \\
 R(1, 1, i_0) & R(1, 1, i_1) & \dots & R(1, 1, i_{nder-1}) \\
 \dots & \dots & \dots & \dots \\
 R(ny-1, 1, i_0) & R(ny-1, 1, i_1) & \dots & R(ny-1, 1, i_{nder-1}) \\
 \\
 \dots & \dots & \dots & \dots
 \end{array}$$

- Storage in *r* for *rhint* = DF\_MATRIX\_STORAGE\_SITES\_DERS\_FUNCS:

$$\begin{array}{cccc}
 R(0, 0, i_0) & R(1, 0, i_0) & \dots & R(ny-1, 0, i_0) \\
 R(0, 0, i_1) & R(1, 0, i_1) & \dots & R(ny-1, 0, i_1) \\
 \dots & \dots & \dots & \dots \\
 R(0, 0, i_{nder-1}) & R(1, 0, i_{nder-1}) & \dots & R(ny-1, 0, i_{nder-1}) \\
 \\
 R(0, 1, i_0) & R(1, 1, i_0) & \dots & R(ny-1, 1, i_0) \\
 R(0, 1, i_1) & R(1, 1, i_1) & \dots & R(ny-1, 1, i_1) \\
 \dots & \dots & \dots & \dots \\
 R(0, 1, i_{nder-1}) & R(1, 1, i_{nder-1}) & \dots & R(ny-1, 1, i_{nder-1}) \\
 \\
 \dots & \dots & \dots & \dots
 \end{array}$$

To speed up Data Fitting computations, use the *datahint* parameter that provides additional information about the structure of the partition and interpolation sites. This data represents a floating-point or a double array with the following structure:

**Structure of the *datahint* Array**

| Element Number | Description                    |
|----------------|--------------------------------|
| 0              | Task dimension                 |
| 1              | Type of additional information |
| 2              | Reserved field                 |

| Element Number | Description                                                         |
|----------------|---------------------------------------------------------------------|
| 3              | The total number $q$ of elements containing additional information. |
| 4              | Element (1)                                                         |
| ...            | ...                                                                 |
| $q+3$          | Element ( $q$ )                                                     |

Data Fitting computation functions support the following types of additional information for `datahint[1]`:

### Types of Additional Information

| Type                        | Element Number | Parameter                                                                   |
|-----------------------------|----------------|-----------------------------------------------------------------------------|
| DF_NO_APRIORI_INFO          | 0              | No parameters are provided. Information about the data structure is absent. |
| DF_APRIORI_MOST_LIKELY_CELL | 1              | Index of the cell that is likely to contain interpolation sites.            |

To compute indices of the cells that contain interpolation sites, provide the pointer to the array of size `nsite` for the results. The library supports the following scheme of cell indexing for the given partition  $\{x_i\}$ ,  $i=1, \dots, nx$ :

$$cell[j] = i, \text{ if } site[j] \in [x_i, x_{i+1}), i = 0, \dots, nx - 2,$$

$$cell[j] = nx - 1, \text{ if } site[j] \in [x_{nx-1}, x_{nx}],$$

$$cell[j] = nx, \text{ if } site[j] \in (x_{nx}, x_{nx+1}],$$

where

- $x_0 = -\infty$
- $x_{nx+1} = +\infty$
- $j = 0, \dots, n_{site}-1$

To perform interpolation computations with spline types unsupported in the Data Fitting component, use the extended version of the routine `df?InterpolateEx1D`. With this routine, you can provide user-defined callback functions for computations within, to the left of, or to the right of interpolation interval  $[a, b]$ . The callback functions compute indices of the cells that contain the specified interpolation sites or can serve as an approximation for computing the exact indices of such cells.

If you do not pass any function for computations at the sites outside the interval  $[a, b]$ , the routine uses the default settings.

### See Also

[Mathematical Conventions for Data Fitting Functions](#)

[df?InterpCallback](#)

[df?SearchCellsCallback](#)

### [df?Integrate1D/df?IntegrateEx1D](#)

*Computes a spline-based integral.*

### Syntax

```
status = dfsIntegrate1D(task, method, nlim, llim, llimhint, rlim, rlimhint, ldatahint,
rdatahint, r, rhint)
```

```
status = dfdIntegrate1D(task, method, nlim, llim, llimhint, rlim, rlimhint, ldatahint,
rdatahint, r, rhint)
```

```
status = dfsIntegrateEx1D(task, method, nlim, llim, llimhint, rlim, rlimhint,
ldatahint, rdatahint, r, rhint, le_cb, le_params, re_cb, re_params, i_cb, i_params,
search_cb, search_params)
```

```
status = dfdIntegrateEx1D(task, method, nlim, llim, llimhint, rlim, rlimhint,
ldatahint, rdatahint, r, rhint, le_cb, le_params, re_cb, re_params, i_cb, i_params,
search_cb, search_params)
```

## Include Files

- mkl.h

## Input Parameters

| Name             | Type                                                                                                                    | Description                                                                                                                                                                                                                                                                                                                     |
|------------------|-------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>      | DFTaskPtr                                                                                                               | Descriptor of the task.                                                                                                                                                                                                                                                                                                         |
| <i>method</i>    | const MKL_INT                                                                                                           | Integration method. The supported value is DF_METHOD_PP.                                                                                                                                                                                                                                                                        |
| <i>nlim</i>      | const MKL_INT                                                                                                           | Number of pairs of integration limits.                                                                                                                                                                                                                                                                                          |
| <i>llim</i>      | const float* for<br>dfsIntegrate1D/<br>dfsIntegrateEx1D<br><br>const double* for<br>dfdIntegrate1D/<br>dfdIntegrateEx1D | Array of size <i>nlim</i> that defines the left-side integration limits.                                                                                                                                                                                                                                                        |
| <i>llimhint</i>  | const MKL_INT                                                                                                           | A flag describing the structure of the left-side integration limits <i>llim</i> . For the list of possible values of <i>llimhint</i> , see table "Hint Values for Integration Limits". If you set the flag to the DF_NO_HINT value, the library assumes that the left-side integration limits define a non-uniform partition.   |
| <i>rlim</i>      | const float* for<br>dfsIntegrate1D/<br>dfsIntegrateEx1D<br><br>const double* for<br>dfdIntegrate1D/<br>dfdIntegrateEx1D | Array of size <i>nlim</i> that defines the right-side integration limits.                                                                                                                                                                                                                                                       |
| <i>rlimhint</i>  | const MKL_INT                                                                                                           | A flag describing the structure of the right-side integration limits <i>rlim</i> . For the list of possible values of <i>rlimhint</i> , see table "Hint Values for Integration Limits". If you set the flag to the DF_NO_HINT value, the library assumes that the right-side integration limits define a non-uniform partition. |
| <i>ldatahint</i> | const float* for<br>dfsIntegrate1D/<br>dfsIntegrateEx1D                                                                 | Array that contains additional information about the structure of partition <i>x</i> and left-side integration limits. For details on the <i>ldatahint</i> array, see table "Structure of the <i>datahint</i> Array" in the description of the <code>df?</code> Interpolate1D function.                                         |

| Name             | Type                                                                                                                    | Description                                                                                                                                                                                                                                                                   |
|------------------|-------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                  | const double* for<br>dfdIntegrate1D/<br>dfdIntegrateEx1D                                                                |                                                                                                                                                                                                                                                                               |
| <i>rdatahint</i> | const float* for<br>dfsIntegrate1D/<br>dfsIntegrateEx1D<br><br>const double* for<br>dfdIntegrate1D/<br>dfdIntegrateEx1D | Array that contains additional information about the structure of partition $x$ and right-side integration limits. For details on the <i>rdatahint</i> array, see table "Structure of the <i>datahint</i> Array" in the description of the <i>df?</i> Interpolate1D function. |
| <i>rhint</i>     | const MKL_INT                                                                                                           | A flag describing the structure of the results. For the list of possible values of <i>rhint</i> , see table "Hint Values for Integration Results". If you set the flag to the DF_NO_HINT value, the library stores the results in row-major format.                           |
| <i>le_cb</i>     | constdfsIntegrCallback<br>for dfsIntegrateEx1D<br><br>constdfdIntegrCallback<br>for dfdIntegrateEx1D                    | User-defined callback function for integration on interval $[llim[i], \min(rlim[i], a))$ for $llim[i] < a$ .<br><br>Set to NULL if you are not supplying a callback function.                                                                                                 |
| <i>le_params</i> | const void*                                                                                                             | Pointer to additional user-defined parameters passed by the library to the <i>le_cb</i> function.<br><br>Set to NULL if there are no additional parameters or if you are not supplying a callback function.                                                                   |
| <i>re_cb</i>     | constdfsInterpCallback<br>for dfsIntegrateEx1D<br><br>constdfdInterpCallback<br>for dfdIntegrateEx1D                    | User-defined callback function for integration on interval $[\max(llim[i], b), rlim[i])$ for $rlim[i] \geq b$ .<br><br>Set to NULL if you are not supplying a callback function.                                                                                              |
| <i>re_params</i> | const void*                                                                                                             | Pointer to additional user-defined parameters passed by the library to the <i>re_cb</i> function.<br><br>Set to NULL if there are no additional parameters or if you are not supplying a callback function.                                                                   |
| <i>i_cb</i>      | constdfsIntegrCallback<br>for dfsIntegrateEx1D<br><br>constdfdIntegrCallback<br>for dfdIntegrateEx1D                    | User-defined callback function for integration on interval $[\max(a, llim[i], ), \min(rlim[i], b))$ .<br><br>Set to NULL if you are not supplying a callback function.                                                                                                        |
| <i>i_params</i>  | const void*                                                                                                             | Pointer to additional user-defined parameters passed by the library to the <i>i_cb</i> function.<br><br>Set to NULL if there are no additional parameters or if you are not supplying a callback function.                                                                    |
| <i>search_cb</i> | constdfsSearchCellsCal<br>lBack for<br>dfsIntegrateEx1D                                                                 | User-defined callback function for computing indices of cells that can contain interpolation sites.<br><br>Set to NULL if you are not supplying a callback function.                                                                                                          |

| Name                 | Type                                                    | Description                                                                                                                                                                                                     |
|----------------------|---------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                      | constdfdSearchCellsCal<br>lBack for<br>dfdIntegrateEx1D |                                                                                                                                                                                                                 |
| <i>search_params</i> | const void*                                             | Pointer to additional user-defined parameters passed by the library to the <i>search_cb</i> function.<br><br>Set to NULL if there are no additional parameters or if you are not supplying a callback function. |

**Output Parameters**

| Name          | Type                                                                                                  | Description                                                                                                                                                                                                                                                                |
|---------------|-------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>status</i> | int                                                                                                   | Status of the routine: <ul style="list-style-type: none"> <li>• DF_STATUS_OK if the routine execution completed successfully.</li> <li>• Non-zero error code if the routine execution failed. See "Task Status and Error Reporting" for error code definitions.</li> </ul> |
| <i>r</i>      | float* for dfsIntegrate1D/<br>dfsIntegrateEx1D<br><br>double* for dfdIntegrate1D/<br>dfdIntegrateEx1D | Array of integration results. The size of the array should be sufficient to hold <i>nlim*ny</i> values, where <i>ny</i> is the dimension of the vector-valued function. The integration results are packed according to the settings in <i>rhint</i> .                     |

**Description**

The df?Integrate1D/df?IntegrateEx1D routine computes spline-based integral on user-defined intervals

$$I(i, j) = \int_{r_i}^{r_i} f_j(x) dx$$

where  $r_i = rlim[i]$ ,  $l_i = llim[i]$ , and  $i = 0, \dots, ny - 1$ .

If  $rlim[i] < llim[i]$ , the routine returns

$$I(i, j) = -\int_{r_i}^{l_i} f_j(x) dx$$

The routine supports the following hint values for integration results:

**Hint Values for Integration Results**

| Value                  | Description                                                                                                                      |
|------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| DF_MATRIX_STORAGE_ROWS | Data is stored in row-major format according to C conventions.                                                                   |
| DF_MATRIX_STORAGE_COLS | Data is stored in column-major format according to Fortran conventions.                                                          |
| DF_NO_HINT             | No hint is provided. By default, the coordinates of vector-valued function <i>y</i> are provided and stored in row-major format. |

A common structure of the storage formats for the integration results is as follows:

- Row-major format

---

|                |     |                       |
|----------------|-----|-----------------------|
| $I(0, 0)$      | ... | $I(0, nlim - 1)$      |
| ...            | ... | ...                   |
| $I(ny - 1, 0)$ | ... | $I(ny - 1, nlim - 1)$ |

---

- Column-major format

---

|                  |     |                       |
|------------------|-----|-----------------------|
| $I(0, 0)$        | ... | $I(ny - 1, 0)$        |
| ...              | ... | ...                   |
| $I(0, nlim - 1)$ | ... | $I(ny - 1, nlim - 1)$ |

---

Using the *llimhint* and *rlimhint* parameters, you can provide the following hint values for integration limits:

### Hint Values for Integration Limits

---

| Value                    | Description                                                                                             |
|--------------------------|---------------------------------------------------------------------------------------------------------|
| DF_SORTED_DATA           | Integration limits are sorted in the ascending order and define a non-uniform partition.                |
| DF_NON_UNIFORM_PARTITION | Partition defined by integration limits is non-uniform.                                                 |
| DF_UNIFORM_PARTITION     | Partition defined by integration limits is uniform.                                                     |
| DF_NO_HINT               | No hint is provided. By default, partition defined by integration limits is interpreted as non-uniform. |

---

To compute integration with splines unsupported in the Data Fitting component, use the extended version of the routine `df?IntegrateEx1D`. With this routine, you can provide user-defined callback functions that compute:

- integrals within, to the left of, or to the right of the interpolation interval  $[a, b]$
- indices of cells that contain the provided integration limits or can serve as an approximation for computing the exact indices of such cells

If you do not pass callback functions, the routine uses the default settings.

### See Also

[Mathematical Conventions for Data Fitting Functions](#)

[df?Interpolate1D/df?InterpolateEx1D](#)

[df?IntegrCallback](#)

[df?SearchCellsCallback](#)

[df?SearchCells1D/df?SearchCellsEx1D](#)

*Searches sub-intervals containing interpolation sites.*

---

### Syntax

```
status = dfsSearchCells1D(task, method, nsite, site, sitehint, datahint, cell)
```

```
status = dfdSearchCells1D(task, method, nsite, site, sitehint, datahint, cell)
```

```
status = dfsSearchCellsEx1D(task, method, nsite, site, sitehint, datahint, cell,
search_cb, search_params)
```

```
status = dfdSearchCellsEx1D(task, method, nsite, site, sitehint, datahint, cell,
search_cb, search_params)
```

## Include Files

- mkl.h

## Input Parameters

| Name                 | Type                                                                                                                            | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|----------------------|---------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>task</i>          | DFTaskPtr                                                                                                                       | Descriptor of the task.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>method</i>        | const MKL_INT                                                                                                                   | Search method. The supported value is DF_METHOD_STD.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <i>nsite</i>         | const MKL_INT*                                                                                                                  | Number of interpolation sites.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <i>site</i>          | const float* for<br>dfsSearchCells1D/<br>dfsSearchCellsEx1D<br><br>const double* for<br>dfdSearchCells1D/<br>dfdSearchCellsEx1D | <p>Array of interpolation sites of size <i>nsite</i>. The structure of the array is defined by the <i>sitehint</i> parameter:</p> <ul style="list-style-type: none"> <li>• If the sites form a non-uniform partition, the array should contain <i>nsite</i> values.</li> <li>• If the sites form a uniform partition, the array should contain two entries that represent the left-most and the right-most interpolation sites. The first entry of the array contains the left-most interpolation point. The second entry of the array contains the right-most interpolation point.</li> </ul> |
| <i>sitehint</i>      | const MKL_INT                                                                                                                   | A flag describing the structure of the interpolation sites. For the list of possible values of <i>sitehint</i> , see table " <a href="#">Hint Values for Interpolation Sites</a> ". If you set the flag to DF_NO_HINT, the library interprets the site-defined partition as non-uniform.                                                                                                                                                                                                                                                                                                       |
| <i>datahint</i>      | const float* for<br>dfsSearchCells1D/<br>dfsSearchCellsEx1D<br><br>const double* for<br>dfdSearchCells1D/<br>dfdSearchCellsEx1D | Array that contains additional information about the structure of the partition and interpolation sites. This data helps to speed up the computation. If you provide a NULL pointer, the routine uses the default settings for computations. For details on the <i>datahint</i> array, see table " <a href="#">Structure of the datahint Array</a> ".                                                                                                                                                                                                                                          |
| <i>search_cb</i>     | constdfsSearchCellsCallBack for dfsSearchCellsEx1D<br><br>constdfdSearchCellsCallBack for dfdSearchCellsEx1D                    | <p>User-defined callback function for computing indices of cells that can contain interpolation sites.</p> <p>Set to NULL if you are not supplying a callback function.</p>                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>search_params</i> | const void*                                                                                                                     | <p>Pointer to additional user-defined parameters passed by the library to the <i>search_cb</i> function.</p> <p>Set to NULL if there are no additional parameters or if you are not supplying a callback function.</p>                                                                                                                                                                                                                                                                                                                                                                         |

## Output Parameters

| Name                | Type                  | Description                                                                                                                                                                                                                                                                                             |
|---------------------|-----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code>      | Status of the routine: <ul style="list-style-type: none"> <li>• <code>DF_STATUS_OK</code> if the routine execution completed successfully.</li> <li>• Non-zero error code if the routine execution failed. See "<a href="#">Task Status and Error Reporting</a>" for error code definitions.</li> </ul> |
| <code>cell</code>   | <code>MKL_INT*</code> | Array of cell indices in the partition that contain the interpolation sites.                                                                                                                                                                                                                            |

## Description

The `df?SearchCells1D/df?SearchCellsEx1D` routines return array `cell` of indices of sub-intervals (cells) in the partition that contain interpolation sites available in array `site`. For details on the cell indexing scheme, see the description of the `df?Interpolate1D/df?InterpolateEx1D` computation routines.

Use the `datahint` parameter to provide additional information about the structure of the partition and/or interpolation sites. The definition of the `datahint` parameter is available in the description of the `df?Interpolate1D/df?InterpolateEx1D` computation routines.

For description of the user-defined callback for computation of cell indices, see `df?SearchCellsCallBack`.

## See Also

[Mathematical Conventions for Data Fitting Functions](#)

[df?Interpolate1D/df?InterpolateEx1D](#)

[df?SearchCellsCallBack](#)

## `df?InterpCallBack`

*A callback function for user-defined interpolation to be passed into `df?InterpolateEx1D`.*

## Syntax

```
status = dfsInterpCallBack(n, cell, site, r, user_params, library_params)
```

```
status = dfdInterpCallBack(n, cell, site, r, user_params, library_params)
```

## Include Files

- `mk1.h`

## Input Parameters

| Name              | Type                                                                                                              | Description                                                                                                                      |
|-------------------|-------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>    | <code>long long*</code>                                                                                           | Number of interpolation sites.                                                                                                   |
| <code>cell</code> | <code>long long*</code>                                                                                           | Array of size <code>n</code> containing indices of the cells to which the interpolation sites in array <code>site</code> belong. |
| <code>site</code> | <code>float*</code> for <code>dfsInterpCallBack</code><br><code>double*</code> for <code>dfdInterpCallBack</code> | Array of interpolation sites of size <code>n</code> .                                                                            |

| Name                        | Type                                        | Description                                                     |
|-----------------------------|---------------------------------------------|-----------------------------------------------------------------|
| <code>user_params</code>    | <code>void*</code>                          | Pointer to user-defined parameters of the callback function.    |
| <code>library_params</code> | <code>dfInterpCallBackLibraryParams*</code> | Pointer to library-defined parameters of the callback function. |

## Output Parameters

| Name                | Type                                                                                                              | Description                                                                                                                                                                                                                                                                                                                                                        |
|---------------------|-------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code>                                                                                                  | <p>The status returned by the callback function:</p> <ul style="list-style-type: none"> <li>• Zero indicates successful completion of the callback operation.</li> <li>• A negative value indicates an error.</li> <li>• A positive value indicates a warning.</li> </ul> <p>See "<a href="#">Task Status and Error Reporting</a>" for error code definitions.</p> |
| <code>r</code>      | <code>float*</code> for <code>dfsInterpCallBack</code><br><code>double*</code> for <code>dfdInterpCallBack</code> | Array of the computed interpolation results packed in row-major format.                                                                                                                                                                                                                                                                                            |

## Description

When passed into the `df?InterpolateEx1D` routine, this function performs user-defined interpolation operation.

The `library_params` parameter allows the library to provide extra parameters. Currently no parameters are provided.

## See Also

[df?Interpolate1D/df?InterpolateEx1D](#)  
[df?SearchCellsCallBack](#)

### [df?IntegrCallBack](#)

*A callback function that you can pass into `df?IntegrateEx1D` to define integration computations.*

## Syntax

```
status = dfsIntegrCallBack(n, lcell, llim, rcell, rlim, r, user_params, library_params)
```

```
status = dfdIntegrCallBack(n, lcell, llim, rcell, rlim, r, user_params, library_params)
```

## Include Files

- `mkl.h`

## Input Parameters

| Name                             | Type                                                                                    | Description                                                                                                            |
|----------------------------------|-----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|
| <i>n</i>                         | long long*                                                                              | Number of pairs of integration limits.                                                                                 |
| <i>lcell</i>                     | long long*                                                                              | Array of size <i>n</i> with indices of the cells that contain the left-side integration limits in array <i>llim</i> .  |
| <i>llim</i>                      | float* for <code>dfsIntegrCallback</code><br>double* for <code>dfdIntegrCallback</code> | Array of size <i>n</i> that holds the left-side integration limits.                                                    |
| <i>rcell</i>                     | long long*                                                                              | Array of size <i>n</i> with indices of the cells that contain the right-side integration limits in array <i>rlim</i> . |
| <i>rlim</i>                      | float* for <code>dfsIntegrCallback</code><br>double* for <code>dfdIntegrCallback</code> | Array of size <i>n</i> that holds the right-side integration limits.                                                   |
| <i>user_para</i><br><i>ms</i>    | void*                                                                                   | Pointer to user-defined parameters of the callback function.                                                           |
| <i>library_p</i><br><i>arams</i> | <code>dfIntegrCallbackLibraryParams*</code>                                             | Pointer to library-defined parameters of the callback function.                                                        |

## Output Parameters

| Name          | Type                                                                                    | Description                                                                                                                                                                                                                                                                                                                                            |
|---------------|-----------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>status</i> | int                                                                                     | The status returned by the callback function: <ul style="list-style-type: none"> <li>• Zero indicates successful completion of the callback operation.</li> <li>• A negative value indicates an error.</li> <li>• A positive value indicates a warning.</li> </ul> See " <a href="#">Task Status and Error Reporting</a> " for error code definitions. |
| <i>r</i>      | float* for <code>dfsIntegrCallback</code><br>double* for <code>dfdIntegrCallback</code> | Array of integration results. For packing the results in row-major format, follow the instructions described in <a href="#">df?Interpolate1D/df?InterpolateEx1D</a> .                                                                                                                                                                                  |

## Description

When passed into the `df?IntegrateEx1D` routine, this function defines integration computations. If at least one of the integration limits is outside the interpolation interval  $[a, b]$ , the library decomposes the integration into sub-intervals that belong to the extrapolation range to the left of  $a$ , the extrapolation range to the right of  $b$ , and the interpolation interval  $[a, b]$ , as follows:

- If the left integration limit is to the left of the interpolation interval ( $llim < a$ ), the `df?IntegrateEx1D` routine passes *llim* as the left integration limit and  $\min(rlim, a)$  as the right integration limit to the user-defined callback function.

- If the right integration limit is to the right of the interpolation interval ( $rlim > b$ ), the `df?IntegrateEx1D` routine passes  $\max(llim, b)$  as the left integration limit and  $rlim$  as the right integration limit to the user-defined callback function.
- If the left and the right integration limits belong to the interpolation interval, the `df?IntegrateEx1D` routine passes them to the user-defined callback function unchanged.

The value of the integral is the sum of integral values obtained on the sub-intervals.

The `library_params` parameter allows the library to provide extra parameters. Currently no parameters are provided.

### Optimization Notice

Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSSE3 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice.

Notice revision #20110804

### See Also

[df?Integrate1D/df?IntegrateEx1D](#)

[df?IntegrCallBack](#)

[df?SearchCellsCallBack](#)

### df?SearchCellsCallBack

A callback function for user-defined search to be passed into `df?InterpolateEx1D`, `df?IntegrateEx1D`, or `df?SearchCellsEx1D`.

### Syntax

```
status = dfsSearchCellsCallBack(n, site, cell, flag, user_params, library_params)
```

```
status = dfdSearchCellsCallBack(n, site, cell, flag, user_params, library_params)
```

### Include Files

- `mkl.h`

### Input Parameters

| Name        | Type                                                                                                        | Description                                                          |
|-------------|-------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------|
| <i>n</i>    | long long*                                                                                                  | Number of interpolation sites or integration limits.                 |
| <i>site</i> | float* for<br><code>dfsSearchCellsCallBack</code><br><br>double* for<br><code>dfdSearchCellsCallBack</code> | Array, size <i>n</i> , of interpolation sites or integration limits. |
| <i>flag</i> | int*                                                                                                        | Array of size <i>n</i> , with values set as follows:                 |

| Name                                      | Type                                        | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|-------------------------------------------|---------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>user_para</code><br><code>ms</code> | <code>void*</code>                          | <ul style="list-style-type: none"> <li>If the cell with index <code>cell[i]</code> contains <code>site[i]</code>, set <code>flag[i]</code> to 1.</li> <li>Otherwise, set <code>flag[i]</code> to zero. In this case, the library interprets the index as an approximation and computes the index of the cell containing <code>site[i]</code> by using the provided index as a starting point for the search.</li> </ul> Pointer to user-defined parameters of the callback function. |
| <code>library_params</code>               | <code>dfSearchCallBackLibraryParams*</code> | Pointer to library-defined parameters of the callback function.                                                                                                                                                                                                                                                                                                                                                                                                                      |

## Output Parameters

| Name                | Type                    | Description                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|---------------------|-------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code>        | The status returned by the callback function: <ul style="list-style-type: none"> <li>Zero indicates successful completion of the callback operation.</li> <li>A negative value indicates an error.</li> <li>The <code>DF_STATUS_EXACT_RESULT</code> status indicates that cell indices returned by the callback function are exact. In this case, you do not need to initialize entries of the <code>flag</code> array.</li> <li>A positive value indicates a warning.</li> </ul> See " <a href="#">Task Status and Error Reporting</a> " for error code definitions. |
| <code>cell</code>   | <code>long long*</code> | Array of size <code>n</code> that returns indices of the cells computed by the callback function.                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |

## Description

When passed into the `df?InterpolateEx1D`, `df?IntegrateEx1D`, or `df?SearchCellsEx1D` routine, this function performs a user-defined search.

The `library_params` parameter allows the library to provide extra parameters. The `df?InterpolateEx1D`, and `df?SearchCellsEx1D` routines do not provide extra parameters and set `library_params` to `NULL`. The `df?IntegrateEx1D` routines use this parameter to specify which type of integration limits, left or right, are provided for the callback. To do this the library declares the `dfSearchCallBackLibraryParams` structure. It currently contains one field, `limit_type_flag`, of type `int`. The field is set by the library to one of two possible values: `DF_INTEGR_SEARCH_CB_LLIM_FLAG` if the left integration limits are provided, or `DF_INTEGR_SEARCH_CB_RLIM_FLAG` if the right integration limits are provided.

## See Also

[df?Interpolate1D/df?InterpolateEx1D](#)  
[df?InterpCallBack](#)

## Data Fitting Task Destructors

---

Task destructors are routines used to delete task descriptors and deallocate the corresponding memory resources. The Data Fitting task destructor `dfDeleteTask` destroys a Data Fitting task and frees the memory.

### `dfDeleteTask`

*Destroys a Data Fitting task object and frees the memory.*

---

### Syntax

```
status = dfDeleteTask(&task)
```

### Include Files

- `mkl.h`

### Input Parameters

| Name              | Type                   | Description                        |
|-------------------|------------------------|------------------------------------|
| <code>task</code> | <code>DFTaskPtr</code> | Descriptor of the task to destroy. |

### Output Parameters

| Name                | Type             | Description                                                                                                                                                                                                                                                                      |
|---------------------|------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>status</code> | <code>int</code> | Status of the routine: <ul style="list-style-type: none"><li>• <code>DF_STATUS_OK</code> if the task is deleted successfully.</li><li>• Non-zero error code if the operation failed. See "<a href="#">Task Status and Error Reporting</a>" for error code definitions.</li></ul> |

### Description

Given a pointer to a task descriptor, this routine deletes the Data Fitting task descriptor and frees the memory allocated for the structure. If the task is deleted successfully, the routine sets the task pointer to `NULL`. Otherwise, the routine returns an error code.

# Deep Neural Network Functions

Intel® Math Kernel Library (Intel® MKL) functions for Deep Neural Networks (DNN functions) is a collection of performance primitives for Deep Neural Networks (DNN) applications optimized for Intel® architecture. The implementation of DNN functions includes a set of primitives necessary to accelerate popular image recognition topologies, such as AlexNet, Visual Geometry Group (VGG), GoogleNet, and Residual Networks (ResNet).

The primitives implement forward and backward passes for the following operations:

- Convolution: direct batched convolution
- Inner product
- Pooling: maximum, minimum, and average
- Normalization: local response normalization across channels (LRN) and batch normalization
- Activation: rectified linear neuron activation (ReLU)
- Data manipulation: multi-dimensional transposition (conversion), split, concat, sum, and scale

Intel MKL DNN primitives implement a plain C application programming interface (API) that can be used in the existing C/C++ DNN frameworks, as well as in custom DNN applications.

In addition to input and output arrays of DNN applications, the DNN primitives work with special opaque data types to represent the following:

- DNN operations.

This data type specifies the operation type (for example: convolution forward propagation, convolution backward filter propagation, and so on) and parameters (for example: the filter size for convolution, alpha and beta for normalization, and so on)

- Layouts of processed data.

This data type specifies relative location of elements of processed arrays in memory.

Input and output arrays of DNN operations are called resources. Each DNN operation requires that the resources have certain data layouts. The application can query DNN operations about the required data layouts and check whether the layouts of the resources are really the required layouts.

An application that calls Intel MKL DNN functions should involve the following stages:

## 1. Setup

Given a DNN topology, the application creates all DNN operations necessary to implement scoring, training, or other application-specific computations. To pass data from one DNN operation to the next one, some applications create intermediate conversions and allocate temporary arrays if the appropriate output and input data layouts do not match.

## 2. Execution

This stage consists of calls to the DNN primitives that apply the DNN operations, including necessary conversions, to the input, output, and temporary arrays.

This chapter describes Intel MKL DNN functions, enumerated types used, as well as array layouts and attributes required to perform DNN operations.

The following table lists Intel MKL DNN functions grouped according to their purpose.

### Intel MKL DNN Functions

| Function Name          | Description |
|------------------------|-------------|
| Handling Array Layouts |             |

| Function Name                                                                 | Description                                                                                        |
|-------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| <code>dnnLayoutCreate</code>                                                  | Creates a plain layout.                                                                            |
| <code>dnnLayoutCreateFromPrimitive</code>                                     | Creates a custom layout.                                                                           |
| <code>dnnLayoutGetMemorySize</code>                                           | Returns the size of the array specified by a layout.                                               |
| <code>dnnLayoutCompare</code>                                                 | Checks whether layouts are equal.                                                                  |
| <code>dnnLayoutDelete</code>                                                  | Deletes a layout.                                                                                  |
| <b>Handling Attributes of DNN Operations</b>                                  |                                                                                                    |
| <code>dnnPrimitiveAttributesCreate</code>                                     | Creates an attribute container.                                                                    |
| <code>dnnPrimitiveAttributesDestroy</code>                                    | Destroys an attribute container.                                                                   |
| <code>dnnPrimitiveGetAttributes</code>                                        | Returns the container with attributes set for an instance of a primitive.                          |
| <b>DNN Operations</b>                                                         |                                                                                                    |
| <code>dnnConvolutionCreate,</code><br><code>dnnGroupsConvolutionCreate</code> | Creates propagation operations for convolution layers.                                             |
| <code>dnnInnerProductCreate</code>                                            | Creates propagation operations for inner product layers.                                           |
| <code>dnnReLUCreate</code>                                                    | Creates propagation operations for rectified linear neuron activation layers.                      |
| <code>dnnLRNCreate</code>                                                     | Creates propagation operations for layers performing local response normalization across channels. |
| <code>dnnPoolingCreate</code>                                                 | Creates propagation operations for pooling layers.                                                 |
| <code>dnnBatchNormalizationCreate</code>                                      | Creates propagation operations for batch normalization layers.                                     |
| <code>dnnBatchNormalizationCreate_v2</code>                                   | Creates propagation operations for batch normalization performed using the specified method.       |
| <code>dnnSplitCreate</code>                                                   | Creates split layers.                                                                              |
| <code>dnnConcatCreate</code>                                                  | Creates concatenation layers.                                                                      |
| <code>dnnSumCreate</code>                                                     | Creates sum layers.                                                                                |
| <code>dnnScaleCreate</code>                                                   | Creates scale layers.                                                                              |
| <code>dnnConversionCreate</code>                                              | Creates conversion operations.                                                                     |
| <code>dnnExecute</code>                                                       | Performs DNN operations.                                                                           |
| <code>dnnConversionExecute</code>                                             | Performs a conversion operation.                                                                   |
| <code>dnnDelete</code>                                                        | Deletes descriptions of DNN operations.                                                            |
| <code>dnnAllocateBuffer</code>                                                | Allocates an array with a given layout.                                                            |
| <code>dnnReleaseBuffer</code>                                                 | Releases an array allocated by <code>dnnAllocateBuffer</code> .                                    |

## Enumerated Types

Intel MKL DNN functions use the following enumerated types:

- `dnnError_t`

- `dnnResourceType_t`
- `dnnAlgorithm_t`
- `dnnBorder_t`
- `dnnBatchNormalizationFlag_t`

### `dnnError_t`

Each DNN function returns a value of type `dnnError_t` that indicates completion status of the function.

| Value                                    | Meaning                                                  |
|------------------------------------------|----------------------------------------------------------|
| <code>E_SUCCESS</code>                   | The operation completed successfully.                    |
| <code>E_INCORRECT_INPUT_PARAMETER</code> | The value of one of input parameters is incorrect.       |
| <code>E_MEMORY_ERROR</code>              | Allocation of some memory failed.                        |
| <code>E_UNSUPPORTED_DIMENSION</code>     | The dimension of input or output array is not supported. |
| <code>E_UNIMPLEMENTED</code>             | The operation is not implemented.                        |

### `dnnResourceType_t`

At the execution stage, DNN operations receive a `resources` array, which contains pointers to the operation resources. The value of type `dnnResourceType_t` specifies the index of the pointer to a specific resource in this array:

| Value                                                                    | Resource pointed by<br><code>resources (Value)</code>                                      |
|--------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| <code>dnnResourceSrc, dnnResourceFrom</code>                             | Input data.                                                                                |
| <code>dnnResourceDst, dnnResourceTo</code>                               | Output data.                                                                               |
| <code>dnnResourceFilter</code>                                           | Filter data.                                                                               |
| <code>dnnResourceScaleShift</code>                                       | Scale and shift data.                                                                      |
| <code>dnnResourceBias</code>                                             | Bias data.                                                                                 |
| <code>dnnResourceDiffSrc</code>                                          | Gradient with respect to input data.                                                       |
| <code>dnnResourceDiffFilter</code>                                       | Gradient with respect to filter data.                                                      |
| <code>dnnResourceScaleShift</code>                                       | Gradient with respect to scale and shift.                                                  |
| <code>dnnResourceDiffBias</code>                                         | Gradient with respect to bias data.                                                        |
| <code>dnnResourceDiffDst</code>                                          | Gradient with respect to output data.                                                      |
| <code>dnnResourceWorkspace</code>                                        | Workspace.                                                                                 |
| <code>dnnResourceMultipleSrc, ... ,<br/>dnnResourceMultipleSrc+ k</code> | For multiple input data, elements of the input, from first to last, where $k \in [0, 8]$ . |
| <code>dnnResourceMultipleDst, ... ,<br/>dnnResourceMultipleDst+ k</code> | For multiple output data, elements of the output, from first to last.                      |
| <code>dnnResourceNumber</code>                                           | The number of elements in the <code>resources</code> array.                                |

## `dnnAlgorithm_t`

At the setup stage, the value of type `dnnAlgorithm_t` species the implementation of convolution and pooling:

| Value                                      | Meaning             |
|--------------------------------------------|---------------------|
| <code>dnnAlgorithmConvolutionDirect</code> | Direct convolution. |
| <code>dnnAlgorithmPoolingMax</code>        | Maximum pooling.    |
| <code>dnnAlgorithmPoolingMin</code>        | Minimum pooling.    |
| <code>dnnAlgorithmPoolingAvg</code>        | Average pooling.    |

## `dnnBorder_t`

At the setup stage, the value of type `dnnBorder_t` specifies the method to pad the input array if padding is necessary:

| Value                               | Meaning                              |
|-------------------------------------|--------------------------------------|
| <code>dnnBorderZeros</code>         | Pad with zeros.                      |
| <code>dnnBorderZerosAsymm</code>    | Asymmetrically pad with zeros.       |
| <code>dnnBorderExtrapolation</code> | Extrapolate based on the input data. |

## `dnnBatchNormalizationFlag_t`

At the setup stage, the value of type `dnnBatchNormalizationFlag_t` specifies the method to perform batch normalization:

| Value                                | Meaning                                                                                                                                                                                                           |
|--------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>dnnUseInputMeanVariance</code> | If this value is set, use externally calculated mean and variance as input for the forward propagation step. Otherwise, for backward propagation, use mean and variance output from the forward propagation step. |
| <code>dnnUseScaleShift</code>        | Use learnable <i>gamma</i> and <i>beta</i> parameters.                                                                                                                                                            |

## Handling Array Layouts

The type `dnnLayout_t` is a pointer to an opaque structure that describes layouts of arrays in the memory. DNN functions distinguish the following kinds of layouts:

- Plain.  
Different DNN operations can use plain layouts for their resources.
- Custom.  
Specific DNN operations require custom layouts for some of their resources.

To create a plain layout, in your application call a `dnnLayoutCreate_<Data Type>` function and pass to it the dimension, sizes, and strides of the array.

To create a custom layout for a resource of a DNN operation, call a `dnnLayoutCreateFromPrimitive_<Data Type>` function and pass to it the description of the operation and index of the resource.

To destroy a layout, call a `dnnLayoutDelete_<Data Type>` function.

## dnnLayoutCreate

*Creates a plain layout.*

---

### Syntax

```
dnnError_t dnnLayoutCreate_F32(dnnLayout_t *pLayout, size_t dimension, const size_t size[], const size_t strides[]);
```

```
dnnError_t dnnLayoutCreate_F64(dnnLayout_t *pLayout, size_t dimension, const size_t size[], const size_t strides[]);
```

### Include Files

- `mkl.h`

### Input Parameters

|                  |                                                                                            |
|------------------|--------------------------------------------------------------------------------------------|
| <i>dimension</i> | The number of dimensions of the array.                                                     |
| <i>size</i>      | The size along each dimension of the array.                                                |
| <i>strides</i>   | The distance (in elements) between consecutive elements along each dimension of the array. |

### Output Parameters

|                |                                       |
|----------------|---------------------------------------|
| <i>pLayout</i> | Pointer to the to the created layout. |
|----------------|---------------------------------------|

### Description

Each `dnnLayoutCreate` function creates a plain layout. The elements of an array that have the layout `*pLayout` are located at offsets  $(X, strides)$  from the beginning of the array where  $(. , .)$  denotes the scalar product of two vectors and the vector  $X$  is such that  $0 \leq X[i] < size[i]$  for  $i$  from 0 to  $dimension-1$ .

## dnnLayoutCreateFromPrimitive

*Creates a custom layout.*

---

### Syntax

```
dnnError_t dnnLayoutCreateFromPrimitive_F32(dnnLayout_t *pLayout, const dnnPrimitive_t primitive, dnnResourceType_t type);
```

```
dnnError_t dnnLayoutCreateFromPrimitive_F64(dnnLayout_t *pLayout, const dnnPrimitive_t primitive, dnnResourceType_t type);
```

### Include Files

- `mkl.h`

### Input Parameters

|                  |                                                 |
|------------------|-------------------------------------------------|
| <i>primitive</i> | The primitive to query for the required layout. |
| <i>type</i>      | The resource with the required layout queried.  |

## Output Parameters

*pLayout* Pointer to the layout to create.

## Description

Each `dnnLayoutCreateFromPrimitive` function creates a layout that a DNN operation requires for its resource.

## `dnnLayoutGetMemorySize`

Returns the size of the array specified by a layout.

## Syntax

```
size_t dnnLayoutGetMemorySize_F32(const dnnLayout_t Layout);
size_t dnnLayoutGetMemorySize_F64(const dnnLayout_t Layout);
```

## Include Files

- `mkl.h`

## Input Parameters

*Layout* The layout for which to compute the size.

## Output Parameters

None.

## Description

Each `dnnLayoutGetMemorySize` function returns the memory size in bytes sufficient to hold an array of a given layout.

## `dnnLayoutCompare`

Checks whether layouts are equal.

## Syntax

```
int dnnLayoutCompare_F32(const dnnLayout_t L1, const dnnLayout_t L2);
int dnnLayoutCompare_F64(const dnnLayout_t L1, const dnnLayout_t L2);
```

## Include Files

- `mkl.h`

## Input Parameters

*L1* The first layout.

*L2* The second layout.

## Output Parameters

None.





To perform a DNN operation, in your application call available Intel MKL DNN functions to do the following:

1. Create a description of a DNN operation
2. Perform the DNN operation
3. Destroy the description of the operation

To create a description of a DNN operation, call a function:

- `dnnConversionCreate_<Data Type>` for conversions
- `dnn<Operation>Create<Direction>[<Resource Name>]_<Data Type>` for other operations

Here

`<Operation>` is Convolution, Pooling, Normalization, or ReLU,

`<Direction>` is Forward or Backward,

`<Resource Name>`, optional, is Bias, Data, Filter, or an empty string,

`<Data Type>` is F32 or F64.

To perform a DNN operation, call a `dnnExecute_<Data Type>` function and pass to it the description of the operation and an array of pointers to operation resources.

Because the `dnnConversionExecute_<Data Type>` function has a simpler interface than `dnnExecute_<Data Type>`, if a DNN operation is a conversion, you can alternatively call the `dnnExecute_<Data Type>` function and pass to it the description of the conversion and pointers to input and output arrays.

To destroy a description of a DNN operation, call a `dnnDelete_<Data Type>` function.

## Notational Conventions

Descriptions of functions that create DNN operations use the following notational conventions:

|         |                                                                                                                               |
|---------|-------------------------------------------------------------------------------------------------------------------------------|
| $x, dx$ | denote <i>dimension</i> -2 spatial indices, where <i>dimension</i> specifies the number of dimensions in the input and output |
| $ic$    | denotes the index of an input channel                                                                                         |
| $oc$    | denotes the index of an output channel                                                                                        |
| $bc$    | denotes the index of an element in a batch                                                                                    |

## dnnConvolutionCreate, dnnGroupsConvolutionCreate

Creates propagation operations for convolution layers.

### Syntax

```
dnnError_t dnnConvolutionCreateForward_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t srcSize[], const size_t dstSize[], const size_t filterSize[], const size_t
convolutionStrides[], const int inputOffset[], const dnnBorder_t borderType);
```

```
dnnError_t dnnConvolutionCreateForwardBias_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t srcSize[], const size_t dstSize[], const size_t filterSize[], const size_t
convolutionStrides[], const int inputOffset[], const dnnBorder_t borderType);
```

```
dnnError_t dnnConvolutionCreateBackwardData_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t srcSize[], const size_t dstSize[], const size_t filterSize[], const size_t
convolutionStrides[], const int inputOffset[], const dnnBorder_t borderType);
```

```

dnnError_t dnnConvolutionCreateBackwardFilter_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t srcSize[], const size_t dstSize[], const size_t filterSize[], const size_t
convolutionStrides[], const int inputOffset[], const dnnBorder_t borderType);

dnnError_t dnnConvolutionCreateBackwardBias_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t dstSize[]);

dnnError_t dnnConvolutionCreateForward_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t srcSize[], const size_t dstSize[], const size_t filterSize[], const size_t
convolutionStrides[], const int inputOffset[], const dnnBorder_t borderType);

dnnError_t dnnConvolutionCreateForwardBias_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t srcSize[], const size_t dstSize[], const size_t filterSize[], const size_t
convolutionStrides[], const int inputOffset[], const dnnBorder_t borderType);

dnnError_t dnnConvolutionCreateBackwardData_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t srcSize[], const size_t dstSize[], const size_t filterSize[], const size_t
convolutionStrides[], const int inputOffset[], const dnnBorder_t borderType);

dnnError_t dnnConvolutionCreateBackwardFilter_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t srcSize[], const size_t dstSize[], const size_t filterSize[], const size_t
convolutionStrides[], const int inputOffset[], const dnnBorder_t borderType);

dnnError_t dnnConvolutionCreateBackwardBias_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t dimension, const
size_t dstSize[]);

dnnError_t dnnGroupsConvolutionCreateForward_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[], const size_t filterSize[],
const size_t convolutionStrides[], const int inputOffset[], const dnnBorder_t
borderType);

dnnError_t dnnGroupsConvolutionCreateForwardBias_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[], const size_t filterSize[],
const size_t convolutionStrides[], const int inputOffset[], const dnnBorder_t
borderType);

dnnError_t dnnGroupsConvolutionCreateBackwardData_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[], const size_t filterSize[],
const size_t convolutionStrides[], const int inputOffset[], const dnnBorder_t
borderType);

dnnError_t dnnGroupsConvolutionCreateBackwardFilter_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[], const size_t filterSize[],
const size_t convolutionStrides[], const int inputOffset[], const dnnBorder_t
borderType);

```

```

dnnError_t dnnGroupsConvolutionCreateBackwardBias_F32 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[]);

dnnError_t dnnGroupsConvolutionCreateForward_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[], const size_t filterSize[],
const size_t convolutionStrides[], const int inputOffset[], const dnnBorder_t
borderType);

dnnError_t dnnGroupsConvolutionCreateForwardBias_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[], const size_t filterSize[],
const size_t convolutionStrides[], const int inputOffset[], const dnnBorder_t
borderType);

dnnError_t dnnGroupsConvolutionCreateBackwardData_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[], const size_t filterSize[],
const size_t convolutionStrides[], const int inputOffset[], const dnnBorder_t
borderType);

dnnError_t dnnGroupsConvolutionCreateBackwardFilter_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t srcSize[], const size_t dstSize[], const size_t filterSize[],
const size_t convolutionStrides[], const int inputOffset[], const dnnBorder_t
borderType);

dnnError_t dnnGroupsConvolutionCreateBackwardBias_F64 (dnnPrimitive_t *pConvolution,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t algorithm, size_t groups, size_t
dimension, const size_t dstSize[]);

```

## Include Files

- mkl.h

## Input Parameters

|                           |                                                                                                                                                                                                                                                                      |
|---------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>algorithm</i>          | The algorithm to be used by the created propagation operation.                                                                                                                                                                                                       |
| <i>attributes</i>         | The set of attributes for the primitive.                                                                                                                                                                                                                             |
| <i>groups</i>             | The number of groups in the input and output.                                                                                                                                                                                                                        |
| <i>dimension</i>          | The number of dimensions of the input and output.                                                                                                                                                                                                                    |
| <i>srcSize</i>            | <i>srcSize</i> [ <i>i</i> ] is the size of the input along the dimension <i>i</i> .                                                                                                                                                                                  |
| <i>dstSize</i>            | <i>dstSize</i> [ <i>i</i> ] is the size of the output along the dimension <i>i</i> .                                                                                                                                                                                 |
| <i>filterSize</i>         | <i>filterSize</i> [ <i>i</i> ] is the size of the filter along the dimension <i>i</i> .                                                                                                                                                                              |
| <i>convolutionStrides</i> | Spatial intervals where to apply the filters to the input.                                                                                                                                                                                                           |
| <i>inputOffset</i>        | An array of size: <ul style="list-style-type: none"> <li>• <i>dimension</i> - 2 if <i>borderType</i> equals <code>dnnBorderZeros</code></li> <li>• <math>2 * (\textit{dimension} - 2)</math> if <i>borderType</i> equals <code>dnnBorderZerosAsymm</code></li> </ul> |

Describes the offset for the spatial domain. Values of *inputOffset* must be non-positive and specify the spatial offset to add to the indices when applying the filter to the input:

- If *borderType* equals `dnnBorderZeros`, *inputOffset*[*i*] specifies equal left and right offsets.
- If *borderType* equals `dnnBorderZerosAsymm`, *inputOffset*[*i*] specifies the left (top) offset, while *inputOffset*[*dimension - 2 + i*] specifies the right (bottom) offset.

*borderType* The method to pad the input when a summation runs out of the input array.

## Output Parameters

|                                                       |                                              |
|-------------------------------------------------------|----------------------------------------------|
| <i>pConvolution</i>                                   | Pointer to the primitive to create:          |
| <code>dnnConvolutionCreateForward</code>              | Forward without bias                         |
| <code>dnnConvolutionCreateForwardBias</code>          | Forward with bias                            |
| <code>dnnConvolutionCreateBackwardData</code>         | Backward with respect to data                |
| <code>dnnConvolutionCreateBackwardFilter</code>       | Backward with respect to filter              |
| <code>dnnConvolutionCreateBackwardBias</code>         | Backward with respect to bias                |
| <code>dnnGroupsConvolutionCreateForward</code>        | Forward without bias, with groups            |
| <code>dnnGroupsConvolutionCreateForwardBias</code>    | Forward with bias, with groups               |
| <code>dnnGroupsConvolutionCreateBackwardData</code>   | Backward with respect to data, with groups   |
| <code>dnnGroupsConvolutionCreateBackwardFilter</code> | Backward with respect to filter, with groups |
| <code>dnnGroupsConvolutionCreateBackwardBias</code>   | Backward with respect to bias, with groups   |

## Description

The `dnnConvolutionCreate` and `dnnGroupsConvolutionCreate` functions create forward and backward propagation operations for batch convolutions.

Dimensions from 0 to *dimension-3* are spatial dimensions.

The dimension *dimension-2* is interpreted as input and output channels.

The dimension *dimension-1* is interpreted as elements of the batch.

Let the input and output arrays have plain layout. The channels are divided into *groups* groups. Assuming *g* is the index of some group from 0 to *groups-1*, the output element *dst*[*x*][*oc*][*b*] is computed as follows:

$$dst[x][oc][b] = \text{sum}(dx, ic; src[x.convolutionStrides+dx+inputOffset][ic][b] * filter[dx][ic][oc][g]) + bias[oc][g],$$

where the summation is restricted to the input channels in the group *g* and '.' denotes the element-wise product of two arrays.

If the number of groups is 1, call `dnnConvolutionCreate`, otherwise, call `dnnGroupsConvolutionCreate`.

Formulas to calculate the sizes of the output spatial domain depend on the value of `borderType` as illustrated for the width:

- If `borderType` equals `dnnBorderZeros`,

$$OW = \text{floor}\left(\frac{IW - KW - 2 \cdot \text{inputOffset}[0]}{S}\right) + 1$$

- If `borderType` equals `dnnBorderZerosAsymm`,

$$OW = \text{floor}\left(\frac{IW - KW - \text{inputOffset}[0] - \text{inputOffset}[\text{dimension} - 2]}{S}\right) + 1$$

In these formulas:

`OW` is the output width (`dstSize[0]`)

`IW` is the input width (`srcSize[0]`)

`KW` is the filter size for width (`filterSize[0]`)

`S` is the convolution stride for width (`convolutionStrides[0]`)

## dnnInnerProductCreate

*Creates propagation operations for inner product layers.*

### Syntax

```
dnnError_t dnnInnerProductCreateForward_F32 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t srcSize[], size_t
outputChannels);
```

```
dnnError_t dnnInnerProductCreateForwardBias_F32 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t srcSize[], size_t
outputChannels);
```

```
dnnError_t dnnInnerProductCreateBackwardData_F32 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t srcSize[], size_t
outputChannels);
```

```
dnnError_t dnnInnerProductCreateBackwardFilter_F32 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t srcSize[], size_t
outputChannels);
```

```
dnnError_t dnnInnerProductCreateBackwardBias_F32 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t dstSize[]);
```

```
dnnError_t dnnInnerProductCreateForward_F64 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t srcSize[], size_t
outputChannels);
```

```
dnnError_t dnnInnerProductCreateForwardBias_F64 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t srcSize[], size_t
outputChannels);
```

```
dnnError_t dnnInnerProductCreateBackwardData_F64 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t srcSize[], size_t
outputChannels);
```

```
dnnError_t dnnInnerProductCreateBackwardFilter_F64 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t srcSize[], size_t
outputChannels);
```

```
dnnError_t dnnInnerProductCreateBackwardBias_F64 (dnnPrimitive_t *pInnerProduct,
dnnPrimitiveAttributes_t attributes, size_t dimentions, const size_t dstSize[]);
```

## Include Files

- mkl.h

## Input Parameters

|                       |                                                                            |
|-----------------------|----------------------------------------------------------------------------|
| <i>attributes</i>     | The set of attributes for the primitive.                                   |
| <i>dimension</i>      | The number of dimensions in the input and output.                          |
| <i>srcSize</i>        | <i>srcSize[i]</i> is the size of the input along the dimension <i>i</i> .  |
| <i>dstSize</i>        | <i>dstSize[i]</i> is the size of the output along the dimension <i>i</i> . |
| <i>outputChannels</i> | The number of output channels.                                             |

## Output Parameters

|                                                      |                                     |
|------------------------------------------------------|-------------------------------------|
| <i>pInnerProduct</i>                                 | Pointer to the primitive to create: |
| <code>dnnInnerProductCreateForward</code>            | Forward without bias                |
| <code>dnnInnerProductCreateForwardBias</code>        | Forward with bias                   |
| <code>dnnInnerProductCreateBackwardData</code>       | Backward with respect to data       |
| <code>dnnInnerProductCreateBackward</code><br>Filter | Backward with respect to filter     |
| <code>dnnInnerProductCreateBackwardBias</code>       | Backward with respect to bias       |

## Description

Each `dnnInnerProductCreate` function creates a forward or backward propagation operation for the inner product. The inner product operation is defined as:

$$IC = \prod_{i=0}^{dimension-2} srcSize[i],$$

$$dst[n][oc] = \sum_{ic=0}^{IC-1} src[n][ic] * filter[oc][ic] + bias[oc].$$

## dnnReLUCreate

Creates propagation operations for rectified linear neuron activation layers.

---

## Syntax

```
dnnError_t dnnReLUCreateForward_F32 (dnnPrimitive_t *pRelu, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t dataLayout, float negativeSlope);

dnnError_t dnnReLUCreateBackward_F32 (dnnPrimitive_t *pRelu, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t diffLayout, const dnnLayout_t dataLayout, float
negativeSlope);

dnnError_t dnnReLUCreateForward_F64 (dnnPrimitive_t *pRelu, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t dataLayout, double negativeSlope);

dnnError_t dnnReLUCreateBackward_F64 (dnnPrimitive_t *pRelu, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t diffLayout, const dnnLayout_t dataLayout, double
negativeSlope);
```

## Include Files

- mkl.h

## Input Parameters

|                      |                                          |
|----------------------|------------------------------------------|
| <i>attributes</i>    | The set of attributes for the primitive. |
| <i>dataLayout</i>    | The layout of the input.                 |
| <i>diffLayout</i>    | The layout of the destination diff.      |
| <i>negativeSlope</i> | The negative slope.                      |

## Output Parameters

|              |                                     |          |
|--------------|-------------------------------------|----------|
| <i>pRelu</i> | Pointer to the primitive to create: |          |
|              | dnnReLUCreateForward                | Forward  |
|              | dnnReLUCreateBackward               | Backward |

## Description

Each `dnnReLUCreate` function creates a forward or backward propagation operation for batch rectified linear neuron activation (ReLU). The ReLU operation is defined as:

$$dst[x] = \max(src[x], 0) + negativeSlope * \min(src[x], 0).$$

## dnnLRNCreate

*Creates propagation operations for layers performing local response normalization across channels.*

## Syntax

```
dnnError_t dnnLRNCreateForward_F32 (dnnPrimitive_t *pLrn, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t dataLayout, size_t kernelSize, float alpha, float beta,
float k);

dnnError_t dnnLRNCreateBackward_F32 (dnnPrimitive_t *pLrn, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t diffLayout, const dnnLayout_t dataLayout, size_t
kernelSize, float alpha, float beta, float k);
```

```
dnnError_t dnnLRNCreateForward_F64 (dnnPrimitive_t *pLrn, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t dataLayout, size_t kernelSize, double alpha, double
beta, double k);
```

```
dnnError_t dnnLRNCreateBackward_F64 (dnnPrimitive_t *pLrn, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t diffLayout, const dnnLayout_t dataLayout, size_t
kernelSize, double alpha, double beta, double k);
```

## Include Files

- mkl.h

## Input Parameters

|                   |                                             |
|-------------------|---------------------------------------------|
| <i>attributes</i> | The set of attributes for the primitive.    |
| <i>dataLayout</i> | The layout of the input.                    |
| <i>diffLayout</i> | The layout of the destination diff.         |
| <i>kernelSize</i> | The number of channels to normalize across. |
| <i>alpha</i>      | The $\alpha$ parameter.                     |
| <i>beta</i>       | The $\beta$ parameter.                      |
| <i>k</i>          | The $k$ parameter.                          |

## Output Parameters

|                      |                                     |
|----------------------|-------------------------------------|
| <i>pLrn</i>          | Pointer to the primitive to create: |
| dnnLRNCreateForward  | Forward                             |
| dnnLRNCreateBackward | Backward                            |

## Description

Each `dnnLRNCreate` function creates a forward or backward propagation operation for local response normalization (LRN) across channels. The LRN operation is defined as:

$$dst[x][oc][b] = \left( k + \frac{\alpha}{kernelSize} \sum_{dc = -kernelSize/2}^{kernelSize/2} (src[x][oc + dc][b])^2 \right)^{-\beta} src[x][oc][b]$$

## dnnPoolingCreate

Creates propagation operations for pooling layers.

### Syntax

```
dnnError_t dnnPoolingCreateForward_F32 (dnnPrimitive_t *pPooling,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t op, const dnnLayout_t srcLayout,
const size_t kernelSize[], const size_t kernelStride[], const int inputOffset[], const
dnnBorder_t borderType);
```

```
dnnError_t dnnPoolingCreateBackward_F32 (dnnPrimitive_t *pPooling,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t op, const dnnLayout_t srcLayout,
const size_t kernelSize[], const size_t kernelStride[], const int inputOffset[], const
dnnBorder_t borderType);
```

```
dnnError_t dnnPoolingCreateForward_F64 (dnnPrimitive_t *pPooling,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t op, const dnnLayout_t srcLayout,
const size_t kernelSize[], const size_t kernelStride[], const int inputOffset[], const
dnnBorder_t borderType);
```

```
dnnError_t dnnPoolingCreateBackward_F64 (dnnPrimitive_t *pPooling,
dnnPrimitiveAttributes_t attributes, dnnAlgorithm_t op, const dnnLayout_t srcLayout,
const size_t kernelSize[], const size_t kernelStride[], const int inputOffset[], const
dnnBorder_t borderType);
```

## Include Files

- mkl.h

## Input Parameters

|                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>op</i>           | The operation used by pooling: min, max, or average.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <i>attributes</i>   | The set of attributes for the primitive.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>srcLayout</i>    | The layout of the input.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <i>kernelSize</i>   | <i>kernelSize[i]</i> is the size of the filter along the dimension <i>i</i> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <i>kernelStride</i> | Spatial intervals by which to shift the pooling region.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| <i>inputOffset</i>  | An array of size: <ul style="list-style-type: none"> <li>• <i>dimension - 2</i> if <i>borderType</i> equals <code>dnnBorderZeros</code></li> <li>• <math>2 * (\textit{dimension} - 2)</math> if <i>borderType</i> equals <code>dnnBorderZerosAsymm</code></li> </ul> Describes the offset for the spatial domain. Values of <i>inputOffset</i> must be non-positive and specify the spatial offset to add to the indices when applying the filter to the input: <ul style="list-style-type: none"> <li>• If <i>borderType</i> equals <code>dnnBorderZeros</code>, <i>inputOffset[i]</i> specifies equal left and right offsets.</li> <li>• If <i>borderType</i> equals <code>dnnBorderZerosAsymm</code>, <i>inputOffset[i]</i> specifies the left (top) offset, while <i>inputOffset[dimension - 2 + i]</i> specifies the right (bottom) offset.</li> </ul> |
| <i>borderType</i>   | The method to pad the input when pooling runs out of the input array.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |

## Output Parameters

|                                       |                                     |
|---------------------------------------|-------------------------------------|
| <i>pPooling</i>                       | Pointer to the primitive to create: |
| <code>dnnPoolingCreateForward</code>  | Forward                             |
| <code>dnnPoolingCreateBackward</code> | Backward                            |

## Description

Each `dnnPoolingCreate` function creates a forward or backward propagation operation for pooling layers. A pooling operation is defined as:

$$op(\{src[x \cdot kernelStride + dx + inputOffset][oc][b] \mid 0 < = dx[I] < kernelSize[i]\}).$$

Formulas to calculate the sizes of the output spatial domain depend on the value of *borderType* as illustrated for the width:

- If *borderType* equals `dnnBorderZeros`,

$$OW = \text{ceil}\left(\frac{IW - KW - 2 \cdot \text{inputOffset}[0]}{S}\right) + 1$$

- If *borderType* equals `dnnBorderZerosAsymm`,

$$OW = \text{floor}\left(\frac{IW - KW - \text{inputOffset}[0] - \text{inputOffset}[\text{dimension} - 2]}{S}\right) + 1$$

In these formulas:

*OW* is the output width

*IW* is the input width

*KW* is the kernel width (`kernelSize[0]`)

*S* is the kernel stride for width (`kernelStride[0]`)

## dnnBatchNormalizationCreate

*Creates propagation operations for batch normalization layers.*

---

### Syntax

```
dnnError_t dnnBatchNormalizationCreateForward_F32 (dnnPrimitive_t *pBatchNormalization,
dnnPrimitiveAttributes_t attributes, const dnnLayout_t dataLayout, float eps);
```

```
dnnError_t dnnBatchNormalizationCreateBackwardScaleShift_F32 (dnnPrimitive_t
*pBatchNormalization, dnnPrimitiveAttributes_t attributes, const dnnLayout_t
dataLayout, float eps);
```

```
dnnError_t dnnBatchNormalizationCreateBackwardData_F32 (dnnPrimitive_t
*pBatchNormalization, dnnPrimitiveAttributes_t attributes, const dnnLayout_t
dataLayout, float eps);
```

```
dnnError_t dnnBatchNormalizationCreateForward_F64 (dnnPrimitive_t *pBatchNormalization,
dnnPrimitiveAttributes_t attributes, const dnnLayout_t dataLayout, double eps);
```

```
dnnError_t dnnBatchNormalizationCreateBackwardScaleShift_F64 (dnnPrimitive_t
*pBatchNormalization, dnnPrimitiveAttributes_t attributes, const dnnLayout_t
dataLayout, double eps);
```

```
dnnError_t dnnBatchNormalizationCreateBackwardData_F64 (dnnPrimitive_t
*pBatchNormalization, dnnPrimitiveAttributes_t attributes, const dnnLayout_t
dataLayout, double eps);
```

### Include Files

- `mkl.h`

### Input Parameters

|                   |                                          |
|-------------------|------------------------------------------|
| <i>dataLayout</i> | The layout of the input.                 |
| <i>attributes</i> | The set of attributes for the primitive. |
| <i>eps</i>        | The <i>eps</i> parameter.                |

## Output Parameters

*pBatchNormalization*

Pointer to the primitive to create:

|                                                   |                                             |
|---------------------------------------------------|---------------------------------------------|
| dnnBatchNormalization<br>CreateForward            | Forward                                     |
| dnnBatchNormalization<br>CreateBackwardData       | Backward with respect to data               |
| dnnBatchNormalization<br>CreateBackwardScaleShift | Backward with respect to scale<br>and shift |

## Description

Each `dnnBatchNormalizationCreate` function creates a forward or backward propagation operation for the batch normalization. Batch normalization is defined as:

$$dst[w][h][k][n] = \gamma[k] \frac{src[w][h][k][n] - M[k]}{\sqrt{S[k] + eps}} + \beta[k],$$

where

$w \in [1, W]$ ,  $h \in [1, H]$ ,  $n \in [1, N]$ ,  $k \in [1, K]$ ,

$\gamma[k]$  is the weight and  $\beta[k]$  is the bias of  $k$ ,

$$M[k] = \frac{1}{WHN} \sum_{whn} src[w][h][k][n],$$

$$S[k] = \frac{1}{WHN} \sum_{whn} (src[w][h][k][n] - M[k])^2,$$

and  $eps$  is the constant to improve numerical stability.

This primitive does the same as the `dnnBatchNormalizationCreate_v2` primitive called with the `dnnUseScaleShift` value of `flags` except that `dnnBatchNormalizationCreate` saves the mean and variance in the workspace buffer instead of output buffers.

## dnnBatchNormalizationCreate\_v2

*Creates propagation operations for batch normalization performed using the specified method.*

### Syntax

```
dnnError_t dnnBatchNormalizationCreate_Forward_v2_F32 (dnnPrimitive_t
*pBatchNormalization, dnnPrimitiveAttributes_t attributes, const dnnLayout_t
dataLayout, float eps, unsigned int flags);
```

```
dnnError_t dnnBatchNormalizationCreate_Backward_v2_F32 (dnnPrimitive_t
*pBatchNormalization, dnnPrimitiveAttributes_t attributes, const dnnLayout_t
dataLayout, float eps, unsigned int flags);
```

```
dnnError_t dnnBatchNormalizationCreate_Forward_v2_F64 (dnnPrimitive_t
*pBatchNormalization, dnnPrimitiveAttributes_t attributes, const dnnLayout_t
dataLayout, double eps, unsigned int flags);
```

```
dnnError_t dnnBatchNormalizationCreate_Backward_v2_F64 (dnnPrimitive_t
*pBatchNormalization, dnnPrimitiveAttributes_t attributes, const dnnLayout_t
dataLayout, double eps, unsigned int flags);
```

## Include Files

- `mkl.h`

## Input Parameters

|                         |                                                                      |
|-------------------------|----------------------------------------------------------------------|
| <code>dataLayout</code> | The layout of the input.                                             |
| <code>attributes</code> | The set of attributes for the primitive.                             |
| <code>eps</code>        | The <code>eps</code> parameter.                                      |
| <code>flags</code>      | The set of flags to define the computation method for the primitive. |

## Output Parameters

|                                  |                                     |          |
|----------------------------------|-------------------------------------|----------|
| <code>pBatchNormalization</code> | Pointer to the primitive to create: |          |
|                                  | <code>dnnBatchNormalization</code>  | Forward  |
|                                  | <code>CreateForward</code>          |          |
|                                  | <code>dnnBatchNormalization</code>  | Backward |
|                                  | <code>CreateBackward</code>         |          |

## Description

Each `dnnBatchNormalizationCreate_v2` function creates a forward or backward propagation operation for batch normalization to be performed using the computation method specified by `flags`.

Batch normalization is defined as:

$$dst[w][h][k][n] = \gamma[k] \frac{src[w][h][k][n] - M[k]}{\sqrt{S[k] + eps}} + \beta[k],$$

where

$$w \in [1, W], h \in [1, H], n \in [1, N], k \in [1, K],$$

$\gamma[k]$  is the weight and  $\beta[k]$  is the bias of  $k$ ,

$$M[k] = \frac{1}{WHN} \sum_{whn} src[w][h][k][n],$$

$$S[k] = \frac{1}{WHN} \sum_{whn} (src[w][h][k][n] - M[k])^2,$$

and `eps` is the constant to improve numerical stability.

`dnnBatchNormalizationCreate_v2` called with the `dnnUseScaleShift` value of `flags` does the same as `dnnBatchNormalizationCreate` except that `dnnBatchNormalizationCreate_v2` saves the mean and variance in separate output buffers instead of the workspace buffer.

## dnnSplitCreate

*Creates split layers.*

### Syntax

```
dnnError_t dnnSplitCreate_F32 (dnnPrimitive_t *pSplit, dnnPrimitiveAttributes_t
attributes, const size_t nDstTensors, dnnLayout_t dataLayout, size_t dstChannelSize[]);

dnnError_t dnnSplitCreate_F64 (dnnPrimitive_t *pSplit, dnnPrimitiveAttributes_t
attributes, const size_t nDstTensors, dnnLayout_t dataLayout, size_t dstChannelSize[]);
```

## Include Files

- `mkl.h`

## Input Parameters

|                             |                                                                                         |
|-----------------------------|-----------------------------------------------------------------------------------------|
| <code>attributes</code>     | The set of attributes for the primitive.                                                |
| <code>nDstTensors</code>    | The number of output tensors.                                                           |
| <code>dataLayout</code>     | The layout of the input tensor.                                                         |
| <code>dstChannelSize</code> | <code>dstChannelSize[i]</code> is the number of channels for the $i$ -th output tensor. |

## Output Parameters

`pSplit` Pointer to the primitive to create.

## Description

Each `dnnSplitCreate` function creates a split primitive. The split of a tensor into  $M$  tensors with  $C_0, C_1, \dots, C_{M-1}$  channels, respectively, and same other sizes is defined as:

$$dst_m[n][c][h][w] = src[n][\sum_{i=0}^{k-1} C_i + c][h][w], c \in [0, C_m - 1],$$

where  $\sum_{i=0}^{M-1} C_i = C$ .

## dnnConcatCreate

*Creates concatenation layers.*

---

## Syntax

```
dnnError_t dnnConcatCreate_F32 (dnnPrimitive_t *pConcat, dnnPrimitiveAttributes_t
attributes, size_t nSrcTensors, const dnnLayout_t *src);
```

```
dnnError_t dnnConcatCreate_F64 (dnnPrimitive_t *pConcat, dnnPrimitiveAttributes_t
attributes, size_t nSrcTensors, const dnnLayout_t *src);
```

## Include Files

- `mkl.h`

## Input Parameters

|                          |                                          |
|--------------------------|------------------------------------------|
| <code>attributes</code>  | The set of attributes for the primitive. |
| <code>nSrcTensors</code> | The number of input tensors.             |
| <code>src</code>         | Pointer to the layout of input tensors.  |

## Output Parameters

`pConcat` Pointer to the primitive to create.

## Description

Each `dnnConcatCreate` function creates a concatenation primitive. The concatenation tensor for  $M$  tensors with  $C_0, C_1, \dots, C_{M-1}$  channels, respectively, and same other sizes is defined as:

$$dst[n][\sum_{i=0}^{m-1} C_i + k][h][w] = src_m[n][k][h][w], k \in [0, C_m - 1],$$

where  $\sum_{i=0}^{M-1} C_i = C$ .

## dnnSumCreate

*Creates sum layers.*

---

### Syntax

```
dnnError_t dnnSumCreate_F32 (dnnPrimitive_t *pSum, dnnPrimitiveAttributes_t attributes,
const size_t nSummands, dnnLayout_t dataLayout, float *coefficients);
```

```
dnnError_t dnnSumCreate_F64 (dnnPrimitive_t *pSum, dnnPrimitiveAttributes_t attributes,
const size_t nSummands, dnnLayout_t dataLayout, float *coefficients);
```

### Include Files

- `mkl.h`

### Input Parameters

|                     |                                                    |
|---------------------|----------------------------------------------------|
| <i>attributes</i>   | The set of attributes for the primitive.           |
| <i>nSummands</i>    | The number of input layouts.                       |
| <i>dataLayout</i>   | Pointer to the layout of the input.                |
| <i>coefficients</i> | Coefficients of input tensors in the weighted sum. |

### Output Parameters

|             |                                     |
|-------------|-------------------------------------|
| <i>pSum</i> | Pointer to the primitive to create. |
|-------------|-------------------------------------|

### Description

Each `dnnSumCreate` function creates a sum primitive. The sum (weighted) of  $N$  tensors of the same sizes is defined as:

$$dst[x] = \sum_{n=0}^{N-1} \alpha_n src_n[x].$$

## dnnScaleCreate

*Creates scale layers.*

---

## Syntax

```
dnnError_t dnnScaleCreate_F32 (dnnPrimitive_t *pScale, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t dataLayout, float alpha);
```

```
dnnError_t dnnScaleCreate_F64 (dnnPrimitive_t *pScale, dnnPrimitiveAttributes_t
attributes, const dnnLayout_t dataLayout, float alpha);
```

## Include Files

- mkl.h

## Input Parameters

|                   |                                          |
|-------------------|------------------------------------------|
| <i>attributes</i> | The set of attributes for the primitive. |
| <i>dataLayout</i> | The layout of the input.                 |
| <i>alpha</i>      | The scaling factor.                      |

## Output Parameters

|               |                                     |
|---------------|-------------------------------------|
| <i>pScale</i> | Pointer to the primitive to create. |
|---------------|-------------------------------------|

## Description

Each `dnnScaleCreate` function creates a scale primitive. A scaling operation is defined as follows:

$$dst[x] = alpha * src[x].$$

The primitive supports the following kinds of scaling:

- In-place: the *src* and *dst* pointers reference the same memory block.
- Out-of-place: the *src* and *dst* pointers reference different non-interleaving memory blocks.

## dnnConversionCreate

*Creates conversion operations.*

---

## Syntax

```
dnnError_t dnnConversionCreate_F32 (dnnPrimitive_t *pConversion, const dnnLayout_t
from, const dnnLayout_t to);
```

```
dnnError_t dnnConversionCreate_F64 (dnnPrimitive_t *pConversion, const dnnLayout_t
from, const dnnLayout_t to);
```

## Include Files

- mkl.h

## Input Parameters

|             |                             |
|-------------|-----------------------------|
| <i>from</i> | The layout to convert from. |
| <i>to</i>   | The layout to convert to.   |

## Output Parameters

*pConversion* Pointer to the primitive to create.

### Description

Each `dnnConversionCreate` function creates a forward or backward propagation operation for conversion layers.

If both layouts are plain, they must have the same number of dimensions and the same size along each dimension. In this case, the following formula is applied:

$$dst[(x, stridesTo)] = src[(x, stridesFrom)],$$

where *stridesFrom* and *stridesTo* are strides in layouts from and to and  $(. , .)$  denotes the scalar product of two vectors.

If one of the layouts is custom, *dst* is the matrix-vector product  $C * src$  for a matrix  $C = C (from, to)$  such that all  $C_{ij}$  are 0 or 1,  $\sum (j; C_{ij}) \leq 1$ , and  $\sum (i; C_{ij}) \geq 1$ .

### dnnExecute

*Performs DNN operations.*

#### Syntax

```
dnnError_t dnnExecute_F32 (dnnPrimitive_t primitive, void *resources[]);
```

```
dnnError_t dnnExecute_F64 (dnnPrimitive_t primitive, void *resources[]);
```

#### Include Files

- `mkl.h`

### Input Parameters

*primitive* The description of a DNN operation to perform.

*resources* An array of pointers to the resources according to the following table:

| DNN Operation                               | Indices of input resources                                                                      |
|---------------------------------------------|-------------------------------------------------------------------------------------------------|
| Convolution forward without bias            | <code>dnnResourceSrc,</code><br><code>dnnResourceFilter</code>                                  |
| Convolution forward with bias               | <code>dnnResourceSrc,</code><br><code>dnnResourceFilter,</code><br><code>dnnResourceBias</code> |
| Convolution backward with respect to data   | <code>dnnResourceDiffDst,</code><br><code>dnnResourceFilter</code>                              |
| Convolution backward with respect to filter | <code>dnnResourceDiffDst,</code><br><code>dnnResourceSrc</code>                                 |
| Convolution backward with respect to bias   | <code>dnnResourceDiffDst</code>                                                                 |
| Inner product forward without bias          | <code>dnnResourceSrc,</code><br><code>dnnResourceFilter</code>                                  |

| <b>DNN Operation</b>                                                                                                                                            | <b>Indices of input resources</b>                                                                                                                                                                                                                  |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Inner product forward with bias                                                                                                                                 | dnnResourceSrc,<br>dnnResourceFilter,<br>dnnResourceBias                                                                                                                                                                                           |
| Inner product backward with respect to data                                                                                                                     | dnnResourceDiffDst,<br>dnnResourceFilter                                                                                                                                                                                                           |
| Inner product backward with respect to filter                                                                                                                   | dnnResourceDiffDst,<br>dnnResourceSrc                                                                                                                                                                                                              |
| Inner product backward with respect to bias                                                                                                                     | dnnResourceDiffDst                                                                                                                                                                                                                                 |
| LRN forward                                                                                                                                                     | dnnResourceSrc,<br>dnnResourceWorkspace                                                                                                                                                                                                            |
| LRN backward                                                                                                                                                    | dnnResourceDiffDst,<br>dnnResourceWorkspace                                                                                                                                                                                                        |
| ReLU forward                                                                                                                                                    | dnnResourceSrc                                                                                                                                                                                                                                     |
| ReLU backward                                                                                                                                                   | dnnResourceDiffDst                                                                                                                                                                                                                                 |
| Pooling forward                                                                                                                                                 | dnnResourceSrc,<br>dnnResourceWorkspace                                                                                                                                                                                                            |
| Pooling backward                                                                                                                                                | dnnResourceDiffDst,<br>dnnResourceWorkspace                                                                                                                                                                                                        |
| Batch normalization forward                                                                                                                                     | dnnResourceSrc,<br>dnnResourceScaleShift,<br>dnnResourceWorkspace                                                                                                                                                                                  |
| Batch normalization backward with respect to data                                                                                                               | dnnResourceDiffDst,<br>dnnResourceScaleShift,<br>dnnResourceWorkspace                                                                                                                                                                              |
| Batch normalization backward with respect to scale and shift                                                                                                    | dnnResourceSrc,<br>dnnResourceDiffDst,<br>dnnResourceWorkspace                                                                                                                                                                                     |
| Batch normalization forward with specified computation method; created in a call to <a href="#">dnnBatchNormalizationCreate_v2</a> with <i>flags</i> parameter. | dnnResourceSrc,<br>dnnResourceScaleShift if <i>flags</i> values include dnnUseScaleShift,<br>dnnResourceMean if <i>flags</i> values include dnnUseInputMeanVariance,<br>dnnResourceVariance if <i>flags</i> values include dnnUseInputMeanVariance |
| Batch normalization backward with specified computation method                                                                                                  | dnnResourceSrc,<br>dnnResourceDiffDst,<br>dnnResourceScaleShift if <i>flags</i> values include dnnUseScaleShift,<br>dnnResourceMean,<br>dnnResourceVariance                                                                                        |

| <b>DNN Operation</b> | <b>Indices of input resources</b>                                                                                              |
|----------------------|--------------------------------------------------------------------------------------------------------------------------------|
| Split                | <code>dnnResourceSrc</code>                                                                                                    |
| Concat               | <code>dnnResourceMultipleSrc,</code><br><code>dnnResourceMultipleSrc+1, ...,</code><br><code>dnnResourceMultipleSrc+N-1</code> |
| Sum                  | <code>dnnResourceMultipleSrc,</code><br><code>dnnResourceMultipleSrc+1, ...,</code><br><code>dnnResourceMultipleSrc+N-1</code> |

## Output Parameters

*resources*

An array of pointers to the resources according to the following table:

| <b>DNN Operation</b>                                         | <b>Indices of output resources</b>     |
|--------------------------------------------------------------|----------------------------------------|
| Convolution forward without bias                             | <code>dnnResourceDst</code>            |
| Convolution forward with bias                                | <code>dnnResourceDst</code>            |
| Convolution backward with respect to data                    | <code>dnnResourceDiffSrc</code>        |
| Convolution backward with respect to filter                  | <code>dnnResourceDiffFilter</code>     |
| Convolution backward with respect to bias                    | <code>dnnResourceDiffBias</code>       |
| Inner product forward without bias                           | <code>dnnResourceDst</code>            |
| Inner product forward with bias                              | <code>dnnResourceDst</code>            |
| Inner product backward with respect to data                  | <code>dnnResourceDiffSrc</code>        |
| Inner product backward with respect to filter                | <code>dnnResourceDiffFilter</code>     |
| Inner product backward with respect to bias                  | <code>dnnResourceDiffBias</code>       |
| ReLU, LRN forward                                            | <code>dnnResourceDst</code>            |
| ReLU, LRN backward                                           | <code>dnnResourceDiffSrc</code>        |
| Pooling forward                                              | <code>dnnResourceDst</code>            |
| Pooling backward                                             | <code>dnnResourceDiffSrc</code>        |
| Batch normalization forward                                  | <code>dnnResourceDst</code>            |
| Batch normalization backward with respect to data            | <code>dnnResourceDiffSrc</code>        |
| Batch normalization backward with respect to scale and shift | <code>dnnResourceDiffScaleShift</code> |

| DNN Operation                                                                                                                                                      | Indices of output resources                                                                                                                                                                                                                             |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Batch normalization forward with specified computation method; created in a call to <code>dnnBatchNormalizationCreate_v2</code> with <code>flags</code> parameter. | <code>dnnResourceDst</code> ,<br><code>dnnResourceMean</code> if <code>flags</code> values include <code>dnnUseInputMeanVariance</code> ,<br><code>dnnResourceVariance</code> if <code>flags</code> values include <code>dnnUseInputMeanVariance</code> |
| Batch normalization backward with specified computation method                                                                                                     | <code>dnnResourceDiffSrc</code> ,<br><code>dnnResourceDiffScaleShift</code> if <code>flags</code> values include <code>dnnUseScaleShift</code>                                                                                                          |
| Split                                                                                                                                                              | <code>dnnResourceMultipleDst</code> ,<br><code>dnnResourceMultipleDst+1</code> , ...,<br><code>dnnResourceMultipleDst+N-1</code>                                                                                                                        |
| Concat                                                                                                                                                             | <code>dnnResourceDst</code>                                                                                                                                                                                                                             |
| Sum                                                                                                                                                                | <code>dnnResourceDst</code>                                                                                                                                                                                                                             |

## Description

Each `dnnExecute` function performs a DNN operation on given resources. All the required resources, including the output resources, must point to arrays having a plain layout or to the arrays allocated with a `dnnAllocateBuffer` function. The `dnnExecute` functions do not use the remaining elements of the `resources` array in dereferencing operations.

The following table explains more usage details of `dnnExecute` for specific operations:

| DNN Operation                | Indices of output resources                                                                                                                                                                                                                                                                                                                |
|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Local response normalization | The resource indexed <code>dnnResourceWorkspace</code> is required on forward and backward propagations. On backward propagation, use the buffer allocated for the workspace on forward propagation.                                                                                                                                       |
| Pooling                      | The resource indexed <code>dnnResourceWorkspace</code> is required on forward and backward propagations. On backward propagation, use the buffer allocated for the workspace on forward propagation.                                                                                                                                       |
| Pooling                      | The resource indexed <code>dnnResourceWorkspace</code> is required on forward and backward propagations. On backward propagation, use the buffer allocated for the workspace on forward propagation.                                                                                                                                       |
| Batch normalization          | Perform backward batch normalization with respect to data before backward batch normalization with respect to scale and shift.<br><br>The resource indexed <code>dnnResourceWorkspace</code> is required on forward and backward propagations. On backward propagation, use the buffer allocated for the workspace on forward propagation. |

## DNN Operation

Batch normalization with specified computation method; created with `dnnBatchNormalizationCreate_v2`

## Indices of output resources

The resources indexed `dnnResourceMean` and `dnnResourceVariance` are required on forward and backward propagations. On backward propagation, use the buffers allocated for the mean and variance on forward propagation.

### See Also

[dnnResourceType\\_t](#)

## dnnConversionExecute

*Performs a conversion operation.*

### Syntax

```
dnnError_t dnnConversionExecute_F32 (dnnPrimitive_t conversion, void *from, void *to);
dnnError_t dnnConversionExecute_F64 (dnnPrimitive_t conversion, void *from, void *to);
```

### Include Files

- `mkl.h`

### Input Parameters

|                   |                                                         |
|-------------------|---------------------------------------------------------|
| <i>conversion</i> | The description of the conversion operation to perform. |
| <i>from</i>       | Pointer to the input.                                   |

### Output Parameters

|           |                        |
|-----------|------------------------|
| <i>to</i> | Pointer to the output. |
|-----------|------------------------|

### Description

Each `dnnConversionExecute` function performs a conversion operation.

## dnnDelete

*Deletes descriptions of DNN operations.*

### Syntax

```
dnnError_t dnnDelete_F32(dnnPrimitive_t primitive);
dnnError_t dnnDelete_F64(dnnPrimitive_t primitive);
```

### Include Files

- `mkl.h`

### Input Parameters

|                  |                          |
|------------------|--------------------------|
| <i>primitive</i> | The primitive to delete. |
|------------------|--------------------------|

### Output Parameters

None.





# Linear Solvers Basics

Many applications in science and engineering require the solution of a system of linear equations. This problem is usually expressed mathematically by the matrix-vector equation,  $Ax = b$ , where  $A$  is an  $m$ -by- $n$  matrix,  $x$  is the  $n$  element column vector and  $b$  is the  $m$  element column vector. The matrix  $A$  is usually referred to as the coefficient matrix, and the vectors  $x$  and  $b$  are referred to as the solution vector and the right-hand side, respectively.

Basic concepts related to solving linear systems with sparse matrices are described in [Sparse Linear Systems](#) and various storage schemes for sparse matrices are described in [Sparse Matrix Storage Formats](#).

## Sparse Linear Systems

In many real-life applications, most of the elements in  $A$  are zero. Such a matrix is referred to as sparse. Conversely, matrices with very few zero elements are called dense. For sparse matrices, computing the solution to the equation  $Ax = b$  can be made much more efficient with respect to both storage and computation time, if the sparsity of the matrix can be exploited. The more an algorithm can exploit the sparsity without sacrificing the correctness, the better the algorithm.

Generally speaking, computer software that finds solutions to systems of linear equations is called a solver. A solver designed to work specifically on sparse systems of equations is called a sparse solver. Solvers are usually classified into two groups - direct and iterative.

**Iterative Solvers** start with an initial approximation to a solution and attempt to estimate the difference between the approximation and the true result. Based on the difference, an iterative solver calculates a new approximation that is closer to the true result than the initial approximation. This process is repeated until the difference between the approximation and the true result is sufficiently small. The main drawback to iterative solvers is that the rate of convergence depends greatly on the values in the matrix  $A$ . Consequently, it is not possible to predict how long it will take for an iterative solver to produce a solution. In fact, for ill-conditioned matrices, the iterative process will not converge to a solution at all. However, for well-conditioned matrices it is possible for iterative solvers to converge to a solution very quickly. Consequently, if an application involves well-conditioned matrices iterative solvers can be very efficient.

**Direct Solvers**, on the other hand, factor the matrix  $A$  into the product of two triangular matrices and then perform a forward and backward triangular solve.

This approach makes the time required to solve a systems of linear equations relatively predictable, based on the size of the matrix. In fact, for sparse matrices, the solution time can be predicted based on the number of non-zero elements in the array  $A$ .

## Matrix Fundamentals

A matrix is a rectangular array of either real or complex numbers. A matrix is denoted by a capital letter; its elements are denoted by the same lower case letter with row/column subscripts. Thus, the value of the element in row  $i$  and column  $j$  in matrix  $A$  is denoted by  $a(i, j)$ . For example, a 3 by 4 matrix  $A$ , is written as follows:

$$A = \begin{bmatrix} a(1, 1) & a(1, 2) & a(1, 3) & a(1, 4) \\ a(2, 1) & a(2, 2) & a(2, 3) & a(2, 4) \\ a(3, 1) & a(3, 2) & a(3, 3) & a(3, 4) \end{bmatrix}$$

Note that with the above notation, we assume the standard Fortran programming language convention of starting array indices at 1 rather than the C programming language convention of starting them at 0.

A matrix in which all of the elements are real numbers is called a real matrix. A matrix that contains at least one complex number is called a complex matrix. A real or complex matrix  $A$  with the property that  $a(i,j) = a(j,i)$ , is called a symmetric matrix. A complex matrix  $A$  with the property that  $a(i,j) = \text{conj}(a(j,i))$ , is called a Hermitian matrix. Note that programs that manipulate symmetric and Hermitian matrices need only store half of the matrix values, since the values of the non-stored elements can be quickly reconstructed from the stored values.

A matrix that has the same number of rows as it has columns is referred to as a square matrix. The elements in a square matrix that have same row index and column index are called the diagonal elements of the matrix, or simply the diagonal of the matrix.

The transpose of a matrix  $A$  is the matrix obtained by “flipping” the elements of the array about its diagonal. That is, we exchange the elements  $a(i,j)$  and  $a(j,i)$ . For a complex matrix, if we both flip the elements about the diagonal and then take the complex conjugate of the element, the resulting matrix is called the Hermitian transpose or conjugate transpose of the original matrix. The transpose and Hermitian transpose of a matrix  $A$  are denoted by  $A^T$  and  $A^H$  respectively.

A column vector, or simply a vector, is a  $n \times 1$  matrix, and a row vector is a  $1 \times n$  matrix. A real or complex matrix  $A$  is said to be positive definite if the vector-matrix product  $x^T Ax$  is greater than zero for all non-zero vectors  $x$ . A matrix that is not positive definite is referred to as indefinite.

An upper (or lower) triangular matrix, is a square matrix in which all elements below (or above) the diagonal are zero. A unit triangular matrix is an upper or lower triangular matrix with all 1's along the diagonal.

A matrix  $P$  is called a permutation matrix if, for any matrix  $A$ , the result of the matrix product  $PA$  is identical to  $A$  except for interchanging the rows of  $A$ . For a square matrix, it can be shown that if  $PA$  is a permutation of the rows of  $A$ , then  $AP^T$  is the same permutation of the columns of  $A$ . Additionally, it can be shown that the inverse of  $P$  is  $P^T$ .

In order to save space, a permutation matrix is usually stored as a linear array, called a permutation vector, rather than as an array. Specifically, if the permutation matrix maps the  $i$ -th row of a matrix to the  $j$ -th row, then the  $i$ -th element of the permutation vector is  $j$ .

A matrix with non-zero elements only on the diagonal is called a diagonal matrix. As is the case with a permutation matrix, it is usually stored as a vector of values, rather than as a matrix.

## Direct Method

For solvers that use the direct method, the basic technique employed in finding the solution of the system  $Ax = b$  is to first factor  $A$  into triangular matrices. That is, find a lower triangular matrix  $L$  and an upper triangular matrix  $U$ , such that  $A = LU$ . Having obtained such a factorization (usually referred to as an  $LU$  decomposition or  $LU$  factorization), the solution to the original  $LU$  problem can be rewritten as follows.

$$\begin{aligned} Ax &= b \\ \Rightarrow LUx &= b \\ \Rightarrow L(Ux) &= b \end{aligned}$$

This leads to the following two-step process for finding the solution to the original system of equations:

1. Solve the systems of equations  $Ly = b$ .
2. Solve the system  $Ux = y$ .

Solving the systems  $Ly = b$  and  $Ux = y$  is referred to as a forward solve and a backward solve, respectively.

If a symmetric matrix  $A$  is also positive definite, it can be shown that  $A$  can be factored as  $LL^T$  where  $L$  is a lower triangular matrix. Similarly, a Hermitian matrix,  $A$ , that is positive definite can be factored as  $A = LL^H$ . For both symmetric and Hermitian matrices, a factorization of this form is called a Cholesky factorization.

In a Cholesky factorization, the matrix  $U$  in an  $LU$  decomposition is either  $L^T$  or  $L^H$ . Consequently, a solver can increase its efficiency by only storing  $L$ , and one-half of  $A$ , and not computing  $U$ . Therefore, users who can express their application as the solution of a system of positive definite equations will gain a significant performance improvement over using a general representation.

For matrices that are symmetric (or Hermitian) but not positive definite, there are still some significant efficiencies to be had. It can be shown that if  $A$  is symmetric but not positive definite, then  $A$  can be factored as  $A = LDL^T$ , where  $D$  is a diagonal matrix and  $L$  is a lower unit triangular matrix. Similarly, if  $A$  is Hermitian, it can be factored as  $A = LDL^H$ . In either case, we again only need to store  $L$ ,  $D$ , and half of  $A$  and we need not compute  $U$ . However, the backward solve phases must be amended to solving  $L^T x = D^{-1} y$  rather than  $L^T x = y$ .

### Fill-In and Reordering of Sparse Matrices

Two important concepts associated with the solution of sparse systems of equations are fill-in and reordering. The following example illustrates these concepts.

Consider the system of linear equation  $Ax = b$ , where  $A$  is a symmetric positive definite sparse matrix, and  $A$  and  $b$  are defined by the following:

$$A = \begin{bmatrix} 9 & \frac{3}{2} & 6 & \frac{3}{4} & 3 \\ \frac{3}{2} & \frac{1}{2} & * & * & * \\ 6 & * & 12 & * & * \\ \frac{3}{4} & * & * & \frac{5}{8} & * \\ 3 & * & * & * & 16 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

A star (\*) is used to represent zeros and to emphasize the sparsity of  $A$ . The Cholesky factorization of  $A$  is:  $A = LL^T$ , where  $L$  is the following:

$$L = \begin{bmatrix} 3 & * & * & * & * \\ \frac{1}{2} & \frac{1}{2} & * & * & * \\ 2 & -2 & 2 & * & * \\ \frac{1}{4} & \frac{1}{-4} & \frac{1}{-2} & \frac{1}{2} & * \\ 1 & -1 & -2 & -3 & 1 \end{bmatrix}$$

Notice that even though the matrix  $A$  is relatively sparse, the lower triangular matrix  $L$  has no zeros below the diagonal. If we computed  $L$  and then used it for the forward and backward solve phase, we would do as much computation as if  $A$  had been dense.

The situation of  $L$  having non-zeros in places where  $A$  has zeros is referred to as fill-in. Computationally, it would be more efficient if a solver could exploit the non-zero structure of  $A$  in such a way as to reduce the fill-in when computing  $L$ . By doing this, the solver would only need to compute the non-zero entries in  $L$ . Toward this end, consider permuting the rows and columns of  $A$ . As described in [Matrix Fundamentals](#) section, the permutations of the rows of  $A$  can be represented as a permutation matrix,  $P$ . The result of permuting the rows is the product of  $P$  and  $A$ . Suppose, in the above example, we swap the first and fifth row of  $A$ , then swap the first and fifth columns of  $A$ , and call the resulting matrix  $B$ . Mathematically, we can express the process of permuting the rows and columns of  $A$  to get  $B$  as  $B = PAP^T$ . After permuting the rows and columns of  $A$ , we see that  $B$  is given by the following:

$$B = \begin{bmatrix} 16 & * & * & * & 3 \\ * & \frac{1}{2} & * & * & \frac{3}{2} \\ * & * & 12 & * & 6 \\ * & * & * & \frac{5}{8} & \frac{3}{4} \\ 3 & \frac{3}{2} & 6 & \frac{3}{4} & 9 \end{bmatrix}$$

Since  $B$  is obtained from  $A$  by simply switching rows and columns, the numbers of non-zero entries in  $A$  and  $B$  are the same. However, when we find the Cholesky factorization,  $B = LL^T$ , we see the following:

$$L = \begin{bmatrix} 4 & * & * & * & * \\ * & \frac{1}{\sqrt{2}} & * & * & * \\ * & * & 2(\sqrt{3}) & * & * \\ * & * & * & \frac{\sqrt{10}}{4} & * \\ \frac{3}{4} & \frac{3}{\sqrt{2}} & \sqrt{3} & \frac{3}{\sqrt{10}} & \frac{\sqrt{3}}{4} \end{bmatrix}$$

The fill-in associated with  $B$  is much smaller than the fill-in associated with  $A$ . Consequently, the storage and computation time needed to factor  $B$  is much smaller than to factor  $A$ . Based on this, we see that an efficient sparse solver needs to find permutation  $P$  of the matrix  $A$ , which minimizes the fill-in for factoring  $B = PAP^T$ , and then use the factorization of  $B$  to solve the original system of equations.

Although the above example is based on a symmetric positive definite matrix and a Cholesky decomposition, the same approach works for a general  $LU$  decomposition. Specifically, let  $P$  be a permutation matrix,  $B = PAP^T$  and suppose that  $B$  can be factored as  $B = LU$ . Then

$$Ax = b$$

$$\Rightarrow PA(P^{-1}P)x = Pb$$

$$\Rightarrow PA(P^T P)x = Pb$$

$$\Rightarrow (PAP^T)(Px) = Pb$$

$$\Rightarrow B(Px) = Pb$$

$$\Rightarrow LU(Px) = Pb$$

It follows that if we obtain an  $LU$  factorization for  $B$ , we can solve the original system of equations by a three step process:

1. Solve  $Ly = Pb$ .
2. Solve  $Uz = y$ .
3. Set  $x = P^Tz$ .

If we apply this three-step process to the current example, we first need to perform the forward solve of the systems of equation  $Ly = Pb$ :

$$Ly = \begin{bmatrix} 4 & * & * & * & * \\ * & \frac{1}{\sqrt{2}} & * & * & * \\ * & * & 2(\sqrt{3}) & * & * \\ * & * & * & \frac{\sqrt{10}}{4} & * \\ \frac{3}{4} & \frac{3}{\sqrt{2}} & \sqrt{3} & \frac{3}{\sqrt{10}} & \frac{\sqrt{3}}{4} \end{bmatrix} * \begin{bmatrix} y1 \\ y2 \\ y3 \\ y4 \\ y5 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

This gives:  $y^T = \frac{5}{4}, 2\sqrt{2}, \frac{\sqrt{3}}{2}, \frac{16}{\sqrt{10}}, \frac{-979\sqrt{3}}{12\sqrt{5}}$ .

The second step is to perform the backward solve,  $Uz = y$ . Or, in this case, since a Cholesky factorization is used,  $L^Tz = y$ .

$$\begin{bmatrix} 4 & * & * & * & * \\ * & \frac{1}{\sqrt{2}} & * & * & * \\ * & * & 2(\sqrt{3}) & * & * \\ * & * & * & \frac{\sqrt{10}}{4} & * \\ \frac{3}{4} & \frac{3}{\sqrt{2}} & \sqrt{3} & \frac{3}{\sqrt{10}} & \frac{\sqrt{3}}{4} \end{bmatrix}^T * \begin{bmatrix} z1 \\ z2 \\ z3 \\ z4 \\ z5 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} \\ 2(\sqrt{2}) \\ \frac{\sqrt{3}}{2} \\ \frac{16}{\sqrt{10}} \\ \frac{-979\sqrt{3}}{12\sqrt{5}} \end{bmatrix}$$

This gives  $z^T = \frac{123}{2}, 983, \frac{1961}{12}, 398, \frac{-979}{3}$ .

The third and final step is to set  $x = P^Tz$ . This gives  $x^T = \frac{-979}{3}, 983, \frac{1961}{12}, 398, \frac{123}{2}$ .

## Sparse Matrix Storage Formats

As discussed above, it is more efficient to store only the non-zero elements of a sparse matrix. There are a number of common storage formats used for sparse matrices, but most of them employ the same basic technique. That is, store all non-zero elements of the matrix into a linear array and provide auxiliary arrays to describe the locations of the non-zero elements in the original matrix.

### Storage Formats for the Direct Sparse Solvers

Storing the non-zero elements of a sparse matrix into a linear array is done by walking down each column (column-major format) or across each row (row-major format) in order, and writing the non-zero elements to a linear array in the order they appear in the walk.

- [DSS Symmetric Matrix Storage](#)
- [DSS Nonsymmetric Matrix Storage](#)
- [DSS Structurally Symmetric Matrix Storage](#)
- [DSS Distributed Symmetric Matrix Storage](#)

### Sparse Matrix Storage Formats for Sparse BLAS Levels 2 and Level 3

These sections describe in detail the sparse matrix storage formats supported in the current version of the Intel MKL Sparse BLAS Level 2 and Level 3.

- [Sparse BLAS CSR Matrix Storage](#)
- [Sparse BLAS CSC Matrix Storage](#)
- [Sparse BLAS Coordinate Matrix Storage](#)
- [Sparse BLAS Diagonal Matrix Storage](#)
- [Sparse BLAS Skyline Matrix Storage](#)
- [Sparse BLAS BSR Matrix Storage](#)

### DSS Symmetric Matrix Storage

For symmetric matrices, it is necessary to store only the upper triangular half of the matrix (upper triangular format) or the lower triangular half of the matrix (lower triangular format).

The Intel MKL direct sparse solvers use a row-major upper triangular storage format: the matrix is compressed row-by-row and for symmetric matrices only non-zero elements in the upper triangular half of the matrix are stored.

The Intel MKL sparse matrix storage format for direct sparse solvers is specified by three arrays: *values*, *columns*, and *rowIndex*. The following table describes the arrays in terms of the values, row, and column positions of the non-zero elements in a sparse matrix.

|                 |                                                                                                                                                                                                                     |
|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>values</i>   | A real or complex array that contains the non-zero elements of a sparse matrix. The non-zero elements are mapped into the <i>values</i> array using the row-major upper triangular storage mapping described above. |
| <i>columns</i>  | Element <i>i</i> of the integer array <i>columns</i> is the number of the column that contains the <i>i</i> -th element in the <i>values</i> array.                                                                 |
| <i>rowIndex</i> | Element <i>j</i> of the integer array <i>rowIndex</i> gives the index of the element in the <i>values</i> array that is first non-zero element in a row <i>j</i> .                                                  |

The length of the *values* and *columns* arrays is equal to the number of non-zero elements in the matrix.

As the *rowIndex* array gives the location of the first non-zero element within a row, and the non-zero elements are stored consecutively, the number of non-zero elements in the *i*-th row is equal to the difference of *rowIndex*[*i*] and *rowIndex*[*i*+1].

To have this relationship hold for the last row of the matrix, an additional entry (dummy entry) is added to the end of *rowIndex*. Its value is equal to the number of non-zero elements plus one. This makes the total length of the *rowIndex* array one larger than the number of rows in the matrix.

**NOTE**

The Intel MKL sparse storage scheme for the direct sparse solvers supports both one-based indexing and zero-based indexing.

Consider the symmetric matrix *A*:

$$A = \begin{pmatrix} 1 & -1 & * & -3 & * \\ -1 & 5 & * & * & * \\ * & * & 4 & 6 & 4 \\ -3 & * & 6 & 7 & * \\ * & * & 4 & * & -5 \end{pmatrix}$$

Only elements from the upper triangle are stored. The actual arrays for the matrix *A* are as follows:

**Storage Arrays for a Symmetric Matrix****one-based indexing**

|                 |   |    |    |    |   |   |     |   |   |     |
|-----------------|---|----|----|----|---|---|-----|---|---|-----|
| <i>values</i>   | = | (1 | -1 | -3 | 5 | 4 | 6   | 4 | 7 | -5) |
| <i>columns</i>  | = | (1 | 2  | 4  | 2 | 3 | 4   | 5 | 4 | 5)  |
| <i>rowIndex</i> | = | (1 | 4  | 5  | 8 | 9 | 10) |   |   |     |

**zero-based indexing**

|                 |   |    |    |    |   |   |    |   |   |     |
|-----------------|---|----|----|----|---|---|----|---|---|-----|
| <i>values</i>   | = | (1 | -1 | -3 | 5 | 4 | 6  | 4 | 7 | -5) |
| <i>columns</i>  | = | (0 | 1  | 3  | 1 | 2 | 3  | 4 | 3 | 4)  |
| <i>rowIndex</i> | = | (0 | 3  | 4  | 7 | 8 | 9) |   |   |     |

**Storage Format Restrictions**

The storage format for the sparse solver must conform to two important restrictions:

- the non-zero values in a given row must be placed into the *values* array in the order in which they occur in the row (from left to right);
- no diagonal element can be omitted from the *values* array for any symmetric or structurally symmetric matrix.

The second restriction implies that if symmetric or structurally symmetric matrices have zero diagonal elements, then they must be explicitly represented in the *values* array.

**DSS Nonsymmetric Matrix Storage**

For a non-symmetric or non-Hermitian matrix, all non-zero elements need to be stored. Consider the non-symmetric matrix *B*:

$$B = \begin{pmatrix} 1 & -1 & * & -3 & * \\ -2 & 5 & * & * & * \\ * & * & 4 & 6 & 4 \\ -4 & * & 2 & 7 & * \\ * & 8 & * & * & -5 \end{pmatrix}$$

The matrix *B* has 13 non-zero elements, and all of them are stored as follows:

**Storage Arrays for a Non-Symmetric Matrix****one-based indexing**

|                 |   |    |    |    |    |    |     |   |   |    |   |   |   |     |
|-----------------|---|----|----|----|----|----|-----|---|---|----|---|---|---|-----|
| <i>values</i>   | = | (1 | -1 | -3 | -2 | 5  | 4   | 6 | 4 | -4 | 2 | 7 | 8 | -5) |
| <i>columns</i>  | = | (1 | 2  | 4  | 1  | 2  | 3   | 4 | 5 | 1  | 3 | 4 | 2 | 5)  |
| <i>rowIndex</i> | = | (1 | 4  | 6  | 9  | 12 | 14) |   |   |    |   |   |   |     |

**zero-based indexing**

|                 |   |    |    |    |    |    |     |   |   |    |   |   |   |     |
|-----------------|---|----|----|----|----|----|-----|---|---|----|---|---|---|-----|
| <i>values</i>   | = | (1 | -1 | -3 | -2 | 5  | 4   | 6 | 4 | -4 | 2 | 7 | 8 | -5) |
| <i>columns</i>  | = | (0 | 1  | 3  | 0  | 1  | 2   | 3 | 4 | 0  | 2 | 3 | 1 | 4)  |
| <i>rowIndex</i> | = | (0 | 3  | 5  | 8  | 11 | 13) |   |   |    |   |   |   |     |

**Storage Format Restrictions**

The storage format for the sparse solver must conform to two important restrictions:

- the non-zero values in a given row must be placed into the *values* array in the order in which they occur in the row (from left to right);
- no diagonal element can be omitted from the *values* array for any symmetric or structurally symmetric matrix.

The second restriction implies that if symmetric or structurally symmetric matrices have zero diagonal elements, then they must be explicitly represented in the *values* array.

**DSS Structurally Symmetric Matrix Storage**

Direct sparse solvers can also solve symmetrically structured systems of equations. A symmetrically structured system of equations is one where the pattern of non-zero elements is symmetric. That is, a matrix has a symmetric structure if  $a_{j,i}$  is not zero if and only if  $a_{i,j}$  is not zero. From the point of view of the solver software, a "non-zero" element of a matrix is any element stored in the *values* array, even if its value is equal to 0. In that sense, any non-symmetric matrix can be turned into a symmetrically structured matrix by carefully adding zeros to the *values* array. For example, the above matrix *B* can be turned into a symmetrically structured matrix by adding two non-zero entries:

$$B = \begin{pmatrix} 1 & -1 & * & 3 & * \\ -2 & 5 & * & * & 0 \\ * & * & 4 & 6 & 4 \\ -4 & * & 2 & 7 & * \\ * & 8 & 0 & * & -5 \end{pmatrix}$$

The matrix *B* can be considered to be symmetrically structured with 15 non-zero elements and represented as:

**Storage Arrays for a Symmetrically Structured Matrix**

**one-based indexing**

|                 |   |    |    |    |    |    |     |   |   |   |    |   |   |   |   |     |
|-----------------|---|----|----|----|----|----|-----|---|---|---|----|---|---|---|---|-----|
| <i>values</i>   | = | (1 | -1 | -3 | -2 | 5  | 0   | 4 | 6 | 4 | -4 | 2 | 7 | 8 | 0 | -5) |
| <i>columns</i>  | = | (1 | 2  | 4  | 1  | 2  | 5   | 3 | 4 | 5 | 1  | 3 | 4 | 2 | 3 | 5)  |
| <i>rowIndex</i> | = | (1 | 4  | 7  | 10 | 13 | 16) |   |   |   |    |   |   |   |   |     |

**zero-based indexing**

|                 |   |    |    |    |    |    |     |   |   |   |    |   |   |   |   |     |
|-----------------|---|----|----|----|----|----|-----|---|---|---|----|---|---|---|---|-----|
| <i>values</i>   | = | (1 | -1 | -3 | -2 | 5  | 0   | 4 | 6 | 4 | -4 | 2 | 7 | 8 | 0 | -5) |
| <i>columns</i>  | = | (0 | 1  | 3  | 0  | 1  | 4   | 2 | 3 | 4 | 0  | 2 | 3 | 1 | 2 | 4)  |
| <i>rowIndex</i> | = | (0 | 3  | 6  | 9  | 12 | 15) |   |   |   |    |   |   |   |   |     |

**Storage Format Restrictions**

The storage format for the sparse solver must conform to two important restrictions:

- the non-zero values in a given row must be placed into the *values* array in the order in which they occur in the row (from left to right);
- no diagonal element can be omitted from the *values* array for any symmetric or structurally symmetric matrix.

The second restriction implies that if symmetric or structurally symmetric matrices have zero diagonal elements, then they must be explicitly represented in the *values* array.

## DSS Distributed Symmetric Matrix Storage

The distributed assembled matrix input format can be used by the Parallel Direct Sparse Solver for Clusters Interface.

In this format, the symmetric input matrix  $A$  is divided into sequential row subsets, or domains. Each domain belongs to an MPI process. Neighboring domains can overlap. For such intersection between two domains, the element values of the full matrix can be obtained by summing the respective elements of both domains.

As in the centralized format, the distributed format uses three arrays to describe the input data, but the *values*, *columns*, and *rowIndex* arrays on each processor only describe the domain belonging to that particular processor and not the entire matrix.

For example, consider a symmetric matrix  $A$ :

$$A = \begin{pmatrix} 6 & -1 & * & -3 & * \\ -1 & 5 & * & * & * \\ * & * & 11 & 5 & 4 \\ -3 & * & 5 & 10 & * \\ * & * & 4 & * & 5 \end{pmatrix}$$

This array could be distributed between two domains corresponding to two MPI processes, with the first containing rows 1 through 3, and the second containing rows 3 through 5.

**NOTE**

For the symmetric input matrix, it is not necessary to store the values from the lower triangle.

$$A_{Domain1} = \begin{pmatrix} 6 & -1 & * & -3 & * \\ -1 & 5 & * & * & * \\ * & * & 3 & * & 2 \end{pmatrix}$$

### Distributed Storage Arrays for a Symmetric Matrix, Domain 1

|                            |   |    |    |    |    |   |    |
|----------------------------|---|----|----|----|----|---|----|
| <b>one-based indexing</b>  |   |    |    |    |    |   |    |
| <i>values</i>              | = | (6 | -1 | -3 | 5  | 3 | 2) |
| <i>columns</i>             | = | (1 | 2  | 4  | 2  | 3 | 5) |
| <i>rowIndex</i>            | = | (1 | 4  | 5  | 7) |   |    |
| <b>zero-based indexing</b> |   |    |    |    |    |   |    |
| <i>values</i>              | = | (6 | -1 | -3 | 5  | 3 | 2) |
| <i>columns</i>             | = | (0 | 1  | 3  | 1  | 2 | 4) |
| <i>rowIndex</i>            | = | (0 | 3  | 4  | 6) |   |    |

$$A_{Domain2} = \begin{pmatrix} * & * & 8 & 5 & 2 \\ -3 & * & 5 & 10 & * \\ * & * & 4 & * & 5 \end{pmatrix}$$

### Distributed Storage Arrays for a Symmetric Matrix, Domain 2

|                            |   |    |   |   |    |    |
|----------------------------|---|----|---|---|----|----|
| <b>one-based indexing</b>  |   |    |   |   |    |    |
| <i>values</i>              | = | (8 | 5 | 2 | 10 | 5) |
| <i>columns</i>             | = | (3 | 4 | 5 | 4  | 5) |
| <i>rowIndex</i>            | = | (1 | 4 | 5 | 6) |    |
| <b>zero-based indexing</b> |   |    |   |   |    |    |
| <i>values</i>              | = | (8 | 5 | 2 | 10 | 5) |
| <i>columns</i>             | = | (2 | 3 | 4 | 3  | 4) |
| <i>rowIndex</i>            | = | (0 | 3 | 4 | 5) |    |

The third row of matrix  $A$  is common between domain 1 and domain 2. The values of row 3 of matrix  $A$  are the sums of the respective elements of row 3 of matrix  $A_{Domain1}$  and row 1 of matrix  $A_{Domain2}$ .

## Storage Format Restrictions

The storage format for the sparse solver must conform to two important restrictions:

- the non-zero values in a given row must be placed into the *values* array in the order in which they occur in the row (from left to right);
- no diagonal element can be omitted from the *values* array for any symmetric or structurally symmetric matrix.

The second restriction implies that if symmetric or structurally symmetric matrices have zero diagonal elements, then they must be explicitly represented in the *values* array.

**Optimization Notice**

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Notice revision #20110804

**Sparse BLAS CSR Matrix Storage Format**

The Intel MKL compressed sparse row (CSR) format is specified by four arrays: the *values*, *columns*, *pointerB*, and *pointerE*. The following table describes the arrays in terms of the values, row, and column positions of the non-zero elements in a sparse matrix *A*.

|                 |                                                                                                                                                                                                                                                         |
|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>values</i>   | A real or complex array that contains the non-zero elements of <i>A</i> . Values of the non-zero elements of <i>A</i> are mapped into the <i>values</i> array using the row-major storage mapping described above.                                      |
| <i>columns</i>  | Element <i>i</i> of the integer array <i>columns</i> is the number of the column in <i>A</i> that contains the <i>i</i> -th value in the <i>values</i> array.                                                                                           |
| <i>pointerB</i> | Element <i>j</i> of this integer array gives the index of the element in the <i>values</i> array that is first non-zero element in a row <i>j</i> of <i>A</i> . Note that this index is equal to <i>pointerB</i> [ <i>j</i> ] - <i>pointerB</i> [0]+1 . |
| <i>pointerE</i> | An integer array that contains row indices, such that <i>pointerE</i> [ <i>j</i> ]- <i>pointerB</i> [0] is the index of the element in the <i>values</i> array that is last non-zero element in a row <i>j</i> of <i>A</i> .                            |

The length of the *values* and *columns* arrays is equal to the number of non-zero elements in *A*. The length of the *pointerB* and *pointerE* arrays is equal to the number of rows in *A*.

**NOTE**

Note that the Intel MKL Sparse BLAS routines support the CSR format both with one-based indexing and zero-based indexing.

The matrix *B*

$$B = \begin{pmatrix} 1 & -1 & * & -3 & * \\ -2 & 5 & * & * & * \\ * & * & 4 & 6 & 4 \\ -4 & * & 2 & 7 & * \\ * & 8 & * & * & -5 \end{pmatrix}$$

can be represented in the CSR format as:

### Storage Arrays for a Matrix in CSR Format

#### one-based indexing

```

values = (1 -1 -3 -2 5 4 6 4 -4 2 7 8 -5)
columns = (1 2 4 1 2 3 4 5 1 3 4 2 5)
pointerB = (1 4 6 9 12)
pointerE = (4 6 9 12 14)

```

#### zero-based indexing

```

values = (1 -1 -3 -2 5 4 6 4 -4 2 7 8 -5)
columns = (0 1 3 0 1 2 3 4 0 2 3 1 4)
pointerB = (0 3 5 8 11)
pointerE = (3 5 8 11 13)

```

This storage format is used in the NIST Sparse BLAS library [Rem05].

### Three Array Variation of CSR Format

The storage format accepted for the direct sparse solvers is a variation of the CSR format. It also is used in the Intel MKL Sparse BLAS Level 2 both with one-based indexing and zero-based indexing. The above matrix  $B$  can be represented in this format (referred to as the 3-array variation of the CSR format or CSR3) as:

### Storage Arrays for a Matrix in CSR Format (3-Array Variation)

#### one-based indexing

```

values = (1 -1 -3 -2 5 4 6 4 -4 2 7 8 -5)
columns = (1 2 4 1 2 3 4 5 1 3 4 2 5)
rowIndex = (1 4 6 9 12 14)

```

#### zero-based indexing

```

values = (1 -1 -3 -2 5 4 6 4 -4 2 7 8 -5)
columns = (0 1 3 0 1 2 3 4 0 2 3 1 4)
rowIndex = (0 3 5 8 11 13)

```

The 3-array variation of the CSR format has a restriction: all non-zero elements are stored continuously, that is the set of non-zero elements in the row  $J$  goes just after the set of non-zero elements in the row  $J-1$ .

There are no such restrictions in the general (NIST) CSR format. This may be useful, for example, if there is a need to operate with different submatrices of the matrix at the same time. In this case, it is enough to define the arrays *pointerB* and *pointerE* for each needed submatrix so that all these arrays are pointers to the same array *values*.

By definition, the array *rowIndex* from the Table "Storage Arrays for a Non-Symmetric Example Matrix" is related to the arrays *pointerB* and *pointerE* from the Table "Storage Arrays for an Example Matrix in CSR Format", and you can see that

```

pointerB[i] = rowIndex[i] for i=0, ..4;
pointerE[i] = rowIndex[i+1] for i=0, ..4.

```

This enables calling a routine that has *values*, *columns*, *pointerB* and *pointerE* as input parameters for a sparse matrix stored in the format accepted for the direct sparse solvers. For example, a routine with the interface:

```

void name_routine(... , double *values, MKL_INT *columns, MKL_INT *pointerB, MKL_INT
*pointerE, ...)

```

can be called with parameters *values*, *columns*, *rowIndex* as follows:

```

name_routine(... , values, columns, rowIndex, rowIndex+1, ...).

```

## Sparse BLAS CSC Matrix Storage Format

The compressed sparse column format (CSC) is similar to the CSR format, but the columns are used instead the rows. In other words, the CSC format is identical to the CSR format for the transposed matrix. The CSR format is specified by four arrays: *values*, *columns*, *pointerB*, and *pointerE*. The following table describes the arrays in terms of the values, row, and column positions of the non-zero elements in a sparse matrix *A*.

|                 |                                                                                                                                                                                                                                                           |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>values</i>   | A real or complex array that contains the non-zero elements of <i>A</i> . Values of the non-zero elements of <i>A</i> are mapped into the <i>values</i> array using the column-major storage mapping.                                                     |
| <i>rows</i>     | Element <i>i</i> of the integer array <i>rows</i> is the number of the row in <i>A</i> that contains the <i>i</i> -th value in the <i>values</i> array.                                                                                                   |
| <i>pointerB</i> | Element <i>j</i> of this integer array gives the index of the element in the <i>values</i> array that is first non-zero element in a column <i>j</i> of <i>A</i> . Note that this index is equal to <i>pointerB</i> [ <i>j</i> ] - <i>pointerB</i> [0]+1. |
| <i>pointerE</i> | An integer array that contains column indices, such that <i>pointerE</i> [ <i>j</i> ]- <i>pointerB</i> [0] is the index of the element in the <i>values</i> array that is last non-zero element in a column <i>j</i> of <i>A</i> .                        |

The length of the *values* and *columns* arrays is equal to the number of non-zero elements in *A*. The length of the *pointerB* and *pointerE* arrays is equal to the number of columns in *A*.

---

### NOTE

Note that the Intel MKL Sparse BLAS routines support the CSC format both with one-based indexing and zero-based indexing.

---

For example, consider matrix *B*:

$$B = \begin{pmatrix} 1 & -1 & * & -3 & * \\ -2 & 5 & * & * & * \\ * & * & 4 & 6 & 4 \\ -4 & * & 2 & 7 & * \\ * & 8 & * & * & -5 \end{pmatrix}$$

It can be represented in the CSC format as:

### Storage Arrays for a Matrix in CSC Format

---

|                            |   |    |    |    |    |     |   |   |   |    |   |   |   |     |
|----------------------------|---|----|----|----|----|-----|---|---|---|----|---|---|---|-----|
| <b>one-based indexing</b>  |   |    |    |    |    |     |   |   |   |    |   |   |   |     |
| <i>values</i>              | = | (1 | -2 | -4 | -1 | 5   | 8 | 4 | 2 | -3 | 6 | 7 | 4 | -5) |
| <i>rows</i>                | = | (1 | 2  | 4  | 1  | 2   | 5 | 3 | 4 | 1  | 3 | 4 | 3 | 5)  |
| <i>pointerB</i>            | = | (1 | 4  | 7  | 9  | 12) |   |   |   |    |   |   |   |     |
| <i>pointerE</i>            | = | (4 | 7  | 9  | 12 | 14) |   |   |   |    |   |   |   |     |
| <b>zero-based indexing</b> |   |    |    |    |    |     |   |   |   |    |   |   |   |     |
| <i>values</i>              | = | (1 | -2 | -4 | -1 | 5   | 8 | 4 | 2 | -3 | 6 | 7 | 4 | -5) |
| <i>rows</i>                | = | (0 | 1  | 3  | 0  | 1   | 4 | 2 | 3 | 0  | 2 | 3 | 2 | 4)  |
| <i>pointerB</i>            | = | (0 | 3  | 6  | 8  | 11) |   |   |   |    |   |   |   |     |
| <i>pointerE</i>            | = | (3 | 6  | 8  | 11 | 13) |   |   |   |    |   |   |   |     |

---

## Sparse BLAS Coordinate Matrix Storage Format

The coordinate format is the most flexible and simplest format for the sparse matrix representation. Only non-zero elements are stored, and the coordinates of each non-zero element are given explicitly. Many commercial libraries support the matrix-vector multiplication for the sparse matrices in the coordinate format.

The Intel MKL coordinate format is specified by three arrays: *values*, *rows*, and *column*, and a parameter *nnz* which is number of non-zero elements in *A*. All three arrays have dimension *nnz*. The following table describes the arrays in terms of the values, row, and column positions of the non-zero elements in a sparse matrix *A*.

|                |                                                                                                                                                               |
|----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>values</i>  | A real or complex array that contains the non-zero elements of <i>A</i> in any order.                                                                         |
| <i>rows</i>    | Element <i>i</i> of the integer array <i>rows</i> is the number of the row in <i>A</i> that contains the <i>i</i> -th value in the <i>values</i> array.       |
| <i>columns</i> | Element <i>i</i> of the integer array <i>columns</i> is the number of the column in <i>A</i> that contains the <i>i</i> -th value in the <i>values</i> array. |

**NOTE**

Note that the Intel MKL Sparse BLAS routines support the coordinate format both with one-based indexing and zero-based indexing.

For example, the sparse matrix *C*

$$C = \begin{pmatrix} 1 & -1 & -3 & 0 & 0 \\ -2 & 5 & 0 & 0 & 0 \\ 0 & 0 & 4 & 6 & 4 \\ -4 & 0 & 2 & 7 & 0 \\ 0 & 8 & 0 & 0 & -5 \end{pmatrix}$$

can be represented in the coordinate format as follows:

**Storage Arrays for an Example Matrix in case of the coordinate format**

| one-based indexing  |   |    |    |    |    |   |   |   |   |    |   |   |   |     |
|---------------------|---|----|----|----|----|---|---|---|---|----|---|---|---|-----|
| <i>values</i>       | = | (1 | -1 | -3 | -2 | 5 | 4 | 6 | 4 | -4 | 2 | 7 | 8 | -5) |
| <i>rows</i>         | = | (1 | 1  | 1  | 2  | 2 | 3 | 3 | 3 | 4  | 4 | 4 | 5 | 5)  |
| <i>columns</i>      | = | (1 | 2  | 3  | 1  | 2 | 3 | 4 | 5 | 1  | 3 | 4 | 2 | 5)  |
| zero-based indexing |   |    |    |    |    |   |   |   |   |    |   |   |   |     |
| <i>values</i>       | = | (1 | -1 | -3 | -2 | 5 | 4 | 6 | 4 | -4 | 2 | 7 | 8 | -5) |
| <i>rows</i>         | = | (0 | 0  | 0  | 1  | 1 | 2 | 2 | 2 | 3  | 3 | 3 | 4 | 4)  |
| <i>columns</i>      | = | (0 | 1  | 2  | 0  | 1 | 2 | 3 | 4 | 0  | 2 | 3 | 1 | 4)  |

**Sparse BLAS Diagonal Matrix Storage Format**

If the sparse matrix has diagonals containing only zero elements, then the diagonal storage format can be used to reduce the amount of information needed to locate the non-zero elements. This storage format is particularly useful in many applications where the matrix arises from a finite element or finite difference discretization. The Intel MKL diagonal storage format is specified by two arrays: *values* and *distance*, and two parameters: *ndiag*, which is the number of non-empty diagonals, and *lval*, which is the declared leading dimension in the calling (sub)programs. The following table describes the arrays *values* and *distance*:

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>values</i> | A real or complex two-dimensional array is dimensioned as <i>lval</i> by <i>ndiag</i> . Each column of it contains the non-zero elements of certain diagonal of <i>A</i> . The key point of the storage is that each element in <i>values</i> retains the row number of the original matrix. To achieve this diagonals in the lower triangular part of the matrix are padded from the top, and those in the upper triangular part are padded from the bottom. Note that the value of <i>distance</i> [ <i>i</i> ] is the number of elements to be padded for diagonal <i>i</i> . |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

*distance* An integer array with dimension *ndiag*. Element *i* of the array *distance* is the distance between *i*-diagonal and the main diagonal. The distance is positive if the diagonal is above the main diagonal, and negative if the diagonal is below the main diagonal. The main diagonal has a distance equal to zero.

The above matrix *C* can be represented in the diagonal storage format as follows:

$$\begin{aligned}
 \text{distance} &= (-3 \ -1 \ 0 \ 1 \ 2) \\
 \text{values} &= \begin{pmatrix} * & * & 1 & -1 & -3 \\ * & -2 & 5 & 0 & 0 \\ * & 0 & 4 & 6 & 4 \\ -4 & 2 & 7 & 0 & * \\ 8 & 0 & -5 & * & * \end{pmatrix}
 \end{aligned}$$

where the asterisks denote padded elements.

When storing symmetric, Hermitian, or skew-symmetric matrices, it is necessary to store only the upper or the lower triangular part of the matrix.

For the Intel MKL triangular solver routines elements of the array *distance* must be sorted in increasing order. In all other cases the diagonals and distances can be stored in arbitrary order.

### Sparse BLAS Skyline Matrix Storage Format

The skyline storage format is important for the direct sparse solvers, and it is well suited for Cholesky or LU decomposition when no pivoting is required.

The skyline storage format accepted in Intel MKL can store only triangular matrix or triangular part of a matrix. This format is specified by two arrays: *values* and *pointers*. The following table describes these arrays:

*values* A scalar array. For a lower triangular matrix it contains the set of elements from each row of the matrix starting from the first non-zero element to and including the diagonal element. For an upper triangular matrix it contains the set of elements from each column of the matrix starting with the first non-zero element down to and including the diagonal element. Encountered zero elements are included in the sets.

*pointers* An integer array with dimension  $(m+1)$ , where  $m$  is the number of rows for lower triangle (columns for the upper triangle).  $\text{pointers}[i] - \text{pointers}[0]+1$  gives the index of element in *values* that is first non-zero element in row (column)  $i$ . The value of  $\text{pointers}[m]$  is set to  $\text{nnz} + \text{pointers}[0]$ , where  $\text{nnz}$  is the number of elements in the array *values*.

For example, consider the matrix *C*:

$$C = \begin{pmatrix} 1 & -1 & -3 & 0 & 0 \\ -2 & 5 & 0 & 0 & 0 \\ 0 & 0 & 4 & 6 & 4 \\ -4 & 0 & 2 & 7 & 0 \\ 0 & 8 & 0 & 0 & -5 \end{pmatrix}$$

The low triangle of the matrix *C* given above can be stored as follows:

```

values = [1 -2 5 4 -4 0 2 7 8 0 0 -5]
pointers = [0 1 3 4 8 12]

```

and the upper triangle of this matrix *C* can be stored as follows:

```

values = [1 -1 5 -3 0 4 6 7 4 0 -5]
pointers = [0 1 3 6 8 11]

```

This storage format is supported by the NIST Sparse BLAS library [Rem05].

Note that the Intel MKL Sparse BLAS routines operating with the skyline storage format do not support general matrices.

## Sparse BLAS BSR Matrix Storage Format

The Intel MKL block compressed sparse row (BSR) format for sparse matrices is specified by four arrays: *values*, *columns*, *pointerB*, and *pointerE*. The following table describes these arrays.

|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|-----------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>values</i>   | A real array that contains the elements of the non-zero blocks of a sparse matrix. The elements are stored block-by-block in row-major order. A non-zero block is the block that contains at least one non-zero element. All elements of non-zero blocks are stored, even if some of them are equal to zero. Within each non-zero block elements are stored in column-major order in the case of one-based indexing, and in row-major order in the case of the zero-based indexing. |
| <i>columns</i>  | Element <i>i</i> of the integer array <i>columns</i> is the number of the column in the block matrix that contains the <i>i</i> -th non-zero block.                                                                                                                                                                                                                                                                                                                                 |
| <i>pointerB</i> | Element <i>j</i> of this integer array gives the index of the element in the <i>columns</i> array that is first non-zero block in a row <i>j</i> of the block matrix.                                                                                                                                                                                                                                                                                                               |
| <i>pointerE</i> | Element <i>j</i> of this integer array gives the index of the element in the <i>columns</i> array that contains the last non-zero block in a row <i>j</i> of the block matrix plus 1.                                                                                                                                                                                                                                                                                               |

The length of the *values* array is equal to the number of all elements in the non-zero blocks, the length of the *columns* array is equal to the number of non-zero blocks. The length of the *pointerB* and *pointerE* arrays is equal to the number of block rows in the block matrix.

### NOTE

Note that the Intel MKL Sparse BLAS routines support BSR format both with one-based indexing and zero-based indexing.

For example, consider the sparse matrix *D*

$$D = \begin{pmatrix} 1 & 0 & 6 & 7 & * & * \\ 2 & 1 & 8 & 2 & * & * \\ * & * & 1 & 4 & * & * \\ * & * & 5 & 1 & * & * \\ * & * & 4 & 3 & 7 & 2 \\ * & * & 0 & 0 & 0 & 0 \end{pmatrix}$$

If the size of the block equals 2, then the sparse matrix *D* can be represented as a 3x3 block matrix *E* with the following structure:

$$E = \begin{pmatrix} L & M & * \\ * & N & * \\ * & P & Q \end{pmatrix}$$

where

$$L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \quad M = \begin{pmatrix} 6 & 7 \\ 8 & 2 \end{pmatrix}, \quad N = \begin{pmatrix} 1 & 4 \\ 5 & 1 \end{pmatrix}, \quad P = \begin{pmatrix} 4 & 3 \\ 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 7 & 2 \\ 0 & 0 \end{pmatrix}$$

The matrix *D* can be represented in the BSR format as follows:

one-based indexing

```
values = (1 2 0 1 6 8 7 2 1 5 4 1 4 0 3 0 7 0 2 0)
columns = (1 2 2 2 3)
pointerB = (1 3 4)
pointerE = (3 4 6)
```

zero-based indexing

```
values = [1 0 2 1 6 7 8 2 1 4 5 1 4 3 0 0 7 2 0 0]
columns = [0 1 1 1 2]
pointerB = [0 2 3]
pointerE = [2 3 5]
```

This storage format is supported by the NIST Sparse BLAS library [[Rem05](#)].

**Three Array Variation of BSR Format**

Intel MKL supports the variation of the BSR format that is specified by three arrays: *values*, *columns*, and *rowIndex*. The following table describes these arrays.

|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <i>values</i>   | A real array that contains the elements of the non-zero blocks of a sparse matrix. The elements are stored block by block in row-major order. A non-zero block is the block that contains at least one non-zero element. All elements of non-zero blocks are stored, even if some of them is equal to zero. Within each non-zero block the elements are stored in column major order in the case of the one-based indexing, and in row major order in the case of the zero-based indexing. |
| <i>columns</i>  | Element <i>i</i> of the integer array <i>columns</i> is the number of the column in the block matrix that contains the <i>i</i> -th non-zero block.                                                                                                                                                                                                                                                                                                                                        |
| <i>rowIndex</i> | Element <i>j</i> of this integer array gives the index of the element in the <i>columns</i> array that is first non-zero block in a row <i>j</i> of the block matrix.                                                                                                                                                                                                                                                                                                                      |

The length of the *values* array is equal to the number of all elements in the non-zero blocks, the length of the *columns* array is equal to the number of non-zero blocks.

As the *rowIndex* array gives the location of the first non-zero block within a row, and the non-zero blocks are stored consecutively, the number of non-zero blocks in the *i*-th row is equal to the difference of *rowIndex*[*i*] and *rowIndex*[*i*+1].

To retain this relationship for the last row of the block matrix, an additional entry (dummy entry) is added to the end of *rowIndex* with value equal to the number of non-zero blocks plus one. This makes the total length of the *rowIndex* array one larger than the number of rows of the block matrix.

The above matrix *D* can be represented in this 3-array variation of the BSR format as follows:

one-based indexing

```
values = [1 2 0 1 6 8 7 2 1 5 4 2 4 0 3 0 7 0 2 0]
columns = [1 2 2 2 3]
rowIndex = [1 3 4 6]
```

zero-based indexing

```
values = [1 0 2 1 6 7 8 2 1 4 5 1 4 3 0 0 7 2 0 0]
columns = [0 1 1 1 2]
rowIndex = [0 2 3 5]
```

When storing symmetric matrices, it is necessary to store only the upper or the lower triangular part of the matrix.

For example, consider the symmetric sparse matrix *F*:

$$F = \begin{pmatrix} 1 & 0 & 6 & 7 & * & * \\ 2 & 1 & 8 & 2 & * & * \\ 6 & 8 & 1 & 4 & * & * \\ 7 & 2 & 5 & 2 & * & * \\ * & * & * & * & 7 & 2 \\ * & * & * & * & 0 & 0 \end{pmatrix}$$

If the size of the block equals 2, then the sparse matrix  $F$  can be represented as a 3x3 block matrix  $G$  with the following structure:

$$G = \begin{pmatrix} L & M & * \\ M' & N & * \\ * & * & Q \end{pmatrix}$$

where

$$L = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, M = \begin{pmatrix} 6 & 7 \\ 8 & 2 \end{pmatrix}, M' = \begin{pmatrix} 6 & 8 \\ 7 & 2 \end{pmatrix}, N = \begin{pmatrix} 1 & 4 \\ 5 & 2 \end{pmatrix}, \text{ and } Q = \begin{pmatrix} 7 & 2 \\ 0 & 0 \end{pmatrix}$$

The symmetric matrix  $F$  can be represented in this 3-array variation of the BSR format (storing only the upper triangular part) as follows:

one-based indexing

```
values = [1 2 0 1 6 8 7 2 1 5 4 2 7 0 2 0]
columns = [1 2 2 3]
rowIndex = [1 3 4 5]
```

zero-based indexing

```
values = [1 0 2 1 6 7 8 2 1 4 5 2 7 2 0 0]
columns = [0 1 1 2]
rowIndex = [0 2 3 4]
```

### Variable BSR Format

A variation of BSR3 is variable block compressed sparse row format. For a trust level  $t$ ,  $0 \leq t \leq 100$ , rows similar up to  $t$  percent are placed in one supernode.



# Routine and Function Arguments

The major arguments in the BLAS routines are vector and matrix, whereas VM functions work on vector arguments only. The sections that follow discuss each of these arguments and provide examples.

## Vector Arguments in BLAS

Vector arguments are passed in one-dimensional arrays. The array dimension (length) and vector increment are passed as integer variables. The length determines the number of elements in the vector. The increment (also called stride) determines the spacing between vector elements and the order of the elements in the array in which the vector is passed.

A vector of length  $n$  and increment  $incx$  is passed in a one-dimensional array  $x$  whose values are defined as

```
x[0], x[|incx|], ..., x[(n-1)* |incx|]
```

If  $incx$  is positive, then the elements in array  $x$  are stored in increasing order. If  $incx$  is negative, the elements in array  $x$  are stored in decreasing order with the first element defined as  $x[(n-1)* |incx|]$ . If  $incx$  is zero, then all elements of the vector have the same value,  $x[0]$ . The size of the one-dimensional array that stores the vector must always be at least

```
idimx = 1 + (n-1)* |incx|
```

### Example. One-dimensional Real Array

Let  $x[0:6]$  be the one-dimensional real array

```
x = [1.0, 3.0, 5.0, 7.0, 9.0, 11.0, 13.0].
```

If  $incx = 2$  and  $n = 3$ , then the vector argument with elements in order from first to last is  $[1.0, 5.0, 9.0]$ .

If  $incx = -2$  and  $n = 4$ , then the vector elements in order from first to last is  $[13.0, 9.0, 5.0, 1.0]$ .

If  $incx = 0$  and  $n = 4$ , then the vector elements in order from first to last is  $[1.0, 1.0, 1.0, 1.0]$ .

One-dimensional substructures of a matrix, such as the rows, columns, and diagonals, can be passed as vector arguments with the starting address and increment specified.

Storage of the  $m$ -by- $n$  matrix can be based on either column-major ordering where the increment between elements in the same column is 1, the increment between elements in the same row is  $m$ , and the increment between elements on the same diagonal is  $m + 1$ ; or row-major ordering where the increment between elements in the same row is 1, the increment between elements in the same column is  $n$ , and the increment between elements on the same diagonal is  $n + 1$ .

### Example. Two-dimensional Real Matrix

Let  $a$  be a real  $5 \times 4$  matrix declared as .

To scale the third column of  $a$  by 2.0, use the BLAS routine `sscal` with the following calling sequence:

```
cblas_sscal (5, 2.0, a[2], 4)
```

To scale the second row, use the statement:

```
cblas_sscal (4, 2.0, a[4], 1)
```

To scale the main diagonal of  $a$  by 2.0, use the statement:

```
cblas_sscal (4, 2.0, a[0], 5)
```

---

**NOTE**

The default vector argument is assumed to be 1.

---

## Vector Arguments in VM

---

Vector arguments of VM mathematical functions are passed in one-dimensional arrays with unit vector increment. It means that a vector of length  $n$  is passed contiguously in an array  $a$  whose values are defined as

$$a[0], a[1], \dots, a[n-1].$$

To accommodate for arrays with other increments, or more complicated indexing, VM contains auxiliary pack/unpack functions that gather the array elements into a contiguous vector and then scatter them after the computation is complete.

Generally, if the vector elements are stored in a one-dimensional array  $a$  as

$$a[m_0], a[m_1], \dots, a[m_n-1]$$

and need to be regrouped into an array  $y$  as

$$y[k_0], y[k_1], \dots, y[k_n-1],.$$

VM pack/unpack functions can use one of the following indexing methods:

### Positive Increment Indexing

$$k_j = \text{incy} * j, m_j = \text{inca} * j, j = 0, \dots, n-1.$$

Constraint:  $\text{incy} > 0$  and  $\text{inca} > 0$ .

For example, setting  $\text{incy} = 1$  specifies gathering array elements into a contiguous vector.

This method is similar to that used in BLAS, with the exception that negative and zero increments are not permitted.

### Index Vector Indexing

$$k_j = \text{iy}[j], m_j = \text{ia}[j], j = 0, \dots, n-1,.$$

where  $\text{ia}$  and  $\text{iy}$  are arrays of length  $n$  that contain index vectors for the input and output arrays  $a$  and  $y$ , respectively.

### Mask Vector Indexing

Indices  $k_j, m_j$  are such that:

$$m_y[k_j] \neq 0, m_a[m_j] \neq 0, j = 0, \dots, n-1,.$$

where  $m_a$  and  $m_y$  are arrays that contain mask vectors for the input and output arrays  $a$  and  $y$ , respectively.

## Matrix Arguments

---

Matrix arguments of the Intel® Math Kernel Library routines can be stored in arrays, using the following storage schemes:

- [conventional full storage](#)
- [packed storage](#) for Hermitian, symmetric, or triangular matrices
- [band storage](#) for band matrices
- [rectangular full packed storage](#) for symmetric, Hermitian, or triangular matrices as compact as the Packed storage while maintaining efficiency by using Level 3 BLAS/LAPACK kernels.

**Full storage** is the simplest scheme. A matrix  $A$  is stored in a one-dimensional array  $a$ , with the matrix element  $a_{ij}$  stored in the array element  $a[i - 1 + (j - 1)*lda]$ , where  $lda$  is the leading dimension of array  $a$ .

If a matrix is triangular (upper or lower, as specified by the argument  $uplo$ ), only the elements of the relevant triangle are stored; the remaining elements of the array need not be set.

Routines that handle symmetric or Hermitian matrices allow for either the upper or lower triangle of the matrix to be stored in the corresponding elements of the array:

if  $uplo = 'U'$ ,  $a_{ij}$  is stored as described for  $i \leq j$ , other elements of  $a$  need not be set.

if  $uplo = 'L'$ ,  $a_{ij}$  is stored as described for  $j \leq i$ , other elements of  $a$  need not be set.

**Packed storage** allows you to store symmetric, Hermitian, or triangular matrices more compactly: the relevant triangle (again, as specified by the argument  $uplo$ ) is packed by columns in a one-dimensional array  $ap$ :

if  $uplo = 'U'$ ,  $a_{ij}$  is stored in  $ap[i - 1 + j(j - 1)/2]$  for  $i \leq j$

if  $uplo = 'L'$ ,  $a_{ij}$  is stored in  $ap[i - 1 + (2*n - j)*(j - 1)/2]$  for  $j \leq i$ .

In descriptions of LAPACK routines, arrays with packed matrices have names ending in  $p$ .

**Band storage** is as follows: an  $m$ -by- $n$  band matrix with  $kl$  non-zero sub-diagonals and  $ku$  non-zero super-diagonals is stored compactly in an array  $ab$  with  $(kl+ku + 1)*n$  elements. Thus,

$a_{ij}$  is stored in  $ab(ku+1+i-j,j)$  for  $\max(1,j-ku) \leq i \leq \min(n,j+kl)$ .

Use the band storage scheme only when  $kl$  and  $ku$  are much less than the matrix size  $n$ . Although the routines work correctly for all values of  $kl$  and  $ku$ , using the band storage is inefficient if your matrices are not really banded.

The band storage scheme is illustrated by the following example, when

$$m = n = 6, \quad kl = 2, \quad ku = 1$$

Array elements marked \* are not used by the routines:

| Banded matrix A                                                                                                                                                                                                                                                                        | Band storage of A                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |          |          |          |          |          |          |          |          |          |          |          |          |          |  |          |          |          |          |          |   |  |          |          |          |          |   |   |  |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|--|----------|----------|----------|----------|----------|---|--|----------|----------|----------|----------|---|---|--|
| $\begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & a_{42} & a_{43} & a_{44} & a_{45} & 0 \\ 0 & 0 & a_{53} & a_{54} & a_{55} & a_{56} \\ 0 & 0 & 0 & a_{64} & a_{65} & a_{66} \end{bmatrix}$ | <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px;">*</td> <td style="padding: 5px;"><math>a_{12}</math></td> <td style="padding: 5px;"><math>a_{23}</math></td> <td style="padding: 5px;"><math>a_{34}</math></td> <td style="padding: 5px;"><math>a_{45}</math></td> <td style="padding: 5px;"><math>a_{56}</math></td> </tr> <tr> <td style="padding: 5px;"><math>a_{11}</math></td> <td style="padding: 5px;"><math>a_{22}</math></td> <td style="padding: 5px;"><math>a_{33}</math></td> <td style="padding: 5px;"><math>a_{44}</math></td> <td style="padding: 5px;"><math>a_{55}</math></td> <td style="padding: 5px;"><math>a_{66}</math></td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;"><math>a_{21}</math></td> <td style="padding: 5px;"><math>a_{32}</math></td> <td style="padding: 5px;"><math>a_{43}</math></td> <td style="padding: 5px;"><math>a_{54}</math></td> <td style="padding: 5px;"><math>a_{65}</math></td> <td style="padding: 5px;">*</td> <td style="padding: 5px;"></td> </tr> <tr> <td style="padding: 5px;"><math>a_{31}</math></td> <td style="padding: 5px;"><math>a_{42}</math></td> <td style="padding: 5px;"><math>a_{53}</math></td> <td style="padding: 5px;"><math>a_{64}</math></td> <td style="padding: 5px;">*</td> <td style="padding: 5px;">*</td> <td style="padding: 5px;"></td> </tr> </table> |          | *        | $a_{12}$ | $a_{23}$ | $a_{34}$ | $a_{45}$ | $a_{56}$ | $a_{11}$ | $a_{22}$ | $a_{33}$ | $a_{44}$ | $a_{55}$ | $a_{66}$ |  | $a_{21}$ | $a_{32}$ | $a_{43}$ | $a_{54}$ | $a_{65}$ | * |  | $a_{31}$ | $a_{42}$ | $a_{53}$ | $a_{64}$ | * | * |  |
|                                                                                                                                                                                                                                                                                        | *                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | $a_{12}$ | $a_{23}$ | $a_{34}$ | $a_{45}$ | $a_{56}$ |          |          |          |          |          |          |          |          |  |          |          |          |          |          |   |  |          |          |          |          |   |   |  |
| $a_{11}$                                                                                                                                                                                                                                                                               | $a_{22}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $a_{33}$ | $a_{44}$ | $a_{55}$ | $a_{66}$ |          |          |          |          |          |          |          |          |          |  |          |          |          |          |          |   |  |          |          |          |          |   |   |  |
| $a_{21}$                                                                                                                                                                                                                                                                               | $a_{32}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $a_{43}$ | $a_{54}$ | $a_{65}$ | *        |          |          |          |          |          |          |          |          |          |  |          |          |          |          |          |   |  |          |          |          |          |   |   |  |
| $a_{31}$                                                                                                                                                                                                                                                                               | $a_{42}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | $a_{53}$ | $a_{64}$ | *        | *        |          |          |          |          |          |          |          |          |          |  |          |          |          |          |          |   |  |          |          |          |          |   |   |  |

When a general band matrix is supplied for *LU factorization*, space must be allowed to store  $kl$  additional super-diagonals generated by fill-in as a result of row interchanges. This means that the matrix is stored according to the above scheme, but with  $kl + ku$  super-diagonals. Thus,

$a_{ij}$  is stored in  $ab(kl+ku+1+i-j, j)$  for  $\max(1, j-ku) \leq i \leq \min(n, j+kl)$ .

The band storage scheme for LU factorization is illustrated by the following example, when  $m = n = 6$ ,  $kl = 2$ ,  $ku = 1$ :

| Banded matrix A |          |          |          |          |          | Band storage of A |          |          |          |          |          |
|-----------------|----------|----------|----------|----------|----------|-------------------|----------|----------|----------|----------|----------|
| $a_{11}$        | $a_{12}$ | 0        | 0        | 0        | 0        | *                 | *        | *        | +        | +        | +        |
| $a_{21}$        | $a_{22}$ | $a_{23}$ | 0        | 0        | 0        | *                 | *        | +        | +        | +        | +        |
| $a_{31}$        | $a_{32}$ | $a_{33}$ | $a_{34}$ | 0        | 0        | *                 | $a_{12}$ | $a_{23}$ | $a_{34}$ | $a_{45}$ | $a_{56}$ |
| 0               | $a_{42}$ | $a_{43}$ | $a_{44}$ | $a_{45}$ | 0        | $a_{11}$          | $a_{22}$ | $a_{33}$ | $a_{44}$ | $a_{55}$ | $a_{66}$ |
| 0               | 0        | $a_{53}$ | $a_{54}$ | $a_{55}$ | $a_{56}$ | $a_{21}$          | $a_{32}$ | $a_{43}$ | $a_{54}$ | $a_{65}$ | *        |
| 0               | 0        | 0        | $a_{64}$ | $a_{65}$ | $a_{66}$ | $a_{31}$          | $a_{42}$ | $a_{53}$ | $a_{64}$ | *        | *        |

Array elements marked \* are not used by the routines; elements marked + need not be set on entry, but are required by the LU factorization routines to store the results. The input array will be overwritten on exit by the details of the LU factorization as follows:

|          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|
| *        | *        | *        | $u_{14}$ | $u_{25}$ | $u_{36}$ |
| *        | *        | $u_{13}$ | $u_{24}$ | $u_{35}$ | $u_{46}$ |
| *        | $u_{12}$ | $u_{23}$ | $u_{34}$ | $u_{45}$ | $u_{56}$ |
| $u_{11}$ | $u_{22}$ | $u_{33}$ | $u_{44}$ | $u_{55}$ | $u_{66}$ |
| $m_{21}$ | $m_{32}$ | $m_{43}$ | $m_{54}$ | $m_{65}$ | *        |
| $m_{31}$ | $m_{42}$ | $m_{53}$ | $m_{64}$ | *        | *        |

where  $u_{ij}$  are the elements of the upper triangular matrix U, and  $m_{ij}$  are the multipliers used during factorization.

Triangular band matrices are stored in the same format, with either  $kl=0$  if upper triangular, or  $ku=0$  if lower triangular. For symmetric or Hermitian band matrices with  $k$  sub-diagonals or super-diagonals, you need to store only the upper or lower triangle, as specified by the argument *uplo*:

if *uplo* = 'U',  $a_{ij}$  is stored in  $ab(k+1+i-j,j)$  for  $\max(1,j-k) \leq i \leq j$

if *uplo* = 'L',  $a_{ij}$  is stored in  $ab(1+i-j,j)$  for  $j \leq i \leq \min(n,j+k)$ .

In descriptions of LAPACK routines, arrays that hold matrices in band storage have names ending in *b*.

In Fortran, column-major ordering of storage is assumed. This means that elements of the same column occupy successive storage locations.

Three quantities are usually associated with a two-dimensional array argument: its leading dimension, which specifies the number of storage locations between elements in the same row, its number of rows, and its number of columns. For a matrix in full storage, the leading dimension of the array must be at least as large as the number of rows in the matrix.

A character transposition parameter is often passed to indicate whether the matrix argument is to be used in normal or transposed form or, for a complex matrix, if the conjugate transpose of the matrix is to be used.

The values of the transposition parameter for these three cases are the following:

'N' or 'n'                      normal (no conjugation, no transposition)

|            |                      |
|------------|----------------------|
| 'T' or 't' | transpose            |
| 'C' or 'c' | conjugate transpose. |

### Example. Two-Dimensional Complex Array

Suppose  $A$  (1:5, 1:4) is the complex two-dimensional array presented by matrix

$$\begin{bmatrix} (1.1, 0.11) & (1.2, 0.12) & (1.3, 0.13) & (1.4, 0.14) \\ (2.1, 0.21) & (2.2, 0.22) & (2.3, 0.23) & (1.4, 0.24) \\ (3.1, 0.31) & (3.2, 0.32) & (3.3, 0.33) & (1.4, 0.34) \\ (4.1, 0.41) & (4.2, 0.42) & (4.3, 0.43) & (1.4, 0.44) \\ (5.1, 0.51) & (5.2, 0.52) & (5.3, 0.53) & (1.4, 0.54) \end{bmatrix}$$

Let  $transa$  be the transposition parameter,  $m$  be the number of rows,  $n$  be the number of columns, and  $lda$  be the leading dimension. Then if

$transa = 'N'$ ,  $m = 4$ ,  $n = 2$ , and  $lda = 5$ , the matrix argument would be

$$\begin{bmatrix} (1.1, 0.11) & (1.2, 0.12) \\ (2.1, 0.21) & (2.2, 0.22) \\ (3.1, 0.31) & (3.2, 0.32) \\ (4.1, 0.41) & (4.2, 0.42) \end{bmatrix}$$

If  $transa = 'T'$ ,  $m = 4$ ,  $n = 2$ , and  $lda = 5$ , the matrix argument would be

$$\begin{bmatrix} (1.1, 0.11) & (2.1, 0.21) & (3.1, 0.31) & (4.1, 0.41) \\ (1.2, 0.12) & (2.2, 0.22) & (3.2, 0.32) & (4.2, 0.42) \end{bmatrix}$$

If  $transa = 'C'$ ,  $m = 4$ ,  $n = 2$ , and  $lda = 5$ , the matrix argument would be

$$\begin{bmatrix} (1.1, -0.11) & (2.1, -0.21) & (3.1, -0.31) & (4.1, -0.41) \\ (1.2, -0.12) & (2.2, -0.22) & (3.2, -0.32) & (4.2, -0.42) \end{bmatrix}$$

Note that care should be taken when using a leading dimension value which is different from the number of rows specified in the declaration of the two-dimensional array. For example, suppose the array  $A$  above is declared as a complex 5-by-4 matrix.

Then if  $transa = 'N'$ ,  $m = 3$ ,  $n = 4$ , and  $lda = 4$ , the matrix argument will be

$$\begin{bmatrix} (1.1, 0.11) & (5.1, 0.51) & (4.2, 0.42) & (3.3, 0.33) \\ (2.1, 0.21) & (1.2, 0.12) & (5.2, 0.52) & (4.3, 0.43) \\ (3.1, 0.31) & (2.2, 0.22) & (1.3, 0.13) & (5.3, 0.53) \end{bmatrix}$$

**Rectangular Full Packed storage** allows you to store symmetric, Hermitian, or triangular matrices as compact as the Packed storage while maintaining efficiency by using Level 3 BLAS/LAPACK kernels. To store an  $n$ -by- $n$  triangle (and suppose for simplicity that  $n$  is even), you partition the triangle into three parts: two  $n/2$ -by- $n/2$  triangles and an  $n/2$ -by- $n/2$  square, then pack this as an  $n$ -by- $n/2$  rectangle (or  $n/2$ -by- $n$  rectangle), by transposing (or transpose-conjugating) one of the triangles and packing it next to the other triangle. Since the two triangles are stored in full storage, you can use existing efficient routines on them.

There are eight cases of RFP storage representation: when  $n$  is even or odd, the packed matrix is transposed or not, the triangular matrix is lower or upper. See below for all the eight storage schemes illustrated:

$n$  is odd,  $A$  is lower triangular

| Full format                                                    | RFP (not transposed)                | RFP (transposed)                                               |
|----------------------------------------------------------------|-------------------------------------|----------------------------------------------------------------|
| $a_{11}$ X X X X X                                             | $a_{11}$ $a_{55}$ $a_{65}$ $a_{75}$ |                                                                |
| $a_{21}$ $a_{22}$ X X X X                                      | $a_{21}$ $a_{22}$ $a_{66}$ $a_{76}$ | $a_{11}$ $a_{21}$ $a_{31}$ $a_{41}$ $a_{51}$ $a_{61}$ $a_{71}$ |
| $a_{31}$ $a_{32}$ $a_{33}$ X X X                               | $a_{31}$ $a_{32}$ $a_{33}$ $a_{77}$ | $a_{65}$ $a_{22}$ $a_{32}$ $a_{42}$ $a_{52}$ $a_{62}$ $a_{72}$ |
| $a_{41}$ $a_{42}$ $a_{43}$ $a_{44}$ X X                        | $a_{41}$ $a_{42}$ $a_{43}$ $a_{44}$ | $a_{65}$ $a_{66}$ $a_{33}$ $a_{43}$ $a_{53}$ $a_{63}$ $a_{73}$ |
| $a_{51}$ $a_{52}$ $a_{53}$ $a_{54}$ $a_{55}$ X X               | $a_{51}$ $a_{52}$ $a_{53}$ $a_{54}$ | $a_{75}$ $a_{76}$ $a_{77}$ $a_{44}$ $a_{54}$ $a_{64}$ $a_{74}$ |
| $a_{61}$ $a_{62}$ $a_{63}$ $a_{64}$ $a_{65}$ $a_{66}$ X        | $a_{61}$ $a_{62}$ $a_{63}$ $a_{64}$ |                                                                |
| $a_{71}$ $a_{72}$ $a_{73}$ $a_{74}$ $a_{75}$ $a_{76}$ $a_{77}$ | $a_{71}$ $a_{72}$ $a_{73}$ $a_{74}$ |                                                                |

$n$  is even,  $A$  is lower triangular

| Full format                                           | RFP (not transposed)       | RFP (transposed)                                               |
|-------------------------------------------------------|----------------------------|----------------------------------------------------------------|
| $a_{11}$ X X X X X                                    | $a_{44}$ $a_{54}$ $a_{64}$ |                                                                |
| $a_{21}$ $a_{22}$ X X X X                             | $a_{11}$ $a_{55}$ $a_{65}$ | $a_{44}$ $a_{11}$ $a_{21}$ $a_{31}$ $a_{41}$ $a_{51}$ $a_{61}$ |
| $a_{31}$ $a_{32}$ $a_{33}$ X X X                      | $a_{21}$ $a_{22}$ $a_{66}$ | $a_{54}$ $a_{55}$ $a_{22}$ $a_{32}$ $a_{42}$ $a_{52}$ $a_{62}$ |
| $a_{41}$ $a_{42}$ $a_{43}$ $a_{44}$ X X               | $a_{31}$ $a_{32}$ $a_{33}$ | $a_{64}$ $a_{65}$ $a_{66}$ $a_{33}$ $a_{43}$ $a_{53}$ $a_{63}$ |
| $a_{51}$ $a_{52}$ $a_{53}$ $a_{54}$ $a_{55}$ X        | $a_{41}$ $a_{42}$ $a_{43}$ |                                                                |
| $a_{61}$ $a_{62}$ $a_{63}$ $a_{64}$ $a_{65}$ $a_{66}$ | $a_{51}$ $a_{52}$ $a_{53}$ |                                                                |
|                                                       | $a_{61}$ $a_{62}$ $a_{63}$ |                                                                |

$n$  is odd,  $A$  is upper triangular

| Full format                                                    | RFP (not transposed)                | RFP (transposed)                                               |
|----------------------------------------------------------------|-------------------------------------|----------------------------------------------------------------|
| $a_{11}$ $a_{12}$ $a_{13}$ $a_{14}$ $a_{15}$ $a_{16}$ $a_{17}$ | $a_{14}$ $a_{15}$ $a_{16}$ $a_{17}$ |                                                                |
| X $a_{22}$ $a_{23}$ $a_{24}$ $a_{25}$ $a_{26}$ $a_{27}$        | $a_{24}$ $a_{25}$ $a_{26}$ $a_{27}$ | $a_{14}$ $a_{24}$ $a_{34}$ $a_{44}$ $a_{11}$ $a_{12}$ $a_{13}$ |
| X X $a_{33}$ $a_{34}$ $a_{35}$ $a_{36}$ $a_{37}$               | $a_{34}$ $a_{35}$ $a_{36}$ $a_{37}$ | $a_{15}$ $a_{25}$ $a_{35}$ $a_{45}$ $a_{55}$ $a_{22}$ $a_{23}$ |
| X X X $a_{44}$ $a_{45}$ $a_{46}$ $a_{47}$                      | $a_{44}$ $a_{45}$ $a_{46}$ $a_{47}$ | $a_{16}$ $a_{26}$ $a_{36}$ $a_{46}$ $a_{56}$ $a_{66}$ $a_{33}$ |
| X X X X $a_{55}$ $a_{56}$ $a_{57}$                             | $a_{11}$ $a_{55}$ $a_{56}$ $a_{57}$ | $a_{17}$ $a_{27}$ $a_{37}$ $a_{47}$ $a_{57}$ $a_{67}$ $a_{77}$ |
| X X X X X $a_{66}$ $a_{67}$                                    | $a_{12}$ $a_{22}$ $a_{66}$ $a_{67}$ |                                                                |
| X X X X X X $a_{77}$                                           | $a_{13}$ $a_{23}$ $a_{33}$ $a_{77}$ |                                                                |

$n$  is even,  $A$  is upper triangular

| Full format |          |          |          |          |          | RFP (not transposed) |          |          | RFP (transposed) |          |          |          |          |          |          |  |  |  |
|-------------|----------|----------|----------|----------|----------|----------------------|----------|----------|------------------|----------|----------|----------|----------|----------|----------|--|--|--|
| $a_{11}$    | $a_{12}$ | $a_{13}$ | $a_{14}$ | $a_{15}$ | $a_{16}$ | $a_{14}$             | $a_{15}$ | $a_{16}$ |                  |          |          |          |          |          |          |  |  |  |
| X           | $a_{22}$ | $a_{23}$ | $a_{24}$ | $a_{25}$ | $a_{26}$ | $a_{24}$             | $a_{25}$ | $a_{26}$ |                  |          |          |          |          |          |          |  |  |  |
| X           | X        | $a_{33}$ | $a_{34}$ | $a_{35}$ | $a_{36}$ | $a_{34}$             | $a_{35}$ | $a_{36}$ | $a_{14}$         | $a_{24}$ | $a_{34}$ | $a_{44}$ | $a_{11}$ | $a_{12}$ | $a_{13}$ |  |  |  |
| X           | X        | X        | $a_{44}$ | $a_{45}$ | $a_{46}$ | $a_{44}$             | $a_{45}$ | $a_{46}$ | $a_{15}$         | $a_{25}$ | $a_{35}$ | $a_{45}$ | $a_{55}$ | $a_{22}$ | $a_{23}$ |  |  |  |
| X           | X        | X        | X        | $a_{55}$ | $a_{56}$ | $a_{11}$             | $a_{55}$ | $a_{56}$ | $a_{16}$         | $a_{26}$ | $a_{36}$ | $a_{46}$ | $a_{56}$ | $a_{66}$ | $a_{33}$ |  |  |  |
| X           | X        | X        | X        | X        | $a_{66}$ | $a_{12}$             | $a_{22}$ | $a_{66}$ |                  |          |          |          |          |          |          |  |  |  |
|             |          |          |          |          |          | $a_{13}$             | $a_{23}$ | $a_{33}$ |                  |          |          |          |          |          |          |  |  |  |

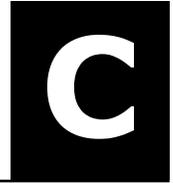
Intel MKL provides a number of routines such as `?hfrk`, `?sfrk` performing BLAS operations working directly on RFP matrices, as well as some conversion routines, for instance, `?tpttf` goes from the standard packed format to RFP and `?trttf` goes from the full format to RFP.

Please refer to the Netlib site for more information.

Note that in the descriptions of LAPACK routines, arrays with RFP matrices have names ending in `fp`.



# FFTW Interface to Intel® Math Kernel Library



Intel® Math Kernel Library (Intel® MKL) offers FFTW2 and FFTW3 interfaces to Intel MKL Fast Fourier Transform and Trigonometric Transform functionality. The purpose of these interfaces is to enable applications using FFTW ([www.fftw.org](http://www.fftw.org)) to gain performance with Intel MKL without changing the program source code.

Both FFTW2 and FFTW3 interfaces are provided in open source as FFTW wrappers to Intel MKL. For ease of use, FFTW3 interface is also integrated in Intel MKL.

## FFTW Notational Conventions

This appendix typically employs path notations for Windows\* OS.

## FFTW2 Interface to Intel® Math Kernel Library

This section describes a collection of C and Fortran wrappers providing FFTW 2.x interface to Intel MKL. The wrappers translate calls to FFTW 2.x functions into the calls of the Intel MKL Fast Fourier Transform interface (FFT interface).

Note that Intel MKL FFT interface operates on both single- and double-precision floating-point data types.

Because of differences between FFTW and Intel MKL FFT functionalities, there are restrictions on using wrappers instead of the FFTW functions. Some FFTW functions have empty wrappers. However, many typical FFTs can be computed using these wrappers.

Refer to [chapter 11 "Fourier Transform Functions"](#), for better understanding the effects from the use of the wrappers.

### Wrappers Reference

The section provides a brief reference for the FFTW 2.x C interface. For details please refer to the original FFTW 2.x documentation available at [www.fftw.org](http://www.fftw.org).

Each FFTW function has its own wrapper. Some of them, which are *not* expressly listed in this section, are empty and do nothing, but they are provided to avoid link errors and satisfy the function calls.

### See Also

[Limitations of the FFTW2 Interface to Intel MKL](#)

### One-dimensional Complex-to-complex FFTs

The following functions compute a one-dimensional complex-to-complex Fast Fourier transform.

```
fftw_plan fftw_create_plan(int n, fftw_direction dir, int flags);
fftw_plan fftw_create_plan_specific(int n, fftw_direction dir, int flags, fftw_complex
*in, int istride, fftw_complex *out, int ostride);

void fftw(fftw_plan plan, int howmany, fftw_complex *in, int istride, int idist,
fftw_complex *out, int ostride, int odist);

void fftw_one(fftw_plan plan, fftw_complex *in , fftw_complex *out);

void fftw_destroy_plan(fftw_plan plan);
```

## Multi-dimensional Complex-to-complex FFTs

The following functions compute a multi-dimensional complex-to-complex Fast Fourier transform.

```
fftwnd_plan fftwnd_create_plan(int rank, const int *n, fftw_direction dir, int flags);
fftwnd_plan fftw2d_create_plan(int nx, int ny, fftw_direction dir, int flags);
fftwnd_plan fftw3d_create_plan(int nx, int ny, int nz, fftw_direction dir, int flags);
fftwnd_plan fftwnd_create_plan_specific(int rank, const int *n, fftw_direction dir, int flags,
fftw_complex *in, int istride, fftw_complex *out, int ostride);
fftwnd_plan fftw2d_create_plan_specific(int nx, int ny, fftw_direction dir, int flags,
fftw_complex *in, int istride, fftw_complex *out, int ostride);
fftwnd_plan fftw3d_create_plan_specific(int nx, int ny, int nz, fftw_direction dir, int flags,
fftw_complex *in, int istride, fftw_complex *out, int ostride);

void fftwnd(fftwnd_plan plan, int howmany, fftw_complex *in, int istride, int idist,
fftw_complex *out, int ostride, int odist);

void fftwnd_one(fftwnd_plan plan, fftw_complex *in, fftw_complex *out);

void fftwnd_destroy_plan(fftwnd_plan plan);
```

## One-dimensional Real-to-half-complex/Half-complex-to-real FFTs

Half-complex representation of a conjugate-even symmetric vector of size  $N$  in a real array of the same size  $N$  consists of  $N/2+1$  real parts of the elements of the vector followed by non-zero imaginary parts in the reverse order. Because the Intel MKL FFT interface does not currently support this representation, all wrappers of this kind are empty and do nothing.

Nevertheless, you can perform one-dimensional real-to-complex and complex-to-real transforms using `rfftwnd` functions with `rank=1`.

### See Also

[Multi-dimensional Real-to-complex/Complex-to-real FFTs](#)

## Multi-dimensional Real-to-complex/Complex-to-real FFTs

The following functions compute multi-dimensional real-to-complex and complex-to-real Fast Fourier transforms.

```
rfftwnd_plan rfftwnd_create_plan(int rank, const int *n, fftw_direction dir, int flags);
rfftwnd_plan rfftw2d_create_plan(int nx, int ny, fftw_direction dir, int flags);
rfftwnd_plan rfftw3d_create_plan(int nx, int ny, int nz, fftw_direction dir, int flags);

rfftwnd_plan rfftwnd_create_plan_specific(int rank, const int *n, fftw_direction dir, int flags,
fftw_real *in, int istride, fftw_real *out, int ostride);
rfftwnd_plan rfftw2d_create_plan_specific(int nx, int ny, fftw_direction dir, int flags,
fftw_real *in, int istride, fftw_real *out, int ostride);
rfftwnd_plan rfftw3d_create_plan_specific(int nx, int ny, int nz, fftw_direction dir, int flags,
fftw_real *in, int istride, fftw_real *out, int ostride);

void rfftwnd_real_to_complex(rfftwnd_plan plan, int howmany, fftw_real *in, int istride, int idist,
fftw_complex *out, int ostride, int odist);

void rfftwnd_complex_to_real(rfftwnd_plan plan, int howmany, fftw_complex *in, int istride, int idist,
fftw_real *out, int ostride, int odist);

void rfftwnd_one_real_to_complex(rfftwnd_plan plan, fftw_real *in, fftw_complex *out);
```

```
void rffftwnd_one_complex_to_real(rffftwnd_plan plan, fftw_complex *in, fftw_real *out);
void rffftwnd_destroy_plan(rffftwnd_plan plan);
```

## Multi-threaded FFTW

This section discusses multi-threaded FFTW wrappers only. MPI FFTW wrappers, available only with Intel MKL for the Linux\* and Windows\* operating systems, are described in [section "MPI FFTW Wrappers"](#).

Unlike the original FFTW interface, every computational function in the FFTW2 interface to Intel MKL provides multithreaded computation by default, with the maximum number of threads permitted in FFT functions (see "Techniques to Set the Number of Threads" in *Intel MKL Developer Guide*). To limit the number of threads, call the threaded FFTW computational functions:

```
void fftw_threads(int nthreads, fftw_plan plan, int howmany, fftw_complex *in, int
istride, int idist, fftw_complex *out, int ostride, int odist);
void fftw_threads_one(int nthreads, rffftwnd_plan plan, fftw_complex *in, fftw_complex
*out);
...
void rffftwnd_threads_real_to_complex(int nthreads, rffftwnd_plan plan, int howmany,
fftw_real *in, int istride, int idist, fftw_complex *out, int ostride, int odist);
```

Compared to its non-threaded counterpart, every threaded computational function has `threads_` as the second part of its name and additional first parameter `nthreads`. Set the `nthreads` parameter to the thread limit to ensure that the computation requires at most that number of threads.

### Optimization Notice

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## FFTW Support Functions

The FFTW wrappers provide memory allocation functions to be used with FFTW:

```
void* fftw_malloc(size_t n);
void fftw_free(void* x);
```

The `fftw_malloc` wrapper aligns the memory on a 16-byte boundary.

If `fftw_malloc` fails to allocate memory, it aborts the application. To override this behavior, set a global variable `fftw_malloc_hook` and optionally the complementary variable `fftw_free_hook`:

```
void>(*fftw_malloc_hook)(size_t n);
void(*fftw_free_hook)(void *p);
```

The wrappers use the function `fftw_die` to abort the application in cases when a caller cannot be informed of an error otherwise (for example, in computational functions that return `void`). To override this behavior, set a global variable `fftw_die_hook`:

```
void(*fftw_die_hook)(const char *error_string);
void fftw_die(const char *s);
```

## Limitations of the FFTW2 Interface to Intel MKL

The FFTW2 wrappers implement the functionality of only those FFTW functions that Intel MKL can reasonably support. Other functions are provided as no-operation functions, whose only purpose is to satisfy link-time symbol resolution. Specifically, no-operation functions include:

- Real-to-half-complex and respective backward transforms
- Print plan functions
- Functions for importing/exporting/forgetting wisdom
- Most of the FFTW functions not covered by the original FFTW2 documentation

Because the Intel MKL implementation of FFTW2 wrappers does not use plan and plan node structures declared in `fftw.h`, the behavior of an application that relies on the internals of the plan structures defined in that header file is undefined.

FFTW2 wrappers define plan as a set of attributes, such as strides, used to commit the Intel MKL FFT descriptor structure. If an FFTW2 computational function is called with attributes different from those recorded in the plan, the function attempts to adjust the attributes of the plan and recommit the descriptor. So, repeated calls of a computational function with the same plan but different strides, distances, and other parameters may be performance inefficient.

Plan creation functions disregard most planner flags passed through the `flags` parameter. These functions take into account only the following values of `flags`:

- `FFTW_IN_PLACE`

If this value of `flags` is supplied, the plan is marked so that computational functions using that plan ignore the parameters related to output (`out`, `ostride`, and `odist`). Unlike the original FFTW interface, the wrappers never use the `out` parameter as a scratch space for in-place transforms.

- `FFTW_THREADSAFE`

If this value of `flags` is supplied, the plan is marked read-only. An attempt to change attributes of a read-only plan aborts the application.

FFTW wrappers are generally not thread safe. Therefore, do not use the same plan in parallel user threads simultaneously.

## Installing FFTW2 Interface Wrappers

Wrappers are delivered as source code, which you must compile to build the wrapper library. Then you can substitute the wrapper and Intel MKL libraries for the FFTW library. The source code for the wrappers, makefiles, and files with lists of wrappers are located in the `.\interfaces\fftw2xc` subdirectory in the Intel MKL directory.

## Creating the Wrapper Library

Two header files are used to compile the C wrapper library: `fftw2_mkl.h` and `fftw.h`. The `fftw2_mkl.h` file is located in the `.\interfaces\fftw2xc\wrappers` subdirectory in the Intel MKL directory.

The file `fftw.h`, used to compile libraries and located in the `.\include\fftw` subdirectory in the Intel MKL directory, slightly differs from the original FFTW ([www.fftw.org](http://www.fftw.org)) header file `fftw.h`.

The source code for the wrappers, makefiles, and files with lists of functions are located in the `.\interfaces\fftw2xc` subdirectory in the Intel MKL directory.

A wrapper library contains wrappers for complex and real transforms in a serial and multi-threaded mode for double- or single-precision floating-point data types. A makefile parameter manages the data type.

Parameters of a makefile also specify the platform (required), compiler, and data precision. The makefile comment heading provides the exact description of these parameters.

To build the library, run the `make` command on Linux\* OS and macOS\* or the `nmake` command on Windows\* OS with appropriate parameters.

For example, on Linux OS the command

```
make libintel64
```

builds a double-precision wrapper library for Intel® 64 architecture based applications using the Intel® C++ Compiler or the Intel® Fortran Compiler (Compilers and data precision are chosen by default.)

Each makefile creates the library in the directory with Intel MKL libraries corresponding to the platform used. For example, `./lib/ia32` (on Linux OS and macOS) or `.\lib\ia32` (on Windows\* OS).

In the names of a wrapper library, the suffix corresponds to the compiler used and the letter preceding the underscore is "c" for the C programming language.

For example,

```
fftw2xc_intel.lib (on Windows OS); libfftw2xc_intel.a (on Linux OS and macOS);
```

```
fftw2xc_ms.lib (on Windows OS); libfftw2xc_gnu.a (on Linux OS and macOS).
```

## Application Assembling

Use the necessary original FFTW ([www.fftw.org](http://www.fftw.org)) header files without any modifications. Use the created wrapper library and the Intel MKL library instead of the FFTW library.

## Running FFTW2 Interface Wrapper Examples

Intel MKL provides examples to demonstrate how to use the MPI FFTW wrapper library. The source code for the examples, makefiles used to run them, and files with lists of examples are located in the `.\examples\fftw2xc` subdirectory in the Intel MKL directory. To build examples, several additional files are needed: `fftw.h`, `fftw_threads.h`, `rfftw.h`, and `rfftw_threads.h`. These files are distributed with permission from FFTW and are available in `.\include\fftw`. The original files can also be found in FFTW 2.1.5 at <http://www.fftw.org/download.html>.

An example makefile uses the `function` parameter in addition to the parameters of the corresponding wrapper library makefile (see [Creating a Wrapper Library](#)). The makefile comment heading provides the exact description of these parameters.

An example makefile normally invokes `examples`. However, if the appropriate wrapper library is not yet created, the makefile first builds the library the same way as the wrapper library makefile does and then proceeds to `examples`.

If the parameter `function=<example_name>` is defined, only the specified example runs. Otherwise, all examples from the appropriate subdirectory run. The subdirectory `.\_results` is created, and the results are stored there in the `<example_name>.res` files.

### Optimization Notice

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## MPI FFTW2 Wrappers

MPI FFTW wrappers for FFTW 2 are available only with Intel® MKL for the Linux\* and Windows\* operating systems.

## MPI FFTW Wrappers Reference

The section provides a reference for MPI FFTW C interface.

### Complex MPI FFTW

#### Complex One-dimensional MPI FFTW Transforms

```
fftw_mpi_plan fftw_mpi_create_plan(MPI_Comm comm, int n, fftw_direction dir, int
flags);

void fftw_mpi(fftw_mpi_plan p, int n_fields, fftw_complex *local_data, fftw_complex
*work);

void fftw_mpi_local_sizes(fftw_mpi_plan p, int *local_n, int *local_start, int
*local_n_after_transform, int *local_start_after_transform, int *total_local_size);

void fftw_mpi_destroy_plan(fftw_mpi_plan plan);
```

*Argument restrictions:*

- Supported values of *flags* are FFTW\_ESTIMATE, FFTW\_MEASURE, FFTW\_SCRAMBLED\_INPUT and FFTW\_SCRAMBLED\_OUTPUT. The same algorithm corresponds to all these values of the flags parameter. If any other *flags* value is supplied, the wrapper library reports an error '*CDFT error in wrapper: unknown flags*'.
- The only supported value of *n\_fields* is 1.

#### Complex Multi-dimensional MPI FFTW Transforms

```
fftwnd_mpi_plan fftw2d_mpi_create_plan(MPI_Comm comm, int nx, int ny, fftw_direction
dir, int flags);

fftwnd_mpi_plan fftw3d_mpi_create_plan(MPI_Comm comm, int nx, int ny, int nz,
fftw_direction dir, int flags);

fftwnd_mpi_plan fftwnd_mpi_create_plan(MPI_Comm comm, int dim, int *n, fftw_direction
dir, int flags);

void fftwnd_mpi(fftwnd_mpi_plan p, int n_fields, fftw_complex *local_data, fftw_complex
*work, fftwnd_mpi_output_order output_order);

void fftwnd_mpi_local_sizes(fftwnd_mpi_plan p, int *local_nx, int *local_x_start, int
*local_ny_after_transpose, int *local_y_start_after_transpose, int *total_local_size);

void fftwnd_mpi_destroy_plan(fftwnd_mpi_plan plan);
```

*Argument restrictions:*

- Supported values of *flags* are FFTW\_ESTIMATE and FFTW\_MEASURE. If any other value of *flags* is supplied, the wrapper library reports an error '*CDFT error in wrapper: unknown flags*'.
- The only supported value of *n\_fields* is 1.

### Real MPI FFTW

#### Real-to-Complex MPI FFTW Transforms

```
rfftwnd_mpi_plan rfftw2d_mpi_create_plan(MPI_Comm comm, int nx, int ny, fftw_direction
dir, int flags);

rfftwnd_mpi_plan rfftw3d_mpi_create_plan(MPI_Comm comm, int nx, int ny, int nz,
fftw_direction dir, int flags);
```

```

rfftwnd_mpi_plan rfftwnd_mpi_create_plan(MPI_Comm comm, int dim, int *n, fftw_direction
dir, int flags);

void rfftwnd_mpi(rfftwnd_mpi_plan p, int n_fields, fftw_real *local_data, fftw_real
*work, fftwnd_mpi_output_order output_order);

void rfftwnd_mpi_local_sizes(rfftwnd_mpi_plan p, int *local_nx, int *local_x_start, int
*local_ny_after_transpose, int *local_y_start_after_transpose, int *total_local_size);

void rfftwnd_mpi_destroy_plan(rfftwnd_mpi_plan plan);

```

#### Argument restrictions:

- Supported values of *flags* are FFTW\_ESTIMATE and FFTW\_MEASURE. If any other value of *flags* is supplied, the wrapper library reports an error 'CDFT error in wrapper: unknown flags'.
- The only supported value of *n\_fields* is 1.

- 
- Function *rfftwnd\_mpi\_create\_plan* can be used for both one-dimensional and multi-dimensional transforms.
  - Both values of the *output\_order* parameter are supported: FFTW\_NORMAL\_ORDER and FFTW\_TRANSPOSED\_ORDER.
- 

## Creating MPI FFTW2 Wrapper Library

The source code for the wrappers, makefiles, and files with lists of wrappers are located in the `.\interfaces\fftw2x_cdft` subdirectory in the Intel MKL directory.

A wrapper library contains C wrappers for Complex One-dimensional MPI FFTW Transforms and Complex Multi-dimensional MPI FFTW Transforms. The library also contains empty C wrappers for Real Multi-dimensional MPI FFTW Transforms. For details, see [MPI FFTW Wrappers Reference](#).

Parameters of a makefile specify the platform (required), compiler, and data precision. Specifying the platform is required. The makefile comment heading provides the exact description of these parameters.

To build the library, run the `make` command on Linux\* OS and macOS\* or the `nmake` command on Windows\* OS with appropriate parameters.

For example, on Linux OS the command

```
make libintel64
```

builds a double-precision wrapper library for Intel® 64 architecture based applications using Intel MPI and the Intel® C++ Compiler (compilers and data precision are chosen by default.).

A makefile creates the wrapper library in the directory with the Intel MKL libraries corresponding to the used platform. For example, `./lib/ia32` (on Linux OS) or `.\lib\ia32` (on Windows\* OS).

In the wrapper library names, the suffix corresponds to the used data precision. For example,

```
fftw2x_cdft_SINGLE.lib on Windows OS;
```

```
libfftw2x_cdft_DOUBLE.a on Linux OS.
```

## Application Assembling with MPI FFTW Wrapper Library

Use the necessary original FFTW ([www.fftw.org](http://www.fftw.org)) header files without any modifications. Use the created MPI FFTW wrapper library and the Intel MKL library instead of the FFTW library.

## Running MPI FFTW2 Wrapper Examples

There are some examples that demonstrate how to use the MPI FFTW wrapper library for FFTW2. The source C code for the examples, makefiles used to run them, and files with lists of examples are located in the `.\examples\fftw2x_cdft` subdirectory in the Intel MKL directory. To build examples, one additional file `fftw_mpi.h` is needed. This file is distributed with permission from FFTW and is available in `.\include\fftw`. The original file can also be found in FFTW 2.1.5 at <http://www.fftw.org/download.html>.

Parameters for the example makefiles are described in the makefile comment headings and are similar to the parameters of the wrapper library makefiles (see [Creating MPI FFTW Wrapper Library](#)).

The table below lists examples available in the `.\examples\fftw2x_cdft\source` subdirectory.

### Examples of MPI FFTW Wrappers

| Source file for the example | Description                                                                                         |
|-----------------------------|-----------------------------------------------------------------------------------------------------|
| <code>wrappers_c1d.c</code> | One-dimensional Complex MPI FFTW transform, using <code>plan = fftw_mpi_create_plan(...)</code>     |
| <code>wrappers_c2d.c</code> | Two-dimensional Complex MPI FFTW transform, using <code>plan = fftw2d_mpi_create_plan(...)</code>   |
| <code>wrappers_c3d.c</code> | Three-dimensional Complex MPI FFTW transform, using <code>plan = fftw3d_mpi_create_plan(...)</code> |
| <code>wrappers_c4d.c</code> | Four-dimensional Complex MPI FFTW transform, using <code>plan = fftwnd_mpi_create_plan(...)</code>  |
| <code>wrappers_r1d.c</code> | One-dimensional Real MPI FFTW transform, using <code>plan = rfftw_mpi_create_plan(...)</code>       |
| <code>wrappers_r2d.c</code> | Two-dimensional Real MPI FFTW transform, using <code>plan = rfftw2d_mpi_create_plan(...)</code>     |
| <code>wrappers_r3d.c</code> | Three-dimensional Real MPI FFTW transform, using <code>plan = rfftw3d_mpi_create_plan(...)</code>   |
| <code>wrappers_r4d.c</code> | Four-dimensional Real MPI FFTW transform, using <code>plan = rffwnd_mpi_create_plan(...)</code>     |

## FFTW3 Interface to Intel® Math Kernel Library

This section describes a collection of FFTW3 wrappers to Intel MKL. The wrappers translate calls of FFTW3 functions to the calls of the Intel MKL Fourier transform (FFT) or Trigonometric Transform (TT) functions. The purpose of FFTW3 wrappers is to enable developers whose programs currently use the FFTW3 library to gain performance with the Intel MKL Fourier transforms without changing the program source code.

The FFTW3 wrappers provide a limited functionality compared to the original FFTW 3.x library, because of differences between FFTW and Intel MKL FFT and TT functionality. This section describes limitations of the FFTW3 wrappers and hints for their usage. Nevertheless, many typical FFT tasks can be performed using the FFTW3 wrappers to Intel MKL.

The FFTW3 wrappers are integrated in Intel MKL. The only change required to use Intel MKL through the FFTW3 wrappers is to link your application using FFTW3 against Intel MKL.

A reference implementation of the FFTW3 wrappers is also provided in open source. You can find it in the `interfaces` directory of the Intel MKL distribution. You can use the reference implementation to create your own wrapper library (see [Building Your Own Wrapper Library](#))

See also these resources:

Intel MKL Release Notes [for the version of the FFTW3 library supported by the wrappers.](#)

|                                                  |                                                   |
|--------------------------------------------------|---------------------------------------------------|
| <a href="http://www.fftw.org">www.fftw.org</a>   | for a description of the FFTW interface.          |
| <a href="#">Fourier Transform Functions</a>      | for a description of the Intel MKL FFT interface. |
| <a href="#">Trigonometric Transform Routines</a> | for a description of Intel MKL TT interface.      |

## Using FFTW3 Wrappers

The FFTW3 wrappers are a set of functions and data structures depending on one another. The wrappers are not designed to provide the interface on a function-per-function basis. Some FFTW3 wrapper functions are empty and do nothing, but they are present to avoid link errors and satisfy function calls.

This document does not list the declarations of the functions that the FFTW3 wrappers provide (you can find the declarations in the `fftw3.h` header file). Instead, this section comments particular limitations of the wrappers and provides usage hints:

- The FFTW3 wrappers do not support long double precision because Intel MKL FFT functions operate only on single- and double-precision floating-point data types (`float` and `double`, respectively). Therefore the functions with prefix `fftwl_`, supporting the long double data type, are not provided.
- The wrappers provide equivalent implementation for double- and single-precision functions (those with prefixes `fftw_` and `fftwf_`, respectively). So, all these comments equally apply to the double- and single-precision functions and will refer to functions with prefix `fftw_`, that is, double-precision functions, for brevity.
- The FFTW3 interface that the wrappers provide is defined in the `fftw3.h` header file. This file is borrowed from the FFTW3.x package and distributed within Intel MKL with permission. Additionally, the `fftw3_mkl.h` header file defines supporting structures and supplementary constants and macros.
- Actual functionality of the plan creation wrappers is implemented in `guru64` set of functions. Basic interface, advanced interface, and `guru` interface plan creation functions call the `guru64` interface functions. So, all types of the FFTW3 plan creation interface in the wrappers are functional.
- Plan creation functions may return a `NULL` plan, indicating that the functionality is not supported. So, please carefully check the result returned by plan creation functions in your application. In particular, the following problems return a `NULL` plan:
  - `c2r` and `r2c` problems with a split storage of complex data.
  - `r2r` problems with `kind` values `FFTW_R2HC`, `FFTW_HC2R`, and `FFTW_DHT`. The only supported `r2r` kinds are even/odd DFTs (sine/cosine transforms).
  - Multidimensional `r2r` transforms.
  - Transforms of multidimensional vectors. That is, the only supported values for parameter `howmany_rank` in `guru` and `guru64` plan creation functions are 0 and 1.
  - Multidimensional transforms with `rank > MKL_MAXRANK`.
- The `MKL_RODFT00` value of the `kind` parameter is introduced by the FFTW3 wrappers. For better performance, you are strongly encouraged to use this value rather than `FFTW_RODFT00`. To use this `kind` value, provide an extra first element equal to 0.0 for the input/output vectors. Consider the following example:

```
plan1 = fftw_plan_r2r_1d(n, in1, out1, FFTW_RODFT00, FFTW_ESTIMATE);
plan2 = fftw_plan_r2r_1d(n, in2, out2, MKL_RODFT00, FFTW_ESTIMATE);
```

Both plans perform the same transform, except that the `in2/out2` arrays have one extra zero element at location 0. For example, if `n=3`, `in1={x,y,z}` and `out1={u,v,w}`, then `in2={0,x,y,z}` and `out2={0,u,v,w}`.

- The `flags` parameter in plan creation functions is always ignored. The same algorithm is used regardless of the value of this parameter. In particular, `flags` values `FFTW_ESTIMATE`, `FFTW_MEASURE`, etc. have no effect.
- For multithreaded plans, use normal sequence of calls to the `fftw_init_threads()` and `fftw_plan_with_nthreads()` functions (refer to FFTW documentation).
- Memory allocation function `fftw_malloc` returns memory aligned at a 16-byte boundary. You must free the memory with `fftw_free`.

- The FFTW3 wrappers to Intel MKL use the 32-bit `int` type in both LP64 and ILP64 interfaces of Intel MKL. Use `guru64` FFTW3 interfaces for 64-bit sizes.
- The wrappers typically indicate a problem by returning a `NULL` plan. In a few cases, the wrappers may report a descriptive message of the problem detected. By default the reporting is turned off. To turn it on, set variable `fftw3_mkl.verbose` to a non-zero value, for example:

```
#include "fftw3.h"
#include "fftw3_mkl.h"
fftw3_mkl.verbose = 0;
plan = fftw_plan_r2r(...);
```

- The following functions are empty:
  - For saving, loading, and printing plans
  - For saving and loading wisdom
  - For estimating arithmetic cost of the transforms.
- Do not use macro `FFTW_DLL` with the FFTW3 wrappers to Intel MKL.
- Do not use negative stride values. Though FFTW3 wrappers support negative strides in the part of advanced and guru FFTW interface, the underlying implementation does not.

## Building Your Own FFTW3 Interface Wrapper Library

The FFTW3 wrappers to Intel MKL are delivered both integrated in Intel MKL and as source code, which can be compiled to build a standalone wrapper library with exactly the same functionality. Normally you do not need to build the wrappers yourself.

The source code for the wrappers, makefiles, and files with lists of functions are located in the `.\interfaces\fftw3xc` subdirectory in the Intel MKL directory.

To build the wrappers,

1. Change the current directory to the wrapper directory
2. Run the `make` command on Linux\* OS and macOS\* or the `nmake` command on Windows\* OS with a required target and optionally several parameters.

The target `libia32` or `libintel64` defines the platform architecture, and the other parameters specify the compiler, size of the default integer type, and placement of the resulting wrapper library. You can find a detailed and up-to-date description of the parameters in the makefile.

In the following example, the `make` command is used to build the FFTW3 C wrappers to Intel MKL for use from the GNU `gcc`\* compiler on Linux OS based on Intel® 64 architecture:

```
cd interfaces/fftw3xc
make libintel64 compiler=gnu INSTALL_DIR=/my/path
```

This command builds the wrapper library and places the result, named `libfftw3xc_gnu.a`, into the `/my/path` directory. The name of the resulting library is composed of the name of the compiler used and may be changed by an optional parameter `INSTALL_LIBNAME`.

## Building an Application With FFTW3 Interface Wrappers

Normally, the only change needed to build your application with FFTW3 wrappers replacing original FFTW library is to add Intel MKL at the link stage (see section "*Linking Your Application with Intel® Math Kernel Library*" in the *Intel MKL Developer Guide*).

If you recompile your application, add subdirectory `include\fftw` to the search path for header files to avoid FFTW3 version conflicts.

Sometimes, you may have to modify your application according to the following recommendations:

- The application requires
 

```
#include "fftw3.h" ,
```

 which it probably already includes.
- The application does not require
 

```
#include "mkl_dfti.h" .
```
- The application does not require
 

```
#include "fftw3_mkl.h" .
```

 It is required only in case you want to use the `MKL_RODFT00` constant.
- If the application does not check whether a `NULL` plan is returned by plan creation functions, this check must be added, because the FFTW3 to Intel MKL wrappers do not provide 100% of FFTW3 functionality.

## Running FFTW3 Interface Wrapper Examples

There are some examples that demonstrate how to use the wrapper library. The source code for the examples, makefiles used to run them, and files with lists of examples are located in the `.\examples\fftw3xc` subdirectory in the Intel MKL directory.

Parameters of the example makefiles are similar to the parameters of the wrapper library makefiles. Example makefiles normally build and invoke the examples. If the parameter `function=<example_name>` is defined, then only the specified example will run. Otherwise, all examples will be executed. Results of running the examples are saved in subdirectory `.\_results` in files with extension `.res`.

For detailed information about options for the example makefile, refer to the makefile.

## MPI FFTW3 Wrappers

This section describes a collection of MPI FFTW wrappers to Intel® MKL.

MPI FFTW wrappers are available only with Intel MKL for the Linux\* and Windows\* operating systems.

These wrappers translate calls of MPI FFTW functions to the calls of the Intel MKL cluster Fourier transform (CFFT) functions. The purpose of the wrappers is to enable users of MPI FFTW functions improve performance of the applications without changing the program source code.

Although the MPI FFTW wrappers provide less functionality than the original FFTW3 because of differences between MPI FFTW and Intel MKL CFFT, the wrappers cover many typical CFFT use cases.

The MPI FFTW wrappers are provided as source code. To use the wrappers, you need to build your own wrapper library (see [Building Your Own Wrapper Library](#)).

See also these resources:

Intel MKL Release Notes                      for the version of the FFTW3 library supported by the wrappers.

[www.fftw.org](http://www.fftw.org)                                      for a description of the MPI FFTW interface.

[Cluster FFT Functions](#)                      for a description of the Intel MKL CFFT interface.

## Building Your Own Wrapper Library

The MPI FFTW wrappers for FFTW3 are delivered as source code, which can be compiled to build a wrapper library.

The source code for the wrappers, makefiles, and files with lists of functions are located in subdirectory `.\interfaces\fftw3x_cdft` in the Intel MKL directory.

To build the wrappers,

1. Change the current directory to the wrapper directory

2. Run the `make` command on Linux\* OS or the `nmake` command on Windows\* OS with a required target and optionally several parameters.

The target `libia32` or `libintel64` defines the platform architecture, and the other parameters specify the compiler, size of the default `INTEGER` type, as well as the name and placement of the resulting wrapper library. You can find a detailed and up-to-date description of the parameters in the makefile.

In the following example, the `make` command is used to build the MPI FFTW wrappers to Intel MKL for use from the GNU C compiler on Linux OS based on Intel® 64 architecture:

```
cd interfaces/fftw3x_cdft
make libintel64 compiler=gnu mpi=openmpi INSTALL_DIR=/my/path
```

This command builds the wrapper library using the GNU `gcc` compiler so that the final executable can use Open MPI, and places the result, named `libfftw3x_cdft_DOUBLE.a`, into directory `/my/path`.

## Building an Application

Normally, the only change needed to build your application with MPI FFTW wrappers replacing original FFTW3 library is to add Intel MKL and the wrapper library at the link stage (see section "*Linking Your Application with Intel® Math Kernel Library*" in the *Intel MKL Developer Guide*).

When you are recompiling your application, add subdirectory `include\fftw` to the search path for header files to avoid FFTW3 version conflicts.

## Running Examples

There are some examples that demonstrate how to use the MPI FFTW wrapper library for FFTW3. The source code for the examples, makefiles used to run them, and files with lists of examples are located in the `.\examples\fftw3x_cdft` subdirectory in the Intel MKL directory.

Parameters of the example makefiles are similar to the parameters of the wrapper library makefiles. Example makefiles normally build and invoke the examples. Results of running the examples are saved in subdirectory `.\_results` in files with extension `.res`.

For detailed information about options for the example makefile, refer to the makefile.

## See Also

[Building Your Own Wrapper Library](#)

# Code Examples

---

This appendix presents code examples of using some Intel MKL routines and functions.

Please refer to respective chapters in the document for detailed descriptions of function parameters and operation.

## BLAS Code Examples

---

### Example. Using BLAS Level 1 Function

---

The following example illustrates a call to the BLAS Level 1 function `sdot`. This function performs a vector-vector operation of computing a scalar product of two single-precision real vectors `x` and `y`.

#### Parameters

|                   |                                                                                 |
|-------------------|---------------------------------------------------------------------------------|
| <code>n</code>    | Specifies the number of elements in vectors <code>x</code> and <code>y</code> . |
| <code>incx</code> | Specifies the increment for the elements of <code>x</code> .                    |
| <code>incy</code> | Specifies the increment for the elements of <code>y</code> .                    |

```
#include <stdio.h>
#include <stdlib.h>

#include "mkl_example.h"

int main()
{
 MKL_INT n, incx, incy, i;
 float *x, *y;
 float res;
 MKL_INT len_x, len_y;

 n = 5;
 incx = 2;
 incy = 1;

 len_x = 1+(n-1)*abs(incx);
 len_y = 1+(n-1)*abs(incy);
 x = (float *)calloc(len_x, sizeof(float));
 y = (float *)calloc(len_y, sizeof(float));
 if(x == NULL || y == NULL) {
 printf("\n Can't allocate memory for arrays\n");
 return 1;
 }

 for (i = 0; i < n; i++) {
 x[i*abs(incx)] = 2.0;
 y[i*abs(incy)] = 1.0;
 }

 res = cblas_sdot(n, x, incx, y, incy);
```

```

 printf("\n SDOT = %7.3f", res);

 free(x);
 free(y);

 return 0;
}

```

As a result of this program execution, the following line is printed:

SDOT = 10.000

### Example. Using BLAS Level 1 Routine

The following example illustrates a call to the BLAS Level 1 routine `scopy`. This routine performs a vector-vector operation of copying a single-precision real vector `x` to a vector `y`.

#### Parameters

|                   |                                                                                 |
|-------------------|---------------------------------------------------------------------------------|
| <code>n</code>    | Specifies the number of elements in vectors <code>x</code> and <code>y</code> . |
| <code>incx</code> | Specifies the increment for the elements of <code>x</code> .                    |
| <code>incy</code> | Specifies the increment for the elements of <code>y</code> .                    |

```

#include <stdio.h>
#include <stdlib.h>

#include "mkl_example.h"

int main()
{
 MKL_INT n, incx, incy, i;
 float *x, *y;
 MKL_INT len_x, len_y;
 n = 3;
 incx = 3;
 incy = 1;

 len_x = 10;
 len_y = 10;
 x = (float *)calloc(len_x, sizeof(float));
 y = (float *)calloc(len_y, sizeof(float));
 if(x == NULL || y == NULL) {
 printf("\n Can't allocate memory for arrays\n");
 return 1;
 }
 for (i = 0; i < 10; i++) {
 x[i] = i + 1;
 }

 cblas_scopy(n, x, incx, y, incy);

/* Print output data */

 printf("\n\n OUTPUT DATA");
 PrintVectorS(FULLPRINT, n, y, incy, "Y");

 free(x);
}

```

```

 free(y);
 return 0;
}

```

As a result of this program execution, the following line is printed:

Y = 1.00000 4.00000 7.00000

### Example. Using BLAS Level 2 Routine

The following example illustrates a call to the BLAS Level 2 routine `sger`. This routine performs a matrix-vector operation

```
a := alpha*x*y' + a.
```

#### Parameters

|              |                                   |
|--------------|-----------------------------------|
| <i>alpha</i> | Specifies a scalar <i>alpha</i> . |
| <i>x</i>     | <i>m</i> -element vector.         |
| <i>y</i>     | <i>n</i> -element vector.         |
| <i>a</i>     | <i>m</i> -by- <i>n</i> matrix.    |

```

#include <stdio.h>
#include <stdlib.h>

#include "mkl_example.h"

int main()
{
 MKL_INT m, n, lda, incx, incy, i, j;
 MKL_INT rmaxa, cmaxa;
 float alpha;
 float *a, *x, *y;
 CBLAS_LAYOUT layout;
 MKL_INT len_x, len_y;

 m = 2;
 n = 3;
 lda = 5;
 incx = 2;
 incy = 1;
 alpha = 0.5;

 layout = CblasRowMajor;

 len_x = 10;
 len_y = 10;
 rmaxa = m + 1;
 cmaxa = n;
 a = (float *)calloc(rmaxa*cmaxa, sizeof(float));
 x = (float *)calloc(len_x, sizeof(float));
 y = (float *)calloc(len_y, sizeof(float));
 if(a == NULL || x == NULL || y == NULL) {
 printf("\n Can't allocate memory for arrays\n");
 return 1;
 }
 if(layout == CblasRowMajor)
 lda=cmaxa;
 else

```

```

 lda=rmaxa;

 for (i = 0; i < 10; i++) {
 x[i] = 1.0;
 y[i] = 1.0;
 }

 for (i = 0; i < m; i++) {
 for (j = 0; j < n; j++) {
 a[i + j*lda] = j + 1;
 }
 }

 cblas_sger(layout, m, n, alpha, x, incx, y, incy, a, lda);

 PrintArrayS(&layout, FULLPRINT, GENERAL_MATRIX, &m, &n, a, &lda, "A");

 free(a);
 free(x);
 free(y);

 return 0;
}

```

As a result of this program execution, matrix *a* is printed as follows:

Matrix A:

```

1.50000 2.50000 3.50000
1.50000 2.50000 3.50000

```

### Example. Using BLAS Level 3 Routine

The following example illustrates a call to the BLAS Level 3 routine `ssymm`. This routine performs a matrix-matrix operation

```
c := alpha*a*b' + beta*c.
```

#### Parameters

|              |                                   |
|--------------|-----------------------------------|
| <i>alpha</i> | Specifies a scalar <i>alpha</i> . |
| <i>beta</i>  | Specifies a scalar <i>beta</i> .  |
| <i>a</i>     | Symmetric matrix                  |
| <i>b</i>     | <i>m</i> -by- <i>n</i> matrix     |
| <i>c</i>     | <i>m</i> -by- <i>n</i> matrix     |

```

#include <stdio.h>
#include <stdlib.h>

#include "mkl_example.h"

int main(int argc, char *argv[])
{
 MKL_INT m, n, i, j;
 MKL_INT lda, ldb, ldc;
 MKL_INT rmaxa, cmaxa, rmaxb, cmaxb, rmaxc, cmaxc;
 float alpha, beta;

```

```

float *a, *b, *c;
CBLAS_LAYOUT layout;
CBLAS_SIDE side;
CBLAS_UPLO upto;
MKL_INT ma, na, typeA;
upto = 'u';
side = 'l';
layout = CblasRowMajor;
m = 3;
n = 2;
lda = 3;
ldb = 3;
ldc = 3;
alpha = 0.5;
beta = 2.0;

if(side == CblasLeft) {
 rmaxa = m + 1;
 cmaxa = m;
 ma = m;
 na = m;
} else {
 rmaxa = n + 1;
 cmaxa = n;
 ma = n;
 na = n;
}
rmaxb = m + 1;
cmaxb = n;
rmaxc = m + 1;
cmaxc = n;
a = (float *)calloc(rmaxa*cmaxa, sizeof(float));
b = (float *)calloc(rmaxb*cmaxb, sizeof(float));
c = (float *)calloc(rmaxc*cmaxc, sizeof(float));
if (a == NULL || b == NULL || c == NULL) {
 printf("\n Can't allocate memory arrays");
 return 1;
}
if(layout == CblasRowMajor) {
 lda=cmaxa;
 ldb=cmaxb;
 ldc=cmaxc;
} else {
 lda=rmaxa;
 ldb=rmaxb;
 ldc=rmaxc;
}
if (upto == CblasUpper) typeA = UPPER_MATRIX;
else typeA = LOWER_MATRIX;
for (i = 0; i < m; i++) {
 for (j = 0; j < m; j++) {
 a[i + j*lda] = 1.0;
 }
}
for (i = 0; i < m; i++) {
 for (j = 0; j < n; j++) {
 c[i + j*ldc] = 1.0;
 b[i + j*ldb] = 2.0;
 }
}

```

```

}

cblas_ssymm(layout, side, uplo, m, n, alpha, a, lda,
 b, ldb, beta, c, ldc);

printf("\n\n OUTPUT DATA");
PrintArrayS(&layout, FULLPRINT, GENERAL_MATRIX, &m, &n, c, &ldc, "C");

free(a);
free(b);
free(c);

return 0;
}

```

As a result of this program execution, matrix *c* is printed as follows:

Matrix C:

5.00000 5.00000

5.00000 5.00000

5.00000 5.00000

The following example illustrates a call from a C program to the Fortran version of the complex BLAS Level 1 function `zdotc()`. This function computes the dot product of two double-precision complex vectors.

### Example. Calling a Complex BLAS Level 1 Function from C

In this example, the complex dot product is returned in the structure *c*.

```

#include <stdio>
#include "mkl_blas.h"
#define N 5
void main()
{
 int n, inca = 1, incb = 1, i;
 MKL_Complex16 a[N], b[N], c;
 void zdotc();
 n = N;
 for(i = 0; i < n; i++){
 a[i].real = (double)i; a[i].imag = (double)i * 2.0;
 b[i].real = (double)(n - i); b[i].imag = (double)i * 2.0;
 }
 zdotc(&c, &n, a, &inca, b, &incb);
 printf("The complex dot product is: (%6.2f, %6.2f)\n", c.real, c.imag);
}

```

#### NOTE

Instead of calling BLAS directly from C programs, you might wish to use the C interface to the Basic Linear Algebra Subprograms (CBLAS) implemented in Intel® MKL. See [C Interface Conventions](#) for more information.

## Fourier Transform Functions Code Examples

This section presents code examples for functions described in the “[FFT Functions](#)” and “[Cluster FFT Functions](#)” sections in the “Fourier Transform Functions” chapter. The examples are grouped in subsections

- [Examples for FFT Functions](#), including [Examples of Using Multi-Threading for FFT Computation](#)
- [Examples for Cluster FFT Functions](#)
- [Auxiliary data transformations](#).

## FFT Code Examples

This section presents examples of using the FFT interface functions described in “[Fourier Transform Functions](#)” chapter.

Here are the examples of two one-dimensional computations. These examples use the default settings for all of the configuration parameters, which are specified in “[Configuration Settings](#)”.

### One-dimensional In-place FFT

```
/* C example, float _Complex is defined in C9X */
#include "mkl_dfti.h"
float _Complex x[32];
float y[34];
DFTI_DESCRIPTOR_HANDLE my_desc1_handle;
DFTI_DESCRIPTOR_HANDLE my_desc2_handle;
MKL_LONG status;
//...put input data into x[0],...,x[31]; y[0],...,y[31]
status = DftiCreateDescriptor(&my_desc1_handle, DFTI_SINGLE,
 DFTI_COMPLEX, 1, 32);
status = DftiCommitDescriptor(my_desc1_handle);
status = DftiComputeForward(my_desc1_handle, x);
status = DftiFreeDescriptor(&my_desc1_handle);
/* result is x[0], ..., x[31]*/
status = DftiCreateDescriptor(&my_desc2_handle, DFTI_SINGLE,
 DFTI_REAL, 1, 32);
status = DftiCommitDescriptor(my_desc2_handle);
status = DftiComputeForward(my_desc2_handle, y);
status = DftiFreeDescriptor(&my_desc2_handle);
/* result is given in CCS format*/
```

### One-dimensional Out-of-place FFT

```
/* C example, float _Complex is defined in C9X */
#include "mkl_dfti.h"
float _Complex x_in[32];
float _Complex x_out[32];
float y_in[32];
float y_out[34];

DFTI_DESCRIPTOR_HANDLE my_desc1_handle;
DFTI_DESCRIPTOR_HANDLE my_desc2_handle;
MKL_LONG status;
//...put input data into x_in[0],...,x_in[31]; y_in[0],...,y_in[31]
status = DftiCreateDescriptor(&my_desc1_handle, DFTI_SINGLE,
 DFTI_COMPLEX, 1, 32);
status = DftiSetValue(my_desc1_handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE);
status = DftiCommitDescriptor(my_desc1_handle);
status = DftiComputeForward(my_desc1_handle, x_in, x_out);
status = DftiFreeDescriptor(&my_desc1_handle);
/* result is x_out[0], ..., x_out[31]*/
status = DftiCreateDescriptor(&my_desc2_handle, DFTI_SINGLE,
 DFTI_REAL, 1, 32);
Status = DftiSetValue(My_Desc2_Handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE);
```

```
status = DftiCommitDescriptor(my_desc2_handle);
status = DftiComputeForward(my_desc2_handle, y_in, y_out);
status = DftiFreeDescriptor(&my_desc2_handle);
/* result is given by y_out in CCS format*/
```

## Two-dimensional FFT

---

```
/* C99 example */
#include "mkl_dfti.h"
float _Complex x[32][100];
float y[34][102];
DFTI_DESCRIPTOR_HANDLE my_desc1_handle;
DFTI_DESCRIPTOR_HANDLE my_desc2_handle;
MKL_LONG status, l[2];
//...put input data into x[j][k] 0<=j<=31, 0<=k<=99
//...put input data into y[j][k] 0<=j<=31, 0<=k<=99
l[0] = 32; l[1] = 100;
status = DftiCreateDescriptor(&my_desc1_handle, DFTI_SINGLE,
 DFTI_COMPLEX, 2, l);
status = DftiCommitDescriptor(my_desc1_handle);
status = DftiComputeForward(my_desc1_handle, x);
status = DftiFreeDescriptor(&my_desc1_handle);
/* result is the complex value x[j][k], 0<=j<=31, 0<=k<=99 */
status = DftiCreateDescriptor(&my_desc2_handle, DFTI_SINGLE,
 DFTI_REAL, 2, l);
status = DftiCommitDescriptor(my_desc2_handle);
status = DftiComputeForward(my_desc2_handle, y);
status = DftiFreeDescriptor(&my_desc2_handle);
/* result is the complex value z(j,k) 0<=j<=31; 0<=k<=99
/* and is stored in CCS format*/
```

The following example demonstrates how you can change the default configuration settings by using the `DftiSetValue` function.

For instance, to preserve the input data after the FFT computation, the configuration of `DFTI_PLACEMENT` should be changed to "not in place" from the default choice of "in place."

The code below illustrates how this can be done:

## Changing Default Settings

---

```
/* C99 example */
#include "mkl_dfti.h"
float _Complex x_in[32], x_out[32];
DFTI_DESCRIPTOR_HANDLE my_desc_handle;
MKL_LONG status;
//...put input data into x_in[j], 0 <= j < 32
status = DftiCreateDescriptor(&my_desc_handle, DFTI_SINGLE,
 DFTI_COMPLEX, 1, 32);
status = DftiSetValue(my_desc_handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE);
status = DftiCommitDescriptor(my_desc_handle);
status = DftiComputeForward(my_desc_handle, x_in, x_out);
status = DftiFreeDescriptor(&my_desc_handle);
/* result is x_out[0], x_out[1], ..., x_out[31] */
```

## Using Status Checking Functions

This example illustrates the use of status checking functions described in [Chapter 11](#).

```

/* C */
DFTI_DESCRIPTOR_HANDLE desc;
MKL_LONG status;
// . . . descriptor creation and other code
status = DftiCommitDescriptor(desc);
if (status && !DftiErrorClass(status,DFTI_NO_ERROR))
{
 printf ('Error: %s\n', DftiErrorMessage(status));
}

```

## Computing 2D FFT by One-Dimensional Transforms

Below is an example where a 20-by-40 two-dimensional FFT is computed explicitly using one-dimensional transforms.

```

/* C */
#include "mkl_dfti.h"
float _Complex x[20][40];
MKL_LONG stride[2];
MKL_LONG status;
DFTI_DESCRIPTOR_HANDLE desc_handle_dim1;
DFTI_DESCRIPTOR_HANDLE desc_handle_dim2;
//...
status = DftiCreateDescriptor(&desc_handle_dim1, DFTI_SINGLE,
 DFTI_COMPLEX, 1, 20);
status = DftiCreateDescriptor(&desc_handle_dim2, DFTI_SINGLE,
 DFTI_COMPLEX, 1, 40);

/* perform 40 one-dimensional transforms along 1st dimension */
/* note that the 1st dimension data are not unit-stride */
stride[0] = 0; stride[1] = 40;
status = DftiSetValue(desc_handle_dim1, DFTI_NUMBER_OF_TRANSFORMS, 40);
status = DftiSetValue(desc_handle_dim1, DFTI_INPUT_DISTANCE, 1);
status = DftiSetValue(desc_handle_dim1, DFTI_OUTPUT_DISTANCE, 1);
status = DftiSetValue(desc_handle_dim1, DFTI_INPUT_STRIDES, stride);
status = DftiSetValue(desc_handle_dim1, DFTI_OUTPUT_STRIDES, stride);
status = DftiCommitDescriptor(desc_handle_dim1);
status = DftiComputeForward(desc_handle_dim1, x);
/* perform 20 one-dimensional transforms along 2nd dimension */
/* note that the 2nd dimension is unit stride */
status = DftiSetValue(desc_handle_dim2, DFTI_NUMBER_OF_TRANSFORMS, 20);
status = DftiSetValue(desc_handle_dim2,
 DFTI_INPUT_DISTANCE, 40);
status = DftiSetValue(desc_handle_dim2,
 DFTI_OUTPUT_DISTANCE, 40);
status = DftiCommitDescriptor(desc_handle_dim2);
status = DftiComputeForward(desc_handle_dim2, x);
status = DftiFreeDescriptor(&desc_handle_dim1);
status = DftiFreeDescriptor(&desc_handle_dim2);

```

The following code illustrates real multi-dimensional transforms with CCE format storage of conjugate-even complex matrix. [Example "Three-Dimensional REAL FFT \(C Interface\)"](#) is three-dimensional out-of-place transform in C interface.

### Three-Dimensional REAL FFT

```

/* C99 example */
#include "mkl_dfti.h"
float x[32][100][19];
float _Complex y[32][100][10]; /* 10 = 19/2 + 1 */
DFTI_DESCRIPTOR_HANDLE my_desc_handle;
MKL_LONG status, l[3];
MKL_LONG strides_out[4];

//...put input data into x[j][k][s] 0<=j<=31, 0<=k<=99, 0<=s<=18
l[0] = 32; l[1] = 100; l[2] = 19;

strides_out[0] = 0; strides_out[1] = 1000;
strides_out[2] = 10; strides_out[3] = 1;

status = DftiCreateDescriptor(&my_desc_handle, DFTI_SINGLE,
DFTI_REAL, 3, l);
status = DftiSetValue(my_desc_handle,
DFTI_CONJUGATE_EVEN_STORAGE, DFTI_COMPLEX_COMPLEX);
status = DftiSetValue(my_desc_handle, DFTI_PLACEMENT, DFTI_NOT_INPLACE);
status = DftiSetValue(my_desc_handle,
DFTI_OUTPUT_STRIDES, strides_out);

status = DftiCommitDescriptor(my_desc_handle);
status = DftiComputeForward(my_desc_handle, x, y);
status = DftiFreeDescriptor(&my_desc_handle);
/* result is the complex value z(j,k,s) 0<=j<=31; 0<=k<=99, 0<=s<=9
and is stored in complex matrix y in CCE format. */

```

### Examples of Using OpenMP\* Threading for FFT Computation

The following sample program shows how to employ internal OpenMP\* threading in Intel MKL for FFT computation.

To specify the number of threads inside Intel MKL, use the following settings:

set MKL\_NUM\_THREADS = 1 for one-threaded mode;

set MKL\_NUM\_THREADS = 4 for multi-threaded mode.

### Using Intel MKL Internal Threading Mode (C Example)

```

#include "mkl_dfti.h"

int main ()
{
 float x[200][100];
 DFTI_DESCRIPTOR_HANDLE fft;
 MKL_LONG len[2] = {200, 100};
 // initialize x
 DftiCreateDescriptor (&fft, DFTI_SINGLE, DFTI_REAL, 2, len);
 DftiCommitDescriptor (fft);
 DftiComputeForward (fft, x);
 DftiFreeDescriptor (&fft);
}

```

```

return 0;
}

```

The following [Example "Using Parallel Mode with Multiple Descriptors Initialized in a Parallel Region"](#) and [Example "Using Parallel Mode with Multiple Descriptors Initialized in One Thread"](#) illustrate a parallel customer program with each descriptor instance used only in a single thread.

Specify the number of OpenMP threads for [Example "Using Parallel Mode with Multiple Descriptors Initialized in a Parallel Region"](#) like this:

set MKL\_NUM\_THREADS = 1 for Intel MKL to work in the single-threaded mode (recommended);

set OMP\_NUM\_THREADS = 4 for the customer program to work in the multi-threaded mode.

### Using Parallel Mode with Multiple Descriptors Initialized in a Parallel Region

Note that in this example, the program can be transformed to become single-threaded at the customer level but using parallel mode within Intel MKL. To achieve this, you need to set the parameter DFTI\_NUMBER\_OF\_TRANSFORMS = 4 and to set the corresponding parameter DFTI\_INPUT\_DISTANCE = 5000.

```

#include "mkl_dfti.h"
#include <omp.h>
#define ARRAY_LEN(a) sizeof(a)/sizeof(a[0])
int main ()
{
 // 4 OMP threads, each does 2D FFT 50x100 points
 MKL_Complex8 x[4][50][100];
 int nth = ARRAY_LEN(x);
 MKL_LONG len[2] = {ARRAY_LEN(x[0]), ARRAY_LEN(x[0][0])};
 int th;
 // assume x is initialized and do 2D FFTs
#pragma omp parallel for shared(len, x)
 for (th = 0; th < nth; th++)
 {
 DFTI_DESCRIPTOR_HANDLE myFFT;

 DftiCreateDescriptor (&myFFT, DFTI_SINGLE, DFTI_COMPLEX, 2, len);
 DftiCommitDescriptor (myFFT);
 DftiComputeForward (myFFT, x[th]);
 DftiFreeDescriptor (&myFFT);
 }
 return 0;
}

```

Specify the number of OpenMP threads for [Example "Using Parallel Mode with Multiple Descriptors Initialized in One Thread"](#) like this:

set MKL\_NUM\_THREADS = 1 for Intel MKL to work in the single-threaded mode (obligatory);

set OMP\_NUM\_THREADS = 4 for the customer program to work in the multi-threaded mode.

### Using Parallel Mode with Multiple Descriptors Initialized in One Thread

```

#include "mkl_dfti.h"
#include <omp.h>
#define ARRAY_LEN(a) sizeof(a)/sizeof(a[0])
int main ()
{
 // 4 OMP threads, each does 2D FFT 50x100 points
 MKL_Complex8 x[4][50][100];
 int nth = ARRAY_LEN(x);
 MKL_LONG len[2] = {ARRAY_LEN(x[0]), ARRAY_LEN(x[0][0])};

```

```

DFTI_DESCRIPTOR_HANDLE FFT[ARRAY_LEN(x)];
int th;

for (th = 0; th < nth; th++)
 DftiCreateDescriptor (&FFT[th], DFTI_SINGLE, DFTI_COMPLEX, 2, len);
for (th = 0; th < nth; th++)
 DftiCommitDescriptor (FFT[th]);
// assume x is initialized and do 2D FFTs
#pragma omp parallel for shared(FFT, x)
for (th = 0; th < nth; th++)
 DftiComputeForward (FFT[th], x[th]);
for (th = 0; th < nth; th++)
 DftiFreeDescriptor (&FFT[th]);
return 0;
}

```

The following [Example “Using Parallel Mode with a Common Descriptor”](#) illustrates a parallel customer program with a common descriptor used in several threads.

## Using Parallel Mode with a Common Descriptor

```

#include "mkl_dfti.h"
#include <omp.h>
#define ARRAY_LEN(a) sizeof(a)/sizeof(a[0])
int main ()
{
 // 4 OMP threads, each does 2D FFT 50x100 points
 MKL_Complex8 x[4][50][100];
 int nth = ARRAY_LEN(x);
 MKL_LONG len[2] = {ARRAY_LEN(x[0]), ARRAY_LEN(x[0][0])};
 DFTI_DESCRIPTOR_HANDLE FFT;
 int th;

 DftiCreateDescriptor (&FFT, DFTI_SINGLE, DFTI_COMPLEX, 2, len);
 DftiCommitDescriptor (FFT);
 // assume x is initialized and do 2D FFTs
#pragma omp parallel for shared(FFT, x)
for (th = 0; th < nth; th++)
 DftiComputeForward (FFT, x[th]);
 DftiFreeDescriptor (&FFT);
 return 0;
}

```

## Examples for Cluster FFT Functions

The following C example computes a 2-dimensional out-of-place FFT using the cluster FFT interface:

### 2D Out-of-place Cluster FFT Computation

```

DFTI_DESCRIPTOR_DM_HANDLE desc;
MKL_LONG len[2], v, i, j, n, s;
Complex *in, *out;

MPI_Init(...);

// Create descriptor for 2D FFT
len[0]=nx;
len[1]=ny;
DftiCreateDescriptorDM(MPI_COMM_WORLD, &desc, DFTI_DOUBLE, DFTI_COMPLEX, 2, len);
// Ask necessary length of in and out arrays and allocate memory

```

```

DftiGetValueDM(desc,CDFT_LOCAL_SIZE,&v);
in=(Complex*)malloc(v*sizeof(Complex));
out=(Complex*)malloc(v*sizeof(Complex));
// Fill local array with initial data. Current process performs n rows,
// 0 row of in corresponds to s row of virtual global array
DftiGetValueDM(desc,CDFT_LOCAL_NX,&n);
DftiGetValueDM(desc,CDFT_LOCAL_X_START,&s);
// Virtual global array globalIN is defined by function f as
// globalIN[i*ny+j]=f(i,j)
for(i=0;i<n;i++)
 for(j=0;j<ny;j++) in[i*ny+j]=f(i+s,j);
// Set that we want out-of-place transform (default is DFTI_INPLACE)
DftiSetValueDM(desc,DFTI_PLACEMENT,DFTI_NOT_INPLACE);
// Commit descriptor, calculate FFT, free descriptor
DftiCommitDescriptorDM(desc);
DftiComputeForwardDM(desc,in,out);
// Virtual global array globalOUT is defined by function g as
// globalOUT[i*ny+j]=g(i,j)
// Now out contains result of FFT. out[i*ny+j]=g(i+s,j)
DftiFreeDescriptorDM(&desc);
free(in);
free(out);
MPI_Finalize();

```

## 1D In-place Cluster FFT Computations

The C example below illustrates one-dimensional in-place cluster FFT computations effected with a user-defined workspace:

```

DFTI_DESCRIPTOR_DM_HANDLE desc;
MKL_LONG len,v,i,n_out,s_out;
Complex *in,*work;

MPI_Init(...);
// Create descriptor for 1D FFT
DftiCreateDescriptorDM(MPI_COMM_WORLD,&desc,DFTI_DOUBLE,DFTI_COMPLEX,1,len);
// Ask necessary length of array and workspace and allocate memory
DftiGetValueDM(desc,CDFT_LOCAL_SIZE,&v);
in=(Complex*)malloc(v*sizeof(Complex));
work=(Complex*)malloc(v*sizeof(Complex));
// Fill local array with initial data. Local array has n elements,
// 0 element of in corresponds to s element of virtual global array
DftiGetValueDM(desc,CDFT_LOCAL_NX,&n);
DftiGetValueDM(desc,CDFT_LOCAL_X_START,&s);
// Set work array as a workspace
DftiSetValueDM(desc,CDFT_WORKSPACE,work);
// Virtual global array globalIN is defined by function f as globalIN[i]=f(i)
for(i=0;i<n;i++) in[i]=f(i+s);
// Commit descriptor, calculate FFT, free descriptor
DftiCommitDescriptorDM(desc);
DftiComputeForwardDM(desc,in);
DftiGetValueDM(desc,CDFT_LOCAL_OUT_NX,&n_out);
DftiGetValueDM(desc,CDFT_LOCAL_OUT_X_START,&s_out);
// Virtual global array globalOUT is defined by function g as globalOUT[i]=g(i)
// Now in contains result of FFT. Local array has n_out elements,
// 0 element of in corresponds to s_out element of virtual global array.
// in[i]==g(i+s_out)
DftiFreeDescriptorDM(&desc);
free(in);

```

```
free(work);
MPI_Finalize();
```

## Auxiliary Data Transformations

This section presents C examples for conversion from the Cartesian to polar representation of complex data and vice versa.

### Conversion from Cartesian to polar representation of complex data

---

```
// Cartesian->polar conversion of complex data
// Cartesian representation: z = re + I*im
// Polar representation: z = r * exp(I*phi)
#include <mkl_vml.h>

void
variant1_Cartesian2Polar(int n,const double *re,const double *im,
 double *r,double *phi)
{
 vdHypot(n,re,im,r); // compute radii r[]
 vdAtan2(n,im,re,phi); // compute phases phi[]
}

void
variant2_Cartesian2Polar(int n,const MKL_Complex16 *z,double *r,double *phi,
 double *temp_re,double *temp_im)
{
 vzAbs(n,z,r); // compute radii r[]
 vdPackI(n, (double*)z + 0, 2, temp_re);
 vdPackI(n, (double*)z + 1, 2, temp_im);
 vdAtan2(n,temp_im,temp_re,phi); // compute phases phi[]
}
```

### Conversion from polar to Cartesian representation of complex data

---

```
// Polar->Cartesian conversion of complex data.
// Polar representation: z = r * exp(I*phi)
// Cartesian representation: z = re + I*im
#include <mkl_vml.h>

void
variant1_Polar2Cartesian(int n,const double *r,const double *phi,
 double *re,double *im)
{
 vdSinCos(n,phi,im,re); // compute direction, i.e. z[]/abs(z[])
 vdMul(n,r,re,re); // scale real part
 vdMul(n,r,im,im); // scale imaginary part
}

void
variant2_Polar2Cartesian(int n,const double *r,const double *phi,
 MKL_Complex16 *z,
 double *temp_re,double *temp_im)
{
 vdSinCos(n,phi,temp_im,temp_re); // compute direction, i.e. z[]/abs(z[])
 vdMul(n,r,temp_im,temp_im); // scale imaginary part
}
```

```
vdMul(n,r,temp_re,temp_re); // scale real part
vdUnpackI(n,temp_re,(double*)z + 0, 2); // fill in result.re
vdUnpackI(n,temp_im,(double*)z + 1, 2); // fill in result.im
}
```



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For a reference implementation of BLAS, sparse BLAS, LAPACK, and ScaLAPACK packages visit [www.netlib.org](http://www.netlib.org).



# Glossary

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|                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
|-----------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $A^H$                       | Denotes the conjugate transpose of a general matrix $A$ . See also conjugate matrix.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| $A^T$                       | Denotes the transpose of a general matrix $A$ . See also transpose.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| band matrix                 | A general $m$ -by- $n$ matrix $A$ such that $a_{ij} = 0$ for $ i - j  > l$ , where $1 < l < \min(m, n)$ . For example, any tridiagonal matrix is a band matrix.                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| band storage                | A special storage scheme for band matrices. A matrix is stored in a two-dimensional array: columns of the matrix are stored in the corresponding columns of the array, and <i>diagonals</i> of the matrix are stored in rows of the array.                                                                                                                                                                                                                                                                                                                                                                                       |
| BLAS                        | Abbreviation for Basic Linear Algebra Subprograms. These subprograms implement vector, matrix-vector, and matrix-matrix operations.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| BRNG                        | Abbreviation for Basic Random Number Generator. Basic random number generators are pseudorandom number generators imitating i.i.d. random number sequences of uniform distribution. Distributions other than uniform are generated by applying different transformation techniques to the sequences of random numbers of uniform distribution.                                                                                                                                                                                                                                                                                   |
| BRNG registration           | Standardized mechanism that allows a user to include a user-designed BRNG into the VSL and use it along with the predefined VSL basic generators.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| Bunch-Kaufman factorization | Representation of a real symmetric or complex Hermitian matrix $A$ in the form $A = PUDU^H P^T$ (or $A = PLDL^H P^T$ ) where $P$ is a permutation matrix, $U$ and $L$ are upper and lower triangular matrices with unit diagonal, and $D$ is a Hermitian block-diagonal matrix with 1-by-1 and 2-by-2 diagonal blocks. $U$ and $L$ have 2-by-2 unit diagonal blocks corresponding to the 2-by-2 blocks of $D$ .                                                                                                                                                                                                                  |
| c                           | When found as the first letter of routine names, $c$ indicates the usage of single-precision complex data type.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| CBLAS                       | C interface to the BLAS. See BLAS.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| CDF                         | Cumulative Distribution Function. The function that determines probability distribution for univariate or multivariate random variable $X$ . For univariate distribution the cumulative distribution function is the function of real argument $x$ , which for every $x$ takes a value equal to probability of the event $A: X \leq x$ . For multivariate distribution the cumulative distribution function is the function of a real vector $x = (x_1, x_2, \dots, x_n)$ , which, for every $x$ , takes a value equal to probability of the event $A = (X_1 \leq x_1 \ \& \ X_2 \leq x_2, \ \& \ \dots, \ \& \ X_n \leq x_n)$ . |

|                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Cholesky factorization                   | Representation of a symmetric positive-definite or, for complex data, Hermitian positive-definite matrix $A$ in the form $A = U^H U$ or $A = L L^H$ , where $L$ is a lower triangular matrix and $U$ is an upper triangular matrix.                                                                                                                                                                                                                                                                 |
| condition number                         | The number $\kappa(A)$ defined for a given square matrix $A$ as follows: $\kappa(A) = \ A\  \ A^{-1}\ $ .                                                                                                                                                                                                                                                                                                                                                                                           |
| conjugate matrix                         | The matrix $A^H$ defined for a given general matrix $A$ as follows: $(A^H)_{ij} = (a_{ji})^*$ .                                                                                                                                                                                                                                                                                                                                                                                                     |
| conjugate number                         | The conjugate of a complex number $z = a + bi$ is $z^* = a - bi$ .                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| d                                        | When found as the first letter of routine names, d indicates the usage of double-precision real data type.                                                                                                                                                                                                                                                                                                                                                                                          |
| dot product                              | The number denoted $x \cdot y$ and defined for given vectors $x$ and $y$ as follows: $x \cdot y = \sum_i x_i y_i$ .<br>Here $x_i$ and $y_i$ stand for the $i$ -th elements of $x$ and $y$ , respectively.                                                                                                                                                                                                                                                                                           |
| double precision                         | A floating-point data type. On Intel® processors, this data type allows you to store real numbers $x$ such that $2.23 \times 10^{-308} <  x  < 1.79 \times 10^{308}$ . For this data type, the machine precision $\epsilon$ is approximately $10^{-15}$ , which means that double-precision numbers usually contain no more than 15 significant decimal digits. For more information, refer to <i>Intel® 64 and IA-32 Architectures Software Developer's Manual, Volume 1: Basic Architecture</i> . |
| eigenvalue                               | See eigenvalue problem.                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| eigenvalue problem                       | A problem of finding non-zero vectors $x$ and numbers $\lambda$ (for a given square matrix $A$ ) such that $Ax = \lambda x$ . Here the numbers $\lambda$ are called the eigenvalues of the matrix $A$ and the vectors $x$ are called the eigenvectors of the matrix $A$ .                                                                                                                                                                                                                           |
| eigenvector                              | See eigenvalue problem.                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| elementary reflector(Householder matrix) | Matrix of a general form $H = I - \tau v v^T$ , where $v$ is a column vector and $\tau$ is a scalar. In LAPACK elementary reflectors are used, for example, to represent the matrix $Q$ in the $QR$ factorization (the matrix $Q$ is represented as a product of elementary reflectors).                                                                                                                                                                                                            |
| factorization                            | Representation of a matrix as a product of matrices. See also Bunch-Kaufman factorization, Cholesky factorization, $LU$ factorization, $LQ$ factorization, $QR$ factorization, Schur factorization.                                                                                                                                                                                                                                                                                                 |
| FFTs                                     | Abbreviation for Fast Fourier Transforms. See Chapter 11 of this book.                                                                                                                                                                                                                                                                                                                                                                                                                              |
| full storage                             | A storage scheme allowing you to store matrices of any kind. A matrix $A$ is stored in a two-dimensional array $a$ , with the matrix element $a_{ij}$ stored in the array element $a(i, j)$ .                                                                                                                                                                                                                                                                                                       |
| Hermitian matrix                         | A square matrix $A$ that is equal to its conjugate matrix $A^H$ . The conjugate $A^H$ is defined as follows: $(A^H)_{ij} = (a_{ji})^*$ .                                                                                                                                                                                                                                                                                                                                                            |
| I                                        | See identity matrix.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| identity matrix                          | A square matrix $I$ whose diagonal elements are 1, and off-diagonal elements are 0. For any matrix $A$ , $AI = A$ and $IA = A$ .                                                                                                                                                                                                                                                                                                                                                                    |

|                    |                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|--------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| i.i.d.             | Independent Identically Distributed.                                                                                                                                                                                                                                                                                                                                                                                                 |
| in-place           | Qualifier of an operation. A function that performs its operation in-place takes its input from an array and returns its output to the same array.                                                                                                                                                                                                                                                                                   |
| Intel MKL          | Abbreviation for Intel® Math Kernel Library.                                                                                                                                                                                                                                                                                                                                                                                         |
| inverse matrix     | The matrix denoted as $A^{-1}$ and defined for a given square matrix $A$ as follows: $AA^{-1} = A^{-1}A = I$ . $A^{-1}$ does not exist for singular matrices $A$ .                                                                                                                                                                                                                                                                   |
| $LQ$ factorization | Representation of an $m$ -by- $n$ matrix $A$ as $A = LQ$ or $A = (L \ 0)Q$ . Here $Q$ is an $n$ -by- $n$ orthogonal (unitary) matrix. For $m \leq n$ , $L$ is an $m$ -by- $m$ lower triangular matrix with real diagonal elements; for $m > n$ ,                                                                                                                                                                                     |
|                    | $L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$                                                                                                                                                                                                                                                                                                                                                                                       |
|                    | where $L_1$ is an $n$ -by- $n$ lower triangular matrix, and $L_2$ is a rectangular matrix.                                                                                                                                                                                                                                                                                                                                           |
| $LU$ factorization | Representation of a general $m$ -by- $n$ matrix $A$ as $A = PLU$ , where $P$ is a permutation matrix, $L$ is lower triangular with unit diagonal elements (lower trapezoidal if $m > n$ ) and $U$ is upper triangular (upper trapezoidal if $m < n$ ).                                                                                                                                                                               |
| machine precision  | The number $\varepsilon$ determining the precision of the machine representation of real numbers. For Intel® architecture, the machine precision is approximately $10^{-7}$ for single-precision data, and approximately $10^{-15}$ for double-precision data. The precision also determines the number of significant decimal digits in the machine representation of real numbers. See also double precision and single precision. |
| MPI                | Message Passing Interface. This standard defines the user interface and functionality for a wide range of message-passing capabilities in parallel computing.                                                                                                                                                                                                                                                                        |
| MPICH              | A freely available, portable implementation of MPI standard for message-passing libraries.                                                                                                                                                                                                                                                                                                                                           |
| orthogonal matrix  | A real square matrix $A$ whose transpose and inverse are equal, that is, $A^T = A^{-1}$ , and therefore $AA^T = A^T A = I$ . All eigenvalues of an orthogonal matrix have the absolute value 1.                                                                                                                                                                                                                                      |
| packed storage     | A storage scheme allowing you to store symmetric, Hermitian, or triangular matrices more compactly. The upper or lower triangle of a matrix is packed by columns in a one-dimensional array.                                                                                                                                                                                                                                         |
| PDF                | Probability Density Function. The function that determines probability distribution for univariate or multivariate continuous random variable $X$ . The probability density function $f(x)$ is closely related with the cumulative distribution function $F(x)$ .<br>For univariate distribution the relation is                                                                                                                     |

$$F(x) = \int_{-\infty}^x f(t)dt .$$

For multivariate distribution the relation is

$$F(x_1, x_2, \dots, x_n) = \int_{-\infty}^{x_1} \int_{-\infty}^{x_2} \dots \int_{-\infty}^{x_n} f(t_1, t_2, \dots, t_n) dt_1 dt_2 \dots dt_n$$

|                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|---------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| positive-definite matrix              | A square matrix $A$ such that $Ax \cdot x > 0$ for any non-zero vector $x$ . Here $\cdot$ denotes the dot product.                                                                                                                                                                                                                                                                                                                                                                                |
| pseudorandom number generator         | A completely deterministic algorithm that imitates truly random sequences.                                                                                                                                                                                                                                                                                                                                                                                                                        |
| QR factorization                      | Representation of an $m$ -by- $n$ matrix $A$ as $A = QR$ , where $Q$ is an $m$ -by- $m$ orthogonal (unitary) matrix, and $R$ is $n$ -by- $n$ upper triangular with real diagonal elements (if $m \geq n$ ) or trapezoidal (if $m < n$ ) matrix.                                                                                                                                                                                                                                                   |
| random stream                         | An abstract source of independent identically distributed random numbers of uniform distribution. In this manual a random stream points to a structure that uniquely defines a random number sequence generated by a basic generator associated with a given random stream.                                                                                                                                                                                                                       |
| RNG                                   | Abbreviation for Random Number Generator. In this manual the term "random number generators" stands for pseudorandom number generators, that is, generators based on completely deterministic algorithms imitating truly random sequences.                                                                                                                                                                                                                                                        |
| Rectangular Full Packed (RFP) storage | A storage scheme combining the full and packed storage schemes for the upper or lower triangle of the matrix. This combination enables using half of the full storage as packed storage while maintaining efficiency by using Level 3 BLAS/LAPACK kernels as the full storage.                                                                                                                                                                                                                    |
| s                                     | When found as the first letter of routine names, $s$ indicates the usage of single-precision real data type.                                                                                                                                                                                                                                                                                                                                                                                      |
| ScaLAPACK                             | Stands for Scalable Linear Algebra PACKage.                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
| Schur factorization                   | Representation of a square matrix $A$ in the form $A = ZTZ^H$ . Here $T$ is an upper quasi-triangular matrix (for complex $A$ , triangular matrix) called the Schur form of $A$ ; the matrix $Z$ is orthogonal (for complex $A$ , unitary). Columns of $Z$ are called Schur vectors.                                                                                                                                                                                                              |
| single precision                      | A floating-point data type. On Intel® processors, this data type allows you to store real numbers $x$ such that $1.18 \cdot 10^{-38} <  x  < 3.40 \cdot 10^{38}$ . For this data type, the machine precision ( $\epsilon$ ) is approximately $10^{-7}$ , which means that single-precision numbers usually contain no more than 7 significant decimal digits. For more information, refer to <i>Intel® 64 and IA-32 Architectures Software Developer's Manual, Volume 1: Basic Architecture</i> . |
| singular matrix                       | A matrix whose determinant is zero. If $A$ is a singular matrix, the inverse $A^{-1}$ does not exist, and the system of equations $Ax = b$ does not have a unique solution (that is, there exist no solutions or an infinite number of solutions).                                                                                                                                                                                                                                                |
| singular value                        | The numbers defined for a given general matrix $A$ as the eigenvalues of the matrix $AA^H$ . See also SVD.                                                                                                                                                                                                                                                                                                                                                                                        |

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|                    |                                                                                                                                                                                                                                                          |
|--------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| SMP                | Abbreviation for Symmetric MultiProcessing. The MKL offers performance gains through parallelism provided by the SMP feature.                                                                                                                            |
| sparse BLAS        | Routines performing basic vector operations on sparse vectors. Sparse BLAS routines take advantage of vectors' sparsity: they allow you to store only non-zero elements of vectors. See BLAS.                                                            |
| sparse vectors     | Vectors in which most of the components are zeros.                                                                                                                                                                                                       |
| storage scheme     | The way of storing matrices. See full storage, packed storage, and band storage.                                                                                                                                                                         |
| SVD                | Abbreviation for Singular Value Decomposition. See <i>also</i> Singular value decomposition section in Chapter 5.                                                                                                                                        |
| symmetric matrix   | A square matrix $A$ such that $a_{ij} = a_{ji}$ .                                                                                                                                                                                                        |
| transpose          | The transpose of a given matrix $A$ is a matrix $A^T$ such that $(A^T)_{ij} = a_{ji}$ (rows of $A$ become columns of $A^T$ , and columns of $A$ become rows of $A^T$ ).                                                                                  |
| trapezoidal matrix | A matrix $A$ such that $A = (A_1 A_2)$ , where $A_1$ is an upper triangular matrix, $A_2$ is a rectangular matrix.                                                                                                                                       |
| triangular matrix  | A matrix $A$ is called an upper (lower) triangular matrix if all its subdiagonal elements (superdiagonal elements) are zeros. Thus, for an upper triangular matrix $a_{ij} = 0$ when $i > j$ ; for a lower triangular matrix $a_{ij} = 0$ when $i < j$ . |
| tridiagonal matrix | A matrix whose non-zero elements are in three diagonals only: the leading diagonal, the first subdiagonal, and the first super-diagonal.                                                                                                                 |
| unitary matrix     | A complex square matrix $A$ whose conjugate and inverse are equal, that is, that is, $A^H = A^{-1}$ , and therefore $AA^H = A^H A = I$ . All eigenvalues of a unitary matrix have the absolute value 1.                                                  |
| VML                | Abbreviation for Vector Mathematical Library. See Chapter 9 of this book.                                                                                                                                                                                |
| VSL                | Abbreviation for Vector Statistical Library. See Chapter 10 of this book.                                                                                                                                                                                |
| $z$                | When found as the first letter of routine names, $z$ indicates the usage of double-precision complex data type.                                                                                                                                          |



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